Utilization, Exogenous Balance Paths and Risk

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Motivating Question

If balances are determine exogenously from credit risk, how should we calculate the appropriate loss rate to apply to those balances?

Summary

- Conventional approach to loss aggregation
- Less conventional; more realistic
- Utilization rate = risk factor

Assumptions

- “PD * LGD * EAD” model of expected loss at the loan level

- Portfolio balance paths are modeled exogenously to credit risk

Conventional Approach

- Credit modeling team calculates a portfolio loss rate

$$PLR_t = \frac{\sum_i \left(PD_{i,t} * LGD_{i,t} * EAD_{i,t} \right)}{\sum_i EAD_{i,t}}$$

  - *PLR="Portfolio Loss Rate"*

- Another team projects portfolio balances

$$PB_t = OB_{t_0} * BG_{t_0,t}$$

  - *PB="Portfolio Balance"; OB="Outstanding Balance"; BG="Balance Growth"*

- Aggregate to get "conventional approach" losses

$$CPL_t^{CA} = PLR_t * PB_t$$

  - *CPL="Portfolio Loss Conditional on Balance Path, Conventional Approach"*

Step Back

- Balance paths are deceptively complex.
- We know something about defaulting and maturing balances. Room for cleverness.
- Unlikely to know about conditional changes in originations and utilization separately
- In that case**, assume a constant utilization rate**

Constant Utilization Rate: Implications I

- **EAD scales** with the balance path
- For concreteness, suppose we have conditional LEQ, then

$$EAD_{i,t} = OB_{i,t_0} + LEQ_t * \left(CB_{i,t_0} - OB_{i,t_0}\right)$$

  - *CB="Committed Balance"*
- Since portfolio balance growth is assumed exogenous, each loan's balances scale identically

$$EAD_t | BG_{t_0,t} = CEAD_{i,t}$$
$$= OB_{i,t_0} * BG_{t_0,t} + LEQ_t * \left(CB_{i,t_0} * BG_{t_0,t} - OB_{i,t_0} * BG_{t_0,t}\right)$$
$$= BG_{t_0,t} * \left(OB_{i,t_0} + LEQ_t * \left(CB_{i,t_0} - OB_{i,t_0}\right)\right)$$
$$= EAD_{i,t} * BG_{t_0,t}$$

  - *CEAD="EAD Conditional on Balance Path"*

Constant Utilization Rate: Implications II

- **Portfolio losses scale** with the balance path
- Plugging in the conditional EAD from previous slide…

$$EL_{i,t}|BG_{t_0,t} = CEL_{i,t} = PD_{i,t} * LGD_{i,t} * CEAD_{i,t}$$
$$= PD_{i,t} * LGD_{i,t} * EAD_{i,t} * BG_{t_0,t}$$

  - *EL="Expected Loss"; CEL="Expected Loss Conditional on Balance Path"*
- And, again, since balance growth is constant across the portfolio, i.e. balances are exogenous

$$CPL_t^{LC} = \sum_i \left(PD_{i,t} * LGD_{i,t} * EAD_{i,t} * BG_{t_0,t}\right)$$
$$= BG_{t_0,t} * \sum_i \left(PD_{i,t} * LGD_{i,t} * EAD_{i,t}\right)$$

  - *CPL="Portfolio Loss Conditional on Balance Path, Less Conventional"*

Constant Utilization Rate: Implications III

- **Loss rate is a function of outstanding**

$$CPL_t^{LC} = BG_{t_0,t} * \sum_i \left(PD_{i,t} * LGD_{i,t} * EAD_{i,t}\right)$$

$$= \frac{\sum_i \left(PD_{i,t} * LGD_{i,t} * EAD_{i,t}\right)}{\boldsymbol{OB_{t_0}}} * \left(OB_{t_0} * BG_{t_0,t}\right)$$

$$= \left(\frac{\sum_i EAD_{i,t}}{OB_{t_0}}\right) * \frac{\sum_i \left(PD_{i,t} * LGD_{i,t} * EAD_{i,t}\right)}{\sum_i EAD_{i,t}} * \left(OB_{t_0} * BG_{t_0,t}\right)$$

$$= \left(\frac{\sum_i EAD_{i,t}}{OB_{t_0}}\right) * CPL_t^{CA}$$

Constant Utilization Rate ➔ Utilization Rate = Risk Factor

- Difference between CPL_t^{CA} and CPL_t^{LC} is the ratio of portfolio EAD to outstanding…

$$\left(\frac{\sum_i \boldsymbol{EAD_{i,t}}}{\boldsymbol{OB_{t_0}}}\right) = \left(\frac{\sum_i OB_{i,t_0} + LEQ_t * \left(CB_{i,t_0} - OB_{i,t_0}\right)}{OB_{t_0}}\right)$$

$$= 1 + \left[LEQ_t * \left(\left(\frac{CB_{t_0}}{OB_{t_0}}\right) - 1\right)\right] > \mathbf{1}$$

- Everything else equal**, portfolios with lower utilization rates are more risky** from a stress testing perspective