Should U.S. Investors Invest Overseas?

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Interest in foreign investment has been high among U.S. investors in recent years. The unprecedented growth of 401k pension plans has greatly increased the number of people who must make their own investment decisions in planning for their retirement. Many investors know that geographic diversification can improve investment returns without increasing risk. However, whether or not to invest abroad and, if so, how much weight to give to foreign investment, are questions often subject to heated debate. Some investment advisors recommend that U.S. investors put as much as one-third of their stock portfolio in foreign stocks to take advantage of the benefits of diversification. Others believe that foreign investment should play only a small role, if any, in a U.S. investor's stock portfolio. They argue that political uncertainties and currency fluctuations make the value of foreign investments far more volatile for the investor without the offsetting benefits of higher returns, and that diversification benefits are not enough to offset this disadvantage.[1] Moreover, U.S. investors can get overseas exposure by investing in the stocks of domestic companies. Many U.S. multinationals that are part of the Dow Jones Industrial Average, such as IBM and Coca-Cola, derive a substantial portion of their revenue from overseas operations.

The question of whether or not to invest abroad is part of the larger question of how to assemble a portfolio that is appropriate for the investor's circumstances and degree of risk tolerance. Modern portfolio theory, introduced by Markowitz in the 1950s, uses optimization techniques and historical data on the returns, risks, and correlations of available securities to construct a portfolio with the lowest possible risk for a given level of return. The theory has been widely accepted for almost half a century, and it has found practical applications among pension funds and other institutional investors in the past 20 years. Because of heavy data demands and computational intensity, however, it has largely been out of reach of individual investors. With the advent of cheap computing power and the Internet, commercial services are beginning to bring portfolio optimization to individual investors participating in 401k plans.

This article examines the question of international investing within the broader context of the use of portfolio optimization by individual investors. It illustrates the concept by constructing portfolios from index funds based on major asset classes, including two foreign indices, European and Pacific, in addition to domestic stocks, bonds, and Treasury bills. Different measures of historical returns on these assets are used to construct optimal portfolios for various levels of risk. We find that the results of portfolio optimization are highly sensitive to input parameters and, thus, to the way historical returns are measured.

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I. Ways to Invest Overseas

In investing overseas, U.S. investors have a number of options, depending on the level of detail and control they want to have over their investment. Some investors buy foreign stocks directly on foreign stock exchanges. However, investors face a number of obstacles in doing this, including lack of information, unfamiliar market practices and tax rules, undependable settlements, and costly currency conversions. High transaction costs associated with direct purchases of securities overseas make this option impractical for many small investors.

One alternative to direct investment is American Depositary Receipts (ADRs), also known as Global Depositary Receipts (GDRs). An ADR is a negotiable certificate that represents a foreign company's publicly traded equity or debt. An ADR is created when a broker purchases the actual shares in the foreign market and deposits them with a local custodian bank. The U.S. depositary bank then issues ADRs representing these shares. ADRs trade freely in the United States, just like any other security, either on an exchange or in the over-the-counter market. They are quoted in U.S. dollars and pay dividends or interest in U.S. dollars. ADRs overcome many of the difficulties associated with foreign investments. While the number of available ADRs is now quite large,[2] they may not be enough to construct a diversified global portfolio.

If an investor is willing to forgo investing in individual securities it is easier to use mutual funds to invest in specific countries or broader regions. Thousands of mutual funds either invest anywhere in the world or concentrate on specific geographical regions. However, few open-ended mutual funds limit themselves to individual countries. An investor who wishes to allocate his portfolio among specific countries would usually have to purchase closed-end country funds. Closed-end mutual funds sell a limited number of shares and invest the proceeds in the stocks of the manager's choosing from the given country or region. Unlike open-end funds, closed end funds generally do not buy their shares back from investors who wish to sell their holdings. Instead, the funds' shares trade on a stock exchange, just like individual stocks.

Another way to invest in specific countries, World Equity Benchmarks or WEBs, which have been available since 1996, are country-specific portfolios that seek to track the performance of a specific Morgan Stanley Capital International (MSCI) country index. Currently 17 WEBS are listed and traded on the American Stock Exchange (AMEX). WEBS are similar to country-specific closed-end funds, which also trade on U.S. exchanges. However, WEBs, unlike the closed-end country funds, do not trade at discounts or premiums to their net asset value, because their shares can be created or redeemed on any business day by institutional investors.

II. Portfolio Optimization

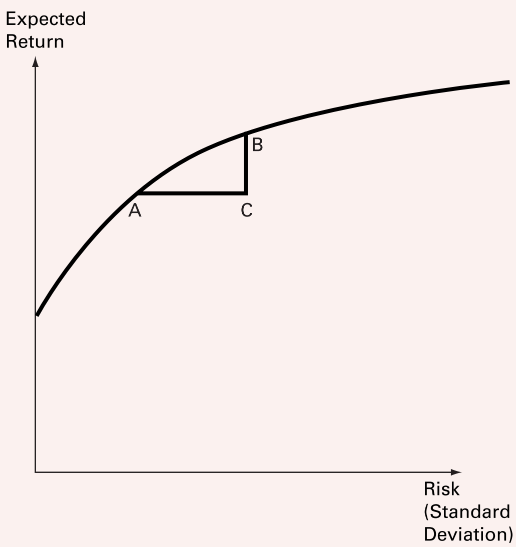
Portfolio theory is based on the premise that investors desire high investment returns and wish to minimize risk. A riskier security must have a higher expected return to compensate investors for assuming the risk. Thus, constructing a portfolio involves a

[1] A recent *Wall Street Journal* column by Jonathan Clemens (1999) reports a good example of two such opposing views.

[2] According to the Bank of New York web site at http://www.bankofny.com/adr/aovrview.htm, there are now over 1600 ADR programs representing companies from 60 countries.

Figure 1

An Efficient Frontier, Showing the Location of Three Portfolios



trade-off between risk and return. A portfolio is said to be efficient if it provides the highest expected (or mean) return for a given level of risk (variance). An "efficient frontier" describes means and standard deviations of returns of all possible efficient portfolios. Figure 1 depicts an efficient frontier with expected return plotted on the vertical axis and risk on the horizontal axis. Portfolios A and B are both on the efficient frontier, with B offering a higher return and greater risk. Portfolio C, on the other hand, is not efficient, because it is possible to increase return without increasing risk by rebalancing the amounts of different assets held in the portfolio. Thus, portfolio optimization can be thought of as a movement along (or to) the efficient frontier, which results from the changing blend of various assets in the portfolio.

Mean/variance optimization can be used for asset allocation. The investor must allocate the funds among broad categories of assets, which may include domestic and foreign equities and corporate and government bonds. The combination of assets in an efficient portfolio depends not just on their means and standard deviations, but on the interrelationships among their returns, as measured by their correlations. Thus, historical data on the returns, standard deviations, and correlations among the assets are an important input into portfolio optimization. Monthly returns going back several years are usually used for the analysis. We will illustrate the use of mean/variance optimization for asset allocation by constructing a portfolio out of index funds that represent five major asset classes and are easily available to individual investors.

III. Five-Asset Example

The indexes used were the Wilshire 5000 (a broad index of U.S. stocks, represented by the Vanguard Total Stock Fund), Morgan Stanley's Capital International (MSCI) Europe Index (over 550 stocks traded in 14 European markets, represented by the Vanguard European Equity Fund), the MSCI Pacific Index (over 400 stocks representing six Pacific region markets represented by the Vanguard Pacific Equity Fund), and the Lehman Brothers Aggregate Government/Corporate Bond Index (represented by the Vanguard Total Bond Market Fund). We use monthly returns on these index funds for the 19-year period from January of 1980 through September of 1998. For the time periods before the return on a fund was available, the return on the index itself was used, adjusted for the management expenses of the fund.[3] In addition to the four indexes, the monthly return on the Vanguard Prime Money Market Fund was used to represent a money market investment. Average annualized monthly returns and standard deviations for this time period are shown in Table 1. Correlation coefficients among the funds' returns are shown in Table 2.

Correlation coefficients describe the extent to which asset returns "move together." Correlation coefficients range in value between negative one (completely negatively correlated) and positive one (completely positively correlated), while a correlation of zero means that there is no correlation. We see that the returns on the money market fund were negatively correlated with stock market returns during this period, reflecting the fact that periods of high interest

[3] Since one cannot invest in an index directly, for those periods before a fund existed, management expenses need to be subtracted from the index return to make it comparable to the fund return. Annual expense ratios for the Vanguard Europe and Vanguard Pacific funds are 0.31 and 0.35, respectively. Vanguard Total Stock and Bond Index funds both have an annual expense ratios of 0.2 percent. Thus, if the Wilshire 5000 index had a return of 1 percent in a certain month, the return would adjusted by subtracting 0.017 percent (the monthly equivalent of the annual 0.2 percent expense ratio) for the net return of 0.983 percent.

Table 1

Average Annual Returns and Risks of Five Funds

Monthly Returns, 1980 to 1998

Fund	Annualized Return (Percent)	Annualized Standard Deviation (Percent)
U.S. Stock	15.49	15.31
European Stock	13.57	16.32
Pacific Stock	9.98	22.45
U.S. Bond	10.15	6.61
U.S. Money Market	7.20	.98

rates generally corresponded to periods of low stock prices. On the other hand, the stock markets exhibited similar movements during this period, as reflected in their positive correlations. Figure 2 plots monthly returns of the three stock indexes from 1980 to 1998. The positive correlation among their returns is easily seen in the plots. One can also see that the Pacific index has been considerably more volatile than the U.S. and European indexes.

Given this information on returns, risks, and correlations of individual assets, we can calculate both the risk and the return on any portfolio consisting of these assets. The return on the portfolio is a weighted average of the returns on the assets in it, with the weight given to each asset equal to its share in the portfolio, as can be seen in Equation 1:

$$r_p = \sum w_i r_i \tag{1}$$

where r_p is the return on the portfolio, r_i is the return on asset i, and w_i is the proportion of asset i in the portfolio (its portfolio weight). Consequently, the mean return on the portfolio is the weighted average of the mean asset returns on its assets, as can be seen in Equation 2:

$$\mu_p = \sum w_i \mu_i \tag{2}$$

where μ_p is the mean return on the portfolio and μ_i is the mean return on asset i.

Equation 3 shows the variance of the portfolio's return as a weighted average of the standard deviations and correlations of the returns for its assets:

$$\sigma_p^2 = \sum_i^n \sum_j^n w_i w_j \sigma_i \sigma_j \rho_{ij} \tag{3}$$

where $\sigma^2{}_p$ is the variance of the portfolio, σ_i and σ_j are standard deviations of assets i and j, w_i and w_j are their respective weights in the portfolio, and ρ_{ij} is the correlation coefficient between the two assets.

The standard deviation of the portfolio's return, which is the square root of variance, is the measure of risk that is used in all subsequent statistical analysis. Note that the risk of the portfolio depends not just on the standard deviations of the returns on assets that constitute it, but also on their correlations. The lower the correlation coefficient among the assets, the lower the risk of the overall portfolio. This is the reason why diversification reduces risk.

The goal of optimization is to find the blend of assets that would minimize the standard deviation of the portfolio's return for any given level of expected return. The optimization problem is usually subject to constraints. In particular, the weights must sum to one (the budget constraint) and cannot be negative (no short-selling). Some institutional investors can, in fact, sell assets short. However, to keep this example realistic from the point of view of an individual investor allocating the portfolio among easily available mutual

Table 2

Correlations Among the Five Funds' Returns

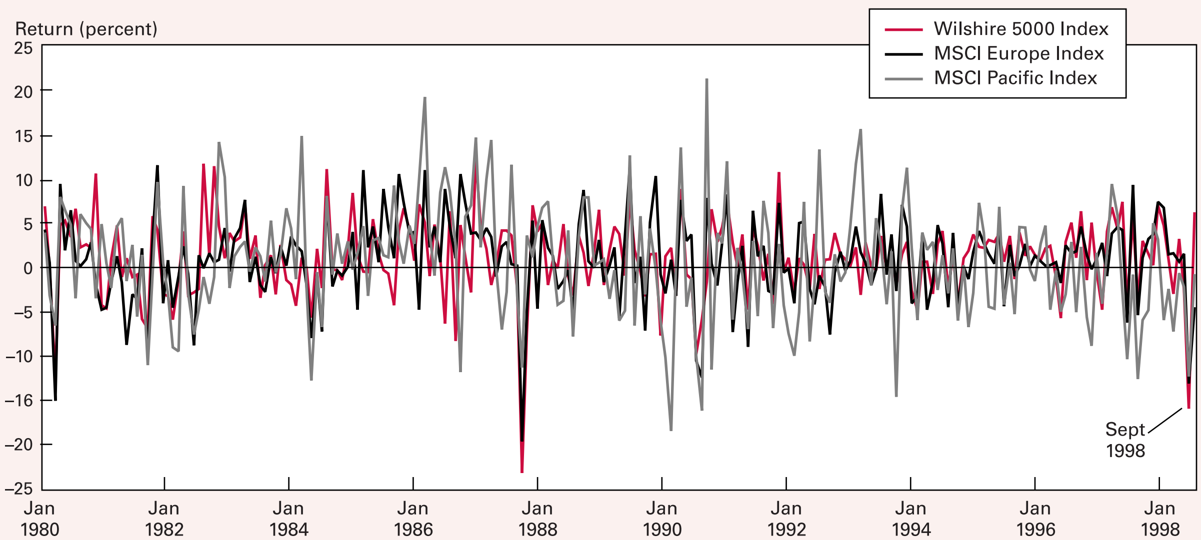
Monthly Returns, 1980 to 1998

Fund	U.S. Stock	European Stock	Pacific Stock	U.S. Bond	U.S. Money Market
U.S. Stock	1.00	.59	.33	.29	−.05
European Stock		1.00	.53	.22	−.13
Pacific Stock			1.00	.14	−.10
U.S. Bond				1.00	.14
U.S. Money Market					1.00

Figure 2

Monthly Returns on Three Index Funds

January 1980 to September 1998



funds, we do not allow short-selling. This means that portfolio weights cannot be negative, and this necessitates using a quadratic programming algorithm. The algorithm used in these calculations is a gradient quadratic programming method from Sharpe (1987). In addition to the constraints on portfolio weights, the expected returns, standard deviations, and correlations of the assets, the algorithm requires a measure of the investor's risk tolerance. This represents the trade-off the investor is willing to make between expected return and risk. The maximization problem can be defined as follows:

$$U = \mu_p - ((\sigma_p^2)/t)$$

where:

U = the utility of the portfolio

μ_p = the portfolio's expected return

σ_p = the portfolio's standard deviation of return

t = the investor's risk tolerance.

Risk tolerance, t, represents the marginal rate of substitution between the expected return of the portfolio and its variance, and the utility measure, U, represents the risk-adjusted return on the portfolio. The objective of the optimizer is to find the portfolio that would maximize the investor's utility given the risk tolerance and the constraints on the portfolio composition.

Table 3 shows 10 optimal portfolios based on the historical data for the five funds shown in Tables 1 and 2. Figure 3 shows the efficient frontier that results if we plot the expected returns and standard deviations of these optimal portfolios that comprise different blends of the five mutual funds. The allocations of each fund (U.S. Stock, European Stock, Pacific Stock, U.S. Bond, and U.S. Money Market) are shown in parentheses next to the expected return and standard deviation of each fund.

The least risky portfolio invests 99 percent of its assets in the Money Market Fund and 1 percent in the European Stock Fund. Interestingly, U.S. stocks and bonds are not included in it. On the other hand, the most risky portfolio is invested fully in U.S. stocks. The two moderate-risk portfolios, numbers 5 and 6, have very "conventional" asset allocations. For example, portfolio number 5 has 51 percent in U.S. stocks, 42 percent in U.S. bonds, and 7 percent in European stocks. A slightly riskier portfolio, number 6, has 63 percent in U.S. stocks, 30 percent in U.S. bonds, and 7 percent in European stocks.

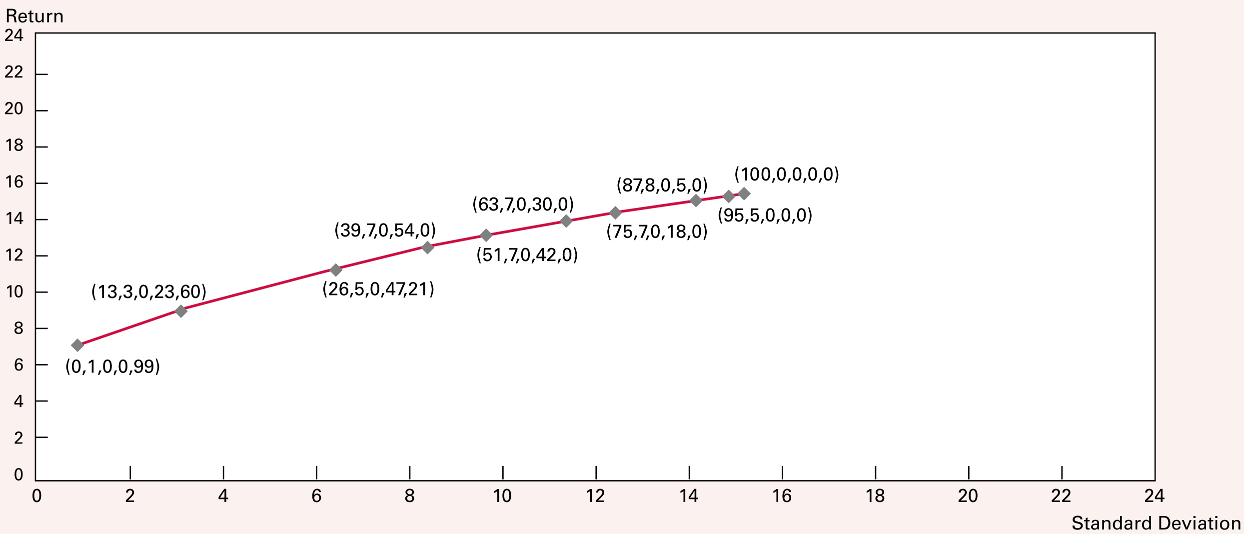
Table 3
Fund Allocation of 10 Optimal Portfolios
Percent

Portfolio Number	Expected Return	Standard Deviation	U.S. Stock	European Stock	Pacific Stock	U.S. Bond	U.S. Money Market
1	7.27	.96	0	1	0	0	99
2	9.19	3.25	13	3	0	23	60
3	11.12	6.28	26	5	0	47	21
4	12.45	8.44	39	7	0	54	0
5	13.10	9.69	51	7	0	42	0
6	13.75	11.10	63	7	0	30	0
7	14.40	12.60	75	7	0	18	0
8	15.05	14.17	87	8	0	5	0
9	15.38	15.02	95	5	0	0	0
10	15.49	15.31	100	0	0	0	0

Perhaps the most striking feature of this exercise is the complete exclusion of the Pacific Stock Fund from all portfolios. From the data in Tables 1 and 2, it is easy to see why. During the period in question, the Pacific Stock Fund had the highest standard deviation, 22 percent, which greatly exceeded that of the next riskiest fund, the European, 16 percent. Despite its high risk, the Pacific Fund also had low returns. The returns on the U.S. and the European Stock Funds, as well as the U.S. Bond Fund, which had a much lower standard deviation, exceeded that on the Pacific Stock Fund. As was already mentioned, portfolio theory

Figure 3

Efficient Frontier
Generated from Historical Data



Note: The order of the portfolio compositions in parentheses is as follows: U.S. Fund, Europe Fund, Pacific Fund, Bond Fund, and Money Market Fund.
Portfolio composition data are in percents.

considers each asset in terms of its contribution to the overall risk and return of the portfolio and not solely in terms of its own performance. Unfortunately, as can be seen from Table 2, the returns on the Pacific Fund were positively correlated both with the U.S. Stock and Bond Funds and with the European Stock Fund, so that the benefits of diversification it could have provided were too small to offset its own poor performance. The European Stock Fund, while not excluded altogether, has a relatively low weight in this efficient frontier. Its highest weighting appears in portfolio number 8, where it has 8 percent of the total allocation, or 9 percent of stock allocation.

IV. Stability of Correlations and Exponential Weights

The analysis above was based on the assumption that volatilities and correlations of the five series of returns were stable during the 10 years used for their measurement. In fact, volatilities change over time. Moreover, they tend to "swing" from high to low values. Thus, observations far in the past may not be as relevant as the more recent observations in estimating expected returns, volatilities, and correlations, especially since structural changes in the economy and financial markets could make the past less meaningful.

One could simply use a shorter time interval and ignore observations that are far in the past. However, as a rule, more data are better than less, and ignoring available data decreases the reliability of the estimate. One popular approach is exponential weighting. Instead of applying the same weight to each data point, as is done in calculating a simple average, the exponential average places relatively more weight on more recent observations.

$$\bar{X}_t = \sum_{j=0}^{\infty} \omega_j X_{t-j}. \tag{4}$$

Equation 4 says that the current estimate of the average return ($\bar{X}_t$) is the weighted average of all returns that came before it. Each observation is given less weight than the one after it and all the weights must sum to 1.

Two Implementations of Exponential Weights

This section shows two ways to implement exponential weights by using decay factors, which determine the weight given to each observation. Usually the decay factor is chosen so that the weight given to each preceding observation is a multiple (less than 1) of the current observation.

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One way to define the weight given to observation j is shown in Equation 5:

$$\omega_j = \lambda^j(1 - \lambda) \text{ where } 0 < \lambda < 1. \tag{5}$$

Substituting Equation 5 into Equation 4, we can write the average return as follows:

$$\bar{X}_t = \sum_{j=0}^{\infty} \lambda^i(1 - \lambda) X_{t-j} = (1 - \lambda) \sum_{j=0}^{\infty} \lambda^i X_{t-j}. \tag{6}$$

The condition that weights must sum to one is approximately satisfied because if the number of observations is large, then

$$lim_{t\to\infty} \sum_{j=0}^{t-1} \omega_j = (1 - \lambda) \lim_{t\to\infty} \sum_{j=0}^{t-1} \lambda^j = 1. \tag{7}$$

The choice of the discount factor λ determines how fast the weight given to past observations declines over time. Lower values of λ mean faster decay and less weight on past observations.

The second way to implement exponential decay is shown in Equation 8:

$$\omega_j = \frac{\frac{2^{j/h}}{k}}{\sum \omega_j} \tag{8}$$

Table 4
Means and Standard Deviations Calculated Using Exponential Weights
Decay Factor = 10

Fund	Annualized Return (Percent)	Annualized Standard Deviation (Percent)
U.S. Stock	8.07	19.16
European Stock	14.75	17.12
Pacific Stock	−27.96	19.01
U.S. Bond	9.09	.12
U.S. Money Market	4.94	.98

where h is the decay factor and k is a constant chosen so that the weights sum to one. In this case the average return will be calculated as follows:

$$\bar{X}_t = \sum_{j=0}^{\infty} \frac{\frac{2^{j/h}}{k}}{\sum 2^{j/h}} X_{t-j}. \qquad (9)$$

As in the first method, the decay factor, in this case h, governs the speed with which past observations decline in importance. Lower values of h lead to faster decay. In this method, the decay factor is sometimes referred to as the "half-life" of an observation, because the weight given to any observation declines by half after h observations.

Appendix Table 1 shows how one can get a very similar weighting pattern using either of these exponential-weighting techniques. The decay factors used in this example both result in a relatively steep decline in the weight assigned to past observations, so that the weights decline to zero after 70 and 77 observations, respectively. This is achieved in this example by using decay factors h=10 and λ = .94. Note also that when h=10, the weight given to an observation declines by half after approximately 10 observations. as would be expected with the half-life of 10.

To continue with our example, we apply the exponential weighting with h=10 to the historical returns on the five assets. Table 4 shows the estimated means and standard deviations of the assets, while Table 5 shows the correlation coefficients. Table 6 shows a new set of efficient portfolios formed from the five assets using the same optimization algorithm and the second set of parameters obtained from the exponential weighting.

Perhaps the most striking feature of this revised efficient frontier is the complete exclusion from it of U.S. stocks, as well as Pacific stocks. Instead, with the exception of the most conservative portfolio, which is completely invested in the money market fund, the optimized portfolios include only U.S. bonds and European stocks. Recall that this set of "efficient" portfolios was produced using the same set of data and the same optimization algorithm as the one shown in Table 3, which included only a very small proportion of European stocks. The dramatic difference in results is due solely to the exponential weighting scheme that gave more weight to recent observations.

V. Statistical Sampling of the Efficient Frontier

The example described in the previous section pinpoints a fundamental limitation of portfolio optimization. It often gives "strange" or unintuitive results, and these results can differ dramatically with

Table 5
Correlations Calculated Using Exponential Weights
Decay Factor = 10

Fund	U.S. Stock	European Stock	Pacific Stock	U.S. Bond	U.S. Money Market
U.S. Stock	1.00	.85	.63	−.05	−.08
European Stock		1.00	.63	−.13	−.06
Pacific Stock			1.00	−.05	−.19
U.S. Bond				1.00	.23
U.S. Money Market					1.00

Table 6
Fund Allocations, Annual Returns, and Standard Deviations of a Set of Efficient Portfolios
Percent
Decay Factor = 10

Portfolio Number	Expected Return	Standard Deviation	U.S. Stock	European Stock	Pacific Stock	U.S. Bond	U.S. Money Market
1	4.90	.12	0	0	0	0	100
2	9.95	3.72	0	15	0	85	0
3	10.44	4.62	0	24	0	76	0
4	10.95	5.81	0	33	0	67	0
5	11.45	7.15	0	42	0	58	0
6	11.94	8.58	0	50	0	50	0
7	12.44	1.05	0	59	0	41	0
8	12.94	11.54	0	68	0	32	0
9	13.43	13.06	0	77	0	23	0
10	14.43	16.13	0	94	0	6	0

very small changes in the data. The example also explains why, despite the wide acceptance of the theories underlying modern finance in the past 50 years, portfolio optimization has proved to be a difficult practical challenge for making real-life investment decisions, even for professional money managers and sophisticated institutional investors. Portfolio theory is very demanding, in that one cannot use simple rules applied to past data to obtain reliable investment strategies. At the very least, one must decide how to blend the past information with expectations about the future. The future means and variances of asset returns do not necessarily evolve from past data in obvious ways. Instead, they depend on technology, current economic conditions, and government policy.

Ultimately, the optimal portfolio is no better than the assessment of future means and variances used to construct it. Furthermore, portfolio optimizers exhibit sensitive dependence on initial conditions. In this case, the "initial conditions" are the parameter estimates of the return distributions. Small variations in these estimates often result in very large changes in the resulting optimal portfolios. Moreover, it is exactly the estimate with the biggest error that is likely to have the biggest influence on the result of the mean-variance optimization. For example, if we overestimate the return on one asset (or underestimate its risk), the optimizer will concentrate the portfolio in that very asset and exclude the others. For this reason, mean variance optimizers have been criticized as "estimation error maximizers" (Michaud 1989).

One interesting approach to quantifying the uncertainty of optimization involves using Monte Carlo simulation to define a "fuzzy" efficient frontier that results from the sampling error of parameter estimation. The idea is to treat the monthly historical returns on assets as a random draw from a probability distribution. The means, standard deviations, and correlations among the returns are parameters of that distribution. The simulation involves drawing another random sample of returns from the same distribution with the same parameters. Here, we use a multivariate normal distribution to draw a random 10-year sample of monthly returns.[4] Briefly, the procedure is as follows:[5]

1. Estimate the means, standard deviations, and correlations of asset returns using historical data.

2. Using an optimization algorithm, such as the one described in the previous section, construct an efficient frontier based on the estimated parameters of the historical data.

3. Using the means, standard deviations, and correlations of historical returns estimated in step 1 as the initial parameters, generate a "new" 10-year set of simulated monthly asset returns.

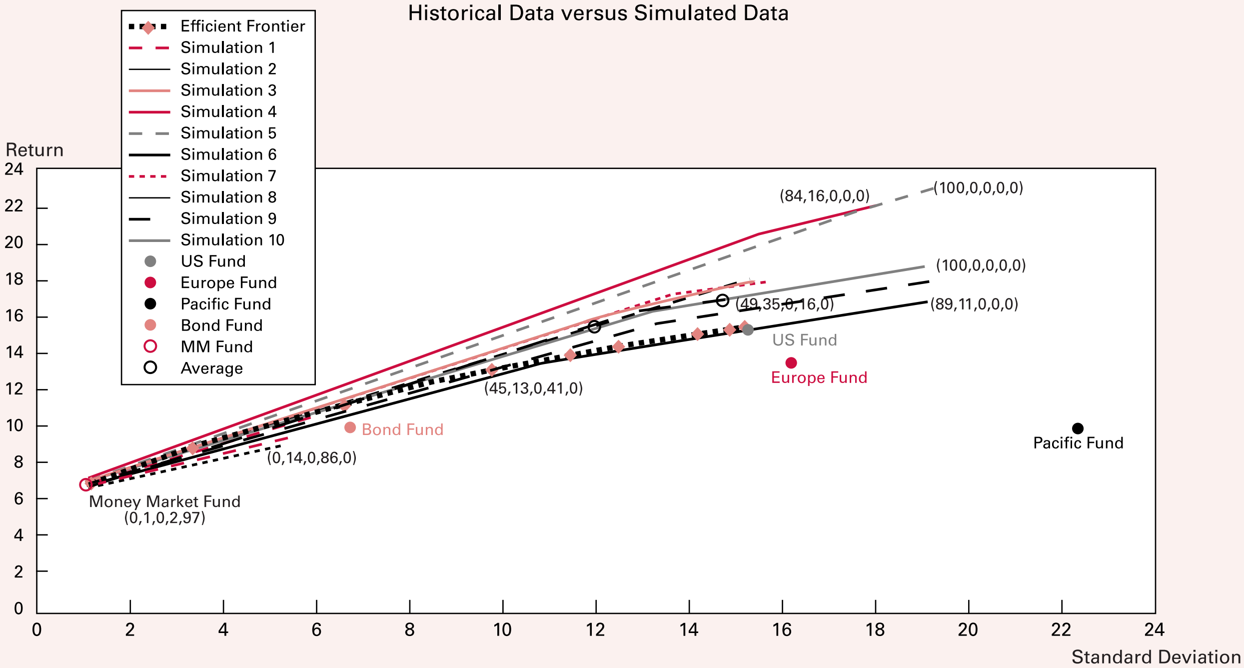
[4] While the normal distribution is routinely used to model asset returns, it has been widely recognized for many years that financial markets exhibit significant non-normalities. In particular, extreme outcomes, such as market crashes, are relatively more likely than would be implied by the normal distribution. Nevertheless, we use normal distribution here because of the ease of computation and because even the normal distribution successfully demonstrates the inherent uncertainty of the efficient frontier.

[5] The statistical sampling of the efficient frontier is described in detail in Michaud (1998).

Figure 4

Efficient Frontier

Historical Data versus Simulated Data



Note: The order of the portfolio compositions in parentheses is as follows: U.S. Fund, Europe Fund, Pacific Fund, Bond Fund, and Money Market Fund.
Portfolio composition data are in percents.

4. Calculate the means, standard deviations, and correlations from the simulated observations. (Note that they will not be the same as the original ones, although they will be similar.)

5. Construct a new efficient frontier based on the new simulated parameters.

6. Repeat the process of generating simulated returns, calculating their parameters and constructing efficient portfolios a few hundred times, to construct a "fuzzy" efficient frontier consisting of all the efficient frontiers constructed from the simulated returns.

On a practical level, the sampling of the efficient frontier is useful in evaluating existing portfolios before they are changed. If an investor's existing portfolio is within the "fuzzy" efficient set, it may not be worth incurring transactions costs, taxes, and the like to switch to a more "efficient" portfolio computed by an optimization program, if the resulting change is likely to be no better than the portfolio the investor already has. While this procedure can be revealing, it still assumes that outcomes are anchored to historical estimates from step 1. It would be of little use if history is an "outlier" in the true distribution of asset returns.

We have created a resampled efficient frontier for our five-asset example. The parameters used in this simulation were the means, standard deviations, and correlations calculated for the five asset return series and shown in Tables 1 and 2. These parameters were used to generate random observations for the returns on the five assets from a jointly normal distribution. Each simulation run generated 120 random observations for each asset, corresponding to the 10 years of monthly returns in the historical sample. The means, standard deviations, and correlations of the simulated observations were then calculated and used as inputs to compute a new efficient frontier.

Figure 4 shows a number of efficient frontiers constructed from such simulations. It also shows (as a thick line) the original frontier based on the historical parameters and the average of the simulated frontiers. In the low-risk low-return range, the original frontier and the simulation lines are bunched close together—the most conservative, or lowest-risk portfolios on all the efficient frontiers are concentrated in the money-market fund. It is among the aggressive portfolios at the high risk/high return end of the range of the efficient frontier that the "fuzziness" becomes strikingly clear. The figure shows that a very large range of possible portfolio compositions are consistent with at least some version of the efficient frontier. In fact, as the figure shows, it would be impossible to rule out any combination of U.S. stocks, bonds, and cash as being efficient, including portfolios concentrated solely in one of these assets. However, a portfolio consisting solely of the Pacific stock index fund appears to be obviously inferior, no matter how "fuzzy" the efficient frontier really is. Figure 4 shows that just by moving horizontally from the Pacific Portfolio to the left toward the frontier, one could reduce risk by about 16 percent without sacrificing any return.

A fundamental limitation of portfolio optimization is that it often gives "strange" or unintuitive results, and these results can differ dramatically with very small changes in the data.

VI. Conclusion

While helping to quantify the uncertainty in mean variance optimization, statistical sampling does not by itself reduce it. The only way to improve optimal portfolios is by improving the quality of parameter estimation, which can include more sophisticated techniques for modeling time-varying volatilities and correlations of returns, as well as utilizing the investor's views on the likely future parameters of the distribution. In particular, the investor can base these forecasts on his fundamental views of the global economic conditions and the way they would affect asset returns. So the issue of if and how much U.S. investors should commit to overseas investments depends ultimately on those investors' view of the future economic prospects of various geographical regions vis-à-vis the domestic economy. In this scenario, the optimizer is simply a computational convenience that helps the investor translate his views of the future asset returns, risks, and correlations into a portfolio that best represents this view. It does not replace the informed judgment that is the ultimate arbiter of investment decisions.

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Appendix Table 1

Exponential Weighting of Observations

Observation	h = 10	λ = .94	Observation	h = 10	λ = .94
0	.067	.060	41	.004	.005
1	.062	.056	42	.004	.004
2	.058	.053	43	.003	.004
3	.054	.050	44	.003	.004
4	.051	.047	45	.003	.004
5	.047	.044	46	.003	.003
6	.044	.041	47	.003	.003
7	.041	.039	48	.002	.003
8	.038	.037	49	.002	.003
9	.036	.034	50	.002	.003
10	.033	.032	51	.002	.003
11	.031	.030	52	.002	.002
12	.029	.029	53	.002	.002
13	.027	.027	54	.002	.002
14	.025	.025	55	.001	.002
15	.024	.024	56	.001	.002
16	.022	.022	57	.001	.002
17	.021	.021	58	.001	.002
18	.019	.020	59	.001	.002
19	.018	.019	60	.001	.001
20	.017	.017	61	.001	.001
21	.016	.016	62	.001	.001
22	.015	.015	63	.001	.001
23	.014	.014	64	.001	.001
24	.013	.014	65	.001	.001
25	.012	.013	66	.001	.001
26	.011	.012	67	.001	.001
27	.010	.011	68	.001	.001
28	.010	.011	69	.001	.001
29	.009	.010	70	.001	.001
30	.008	.009	71		.001
31	.008	.009	72		.001
32	.007	.008	73		.001
33	.007	.008	74		.001
34	.006	.007	75		.001
35	.006	.007	76		.001
36	.006	.006	77		.001
37	.005	.006	78		
38	.005	.006	79		
39	.004	.005	80		
40	.004	.005			