Cross-Sectional Asset Pricing Puzzles: A Long-Run Perspective†

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ABSTRACT

This paper proposes an intertemporal asset pricing model within a long-run risk economy featuring a formal cross section of firms characterized by mean-reverting expected dividend growth. We find considerable empirical support for the cross-sectional implications of the model, as cash flow- and return-based measures of long-run risk exposure are both positively related to returns and offer a partial explanation of the size, value, and momentum anomalies. Interestingly, the model implies a negative relation between exposures to systematic and firm-specific risks in the cross section. Higher cash-flow duration firms exhibit higher exposure to economic growth shocks while they are less sensitive to firm-specific news. Such firms command higher risk premiums but exhibit lower analyst forecast dispersion, idiosyncratic volatility, and distress risk. We find theoretical and empirical support of a long-run risk explanation of these anomalies.

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Introduction

The past three decades of empirical asset pricing literature suggests that patterns in returns, left unexplained by the Sharpe–Lintner CAPM, are pervasive throughout the cross section. In addition to anomalies related to size and book-to-market (e.g., Fama and French (1992)) and momentum (Jegadeesh and Titman (1993)), recent empirical works document further counterintuitive relations. Specifically, Diether, Malloy, and Scherbina (2002) demonstrate that firms with more uncertain earnings (higher forecast dispersion) underperform lower dispersion firms. Ang, Hodrick, Xing, and Zhang (2006) document a negative relation between average return and idiosyncratic volatility (IV). Dichev (1998) and Campbell, Hilscher, and Szilagyi (2008) find evidence that financially distressed stocks deliver abnormally low returns, and Avramov, Chordia, Jostova, and Philipov (2009a) show that high credit rating firms considerably outperform their lower-rated counterparts. These three effects are anomalous because, while investors may be expected to discount uncertainty about firm fundamentals (e.g., Merton (1987)), they appear to pay a premium for bearing such uncertainty. Past work offers potential explanations for the puzzling effects. Johnson (2004) interprets forecast dispersion as a proxy for unpriced information risk, and shows that in the presence of leverage, equity value rises (and expected return falls) as unpriced risk increases. Garlappi, Shu, and Yan (2008) and Garlappi and Yan (2011) attribute the distress risk effect to shareholders' ability to extract rents through strategic renegotiation of debt obligations, while Johnson, Chebonenko, Cunha, D'Almeida, and Spencer (2011) propose that the endogenous choice of leverage coupled with heterogeneity in business risk explains the distress risk effect.

This paper proposes a unified resolution of these puzzling return patterns within a long-run risk equilibrium. Two ingredients pertaining to the dynamics of aggregate and firm fundamentals underly our model. The aggregate economy is formulated based on Bansal and Yaron (2004) in that long-run risk arises through the interaction of the stochastic differential utility of Duffie and Epstein (1992) with persistent consumption and dividend growth rates. In the cross section, we employ a tractable mean-reverting process to formulate firm dividend share, which is the fraction of the dividend paid by the firm relative to aggregate dividend. Firm dividend growth depends on the long-run share ratio, which is the long-run expected dividend share of a firm as a proportion of its current dividend share.[1] This framework allows us to solve for cross-sectional asset pricing quantities while ensuring that the cross section of firms aggregates to form the economy. Ultimately, we successfully merge the long-run risk literature with the shares-based cross section literature advocated by Menzly, Santos, and Veronesi (2004) and Santos and Veronesi (2006, 2010). Moreover, we develop an intertemporal asset pricing model in the spirit of Merton (1973), as a firm's expected return is affected by its beta with respect to an economic growth hedge portfolio.

In a long-run risk economy, the cross section of expected returns is determined by firms' cash flow duration. Long-run cash flows have pronounced sensitivities to the persistent economic growth rate and thus command large risk premiums. Therefore, high duration firms have high expected returns due to their reliance on long-run cash flows. The duration of a firm is positively related to its share ratio, which determines expected dividend growth.

We demonstrate that the long-run risk model has both cash flow- and return-based implications. Expected returns are positively related to share ratio as well as economic growth betas. Empirically, we find support for both predictions. Average returns are significantly positively related to the cash flow-based share ratio measure and the return-based economic growth beta measure.[2] Furthermore, there is some evidence that the model

[1] Share ratio is similar to the relative share characteristic of Menzly, Santos, and Veronesi (2004).

[2] Our return-based results are complementary to those of Ferson, Nallareddy, and Xie (2011), who examine the out-of-sample performance of the long-run risk model for a smaller set of anomaly-based and bond portfolios.

can help explain the book-to-market, size, and momentum anomalies, as the cash flows of these portfolios are consistent with a long-run risk explanation.[3] However, economic growth betas do not appear to fully explain these anomalies in the return-based tests.

We then explore an interesting feature of our model – the negative cross-sectional relation between exposures to systematic and firm-specific risks – as a potential explanation for the dispersion, IV, and distress risk effects. In the model, low share ratio firms exhibit low duration as they derive their values primarily from short-run cash flows. Whereas such firms display low expected returns, they have high sensitivities to firm-specific shocks leading to high dispersion, IV, and credit risk. In contrast, high share ratio firms display high duration and are particularly sensitive to aggregate shocks but comparatively less susceptible to firm-specific dividend shocks.

The empirical evidence broadly supports our model predictions. Dispersion, IV, and credit risk are closely related in the cross section, exhibiting high cross-sectional correlation. More importantly, we find empirical support for a long-run risk explanation of these effects, as portfolios of high (low) dispersion, IV, and credit risk firms have low (high) estimated share ratio as predicted by our model. Furthermore, introducing the economic growth factor drives out the ability of these characteristics to forecast returns in the cross section. Overall, the evidence suggests that long-run risk can explain the dispersion, IV, and distress risk effects.

Finally, we offer a potential explanation for the disagreement in the literature regarding the IV effect. Depending on the specification, researchers have demonstrated a positive (e.g., Fu (2009)), negative (e.g., Ang, Hodrick, Xing, and Zhang (2006)), or non-existent (e.g., Bali and Cakici (2008)) relation between IV and average returns. In our model, IV is decreasing in both dividend share and share ratio. While the true negative IV–

[3]See also Bansal, Dittmar, and Lundblad (2005) who show a similar result using cash flow betas to measure aggregate consumption risk.

return relation is exclusively determined by share ratio, sorting firms by dividend share appears to produce a positive IV–return relation. We find that Ang, Hodrick, Xing, and Zhang's (2006) measure is closely related to share ratio, explaining its negative relation with returns. However, alternative measures and sorting procedures can result in portfolios which primarily reflect dividend share's effect on IV, producing an apparently flat or positive IV–return relation in the cross section.

In sum, recent work utilizes the long-run risk framework to offer explanations for aggregate and cross-sectional asset pricing puzzles including the equity premium, risk-free rate, and excess volatility puzzles (Bansal and Yaron (2004)), as well as the value premium (e.g., Hansen, Heaton, and Li (2008) and Ai and Kiku (2011)) and the momentum effect (Avramov and Hore (2008)). We enhance the long-run risk literature in both methodology and substantive findings. In particular, we derive formal cross-sectional asset pricing restrictions and are able to express our model through an intuitive ICAPM relationship. Moreover, our model simultaneously resolves three apparently counterintuitive regularities – the dispersion, IV, and credit risk effects.

The remainder of the paper is organized as follows. In Section 1, we introduce the aggregate economy, develop our formulation of the cross section of firms, and establish asset pricing results. We then test the primary cross-sectional predictions of the model in Section 2. In Section 3, we examine the dispersion, IV, and credit risk effects within our framework and provide empirical evidence. Section 4 concludes.

1 The Model

Two essential ingredients pertaining to the dynamics of aggregate and firm fundamentals underly our equilibrium setup. The aggregate economy is based on a simple version of Bansal and Yaron's (2004) long-run risk framework in which an economic agent with recursive preferences is exposed to persistent consumption and dividend growth rates. This economy generates an equity premium, market volatility, and a risk-free rate that match the data. Within the aggregate economy, we formulate a cross section of firms whose dividends sum to the aggregate dividend. Firms are differentiated by expected cash flow timing. In a long-run risk equilibrium, investors require larger risk premiums to hold firms weighted toward long-run cash flows.

1.1 The Aggregate Economy

The representative investor is endowed with the stochastic differential utility of Duffie and Epstein (1992), a continuous-time equivalent to Epstein–Zin (1989) preferences, with

$$J_t = E_t \left[\int_t^{\infty} f(C_s, J_s) ds \right], \tag{1}$$

where J_t is the value function and $f(C_s, J_s)$ is a normalized aggregator of current consumption and continuation utility. Assuming that the elasticity of intertemporal substitution (EIS) equals one, the normalized aggregator takes the form

$$f(C, J) = \beta(1-\gamma)J \left[\log C - \frac{\log((1-\gamma)J)}{1-\gamma} \right], \tag{2}$$

where γ is the coefficient of relative risk aversion and β is the time preference parameter.[4]

[4]Empirically, there is a wide debate about the value of EIS. See Bansal, Kiku, and Yaron (2010) and Beeler and Campbell (2011) for discussion and estimation of EIS in the context of the long-run risk model.

Next, we formulate aggregate consumption and dividend growth rates as[5]

$$\frac{dC_t}{C_t} = (\mu_C + \lambda X_t)dt + \sigma_C dW_C, \quad (3)$$

$$\frac{dD_t}{D_t} = (\mu_D + X_t)dt + \sigma_D dW_D, \quad (4)$$

where X_t follows the mean-reverting dynamics

$$dX_t = -\kappa X_t dt + \sigma_X dW_X. \quad (5)$$

The time-varying component of economic growth, X_t, reverts to zero, with the speed of reversion governed by κ. Shocks to X_t propagate through several periods of consumption and dividend growth rates due to slow mean reversion, which combined with recursive preferences gives rise to long-run risk. Since expected consumption growth is less volatile than expected dividend growth we allow for a differential effect of X_t on consumption and dividend growth rates, with λ controlling the relative strength of shocks. In the data we find $\lambda < 1$ and we impose this restriction in our analysis. Thus consumption growth is smoother than dividend growth, capturing the insight of Abel's (1999) levered economy. Following Bansal and Yaron (2004), we assume the processes dW_C, dW_D, and dW_X are uncorrelated for simplicity of presentation.[6]

We present asset pricing quantities for the aggregate setup in the proposition below and refer the interested reader to Hore (2010) for complete derivation and Appendix A.1 for

[5] Consumption and dividend growth are the continuous-time versions of the discrete-time processes employed by Bansal and Yaron (2004). We adopt a model with homoskedastic shocks to dividend growth, consumption growth, and the aggregate growth rate. Bansal and Yaron (2004) investigate both homoskedastic and heteroskedastic cases. The homoskedastic model is simpler and sufficient for our purposes.

[6] Introducing correlation between the consumption and dividend processes leads to an additional term in expected returns taking the form $\rho\sigma_C\sigma_D\gamma$, which is the familiar expected return in an *i.i.d.* growth economy with CRRA preferences. This term is economically small for reasonable values of risk aversion (e.g. Mehra and Prescott (1985)). We assume zero correlation to simplify the model and focus on the effects of long-run risk.

additional details.[7]

Proposition 1. *The price-dividend ratio is*

$$\frac{P_t}{D_t} = G(X_t) = \int_t^\infty S(X_t, \tau) ds \qquad [\tau = s - t], \tag{6}$$

where the $S(X_t, \tau)$ function is described in Appendix A.1. The risk premium is

$$\mu_t = \frac{\lambda(\gamma - 1)}{\kappa + \beta} \sigma_X^2 \frac{G_X}{G}. \tag{7}$$

The essential feature of our aggregate economy is the term structure of risk premiums, which is formulated in the following theorem.

Theorem 1. *Expected return is increasing in duration for any asset that pays a non-negative portion of the aggregate dividend at all times.*

Proof. See Appendix A.1. □

Theorem 1 establishes that longer-run cash flows are riskier, thus commanding higher risk premiums. Figure 1 exhibits the representative investor's required discount rate on cash flows as a function of duration. Risk premiums on low duration cash flows are negligible while high duration cash flows command large risk premiums.

Why do agents require higher risk premiums on longer-run cash flows? Investors endowed with Duffie–Epstein preferences prefer early resolution of uncertainty if $\gamma > 1$ with unit EIS ($\psi = 1$), or if $\gamma > \frac{1}{\psi}$ when ψ is unrestricted. Even with persistent consumption and dividend growth dynamics, shocks to growth rates have little effect on short-run cash flows. Short-run cash flows are thus relatively safe. On the other hand, the effect of these

[7] We impose the following parameter restrictions: $\gamma > 1$, $\lambda < 1$, $\beta > 0$, $\sigma_X > 0$, and $\kappa > 0$.

shocks builds over time and heightens uncertainty about the magnitudes of long-run dividends and consumption. Much of this uncertainty remains unresolved for a long period. Thus, investors heavily discount long-run cash flows giving rise to a long-run risk premium.

1.2 The Cross Section of Firms

We consider a cross section of N firms that aggregates to the economy derived above. Explicitly modeling the cross section ensures the reasonable long-term evolution of firm cash flows, which is particularly important in a long-run risk framework. Our tractable framework facilitates drawing cross-sectional inferences by pricing a single firm.

In the cross section, firm i contributes a time-varying portion of the aggregate dividend, defined as the firm's dividend share θ_t^i. Thus, if the aggregate dividend is D_t, firm i contributes $D_t^i = \theta_t^i D_t$, whereas the remaining firms contribute a total of $(1 - \theta_t^i) D_t$. The dividend share is formulated using the mean-reverting Wright–Fisher (WF) process[8]

$$d\theta_t^i = \alpha(\bar{\theta}^i - \theta_t^i)dt + \delta\sqrt{(1 - \theta_t^i)\theta_t^i}dW_{\theta^i}. \tag{8}$$

Moreover, we parameterize long-run dividend shares such that $\sum_{i=1}^N \bar{\theta}^i = 1$ and assume the covariance structure of the $(N \times 1)$ vector dW_θ satisfies

$$\rho_t(dW_{\theta^i}, dW_{\theta^j}) = -\sqrt{\frac{\theta_t^i \theta_t^j}{(1 - \theta_t^i)(1 - \theta_t^j)}} \text{ for all } i,\ j. \tag{9}$$

The negative correlation between dividend share shocks naturally arises from the aggregation identity $\sum_{i=1}^N D_t^i = D_t$. Essentially, as one firm's dividend share increases through a firm-specific shock, dividend shares of all other firms are "crowded out" to maintain proper

[8] A WF process for dividend share arises endogenously in an endowment economy with two trees, as shown by Cochrane, Longstaff, and Santa-Clara (2008). WF processes originate in genetics literature by Fisher (1930) and Wright (1931). See Crow and Kimura (1970) for examples and further discussion of WF processes. We impose the parameter restrictions $\alpha > 0$ and $\delta > 0$.

aggregation. In a typical economy, dividend shares of all firms are small so $\rho_t(dW_{\theta^i}, dW_{\theta^j})$ is negligible (e.g. correlation of -0.01 between firms each paying 1% of the aggregate dividend).

Several features of the dividend share process are desirable. First, the dividend share of all firms is bounded between zero and one.[9] Second, as we formally show below, $\sum_{i=1}^{N} \theta_t^i = 1$ for all t, so the cross section of firms does aggregate to form the economy described above. Third, firm dividend shares revert towards their long-run means. Indeed, mean reversion in dividend (or consumption) share is a common assumption in the shares-based literature (e.g. Menzly, Santos, and Veronesi (2004), Santos and Veronesi (2006, 2010), and Da (2009)). Da (2009) finds empirical support for a similar mean-reverting cash flow share process for book-to-market, size, and reversal portfolios. Also common are the assumptions of mean reversion in expected dividend growth (e.g. Campbell and Shiller (1988)) as well as long-run convergence to an economy-wide steady state dividend growth (e.g. Pástor, Sinha, and Swaminathan (2008) and Da and Warachka (2009)). Mean reversion ensures that no firm eventually dominates the economy, and it captures the intuitive notion that expectations of future dividends are revised less than current dividends after a shock to the firm's dividend.

We refer to $\frac{\bar{\theta}^i}{\theta_t^i}$ as the long-run share ratio of the firm. The share ratio is the long-run expected dividend share of the firm as a proportion of its current dividend share. Several of our model implications depend on dividend share θ_t^i through the share ratio. To briefly illustrate the dividend share and share ratio characteristics, suppose that a firm is currently paying 1% of the total market dividend but is expected to exhibit high dividend growth so that it eventually pays 5% of the market dividend. The dividend share of this firm is 0.01 and the long-run share ratio is 5 $\left(\frac{5\%}{1\%}\right)$. Consider another firm that is currently paying 1%

[9] Assuming that the initial dividend share of firm i is between zero and one, i.e. $\theta_0^i \in (0,1)$, then the boundary points zero and one are unattainable within finite time if $2\alpha\bar{\theta}^i > \delta^2$ (see Karlin and Taylor (1981), pp 239–241). If the restriction $2\alpha\bar{\theta}^i > \delta^2$ is violated, we propose a reflecting barrier at each of the boundaries to maintain proper aggregation.

of the market dividend but is expected to eventually pay only 0.2% of the market dividend. This firm has a dividend share of 0.01 and a share ratio of 0.2 $\left(\frac{0.2\%}{1\%}\right)$.

We now turn to demonstrating how dividend shares and share ratios aggregate at the portfolio level. Theorem 2 provides a convenient aggregation result where the sum of any two dividend shares also follows a WF process.

Theorem 2. *Assume that the dividend share of each firm $i = 1, ..., N$ follows a WF process, as in equation (8), with the correlation structure described in equation (9). Then the dividend share of a portfolio of any two firms i and j follows a WF process,*

$$d\theta_t^p = \alpha(\bar{\theta}^p - \theta_t^p)dt + \delta\sqrt{(1-\theta_t^p)\theta_t^p}dW_{\theta^p}, \tag{10}$$

where $\bar{\theta}^p = \bar{\theta}^i + \bar{\theta}^j$ and $\theta_t^p = \theta_t^i + \theta_t^j$.

Proof. See Appendix A.2. □

Theorem 2 leads to proper aggregation of the cross section. Since the sum of any two WF processes also follows a WF process, it is trivial to show that the dividend shares of the N firms in the economy sum to one. Hence, firm dividends also sum to the aggregate dividend. Further, our model's implications for firm-level asset pricing carry over to the portfolio level. A value-weighted portfolio's share ratio, which determines exposure to long-run risk, is the dividend-weighted average of firm share ratios,

$$\frac{\bar{\theta}^p}{\theta_t^p} = \sum_{i\in\mathcal{P}} \frac{D_t^i}{\sum_{i\in\mathcal{P}} D_t^i} \frac{\bar{\theta}^i}{\theta_t^i}, \tag{11}$$

where $\mathcal{P}$ is the set of firms held in the portfolio. See Appendix A.2 for additional details on aggregation.

We assume throughout that $Cov\left(d\theta_t^i, \frac{dC_t}{C_t}\right) = Cov\left(d\theta_t^i, dX_t\right) = 0$, so the dividend share does not covary with the pricing kernel. Hence, no asset in this economy hedges against transient consumption shocks from dW_C. Even then, as we show below, firm share ratios interact with the persistent component of consumption growth to play a powerful role in determining time-varying exposures to systematic risk for each firm.

Next, firm dividend growth, obtained by applying Itô's Lemma to $D_t^i = \theta_t^i D_t$, is given by

$$\frac{dD_t^i}{D_t^i} = \left(\alpha\left(\frac{\bar{\theta}^i}{\theta_t^i} - 1\right) + \mu_D + X_t\right)dt + \delta\sqrt{\frac{1-\theta_t^i}{\theta_t^i}}dW_\theta^i + \sigma_D dW_D. \tag{12}$$

Firm-level dividend growth absorbs the expected aggregate dividend growth drift component $\mu_D + X_t$ as well as the aggregate dividend shock $\sigma_D dW_D$. The firm share ratio also affects firm dividend growth. To illustrate, if $\bar{\theta}^i > \theta_t^i$, then the dividend share is likely to increase towards $\bar{\theta}^i$. In this case, $\frac{\bar{\theta}^i}{\theta_t^i} > 1$ and expected dividend growth is higher than aggregate dividend growth. On the other hand, if $\bar{\theta}^i < \theta_t^i$, the dividend share is expected to decrease, leading to low expected dividend growth relative to the aggregate dividend. Finally, if $\bar{\theta}^i = \theta_t^i$ then the expected dividend growth rate is equal to the expected aggregate dividend growth rate. Still, firm dividend growth is more volatile due to the firm-specific shock dW_θ^i.

1.3 Cross-Sectional Asset Pricing

The cross section of returns is driven by the interaction between two properties of the economy: (i) long-run cash flows carry a higher risk premium than short-run cash flows and (ii) firms have different expected cash flow timing, reflected by the long-run share ratio characteristic. Firms with dividends that are concentrated in the short run primarily derive their value from relatively safe cash flows, while firms with primarily long-run dividends are risky investments. Share ratio is positively related to expected dividend growth and cash flow duration. Hence, share ratio is positively related to systematic risk exposure and expected return.

Below, we formalize asset pricing properties at the firm level.

Theorem 3. *The firm price-dividend ratio is*

$$\begin{aligned}\frac{P_t^i}{D_t^i} \equiv G^i(X_t, \theta_t^i; \bar{\theta}^i, \alpha) &= \int_t^\infty S(X_t, \tau) E_t[\theta_s^i] ds \qquad [\tau = s - t] \\ &= \frac{P_t}{D_t} + \left(\frac{\bar{\theta}^i}{\theta_t^i} - 1\right) \int_t^\infty S(X_t, \tau)\left(1 - e^{-\alpha\tau}\right) ds, \end{aligned} \tag{13}$$

where $S(X_t, \tau)$ is defined in Proposition 1.[10] *The firm price-dividend ratio is increasing in share ratio $\frac{\bar{\theta}^i}{\theta_t^i}$. Next, the firm-level risk premium is*

$$\mu_t^i = \frac{\lambda(\gamma - 1)}{\kappa + \beta} \sigma_X^2 \frac{G_X^i}{G^i}, \tag{14}$$

and μ_t^i is increasing in share ratio. Finally, the firm-level instantaneous variance is

$$\sigma_{i,t}^2 = \sigma_D^2 + \left(\frac{G_X^i}{G^i}\right)^2 \sigma_X^2 + \left(1 + \frac{\theta_t^i G_\theta^i}{G^i}\right)^2 \delta^2 \frac{1 - \theta_t^i}{\theta_t^i}. \tag{15}$$

Proof. See Appendix A.3. □

The firm price-dividend ratio is affected by aggregate and firm-specific conditions. In particular, all firms are affected by expected economic growth, and all price-dividend ratios increase when X_t increases. If $\theta_t^i = \bar{\theta}^i$ (i.e. the dividend share is at its long-run mean), then the firm price-dividend ratio equals that of the market. If $\frac{\bar{\theta}^i}{\theta_t^i} > 1$ ($\frac{\bar{\theta}^i}{\theta_t^i} < 1$), then firm expected dividend growth is higher (lower) than aggregate expected dividend growth, which pushes the firm price-dividend ratio higher (lower) than the aggregate price-dividend ratio. However, high share ratio also implies that firm dividends are weighted towards the long-run and agents exposed to long-run risk discount these cash flows more heavily. Still, the higher discount rate for these cash flows is not sufficient to offset the increase in the

[10] Integrability of (13) is trivially satisfied due to the transversality condition of $S(X_t, \tau)$ with $\alpha > 0$.

price-dividend ratio stemming from the higher expected dividend growth.

Finally, observe from equation (15) that firm return variance has both systematic and idiosyncratic components. Shocks to the aggregate dividend and growth rate are market-wide risks to which all firms are exposed. The systematic portion of firm return volatility varies in the cross section only to the extent that firms have differential exposures to systematic risk through $\frac{G^i_X}{G^i}$. On the other hand, the $\left(1 + \frac{\theta_t G^i_\theta}{G^i}\right)^2 \delta^2 \frac{1-\theta^i_t}{\theta^i_t}$ component arises solely from firm-specific shocks to dividend share. Later, we discuss properties of IV as well as describe the cross-sectional relation between expected return and IV.

1.3.1 A Parsimonious Beta Pricing Model

Comparing the aggregate and firm risk premiums in equations (7) and (14), respectively, we are able to formulate a conditional single-factor model. The firm expected return is related to an economic growth hedge portfolio through the beta pricing relation

$$\mu^i_t = \beta^i_{X,t} \mu_{X,t}, \tag{16}$$

where

$$\beta^i_{X,t}\left(X_t, \frac{\bar{\theta}^i}{\theta^i_t}\right) = \frac{G^i_X}{G^i} \frac{G}{G_X} \tag{17}$$

is the (time-varying) firm beta with respect to the hedging portfolio. Here, the hedging portfolio is constructed to be exposed solely to economic growth shocks through dW_X and to have expected return $\mu_{X,t} = \mu_t$. Additional information about the hedging portfolio is available in Appendix A.3.

It follows from Theorem 3 that beta is positively related to share ratio, as is illustrated in Figure 2. In fact, beta and duration are closely linked in the cross section. Long-run cash flows are more sensitive to growth rate shocks, leading to higher exposures to hedging portfolio returns for high duration firms. Therefore, firms with higher (lower) than average duration have betas greater (less) than one. There is an economically significant spread in expected returns in the cross section between high and low share ratio firms arising from these differential exposures to long-run risk.

As previously noted, we concentrate on a version of the model with uncorrelated consumption and dividend growth shocks. In the presence of non-zero correlation, we can derive an intertemporal asset pricing model in the spirit of Merton (1973), as expected return can be written

$$\mu_t^i = \beta_{M,t}^i \mu_{M,t} + \beta_{X,t}^i \mu_{X,t}, \tag{18}$$

where $\beta_{M,t}^i$ and $\mu_{M,t}$ are the firm beta and expected return for the market portfolio paying the aggregate dividend and $\beta_{X,t}^i = \left(\frac{G_X^i}{G^i} \frac{G}{G_X} - 1 \right)$. Market returns are exposed to both short-run and long-run consumption shocks, so the two factors are positively correlated. To the extent that firms have differing exposures to economic growth shocks, however, the cross-sectional implications of the long-run risk model and CAPM will differ. In Section 2.3, we test this model and generally find support for its predictions.

1.3.2 Asset Pricing with Leverage

While the dispersion and IV effects can be analyzed whether or not firms employ leverage, the credit risk effect is exclusively studied in the context of leverage. We adopt a simple specification for firm debt from Merton (1974). In this framework, a firm takes on zero-coupon debt with a face value of B payable at time T. The firm defaults if its value at time T falls below the face value of debt. If the firm does default, all its value is distributed to debtholders. Otherwise, shareholders receive the residual firm value, the value remaining after the principal payment to debtholders. Despite its simplicity, Merton's (1974) framework achieves empirical success in forecasting firm defaults. Duffie, Saita, and

Wang (2007) establish that distance to default, a key determinant of credit risk in the Merton model, is an important predictor of bankruptcies and defaults.

To formalize, the value of levered equity is given by

$$V_t^i = E_t \int_t^T \frac{\Lambda_s}{\Lambda_t} D_s^i ds + E_t \left[\frac{\Lambda_T}{\Lambda_t} \max\left(P_T^i - B, 0\right) \right]. \tag{19}$$

The first component is the expected discounted dividend stream paid prior to bond maturity, while the second is the expected discounted payoff at bond maturity. Closed-form solutions for equity value, beta, and expected return are unavailable in the presence of leverage. However, these quantities can be estimated through simulation. Appendix B.2 describes the simulation details.

In the presence of leverage, equity and debt expected returns are still determined by exposures to market and long-run risk, as in equation (18).[11] Simulations show that equity value is more sensitive to economic growth shocks than debt value. Since the influence of economic growth shocks on equity is amplified by leverage, levered equity beta is generally higher than unlevered beta. Figure 3 exhibits estimated conditional equity betas for firms with one year to debt maturity and a market debt ratio of 0.5, confirming that levered equity betas are indeed higher than their unlevered counterparts (Figure 3 versus Figure 2). Therefore, expected excess equity returns for levered firms are higher than for unlevered firms (Figure 3 versus Figure 2), while the positive expected return–share ratio relation persists among levered firms.

[11] Firm levered equity return follows the process $dR_t^{V_i} = \mu_t^{V_i} dt + \phi_X^i dW_X + O(\sqrt{dt})$, where ϕ_X^i is the elasticity of firm equity value with respect to aggregate growth shocks. The $O(\sqrt{dt})$ term contains all other diffusion terms (e.g. dW_θ and dW_D) that are uncorrelated with growth shocks. Then $\mu_t^{V_i} = \frac{(\gamma-1)\lambda}{\kappa+\beta}\sigma_X \phi_X^i = \phi_X^i \frac{G}{\sigma_X G_X}\mu_t \equiv \beta_t^{V_i}\mu_t$. We do not have an analytical expression for ϕ_X^i, which prevents us from deriving a closed-form solution for $\beta_t^{V_i}$.

2 Cross-Sectional Tests

In this section, we test the cross-sectional implications of the long-run risk model. Section 1.3 derives two cross-sectional predictions: (i) expected return is positively related to long-run share ratio, and (ii) expected return is positively related to economic growth beta. Since share ratios are based on cash-flow data while economic growth betas are based on return covariances, these predictions are complementary in nature.

2.1 Data

Aggregate parameters and the time series of expected economic growth (X_t) are estimated using quarterly US dividend and consumption data from 1948 to 2008. Aggregate dividends are calculated for the CRSP value-weighted index following Cochrane (2008).[12] Aggregate consumption is non-durable goods plus services from the NIPA tables of the Bureau of Economic Analysis. We convert the data to a real, per capita basis by adjusting for CPI inflation and the US population.

We test the implications of the long-run risk model using the cross section of U.S. stocks. Stock return data is from CRSP and financial information is from Compustat unless otherwise noted. We construct 20 value-weighted industry portfolios following Menzly, Santos, and Veronesi (2004). We also form value-weighted decile portfolios based on several firm characteristics previously shown to be related to average returns. Construction of the book-to-market, size, and momentum characteristics follows Fama and French (2008). Dispersion is calculated following Avramov, Chordia, Jostova, and Philipov (2009b) as the standard deviation of analyst fiscal year one earnings estimates from I/B/E/S divided by the absolute value of the mean estimate. As in Ang, Hodrick, Xing, and Zhang (2006), IV is the standard deviation of pricing errors relative to the Fama–French (1993) three-factor

[12] Cochrane (2008) points out that CRSP dividends capture all payments to investors including cash mergers, liquidations, and repurchases.

model in a regression using daily returns over the past month.[13] Our proxy for credit risk is the Campbell, Hilscher, and Szilagyi (2008) measure which models the probability of corporate failure over the next year as a function of accounting and market variables.

Portfolio dividend shares are calculated in each quarter by dividing the portfolio dividend by the sum of the dividends from all portfolios. Portfolio dividend formation follows Bansal, Dittmar, and Lundblad (2005). In particular, each dividend series assumes that an investor extracts the dividends and reinvests the capital gains. Dividends are aggregated across months within a quarter to form quarterly dividends, and the final dividend figure for quarter t is the average dividend in quarters $t-3$ to t to diminish the effects of seasonality in dividends.

The sample periods vary across sets of test assets based on data availability. For tests based on industry, book-to-market, size, and momentum portfolios or individual firms, the sample period is July 1948 to June 2008. IV portfolios have a sample period of July 1965 to June 2008, while dispersion and distress risk portfolios are examined from July 1981 to June 2008.

Our tests in this section focus on industry, book-to-market, size, and momentum portfolios. Table I displays average excess quarterly returns for these test assets. There is relatively little dispersion in the average industry returns in Panel A, as half of the portfolio returns are between 2.00% and 2.20% per quarter. Overall, average returns range from 1.50% (Apparel) to 2.57% (Railroads). Panel B shows that anomaly-based portfolios capture much larger return spreads. The high-minus-low portfolios for book-to-market, size, and momentum earn average returns of 1.71%, −2.35%, and 1.77%, respectively.

[13] We thank Kenneth French for making factor returns available through his website at http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html.

2.2 Cash Flow-Based Evidence

We now test the model-implied positive relation between share ratio and expected return shown in Theorem 3. Share ratio, which is an unobserved characteristic, is estimated by first estimating $\bar{\theta}^i$ for each portfolio using a Bayesian approach. The Markov chain Monte Carlo (MCMC) procedure imposes the parameter restrictions $\sum_{i=1}^{N} \bar{\theta}^i = 1$, $\alpha > 0$, and $\delta > 0$ while ensuring that Σ_t captures the dividend share dynamics from equations (8) and (9). Since direct sampling is difficult under these restrictions, we develop a Metropolis–Hastings algorithm to estimate $\bar{\theta}^i$. The share ratio time series is then calculated by dividing $\bar{\theta}^i$ by θ_t^i in each quarter. Finally, we find the full-period relation between average returns and log share ratio, $\overline{\lambda}_{\bar{\theta}/\theta}$, by estimating

$$\overline{R}_t^i = \lambda_{0,t} + \lambda_{\bar{\theta}/\theta,t} \log \frac{\bar{\theta}^i}{\theta_{t-1}^i} + \epsilon_t^i, \qquad \epsilon_t^i \sim N(0, \sigma_t^2), \tag{20}$$

$$\lambda_t = \overline{\lambda} + \nu_t, \qquad \nu_t \sim N(0, V_\lambda), \tag{21}$$

using a hierarchical Bayesian regression in the spirit of a Fama–MacBeth (1973) approach. We relate average returns to log share ratio to capture the concave theoretical relation between the two variables. Since we include a new draw of $\bar{\theta}$ in each iteration, the resulting estimates reflect uncertainty about estimated share ratios. Further estimation details are provided in Appendix B.1.1.[14]

Table II displays estimates of long-run dividend share and share ratio. Panel A shows figures for industry portfolios, and the industries are sorted by estimated share ratio. According to the long-run dividend share estimates, future dividends to the investment strategy are expected to be dominated by a few industries. For example, the Petroleum industry has a long-run dividend share of 0.385, implying this industry is expected to eventually pay

[14] Discussion of statistical significance is based upon Bayesian credible intervals. The reported cross-sectional R^2 follows Gelman and Pardoe (2006) who develop this statistic for Bayesian hierarchical models.

nearly two-fifths of the total market dividend. This estimate reflects not only the large initial size of the Petroleum industry, which constituted 16.3% of the total market capitalization and paid 11.4% of the aggregate dividend at the beginning of the sample period, but also high expected dividend growth. The share ratio estimate of 1.58 implies that Petroleum expected dividend growth was expected to outpace market dividend growth throughout the sample period. In fact, the Petroleum portfolio dividend grew faster than the market dividend by about 0.4% per quarter. Thus, relative to other industries, Petroleum industry cash flows has high duration and consequently higher exposure to long-run risk during the sample period.

Panel B of Table II shows estimates for book-to-market, size, and momentum portfolios. The cross-sectional patterns in share ratio are pronounced. Value firms have high estimated share ratio (8.75 for portfolio ten) while growth firms have low share ratio (0.26 for portfolio one). Similarly, share ratio estimates for size portfolios show high exposure to long-run risk for small firms (921.59), while big firms are less exposed (0.41). Finally, winner stocks display high share ratio (2.24) relative to losers (0.06) in the cross section. All three sets of portfolios display a general pattern in share ratio which is consistent with a long-run risk explanation for the return spreads in Table I.

Table III presents the results of the formal tests. Panel A shows a significant positive relation between share ratio and average returns for industry portfolios, consistent with the prediction of Theorem 3. An increase of one in log share ratio corresponds to an increase in average return of about 0.63% per year. In Panel B, average returns are significantly related to share ratio for book-to-market, size, and momentum portfolios. Furthermore, the effects are economically large. Based on the estimated coefficients and the average differences in log share ratio, share ratio explains quarterly average return spreads for High-Low portfolios of 0.54% (book-to-market), −1.27% (size), and 1.25% (momentum). Our cash-flow based results are consistent with the findings of Bansal, Dittmar, and Lundblad (2005), who find that the cash-flow characteristics of book-to-market, size, and momentum portfolios are consistent with a long-run risk explanation using measures different from our own. In sum, there is generally strong evidence in favor of the primary cash flow-based prediction of the long-run risk model, as share ratio is positively related to average returns across a broad set of test assets.

2.3 Return-Based Evidence

Equation (16) predicts that expected returns are positively related to economic growth betas, which capture exposure to unexpected innovations in X_t. To test this model implication, we utilize a Bayesian approach. Since X_t is a latent variable, we first estimate the unexpected innovations using a forward-filtering, backward-sampling (FFBS) technique. We then simultaneously estimate the system of equations,

$$R^i_{t,y} = \alpha^i_y + \beta^i_{m,y} R^m_{t,y} + \beta^i_{X,y}(\Delta X_{t,y}) + \epsilon^i_{t,y}, \quad \epsilon^i_{t,y} \sim N(0, \sigma^2_{i,y}), \tag{22}$$

$$\overline{R}^i_y = \lambda_{0,y} + \lambda_{m,y}\beta^i_{m,y} + \lambda_{X,y}\beta^i_{X,y} + \epsilon^i_y, \quad \epsilon^i_t \sim N(0, \sigma^2_y), \tag{23}$$

$$\lambda_y = \overline{\lambda} + \nu_t, \quad \nu_t \sim N(0, V_\lambda), \tag{24}$$

using a hierarchical Bayesian regression. Similar to a Fama–MacBeth approach, equation (22) is a time-series regression for each firm-period which captures exposures to the risk factors for firm i in the three-year period y, while equation (23) is a cross-sectional regression of average returns on factor loadings within each period y. Finally, equation (24) aggregates the period-by-period estimates to find a full-period estimate $\overline{\lambda}$ for the prices of risk. The primary advantage of this simultaneous estimation approach is that measurement error biases, which typically arise in the cross-sectional regression of average returns on estimated betas, are mitigated (see Davies (2011) for further discussion of this method). Appendix

B.1.1 further discusses our approach.

Table IV shows the cross-sectional test results for the long-run risk model with the CAPM included for comparison. Among industry portfolios in Panel A, the CAPM and long-run risk model both perform poorly. There is little evidence that exposure to either market risk or long-run risk is priced in the cross section, and the intercepts for both models are significantly positive. However, valuable information is potentially lost during portfolio formation (e.g., Litzenberger and Ramaswamy (1979)) and tests of factor models are quite sensitive to the choice of test portfolios (e.g., Ahn, Conrad, and Dittmar (2009) and Lewellen, Nagel, and Shanken (2010)). Individual firms may be a particularly attractive set of test assets, as argued by Avramov and Chordia (2006). Ang, Liu, and Schwarz (2010) show there is a loss of efficiency from forming portfolios and note that tests of whether a factor carries a significant price of risk can differ greatly using stocks or portfolios. They conclude that "there should be no reason to create portfolios and the asset pricing tests should be run on individual stocks instead." We therefore also test the long-run risk model among individual firms.

Panel B of Table IV shows that the CAPM and long-run risk model perform relatively well in the cross section of stocks. Exposure to the market and long-run risk factors are significantly and positively priced in the cross section of stocks, as predicted by the model. For the long-run risk model, the market factor carries a quarterly risk premium of 2.42%, while the premium for the long-run risk factor is 0.26%. The cross-sectional dispersion in economic growth betas is higher than for market betas, with a cross-sectional standard deviation of 3.32 for estimated β_x versus 1.84 for β_m. Both sources of risk therefore generate economically large return spreads in the cross section of stocks. Further, the intercept for the long-run risk model is -0.29% and not different from zero in a statistical sense.

We find support for the long-run risk model, as the prices of risk for the market and economic growth factors are positive and the intercept is not different from zero. However, there is some evidence that the model is misspecified. When characteristics are included in the model, book-to-market carries a significant incremental ability to forecast returns. Size contains no additional information about returns. Past returns associated with momentum are insignificantly negatively related to returns. Our base procedure requires the characteristics to forecast average returns over a three-year period and momentum is a short-run phenomenon. In unreported results, we find that when using one-year periods, momentum has a coefficient of 0.86% (standard deviation of 0.30%) when added to the CAPM versus 0.57% (standard deviation of 0.26%) after including the economic growth factor. The misspecification has several potential sources, including the imperfect measurement of economic growth innovations or a shortcoming of the long-run risk model. It is important to note, however, that economic growth betas continue to have a large and significant impact on average returns after controlling for characteristics which are often considered to be primary determinants of cross-sectional returns (e.g. Daniel, Grinblatt, Titman, and Wermers (1997)).

2.4 Discussion

The empirical evidence in Sections 2.2 and 2.3 supports the primary cross-sectional predictions of the long-run risk model. Measures of exposure to long-run risk based on either cash flows or return covariances are positively related to average returns. Long-run risk exposure does, therefore, appear to be an important determinant of expected returns.

The evidence with respect to cross-sectional anomalies based on book-to-market, size, and momentum is less clear. We do find evidence that the cash flows of portfolios sorted on these measures display patterns in risk exposure consistent with a long-run risk explanation. However, important shortcomings of the basic model should be noted. For example, taking dividend-to-price as a proxy for book-to-market, the model actually produces a growth premium. High long-run share ratio firms have high expected returns matched with low dividend-to-price ratios. Similarly, the high valuations of high share ratio firms is somewhat at odds with the size premium, although variation in dividend share which is unrelated to expected returns can also account for firm size. Modifications to the basic model appear necessary to overcome these issues. Potential model modifications to avoid these problems are offered in a long-run risk context by Croce, Lettau, and Ludvigson (2010) and Ai and Kiku (2011). Santos and Veronesi (2010) show a similar issue in the habit formation framework, and note that substantial heterogeneous cash flow risk is a potential, albeit imperfect, solution.

3 Dispersion, Idiosyncratic Volatility, and Credit Risk Effects

Thus far, we have laid out the cross-sectional predictions of the long-run risk model and demonstrated the model's considerable success in explaining the cross section of returns. We now turn to examining an intriguing feature that arises within the model, which is a negative cross-sectional relation between exposures to firm-specific and systematic risks. In essence, uncertainty about firm prospects tends to be inversely related to long-run risk exposure.

We investigate this inverse relation in the context of analyst forecast dispersion, IV, and credit risk, three characteristics related to firm-specific uncertainty which are each negatively related to average returns as shown in Table V. Consistent with the prior literature, we find that average returns are significantly negatively related to dispersion (Diether, Malloy, and Scherbina (2002)), IV (Ang, Hodrick, Xing, and Zhang (2006)), and credit risk (Dichev (1998) and Campbell, Hilscher, and Szilagyi (2008)). The average quarterly return spreads between the high and low portfolios are economically large at −2.31% for dispersion, −3.96% for IV, and −3.20% for credit risk. In this section, we develop theoretical relations between expected returns and the dispersion, IV, and credit risk characteristics. After showing that the three anomalies are closely related both theoretically and empirically, we test whether the cash flows and returns of portfolios based on the three characteristics are consistent with a long-run risk explanation.

3.1 Theoretical Relations

We first establish our proxies for each characteristic within the model. Based on the firm dividend growth process formulated in equation (12), dispersion is proxied by a measure of firm cash-flow volatility,

$$Dispersion_t^i = \sigma(dD_t^i) = D_t\sqrt{\theta_t^{i2}\sigma_D^2 + \delta^2\theta_t^i(1-\theta_t^i)}. \tag{25}$$

This interpretation of dispersion is consistent with Barron, Stanford, and Yu's (2009) finding that the dispersion effect is driven by earnings uncertainty rather than information asymmetry. Further, our measure of cash-flow variability is closely related to the measures of Zhang (2006) and Huang (2009) which are also negatively related to average returns.

While our dispersion measure is solely based on cash-flow characteristics, the IV and credit risk measures are endogenously determined through equilibrium pricing. The idiosyncratic component of instantaneous return volatility formulated in equation (15) is

$$IV_t^i = \delta\sqrt{\frac{1-\theta_t^i}{\theta_t^i}}\left(1+\frac{\theta_t^i G_\theta^i}{G^i}\right). \tag{26}$$

Finally, we examine credit risk in a Merton (1974) framework, so default occurs if $V_T < B$. Default risk in this context is heavily dependent on the probability of large decreases in firm value, which suggests a close link with firm return volatility. A closed-form solution for the probability of default is not available, but we can easily approximate credit risk using simulation. Indeed, we find a close relation between volatility and credit risk.

Our model produces a negative cross-sectional relation between expected returns and $Dispersion^i$ from equation (25). Under the sufficient condition $\theta_t^i < 1/2$ (which must hold for at least $N-1$ of the N firms), dispersion is negatively related to share ratio (see Appendix A.4). This relation, in turn, produces the dispersion effect in returns. Figure 4 confirms this negative relation among simulated firms.

We note that, since dispersion is defined solely with respect to cash-flow uncertainty, this result is dependent on our assumptions about dividend dynamics. Like our theoretical measures, dispersion is closely related to IV and credit risk in the data. However, the IV and credit risk effects in our framework are perhaps more interesting, as they arise through equilibrium prices determined by investors who are exposed to firm-specific and systematic risks.

To examine the IV effect, note that the elasticity of firm value with respect to dividend share, $\frac{\theta_t^i P_\theta^i}{P^i}$, can be written

$$\begin{aligned} \frac{\theta_t^i P_\theta^i}{P^i} &= 1 + \frac{\theta_t^i G_\theta^i}{G^i} \\ &= \frac{\int_t^\infty S(X_t,\tau) e^{-\alpha\tau} ds}{\int_t^\infty S(X_t,\tau)\left(e^{-\alpha\tau} + \frac{\bar{\theta}^i}{\theta_t^i}\left(1-e^{-\alpha\tau}\right)\right) ds}. \end{aligned} \tag{27}$$

Therefore, the term in parentheses from equation (26) simply reflects the sensitivity of firm value to firm-specific cash-flow shocks. This term only depends on dividend share through share ratio and is sharply decreasing in share ratio. In essence, the value of any firm increases with positive movement in dividend share, but this increase is greatest for low duration stocks. Low share ratio firms are strongly impacted due to their reliance on short-run cash flows. For high share ratio firms, on the other hand, the expected mean reversion in dividend share diminishes the impact of short-run shocks on their high-duration cash flows. IV arises from these short-run shocks and is thus negatively related to share ratio under the sufficient conditions in Theorem 4.

Theorem 4. *IV decreases in share ratio under the restriction on the* $(\bar{\theta}, \alpha)$*-plane* $\left(\frac{\bar{\theta}^i}{\theta_t^i} - 2\bar{\theta}^i\right)(\sqrt{2\alpha}M - 1) > 1$ *where* $M = \frac{|S(X_t,\tau)|_1}{|S(X_t,\tau)|_2}$ *is independent of* α*, and* $|\cdot|_i$ *represents the* i*-th norm.*

Proof. See Appendix A.4. □

This negative relation translates to a negative IV–expected return relation in our model, which is shown in Figure 4.

The credit risk effect is closely related to the IV effect. Intuitively, the large majority of a firm's return volatility is idiosyncratic (e.g. Campbell, Lettau, Malkiel, and Xu (2001)). Therefore, one would expect IV to be the primary cause of defaults.[15] Low share ratio firms with high levels of IV are most likely to experience this type of default. Meanwhile, high share ratio firms have low IV and do not often default for firm-specific reasons. Default risk models designed to predict bankruptcies and defaults on a year-to-year basis (e.g., Altman (1968), Ohlson (1980), and Campbell, Hilscher, and Szilagyi (2008)) are likely to identify firms with high IV as high default risk firms. However, in equilibrium these firms have low systematic risk exposures leading to a negative relation between expected return and distress risk estimates.

Simulations confirm this intuition for the credit risk effect. Most defaults occur primarily due to firm-specific shocks, while default waves accompany large economic growth shocks.

[15] Consistent with this intuition, Campbell and Taksler (2003) find that the IV of firm equity is as effective as credit rating in explaining cross-sectional variation in credit spreads. Additionally, the time series of credit spreads generally matches up with average IV, further suggesting IV plays an important role in defaults.

A negative expected return–credit risk relation therefore appears across a wide variety of parameterizations (see Appendix A.4). Figure 4 demonstrates the credit risk effect in our model.

In sum, we have shown theoretical connections between the dispersion, IV, and credit risk effects. Cross-sectional differences in cash flow timing tend to produce a negative relation between firm-specific risk and exposure to systematic risk in the cross section. Expected returns are negatively related to each of the three firm characteristics as a result.

3.2 Empirical Evidence

Prior research suggests positive relations between dispersion and IV (e.g., Barron, Stanford, and Yu (2009)), dispersion and credit risk (e.g., Avramov, Chordia, Jostova, and Philipov (2009b)), and IV and credit risk (e.g., Campbell and Taksler (2003)). Table VI confirms that all three characteristics are closely related. Panel A exhibits time-series averages of the monthly cross-sectional Spearman rank correlations between dispersion, IV, and credit risk. The evidence shows that a firm with high IV is quite likely to have high distress risk as evidenced by the correlation of 0.47 between the two measures. Similarly, the dispersion–IV correlation is 0.31 and the dispersion–default probability correlation is also 0.31.

Panel B of Table VI examines the characteristics of firms in three groups of decile portfolios sorted on dispersion, IV, and distress risk. It displays the mean dispersion, IV, and probability of default across firms in each portfolio. In the ten dispersion portfolios, both IV and the probability of default are nearly monotonically increasing as the dispersion ranking increases. Similar results hold for the IV and credit risk portfolios, with all three characteristics having nearly monotonic patterns across the portfolios.

More specifically, moving from the lowest to highest dispersion portfolio, the mean IV increases from 2.29% to 3.51%, and the mean default probability rises from 0.08% to

0.40%. Similarly, moving from the lowest to highest IV portfolio, the mean dispersion rises from 0.11 to 1.49, and the mean default probability advances from 0.11% to 3.29%. Finally, moving from the lowest to highest distress risk portfolio, the mean dispersion rises from 0.18 to 1.75, and the mean IV increases from 2.15% to 9.89%. Overall, the evidence exhibited in Table VI illustrates that a strong empirical link exists between the dispersion, IV, and credit risk measures. The remaining task is to show that all three effects are explained by exposures to long run risk.

We begin with share ratio, the cash-flow based measure of long-run risk exposure. Table VII shows estimates of share ratio for dispersion, IV, and distress risk portfolios. An overall negative pattern exists between share ratio and each of the characteristics. The lowest dispersion portfolio displays a share ratio estimate of 6.68 versus just 0.72 for the highest dispersion portfolio. Similarly, the high IV portfolio's share ratio (0.79) is much larger than the low IV share ratio (0.02). The share ratio difference between the low and high default risk portfolios is smaller (1.93 versus 1.75), while a general negative pattern appears among portfolios one through nine. Further investigation reveals considerable volatility in the dividend share paid by the top default risk deciles. In fact, the combined dividend of the two highest distress risk portfolios regularly fluctuates between high points greater than 1% of the aggregate dividend to low points of paying less than \$70 of every \$1,000,000 dollars of market dividend. The proposed dividend share process may not provide an adequate description of the extremely low dividends of distressed firms during bad times, but the evidence in Table VII is suggestive of a negative relation between default risk and share ratio. Indeed, share ratio estimates of the portfolios imply that each of the three characteristics should be negatively related to expected returns.

Table VIII formally tests the relation between average returns and share ratio among dispersion, IV, and distress risk portfolios. Dispersion portfolio returns are positively related to share ratio at a 5% significance level with a coefficient of 0.22% per quarter. The relation between returns and share ratio among IV portfolios is even stronger, with a coefficient of 0.64% which is significant at a 1% level. Distress risk portfolios, in contrast to dispersion and IV portfolios, display only a small, insignificantly positive relation as a result of the previously mentioned issues with the dividend shares of the high default risk portfolios. Excluding these two portfolios, we find a larger positive, though insignificant, coefficient of 0.33%.

Finally, we examine whether the dispersion, IV, and distress risk characteristics maintain their abilities to forecast returns after controlling for exposures to market risk and long-run risk. Table IX shows the test results for the period 1981 to 2008. Estimates for the CAPM and long-run risk model are shown for the shorter sample period. As in the previous results in Table IV, market risk is significantly positively rewarded in both models. The coefficient for the economic growth factor is 0.23% and over 90% of the posterior is positive. The sample is smaller both in length and breadth due to the characteristics data requirement, perhaps accounting for the loss of statistical significance.

When the three characteristics are introduced into the CAPM, dispersion and IV appear to help explain the cross section even after controlling for market risk. The coefficient on dispersion is −0.48% (over 94% of the posterior is negative) and the corresponding figure for IV is significant at the 5% level at −0.52%. Including default risk as a characteristic does not appear to help explain the cross section, as the coefficient is only −0.03%.[16]

After the economic growth factor is included, the explanatory abilities of dispersion and IV are diminished. The magnitude of the dispersion coefficient is about 40% smaller in the long-run risk model compared to the CAPM. Similarly, the IV coefficient drops from −0.52% to −0.36% and loses significance after including the economic growth factor. These results provide evidence that the long-run risk model (at least partially) captures

[16] In unreported results, we find an insignificant coefficient on default risk of −0.44% in tests using one-year periods, suggesting the one-year holding period may be important for the credit risk effect.

the cross-sectional effects related to dispersion and IV.

In sum, we find evidence consistent with our model's prediction that the dispersion, IV, and distress risk anomalies can be explained by long-run risk. The three variables are closely linked, and their cash-flow characteristics appear to be largely consistent with a long-run risk explanation. Finally, there is some evidence that a long-run risk factor captures much of the cross-sectional variation related to dispersion and IV.

3.2.1 Reconciling the Empirical IV Evidence

The empirical literature is conflicted about the sign of the IV–return relation. For example, Ang, Hodrick, Xing, and Zhang (2006, 2009) and Jiang, Xu, and Yao (2009) find a negative relation between returns and IV. On the other hand, Malkiel and Xu (2002) and Fu (2009) show a positive IV–return relation. Accordingly, Bali and Cakici (2008) and Huang, Liu, Rhee, and Zhang (2010) question whether a robust IV–return relation exists. This disagreement is perhaps unsurprising in light of our theoretical results.

In our model, IV is determined by two firm characteristics – dividend share and long-run share ratio. IV is decreasing in both measures. However, the relation between IV and expected returns is fully driven by share ratio. High share ratio firms have high expected returns, while share ratio negatively influences IV which produces a negative cross-sectional relation between IV and expected returns. As previously noted, however, IV is also negatively related to dividend share. Dividend share tends to be negatively related to expected returns in the cross section since high θ_t tends to be associated with low $\frac{\bar{\theta}}{\theta_t}$. Thus, a spurious positive IV–return relation can appear if the high (low) IV portfolios from a given sorting procedure have low (high) dividend share.

Tables X and XI illustrate this possibility. Panel A of Table X shows average characteristics and returns for portfolios sorted by firm dividend share. Dispersion, IV, default probability, and average returns are each negatively associated with θ_t, as predicted by the model. These patterns are suggestive of positive relations between expected returns and each characteristic. Panel B, on the other hand, examines portfolios sorted by dividend growth over the past year. Realized dividend growth is a rough proxy for $\frac{\bar{\theta}}{\theta_t}$ which is available at the firm level.[17] While dispersion, IV, and default risk are negatively related to $\frac{\bar{\theta}}{\theta_t}$, average returns are increasing to produce negative relations between returns and the characteristics.

When stocks are double-sorted by dividend share and share ratio in Table XI, negative relations between the characteristics and returns dominate the evidence. Consistent with model predictions, dispersion, IV, and default risk are decreasing in both θ_t and $\frac{\bar{\theta}}{\theta_t}$. Average returns are generally increasing in $\frac{\bar{\theta}}{\theta_t}$ and decreasing in θ_t, as in Table X. However, the return spreads in the θ_t dimension appear to be largely unrelated to spreads in characteristics. The largest differences in characteristics occur among low dividend growth firms, matched by the smallest average return spread (-0.46%). Conversely, the largest average return spread (-2.24%) is associated with the smallest differences in characteristics. In contrast, average return spreads in the $\frac{\bar{\theta}}{\theta_t}$ dimension match the patterns in characteristic spreads, suggesting a closer link between the characteristics and share ratio.

The nature of the observed IV–return relation will therefore depend on the IV proxy and sorting procedure. An IV proxy which is more closely related to θ_t than to $\frac{\bar{\theta}}{\theta_t}$ may exhibit a positive IV–return relation. In contrast, the IV proxy of Ang, Hodrick, Xing, and Zhang (2006) is closely (and negatively) related to $\frac{\bar{\theta}}{\theta_t}$, as demonstrated in Table VII, producing a negative IV–return relation. Furthermore, our model reconciles the evidence that the IV effect is strong in value-weighted portfolios and essentially non-existent in equally weighted

[17] An unpleasant feature of using realized dividend growth to proxy for $\frac{\bar{\theta}}{\theta_t}$ is that the realization is the sum of expected dividend growth (which is positively related to $\frac{\bar{\theta}}{\theta_t}$) and an unexpected shock. Firms with high uncertainty are more likely to experience extreme shocks, so we are more likely to misclassify high dispersion, IV, and default risk firms as having exceptionally low or high $\frac{\bar{\theta}}{\theta_t}$. The general cross-sectional patterns appear to be unaffected by this shortcoming of our proxy.

portfolios (Bali and Cakici (2008)). The returns of equally weighted portfolios can be dominated by small stocks while value-weighted portfolios draw more information from relatively large stocks. High IV portfolios tend to be populated with many small stocks, many of which have high IV due to low dividend share. On the other hand, the high IV of relatively large stocks is almost always driven by low share ratio. Thus, value-weighted returns are driven by variation in share ratio, while the IV–return relation is clouded by dividend share with equal weights. In sum, our model's predictions are consistent with the observed sensitivity of the IV effect.

4 Conclusion

This paper develops and applies an intertemporal asset pricing model in a long-run risk economy with a formal cross section of firms. Expected returns in the economy are positively related to cash flow duration, while firms in the cross section are characterized by expected cash flow timing. Firms with high share ratio, which corresponds to high expected dividend growth, are more sensitive to shocks to the persistent economic growth rate and hence command higher risk premiums.

The model predictions are broadly supported in the data. Exposures to long-run risk measured using either cash flows or return covariances are positively related to returns in the cross section. Portfolios sorted on book-to-market, size, and momentum display cash-flow characteristics consistent with a long-run risk explanation, although challenges remain in fully explaining these return patterns.

Finally, our model suggests that the puzzling negative cross-sectional relations between expected stock returns and analysts' forecast dispersion, IV, and credit risk emerge as an equilibrium response to long-run risk. While high duration firms are highly exposed to systematic shocks, low duration firms are particularly sensitive to firm-specific dividend shocks. As a result, firms with high measures of idiosyncratic risk tend to have low systematic risk and low expected returns, which explains the observed patterns. The evidence supports a long-run risk explanation of these effects, and we provide a potential resolution of the empirical sensitivity of the IV effect.

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Table I: **Average Quarterly Portfolio Returns**

This table reports average quarterly excess returns for value-weighted portfolios. Panel A shows returns of 20 industry portfolios formed following Menzly, Santos, and Veronesi (2004). Panel B displays portfolios based on book-to-market, size, and momentum. These three characteristics are formed following Fama and French (2008). The book-to-market and size portfolios are rebalanced annually at the end of June, while the momentum portfolios are rebalanced monthly. The table reports t-stats in parentheses. The sample period is July 1948 to June 2008.

Panel A: Average Returns of Industry Portfolios

Industry	Avg. Ret.	(t-stat)
Railroads	2.57	(3.76)
Petroleum	2.55	(4.85)
Mining	2.36	(3.55)
Fab. Metals	2.24	(3.76)
Food	2.20	(4.16)
Manufacturing	2.20	(3.38)
Machinery	2.18	(3.08)
Electrical Eq.	2.13	(2.78)
Financial	2.13	(3.50)
Transport Eq.	2.11	(3.20)
Retail	2.06	(3.14)
Chemical	2.04	(3.80)
Dept. Stores	2.03	(3.00)
Paper	2.02	(3.27)
Other Transport.	1.81	(2.28)
Prim. Metals	1.78	(2.47)
Utilities	1.72	(3.69)
Construction	1.58	(2.14)
Other	1.57	(2.75)
Apparel	1.50	(2.00)

Panel B: Average Returns of Anomaly Portfolios						
	Book-to-Market		Size		Momentum	
Portfolio	Avg. Ret.	(t-stat)	Avg. Ret.	(t-stat)	Avg. Ret.	(t-stat)
Low	1.46	(2.16)	4.09	(4.50)	1.07	(1.00)
2	1.48	(2.59)	2.93	(3.43)	1.43	(1.84)
3	1.82	(3.31)	2.58	(3.08)	1.26	(1.88)
4	1.73	(3.25)	2.38	(2.98)	1.82	(3.16)
5	2.13	(4.16)	2.26	(2.94)	1.92	(3.65)
6	2.23	(4.58)	2.21	(2.99)	2.91	(3.64)
7	2.22	(4.06)	2.13	(3.01)	2.14	(4.13)
8	2.45	(4.37)	2.15	(3.22)	2.03	(3.96)
9	2.78	(4.89)	2.21	(3.64)	2.40	(4.02)
High	3.17	(4.34)	1.73	(3.47)	2.84	(3.66)
High-Low	1.71	(2.71)	-2.35	(-3.13)	1.77	(1.98)

Table II: **Long-Run Dividend Share and Share Ratio Estimates**

This table reports the average draws of long-run dividend share, $\bar{\theta}$, and share ratio, $\bar{\theta}/\theta_t$, for the test portfolios. Panel A reports these statistics for 20 industry portfolios while Panel B shows statistics for ten portfolios sorted on book-to-market, size, and momentum. Industries are defined following Menzly, Santos, and Veronesi (2004). Long-run dividend share is estimated using a Markov chain Monte Carlo (MCMC) technique with Metropolis–Hastings steps as discussed in Appendix B.1.1. We display the mean and standard deviation of $\bar{\theta}$ and the time-series median of mean and standard deviation $\bar{\theta}/\theta_t$ draws from the MCMC procedure. The sample period is July 1948 to June 2008.

Panel A: Industry Portfolios

Industry	$\bar{\theta}$	$\sigma(\bar{\theta})$	$\bar{\theta}/\theta_t$	$\sigma(\bar{\theta}/\theta_t)$
Food	0.098	(0.041)	1.865	(0.785)
Petroleum	0.385	(0.071)	1.577	(0.291)
Chemical	0.173	(0.057)	1.537	(0.503)
Machinery	0.057	(0.028)	1.433	(0.699)
Railroads	0.046	(0.026)	1.402	(0.774)
Financial	0.030	(0.017)	1.293	(0.756)
Paper	0.030	(0.018)	1.216	(0.737)
Retail	0.016	(0.011)	1.048	(0.736)
Mining	0.021	(0.014)	1.030	(0.673)
Fabricated Metals	0.012	(0.009)	0.858	(0.653)
Manufacturing	0.015	(0.011)	0.786	(0.560)
Dept Stores	0.029	(0.019)	0.717	(0.471)
Other Trans	0.002	(0.002)	0.465	(0.375)
Transport Equipment	0.036	(0.023)	0.422	(0.271)
Other	0.023	(0.017)	0.353	(0.260)
Electrical Equipment	0.010	(0.008)	0.271	(0.215)
Apparel	0.001	(0.001)	0.186	(0.151)
Utilities	0.011	(0.009)	0.177	(0.142)
Primary Metals	0.007	(0.005)	0.172	(0.139)
Construction	0.001	(0.000)	0.039	(0.031)

Panel B: Anomaly Portfolios				
Industry	$\bar{\theta}$	$\sigma(\bar{\theta})$	$\bar{\theta}/\theta_t$	$\sigma(\bar{\theta}/\theta_t)$
		Book-to-Market		
Low	0.033	(0.022)	0.263	(0.177)
2	0.037	(0.024)	0.313	(0.205)
3	0.026	(0.019)	0.219	(0.162)
4	0.032	(0.022)	0.281	(0.189)
5	0.143	(0.061)	1.219	(0.518)
6	0.168	(0.065)	1.660	(0.639)
7	0.073	(0.041)	0.995	(0.559)
8	0.049	(0.029)	0.779	(0.452)
9	0.193	(0.066)	4.279	(1.466)
High	0.245	(0.073)	8.751	(2.618)
		Size		
Low	0.178	(0.110)	921.593	(570.481)
2	0.061	(0.044)	79.043	(56.262)
3	0.030	(0.024)	17.035	(13.469)
4	0.030	(0.023)	9.320	(7.329)
5	0.029	(0.021)	4.692	(3.382)
6	0.031	(0.026)	2.885	(2.367)
7	0.047	(0.039)	2.345	(1.921)
8	0.084	(0.064)	2.085	(1.602)
9	0.179	(0.116)	1.693	(1.097)
High	0.331	(0.182)	0.408	(0.224)
		Momentum		
Low	0.001	(0.000)	0.060	(0.030)
2	0.006	(0.002)	0.134	(0.052)
3	0.016	(0.004)	0.194	(0.052)
4	0.067	(0.007)	0.620	(0.066)
5	0.132	(0.010)	1.027	(0.077)
6	0.149	(0.011)	1.113	(0.078)
7	0.177	(0.011)	1.179	(0.074)
8	0.178	(0.011)	1.316	(0.083)
9	0.184	(0.011)	1.649	(0.100)
High	0.090	(0.008)	2.235	(0.193)

Table III: **Relation between Share Ratio and Average Return**

This table reports the cross-sectional relations between average returns and share ratio. Panel A shows the relation among industry portfolios, while Panel B displays results for portfolios formed on book-to-market, size, and momentum. The reported cross-sectional relations between returns and lagged share ratios are estimated from equations (20) to (21) using a hierarchical Bayesian regression. The portfolio share ratios are estimated using a Markov chain Monte Carlo (MCMC) technique. Further discussion of the estimation technique appears in Appendix B.1.1. Standard errors are reported in parentheses and an * (**) denotes statistical significance at the 5% (1%) level. The sample period is July 1948 to June 2008.

Panel A: Industry Portfolios			
Portfolios	$\overline{\lambda}_0$	$\overline{\lambda}_{\bar{\theta}/\theta}$	R^2
Industries	2.183**	0.157**	0.013
	(0.523)	(0.067)	

Panel B: Anomaly Portfolios			
Portfolios	$\overline{\lambda}_0$	$\overline{\lambda}_{\bar{\theta}/\theta}$	R^2
Book-to-Market	2.230**	0.158*	0.087
	(0.527)	(0.077)	
Size	2.188**	0.171*	0.268
	(0.653)	(0.084)	
Momentum	2.035**	0.353*	0.170
	(0.553)	(0.165)	

Table IV: **Cross-Sectional Test of the Long-Run Risk Beta-Pricing Model**

This table reports the results of cross-sectional tests of the CAPM and long-run risk factor model. Panel A shows the relation among industry portfolios, while Panel B displays results for individual firms. The reported relations are estimated from equations (22) to (24) using a hierarchical Bayesian regression. The innovations in X_t are estimated using a forward filtering–backward sampling (FFBS) algorithm. The size, book-to-market, and momentum characteristics in Panel B are measured at the beginning of each three-year period. Further discussion of the estimation technique appears in Appendix B.1.1. Standard errors are reported in parentheses and an * (**) denotes statistical significance at the 5% (1%) level. The sample period is July 1948 to June 2008.

Panel A: Industry Portfolios

Model	$\overline{\lambda}_0$	$\overline{\lambda}_m$	$\overline{\lambda}_x$	R^2
CAPM	1.377**	0.627		0.201
	(0.335)	(0.469)		
LRR	1.278**	0.691	0.006	0.283
	(0.333)	(0.443)	(0.198)	

Panel B: Individual Firms

Model	$\overline{\lambda}_0$	$\overline{\lambda}_m$	$\overline{\lambda}_x$	$\overline{\lambda}_{size}$	$\overline{\lambda}_{bm}$	$\overline{\lambda}_{mom}$	R^2
CAPM	-0.488	2.513**					0.414
	(0.394)	(0.252)					
LRR	-0.287	2.424**	0.263**				0.496
	(0.372)	(0.240)	(0.099)				
CAPM (w/ Char.)	-0.480	2.502**		-0.037	0.751**	-0.414	0.470
	(0.466)	(0.311)		(0.114)	(0.189)	(0.350)	
LRR (w/ Char.)	-0.388	2.517**	0.372**	-0.033	0.681**	-0.490	0.548
	(0.465)	(0.262)	(0.117)	(0.117)	(0.186)	(0.356)	

Table V: **Anomaly Portfolio Return Characteristics**

This table reports average quarterly excess returns for portfolios sorted on analysts' forecast dispersion, idiosyncratic volatility (IV), and distress risk (Def Prob). Dispersion is the standard deviation of earnings forecasts divided by the absolute value of the mean forecast. IV is the standard deviation of the residual from a Fama–French (1993) three-factor regression using daily returns from the month prior to portfolio formation. The probability of default is based on the Campbell, Hilscher, and Szilagyi (2008) measure. Portfolios are value weighted. The dispersion and IV portfolios are rebalanced monthly, while the distress risk portfolio is rebalanced annually in July. The table reports t-stats in parentheses. The sample period is July 1980 to June 2008 for dispersion and distress risk July 1965 to June 2008 for idiosyncratic volatility.

Average Returns of Anomaly Portfolios

	Dispersion		Idiosyncratic Volatility		Distress Risk	
Portfolio	Avg. Ret.	(t-stat)	Avg. Ret.	(t-stat)	Avg. Ret.	(t-stat)
Low	3.04	(3.87)	1.42	(2.76)	2.48	(3.17)
2	1.69	(2.29)	1.44	(2.46)	1.95	(2.82)
3	1.39	(1.82)	1.55	(2.35)	1.92	(2.74)
4	1.93	(2.32)	1.84	(2.48)	1.79	(2.22)
5	1.44	(1.61)	1.81	(2.17)	1.87	(2.24)
6	2.19	(2.26)	1.98	(2.07)	1.50	(1.61)
7	1.48	(1.62)	1.32	(1.19)	0.85	(0.73)
8	1.92	(1.78)	0.66	(0.56)	0.13	(0.09)
9	0.86	(0.78)	-0.27	(-0.20)	0.41	(0.23)
High	0.73	(0.60)	-2.55	(-1.86)	-0.72	(-0.36)
High-Low	-2.31	(-2.45)	-3.96	(-3.33)	-3.20	(-2.03)

Table VI: **Relation between Dispersion, Idiosyncratic Volatility, and Distress Risk**

This table reports statistics on the relation between dispersion, idiosyncratic volatility (IV), and distress risk (Default Probability). Dispersion is the standard deviation of earnings forecasts divided by the absolute value of the mean forecast. IV is the standard deviation of the residual from a Fama–French (1993) three-factor regression using daily returns from the month prior to portfolio formation. The probability of default is based on the Campbell, Hilscher, and Szilagyi (2008) measure. Panel A reports the correlations between dispersion, IV, and default probability. The reported correlations are time-series averages of the monthly Spearman cross-sectional correlations between the three measures. Panel B shows the average dispersion, IV, and default probability for firms appearing in decile portfolios sorted on each of the three variables. The dispersion and IV portfolios are rebalanced monthly, while the distress risk portfolio is rebalanced annually in July. The sample period is July 1981 to June 2008.

Panel A: Spearman Rank Correlation

	Dispersion	Idiosyncratic Volatility	Default Probability
Dispersion	1.00	0.31	0.31
Idiosyncratic Volatility		1.00	0.47
Default Probability			1.00

Panel B: Average Characteristics of Firms in Portfolios

	Portfolio									
Characteristic	Low	2	3	4	5	6	7	8	9	High
					Dispersion Portfolios					
Dispersion	0.00	0.02	0.02	0.03	0.04	0.06	0.08	0.12	0.23	3.78
Idiosyncratic Volatility	2.29	1.88	1.98	2.09	2.23	2.37	2.69	2.78	3.08	3.51
Default Probability	0.08	0.06	0.07	0.09	0.11	0.13	0.18	0.26	0.32	0.40
					Idiosyncratic Volatility Portfolios					
Dispersion	0.11	0.14	0.22	0.31	0.42	0.53	0.72	0.90	1.12	1.49
Idiosyncratic Volatility	0.69	1.24	1.61	1.98	2.38	2.84	3.40	4.17	5.42	9.89
Default Probability	0.11	0.05	0.07	0.10	0.13	0.20	0.33	0.58	1.09	3.29
					Distress Risk Portfolios					
Dispersion	0.18	0.17	0.19	0.26	0.40	0.42	0.62	0.98	1.47	1.75
Idiosyncratic Volatility	2.15	2.18	2.24	2.36	2.46	2.65	3.07	3.76	4.91	7.14
Default Probability	0.02	0.02	0.03	0.04	0.05	0.06	0.07	0.11	0.24	5.60

Table VII: **Long-Run Dividend Share and Share Ratio Estimates**

This table reports the average draws of long-run dividend share, $\bar{\theta}$, and share ratio, $\bar{\theta}/\theta_t$, for the test portfolios. These statistics are reported for ten portfolios sorted on dispersion, idiosyncratic volatility, and momentum. Long-run dividend share is estimated using a Markov chain Monte Carlo (MCMC) technique with Metropolis–Hastings steps as discussed in Appendix B.1.1. We display the mean and standard deviation of $\bar{\theta}$ and the time-series median of mean and standard deviation $\bar{\theta}/\theta_t$ draws from the MCMC procedure. The sample period is July 1980 to June 2008 for dispersion and distress risk July 1965 to June 2008 for idiosyncratic volatility.

Portfolio	$\bar{\theta}$	$\sigma(\bar{\theta})$	$\bar{\theta}/\theta_t$	$\sigma(\bar{\theta}/\theta_t)$
		Dispersion		
Low	0.633	(0.094)	6.675	(0.986)
2	0.066	(0.042)	0.358	(0.226)
3	0.059	(0.039)	0.364	(0.244)
4	0.060	(0.038)	0.469	(0.298)
5	0.031	(0.023)	0.266	(0.196)
6	0.085	(0.046)	0.808	(0.435)
7	0.019	(0.014)	0.222	(0.170)
8	0.030	(0.021)	0.530	(0.379)
9	0.006	(0.005)	0.178	(0.141)
High	0.012	(0.008)	0.720	(0.502)
		Idiosyncratic Volatility		
Low	0.290	(0.015)	0.789	(0.041)
2	0.180	(0.013)	0.649	(0.047)
3	0.265	(0.015)	1.732	(0.097)
4	0.133	(0.011)	1.629	(0.137)
5	0.071	(0.008)	1.371	(0.160)
6	0.052	(0.007)	1.681	(0.234)
7	0.010	(0.003)	0.481	(0.137)
8	0.001	(0.001)	0.079	(0.054)
9	0.000	(0.000)	0.020	(0.016)
High	0.000	(0.000)	0.018	(0.014)
		Distress Risk		
Low	0.258	(0.016)	1.932	(0.119)
2	0.261	(0.016)	1.223	(0.077)
3	0.170	(0.014)	0.897	(0.071)
4	0.108	(0.012)	0.715	(0.076)
5	0.091	(0.011)	0.736	(0.085)
6	0.072	(0.010)	0.862	(0.118)
7	0.033	(0.006)	0.982	(0.195)
8	0.005	(0.002)	0.494	(0.218)
9	0.001	(0.001)	0.329	(0.201)
High	0.001	(0.000)	1.750	(0.371)

Table VIII: **Relation between Share Ratio and Average Return**

This table reports the relations between average returns and share ratio. The table displays results for portfolios formed on dispersion, idiosyncratic volatility, and distress risk. The reported relations are time-series average of the coefficients from cross-sectional regressions of excess returns on share ratio. The portfolio share ratios are estimated using the Markov chain Monte Carlo (MCMC) technique discussed in Appendix B.1.1. Standard errors are reported in parentheses and an * (**) denotes statistical significance at the 5% (1%) level. The sample period is July 1948 to June 2008.

Portfolios	$\overline{\lambda}_0$	$\overline{\lambda}_{\bar{\theta}/\theta}$	R^2
Dispersion	1.992*	0.215*	0.023
	(0.834)	(0.131)	
Idiosyncratic Volatility	0.349	0.639**	0.222
	(1.680)	(0.270)	
Distress Risk	1.466	0.051	0.053
	(0.920)	(0.297)	

Table IX: **Cross-Sectional Test of the Long-Run Risk Beta-Pricing Model**

This table reports the results of cross-sectional tests of the CAPM and long-run risk factor model among individual firms. The reported relations are estimated from equations (22) to (24) using a hierarchical Bayesian regression. The innovations in X_t are estimated using a forward filtering–backward sampling (FFBS) algorithm. The dispersion, IV, and default probability characteristics are measured at the beginning of each period. Further discussion of the estimation technique appears in Appendix B.1.1. Standard errors are reported in parentheses and an * (**) denotes statistical significance at the 5% (1%) level. The sample period is July 1980 to June 2008.

Model	$\overline{\lambda}_0$	$\overline{\lambda}_m$	$\overline{\lambda}_x$	$\overline{\lambda}_{disp}$	$\overline{\lambda}_{iv}$	$\overline{\lambda}_{def}$	R^2
CAPM	-0.373	2.291**					0.375
	(0.481)	(0.487)					
LRR	-0.053	2.185**	0.228				0.563
	(0.429)	(0.485)	(0.182)				
CAPM (w/ Char.)	-0.658	2.515**		-0.475	-0.522*	-0.032	0.422
	(0.610)	(0.581)		(0.320)	(0.234)	(0.294)	
LRR (w/ Char.)	-0.260	2.309**	0.274	-0.295	-0.362	0.023	0.582
	(0.600)	(0.627)	(0.228)	(0.325)	(0.237)	(0.281)	

Table X: **Anomaly Portfolio Return Characteristics**

This table reports the average dispersion, idiosyncratic volatility, default probability, and quarterly excess return for portfolios sorted on dividend size (θ) in Panel A and realized dividend growth (which is a proxy for $\bar{\theta}/\theta$ in Panel B. The sample period is July 1980 to June 2008 for dispersion and distress risk July 1965 to June 2008 for idiosyncratic volatility and average returns. The return patterns are similar for the July 1980 to June 2008 period.

Panel A: Dividend Size (θ) Portfolios				
Portfolio	Dispersion	Idio. Vol.	Def. Prob.	Avg. Ret.
0	0.81	3.89	0.76	2.88
Low	0.32	2.72	0.10	3.77
2	0.29	2.43	0.07	3.18
3	0.23	2.25	0.07	3.06
4	0.23	2.09	0.06	2.92
5	0.15	1.95	0.06	2.79
6	0.15	1.83	0.05	2.71
7	0.15	1.71	0.04	2.40
8	0.12	1.59	0.06	2.31
9	0.13	1.46	0.04	2.08
High	0.10	1.30	0.04	1.61
Panel B: Dividend Growth ($\bar{\theta}/\theta$ Proxy) Portfolios				
Portfolio	Dispersion	Idio. Vol.	Def. Prob.	Avg. Ret.
Cut	1.35	3.49	0.48	1.90
Low	0.66	2.37	0.11	1.76
2	0.27	1.97	0.05	2.46
3	0.21	1.89	0.05	2.44
4	0.15	1.74	0.05	2.44
5	0.10	1.66	0.04	2.34
6	0.08	1.67	0.05	2.45
7	0.07	1.71	0.04	2.61
8	0.08	1.79	0.04	2.65
9	0.12	1.93	0.05	3.09
High	0.11	2.16	0.05	3.46

Table XI: **Characteristics and Returns of Portfolios Sorted on θ and $\bar{\theta}/\theta$**

This table reports the average dispersion, idiosyncratic volatility, default probability, and quarterly excess return for portfolios sorted on first on dividend size (θ) and then on realized dividend growth (which is a proxy for $\bar{\theta}/\theta$. The sample period is July 1980 to June 2008 for dispersion and distress risk July 1965 to June 2008 for idiosyncratic volatility and average returns. The return patterns are similar for the July 1980 to June 2008 period.

Panel A: Average Dispersion

		Dividend Growth ($\bar{\theta}/\theta$ Proxy)					
		Low	2	3	4	High	High-Low
Dividend (θ)	Low	0.68	0.36	0.18	0.13	0.12	-0.56
	2	0.56	0.20	0.13	0.10	0.15	-0.41
	3	0.38	0.17	0.07	0.07	0.10	-0.28
	4	0.34	0.14	0.08	0.06	0.08	-0.26
	High	0.24	0.10	0.06	0.05	0.09	-0.15
	High-Low	-0.44	-0.26	-0.12	-0.08	-0.03	

Panel B: Average Idiosyncratic Volatility

		Dividend Growth ($\bar{\theta}/\theta$ Proxy)					
		Low	2	3	4	High	High-Low
Dividend (θ)	Low	2.71	2.52	2.39	2.38	2.54	-0.17
	2	2.27	2.10	2.04	2.07	2.24	-0.03
	3	2.00	1.83	1.77	1.80	1.99	-0.01
	4	1.80	1.58	1.52	1.58	1.78	-0.02
	High	1.50	1.35	1.30	1.36	1.48	-0.02
	High-Low	-1.20	-1.17	-1.10	-1.03	-1.06	

Panel C: Average Default Probability

		Dividend Growth ($\bar{\theta}/\theta$ Proxy)					
		Low	2	3	4	High	High-Low
Dividend (θ)	Low	0.12	0.07	0.06	0.06	0.06	-0.06
	2	0.08	0.05	0.08	0.05	0.05	-0.03
	3	0.07	0.05	0.04	0.04	0.05	-0.02
	4	0.06	0.04	0.04	0.04	0.04	-0.02
	High	0.05	0.04	0.04	0.04	0.04	-0.01
	High-Low	-0.07	-0.03	-0.02	-0.02	-0.02	

Panel D: Average Excess Return

		Dividend Growth ($\bar{\theta}/\theta$ Proxy)					
		Low	2	3	4	High	High-Low
Dividend (θ)	Low	2.56	3.45	3.18	3.47	3.83	1.27
	2	2.28	2.96	2.75	3.05	3.17	0.89
	3	2.25	2.28	2.71	2.76	3.00	0.75
	4	2.15	1.90	2.32	2.51	2.63	0.48
	High	2.10	1.83	1.81	1.76	1.59	-0.51
	High-Low	-0.46	-1.62	-1.37	-1.71	-2.24	

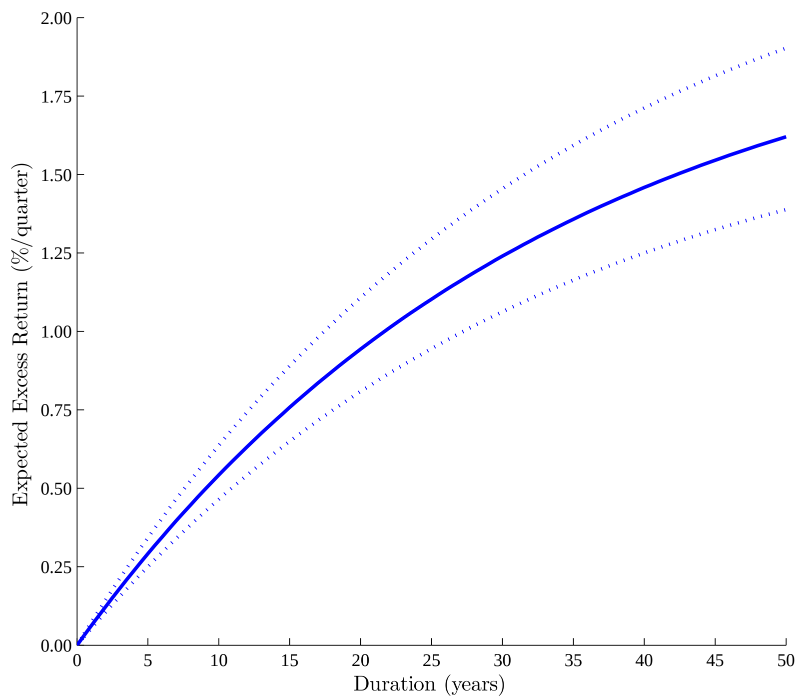

Figure 1: Cash Flow Duration and Expected Returns

This figure shows the expected quarterly excess return of an asset as a function of its cash flow duration. The asset considered pays a non-negative portion of the aggregate dividend at all times t. The dotted lines show the 90% confidence band for expected excess returns given the posterior distribution of aggregate parameters.

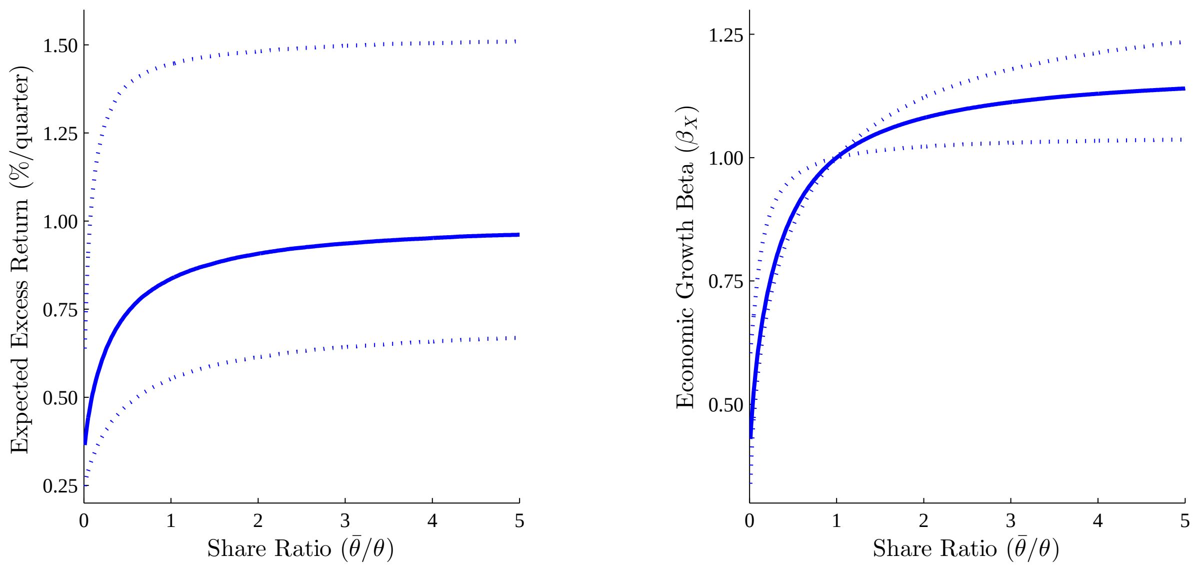

Figure 2: Expected Excess Returns and Economic Growth Betas for Unlevered Firms
This figure shows expected excess returns and economic growth betas as a function of firm share ratio for an unlevered firm. The first figure graphs the expected monthly excess return against share ratio. The expected excess return of firm i is given by equation (14). The second figure plots the firm's beta relative to an economic growth hedge portfolio, given by equation (16), as a function of share ratio. The dotted lines show the 90% confidence bands given the posterior distribution of aggregate and portfolio parameters. See Appendix B for details on aggregate and portfolio parameter estimation.

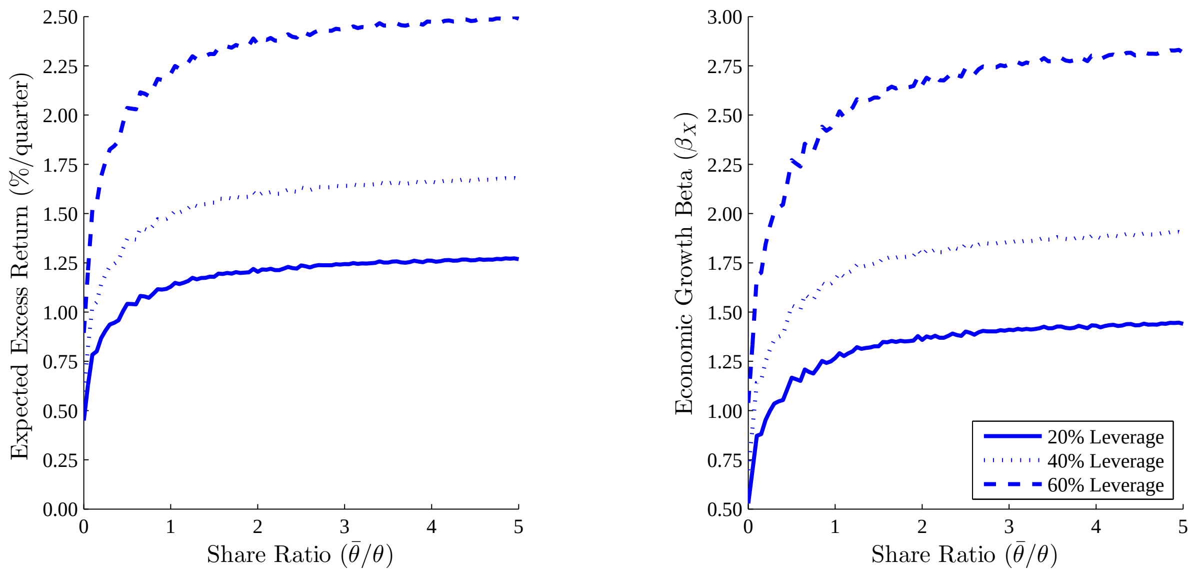

Figure 3: Expected Excess Returns and Economic Growth Betas for Levered Firms
This figure shows expected excess returns and economic growth betas as a function of firm share ratio for a levered firm. The firm is assumed to have a market debt ratio of 0.5 with one year to debt maturity. Following the Merton (1974) framework, a firm is assumed to default if its firm value at debt maturity is less than the face value of its debt. The first figure graphs the expected monthly excess return against share ratio. The second figure plots the firm's beta relative to an economic growth hedge portfolio as a function of share ratio. The dotted lines show the 90% confidence bands of simulated values from 5,000 iterations of the simulation, where the aggregate parameters in each iteration are drawn from their posterior distribution. See Appendix B for details on aggregate and portfolio parameter estimation and the simulation procedure.

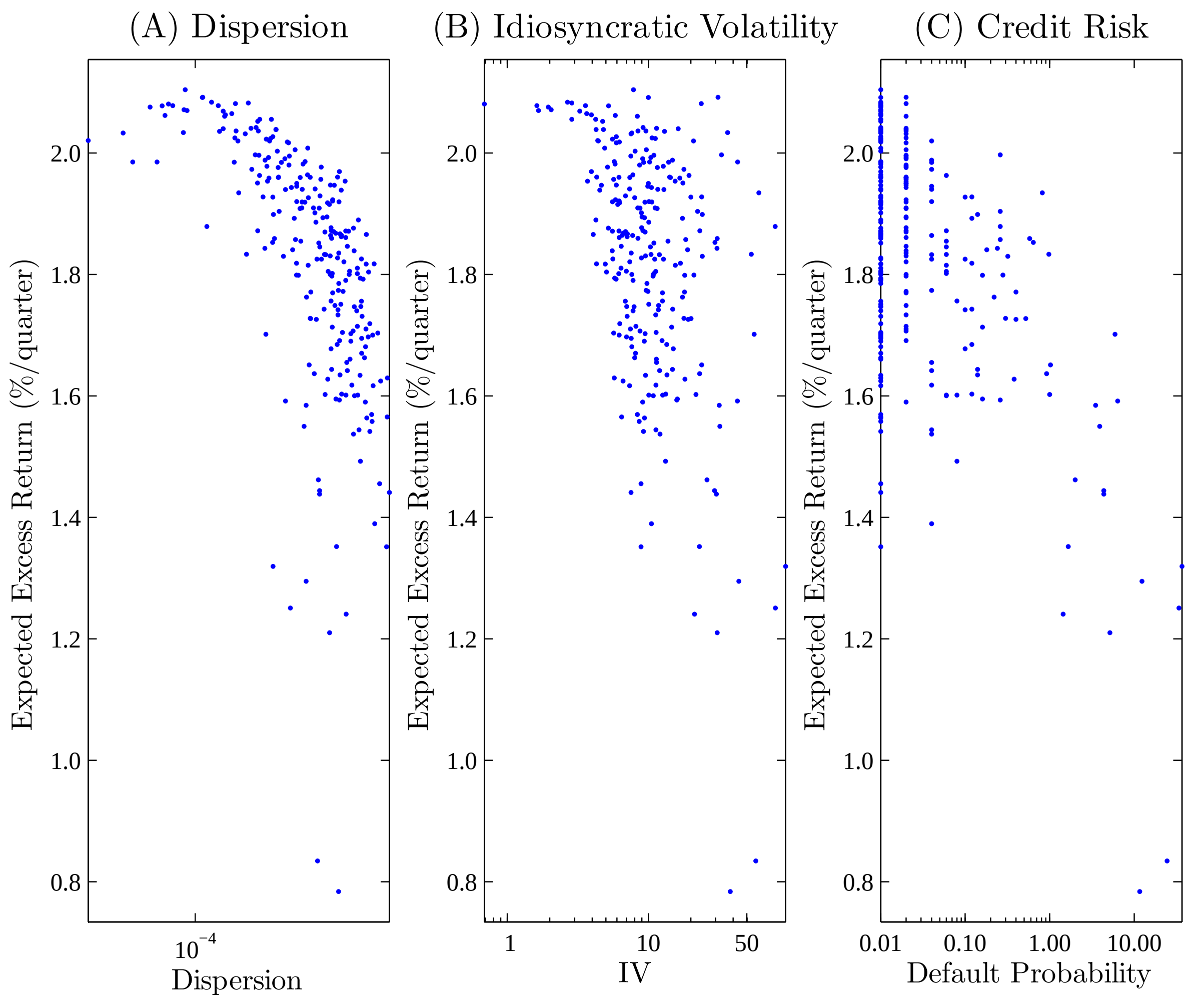

Figure 4: Expected Returns and Firm Characteristics

This figure plots the cross-sectional relations between expected excess return and each of dispersion, idiosyncratic volatility (IV), and credit risk. The plots are based on a cross section of 250 simulated firms. For each firm, dispersion and IV are calculated based on equations (25) and (26), respectively. Credit risk is estimated from the percentage of simulated draws in which the firm defaults. The firm is assumed to have a market debt ratio of 0.5 with one year to debt maturity. Following the Merton (1974) framework, a firm is assumed to default if its value at debt maturity is less than the face value of its debt. See Appendix B.2 for full details on the simulation procedure.

Appendix

A Model Appendix

A.1 Aggregate Asset Pricing

Expanding on Proposition 1, we have the following aggregate asset pricing results.

- The value function is given by

$$J(C_t, X_t) = \frac{C_t^{1-\gamma}}{1-\gamma} \exp\left(\frac{\lambda(1-\gamma)}{\kappa+\beta} X_t + \frac{1-\gamma}{\beta} \left[\mu_C - \frac{1}{2}\gamma\sigma_C^2 + \frac{\lambda^2(1-\gamma)\sigma_x^2}{2(\kappa+\beta)^2} \right] \right). \quad \text{(A.1)}$$

- The pricing kernel is given by

$$\frac{d\Lambda}{\Lambda} = -r_t^f dt - \gamma\sigma_C dW_C - \frac{(\gamma-1)\lambda}{\kappa+\beta}\sigma_X dW_X, \quad \text{(A.2)}$$

$$r_t^f = \beta + \mu_C + \lambda X_t - \gamma\sigma_C^2, \quad \text{(A.3)}$$

where r_t^f is the risk-free rate.

- The price-dividend ratio, from equation (6), is

$$\frac{P_t}{D_t} = G(X_t) = \int_t^\infty S(X_t, \tau) ds \qquad [\tau = s - t],$$

where

$$S(X_t, \tau) = \exp(P_1(\tau)X_t + P_2(\tau)), \quad \text{(A.4)}$$

with P_1 and P_2 given by

$$
\begin{aligned}
P_1(\tau) &= \frac{1-\lambda}{\kappa}\left(1-e^{-\kappa\tau}\right) && \text{(A.5)}\\
P_2(\tau) &= a\tau + b(e^{-\kappa\tau}-1) + c(1-e^{-2\kappa\tau}) && \text{(A.6)}\\
a &= \left(\mu_D - \mu_C + \gamma\sigma_C^2 - \beta + \frac{1-\lambda}{\kappa}\sigma_X^2\left(\frac{1-\lambda}{2\kappa} - \frac{(\gamma-1)\lambda}{\kappa+\beta}\right)\right) && \text{(A.7)}\\
b &= \frac{(1-\lambda)\sigma_X^2}{\kappa^2}\left(\frac{1-\lambda}{\kappa} - \frac{(\gamma-1)\lambda}{\kappa+\beta}\right) && \text{(A.8)}\\
c &= \frac{(1-\lambda)^2\sigma_X^2}{4\kappa^3}. && \text{(A.9)}
\end{aligned}
$$

Proof of Theorem 1:

Proof. Consider an asset that pays a single cash flow equal to the aggregate dividend at time T, D_T. The value of this asset at time t is given by

$$
\begin{aligned}
p_t(\tau, D_t, X_t) &= E_t\left[\frac{\Lambda_T}{\Lambda_t}D_T\right]\\
&= D_t S(X_t, \tau) \qquad [\tau = T - t],
\end{aligned}
$$

where the second step includes the equivalence $E_t[\Lambda_T D_T] = \Lambda_t D_t S(X_t, \tau)$ where $S(X_t, \tau)$ is defined above. Then the risk premium on this asset is

$$
\begin{aligned}
-Cov\left(\frac{d\Lambda_t}{\Lambda_t}, \frac{dp_t}{p_t}\right) &= \frac{(\gamma-1)\lambda}{\kappa+\beta}\sigma_X^2\frac{S_X(X_t,\tau)}{S(X_t,\tau)}\\
&= \frac{(\gamma-1)\lambda}{\kappa+\beta}\sigma_X^2\frac{S(X_t,\tau)P_1(\tau)}{S(X_t,\tau)}\\
&= \frac{(\gamma-1)\lambda}{\kappa+\beta}\sigma_X^2\frac{1-\lambda}{\kappa}(1-e^{-\kappa\tau}), && \text{(A.10)}
\end{aligned}
$$

which is increasing in τ since $\gamma > 1$, $\lambda < 1$, $\kappa > 0$, and $\beta > 0$. The asset pays a single cash flow with duration τ. The duration and expected return of a portfolio of cash flows are value-weighted averages of the duration and expected returns of its cash flows, respectively. Therefore, the positive duration–expected return relation generalizes to any portfolio whose payoffs are non-negative fractions of the aggregate dividend. □

A.2 Cross Section of Dividends

Proof of Theorem 2:

Proof. The sum of two dividend shares $\theta^p_t = \theta^i_t + \theta^j_t$ follows a WF process. Applying Itô's Lemma with $\rho_t(dW_{\theta^i}, dW_{\theta^j}) = -\sqrt{\frac{\theta^i_t\theta^j_t}{(1-\theta^i_t)(1-\theta^j_t)}}$ and defining $\bar{\theta}^p \equiv \bar{\theta}^i + \bar{\theta}^j$,

$$\begin{aligned}
d\theta^p_t &= d\theta^i_t + d\theta^j_t \\
&= \alpha(\bar{\theta}^i + \bar{\theta}^j - \theta^i_t - \theta^j_t)dt + \delta\sqrt{(1-\theta^i_t)\theta^i_t}dW_{\theta^i} + \delta\sqrt{(1-\theta^j_t)\theta^j_t}dW_{\theta^j} \\
&= \alpha(\bar{\theta}^p - \theta^p_t)dt \\
&\quad + \delta\sqrt{(1-\theta^i_t)\theta^i_t + (1-\theta^j_t)\theta^j_t - 2\sqrt{(1-\theta^i_t)\theta^i_t(1-\theta^j_t)\theta^j_t\frac{\theta^i_t\theta^j_t}{(1-\theta^i_t)(1-\theta^j_t)}}}dW_{\theta^p} \\
&= \alpha(\bar{\theta}^p - \theta^p_t)dt + \delta\sqrt{(1-\theta^p_t)\theta^p_t}dW_{\theta^p}. \qquad \text{(A.11)}
\end{aligned}$$

□

Additional Aggregation Results:

We now show that the portfolio share ratio is given by equation (11) and that the cross section of firms aggregates properly. From Theorem 2, the dividend share of a portfolio p^* holding the full value of all firms in $\mathcal{P}$ follows the WF process,

$$d\theta^{p^*}_t = \alpha(\bar{\theta}^{p^*} - \theta^{p^*}_t)dt + \delta\sqrt{(1-\theta^{p^*}_t)\theta^{p^*}_t}dW_{\theta^{p^*}}, \qquad \text{(A.12)}$$

where $\bar{\theta}^{p^*} = \sum_{i\in\mathcal{P}}\bar{\theta}^i$ and $\theta^{p^*}_t = \sum_{i\in\mathcal{P}}\theta^i_t$. Then consider a portfolio p which holds the proportion of firm value ω of each firm in $\mathcal{P}$. By Itô's lemma, the process for the portfolio's dividend share is

$$d\theta_t^p = \alpha(\bar{\theta}^p - \theta_t^p)dt + \delta\sqrt{(\omega - \theta_t^p)\theta_t^p}dW_{\theta^p}, \tag{A.13}$$

where $\bar{\theta}^p = \sum_{i\in\mathcal{P}} \omega\bar{\theta}^i$ and $\theta_t^p = \sum_{i\in\mathcal{P}} \omega\theta_t^i$. Asset pricing results must be equivalent for $\omega < 1$ and $\omega = 1$ to prevent the existence of arbitrage opportunities. Equation (11) is derived by

$$\frac{\bar{\theta}^p}{\theta_t^p} = \frac{\sum_{i\in\mathcal{P}} \omega\bar{\theta}^i}{\sum_{i\in\mathcal{P}} \omega\theta_t^i} = \sum_{i\in\mathcal{P}} \frac{\theta_t^i}{\sum_{i\in\mathcal{P}} \theta_t^i}\frac{\bar{\theta}^i}{\theta_t^i}. \tag{A.14}$$

For the aggregation result, let all N firms be included in $\mathcal{P}$ and $\omega = 1$ so the portfolio represents ownership of all assets in the economy. Then $\sum_{i\in\mathcal{P}} \theta_0^i = 1$ at time 0 and $d(\sum_{i\in\mathcal{P}} \theta_t^i) = 0$ by equation (A.13), so $1 = \sum_{i\in\mathcal{P}} \theta_t^i = \sum_{i=1}^N \theta_t^i$ for all t and $D_t = \sum_{i=1}^N D_t^i$.

A.3 Cross-Sectional Asset Pricing

Proof of Theorem 3:

Proof. The value of a firm is the expected discounted value of firm dividends,

$$\begin{aligned} P_t^i &= E_t \int_t^\infty \frac{\Lambda_s}{\Lambda_t} D_s^i ds \\ &= E_t \int_t^\infty \frac{\Lambda_s}{\Lambda_t} D_s \theta_s^i ds. \end{aligned} \tag{A.15}$$

In order to value future dividends, we need to compute $E_t[\theta_s^i]$, which has a solution

$$E_t[\theta_s^i] = \theta_t^i e^{-\alpha\tau} + \bar{\theta}^i(1 - e^{-\alpha\tau}). \tag{A.16}$$

This expectation is simply a weighted average of the current dividend share θ_t^i and the long-run dividend share $\bar{\theta}^i$, with $E_t[\theta_s^i] = \theta_t^i$ at $\tau = 0$ and $\lim_{\tau\to\infty} E_t[\theta_s^i] = \bar{\theta}^i$ as our expectation of θ_t^i converges to the long-run dividend share.

The price of security i is

$$
\begin{aligned}
P_t^i &= \frac{1}{\Lambda_t}\int_t^{\infty} E_t[\Lambda_s \theta_s^i D_s]ds \\
&= \frac{1}{\Lambda_t}\int_t^{\infty} E_t[\Lambda_s D_s]E_t[\theta_s^i]ds \qquad\qquad [independence] \\
&= \frac{1}{\Lambda_t}\int_t^{\infty} \Lambda_t D_t S(X_t,\tau)[\theta_t^i e^{-\alpha\tau} + \bar{\theta}^i(1-e^{-\alpha\tau})]ds \qquad [\tau = s-t], \quad \text{(A.17)}
\end{aligned}
$$

where $S(X_t,\tau)$ is defined above. Dividing both sides by $D_t^i = \theta_t^i D_t$,

$$
\frac{P_t^i}{D_t^i} \equiv G^i(X_t,\theta_t^i;\bar{\theta}^i,\alpha) = \int_t^{\infty} S(X_t,\tau)\left[e^{-\alpha\tau} + \frac{\bar{\theta}^i}{\theta_t^i}(1-e^{-\alpha\tau})\right]ds. \qquad \text{(A.18)}
$$

Firm i's price-dividend ratio is increasing in share ratio given $\alpha > 0$ since $(1-e^{-\alpha\tau}) > 0$ and $S(X_t,\tau) > 0$ for all X_t and τ.

Individual firm excess return obeys

$$
dR_t^i = \mu_t^i dt + \sigma_D dW_D + \frac{G_X^i}{G^i}\sigma_X dW_X + \delta\sqrt{\frac{1-\theta_t^i}{\theta_t^i}}\left(1+\frac{\theta_t^i G_\theta^i}{G^i}\right)dW_{\theta^i}, \qquad \text{(A.19)}
$$

where μ_t^i is the expected excess return given by

$$
\begin{aligned}
\mu_t^i &= -Cov\left(\frac{dP^i}{P^i},\frac{d\Lambda}{\Lambda}\right) \\
&= \frac{\lambda(\gamma-1)}{\kappa+\beta}\sigma_X^2\frac{G_X^i}{G^i}. \qquad \text{(A.20)}
\end{aligned}
$$

We now establish conditions under which μ_t^i is increasing in share ratio. First, define share ratio $s_t^i = \frac{\bar{\theta}^i}{\theta_t^i}$. Assuming $\gamma > 1$, μ_t^i is increasing in share ratio if $\frac{G_X^i}{G^i}$ is increasing in share ratio, where

$$G_X^i = s_t^i \int_t^\infty S(X_t,\tau)P_1(\tau)(1-e^{-\alpha\tau})ds + \int_t^\infty S(X_t,\tau)P_1(\tau)e^{-\alpha\tau}ds \tag{A.21}$$

$$G^i = s_t^i \int_t^\infty S(X_t,\tau)(1-e^{-\alpha\tau})ds + \int_t^\infty S(X_t,\tau)e^{-\alpha\tau}ds. \tag{A.22}$$

Since G^i and G_X^i are both linear in s_t^i and substituting in $P_1(\tau) = \frac{1-\lambda}{\kappa}(1-e^{-\kappa\tau})$, the sign of $\frac{\partial(G_X^i/G^i)}{\partial s_t^i}$ depends on

$$\text{sgn}(c) = \text{sgn}\left(\int_t^\infty S(X_t,\tau)ds \int_t^\infty S(X_t,\tau)e^{-(\alpha+\kappa)\tau}ds - \int_t^\infty S(X_t,\tau)e^{-\alpha\tau}ds \int_t^\infty S(X_t,\tau)e^{-\kappa\tau}ds\right). \tag{A.23}$$

In order to show $\text{sgn}(c) > 0$, we need to show that

$$\int_t^\infty S(X_t,\tau)ds \int_t^\infty S(X_t,\tau)e^{-(\alpha+\kappa)\tau}ds > \int_t^\infty S(X_t,\tau)e^{-\alpha\tau}ds \int_t^\infty S(X_t,\tau)e^{-\kappa\tau}ds, \tag{A.24}$$

which follows by Chebyshev's algebraic inequality since $e^{-\alpha\tau}$ and $e^{-\kappa\tau}$ are monotonic functions of τ (Mitrinović, Pečarić, and Fink (1993, p. 239)).

Finally, given the process for returns in equation (A.19), the instantaneous variance of firm returns in equation (15) follows from the assumption of zero correlation between the dW_X, dW_D, and dW_{θ^i} processes. □

From equation (16), we have the beta-pricing relation, $\mu_t^i = \beta_t^i \mu_t$, where $\beta_t^i\left(X_t, \frac{\bar{\theta}^i}{\theta_t^i}\right)$ is firm i's beta on an economic growth hedge portfolio with expected return μ_t. The economic growth hedge portfolio is constructed to have returns that follow

$$dR_t = \mu_t dt + \frac{G_X}{G}\sigma_X dW_X, \tag{A.25}$$

where μ_t is defined in Proposition 1. Firm i's beta is

$$\begin{aligned}\beta_t^i\left(X_t, \frac{\bar{\theta}^i}{\theta_t^i}\right) &= \frac{Cov(dR_t^i, dR_t)}{Var(dR_t)} \\ &= \frac{\sigma_X^2 \frac{G_X^i}{G^i}\frac{G_X}{G}}{\sigma_X^2\left(\frac{G_X}{G}\right)^2} \\ &= \frac{G_X^i}{G^i}\frac{G}{G_X}. \end{aligned} \tag{A.26}$$

Then substituting $\frac{G_X^i}{G^i} = \beta_t^i \frac{G_X}{G}$ into the equation for μ_t^i gives

$$\begin{aligned}\mu_t^i &= \frac{\lambda(\gamma-1)}{\kappa+\beta}\sigma_X^2 \frac{G_X^i}{G^i} \\ &= \beta_t^i \mu_t. \end{aligned} \tag{A.27}$$

A.4 Dispersion and Idiosyncratic Volatility

A negative share ratio–dispersion relation exists when $\theta_t^i < 1/2$. Since dispersion is positive, the derivatives of both dispersion and dispersion squared with respect to share ratio will have the same sign. We have

$$\frac{\partial(Dispersion_t^i)^2}{\partial s_t^i} = \frac{\partial(D_t^2\bar{\theta}^{i2}(s_t^i)^{-2}\sigma_D^2 + D_t^2\delta^2\bar{\theta}^i(s_t^i)^{-1}(1-\bar{\theta}^i(s_t^i)^{-1}))}{\partial s_t^i} \tag{A.28}$$

$$= D_t^2\bar{\theta}^i(s_t^i)^{-2}(2\bar{\theta}^i(\delta^2-\sigma_D^2)(s_t^i)^{-1}-\delta^2), \tag{A.29}$$

which is negative under the condition

$$2\theta_t^i(\delta^2-\sigma_D^2)-\delta^2 < 0. \tag{A.30}$$

A sufficient condition for the negative share ratio–dispersion relation is therefore $\theta_t^i < 1/2$.

From a technical perspective, IV decreases in share ratio under the sufficient condition in Theorem 4.

Proof. IV of asset i is given by

$$\begin{aligned} IV_t^i &= \delta\sqrt{\left(\frac{1}{\theta_t^i}-1\right)}\left(1+\frac{\theta_t^i G_\theta^i}{G^i}\right) \\ &= \delta\sqrt{\left(\frac{1}{\theta_t^i}-1\right)}\left(\frac{\int_t^\infty S(X_t,\tau)e^{-\alpha\tau}ds}{G+\int_t^\infty S(X_t,\tau)\left(\frac{\bar{\theta}^i}{\theta_t^i}-1\right)(1-e^{\alpha\tau})ds}\right) \\ &= \delta\left(\int_t^\infty S(X_t,\tau)e^{\alpha\tau}ds\right)\frac{\sqrt{\frac{s_t^i}{\bar{\theta}^i}-1}}{G+(s_t^i-1)\int_t^\infty S(X_t,\tau)(1-e^{-\alpha\tau})ds}. \qquad \text{(A.31)} \end{aligned}$$

Notice the change of variable $\frac{\bar{\theta}^i}{\theta_t^i}=s_t^i$ which implies that $\frac{1}{\theta_t^i}=\frac{s_t^i}{\bar{\theta}^i}$. By construction, $\frac{s_t^i}{\bar{\theta}^i}>1$.

The sign of the derivative $\frac{\partial IV_t^i}{\partial s_t^i}$ depends on whether the quantity

$$c(r_t^i)=\frac{\sqrt{\frac{s_t^i}{\bar{\theta}^i}-1}}{G+(s_t^i-1)\int_t^\infty S(X_t,\tau)(1-e^{-\alpha\tau})ds} \qquad \text{(A.32)}$$

is increasing or decreasing in s_t^i. By taking the derivative of $c(s_t^i)$ with respect to s_t^i, $\text{sgn}\left(\frac{\partial IV_t^i}{\partial s_t^i}\right)$ is determined by the sign of the quantity

$$\bar{c}=(2\bar{\theta}^i-s_t^i)\int_t^\infty S(X_t,\tau)(1-e^{-\alpha\tau})ds+\int_t^\infty S(X_t,\tau)e^{-\alpha\tau}ds. \qquad \text{(A.33)}$$

Rewrite $\bar{c}$ as

$$\begin{aligned} \bar{c} &= (2\bar{\theta}^i-s_t^i)\int_t^\infty S(X_t,\tau)ds+(s_t^i-2\bar{\theta}^i+1)\int_t^\infty S(X_t,\tau)e^{-\alpha\tau}ds \qquad \text{(A.34)} \\ &\le (2\bar{\theta}^i-s_t^i)|S(X_t,\tau)|_1+\frac{(s_t^i-2\bar{\theta}^i+1)}{\sqrt{2\alpha}}|S(X_t,\tau)|_2, \qquad \text{(A.35)} \end{aligned}$$

where the last inequality is due to the Cauchy–Schwarz inequality and $|\cdot|_i$ is the i-th norm. The success of equation (A.35) is to separate α from inside the integral, thus disentangling any non-linear dependence from other variables inside the integral. Now, we can enforce a condition for the IV result in the $(\bar{\theta}^i, \alpha)$-plane that is independent from the other variables in the system. Since, $S(X_t, \tau)$ is positive $\forall \tau$, the first norm is much greater than the second norm because the cross-terms are all positive. Thus, for $\bar{c} < 0$ in (A.35), we need the following parameter restriction to hold:

$$(s_t^i - 2\bar{\theta}^i)(\sqrt{2\alpha}M - 1) > 1, \tag{A.36}$$

where $M = \frac{|S(X_t,\tau)|_1}{|S(X_t,\tau)|_2} >> 1$ for reasonable parameter values that we consider. Simulations show that the bound $\bar{c} < 0$ is easily achieved for a wide variety of parameters. □

B Estimation and Simulation Appendix

B.1 Estimation

We estimate the aggregate parameters of the Euler approximations of equations (3), (4), and (5) using a Bayesian approach. We refer the reader to Hore (2010) for estimation details and discussion of priors. Below, we describe the estimation of cross-sectional parameters and the methodology for the cross-sectional tests.

B.1.1 Portfolio Parameter Estimation

For each of the three sets of decile portfolios, we estimate the parameter $\bar{\theta}^i$ for each portfolio and the parameters α and δ using a Bayesian MCMC approach. Using the Euler approximation of equation (8) along with the instantaneous correlation in equation (9), we have a system of N equations with one equation for each portfolio i of the form,

$$\theta_{t+1}^i = \theta_t^i + \alpha(\bar{\theta}^i - \theta_t^i) + \delta\epsilon_{t+1}^i, \quad i = 1, ..., N, \quad \epsilon_{t+1} \sim N(0, \Sigma_t), \tag{B.1}$$

where

$$\Sigma_t = \begin{bmatrix} \theta_t^1(1-\theta_t^1) & \cdots & -\theta_t^1\theta_t^i & \cdots & -\theta_t^1\theta_t^N \\ \vdots & \ddots & & & \vdots \\ -\theta_t^i\theta_t^1 & \cdots & \theta_t^i(1-\theta_t^i) & \cdots & -\theta_t^i\theta_t^N \\ \vdots & & & \ddots & \vdots \\ -\theta_t^N\theta_t^1 & \cdots & -\theta_t^N\theta_t^i & \cdots & \theta_t^N(1-\theta_t^N) \end{bmatrix}. \tag{B.2}$$

Further, we have the parameter restrictions that $\bar{\theta}^j > 0$ for all portfolios j, $\alpha > 0$, $\delta > 0$, and $\sum_{j=1}^n \bar{\theta}^j = 1$. Note that the full variance-covariance matrix Σ is not invertible. To see this, consider the variance of θ_{t+1}^N given $\{\theta_{t+1}^i\}_{i=1}^{N-1}$, which is

$$\delta^2\theta_t^N(1-\theta_t^N) - \delta^2\Sigma_{(N,1:N-1)}\Sigma_{(1:N-1,1:N-1)}^{-1}\Sigma_{(1:N-1,N)} = 0, \tag{B.3}$$

while the mean of θ_{t+1}^N is $1 - \sum_{i=1}^{N-1}\theta_{t+1}^i$. Therefore, the variance-covariance matrix Σ automatically enforces the constraint $\sum_{j=1}^N \bar{\theta}^j = 1$. In practice, a combination of the variance-covariance matrix of the first $N-1$ dividend shares, $\Sigma_{(1:N-1,1:N-1)}$, combined with the constraint $\sum_{j=1}^N \bar{\theta}^j = 1$ are used to estimate the system.

The parameters in equation (B.1) do not have a convenient posterior distribution, so we employ a Metropolis–Hastings algorithm to draw from the posterior distribution. Below, we outline the MCMC chain. Defining the vector of parameters $\Omega = [\bar{\theta}, \alpha, \delta]$, the draw of Ω in iteration m of the MCMC chain is as follows:

1. Draw a vector $\hat{\theta} \sim N_{[0,\infty)}(\bar{\theta}^{(m-1)}, A)$, where A is a diagonal matrix scaled to provide for sufficient mixing and the multivariate normal distribution is truncated at zero in all N dimensions because of the restrictions $\bar{\theta}^j > 0$ for $j = 1, ..., N$. The candidate vector is then normalized to sum to one, $\bar{\theta}^* = \frac{\hat{\theta}}{\hat{\theta}'\iota}$, to satisfy the restriction $\sum_{j=1}^N \bar{\theta}^j = 1$.

Finally, $\Omega^* = [\bar{\theta}^*, \alpha^{(m-1)}, \delta^{(m-1)}]$ is accepted with probability

$$\min\left\{\frac{p(\Omega^*)/q(\Omega^*|\Omega^{(m-1)})}{p(\Omega^{(m-1)})/q(\Omega^{(m-1)}|\Omega^*)}, 1\right\}, \tag{B.4}$$

where

$$p(\Omega) \propto \prod_{t=1}^{T} \exp\left(-\frac{\left(\theta_{t+1}^{-N} - (1-\alpha)\theta_t^{-N} - \alpha\bar{\theta}^{-N}\right)' \Sigma_{t,-N}^{-1} \left(\theta_{t+1}^{-N} - (1-\alpha)\theta_t^{-N} - \alpha\bar{\theta}^{-N}\right)}{2\delta^2}\right), \tag{B.5}$$

is the kernel of a multivariate normal distribution, θ_{t+1}^{-N} and $\bar{\theta}^{-N}$ are vectors of dividend shares and long-run means of dividend shares of the first $N-1$ firms, Σ_{-N} is the $(N-1 \times N-1)$ block of the variance-covariance matrix defined in equation (B.2) formed by dropping the Nth row and column, and $q(\Omega^a|\Omega^b)$ can be replaced by the probability that all elements of a draw $\bar{\theta}^b \sim N(\bar{\theta}^a, A)$ are greater than zero.[18] If the transition density was not truncated, this step would be a random-walk Metropolis–Hastings step and the $q(\Omega^*|\Omega^{(m-1)})$ and $q(\Omega^{(m-1)}|\Omega^*)$ terms would cancel. Scaling the likelihoods of Ω^* and $\Omega^{(m-1)}$ by the probabilities of positive draws adjusts for the differences in transition probabilities arising from the truncation. Transition probabilities are unaffected by the rescaling of $\hat{\theta}$ to $\bar{\theta}^*$ since A is diagonal. If the draw $\bar{\theta}^*$ is accepted then $\bar{\theta}^{(m)} = \bar{\theta}^*$. Otherwise, $\bar{\theta}^{(m)} = \bar{\theta}^{(m-1)}$.

2. Draw a candidate $\alpha^* \sim N_{[0,\infty)}(\alpha^{(m-1)}, \sigma_\alpha)$. The normal distribution is truncated at zero because of the restriction $\alpha > 0$. Then the draw $\Omega^* = [\bar{\theta}^{(m)}, \alpha^*, \delta^{(m-1)}]$ is accepted with probability

$$\min\left\{\frac{p(\Omega^*)/q(\Omega^*|\Omega^{(m-1)})}{p(\Omega^{(m-1)})/q(\Omega^{(m-1)}|\Omega^*)}, 1\right\}, \tag{B.6}$$

[18]Note that the constants multiplying the kernels to arrive at likelihoods are the same for $p(\Omega^*)$ and $p(\Omega^{(m-1)})$ so they can safely be ignored.

where

$$p(\Omega) \propto \prod_{t=1}^{T} \exp\left(-\frac{\left(\theta_{t+1}^{-N} - (1-\alpha)\theta_t^{-N} - \alpha\bar{\theta}^{-N}\right)' \Sigma_{t,-N}^{-1} \left(\theta_{t+1}^{-N} - (1-\alpha)\theta_t^{-N} - \alpha\bar{\theta}^{-N}\right)}{2\delta^2}\right), \tag{B.7}$$

and $q(\Omega^a|\Omega^b)$ can be replaced by the probability that a draw $\alpha^b \sim N(\alpha^a, \sigma_\alpha)$ is greater than zero. If the draw α^* is accepted then $\alpha^{(m)} = \alpha^*$, else $\alpha^{(m)} = \alpha^{(m-1)}$.

3. Draw a candidate $\delta^* \sim N_{[0,\infty)}(\delta^{(m-1)}, \sigma_\delta)$. Then the draw $\Omega^* = [\bar{\theta}^{(m)}, \alpha^{(m)}, \delta^*]$ is accepted with probability

$$\min\left\{\frac{p(\Omega^*)/q(\Omega^*|\Omega^{(m-1)})}{p(\Omega^{(m-1)})/q(\Omega^{(m-1)}|\Omega^*)}, 1\right\}, \tag{B.8}$$

where

$$p(\Omega) \propto \prod_{t=1}^{T} \frac{1}{\delta^{N-1}} \exp\left(-\frac{\left(\theta_{t+1}^{-N} - (1-\alpha)\theta_t^{-N} - \alpha\bar{\theta}^{-N}\right)' \Sigma_{t,-N}^{-1} \left(\theta_{t+1}^{-N} - (1-\alpha)\theta_t^{-N} - \alpha\bar{\theta}^{-N}\right)}{2\delta^2}\right), \tag{B.9}$$

and $q(\Omega^a|\Omega^b)$ can be replaced by the probability that a draw $\delta^b \sim N(\delta^a, \sigma_\delta)$ is greater than zero. If the draw δ^* is accepted then $\delta^{(m)} = \delta^*$, else $\delta^{(m)} = \delta^{(m-1)}$.

We draw 250,000 parameter vectors from the posterior distribution and discard the first 50,000 as a burn-in period. Due to the relatively strong autocorrelation in draws that can arise when using a Metropolis–Hastings algorithm, we use every fortieth draw from the posterior as the 5,000 draws that are used in the simulation below. The resulting draws are nearly serially uncorrelated.

B.1.2 Cross-Sectional Tests

We perform asset-pricing tests using an approach similar to Fama–MacBeth. Our Bayesian hierarchical regressions estimate the system of equations simultaneously, mitigating measurement error biases (see Davies (2011) and Cederburg (2011) for additional details). In the tests, we examine 20 three-year periods y using data from quarterly sub-periods t within each period. The MCMC steps for the tests in Section 2.3 follow.

1. Draw $\beta_y^i|\sigma_{i,y}^2, \{\Delta X_{t,y}\}_{t=1}^T, \lambda_y, \sigma_y^2, \underline{\beta}, \underline{\mathbf{V}}_\beta \sim N(\overline{\beta}_y^i, \overline{\mathbf{V}}_{\beta,y}^i)$ for $i = 1, ..., N_y$ and $y = 1, ..., Y$, where N_y is the number of assets in period y, $\beta_y^i = [\alpha_y^i \quad \beta_{m,y}^i \quad \beta_{X,y}^i]'$,

$$\overline{\beta}_y^i = \overline{\mathbf{V}}_{\beta,y}^i \left(\underline{\mathbf{V}}_\beta^{-1} \underline{\beta} + \sigma_{i,y}^{-2} Z_{1,y}' R_y^i + \sigma_y^{-2} Z_{2,y}' (\overline{R}_y^i - \lambda_{0,y}) \right), \tag{B.10}$$

$$\overline{\mathbf{V}}_{\beta,y}^i = \left(\underline{\mathbf{V}}_\beta^{-1} + \sigma_{i,y}^{-2} Z_{1,y}' Z_{1,y} + \sigma_y^{-2} Z_{2,y}' Z_{2,y} \right)^{-1}, \tag{B.11}$$

$Z_{1,y} = \left[\iota_T \quad \{R_{m,t,y}^m\}_{t=1}^T \quad \{\Delta X_{t,y}\}_{t=1}^T \right]$, and $Z_{2,y} = [0 \quad \lambda_{m,y} \quad \lambda_{X,y}]$. We set the prior parameters of β_y^i to $\underline{\beta} = [0 \quad 1 \quad 0]'$ and $\underline{\mathbf{V}}_\beta = 100I$ to specify proper but diffuse priors.

2. Draw $\sigma_{i,y}^2|\beta_y^i, \{\Delta X_{t,y}\}_{t=1}^T, \underline{\nu}, \underline{s}^2 \sim$ Inverse Gamma$(\frac{\overline{s}^2}{2}, \frac{\overline{\nu}}{2})$ for $i = 1, ..., N_y$ and $y = 1, ..., Y$, where $\overline{\nu} = \underline{\nu} + T$, $\overline{s}^2 = \underline{\nu}\underline{s}^2 + s^2$, and

$$s^2 = \sum_{t=1}^T \left(R_{t,y}^i - \alpha_y^i - \beta_{m,y}^i R_{t,y}^m - \beta_{X,y}^i \Delta X_{t,y} \right)^2. \tag{B.12}$$

We set $\underline{\nu} = 4$ and $\underline{s}^2$ equal to the sample variance of firm i's returns in period y.

3. Draw $\lambda_y|\{\beta_y^i\}_{i=1}^{N_y}, \sigma_y^2, \overline{\lambda}, \mathbf{V}_\lambda \sim N(\hat{\lambda}_y, \hat{\mathbf{V}}_{\lambda y})$ for $y = 1, ..., Y$, where

$$\hat{\lambda}_y = \hat{\mathbf{V}}_{\lambda y} (\mathbf{V}_\lambda^{-1} \overline{\lambda} + \sigma_y^{-2} Z_y' \overline{R}_y), \tag{B.13}$$

$$\hat{\mathbf{V}}_{\lambda y} = (\mathbf{V}_\lambda^{-1} + \sigma_y^{-2} Z_y' Z_y)^{-1}, \tag{B.14}$$

$\lambda_y = [\lambda_{0,y} \quad \lambda_{m,y} \quad \lambda_{X,y}]'$, and $Z_y = \left[\iota_{N_y} \quad \{\beta_{m,y}^i\}_{i=1}^{N_y} \quad \{\beta_{X,y}^i\}_{i=1}^{N_y} \right]$.

4. Draw $\sigma_y^2|\lambda_y, \{\beta_y^i\}_{i=1}^{N_y}, \underline{\nu}_\lambda, \underline{s}_\lambda^2 \sim$ Inverse Gamma$(\frac{\overline{s}_\lambda^2}{2}, \frac{\overline{\nu}_\lambda}{2})$ for $y = 1, ..., Y$, where $\overline{\nu}_\lambda = \underline{\nu}_\lambda + N_y$, $\overline{s}_\lambda^2 = \underline{\nu}_\lambda \underline{s}_\lambda^2 + s_\lambda^2$, and $s_\lambda^2 = \sum_{i=1}^{N_y} \left(\overline{R}_y^i - \lambda_{0,y} - \lambda_{m,y}\beta_{m,y}^i - \lambda_{X,y}\beta_{X,y}^i \right)^2$. We set

$\underline{\nu}_\lambda = 4$ and $\underline{s}^2_\lambda$ to the variance of errors in a period-y regression of average returns on OLS betas.

5. Draw $\mathbf{V}_\lambda | \overline{\lambda}, \{\lambda_y\}_{y=1}^{Y}, g, G \sim$ Inverse Wishart$(Y+g, \sum_{y=1}^{Y}(\lambda_y - \overline{\lambda})(\lambda_y - \overline{\lambda})' + G)$, where $g = 6$ and $G = g \begin{bmatrix} 0.5^2 & 0 & 0 \\ 0 & 0.5^2 & 0 \\ 0 & 0 & 0.2^2 \end{bmatrix}$.

6. Draw $\overline{\lambda} | \{\lambda_y\}_{y=1}^{Y}, \mathbf{V}_\lambda, \underline{V}, \underline{\lambda} \sim N(\tilde{\lambda}, \tilde{\mathbf{V}}_\lambda)$, where

$$\tilde{\lambda} = \tilde{\mathbf{V}}_\lambda \left((\mathbf{V}_\lambda / Y)^{-1} \sum_{y=1}^{Y} \frac{\lambda_y}{Y} + \underline{V}^{-1} \underline{\lambda} \right), \tag{B.15}$$

$$\tilde{\mathbf{V}}_\lambda = \left((\mathbf{V}_\lambda / Y)^{-1} + \underline{V}^{-1} \right)^{-1}, \tag{B.16}$$

$\underline{V} = 100\mathbf{I}$, and $\underline{\lambda} = [0 \quad 0 \quad 0]'$.

The tests in Section 2.2 follow a similar procedure. The first two steps, concentrating on drawing firm betas, are omitted. In addition, the betas in steps three and four are replaced by firm share ratio and the λ parameters are vectors with two, rather than three, elements. The remaining steps follow as in the previous case.

B.2 Simulation

In each iteration of the simulation, we use a draw from the posterior distributions of the aggregate and portfolio parameters. We have closed-form solutions for several quantities in the unlevered case, such as firm value, expected returns, betas, dispersion, and IV. For the levered case we use simulations to estimate expected returns, betas, and the probability of bankruptcy. We run the simulation for 5,000 iterations.

We work within the Merton (1974) framework, where firms default if firm value at debt maturity is less than the debt face value. In each iteration, we simulate one year of D_t, X_t, Λ_t, and θ_t^i from the Euler approximations of equations (3), (4), (5), and (8), respectively, using 4,000 subperiods during the year while simulating the diffusions. Using these simulated time series, we calculate the end-of-year firm value,

$$P_T^i = D_T \theta_T^i \int_T^\infty S(X_T, \tau) \left[e^{-\alpha\tau} + \frac{\bar{\theta}^i}{\theta_t^i}(1 - e^{-\alpha\tau}) \right] ds \qquad [\tau = s - t], \tag{B.17}$$

using numerical integration. We estimate bankruptcy probabilities by comparing firm value, P_T^i, with the face value of debt, B. The proportion of iterations in which $B > P_T^i$ provides a proxy for default probability.

Expected returns on levered equity is estimated by calculating the average return to levered equity across iterations. The (one-year) return from time t to time T in each iteration is

$$R_T^i = \frac{V_T^i + \int_t^T D_s \theta_s^i ds}{V_t^i}, \tag{B.18}$$

with V_t^i representing the value of levered equity and $\int_t^T D_s \theta_s^i ds$ providing the cumulative firm dividend paid over the period. Given the series of D_t, X_t, Λ_t, and θ_t^i simulated above for this iteration,

$$V_T^i = \max\left\{P_T^i - B, 0\right\}, \tag{B.19}$$

which can be explicitly calculated and $\int_t^T D_s \theta_s^i ds$ is estimated using the discrete approximation $\frac{1}{4000}\sum_{s=1}^{4000} D_{s/4000}\theta_{s/4000}^i$. Finally, the value of levered equity at time t, V_t^i, must be estimated. Within each iteration in the simulation, we run a second loop for 500 iterations to estimate V_t^i. In each subiteration, we simulate paths for D_t, X_t, Λ_t, and θ_t^i using the same posterior draws of aggregate and portfolio parameters as in the main iteration. The estimate for V_t^i is the average across the 500 subiterations of

$$V_t^i = \int_t^T D_s \theta_s^i \Lambda_s ds + \Lambda_T \max\left\{P_T^i - B, 0\right\}, \tag{B.20}$$

where the integral is again estimated using a discrete approximation. The expected levered return is estimated using these quantities. In the last step, we estimate the levered equity beta by dividing the estimated expected levered return by the expected return on the economic growth hedge portfolio.

To examine the cross-sectional relation between expected return and dispersion, IV, and credit risk in Figure 4, we first develop a cross section of 250 firms. For each firm, we draw a random number from a χ^2 distribution with two degrees of freedom, then scale this number by the sum of the 250 random draws to create the firm's long-run dividend share $\bar{\theta}^i$. This procedure creates a cross section with more small firms than large firms while ensuring proper aggregation of the cross section. Each firm's dividend share at time 0, θ_0^i, is initially set to its long-run dividend share before simulating each dividend share process for a period of 100 years. This procedure creates a cross section of firms which is close to the steady-state distribution in terms of dividend share θ_t^i and share ratio $\bar{\theta}^i/\theta_t^i$, with minimal dependence on our initial assumptions about θ_0^i. We then calculate dispersion, IV, and credit risk as discussed above.