Stocks, Bonds, Options, Futures, and Portfolio Insurance: A Rose by Any Other Name . . . .

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Trading volume and open interest in options and futures contracts on stock indices, equities, and interest rate instruments traded on world exchanges have experienced remarkable growth. From 1986 through 1991, the open interest in exchange-traded derivatives grew by 36 percent per year, reaching $3.5 billion at the end of 1991. The notional principal of financial derivatives traded in the even larger over-the-counter market (mostly on interest rates, in the form of swaps, forward agreements, and option-like caps, collars, and floors) grew at an annual rate of 40 percent.[1]

This rapid growth has been accompanied by controversy about the proper role of financial derivatives and the potential for abuse. Prominent attention has been given to losses by major corporations (for example, Procter & Gamble and Gibson Greetings on interest rate swaps), to the losses of broker-related short-term mutual funds (Piper Jaffray and Paine Webber on mortgage-backed securities), and to losses experienced by municipal agencies (Orange County, California, on just about everything).

Financial managers who believe they know the future course of interest rates and asset prices have always found ways to lose big, using traditional investments. Rightly, we have always put the responsibility for being "often wrong but never in doubt" on hubris, absolving the financial instruments themselves from responsibility. But now we are told by the media—a group notoriously ill-equipped to understand derivatives—that these "new things" are so complex that they are unknowable to all but a few. And those few are destined to misuse derivatives, it is claimed, because we do not understand what is going on and, not knowing how to ask the right questions, cannot limit the misuse. We now blame the instruments, not the arrogance of the owners or money managers. This counsel of despair leads the uninformed to believe that derivatives must be eliminated or regulated in order to prevent them from doing their damage.

Thus, the public debate about "derivatives" has promoted the impression that the heart of the problem has been a proliferation of brand new ways of making bets on future stock prices, interest rates, and exchange rates. The positive functions of derivatives as means of risk management are almost forgotten.

This article demonstrates how prices of *exchange-traded* stock index and equity options, as well as futures contracts, can be derived from information on an "option-replicating" portfolio of stocks and bonds that mimics the behavior of the option's premium. Using the equivalence between an option or futures contract and its replicating portfolio, the article demonstrates that exchange-traded options are really nothing new. Rather, they are repackages of the same traditional financial instruments. The article pursues this point by outlining several related risk-management strategies using options and futures contracts. These include dynamic hedging and its related strategy, portfolio insurance. Finally, the article addresses some circumstances in which "derivatives" are not equivalent to traditional instruments. These limitations are most common in the over-the-counter markets where custom-made derivatives are designed for specific uses.

I. The Pricing of Options and Futures

An equity option is a contract allowing the holder to buy or sell a fixed number of shares at a fixed price (the strike price) on or before an expiration date. The holder will exercise the option only if it is in his interest to do so. Thus, an equity option gives its holder the right, not the obligation, to buy or sell at a fixed price. The person who gives the option is called the *writer*, and for every option held an option must be written. The option is a call if the holder has the right to buy (take delivery of) the shares upon payment of the strike price; the writer must deliver the shares if the option is exercised. The option is a put if the holder has the right to sell (deliver) the shares upon receipt of the strike price; the writer of the put must take delivery if the put is exercised. The option is "European" if it can be exercised only on the expiration date, and "American" if it can be exercised at any time up to the expiration date. All equity options traded on U.S. exchanges are American-style options.

A stock index option is similar to an equity option with two important differences. First, the underlying security is a stock price index (for example, the S&P 500, the S&P 100), not a traded security. Secondly, the settlement is in cash rather than in securities. The owner of a call option on the S&P 500 will, upon choosing to exercise, receive the cash equivalent of the excess of the S&P 500 over the strike price rather than take delivery of the securities. All stock index options traded on U.S. exchanges are American-style with the exception of the S&P 500 index option.

An option's market price, or *premium*, is the sum of two components. The *intrinsic value* is the amount that will be received if the holder chooses to exercise the option immediately.[2] The intrinsic value of the option cannot be negative, for the holder would never choose to exercise the option if it reduces his wealth. Hence, the intrinsic value of a call option is denoted as $max(S - X, 0)$, where S is the current stock price and X is the strike price specified in the option contract. This notation simply means that the payoff is the larger of two values, $S - X$ or zero. The intrinsic value of a put option is $max(X - S, 0)$, which also cannot be negative, for the holder of a put will never choose to exercise it if the amount he receives (the strike price) is less than the current stock price.

The value of an option (its premium) will equal the intrinsic value only at the moment of expiration. Prior to expiration, the option will have a *time value*, which reflects the potential for the profitability of the option to change. Thus, an *out-of-the-money* call option, which has zero intrinsic value because the stock price (S) is less than the strike price (X), will still sell at a positive premium because investors realize that the option might become in-the-money at a later date, should the stock perform sufficiently well.

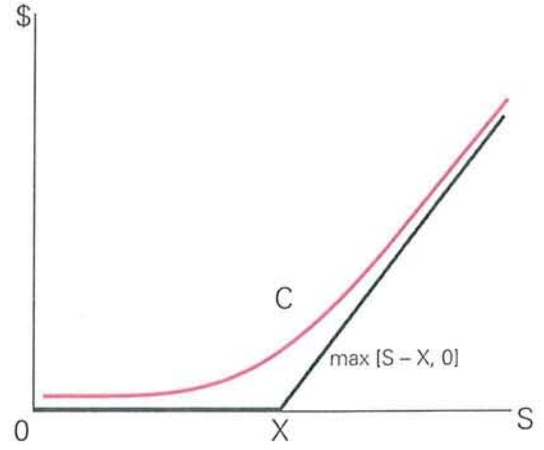
Figure 1 shows the typical relationship between the premium on a call option and its intrinsic value. The intrinsic value is the black line, which has a zero value when the stock price is at or below the strike price but increases dollar for dollar with the stock price when the call is in-the-money. The call premium, denoted by the red curved line labeled C, increases

[1] For a description of this growth, see Remolona (1993). Notional principal is the value of the contract upon which payments are based. It is considerably greater than the market value of the contracts with which it is associated. First, for most contracts the market price is well below the notional principal upon which payments are based. Second, notional principal involves double-counting. If, say, the holder of an interest rate swap for $1,000,000 (which has a net market value of zero) offsets it by selling a similar contract, the "true" net value is zero, but the reported notional principal will be $2,000,000.

[2] Immediate exercise of a call option will require the holder to pay the strike price (X) in exchange for shares valued at the market price (S). The profit is $S - X$ when S exceeds X. If S is less than X, the holder will not exercise the option and it will expire without value.

Figure 1

Call Premium and Intrinsic Value



from almost zero when the stock is without value and approaches the intrinsic value as the stock price gets very high. The vertical distance between the premium and the intrinsic value is the time value for that stock price. Time value is at its maximum for an at-the-money option.

The convex shape of the call premium–stock price relationship plays an important role in understanding option pricing. The intuition underlying this convexity is straightforward. If the stock price is very low, the option will be so out-of-the-money that it would take a rare boom in stock prices to become in-the-money; very little time value would be given to such an option. On the other hand, if the option is very deep in-the-money, it would take a rare downdraft to put it out-of-the-money, and the value of the option will be close to the intrinsic value.

The Pricing of European-Style Options

This section lays out the logic of option pricing in its simplest form. It relies on a standard assumption of economics, the "no free lunch" assumption: Riskless arbitrage opportunities arise only in disequilibrium and will not exist when security prices are in equilibrium. This means that if a portfolio of stocks and bonds can be constructed to match movements in the option premium, profit-seeking traders will ensure that, when markets are in equilibrium, no profitable arbitrage between the option and its *replicating portfolio* can occur. This assumption allows the option premium to be inferred from the value of its replicating portfolio.

Assume that the option in question is a European equity option on a stock whose daily price movements are binomial; that is, if the current stock price is S, the price on the next day will be either μS if the price goes up or δS if it goes down. Thus, μ is one plus the rate of increase and δ is one plus the rate of decrease.

The analysis of a European option can be summarized in a binomial tree. Consider a simple two-period call option, one that has a premium of C dollars in the first period and expires in the second period; the value of the premium is to be determined. Because the underlying stock price will increase to μS or decrease to δS, the value of the option at expiration on the second day will be either $max(\mu S - X, 0)$ or $max(\delta S - X, 0)$. To be specific, assume an option with a strike price of \$48 on a stock with a price of \$50 ($X = 48$, $S = 50$). If the stock price will either increase or decrease by 5 percent ($\mu = 1.05$ and $\delta = 0.95$), the next-day stock price will be either \$52.50 or \$47.50, and the payoff of the option will be either \$4.50 or zero.

If the option premium and stock price after an "up" day and a "down" day are denoted C_u and C_d, and S_u and S_d, respectively, the binomial tree for a two-period option is

$$C \begin{cases} C_u = max[S_u - X, 0], \quad \text{where } S_u = \mu S \\ C_d = max[S_d - X, 0], \quad \text{where } S_d = \delta S \end{cases} \qquad (1)$$

The market price of the option on the first day cannot be determined without further information. That information is provided by noting that exactly the same final values could be achieved by investing in a portfolio of stocks and bonds; this is the option-replicating portfolio. Because the option-replicating portfolio is designed to have exactly the same payoff structure as the option, and because we know the final payoff structure of the option, the option must have exactly the same value as the option-replicating portfolio. The reason is that smart money knows that two assets worth exactly the same at any future time must be worth the same in the present, if arbitrage opportunities are to be eliminated. (The assumption that economic agents will act to eliminate arbitrage profits is a crucial foundation of finance theory.)

Suppose that the option-replicating portfolio consists of Δ ("delta") shares of the stock plus an investment of \$$B$ in bonds. Thus, a portfolio is simply a choice of the values of Δ and B. If bonds pay $\$rB$ on the following day (r is 1.0 plus the riskless interest rate), the binomial tree for the option-replicating portfolio is

$$(\Delta S + B) \begin{cases} (\Delta S_u + Br), & \text{where} \quad S_u = \mu S \\ (\Delta S_d + Br), & \text{where} \quad S_d = \delta S \end{cases} \tag{2}$$

The next step in determining the call premium is to find the values of Δ and B that represent a portfolio of stocks and bonds with final values exactly matching the final values of the call option. That means that the option-replicating portfolio must satisfy the two equations describing the end-points of (1) and (2): $(\Delta S_u + Br) = C_u$ and $(\Delta S_d + Br) = C_d$, where C_u and C_d are known from the option's characteristics and S_u and S_d are known from the assumed values of S, μ, and δ. The required values of Δ and B are

$$\begin{aligned} &\Delta = (C_u - C_d)/(S_u - S_d), \\ &B = (C_u - \Delta S_u)/r, \quad \text{and} \\ &C = \Delta S + B, \\ &\text{where} \quad S_u = \mu S \quad \text{and} \quad S_d = \delta S. \end{aligned} \tag{3}$$

Suppose, as before, that a call option in question has a strike price of \$48 and that the current stock price is \$50, putting the option in-the-money with an intrinsic value of \$2. Assuming a 5 percent increase or decrease, the stock price will go to either \$52.50 or \$47.50 the next day. Under these assumptions, we have seen that the payoffs for this option must be either \$4.50 if the stock goes up or zero if it goes down. If the riskless interest rate is 1 percent (r = 1.01), the option-replicating portfolio will have a delta of 0.90 and the option-replicating portfolio will be a leveraged purchase of \$45 of stock financed by \$42.33 of debt, with a net value of \$2.67.

The final step requires another "no free lunch" assumption. If any two securities are known to have the same values at any future point in time, they must, in equilibrium, have the same values at every point in time. If they did not, traders would find profitable arbitrage opportunities and their actions would eliminate those opportunities, forcing prices into conformity. For example, we have seen that the option and its replicating portfolio are both worth either \$4.50 if the day is "up" or zero if it is down, and that the option-replicating portfolio is worth \$2.67 at the outset. Suppose that the call premium is only \$2. In this case, traders would buy the call and short the replicating portfolio, receiving a net amount of 67 cents. Because the final values of the option and the portfolio are equal, they will lose nothing at the end of the first period—increases in one are matched by declines in the other. Thus, they make a net profit of 67 cents with no risk. Traders would take advantage of this opportunity unless the call premium rose to \$2.67, exactly matching the value of the replicating portfolio. If, on the other hand, the initial value of the call had been \$3, traders would have sold the call and bought the option-replicating portfolio for \$2.67. They would make 33 cents with absolutely no risk, because at the end of the first period any profit or loss on the call is offset by loss or profit on the replicating portfolio.

This three-step analysis shows that, in an equilibrium with no arbitrage profits, the call premium at the outset must be equal to the value of the option-replicating portfolio. This simple example illustrates a key point of this article: A European call option is precisely equivalent to a portfolio of traditional securities, specifically, to a leveraged purchase of the underlying security. All that can be done with one can also be done with the other. Thus, caution must be used when interpreting statements that attribute some special qualities to options. For example, when options are described as allowing "high leverage," we should see that they have no special advantage in providing leverage; they are an alternative way of achieving a leveraged position, and in equilibrium they should cost about the same as the traditional way.

The full derivation of a binomial option pricing model for multiple time periods is given in Box 1. This involves solving the full binomial tree *backward* in the manner just outlined: From any two adjacent final payoffs, the value of the option at the preceding node can be computed, allowing derivation of the full range of option values on the day before expiration. Then, armed with those data, the option values at each node on the second day before expiration can be constructed. As developed in Box 1, at each node the value of the call option can be derived as

$$C = [qC_u + (1 - q)C_d]/r, \tag{4}$$

$$\text{where} \quad q = (r - \delta)/(\mu - \delta)$$

This recursion formula can be shown to be equivalent to the option-replicating formula $C = \Delta S + B$, but it puts the call premium into a recursive format that reveals the connection between current and future

Box 1: The General Binomial Option Pricing Model

The logic of the binomial pricing model is laid out in the text for a two-period option. Here we derive its general form for a multi-period European call option.

At time t a European call option with strike price X is written on an underlying stock with price S. The option expires at time T, at which time it will pay the holder $max[S_T - X, 0]$, the excess of the stock price over the strike price, if positive, or zero.[3] The stock's price follows a Bernoulli process: On each day it either increases to μ ($\mu > 1$) times the previous day's price with probability π, or falls to δ ($0 < \delta < 1$) times the previous day's price with probability $1 - \pi$. Thus, μ is 1 plus the rate of increase and δ is 1 plus the rate of decrease. The statistical expected value and variance of the one-period rate of return are $\pi(\mu - 1) + (1 - \pi)(\delta - 1)$ and $\pi(1 - \pi)(\mu - \delta)^2$, respectively.

We first consider the final payoffs at expiration. An option with $T - t$ periods will have $T - t + 1$ payoffs. If x is the number of "up" days in the remaining $T - t$ days, the payoff will be $max(\mu^x\delta^{T-t-x}S - X, 0)$. If there are too few "up" days, the payoff will be zero because the option will expire out-of-the-money. We can derive the critical number of good days, defined as the minimum number of "up" days required to put the option just at-the-money. This occurs when $x < x^*$, where $x^* = ln(X/S\delta^T)ln(\mu/\delta)$ is the minimum number of "ups" required to put the option at-the-money.

Consider any two adjacent final payoffs valued at $C(x, T) - max(\mu^x\delta^{T-x}S - X, 0)$ and $C(x + 1, T) = max(\mu^{x+1}\delta^{T-(x+1)}S - X, 0)$. These differ only because of the presence or absence of an "up" on day $T - 1$. Because the call can be hedged by an option-replicating portfolio, it can be shown that that arbitrage ensures that the option's value on the previous day is

$$C(x, T - 1) = r^{-1}[qC(x + 1, T) + (1 - q)C(x, T)] \quad \text{(B1.1)}$$

where q is the risk-neutral probability of an "up." This is the fundamental recursion formula described in the text. The analysis of the hedged position reveals that the probability parameter is $q = (r - \delta)(\mu - \delta)$, which does not depend upon the statistical probability (π) or on the expected return on stocks.

This recursive equation can be used to solve the whole binomial tree back to the beginning. Thus, starting with final values $C(x, T)$ and $C(x + 1, T)$ we can solve for the previous node $C(x, T - 1)$. Doing the same for the next two adjacent final payoffs we can find $C(x - 1, T - 1)$, allowing us to find $C(x - 1, T - 2)$, and so on.

Suppose that we are on day t of the option's life. Defining $(T - t, i)$ as the number of ways that there can be i "ups" in the remaining $T - t$ days, we can see that the call premium at any day after x "ups" (and $n - x$ "downs") is

$$C_t = r^{-(T-t)} \sum_{i=0}^{T-t} q^i(1 - q)^{T-t-i} max(S\mu^i\delta^{T-t-i} - X, 0) \quad \text{(B1.2)}$$

This says that the call premium is the present value of the expected final payoffs, using risk-neutral analysis, that is, using the risk-neutral interest rate as the discount rate, and computing expectations using the "objective" risk-neutral probability of an "up." The probability distribution used is the binomial distribution, hence the name binomial option pricing.

option prices. It states that the call premium at any node is the present value of the expected call premium at the next node. The expected call premium is a weighted average of the known call premiums in the "up" and "down" states of the stock. The parameter "q," called the *risk-neutral probability* of a stock price increase, sets the weights given to the "up" and "down" states. For our example (with $\mu = 1.05$, $\delta = 0.95$ and $r = 1.01$), this probability is $q = 0.60$ and the option premium implied by equation (4) is \$2.67.

Somewhat paradoxically, the option is valued as if investors are risk-neutral, that is, the premium depends upon the expected present value defined by the risk-neutral probability and the riskless interest rate. It is not that investors are truly risk-neutral, but rather that in pricing options they can be treated as if

[3] An essential feature of the binomial model is that time is divided into discrete intervals. We call these "days," with no necessary connection to our circadian rhythms.

Table 1
Value Matrix for Ten-Period European Call Option
No Cash Dividends

	Number of Ups (x)										
Period	0	1	2	3	4	5	6	7	8	9	10
0	10.91										
1	6.77	14.27									
2	3.60	9.29	18.36								
3	1.48	5.26	12.51	23.22							
4	.36	2.34	7.54	16.52	28.85						
5	.00	.63	3.66	10.60	21.37	35.24					
6	.00	.00	1.10	5.63	14.55	27.01	42.39				
7	.00	.00	.00	1.94	8.49	19.43	33.41	50.32			
8	.00	.00	.00	.00	3.41	12.44	25.15	40.52	59.12		
9	.00	.00	.00	.00	.00	5.98	17.53	31.51	48.42	68.88	
10	.00	.00	.00	.00	.00	.00	10.50	23.21	38.58	57.18	79.69

Parameters: μ = 1.10, δ = 0.9091, ρ = 1.02, S = X = 50. Note that q = 0.5809.

they are. As a result, the option premium is independent of the *statistical* probability of a stock price increase, and of the *statistical* expected rate of return on the stock. Rather, the option is valued using the riskless rate of interest, not an interest rate containing market risk.

The disconnect between option prices and the expected returns on the underlying assets appears paradoxical, for how can the value of a call option not be higher when the expected rate of increase of the stock price is higher? The answer lies in the ability to create a riskless arbitrage by buying a call and selling its option-replicating portfolio. Smart money will realize that a call option combined with a short position in its option-replicating portfolio is a perfect hedge, creating a riskless position requiring no net investment. The option premium will not contain any reward for risk, for while the option is risky in isolation, it has a perfect hedge and holding the option carries no inherent risks. The investor who decides to hold an unhedged option must do so without any expectation of reward, for the risk he bears is a matter of individual choice and is not inherent in the option itself. In the language of portfolio theory, any risks borne by the option holder are idiosyncratic, not systematic, and can earn no reward.

An example of the multiperiod valuation model illustrated in Box 1 is given in Table 1, which assumes a 10-period at-the-money European call option on a \$50 stock, with $\mu = 1.10$, $\delta = 1/\mu = 0.9091$, and $r = 1.02$. Each cell, equivalent to a node on the binomial tree, shows the value of the call on the *nth* trading day after x "ups" and $10 - x$ "downs"; this is denoted as $C(x, n)$. To compute the option premium in each cell we begin at the end, with the possible payoffs at expiration on day 10. These possible payoffs are computed as $max(\mu^i\delta^{10-i}S - X, 0)$; hence, each differs because of the different numbers of "up" and "down" days over the 10-day lifetime of the option. We see that the expiration-day values are zero for five or fewer ups and rise to \$79.69 for 10 consecutive "ups." These intrinsic values must be the call premiums at expiration because no time remains to receive a time value.

The day-9 option premiums can then be constructed using the known day-10 payoffs along with equation (4), using $q = 0.5809$. For example, if on day 9 there have been eight "ups," then by day 10 there must have been either eight or nine "ups," with payoffs of \$38.58 or \$57.18, respectively. Following equation (4), the premium at the (8,9) node must be $C(8,9)$ = \$48.42. Computing all the possible call premiums on day 9 allows the day-8 premiums to be computed, and so on. Tracing the values back to the beginning, we see that the initial premium on this call option will be \$10.91.

Tables 2 and 3 show the option-replicating portfolio for our hypothetical 10-day European call option whose values are shown in Table 1. Table 2 reports the value of delta (Δ) for our hypothetical 10-day call option, while Table 3 shows the investment in bonds, $B(x, n)$, required to replicate the call option; this

Table 2
Option-Replicating Number of Shares

	Number of Ups (x)									
Day	0	1	2	3	4	5	6	7	8	9
0	.79									
1	.65	.86								
2	.48	.76	.93							
3	.28	.60	.86	.97						
4	.10	.38	.73	.93	.99					
5	.00	.15	.52	.85	.98	1.00				
6	.00	.00	.25	.69	.95	1.00	1.00			
7	.00	.00	.00	.39	.86	1.00	1.00	1.00		
8	.00	.00	.00	.00	.63	1.00	1.00	1.00	1.00	
9	.00	.00	.00	.00	.00	1.00	1.00	1.00	1.00	1.00

Parameters: $\mu = 1.10$, $\delta = 0.9091$, $\rho = 1.02$, $S = X = 50$.

Note: The option-replicating number of shares is the number of shares that results in value changes that match the change in the value of one European call option.

Table 3
Option-Replicating Investment in Bonds per European Call

	Number of Ups (x)									
Day	0	1	2	3	4	5	6	7	8	9
0	−28.40									
1	−22.99	−33.28								
2	−16.18	−28.70	−37.73							
3	−8.92	−21.98	−34.54	−41.33						
4	−2.94	−13.53	−28.82	−39.85	−43.82					
5	.00	−5.16	−20.04	−36.15	−43.89	−45.29				
6	.00	.00	−9.06	−28.66	−42.79	−46.19	−46.19			
7	.00	.00	.00	−15.90	−38.84	−47.12	−47.12	−47.12		
8	.00	.00	.00	.00	−27.92	−48.06	−48.06	−48.06	−48.06	
9	.00	.00	.00	.00	.00	−49.02	−49.02	−49.02	−49.02	−49.02

Parameters: $\mu = 1.10$, $\delta = 0.9091$, $\rho = 1.02$, $S(0) = 50$, $X = 50$, $N = 10$.

Note: The replicating investment in bonds is defined as $B(x, n) = C(x, n) - \delta(x, n) * S(n)$. A negative value indicates borrowing.

depends on the number of "ups" and "downs."[4] From these tables we see that at the outset the call is equivalent to 0.79 shares plus borrowing of \$28.40. However, if 3 "ups" have occurred by day 5, the option-replicating portfolio consists of \$36.15 in debt plus 0.85 shares. In the lower left portion of the matrices, the option is so far out-of-the-money that no shares are bought and no debt is incurred. Thus, the option is worthless because it cannot end in-the-money. In the lower right portion, the option is so deep in-the-money that one share is required to replicate one option.

Pricing of Put Options

From the call option pricing model it is easy to construct a pricing model for a European put option by invoking the *put–call parity theorem*. According to this theorem, arbitrage enforces a simple relationship between put and call premiums. A put and a call for the same stock, each with the same strike price and expiration date, must be priced so that at any time t the following is satisfied (P is the put premium):

[4] The cells are computed as $\Delta(x, n) = [C(x + 1, n + 1) - C(x, n + 1)]/[(\mu - \delta)S(n)$ and $B(x, n) = [\mu C(x, n + 1) - \delta C(x + 1, n + 1)]/[(\mu - \delta)S(n)]$.

$$P_t + S_t = C_t + Xr^{-(T-t)} \qquad (5)$$

A simultaneous investment in a put and one share of the stock must be equal to an investment in a call plus bonds equal to the present value of the strike price. Arbitrage forces this to be true because the final values of the two positions are equal: At expiration on day T the stock *cum* put will be worth $S_T + max(X - S_T, 0)$, which is the greater of the exercise price or the stock price. On that same date the call *cum* bond position will be worth $max(S_{T-X}, 0) + X$, also equal to the larger of the stock price or the exercise price. Because two positions worth the same amount at one time must, in equilibrium, be worth the same at any other time, relationship (5) must hold.

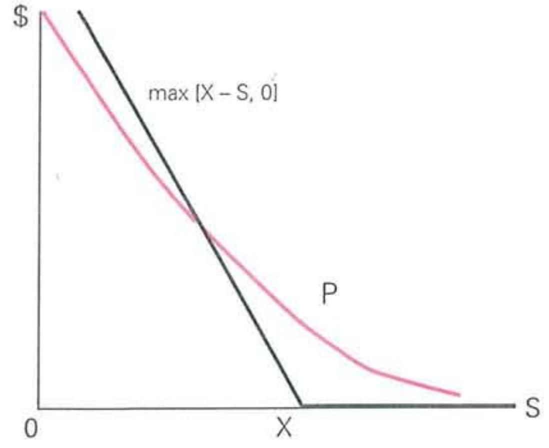
From put–call parity we see that a put is equivalent to a call plus bonds equal to the present value of the strike price plus a *short* position in the stock. Once the equilibrium call premium is known, the equilibrium put premium is also known. Thus, in the case of *European* options, put pricing reduces to a simple transformation of call pricing. This is not true of American put options, for which there is no put–call parity relationship.

Figure 2 shows the typical relationship between the put premium and the stock price. The intrinsic value is shown by the black line and the put premium is shown by the red convex curve. There is one notable difference between the call and put relationships shown in Figures 1 and 2: For a call, the time value is always positive, but for a put it can be negative if the stock price is sufficiently low. That is, the premium described by put–call parity for a European put can be less than the intrinsic value if the put is deep-in-the-money. This anomaly of a negative time value for deep-in-the-money puts means that there can be an incentive to exercise the put early. For example, if the intrinsic value of the put is \$10 and the put premium is at the put–call parity level of, say, \$8, traders will want to buy the put at \$8 and receive \$10 by exercising it. This behavior is not possible for a European put, which cannot be exercised early. But it does present problems for an American put because the American put premium cannot go below its intrinsic value; if it did, traders would make a riskless profit by buying the put and immediately exercising it. Therefore, it is more accurate to say that the observed American put premium will be the higher of the intrinsic value or the put–call parity value.

Thus, because no value is attached to the ability to exercise an American option early, the American call option must be priced as if it were a European call option, and the pricing model just outlined should work for both American and European call options. But an American put option can be worth more than its European counterpart because of the possibility that it will go so deeply into the money that early exercise will be profitable.

Figure 2

Put Premium and Intrinsic Value



The Effects of Cash Dividends: A Digression

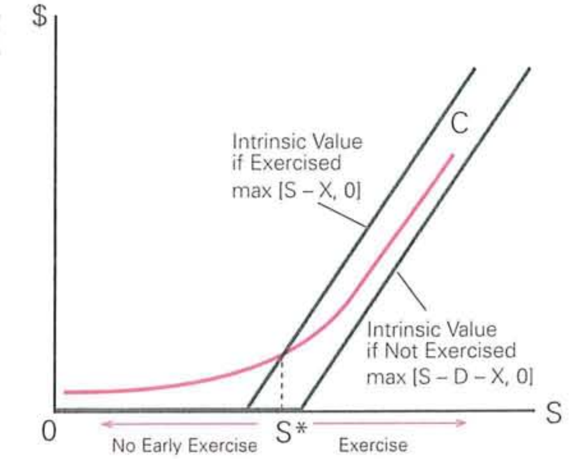
The previous sections assume that the stock underlying the equity option pays no cash dividends during the life of the option. While the exposition that follows maintains this assumption, it is clearly not universally valid. Therefore we briefly extend the option pricing model to acknowledge cash dividends.

Cash dividends do not complicate the story for a European option because it must be held to expiration, but they do require modification of the pricing of American options. Under certain circumstances it is profitable to engage in "dividend capture" strategies, which require early exercise of American options. These strategies involve buying a call option and converting it into stock in order to receive the dividend, then selling the stock. Note that since a call option is equivalent to a leveraged purchase of stocks, the same result can be obtained by borrowing money to buy the stock, then closing that position out after the ex-dividend date.

It is clear that the only incentive for early exercise for dividend capture occurs just before the stock goes

Figure 3

Early Exercise of American Call on Dividend-Paying Stock



ex-dividend. The reason is that exercise before the ex-dividend date requires payment of the strike price before the rights to the dividend are received, hence exercise before the ex-dividend date means that you pay the strike price earlier with no offsetting benefit. It would be wiser to delay the exercise until the last moment before the stock goes ex-dividend. Thus, early exercise will be prompted by dividends only *at* ex-dividend dates.

The decision at each ex-dividend date involves a simple calculation. If the option is exercised, the holder receives the subsequent stream of cash dividends but, because exercise gives only the intrinsic value of the option, the time value is lost. Hence, the present value of the dividend is received in exchange for a capital value loss. Early exercise for dividend capture is optimal only when the value of the dividends exceeds the time value of the option. The "break-even" stock price for triggering dividend capture occurs when the option is in-the-money. This is illustrated in Figure 3. The horizontal axis is the observed (*cum*-dividend) price, which includes the value of the dividend to be received. The higher kinked line shows the intrinsic value if the option is exercised early; this includes the value of the cash dividend. The lower kinked line shows the intrinsic value if the option is *not* exercised, in which case the option's value depends on the *ex*-dividend price, defined as the observed (cum-dividend) price less the value of the dividend. Thus, the vertical distance between the two intrinsic value lines is the value of the cash dividend. The red convex curve shows the call premium as a function of the *ex*-dividend price. This is the value of the call if it is not exercised.

A dividend-capture incentive to exercise the option is present when the intrinsic value upon exercise exceeds the value of the call if it is not. In that case, the dividends received exceed the value of the call sacrificed. In Figure 3 the "break-even" occurs at a stock price of S^*. Dividend capture is profitable at any stock price above S^*, and carries a net loss at any lower stock price. Note that at that break-even point the option must be in-the-money, for the observed (cum-dividend) stock price must exceed the strike price.

We see that while early exercise of an American call option is never optimal in the absence of a cash dividend, it will be optimal if the option is sufficiently in-the-money. As a result, American call options on dividend-paying stocks will have premiums above those on equivalent non-dividend stocks.

Determinants of Call Premiums

The call premium depends on several variables, and any changes in the premium must be due to changes in one or several of these variables. While calculation of the parameters describing the effect of these variables is cumbersome in a binomial model, it is very straightforward in the familiar Black-Scholes model, upon which this section is based. The comparison of the two models, and the calculation of the relevant parameters in the Black-Scholes model, is developed in Box 2.

The most prominent underlying variable is the *stock price*. As shown in Figure 1, a convex relationship exists between the call premium and the contemporaneous stock price, so changes in price affect the premium in the same direction and with increasing force. The parameters that describe this relationship are its slope, called the option's *delta*, denoted by Δ, and the rate of change in the slope, called the *gamma* and denoted by Γ.[5] Because both delta and gamma are positive, the relationship is convex. This convexity arises from the fact that at a very low stock price the option is almost certain to expire out-of-the-money, so

[5] This delta is the same as the delta in the binomial model. Both represent the change in the call premium when the stock price changes. For the binomial model, with discrete price movements, the value of delta is given by the first equation in (3).

Box 2: Relationship of the Binomial Model with the Black-Scholes Model

The binomial pricing model can be written in a way that more accurately conveys the intuition underlying it. Equation (B1.1) can be rewritten as

$$C = SN_1 - Xr^{-(T-t)}N_2 \qquad \text{(B2.1)}$$

$$N_1 = \sum_{i=0}^{T-t} q^i(1-q)^{T-t-i}[\mu^i\delta^{T-t-i}/r^{T-t}]$$

$$N_2 = \sum_{i=0}^{T-t} q^i(1-q)^{T-t-i} \quad i = max(x^*,0),\ldots,T-t$$

$$x^* = \ln(X/S\delta^{T-t})/\ln(\mu/\delta)$$

The value of N_1 is the expected present value of a dollar invested in the stock conditional on the option ending in-the money, hence the term SN_1 is the expected present value of the stock obtained if the option is exercised. The value of N_2 is the probability that the option will end in-the-money, hence the second term is the expected present value of the strike price. Therefore, the call premium is simply the expected net present value of the option, if it is worth exercising it.

Cox, Ross, and Rubinstein (1977) have shown that the binomial model converges to the more familiar Black-Scholes (1973) model when time is continuous rather than discrete, and when the probability distribution of stock prices is log-normal. The Black-Scholes pricing model for a European call option on a stock having the value for the standard deviation of its rate of price change is

$$C = SN(d_1) - Xe^{-r(T-t)}N(d_2) \qquad \text{(B2.2)}$$

$$d_1 = [ln(S/X) + (r + ½\sigma^2)(T-t)]/\sigma\sqrt{(T-t)}$$

$$d_2 = d_1 - \sigma\sqrt{(T-t)}$$

where $N(d)$ is the probability that a standard normal random variable is less than d. The interpretation is precisely the same as that of the binomial model: The value of the call is the value of the option replicating portfolio, which consists of $N(d_1)$ shares of the stock of which the amount $Xe^{-r(T-t)}$ $N(d_2)$ is financed by borrowing at the riskless interest rate. As in the binomial case, the first term, $SN(d_1)$, is the expected present value of the stock obtained by exercising the option and $Xe^{-r(T-t)}$ $N(d_2)$ is the expected present value of the strike price paid, both expectations formed conditional on the option ending in-the-money.

The Black-Scholes model is remarkably simple, given its rather arcane foundations, and it has the advantage of being very specific about the effects of changes in parameters on call premiums. In particular, it states that the response of the call premium to changes in the parameters is

(a) Delta $(\Delta_c) = \partial C/\partial S = N(d_1)$

(b) Rho $(\rho_c) = \partial C/\partial r = (T-t)Xe^{-r(T-t)}N(d_2)$

(c)Gamma $(\Gamma_c) = \partial^2 C/\partial^2 S = N'(d_1)/[S\sigma(T-t)^{1/2}]$

(d) Vega $(\Lambda_c) = \partial C/\partial\sigma = (T-t)^{1/2}Xe^{-r(T-t)}N'(d_2)$

(e) Theta $(\theta_c) = \partial C/\partial t = -Xe^{-r(T-t)}\{[\sigma/2(T-t)^{1/2}]N'(d_2) + rN(d_2)\}$

(f) Chi $(\chi_c) = \partial C/\partial X = -e^{-r(T-t)}N(d_2)$

Thus, the delta and gamma (or first and second derivatives of C with respect to S) are both positive, reflecting the convexity of the premium as seen in Figure 1. The derivative of C with respect to r (the Rho) is positive, so that a rise in r will increase the call premium. The response of C to a change in volatility is also direct, so that call premiums rise when volatility increases. The negative Theta indicates that call premiums are higher for options with longer remaining lives, and that as the life of an option shortens, its premium declines. The negative Chi says that the call premium is negatively related to the strike price.

Using put–call parity the parameters describing the put option can be expressed as follows:

$$\Delta_p = \Delta_c - 1 \qquad \Gamma_p = \Gamma_c$$

$$\rho_p = \rho_c - (T-t)Xe^{-r(T-t)} \qquad \theta_p = \theta_c - rXe^{-r(T-t)}$$

$$\Lambda_p = \Lambda_c \qquad \chi_p = \chi_c + e^{-r(T-t)}$$

any change in S has little chance of affecting the final payoff and the slope approaches zero. As the stock price falls, the option-replicating portfolio approaches a bonds-only portfolio. At a very high stock price, the option is so far in-the-money that it is unlikely to expire without value, so the slope approaches one.

As the stock price increases, the option-replicating portfolio approaches a stocks-only portfolio; that is, the option is equivalent to holding a share of common stock.

The premium on a call option will increase with the *time remaining until expiration*. The parameter describing this, the option's *theta*, measures the response of the premium to the passage of time.[6] As the time to expiration increases, the distribution of the stock price at expiration widens in response to the prospect of longer runs in "ups" and "downs." Because the payoffs of the option are truncated at zero, the investor will benefit from longer runs of "ups" but not be harmed by longer runs of "downs"; hence, a longer time to expiration will confer potential gains that offset the potential losses.

The parameter describing the effect of a change in the *riskless interest rate* is the option's *rho*. While changes in r operate through several channels (the most direct being the present value of the exercise price), the net result is a positive rho. Thus, increases in interest rates, other things equal, will raise the call premium, and the greater is the rho the stronger is the effect.

Finally, *the variability of the stock's rate of return* will also affect the value of a call option. In a binomial model, the variability can be measured by the range of stock returns, or $(\mu - \delta)$. The parameter measuring the response of the call premium to changes in variability is called the option's *vega*. Vega, also called Lambda because it is denoted by Λ, is positive because a greater range of returns improves the prospects of being in-the-money, hence raising the value of the call.

Options and Futures

We have seen that any option has a replicating portfolio of stocks and bonds, and that the option-pricing formula states the characteristics of that portfolio. We now show that futures contracts are also equivalent to a portfolio of options, stocks, and bonds. Specifically, a futures contract is equivalent to a leveraged purchase of one full share of stock combined with borrowing the present value of the futures price.

A futures contract provides a zero payoff if, at expiration, the stock price is equal to the futures price determined by the market at the time the contract is initiated. For each one-dollar deviation in stock price from the contractual futures price, a one-dollar change in payoff occurs. Thus, if the futures price at time t for delivery of a share at time T is F_t, a futures contract on common stock entered into at time t has a profit or loss of $S_T - F_t$ at expiration. Buying this futures contract and simultaneously investing the amount $r^{-(T-t)}F_t$ in bonds has a value of S_T at expiration. The value of a share of the stock owned outright at time t, S_t, will also be S_T at expiration. Because arbitrage will ensure that the amount required to buy two perfectly equivalent claims on a future share must be equal at the outset, the equilibrium requires $S_t = r^{-(T-t)}F_t$. Hence, in equilibrium

$$F_t = S_t\, r^{(T-t)} \tag{6}$$

Thus, the futures price is equal to the price of a share accumulated to the expiration date at the *riskless* rate of interest.[7] Note that the accumulation factor is the riskless rate of interest, not the expected rate of return on the stock. Note also that, just as we have seen that options are equivalent to a portfolio of stocks and bonds, futures contracts are also equivalent to a (different) portfolio of stocks and bonds. From these equivalences, others emerge. For example, a futures contract must be equivalent to a suitably constructed portfolio of options and bonds. In particular, by put–call parity, it must be equivalent to simultaneously buying a call, writing a put with the same strike price and term, and investing the difference between the strike price and the futures price in bonds.

Futures contracts are also equivalent to a portfolio of options, stocks, and bonds. Specifically, a futures contract is equivalent to a leveraged purchase of one full share of stock combined with borrowing the present value of the futures price.

[6] Theta is defined as $-(\partial C/\partial t)$, the negative of the change in the premium as the time to expiration *shortens* slightly; hence theta is positive.

[7] If the security underlying the futures contract pays a cash dividend proportional to the security price, the discount rate is the riskless rate *less* the dividend yield.

Table 4
Hedging Ratio: European Calls per Share of Stock

	Number of Ups (x)									
Day	0	1	2	3	4	5	6	7	8	9
0	−1.27									
1	−1.53	−1.16								
2	−2.09	−1.32	−1.08							
3	−3.61	−1.67	−1.17	−1.03						
4	−10.36	−2.60	−1.37	−1.07	−1.01					
5	.00	−6.49	−1.92	−1.18	−1.02	−1.00				
6	.00	.00	−4.07	−1.46	−1.05	−1.00	−1.00			
7	.00	.00	.00	−2.55	−1.16	−1.00	−1.00	−1.00		
8	.00	.00	.00	.00	−1.60	−1.00	−1.00	−1.00	−1.00	
9	.00	.00	.00	.00	.00	−1.00	−1.00	−1.00	−1.00	−1.00

Parameters: $\mu = 1.10$, $\delta = 0.9091$, $\rho = 1.02$, $S = X = 50$.
Note: The hedging ratio is defined as $h(x, n) = -1/\text{delta}(x, n)$. It is the negative of the reciprocal of the change in call premium per dollar change in stock price.

II. Risk Management with Options and Futures

Just as a combination of stocks and bonds can replicate an option's price movements, options can be used to hedge against movements in stock prices. Hedges can be static or dynamic. The static hedge adopts a hedging ratio and adheres to it as the future of stock prices unfolds. Dynamic hedges adjust the hedging ratio as new information comes in. Maintaining the proper hedge requires information about the sensitivity of the option premium to changes in the stock price, that is, about the option's delta and gamma.

Dynamic Hedging

Delta hedging is a form of dynamic hedging that provides a short-term hedge against relatively small stock price movements. This hedging strategy requires computation of the *hedging ratio,* defined as minus the inverse of the delta ($h = -1/\Delta$). The hedging ratio, h, is the number of calls that must be *written* to match one long share, or the number of calls that must be purchased to match one short share. A position in calls equal to the hedging ratio will ensure that if the stock price rises (falls), the value of the call position will fall (rise) by just enough to provide a complete hedge.

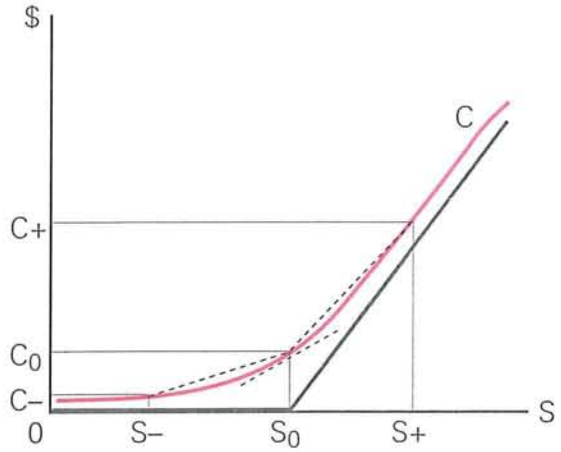
The hedging ratio implied by the hypothetical call option of Tables 1 to 3 is shown in Table 4. Each cell is the reciprocal of the associated entry in Table 2. At the outset, the investor must *write* 1.27 calls to match one long share. As time passes, the hedging ratio rises if the stock price falls, and falls if the stock price rises. As the stock price rises, the call goes deeper in-the-money and its delta increases, hence fewer calls must be written to match a dollar of stock price change.

Examination of the formula for delta reveals that it is simply the slope of the call premium–stock price relationship, as shown in Figure 1. For some derivatives, like stock index futures, the delta is constant so the gamma is zero. For these "linear" derivatives, the appropriate hedge does not change with the underlying stock price and a static hedge can be adopted. However, the convexity of the call premium–stock price relationship arising from a positive gamma means that the delta changes with stock prices. The convexity of the call premium–stock price relationship has two important implications for hedging. First, the higher the gamma, the more frequently a delta hedge must be altered as stock prices change. The reason is, of course, that the delta itself will respond more sensitively to changes in stock prices when the gamma is high. This is particularly pronounced when the option is just at-the-money, for the gamma is greatest at this point. The frequency of position adjustments can be reduced by creating both delta and gamma neutrality. Just as a position in a certain number of options can create a delta-neutral portfolio, so a portfolio that is both delta- and gamma-neutral can be constructed by using two different options on the stock.[8]

[8] Let Δ_1 and Γ_1 be the delta and gamma of one option, and Δ_2 and Γ_2 be the delta and gamma of a second option, differing in strike price or time to expiration. Let there be m and n numbers of options of each type, respectively, per share of stock held. Then the value of

Figure 4

The Role of Delta and Gamma in Dynamic Hedging



A second implication is that when gamma is high, delta hedging is more likely to fail when large stock price jumps occur. With large price changes, the optimal hedging ratio will differ from the hedging ratio designed for small changes. An example is shown in Figure 4, where the delta at stock price S_0 is shown as the dashed line tangent to the convex curve at S_0. A delta hedge can be placed by writing calls equal to the reciprocal of that delta at S_0. This will suffice to protect against small price fluctuations. Suppose, however, the stock price jumps to $S+$. The reciprocal of the slope of the line connecting C_0 and $C+$ represents the average delta, and the hedging ratio that would protect against that price jump is lower: Fewer calls need to be written to protect against large price jumps.

Suppose instead that the stock price fell to $S-$. The reciprocal of the slope of the line connecting C_0 and $C-$ will measure the correct hedging ratio. This slope is lower than the local delta at S_0, so the delta hedge will require writing fewer calls than would be necessary to protect against the fall. The value of the written calls will rise less than the decline in the stock price, leaving the delta-hedging investor exposed to stock market crashes.

Thus, a hedger will be exposed to jumps in the underlying stock's price: He will write too many calls when prices jump up, and too few calls when prices jump down. There is no way out of this problem. One compromise is to use the average delta between the original position and a chosen stock price. Thus, in Figure 4 the hedging ratio for a jump to $S+$ would be the reciprocal of the line connecting C_0 and $C+$. However, this approach can increase risk exposure if the price forecast is wrong: If the price should fall rather than rise, the compromise moves the portfolio in the wrong direction.

Portfolio Insurance

Portfolio insurance is a hedging method closely related to dynamic hedging. The problem posed for portfolio insurance is to construct the equivalent of a put option on a portfolio by dynamically varying the stock-bond composition of the portfolio. Thus, a synthetic put is created by portfolio allocations that mimic the option-replicating portfolio.

Consider a financial institution with a portfolio of \$100 million. Suppose that the manager wants to ensure that at the end of 10 days the portfolio is not worth less than its starting value of \$100 million. A direct approach would be to buy a 10-day put on the portfolio with a strike price of \$100 million. However, this might not be feasible because puts generally are not available for portfolios, because no puts are available with matching strike prices or expiration dates, or because regulation inhibits the use of derivatives. Even if an appropriate put option is available, it is more likely to be a custom-made option provided by a dealer than a standardized option traded in open markets. Hence, the put premium might be excessive, reflecting the bargaining position of the dealer.

An alternative approach is to buy "portfolio insurance" by creating a synthetic put on the portfolio. This is possible because, as we have seen, any position in calls (or puts) can be replicated using stocks and bonds. An application of portfolio insurance is demonstrated in Table 5. If the entire portfolio of \$100 million is invested entirely in stocks for 10 periods, assuming the binomial stock price process with parameter values used in previous examples, the final value will range from a low of \$38.554 million (if 10 "downs" occur) to a high of \$259.374 million (if 10 "ups" occur). These final values are shown in the "Final Value: If 100 percent stock" row at the bottom of the table.

Suppose instead that our manager wants to estab-

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the portfolio is given by $V = mC + nC + S$. A delta-neutral portfolio ($\partial V/\partial S = 0$) requires $m\Delta_1 + n\Delta_2 + 1 = 0$, while a gamma-neutral portfolio ($\partial^2 V/\partial S^2 = 0$) requires $m\Gamma_1 + n\Gamma_2 = 0$. These two equations can be solved for the values of m and n.

Table 5
Portfolio Insurance Via Portfolio Allocation

		Number of Ups (x)										
Day		0	1	2	3	4	5	6	7	8	9	10
0	Shares (# mill)	1.573										
	stocks ($mill)	78.627										
	bonds ($mill)	25.231										
	Insured Value ($mill)	103.858										
1	Shares (# mill)	1,309.588	1,729.303									
	stocks ($mill)	59.527	95.112									
	bonds ($mill)	37.688	17.114									
	Insured Value ($mill)	97.215	112.226									
2	Shares (# mill)	957.546	1,519.449	1,854.403								
	stocks ($mill)	39.568	75.972	112.191								
	bonds ($mill)	52.989	27.949	9.888								
	Insured Value ($mill)	92.557	103.922	122.079								
3	Shares (# mill)	553.587	1,198.357	1,170.860	1,939.972							
	stocks ($mill)	20.796	54.471	94.097	129.105							
	bonds ($mill)	69.224	43.103	17.981	4.391							
	Insured Value ($mill)	90.020	97.574	112.078	133.496							
4	Shares (# mill)	193.061	768.505	1,454.602	1,863.621	1,985.487						
	stocks ($mill)	6.593	31.756	72.730	112.749	145.348						
	bonds ($mill)	82.921	61.728	31.153	9.098	1.147						
	Insured Value ($mill)	89.514	93.484	103.883	121.847	146.495						
5	Shares (# mill)	.000	308.150	1,042.934	1,700.008	1,961.156	1,999.992					
	stocks ($mill)	.000	11.576	47.406	93.500	130.515	161.050					
	bonds ($mill)	90.573	80.256	50.488	18.279	2.789	.002					
	Insured Value ($mill)	90.573	91.832	97.894	111.779	133.304	161.052					
6	Shares (# mill)	.000	.000	491.846	1,371.451	1,895.869	2,000.075	1,999.942				
	stocks ($mill)	.000	.000	20.324	68.573	114.700	146.415	177.151				
	bonds ($mill)	92.385	92.385	74.270	35.072	6.795	(.004)	.006				
	Insured Value ($mill)	92.385	92.385	94.594	103.645	121.495	146.411	177.157				
7	Shares (# mill)	.000	.000	.000	785.048	1,721.021	2,000.100	2,000.060	1,999.872			
	stocks ($mill)	.000	.000	.000	35.684	94.656	133.107	161.056	194.859			
	bonds ($mill)	94.232	94.232	94.232	62.428	16.547	(.006)	(.003)	.013			
	Insured Value ($mill)	94.232	94.232	94.232	98.112	111.203	133.101	161.053	194.872			
8	Shares (# mill)	.000	.000	.000	.000	1,253.035	2,000.000	2,000.159	2,000.001	1,999.795		
	stocks ($mill)	.000	.000	.000	.000	62.652	121.000	146.422	177.156	214.337		
	bonds ($mill)	96.117	96.117	96.117	96.117	40.278	.000	(.010)	.002	.022		
	Insured Value ($mill)	96.117	96.117	96.117	96.117	102.929	121.000	146.411	177.158	214.359		
9	Shares (# mill)	.000	.000	.000	.000	.000	2,000.000	2,000.000	2,000.254	1,999.850	1,999.762	
	stocks ($mill)	.000	.000	.000	.000	.000	110.000	133.100	161.071	194.857	235.767	
	bonds ($mill)	98.039	98.039	98.039	98.039	98.039	.000	.000	(.018)	.017	.026	
	Insured Value ($mill)	98.039	98.039	98.039	98.039	98.039	110.000	133.100	161.053	194.874	235.793	
Final	If insured ($mill)	100.000	100.000	100.000	100.000	100.000	100.000	121.000	146.410	177.160	214.360	259.370
Value:	If 100% stock ($mill)	38.554	46.651	56.447	68.301	82.645	100.000	121.000	146.410	177.156	214.359	259.374

lish a floor of $100 million at the end of 10 periods, while enjoying *all* the benefits of a 100 percent stock position if stocks increase. This means that he wants to achieve the final values shown by the "Final Value: If insured" row, computed as $max(1.10^x 0.9091^{10-x} 100, 100)$ with x "up" periods. Table 5 shows that the 100 percent stock fund will be worth at least $100 million if five or more "ups" occur in the 10 days. These "Final Value: If insured" values establish the endpoints of a dynamic option-replicating strategy. By solving backward from these final values, we can construct the dynamic path of the option-replicating portfolio of stocks and bonds that leads to these "insured" values. These computations form the cells of Table 5.

Portfolio insurance is a hedging method closely related to dynamic hedging.

Suppose that only two "ups" occur in the first nine days. Table 5 shows that the tenth day's goal is $100 million, regardless of the movement of stock prices on the tenth day. The only way this can be achieved is by putting $98.039 million into bonds, nothing into stocks, on the ninth day. Stocks are avoided because any investment in stocks will potentially break the $100 million floor. However, if six "ups" have occurred in nine days, the tenth-day goal is $146.41 million if an "up" occurs on the tenth day and $121 million if a "down" occurs. In order to achieve this, the replicating portfolio must be worth $133.1 million, all invested in stocks because there is no chance the floor will be broken.

Working all the way back we can compute the total value and composition of the portfolio required to replicate the desired final results. This leads us to an apparent paradox: In order to achieve the insurance goals, the portfolio must have a starting value of $103.86 million, an impossibility in light of the postulated $100 million starting value.

What this paradox reveals is that you cannot simultaneously enjoy the full benefits of 100 percent investment in stocks when prices increase and establish a floor that avoids price decreases. The first goal requires full investment in stocks, while the second requires some investment in bonds. The synthetic put of portfolio insurance has a hidden premium. This "insurance" premium, which should be equal to the premium that would have been paid for a standardized put option if it were available, arises from the necessary sacrifice of upside potential in order to reduce downside risk. In our example, the missing $3.86 million measures the insurance premium. Indeed, this is the premium for the synthetic put and (in the absence of transactions costs) it would also be the premium for an actual put option with a $100 million strike price.

The insurance premium in this example is 3.86 percent of the "face value" of the insurance. The institution with an initial portfolio of $100 million can do no better than establish a floor of $96.14 million, the remaining $3.86 million being the required sacrifice necessary to achieve that floor. Indeed, each number in Table 5 will be reduced by 3.86 percent to reflect the insurance premium required to keep to the desired floor. Thus, the achievable final values "if insured" will range from the floor of $96.14 million to a maximum of $249.36 million.

Portfolio Insurance with Stock Index Futures

The strategy just outlined requires frequent portfolio reallocations as stock prices change. These can entail considerable expense in terms of commissions as well as bid-asked spreads. Because transactions in stock index futures carry relatively low costs, Rubinstein (1985) suggests that an institution with a diversified portfolio might find it appropriate to use stock index futures rather than stock transactions to achieve the desired insurance.

The use of stock index futures can carry its own costs. While transactions costs are low and margin requirements are both low and often not binding,[9] there is the possibility of "basis risk" when the portfolio of stocks does not exactly match the stock index future being used. This is discussed in the next section. Even so, the net costs of the insurance can be reduced using futures.

We have seen that a futures contract should be priced so that $F = Sr^{(T-t)}$. The equivalence between futures and spot stock prices suggests a simple approach to portfolio rebalancing: Fully invest in stocks at the outset and maintain the original position in shares, using futures contracts rather than stock trades to achieve the insurance goals, and finance any profits

[9] Margin can be provided in the form of securities already held by the institution.

Table 6
Portfolio Insurance Via Stock Index Futures

Day		Number of Ups (x) 0	1	2	3	4	5	6	7	8	9	10
0	Insured Value ($mill)	103.858										
	Share Price ($)	50.000										
	Futures Price ($)	60.950										
	Shares (# mill)	2.077										
	Futures (# mill)	−.422										
	Shares ($ mill)	103.858										
	Bonds ($mill)	−.000										
	Actual Value ($mill)	103.858										
1	Insured Value ($mill)	97.215	112.226									
	Share Price ($)	45.455	55.000									
	Futures Price ($)	54.322	65.730									
	Shares (# mill)	2.077	2.077									
	Futures (# mill)	−.655	−.297									
	Shares ($mill)	94.417	114.244									
	Bonds ($mill)	2.798	−2.018									
	Actual Value ($mill)	97.215	112.226									
2	Insured Value ($mill)	92.557	103.922	122.079								
	Share Price ($)	41.322	50.000	60.500								
	Futures Price ($)	48.416	58.583	70.885								
	Shares (# mill)	2.077	2.077	2.077								
	Futures (# mill)	−.975	−.486	−.194								
	Shares ($mill)	85.833	103.358	125.669								
	Bonds ($mill)	6.724	.063	−3.589								
	Actual Value ($mill)	92.557	103.922	122.079								
3	Insured Value ($mill)	90.020	97.574	112.078	133.496							
	Share Price ($)	37.566	45.455	55.000	66.550							
	Futures Price ($)	43.151	52.213	63.178	76.445							
	Shares (# mill)	2.077	2.077	2.077	2.077							
	Futures (# mill)	−1.353	−.780	−.325	−.122							
	Shares ($mill)	78.030	94.417	114.244	138.236							
	Bonds ($mill)	11.990	3.157	−2.166	−4.739							
	Actual Value ($mill)	90.020	97.574	112.078	133.496							
4	Insured Value ($mill)	89.514	93.484	103.883	121.847	146.495						
	Share Price ($)	34.151	41.322	50.000	60.500	73.205						
	Futures Price ($)	38.459	46.536	56.308	68.133	82.441						
	Shares (# mill)	2.077	2.077	2.077	2.077	2.077						
	Futures (# mill)	−1.706	−1.185	−.564	−.193	−.083						
	Shares ($mill)	70.937	85.833	103.858	125.669	152.059						
	Bonds ($mill)	18.577	7.651	.025	−3.822	−5.565						
	Actual Value ($mill)	89.514	93.484	103.883	121.847	146.495						

Table 6 continued

Portfolio Insurance Via Stock Index Futures

Day		Number of Ups (x) 0	1	2	3	4	5	6	7	8	9	10
5	Insured Value ($mill)	90.573	91.832	97.894	111.779	133.304	161.052					
	Share Price ($)	31.046	37.566	45.455	55.000	66.550	80.526					
	Futures Price ($)	34.277	41.476	50.185	60.724	73.477	88.907					
	Shares (# mill)	2.077	2.077	2.077	2.077	2.077	2.077					
	Futures (# mill)	−1.919	−1.634	−.955	−.348	−.107	−.071					
	Shares ($mill)	64.488	78.030	94.417	114.244	138.236	167.265					
	Bonds ($mill)	26.085	13.301	3.478	−2.465	−4.932	−6.213					
	Actual Value ($mill)	90.573	91.832	97.894	111.779	133.304	161.052					
6	Insured Value ($mill)	92.385	92.385	94.594	103.645	121.495	146.411	177.157				
	Share Price ($)	28.224	34.151	41.322	50.000	60.500	73.205	88.578				
	Futures Price ($)	30.550	36.966	44.729	54.122	65.487	79.239	95.880				
	Shares (# mill)	2.077	2.077	2.077	2.077	2.077	2.077	2.077				
	Futures (# mill)	−1.957	−1.957	−1.494	−.665	−.171	−.073	−.073				
	Shares ($ mill)	58.625	70.937	85.833	103.858	125.669	152.059	183.992				
	Bonds ($mill)	33.759	21.448	8.761	−.214	−4.174	−5.648	−6.834				
	Actual Value ($mill)	92.385	92.385	94.594	103.645	121.495	146.411	177.157				
7	Insured Value ($mill)	94.232	94.232	94.232	98.112	111.203	133.101	161.053	194.872			
	Share Price ($)	25.658	31.046	37.566	45.455	55.000	66.550	80.526	97.436			
	Futures Price ($)	27.228	32.946	39.865	48.237	58.366	70.623	85.454	103.400			
	Shares (# mill)	2.077	2.077	2.077	2.077	2.077	2.077	2.077	2.077			
	Futures (# mill)	−1.997	−1.997	−1.997	−1.242	−.342	−.074	−.074	−.074			
	Shares ($mill)	53.296	64.488	78.030	94.417	114.244	138.236	167.265	202.391			
	Bonds ($mill)	40.936	29.744	16.202	3.696	−3.041	−5.135	−6.212	−7.518			
	Actual Value ($mill)	94.232	94.232	94.232	98.112	111.203	133.101	161.053	194.872			
8	Insured Value ($mill)	96.117	96.117	96.117	96.117	102.929	121.000	146.411	177.158	214.359		
	Share Price ($)	23.325	28.224	34.151	41.322	50.000	60.500	73.205	88.578	107.179		
	Futures Price ($)	24.268	29.364	35.530	42.992	52.020	62.944	76.162	92.157	111.509		
	Shares (# mill)	2.077	2.077	2.077	2.077	2.077	2.077	2.077	2.077	2.077		
	Futures (# mill)	−2.036	−2.036	−2.036	−2.036	−.808	−.076	−.076	−.076	−.076		
	Shares ($mill)	48.451	58.625	70.937	85.833	103.858	125.669	152.059	183.992	222.630		
	Bonds ($mill)	47.666	37.491	25.180	10.283	−.929	−4.669	−5.648	−6.833	−8.271		
	Actual Value ($mill)	96.117	96.117	96.117	96.117	102.929	121.000	146.411	177.158	214.359		
9	Insured Value ($mill)	98.039	98.039	98.039	98.039	98.039	110.000	133.100	161.053	194.874	235.793	
	Share Price ($)	21.205	25.658	31.046	37.566	45.455	55.000	66.550	80.526	97.436	117.897	
	Futures Price ($)	21.629	26.171	31.667	38.317	46.364	56.100	67.881	82.136	99.385	120.255	
	Shares (# mill)	2.077	2.077	2.077	2.077	2.077	2.077	2.077	2.077	2.077	2.077	
	Futures (# mill)	−2.077	−2.077	−2.077	−2.077	−1.851	−.077	−.077	−.077	−.077	−.077	
	Shares ($mill)	44.046	53.296	64.488	78.030	94.417	114.244	138.236	167.265	202.391	244.893	
	Bonds ($mill)	53.993	44.743	33.551	20.009	3.622	−4.244	−5.136	−6.212	−7.517	−9.100	
	Actual Value ($mill)	98.039	98.039	98.039	98.039	98.039	110.000	133.100	161.053	194.874	235.793	
10	Insured Value (=Actual)	98.039	98.039	98.039	98.039	98.039	100.000	121.000	146.410	177.160	214.360	259.370
	Share Price	19.277	23.325	28.224	34.151	41.322	50.000	60.500	73.205	88.578	107.179	129.687
	Futures Price	19.277	23.325	28.224	34.151	41.322	50.000	60.500	73.205	88.578	107.179	129.687

or losses on the futures contracts by lending or borrowing. Alternatively, one could fully invest in bonds at the outset and vary the positions in stocks and futures contracts to achieve the goals.

Table 6 demonstrates the first approach. An initial amount of $103.86 million is assumed so that the portfolio insurance results of Table 5 can be used as a guide. Variations in bonds and futures contracts are constructed to replicate the entire path of "insured value" outcomes in Table 5. This is done by buying futures contracts after stock price increases and selling them following a decline in prices, thereby realizing some of the futures profits. The amount of futures profits to realize is just enough so that, when invested in bonds, the value of the portfolio is equal to the insurance goal.

For example, on day zero the institution has $103.858 million to invest. It holds all of this in stocks, none in bonds, and *writes* 10-day futures contracts for 0.422 million shares. The futures price is determined as $F = 50(1.02)^{10} = \$60.95$. If the stock price goes up to $55, the value of shares on day 1 is $114.244 million and paper losses of $2.018 million occur on the futures contracts, arising from the increase in the futures price to $65.73. These futures losses are all realized and are paid for by borrowing $2.018 million. The net value of the portfolio is $112.226 million, which is the insurance goal under the plan established in Table 5. Then, in order to position itself for the second day, the institution will write 0.125 futures contracts. Thus, it will conclude the first day with a portfolio worth $112.226 million, of which $114.244 million is stock financed by borrowing $2.018 million, plus a short position of 0.125 futures.[10]

Had prices gone down on the first day, the manager would have experienced a gain of $6.628 per futures contract, for a total profit of $2.798 million. This profit would be realized and invested in bonds so that the total value of stocks and bonds would be the insurance goal of $97.215 million. The institution would then write $0.655 million of new 9-day contracts.

The portfolio insurance strategy using futures can be summarized as follows. Determine your insurance goals for each period; these are the actual portfolio values necessary to end up with the portfolio worth at least the desired floor. Then invest all your initial money in stocks and write the appropriate number of futures contracts. On each day realize enough profit or loss on your futures contract to achieve your insurance goal, investing all realized futures profits in bonds (or borrowing to finance realized futures losses). At the end of the 10-day period you will have the outcomes you selected when you undertook the portfolio insurance.

Stock Price Dynamics and Portfolio Insurance

Following the Crash of 1987, portfolio insurance became unfashionable, for two fundamental reasons. First, the dynamic interactions between portfolio insurance and stock prices suggested to some that portfolio insurance contributed to the magnitude of the Crash. This view was most forcefully presented in the Brady Commission's 1988 analysis of that event (Brady 1989). Second, institutions that had bought portfolio insurance strategies found that they were not protected from the short, but violent, free fall in stock prices.

The first criticism, that portfolio insurance magnifies stock price movements, is demonstrated in Table 5. A glance across any row (or down any column) shows that as stock prices increase (decrease), the portfolio insurer will buy more (fewer) shares. The reason for this "buy high–sell low" feature of portfolio insurance is that the need for insurance is inversely related to the stock price. An increase in stock prices reduces the likelihood that the floor will be reached, hence requiring that less money be put into riskless bonds and a correspondingly greater number of shares held. If, on the other hand, prices have been falling, the likelihood that the final result will exceed the floor is high, and the portfolio must be more heavily weighted with bonds to ensure that the floor will not be broken.

The use of futures markets as a substitute for the spot market will not eliminate the unfortunate market dynamics of portfolio insurance. While Table 6 suggests that the dynamics are absent because there are no transactions in stocks, the pressure remains but is hidden from view. The strategy underlying Table 6 has futures contracts being bought (sold) after the stock price increases (declines). As the futures price increases in response to buying pressure in that market, index arbitrageurs will find it advantageous to buy in the spot market and sell futures short, thereby transferring the pressure back to the spot market.

The second problem, failure to protect, reflected less on the merits of portfolio insurance and more on the specific circumstances of the Crash. Any insurance

[10] Of course, the institution will not buy back 0.422 futures then sell 0.125 futures. Instead, it will simply buy the difference, or 0.297 futures, leaving the remaining 0.125 short futures on the books.

program rests on assumptions about the normal range of outcomes an insurer will experience. Insurance rests on actuarial tables that work well for a pool of unrelated risks, but companies are exposed to losses from extreme outcomes, especially those that create correlated risks, like major earthquakes. Wind insurance can also fail to protect when extreme storms occur, such as Hurricane Andrew, rather than microstorms, like tornadoes.

Portfolio insurance failed during the 1987 Crash because the magnitude of the event was so extreme, because trading halts prevented execution of the insurance program, and because reported stock prices gave an incorrect signal about the state of the markets.

The Crash of '87 was an extreme outcome, and we should not be surprised that financial institutions did not find the protection they sought. The heart of portfolio insurance is the frequent adjustment of hedge positions. During a crash, a portfolio insurer would want to sell stocks and buy bonds. But, as noted in a previous article in this *Review* (Fortune 1993), this could not be done during the 1987 Crash, for a number of reasons. Trading in many stocks, as well as trading in stock index futures, was suspended for a significant portion of the time, and the backlog of unexecuted orders for those stocks that were trading was unusually long. A portfolio insurer could not maintain his program in this environment.

A problem related to this is the existence of "stale" prices in the cash market for stocks. During the Crash the *reported* levels of stock prices and stock indices exceeded the true level. This happened for two reasons, trading halts and limit orders. During a halt, the reported price is the price of the last trade, which becomes more "stale" the longer the halt. Because stock indices use these stale prices, halts give the appearance of a smaller price decline and discourage the sales that should be made to provide portfolio insurance. Probably more important were limit orders. If the specialist's book is filled with limit buy orders, a selling panic will not have its full effect on prices, because the surge of sell orders will be matched with the book, giving the appearance of a more gradual price decline. Not until the limit orders are exhausted will a free fall in prices show up.

Thus, portfolio insurance failed during the Crash because the magnitude of the event was so extreme, because trading halts prevented execution of the insurance program, and because reported stock prices gave an incorrect signal about the state of the markets.

III. Further Considerations

We have shown that "plain-vanilla" derivative instruments like equity options, index options, and index futures can be viewed as equivalent to a traditional portfolio of stocks and bonds. The analysis focused on the market risks arising from derivatives and abstracted from other types of risk. It also assumed that the statistical properties of the price of the underlying security are known. For example, the binomial pricing model assumes that prices are binomially distributed, while the Black-Scholes model assumes a log-normal distribution. In this section we address some of those loose ends.

The Limits of the Equivalence between Derivatives and Traditional Instruments

The equivalence we have demonstrated applies to a wide range of derivative products. For example, a plain-vanilla *interest rate swap* is equivalent to purchasing a fixed-rate instrument and financing it with floating-rate debt. The swap is designed so that it has a zero net present value. The swap is equivalent to buying a fixed-rate bond and financing it with a floating-rate bond. The incentive to engage in a swap is related to the comparative advantage arising from the gap between what the two parties would pay on the fixed-rate instrument and the gap on floating-rate instruments. If one company (AAA) can borrow both floating and fixed at lower rates than another (BBB), AAA has an *absolute* advantage over BBB in credit markets. Even so, AAA might have a *comparative* advantage in the fixed-rate market, while BBB has a *comparative* advantage in the floating-rate market. Hence both AAA and BBB might gain from having AAA borrow at a fixed rate and engage in a receive-fixed swap with a floating-rate borrower like BBB.

This effectively converts AAA's fixed-rate liability to a floating rate, and BBB's floating-rate liability to a fixed rate, both at more advantageous terms.[11]

To extend the metaphor, the notorious *inverse floater* is equivalent to lending at a fixed rate and borrowing a fraction or multiple of that notional value at a floating rate. A complex variation, stepped inverse bonds are equivalent to a series of forward contracts.

For some derivatives it is difficult to establish an equivalence, because no replicating portfolio, or closed form solution, can be found.

An example is the stepped inverse bond issued by the Federal National Mortgage Association (FNMA) and bought by Orange County, California, in February of 1994.[12] These bonds paid a rate of 7 percent for the first three months, then paid 10 percent minus the three-month LIBOR rate at each three-month interval until 1996 (this amounted to 5.1 percent in the first three months). In 1996 the terms changed to 11.25 percent less three-month LIBOR until maturity in 1999. This was equivalent to buying a FNMA bond due in 1996 at 10 percent and borrowing at LIBOR, for a net return of 5.1 percent in the first three months, while simultaneously engaging in a forward contract to buy a three-year 11.25 percent FNMA bond in 1996 and to sell a three-year floating rate note at three-month LIBOR.

These derivatives have a property shared by all derivatives that satisfy the equivalence property—they have replicating portfolios which allow the derivation of closed-form solutions for the derivative price. A closed-form solution means that the price can be expressed as a function of relevant variables and parameters. For example, the Black-Scholes model states that a European call option is equivalent to a fractional share of the underlying security plus borrowing a fractional share of the exercise price; this is a closed-form solution.

However, for some derivatives it is difficult to establish an equivalence, because no replicating portfolio, or closed-form solution, can be found. This can occur for several reasons. One reason might be that the standard assumptions about the probability distribution of final payoffs do not apply. Typically, this means that there is no precise replicating portfolio and valuation must be done by numerical methods. An example is a *lookback option*, for which the exercise price depends upon the history of the underlying security's price. In one form of lookback option, the Asian-style option, the strike price is the (arithmetic) average price of the underlying security over the life of the option. In this case the distribution of payoffs is not normal even though the returns on the underlying security might be normally distributed.[13] Closed-form solutions based on the normal distribution will not apply, and numerical simulation methods are the only alternative approach to valuation. Hence, we cannot treat an Asian option as equivalent to a portfolio of stocks and riskless bonds. We can only recognize the fuzzy correspondence between the lookback option and a portfolio.

Basis Risk

Another reason for absence of a replicating portfolio is transactions costs. These can make it expensive to engage in the portfolio-option arbitrage which allows a closed-form solution. Often this gives rise to *basis risk*. The problem of basis risk arises when an option or futures contract based on an index of securities is used to hedge a portfolio that is not precisely mimicked by the derivative. For example, our financial institution's portfolio insurance scheme employed stock index futures to adjust the effective portion of the portfolio devoted to stocks. If the institution was an index fund holding, say, the S&P 500 portfolio of

[11] For example, if AAA can borrow at 10 percent fixed and LIBOR + 1 percent floating, while BBB can borrow at 12 percent fixed and LIBOR + 2 percent floating, then AAA has a comparative advantage as a fixed-rate borrower. Suppose AAA borrows fixed at 10 percent and engages in a fixed-rate swap in which it pays LIBOR + 1.5 percent floating in exchange for 11 percent fixed. BBB, of course, borrows floating at LIBOR + 2 percent and takes the other side of the swap, paying 11 percent fixed and receiving LIBOR + 1.5 percent floating. The net effect is that AAA has a floating-rate liability at LIBOR + 0.5 percent and BBB has a fixed-rate liability at 11.5 percent. Both parties are better off.

[12] In a structured note, a government agency issues a bond whose rate of return is set according to a specified relationship with a short-term interest rate, with the payments reset at the interval of the short rate. The relationship between the rate paid and the short rate can be direct or inverse, fractional or multiple. By selling offsetting structured notes, the agency's obligation can be equivalent to a fixed-rate bond.

[13] An Asian call option based on the average price has the payoff $max(S - S_{avg}, 0)$. Even though S might be log-normally distributed, S_{avg} is not. Indeed, $S - S_{avg}$ has no known distribution.

stocks, the use of an S&P 500 futures contract involves no basis risk. But in most cases the stock portfolio will differ in composition from the portfolio upon which the futures contract is based. As a result, the correlation between the derivative price and the value of the stocks in the portfolio is imperfect.

Basis risk clearly means that hedges of the type outlined above are imperfect. However, stocks have been shown to have a significant common factor (called the market factor) in their price movements. Because idiosyncratic risks attached to specific stocks can largely be diversified away, the major source of price variation is the market factor. In short, funds do not have to be index funds to be highly correlated with stock market movements, and the higher this correlation the smaller is the basis risk.

Counterparty Risk

Counterparty risk is the prospect that the counterparty to a derivative transaction might default. For example, the writer of a call option is obligated to deliver shares in the event that the call is in-the-money and the holder exercises it. If the writer reneges, and if no insurance pool or other means is available to enforce the contract, the value of the holder's call is reduced or disappears.

Counterparty risk is a relatively small problem in the exchange-traded options and futures we have focused on here. The reason is that the transaction is not with a specific counterparty, but with a clearing house. The functions of the clearing house are to determine to which writer an exercised contract is assigned, to receive payments for and make transmittal of underlying securities, and to ensure that all contracts are honored. Any default is with the clearing house, which has adequate resources to meet the obligations of the contract. The clearing house provides an extremely important function in the formation of actively traded markets for standardized contracts, for it removes the specific names on the contract from each party's consideration. In the absence of the clearing house, any buyer of an option contract, or participant in a futures contract, would require detailed information on the financial position of the counterparty, a requirement that would inhibit use of the instruments.

Counterparty risk is considerably more important in the over-the-counter (OTC) markets for derivatives. These dealer markets create custom-made contracts between parties, which have less liquidity (that is, they are more difficult to reverse) and for which no guaranteeing agency is present to ensure payments. Thus, the pricing of OTC products ranging from plain-vanilla interest rate swaps to exotica like diff swaps[14] involves counterparty risk, which affects the value, or terms, of the derivative. Counterparty risk can be reduced in a variety of ways, from compensatory interest rates (in the form of a premium paid over LIBOR or over Treasury bonds) through credit enhancements (for example, collateral) to netting agreements between the parties. Recently, efforts have been made to organize a clearing-house arrangement for OTC derivatives, although none has been established as of this writing.

IV. Summary and Conclusions

This article demonstrates that such simple derivatives as exchange-traded options can be easily understood if one is just willing to spend some time at it. Many, but not all, of the new derivatives are, in fact, old instruments in new clothing. As such, they may represent more complicated ways to speculate or to hedge, but in most cases they can be understood as equivalent to traditional instruments. Indeed, one approach to evaluating the risk exposure of financial institutions holding derivatives is to convert them to their equivalence in more traditional financial instruments.

In order to understand the equivalence between many derivatives and traditional portfolios of stocks and bonds, we first explain the technology of derivatives. This is done for the simplest forms of derivatives—plain-vanilla equity options, stock index options, and stock index futures. We show that these derivatives are equivalent to an underlying portfolio of stocks and bonds. Hence our subtitle "A Rose by Any Other Name . . . ." This equivalence is illustrated in several ways. We first demonstrate the equivalence using a simple formal model of option price determination, the *binomial option pricing model*, in which options premiums are directly derived from the prices of stocks and bonds. Because the binomial model serves as a first approximation to option premiums described by more sophisticated theory (for example, the Black-Scholes model), it is a particularly fruitful starting point.

[14] A diff swap involves payment of a foreign interest rate by one party in exchange for a U.S. interest rate by the other, all payments in U.S. dollars. Hence, it is equivalent to borrowing at the U.S. rate to buy foreign securities, combined with an exchange rate guarantee.

After discussing the pricing of stock options and stock futures, and their equivalence to stocks and bonds, we then turn the problem on its head and show how stocks, bonds and futures can be used to mimic options; that is, traditional stock-bond portfolios can be used to construct "synthetic" options. Thus, in the second section we focus on aspects of risk management using synthetic options constructed by taking positions in traditional instruments. Our particular interest is in dynamic hedging, and in one of its more interesting manifestations, portfolio insurance. We point out why portfolio insurance seemed such a promising strategy, and what led to its decline after the Crash of 1987.

No metaphor precisely fits all circumstances, and our analogy between traditional instruments and exchange-traded options does not apply to all derivative securities. We discuss several circumstances in which there is no replicating portfolio and, therefore, the equivalence between a derivative and traditional instruments breaks down. Among these are basis risk, counterparty risk, and discontinuities in the price of the underlying security. In these cases, derivative securities do introduce something new because they do not precisely replicate the movements in prices of traditional instruments. These situations are most often found in over-the-counter derivatives, though they can also apply to exchange-traded derivatives.

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