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Do Real-Time Okun's Law Errors Predict GDP Data Revisions?

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Abstract:

Using U.S. real-time data, we show that changes in the unemployment rate unexplained by Okun's Law have significant predictive power for GDP data revisions. A positive (negative) error in Okun's Law in real time implies that GDP will be later revised to show less (more) growth than initially estimated by the statistical agency. The information in Okun's Law errors about the true state of real economic activity also helps to improve GDP forecasts in the near term. Our findings add a new dimension to the interpretation of real-time Okun's Law errors, as they show that these errors can convey information other than a change in potential GDP, the equilibrium unemployment rate, or the use of labor's intensive margin.

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1. Introduction

The empirical regularity of a negative relationship between movements in the unemployment rate and GDP, first established by Arthur Okun (1962) and named after him as Okun's Law, is an important tool for the conduct of monetary policy. Errors in Okun's Law, if persistent, tend to be heavily scrutinized, as they may convey information about changes in potential GDP and (or) the natural rate of unemployment. Having a reliable estimate of the size of the output and unemployment rate gaps is of crucial importance for the good conduct of monetary policy. It is then not very surprising to see policymakers continually reverse-engineering Okun's Law as one way of drawing inference about these gaps. Under current circumstances, for example, the noticeable decline in the unemployment rate from a peak of 10 percent in October 2009 to 7.9 percent in October 2012, with GDP growth averaging 2.2 percent over the period 2009:Q4 to 2012:Q2 could be taken as evidence of a decline in potential GDP growth from the 2 to 2.5 percent estimates prevailing before the onset of the 2008–'09 recession.[1]

Real-time errors in Okun's Law, however, carry other economic information that is not related to changes in potential GDP and (or) the natural rate of unemployment. Specifically, this paper shows that real-time errors in Okun's Law contain information about future revisions to GDP. If the unemployment rate increases (decreases) by more than the amount that Okun's Law predicts on the basis of real-time GDP readings, then those GDP readings will later be revised to show less (more) growth than the statistical agency was first assessing. In this respect, it is interesting to note that during the 2008–'09 recession the unemployment rate increased by more than the amount that Okun's Law would have predicted on the basis of real-time GDP data. Later, those GDP data were revised to show substantially less growth than initially thought.[2]

[1] Such an interpretation of the recent developments in Okun's Law has drawn some criticism, as Okun's Law generated positive errors during the recession, with the unemployment rate increasing faster than was being predicted by GDP readings. While it is possible that the natural rate of unemployment increased somewhat during that period, other interpretations have been advanced to explain the failure of Okun's Law during the Great Recession and the subsequent recovery that do not rely on significant changes in structural unemployment and potential GDP growth (see, among others, Romer 2012, and Yellen 2012).

[2] These GDP revisions have lessened, but not completely eliminated, errors in Okun's Law during the Great Recession.

While we still do not have much information in terms of GDP revisions for the ensuing recovery period, this paper's findings raise the possibility that future GDP revisions will show more GDP growth than is currently being estimated.

The present paper contributes to a vast literature on data revisions.[3] While the ability of the unemployment rate—which, unlike GDP, is not subject to revision aside from minor seasonal adjustments—to predict future GDP revisions has already been noted, this is, to our knowledge, the first paper showing that it is the portion of the change in the unemployment rate not explained by Okun's Law in real time that has predictive power for future GDP revisions. Moreover, we show that this predictive power is significant from an economic standpoint. The paper also shows that information contained in real-time Okun's Law errors can produce some improvement in the near-term forecasts of GDP growth in the Survey of Professional Forecasters. Thus, the paper provides additional evidence on the predictability of the Survey of Professional Forecasters' forecast errors.[4] The paper also relates to the literature on the relevance and stability of Okun's Law itself, as it provides a caveat against drawing overly strong conclusions about potential output or the natural rate of unemployment, when working with real-time GDP data.[5] Ultimately, however, we view the main contribution of the paper as adding another dimension to the policy debate on what signal to extract from Okun's Law errors in real time. While typically the policy discussion around these real-time errors is framed in terms of changes to potential GDP, the equilibrium unemployment rate, or transitory changes in labor's intensive margin (possibly via changes in effort), this paper stresses the information content of these errors for GDP data revisions and for assessing the true pace of output growth in real time.

The rest of the paper is structured as follows. Section 2 provides a description of how we estimate errors in Okun's Law from real-time data. Section 3 illustrates the predictability of GDP data revisions from Okun's Law errors in real time, and Section 4 provides an application to the Great Recession and subsequent recovery to illustrate the economic relevance of this

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[3] See Croushore (2011) for a recent review of the literature.
[4] See, for example, Adam and Padula (2011) and the references therein.
[5] See, among others, Ball, Leigh, and Loungani (2012) and Owyang and Sekhposyan (2012).

finding. Section 5 discusses the robustness of the results, while Section 6 briefly shows that explicitly taking into account real-time errors in Okun's Law can produce some improvement in near-term forecasts of GDP growth. Section 7 offers concluding remarks and directions for future research.

2. Okun's Law: Specification and Estimation

Our baseline findings are established by estimating a first-difference specification of Okun's Law taking the following form:

$$(1) \quad \Delta U_t = \alpha + \beta(L)\Delta y_t + \varepsilon_t,$$

where U is the unemployment rate, y is the natural log of real GDP, and Δ denotes the first-difference operator. Since the relationship is estimated at the quarterly frequency, we include current and lagged growth rates (as log differences) of real GDP. These lags allow, for instance, for the possibility of a delayed adjustment of labor's extensive margin to changes in GDP. The term ϵ in the above equation is the error in the Okun's Law relationship, and is the variable of interest for our analysis.

The first-difference specification featured in (1) assumes both a constant equilibrium unemployment rate and a constant rate of potential GDP growth. These assumptions can prove unrealistic over extended time periods. For this reason, we consider estimates of equation (1) using a rolling-sample with a relatively short (60-quarter) window. In addition, to account for the possibility of changes in the equilibrium unemployment rate and in potential growth over a relatively short time frame, we also consider the following variant:

$$(2) \quad \Delta U_t = \alpha_t + \beta(L)\Delta y_t + \varepsilon_t,$$

where now the intercept is allowed to change over time. We assume that the intercept evolves according to an autoregressive process of the form:

(3) $\alpha_t = \rho\alpha_{t-1} + u_t,\ |\rho| \leq 1.$

We view the specification in (2)–(3) with the unobserved component as a more plausible alternative to estimating the equilibrium rate of unemployment and potential GDP using a Hodrick-Prescott (or similar) filter. Our methodology relies on estimating the Okun's law relationship in real time and considering at each point in time the most recent error in the relationship. Since endpoints are the focus of our analysis, the use of band-pass filters is problematic, as these filters can be highly sensitive to endpoint conditions.[6] Nonetheless, in the robustness section we report, for completeness, results based on estimating trend GDP growth and the trend unemployment rate in real time via Hodrick-Prescott filters.

The relationship in (1), like its variant in (2)–(3), does not keep track of levels. Therefore, it may present shortcomings relative to a level Okun's law relationship, which explicitly imposes a co-movement between the unemployment rate gap and the output gap. There is, however, a tradeoff between fully accounting for the co-movements in the gaps and the need to estimate these gaps in real time, a need that cannot be avoided when one is confronted with the level relationship. In all, specifying the relationship in first differences raises issues related to over-differencing, but as long as the level adjustment is sufficiently slow, the difference specification should provide a reasonable approximation to the short-run dynamics of the unemployment rate. Still, in the robustness section we will discuss results based on a level specification as well as those based on a first-difference specification.

We use real-time data when estimating the above Okun's Law relationships. This implies that in each quarter over the period 1965:Q4 to 2012:Q4 we take the latest vintage of GDP data

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[6] At endpoints, the Hodrick-Prescott filter cannot be two-sided anymore and therefore can have difficulty in selecting the appropriate gain. The Kalman filter is the optimal one-sided filter (see Hamilton 1994).

available on the 15th day of the middle month of that quarter. This latest vintage will contain information up to the previous quarter. However, we consider as real-time the information encompassing at least the third GDP release (the so-called "final" release) from the Bureau of Economic Analysis (BEA). In practice, this means that at any given quarter t we take the latest GDP vintage available then, up to quarter $t-2$. Thus, for the purpose of estimating Okun's Law in real time, we do not consider the real-time t information about quarter $t-1$, as the BEA's estimate of $t-1$ GDP available 45 days thence is based on more limited and preliminary information. For the real-time unemployment rate, we follow the same timing as for real-time GDP. Still, revisions to the unemployment rate are minor and relate only to adjustments in the seasonal factors. This is an essential feature for the purpose of our exercise, as the premise is that the unemployment rate series is not subject to material revision. Therefore, it may feature information about the state of the real economy, as filtered through Okun's Law, which the real-time GDP data do not fully capture.

The real-time vintages of data are used in regressions with a rolling window of 60 quarters. [7] The rolling window at any given point in time t when estimating (1) or (2)–(3) consists of the following information set:

$$(4) \quad \begin{bmatrix} \Delta U_t^{t+2} & \Delta y_t^{t+2} \\ \Delta U_{t-1}^{t+2} & \Delta y_{t-1}^{t+2} \\ \dots & \dots \\ \dots & \dots \\ \Delta U_{t-60}^{t+2} & \Delta y_{t-60}^{t+2} \end{bmatrix},$$

where ΔU_{t-j}^{t+2} and Δy_{t-j}^{t+2} are the vintage $t+2$ values of the change in the unemployment rate and the change in real GDP at time $t-j$, with $j \le t$.[8] We label the vintage for real-time information related to time t and earlier with the time superscript $t+2$ because, as we discussed earlier, we

[7] In the robustness section we also discuss results using expanded and contracted rolling estimation windows.

[8] Since (1) and (2) feature lags of GDP growth, in practice the information set at each point in time will go back slightly further than 60 quarters.

consider GDP observations for which the BEA's third release has already been published. For the most recent quarter *t*, such information is available two quarters later. The object of interest for our analysis is the last estimated Okun's Law error $\hat{\varepsilon}_t^{t+2}$ from each rolling regression, with $t \in [1965:Q2\ ,\ 2012{:}Q2]$. Our exercise mimics a real-time exercise where, at each point in time, the econometrician estimates Okun's Law with the most up-to-date information available. We take the most recent error in that estimated relationship as providing a real-time assessment of how closely the real-time measurement of GDP growth is reflected in movements in the unemployment rate. We then consider the informational content in Okun's Law error as useful for assessing the true strength of real economic activity.

Our real-time data are taken from the "Real-Time Dataset for Macroeconomists" maintained by the Federal Reserve Bank of Philadelphia.[9] The vintage of data in each quarter is the most recent available in the mid-month of that quarter. For example, for observations pertaining to quarter 1998:Q2 and earlier, we take the most up-to-date vintage available as of October 15, 1998. For observations pertaining to quarter 1998:Q3 and earlier, we take the most up-to-date vintage as of February 15, 1999, and so on. This holds true for both the GDP and the unemployment rate data, again with the understanding that, aside from adjustments in the seasonal factors, the unemployment rate series is not subject to revisions while the GDP series is. The monthly unemployment rate series is averaged in each quarter to convert it to a quarterly frequency.

We use OLS to estimate (1), while for the unobserved component specification in (2)–(3) we use a maximum-likelihood-based Kalman filter.[10] The maximum likelihood procedure also estimates the variance σ_u^2 in the law of motion (3) for α_t. In (1) and (2), we use the contemporaneous real-time value of GDP growth and three lags. Figure 1 depicts the series of estimated Okun Law's errors $\left\{\hat{\varepsilon}_t^{t+2}\right\}$ over the rolling window 1965:Q2 to 2012:Q2, using OLS

[9] See: http://www.philadelphiafed.org/research-and-data/real-time-center/real-time-data/data-files/

[10] The maximum likelihood procedure also estimates the variance σ_u^2 in the law of motion (3) for α_t.

and the Kalman filter. The two estimation procedures generate, overall, similar errors, with the error from the unobserved component estimation being somewhat less volatile, as a portion of the variation is being absorbed by the time-varying intercept in (2). One noticeable difference in the pattern of the two errors occurs in the mid-to-late 'seventies, when the simple OLS procedure generates much larger negative errors. It is possible that the Kalman filter is capturing a decline in potential GDP growth over that period that is not detected via OLS, despite the relatively narrow estimation window. In the late 'nineties, a somewhat less striking difference also emerges, which could reflect the fact that the unobserved component method is capturing an increase in potential GDP growth. According to both estimation procedures, in the most recent recession the unemployment rate increased significantly more than would have been predicted by Okun's Law in real time. The errors have since become preponderantly negative over the course of the ensuing recovery. Such a pattern, although not as pronounced, emerges in other, but not all, recession and recovery episodes.

3. Okun's Law Errors and Data Revisions

We now turn to evaluating whether the series of estimated Okun Law errors $\left\{\hat{\varepsilon}_t^{t+2}\right\}$ predicts revisions to GDP. For this purpose we begin by estimating the simple relationship:

$$(5) \qquad \Delta y_t^{t+2+8} - \Delta y_t^{t+2} = \kappa + \delta \hat{\varepsilon}_t^{t+2} + \eta_t ,$$

where Δy_t^{t+2+8} is quarterly GDP growth at time *t* as of the vintage available eight quarters after the release of the vintage that we take as real-time. Consequently, the left-hand side of (5) represents the revision to GDP growth at time *t* two years hence. The series η in (5) is an error term, and κ is a constant parameter. Results from estimating (5) by OLS over the sample period 1965:Q2 to 2010:Q2, using the two alternative measures of Okun's Law error, are reported in

Table 1.[11] In interpreting the results, it should be kept in mind that the series $\left\{\hat{\varepsilon}_t^{t+2}\right\}$ is a generated regressor. In this linear univariate setting the estimated slope will be attenuated towards zero, but so will its estimated standard error.[12] Nonetheless, the estimated coefficient is economically meaningful. It is negative, as expected, in order for the exercise to have a sensible economic interpretation. If movements in the unemployment rate suggest a weaker economy than the real-time signal from GDP does, then the BEA will later revise GDP to reflect slower growth than initially thought. According to estimates, a change in the unemployment rate that is one percentage point greater than predicted by Okun's Law in real time is associated with a downward revision to GDP growth two years hence of roughly 2 percent. The estimated revision is somewhat larger with the Okun's Law errors generated via the unobserved components method, but the portion of the variance of the revision explained by the two approaches is essentially the same.

The fit of the estimated relationship between revisions to GDP growth and errors in Okun's Law is far from tight—also as a consequence of the attenuation bias stemming from the generated regressor—but it is still relevant when placed in the context of the literature on the ability to forecast data revisions (see, for example, Aruoba 2008). As we show later, most of the ability to forecast GDP data revisions from Okun's Law errors and their economic relevance comes from the post-1983 period. Here, we relate our findings to the extant literature by considering the following variant of (5):

$$(6)\ \Delta y_t^{t+2+8} = \kappa + \gamma \Delta y_t^{t+2} + \zeta(\Delta y_t^{t+2} - \Delta y_t^{t+1}) + \sum_{s=1}^{4} \lambda_i Q_t^i + \theta t + \delta \hat{\varepsilon}_t^{t+2} + \phi \Delta U_t^{t+2} + \eta_t .$$

This kind of relationship, without the inclusion of the error from the real-time Okun's Law, was

[11] The sample ends in 2010:Q2, as this is the last observation for which Δy_t^{t+2+8} was available at the time of writing.

[12] See Murphy and Topel (1985). The correction proposed by Murphy and Topel is not immediate in this setting, as each element of the series $\left\{\varepsilon_t^{t+2}\right\}$ is obtained from a different conditioning set of information.

considered in Aruoba (2008), when illustrating the predictability of data revisions.[13] The term $\Delta y_t^{t+2} - \Delta y_t^{t+1}$ in (6) is the revision of time t GDP growth based on the new information available at time $t + 2$ relative to the $t + 1$ release. Other explanatory variables include the quarterly dummy variables Q^i, $i = 1,...,4$, and a time trend. The explanatory variables are all known as of time $t + 2$, and from such a perspective the regression can be viewed as an exercise in forecasting revisions to GDP growth. The usefulness of information about the unemployment rate in terms of predicting future revisions to GDP growth was already noted by Aruoba. Here, we show that it is the portion of the change in the unemployment rate that is not explained by Okun's Law in real time, that is, $\hat{\varepsilon}_t^{t+2}$, that helps to predict GDP revisions.

Results from this exercise are shown in Table 2, for the sample period covering 1965:Q2 to 2010:Q2. The first column in the table reports estimation results without the inclusion of the Okun's Law error $\hat{\varepsilon}_t^{t+2}$. These results update and confirm Aruoba's findings on the predictability of GDP revisions, and the usefulness of changes in the unemployment rate to predict such revisions. Columns (2) and (3) in the table include $\hat{\varepsilon}_t^{t+2}$ as an additional regressor, with the error obtained from the two different estimation methods described in the previous section. With the inclusion of the real-time error in Okun's Law, the coefficient on the change in the unemployment rate loses in statistical significance and economic relevance. In contrast, the coefficient on the Okun's Law error term is economically relevant, the generated regressor bias notwithstanding. Relative to the estimates in Table 1, the coefficient on Okun's Law error obtained via the unobserved components method shows more stability. Overall, these findings support the notion that real-time errors in Okun's Law forecast revisions to GDP not only in univariate regressions, but also when controlling for other factors that previous literature has shown to be relevant for forecasting GDP revisions.

Given that, as already mentioned, GDP revisions are predictable but the extent of predictability is small, it is important to assess the stability of the relationship over time. For this purpose, we

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[13] See equation (4) on page 329 in Aruoba (2008). Aruoba's timing is slightly different, as his considers the real-time vintage $t + 1$ rather than $t + 2$.Also, we include only the time t GDP growth revision, but our results are not affected by considering earlier revisions.

first re-run the simple univariate regression in Table 1 splitting the sample over two subperiods, 1965:Q2 to 1983:Q4, and 1984:Q1 to 2010:Q2. Results from this exercise are reported in Table 3, and they indicate that errors in Okun's Law are useful at forecasting revisions to GDP only in the most recent subsample. Similar results, shown in Table 4, hold for the specification in (6) that controls for other factors affecting the ability to forecast revisions.[14] A comparison of the goodness of fit in this table and in Table 3 also makes clear that in the most recent subsample Okun's Law errors account for most of the predictability in GDP revisions, with other factors in (6) yielding only a marginal contribution.

4. An Application to the 2008–'09 Recession and Subsequent Recovery

We now assess the ability of Okun's Law errors in real time to predict revisions to GDP in the post-2007 period. Okun's Law came under scrutiny over this period, as during the 2008–'09 recession the unemployment rate climbed by more than the amount that real-time readings of GDP growth would have suggested. A similar phenomenon in reverse occurred in the ensuing recovery, as illustrated in Figure 1. One possible interpretation for the failure of Okun's Law during the recession and the ensuing recovery is that during the recession firms became very risk averse and cut employment sharply. This resulted in an increase in output per hour, as more effort was being obtained from the existing workforce. However, the existing workforce could be pushed harder only for a limited amount of time, and as activity recovered firms had to increase hiring so as to bring output per hour back to a more sustainable level, even if advances in real activity were only modest.

An alternative interpretation for this period is that real-time GDP readings underestimated the depth of the recession and subsequently underestimated the strength of the recovery. For this

[14] Rolling regression estimates of (6) with a window of 40 quarters (not reported) also indicate that the real-time error in Okun's Law becomes relevant for predicting GDP revisions as the rolling window moves into the post-1983 period.

purpose, we consider estimates of (5) over the period 1984:Q1 to 2007:Q4 with Okun's Law errors estimated via OLS.[15] Since, as we have already discussed, the errors are generated regressors, we also consider instrumental variable estimates of (5). We instrument Okun's Law errors with the contemporaneous percentage change in workweek hours and the lagged first difference in the unemployment rate. These variables may capture some short-run dynamics that are not detected by Okun's Law. Thus, they are correlated with our estimated error in (5). Over the sample period that we consider, a regression of Okun's Law error on these two variables yields an adjusted R^2 of 0.17. At the same time, both the workweek hours and the unemployment rate lack explanatory power for revisions to GDP growth once Okun's Law errors are controlled for. Thus, they are plausible instruments.

Results from this exercise are shown in Panel (a) of Table 5. When instrumented, the estimated effect of Okun's Law error on GDP revisions becomes larger in absolute value, suggesting that the economic relevance of Okun's Law errors may be even greater than the estimates reported in the previous tables. Panel (b) of the table reports actual revisions and the predicted values for the revisions from the estimates in Panel (a) over the post-2007 period. With both estimation procedures, but clearly more so when using instrumental variables, Okun's Law errors in real time explain a nontrivial portion of the downward revisions to GDP growth in 2008 and 2009. At the time of writing, we lack two-year revisions to GDP growth for most of the ongoing recovery. The estimates, however, suggest a potentially large upward revision to GDP growth in 2011 relative to the real-time reading, amounting to roughly 1.4 percentage points when using instrumental variables. These findings do not fully explain the failure of Okun's Law in the most recent recession and recovery. It is indeed possible to show that there continue to be sizable errors in Okun's Law during the recession period even when using the most up-to-date vintage of GDP data. But the results illustrate that at least a portion of the failure of Okun's Law

[15] We report only results based on estimating (5) rather than (6) because in the post-1983 sample, as already illustrated, Okun's Law errors account for most of the predictability of GDP revisions, with the other regressors in (6) playing only a minor role.

to account in real-time for movements in the unemployment rate, given real-time readings of GDP growth, can be subsequently explained away by revisions to GDP growth.

5. Robustness

We discuss here the robustness of our results along some relevant dimensions. First, we show that the findings are not sensitive to the date of the revision. In our benchmark estimates reported in Section 3, we examined the revision to GDP growth two years after the real-time reading. Table 6 shows estimation results for (5) when Δy_t^{t+2+8} is replaced by Δy_t^{t+2+12} or by Δy_t^{f}, where the superscript f denotes the most recent vintage of GDP data available as of 2012:Q4. In other words, we compare results when considering the GDP revision three years after the real-time reading, and when considering the GDP revision to the real-time data from today's vantage point. In order to compare results, we use the same sample of observations, which spans the period 1984:Q1 to 2009:Q2. We start the sample in 1984 because, as shown earlier, there is more evidence of predictability of revisions to GDP growth from Okun's Law since then.[16] As for the benchmark findings in Table (2), we report results based on OLS estimates and unobserved components estimates, respectively, of $\left\{\hat{\varepsilon}_t^{t+2}\right\}$. The results illustrate that the point estimates are, in absolute value, even larger when considering the three-year and the current-vintage revisions. Another recurring pattern is that the economic relevance of the estimates increases when Okun's Law error is obtained via the unobserved components method in (2) and (3).[17]

We now consider the ability to forecast GDP revisions from real-time Okun's Law errors when Okun's Law takes the following form:

$$(7) \qquad \Delta U_t = \alpha + \beta(L)(\Delta y_t - \Delta y_t^{POT}) + \varepsilon_t .$$

[16] The results, however, are qualitatively similar when considering the sample starting in 1965:Q2.

[17] When estimating (6) instead of (5), results (not reported but available upon request) are qualitatively very similar to the ones reported in Table 6.

In the first-difference specification (7), GDP growth is expressed as a deviation from a time-varying estimate of potential GDP growth, Δy^{POT}. We estimate potential GDP growth in real time by filtering the available real-time GDP data at each point in time via a Hodrick-Prescott filter. We show results based on assigning a value to the filter's smoothing parameter equal to 1600, but extracting a more rigid trend would not alter the findings. Panel (a) of Table 7 reports estimation results for equation (5), where the errors $\left\{\hat{\varepsilon}_t^{t+2}\right\}$ are estimated from (1′) in real time. The first column considers the full sample 1965:Q2 to 2010:Q2, while the second column looks only at the post-1983 period. Overall, the estimates are similar to the ones reported in Tables 1 and 3, although somewhat smaller in absolute value. As before, each element in $\left\{\hat{\varepsilon}_t^{t+2}\right\}$ is the last observation from a rolling regression of (7) in real time with a window of 60 quarters. With an explicit estimate for time-varying potential GDP growth, there is less need for using a rolling estimation window, although the adjustment of the unemployment rate to the output dynamics, captured by $\beta(L)$ in (7) could still change over time. Results (not shown) for the post-1983 period based on an expanding window with fixed starting point in 1966:Q1 remain very similar to the findings based on the rolling estimation window. We note here also that the results reported so far are not affected by modifying the dynamics in Okun's Law (1), (2), or (7) to include lags of the change in the unemployment rate.[18]

Similar results hold when, instead of a first-difference specification, we consider an Okun's Law relationship in levels, whose dynamics are given by:

(8) $$U_t - U_t^* = \beta_1(y_t - y_t^{POT}) + \beta_2\Delta(y_t - y_t^{POT}) + \rho_1(U_{t-1} - U_{t-1}^*) + \rho_1\Delta(U_{t-1} - U_{t-1}^*) + \varepsilon_t,$$

[18] Knotek (2007) and Owyang and Sekhposyan (2012), among others, consider first-difference versions of Okun's Law with lags of the change in the unemployment rate as an additional explanatory variable. Results based on this type of specification are available upon request.

with U^* denoting the equilibrium unemployment rate. The explanatory variables other than the contemporaneous output gap are included to better account for the fact that the adjustment of the unemployment rate gap to the output gap does not occur immediately. [19] Indeed, at a quarterly frequency the estimated coefficient on the lagged unemployment rate gap in (8), ρ_1, is typically above 0.9. We use Hodrick-Prescott filters on real-time data to obtain a real-time estimate of the equilibrium unemployment rate and of the level of potential GDP. This allows us to compute estimates of the unemployment rate gap and of the output gap in real time, which we then use to obtain estimates of $\left\{\hat{\varepsilon}_t^{t+2}\right\}$ from (8). As before, results are not especially sensitive to the chosen value for the filter's smoothing parameter. Panel (b) of Table (7) reports estimates of (5) using (8) to obtain Okun's Law errors in real time. The estimates continue to suggest a role for Okun's Law errors in real time when predicting future revisions to GDP growth. They also confirm that extracting trends in real time using the Hodrick-Prescott filters lowers the estimated slope δ in (5), in absolute value, relative to the unobserved components method (see Tables 1 and 3). Still, the explanatory power for GDP revisions is essentially the same under these different methods.[20] When considering (6) instead of (5), results continue to be qualitatively similar to the estimates reported in Tables 2 and 4. The fit of these equations, however, tends to be better when using the unobserved components specification in (2) and (3) to extract the real-time errors, rather than relying on Hodrick-Prescott estimates of the trend variables in (7) or (8).[21]

We conclude this section by mentioning that our findings are not sensitive to the width of the rolling window used for estimating Okun's Law in real time. It can be shown that differences in fit for equations (5) or (6) tend to be minor whether the estimation window to obtain the series $\left\{\hat{\varepsilon}_t^{t+2}\right\}$ is shortened to 40 quarters or expanded to 80 quarters. For the post-1983 sample, as

[19] The level Okun's Law relationship (8) is used in the Federal Reserve Board's large-scale econometric model of the U.S. economy (FRB/US). See http://www.petertulip.com/FRBUS_equation_documentation.pdf.

[20] Real-time errors obtained from (7) or (8) are more volatile than the corresponding errors estimated from the unobserved components specification in (2) and (3).

[21] Results are available upon request.

already indicated, having an expanding window with a fixed starting point does not materially alter the results.

6. Okun's Law Errors and GDP Forecasts

We have shown so far that movements in the unemployment rate not accounted for by real-time changes in GDP have predictive power for future revisions to GDP growth. For example, if the unemployment rate is increasing by more than the amount that Okun's Law would have predicted in real time, then revised GDP data will later show slower GDP growth than initially reported. This means that errors in Okun's Law contain information about the true underlying state of the economy as measured by GDP that is not embedded in the real-time GDP readings. It is then natural to ask whether the information in Okun's Law real-time errors can help to achieve more accurate GDP forecasts. Suppose, to continue the previous example, that the most recent Okun's Law errors in real time have been positive and thus are signaling less strength in GDP than the real-time data indicate. Given that economic activity typically evolves with some inertia, such information could be useful to predict near-term developments in GDP.

It is possible that forecasters take into account discrepancies between the unemployment rate and real-time GDP readings through the lens of Okun's Law when projecting future activity. A higher unemployment rate than predicted by Okun's Law in real time could, other things being equal, make forecasters more pessimistic about their assessment of the near-term prospects of the economy. To ascertain whether this is indeed the case, we estimate the following relationship:

$$(9) \qquad \Delta y_{t+i,t-1}^{t+8+i} - \Delta y_{t+i,t-1}^{SPF,t} = a_0 + a_1 \Delta y_{t+i,t-1}^{SPF,t} + b(L)\hat{\varepsilon}_{t-2}^{t} + e_t ,$$

where $\Delta y_{t+i,t-1}^{SPF,t}$ denotes the Survey of Professional Forecasters' median value of projected GDP growth from $t-1$ to $t+i$, with $i \geq 0$. The superscript t indicates that this median forecast is as of time t. The variable $\Delta y_{t+i,t-1}^{t+8+i}$ denotes the actual value of GDP growth over the same period that is being forecasted, with the superscript $t + 8 + i$ indicating that we take as actual the GDP vintage prevailing $8 + i$ quarters after the date of the forecast. The left-hand side of (9) is thus the forecast error, which is regressed on a constant, the Survey of Professional Forecasters' median forecast itself, and lags of Okun's Law real-time error. Since the real-time information that we use at any given quarter t to back out Okun's Law errors goes up to $t - 2$, we include in (9) only real-time Okun's Law errors dated $t - 2$ and earlier.

Results from estimating (9) over the period from 1984:Q1 to the present are reported in Table 8. We analyze the most recent sample only, as this is the time when Okun's Law errors appear to have information about the true pace of GDP growth. In the table, we report estimates of (9) for $i = 0, 1,$ and 3. In other words, we take the dependent variable to be in turn the current-quarter GDP growth forecast error, the current and next quarter GDP growth forecast error, and the current and next three quarters GDP growth forecast error. Median GDP growth forecasts from the Survey of Professional Forecasters are taken from the "Real-Time Dataset for Macroeconomists" maintained by the Federal Reserve Bank of Philadelphia. Standard errors for the estimates in the table are corrected to account for a moving average error term when $i > 0$. The Okun's Law errors are derived from estimating (1) in real time, as described in Section 2. We include three lags of Okun's Law error in equation (9). The results in the table show that the estimated sum of Okun's Law errors is negative, in accordance with the notion that GDP growth forecasts from the Survey of Professional Forecasters tend to be overstated (understated) when real-time Okun's Law errors are suggesting weaker (stronger) growth than the real-time signal from GDP. While these results illustrate that the Survey of Professional Forecasters' GDP forecast could be improved by incorporating the information in real-time Okun's Law errors, the estimated fit for equation (9) also illustrates that the improvement in the forecast would be small.

7. Conclusions

We have shown that real-time errors in Okun's Law have predictive power for future revisions to GDP growth. A positive (negative) error signals revisions to GDP growth to show less (more) growth than the BEA is initially estimating in real time. Some of our results for the post-1983 period imply that a one percentage point positive (negative) error in Okun's law translates into a downward (upward) revision to GDP growth in that quarter of 4 percent at an annual rate. On a yearly basis this implies that the estimated slope between Okun's Law real-time error and revisions to GDP growth is *-1*, which is about half of the slope typically associated with Okun's Law. Such an effect is economically relevant, and we have shown some of its economic implications for the most recent recession and recovery—a period when real-time errors in Okun's Law have been large by historical standards.

This is not the first paper to show that the unemployment rate contains information about future GDP revisions. However, it is the first to show that it is the portion of the movement in the unemployment rate that is not accounted for by real-time Okun's Law, that is, by movements in GDP as measured in real time, that signals future revisions to GDP. The paper also illustrates that this information can be used to improve forecasts of GDP growth in the near term.

We adopted a reduced-form approach to establish a relationship between Okun's Law errors in real time and revisions to GDP. Future research could use more structure to try to tease out the portion of Okun's Law error in real time that relates to GDP revisions versus the portion that signals a change in potential GDP (and/or in the natural rate of unemployment). A real-time error in Okun's Law contains information about both of these factors. In this paper we have stressed the information in the error that pertains to GDP revisions. This aspect tends to be neglected in policy discussions, where Okun's Law errors in real time are typically viewed only through the lens of the information that they may convey about changes in potential GDP. But in practice both of these aspects are likely to matter, and future research should explore avenues to analyze them jointly.

Future research could also refine inquiry into the source of Okun's Law errors in real time that provide explanatory power for future GDP revisions. Suppose, for example, that the unemployment rate drops by more than expected given the evolution of real-time GDP. For subsequent GDP revisions, it could in principle matter whether the decline in the unemployment rate occurs via an increase in the employment to population ratio or via a drop in labor force participation. Such information could also be valuable to the statistical agency in order to understand how to improve estimates of GDP in real time.

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Figure 1. Okun's Law Errors in Real Time

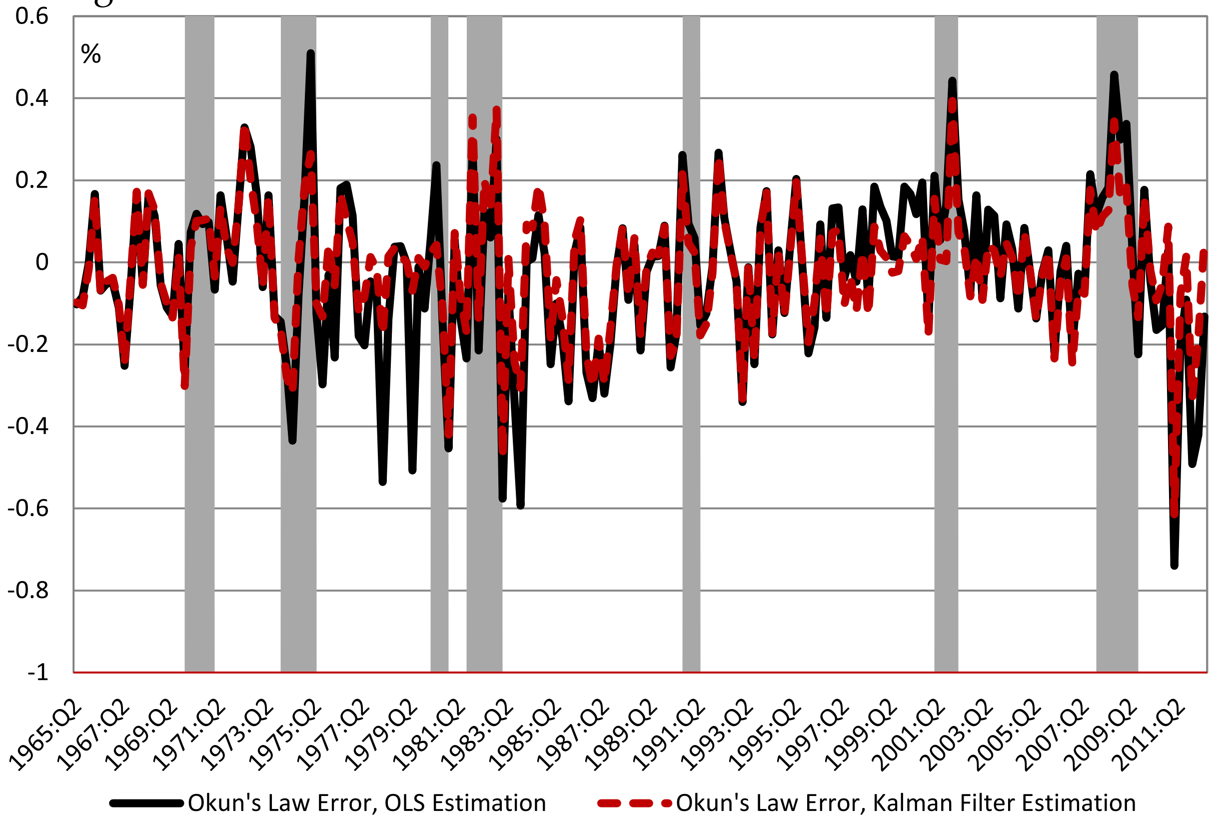

Table 1. GDP Revisions and Errors in Okun's Law

Eq. (5): $\Delta y_t^{t+2+8} - \Delta y_t^{t+2} = \kappa + \delta \hat{\varepsilon}_t^{t+2} + \eta_t$

	$\hat{\varepsilon}_t^{t+2}$ from OLS	$\hat{\varepsilon}_t^{t+2}$ from KF
δ	**-1.934**	**-2.432**
	(0.5126)	(0.6494)
N	*181*	*181*
$\bar{R}^2$	0.068	0.067

Note: Standard errors in parentheses. Boldface denotes significance at the 5 percent significance level or lower.

Table 2. GDP Revisions and Errors in Okun's Law, Other Controls Included

Eq. (6): $\Delta y_t^{t+2+8} - \Delta y_t^{t+2} = \kappa + \gamma \Delta y_t^{t+2} + \zeta (\Delta y_t^{t+2} - \Delta y_t^{t+1}) + \sum_{s=1}^{4} \lambda_i Q_t^i + \theta t + \delta \hat{\varepsilon}_t^{t+2} + \phi \Delta U_t^{t+2} + \eta_t$

	No $\hat{\varepsilon}_t^{t+2}$	$\hat{\varepsilon}_t^{t+2}$ from OLS	$\hat{\varepsilon}_t^{t+2}$ from KF
γ	**-0.214**	**-0.154**	**-0.127**
	(0.044)	(0.053)	(0.048)
ζ	**0.389**	**0.374**	**0.416**
	(0.132)	(0.131)	(0.130)
δ		**-1.479**	**-2.596**
		(0.729)	(0.802)
φ	**-1.441**	-0.563	-0.197
	(0.399)	(0.586)	(0.510)
N	*180*	*180*	*180*
$\bar{R}^2$	0.163	0.178	0.190

Note: Standard errors in parentheses. Boldface denotes significance at the 5 percent significance level or lower.

Table 3. GDP Revisions and Errors in Okun's Law: Subsample Analysis

Eq. (5): $\Delta y_t^{t+2+8} - \Delta y_t^{t+2} = \kappa + \delta \hat{\varepsilon}_t^{t+2} + \eta_t$

	1965:Q2 to 1983:Q4		1984:Q1 to 2010:Q2	
	$\hat{\varepsilon}_t^{t+2}$ from OLS	$\hat{\varepsilon}_t^{t+2}$ from KF	$\hat{\varepsilon}_t^{t+2}$ from OLS	$\hat{\varepsilon}_t^{t+2}$ from KF
δ	-0.703 (0.741)	-1.179 (0.964)	**-3.186** (0.708)	**-3.731** (0.855)
N	*75*	*75*	*106*	*106*
$\overline{R}^2$	0.01	0.01	0.154	0.146

Note: Standard errors in parentheses. Boldface denotes significance at the 5 percent significance level or lower.

Table 4. GDP Revisions and Errors in Okun's Law, Other Controls Included: Subsample Analysis

Eq. (6): $\Delta y_t^{t+2+8} - \Delta y_t^{t+2} = \kappa + \gamma \Delta y_t^{t+2} + \zeta(\Delta y_t^{t+2} - \Delta y_t^{t+1}) + \sum_{s=1}^{4} \lambda_i Q_t^i + \theta t + \delta \hat{\varepsilon}_t^{t+2} + \phi \Delta U_t^{t+2} + \eta_t$

	1965:Q2 to 1983:Q4		1984:Q1 to 2010:Q2	
	$\hat{\varepsilon}_t^{t+2}$ from OLS	$\hat{\varepsilon}_t^{t+2}$ from KF	$\hat{\varepsilon}_t^{t+2}$ from OLS	$\hat{\varepsilon}_t^{t+2}$ from KF
γ	**-0.185** (0.070)	**-0.153** (0.059)	-0.111 (0.089)	-0.109 (0.085)
ζ	**0.699** (0.198)	**0.709** (0.195)	0.184 (0.185)	0.195 (0.185)
δ	-0.250 (1.042)	-1.213 (1.138)	**-2.910** (1.128)	**-3.393** (1.186)
φ	-0.802 (0.860)	-0.358 (0.694)	-0.057 (0.881)	-0.130 (0.781)
N	*74*	*74*	*106*	*106*
$\overline{R}^2$	0.204	0.210	0.197	0.204

Note: Standard errors in parentheses. Boldface denotes significance at the 5 percent significance level or lower.

Table 5. GDP Revisions and Errors in Okun's Law: Out-of-Sample Analysis for Post-2007 Period

Eq. (5): $\Delta y_t^{t+2+8} - \Delta y_t^{t+2} = \kappa + \delta \hat{\varepsilon}_t^{t+2} + \eta_t$

(a) 1984:Q1 to 2007:Q4

	OLS Estimation	IV Estimation
δ	**-2.904**	**-3.983**
	(0.800)	(1.865)
N	*96*	*96*
R^2	0.113	0.113

(b) Out-of-Sample Predicted Real GDP Growth Revisions

	Actual	Predicted from OLS	Predicted from IV
2008:Q1	-1.59	-0.58	-0.76
2008:Q2	-2.23	-0.65	-0.85
2008:Q3	-3.48	-1.45	-1.95
2008:Q4	-0.42	-0.99	-1.32
Average for 2008	***-1.93***	***-0.92***	***-1.22***
2009:Q1	-0.23	-1.10	-1.47
2009:Q2	0.04	-0.20	-0.23
2009:Q3	-0.54	0.52	0.76
2009:Q4	-1.75	-0.63	-0.83
Average for 2009	***-0.62***	***-0.35***	***-0.44***
2010:Q1	-1.39	0.00	0.04
2010:Q2	0.52	0.35	0.52
2010:Q3	.	0.32	0.47
2010:Q4	.	-0.20	-0.26
Average for 2010	***.***	***0.11***	***0.20***
2011:Q1	.	2.02	2.81
2011:Q2	.	0.44	0.65
2011:Q3	.	0.13	0.23
2011:Q4	.	1.30	1.82
Average for 2011	***.***	***0.97***	***1.38***
2012:Q1	.	1.09	1.54
2012:Q1	.	0.25	0.39

Note: Standard errors in parentheses. Boldface denotes significance at the 5 percent significance level or lower.

Table 6. GDP Revisions and Errors in Okun's Law: Alternative Revision Dates, Post-1983 Sample

Eq. (5): $\Delta y_t^{t+2+12} - \Delta y_t^{t+2} = \kappa + \delta \hat{\varepsilon}_t^{t+2} + \eta_t$ or $\Delta y_t^f - \Delta y_t^{t+2} = \kappa + \delta \hat{\varepsilon}_t^{t+2} + \eta_t$

	Revision is: $\Delta y_t^{t+2+8} - \Delta y_t^{t+2}$		Revision is: $\Delta y_t^{t+2+12} - \Delta y_t^{t+2}$		Revision is: $\Delta y_t^f - \Delta y_t^{t+2}$	
	$\hat{\varepsilon}_t^{t+2}$ OLS	$\hat{\varepsilon}_t^{t+2}$ KF	$\hat{\varepsilon}_t^{t+2}$ OLS	$\hat{\varepsilon}_t^{t+2}$ KF	$\hat{\varepsilon}_t^{t+2}$ OLS	$\hat{\varepsilon}_t^{t+2}$ KF
δ	**-3.231**	**-3.685**	**-3.536**	**-4.007**	**-3.881**	**-4.173**
	(0.727)	(0.874)	(0.826)	(0.994)	(0.903)	(1.095)
N	*102*	*102*	*102*	*102*	*102*	*102*
R^2	0.156	0.142	0.146	0.131	0.147	0.118

Note: Standard errors in parentheses. Boldface denotes significance at the 5 percent significance level or lower.

Table 7. GDP Revisions and Errors in Okun's Law: Alternative Okun's Law Specifications

Eq. (5): $\Delta y_t^{t+2+8} - \Delta y_t^{t+2} = \kappa + \delta \hat{\varepsilon}_t^{t+2} + \eta_t$

(a) Okuns' Law is: $\Delta U_t = \alpha + \beta(L)(\Delta y_t - \Delta y_t^{POT}) + \varepsilon_t$

	1965:Q2 to 2010:Q2	1984:Q1 to 2010:Q2
δ	**-1.656**	**-2.691**
	(0.440)	(0.606)
N	*181*	*106*
R^2	0.068	0.151

(b) Okuns' Law is:

$$U_t - U_t^* = \beta_1(y_t - y_t^{POT}) + \beta_2\Delta(y_t - y_t^{POT}) + \rho_1(U_{t-1} - U_{t-1}^*) + \rho_1\Delta(U_{t-1} - U_{t-1}^*) + \varepsilon_t$$

	1965:Q2 to 2010:Q2	1984:Q1 to 2010:Q2
δ	**-1.819**	**-2.511**
	(0.427)	(0.577)
N	*181*	*106*
R^2	0.086	0.145

Note: Standard errors in parentheses. Boldface denotes significance at the 5 percent significance level or lower.

Table 8. Survey of Professional Forecasters' GDP Forecast Errors and Errors in Okun's Law Eq. (9): $\Delta y_{t+i,t-1}^{t+8+i} - \Delta y_{t+i,t-1}^{SPF,t} = a_0 + a_1 \Delta y_{t+i,t-1}^{SPF,t} + b(L)\hat{\varepsilon}_{t-2}^{t} + e_t$, Okun's Law Error Estimated from (1) via OLS

	$i=0$	$i=1$	$i=3$
a_0	-0.790	-1.017	-0.652
	(0.541)	(1.011)	(1.375)
a_1	0.271	0.320	0.131
	(0.194)	(0.337)	(0.428)
$b(1)$	**-4.267**	**-4.045**	**-3.561**
	(1.704)	(1.969)	(1.766)
N	*106*	*105*	*103*
R^2	0.086	0.077	0.042

Note: Standard errors in parentheses. Boldface denotes significance at the 5 percent significance level or lower.