Downward Nominal Wage Rigidities Bend the Phillips Curve

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We show that the existence of downward nominal wage rigidities bends the short-run wage Phillips curve. We introduce a model of monetary policy with downward nominal wage rigidities and show that both the slope and curvature of the Phillips curve depend on the level of inflation and the extent of downward nominal wage rigidities. This is true for the both the long-run and the short-run Phillips curve. Comparing simulation results from the model with data on U.S. wage changes since the onset of the Great Recession, we show that downward nominal wage rigidities have likely played a role in shaping the dynamics of unemployment and wage growth from 2006 through 2012.

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1. Introduction

Individual-level data on wage changes as well as survey-based evidence on wage setting show that nominal cuts to pay are rare.[2] This is interpreted as evidence that wages are downwardly rigid. Tobin (1972) argued that the existence of downward nominal wage rigidities induces a long-run, or steady state, trade-off between inflation and unemployment. Subsequent studies have focused on understanding the extent of the trade-off and how it relates to the central bank's inflation target and the natural rate of unemployment.[3,4]

Two key findings have emerged from this literature. First, for downward nominal wage rigidities to have a meaningful impact on unemployment in standard macroeconomic models, the models need to include heterogeneous shocks to the labor supply or productivity of workers. Such shocks are necessary to make nominal wage cuts desirable when the economy is in steady state (Benigno and Ricci, 2011).

Second, when such idiosyncratic shocks are included, the unemployment-inflation trade-off is nearly vertical at high inflation and flattens at low inflation, implying progressively larger output costs of reducing inflation (Akerlof, Dickens, and Perry, 1996, Benigno and Ricci, 2011). That said, even at low inflation, the long-run trade-off is not very big, at least for levels of downward wage rigidities commonly observed in the U.S.

In this paper we extend this literature and consider to what extent downward nominal wage rigidities affect the short-run Phillips curve.[5] Our hypothesis that downward nominal wage rigidities affect the shape of the short-run unemployment-inflation trade-off is as old as the Phillips curve itself. Phillips (1958) conjectured that part of the curvature in the historical relationship between money wage growth and unemployment that he documented for the U.K. was because "…workers

[2] For individual-level evidence for the U.S. see Akerlof, Dickens, and Perry (1996), Kahn (1997), Card and Hyslop (1997), Altonji and Devereux (2000), Lebow, Saks, and Wilson (2003), Elsby (2009), Daly, Hobijn, and Lucking (2012). Individual-level evidence for a large number of countries is in Dickens et al. (2007). Kahnemann, Knetsch, and Thaler (1986) and Bewley (1995, 1999) provide extensive anecdotal and survey evidence on downward nominal wage rigidities in the U.S. Bonin and Radowski (2011) present survey results from Germany.

[3] Examples of such studies are Akerlof, Dickens, and Perry (1996), Kim and Ruge-Murcia (2008), Fagan and Messina (2009), Benigno and Ricci (2011), Coibion et al. (2012).

[4] Ball (1994) provides cross-country evidence on how the shape of the output-inflation trade-off depends on wage flexibility.

[5] We define long-run as the steady state and short-run as deviations from the steady state.

are reluctant to offer their services at less than prevailing rates when the demand for labour is low and unemployment is high so that wage rates fall only very slowly."[6]

In this paper we confirm Phillips' (1958) original conjecture and show how downward nominal wage rigidities bend the (wage) Phillips curve. We illustrate this in the context of a simple discrete-time version of the model introduced by Benigno and Ricci (2011). Our model is deliberately simplified to make it possible to solve its full non-linear transitional dynamics, taking into account the evolution of the distribution of wages along the equilibrium path.

Our model simulations show that downward nominal wage rigidities bend the Phillips curve in two ways. First, during deep recessions the rigidities become more binding and more of the labor market adjustment happens through unemployment rather than through wages. That is, the deeper the recession the higher the short-run sacrifice ratio between unemployment and inflation. Second, deep recessions, in which downward nominal wage rigidities are particularly binding, result in substantial pent up wage deflation. This leads to a simultaneous deceleration of wage inflation and a decline in the unemployment rate during the ensuing recovery period.

Although our model simulations are not intended to match the data, they are remarkably consistent with data on both individual level wage changes and the dynamics of unemployment and wage growth observed for the U.S. since the 2007 recession. To show this, we update the results from Card and Hyslop (1997) and compare the changes in the histogram of log nominal wage changes between before the recession in 2006 and after the recession in 2011 with the changes in the distribution of log nominal wage changes implied by our model along its transition path.

In addition, we construct a U.S. wage Phillips curve for 1986-2012 and show that, since 2007, (*i*) unemployment first rose significantly while wage growth remained flat and (*ii*) subsequently fell while wage growth decelerated. This is consistent with the two ways in which downward nominal wage rigidities bend the Phillips curve following a large negative demand shock in our model.

We interpret these results as evidence that downward nominal wage rigidities have been an important force shaping the dynamics of unemployment, wage growth, and inflation over the period from 2006-2012.

[6] Samuelson and Solow (1960) replicate Phillips' results for the U.S. and find similar curvature in the U.S. (wage) Phillips curve. They, instead, emphasize that the curvature might reflect an increase in the natural rate of unemployment rather than a bending due to downward nominal wage rigidities.

The remainder of this paper is structured as follows. In Section 2 we present an update of previous evidence on downward nominal wage rigidities in the U.S. and construct the U.S. wage Phillips curve for 1986-2012. In Section 3 we describe our simplified off-the-shelf model of downward nominal wage rigidities and monetary policy. In Section 4 we show how the steady state of the model generates the same long-run unemployment-inflation trade-off as the more complicated models from which it is derived. In Section 5 we solve the transitional dynamics of the model and show how along the transition path, the shift in the distribution of log wage changes matches that observed in the data. We also illustrate the two ways in which downward nominal wage rigidities bend the Phillips curve again highlighting patterns observed in the data. We conclude in Section 6.

2. Downward nominal wage rigidities and the U.S. wage Phillips curve

Before describing our model we provide new evidence on the importance of downward nominal wage rigidities in the U.S. and on the dynamics of unemployment and wage growth, as captured by the wage Phillips curve. We use this evidence to establish three stylized facts about downward nominal wage rigidities, the dynamics of wage growth, and unemployment, during the 2006-2012 period. We subsequently compare these facts to the properties of the transition path of our model in Section 5.

Distortion of log nominal wage changes

Our first stylized fact is related to the distribution of individual-level wage changes. For our analysis we follow Card and Hyslop (1997) and track 12-month log changes in the nominal wages of individuals using micro data from the Current Population Survey (CPS).[7] To identify whether nominal wage rigidities we, like previous researchers, focus the distribution of wage changes across individuals as displayed in Figure 1. The histogram plots the distribution of wage changes in the

[7] Earlier studies (Akerlof, Dickens, and Perry, 1996, Kahn, 1997, Altonji and Devereux, 2000, and Elsby, 2009) used data from the PSID for this type of analysis. However, the PSID has gone from an annual to a biannual frequency and thus does not allow for the analysis of 12-month wage changes anymore. Another data source that would allow for a similar analysis is the Survey of Income and Program Participation (SIPP). In results not shown we replicated our analysis using the SIPP. The key results were qualitatively similar to those for the CPS although the prevalence of nominal rigidities is higher in the SIPP than in the CPS (see Barattieri, Basu, and Gottschalk (2010) and Gottshalk (2005) for analyses of nominal wage rigidities using the SIPP).

CPS data for 2006 and 2011, reflecting the distributions in two different points in the business cycle. We take the distribution of wage changes in 2006 to represent the steady state distribution and the distribution in 2011 to represent the additional distortions that arise in business cycle downturns.[8]

As the histograms in Figure 1 show, actual wage changes exhibit considerable discontinuity with a prominent spike at zero. This is true in both 2006 and 2011. Consistent with the view that individuals do not like nominal wage cuts (Kahneman et al. 1986, Bewley 1995,1999), the spike at zero is produced asymmetrically, with most of the mass coming from the left of zero rather than from the right. This suggests that a sizeable fraction of desired negative wage changes are being swept up to zero (Altonji and Devereux 2000; Card and Hyslop 1996; Kahn 1997; Lebow, Saks, Wilson 2003).

When we compare the 2006 histogram with that for 2011, three things stand out. First, the fraction of workers with no wage change increased substantially in 2011 relative to 2006. In 2006 about 12 percent of workers reported zero wage change; in 2011 the share had risen to about 16 percent. This 16 percent is the highest level of the spike at zero for all the 32 years for which we have CPS data. Second, the fraction of workers getting a wage increase declined noticeably over the period and the size of wage increases, conditional on getting one, was substantially lower in 2011 than in 2006. Thus, there is notable compression of wage gains near zero, suggesting that the inability to adjust nominal wages downward may influence the magnitude of wage increases. This is a point made by Elsby (2009). Finally and surprisingly, there is little difference in the fraction of workers that get wage cuts between 2006 and 2011.

Time-series path of spike at zero

Another important stylized fact, hinted at in Figure 1, is that the prevalence of zero wage changes varies over the business cycle. This can be seen in Figure 2 which plots the 12-month moving average of the fraction of workers in the CPS data reporting zero nominal wage changes. As the figure shows, there is always a non-trivial fraction of workers receiving zero wage changes in the U.S. economy. This fraction increases around business cycle downturns and has risen to historical

[8] Most studies of downward nominal wage rigidities focus on the wage changes of those who remain in the same job. Since our model does not distinguish between job stayers and job switchers, however, we present the evidence for all workers who earned a wage at the beginning and end of the year.

highs since the 2007 recession. Notably, the prevalence of zero wage changes has remained high during the recovery, coming down only slightly from its 2009 peak, despite notable improvement in the labor market and associated declines in the unemployment rate.

Wage Phillips curve

The puzzling behavior of nominal wage growth relative to the unemployment rate since the 2007 recession is the final stylized fact we seek to understand with our model. The puzzle is easily seen in a plot of the wage Phillips curve (Figure 3). The figure plots the wage Phillips curve for the U.S. using a composite measure of U.S. nominal wage growth and a measure of the unemployment gap for 1986 through 2012. The composite measure of wage growth is the first principal component of the four major wage series for the U.S.[9] The unemployment gap is defined as the percentage point difference between the long-term natural rate of unemployment estimated by the Congressional Budget Office and the civilian unemployment rate. For ease of comparison we identify two separate periods: the first period covers 1986 until the start of the 2007 recession, and the second period includes the recession to the end of 2012.

Figure 3 shows the expected (Phillips, 1958, Solow and Samuelson, 1960, Galí, 2011) relationship between wage growth and labor market slack; higher slack means slower wage growth, lower slack means faster wage growth. Notice the two distinct non-linearities in the curve that have arisen since the onset of the 2007 recession.

First, as slack increased wage growth slowed but not nearly as much as a linear model would predict. Starting in 2008 wage growth leveled off at around 2 percent while unemployment continued to rise. Second, during the recovery from the recession the reverse has occurred. As unemployment came down, wage growth continued to decelerate, dropping below 2 percent. As we will show in our model, these dynamics of wage growth and unemployment between 2007 and 2012 are consistent with downward nominal wage rigidities preventing wage adjustments so as to bend the short-run Phillips curve.

[9] Appendix B contains a brief description of how this composite measure is constructed.

3. Model of monetary policy with DNWR

In this section we introduce a simplified version of a model with downward nominal wage rigidities as the sole source of nominal rigidities. This model is a simplified discrete-time version of Benigno and Ricci (2011). Our simplifications make it possible to solve the non-linear transitional dynamics of the model, without sacrificing the key long-run implications identified in more complicated models.

Firms

Firms operate in a perfectly competitive goods market and produce output using the production function

$$Y_t = L_t. \tag{1}$$

The labor input, L_t, consists of a measure one of different types of labor. Each type is indexed by i and its type-specific input by L_{it}. Each type of labor is hired at the nominal wage rate, W_{it}. The labor aggregate, L_t, is a Dixit-Stiglitz (1977) aggregate over the different types of labor inputs and is of the form.

$$L_t = \left[\int_0^1 L_{it}^{\frac{\eta-1}{\eta}} di\right]^{\frac{\eta}{\eta-1}}, \text{ where } \eta > 1. \tag{2}$$

Cost minimization by the firms results in the labor demand function

$$L_{it} = \left(\frac{W_t}{W_{it}}\right)^{\eta} L_t, \tag{3}$$

Where the nominal wage aggregate, which for the production function (1) is the unit production cost, is of the form

$$W_t = \left[\int_0^1 \left(\frac{1}{W_{it}}\right)^{\eta-1} di\right]^{-\frac{1}{\eta-1}}. \tag{4}$$

Because firms operate in a perfectly competitive goods market, the equilibrium price level, P_t, equals the unit production cost, W_t. Thus, equilibrium in the goods market implies that real wage aggregate in this economy is constant and equal to one. That is,

$$1 = \frac{W_t}{P_t} = \left[\int_0^1 \left(\frac{P_t}{W_{it}}\right)^{\eta-1} di\right]^{-\frac{1}{\eta-1}} = \left[\int_0^1 \left(\frac{1}{w_{it}}\right)^{\eta-1} di\right]^{-\frac{1}{\eta-1}}. \tag{5}$$

Here, $w_{it} = W_{it}/P_t$ is the real wage rate of labor of type i at time t.

Households

We follow Fagan and Messina (2009) and Benigno and Ricci (2011) and model downward nominal wage rigidities based on the staggered wage setting model of Erceg, Henderson, and Levin (2000). That is, we consider a representative household in which the members share their consumption risk and individually set the wage they charge for their labor services. They then supply as much labor as is demanded by the firms.

The representative household chooses its path of consumption, Y_t,[10] and wages and labor supply $\{W_{it}, L_{it}\}_{i=0,t=0}^{1,\infty}$ to maximize the present discounted of utility

$$\sum_{t=0}^{\infty} \beta^t e^{-\sum_{s=0}^{t-1} D_s} \left\{\ln Y_t - \frac{\gamma}{\gamma+1}\int_0^1 Z_{it} L_{it}^{\frac{\gamma+1}{\gamma}} di\right\} \text{ where } \gamma > 0. \tag{6}$$

Here Z_{it} is the disutility from working for household member i, which varies over time; D_s is a demand "shock" that affects the household's discount factor.[11] The household maximizes this objective function subject to five constraints.

The first is the budget constraint. Defining the level of nominal assets held by the household at the end of period t as A_t and the nominal interest paid during period t on the assets held at the end of period $t-1$ as i_{t-1}, we write this constraint as

$$A_t = (1 + i_{t-1})A_{t-1} + \int_0^1 W_{it} L_{it} di, \tag{7}$$

The second constraint is that each of the members of the household sets their wage taking as given the labor demand function in equation (3). Given that in equilibrium the aggregate real wage equals one, this labor demand function can be written in terms of the real wage charged by household member i as

[10] The absence of capital goods in the economy also means that, in equilibrium, output, Y_t, equals the level of consumption of the representative household, C_t. Throughout this paper, we substitute in this equilibrium condition and use Y_t.

[11] We put "shock" here in quotation marks because we assume that the household knows the path $\{D_t\}_{t=0}^{\infty}$. In addition, we assume that D_t in the long run is zero.

$$L_{it} = \left(\frac{1}{w_{it}}\right)^{\eta} L_t. \tag{8}$$

The third constraint is that, because every household is infinitesimally small, it takes as given the path of the aggregate wage, W_t, and labor input, L_t, the price level, P_t, and the nominal interest rate, i_t.

The fourth constraint is that the household takes the path of its discount rate D_t shock, as well as the stochastic process that drives the member-specific disutility of working, Z_{it}, as given. In particular, we assume that in each period a member's disutility from working is drawn from a log-normal distribution, where $ln(Z) \sim N\left(-\frac{\sigma^2}{2}, \sigma\right)$ such that $E[Z] = 1$.[12] This means that there is no persistence in the disutility shocks, Z_{it}. We make this assumption to keep the equilibrium dynamics of the model tractable and solvable.

The final constraint is that of downward nominal wage rigidities. We model these rigidities along the lines of the time-dependent price stickiness of Calvo (1983) commonly used in models with price rigidities. In particular, we assume that, every period, a randomly selected fraction $\lambda \in [0,1)$ of household members is not allowed to change their nominal wage downward and have to set their wage subject to the constraint $W_{it} \geq W_{it-1}$.

Optimal savings decision

Defining the inflation rate as π_t, the household's saving decision yields the Euler equation

$$\frac{1}{Y_t} = \beta e^{-D_t}(1 + r_t)\frac{1}{Y_{t+1}}, \tag{9}$$

where r_t is the real interest rate and is given by $r_t = (1 + i_t)/(1 + \pi_{t+1}) - 1$. Since our model economy does not exhibit any aggregate uncertainty, there is no uncertainty about future inflation, π_{t+1}.

[12] Our setup with disutility shocks follows Benigno and Ricci (2011). Alternatively, one can introduce household member specific productivity shocks, as in Fagan and Messina (2009). Both these approaches lead to similar equilibrium dynamics. The latter, however, results in a more complicated expression for the aggregates in equilibrium. To keep our illustrative model as simple as possible, we chose the former. Some technical details about the log-normal distribution, which we use for our derivations, are described in Appendix A.

Wage setting under flexible wages: $\boldsymbol{\lambda = 0}$.

Similar to Erceg, Henderson, and Levin (2000) and Benigno and Ricci (2011), each household member chooses the sequence of real wages, w_{it}, to maximize the expected present value of the difference between the utility value of the generated labor income and the disutility from providing the labor services demanded by firms.

As we show in Appendix A, this reduces to the household choosing the path of wages of member i to maximize

$$\mathrm{E}_t\left[\sum_{t=0}^{\infty}\beta^t e^{-\sum_{s=0}^{t-1}D_s}\Omega(Z_{it};w_{it},L_t)\right], \tag{10}$$

where

$$\Omega(Z_{it};w_{it},L_t) = w_{it}^{1-\eta} - \frac{\gamma}{\gamma+1}Z_{it}\left[w_{it}^{-\eta}L_t\right]^{\frac{\gamma}{1+\gamma}}. \tag{11}$$

Note that the objective here is an *expected* present discounted value because, even though the household as a whole does not face any aggregate uncertainty, each individual member of the household faces uncertainty about the future path of the idiosyncratic shocks to its disutility from working, Z_{it}.

When wages can be flexibly adjusted then, in every period, the real wage w_{it} is chosen to maximize $\Omega(Z_{it};w_{it},L_t)$. The resulting optimal real wage schedule is

$$\widehat{w}(Z_{it};L_t) = \widehat{w}_{it} = \left(\frac{\eta}{\eta-1}\right)^{\frac{\gamma}{\gamma+\eta}} Z_{it}^{\frac{\gamma}{\gamma+\eta}} L_t^{\frac{1+\gamma}{\gamma+\eta}}. \tag{12}$$

This turns out to be an important benchmark to compare the wage-setting under downward nominal wage rigidities to.

Wage setting under downward nominal wage rigidities: $\boldsymbol{\lambda > 0}$.

When household members' wage setting choices are subject to downward nominal wage rigidities, their decisions depend on L_t and Z_{it}, as well as the future path of these aggregates, the future path of inflation, π_t, and the demand shock, D_t,. Two things produce this dependency. First, each of these variables influences the likelihood that a household member will want to reduce the current wage at some point in the future. Second, together these variables determine the present discounted value associated with the potential inability to cut wages in future periods.

In order to formalize the optimal wage setting problem under downward nominal wage rigidities, we define the value of the objective (10) for a household member who gets paid a real wage w at the beginning of period t as $V_t(w)$. We then hit this worker with a disutility shock, i.e. draws a value of Z_{it}, after which the worker decides whether and how to change the charged real wage, w_{it}.

With probability $(1-\lambda)$ the worker is able to choose any non-negative wage desired in period t. With probability λ, the worker is restricted from choosing a real wage smaller than w. In other words, the worker can set a real wage w_{it} such that $w_{it} \geq w$. Because of inflation, once the worker has chosen a real wage rate w_{it} in period t, the period $t+1$ real wage is equal to $w' = w_{it}/(1+\pi_{t+1})$.

The resulting optimization problem can be written in the form of the following Bellman equation

$$
\begin{aligned}
V_t(w) = (1-\lambda)\int_0^\infty \max_{w_{it}\geq 0}\{\Omega(w_{it}; Z_{it}, L_t) + \beta e^{-D_t} V_{t+1}(w_{it}/(1+\pi_{t+1}))\}\, dF(Z_{it}) \\
+\lambda \int_0^\infty \max_{w_{it}\geq w}\{\Omega(w_{it}; Z_{it}, L_t) + \beta e^{-D_t} V_{t+1}(w_{it}/(1+\pi_{t+1}))\}\, dF(Z_{it})\,.
\end{aligned}
\tag{13}
$$

Here, $F(.)$ denotes the distribution function of the idiosyncratic preference shocks. We show in Appendix A that all workers who are not constrained in their wage setting in period t and have the same productivity level, Z_{it}, choose the same real wage $w_t(Z_{it})$. Moreover, the wage they set is strictly increasing in Z_{it}. This means that $w_t(Z_{it})$ is invertible and we denote its inverse as $z_t(w)$. This function gives the disutility from working value at which workers set their real wage to w.

The existence of downward nominal wage rigidities affects the wage-setting decisions of workers who do and do not face the constraint in the current period. For those who face the constraint the statement is obvious. For those who do not face the constraint, the effect works as follows. The wage that non-constrained workers set in the current period, t, determines the likelihood of being constrained in the next period. The higher the wage they set the higher this probability. Thus, relative to the flexible wage-setting case, downward nominal wage rigidities add an additional marginal cost to raising current wages. The result implies that workers will set their wage less than or equal to what they would have done in the absence of DNWR, that is $w_t(Z_{it}) \leq \hat{w}_{it}$. This is reminiscent of Elsby (2009) who emphasizes that downward nominal wage rigidities not only prevent wage declines but they also dampen wage increases.

The workers who set a real wage w and face the DNWR constraint in the current period consist of two groups. The first group are those who get a shock, $Z_{it} \geq z_t(w)$, such that they would like to increase their wage anyway and for whom the constraint is thus not binding. The second group get a productivity shock such that the wage they currently charge, w, is higher than the wage they would have set under flexible wage setting, $\widehat{w}_{it}$. We show in Appendix A, that these workers will keep their wage fixed at w.

Monetary policy rule

Because wages in this economy are sticky due to the downward nominal wage rigidities, there is room for monetary policy to affect the allocation of resources in equilibrium. Since our emphasis in this paper is not on optimal policy, but rather on the shape of the Phillips curve for a given monetary policy rule, we assume that the central bank, which targets an inflation rate equal to $\bar{\pi}$, sets the nominal interest rate according to a standard Taylor (1993) rule.[13] That is, the central bank's policy rule is

$$i_t = \frac{1+\bar{\pi}}{\beta}\left(\frac{Y_t}{\bar{Y}}\right)^{\varphi_Y}\left(\frac{1+\pi_t}{1+\bar{\pi}}\right)^{1+\varphi_\pi} - 1 \text{ where } \varphi_Y, \varphi_\pi > 0. \tag{14}$$

Here, $\bar{Y}$ is the steady-state level of output, such that $Y_t/\bar{Y}$ is one plus the output gap.

Equilibrium

Equilibrium in this economy is a path of $\{L_t, Y_t, i_t, \pi_t, r_t\}_{t=0}^{\infty}$ that satisfies (*i*) the production function, (1), (*ii*) the consumption Euler equation, (9), (*iii*) the Fisher equation, and (*iv*) the monetary policy rule, (14). In addition, *(v)* the wage setting decisions satisfy that the real wage equals 1, i.e. (5).

Under downward nominal wage rigidities the current wage setting by workers depends not only on current economic conditions and their expectations of future economic outcomes but also on their past wage-setting decisions. Thus, the distribution of real wages is a state variable that determines the equilibrium dynamics of the economy. As a result, the equilibrium dynamics of this economy cannot be solved in closed form and need to be simulated numerically. In this section we

[13] An obvious extension to our results would be to include a zero lower bound constraint on monetary policy accommodation.

describe the main recursive equations that drive these equilibrium dynamics. Before we do so, however, we first consider the special case in which wages can be adjusted flexibly, $\lambda = 0$.

Equilibrium under flexible wages, $\boldsymbol{\lambda = 0}$.

This is a useful benchmark to compare the more general case to and can be solved analytically. Because there are no nominal rigidities, the Classical Dichotomy holds and monetary policy does not affect the allocation of goods and labor in the economy. We thus limit our analysis to the real equilibrium variables under flexible wages.

As we show in Appendix A, when wages can adjust flexibly, the equilibrium level of output is given by

$$Y_t = L_t = \left(\frac{\eta - 1}{\eta}\right)^{\frac{\gamma}{1+\gamma}} \left(\frac{1}{Z_t}\right)^{\frac{\gamma}{1+\gamma}}, \tag{15}$$

where

$$Z_t = \left[\int_0^1 \left(\frac{1}{Z_{it}}\right)^{\frac{\gamma(\eta-1)}{\eta+\gamma}} di\right]^{-\frac{\eta+\gamma}{\gamma(\eta-1)}} = e^{-\frac{1}{2}\frac{\eta(1+\gamma)}{\gamma+\eta}\sigma^2} \tag{16}$$

is the aggregate level of the disutility from working. The first factor on the right hand side of equation (15) reflects the output loss due to the workers charging a wage markup and setting their wages inefficiently high. This reduces the equilibrium level of labor demanded and thus of output.

If the elasticity of substitution between the different types of labor, η, goes to infinity then this markup disappears and the equilibrium level of output under flexible wages coincides with that obtained in the first-best where both the goods and labor markets are perfectly competitive.

Equilibrium under downward nominal wage rigidities, $\boldsymbol{\lambda > 0}$.

The equilibrium dynamics in the general case, where $\lambda > 0$, depend on the evolution of the distribution of real wages across workers. We denote the distribution of real wages in period t after each of the workers has set their wage by $G_t(w)$ and the corresponding density function by $g_t(w)$. The evolution of this distribution function is driven by three types of workers.

Figure 4 illustrates these three groups. Consider those that end up with a real wage w' or lower in period t. There are three types of wage setters for whom this happens.

The first type consists of those who are not subject to the DNWR constraint in period t and draw a productivity shock, $Z_{it} < z_t(w')$, such that they would like to set their wage lower than or equal to w'. In Figure 4 this a fraction $(1-\lambda)$ of all household members in the areas A, B, and C. The share of workers in these areas is equal to $F\left(z_t(w')\right)$.

The second type is those workers who are subject to the DNWR constraint in period t, started the period at a real wage $w \leq w'$, and drew a shock, $z_t(w) \leq Z_{it} \leq z_t(w')$ that makes them want to raise their real wage to a level lower than or equal to w'. These workers make up a fraction λ of those in the area B in Figure 4. They are the ones with real wages lower or equal to w' for which the DNWR constraint is not binding.

The final type of worker is those who are constrained by downward nominal wage rigidities. These workers started the period with a real wage $w \leq w'$ and drew a shock $Z_{it} < z_t(w)$. That is, they would like to lower their wage but are not able to. These individuals make up a fraction λ of workers in the area C in Figure 4.

Together, the latter two types of workers make up a fraction λ of those in areas B and C. Because, due to inflation, a worker who started the period with a real wage w paid a real wage $w(1+\pi_t)$ in the previous period, the share of workers in these two areas is equal to $G_{t-1}\left(w'(1+\pi_t)\right)F\left(z_t(w')\right)$.

Adding the mass for these three types of workers, we obtain the following recursive dynamic equation for the distribution function of wages

$$G_t(w) = (1-\lambda)F\left(z_t(w)\right) + \lambda G_{t-1}\left(w(1+\pi_t)\right)F\left(z_t(w)\right). \tag{17}$$

Since under flexible wages $G_t(w) = F\left(z_t(w)\right)$, this equation illustrates how inflation "greases" the wheels of the labor market in this economy. The higher inflation, the fewer workers are stuck at a real wage that they cannot adjust downwards. That is, $G_{t-1}\left(w(1+\pi_t)\right)$ is increasing in the inflation rate and, in the limit, goes to one. In that limit, the distribution of real wages in this economy would be the same as under flexible wage setting. Of course, as we showed above, the equilibrium under flexible wages is distorted by the wage markup that workers charge.

Given the distribution of real wages across workers at the end of the previous period, $G_{t-1}\left(w(1+\pi_t)\right)$, and the wage-setting schedule, $w_t(Z)$ and $z_t(w)$, we can integrate out wage-setting decisions in (5) to solve for the equilibrium level of output, Y_t, and employment, L_t. As we show in Appendix A, this yields

$$Y_t = L_t = \left(\frac{\eta - 1}{\eta}\right)^{\frac{\gamma}{1+\gamma}} \left(\frac{1}{Z_t^*}\right)^{\frac{\gamma}{1+\gamma}}, \tag{18}$$

where the distorted aggregate disutility level is given by

$$\begin{aligned} Z_t^* = \Bigg\{ &(1-\lambda) \int_0^\infty \left(\frac{1}{Z}\right)^{\frac{\gamma(\eta-1)}{\eta+\gamma}} \left(\frac{\hat{w}_t(Z)}{w_t(Z)}\right)^{\eta-1} dF(Z) \\ &+\lambda \int_0^\infty \left(\frac{1}{Z}\right)^{\frac{\gamma(\eta-1)}{\eta+\gamma}} G_{t-1}\big(w_t(Z)(1+\pi_t)\big) \left(\frac{\hat{w}_t(Z)}{w_t(Z)}\right)^{\eta-1} dF(Z) \\ &+\lambda \int_0^\infty \left(\frac{1}{Z}\right)^{\frac{\gamma(\eta-1)}{\eta+\gamma}} \left[\int_{w_t(Z)}^\infty (1+\pi_t) g_{t-1}\big(w(1+\pi_t)\big) \left(\frac{\hat{w}_t(Z)}{w}\right)^{\eta-1} dw\right] dF(Z) \Bigg\}^{-\frac{\eta+\gamma}{\gamma(\eta-1)}}. \end{aligned} \tag{19}$$

Each of the three lines in this equation corresponds to a different type of worker. The first line consists of workers who are not subject to the downward nominal wage rigidities constraint in period t. The second line represents those who are subject to DNWR but for whom the constraint is not binding. For a given level of L_t, both these groups set their wage equal to $w_t(Z) < \hat{w}_t(Z)$ and end up supplying more labor than they would have done under flexible wages. The third line is the workers who are constrained by the downward nominal wage rigidities. Many of these workers end up supplying less labor than they would have done under flexible wages.

Note that the fact that wages are set such that $w_t(Z) < \hat{w}_t(Z)$ partially offsets the effect of the downward rigidities. This is a point Elsby (2009) emphasizes as limiting the welfare cost of downward nominal wage rigidities.[14] Comparing (18) with (15) one can see that the effect of downward nominal wage rigidities is captured by the ratio Z_t/Z_t^*. This is the labor wedge introduced by the rigidities.

Unfortunately, neither the steady-state value nor the equilibrium path of this wedge can be solved in closed form. Therefore, we use numerical examples in the next two sections to illustrate the main features of the equilibrium of our model. In the next section we focus on the steady state of the model and its implications for the shape of the long-run Phillips curve. The subsequent section considers the transitional dynamics and the short-run output-inflation trade-off.

[14] Ball and Mankiw (1994) make a similar point about the welfare costs of asymmetric costs to price adjustments.

4. Steady state: Long-run unemployment-inflation trade-off

That downward nominal wage rigidities cause a long-run unemployment-inflation trade-off is a well-known argument, made by Tobin (1972). Tobin's main argument has been formalized in several theoretical models (Akerlof, Dickens, and Perry, 1996, Fagan and Messina, 2008, and Benigno and Ricci, 2011).

In this section we show that our simplified version of previous models produces the same long-run properties emphasized in more sophisticated versions. In particular, we focus on two main properties of the steady-state equilibrium of our model.

The first is that the central bank's choice of inflation target affects the level of the natural rate of unemployment. Thus, there is a long-run trade-off between unemployment and inflation in our model due to the downward rigidity of wages. For a given level of rigidities this generates a long-run Phillips curve that captures the downward sloping relationship between the target inflation rate and the natural rate of unemployment.

The second steady-state property is the distortion in the distribution of nominal wage changes due to downward nominal rigidities and its implication for the imputation of the importance of these rigidities constructing counterfactual wage change distributions using individual level data on nominal wage changes.

These two exercises show that our simplifications, made to solve the transitional dynamics of the model, did not come at the cost of missing the long-run or steady-state trade-off documented in previous research.

Long-run Phillips curve

To understand how the choice of the inflation target, $\bar{\pi}$, affects the steady-state level of unemployment,[15] we follow Benigno and Ricci (2011) and define the unemployment rate in our model as the percentage shortfall in employment under downward nominal wage rigidities compared to under flexible wages.[16]

[15] Which we interpret as the natural rate of unemployment in the rest of this paper.

[16] The unemployment gap is defined as the percentage point deviation of the unemployment rate from its steady-state level. Since output equals employment in our model due to, (1), Okun's (1962) Law holds by definition with coefficient -1.

The reason that the level of employment is lower under downward nominal wage rigidities is that some workers end up stuck charging a wage higher than they initially intended because they drew an unexpectedly low disutility shock, Z. Stuck at this wage, they then end up providing fewer labor services than they wanted to supply.

The existence of the idiosyncratic shocks is thus a central part of this argument. Without them, downward nominal wage rigidities are unlikely to cause a substantial distortion in equilibrium. In fact, most studies of downward nominal wage rigidities that do not include such shocks find that these rigidities have a very small effect on labor market allocations and welfare.[17]

Even with idiosyncratic shocks, the employment loss due to workers being stuck at a high wage is partly offset by the change in wage setting behavior for the workers who do adjust their wage. As we showed in the previous section, in order to decrease the likelihood of the downward rigidities being binding in the future, they set their wage, $w_t(Z)$, lower than under flexible wages, $\hat{w}_t(Z)$. At this lower real wage they end up supplying more labor than they would have under flexible wages.

In our model, the likelihood that, in steady state, a worker gets stuck charging a wage he would like to lower but can't depends on three things: (i) λ, the probability of being subject to the downward nominal wage rigidities constraint in a period, (ii) the variance of the idiosyncratic shocks, σ^2, and (iii) the target inflation level, $\bar{\pi}$, and degree to which it greases the labor market in the sense of (17). In our examples in this section, we mainly focus on the effects of λ and $\bar{\pi}$.[18]

In terms of the other parameters, we follow Benigno and Ricci (2011) and set the wage elasticity of labor demand to $\eta = 2.5$ and the Frisch elasticity of the labor supply, $\gamma = 0.5$. In addition, we set $\sigma = 0.2$. Since, in steady state, the inflation rate is equal to its target, $\bar{\pi}$, and the output gap is zero, the monetary policy parameters, φ_Y and φ_π, as well as the discount factor, β, do not affect the steady-state outcome. Throughout, our parameter choices are based on a time period being a quarter.

Figure 5 plots the long-run Phillips curves for three different degrees of downward nominal wage rigidities, $\lambda = \{0.2, 0.5, 0.8\}$. On the vertical axis is the central bank's inflation target, while on the horizontal axis is the natural rate of unemployment that results from the inflation-target choice.

[17] See Kim and Ruge-Murcia (2008) and Coibion, Gorodnichenko, and Wieland (2012), for example. Carlsson and Westermark (2008) even find that DNWR can be welfare improving because they reduce wage fluctuations.

[18] See Benigno and Ricci (2011) for an extensive analysis of the effect of changes in σ^2.

For all three degrees of rigidities, the long-run Phillips curve is downward sloping. That is, because inflation greases the wheels of the labor market, the lower the central bank's inflation target the less lubricated the labor market and the higher the natural rate of unemployment. The higher λ the more unemployment increases for a given reduction in the target inflation rate. That is, the long-run sacrifice ratio is increasing in the degree to which the downward nominal wage rigidities are binding.[19]

The average long-run sacrifice ratio for $\lambda = 0.8$ is similar to that in Akerlof, Dickens, and Perry (1996, Figure 3) in that an increase of the inflation target from 0 to 10 results in an approximately 2 percentage point drop in the natural rate of unemployment. Just like Akerlof, Dickens, and Perry (1996, Figure 3) and Benigno and Ricci (2011, Figure 2), we find that the long-run Phillips curve is bent such that the long-run sacrifice ratio is higher for lower target inflation rates.

Distortion of log nominal wage changes

The long-run unemployment-inflation trade-off that downward nominal wage rigidities induce is the result of the rigidities distorting wage-setting behavior. To illustrate this distortion, we plot the theoretical equivalent of the distribution of log nominal wage changes, depicted in Figure 1, for $\bar{\pi} = 2$ percent (annualized) and for $\lambda = 0.8$ as well as for the case of flexible wages, $\lambda = 0$. The distribution of quarterly log nominal wage changes in our model is plotted in Figure 6.

Log-normality of the idiosyncratic shocks, Z, together with the optimal wage setting under flexible wages, (12), implies that, under flexible wages, the steady-state distribution of quarterly log nominal wage changes will be normal with a mean equal to the quarterly steady-state inflation rate, in this case 0.5 percent.[20] This is the 'flexible' distribution plotted in figure 6.

The distribution of quarterly log wage changes under downward nominal wage rigidities looks surprisingly different. The presence of downward nominal wage rigidities distorts this distribution in three ways. First of all, there are many fewer negative wage changes due to the rigidities. Secondly, there are many fewer positive wage changes as well and those that do occur are smaller

[19] Of course, our model solely focuses on downward nominal wage rigidities. A more general analysis of the optimal inflation target, like and Coibion, Gorodnichenko, and Wieland (2012), would take into account the many different costs and benefits of (steady-state) inflation (see, Fischer and Modigliani, 1978).

[20] Our model does not include labor productivity growth. In the case of positive trend labor productivity growth, the mean log wage change under flexible wages would be equal to the quarterly steady-state inflation rate plus the productivity growth rate.

than under flexible wages. That the size of the positive wage changes under downward nominal wage rigidities is smaller has been emphasized by Elsby (2009). What is more surprising is that the number of workers with wage increases also goes down due to the rigidities. Under flexible wages 48% of workers get a quarterly wage increase of 0.25 percent or higher. While under downwardly rigid wages, this fraction is only 27 percent. Third, and finally, the rigidities result in a spike at zero nominal wage changes. For our parameter combination, 57 percent of workers have the same wage as in the previous quarter.

This spike is much lower when calculated for 4-quarter wage changes. Over four quarters, the spike at zero in our model is only 1.4 percent. This is in large part due to our assumption that there is no persistence in the idiosyncratic shocks, Z. For a worker in our model to be stuck at a nominal wage for four quarters in a row, eight things need to happen. For the four quarters in a row the worker needs to be subject to the downward wage rigidities constraint. This happens with probability λ^4. In addition, the worker needs to draw four consecutive idiosyncratic shocks that do not want him to increase his wage. The joint probability of these eight events occurring is low. If one takes the 4-quarter spike at zero from the model at face value, then our model exhibits relatively low downward nominal rigidities compared to the data, where this spike is higher than 10 percent (Figure 2).

The result that the existence of downward nominal rigidities affects positive wage increases and that part of the spike at zero reflects wage increases that did not take place has an important implication for interpreting the patterns in Figure 1. Our theoretical model suggests that some of the spike at zero in the data reflects a pulling back of wage increases. Our model results also suggest that empirical studies that estimate the importance of downward wage rigidities by comparing the actual distribution of wage changes with a constructed counterfactual (Card and Hyslop, 1997, and Lebow, Saks, and Wilson, 2003) may be underestimating the true magnitude. These studies construct counterfactuals assuming that the observed frequency and size of positive wage changes are not affected by the existence of the rigidities. This assumption is clearly violated in our theoretical model.

5. Transitional dynamics: Slope and curvature of the Phillips curve

Of course, the experience in the wake of the Great Recession, about which we presented facts in Section 2, does not reflect a movement of the steady state but instead a shock that causes a deviation from steady state. To interpret this recent evidence on downward nominal wage rigidities and the wage Phillips curve in the context of our model, we thus need to solve its transitional dynamics. We do so in this section. In particular, we are interested in whether these dynamics are qualitatively consistent with the facts we documented in Section 2.

To isolate the effect of downward nominal wage rigidities we consider the response of the economy to a negative persistent discount rate shock

$$D_t = \rho D_{t-1}, \text{ where } \rho = 0.8,\ D_0 = 0, \text{ and } D_1 < 0. \tag{20}$$

In the absence of downward nominal wage rigidities such a shock would be perfectly offset by a decline in the real interest rate, leaving the real allocation of resources in the economy unaffected.[21]

The monetary policy rule that replicates the flexible wage equilibrium in response to this shock is one where the nominal interest rate moves in lockstep with the discount rate shock, i.e.

$$i_t = \frac{1+\bar{\pi}}{\beta} e^{D_t} - 1. \tag{21}$$

We, however, consider the case in which the central bank follows a standard Taylor rule. We use Taylor's (1993) parameter estimates for the policy rule and fix $\varphi_Y = \varphi_\pi = 0.5$ and assume that the natural real interest rate is 2 percent annualized. The latter pins down the discount factor, β.[22] The target inflation rate, $\bar{\pi}$, is set to 2 percent annualized.

In the rest of this section we show that the transition path of the model, in response to such a discount rate shock and under the standard Taylor monetary policy rule, (*i*) matches the shift in the histogram of log nominal wage changes, presented in Figure 1, (*ii*) exhibits a run up in the spike at zero nominal wage changes, as in Figure 2, and (*iii*) generates the non-linearity in the empirical wage Phillips curve shown in Figure 3.

[21] Alternative shocks, like a productivity shock or a shock to the marginal of substitution between consumption and leisure will also affect output and inflation in the absence of downward nominal wage rigidities.

[22] Of course, either of these policy rules might not be feasible for $D_t \ll 0$ if the central bank's policy rule is subject to the zero nominal lower bound (ZLB).

To illustrate this, we solve for the model's non-linear transitional dynamics keeping track of the distribution of real wages along the equilibrium path. We do so by using the extended path method, introduced by Fair and Taylor (1983).[23] Our main example is the transitional dynamics after a large discount rate shock equal to $D_1 = -0.03$.

Distortion of log nominal wage changes

Figure 7 shows the distribution of quarterly log wage changes right before the shock hits, at $t = 0$ when the economy is in steady state, as well as the distribution right after the shock hits, at $t = 2$. It is our model equivalent of Figure 1, interpreting 2006 as the steady state and 2011 as after the shock.

Figure 7 shows that the model replicates three main features from the data. First, the spike at zero nominal wage changes increases in response to the negative demand shock. This occurs because the decline in inflation, in response to the shock, makes DNWR bind for more workers who are not able to reduce their wages. In addition, the likelihood of being constrained by DNWR in the future increases for workers who are raising their wages, making them subdue their wage increases. This results in a compression of wage increases similar to that shown in Figure 1. Note that this compression is fully driven by the wage rigidities and is not due to a change in labor productivity, which is constant in our model. Finally, just like in the data, there does not seem to be a large increase in the fraction of workers with wage declines, though conditional on getting a reduction in the nominal wage this decline is slightly larger after the shock than before.

Time-series path of spike at zero

Figure 8 shows that the model also generates a persistent increase in the spike at zero wage changes, just like in the data in Figure 2. The negative discount rate shock, in combination with the monetary policy response, leads to a decline in the inflation rate. This results in an increase in the fraction of workers who are stuck charging an undesirably high wage, making them sell fewer labor services than they initially planned. As a consequence, the unemployment rate increases.

[23] Examples of other applications of this solution method are Hobijn, Ravenna, and Tambalotti (2006), who solve the transitional dynamics of a model with nominal rigidities, and Carriga, Manuelli, and Peralta-Alva (2012) who consider the non-linear transitional dynamics to a large shock that affects house prices. A more detailed description of how we implement the method can be found in Appendix A.

Wage Phillips curve

The joint movement of inflation and unemployment along the transition path is depicted in Figure 9. The figure shows the path for three different sizes of initial discount rate shocks, $D_1 \in \{-0.01, -0.02, -0.03\}$. Plotting the model's short-run wage Phillips curve for three different sizes of shocks allows us to illustrate the two ways in which downward nominal wage rigidities bend the Phillips curve.

The first way in which downward nominal wage rigidities bend the Phillips curve is through the non-linear response of unemployment and inflation to the initial shock. In Figure 9 this shows up through the first arrows that go from the steady state, of 2 percent inflation and 5 percent unemployment, to the lower right. These are the responses of the model to the initial shocks. As can be seen from the figure, the bigger the initial shock, the more the adjustment occurs through unemployment rather than through inflation. This suggests that the short-run sacrifice ratio is higher in deep recessions than in shallow ones, because in those recessions downward nominal wage rigidities are more binding.

This mimics the behavior of the actual U.S. wage Phillips curve, plotted in Figure 3. During the depth of the Great Recession nominal wage growth decelerated much less than one would have expected based on a linear relationship between wage inflation and the unemployment gap.

Of course, in reality monetary policy during that period hit the zero nominal lower bound (ZLB), while the transition path in Figure 9 is simulated in absence of that constraint. Under the smallest shock, $D_1 = -0.01$, the standard Taylor rule in our model implies that monetary policy skirts the ZLB. In case of the larger shocks, $D_1 = -0.02$ and $D_1 = -0.03$, the nominal interest rate bottoms out at -4 and -8 percent (annualized) respectively. If these transition paths were calculated under a monetary policy subject to the ZLB the non-linearity in the response to the initial shock would be larger.

The second way in which downward nominal wage rigidities bend the Phillips curve is through their effect on the path of unemployment and wage inflation back to the steady state following a shock. As can be seen from Figure 9, the initial response to all three shocks is followed by a period during which wage growth continues to decelerate in spite of the unemployment rate coming down. This results from the fact that the initial shock causes a large fraction of workers to be stuck charging a wage higher than desired. Because $\lambda < 1$, in subsequent quarters following the initial shock, these workers slowly adjust their wages downward, slowing down wage inflation and reducing the level of unemployment.

This dynamic is remarkably similar to that of wage inflation and the unemployment rate during the recovery after the Great Recession that we showed in Figure 3. Even though the unemployment rate declined by more than 2 percentage points in the two and a half years between the end of the recession and the end of the sample plotted in 2012, wage inflation declined by 0.2 percentage points. Our model implies that this pattern is indicative of a substantial amount of pent up wage deflation, which, as it is slowly realized, contributes to the decline of the unemployment rate.

Given the impact that downward nominal wage rigidities have on the short-run dynamics of the labor market and evidence that in the long run, under downward nominal wage rigidities, inflation greases the wheels of the labor market, the final question we address is whether the short-run bending of the Phillips curve is very different under different levels of the target inflation rate, $\bar{\pi}$. Figure 10 plots the short-run wage Phillips curve in response to a large discount rate shock, $D_1 = -0.03$, for two different levels of the target inflation rate, namely $\bar{\pi} = 2$ percent annualized (as in Figure 9) and $\bar{\pi} = 10$ percent. For comparison purposes, the figure also includes the long-run Phillips curve for $\lambda = 0.8$ from Figure 5.

The first thing to note from Figure 10 is that under the high target inflation rate of $\bar{\pi} = 10$ percent, when downward nominal wage rigidities are less binding, the short-run sacrifice ratio is smaller than under $\bar{\pi} = 2$ percent. That is, the response to the initial shock under the high inflation target is more through (wage) inflation and less through the unemployment rate than under the low inflation target. Under $\bar{\pi} = 10$ percent the initial increase in the unemployment rate is 2.8 percentage points while wage inflation decelerates by 5.6 percentage points. At $\bar{\pi} = 2$ percent this initial response is 3.1 percentage points on the unemployment rate and 3.9 percentage points on inflation respectively.

In addition to the response to the initial shock we can also compare the speed of the transition back to steady state. If inflation greases the wheels then it might accelerate the pace with which the economy returns to steady state in response to a shock. Surprisingly, the speeds of adjustment along the two transitional paths plotted in Figure 10 are very similar. Hence, our simulation results suggest that increasing the target inflation rate, $\bar{\pi}$, to accelerate the pace of labor market adjustments might not have a particularly large impact. Of course, this result is based on our simple model of downward nominal wage rigidities and might change when these rigidities are included in a richer macroeconomic framework.

6. Conclusion

In this paper we made two main contributions. Our first contribution was empirical. We updated existing studies on the prevalence of downward nominal wage rigidities to show that a record high fraction of U.S. workers did not get any wage increases in the wake of the Great Recession. In addition, we constructed the U.S. wage Phillips curve for 1986-2012. We used it to show that the period from mid-2009 through 2012, in which downward nominal wage rigidities were as binding as any time in the last 32 years, the over 2 percentage point decline in the unemployment rate coincided with a continued deceleration of nominal wage growth.

Our second contribution was to show that the transitional dynamics of a simple model of downward nominal wage rigidities, based on Benigno and Ricci (2011), qualitatively replicate the facts we documented. The model by Benigno and Ricci (2011), as well as the related models by Akerlof, Dickens, and Perry (1996) and Fagan and Messina (2008), were developed to analyze Tobin's (1972) claim that downward nominal wage rigidities induce a long-run trade-off between the central bank's inflation target and the natural rate of unemployment. Analysis of the transitional dynamics of our model uncovered that downward nominal wage rigidities also shape the short-run unemployment-inflation trade-off.

In particular, consistent with Phillips' (1958) conjecture, downward nominal wage rigidities bend the Phillips curve. They do so in two ways. First, during deeper recessions the rigidities become more binding and more of the adjustment happens through unemployment rather than through wage deflation. That is, the deeper the recession the higher the short-run sacrifice ratio. Second, deep recessions in which downward nominal wage rigidities are particularly binding result in substantial pent up wage deflation. As a consequence, the recovery of such a recession involves continued wage cuts by firms, alleviating the wage rigidities constraint, resulting in more hiring, and lowering the unemployment rate. During the recovery from such a recession this leads to a simultaneous deceleration of wage inflation and decline in the unemployment rate.

Both of these types of curvature are present during the period from mid-2009 through 2012 in the wage Phillips curve that we constructed. This, together with the micro level data on the spike at zero in the distribution of nominal wage changes, leads us to conclude that downward nominal wage rigidities have played a role in shaping the dynamics of wage growth and unemployment during the recovery from the Great Recession.

Of course, the simplified nature of our model, which allowed us to solve for its non-linear transition path, means that, at best, we were able to qualitatively match the facts on the distribution of nominal wage changes and the wage Phillips curve. Our results suggest that inclusion of downward nominal wage rigidities with idiosyncratic shocks in more elaborate macroeconomic models[24] is a challenging but promising area for future research.

[24] Like the models used to study the likelihood of hitting the ZLB in Chung et. al (2012), or the one used by Coibion et. al (2012).

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Appendix A: Mathematical details

The household's wage setting decision

The household maximizes

$$\sum_{t=0}^{\infty} \beta^t e^{-\sum_{s=0}^{t-1} D_s} \left\{ \ln Y_t - \frac{\gamma}{\gamma+1} \int_0^1 Z_{it} L_{it}^{\frac{\gamma+1}{\gamma}} di \right\} \text{ where } \gamma > 0, \tag{22}$$

It does so subject to the dynamic budget constraint

$$A_t = (1 + i_{t-1}) A_{t-1} + \int_0^1 (1 - u_{it}) W_{it} L_{it}^s di - P_t Y_t. \tag{23}$$

and the labor demand function

$$L_{it} = w_{it}^{-\eta} L_t. \tag{24}$$

The first-order necessary conditions

The discrete-time Hamiltonian associated with this problem is

$$H = \sum_{t=0}^{\infty} \beta^t e^{-\sum_{s=0}^{t-1} D_s} \left[\ln Y_t - \frac{\gamma}{\gamma+1} \int_0^1 Z_{it} L_{it}^{\frac{\gamma+1}{\gamma}} di + \mu_t \left\{ (1 + i_{t-1}) A_{t-1} + \int_0^1 W_{it} w_{it}^{-\eta} L_t di - P_t Y_t - A_t \right\} \right]. \tag{25}$$

Here μ_t is the discounted co-state variable.

Optimal consumption condition. The condition that determines the household's optimal consumption level, Y_t, is given by

$$0 = \frac{\partial H}{\partial Y_t} = \beta^t e^{D_t} \left[\frac{1}{Y_t} - \mu_t P_t \right]. \tag{26}$$

Such that the marginal utility of consumption per dollar spent is equal to the co-state variable, in the sense that

$$\mu_t = \frac{1}{P_t Y_t}. \tag{27}$$

Optimal savings condition. The condition that determines the household's optimal savings level, A_t, is given by

$$0 = \frac{\partial H}{\partial A_t} = -\beta^t e^{D_t} \mu_t + \beta^{t+1} e^{D_{t+1}} (1 + i_t) \mu_{t+1}. \tag{28}$$

Rearranging the terms in this condition yields

$$\mu_t = \beta e^{\Delta D_{t+1}} (1 + i_t) \mu_{t+1}. \tag{29}$$

Combining (27) and (29) gives the consumption Euler equation, (9), in the main text.

Wage-setting objective, equation (10)

We can isolate the part of the household's Hamiltonian that involves the wage-setting of household member i. That part is

$$\widetilde{H}_i = \mathrm{E}_t\left[\sum_{t=0}^{\infty}\beta^t e^{-\sum_{s=0}^{t-1}D_s}\left[\mu_t W_{it} w_{it}^{-\eta} L_t - \frac{\gamma}{\gamma+1} Z_{it} w_{it}^{-\eta\frac{\gamma+1}{\gamma}} L_t^{\frac{\gamma+1}{\gamma}}\right]\right]. \quad (30)$$

Substituting in the result that $\mu_t = 1/P_t Y_t = 1/P_t L_t$, this objective can be simplified to

$$\widetilde{H}_i = \mathrm{E}_t\left[\sum_{t=0}^{\infty}\beta^t e^{-\sum_{s=0}^{t-1}D_s}\left[w_{it}^{1-\eta} L_t - \frac{\gamma}{\gamma+1} Z_{it} w_{it}^{-\eta\frac{\gamma+1}{\gamma}} L_t^{\frac{\gamma+1}{\gamma}}\right]\right] = \mathrm{E}_t\left[\sum_{t=0}^{\infty}\beta^t e^{-\sum_{s=0}^{t-1}D_s}\Omega(Z_{it}; w_{it}, L_t)\right]. \quad (31)$$

This is equation (10) from the main text.

Wage setting under flexible wages, equation (12)

Under flexible wages, in each period household member i chooses its real wage to maximize $\Omega(Z_{it}; w_{it}, L_t)$. The associated first order condition is

$$0 = \frac{\partial}{\partial w_{it}}\Omega(Z_{it}; w_{it}, L_t) = \frac{1}{w_{it}}\left[-(\eta-1) w_{it}^{1-\eta} L_t + \eta Z_{it} w_{it}^{-\eta\frac{\gamma+1}{\gamma}} L_t^{\frac{\gamma+1}{\gamma}}\right]. \quad (32)$$

Solving this condition for the real wage, w_{it}, yields

$$w_{it} = \left(\frac{\eta}{\eta-1}\right)^{\frac{\gamma}{\eta+\gamma}} Z_{it}^{\frac{\gamma}{\eta+\gamma}} L_t^{\frac{\gamma+1}{\eta+\gamma}}. \quad (33)$$

This is equation (12) from the main text.

Properties of the optimal wage setting schedule, $\boldsymbol{w_t(Z_{it})}$

The Bellman equation associated with the dynamic wage-setting problem of the workers under downward nominal wage rigidities is given by (13).

Workers that adjust their wage, are not subject to the DNWR constraint, and have the same Z_{it} set the same wage $w_t(Z_{it})$.

These workers choose their wage w_{it} to solve

$$w_{it} = \underset{w\geq 0}{\operatorname{argmax}}\{\Omega(w; Z_{it}, L_t) + \beta e^{-D_t} V_{t+1}(w/(1+\pi_{t+1}))\}. \quad (34)$$

The only respect with which workers differ is in terms of their productivity level. Hence, workers with the same productivity level choose the same wage.

The wage-setting schedule, $w_t(Z_{it})$, is such that $w_t(Z_{it}) \leq \hat{w}_t(Z_{it})$.

This is easiest seen by a transformation of variables. Instead of having the worker choose, w_{it}, we write its decision variable as $x_{it} = (1/w_{it})^{\eta-1}$. This allows us to write

$$\Omega(x_{it}; Z_{it}, L_t) = x_{it} - \frac{\gamma}{\gamma+1} Z_{it} x_{it}^{\left(\frac{\eta}{(\eta-1)}\frac{(1+\gamma)}{\gamma}\right)} L_t^{\frac{1+\gamma}{\gamma}}. \tag{35}$$

In addition, the transformed Bellman equation, (13), can be written as

$$\begin{aligned} V_t(x) = (1-\lambda)\int_0^\infty \max_{w_{it}\geq 0}\{\Omega(x_{it}; Z_{it}, L_t) + \beta e^{-D_t} V_{t+1}(x_{it}(1+\pi_{t+1})^{\eta-1})\}\, dF(Z_{it}) \\ +\lambda \int_0^\infty \max_{x_{it}\leq x}\{\Omega(x_{it}; Z_{it}, L_t) + \beta e^{-D_t} V_{t+1}(x_{it}(1+\pi_{t+1})^{\eta-1})\}\, dF(Z_{it})\,. \end{aligned} \tag{36}$$

From this Bellman equation it can be seen that $V_t{}'(x) \geq 0$. That is, the higher, x (the lower the real wage), the less binding the constraint of downward nominal wage rigidities and the higher the value of starting the period with a high x.

Note that $\Omega(x_{it}; Z_{it}, L_t)$ is concave, in that

$$\frac{\partial}{\partial x_{it}}\Omega(x_{it}; Z_{it}, L_t) = 1 - \frac{\eta}{(\eta-1)} Z_{it} x_{it}^{\left(\frac{\eta}{(\eta-1)}\frac{(1+\gamma)}{\gamma}-1\right)} L_t^{\frac{1+\gamma}{\gamma}}. \tag{37}$$

and

$$\frac{\partial}{\partial x_{it}}\Omega(x_{it}; Z_{it}, L_t) = -\frac{\eta}{(\eta-1)}\left(\frac{\eta}{(\eta-1)}\frac{(1+\gamma)}{\gamma} - 1\right) Z_{it} x_{it}^{\left(\frac{\eta}{(\eta-1)}\frac{(1+\gamma)}{\gamma}-2\right)} L_t^{\frac{1+\gamma}{\gamma}} < 0. \tag{38}$$

Under flexible wages, the wage set, $\hat{x}_{it}$, satisfies the first-order necessary condition

$$\frac{\partial}{\partial x_{it}}\Omega(\hat{x}_{it}; Z_{it}, L_t) = 0. \tag{39}$$

Under downward nominal wage rigidities, this condition is

$$\frac{\partial}{\partial x_{it}}\Omega(x_{it}; Z_{it}, L_t) + \beta e^{-D_t}(1+\pi_{t+1})^{\eta-1} V'_{t+1}(x_{it}(1+\pi_{t+1})^{\eta-1}) = 0. \tag{40}$$

Since the second term in this equation is non-negative, this means that, at the optimal wage-setting choice under downward nominal wage rigidities it is the case that

$$\frac{\partial}{\partial x_{it}} \Omega(x_{it}; Z_{it}, L_t) \leq 0. \tag{41}$$

Concavity of $\Omega(x_{it}; Z_{it}, L_t)$ then implies that the worker will choose $x_{it} \geq \hat{x}_{it}$. Converting this result back into the real wage that the worker chooses yields that $w_t(Z_{it}) \leq \hat{w}_t(Z_{it})$.

The wage-setting schedule, $w_t(Z_{it})$, is strictly increasing in Z_{it}.

Suppose that $\mathrm{w_t(Z_{it})}$ is not strictly increasing in Z_{it}. Then the optimal choice of x_{it} is not strictly decreasing in Z_{it}. This means that $\exists$ Z and Z' such that $Z' > Z$ and

$$\hat{x}(Z') \leq \hat{x}(Z) \leq x(Z) \leq x(Z'). \tag{42}$$

Here, $\hat{x}(Z)$ is the level of x chosen by a worker with shock Z when wages can be adjusted flexibly. For all $x \geq 0$ it is the case that

$$\begin{gathered} \Omega(x(Z); Z, L_t) + \beta e^{-D_t} V_{t+1}(x(Z)(1+\pi_{t+1})^{\eta-1}) \geq \\ \Omega(x; Z, L_t) + \beta e^{-D_t} V_{t+1}(x(1+\pi_{t+1})^{\eta-1}). \end{gathered} \tag{43}$$

and

$$\begin{gathered} \Omega(x(Z'); Z', L_t) + \beta e^{-D_t} V_{t+1}(x(Z')(1+\pi_{t+1})^{\eta-1}) \geq \\ \Omega(x; Z', L_t) + \beta e^{-D_t} V_{t+1}(x(1+\pi_{t+1})^{\eta-1}). \end{gathered} \tag{44}$$

The last equation implies that

$$\begin{gathered} x(Z') - x(Z') + \beta e^{-D_t}[V_{t+1}(x(Z')(1+\pi_{t+1})^{\eta-1}) - V_{t+1}(x(Z)(1+\pi_{t+1})^{\eta-1})] \geq \\ \frac{\gamma Z'}{\gamma+1} L_t^{\frac{1+\gamma}{\gamma}} \left[x(Z')^{\left(\frac{\eta}{(\eta-1)} \frac{(1+\gamma)}{\gamma}\right)} - x(Z)^{\left(\frac{\eta}{(\eta-1)} \frac{(1+\gamma)}{\gamma}\right)} \right]. \end{gathered} \tag{45}$$

Since $Z' > Z$, this thus implies that

$$\begin{gathered} x(Z') - x(Z') + \beta e^{-D_t}[V_{t+1}(x(Z')(1+\pi_{t+1})^{\eta-1}) - V_{t+1}(x(Z)(1+\pi_{t+1})^{\eta-1})] > \\ \frac{\gamma Z}{\gamma+1} L_t^{\frac{1+\gamma}{\gamma}} \left[x(Z')^{\left(\frac{\eta}{(\eta-1)} \frac{(1+\gamma)}{\gamma}\right)} - x(Z)^{\left(\frac{\eta}{(\eta-1)} \frac{(1+\gamma)}{\gamma}\right)} \right]. \end{gathered} \tag{46}$$

Rearranging terms, this means that

$$\begin{gathered} \Omega(x(Z'); Z, L_t) + \beta e^{-D_t} V_{t+1}(x(Z')(1+\pi_{t+1})^{\eta-1}) > \\ \Omega(x(Z); Z, L_t) + \beta e^{-D_t} V_{t+1}(x(Z)(1+\pi_{t+1})^{\eta-1}). \end{gathered} \tag{47}$$

But this implies that $x(Z)$ cannot be the optimal choice for a worker with shock Z. Hence, the optimal choice of x_{it} is strictly decreasing in Z_{it}. Consequently, $\mathrm{w_t(Z_{it})}$ is strictly increasing in Z_{it}.

Workers that are subject to DNWR and for whom $w_t(Z_{it}) < w$ will keep wage fixed.

The easiest way to show this is to show that if DNWR are binding at Z, which is the case when $\mathrm{w}_t(Z) = \mathrm{w}$, then they are binding at all $Z' < Z$. Suppose not, then $\forall x' < x$

$$\begin{gathered}\Omega(x; Z, L_t) + \beta e^{-D_t} V_{t+1}(x(1+\pi_{t+1})^{\eta-1}) \geq \\ \Omega(x'; Z, L_t) + \beta e^{-D_t} V_{t+1}(x'(1+\pi_{t+1})^{\eta-1}).\end{gathered} \tag{48}$$

while there $\exists\, x^* < x$ and $Z' < Z$ such that $\forall x' < x$

$$\begin{gathered}\Omega(x^*; Z', L_t) + \beta e^{-D_t} V_{t+1}(x^*(1+\pi_{t+1})^{\eta-1}) \geq \\ \Omega(x'; Z', L_t) + \beta e^{-D_t} V_{t+1}(x'(1+\pi_{t+1})^{\eta-1}).\end{gathered} \tag{49}$$

But, using the similar argument as in equations (45) and (46) these two equations can be shown to contradict each other. Thus, if DNWR are binding at Z, then they are binding for all $Z' < Z$. Hence, any worker subject to DNWR at a wage $w < \mathrm{w}_t(Z)$ will keep their wage fixed.

The log normal distribution

For the disutility from working shock, Z, we assume that it is drawn from a log-Normal distribution, such that $\ln Z \sim N\left(-\frac{1}{2}\sigma^2, \sigma\right)$. Throughout, we denote the standard Normal distribution function by $\Phi(.)$ and the corresponding density by $\phi(.)$.

For our derivations, we extensively use the properties of the log-Normal distribution that make make expectations of various exponentials of the random variable easily tractable. First note that we choose the mean $-\frac{1}{2}\sigma^2$ such that

$$E(Z) = e^{-\frac{1}{2}\sigma^2 + \frac{1}{2}\sigma^2} = 1. \tag{50}$$

Hence, the average productivity level across firms equals one.

Moreover,

$$E(Z^r) = e^{\frac{1}{2}r(r-1)\sigma^2} \tag{51}$$

and, consequently, we find that

$$[E(Z^r)]^{\frac{1}{r}} = e^{\frac{1}{2}(r-1)\sigma^2}. \tag{52}$$

The final property that we use is that of the truncated log normal. In particular, we apply that for $b > a > 0$

$$Q(r;a,b) \equiv \int_a^b Z^r \, dF(Z) = \left(\Phi(r\sigma - \tilde{a}) - \Phi\left(r\sigma - \tilde{b}\right)\right) e^{\frac{1}{2}r(r-1)\sigma^2}, \tag{53}$$

where

$$\tilde{a} = \left(\ln a + \frac{1}{2}\sigma^2\right)/\sigma \text{ and } \tilde{b} = \left(\ln b + \frac{1}{2}\sigma^2\right)/\sigma. \tag{54}$$

Equilibrium

Dynamics of density function of real wages

Differentiation of (17) yields the recursive equation for the density function

$$\begin{aligned} g_t(w) = {} & (1-\lambda) z_t{}'(w) f\left(z_t(w)\right) \\ & +\lambda G_{t-1}\left(w(1+\pi_t)\right) z_t{}'(w) f\left(z_t(w)\right) \\ & +\lambda (1+\pi_t) g_{t-1}\left(w(1+\pi_t)\right) F\left(z_t(w)\right). \end{aligned} \tag{55}$$

Each of the lines in this expression corresponds to the respective type of worker discussed in the main text.

Equilibrium under flexible wages, $\boldsymbol{\lambda = 0}$, equation (15)

When wages can be adjusted flexibly they are set such that

$$\widehat{w}_{it} = \left(\frac{\eta}{\eta-1}\right)^{\frac{\gamma}{\eta+\gamma}} Z_{it}^{\frac{\gamma}{\eta+\gamma}} L_t^{\frac{\gamma+1}{\eta+\gamma}}. \tag{56}$$

The corresponding level of labor demand is given by

$$L_{it} = \left(\frac{1}{w_{it}}\right)^{\eta} L_t = \left(\frac{\eta-1}{\eta}\right)^{\frac{\eta\gamma}{\eta+\gamma}} \left(\frac{1}{Z_{it}}\right)^{\frac{\eta\gamma}{\eta+\gamma}} L_t^{-\frac{\gamma(\eta-1)}{\eta+\gamma}}. \tag{57}$$

Such that

$$L_t = \left[\int_0^1 L_{it}^{\frac{\eta-1}{\eta}} di\right]^{\frac{\eta}{\eta-1}} = \left(\frac{\eta-1}{\eta}\right)^{\frac{\eta\gamma}{\eta+\gamma}} L_t^{-\frac{\gamma(\eta-1)}{\eta+\gamma}} \left[\int_0^1 \left(\frac{1}{Z_{it}}\right)^{\frac{\gamma(\eta-1)}{\eta+\gamma}} di\right]^{\frac{\eta}{\eta-1}}. \tag{58}$$

Solving this for $Y_t = L_t$, this yields

$$L_t = \left(\frac{\eta-1}{\eta}\right)^{\frac{\gamma}{1+\gamma}} \left[\int_0^1 \left(\frac{1}{Z_{it}}\right)^{\frac{\gamma(\eta-1)}{\eta+\gamma}} di\right]^{\frac{\gamma+\eta}{(\eta-1)(1+\gamma)}}. \tag{59}$$

This expression simplifies to (15).

Equilibrium under downwardly rigid wages, $\lambda > 0$, equation (18)

In equilibrium the real wage equals one. This means that

$$\begin{aligned} 1 &= \left[\int_0^1 \left(\frac{1}{w_{it}}\right)^{\eta-1} di\right]^{-\frac{1}{\eta-1}} = \left[\int_0^1 \left(\frac{1}{w_{it}}\right)^{\eta-1} di\right] = \left[\int_0^1 \left(\frac{1}{\widehat{w}_{it}}\right)^{\eta-1} \left(\frac{\widehat{w}_{it}}{w_{it}}\right)^{\eta-1} di\right] \\ &= \left(\frac{\eta-1}{\eta}\right)^{\frac{\gamma(\eta-1)}{\eta+\gamma}} L_t^{\frac{(1+\gamma)(\eta-1)}{\eta+\gamma}} \left[\int_0^1 \left(\frac{1}{Z_{it}}\right)^{\frac{\gamma(\eta-1)}{\eta+\gamma}} \left(\frac{\widehat{w}_{it}}{w_{it}}\right)^{\eta-1} di\right]^{-\frac{1}{\eta-1}}. \end{aligned} \tag{60}$$

We divide the integral in this equation into three parts, corresponding to (*i*) workers that are not subject to the DNWR constraint, (*ii*) workers that are subject to it but for whom it is not binding, and (*iii*) workers for whom the constraint is binding. Doing so, we can write

$$\begin{aligned} 1 = &\left(\frac{\eta-1}{\eta}\right)^{\frac{\gamma(\eta-1)}{\eta+\gamma}} L_t^{-\frac{(1+\gamma)(\eta-1)}{\eta+\gamma}} \\ &\times \left\{(1-\lambda)\int_0^\infty \left(\frac{1}{Z_{it}}\right)^{\frac{\gamma(\eta-1)}{\eta+\gamma}} \left(\frac{\widehat{w}_t(Z)}{w_t(Z)}\right)^{\eta-1} dF(Z)\right. \\ &+\lambda \int_0^\infty \left(\frac{1}{Z_{it}}\right)^{\frac{\gamma(\eta-1)}{\eta+\gamma}} G_{t-1}\big(w_t(Z)(1+\pi_t)\big) \left(\frac{\widehat{w}_t(Z)}{w_t(Z)}\right)^{\eta-1} dF(Z) \\ &\left.+\lambda \int_0^\infty \left[\int_{w_t(Z)}^\infty (1+\pi_t) g_{t-1}\big(w(1+\pi_t)\big) \left(\frac{\widehat{w}_t(Z)}{w}\right)^{\eta-1} dw\right] dF(Z)\right\}. \end{aligned} \tag{61}$$

When we define

$$\begin{aligned} Z_t^* = &\left\{(1-\lambda)\int_0^\infty \left(\frac{1}{Z}\right)^{\frac{\gamma(\eta-1)}{\eta+\gamma}} \left(\frac{\widehat{w}_t(Z)}{w_t(Z)}\right)^{\eta-1} dF(Z)\right. \\ &+\lambda \int_0^\infty \left(\frac{1}{Z}\right)^{\frac{\gamma(\eta-1)}{\eta+\gamma}} G_{t-1}\big(w_t(Z)(1+\pi_t)\big) \left(\frac{\widehat{w}_t(Z)}{w_t(Z)}\right)^{\eta-1} dF(Z) \\ &\left.+\lambda \int_0^\infty \left(\frac{1}{Z}\right)^{\frac{\gamma(\eta-1)}{\eta+\gamma}} \left[\int_{w_t(Z)}^\infty (1+\pi_t) g_{t-1}\big(w(1+\pi_t)\big) \left(\frac{\widehat{w}_t(Z)}{w}\right)^{\eta-1} dw\right] dF(Z)\right\}^{-\frac{\eta+\gamma}{\gamma(\eta-1)}}, \end{aligned} \tag{62}$$

Then the equilibrium condition can be written as

$$1 = \left(\frac{\eta - 1}{\eta}\right)^{\frac{\gamma(\eta-1)}{\gamma+\eta}} \left(\frac{1}{Z_t^*}\right)^{\frac{\gamma(\eta-1)}{\gamma+\eta}} L_t^{-\frac{(1+\gamma)(\eta-1)}{\eta+\gamma}}. \tag{63}$$

Solving this with respect to $Y_t = L_t$ yields that

$$Y_t = L_t = \left(\frac{\eta - 1}{\eta}\right)^{\frac{\gamma}{1+\gamma}} \left(\frac{1}{Z_t^*}\right)^{\frac{\gamma}{1+\gamma}}. \tag{64}$$

This is equation (18) in the main text.

Numerical method

Approximation of the distribution and density functions of real wages, $\boldsymbol{G_t(w)}$ and $\boldsymbol{g_t(w)}$

Throughout, we use polynomial approximations for the value function $V_t(w)$ and the wage-setting schedule, $w_t(Z)$ and $z_t(w)$. We truncated the log-normal distribution of Z at $\underline{Z}$ and $\bar{Z}$, such that $F(\underline{Z}) = 1 - F(\bar{Z}) = 0.005$. Truncating the distribution of the disutility shocks means that, at a non-negative target inflation rate, $\bar{\pi}$, the support of the steady-state real wage distribution is bounded. This, in turn, means that the support of the real wage distribution is bounded at any point along the transition path. We then use Piecewise Cubic Hermite Interpolating Polynomials to approximate $G_t(w)$ on this bounded support. Since these polynomials are differentiable, they also allow us to calculate $g_t(w)$.

Steady state solution

In the steady state the inflation rate equals the inflation target, $\bar{\pi}$. Steady state is solved by iterating over the following steps: (*i*) For a given level of output $Y_t = L_t$, iterate over the Bellman equation until convergence. This gives the wage-setting schedule, $w_t(Z)$ and $z_t(w)$. (*ii*) Given the wage-setting schedule, iterate over the recursive equation for the distribution function of real wages $G_t(w)$ to obtain the corresponding real wage distribution. (*iii*) Give the wage-setting schedule and the distribution of real wages, solve for the level of L_t that equates the real wage to zero using the equilibrium condition under downwardly rigid nominal wages.

Transition path

The application of calculating the transition path using the extended path method involves the following steps. The path is calculated for $t = 1 \ldots T$ under the assumption that the economy is in steady state at time $t = 0$ and at time $T + 1$. We start off assuming the economy is in steady state along the whole path, except that the discount rate shock is not zero along that path. (*i*) In the first step we use a backward recursion for the policy rule to obtain the nominal interest rate and inflation rate that the central bank would set in response to the path of $\{Y_t, D_t\}_{t=1}^{T}$. (*ii*) For the path of the inflation rate $\{\pi\}_{t=1}^{T}$ we then solve the path of the optimal wage setting schedule $\{w_t(Z)\}_{t=1}^{T}$ through backward recursion using the assumption that the economy is in steady state for $t > T$. (*iii*) Given the path of $\{w_t(Z)\}_{t=1}^{T}$ we then use forward recursion to solve the path of $\{G_t(w)\}_{t=1}^{T}$. (*iv*) Given the paths of $\{w_t(Z)\}_{t=1}^{T}$ and $\{G_t(w)\}_{t=1}^{T}$ we then calculate the implied path of $\{Y_t\}_{t=1}^{T}$. We iterate over steps (*i*) through (*iv*) until convergence.

Appendix B: Data details

Current Population Survey (CPS)

The CPS is a monthly survey conducted by the Bureau of Labor Statistics. CPS participants stay with the survey for 18 months and over the course of that period are asked to report on earnings twice, once towards the beginning of their time and once towards the end-with the intervening time span equaling 12 months. Since we are interested in wage changes among all workers in the economy, our sample includes both workers who remain with their employer and those who change jobs during the period over which they report their earnings. Similarly, we include both salaried workers and workers paid at an hourly rate. For those paid at an hourly rate we use reported hourly earnings; for salaried workers we impute an hourly wage by dividing weekly earnings by hours worked per week. Nominal wage changes are computed for each worker as the difference in their reported log wages. Our final CPS dataset contains monthly information on the distribution of nominal wage changes for a representative sample of U.S. workers for the period from January 1980 to February 2013. For more details regarding using the CPS for this purpose see Daly, Hobijn and Wiles (2011).

Matching individuals across CPS interviews in different months

Our CPS dataset is constructed using the outgoing CPS monthly files from the BLS. We match individuals across different months using the matching algorithm detailed in Daly, Hobijn, and Wiles (2011). Specifically, individuals must match on age (+3 or -1 years between subsequent interviews), race, household ID, and line number. The Census scrambled household identifiers in July 1985 and June 1995, so we cannot match individuals one year forward from this date. This means that our dataset contains no wage change data from July 1985 to June 1986 and from June 1995 to May 1996. We use all 8 interview months (IMs) in order to match individuals and to determine if they are job stayers, but our final dataset contains only the outgoing rotation groups (IM 4 and IM 8) which are the only rotation groups with earnings information.

Constructing wage variables

When collecting a worker's earnings data, the BLS ascertains if the worker in question is paid at an hourly rate. If so, the worker's hourly rate of pay excluding overtime, tips, and commissions is recorded. We use this measure of earnings when available. Otherwise, we divide usual weekly earnings by usual hours worked per week to create an imputed hourly rate of pay. The BLS collects usual weekly earnings data by asking respondents to report their earnings in the easiest way, and then converting to a weekly rate of pay. Workers that are paid at an hourly rate and do not make more than the binding minimum wage in their state of residence (in either IM 4 or IM 8) are dropped from the sample. We also drop workers with any top-coded earnings data and restrict our analysis to workers age 16 and older. Our final dataset consists of 12-month rolling panels of hourly wage data from January 1980 to February 2013. Wage changes are calculated as the difference in the log of hourly wages from one year ago. In other words, in our dataset a wage change of 0.1 in January 2013 means that the log of a worker's reported or imputed hourly wage in January 2013 was 0.1 greater than the log of his or her hourly wage in January 2012.

Wage composite for wage Phillips curve

The four wage series we use to construct our composite index of wage growth are Compensation per hour (CPH), the Employment Cost Index (ECI), Median usual Weekly Earnings (MWE), and Average Hourly Earnings (AHE). Galí (2011) plots separate wage Phillips curves for CPH and AHE. Combining the four most commonly used wage measures follows the idea of Justiniano, Primiceri, and Tambalotti (2011) who include separate measurement equations for CPH and AHE in the state-space representation of their model and estimate the time-series as wages as the, structurally identified, factor that drives both. Adding the ECI and MWE turns out to give a lot of additional information. Figure B1 plots the four series and their first principle component which we use as our composite measure of nominal wage growth.

Figure 1. Distribution of 12-month log wage changes in 2006 and 2011.

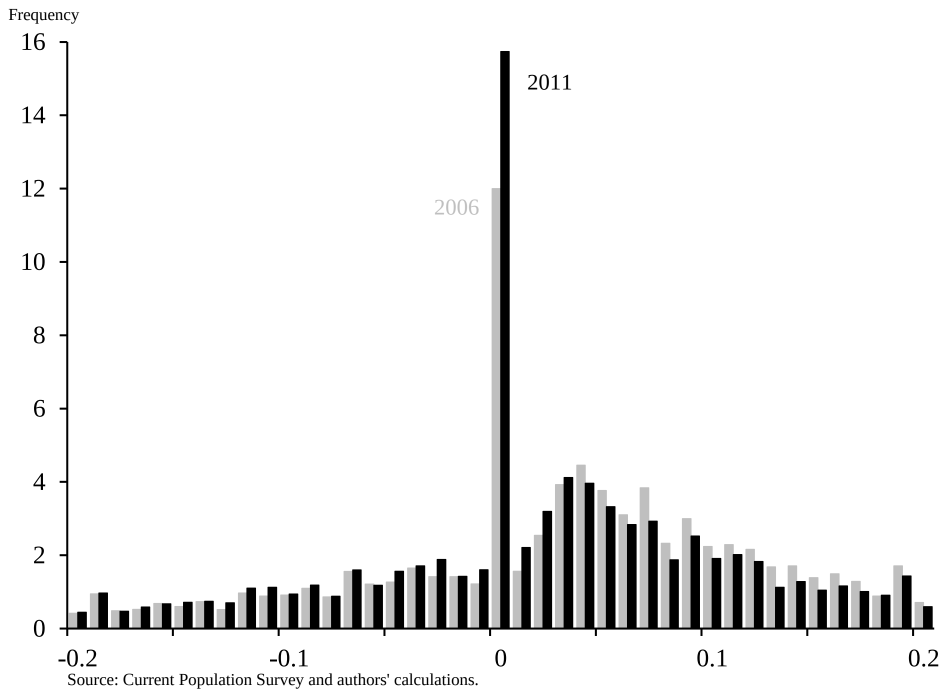


Figure 2. Fraction of workers reporting the same wage as one year prior.

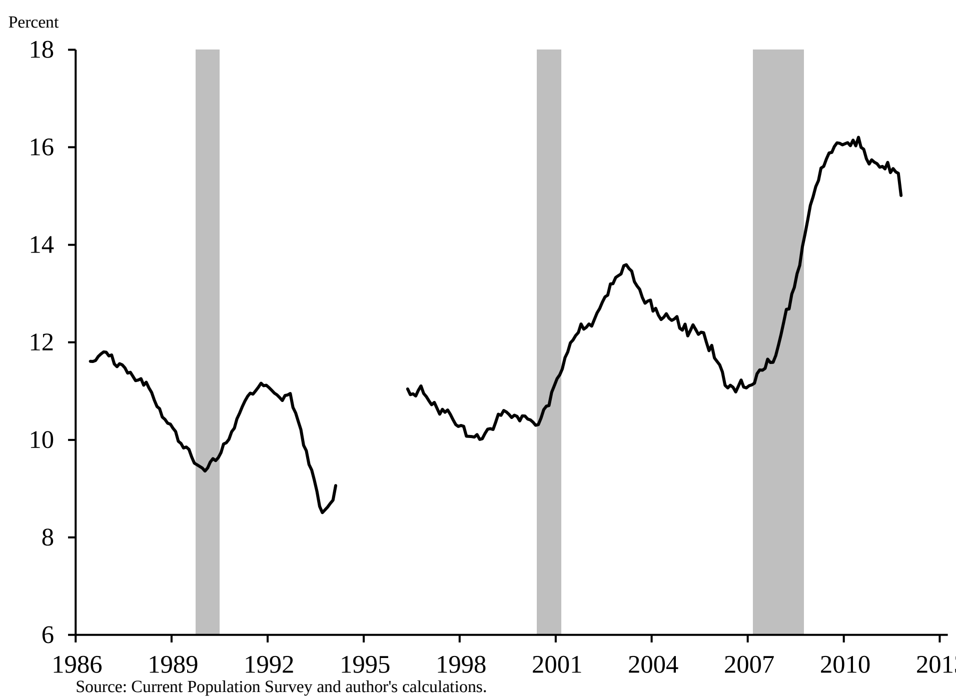
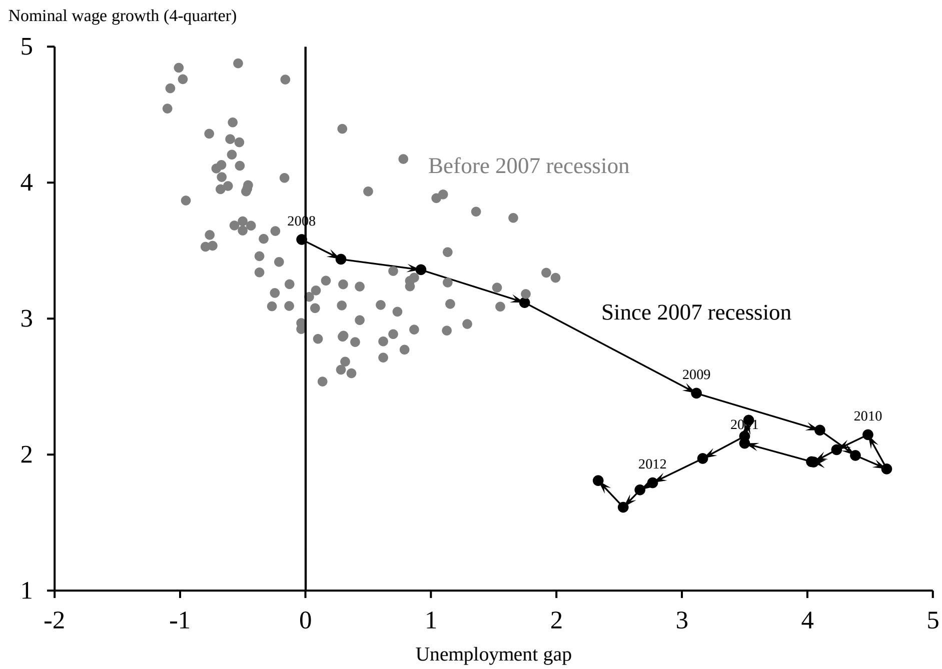


Figure 3. The U.S. wage Phillips curve: 1986-2012.



Source: Bureau of Labor Statistics, Congressional Budget Office, and author's calculations

Figure 4. Different groups of workers that determine dynamics of distribution of real wages.

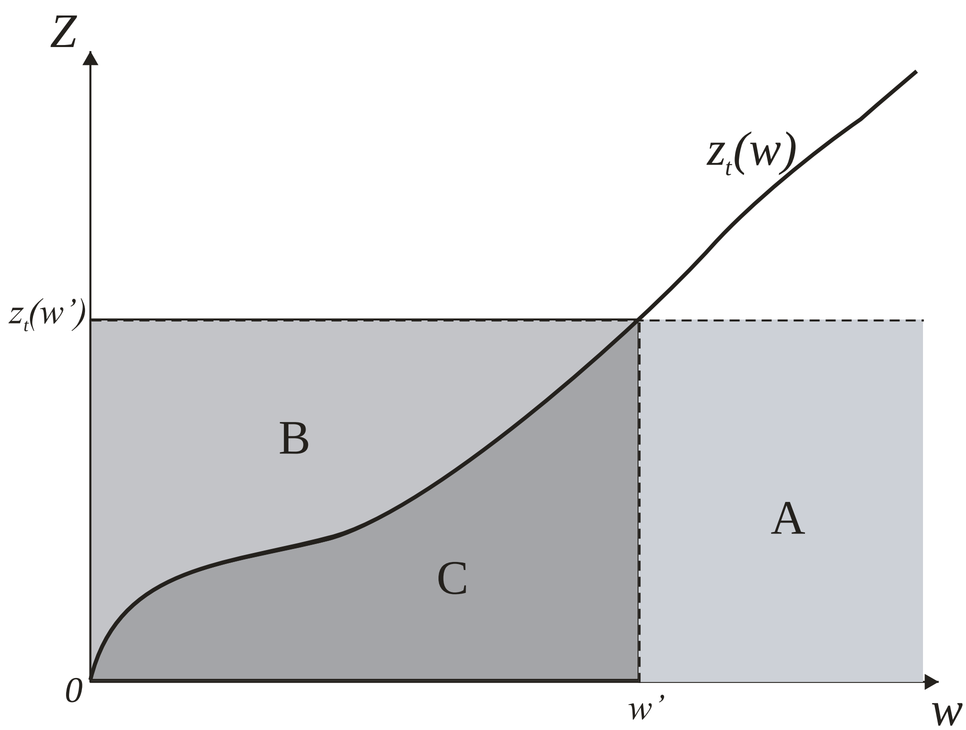


Figure 5. Long-run Phillips curve for parameter calibration.

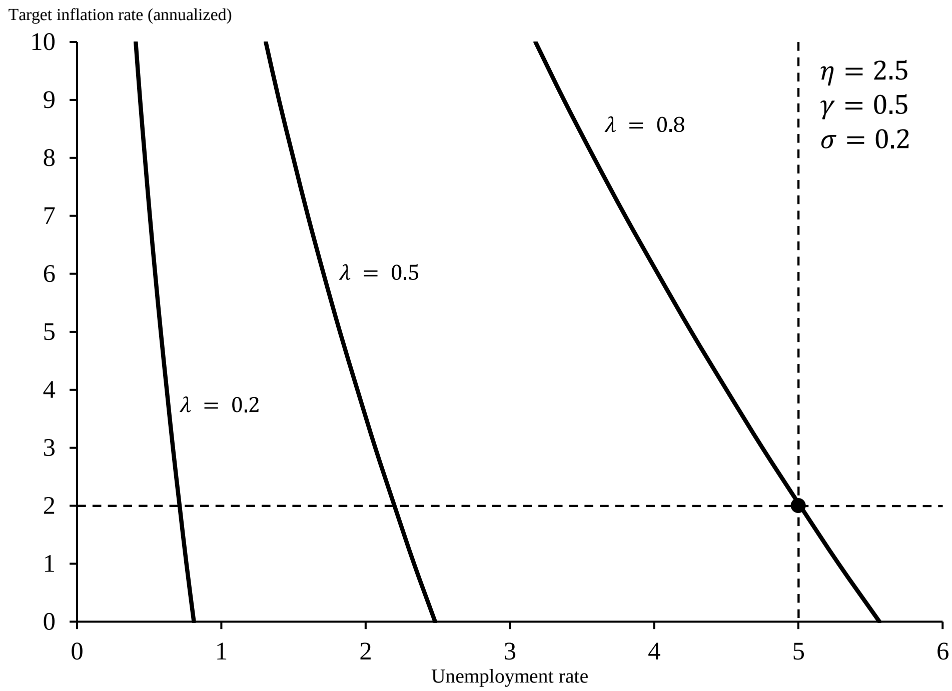


Figure 6. Steady-state distribution of quarterly $\Delta \ln W_{it}$ under flexible wages and DNWR.

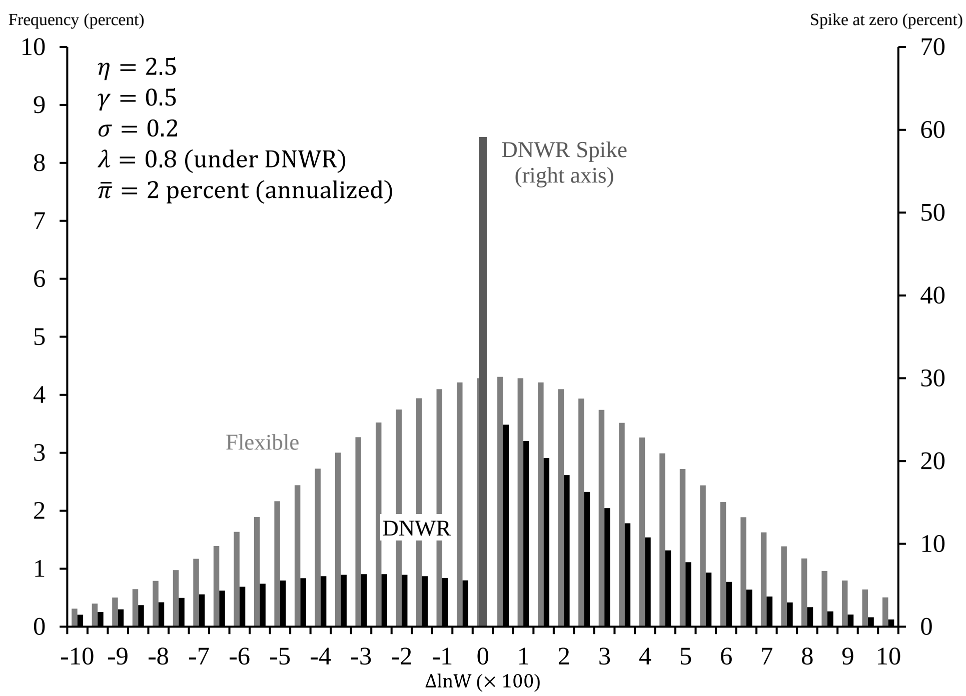


Figure 7. Distribution of quarterly $\Delta \ln W_{it}$ in steady state and after shock $D_1 = -0.03$ at $t = 2$.

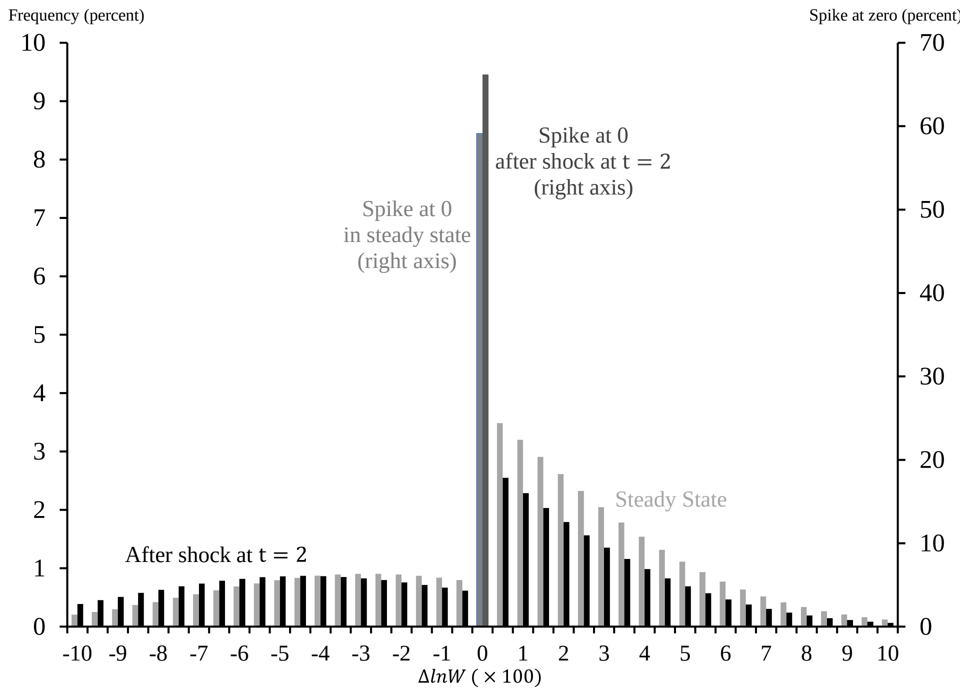


Figure 8. Fraction of workers with same wage as 4 quarters ago, for $D_1 = -0.03$.

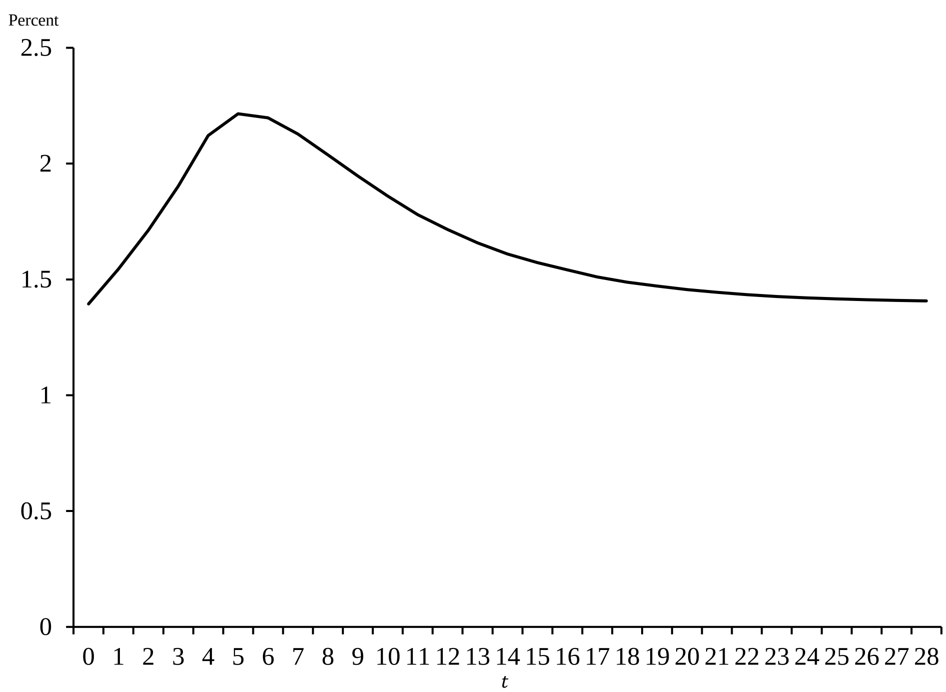


Figure 9. Short-run (wage) Phillips curve in response to 3 different discount rate shocks.

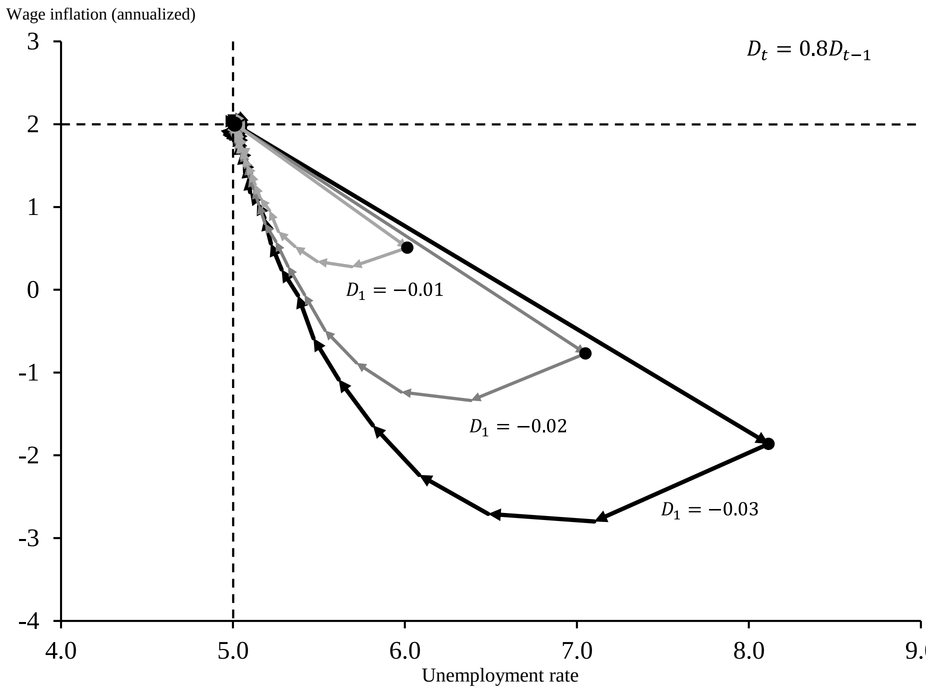


Figure 10. Short-run (wage) Phillips curve at different target inflation rates.

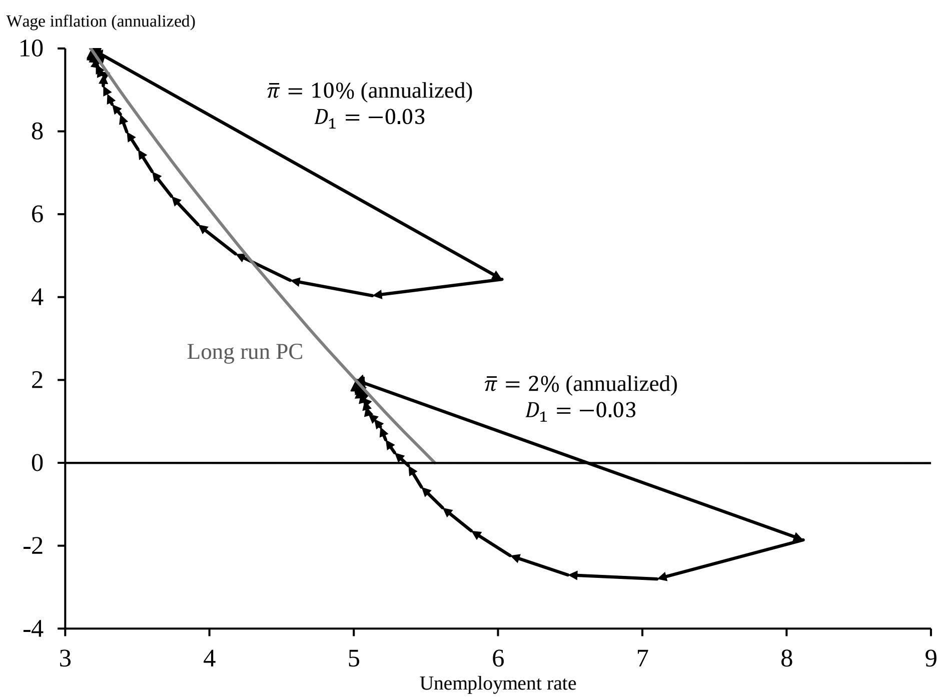
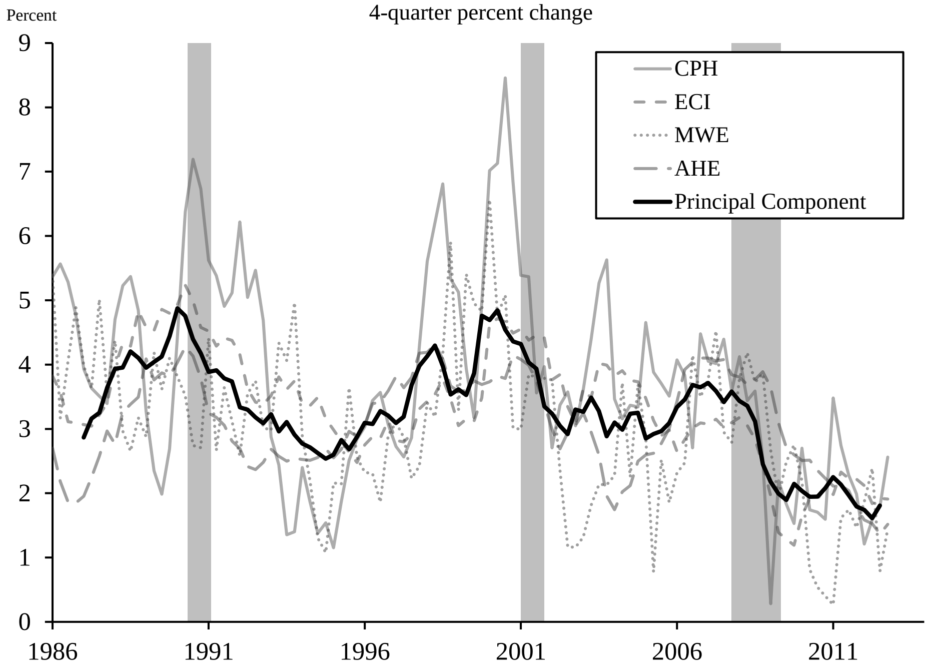


Figure B1. Four measures of nominal wage growth and their first principle component.



Source: Bureau of Labor Statistics and authors' calculations