Ex Ante Optimal Market Design and Financial Sector Regulation

Robert Townsend

Massachusetts Institute of Technology

The Federal Reserve Bank of Boston – April 2014
The views expressed in this presentation do not necessarily reflect the views of the Federal Reserve System.
Introduction

- How should we think about situations in which multiple participants interact with one another
  - ex: willingness to join platforms or trading/communication networks depends on the number and composition of buyers and sellers (two sided markets)
  - ex: there are fire sales in financial markets where downward pressure on prices cause mark-to-market, further tightening, adverse cascades

- Are these “externalities" something which (only) regulation can deal with?
  - i.e. is there a role of Federal Reserve in:
    - regulating exchange fees?
    - regulating interest rates, portfolios, capitalization?
Answer: under assumptions, general equilibrium theory has a way of dealing with the problems, in a manner suggested by Meade (1952) and Arrow (1969), internalize the externality

- ex ante, via contracting/pricing in the first example
- in the creation of new markets for clearing collateral in the second
Related Literature

The Economics of Platforms in a Walrasian Framework

Anil Jain (MIT)

Robert Townsend (MIT)
Motivating Examples

- A successful platform often needs to intermediate between buyers and sellers
- We are interested in platforms where buyers and sellers care about the composition of the platform’s users.
- A credit card company must attract both consumers and merchants
- Dark pools and exchanges must attract buyers and sellers
Question

- In the context of multiple competing platforms is there a Walrasian equilibrium?
- Is the Walrasian equilibrium efficient?
- Is there a role for regulation due to a possible network externality?
Set-up

- There are two types – buyers (A) and sellers (B)
- Buyers and sellers can trade only on a platform
- Buyers and sellers care about the composition of the platform’s users
- Buyers and sellers each have some capital endowment ($\kappa$)
- There are buyer and seller subtypes who may vary in their capital endowment
Utility

- A buyer’s (A) utility function is:

\[ U_A(N_A, N_B) = U_A(N_A, N_B) = \begin{cases} 0 & \text{if } N_A \text{ or } N_B = 0 \\ \left( \frac{N_B}{N_A} \right)^{\gamma_A} + N_B^{\epsilon_A} & \text{else} \end{cases} \]

- A buyer wants a higher ratio of sellers to buyers, and larger platforms.

- Symmetrically, the seller’s (B) utility function is:

\[ U_B(N_A, N_B) = U_B(N_A, N_B) = \begin{cases} 0 & \text{if } N_A \text{ or } N_B = 0 \\ \left( \frac{N_A}{N_B} \right)^{\gamma_B} + N_A^{\epsilon_B} & \text{else} \end{cases} \]
Cost of making a platform

- A platform is costly to manufacture:

\[ C(N_A, N_B) = \begin{cases} 
0 & \text{if } N_A = 0 \text{ or } N_B = 0 \\
c_A N_A + c_B N_B + c N_A N_B & \text{else}
\end{cases} \]

- Larger platforms are more than proportionally more expensive
Consumer’s problem

- Agent type \( T \), subtype \( i \) buys contracts \( b_T(N_A, N_B) \) to join a platform of size and composition \( (N_A, N_B) \).
- Agent \( (T, i) \) can only buy contract indexed by their type \( b_T(N_A, N_B) \).
- Subtype \( i \) vary in the wealth.
Consumer’s problem

- Agent \((T, i)\) takes prices \(p[b_T(N_A, N_B)]\) as given and solve the following maximization problem:

\[
\max_{x_{T,i} \in \{0,1\}} \sum_{N_A, N_B} x_{T,i}[b_T(N_A, N_B)] U_T[b_T(N_A, N_B)] \tag{1}
\]

s.t. \[
\sum_{N_A, N_B} x_{T,i}[b_T(N_A, N_B)] p[b_T(N_A, N_B)] \leq \kappa_{T,i} \tag{2}
\]

\[
\sum_{N_A N_B} x_{T,i}[b_T(N_A, N_B)] = 1 \tag{3}
\]

- Where for now consider \(x_{T,i}\) is an indicator function for which platform an agent type \((T, i)\) joins
Competitive Equilibrium

- A competitive equilibrium in this economy is $(p, x, y) \in L \times X \times Y$ such that
  - For given prices, the allocation solves the consumer and platform maximization problems.
  - All markets clear: the demand for each contract equals the supply of each contract.
  - Active platforms are populated by numbers of buyers and sellers as anticipated (stipulated)
Summary of results

- A competitive equilibrium exists
- The competitive equilibrium is efficient
  - The first and second welfare theorems hold in our modified environment
- The endogenous pricing internalizes the effect of changing the composition of the platform – overcoming any network externality – as in Arrow (1969)
Examples

- Consider a platform with 2 subtypes of buyers, and 2 subtypes of sellers
- There is a measure 1 of each type, a measure 0.5 of each subtype
- Each agent is equally wealthy

<table>
<thead>
<tr>
<th>Platform Size</th>
<th>Number of Platforms created</th>
<th>Cost of Production</th>
</tr>
</thead>
<tbody>
<tr>
<td>((N_A, N_B))</td>
<td>0.5</td>
<td>8</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Type ((T, i))</th>
<th>Wealth</th>
<th>Platform joined ((N_A, N_B))</th>
<th>Price of joining</th>
<th>Pr((\text{joining}))</th>
<th>Utility on Platform</th>
</tr>
</thead>
<tbody>
<tr>
<td>(A, 1)</td>
<td>2</td>
<td>((2, 2))</td>
<td>2</td>
<td>1</td>
<td>2.41</td>
</tr>
<tr>
<td>(A, 2)</td>
<td>2</td>
<td>((2, 2))</td>
<td>2</td>
<td>1</td>
<td>2.41</td>
</tr>
<tr>
<td>(B, 1)</td>
<td>2</td>
<td>((2, 2))</td>
<td>2</td>
<td>1</td>
<td>2.41</td>
</tr>
<tr>
<td>(B, 2)</td>
<td>2</td>
<td>((2, 2))</td>
<td>2</td>
<td>1</td>
<td>2.41</td>
</tr>
</tbody>
</table>
What happens if we increase one type’s wealth \((B,2)\)?

- Now \((B,2)\) are willing and able to pay to join larger platforms and with better ratio of participants
- \((B,2)\) “sponsor” larger platforms – lower prices for Type \((A)\)

<table>
<thead>
<tr>
<th>Platform Size ((N_A, N_B))</th>
<th>Number of Platforms created</th>
<th>Cost of Production</th>
</tr>
</thead>
<tbody>
<tr>
<td>((3, 2))</td>
<td>0.25</td>
<td>11</td>
</tr>
<tr>
<td>((1, 2))</td>
<td>0.25</td>
<td>5</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Type ((T, i))</th>
<th>Wealth</th>
<th>Platform joined ((N_A, N_B))</th>
<th>Price of joining</th>
<th>(\Pr(\text{joining}))</th>
<th>Utility on Platform</th>
<th>Expected Utility</th>
</tr>
</thead>
<tbody>
<tr>
<td>(A, 1)</td>
<td>1.37</td>
<td>((3, 2))</td>
<td>1.37</td>
<td>1</td>
<td>2.23</td>
<td>2.23</td>
</tr>
<tr>
<td>(A, 2)</td>
<td>1.64</td>
<td>((3, 2))</td>
<td>1.37</td>
<td>0.5</td>
<td>2.23</td>
<td>2.53</td>
</tr>
<tr>
<td></td>
<td></td>
<td>((1, 2))</td>
<td>1.91</td>
<td>0.5</td>
<td>2.8</td>
<td></td>
</tr>
<tr>
<td>(B, 1)</td>
<td>1.54</td>
<td>((1, 2))</td>
<td>1.54</td>
<td>1</td>
<td>1.7</td>
<td>1.7</td>
</tr>
<tr>
<td>(B, 2)</td>
<td>3.45</td>
<td>((3, 2))</td>
<td>3.45</td>
<td>1</td>
<td>2.96</td>
<td>2.96</td>
</tr>
</tbody>
</table>
How does the equilibrium change as redistribute wealth?

- Consider an economy with a fixed amount of resources.
- Let us not change \((A, 2)\) or \((B, 2)\)'s wealth. But reallocate wealth across \((A, 1)\) and \((B, 1)\). How does the equilibrium change?
- As we increase \((B, 1)\)'s wealth, the price for type B contracts rise (as we are increasing demand for type B contracts)
- This causes negative effects on \((B, 2)\)'s utility
Conclusion

▶ Even when agents’ preferences are dependent on a platform’s composition – the competitive equilibrium is efficient.

▶ The endogenous pricing internalizes the benefits form changing a platform’s composition.

▶ Using the GE framework we can examine how changes in wealth, preferences, affect the resulting equilibrium.

▶ A decrease in the cost of creating platforms may help the poorest agents the most.

▶ A change in a buyers’ preferences or wealth, can have substantial effects on the size of platforms and subsequently sellers’ utility.
Segregated Security Exchanges with Ex-Ante Rights to Trade

A Market Based Solution to Collateral Constrained Externalities

Weerachart T. Kilenthong (UTCC)

Robert Townsend (MIT)
Collateral Model and Inefficiency Problem

- This paper studies collateral and default in a competitive general equilibrium model with:
  - Directly-collateralized.
  - Asset-backed securities.

- The interaction between the endogenous valuation of a collateral and corresponding default decisions creates a “pecuniary externality.”
Main Contribution

▶ We view this externality problem as a missing market problem (Arrow, 1969)
  ▶ The markets for contracts over the “market fundamentals”, which determine the market-clearing price of collateral, are missing.
▶ We internalize the externality by creating security exchanges at which the value of collateral used to unwind commitments is pre-determined, for the entire range of values for collateral, including out of equilibrium values.
▶ Household types will pay or be paid for their influence on the spot market prices determining the value of collateral
Timing, States, and Commodities

- Two periods: \( t = 0, 1 \).
- In period 1: the probability of state \( s \) is \( \pi_s \), for \( s = 1, 2, \ldots, S \).
- Two physical goods: good 1 and good 2.
  - Good 1 is not storable, used as a numeraire good.
  - Good 2 is storable, and can be used as collateral \( \Rightarrow \) endogenous collateral.
- There are \( H \) types each with continuum agents of mass \( \alpha^h \).
- Endowment of agent \( h \): \((e_{10}^h, e_{20}^h, e_{1s}^h, e_{2s}^h)\).
- Storage technology: One unit of good 2 will become \( R_s \) units of good 2 in state \( s \).
- Expected utility:

\[
u \left( c_{10}^h, c_{20}^h \right) + \beta \sum_{s=1}^{S} \pi_s u \left( c_{1s}^h, c_{2s}^h \right)
\]

\( u \left( c_1^h, c_2^h \right) \) is homothetic.
Market Fundamentals

- **Market fundamental** in state $s$, $z_s$, determines the spot price of good-2 relative to good 1 (the numeraire). It is a sufficient statistic for the spot price.

- With homothetic preferences, $z_s$ is the aggregate ratio of good-1 to good-2: $\frac{p_2}{p_1}(z_s)$ where

$$z_s = \frac{\sum_h \alpha^h e^h_{1s}}{R_s K + \sum_h \alpha^h e^h_{2s}}$$

where $k^h$ is the collateral/storage held by agent $h$ in period-0, and $K = \sum_h \alpha^h k^h$ is the average collateral.

- With market fundamental $z_s$, spot-market-clearing price is $p(z_s)$.

- With collateral constraint:

$$p(z_s)R_sk^h + \hat{\theta}_s + p(z_s)\theta^h_s \geq 0, \forall s$$
Definition (Competitive Collateral Equilibrium)

A competitive collateral equilibrium is a specification of prices of good 2 in period \( t = 0 \), \( P_{20} \), the prices of securities paying in good 1, \( \hat{P}_a \), and the prices of securities paying in good 2, \( P_a \), the spot price of good 2 in each state \( s \), \( p(z_s) \), and an allocation \( (c^h_0, k^h, \hat{\theta}^h, \theta^h) \), such that

- for any agent type \( h \), \( (c^h_0, k^h, \hat{\theta}^h, \theta^h) \) solves
  \[
  \max \left( c^{10}_h, c^{20}_h \right) + \beta \sum_s \pi_s u \left( e^{1s}_h + \hat{\theta}^h_s, e^{2s}_h + R_k^h + \theta^h_s \right)
  \]
  subject to the collateral and budget constraints:
  \[
  p(z_s) R_k^h + \hat{\theta}^h_s + p(z_s) \theta^h_s \geq 0, \forall s,
  \]
  \[
  c^{10}_h + P_{20} \left( c^{20}_h + k^h \right) + \sum_s \hat{P}_a \hat{\theta}^h_s + \sum_s P_a \theta^h_s \leq e^{10}_h + P_{20} e^{20}_h,
  \]
  taking prices \( (P_{20}, \hat{P}_a, P_a) \) as given;
- Goods and security markets clear.
The Externality: Inefficiency of A Competitive Collateral Equilibrium

- A competitive collateral equilibrium allocation satisfies:

\[
\frac{u^h_{20}}{u^h_{10}}\bigg|_{ce} = \sum_s \pi_s \beta \frac{u^h_{2s}}{u^h_{10}} R_s + \sum_s \gamma^h_{cc-s} p(z_s) R_s, \forall h.
\]

- A collateral-constrained optimal allocation satisfies:

\[
\frac{u^h_{20}}{u^h_{10}}\bigg|_{po} = \sum_s \pi_s \beta \frac{u^h_{2s}}{u^h_{10}} R_s + \sum_s \frac{\mu^h_{cc-s}}{\mu^h_{u} u^h_{10}} p(z_s) R_s
\]

\[- \sum_s \frac{\alpha^h}{\mu^h_{u} u^h_{10}} \frac{p'(z_s)}{p(z_s)} \partial z_s \sum_{\hat{h}} \mu^h_{cc-s} \hat{\theta}^h_s, \forall h.\]
The Deviation from the Market Fundamental as the Rights to Trade

- A key object is the type \( h \)'s deviation from the market fundamental in state \( s \):
  \[
  \Delta^h_s (z_s) = \left( e^h_{2s} + R_s k^h \right) \left( z_s - \frac{e^h_{1s}}{e^h_{2s} + R_s k^h} \right), \forall s,
  \]
  where \( z_s \) is the market fundamentals in state \( s \). This object reflects the “marginal impact” of each type on others at an equilibrium price.

- To be able to trade in a security exchange \( z_s \) in state \( s \), an agent type \( h \) needs to hold exactly \( \Delta^h_s (z_s) \) units of the rights to trade.

- Price Time Quantity equals Value
  \[
P_{\Delta} (z_s, s) \Delta^h_s (z_s)
  \]
Environment I: Intertemporal Smoothing

- No uncertainty $S = 1$. Discount factor $\beta = 1$.
- Two types of agents $H = 2$, with $\alpha^h = \frac{1}{2}$ for all $h$.
- Storage technology is constant with $R = 1$.
- Utility function:

$$u(c_1, c_2) = -\frac{1}{c_1} - \frac{1}{c_2}, \forall h.$$ 

- Endowments: agent $h = 1$ is relatively well-endowed in period $t = 0$, and vice versa. Think of agent $h = 1$ as a potential “lender” and $h = 2$ as a potential “borrower”.

<table>
<thead>
<tr>
<th>Type of Agent</th>
<th>$t = 0$</th>
<th>$t = 1$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$e_{10}^h$</td>
<td>$e_{20}^h$</td>
</tr>
<tr>
<td>$h = 1$</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>$h = 2$</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>
Competitive Collateral Equilibrium versus Competitive Equilibrium with Segregated Security

<table>
<thead>
<tr>
<th></th>
<th>with externality</th>
<th>without externality</th>
</tr>
</thead>
<tbody>
<tr>
<td>$k$</td>
<td>1.3595</td>
<td>1.1753</td>
</tr>
<tr>
<td>$p_{20}$</td>
<td>2.2948</td>
<td>2.0073</td>
</tr>
<tr>
<td>$z$</td>
<td>0.7463</td>
<td>0.7729</td>
</tr>
<tr>
<td>$U^1$ (unconstrained)</td>
<td>-2.2527</td>
<td>-2.2936</td>
</tr>
<tr>
<td>$U^2$ (constrained)</td>
<td>-2.5724</td>
<td>-2.3905</td>
</tr>
</tbody>
</table>

- There are no security trade.
- Agent 2 is at binding collateral constraint with $k^2 = 0$ and agent $h = 1$ is not lending but saving.
- The collateral distortion causes too much savings, as proved in Proposition 2. The collateral distortion makes the price of good 2 higher in the first period and lower in the second period relative to the constrained optimality.
- Distributional general equilibrium effect: Internalizing the externality improves efficiency of the economy, but also redistributes wealth.
Trading in Rights to Trade Generates Transfers

- At the equilibrium, the budget constraint in $t = 0$ of a constrained agent ($h = 2$) can be rewritten as follows:

$$c_{10}^h(z^{op}) + P_{20}c_{20}^h(z^{op}) \leq e_{10}^h + P_{20}e_{20}^h + \left[-P_\Delta(z^{op}) \Delta^h(z^{op})\right]$$

Receiving Transfer $= T$

- In equilibrium, $P_\Delta(z^{op}) = 0.5375$, and $\Delta^2(z^{op}) = -0.6813$.

- The constrained agent ($h = 2$) is receiving a transfer of $-P_\Delta(z^{op})\Delta^2(z^{op}) = 0.3662$ in period $t = 0$ for being in the security exchange $z^{op} = 0.7729$.

- This shifts her budget line outward by $T = 0.3662$ as shown in the next figure.
Environment II: Heterogeneous Borrowers and the Role of Randomization

- No uncertainty, $S = 1$. Discount factor $\beta = 1$.
- Three types of agents $H = 3$, with $\alpha^h = \frac{1}{3}$ for all $h$.
- Storage technology is constant with $R = 1$.
- Utility function:

$$ u(c_1, c_2) = -\frac{1}{c_1} - \frac{1}{c_2}, \forall h. $$

<table>
<thead>
<tr>
<th>Type of Agents</th>
<th>$e_{10}^h$</th>
<th>$e_{20}^h$</th>
<th>$e_{11}^h$</th>
<th>$e_{21}^h$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$h = 1$</td>
<td>4.26</td>
<td>11.5</td>
<td>0.5</td>
<td>0.5</td>
</tr>
<tr>
<td>$h = 2$</td>
<td>3.92</td>
<td>0.5</td>
<td>7</td>
<td>5</td>
</tr>
<tr>
<td>$h = 3$</td>
<td>4.32</td>
<td>0.5</td>
<td>5</td>
<td>7</td>
</tr>
</tbody>
</table>
Kilenthong and Townsend (2014)

Competitive Collateral Equilibrium versus Competitive Equilibrium with Segregated Security

<table>
<thead>
<tr>
<th></th>
<th>$h = 1$</th>
<th>$h = 2$</th>
<th>$h = 3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$k$</td>
<td>6.3082</td>
<td>4.6072</td>
<td>0.0000</td>
</tr>
<tr>
<td>$\hat{\theta}$</td>
<td>1.3892</td>
<td>1.6384</td>
<td>-1.3159</td>
</tr>
<tr>
<td>$\tau$</td>
<td>-3.7176</td>
<td>-2.4776</td>
<td>1.9898</td>
</tr>
<tr>
<td>$c_{10}$</td>
<td>5.6204</td>
<td>5.6204</td>
<td>4.4835</td>
</tr>
<tr>
<td>$c_{20}$</td>
<td>3.3982</td>
<td>3.3982</td>
<td>2.7106</td>
</tr>
<tr>
<td>$c_{11}$</td>
<td>1.8892</td>
<td>2.1384</td>
<td>5.6841</td>
</tr>
<tr>
<td>$c_{21}$</td>
<td>3.0905</td>
<td>2.6296</td>
<td>6.9898</td>
</tr>
<tr>
<td>$z$</td>
<td>$0.6113$</td>
<td>$0.8132$</td>
<td>$0.8132$</td>
</tr>
<tr>
<td>$\Delta$</td>
<td>3.6618</td>
<td>3.6532</td>
<td>-2.9340</td>
</tr>
<tr>
<td>$x^h$</td>
<td>0.1969</td>
<td>0.8031</td>
<td>1.0000</td>
</tr>
<tr>
<td>$U^h$</td>
<td>-1.3211</td>
<td>-0.9110</td>
<td>-1.4483</td>
</tr>
</tbody>
</table>
Conclusion

- General equilibrium theory can price “externalities”
- In Jain and Townsend (2014) we show that perfect competition will internalise the network externalities into the pricing mechanism
  - Limitations:
    - We assume no platform has any market power.
    - Our model is purely static – no consideration for entry or innovation in the space of platforms.
- In Kilenthong and Townsend (2014) we can overcome the problem of externalities by the creation of security exchanges where the value of collateral is pre-determined.
  - This method generalizes to other environments, such as incomplete markets and information problems.
  - We do assume exclusivity but given technology underlying current financial markets this is feasible.