Housing as a Measure for Long-Run Risk in Asset Pricing

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Abstract

I evaluate the effects of long-run consumption growth risk and housing consumption risk on asset prices. Current asset values are affected by the risk-return tradeoff in the long-run. Housing plays an important role in the economy. As an asset, it is particularly sensitive to long-run risk-return trade off; as a consumption component, it accounts for one fifth of the total expenditures in non durable goods and services. The investment horizon for housing is usually distant in the future. Investors fear shocks that can affect the value of their house for a long period of time. Such shocks affect substantially the services obtained from the house and its price as an asset as well. I use a non-separable utility function with non-housing consumption and consumption of housing services, which generates an intertemporal composition risk, besides the traditional consumption growth risk. The composition risk has effects for the valuation of cash flow growth fluctuations far into the future due to the persistence of consumption growth. I provide a closed form solution for the valuation function despite the non-separability. This allows me to quantify the price of risk in the long-run with inputs from vector autoregressions. I evaluate the different exposure to long-run risk of a cross section of portfolios of securities and characterize the price of risk for different investment horizons. The model also explains the spread of the returns to different portfolios sorted by book to market and housing returns, at different investment horizons.

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Introduction

Equilibrium prices and expected returns under the consumption-based capital asset pricing model (CCAPM) are determined by consumption growth risk. Recent research[1] suggests that the use of long-run aggregate consumption risk helps explain cross-sectional and aggregate stock returns.

I propose a consumption based model that exploits the pricing implications of risk in the long-run and generates a time-varying risk premium. I use recursive preferences over non-housing consumption and consumption of housing services. Risk premia depend on the exposure of assets' cash flows to risk and change with the investment horizon considered. My model captures the spread between portfolios sorted by book-to-market in the short-run, as well as in the long-run.

The main contribution of the paper is the use of housing services as a source for persistence in consumption growth. When consumption growth is assumed to be independently distributed over time, as in the textbook CCAPM, agents do not fear shocks sufficiently to explain the excess returns that we observe in reality. In part, the temporary nature of the shocks helps agents not to fear them. Alternatively, consumption growth could be modeled with a persistent component. Therefore, when consumption growth is lower consumers fear that the bad times will last for a while. There is little empirical evidence supporting the existence of a persistent component that determines the expected growth rate of consumption every period. I construct a measure for consumption of non-durable goods and services with a constant elasticity of substitution aggregation of housing services and non-housing consumption of goods and services. Housing services are unambiguously persistent. Hence, it is well motivated to model housing services with a persistent component. An increase in the crime rate of the neighborhood is an example of persistent shock. When it happens it takes time to decrease crime rate: it requires coordination of the neighbors, deployment of

[1] Bansal and Yaron (2004) and Hansen, Heaton, and Li (2008) are examples of empirical success of long-run risk models.

police patrols, investment in education, etc. During that recovery period, the inhabitants of that area will enjoy less living in their houses. The market will reflect the situation in lower rental rates (which ultimately represent most of the measure of housing services in the BEA data).

There are two main features of the model. First, I propose an alternative constant elasticity of substitution aggregation between non-durable goods and housing services in the utility function. This measure differs from the measure of aggregate consumption of non-durable goods and services obtained directly from the Bureau of Economic Analysis. Second, I consider recursive preferences as first introduced in Epstein and Zin (1991). Individuals are concerned about three types of risk or, in finance terms, there are three priced factors: consumption growth risk, composition risk —that arises from the non-separable utility function— and long-run consumption growth risk, result of the recursive utility. I obtain an explicit solution for the pricing function (stochastic discount factor), identify the three factors of risk, and evaluate the response of the factors' pricing to shocks that have an effect in the long-run.

The representative consumer derives utility from consumption of non-housing goods and housing services, which are imperfect substitutes. Fluctuations in the expenditure shares of these goods have not only an effect on the expected returns but also on the long-run valuation of an asset's risky cash flows. Piazzesi, Schneider, and Tuzel (2007) have shown that a model with housing services offers an explanation for the long-horizon predictability of excess stock returns. As in the standard model, investors highly value consumption when a recession occurs and they sell claims on future consumption expecting it to be higher than consumption today. This is defined as the consumption growth risk, which appears in the standard consumption-based CAPM. Furthermore, investors are more fearful of changes in expenditure shares on housing —changes in the *composition* of their consumption bundle—. Claims on future streams of consumption are sold desperately in periods where the relative quantity of housing services consumption is low due to the substitutability. Hence, in very bad times, when the housing expenditures share is low, the intra-temporal substitution effect leads to even lower prices. A non-separable utility function allows us to identify better the links between risky asset returns and macroeconomic factors. Lustig and Nieuwerburgh (2004) present a similar model, where the housing collateral plays the role of the variable that predicts expected returns, since constrained homeowners, whose collateral value declines, become effectively more risk averse.

I use the class of "generalized expected utility" preferences proposed by Epstein and Zin (1991) in order to parameterize independently intertemporal elasticity of substitution and risk aversion. This class of representation of preferences has the major advantage of discerning between risk aversion and eagerness to smooth consumption over time, keeping the risk free rate close to observed values. The Euler equation obtained from the power utility model states that differences in risk across portfolios are due to contemporaneous covariances with consumption. Within a recursive utility framework, the agent does not need to perfectly smooth expected marginal utility over time and long-run consumption growth determines differences in risk. Mehra and Prescott (2003) acknowledge that this class of preferences could potentially solve the equity premium puzzle, with an empirical caveat. The original t analysis in Epstein and Zin (1991) hinges on the return on all invested wealth, which is not observable, and uses the market portfolio as a proxy for this variable. Instead of following the approximation approach, I solve explicitly for the value function as a function of the underlying state of the economy. There is recent empirical success of models that investigate the effects of the long-run consumption growth risk generated by the recursive preferences specification. Bansal and Yaron (2004), Bansal, Dittmar, and Lundblad (2005), and Hansen, Heaton, and Li (2008) present the theoretical background and the empirical findings. Colacito and Croce (2008) use the recursive utility framework with persistent shocks to consumption growth to analyze the international equity premium puzzle. In particular, Hansen, Heaton, and Li (2008) finds a closed form solution for the special case of unitary elasticity of intertemporal substitution, also avoiding the use of an approximation for the wealth portfolio[2]. They measure the long-run risk-return trade-off for the valuation of cash flows and solve a model where the price of risk is decomposed in one-period price of risk and long-run price of risk. Their model is successful in explaining the cross section of returns, in particular the spread between high and low book-to-market portfolio returns in the long-run. In addition, my model also offers an explanation for the particular dynamics of housing returns.

In summary, I obtain a three factor model, where the factors are three aggregate sources of macroeconomic risk: consumption growth risk, composition risk, and long-run risk.

The housing expenditure shares have a non-negligible effect on the long-run price of risk, in addition to the price of long-run consumption growth risk. This is imputable to the use of a non-separable utility function and recursive preferences at the same time. Different exposures of the assets to the long-run consumption risk help explain the cross-section of returns. Most importantly, it justifies to some extent the existence of a persistent component in consumption of non-durable goods and services. The autocorrelation of the consumption of housing services is 61% with aggregate data from the National Income and Product Accounts (NIPA). The autocorrelation of aggregate consumption of non-durable goods and services, which includes housing services, is only 24%. In this paper, I compute consumption of non-durable goods and services by aggregating 2 components following a generic constant elasticity of substitution utility function over the two components, which are housing services and non-housing consumption. Aggregate consumption is not assumed to be identically and independently distributed, since the persistence of the housing services component does not justify such assumption.

The time variation of risk premia is also an important element in asset pricing models. Expected returns are high in recessions, when people might be less willing to hold risky assets. It is a well established fact that stock returns can be predicted by instruments that are informative about the business cycle. Pakos (2006) and Yogo (2006) use the consumption

[2]Epstein and Zin (1991) relies on it, fact that is criticized in Mehra and Prescott (2003).

of durable goods for the purpose of obtaining a factor of risk that implies time-varying risk premia. In Piazzesi, Schneider, and Tuzel (2007) consumers fear recessions when consumption is low. However they fear severe recessions when, in addition to consumption being low, housing expenditures are low relative to total consumption expenditures. This risk also implies a time-varying risk premium, which is higher at business cycle troughs than at peaks. Another well-established family of models are those featuring habit formation[3], which generates countercyclical variation of the price of risk as well. The habits models result in time variation of risk aversion, which ultimately causes time variation in risk prices. Return dynamics and the cross-section have diverted attention from the equity premium, which remains largely unexplained, or simply explained by higher levels of risk aversion. In this paper, price of risk is time varying due to the non-linearity of the value function and the stochastic discount factor. When the conditional expected growth rate of consumption is low prices of risk will be higher. On the other hand, when the conditional expected growth rate is high, agents will be willing to pay more for a risky position exposed to the shocks that affect the economy.

By finding a closed form solution for the model with non-separability across non-housing and housing consumption, I can identify the effect of the expenditure shares on the price of risk. I consider an endowment economy as in Lucas (1978), with two *trees*. One delivers non-housing consumption and the other delivers housing services. I propose a statistical model for the exogenously given consumption process and for the housing consumption. The latter is specified indirectly, through expenditure shares in non-housing. To ensure it is bounded between zero and one, expenditure shares follow a log-linear-quadratic process. This strategy implies solution forms related to the risk-sensitive optimal control problems[4]. I consider corporate earnings and aggregate current net stock of private residential structures growth as determinants of the state of the economy. The proposed model generates a heteroskedastic

[3] Abel (1990), Constantinides (1990), and Campbell and Cochrane (1999) are examples featuring habit formation and Chen and Ludvigson (2004) and Fillat and Garduño (2006) are examples of empirical estimation of the Campbell and Cochrane model.

[4] See Whittle (1990).

stochastic discount factor that implies countercyclical price of risk. When the state of the economy is low, either because corporate earnings or the growth in private residential stock are low relative to consumption, marginal utility is high, therefore risk prices are high. This is somehow related to the habits models, where the state variable determines risk aversion and therefore, the price of risk. The state of the economy determines the magnitude of the response of asset prices to shocks to consumption growth. I estimate the parameters of the statistical model for the endowment of consumption and housing services as well as evaluate prices and risk premia for different investment horizons with 5 portfolios sorted by book-to-market, housing, a claim to consumption, and a 3-month T-bill.

Several studies in the housing literature analyze the effects of adjustment costs in consumption and the fact that one cannot freely adjust the size of the house from period to period. Examples are found in Stokey (2007), Chetty and Szeidl (2005), and Flavin and Nakagawa (2004). Adjustment costs do not affect asset prices, only endogenous relative quantities, which are an outcome of intra-temporal first order condition. In this paper, quantities are taken as given, and the objective is to find market clearing prices. As long as relative expenditures are measured with aggregate data, we can safely compute prices from the Euler equation, imposing the equilibrium conditions.

Chapter 1 describes asset returns used in the estimation and the covariation with the relevant variables of the model. In Chapter 2, I present the model, the intra and inter-temporal first order conditions, and the solution of the model. I also discuss possible interpretations of the parameters. In Chapter 3 I estimate the dynamics of the system of variables and I discuss the pricing results obtained.

1 Data

Data on consumption is obtained from the Personal Consumption Expenditures of the National Income and Product Accounts (NIPA). I use quarterly post-war data from the tables of chapter 2, specifically from 1953 to 2005. Data are obtained from Table 2.3.5 of the NIPA, which presents total expenditures. For non-housing consumption goods I use expenditures on non-durable goods and services. For housing services, I use line 14, "Housing Services". It has been pointed out in Prescott (1997) that several components of consumption have been badly defined. Among these components is owner-occupied housing. Since there are no market prices for owner-occupied housing services, price indexes must be constructed. The approximation used by the Bureau of Economic Analysis is based on imputing rental prices of similar houses. The price of a commodity should account for what it costs to the household consuming it, which depends on many other factors like tax situation (for the deductions), size of mortgage, etc. which the current methodology does not take into account. An indication of this type of bias can be seen in the Consumer Expenditure Survey (CEX). In 2004, the shares of expenditure on shelter over total expenditures were computed to be 18.4%[5]. That share is 16.9% for homeowners and 23.8 for renters. Acknowledged this, NIPA statistics separate the dollar expenditures on housing services into a price index and a quantity index.

Besides the problem mentioned above, these series accumulate new problems, observed by the Boskin Report documents[6]. The quality of the houses today is considerably better than several years ago and have several features that were not present in the beginning of the series. There exists a reasonable measurement error in the CPI component p_t^s that affects inversely the quantity index s_t. There are no data on houses' quality improvement, therefore no data either on the quality improvement of housing services. The presence of w_t in the preferences specification is meant to overcome the bias and capture the secular movement in relative quantities consumed of housing services, perhaps provided from housing units with an improved set of characteristics, rather than in raw measures of square meters and number of rooms.

[5]This share from the CEX includes some expenses like maintenance, repairs, insurance and other expenses that do not enter in the definition of housing services used throughout the paper.

[6]Boskin et al. (1996) and more recently Gordon and van Goethem (2005) report that there has been a downward bias in the CPI for shelter since its computation began.

Table 1: **Descriptive Statistics.** Descriptive statistics of consumption growth of non-durable goods and services (Δc^{NIPA}), consumption growth of non-housing non-durable goods and services (Δc^{n-h}), and consumption growth of housing services (Δs^{h}). c^{CES} represents the aggregation of housing and non-housing consumption according to the constant elasticity of substitution functional form in (2).

				ε	
				1.4	1.8
	Δc^{NIPA}	Δc^{n-h}	Δs^{h}	Δc^{CES}	Δc^{CES}
Mean	2.12	2.10	2.52	2.45	2.32
Autocorr.	0.24	0.23	0.61	0.45	0.38
Δc^{NIPA}	2.06	0.98	0.22	0.74	0.90
Δc^{n-h}	0.98	2.45	0.07	0.66	0.85
Δs^{h}	0.22	0.07	1.80	0.30	0.23
Δc^{CES}	0.74	0.66	0.30	2.35	0.95
Δc^{CES}	0.90	0.85	0.23	0.95	2.19

While data about relative prices and relative quantities have been subject to criticism, data on expenditures seem less likely to be subject to those flaws. The observed total expenditures in housing services include the quality measure. Conversely, the quantity index constructed from the expenditures does not. Since the stochastic discount factor can be written as a function of non-housing consumption growth and expenditure shares solely, the problem of the quality improvement disappears for that matter. It becomes irrelevant for the purpose of pricing a set of assets. It remains crucial for estimating the elasticity of substitution between the two types of goods but that is not the main focus of this paper. The sample starts in January 1953, which appears to be the period when the high re-stocking after the war slowed down. The aim of the model is not to explain the response of prices to large events, like the WWII, which triggered a period of restocking of durable goods.

Figure 1 shows that expenditure shares do not converge to either 1 or zero. Data on expenditure shares, more reliable than quantity and price indexes, show that the expenditure shares are stationary. Over the last 50 years, the average consumer has spent his money 80/20 in non-housing goods and services and housing services respectively.

Figure 2 shows the evolution of expenditure shares and consumption growth. Figure 3 shows the evolution of the expenditure shares with the long-run discounted consumption

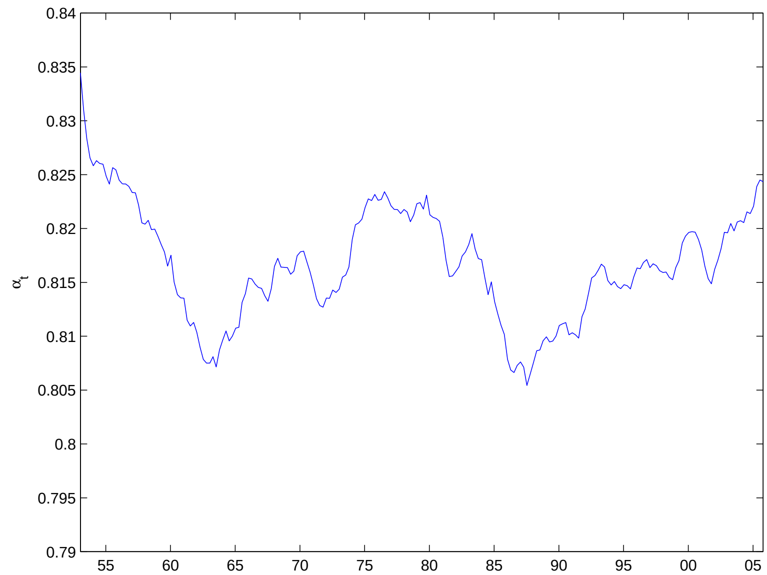

Figure 1: **Expenditure Shares.** Expenditure shares of non-housing consumption over total expenditure in non-durable goods and services, corresponding to α in the model. Source: NIPA Personal Income and Outlays, Table 2.3.5. Quarterly data, 1953-2006.

growth. It can be observed that a decline of the expenditure shares is accompanied by a decrease in the upcoming consumption growth of the next 16 quarters. The correlation of long-run discounted consumption growth and expenditure shares is higher for a longer horizon than for the contemporaneous. This shows potential improvements as a pricing factor, since it is added to the contemporaneous correlation studied by Piazzesi, Schneider, and Tuzel (2007), which does not exploit the long-run relationships.

The price index of the non-housing composite good has been computed using the expenditure shares of each of the categories that compose the aggregate as weights.

Homotheticity between housing and non-housing consumption is assumed throughout the paper. Pakŏs (2004) argues against its validity. Figure 1 shows that the expenditure shares did not increase with an increase of the income during the last decades. It is not evidence against non-homotheticity but clearly shows that there is no steady decline in relative expenditures. If relative quantities are increasing over time, relative expenditures should also trend over time. In reality, that is not the case, since expenditure shares are fairly stable over the year, clearly not converging to 1 or 0. Therefore the preference tilt

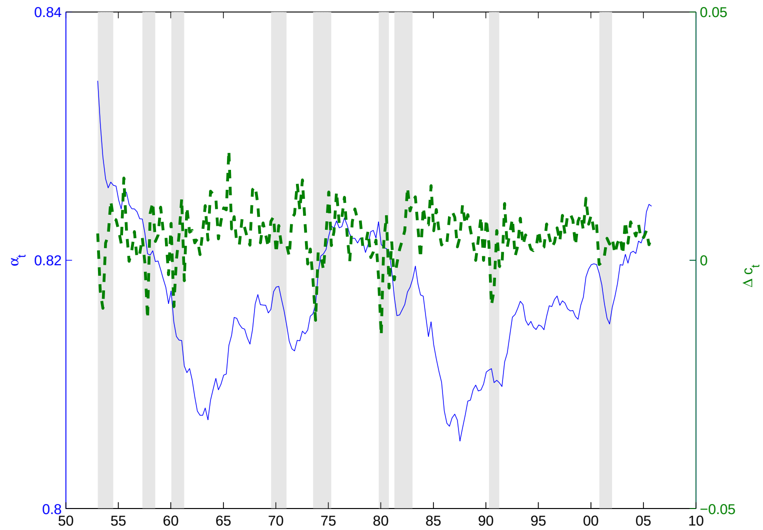

Figure 2: **Expenditure shares and consumption growth.** Non-housing expenditure share over total expenditure (left axis and solid line) and consumption growth series (right axis and dashed line). Shaded areas are NBER recessions.

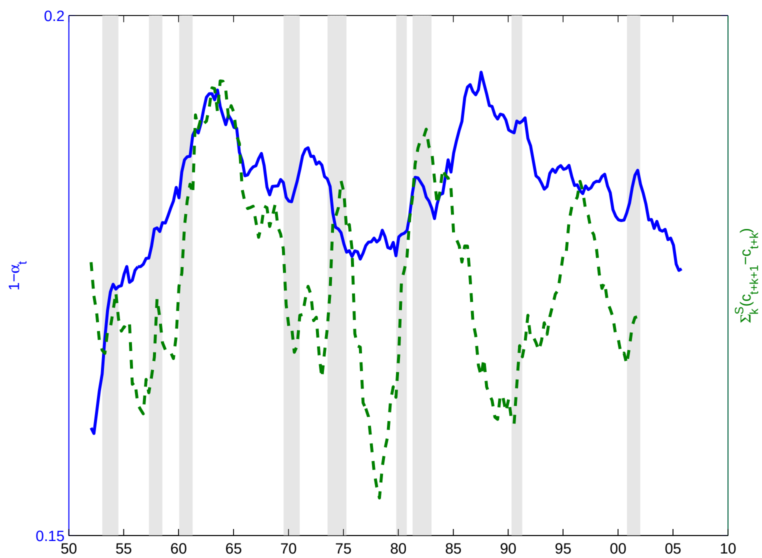

Figure 3: **Expenditure shares and long-run discounted consumption growth.** Non-housing expenditure share over total expenditure (left axis and solid line) and long-run discounted consumption growth series (right axis and dashed line) assuming $\beta = 0.95^{1/4}$ and a 4-year horizon. Shaded areas are NBER recessions.

towards housing services acts so that the equations relative prices and relative quantities in the first order conditions of the model are both consistent with the data.

The state of the economy is characterized by two variables which are corporate earnings and current-cost net value of residential stock. Corporate earnings are a predictor of consumption and a source of aggregate risk, as has been motivated in Hansen, Heaton, and Li (2008). Earnings are very persistent, more substantially more than consumption growth, with an autocorrelation coefficient of 0.96. Therefore high earnings today predict high earnings tomorrow and higher earnings in the future will be translated in higher consumption, which will catch up eventually. A similar rationale justifies the inclusion of the second variable in the state vector. Current-cost net value of residential stock predicts the future stream of housing services, as a higher stock of housing delivers a higher level of housing services. Conversely, a slowdown in construction of new residential structures decreases the number of houses available to extract services from, lowering the expenditures in housing services, all else equal. Therefore, it is a well motivated predictor of future measured housing expenditure shares. The predictive power of this two variables is tested in the empirical section. Figure 4 shows the evolution of the two variables that characterize the state of the economy. Two restrictions are imposed on these state variables. Corporate earnings are assumed to be cointegrated with consumption of non-durable goods and services and the value of residential stock is assumed to be cointegrated with housing services, following the above argument. The null hypothesis of unit root for each of the variables is rejected at a 5% confidence level, using the augmented Dickey-Fuller test for stationarity.

1.1 Stock Returns

Table 2 reports descriptive statistics for the value weighted market from the CRSP, which contains the NYSE-AMEX-NASDAQ stocks, 5 portfolios sorted by book-to-market (book equity over market equity) from the data library of K. French, and returns on housing.

Dividends for the market portfolio and the book-to-market were created from data on returns with and without dividends. The difference between the two results in the dividend yield, while the composition of the returns ex-dividend results in the prices relative to the price at $t = 0$. It remains to set the dividends at the beginning of the sample to construct

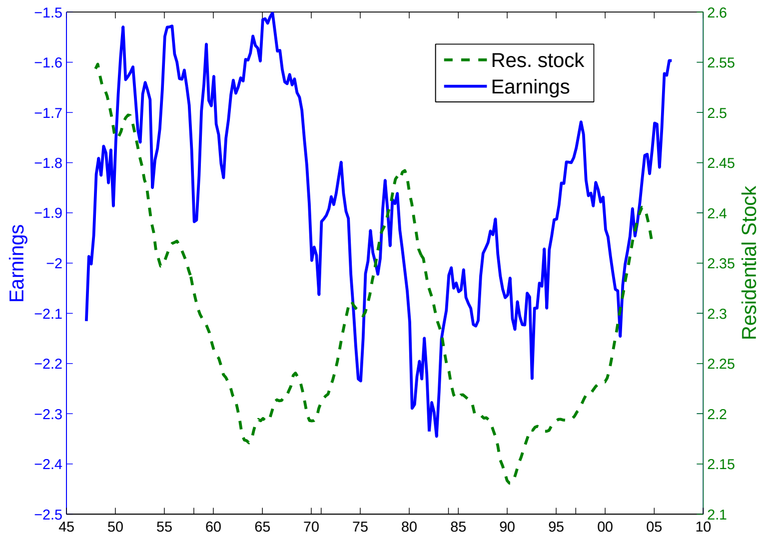

Figure 4: **State Variables.** Log of corporate earnings over non-durable goods and services consumption in the left axis, solid line. Log of current-cost net value of residential stock over housing services consumption on the left scale, dashed line.

the entire series for dividend price ratio and dividends recursively. I initialize the price at time zero so that the dividends are equal to aggregate earnings.

I compose the returns quarterly and compute a 4-quarters moving-average of the dividend-price ratio to eliminate the strong seasonal component that this variable presents.

I use the 3-month T-Bill as the risk free rate, given that agents' decision horizon is one quarter. Data on consumption is also quarterly. As we can observe in Table 2, there is a significant correlation between housing returns and consumption growth, long-run consumption growth, and with expenditure shares. Note that the correlation of future discounted expenditures shares in non-housing with stock returns is larger than the contemporaneous correlation. This partly motivates the use of a non-separable function between housing and non-housing consumption, that is specified below. As argued in Piazzesi, Schneider, and Tuzel (2007), agents will suffer when marginal utility of consumption is low, but even more when marginal utility of housing services is also low. I will argue that this effect in the valuation of future consumption comes entirely from the long run predictability of expenditure shares rather than the contemporaneous relationship. The higher correlation with the latter

Table 2: **Descriptive Statistics.** Sample mean and standard deviation, annualized, for the real returns of the market, 5 portfolios sorted by book-to-market, housing and 3 month T-bill. Column Δc_{t+1} shows the correlation of the corresponding returns with consumption growth, column $\sum_{j=1}^{J} \Delta c_{t+j}$ shows the correlation of the returns with consumption growth during the subsequent 24 quarters and the last column shows the correlation between the returns and the growth in the expenditure shares of non-housing consumption over the aggregate consumption during the subsequent 24 quarters.

	Mean	St.Dev.	Δc_{t+1}	$\sum_{j=1}^{J} \Delta c_{t+j}$	α_t	$\sum_{j=1}^{J} \alpha_{t+j}$
R^{mkt}	7.62	32.25	0.22	0.10	-0.01	-0.06
R^1	6.59	36.42	0.22	0.08	-0.02	-0.04
R^2	7.54	32.51	0.20	0.09	-0.02	-0.05
R^3	9.43	29.47	0.18	0.09	-0.02	-0.09
R^4	10.04	31.17	0.23	0.13	0.02	-0.03
R^5	11.15	36.12	0.22	0.11	0.01	-0.06
R^h	2.35	6.01	0.19	-0.11	0.05	-0.06
R^f	1.12	3.05	0.27	0.37	-0.28	0.10

measure motivates this approach. The correlation between returns on housing and future consumption growth is on average twice as much as the rest of the portfolios. This indicates that returns on housing are sensitive to shocks that affect consumption or the composition of consumption in a persistent manner. The last two columns describe the correlation of the long run consumption growth and long run non-housing expenditure shares with returns.

2 Model

Consider an economy with a representative agent who derives utility from a consumption bundle, $\mathcal{C}_t$. The utility function is recursive as in Epstein and Zin (1989) and can be written as follows:

$$V_t = \left[(1-\beta)\mathcal{C}_t^{(1-\rho)} + \beta E_t \left[V_{t+1}^{1-\gamma} \right]^{\frac{1-\rho}{1-\gamma}} \right]^{\frac{1}{1-\rho}}. \quad (1)$$

The intertemporal elasticity of substitution is measured by $1/\rho$. A higher value of ρ implies that agents are less willing to substitute consumption over time. In addition, a constant elasticity of substitution aggregation is used for the risk adjustment term. The parameter γ is the coefficient of risk aversion, which determines the curvature of the value function. Recursive preferences allow us to modify the willingness of agents to smooth consumption over time independently from their willingness to smooth consumption over different states of the world. The subjective discount factor is represented by β. V_{t+1} is the continuation value of a consumption plan from $t+1$ onwards. The conditional expected value operator, $E_t[\cdot]$, is defined as the expected value conditional on the set of information $\mathcal{F}_t$ that the agent has at time t.

The consumption bundle $\mathcal{C}_t$ is composed of two goods: consumption of housing services, S_t, and non-housing consumption, C_t. The latter represents consumption of non-durable goods and services except housing services.

$$\mathcal{C}_t = \left(C_t^{\frac{\varepsilon-1}{\varepsilon}} + w_t S_t^{\frac{\varepsilon-1}{\varepsilon}} \right)^{\frac{\varepsilon}{\varepsilon-1}}, \tag{2}$$

where ε is the elasticity of substitution between housing and non-housing consumption and w_t represents a preference shift, which could be time varying. It shifts the preferences towards housing services within a period.

There are three assets in the economy: a house, a stock, and a risk-free bond. I follow an endowment economy approach, as introduced by Lucas (1978). In this economy, there are two *trees* with a positive supply: the house and the stock. The bond is in zero net supply. The house pays a stream of housing services and the stock pays a stream of non-housing consumption goods and services. The agent chooses consumption of non-housing goods and services, housing services, and asset holdings subject to

$$p_t^C C_t + p_t^S S_t + q_t^C \theta_t^C + q_t^S \theta_t^S = (q_t^C + p_t^C \bar{C}_t)\theta_{t-1}^C + (q_t^S + p_t^S \bar{S}_t)\theta_{t-1}^S, \tag{3}$$

where q_t^C and q_t^S are the prices at which the two assets trade and θ_t^C and θ_t^S are holdings of the two assets.

2.1 Intra-temporal First Order Condition

The static first order condition results in a relationship between the marginal rate of substitution and relative prices which takes the form

$$\frac{p_t^C}{p_t^S} = \frac{1}{w_t}\left(\frac{C_t}{S_t}\right)^{-\frac{1}{\varepsilon}}. \tag{4}$$

Multiplying both sides of (4) by relative quantities, C_t/S_t, we obtain relative expenditures on the left hand side as a function of the relative quantities,

$$\frac{p_t^C C_t}{p_t^S S_t} = \frac{1}{w_t}\left(\frac{C_t}{S_t}\right)^{\frac{\varepsilon-1}{\varepsilon}}. \tag{5}$$

The left hand side of (5) is the ratio of expenditures of non-housing consumption relative to expenditures of housing services consumption. This ratio is well measured in the data. On the right hand side, we have the relative quantities of non-housing to housing services consumption. I define the non-housing expenditure shares as the fraction of total expenditures spent in non-housing goods and services. This definition will prove useful in solving the model:

$$\alpha_t \equiv \frac{p_t^C C_t}{p_t^C C_t + p_t^S S_t}; \qquad \frac{1}{\alpha} = 1 + w_t\left(\frac{S_t}{C_t}\right)^{\frac{\varepsilon-1}{\varepsilon}}. \tag{6}$$

If the two goods are substitutes, $\varepsilon \geq 1$, an increase in housing consumption relative to non-housing causes a decrease in non-housing expenditure shares, α_t. Conversely, if they are complements, or $\varepsilon \leq 1$, an increase in housing consumption relative to non-housing implies a decrease in non-housing expenditure shares. Expenditure shares in non-housing consumption are stationary over time. Although the autocorrelation is 0.98 with quarterly data, assuming that the expenditure shares are not stationary would imply convergence to 1 or zero with probability 1, which is not realistic. Furthermore, under that assumption one of the two types of goods would vanish from the utility function, feature that is difficult to infer from the data.

Figure 1 shows the evolution of the expenditure shares in the last 55 years and Figure 5 shows the prices and quantities of housing services relative to non-housing consumption.

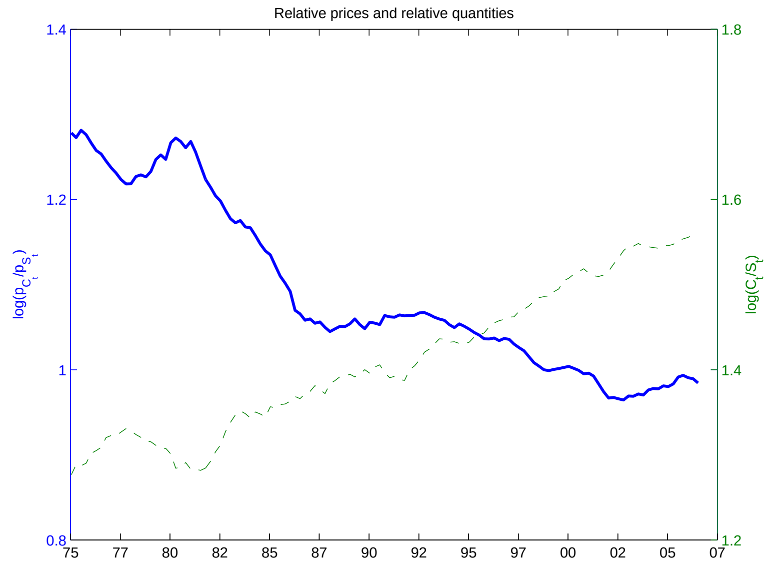

Figure 5: **Relative prices and relative quantities.** On the left axis, the bold line shows the evolution of relative prices in the last 30 years. On the right axis, the dashed line shows the BEA figure for the ratio of quantity indexes for housing services over non-housing consumption. Source: NIPA Personal Income and Outlays, Tables 2.3.4 and 2.3.6. Quarterly data, 1975-2006.

While we do not observe that expenditure shares converge to 1 or zero, in the last 30 years, measured prices of housing services relative to non-housing consumption have risen. The construction of the quantity index from NIPA implies that the relative quantities of non-housing to housing have decreased over the last 30 years as well. The reverse pattern is observed prior to 1975. A low frequency component must drive measured relative prices and relative quantities but relative expenditures remain stationary, though not constant. The process w_t captures this low frequency component that drives relative prices and relative quantities and keeps relative expenditures and expenditure shares stationary over time. This can be seen in (4) and (5). If w_t was to be kept constant, relative prices and relative quantities being integrated of order 1, I(1), would imply a decrease in the left hand side of both (4) and (5). While the data show a decrease in relative prices, the effect on relative expenditures following (5) is not observed. The only case where a time trend in relative quantities could imply a decreasing trend in prices but not in expenditure shares corresponds to unitary elasticity of substitution between housing services and non-housing consumption. However, that case implies a constant expenditure share since the CES specification coincides with the Cobb-Douglas. The data do not support the assumption of constant expenditure shares. The functional form for the aggregation between non-housing and housing consumption assumes homogeneity, which is supported by the fact that expenditure shares remained stable in a period were income increased substantially.

It is not possible to identify the elasticity of substitution between non-housing and housing consumption without a good measure for w_t. Equation (5) implies a cointegration relationship between w_t and S_t/C_t, with the cointegrating vector $[1, 1 - 1/\varepsilon]$, so that both the right hand side and the left hand side are stationary. Since there is no good measure for either relative quantities or for w_t, I evaluate asset prices for a range of elasticities of substitution.

2.2 Inter-temporal First Order Condition and Pricing

In this section, I find a closed-form solution for both the value function, expressed recursively in (1), and for the stochastic discount factor implied by the model.

Equilibrium in the endowment economy is characterized by stochastic processes for aggregate output of the two goods $\{\bar{C}_t, \bar{S}_t\}$, which, in equilibrium, must be equal to C_t and S_t respectively, a vector of prices $\{p_t^C, p_t^S\}$, and a vector of portfolio holdings $\theta_t^S = \theta_t^C = 1$, that maximize (1), subject to the budget constraint (3). I propose a model for the evolution of consumption and housing and compute the prices that support them as equilibrium quantities.

2.3 Endowment

I assume that consumption follows an infinite moving-average process $MA(\infty)$. In particular, I express it as in Bansal and Yaron (2004). Consumption growth follows a random walk plus a state variable that causes persistent and predictable changes. The state variable determines the conditional expectation of consumption growth.

$$c_{t+1} - c_t = \mu^c + \phi^c x_t + \sigma_0^c \nu_{t+1} \tag{7}$$

$$x_{t+1} = \delta x_t + \sigma_0^x \nu_{t+1}, \tag{8}$$

where ν_{t+1} is a multivariate normally distributed vector, with identity variance matrix. Lower-case letters represent the natural logarithm of upper-case ones, so $\log C_t = c_t$. The unconditional long-run average of consumption growth is represented by μ^c.

The predictable component of consumption growth, x_t, follows a persistent autoregressive process. It represents the state of the economy and determines the long-run expected consumption growth. Changes in the state of the economy determine whether the current economy has high or low long-run consumption growth, leveraged by ϕ^c. If consumption follows a random walk, there would be no persistent shocks to consumption growth and the current shock would vanish in one period. Conversely, in this model, consumers fear shocks because shocks affect their conditional expectation of consumption growth for several periods through $\phi^c x_t$. There is no strong evidence against either of these possibilities. The purpose of my work is to observe how returns behave when investors do indeed fear shocks that may lower consumption growth of non-housing goods and also change their optimal mix of housing and non-housing consumption for more than a period. Therefore the model relies on the assumption that consumption has a small predictable component that imposes the persistence of the shocks. The conditional specification allows to identify the long-run response of consumption to innovations and to estimate it empirically using VAR analysis as in Hansen, Heaton, and Li (2008) and Malloy, Moskowitz, and Vissing-Jørgensen (2006).

I model the endowment of housing indirectly through expenditure shares, rather than quantity of housing services. Expenditure shares are an intra-period equilibrium result, as shown in (4). Therefore, there is a one-to-one relationship between real quantities and expenditure shares for a given value of w_t. There exists an endowment process of housing services for which a given vector of prices can support the process of expenditure shares. In particular, I model the log expenditure shares process as a linear quadratic function of the state of the economy x_t. As shares, they must lie in the unit interval, which is ensured by a linear quadratic function of a normally distributed process:

$$-\log \alpha_t = \mu^\alpha + \phi^\alpha x_t + x_t' \Psi x_t. \tag{9}$$

This specification imposes conditions on ϕ^α and Ψ that keep the expenditure shares between 0 and 1. Equation (9) implies that expenditure shares are persistent if x_t is also persistent. The quarterly autocorrelation of non-housing expenditure shares is 0.98, supporting the persistent specification in (9).

2.4 Valuation

Define the stochastic discount factor as the marginal valuation of a stream of future value expressed in terms of non-housing consumption. The choice of numeraire is not innocuous. Empirically, it allows to use data on non-housing consumption and expenditure shares. It has been argued that these data are well measured, relative to the constructed quantity and price indexes for housing services.

By scaling the value function by the marginal valuation of non-housing consumption, I obtain the shadow valuation of a stream of future value expressed in terms of marginal value of non-housing consumption. Therefore it is a valid one-period stochastic discount factor (SDF_{t+1}), which is simplified to the following expression:

$$SDF_{t+1} = \beta \left(\frac{V_{t+1}}{E_t \left[V_{t+1}^{1-\gamma} \right]^{\frac{1}{1-\gamma}}} \right)^{\rho-\gamma} \left(\frac{C_{t+1}}{C_t} \right)^{-\rho} \left(\frac{1 + \left(w_{t+1} \frac{S_{t+1}}{C_{t+1}} \right)^{\frac{\varepsilon-1}{\varepsilon}}}{1 + \left(w_t \frac{S_t}{C_t} \right)^{\frac{\varepsilon-1}{\varepsilon}}} \right)^{\frac{1-\varepsilon\rho}{\varepsilon-1}}. \tag{10}$$

The stochastic discount factor is composed of three factors. The first term corresponds to the risk adjustment, originated by the recursive utility function. It can be also interpreted, following the literature on robustness, as the fear that consumers have of wrongly specifying the model that guides their consumption decision, as in Anderson, Hansen, and Sargent (2003). If agents are risk neutral, the first term captures the innovation or disparity between the realized value and expected value of a stream of future consumption.

The second factor captures consumption growth risk, as in the consumption-based CAPM. Risk averse consumers fear drops in consumption growth. The correlation of this factor with the excess returns is what constitutes the CCAPM. Assets whose returns are high when consumption growth is low will be highly regarded, therefore their price in equilibrium will be high. Conversely, assets with low returns when consumption growth is low will have lower prices due to the positive correlation.

The third factor captures composition risk. It reflects the fact that investors fear severe recessions that happen when the consumption of housing services falls relative to aggregate consumption. When consumption growth is low, agents' marginal utility increases due to the second factor. However, if low consumption growth is compounded by a decline in housing services relative to non-housing consumption, marginal utility increases even more when the two goods are substitutes ($\varepsilon \geq 1$). Therefore, if consumption of non-housing is declining *and* housing consumption also declines relative to non-housing, the substitutability between the two types of goods causes a greater increase in marginal utility. Shocks causing either of these increases of marginal utility are even more feared if they have a persistent effect, which is captured by the first term in equation (10).

With the conditions in (4)-(6), we can express the third term, corresponding to the *composition* risk, as a function of non-housing consumption expenditures as a share of total expenditures, α. Therefore the composition risk is fully described by the change in expenditure shares. I relegate to the Appendix the detailed algebra to obtain it. The stochastic discount factor that results is

$$SDF_{t+1} = \beta \left(\frac{V_{t+1}}{E_t \left[V_{t+1}^{1-\gamma} \right]^{\frac{1}{1-\gamma}}} \right)^{\rho-\gamma} \left(\frac{C_{t+1}}{C_t} \right)^{-\rho} \left(\frac{\alpha_{t+1}}{\alpha_t} \right)^{\frac{1-\varepsilon\rho}{1-\varepsilon}} . \tag{11}$$

Once expenditure shares are substituted into this equation, the role of the preference process w_t disappears. The question whether w_t corrects the measurement error in the quantity and price indexes or accounts for quality improvements in housing services becomes irrelevant once the discount factor is expressed as a function of the expenditure shares. This is because expenditure shares are less subject to measurement error and implicitly include improvements in quality since they are an intra-temporal equilibrium result.

The risk adjustment is not directly observable since the value function depends non-linearly on the two types of goods and the continuation value. Alternatively, the model could be solved as a dynamic asset allocation problem subject to the law of motion of total wealth. By doing so we need to use the return to the total wealth, which in general is not observable. Instead of following the portfolio choice approach approximating the returns on the wealth portfolio by the market returns[7], I focus on the explicit solution of the value function. For the purpose of finding an analytical solution of the problem, I consider the case where the elasticity of intertemporal substitution, ρ, is fixed at 1. For this particular case I can obtain a closed form solution for the value function and for the stochastic discount factor. Fixing intertemporal elasticity of substitution at 1 is key in this paper, as in Hansen, Heaton, and Li (2008), to obtain a closed form solution for the value function and the stochastic discount factor. They explore the consequences of deviations from this assumption in a variational analysis. Log intertemporal utility will impose a fixed wealth consumption ratio, which is a result also for the log-linear approximation approach. There is mixed evidence regarding the value of the intertemporal elasticity of substitution. Jones, Manuelli, and Siu (2000) examine a real business cycle model and conclude that it should be calibrated between 0.8

[7]This approach is followed in Bansal and Yaron (2004), among others.

and 1. Hansen and Singleton (1983) and Hall (1988) estimate the value of intertemporal elasticity of substitution using the conditional Euler equation with stock returns and find its value smaller than 1 and closer to 0. Heterogeneity of agents is also explored in the literature. Attanasio, Banks, and Tanner (2002) and Vissing-Jørgensen (2002) conclude that the elasticity of substitution is closer to 1 for stockholders and smaller for non-stockholders. I consider the case of $\rho = 1$, which gives the exact solution for the value function.

Next, I consider the value function scaled by the consumption of non-housing goods and services, as I have done above for the stochastic discount factor. Non-housing consumption becomes the numeraire, for analytical tractability, but also for empirical reasons regarding the price indexes[8]. The price index that we use to deflate returns is the price index corresponding to the non-housing consumption. Thus, the scaled value function is:

$$\frac{V_t}{C_t} = \left((1-\beta)\left(\frac{1}{\alpha_t}\right)^{\frac{\varepsilon(1-\rho)}{\varepsilon-1}} + \beta E_t\left[\left(\frac{V_{t+1}}{C_{t+1}}\right)^{1-\gamma}\left(\frac{C_{t+1}}{C_t}\right)^{1-\gamma}\right]^{\frac{1-\rho}{1-\gamma}}\right)^{\frac{1}{1-\rho}}. \tag{12}$$

Define $v_t = \log\left(V_t/C_t\right)$. Taking logs and rewriting the term inside the expectation,

$$v_t = \frac{1}{1-\rho}\log\left((1-\beta)\left(\frac{1}{\alpha_t}\right)^{\frac{\varepsilon(1-\rho)}{\varepsilon-1}} + \beta E_t\left[e^{(1-\gamma)(v_{t+1}+c_{t+1}-c_t)}\right]^{\frac{1-\rho}{1-\gamma}}\right). \tag{13}$$

As I mentioned above, I focus on the limiting case of $\rho = 1$. In that case, the value function becomes

$$\lim_{\rho\to 1} v_t = (1-\beta)\frac{\varepsilon}{\varepsilon-1}\log\frac{1}{\alpha_t} + \frac{\beta}{1-\gamma}\log E_t\left[e^{(1-\gamma)(v_{t+1}+c_{t+1}-c_t)}\right]. \tag{14}$$

Note that in this particular case of unitarian intertemporal elasticity of substitution, the expenditure shares do not play a significant role as a pricing factor on its own. Equation (11) shows that when $\rho = 1$, the third term is risen to the power of 1. Taking into account the evidence shown in the data section, the low variability of the expenditure shares growth

[8]See Piazzesi, Schneider, and Tuzel (2007) for the analysis of aggregate consumption as a numeraire.

implies that all the pricing power of the expenditure shares appears in the continuation value, the first term in (11). Therefore, expenditure shares have a permanent effect on the recursive log-value function. Also note that if agents become patient and β goes towards 1, the effect of the expenditure shares in the continuation value will disappear, and the model will converge to Hansen, Heaton, and Li (2008). Now we have the ingredients to solve the continuation value as a function of the state:

Proposition 1. *The value of a consumption plan of future housing and non-housing consumption at time t, expressed in terms of non-housing consumption, is a log-linear quadratic function of the state of the economy, x_t. The value function depends linearly on the log of expenditure shares and risk-adjusted linearly on the consumption growth. The solution of the value function as a function of the state x_t is*

$$v_t = D + Fx_t + x_t' H x_t \tag{15}$$

Proof. See Appendix A.1 for the verification of the functional form and the values of the coefficients. □

The values of D, F, and H are functions of the underlying parameters. The matrix H in the quadratic term is the solution of a well defined Riccati equation, a result familiar in the risk-sensitive optimal control literature[9]. Naturally, H is the only set of parameters related to Ψ, the quadratic form for the logarithm of the shares that ensures the permanence of the level of the shares between 0 and 1. If the shares were approximated by a linear process, H would be a matrix of zeros and the value function would be a linear function of the state.

Using (15) into (11), the stochastic discount factor at $t+1$ can be expressed in a linear-quadratic heteroskedastic function of the state and the shocks. I also leave the details of the derivation and the explicit solution for each of the coefficients to Section A.2 in the Appendix. The heteroskedasticity arises because the quadratic term in (15) implies an inter-

[9]See Whittle (1990) for an extensive review.

action term between the state, x_{t+1}, and the vector of innovations, ν_{t+1}. Taking logarithms of the expression in (11), we obtain the discount factor as function of the risk-adjusted expected continuation value, the consumption growth and the level of expenditure shares in non-housing consumption. Innovations in the stochastic discount factor are determined by innovations in consumption growth, in expenditure shares growth, and in the risk adjustment:

$$\begin{aligned} sdf_{t+1} &= \log\beta + (1-\gamma)\left[v_{t+1} - \frac{1}{1-\gamma}\log E_t\left[e^{(1-\gamma)(v_{t+1}+c_{t+1}-c_t)}\right]\right] \\ &\quad - \gamma(c_{t+1}-c_t) + \log\alpha_{t+1} - \log\alpha_t. \end{aligned} \tag{16}$$

Proposition 2. *The solution of the logarithm of the discount factor is a log-linear quadratic function of the state and of the vector of shocks to the economy.*

$$sdf_{t+1} = J + Kx_t + x_t'Lx_t + M(x_t)\nu_{t+1} + \nu_{t+1}'N\nu_{t+1}. \tag{17}$$

Proof. See Appendix A.2. □

Equation (16) explicitly expresses the logarithm of the stochastic discount factor as a function of the consumption growth, the growth of the expenditure shares, and the long-run discounted impulse response of both processes, with a level term and a quadratic adjustment. Substituting in the processes for consumption growth and expenditure shares growth, the stochastic discount factor results in a linear-quadratic heteroskedastic function of the value for the state x_t and of the underlying parameters. I leave the details for the Appendix, as well as the precise functional form for the coefficients. The time-varying coefficient corresponding to the first order effect of the shock in the pricing function, $M(x_t)$, is specially important, because it determines the time variation in the risk prices.

2.5 Risk Prices

The analysis that follows focuses on risk prices. I obtain the local[10] risk prices by computing the change of the risk premia of the different portfolios when we increase marginally the risk exposure of those portfolios. Similarly, the long-run risk prices are computed by taking the derivative of the asymptotic rate of return in excess of the long-run risk free asset with respect to the risk exposure to long-run risk. This gives us an idea of how agents will price the different portfolios at different investment horizons. I also present the results for the entire term structure of the risk prices at different horizons.

I first define the evolution of cash flows for each individual portfolio, or more generically any asset under consideration, as a geometric random walk with a predictable component, similar to (7).

$$d^i_{t+1} - d^i_t = \mu^d_i + \phi^d_i x_t + \sigma^d_{0i} \nu_{t+1}. \tag{18}$$

where μ^d is the drift, ϕ^d captures the exposure to the persistent component, and σ^d_0 represents the instantaneous exposure to the shock ν_{t+1} which is normally distributed with zero mean. Cash flows of different assets will differ in their unconditional expected growth μ^d, their exposure to the persistent component ϕ^d, and their unconditional volatility σ^d_0.

One of the mechanisms that I use is the intertemporal counterpart to one-period returns. Traditionally, interest is focused on one-period returns computed as the discounted payoff of a one period state contingent Arrow-Debreu security. I turn the attention here, following Hansen, Heaton, and Li (2008), to the price of bundled claims across states and periods, yielding to the term structure of risk prices, rather than one period prices of risk

The t-period risk premium associated to a buy-and-hold strategy of any cash flow with the above dynamics can be expressed as the expected growth of the payoff over the price

[10]Strictly speaking, they are not local, since it is a discrete time model. The analogous for discrete time would be the one-period risk prices.

minus the risk free.

$$\frac{E_t(e^{d_{t+1}-d_t})}{E_t(e^{d_{t+1}-d_t}e^{sdf_t^{t+1}})} - \frac{1}{E_t(e^{sdf_t^{t+1}})} \tag{19}$$

where sdf_t^{t+1} is the one-period intertemporal marginal rate of substitution or discount factor. The price of risk is defined mathematically as the derivative of the risk premium with respect to the risk exposure. Price of a particular risk is how much utility the agent gives up when a payoff (consumption claim, security, or portfolio) becomes infinitesimally exposed to that risk.

In the case of non-separable recursive utility over housing and non-housing consumption, the one period price of risk happens to be

$$-\sigma_0^c + (1-\gamma)(F\sigma_0^x - \sigma_0^c) - \phi^\alpha \sigma_0^x + 2x_t'\delta((1-\gamma)H - \Psi)\sigma_0^x \tag{20}$$

plus a second order effect caused by the quadratic term, N. This second order effect implies a non-linearity of risk prices with risk exposure. Focusing on the first order effects, there are some interesting insights. The first term, $-\sigma_0^c$ is the one-period exposure of consumption to risk. It captures the one-period response of consumption growth to a shock. The second term, $(1-\gamma)(F\sigma_0^x - \sigma_0^c)$, captures the long-run response of consumption and expenditure shares to a shock today. It is the component of the price implied by the recursive formulation. The third term, $2x_t'\delta((1-\gamma)H - \Psi)\sigma_0^x$, accounts for the consumption risk implied by the non-separability between housing services and non-housing consumption. Larger sensitivity of the expenditure shares to the state x_t is captured by ϕ^α and reflected in higher prices. The intuition for this extra term comes from the effect of the shock in the composition of consumption. The higher the ϕ^α, the more negative is the effect that a shock causes in the expenditure shares, therefore, the counter-cyclicality of the non-housing expenditure shares becomes more acute. Expression (20) becomes the mean of the normally distributed shock ν_{t+1} under the risk-neutral probabilities. The last term captures the heteroskedasticity of the pricing function. It arises from the quadratic form and introduces time variation of the price of risk, or equivalently time-varying expected returns. If we eliminate the long-run risk by setting δ to a matrix of zeroes, the only effect that remains is the response of current consumption growth, which is basically the original consumption-based CAPM, plus the contemporaneous composition risk.

In summary, there are three sources of aggregate risk in this economy, identified in (16). As in the CCAPM investors care about the covariance between returns and consumption growth, investors in this economy care about covariance between (1) consumption growth, (2) expenditure shares growth, and (3) long-run consumption growth and expenditure shares responses to a shock. If the NIPA aggregation of non-durable goods and services were representing correctly preferences, the term $-\sigma_0^c + (1-\gamma)(F\sigma_0^x - \sigma_0^c) - \phi^\alpha \sigma_0^x$ would be captured by the response of the aggregate consumption, $\mathcal{C}_t$, and the quadratic adjustment would also disappear from the picture[11]

Once the one-period price of risk has been analyzed, I turn to the term structure of risk prices. That is, how agents price the risk associated with different investment horizons. To do so, I take as reference Hansen (2008), and proceed to decompose the growth rate of cash flows in a convenient way to distinguish the underlying risks that influence values over long horizons from those that affect values only in the short run, and identified above. Once the components are identified, risk prices are defined as before, as the effect on risk premia of a marginal increase in exposure to each of the components.

2.6 Valuation of Long-Run Cash Flows

After obtaining the stochastic discount factor and the one-period risk prices, we are interested in the valuation of cash flows, or dividends, that are generated by the portfolios at longer horizons. Dividend growth is modeled as (18), an exponential of a random walk with time trend and a persistent, predictable component that, like consumption, affects the long-run expected growth rate of dividends.

[11] That is precisely the framework in Hansen, Heaton, and Li (2008).

The logarithm of the dividends can be expressed as an additive process, composed of a time trend, an additive martingale, and a transitory component, function of the stationary state variable x_t in differences. It can be expressed as

$$d_{t+1} - d_t = \mu^d + (\sigma_0^d + \phi^d (I-\delta)^{-1}\sigma_0^x)\nu_{t+1} - \phi^d (I-\delta)^{-1}(x_{t+1} - x_t), \tag{21}$$

I will denote the coefficient of the shock ν_{t+1} as

$$\pi = \sigma_0^d + \phi^d (I-\delta)^{-1}\sigma_0^x, \tag{22}$$

which captures the exposure of cash flows to long-run risk and corresponds to the long-run impulse response to a cash flow shock. To determine the price of such additive process for dividends, we have to take into account the pricing of both the growth component and the permanent component. In the limit, when horizon tends to infinity, the pricing of the transitory component goes to zero. The expected growth of a dividend process that follows (21) is the expectation of a log-normal variable with the variance correction, $\eta = \mu^d + \frac{\pi\pi'}{2}$.

Assets differ precisely in the long-run response to the shock, denoted by π. They may differ in the temporary part but when computing the limiting prices and limiting returns, where cash flows are discounted far into the future, the temporary effect vanishes. Thus, the risk premium is described by the *exposure* to long-run risk and the growth rate component and the *price* of such risk. Limiting results are invariant to the choice of the temporary component. The differences in exposure of portfolio cash flows to long-run risk are reflected in different risk premia. The purpose here is to disentangle what part of the premia is risk exposure and what is risk price. It has been analyzed above the one-period case. For the long-run, two concepts are key: on the one hand, there is the *long-run exposure* of the cash flows to the risk, which determines how cash flows evolve in the future, given current aggregate shocks. The other concept is the *price of long-run risk*, which corresponds to the value that agents assign to cash flows that offer a persistent exposure to risk. I have expressed the stochastic discount factor, or pricing function, and the dividends as functions of the underlying state process for x_t. Chapter 3 starts with the estimation of the consumption and expenditure shares dynamics (7), (8), and (9) in order to quantify how exposure to risk, as in (21), is priced.

The state follows a continuous-state Markov process. Therefore, the pricing function maps states at period t into valuations of cash flows at $t+j$. In a discrete state space, a Markov transition matrix would do the job of mapping functions of the state into functions of the state in the future. To obtain valuations of more than one period it suffices to raise the transition matrix that maps states into valuations to the power of the horizon. In the limit, the valuation of a perpetuity would be given by the solution to an eigenvalue problem. The analogy for the continuous state is the valuation operator, instead of the transition matrix, and the eigenfunction problem, instead of the eigenvalue problem.

Once the cash flows have been decomposed in trend, martingale, and temporary component, the logarithm for the expected risk premium (or excess return), at horizon t is given by

$$\frac{1}{t}\log(RP(t)) = \frac{1}{t}\log(E_0(e^{d_t-d_0})) - \frac{1}{t}\log(E_0(e^{d_t-d_0}e^{sdf_0^t})) + \frac{1}{t}\log(E_0(e^{sdf_0^t})). \tag{23}$$

What we are interested is not only in the limit of the above expression as t increases to infinity, but also in the values along the path to the limit, to understand the term structure of the risk prices. Following Hansen, Heaton, and Li (2008), define $\mathcal{P}\varphi(x_t)$ as the one-period valuation operator that assigns value to the cash flow that is received at time $t+1$, as function of the state of the economy at time t. φ is the temporary component of the cash flow growth that will disappear in the limit. As explained above, the process for the dividends is decomposed in a growth component, a permanent component, and a transitory component. The valuation operator assigns a value to each of them and in the limit, the contribution from the transitory component vanishes. The stochastic discount factor can be expressed as a multiplicative process and the cash-flows processes can be expressed as a growth process and a temporary process. The one-period valuation operator can be written as

$$\mathcal{P}\varphi(x) = E\left[e^{sdf_t^{t+1}}\ e^{\mu^d+\pi\nu_{t+1}}\ \varphi(x_{t+1})\middle| x_t = x\right] \tag{24}$$

which determines the one-period price of the cash-flow that occurs one period ahead, as discussed in the previous section.

To obtain the valuation of a cash flow that occurs at any time t in the future, it suffices to apply the operator as many times as periods until the payment is realized. This methodology has been extensively developed in Hansen (2008). Taking the derivative of this expression at every time t with respect to the exposure of the cash flow to the shocks that drive the economy will result in a term structure of the risk prices. The expected growth rate of the cash flows is $\eta = \mu^d + (\pi \cdot \pi')/2$, with μ^d and π different for each asset. Therefore, applying the valuation operator j times and taking limits results in

$$\lim_{j\to\infty} E\left[\left(\prod_{s=1}^{j} S_{t+s}^{t+s+1}\right)\ e^{\eta j+\sum_{s=0}^{j}\pi\nu_{t+1+s}}\ \varphi(x_{t+j+1})\middle| x_t = x\right] =$$

$$= \lim_{j\to\infty} e^{\eta j}\ \mathcal{P}^j\ \varphi(x) = e^{\eta}e^{-\kappa}\varphi(x) \tag{25}$$

The last equality corresponds to the eigenvalue problem and κ is the dominant eigenvalue, which remains different than zero in the limit, when $j \to \infty$. The value of cash flows that are originated in several periods in the future corresponds to the addition of the values of the contribution of each of the periods where cash flows occur.

The functional form of the operator is maintained after being applied recursively period by period. In the limit, the function φ that solves recursively the problem is the eigenfunction. The eigenvalue that solves the functional equation is the rate at which the valuation decays over time due to two competing forces: on the one hand the growth rate of the cash flow and on the other hand, the required discount rate, or limiting rate of return. The growth rate is given by the deterministic trend of the dividends process so the asymptotic risk adjusted rate of return, r, can be obtained from the difference between the growth rate, η, and the valuation decay rate, κ as explained in (23). Thus, in the limit, when $j \to \infty$, the decay rate is a fixed point of the equation

$$\mathcal{P}\varphi(x) = e^{-\kappa}\varphi(x) \tag{26}$$

that implies that the valuation of a temporary component is just the temporary component discounted at the limiting rate for one period. The solution for the above eigenfunction consists in finding the functional form of the transient component, $\varphi(x)$. It has been shown in Proposition 2 that if the discount factor follows a linear quadratic process in the state and in the vector shocks, the eigenfunction is an exponential quadratic function of the state,

$$\varphi(x_t) = e^{-ax_t - \frac{1}{2}x_t' b x_t} \tag{27}$$

where b solves a Riccati equation that is detailed in the Appendix and a is found by collecting terms that are interacting with x_t. The solution for a and b is given in the Appendix.

Combining (24), (26), and (27), the decay rate is obtained as a function of the underlying parameters. κ, the eigenvalue, is the asymptotic decay rate of the value of the cash flow. The asymptotic decay rate can be expressed as a function of the underlying parameters:

$$\begin{aligned}
-\kappa = & -J - \mu^d + \frac{1}{2}\log|I - 2\Sigma(N + \sigma_0^{x\prime} b \sigma_0^x)| \\
& + (\pi + \sigma_0^c + \phi^\alpha \sigma_0^x - (1-\gamma)(F\sigma_0^x + \sigma_0^c) - a\sigma_0^x)' \cdot \\
& \left(\Sigma^{-1} - 2(N + \sigma_0^{x\prime} b \sigma_0^x)\right)^{-1} (\pi + \sigma_0^c + \phi^\alpha \sigma_0^x - (1-\gamma)(F\sigma_0^x + \sigma_0^c) - a\sigma_0^x).
\end{aligned} \tag{28}$$

The limiting risk premia is the difference between r and the risk free given by the limiting discount factor. Therefore, to obtain risk prices, it suffices to differentiate the risk premia given by

Once we have the risk premia in the limit, given by κ, $\frac{\partial(\eta(\pi)-\kappa(\pi))}{\partial\pi}$ results in the risk prices. Intuitively, the derivative illustrates how much less the agent is willing to pay if the considered cash flow becomes marginally more exposed to the long-run risks.

This is an important result. π captures the long-run impulse response of the cash flows to current fluctuations, the exposure of cash flows to long-run risk. The price of risk is determined by the derivative of $\eta - \kappa$ with respect to the risk exposure. A linear dependence between price of risk and exposure has been derived in Hansen, Heaton, and Li (2008). For the quadratic case I compute a numerical derivative of the limiting risk premia with respect to π.

The special case of $\pi = 0$, where the asset cash flows do not fluctuate in response to shocks, corresponds to the risk free asset which has no exposure to risk and its cash flows do not grow over time either, $\mu^d = 0$, which is the long-run riskless return studied in Alvarez and Jermann (2001).

Having derived the relevant variables like one-period and t-periods risk prices and returns, limiting returns, decay rates, and risk-free rates, I proceed to describe the empirical strategy to estimate the underlying parameters of the model.

3 Results

3.1 Estimating Long-run Risk

The endowment of goods and services is governed by the system defined in (7), (6), and (8), rewritten here:

$$\begin{aligned} c_{t+1} - c_t &= \mu^c + \phi^c x_t + \sigma_0^c \nu_{t+1} \\ -\log \alpha_t &= \mu^\alpha + \phi^\alpha x_t + x_t' \Psi x_t \\ x_{t+1} &= \delta x_t + \sigma_0^x \nu_{t+1}. \end{aligned}$$

The estimation of the structural model follows a two-stage procedure. The quadratic process for the expenditure shares, complicates the likelihood function and it is not feasible to estimate directly the parameters. I use the *indirect inference* method first introduced by Smith (1993) and Gourieroux, Monfort, and Renault (1993). Indirect inference is useful when the likelihood function of the problem is intractable. It consists of estimating the exact likelihood function of an approximated model in the first stage and the objective is to minimize the distance between the estimates of the approximated model with real data and the estimates obtained from simulated data according to the structural model. The method yields consistent and asymptotically normal estimates of structural parameters.

Expenditure shares do not determine or are determined in the dynamics of consumption growth. Thus, non-housing consumption growth only captures first order effects, with no quadratic terms. A linear system with consumption growth and the state vector is estimated according to the following vector autoregressive specification.

$$A_0 \Gamma_{t+1} = A_1 \Gamma_t + A_2 \hat{\nu}_{t+1} \tag{29}$$

with the corresponding restrictions in the matrices A_0, A_1, and A_2 to identify the structural coefficients in (7) and (8). Table 3 shows the results for the predictable consumption growth process. It is noteworthy the highly persistence of both state variables. This result implies that whenever a shock affects either of them, the conditional expected growth in consumption will be affected for a longer period of time.

The estimation of the expenditure shares evolution is trickier. As expenditure shares, the process must be contained within the unit interval, as it has been explained above. Equation (6) enforces the constraint. A quadratic structure is also imposed in the error term, and the necessary restrictions on the parameters are imposed, in order to have a positive right hand side of the negative logarithm of the expenditure shares.

Table 3: **VAR results.** Estimation of the consumption growth and state formed by corporate earnings and housing stock growth. Standard errors in parenthesis.

	Constant	x_t^1	x_t^2
Δc_{t+1}	0.0058	0.0015	0.0101
	0.0003	0.0016	0.0032
x_{t+1}^1	0.0021	0.9581	-0.0362
	0.0014	0.0050	0.0100
x_{t+1}^2	0.0007	-0.0010	0.9851
	0.0005	0.0066	0.0037

$$-\log \alpha_{t+1} = \mu^\alpha + \phi^\alpha x_{t+1} + x'_{t+1}\Psi x_{t+1} + u_{t+1} \tag{30}$$

$$u_{t+1} = \lambda_1 + \lambda_2 z_{t+1} + \lambda_3 z_{t+1}^2 \tag{31}$$

$$z_{t+1} = \lambda_4 z_t + v_{t+1}. \tag{32}$$

Table 4 lists the parameter results and includes the linear and the quadratic components of the model dynamics, as well as the autocorrelation of the hidden shock z_t that has been filtered out. The state vector becomes an 'augmented' state vector, since I add the hidden state component. Therefore there are 4 shocks driving the economy: the consumption shock, the consumption growth shock, the housing growth shock, and the hidden state component, which does not have an intuitive interpretation, and it serves the purpose of give the quadratic structure to the error term. The standard deviations are computed from the asymptotic variance of the structural parameters, developed in Smith (1993). The standard deviations for the expenditure shares equation is obtained by a montecarlo simulation.

To estimate the dynamics of cash flows and the exposure of the cash flows to the shocks in the economy, I add to the system an extra equation, given by (18). It is necessary to add an additional idiosyncratic shock per asset. The model with consumption growth, expenditure shares, and the state vector remains autonomous since the innovations of the dividends equation has been orthogonalized. The value of the parameters corresponding to dividends

Table 4: **Indirect Inference results.** Log of expenditure shares, with a quadratic structure in the error term explained by the hidden state z_t. Standard errors in parenthesis.

	Estimate	Std. Error
μ^α	0.1047	0.0980
ϕ_1^α	0.0014	0.0118
ϕ_2^α	0.0788	0.0450
Ψ_{11}	0.0230	0.0302
Ψ_{12}	0.0792	0.0287
Ψ_{22}	-0.5287	0.0526
λ_4	0.0000	0.0000
λ_1	-0.0000	0.0000
λ_2	0.9924	0.0128

Table 5: **Dividends Equations.** Estimates of the sensitivity of dividend growth to changes in the state variable, given by ϕ^d. σ_0^d captures the immediate response of dividend growth to a shock.

	$R^{\mathbf{mkt}}$	R^1	R^2	R^3	R^4	R^5	R^h
ϕ_1^d	0.0059	0.0069	-0.1018	0.1061	0.0727	0.1366	-0.0138
ϕ_2^d	0.7843	1.2104	-0.4893	-0.1170	1.4841	2.1290	-0.5104
σ_{011}^d	-0.0111	0.0573	-0.1405	0.1428	0.0754	0.1114	-0.0751
σ_{012}^d	0.0011	0.0024	-0.0206	0.0161	0.0160	0.0270	-0.0093
σ_{013}^d	0.0002	-0.0008	0.0023	-0.0024	-0.0012	-0.0019	0.0013
σ_{014}^d	0.0196	0.0515	0.0446	0.0293	0.0332	0.0522	0.0094

are summarized in Table 5. These values represent the exposure of the cash flows in the long-run, determined by the discounted impulse response functions, $\pi = \sigma_0^d + \phi^d(I-\delta)^{-1}\sigma_0^x$.

Figure 6 shows the impulse responses of non-housing consumption and expenditure shares to a shock which maximizes the permanent effect on consumption. For comparison, it is shown the effect of the same shock when aggregate consumption is used (both non-housing and housing), as in Hansen, Heaton, and Li (2008). The impact of a permanent shock is more than twice in the long-run than in the immediate period of the shock. Regarding the housing expenditure shares there is no immediate response. In the long-run it picks up and ends up being non-negligible, about one fifth of the consumption impact. The figure shows that there is a long-run effect of shocks in housing services, and that these are non-negligible. The combination of the two long-run effects determines the numerical results below.

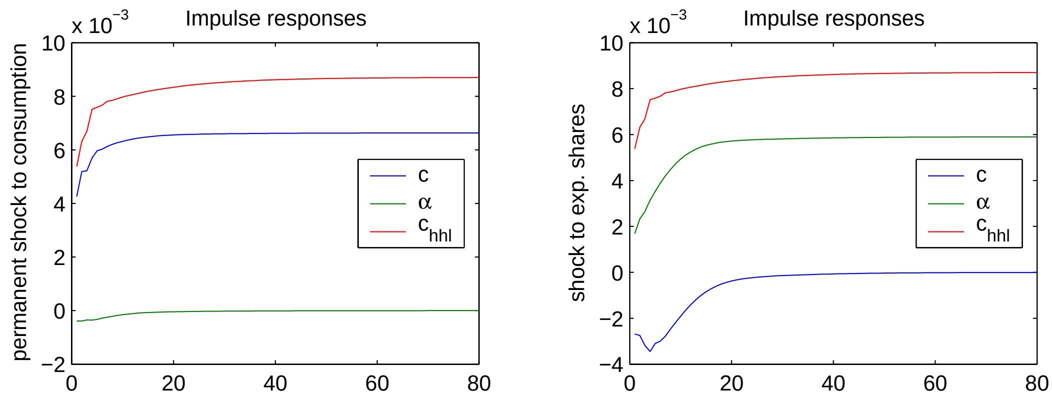

Figure 6: **Impulse response.** Responses of non-housing consumption and non-housing expenditure shares to a shock that has permanent effects on non-housing consumption on the left panel and to a shock that has permanent effects on both non-housing consumption and expenditure shares on the right panel. The shocks are normalized to have a unit standard deviation. The comparison with aggregate consumption is done with a two-dimensional VAR including aggregate consumption growth and earnings, under the cointegration assumption between earnings and consumption.

3.2 Pricing

As a first exploration of the model performance, Figure 7 displays the stochastic discount factor as in (10) and its three components: risk adjustment, consumption growth, and expenditure shares growth. Most of the variation comes from the consumption growth and from the continuation value term. To obtain the closed form solution for the value function and the stochastic discount factor, the elasticity of intertemporal substitution has been set to 1. For that special case, the factor corresponding to expenditure shares growth is raised to the power of 1. For ρ different than 1, elasticity of substitution between non-housing and housing consumption, ε, plays a crucial role: a value close to 1 amplifies the volatility of this factor. The figure displays the case of $\varepsilon = 1.4$. Few outlier observations might be the driving forces of the stochastic discount factor process if raised to a very high power. That happens when ε is very close to 1 and the intertemporal elasticity of substitution is different than 1. With $\rho = 1$, this problem disappears because the only channel through which ε plays a role in this special case is the continuation value, as it can be seen in (14).

The model is not able to deliver a reasonably low risk free rate. That is due to the

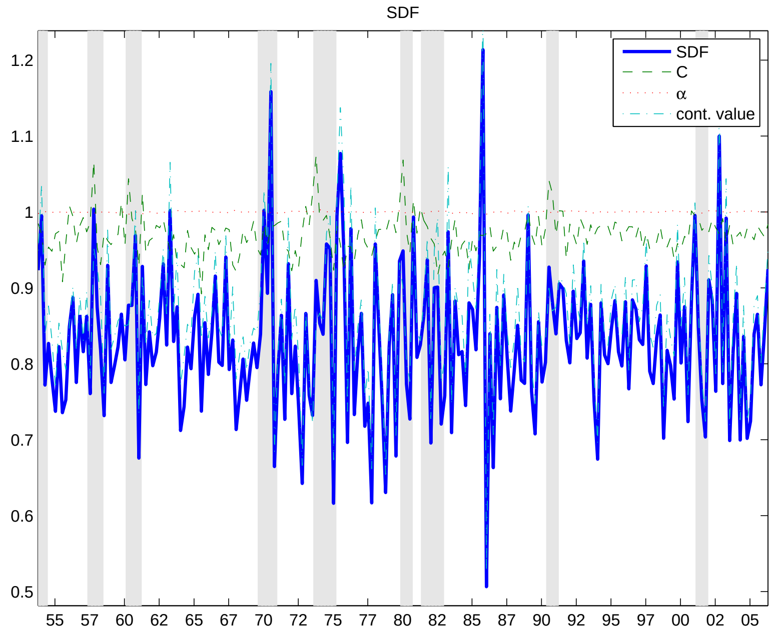

Figure 7: **Stochastic Discount Factor.** SDF with the data, for $\gamma = 2$.

fact that intertemporal elasticity of substitution has been set equal to 1 in order to obtain analytical closed form solutions for risk prices. Nevertheless, the objective of the paper is to estimate the exposure of different risky portfolios to long-run risks, the sources of the long-run risks themselves, and the prices of these long-run risks. Table 7 shows the expected 1 period and limiting risk premia for these two extreme portfolios.

I present the results for three models. First, the model *RU+housing* corresponds to the model developed in Section 2, where agents have recursive preferences over a non-separable bundle of housing and non-housing consumption modeled separately. The second model, *RU* corresponds to a model where agents have recursive preferences over non-housing consumption. It is a slight modification of Bansal and Yaron (2004) and Hansen, Heaton, and Li (2008), since they consider preferences over non-durables and services. The third model is the standard Lucas-Breeden power utility model. The risk prices associated with the shocks that drive the economy are summarized in table 6. The first two columns 6 show the local risk prices. Local risk prices are computed as the limit when the horizon shrinks to zero of the derivative of the risk premia with respect to the risk exposure; intuitively, is the price

Table 6: **Local and Limiting Risk Prices.**Risk Prices associated to the shocks on consumption growth and on expected value of consumption growth for $\gamma = 20$.

	Local			Limiting		
	Δc_{t+1}	$E_t^1(\Delta c_{t+1})$	$E_t^2(\Delta c_{t+1})$	Δc_{t+1}	$E_t^1(\Delta c_{t+1})$	$E_t^2(\Delta c_{t+1})$
RU + housing	0.1176	0.0174	0.0540	0.1177	0.0184	0.0559
Rec. Ut.	0.1176	0.0174	0.0534	0.1177	0.0186	0.0555
P. Ut	0.1163	0.0000	0.0000	0.1188	0.0239	0.0427

that agents put to a cash-flow that they will hold for an infinitesimal period of time if the risk exposure of such cash flow increases marginally. As it can be inferred from the two tables, the average slope of the term structure of risk prices will be positive. The more agents hold an asset, the higher price of the risk. The first shock, the temporary shock to consumption shows this fact. On the other hand, across assets the term structure is very similar with respect to the first shock. The last two columns of table 6 show the consumption and consumption growth risk prices in the long run, given by the derivative of the limiting risk premia with respect to the long run risk exposure, detailed in equation (28). High book to market is particularly sensitive to this risk in the long-run. This is consistent with a general equilibrium framework and the evidence that high book to market portfolios deliver a higher expected return. The valuation of these types of stocks is very sensitive to changes in the exposure of risk.

Figure 8 show the term structure of the risk prices. Intuitively, risk prices are given by how the price of the asset changes with a small change in the exposure of that asset to the underlying risk. That boils down to the derivative of the pricing operator with respect to the exposure of the cash flow. The plot shows how the representative agent prices the risk involved in cash flows that happen in the future. The recursive utility formulation causes the agent to price higher *today* cash flows that are riskier in the future. The continuation utility in the pricing kernel is the factor that captures this forward looking behavior of the agents. That is the reason why shocks to the expected growth rate in consumption have zero local price. The agent does not price the persistent risks in period zero, because there is no forward looking term in the pricing kernel. A shock to the expected consumption growth does not affect consumption growth immediately and the agent does not assign a positive price to it today for a 1-period horizon investment. A recursive-utility agent sees his continuation value affected today, and therefore assigns a positive price to such risk exposure. The non-separability adds an extra source of persistent risk: that it affects the consumption bundle formed by housing and non-housing consumption. Notice how the last shock, which captures the persistent effect in the housing expenditure shares (though it also affect expected consumption growth) has an effect on risk prices which is one order of magnitude bigger than the previous two. Agents in this economy assign a higher price than in an economy without separabilities due to the risk of having their optimal consumption bundle affected by these shocks for a long period of time. That is reflected in a higher risk premium associated to portfolios that have higher exposure to these type of shocks. As a reminder, in the empirical section the state variable that is affected by these shocks is the value of residential stock. Shocks to the value of residential stock predicts a lower growth in future housing services, which in turn affects the composition of the consumption bundle. And as it has been shown in (11), changes in this composition are a pricing factor in discounting future payoffs and also affect the continuation value, which is another pricing factor at the same time.

Alternatively, Table 7 displays the annualized one-year risk premia, for the two extreme portfolios under the two models with recursive utility and the power utility. The model with non-separabilities implies that the one-quarter returns required for the low and high book-to-market portfolios are lower than in the long-run, which can be inferred comparing the first two columns with the last two. They are assets whose exposure is lower in the short-run. The exposure of the high book-to-market portfolio, R^5, is relatively larger than the one for R^1. The spread is much higher in the short-run than in the long-run. Risk premia of the house that pays aggregate housing services has a negative risk premium, which becomes more negative in the future. That can be interpreted as the house being a hedge for the risks that are driving this economy. The longer the horizon we invest in the house, the better

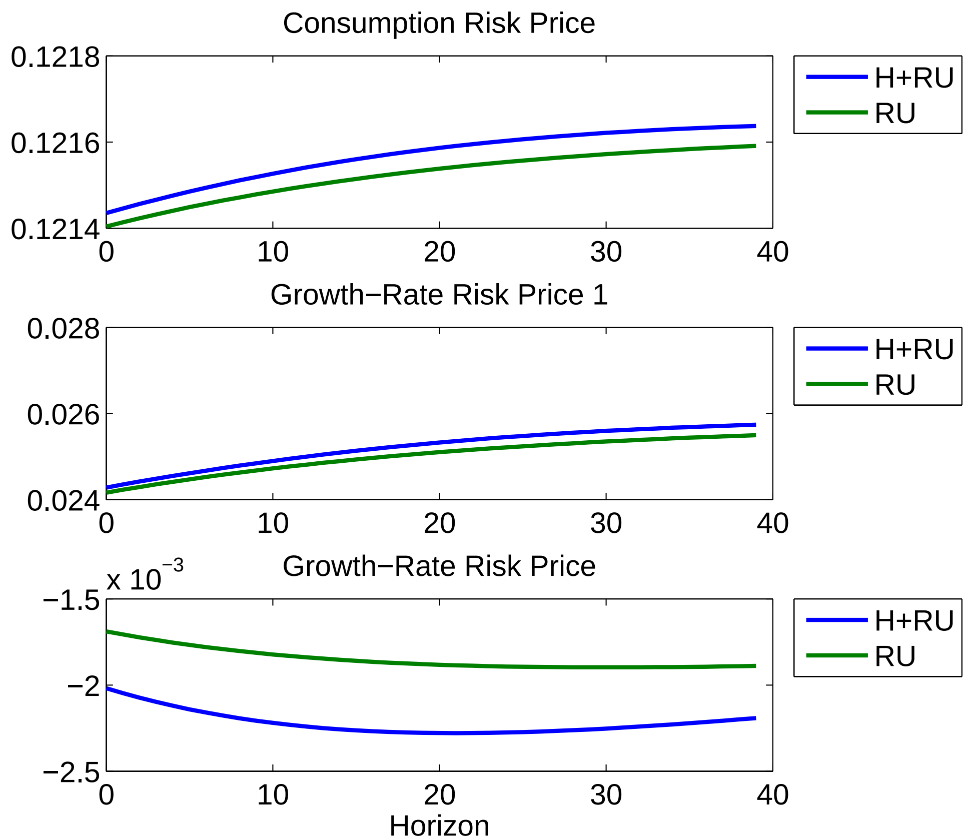

Figure 8: **Risk Prices.** Risk prices as a function of the horizon for each of the 3 shocks. On the y-axes, annualized risk prices of each of the shocks.

the hedge that the house provides, and therefore the more negative risk premium it carries. The returns as a function of the investment horizon are plotted in figure 9 for risk aversion 20, for the two extreme portfolios and also for an asset that pays real housing services as dividends. We can observe how the model with non-separabilities requires a higher return for the long-run holdings of low and high book-to-market, as expected. It is also noteworthy the fact that the value premium, that is, the difference between high and low book-to-market returns is the highest at horizons of 10 quarters, consistent with the evidence that the value premium arises at those horizons. The returns on the asset that pays housing services start higher at short horizons, but eventually becomes a safer asset when horizon increases, to the extent that it can be viewed as a hedge to riskier portfolios since the risk exposure is decreasing in time.

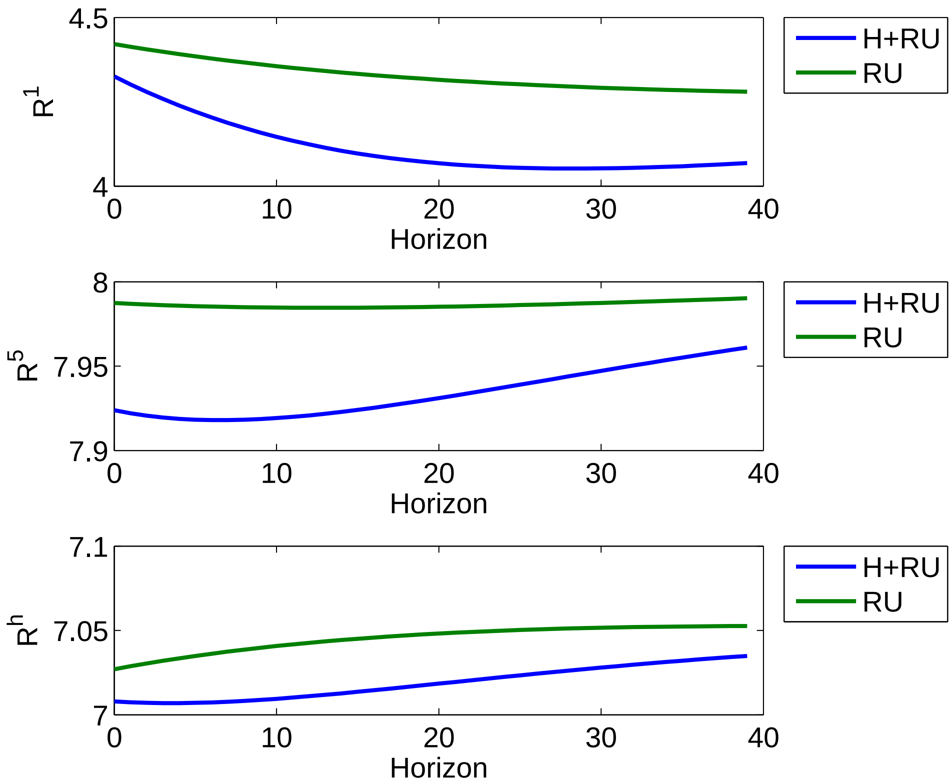

Figure 9: **Returns.** Term structure of returns for the lowest and highest book-to-market portfolios (R^1 and R^5 respectively) for the two models with recursive preferences.

Table 7: **One-Year and Limiting Risk Premia**. Annualized one-year returns and limiting returns, assuming the state of the economy x_t is at $x_t = [0, 0]$, in percentage. Risk aversion $\gamma = 20$. Results are shown for the low book-to-market, high book-to-market portfolios, and housing, R^1, R^5, and R^h respectively

	1 Period			Limiting		
	R^1	R^5	R^h	R^1	R^5	R^h
RU + housing	5.2780	7.1620	-5.8615	5.6000	7.4894	-6.4424
Rec. Ut.	5.2143	7.1017	-5.7907	5.5893	7.4854	-6.4614
P. Ut	-0.0000	0.9801	0.1941	0.4011	0.7897	0.8834

In figure 10 we observe how changes in exposure to the consumption shock (on the horizontal axes) will be reflected in changes of risk premia delivered by such an asset. The risk premia determined by the model explained in this paper is non-linear in the exposure (which is not observable in the scale of the figure) and are above the risk premia implied by the alternative benchmark models. Therefore, an agent that behaves according to the preferences proposed in my model will require higher extra return for an increase in risk exposure to consumption risk, than an agent under the two alternative models.

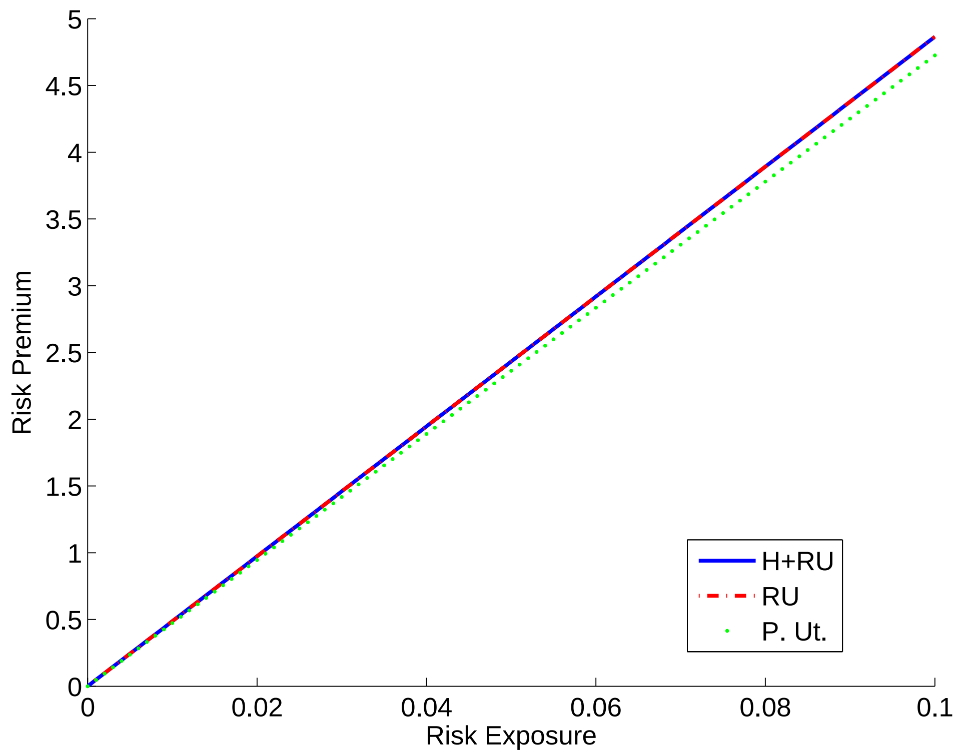

Figure 10: **Limiting Risk Premia.** Risk premia for the tree models considered as a function of exposure to the sources of risk, for $\gamma = 20$.

4 Conclusions

I have proposed a consumption based model that exploits the pricing implications of risk in the long-run and implies a time-varying risk premium. I use recursive preferences over a non-separable aggregation of non-housing goods and services consumption and consumption of housing services. I claim that housing services growth shows evident persistence. Persistence in consumption of housing services can be seen as a motivation for using long-run risk models. Consumers fear persistent shocks and, empirically, shocks to housing services seem to be much more persistent than shocks to non-housing consumption. I find that risk premia depend on the long-run exposure of assets' cash flows to risk and the premia change with the investment horizon considered. Housing is more exposed to risks that arise in the long-run rather than in the short-run, while the high and low book-to-market portfolios are more exposed —therefore highly rewarded— when investment horizons of 10 to 15 quarters are considered.

In this paper I have accomplished a theoretical challenge. Solving the value function and the pricing function with non-separabilities allows to identify three sources of risk and consequently estimate the prices associated to them. There are three main driving forces, parameterized and identified: (1) contemporaneous consumption growth risk, as in the traditional consumption based model; (2) composition risk, that arises from the fact that consumers decide a basket of non-separable consumption goods and services; and (3) long-run consumption growth, which captures the inter-temporal composition risk. The long-run risk requires a highly structured but flexible and interpretable model. The solution of the model with non-separability opens a new line of research. In particular, I have solved the stochastic discount factor, where I find that the price of risk depends on the response of consumption growth, the response of the composition of the consumption bundle of housing and non-housing consumption, and a term that is state dependent. It is shown that there is information about future consumption growth in the continuation value that it is not present in the current consumption but in the conditional expectation of consumption growth and expenditures in housing-services. The heteroskedasticity has another desirable implication. The risk premia are time-varying. In states where housing stock and/or corporate earnings predict lower future housing services or consumption growth, prices of risk are higher.

I have also shown that risk premium is sensitive to the investment horizon that investors consider. Investors with a short investment horizon face higher risks and require higher returns. In the long-run, the dividends that the house pays become safer and safer and the returns on holding the house decrease as investment horizon increase. Investing in housing with the purpose of enjoying the dividends of it, buy-and-hold strategy, provides a hedge against the underlying risks of the economy, particularly the risk coming from the shock to the current net value of residential stock, that predicts future consumption growth and future negative movements in non-housing expenditure shares.

There is yet another implication of using Epstein-Zin preferences, stated by Uhlig (2006). We have to consider all components of consumption believed to contribute to the utility of the agent. Non-separability across states in the Epstein-Zin framework induces a non-separability across goods. In this paper, I have considered non-separabilities where the two goods are non-housing and housing consumption. My approach is promising for capturing the implications of non-separability between consumption and leisure and evaluate the consequences of labor market risks on asset prices. The methodology presented here allows to evaluate the exact model empirically, without the necessity of approximating the return on the wealth portfolio.

Finally, this methodology avoids the criticism of modifying conditional moments of consumption growth to match aggregate facts, besides assuming a non-random walk model. This fact, along with using utility functions with parameters changing with the economic environment is heavily criticized in Zin (2001). I obtain endogenously a time-varying price of risk, since the stochastic discount factor is heteroskedastic. If the random walk versus the small predictable component for consumption growth are statistically not distinguishable, the use of either is equally well founded.

The risk pricing results and risk premia results are promising, in the sense that it is shown that the proposed model and proposed evolution for the endowment predicts higher prices for the risks that drive the economy.

Appendix

A.1 Value Function

Substituting the guess and the processes of equilibrium defined in equations (7) and (8) into 14, we have

$$
\begin{aligned}
v_t =&(1-\beta)\frac{\varepsilon}{\varepsilon-1}\left(\mu^\alpha+\phi^\alpha x_t+x_t'\Psi x_t\right)+\frac{\beta}{1-\gamma}\log E_t\left[e^{(1-\gamma)(v_{t+1}+\mu^c+\phi^c x_t+\sigma_0^c\nu_{t+1})}\right]\\
=&(1-\beta)\frac{\varepsilon}{\varepsilon-1}\left(\mu^\alpha+\phi^\alpha x_t+x_t'\Psi x_t\right)\\
&+E_t\left[e^{(1-\gamma)(D+F\delta x_t+F\sigma_0^x\nu_{t+1}+(\delta x_t+\sigma_0^x\nu_{t+1})'H(\delta x_t+\sigma_0^x\nu_{t+1})+\mu^c+\phi^c x_t+\sigma_0^c\nu_{t+1})}\right]\\
=&(1-\beta)\frac{\varepsilon}{\varepsilon-1}\mu^\alpha+\beta(D+\mu^c)\\
&+\left((1-\beta)\frac{\varepsilon}{\varepsilon-1}\phi^\alpha+\beta F\delta+\beta\phi^c\right)x_t\\
&+x_t'\left((1-\beta)\frac{\varepsilon}{\varepsilon-1}\Psi+\beta\delta'H\delta\right)x_t\\
&+\frac{\beta}{1-\gamma}\log E_t\left[e^{\underbrace{(1-\gamma)(F\sigma_0^x+\sigma_0^c+2x_t'\delta'H\sigma_0^x)}_{a'}\nu_{t+1}+\nu_{t+1}'\underbrace{(1-\gamma)\sigma_0^x{}'H\sigma_0^x}_{\Lambda}\nu_{t+1}}\right]
\end{aligned}
$$

To solve the expectation I use the fact that ν_{t+1} is normally distributed and solve the expected value:

$$
\begin{aligned}
E_t\left[e^{a'\nu_{t+1}+\nu_{t+1}'\Lambda\nu_{t+1}}\right]=&\int e^{a'\nu_{t+1}+\nu_{t+1}'\Lambda\nu_{t+1}}\frac{1}{(2\pi)^{n/2}}e^{-\frac{1}{2}\nu_{t+1}\nu_{t+1}'}d\nu_{t+1}\\
=&\frac{1}{(2\pi)^{n/2}}\int e^{a'\nu_{t+1}+\nu_{t+1}'(\Lambda-\frac{1}{2}I)\nu_{t+1}}d\nu_{t+1}
\end{aligned}
$$

By completing squares, I have to obtain inside the exponential $-\frac{1}{2}(\nu_{t+1}'+m')S^{-1}(\nu_{t+1}+m)$ to have a normal distribution with mean $-m$ and variance S. For

$$
\begin{aligned}
S=&-(2\Lambda-I)^{-1}\\
m'=&a'(2\Lambda-I)^{-1}
\end{aligned}
$$

the only part to complete the square is $-\frac{1}{2}m'S^{-1}m$, which I add and subtract and remains outside of the integral since it does not depend on the shock ν_{t+1}:

$$
\begin{aligned}
E_t\left[e^{a'\nu_{t+1}+\nu'_{t+1}\Lambda\nu_{t+1}}\right] =& \frac{1}{(2\pi)^{n/2}}\int e^{a'\nu_{t+1}+\nu'_{t+1}(\Lambda-1/2I)\nu_{t+1}}d\nu_{t+1} \\
& \times\int e^{\frac{1}{2}a'(2\Lambda-I)^{-1}a-\frac{1}{2}a'(2\Lambda-I)^{-1}a}d\nu_{t+1} \\
=& \frac{1}{(2\pi)^{n/2}}\int e^{-\frac{1}{2}(\nu'_{t+1}-\underbrace{a'(2\Lambda-I)^{-1}}_{\mu})'S^{-1}(\nu'_{t+1}-a'(2\Lambda-I)^{-1})}d\nu_{t+1} \\
& \times\int e^{-\frac{1}{2}a'(2\Lambda-I)^{-1}a}d\nu_{t+1} \\
=& \left|S\right|^{1/2}e^{-\frac{1}{2}a'(2\Lambda-I)^{-1}a}\frac{1}{(2\pi)^{n/2}\left|S\right|^{1/2}}\int e^{-\frac{1}{2}(\nu_{t+1}-\mu)'S^{-1}(\nu_{t+1}-\mu)}d\nu_{t+1} \\
=& \left|S\right|^{1/2}e^{-\frac{1}{2}a'(2\Lambda-I)^{-1}a}
\end{aligned}
$$

Using this result in the value function, I obtain

$$
\begin{aligned}
v_t =& (1-\beta)\frac{\varepsilon}{\varepsilon-1}\mu^{\alpha}+\beta(D+\mu^{c}) \\
& +\left((1-\beta)\frac{\varepsilon}{\varepsilon-1}\phi^{\alpha}+\beta F\delta+\beta\phi^{c}\right)x_t \\
& +x'_t\left((1-\beta)\frac{\varepsilon}{\varepsilon-1}\Psi+\beta\delta'H\delta\right)x_t \\
& +\frac{\beta}{1-\gamma}\log\left(\left|S\right|^{1/2}e^{-\frac{1}{2}a'(2\Lambda-I)^{-1}a}\right)
\end{aligned}
\tag{A-1}
$$

Finally, substituting in (a', S, Λ), I obtain the implicit expression for the value function, linear-quadratic in the state, as the guess was implying:

$$
\begin{aligned}
v_t =&(1-\beta)\frac{\varepsilon}{\varepsilon-1}\mu^\alpha+\beta(D+\mu^c)+\beta\frac{1-\gamma}{2}\log|(I-2(1-\gamma)\sigma_0^{x\prime}H\sigma_0^x)^{-1}| \\
&-\frac{\beta(1-\gamma)}{2}(F\sigma_0^x+\sigma_0^c)(2(1-\gamma)\sigma_0^{x\prime}H\sigma_0^x-I)^{-1}(F\sigma_0^x+\sigma_0^c)' \\
&+\left(\frac{(1-\beta)\varepsilon}{\varepsilon-1}\phi^\alpha+\beta(F\delta+\phi^c-2(1-\gamma)(F\sigma_0^x+\sigma_0^c)(2(1-\gamma)\sigma_0^{x\prime}H\sigma_0^x-I)^{-1}\sigma_0^{x\prime}H'\delta)\right)x_t \\
&+x_t'\left(\frac{(1-\beta)\varepsilon}{\varepsilon-1}\Psi+\beta\delta'H\delta-2\beta(1-\gamma)\delta'H\sigma_0^x(2(1-\gamma)\sigma_0^{x\prime}H\sigma_0^x-I)^{-1}\sigma_0^{x\prime}H'\delta\right)x_t
\end{aligned}
\tag{A-2}
$$

The parameters of the value function guess result as follow (implicitly defined):

$$
\begin{aligned}
D =&\frac{\varepsilon}{\varepsilon-1}\mu^\alpha+\frac{\beta\mu^c}{1-\beta}+\frac{1}{2}\frac{\beta}{1-\beta}\frac{1}{1-\gamma}\log|(I-2(1-\gamma)\sigma_0^{x\prime}H\sigma_0^x)^{-1}| \\
&+\frac{\beta}{1-\beta}\frac{(1-\gamma)}{2}(F\sigma_0^x+\sigma_0^c)(I-2(1-\gamma)\sigma_0^{x\prime}H\sigma_0^x)^{-1}(F\sigma_0^x+\sigma_0^c)'
\end{aligned}
\tag{A-3}
$$

$$
\begin{aligned}
F =&\left((1-\beta)\frac{\varepsilon}{\varepsilon-1}\phi^\alpha+\beta\phi^c+2\beta(1-\gamma)\sigma_0^c(I-2(1-\gamma)\sigma_0^{x\prime}H\sigma_0^x)^{-1}\sigma_0^{x\prime}H'\delta\right) \\
&\times\left(I-\beta\delta-2\beta(1-\gamma)\sigma_0^x(I-2(1-\gamma)\sigma_0^{x\prime}H'\sigma_0^x)^{-1}\sigma_0^{x\prime}H'\delta\right)^{-1}
\end{aligned}
\tag{A-4}
$$

$$
\begin{aligned}
H =&(1-\beta)\frac{\varepsilon}{\varepsilon-1}\Psi+\beta\delta'H\delta \\
&+2\beta(1-\gamma)\delta'H\sigma_0^x(I-2(1-\gamma)\sigma_0^{x\prime}H'\sigma_0^x)^{-1}\sigma_0^{x\prime}H'\delta
\end{aligned}
\tag{A-5}
$$

Working out a bit more the expression for F, we can obtain a simpler expression. The term

$$(I-2(1-\gamma)\sigma_0^{x\prime}H\sigma_0^x)^{-1}$$

will appear repeated times so it is convenient to define

$$\zeta\equiv(I-2(1-\gamma)\sigma_0^{x\prime}H\sigma_0^x)^{-1},$$

and D, F, and H become:

$$D = \frac{\varepsilon}{\varepsilon-1}\mu^{\alpha} + \frac{\mu^{c}}{1-\beta} + \beta\frac{1-\gamma}{2}\log|\zeta| + \frac{\beta}{1-\beta}\frac{(1-\gamma)}{2}(F\sigma_0^x + \sigma_0^c)\zeta(F\sigma_0^x + \sigma_0^c)' \tag{A-6}$$

$$F = \left((1-\beta)\frac{\varepsilon}{\varepsilon-1}\phi^{\alpha} + \beta\phi^{c} + 2\beta(1-\gamma)\sigma_0^c\zeta\sigma_0^{x\prime}H'\delta\right)(I-\beta\zeta\delta)^{-1} \tag{A-7}$$

$$H = (1-\beta)\frac{\varepsilon}{\varepsilon-1}\Psi + \beta\delta' H\delta + 2\beta(1-\gamma)\delta' H\sigma_0^x\zeta\sigma_0^{x\prime}H'\delta \tag{A-8}$$

A.2 Stochastic Discount Factor

I derive the stochastic discount factor from the shadow valuation of a stream of future value expressed in terms of marginal value of non-housing consumption:

$$\frac{V_t}{\frac{\partial V_t}{\partial C_t}} = \frac{\frac{\partial V_t}{\partial \mathcal{C}_t}\mathcal{C}_t}{\frac{\partial V_t}{\partial C_t}\frac{\partial \mathcal{C}_t}{\partial C_t}} + E_t\left[\frac{\frac{\partial V_t}{\partial V_{t+1}}\frac{\partial V_{t+1}}{\partial \mathcal{C}_{t+1}}\frac{\partial \mathcal{C}_{t+1}}{\partial C_{t+1}}}{\frac{\partial V_t}{\partial \mathcal{C}_t}\frac{\partial \mathcal{C}_t}{\partial C_t}}\frac{V_{t+1}}{\frac{\partial V_{t+1}}{\partial C_{t+1}}}\right]. \tag{A-1}$$

The first term inside the expected value is the shadow valuation of an stream of future value expressed in terms of marginal value of non-housing consumption. Therefore it is a valid stochastic discount factor and can be expressed as (10).

To collect the different derivatives,

$$\frac{\partial V_t}{\partial \mathcal{C}_t} = (1-\beta)V_t^{\rho}\mathcal{C}_t^{-\rho} \tag{A-2}$$

$$\frac{\partial V_t}{\partial V_{t+1}} = \beta V_t^{\rho} E_t\left[V_{t+1}^{1-\gamma}\right]^{\frac{\gamma-\rho}{1-\gamma}} V_{t+1}^{-\gamma} \tag{A-3}$$

$$\frac{\partial \mathcal{C}_t}{\partial C_t} = \left(\frac{\mathcal{C}_t}{C_t}\right)^{\frac{1}{\varepsilon}} \tag{A-4}$$

Now I can express (A-1) as

$$
\begin{aligned}
\frac{V_t}{\frac{\partial V_t}{\partial C_t}} &= \frac{\frac{\partial V_t}{\partial \mathcal{C}_t} C_t}{\frac{\partial V_t}{\partial C_t}} + E_t\left[\frac{\frac{\partial V_t}{\partial V_{t+1}} V_{t+1}}{\frac{\partial V_t}{\partial C_t}}\right] \\
&= \frac{\frac{\partial V_t}{\partial \mathcal{C}_t} C_t}{\frac{\partial V_t}{\partial \mathcal{C}_t}\frac{\partial \mathcal{C}_t}{\partial C_t}} + E_t\left[\frac{\frac{\partial V_t}{\partial V_{t+1}}\frac{\partial V_{t+1}}{\partial C_{t+1}}}{\frac{\partial V_t}{\partial C_t}}\frac{V_{t+1}}{\frac{\partial V_{t+1}}{\partial C_{t+1}}}\right] \\
&= \frac{\frac{\partial V_t}{\partial \mathcal{C}_t} C_t}{\frac{\partial V_t}{\partial \mathcal{C}_t}\frac{\partial \mathcal{C}_t}{\partial C_t}} + E_t\left[\frac{\frac{\partial V_t}{\partial V_{t+1}}\frac{\partial V_{t+1}}{\partial \mathcal{C}_{t+1}}\frac{\partial \mathcal{C}_{t+1}}{\partial C_{t+1}}}{\frac{\partial V_t}{\partial \mathcal{C}_t}\frac{\partial \mathcal{C}_t}{\partial C_t}}\frac{V_{t+1}}{\frac{\partial V_{t+1}}{\partial C_{t+1}}}\right] \qquad \text{(A-5)}
\end{aligned}
$$

The first term inside the expected value is the shadow valuation of an stream of future value expressed in terms of marginal value of non-housing consumption, therefore it is a valid one-period stochastic discount factor (SDF_{t+1}):

$$
\begin{aligned}
SDF_{t+1} = \frac{\frac{\partial V_t}{\partial V_{t+1}}\frac{\partial V_{t+1}}{\partial \mathcal{C}_{t+1}}\frac{\partial \mathcal{C}_{t+1}}{\partial C_{t+1}}}{\frac{\partial V_t}{\partial \mathcal{C}_t}\frac{\partial \mathcal{C}_t}{\partial C_t}} &= \frac{\beta V_t^{\rho} E_t\left[V_{t+1}^{1-\gamma}\right]^{\frac{\gamma-\rho}{1-\gamma}} V_{t+1}^{-\gamma}(1-\beta)V_{t+1}^{\rho}\mathcal{C}_{t+1}^{-\rho}\left(\frac{\mathcal{C}_{t+1}}{C_{t+1}}\right)^{\frac{1}{\varepsilon}}}{(1-\beta)V_t^{\rho}\mathcal{C}_t^{-\rho}\left(\frac{\mathcal{C}_t}{C_t}\right)^{\frac{1}{\varepsilon}}} \\
&= \beta\left(\frac{V_{t+1}}{E_t\left[V_{t+1}^{1-\gamma}\right]^{\frac{1}{1-\gamma}}}\right)^{\rho-\gamma}\left(\frac{\mathcal{C}_{t+1}}{\mathcal{C}_t}\right)^{\frac{1}{\varepsilon}-\rho}\left(\frac{C_{t+1}}{C_t}\right)^{-\frac{1}{\varepsilon}} \\
&= \beta\left(\frac{V_{t+1}}{E_t\left[V_{t+1}^{1-\gamma}\right]^{\frac{1}{1-\gamma}}}\right)^{\rho-\gamma}\left(\frac{\mathcal{C}_{t+1}}{\mathcal{C}_t}\right)^{-\rho}\left(\frac{1+\left(w_{t+1}\frac{S_{t+1}}{C_{t+1}}\right)^{\frac{\varepsilon-1}{\varepsilon}}}{1+\left(w_t\frac{S_t}{C_t}\right)^{\frac{\varepsilon-1}{\varepsilon}}}\right)^{\frac{1}{\varepsilon-1}} \\
&= \beta\left(\frac{V_{t+1}}{E_t\left[V_{t+1}^{1-\gamma}\right]^{\frac{1}{1-\gamma}}}\right)^{\rho-\gamma}\left(\frac{C_{t+1}}{C_t}\right)^{-\rho}\left(\frac{1+\left(w_{t+1}\frac{S_{t+1}}{C_{t+1}}\right)^{\frac{\varepsilon-1}{\varepsilon}}}{1+\left(w_t\frac{S_t}{C_t}\right)^{\frac{\varepsilon-1}{\varepsilon}}}\right)^{\frac{1-\varepsilon\rho}{\varepsilon-1}} \\
&= \beta\left(\frac{V_{t+1}}{E_t\left[V_{t+1}^{1-\gamma}\right]^{\frac{1}{1-\gamma}}}\right)^{\rho-\gamma}\left(\frac{C_{t+1}}{C_t}\right)^{-\rho}\left(\frac{\alpha_{t+1}}{\alpha_t}\right)^{\frac{1-\varepsilon\rho}{1-\varepsilon}} \qquad \text{(A-6)}
\end{aligned}
$$

Multiplying and dividing by C_{t+1} and C_t, we re-scale V_{t+1} and V_t so that we can use the solution we obtained in (A-5).

$$SDF_{t+1} = \beta \left(\frac{\frac{V_{t+1}}{C_{t+1}} \frac{C_{t+1}}{C_t}}{E_t \left[\left(\frac{V_{t+1}}{C_{t+1}} \frac{C_{t+1}}{C_t} \right)^{1-\gamma} \right]^{\frac{1}{1-\gamma}}} \right)^{\rho-\gamma} \left(\frac{C_{t+1}}{C_t} \right)^{-\rho} \left(\frac{\alpha_{t+1}}{\alpha_t} \right)^{\frac{1-\varepsilon\rho}{1-\varepsilon}} \tag{A-7}$$

and the consumption growth factor remains to the power of $-\gamma$ after rearranging and canceling

$$SDF_{t+1} = \beta \left(\frac{\frac{V_{t+1}}{C_{t+1}}}{E_t \left[\left(\frac{V_{t+1}}{C_{t+1}} \frac{C_{t+1}}{C_t} \right)^{1-\gamma} \right]^{\frac{1}{1-\gamma}}} \right)^{\rho-\gamma} \left(\frac{C_{t+1}}{C_t} \right)^{-\gamma} \left(\frac{\alpha_{t+1}}{\alpha_t} \right)^{\frac{1-\varepsilon\rho}{1-\varepsilon}} \tag{A-8}$$

Taking logarithms of (A-8), we obtain the following log stochastic discount factor:

$$\begin{aligned} sdf_{t+1} =& \log\beta - \gamma(c_{t+1} - c_t) + (\rho - \gamma)v_{t+1} - \\ & - \frac{\rho - \gamma}{1 - \gamma} \log E_t \left[e^{(1-\gamma)(v_{t+1} + c_{t+1} - c_t)} \right] + \frac{1 - \varepsilon\rho}{1 - \varepsilon} (\log \alpha_{t+1} - \log \alpha_t) \end{aligned} \tag{A-9}$$

Now, taking the approximation $\rho \to 1$, (A-9) becomes

$$\begin{aligned} sdf_{t+1} =& \log\beta - \gamma(c_{t+1} - c_t) + (1 - \gamma)v_{t+1} - \\ & - E_t \left[e^{(1-\gamma)(v_{t+1} + c_{t+1} - c_t)} \right] + \log \alpha_{t+1} - \log \alpha_t \end{aligned} \tag{A-10}$$

Following the above steps to solve for the expectation, we finally obtain

$$
\begin{aligned}
sdf_{t+1} = & \log\beta - \gamma(c_{t+1}-c_t) + \log\alpha_{t+1} - \log\alpha_t - \frac{1}{2}\log|\zeta| \\
& - \frac{(1-\gamma)^2}{2}(F\sigma_0^x+\sigma_0^c)\zeta(F\sigma_0^x+\sigma_0^c)' - (1-\gamma)\mu^c - (1-\gamma)\phi^c x_t \\
& - 2(1-\gamma)^2(F\sigma_0^x+\sigma_0^c)\zeta\sigma_0^{x\prime}H'\delta x_t - x_t'(2(1-\gamma)^2)\delta' H\sigma_0^x\zeta\sigma_0^{x\prime}H'\delta x_t \\
& + (1-\gamma)F\sigma_0^x\nu_{t+1} + 2(1-\gamma)x_t'\delta' H\delta\nu_{t+1} + \nu_{t+1}'(1-\gamma)\sigma_0^{x\prime}H\sigma_0^x\nu_{t+1} = \\
= & \log\beta - \frac{(1-\gamma)^2}{2}(F\sigma_0^x+\sigma_0^c)\zeta(F\sigma_0^x+\sigma_0^c)' - \frac{1}{2}\log|\zeta| \\
& - 2(1-\gamma)^2(F\sigma_0^x+\sigma_0^c)\zeta\sigma_0^{x\prime}H'\delta x_t - x_t' 2(1-\gamma)^2\delta' H\sigma_0^x\zeta\sigma_0^{x\prime}H'\delta x_t \\
& - (c_{t+1}-c_t) + \log\alpha_{t+1} - \log\alpha_t \\
& + \left((1-\gamma)(F\sigma_0^x+\sigma_0^c) + 2(1-\gamma)x_t'\delta' H\delta\right)\nu_{t+1} + \nu_{t+1}'(1-\gamma)\sigma_0^{x\prime}H\sigma_0^x\nu_{t+1} \qquad \text{(A-11)}
\end{aligned}
$$

This is the form where the factors can be identified. If we substitute in the processes to leave the sdf_{t+1} as a function of the state x_t and ν_{t+1}, to evaluate the price of risk, this is what is obtained:

$$
\begin{aligned}
=&\log\beta-\mu^c-\frac{(1-\gamma)^2}{2}(F\sigma_0^x+\sigma_0^c)(I-2(1-\gamma)\sigma_0^{x\prime}H\sigma_0^x)^{-1}(F\sigma_0^x+\sigma_0^c)'-\frac{1}{2}\log|\zeta^{-1}|+\\
&+\left(-\phi^c-(1-\gamma)^2(F\sigma_0^x+\sigma_0^c)\zeta\sigma_0^{x\prime}H'\delta+\phi^\alpha(I-\delta)\right)x_t+\\
&+x_t'\left(-2(1-\gamma)^2\delta'H\sigma_0^x\zeta\sigma_0^{x\prime}H'\delta+\Psi-\delta'\Psi\delta\right)x_t+\\
&+\left((1-\gamma)(F\sigma_0^x+\sigma_0^c)-\sigma_0^c+2x_t'\delta((1-\gamma)H-\Psi)\sigma_0^x-\phi^\alpha\sigma_0^x\right)\nu_{t+1}+\\
&+\nu_{t+1}'\sigma_0^{x\prime}\left((1-\gamma)H-\Psi\right)\sigma_0^x\nu_{t+1} \qquad \text{(A-12)}
\end{aligned}
$$

This is a summary of the variables' impulse responses. $\sigma(1)$ means the long-run impulse response, i.e. the sum of all the impulse responses in the infinite horizon, whilst $\sigma(\beta)$ is the discounted infinite sum of all the responses.

$$\sigma^x(1)=(I-\delta)^{-1}\sigma_0^x \qquad \text{(A-13)}$$

$$\sigma^x(\beta)=(I-\beta\delta)^{-1}\sigma_0^x \qquad \text{(A-14)}$$

$$\sigma^c(1)=\sigma_0^c+\phi^c(I-\delta)^{-1}\sigma_0^x \qquad \text{(A-15)}$$

$$\sigma^c(\beta)=\sigma_0^c+\beta\phi^c(I-\beta\delta)^{-1}\sigma_0^x \qquad \text{(A-16)}$$

$$\sigma^\alpha(1)=\phi^\alpha\sigma^x(1)+\sigma_0^{x\prime}S\sigma_0^x, \quad \text{where } S-\delta'S\delta=\Psi \qquad \text{(A-17)}$$

$$\sigma^\alpha(\beta)=\phi^\alpha\sigma^x(\beta)+\sigma_0^{x\prime}S_\beta\sigma_0^x, \quad \text{where } S_\beta-\beta^2\delta'S_\beta\delta=\Psi \qquad \text{(A-18)}$$

The term $F\sigma_0^x + \sigma_0^c$ can be expressed as

$$\begin{aligned} F\sigma_0^x + \sigma_0^c =& \sigma^c(\beta\zeta) + (1-\beta)\frac{\varepsilon}{\varepsilon-1}\sigma^\alpha(\beta\zeta) \\ & - (1-\beta)\frac{\varepsilon}{\varepsilon-1}\sigma_0^{x\prime} S_{\beta\zeta}\sigma_0^x + 2\beta(1-\gamma)\sigma_0^c\zeta\sigma_0^{x\prime} H\delta(I-\beta\zeta\delta)^{-1}\sigma_0^x \end{aligned} \tag{A-19}$$

A.3 Eigenfunction

The solution to the eigenfunction problem

$$\varphi(x_t) = e^{-ax_t - \frac{1}{2}x_t' b x_t} \tag{A-1}$$

is given by

$$\begin{aligned} a =& \left(K + 2(-\sigma_0^c - \phi^\alpha\sigma_0^x + (F\sigma_0^x + \sigma_0^c) + \pi)(\Sigma^{-1} - 2(N + \sigma_0^{x\prime} b\sigma_0^x))^{-1}\sigma_0^{x\prime}((1-\gamma)H - \Psi + b)'\delta\right) \\ & \left(I - \delta - 2\sigma_0^x(\Sigma^{-1} - 2(N + \sigma_0^{x\prime} b\sigma_0^x))^{-1}\sigma_0^{x\prime}((1-\gamma)H - \Psi + b)\delta\right)^{-1} \end{aligned} \tag{A-2}$$

$$b = L + \delta' b\delta + 2\delta'((1-\gamma)H - \Psi + b)\sigma_0^x(\Sigma^{-1} - 2(N + \sigma_0^{x\prime} b\sigma_0^x))^{-1}\sigma_0^{x\prime}((1-\gamma)H - \Psi + b)'\delta \tag{A-3}$$

which is solved numerically.

A.4 Cointegration Restrictions

Two cointegration restrictions have been imposed in the estimation of the system. Corporate earnings are cointegrated with non-durable goods and services consumption and current-cost net value of residential stock is imposed to be cointegrated with expenditures in non-durables and services.

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