Optimal Severity of Stress-Test Scenarios[*]

Johannes J. Fischer[†] **Natalie Kessler**[‡]

September, 2023

Abstract

Bank stress tests, regularly conducted to ensure stable lending, constitute a de facto constraint on balance sheets: equity must be sufficient to maintain current lending also in the future, even after absorbing severe loan losses. We study the effects of such forward looking constraints in a representative bank model. More severe-stress test scenarios lead to lower dividends, higher equity levels, and universally lower, albeit less volatile, lending. We calibrate our model to large U.S. banks, subject to Federal Reserve stress tests, and compute the optimal, state-dependent severity of stress tests and implied capital buffers (up to 6% during normal times). Finally, we complement stress tests with three macro-prudential policies: the Covid-19 dividend ban, the counter-cyclical capital buffer (CCyB), and the proposed dividend prudential target (DPT). We find that combing stress tests with a dividend ban or DPT improves supervisor welfare equally. Due to its discontinuous nature, however, relaxing the CCyB falls short.

JEL Classification: E61, G18, G21, G32
Keywords: bank stress-tests, forward-looking equity constraints, optimal lending, micro- versus macro-prudential policy
Declarations of Interest: none

[*]We thank Klaus Adam, Thorsten Beck, Giacomo Calzolari, Edouard Challe, Michael Koetter, Iman van Lelyveld, Györgyi Lóránth, Elena Loutskina, Ramon Marimon, Silvia Miranda-Agrippino, Manuel A. Muñoz, Jean-Charles Rochet and Marina Sanchez for their detailed comments. Further, we would like to thank the participants at various seminars, conferences and the two anonymous referees for the DGF-conference.

[†]Bank of England; e-mail: johannes.fischer@bankofengland.co.uk

[‡]De Nederlandsche Bank, and Department of Economics, European University Institute; e-mail: n.kessler@vu.nl

1. Introduction

The financial crisis of 2008-09 has highlighted how crucial bank health is for economic stability and growth. To promote a safe and sound financial system going forward, supervisory authorities around the world have since introduced a wide range of new regulatory measures. As part of this policy package, the Federal Reserve (Fed), the European Central Bank (ECB), and many other authorities have begun to subject banks to regular, usually annual stress tests. The objective of regular bank stress tests is to ensure that banks are sufficiently capitalized to maintain their current lending activities even under severely adverse macroeconomic conditions in the future.[1] In the U.S., banks found to be insufficiently capitalized in a hypothetical downturn are consequently restricted in their dividend payments: depending on the severity of violation, an increasing amount of net-income must be retained to boost equity levels.[2]

This regulatory pressure on dividend payments clashes with the banks' apparent objective to generate stable dividends that compensate shareholders for their investments (Koussis and Makrominas, 2019; Larkin et al., 2017).[3] These dividends are paid from both accumulated equity and returns on assets that are financed via equity capital and debt. To keep dividends smooth across the business cycle, banks deplete capital reserves when facing negative earning shocks (see Figure 1). Unregulated, simultaneously maintaining stable dividend levels and minimum capital ratio requirements may lead to asset shrinkage during crisis periods. Thus, intuitively, supervisory restrictions on dividend payments via stress tests seem warranted to maintain equity capital and thereby to ensure lending to viable firms.

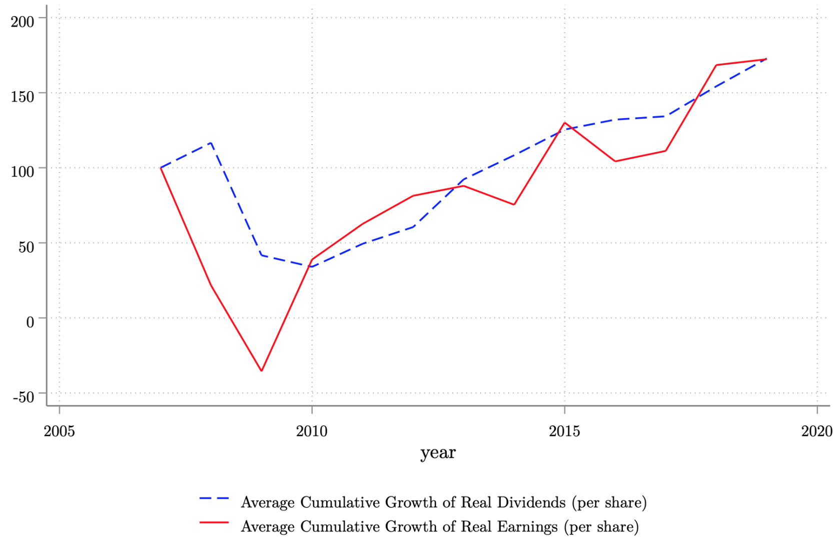
This argument, however, ignores how banks might change their behavior in anticipation of stress-test constrained dividend payments. To the banks' risk-averse shareholders, a safe payment today is worth more than an expected equal amount tomorrow that is subject to uncertainty. To pass the stress tests, banks therefore may avoid cutting dividends and instead reduce lending levels. Hence, one must account for the bank's margin of adjustment when evaluating the efficiency of stress tests. Thus far, the existing literature on stress tests provides little insights on *ex ante* dividend and lending choices by stress-tested banks, as it focuses mainly on the announcement effect of bank stress-test results and the subsequent immediate stock-price responses (Beck et al., 2020; Goldstein and Leitner, 2018a; Sahin et al.,

[1]Thus, stress tests extend the existing macro-prudential framework by going beyond point-in-time-estimates.

[2]A detailed description of the U.S. regulatory framework can be found in Appendix A.

[3]There is no shortage of potential explanations for banks' dividend smoothing policies, ranging from investor interests to managerial pay-out schemes directly linked to dividend stability (Lambrecht and Myers, 2012; Wu, 2018). In this paper we do not take a stand on the cause of this behaviour but rather take it as a given bank objective.

Figure 1: Cumulative Growth of BHC Earnings and Dividends (2007=100)



Note: Sample includes all banks that were registered as Bank Holding Companies (BHCs) in 2007 and were at any subsequent point subject to the stress tests of the Comprehensive Capital Analysis and Review (CCAR) regulatory framework.

2020).

Research Agenda In this paper, we compute the optimal tightness of a forward-looking stress-test constraint, endogenizing its effect on banks' dividend, equity, and lending policies. For this, we build a partial equilibrium framework that characterizes these three bank choices, given different return realizations and varying tightness of the stress-test constraint. We then derive the optimal tightness of the stress-test constraint for a supervisor who seeks to maximize lending levels while avoiding lending volatility. For this derivation we partially rely on a calibration of our model for a quantitatively meaningful discussion. Finally, we investigate how stress tests perform in unison with other policies, such as the Covid-19 dividend ban, the countercyclical capital buffer, and the dividend prudential target by Muñoz (2020) (banks must pay a punishment fee when dividends deviate from a regulatory target).

Theoretical Framework To illustrate the effects of a stress-test constraint on bank balance sheet choices, we propose a three-period, partial equilibrium framework. The model is populated by a supervisor with mean-variance welfare over bank lending and a representative investor with mean-variance preferences over dividends received from investments in said bank loans.[4] The objective of the supervisor is, thus, in conflict with objective of the investor: while the investor prefers high and stable dividends, the supervisor prefers

[4] Assuming mean-variance preferences introduces the above described bank preference for smooth dividends. Lambrecht and Myers (2012) provide a micro-foundation for such an objective function.

high and stable lending.

The environment is characterized by a single source of uncertainty: loan returns evolve over all three periods following an AR(1) process. This ensures that the supervisor is uncertain over next period's bank lending when designing stress tests and, subsequently, the bank is uncertain over loan returns when investing the next period. The parameter space additionally contains an initial bank equity endowment, an interest rate on bank deposits, and an exogenously given minimum equity-to-loan constraint. As our key novelty in this paper, we model the stress test as a forward-looking constraint of the bank's balance sheet choices: The bank's retained equity must be sufficient to absorb (simulated) severely adverse losses from the chosen lending levels without violating the minimum equity-to-loan ratio. It is, thus, similar to a minimum equity-to-loan ratio reflecting that the frequency with which stress tests are performed does not allow banks to temporarily adjust the balance sheet simply for the purpose of passing the stress test.

The model evolves as follows. In period 0, an initial loan return state realizes and the representative investor is endowed with the equity holding in the bank. Observing both, the supervisor decides on the tightness of the forward looking stress-test constraint with the objective to maximize welfare as a function of next period's lending. In period 1, the bank observes the initial equity and an evolved loan return state. With the objective to maximize the shareholder's total expected dividends, the bank first decides how much equity to retain versus to pay out as period 1 dividends. The retained equity and additional external debt are used to invest in risky loans. Here, the degree of debt financing of loans is constrained by both the stress-test and the minimum equity-to-loan ratio. In period 2, a further evolved loan return rate realizes and, together with last period's equity, lending, and debt choices, determines period 2 dividends. After paying out such to the investor, the bank ceases to exist.

Bank Choices First, we show that any meaningful stress-test scenario results in a de facto increased minimum equity-to-asset ratio (capital buffer). Hence, the forward looking stress-test constraint always binds before the minimum equity-to-loan constraint. Moreover, the bank always lends as much as the stress-test constraint allows, given the level of optimal equity. The optimal equity follows a step function in return states: in bad return states no equity is retained as loans are very risky and investments not profitable; in medium states a portion of equity is retained for risky investments and a portion is paid out as dividend; only in high return states all inherited equity is retained to be fully invested in loans. Performing comparative statics over the stress-test constraint tightness (the severity of the adverse scenario) highlights the core supervisory trade-off: an increase in tightness leads to higher retained equity in (almost) all states of the world, but always reduces lending levels.

At the same time, however, a tighter stress-test constraint leads to less volatile lending.

Optimal Tightness A fully analytical expression of the optimal stress-test tightness that would navigate this trade-off is unfortunately not available due to the underlying stochastic process together with the kinks in optimal lending and equity policies. To nevertheless provide a quantitative estimate, we calibrate the model using balance sheet data of U.S. bank holding companies that are subject to the stress tests implemented by the Comprehensive Capital Analysis and Review (CCAR) regulatory framework.[5] We then numerically derive the ex-ante optimal tightness of the stress-test constraint that maximizes the supervisor's mean-variance preferences over expected lending. We find that the optimal tightness typically leads to additional capital buffers of up to 6%, depending on the supervisor's aversion to lending volatility: a supervisor more (less) concerned about the volatility than the level of lending imposes a tighter (looser) stress-test scenario. Furthermore, in a higher (lower) initial return state a supervisor imposes a even looser (stricter) stress-test scenario. This numerical result closely matches the Federal Reserves' recently announced stress-test buffers for 2021 which are reported to lie between 2.5% to 7.5% (Federal Reserve Board, 2021), indicating that we are able to capture well the magnitude of bank balance sheet choices under stress tests.

Sensitivity We then perform two sensitivity analyses to investigate the robustness of our model assumptions. First, we show that it would be optimal for large U.S. banks to violate stress tests in high return states as long as there is no cost associated with this violation. We can, thus, rule out the common concern of strategic violation of the weakest banks. Furthermore, we show that a supervisor would have to put extraordinarily large weight on the investor's welfare for it to make a quantitatively meaningful difference for the optimal stress-test design.

Policy Complements In a final step, we utilize our model to evaluate the joint ability of stress tests and several macro-prudential policies to maintaining stable lending. First, we investigate how a blanket dividend ban, as many supervisory agencies either introduced or contemplated at the beginning of the Covid-19 pandemic, impacts the lending of stress-tested banks. Here, we find that a ban successfully increases lending in all states, even though banks refrain from using as much debt financing as the stress-test constraint allows. At the same time the lending volatility is lower under a ban, leading to overall supervisory welfare improvements. Next, we show that relaxing a counter-cyclical capital buffer (CCyB) only marginally increases lending in bad states. Thus, the CCyB activation is less effective than the dividend ban and, when introduced on top of the ban, the CCyB has no

[5] See Appendix A for a detailed description of the regulatory environment.

further effects.[6] Due to its inherent discontinuity, the CCyB further increases the variance of lending, leading to an overall welfare reduction in comparison to stand-alone stress tests. Finally, we study the proposed dividend prudential target (DPT) of Muñoz (2020): Here, the banks must pay a fine for deviations from a state-dependent dividend target that decreases in return states. It is, thus, a smoother alternative to an outright ban. While it substantially reduces lending volatility, this comes at a high cost of reduced lending in good states. Overall, these two effects cancel out and, surprisingly, we find the DPT and the dividend ban to be equally welfare improving when introduced on top of stress tests. However, the welfare gains of a dividend ban for the supervisor are borne by the bank's shareholders so that the dividend prudential target would perform better in welfare terms once bank owner welfare is taken into account. When considering the investors welfare under DPT as well, we, thus, validate that the findings of Ampudia et al. (2022) also hold when accounting for the micro-prudential stress-test constraint.

Literature To the best of our knowledge, we are the first to explicitly model the forward looking stress-test constraint and, thus, theoretically study its impact on banks' joined decision over lending, equity, and dividend payments. For this purpose, we extend the two-period model by Gollier et al. (1997), in which a risk-averse decision maker chooses how much to invest in a risky and in a risk-free asset, respectively. All consumption takes place after returns have realized. We slightly deviate from this timing and ask how much a risk-averse bank shareholder would consume as dividends today (risk-free) versus how much to invest in loans for consumption tomorrow (risky). Further, we add an initial period 0, in which a risk averse supervisor determines the stress-test scenarios constraining such choice, but abstract from the possibility of bank default as originally studied in Gollier et al. (1997).

To characterize our bank problem in this setting, we borrow several elements from the dynamic banking literature. For our bank objective function (mean-variance utility in dividends), we rely on Lambrecht and Myers (2012), who provide a micro-foundation for the observed dividend smoothing behavior of banks. They show that agency frictions between managers and shareholders lead to a risk-averse bank objective, even when shareholders are initially risk-neutral and well-diversified. Further, we extend the uncertainty of the asset to span all three periods by utilizing the AR(1) process describing loan returns in Bolton et al. (2020). Bolton et al. (2020) study the impact of deposit changes in an environment where banks are price-takers on the asset side. Similar to their model, our representative bank is, thus, subject to market uncertainty. Further, assuming an AR(1) allows us to extend the uncertainty to the supervisor when determining the stress-test tightness. The result is an

[6] Here, we are thus able to provide an explanation for the current policy puzzle of unused CCyB buffers during the Covid-19 crisis (FSB, 2021).

easily extendable model that not only highlights the balance sheet effects of stress tests, but allows us to (numerically) compute the optimal scenario tightness.

By characterizing the optimal tightness of a forward-looking stress-test constraint, our paper primarily contributes to the scarce theoretical literature on optimal stress-test design. Complementary papers, such as those by Bouvard et al. (2015) and Goldstein and Leitner (2018b), explore the time-inconsistencies between ex ante strict stress-test scenarios and ex post lenience in application. Perhaps most closely related to ours is the paper by Orlov et al. (2021), who study the ex ante optimal macro-prudential stress test — applied to the whole banking system — and relaxed individual bank stringency ex post. Further related is also a study by Shapiro and Zeng (2019), who show how banks optimally risk-adjust their portfolio in response to stress tests with uncertain supervisor lenience. However, these papers typically hold dividends, equity, and debt levels fixed and instead study a choice at the intensive margin between a more and a less risky asset.

We complement these studies by endogenizing the banks' balance sheet size, abstracting from portfolio risk-adjustments at the intensive margin. Further, we abstract from regulatory uncertainty over lenience. Instead, our trade-off is driven by the clash between objective functions: while the supervisor prefers stable lending, the bank prefers to pay stable dividends. By calibrating our model, we are able to obtain optimal stress test implied equity buffers that are numerically close to those applied by the Fed in 2021 (Federal Reserve Board, 2021). Going one step further, we extend our model to include additional macro-prudential policies (dividend ban, CCyB and DPT). We study the complementary effect of these policies and stress tests in stabilizing lending, both in absolute and relative terms. We are, thus, able to contribute to similar welfare analyses by e.g Ampudia et al. (2022) through the combination of macro-prudential and micro-prudential policies.

On the empirical side, there exist a few papers, such as e.g. Cappelletti et al. (2019) and Cornett et al. (2020), that study the effect of stress tests on bank balance sheets. Here, we would like to highlight the recent study by García and Steele (2022) that directly explore the CCAR stress tests, used to calibrate our model, in a regression discontinuity design. They find that stress tests leads to asset shrinkage particularly in riskier loans in favor of less risky loans. This finding is resounded in similar studies using U.S. loan-level data. As Acharya et al. (2018), Cortés et al. (2020), and Doerr (2021) document, stress-tested banks reduce credit supply, especially to risky borrowers. Finally, we would like to briefly mention the relatively unrelated yet extensive literature that studies the information revealing mechanism of stress tests and their immediate impact on stock prices (Bird et al., 2020; Morgan et al., 2014; Petrella and Resti, 2013; Quijano, 2014).

Overview The remainder of the paper is organized as follows. In Section 2, we describe the baseline model environment and state the bank's optimal dividend, equity, and lending choices. In Section 3, we calibrate the model, quantify the marginal responses of equity and lending to changes in the stress-test tightness, and finally numerically establish the optimal stress-test tightness. Section 4.1 addresses the possibility for banks to voluntary violate stress tests and consider the behavior of a supervisor who also takes into account the welfare of the investor. In Section 5, we discuss several policy extensions, such as the Covid-19 dividend ban, the CCyB, and the DPT. Section 6 concludes and puts the theoretical and calibration exercise in perspective. The appendix contains a detailed description of the regulatory framework and all proofs.

2. Theoretical Analysis

The following section contains the representative bank problem and is structured in the three following sub-sections: Section 2.1 describes the baseline partial equilibrium framework inspired by the dynamic banking models of Bolton et al. (2020) and Lambrecht and Myers (2012), but modified to a three-period environment to allow for a tractable introduction of bank stress tests;[7] Section 2.2 subsequently derives the lending and equity choices by a stress-tested bank and, relying on this, Section 2.3 performs comparative statistics to study the response of equity and lending to the introduction of a stress test.

2.1. Three-period Model

The model is populated by a representative risk-averse investor owning a bank, or a representative bank for short, and a welfare-maximizing supervisor. Both agents live for three periods, denoted with $t = \{0, 1, 2\}$ respectively, and share a common discount factor β.[8] Each period t is characterized by the stochastic return on loans $r_{l,t}$ which follows an AR(1) process (more below). In period $t = 0$, an initial bank equity endowment $E_0 > 0$ and initial

[7]We rely on the serially auto-correlated loan returns from Bolton et al. (2020), but abstract from bank default and investments in risk-free bonds for tractability, as these play a subordinate role in a three-period model, where the choice is only between consuming today versus tomorrow. Similarly to Lambrecht and Myers (2012), we further assume that deposit rates are fixed and we rely on their Proposition 1 that provides a micro-foundation for the bank objective function proposed here. Here, we take advantage of the fact that normally distributed future loan returns simplify their exponential utility function to mean-variance utility. We additionally include a supervisor constraining bank choices via stress tests.

[8]We make this assumption for simplicity but it does not affect the model outcomes. As will be discussed in more detail in Section 3.3, the supervisor has preferences only about the expected level and variance of lending in period $t = 1$. Therefore, there is no intertemporal trade-off for the supervisor that is influenced be the discount factor.

return state $r_{l,0}$ realize. Observing these, the supervisor decides on the optimal stress-test tightness τ. In period $t = 1$, the representative bank observes an evolved loan return $r_{l,1}$ and E_0, and decides how much of the inherited equity to pay out as dividends versus to retain for loan investments. Here, the additional deposit financing of loans is constrained by both the stress test and a minimum equity-to-asset ratio requirement. In period $t = 2$, a further evolved loan return state $r_{l,2}$, together with inherited loan, deposit, and equity levels, determines the final dividend payment by the bank to the investor.

$t = 0$	$t = 1$	$t = 2$
• Equity E_0 and return state $r_{l,0}$ realize • Supervisor sets τ	• Return state $r_{l,1}$ realizes • Bank decides on dividends d_1, retained equity E_1, loans L_1 and deposits D_1	• Return state $r_{l,2}$ realizes • Dividends d_2 are determined given $r_{l,2}$, E_1, L_1 and D_1

Loan Returns The underlying uncertainty in our model stems from an AR(1) process in loan returns spanning all three periods. The process is initialized at with an initial return $r_{l,0}$ that is public knowledge at $t = 0$. Each subsequent period $t \in 1, 2$ is characterized by an evolved return $r_{l,t}$ that depends on a constant μ_l, past returns $r_{l,t-1}$ multiplied by the autocorrelation coefficient ρ_l and an iid shock ϵ_t, drawn from a standard-normal distribution, that is amplified by volatility parameter σ_l:

$$r_{l,t} = \mu_l + \rho_l r_{l,t-1} + \sigma_l \epsilon_t \qquad \text{where} \quad \epsilon_t \sim \mathcal{N}(0,1), \quad \mu_l > 0, \quad \rho_l \in (0,1), \quad \sigma_l > 0. \tag{1}$$

The initial state $r_{l,0}$ serves as the information set for the supervisor when setting the stress-test constraint (see Section 3 below). The supervisor, thus, experiences uncertainty over both $r_{l,1}$ and $r_{l,2}$. The representative bank takes actions in $t = 1$, when only uncertainty about period-2 returns $r_{l,2}$ remains. For ease of notation throughout the paper, we denote the unconditional mean of the return process with $\overline{\mu}_l$:

$$\mathbf{E}[r_{l,t}] = \mu_l + \rho_l \mathbf{E}[r_{l,t-1}] + \sigma_l \mathbf{E}[\epsilon_t], \tag{2}$$

$$\overline{\mu}_l = \mu_l + \rho_l \overline{\mu}_l, \tag{3}$$

$$\overline{\mu}_l = \frac{\mu_l}{1 - \rho_l}. \tag{4}$$

The Investor There exists a representative investor who is hand-to-mouth and subject to mean-variance utility $u(\cdot)$ from received time t dividends d_t.[9] [10] We denote the

[9]This assumption is micro-founded by Lambrecht and Myers (2012), who show that payout smoothing naturally arises when insiders are risk averse and/or subject to habit formation. Here, we rely on their result from Proposition 1 and directly model an objective function over dividends rather than over managerial rents subject to investor participation constraints.

[10]Because the investor always maximizes expected utility given normally distributed returns, we directly maximize mean-variance utility, whose solutions are exactly equal those given exponential utility and Taylor-

resulting aversion to risk with γ, such that:

$$u(d_t) = \mathbb{E}[d_t] - \frac{\gamma}{2}\mathbb{VAR}[d_t] \tag{5}$$

The Bank's Balance Sheet The investor dividends are financed through an initial equity endowment E_0 in a representative bank. At time $t = 1$, the bank observes E_0, a loan return state $r_{l,1}$, and the two regulatory constraints (more below). Given these states, the bank first decides how much initial dividends d_1 to pay versus how much equity E_1 to retain.

$$d_1 = E_0 - E_1 \tag{6}$$

Subsequently, the bank additionally sources costly deposits D_1, at the exogenous interest rate $r_d < \mu_l$, to finance investments in the risky loans L_1:

$$L_1 = E_1 + D_1. \tag{7}$$

In period $t = 2$, a new loan return $r_{l,2}$ realizes, where we assume that the loan returns follow an AR(1) process:

$$r_{l,2} = \mu_l + \rho_l r_{l,1} + \sigma_l \epsilon_2 \qquad \text{where} \quad \epsilon_2 \sim \mathcal{N}(0,1) \tag{8}$$

Then the combined choices of equity E_1, deposits D_1, and lending L_2 determine dividends d_2. Accounting for the underlying AR(1) process and the loan return state $r_{l,1}$, this implies:

$$d_2 = r_{l,2}L_1 - r_d D_1 + E_1 \qquad \text{where} \quad d_2 \sim \mathcal{N}\left(\left(\mu_l + \rho_l r_{l,1}\right)L_1 - r_d D_1 + E_1,\ L_1^2\sigma_l^2\right). \tag{9}$$

The Supervisory Constraints The choices of E_1, D_1, and L_1 are restricted by two supervisory constraints: a minimum equity-to-asset ratio constraint and a stress-test constraint. The first defines a minimum equity-to-asset ratio χ that effectively restricts the bank's debt financing of loans. Here, we assume that the minimum ratio χ is given exogenously.[11] For the choices E_1 and L_1 this implies:

approximate those of all other concave (risk-averse) utility functions (Levy and Markowitz, 1979; Markowitz, 2014).

[11] This follows the narrative that global minimum capital standards, such at the Basel III requirements, are not quickly and easily adjustable by a national authority without severe costs. Furthermore, it allows us to focus on the effect of the forward looking constraint.

$$\frac{E_1}{L_1} \geq \chi. \tag{10}$$

The stress-test constraint is forward looking instead, and requires that the bank's available equity at time $t = 2$ cannot drop below χ even under a severely adverse loan return state realization $r_{l,2}$. Here, the expected available equity is the sum of the retained equity E_1 and next period profits $\Pi_2(\tau)$ simulated for stress-test scenario τ:

$$\Pi_2(\tau) = (\overline{\mu}_l - \tau\sigma_l)L_1 - r_d D_1 \qquad \text{where} \quad \overline{\mu}_l = \frac{\mu_l}{1-\rho_l}. \tag{11}$$

Here, $\overline{\mu}_l$ denotes the unconditional mean of the AR(1) process and τ defines the number of standard deviations below $\overline{\mu}_l$ that describe the adverse scenario of $r_{l,2}$. As τ defines the severity of the adverse scenario, we will refer to it as stress-test constraint tightness throughout the paper. For now, tightness $\tau \geq 0$ is taken as given and can be interpreted as a model parameter. In Section 3, we relax the latter assumption and explicitly determine the optimal τ numerically. With the definition of $\Pi_2(\tau)$ in mind, the stress-test constraint thus takes the following shape:

$$\frac{E_1 + \Pi_2(\tau)}{L_1} \geq \chi. \tag{12}$$

The Bank's Optimization Problem The above described constraints complete the bank optimization problem in period $t = 1$. For this, we denote the investor's total utility from d_1 and d_2 with $U(d_1, d_2)$. The bank's optimization problem is thus:

$$U(d_1, d_2) = \max_{E_1,\, L_1} \quad d_1 + \beta\left[\mathbb{E}[d_2] - \frac{\gamma}{2}\mathbb{VAR}(d_2)\right], \tag{13}$$

$s.t.$

$$d_1 = E_0 - E_1, \tag{14}$$

$$L_1 = E_1 + D_1, \tag{15}$$

$$d_2 = r_{l,2}L_1 - r_d D_1 + E_1 \qquad \sim \mathcal{N}\Big((\mu_l + \rho_l r_{l,1})L_1 - r_d D_1 + E_1,\ \sigma_l^2 L_1^2\Big), \tag{16}$$

$$E_1 \geq \chi L_1, \tag{17}$$

$$E_1 + \Pi_2(\tau) \geq \chi L_1 \qquad \text{where} \quad \Pi_2(\tau) = (\overline{\mu}_l - \tau\sigma_l)L_1 - r_d D_1, \tag{18}$$

$$L_1 \geq 0, \tag{19}$$

$$E_1 \in [0, E_0]. \tag{20}$$

Here, equations (14) - (16) are the bank's balance sheet constraints, inequalities (17) and (18) denote the two supervisory constraints on equity, and constraints (19) and (20) are the feasibility constraints on lending and equity.[12]

Parameter Restrictions For the AR(1) process on loan returns, we assume that $\mu_l > 0$, $\rho_l \in (0,1)$ and $\sigma_l > 0$. For the supervisory constraints, we assume $\chi \in (0,1)$ and $\tau \geq 0$. For the risk-aversion we assume that $\gamma > 0$. For the initial equity endowment, we assume that $E_0 >> 0$, reflecting that we are dealing with large banks. Finally, for the deposit rate we assume that $r_d < \mu_l$ and $1 + r_d < 1/\beta$, jointly ensuring that debt financing of loans is desirable.[13]

2.2. The Bank's Optimal Choices

We now turn to solving the bank optimization, starting with simplifying the two supervisory constraints: the minimum equity-to-asset ratio (17) and the stress-test constraint (18). First, we use the budget constraint in (15) and the definition of $\Pi_2(\tau)$ to rearrange the stress-test constraint:

$$E_1 + (\overline{\mu}_l - \tau\sigma_l)L_1 - r_d(L_1 - E_1) \geq \chi L_1, \tag{21}$$

$$E_1 \geq \frac{\chi - \overline{\mu}_l + \tau\sigma_l + r_d}{1 + r_d} L_1. \tag{22}$$

Comparing this to the minimum equity-to-asset ratio constraint in (17), it is easy to see that, for sufficiently large τ, the stress-test constraint always binds first:

$$\frac{\chi - \overline{\mu}_l + \tau\sigma_l + r_d}{1 + r_d} \geq \chi, \tag{23}$$

$$\tau \geq \frac{\overline{\mu} - r_d(1+\chi)}{\sigma_l} = \tilde{\tau}. \tag{24}$$

And for τ below $\tilde{\tau}$, the minimum equity-to-asset ratio constraint binds first. In either case, the second constraint is binding exclusively in states where the first one is binding too.

***Lemma* 1.** *There exists a stress-test tightness threshold $\tilde{\tau}$, such that :*

(i) If $\tau < \tilde{\tau}$, the minimum equity-to-asset ratio constraint always binds first.
(ii) If $\tau \geq \tilde{\tau}$, the stress-test constraint always binds first.

The results from *Lemma 1* allow us to generalize the bank optimization problem to nest both supervisory constraints in a single equity constraint:

[12] Constraint (19) implies that the bank cannot short-sell loans. In (20), the lower bound implies that the bank cannot debt-finance dividends and the upper bound rules out additional equity injections.

[13] The latter implies that shareholders are less patient than depositors and thus have a preference for debt-financing of loans. As Gollier et al. (1997) discuss, this is a necessary assumption for this type of banking models and thus commonly found. The alternatives with $1/\beta = 1 + r_d$ and $1/\beta < 1 + r_d$ would respectively imply that the Modigliani Miller theorem holds or that the bank exclusively equity-finances loans.

$$E_1 \geq \chi(\tau) L_1 \qquad \text{where} \qquad \chi(\tau) = \begin{cases} \chi & \tau < \tilde{\tau} \\ \frac{\chi - \overline{\mu}_l + \tau\sigma_l + r_d}{1+r_d} & \tau \geq \tilde{\tau} \end{cases}. \tag{25}$$

Relying on this, we then derive the bank's optimal equity, dividend, and lending choices as a function of $\chi(\tau)$. The proof is described in detail in Appendix B, but follows a few very intuitive steps. First, it can be shown that, given the parameter assumptions, equity-financing loans is never desirable. Thus, the revised minimum equity constraint is always binding at the optimum. Denote the optimal loan level with L_1^*. Then, this implies:

$$L_1^* = \frac{E_1}{\chi(\tau)}. \tag{26}$$

This result can be substituted into the bank optimization problem to simplify it further. Temporarily ignoring the feasibility constraints on equity, equating the first-order-condition with respect to retained equity with zero, yields the following optimal equity level E_1^*:

$$E_1^* = \frac{\chi(\tau)}{\gamma\sigma_l^2}\left[\mu_l + \rho_l r_{l,1} - r_d - \chi(\tau)\left(\frac{1}{\beta} - 1 - r_d\right)\right]. \tag{27}$$

However, E_1 is feasibility-constrained from below at zero and from above at E_0. Inserting these bounds in the above Equation (27) and rearranging allows us to derive two thresholds $\underline{r_l}$ and $\overline{r_l}$:

$$\underline{r_l} = \frac{1}{\rho_l}\left[r_d - \mu_l + \chi(\tau)\left(\frac{1}{\beta} - 1 - r_d\right)\right], \tag{28}$$

$$\overline{r_l} = \frac{1}{\rho_l}\left[\frac{\gamma\sigma_l^2}{\chi(\tau)}E_0 + r_d - \mu_l + \chi(\tau)\left(\frac{1}{\beta} - 1 - r_d\right)\right]. \tag{29}$$

Here, threshold $\underline{r_l}$ denotes the return state $r_{l,1}$ below which no equity is retained and $d_1^* = E_0$. $\overline{r_l}$ denotes the return threshold above which equity is fully retained and $E_1^* = E_0$. With this, the optimal choices are fully characterized for a given $\chi(\tau)$, and summarized in *Proposition 1.*

***Proposition* 1.** *A given constraint tightness τ, equity endowment E_0, and return state $r_{l,1}$ imply the following optimal bank choices:*

(i) If $r_{l,1} \leq \underline{r_l}$ all initial equity is paid out, such that:

$$d_1^* = E_0, \tag{30}$$

$$E_1^* = L_1^* = d_2^* = 0. \tag{31}$$

(ii) If $r_{l,1} \in (\underline{r_l}, \overline{r_l})$, some equity is paid out and some retained, such that:

$$E_1^* = \frac{\chi(\tau)}{\gamma\sigma_l^2}\left[\mu_l + \rho_l r_{l,1} - r_d - \chi(\tau)\left(\frac{1}{\beta} - 1 - r_d\right)\right], \tag{32}$$

$$d_1^* = E_0 - E_1^*, \tag{33}$$

$$L_1^* = \frac{E_1^*}{\chi(\tau)}, \tag{34}$$

$$d_2^* = \frac{E_1^*}{\chi(\tau)}(r_{l,2} - r_d) + E_1^*(1 + r_d). \tag{35}$$

(iii) If $r_{l,1} \geq \overline{r_l}$, the initial equity is fully retained, such that:

$$E_1^* = E_0, \tag{36}$$

$$d_1^* = 0, \tag{37}$$

$$L_1^* = \frac{E_0}{\chi(\tau)}, \tag{38}$$

$$d_2^* = \frac{E_0}{\chi(\tau)}(r_{l,2} - r_d) + E_0(1 + r_d). \tag{39}$$

It is important to note that the kinks in the lending function are not just outliers of the return distribution but are quantitatively important: For an initial equity level equal to the optimal equity level at the unconditional mean of the return process (i.e. $E_0 = E^{ss}(\tau)$), the full-retainment return level is exactly equal to the unconditional mean of the return process. To see this, first define the steady state equity level for a given stress-test tightness τ;

$$E^{ss}(\tau) = \frac{\chi(\tau)}{\gamma\sigma_l^2}\left[\bar{\mu}_l - r_d - \chi(\tau)\left(\frac{1}{\beta} - 1 - r_d\right)\right], \tag{40}$$

and substitute it into the full-retainment return level:

$$\overline{r_l} = \frac{1}{\rho_l}\left[\frac{\gamma\sigma_l^2}{\chi(\tau)}\left(\frac{\chi(\tau)}{\gamma\sigma_l^2}\left[\bar{\mu}_l - r_d - \chi(\tau)\left(\frac{1}{\beta} - 1 - r_d\right)\right]\right) + r_d - \mu_l + \chi(\tau)\left(\frac{1}{\beta} - 1 - r_d\right)\right], \tag{41}$$

which simplifies to

$$\overline{r_l} = \frac{1}{\rho_l}\left(\bar{\mu}_l - \mu_l\right) = \bar{\mu}_l. \tag{42}$$

Therefore, the bank will retain all its initial equity for all return states equal to or larger than the unconditional mean of the return process. The associated lending function will, thus, be also flat for all return states above the unconditional mean. This discontinuity prevents us from deriving a closed-form solution for the optimal stress-test tightness τ^* so that we rely on a numerically solution in Section 3.3 instead.

2.3. The Effect of Stress Tests

In this section, we analyze how E_1^* and L_1^* change when the supervisor decides to introduce a stress-test constraint by raising τ above $\tilde{\tau}$. For this purpose, we introduce two additional superscripts ^{e} and ^{s}, denoting the equilibrium outcomes under a binding minimum equity-to-asset ratio and a binding stress-test constraint, respectively.

First it can be shown that raising τ implies a higher $\chi(\tau) > \chi$, which consequently results in a higher no-retainment state $\underline{r_l}$. We thus have that:

$$\underline{r_{l,1}^s} > \underline{r_{l,1}^e}. \tag{43}$$

An introduction of a τ above $\tilde{\tau}$ also implies that the full-retainment state is reached earlier:

$$\overline{r_{l,1}^s} < \overline{r_{l,1}^e}. \tag{44}$$

This implies that at the low end of the return distribution, a stress-test constraint incentivizes banks to retain equity only in relatively better states. At the high end of the return state distribution, full retainment is reached already at relatively worse states. Complementing this, it can be shown that for all $r_{l,1}$ above $\underline{r_l}$ and below $\overline{r_l}$, the optimal retained equity E_1^* increases linearly in $r_{l,1}$ but with a steeper slope, the higher the τ:

$$\frac{\partial E_1^*}{\partial r_{l,1}} = \frac{\chi(\tau)}{\gamma\sigma_l^2}\rho_l \qquad\qquad \frac{\partial^2 E_1}{\partial r_{l,1}\partial\tau} = \frac{\rho_l}{(1+r_d)\gamma\sigma_l} > 0. \tag{45}$$

Therefore, there exists a return state $\tilde{r} \in (\underline{r_{l,1}^e}, \overline{r_{l,1}^e})$, below (above) which a stress-test constrained bank retains less (more) equity than if it was constrained by the minimum-equity constraint only. Using Equation (27) we can characterize this threshold $\tilde{r}$ as:

$$\tilde{r}_l = \frac{1}{\rho_l}\left[r_d - \mu_l + \left(\chi(\tau) + \chi\right)\left(\frac{1}{\beta} - 1 - r_d\right)\right], \tag{46}$$

$$= \underline{r_{l,1}^s} + \frac{\chi}{\rho_l}\left(\frac{1}{\beta} - 1 - r_d\right). \tag{47}$$

Here, Equation (47) rearranges $\tilde{r}_l$ as a function of the no-retainment state, showing it to be only marginally higher. Thus, in most return states (and definitely the positive states) more equity is retained under stress tests. However, for very low return states, where loans are a particularly unfavorable investment, the bank does not find it optimal to increase its relative equity exposure to loans. Thus, it retains less than in the absence of stress tests.[14] Figure 2a below illustrates this effect of a stress-test constraint on retained equity.

Corollary 1. *Raising τ above $\tilde{\tau}$ leads to more retained equity in almost all states of the world.*

Figure 2: Minimum Equity-to-asset Ratio Versus Stress-test Constraint

(a) Retained Equity Levels (b) Lending Volume

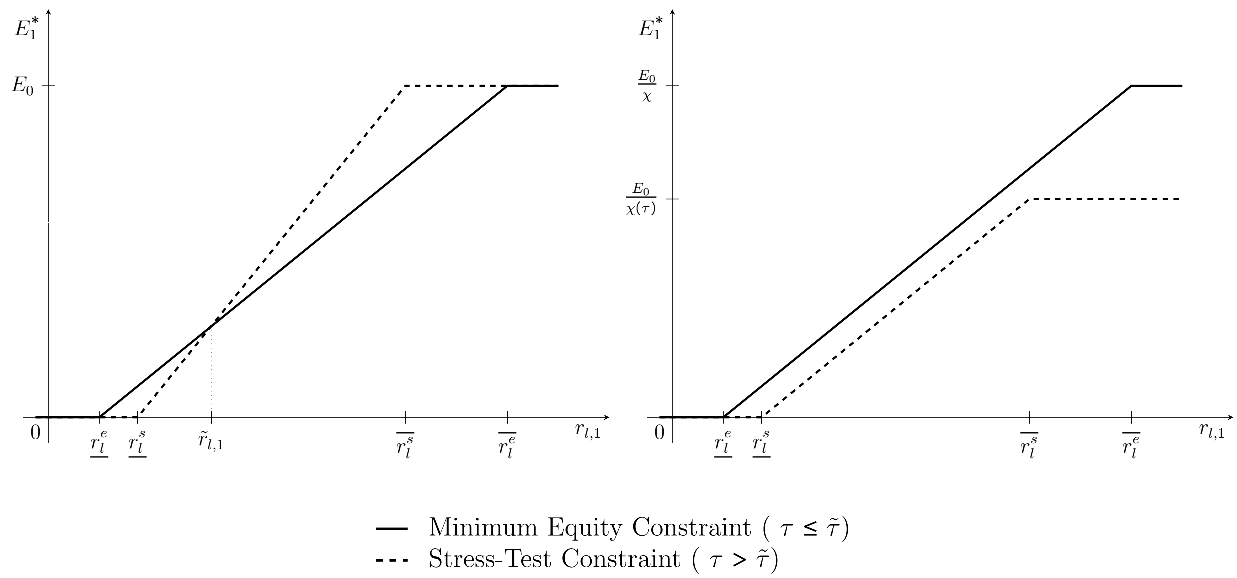


Figure 2b complements the comparison, by illustrating the effect of the stress-test constraint on lending. Here, we can see that the higher retained equity levels between $\tilde{r}_{l,1}$ and $r_{l,1}^s$ never translate into higher lending volumes. The extra equity is lower than the equity level that would be required to maintain the same level of lending under the tighter equity ratio constraint which is implied by the stress-test constraint. Thus:

$$L_1^{*,s} < L_1^{*,e} \quad \forall r_{l,1} > \underline{r_{l,1}^e}. \tag{48}$$

Furthermore, the volatility of lending also decreases under the stress-test constraint, given that equity retainment starts only at a relatively better state but the full-retainment state is reached earlier.

Corollary 2. *Raising τ above $\tilde{\tau}$ implies strictly lower but less volatile lending.*

[14] This provides a micro-founded support for several supervisors' decisions to pause regular bank stress tests during the Covid19 crisis (Baudino, 2020).

3. Calibration & Optimal Stress-test Tightness τ

We now turn to the supervisory choice of τ in period 0 and the resulting impact on lending and equity levels. Since this analysis requires a realistic model calibration, we use balance sheet data of stress-tested U.S. banks and discuss our model calibration in Section 3.1. We then use the calibrated model to quantify the marginal effects of adjusting the stress-test tightness τ on lending and equity in Section 3.2. In a final step, we compute the optimal choice of τ in Section 3.3.

3.1. Model Calibration

To provide a quantitative estimate of the optimal τ, we calibrate our model with two sets of parameters (see Table 1).

The first set of parameters (Panel A. of Table 1) consists of the discount factor, the risk aversion parameter, and the minimum equity-to-asset ratio. We pick a discount factor β equal to 0.99, which corresponds to an annualized real interest rate of 1%. We take the risk aversion parameter from Eisfeldt et al. (2020) and set it to 4.37. Furthermore, we take a minimum equity-to-asset ratio of 7% as given (see Appendix A).

The second set of parameters (Panel B. of Table 1) describes the loan return process as well as the return on deposits. For these parameters we use balance sheet data of U.S. Bank Holding Companies with more than \$10bn in assets between 2009 - 2019 (i.e. banks subject to CCAR stress tests) as reported in the FR Y–9C reports to calibrate the parameters of the loan return process as well as the return on deposits.

To calibrate the return process, we follow De Nicolò et al. (2014) and estimate an AR(1) process on the mean excess return on assets. We use the excess return over the risk-free interest rate to make sure that return movements are not driven by movements in the risk-free rate. We compute the excess return on assets as the ratio of *Total Interest and Noninterest Income* (item *bhcp4000*) to lagged *Total Assets* (item *bhck2170*) minus the 1-year Treasury rate.. We then add this excess return to our implied (time-invariant) risk-free rate $1/\beta - 1$ to arrive at the mean of the return process. The calibrated return process has a mean off 1.02% with a standard deviation of 0.52% and an autocorrelation of $\rho_l = 0.62$,, which implies an unconditional mean return of 2.66%.

To calibrate the deposit rate r_d, we again start by eliminating the movements of the risk-free rate and first estimate the deposit spread. We compute the deposit spread as the mean difference between the 1-year Treasury rate and the mean deposit rate, given by the ratio of interest paid on deposits (the sum of items *bhckhk03*, *bhckhk04*, *bhck6761*, and *bhck4172*) to lagged deposits (the sum of items *bhdm6631*, *bhdm6636*, *bhfn6631*, *bhfn6636*). We then

Table 1: Calibration

Description	Parameter	Value
A. Parameters assumed / obtained from literature		
Discount Factor	β	0.99
Risk Aversion	γ	4.370
Minimum Equity-to-asset Ratio	χ	0.07
B. Parameters estimated from data		
Mean Return of Risky Asset (%)	μ_l	1.02
		(0.03)
AR(1) of Risky Asset	ρ_l	0.62
		(0.04)
SD of Risky Asset (%)	σ_l	0.52
		(0.03)
Lending Spread (%)	$1/\beta - 1 - r_d$	0.39
		(0.01)
Return on Deposits (%)	r_d	0.62
		(0.01)

Note: Bootstrapped standard deviations reported in parenthesis.

subtract this deposit spread from our implied risk-free rate $1/\beta - 1$ to arrive at the deposit rate. Over our sample period, bank deposits yielded on average 0.39 percentage points less than the 1-year Treasury rate, yielding a return on deposits of 0.62% for our implied risk-free rate of 1%.

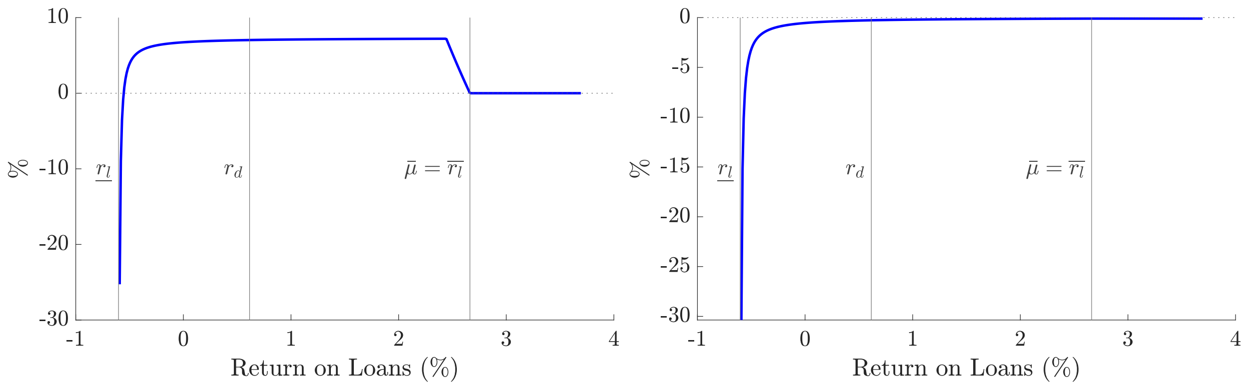
Alongside the calibrated parameters, Panel B. of Table 1 also reports the respective bootstrapped standard deviations (in parenthesis). We use these estimates to bootstrap the confidence intervals for the supervisory choice of τ^*.

3.2. Effect of Stress Tests on Equity and Lending

To illustrate the effect of stress tests, we use the calibrated model and plot the marginal responses of equity and loan levels (in %) to a one unit increase in the tightness of the stress-test constraint τ in Figure 3. It is clear that the effect of a higher stress-test constraint is highly non-linear in the state of the business cycle, i.e. the return state.

Following an increase of the stress-test tightness τ, equity (left panel) is lower for very bad states of the world due to an increased no-retainment threshold (see Equation 47). However, for most of the return realizations below the unconditional mean return $\overline{\mu}_l$, equity

Figure 3: Marginal Response of Equity and Lending to a Unit Increase in τ



(a) Marginal Response of Equity (%)   (b) Marginal Response of Lending (%)

is higher following the increase of τ. For return realizations above $\overline{\mu}_l$ the increase of τ does not lead to higher equity retainment, since banks retain all of their equity either way.

Lending volumes (right panel) are affected by changes of both retained equity as well as the minimum equity constraint in response to an increase of the stress-test tightness τ. For all return states below the unconditional mean return $\overline{\mu}_l$, an increase in τ reduces lending because the the increase in the minimum equity constraint offsets the increase in retained equity. For all return states above the unconditional mean return $\overline{\mu}_l$, retained equity is unchanged but the increased minimum equity constraint leads to lower lending. However, this effect is marginal because a unit increase in τ increases the implied equity constrained only by $\sigma_l/_{1+r_d}$.

This demonstrates that in all but very bad states of the world, the increase of τ can weakly enhance the safety of banks, but this unequivocally comes at the cost of lower lending levels, as the right panel shows. This reduction in lending, however, approaches zero as the return realisations increase.

3.3. The Supervisory Choice of τ

We now investigate how a supervisor optimally sets the severity of simulated losses used in the stress test (i.e. the number of standard deviationsτ below the mean return $\bar{\mu}$) with the objective to ensure stable lending levels.[15] Here, Corollary 2 highlights the supervisory trade-off between reduced but consequently less volatile lending. To capture this trade-off, we assign the welfare weight $\omega \geq 0$ to the expected variance of optimal bank lending L_1^*, i.e. a parameter of supervisor risk aversion (we explore the implications of an alternative supervisor welfare function in Section 4.2). Then, observing E_0 and $r_{l,0}$, the supervisor

[15] Note that this supervisory objective is taken directly from the Federal Reserve Board (2020c).

solves:

$$\max_{\tau} \quad \mathbb{E}[L_1^* \mid r_{l,0}, E_0] - \omega \mathbb{VAR}_0[L_1^* \mid r_{l,0}, E_0], \tag{49}$$

s.t.

$$\chi(\tau) \in [\chi, 1), \tag{50}$$

where

$$r_{l,1} \leq \underline{r_l}: \quad L_1^* = 0, \tag{51}$$

$$r_{l,1} \in (\underline{r_l}, \overline{r_l}): \quad L_1^* = \frac{\mu_l + \rho_l r_{l,1} - r_d - \chi(\tau)(1/\beta - 1 - r_d)}{\gamma \sigma_l^2}, \tag{52}$$

$$r_{l,1} \geq \overline{r_l}: \quad L_1^* = \frac{E_0}{\chi(\tau)}. \tag{53}$$

Equations (51) to (53) show that the supervisor anticipates a rectified normally distributed L_1^* with upper and lower bounds: (51) states that below $\underline{r_l}$, lending L_1^* is set to zero; (52) implies that between $\underline{r_l}$ and $\overline{r_l}$ lending is normally distributed with $\mathcal{N}\left(\mu_{L_1}, \sigma_{L_1}^2\right)$;[16] (53) states that above $\overline{r_l}$, lending is set to $E_0/\chi(\tau)$.

To identify the optimal stress-test tightness τ^*, we utilize our parameterization from Section 3.1 and computationally maximize the supervisor's welfare directly, subject to the respective constraints. As argued previously, the fact that loans follow a two-sided rectified distribution prevents us from deriving a closed-form expression for the optimal stress-test tightness so that we solve this problem numerically. Since the results depend to a large degree on the amount of initial equity E_0, we first define the steady state level E_1^{ss} in the absence of stress tests as

$$E_1^{ss} = \frac{\chi}{\gamma \sigma_l^2}\left[\overline{\mu}_l - r_d - \chi\left(\frac{1}{\beta} - 1 - r_d\right)\right], \tag{54}$$

and fix the initial equity endowment E_0 at this level to ensure comparable results.

To examine the supervisor's decision in more detail, we compute the optimal τ^* for different relative welfare weights ω. In particular, we compute the optimal stress-test tightness for a supervisor who does not care about lending volatility (i.e. $\omega = 0$), a supervisor who cares as much about lending volatility as about lending levels (i.e. $\omega = 1$), a supervisor who is as risk averse as the investor (i.e. $\omega = \gamma/2$), and a supervisor who is twice as risk averse as the investor (i.e. $\omega = \gamma$).

Table 2 below states the resulting, numerically derived optimal stress-test tightness τ^*, the implied minimum equity to asset ratio $\chi(\tau)^*$ (see equation (25)), and the associated supervisory welfare for the different welfare weights given an initial return realization of $r_{l,0} = \overline{\mu}_l = 2.66\%$. Table 2 also reports the 95% confidence intervals of each optimal policy

[16] Closed form expressions for μ_{L_1} and $\sigma_{L_1}^2$ can be found in Appendix C.

in square brackets. These confidence intervals are obtained by taking 10,000 draws from the distribution of parameters reported in Table 1 and computing the associated optimal supervisory policy.

Based on the implied welfare for the respective τ^*, it is clear that the supervisory welfare function is decreasing in the weight given to the variance of loans. The supervisor therefore optimally sets $\tau^* = 4.05$ such that $\chi(\tau^*) = \chi$ when she does not derive any disutility from the variance of loans (i.e. when $\omega = 0$) to maximize the level of loans. However, as ω increases and she derives more disutility from the variance of loans, she optimally sets a higher τ^* to reduce that variance. In the other extreme case, i.e. when the supervisor is twice as risk averse as the investor, she forces the bank to retain an additional stress-test capital buffer of 4%. These estimates generally are associated with confidence bands of up to 2%, indicating that it might be optimal for a very risk averse supervisor to require additional stress-test capital buffers of up to 6%. This matches well the Federal Reserve's publicly announced stress-test buffers, reported to be between 2.5% to 7.5% in the 2021 CCAR report (Federal Reserve Board, 2021) and indicates that we are able to capture well both the mechanism behind and the magnitude of bank balance sheet choices under stress tests.

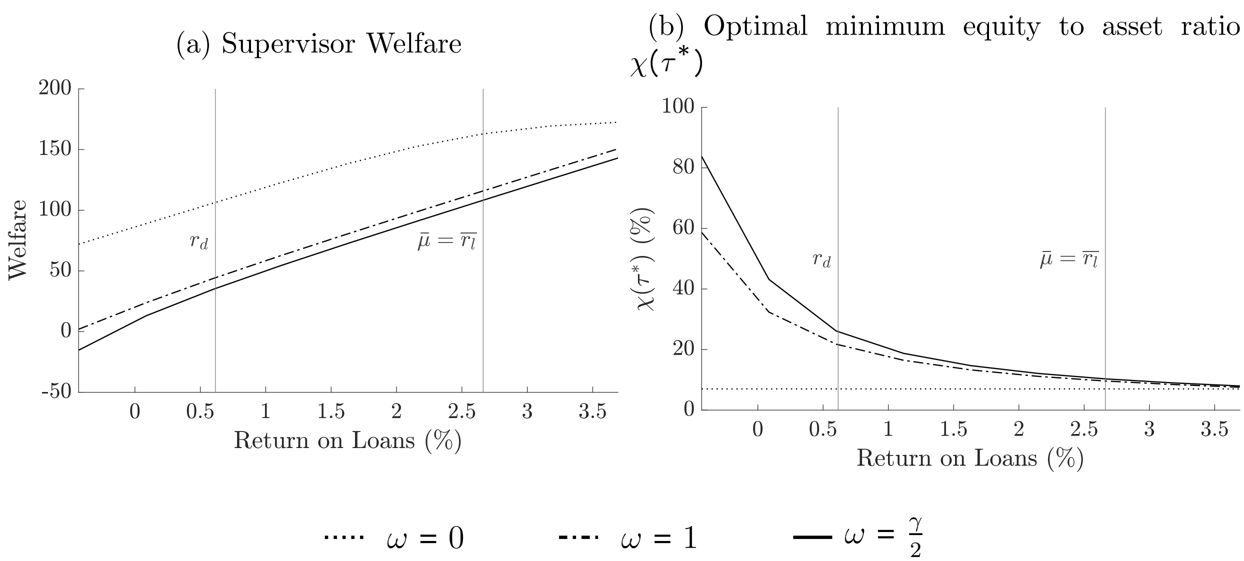
Table 2: Optimal Stress-test Tightness and Supervisor Welfare

Welfare Weight	Optimal Tightness τ^* (%)	$\chi(\tau^*)$	Welfare
$\omega = 0$	4.05	7.00	162.96
	[4.05 4.05]	[7.00 7.00]	[123.28 209.56]
$\omega = 1$	9.16	9.62	115.93
	[6.90 13.15]	[8.46 11.66]	[95.22 143.09]
$\omega = \gamma/2$	10.53	10.32	108.25
	[8.23 14.57]	[9.14 12.39]	[88.45 134.51]
$\omega = \gamma$	11.81	10.97	101.84
	[9.48 15.93]	[9.78 13.08]	[82.78 127.40]

Note: This table shows the results of computationally maximizing the supervisor's welfare, subject to the respective constraints (see Equation 49-53). We rely on the calibration from Section 3.1 to derive the optimal stress-test tightness τ^*, the implied minimum equity to asset ratio $\chi(\tau^*)$ (see Equation 25) and the associated supervisor welfare for different welfare weights ω. Values in square brackets indicate the 95% confidence intervals for each estimate constructed by taking 10,000 draws from the distribution of parameters reported in Table 1 and computing the associated optimal supervisory policy.

To further illustrate how these findings vary over different realizations of the initial return state r_0, Figure 4 displays the supervisor welfare (left panel) as well as the minimum equity to asset ratio $\chi(\tau^*)$ (right panel) implied by the optimal stress-test severity τ^* as a function of $r_{l,0}$ for different welfare weights ω. It is clear that the supervisory welfare function is increasing in the initial return realization $r_{l,0}$ and decreasing in the weight given to the variance of loans. However, the differences in welfare and the minimum equity to asset ratio decrease for higher realizations of $r_{l,0}$. Therefore, the supervisor would find it optimal to conduct significantly more severe stress tests during crises, imposing additional capital buffers of around 15% when loan returns fall below funding costs (i.e. for $r_{l,0}$ just below r_d).

Figure 4: Welfare and Minimum Equity to Asset Ratio under Optimal Stress Tests



4. Sensitivity Analysis

In this section, we perform two sensitivity analyses to investigate the robustness of the model and findings. In Section 4.1, we study whether banks would ever voluntarily fail stress tests. In Section 4.2, we derive the optimal τ^* under the assumption that the supervisor also considers bank investor utility.

4.1. Voluntary Stress-test Violation

In our baseline model environment, banks can neither violate the minimum equity-to-asset ratio nor the stress-test constraint. The U.S. stress test framework, however, allows for voluntary violation of the stress-test constraint, albeit automatically triggering a (partial) ban on dividend payments (see Appendix A for details). This violation allows the bank to invest up to a binding minimum-equity-to-asset ratio constraint instead. In this section, we investigate when a bank might find it optimal to purposely violate the stress-test constraint.

For simplicity, we assume that this immediately triggers a total ban on dividend payments in that period. Then, voluntary violation implies the following equalities:

$$d_1 = 0, \tag{55}$$

$$E_1 = E_0, \tag{56}$$

$$D_1 = L_1 - E_0. \tag{57}$$

Inserting these equalities in the original maximization problem results in the following revised bank objective:

$$\max_{L_1} \quad (\mu_l + \rho_l r_{l,1})L_1 - r_d(L_1 - E_0) + E_0 - \frac{\gamma}{2}\sigma_l^2 L_1^2, \tag{58}$$

$$s.t.$$

$$L_1 \in \left[E_0, \frac{E_0}{\chi} \right]. \tag{59}$$

The upper feasibility limit in (59), where now χ applies instead of $\chi(\tau)$, reflects that the violation replaces minimum equity requirements.

Quite intuitively, the upper feasibility limit is binding in very high return states above a threshold $\overline{r_{l,1}^V}$, where the bank would like to invest more in loans than the minimum equity requirements allow. Hence:

$$L_1^{*V} = \frac{E_0}{\chi} \qquad \forall r_{l,1} \geq \overline{r_l^V} = \frac{1}{\rho_l}\left[\frac{\gamma\sigma_l^2}{\chi} E_0 + r_d - \mu_l \right]. \tag{60}$$

On the contrary, the lower feasibility limit is binding in bad return states, where the bank would like to invest nothing but must at least invest E_0. This applies to all return states below threshold $\underline{r_{l,1}^V}$:

$$L_1^{*V} = 0 \qquad \forall r_{l,1} \leq \underline{r_l^V} = \frac{1}{\rho_l}\left[\sigma_l^2 E_0 + r_d - \mu_l \right]. \tag{61}$$

In between the two return thresholds, full retainment implies sub-optimally high equity levels. Hence, the bank no longer chooses to debt-finance as much as possible. Instead, the bank equity-finances loans with a share strictly above χ but below one. The optimal loan level is determined by the first-order-condition of the objective function (58), when both feasibility constraint multipliers are zero. For $r_{l,1}$ above $\underline{r_{l,1}^V}$ and below $\overline{r_{l,1}^V}$, this implies an optimal lending:

$$L_1^{*V} = \frac{\mu_l - \rho_l r_{l,1} - r_d}{\gamma\sigma_l^2} \qquad \forall r_{l,1} \in \left(\underline{r_{l,1}^V}, \overline{r_{l,1}^V} \right). \tag{62}$$

To derive when voluntary violation is optimal, we must compare the total shareholder utility from voluntary violation, denoted with $U^V(d_1, d_2)$, to the one from the baseline analysis, denoted $U(d_1, d_1)$.

Total utility under voluntary violation:

$$r_{l,1} < \underline{r_l^V}: \quad U^V(d_1, d_2) = \beta(\mu_l + \rho_l r_{l,1} + 1 - \gamma\sigma_l^2 E_0)E_0, \tag{63}$$

$$r_{l,1} \in [\underline{r_l^V}, \overline{r_l^V}]: \quad U^V(d_1, d_2) = \beta\left[\left(\mu_l + \rho_l r_{l,1} - r_d\right) L_1^{*V} - \frac{\gamma\sigma_l^2}{2}\left(L_1^{*V}\right)^2 + \left(1 + r_d\right) E_0\right], \tag{64}$$

$$\text{where} \quad L_1^{*V} = \frac{\mu_l + \rho_l r_{l,1} - r_d}{\gamma\sigma_l^2}, \tag{65}$$

$$r_{l,1} > \overline{r_l^V}: \quad U^V(d_1, d_2) = \beta\left[\left(\mu_l + \rho_l r_{l,1} - r_d\right)\frac{E_0}{\chi} - \frac{\gamma\sigma_l^2}{2}\frac{E_0^2}{\chi^2} + E_0\left(1 + r_d\right)\right], \tag{66}$$

Total utility under compliance (baseline):,

$$r_{l,1} < \underline{r_l}: \quad U(d_1, d_2) = E_0, \tag{67}$$

$$r_{l,1} \in \left[\underline{r_l}, \overline{r_l}\right]: \quad U(d_1, d_2) = E_0 - E_1^* + \beta\left[\left(\mu_l + \rho_l r_{l,1} - r_d\right) L_1^* - \frac{\gamma\sigma_l^2}{2}\left(L_1^*\right)^2 + E_1^*(1 + r_d)\right], \tag{68}$$

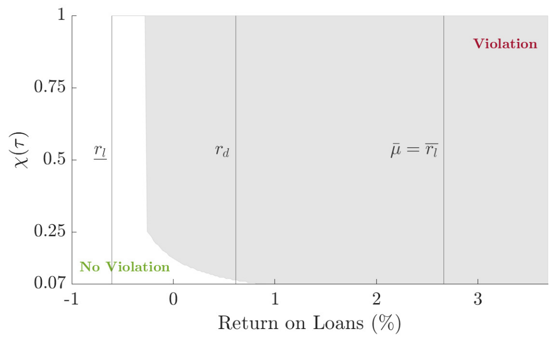
$$\text{where} \quad L_1^* = \frac{E_1^*}{\chi(\tau)} = \frac{\mu_l + \rho_l r_{l,1} - r_d - \chi(\tau)(1 - 1/\beta + r_d)}{\gamma\sigma_l^2}, \tag{69}$$

$$r_{l,1} > \overline{r_l}: \quad U(d_1, d_2) = \beta\left[\left(\mu_l + \rho_l r_{l,1} - r_d\right)\frac{E_0}{\chi(\tau)} - \frac{\gamma\sigma_l^2}{2}\left(\frac{E_0}{\chi(\tau)}\right)^2 + E_0(1 + r_d)\right]. \tag{70}$$

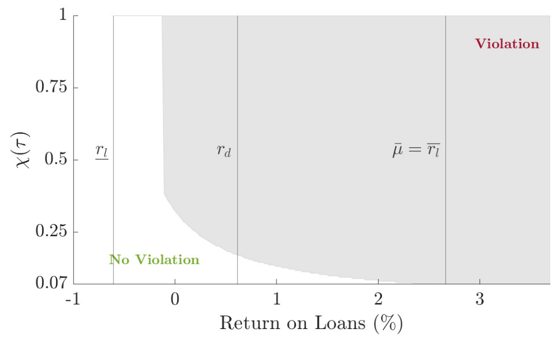
To prove when $U^V(d_1, d_2)$ exceeds $U(d_1, d_1)$ is cumbersome, as the sizes of return thresholds $\underline{r_l^V}$ and $\overline{r_l^V}$ relative to $\underline{r_l}$ and $\overline{r_l}$ strongly depend on the initially inherited equity E_0 relative to other model parameters. Hence, a large number of different utility functions would have to be compared to cover all cases. Instead, we provide insights for a meaningful parameter space and numerically study the voluntary violation decision for large US banks, given our calibration. Figure 5 (below) illustrates when a bank violates the stress tests voluntarily for the above presented calibration and three levels of initial equity as a function of the steady state equity level E_1^{ss}.

Each of the Panels 5a - 5c has the continuum of loan returns $r_{l,1}$ on the x-axis and the range of possible stress-test-implied minimum equity-to-loan ratio requirements on the y-axis. The gray shaded areas indicate when the bank finds it optimal to voluntarily violate the stress-test constraint. Here, we can see that this is generally the case for higher $\chi(\tau)$ and higher return states $r_{l,1}$. This should come as no surprise: the higher $\chi(\tau)$, the lower the total loans a stress-test compliant bank may issue and the more it can increase the loan

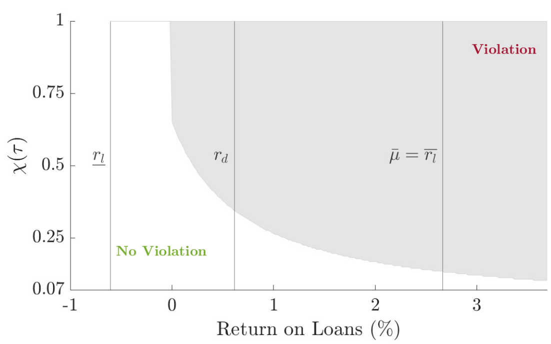
Figure 5: Optimal Choice of Stress-test Violation



(a) $E_0 = 0.5 \cdot E_1^{SS}$



(b) $E_0 = E_1^{SS}$



(c) $E_0 = 2 \cdot E_1^{SS}$

capacity by voluntarily violating. Further, expanding loan capacity is more attractive in good states of the world, where risky loan investment is desirable. On the contrary, exposing (sub-optimally high) equity levels to risky loans in bad states by violating the stress-test constraint is not desirable. Therefore, the desirability of violation also decreases with the size of the initial equity endowment.

Remark 1. *For U.S. stress-tested banks, violation is optimal for higher tightness τ, higher loan return states $r_{l,1}$, and lower initial equity E_0.*

It should be noted, however, that the voluntary violation of the stress-test constraint in our model does not incur any costs above and beyond the restriction on dividend payouts, such as financial market stigma or increased supervisory scrutiny. This explains why in reality, unlike our model would predict, large banks almost never violate the stress-test constraint.

4.2. *Alternative Supervisor Welfare Function*

In our baseline model environment we assumed that the supervisor only cares about the level (and potentially variance) of lending. We now allow the supervisor to also place weight on the investor's utility. Therefore, the supervisor sets the optimal stress-test severity to ensure high and stable levels of lending *and* of dividends so that the bank is able to meet its obligation to its shareholder. To capture the trade-off between the supervisor's and the investor's preferences, we assign the welfare weight $\phi \geq 0$ to the time 0 expected utility of the bank's shareholder. For simplicity, we, furthermore, assume that the supervisor and the investor are equally risk averse, i.e. that $\omega = \frac{\gamma}{2}$. Then, observing E_0 and $r_{l,0}$, the supervisor solves:

$$\max_{\tau} \quad \mathbb{E}[L_1^* \mid r_{l,0}, E_0] - \frac{\gamma}{2}\mathbb{VAR}_0[L_1^* \mid r_{l,0}, E_0] + \phi\Big(\mathbb{E}[d_1^* \mid r_{l,0}, E_0] + \beta\mathbb{E}[d_2^* \mid r_{l,0}, E_0] - \beta\frac{\gamma}{2}\mathbb{VAR}_0[d_2^* \mid r_{l,0}, E_0]\Big), \tag{71}$$

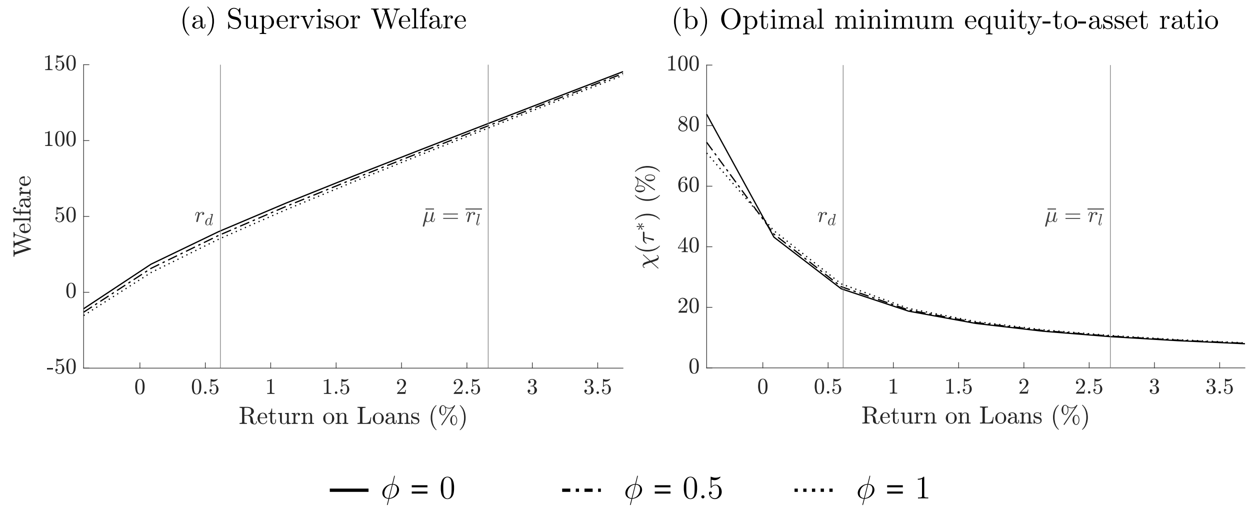
s.t.

$$\chi(\tau) \in [\chi, 1). \tag{72}$$

As Section D.5 in the Appendix shows, the supervisor anticipates rectified normally distributed L_1^*, d_1^*, and d_2^*. To identify the optimal stress-test tightness τ^*, we utilize our parameterization from Section 3.1 and computationally maximize the supervisor's welfare directly, subject to the respective constraints. As argued previously, the fact that loans and dividends follow rectified distributions prevents us from deriving a closed-form expression for the optimal stress-test tightness so that we solve this problem numerically. Figure 6 plots the corresponding supervisor welfare function (left panel) and the minimum equity to asset ratio $\chi(\tau^*)$ (right panel) implied by the optimal stress-test severity τ^* as a function of the initial return realization $r_{l,0}$ for different welfare weights ϕ.

As in the original welfare function, the supervisor's welfare is increasing in the initial return realization $r_{l,0}$ (note that the solid line here corresponds to the solid line in Figure 4). Furthermore, the supervisor's welfare is also increasing in the weight given to the investors utility. Interestingly, the optimal equity to asset ratio $\chi(\tau^*)$ is decreasing in the initial return realization $r_{l,0}$ but not in the welfare weight ϕ: For low levels of $r_{l,0}$ the stress-test severity is *decreasing* in the weight she gives to the investor's preferences, whereas for high levels of $r_{l,0}$ the stress-test severity is *increasing* in the weight given to the investor's preferences. However, in general the differences in the supervisor welfare and the minimum equity to asset ratio for different welfare weights ϕ are quantitatively small. This indicates that a supervisor would have to put extraordinarily large weight to the investor's welfare to make

Figure 6: Welfare and Minimum Equity to Asset Ratio under Optimal Stress Tests



a quantitatively meaningful difference for the optimal stress-test design.

5. Stress Tests in the Wider Regulatory Environment

Stress tests are a micro-prudential policy tool that complements a rich set of macro-prudential policies. It is, thus, crucial to understand their combined effectiveness in stabilizing lending when applied simultaneously. For this purpose, we extend the baseline model to include two currently utilized policy tools: the Covid-19 dividend ban and the counter-cyclical capital buffer (CCyB). We provide an additional welfare comparison to the dividend prudential target proposed as alternative by Muñoz (2020). In the latter, dividends are regulated directly, but less intensely than in an outright ban. In all three cases, we assume a single supervisor that simultaneously sets the optimal stress-test severity and macro-prudential policy rule. A study of supervisory conflict goes beyond the scope of this paper.

5.1. *Covid-19 Dividend Restrictions*

At the onset of the Covid-19 crisis, several jurisdictions introduced either an outright ban on dividend payments or a strong recommendation to stop payments temporarily (Beck et al., 2020). The goal was to boost equity and thereby counteract the procyclicality of lending. Here, we abstract from any moral suasion frictions between supervisors and banks, and analyze the effect of an outright dividend ban on bank lending levels.[17] A ban on dividends

[17]This is without loss of generality. As Beck et al. (2020) show, most European banks did indeed stop dividend payments following the ECB's recommendation.

implies full equity retainment, such that:

$$
\begin{aligned}
d_1 =& 0, && (73)\\
E_1 =& E_0, && (74)\\
D_1 =& L_1 - E_0. && (75)
\end{aligned}
$$

Inserting these into the bank's original optimization problem results in a revised maximization similar to the one under voluntary violation:

$$
U^B(d_1, d_2) = \max_{L_1} \quad (\mu_l + \rho_l r_{l,1})L_1 - r_d(L_1 - E_0) + E_0 - \frac{\gamma}{2}\sigma_l^2 L_1^2, \tag{76}
$$

$$
s.t.
$$

$$
L_1 \in \left[E_0, \frac{E_0}{\chi(\tau)} \right]. \tag{77}
$$

However, here the stress-test constraint still applies and determines the upper bound of loan investments in (77). Stress tests, thus, act as a feasibility constraint for the revised bank maximization problem. Again, the lower and upper feasibility bounds on L_1 imply two return thresholds denoted with $\underline{r_l^B}$ and $\overline{r_l^B}$ respectively:

$$
\underline{r_l^B} = \frac{1}{\rho_l}\left[\gamma\sigma_l^2 E_0 + r_d - \mu_l\right] \qquad \overline{r_l^B} = \frac{1}{\rho_l}\left[\frac{\gamma\sigma_l^2}{\chi(\tau)}E_0 + r_d - \mu_l\right]. \tag{78}
$$

Unlike in the baseline model, however, the two thresholds determine the share of debt financing instead of the degree of equity retainment: for return states below $\underline{r_{l,1}^C}$, the bank fully equity-finances L_1 now equal to E_0. Intuitively, in these bad return states, the shareholder would prefer to liquidate the bank but this is prevented by the dividend ban. Thus the only remaining option is to invest the existing equity in loans.

$$
L_1^{*B} = E_0 \qquad \forall\ r_{l,1} \leq \underline{r_l^B}. \tag{79}
$$

For intermediate return states, the bank sets an optimal loan level L_1^{*B} that requires a share of equity financing strictly below one but strictly above $\chi(\tau)$. Intuitively, in these return states the shareholder would actually prefer some dividends in period 1 but this is prevented by the dividend ban. At the same time, the loans are still relatively risky, limiting the attractiveness of investing in them. Thus the bank utilizes all its equity, but does not leverage up as much as it could. In this case, the level of lending is:

$$
L_1^{*B} = \frac{\mu_l + \rho_l r_{l,1} - r_d}{\gamma\sigma_l^2} \qquad \forall r_{l,1} \in \left(\underline{r_l^B}, \overline{r_l^B}\right). \tag{80}
$$

For high return states above $\overline{r_l^B}$, the bank debt-finances as much as possible given E_0 and $\chi(\tau)$, where the stress-test constraint now becomes the upper feasibility limit:

$$L_1^{*B} = \frac{E_0}{\chi(\tau)} \qquad \forall r_{l,1} \geq \overline{r_l^B}. \tag{81}$$

First, we analytically compare the optimal lending of a bank with free reign over the dividend payments (Section 2.2) with the one subject to a ban. We show that lending is higher under the ban for all return states below $\overline{r_l}$ for a given τ. Only for return states above $\overline{r_l}$ is the feasibility constraint on total lending binding under both regimes and, thus, lending is identical.[18]

***Proposition* 2.** *For a given τ, a dividend ban leads to strictly higher lending during crises.*

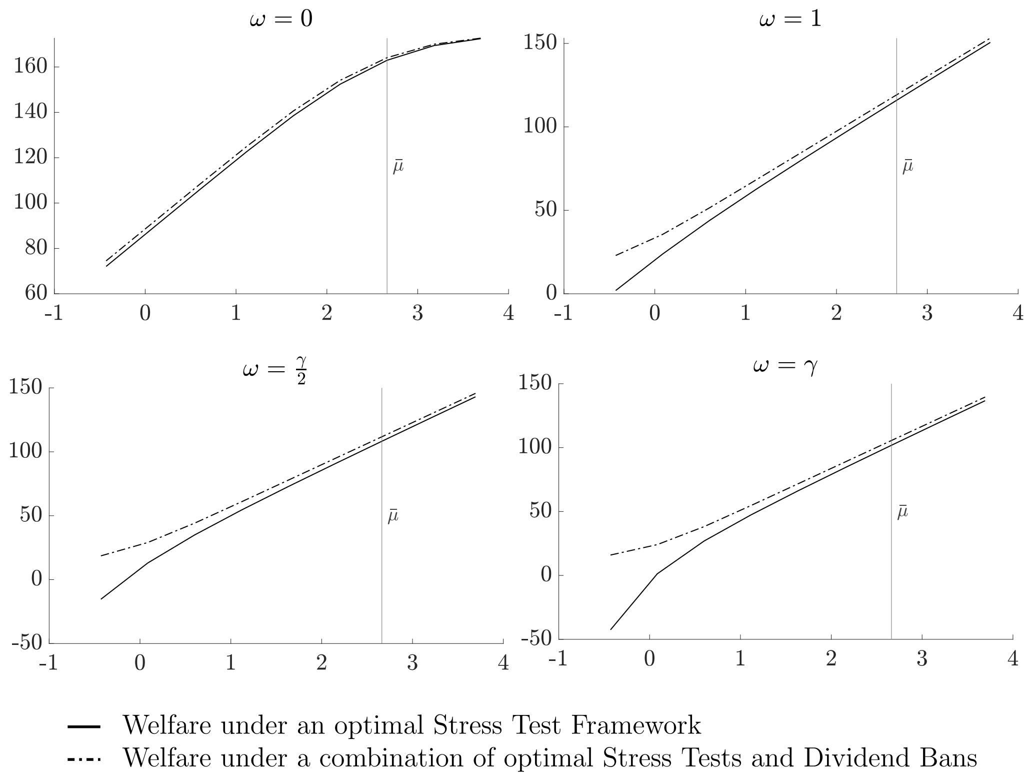
Of course, when setting the optimal τ^{*b} under a dividend ban, a unified supervisor takes the revised lending function into account. This results in strictly lower stress-test buffers under the ban where $\chi(\tau^{*b}) \leq \chi(\tau^*)$ (see Appendix E).

To illustrate the welfare implications of the Covid-19 dividend ban, Figure 7 plots the supervisor's welfare given the optimal stress-test severity τ^{*b} (τ^*) with (without) a dividend ban in place. The four panels of Figure 7 illustrate the associated welfare for different realizations of the initial return on loans $r_{l,0}$ and different degrees of risk aversion ω.

We find that stress tests in combination with a dividend ban yield a higher welfare than as a stand-alone regulation in all except very high initial return states. Intuitively, the dividend ban increases the mean of lending, while the stress test mainly reduces the variance. Each policy tool, thus, impacts a separate element of the supervisory welfare function favorably, thereby leading to overall higher welfare. Only in very high return states does the supervisor expect full-retainment both under a ban and under the stand-alone stress tests. Thus, the dividend ban cannot incentivize anymore lending and welfare is not impacted by a ban. Of course, the welfare gains of this policy combination comes to the detriment of the bank's shareholders, so the welfare comparison would look less favorable if the supervisor also took into account the investor's utility.

[18] Note here that for the formal proof, we account for the fact that the thresholds $\underline{r_l}$ may be above or below $\underline{r_l^B}$. However, $\overline{r_l^B}$ is always below $\overline{r_l}$.

Figure 7: Welfare under a Combination of Optimal Stress Tests and Dividend Bans



5.2. *Counter-Cyclical Capital Buffer*

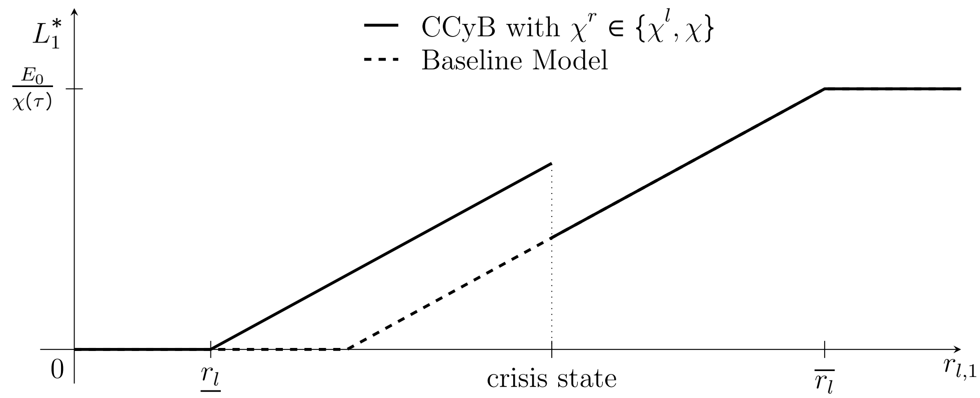
A complementary policy tool to the dividend ban is the relaxation of the counter-cyclical capital buffer (CCyB) during times of crises. In the baseline model we have assumed a constant χ that is state-independent. Instead, a CCyB implies a state-dependent χ^r that takes on a value $\chi^l < \chi$ for low return states. This relaxes the stress-test constraint in bad states via a reduction in $\chi(\tau)$. Relying on insights from Section 2.2, we know that this triggers an increase in lending and lowers the return thresholds below which no equity is retained. Figure 8 below illustrates this.

***Proposition* 3.** *For a given τ, a relaxed CCyB increases lending during crises. However, the CCyB is less effective than a dividend ban.*

To illustrate the welfare implications of the CCyB, we numerically solve for the optimal stress-test tightness under counter-cyclical capital buffers that decrease the stress-test implied equity-to-loan ratio by one percentage point[19] whenever $r_{l,1} < r_d$, i.e. when the return on assets drops below the bank's refinancing costs. As before, when setting the optimal τ^* under CCyBs, a unified supervisor takes the revised lending function into account. The four

[19]That is, we allow the supervisor to temporarily deviate from the absolute minimum equity-to-asset ratio χ in times of crises.

Figure 8: The Impact of CCyBs on Lending



panels of Figure 9 illustrate the associated supervisor welfare for different realizations of the initial return on loans $r_{l,0}$ and different degrees of risk aversion ω.

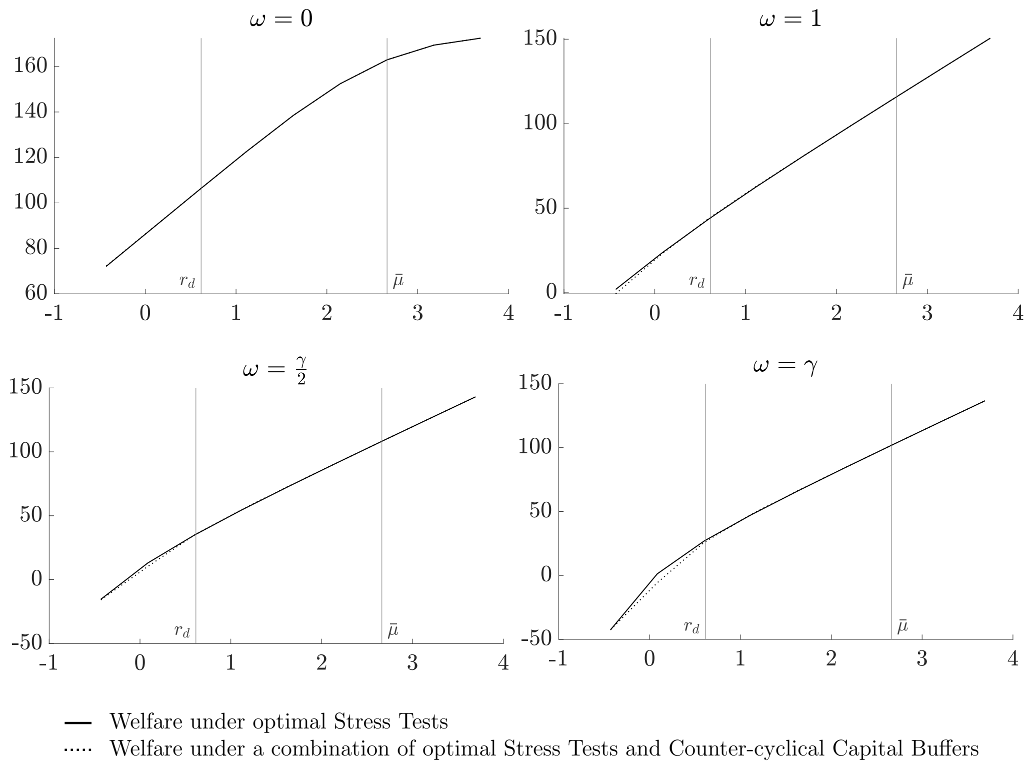
As Figure 9 shows, the two different regulatory frameworks behave extremely similar in welfare terms. In general, even though visually hardly discernibly, the combination of stress tests and CCyBs yields higher welfare except for the case of low r_0 and $\omega > 0$. This is the case because CCyBs increase the left tail of the lending distribution by reducing the no retainment state $\underline{r_l}$ and increasing the associated lending choices. The resulting increase in the expected variance of lending outweighs the resulting increase in the expected level of lending in low return states. This is not the case in higher initial return states r_0, where the no retainment state is a very unlikely realization so that the associated increase in lending volatility matters less than the lending increase around the crisis state. Therefore, in these low realizations of r_0 a risk averse supervisor would prefer to adjust the stress-test constraint only and not having to implement a CCyB on top of it.

Finally, during the Covid-19 crisis many jurisdiction combined the relaxation of the CCyB with the dividend ban discussed above. Analytically comparing the increased lending under relaxed CCyBs to those under a dividend ban for a given τ, it can be shown that the latter results in strictly higher lending in bad states. Intuitively, the main driver of lower loan levels in bad return states is equity withdrawal, which is not adequately addressed by relaxing the CCyB but completely eliminated via the ban.

Additionally, we can show in our model that the CCyB has no additional effect once a dividend ban is put in place. A bank subject to a ban already holds sub-optimally high equity and debt-finances less is potentially than allowed. Therefore, a relaxed CCyB does not change the optimal loan levels when activated on top of a dividend ban during a crisis.

***Proposition* 4.** *When introduced as a stand-alone, the relaxing of CCyB buffers is less effective in increasing lending than a stand-alone ban. It has no further effect on lending,*

Figure 9: Welfare under a Combination of Optimal Stress Tests and Counter-cyclical Capital Buffers



when a dividend ban is already in place.

We are, thus, able to provide an explanation for the recent policy puzzle regarding banks not using their CCyB buffers to finance lending during the Covid-19 crisis (FSB, 2021): additionally relaxing of CCyBs simply does not impact lending choices of already dividend restricted banks.

5.3. *Dividend Prudential Target*

Finally, we discuss the dividend prudential target (DPT), initially suggested by Muñoz (2020). The DPT restricts dividends directly by encouraging retainment in bad states and pay-outs in good states. It, thereby, attempts to directly offset the banks' dividend smoothing behavior to avoid capital depletion in bad states and reduce the pro-cyclicality of lending. In a first step, a DPT defines an ideal dividend pay-out – usually the pay-out made by an unrestricted bank in steady-state. We follow this tradition and evaluate our baseline model at the unconditional mean $\overline{\mu}_l$ of the AR(1) process. The dividends in steady-state, denoted with d_1^{SS}, take on the following value:

$$
\begin{aligned}
d_1^{SS} = & E_1^{SS} + \bar{\mu}\frac{E_1^{SS}}{\chi(\tau)} - r_d\left(\frac{E_1^{SS}}{\chi(\tau)} - E_1^{SS}\right) - E_1^{SS}, & (82)\\
= & \overline{\mu}_l\frac{E_1^{SS}}{\chi(\tau)} - r_d\left(\frac{E_1^{SS}}{\chi(\tau)} - E_1^{SS}\right), & (83)\\
= & \left[\frac{\overline{\mu}_l - r_d}{\chi(\tau)} + r_d\right]\frac{\chi(\tau)}{,}\gamma\sigma_l^2\left[\overline{\mu}_l - r_d - \chi(\tau)\left(\frac{1}{\beta} - 1 - r_d\right)\right]. & (84)
\end{aligned}
$$

Consequently, a state-dependent target dividend level d_1^T is defined that increases in the return state. The goal is to incentive more pay-outs in good and less pay-outs in bad states, thereby stabilizing both retained equity and lending. Intuitively, the DPT is designed to directly counteract dividend smoothing that triggers higher pay-outs in bad and lower pay-outs in good states.

To define such state-dependent DPT, we opt for the simplest possible option by scaling d_1^{SS} with the factor $r_{l,1}/\overline{\mu}_l$. This choice ensures that the target pay-out is increasing in the loan return states and is exactly equal to the steady-state level in steady state:

$$
d_1^T = \frac{r_{l,1}}{\overline{\mu}_l} d_1^{SS}. \tag{85}
$$

Consequently, any (squared) deviations in dividend pay-outs d_1 from the target d_t^T are punished with a cost proportional to the deviation by factor κ:

$$
\begin{aligned}
&\frac{\kappa}{2}\left(d_1 - d_1^T\right)^2, & (86)\\
&\frac{\kappa}{2}\left(E_0 - E_1 - \frac{r_{l,1}}{\overline{\mu}_l} d_1^{SS}\right)^2. & (87)
\end{aligned}
$$

The cost κ is set by the supervisor at $t = 0$ and, similar to Jermann and Quadrini (2012), accounts for both fines to be paid and reputation costs from non-compliance. It is taken as given by the bank at $t = 1$ and enters the optimization problem in the following fashion:

$$
\begin{aligned}
U(d_1, d_2) = \quad & \max_{L_1, E_1} \quad E_0 - E_1 - \frac{\kappa}{2}\left(E_0 - E_1 - \frac{r_{l,1}}{\overline{\mu}_l} d_1^{SS}\right)^2 \\
& + \beta E_1(1 + r_d) + \beta\left[L_1(\mu_l + \rho_l r_{l,1}) - L_1 r_d - \frac{\gamma\sigma_l^2}{2} L_1^2\right], & (88)\\
& s.t. \\
\lambda_1: \quad & L_1 \in \left[E_1, \frac{E_1}{\chi(\tau)}\right], & (89)\\
\lambda_2: \quad & E_1 \in \left[0, E_0\right]. & (90)
\end{aligned}
$$

An important feature to note from condition (89) in the maximization problem is that the optimal choice of E_1 impacts directly the feasibility constraints of L_1. Thus, both when deriving the the optimal equity and lending choices under the DPT, we need to take this co-dependency into account. We, nevertheless, start by deriving the optimal equity level assuming away its impact on lending. After taking the FOC condition with respect to E_1, equating it to zero, and checking feasibility, we get the following constrained-optimal equity levels:

$$E_1^* = 0 \qquad \forall\ r_{l,1} \leq r_l^* = \frac{\overline{\mu}_l}{d_1^{SS}} \frac{1}{\kappa} \Big(\beta \left(1 + r_d\right) - 1\Big), \tag{91}$$

$$E_1^* = \frac{1}{\kappa} \Big(\beta \left(1 + r_d\right) - 1\Big) + E_0 - \frac{r_{l,1}}{\overline{\mu}_l} d_1^{SS} \qquad \forall\ r_{l,1} \in (r_l^*, r_l^{**}], \tag{92}$$

$$E_1^* = 0 \qquad \forall\ r_{l,1} > r_l^{**} = \frac{\overline{\mu}_l}{d_1^{SS}} \left[\frac{1}{\kappa} \Big(\beta \left(1 + r_d\right) - 1\Big) + E_0\right]. \tag{93}$$

Here, we would immediately like to point out that equity now behaves quite differently than under stress tests: more equity is retained in bad states and less in good. This also impacts the optimal lending. Abstracting from feasibility constraints, taking the FOC with respect to L_1 and consequently equating it to zero yields the following optimal lending level:

$$L_1^* = \frac{\mu_l + \rho_l r_{l,1} - r_d}{\gamma \sigma_l^2}. \tag{94}$$

The Figure 10 illustrates both the optimal equity described in equations (91)-(93) and the unconstrained optimal lending in (94). Here, it is immediately visible that L_1^* in (94) is not feasible for low return states $r_{l,1}$, where the bank would ideally like to lend out less than the equity it would like to retain.

Further, we can observe two cases: For low E_0 the feasibility constraint only binds for return states below the full-retainment state (Figure 10a); for high E_0, the feasibility constraint already binds above the full retainment state (Figure 10b). The threshold level on initial equity $\overline{E_0}$ distinguising the two cases is:

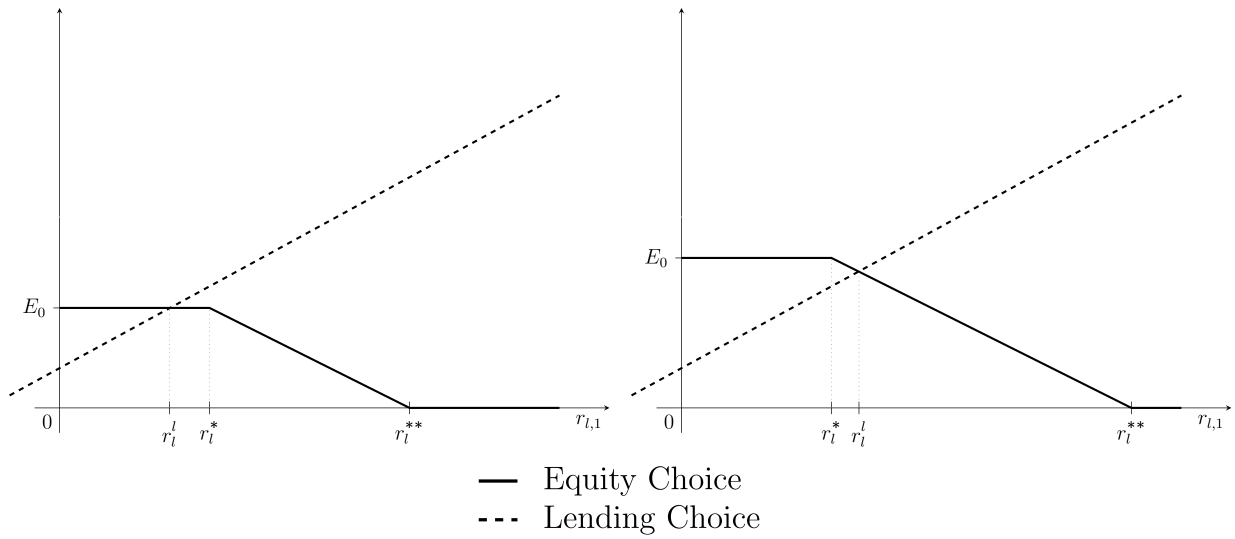
$$\overline{E_0} = \frac{\rho_l \overline{\mu}_l}{\gamma \sigma_l^2 d_1^{SS}} \frac{1}{\kappa} \Big(\beta \left(1 + r_d\right) - 1\Big) + \frac{\mu_l - r_d}{\gamma \sigma_l^2}. \tag{95}$$

We denote the return state below which the lower feasibility limit on L_1^* binds with r_l^l. For the case of low $E_0 \leq \overline{E_0}$ it can be shown, after some cumbersome re-arranging, that the bank is not willing to reduce the equity level in any return state below r_l^l to relax the lower limit on lending. Hence, the optimal equity choice is as defined as:

$$\text{If} \quad E_0 \leq \overline{E_0}\ :$$

Figure 10: Optimal Equity for Unrestricted Lending under the DPT

(a) Low E_0 (b) High E_0



$$r_l^l = \frac{\gamma\sigma_l^2 E_0 - \mu_l + r_d}{\rho_l}, \tag{96}$$

$$L_1^* = E_1^* = E_0 \quad \forall\ r_{l,1} \leq r_l^l. \tag{97}$$

.

For the case with high $E_0 \geq \overline{E_0}$, the bank does find it optimal to take the impact on lending into account, when deciding how much to retain in low return states. Here, we find that for all states below r_l^l, the bank solves a slightly revised optimization problem, where $E_1 = L_1$. Taking again FOCs with respect to E_1 and equating it to zero allows us to derive a slightly different optimal equity below r_l^l (see equation (100)). The bank can of course only retain additional equity as long as it is below E_0. Even for high E_0, the upper-feasibility constraint is eventually binding below return states r_l^{ll}:

If $\quad E_0 \geq \overline{E_0}$:

$$r_l^{ll} = \frac{\overline{\mu}_l}{\beta\rho_l\overline{\mu}_l - \kappa d_1^{SS}}\left[\beta\gamma\sigma_l^2 E_0 + 1 - \beta(1+\mu_l)\right] \tag{98}$$

$$r_l^l = \frac{\overline{\mu}_l}{\rho_l\overline{\mu}_l + \gamma\sigma_l^2 d_1^{ss}}\left[\frac{\gamma\sigma_l^2}{\kappa}\left(\beta(1+r_d) - 1\right) + \gamma\sigma_l^2 E_0 + r_d - \mu_l\right] \tag{99}$$

$$L_1^* = E_1^* = \frac{1}{\kappa + \beta\gamma\sigma_l^2}\left[-1 + \kappa E_0 - \kappa\frac{r_{l,1}}{\overline{\mu}_l}d_1^{SS} + \beta(1 + \mu_l + \rho_l r_{l,1})\right] \quad \forall\ r_{l,1} \in (r_l^{ll}, r_l^l] \tag{100}$$

$$L_1^* = E_1^* = E_0 \qquad \forall\ r_{l,1} \leq r_l^{ll}. \tag{101}$$

Regardless whether E_0 is high or low, the banks preferred lending level will ultimately violate the minimum equity-to-asset ratio in high return states: while optimal equity decreases in returns, optimal lending increases. Lending exceed the feasible amount, given equity, in all return states above a threshold r_l^h. From there on-wards the bank takes into account that (costly) retainment allows for more (profitable) lending. Nevertheless, there exists a threshold r_l^{hh} above which the bank never retains any equity no matter how profitable lending would be:

$$r_l^h = \frac{\chi(\tau)\overline{\mu}_l}{\chi(\tau)\rho_l\overline{\mu}_l + \gamma\sigma_l^2 d_1^{SS}}\left[\frac{\gamma\sigma_l^2}{\chi(\tau)\kappa}\left(\beta(1+r_d)-1\right) + \frac{\gamma\sigma_l^2}{\chi(\tau)}E_0 - \mu_l + r_d\right], \tag{102}$$

$$r_l^{hh} = \frac{\overline{\mu}_l\chi(\tau)}{\kappa d_1^{SS}\chi(\tau) - \overline{\mu}_l\beta\rho_l}\left[-1 + \kappa E_0 + \beta(1+r_d) + \beta\frac{\mu_l - r_d}{\chi(\tau)}\right]. \tag{103}$$

Inserting $L_1 = E_1/\chi(\tau)$ and, again, taking FOCs yields the following optimal lending and equity in high return states.

$$E_1^* = \chi(\tau)L_1^* = \frac{\chi(\tau)^2}{\chi(\tau)^2\kappa + \beta\gamma\sigma_l^2}\left[-1 + \kappa E_0 - \kappa\frac{r_{l,1}}{\overline{\mu}_l}d_1^{SS} + \beta(1+r_d) + \beta\frac{\mu_l + \rho_l r_{l,1} - r_d}{\chi(\tau)}\right] \quad \forall\ r_{l,1} \in (r_l^h, r_l^{hh}] \tag{104}$$

$$E_1^* = \chi(\tau)L_1^* = 0 \qquad \forall\ r_{l,1} > r_l^{hh} \tag{105}$$

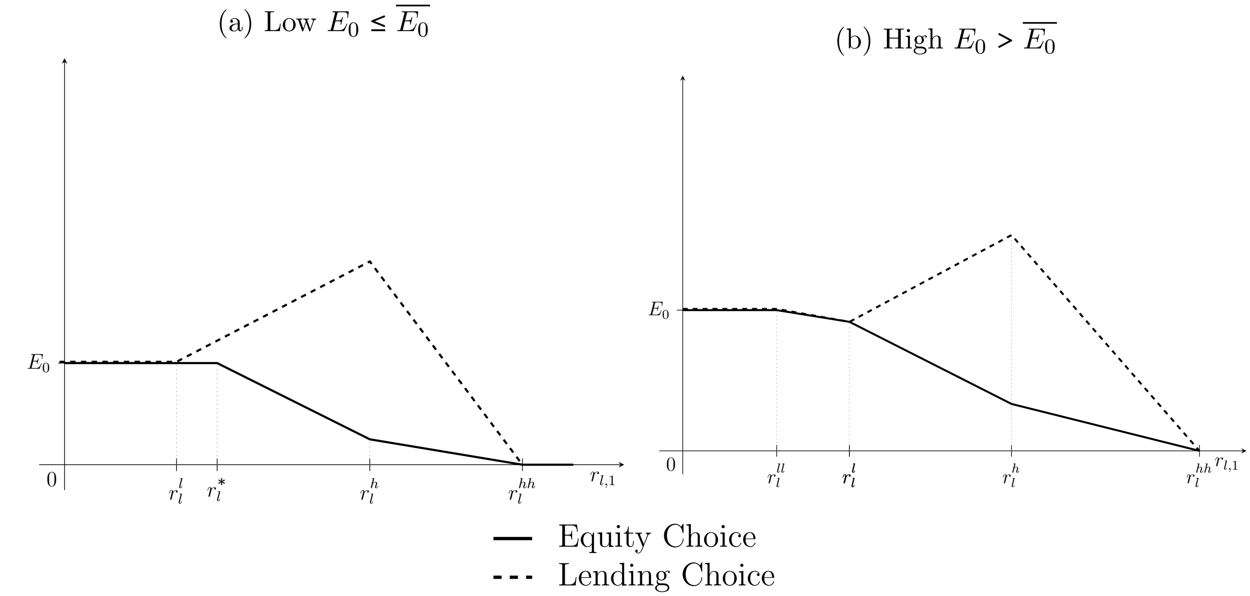
Summarizing the just derived solutions is a bit cumbersome and, we believe, not very informative for the reader. Therefore, we rather display the functional forms of L_1^* and E_1^* for both the low and high initial equity case in Figure 11 below. For the full set of analytical expressions on the return thresholds, the reader is kindly asked to refer to the Appendix D.4.

It can be seen in Figure 11 that a DPT results in a hump shaped policy function over the state-space for both equity and lending. The punishment parameter κ influences the mean and variance of both by affecting equity choices directly and, furthermore, affecting the threshold interest rate levels. Recall that the supervisory authority sets κ in period 0 with the objective to stabilize lending, putting welfare weight ω on the expected lending variance.

Unfortunately, a full closed-form characterization of the mean and variance of lending is cumbersome and provides few general insights. We therefore again immediately rely on the calibrated model (assuming $E_0 = E^{ss}$) to jointly derive the optimal κ^* & τ^* and resulting supervisory welfare.[20] Again, we derive the optimal κ for a range of different initial return

[20]When numerically maximizing the supervisor's welfare function we impose that the supervisor cannot set κ in such a way that $r_l^{hh} < r_l^l$. That way loans would be set to zero for basically all loan return states which would of course minimize the volatility of lending. The condition that loans cannot be zero resembles the standard Inada condition.

Figure 11: Feasibility-Constrained Optimal Equity and Lending under the DPT



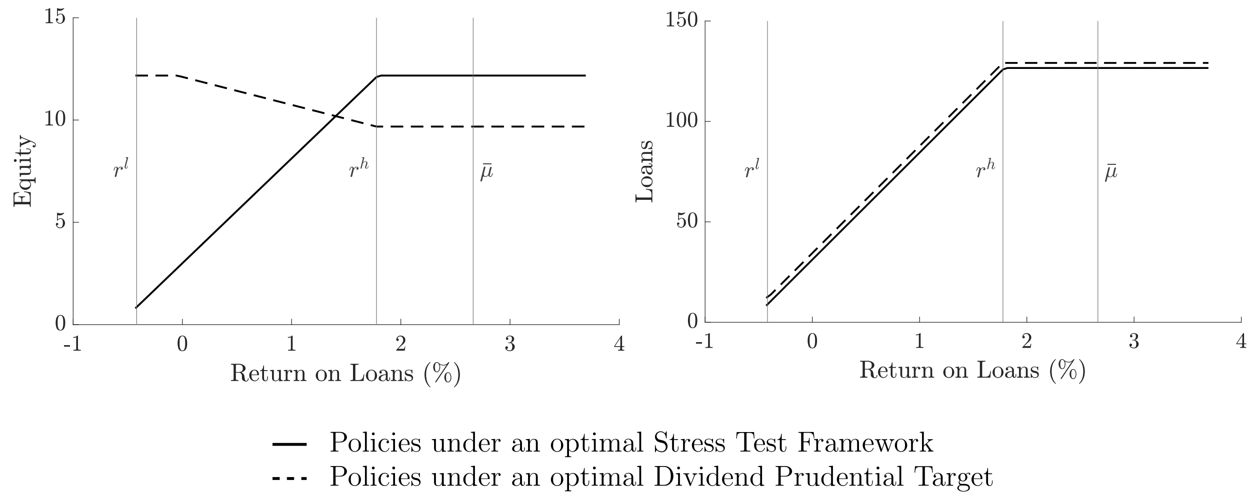
states r_0 and welfare weights ω (see Appendix D.4).

For intuition, we first plot the resulting policy functions for equity and loans for the optimal κ that maximizes the supervisor's welfare function (see Equation 49), assuming $r_{l,0} = \bar{\mu}$ and $\omega = 1$.[21] The left panel of Figure 12 shows that, relative to the stress-test framework, retained equity under the DPT is higher (lower) for bad (good) states. The DPT, thus, successfully addresses the pro-cyclical retainment of equity and dividend smoothing. Further, the right panel shows that, for most return states, the bank uses this equity to lever up slightly more than under the stress-test framework. Only for very high return states, substantially above 4% (not pictured), the DPT leads to lower loan levels than the stress-testing framework. In general, the DPT trades off lower expected lending in good states for higher lending in bad states (see Figure E.3) and generally lower lending volatility E.4) by inducing banks to retain more (less) equity in bad (good) states.

Therefore, it is no surprise that the combination of stress tests and a DPT is more likely to be welfare improving the more the supervisor dislikes lending volatility, as Figure 13 illustrates. For a supervisor who only cares about the level of lending, a combination of DPT and stress tests is naturally welfare improving only for relatively bad return states. In better states, the supervisor would prefer to set $\kappa = 0$, i.e. revert to a framework of stress tests alone, which we have ruled out here for the sake of the welfare comparison. However, as the supervisor becomes more risk averse, the return state for which the combination of DPT and stress tests improves over stress tests alone shifts upwards. Therefore, the suitability of a

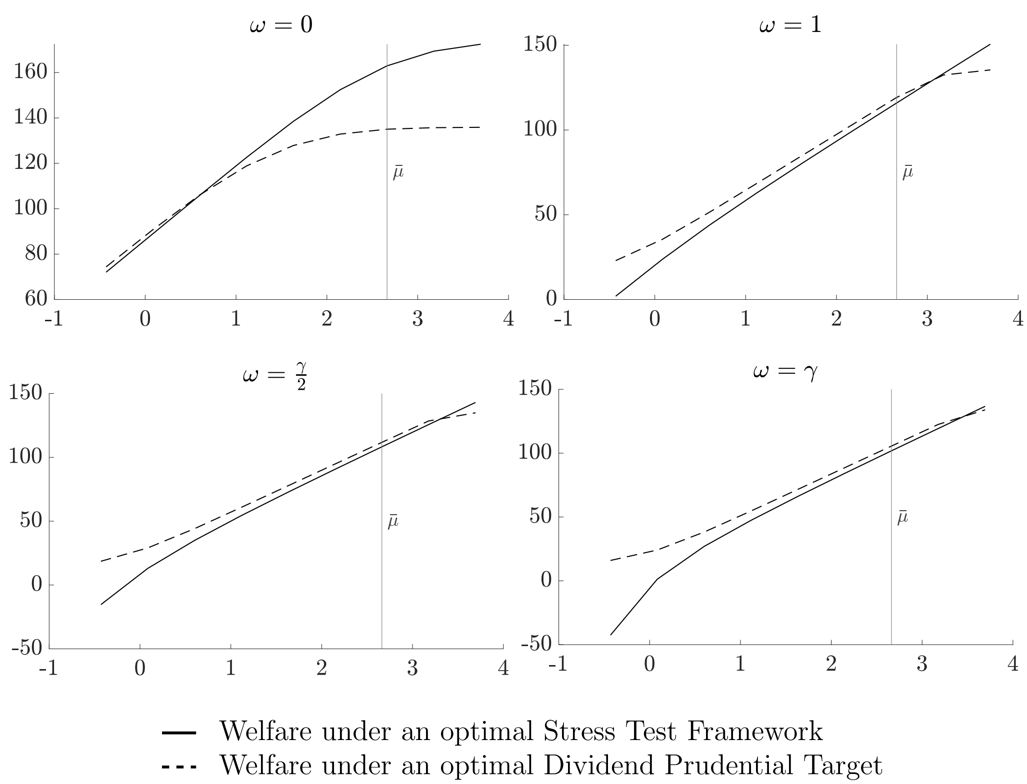
[21] in this case $\kappa^* = 0.06$ & $\chi(\tau^*) = 7.5\%$ - see Figure E.1 for a full illustration of the optimal κ^*

Figure 12: Optimal Policies under Stress Tests and a Dividend Prudential Target



DPT to stabilize lending on top of stress tests increases in the risk aversion of the supervisor.

Figure 13: Welfare under Optimal Stress Tests and a Dividend Prudential Target



5.4. Policy Comparison

To round off our discussion of stress tests in the wider regulatory context, we compare the supervisory welfare between the three different regulatory frameworks presented above. To begin with, Table 3 presents the supervisor welfare (Panel A) and implied optimal equity-to-asset ratio $\chi(\tau^*)$ (Panel B) for the respective optimal stress-test tightness τ^* at $r_0 = \bar{\mu}$.

Table 3: Welfare Comparison

Policy Framework	$\omega = 0$	$\omega = 1$	$\omega = \gamma/2$	$\omega = \gamma$
A. Supervisor Welfare				
Stress Test	162.96	115.93	108.25	101.84
Stress Test + Dividend Ban	164.11	119.21	111.76	105.57
Stress Test + CCyB	162.96	115.93	108.25	101.84
Stress Test + DPT	135.04	119.20	111.76	105.56
B. Minimum equity-to-asset ratio $\chi(\tau^*)$ in %				
Stress Test	7.00	9.62	10.32	10.97
Stress Test + Dividend Ban	7.00	9.40	10.05	10.65
Stress Test + CCyB	7.00	9.62	10.32	10.97
Stress Test + DPT	7.00	7.50	8.12	8.74

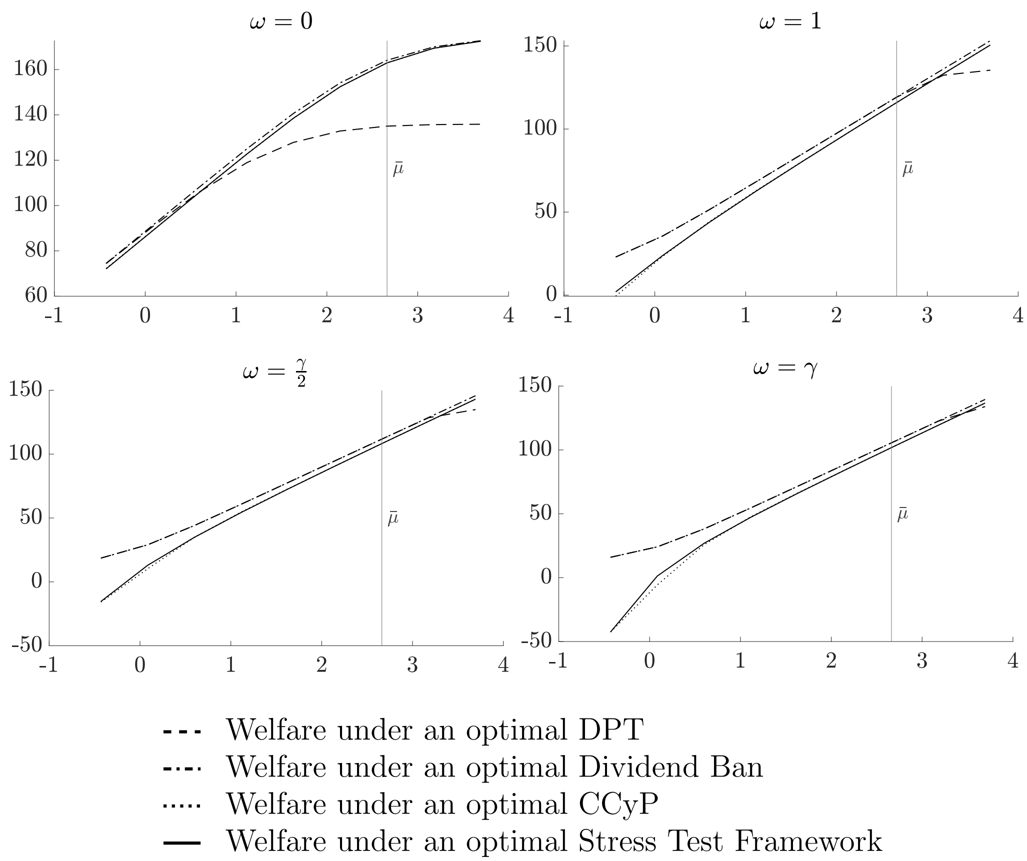
As Table 3 illustrates, a risk neutral supervisor (i.e. $\omega = 0$) would always set the stress-test implied equity buffer equal to zero. Additionally, she would clearly attain a higher welfare than by instituting a blanket dividend ban relative to any other frameworks. This comes at no surprise, given that the ban universally boosts lending levels.

For a risk averse supervisor, on the other hand, the DPT has almost identical welfare implications as a dividend ban (even though the dividend ban leads to a higher supervisor welfare in the second decimal point) and both improve in welfare terms over the other frameworks. Both policy combinations achieve these higher supervisor welfare values even though they imply lower minimum equity-to-asset ratios $\chi(\tau^*)$ as Panel B of Table 3 illustrates. This is the case because they simultaneously make equity withdrawal costly.[22] Furthermore, Figure 14 illustrates, that the dividend ban and the DPT diverge in welfare terms for higher initial return states because the dividend ban leads to relatively higher expected lending in these states. The point of divergence is increasing in the supervisor's degree of risk aversion and thus becomes less likely ex ante. A stress test on its own or in combination with a CCyB always yields relatively lower welfare than the ban and the DPT.

[22] The dividend ban of course puts an infinite punishment fee on deviating from its imposed dividend target of zero.

Finally, we would like to note that the welfare gains of the dividend ban of course come at an absolute cost of the bank's shareholder. This is not the case under the DPT, where in all but very high return states some dividends are paid. Thus, the DPT is uniquely well positioned to stabilize lending, when considering overall welfare. A complementary study by Ampudia et al. (2022), comparing the welfare under a ban and a DPT in a dynamic DSGE framework, equally find the DPT to be welfare improving. Here, we are able to validate their findings to hold even when taking the micro-prudential stress-test constraint into account.

Figure 14: Supervisor Welfare under Optimal Stress Tests + Macroprudential Policies



6. Conclusion

Regularly conducted bank stress tests have become an increasingly important policy tool designed with the intent to ensure stable lending and, thereby, to foster financial stability. In this paper, we derive the optimal bank balance sheet choices subject to a forward-looking stress test-constraint: equity levels should be sufficient to maintain current lending tomorrow, even after absorbing severe losses from said lending. We find that stress tests influence the banks' joint decision over (retained) equity, dividends, and lending. Here, we document the core supervisory trade-off: the more severe the assumed losses, the lower are both expected lending and lending volatility.

To quantitatively assess how such a trade-off plays out in practice, we calibrate our model to the U.S. banks subject to the CCAR stress tests. We derive the optimal stress-test tightness (severity of the adverse scenario) and the implied stress-test capital buffer. We find that a supervisor who prefers to maximize lending levels while minimizing lending volatility finds stress-test equity buffers of up to 6% to be optimal. This matches well the Federal Reserves' publicly announced stress-test buffers, reported to be between 2.5% to 7.5% in the 2021 CCAR report (Federal Reserve Board, 2021). This indicates that we are able to capture well both the mechanism behind and the magnitude of bank balance sheet choices under stress tests. We, further, confirm that these buffers do not incentivize banks to voluntarily violate stress tests in bad times and are largely unaffected by welfare concerns for bank shareholders.

Finally, we place the stress-test framework in the wider net of macro-prudential policies. Here, we highlight in particular the welfare effect of complementing stress tests with a dividend ban, a relaxation of the CCyB in crisis periods, and a dividend prudential target increasing in returns. We find that separately introduced, both relax lending of stress-tested banks in bad states of the world. They can, thus, be utilized to dampen the stress-test induced decrease in lending during downturns. However, CCyB activation is less effective than the dividend ban and, when introduced on top of the ban, has no further effects. We are thus able to rationalize why the relaxation of the CCyB during the onset of the Covid-19 pandemic had no measurable effect on lending by stress-tested banks subject to the dividend bans (FSB, 2021). Using our calibrated model, we compare these different macro-prudential policy complements in terms of supervisor welfare. We conclude that a dividend ban and a dividend prudential target are both very well suited for a risk-averse supervisor to stabilize lending, whereas the CCyB barely makes a difference compared to the stress test on its own.

References

Acharya, V. V., A. N. Berger, and R. A. Roman (2018). Lending implications of U.S. bank stress tests: Costs or benefits? *Journal of Financial Intermediation 34*(January), 58–90.

Ampudia, M., M. A. Muñoz, F. Smets, and A. Van der Ghote (2022). System-wide dividend restrictions: Evidence and theory. *ECB Working Paper Series forthcomin*.

Baudino, P. (2020). Stress-testing banks during the Covid-19 pandemic. *FSI Briefs No 11*.

Beck, T., F. Mazzaferro, R. Portes, J. Quin, and C. Schett (2020). Preserving capital in the financial sector to weather the storm. *VoxEU. org June*.

Berrospide, J. M. and R. M. Edge (2019). The Effects of Bank Capital Buffers on Bank Lending and Firm Activity: What Can We Learn from Five Years of Stress-Test Results?

Bird, A., S. A. Karolyi, and T. G. Ruchti (2020). Bias and the Efficacy of Stress Test Disclosures.

BIS (2019). The capital buffers in Basel III - Executive Summary. Technical report, BIS2021.

Bolton, P., Y. Li, N. Wang, and J. Yang (2020, 12). Dynamic Banking and the Value of Deposits.

Bouvard, M., P. Chaigneau, and A. D. Motta (2015). Transparency in the Financial System: Rollover Risk and Crises. *Journal of Finance 70*(4), 1805–1837.

Cappelletti, G., C. Melo Fernandes, and A. Ponte Marques (2019). The Effects of the Stress-Testing Exercises on Banks' Lending, Profitability and Risk-Taking: Are There Unintended Side Effects? *SSRN Electronic Journal*.

Cornett, M. M., K. Minnick, P. J. Schorno, and H. Tehranian (2020). An examination of bank behavior around Federal Reserve stress tests. *Journal of Financial Intermediation 41*(April 2018), 100789.

Cortés, K. R., Y. Demyanyk, L. Li, E. Loutskina, and P. E. Strahan (2020). Stress tests and small business lending. *Journal of Financial Economics 136*(1), 260–279.

De Nicolò, G., A. Gamba, and M. Lucchetta (2014). Microprudential Regulation in a Dynamic Model of Banking. *The Review of Financial Studies 27*(7), 2097–2138.

Doerr, S. (2021). Stress Tests, Entrepreneurship, and Innovation. *Review of Finance 25*(5), 1609–1637.

Eisfeldt, A. L., B. Herskovic, S. Rajan, and E. Siriwardane (2020). OTC Intermediaries.

Federal Reserve Board (2013, 7). Federal Reserve Board approves final rule to help ensure banks maintain strong capital positions.

Federal Reserve Board (2014). Comprehensive Capital Analysis and Review 2014: Assessment Framework and Results. Technical report.

Federal Reserve Board (2015). Comprehensive Capital Analysis and Review 2015: Assessment Framework and Results. Technical report.

Federal Reserve Board (2016). Comprehensive Capital Analysis and Review 2016: Assessment Framework and Results. Technical report.

Federal Reserve Board (2019a, 3). Federal Reserve Board votes to affirm the Countercyclical Capital Buffer (CCyB) at the current level of 0 percent.

Federal Reserve Board (2019b, 7). Quantitative Assessment Framework and Summary of Results.

Federal Reserve Board (2020a). Amendments to the Regulatory Capital, Capital Plan, and Stress Test Rules. Technical report.

Federal Reserve Board (2020b, 8). Federal Reserve Board announces individual large bank capital requirements, which will be effective on October 1.

Federal Reserve Board (2020c, 8). Stress Tests and Capital Planning.

Federal Reserve Board (2021, 8). Large Bank Capital Requirements. Technical report.

FSB (2021). Lessons Learnt from the COVID-19 Pandemic from a Financial Stability Perspective Interim report. Technical report, Financial Stability Board.

García, R. E. and S. Steele (2022, 2). Stress testing and bank business patterns: A regression discontinuity study. *Journal of Banking & Finance 135*, 105964.

Goldstein, I. and Y. Leitner (2018a). Stress tests and information disclosure. *Journal of Economic Theory 177*(C), 34–69.

Goldstein, I. and Y. Leitner (2018b). Stress tests and information disclosure. *Journal of Economic Theory 177*, 34–69.

Gollier, C., P.-F. Koehl, and J.-C. Rochet (1997). Risk-Taking Behavior with Limited Liability and Risk Aversion. *The Journal of Risk and Insurance 64*(2), 347–370.

Jermann, U. and V. Quadrini (2012). Macroeconomic Effects of Financial Shocks . *American Economic Review 102*(1), 238–271.

Koussis, N. and M. Makrominas (2019). What factors determine dividend smoothing by US and EU banks? *Journal of Business Finance and Accounting 46*(7-8), 1030–1059.

Lambrecht, B. M. and S. C. Myers (2012, 10). A Lintner Model of Payout and Managerial Rents. *The Journal of Finance 67*(5), 1761–1810.

Larkin, Y., M. T. Leary, and R. Michaely (2017, 12). Do investors value dividend-smoothing stocks differently? *Management Science 63*(12), 4114–4136.

Levy, H. and H. M. Markowitz (1979). Approximating Expected Utility by a Function of Mean and Variance. *The American Economic Review 69*(3), 308–317.

Markowitz, H. (2014, 4). Mean–variance approximations to expected utility. *European Journal of Operational Research 234*(2), 346–355.

Morgan, D. P., S. Peristiani, and V. Savino (2014). The Information Value of the Stress Test. *Journal of Money, Credit and Banking 46*(7), 1479–1500.

Muñoz, M. A. (2020). Rethinking capital regulation: the case for a dividend prudential target.

Office of Financial Research (2021). OFR Bank Systemic Risk Monitor.

Orlov, D., P. Zryumov, and A. Skrzypacz (2021, 11). Design of Macro-Prudential Stress Tests. *Simon Business School Working Paper No. FR 17-14.*

Petrella, G. and A. Resti (2013). Supervisors as information producers: Do stress tests reduce bank opaqueness? *Journal of Banking and Finance 37.*

Quijano, M. (2014). Information asymmetry in US banks and the 2009 bank stress test. *Economics Letters 123*(2), 203–205.

Sahin, C., J. de Haan, and E. Neretina (2020, 8). Banking stress test effects on returns and risks. *Journal of Banking and Finance 117*(C), 105843.

Shapiro, J. and J. Zeng (2019). Stress Testing and Bank Lending.

Wu, Y. (2018, 10). What's behind Smooth Dividends? Evidence from Structural Estimation. *The Review of Financial Studies 31*(10), 3979–4016.

Appendix A. Regulatory Framework

Following the financial crisis 08/09, the Federal Reserve Board (FED) was mandated to perform two complementary stress tests: the Comprehensive Capital Analysis and Review (CCAR) and the Dodd-Frank Act stress testing (DFAST). The CCAR is a forward-looking exercise and assesses bank holding companies' (BHC) capital adequacy accounting for individual dividend payment plans. Banks with assets of 10$bn and above are required to take part in the CCAR. The DFAST takes the last three quarters' dividend policy as given and mainly focuses on the sufficiency of loss-absorbing capital (Federal Reserve Board, 2020c). Banks with assets of 250$bn and above are required to take part in the DFAST. For the purpose of this study (apart from the calibration), we focus on the CCAR stress test framework, which is described in detail in the following paragraphs.

CCAR Stress Test As part of the CCAR stress test, the FED calculates the individual BHCs' capitalization under three scenarios: baseline, supervisory adverse, supervisory severely adverse. Here, they account for the BHCs' proposed future dividend payments and capital repurchase plans. Subsequently, the FED decides whether to approve a BHC's planned capital actions by compare the post-stress capital levels under the severely adverse scenarios to the minimum capital requirements plus surcharges (Berrospide and Edge, 2019; Federal Reserve Board, 2019b).

Minimum Capital Requirements From 2009-2013, all stress-test eligible BHCs were subject to a minimum tier 1 common ratio of 5%. In 2014, all banks with at least $250 billion total assets or more than $10 billion foreign asset exposure were subject to a 4% minimum common equity tier 1 ratio (CET1) instead. The remaining banks continued to be subject to the 5% minimum tier 1 common ratio for one more year. From 2015 onward, all BHCs were subject to a 4.5% minimum common equity tier 1 ratio (Federal Reserve Board, 2015, 2016). This change in minimum capital measures was part of the phase-in of the Basel III framework, which also introduced additional capital surcharges.

Capital Surcharge BHCs identified as globally systemically important banks (G-SIB) are subject to additional minimum risk-adjusted capital measures of 1%-3.5%. From 2014 to 2016, the Basel Committee on Banking Supervisisons capital add-on is applied. Since 2017, the maximum of the surcharges calculated under the Basel capital framework and the Federal Reserve Board's assessment methodology titled "Method II" applies (Office of Financial Research, 2021). Additionally, a 2.5% conservation buffer was phased in from 2016-2019 (Federal Reserve Board, 2013, 2014). For our sample period, the banks are not subject to any countercyclical capital buffer (Federal Reserve Board, 2019a).

Table A.1: Maximum Dividend to Net-income Ratio Given CET1

CET1	Maximum Pay-out Ratio
< 5.125%	0%
5.125% − 5.75%	20%
5.75% − 6.375%	40%
6.375% − 7%	60%
> 7%	no limitations

Source: BIS (2019)

Supervisory Power over Dividend Payments Stress-test eligible BHCs are prohibited from any dividend payments and share repurchases until the FED has approved of the capital distribution plan in writing. As mentioned above, such approval is based on the stress test performance and follows in three steps. First, the FED performs an initial set of stress tests given the BHCs' original dividend payout plan. The resulting (preliminary) stress-test results are communicated to the BHC. All BHCs, both insufficiently and sufficiently capitalized, are allowed once to submit an adjusted capital plan (Berrospide and Edge, 2019; Federal Reserve Board, 2019b).

Then either the original or, if submitted, adjusted capital plan forms the base for the FED's payout policy interventions. Capital levels below the minimum tier 1 common ratio or CET1 (plus G-SIB surcharge) respectively, automatically trigger a payout ban. A violation of the capital conservation buffer automatically results in dividend payments to be restricted to a percentage of net income (see Table A.1). Sufficient capital levels do not result in automatic restrictions. The Fed, however, reserves the right to require a BHC to reduce or cease all capital distributions if it felt that the weaknesses in the BHC's capital planning warranted such a response (Federal Reserve Board, 2014). Thus BHCs may feel supervisory pressure especially when close to but not yet violating their respective minimum capital requirements.

Recent Developments In 2020, the Federal Reserve Board decided to replace the 2.5% capital conservative buffer by an individual stress test buffer for each BHC (Federal Reserve Board, 2020b,a). This falls outside our sample period.

Appendix B. Proofs for Section 2

B.1. Solving the Bank's Optimization Problem

1. We start by defining dividend payments at $t = 1$ and $t = 2$.

$$\begin{aligned} d_1 =& E_0 - E_1 && \text{(B.1)} \\ d_2 =& L_1 r_{l,2} - r_d D_1 + E_1 \quad \sim N(\mu, \sigma^2) && \text{(B.2)} \\ \text{where} \quad \mu =& (\mu_l + \rho_l r_{l,1}) L_1 - r_d D_1 + E_1 && \text{(B.3)} \\ \text{and} \quad \sigma^2 =& \sigma_l^2 L_1^2 && \text{(B.4)} \end{aligned}$$

Further note that D_1 is perfectly determined by E_1 and L_1 via the budget constraint:

$$D_1 = L_1 - E_1 \tag{B.5}$$

Finally, note that plugging this into the stress-test constraint yields:

$$\chi L_1 \leq E_1 + L_1(\overline{\mu}_l - \tau\sigma_l - r_d(L_1 - E_1)) \tag{B.6}$$

$$(\chi - \overline{\mu}_l + \tau\sigma_l + r_d)L_1 - (1 + r_d)E_1 \leq 0 \tag{B.7}$$

2. Using the above stated equations and standard properties of a normal distributions, allows us to reduce the bank optimization problem to:

$$U(d_1, d_2) = \max_{E_1, L_1} E_0 - E_1 + \beta \left[L_1(\mu_l + \rho_l r_{l,1} - r_d(L_1 - E_1) + E_1 - \frac{\gamma\sigma_l^2}{2} L_1^2) \right] \tag{B.8}$$

$s.t.$

$$\begin{aligned} \lambda_1 : & \quad \chi L_1 - E_1 \leq 0 && \text{(B.9)} \\ \lambda_2 : & \quad (\chi - \overline{\mu}_l + \tau\sigma_l + r_d)L_1 - (1 + r_d)E_1 \leq 0 && \text{(B.10)} \\ \lambda_3 : & \quad E_1 - E_0 \leq 0 && \text{(B.11)} \\ \lambda_4 : & \quad E_1 - L_1 \leq 0 && \text{(B.12)} \\ \lambda_5 : & \quad E_1 \geq 0 && \text{(B.13)} \end{aligned}$$

We denote the multipliers associated with constraints (B.9)- (B.13) with λ_1 through λ_5 respectively.

3. Before taking any first order conditions, two comments on the constraints.

3.1. Notice that multipliers λ_3 and λ_5 can never be simultaneously be positive. They describe each their own corner solution: full retainment of equity and no retainment of equity.

3.2. Depending on τ, either minimum-equity and stress-test test constraint binds first.

The other one consequently only binds in states in which the first one is already binding.

We start by rearranging the stress-test constraint:

$$\frac{(\chi - \overline{\mu}_l + \tau\sigma_l + r_d)}{(1 + r_d)} L_1 \leq E_1 \tag{B.14}$$

Then notice that the multiplier in front of L_1 in the above equation is determined fully by model parameters and does not depend on equilibrium choices. Further, it enters multiplicatively into the constraint in the same fashion as χ.

Then, logically, the stress-test constraint binds first whenever:

$$\frac{(\chi - \overline{\mu}_l + \tau\sigma_l + r_d)}{(1 + r_d)} \geq \chi \tag{B.15}$$

$$\tau \geq \frac{r_d\chi + \overline{\mu}_l - r_d}{\sigma_l} = \tau^* \tag{B.16}$$

And in reverse logic, the minimum equity constraint binds first, whenever $\tau < \tau^*$. This concludes the proof for *Lemma 1.*

4. The above described result of 3.2. allows us actually to combine the two supervisory constraints in the following fashion:

$$\chi(\tau) = \begin{cases} \chi & \tau < \tau^* \\ \frac{r_d\chi + \overline{\mu}_l - r_d}{\sigma_l} & \tau \geq \tau^* \end{cases} \tag{B.17}$$

And the revised constraint, which nests both cases, is:

$$\chi(\tau) L_1 \leq E_1 \tag{B.18}$$

5. Then, we start solving the simplified maximization problem by assuming the bank has choosen a feasible level $E_1 \in [0, E_0]$. Taking E_1 as given reduces the bank optimization problem to:

$$U(E_0 - E_1, d_2) = E_0 - E_1 + \beta E_1(1 + r_d) + \max_{L_1} \beta \left[L_1(\mu_l + \rho_l r_{l,1}) - L_1 r_d - \frac{\gamma\sigma_l^2}{2} L_1^2 \right] \tag{B.19}$$

$$s.t.$$

$$\lambda_{1+2}: \quad \chi(\tau)L_1 - E_1 \leq 0 \tag{B.20}$$

$$\lambda_4: \quad -L_1 + E_1 \leq 0 \tag{B.21}$$

Then, the FOC wrt to L_1 becomes:

$$(\mu_l + \rho_{1,l} r_{l,1}) - r_d - \gamma\sigma_l^2 L_1 - \lambda_{1+2}\chi(\tau) + \lambda_4 = 0 \tag{B.22}$$

6. We now discuss the different cases for the multipliers. Here, notice that λ_{1+2} and λ_4 can never bind simultaneously: one would bind if the bank would like to set significantly lower L_1 than E_1 and one would bind if the bank would like set significantly higher than E_1/χ.

6.1. With this in mind, we start with (temporarily) ignoring both constraints. Then, the optimal loan level is:

$$L_1 = \frac{\mu_l + \rho_l r_{l,1} - r_d}{\gamma \sigma_l^2} \tag{B.23}$$

6.2. Then for a given E_1, logically there exists a lower threshold level $r_{l,1}^*$ for which investing $L_1 = E_1$ is optimal. And for all lower levels, the bank would like to set $L_1 < E_1$ but cannot due to its constraint choice.

Following a similar logic there exist a second threshold $r_{l,1}^{**}$, for which the bank would like to invest E_1/χ units into loans. And for any higher level, it would like to invest more, but cannot due to the minimum equity constraint.

6.3. However, as we will see later, these two thresholds are not really playing a core role, because E_1 is chosen by the bank and not taken as given. Here, it is important to take away from Equation (B.23) that any interior solution of L_1 without either constraints binding is independent of the level of equity E_1.

7. Lets start with assuming that $\lambda_{1+2} = \lambda_4 = 0$. This implies that the bank indeed finances some loans, but that these loans are more equity-financed than strictly required.

7.1. Recall then that L_1 is independent of E_1 and thus, the optimal level of E_1 can be chosen by the following optimization problem:

$$U(d_1, d_2) = \max_{E_1} E_0 - E_1 + \beta(1 + r_d)E_1 \tag{B.24}$$

$$s.t.$$

$$\lambda_3: \qquad E_1 - E_0 \le 0 \tag{B.25}$$

Abstracting for now from constraint λ_3 this implies a FOC wrt E_1:

$$-1 + \beta(1 + r_d) \tag{B.26}$$

Relying on parameter assumptions, it can be shown that this FOC is always negative:

$$-1 + \beta(1 + r_d) < 0 \tag{B.27}$$

$$(1 + r_d) \le \frac{1}{\beta} \quad \text{True by assumption} \tag{B.28}$$

Hence, any interior solution with only partial debt-financing cannot be sustained. Any solution with positive loan levels is characterized by $E_1 = \chi(\tau)L_1$.

8. With this in mind, we can now derive the optimal equity level E_1 by solving the following maximization problem:

$$U(d_1, d_2) = \max_{E_1} E_0 - E_1 + \beta\left[\frac{E_1}{\chi(\tau)}(\mu_l + \rho_l r_{l,1}) - \frac{\gamma\sigma_l^2}{2\chi(\tau)^2}E_1^2 - r_d\frac{1-\chi(\tau)}{\chi(\tau)}E_1 + E_1\right] \tag{B.29}$$

$$s.t.$$

$$\lambda_4: \quad E_1 - E_0 \leq 0 \tag{B.30}$$

$$\lambda_5: \quad -E_1 \leq 0 \tag{B.31}$$

8.1. Again, we will for now ignore the two feasibility constraints. Then the FOC wrt E_1:

$$-1 + \beta\left[\frac{\mu_l + \rho_l r_{l,1}}{\chi(\tau)} - \frac{\gamma\sigma_l^2}{\chi(\tau)^2}E_1 - r_d\frac{1-\chi(\tau)}{\chi} + 1\right] = 0 \tag{B.32}$$

$$E_1^* = \frac{\chi(\tau)^2}{\gamma\sigma_l^2}\left[\frac{\mu_l + \rho_l r_{l,1}}{\chi(\tau)} - r_d\frac{1-\chi(\tau)}{\chi} + 1 - \frac{1}{\beta}\right] \tag{B.33}$$

8.2. Now recall that an constraint solution requires $E_1 \leq E_0$. This holds up until:

$$\frac{\chi(\tau)^2}{\gamma\sigma_l^2}\left[\frac{\mu_l + \rho r_{l,1}}{\chi(\tau)} - r_d\frac{1-\chi(\tau)}{\chi} + 1 - \frac{1}{\beta}\right] \geq E_0 \tag{B.34}$$

$$r_{l,1} \geq \frac{1}{\rho_l}\left[\frac{\gamma\sigma_l^2}{\chi(\tau)}E_0 + \chi(\tau)\left(\frac{1}{\beta} - 1\right) + r_d(1-\chi(\tau)) - \mu_l\right] = \overline{r_l} \tag{B.35}$$

Or in other words, for any level of $r_{l,1}$ exceeding the threshold $\overline{r_l}$ equity is fully retained and invested in loans. The optimal bank choices and (expected) dividends are thus:

$$E_1^* = E_0 \tag{B.36}$$

$$L_1^* = \frac{E_0}{\chi(\tau)} \tag{B.37}$$

$$d_1^* = 0 \tag{B.38}$$

$$\mathbf{E}[D_1^*] = E_0\left[\frac{\mu_l + \rho_l r_{l,1}}{\chi\tau} - r_d\frac{(1-\chi(\tau))}{\chi(\tau)} + 1\right] \tag{B.39}$$

8.3. A similar logic can be applied for the lower bound such that:

$$\frac{\chi(\tau)^2}{\gamma\sigma_l^2}\left[\frac{\mu_l + \rho r_{l,1}}{\chi(\tau)} - r_d\frac{1-\chi(\tau)}{\chi} + 1 - \frac{1}{\beta}\right] \leq 0 \tag{B.40}$$

$$r_{l,1} \leq \frac{1}{\rho_l}\left[\chi(\tau)\left(\frac{1}{\beta} - 1\right) + r_d(1-\chi(\tau)) - \mu_l\right] = \underline{r_l} \tag{B.41}$$

Or put differently, for any realized stated $r_{l,1}$ weakly below $\underline{r_l}$ no equity is retained. The bank's equilibrium choices and (expected) dividends are thus:

$$L_1^* = E_1^* = D_1^* = 0 \tag{B.42}$$
$$d_1 = E_0 \tag{B.43}$$

8.4. For intermediate levels $r_{l,1} \in (\underline{r_l}, \overline{r_l})$ and interior solution exists with:

$$E_1^* = \frac{\chi(\tau)^2}{\gamma\sigma_l^2}\left[\frac{\mu_l + \rho_l r_{l,1}}{\chi(\tau)} - r_d\frac{1-\chi(\tau)}{\chi(\tau)} + 1 - \frac{1}{\beta}\right] \tag{B.44}$$
$$L_1^* = \frac{E_1^*}{\chi(\tau)} \tag{B.45}$$
$$d_1^* = E_0 - E_1^* \tag{B.46}$$
$$\mathbf{E}[D_1^*] = E_1^*\left[\frac{\mu_l + \rho_l r_{l,1}}{\chi(\tau)} - r_d\frac{1-\chi(\tau)}{\chi(\tau)} + 1\right] \tag{B.47}$$

B.2. Comparative Statics Over τ

We now compare an environment where $\tau < \tilde{\tau}$ such that $\chi(\tau) = \chi$ with an environment, where $\tau > \tilde{\tau}$ such that $\chi(\tau \geq \tau) > \chi$.

1. We start by showing that that $\underline{r_l^s} < \underline{r_l^{n,e}}$.

$$\underline{r_l^s} < \underline{r_l^{n,e}}\chi\left(\frac{1}{\beta} - 1 - r_d\right) < \qquad \chi(\tau)\tilde{\tau} < \tau \tag{B.48}$$

2. Further, we can show that $\overline{r_l^s} > \overline{r_l^{n,e}}$:

$$\overline{r_l^s} > \overline{r_l^{n,e}} \tag{B.49}$$
$$\frac{\gamma\sigma_l^2}{\chi}E_0 + \chi\left(\frac{1}{\beta} - 1 - r_d\right) > \frac{\gamma\sigma_l^2}{\chi(\tau)}E_0 + \chi(\tau)\left(\frac{1}{\beta} - 1 - r_d\right) \tag{B.50}$$
$$\gamma\sigma_l^2 E_0\left(\frac{1}{\chi} - \frac{1}{\chi(\tau)}\right) > (\chi(\tau) - \chi)\left(\frac{1}{\beta} - 1 - r_d\right) \tag{B.51}$$

Notice that the right hand side is a term very close to zero, and thus the inequality holds true under the assumption that $E_0 >> 0$.

3. With this, we know the upper and lower feasibility implied thresholds for equity and thus lending. Now, we turn to the slope of the optimal equity and lending policies.

$$\frac{\partial E_1^*}{\partial r_{l,1}} = \frac{\chi(\tau)}{\gamma\sigma_l^2}\rho_l \tag{B.52}$$

$$\frac{\partial^2 E_1}{\partial r_{l,1}\partial\chi(\tau)} = \frac{\rho_l}{\gamma\sigma_l^2} > 0 \tag{B.53}$$

3.1. It can be shown that E_1^* increases linearly in $r_{l,1}$:

$$\frac{\partial E_1^*}{\partial r_{l,1}} = \frac{\chi(\tau)}{\gamma\sigma_l^2}\rho_l \tag{B.54}$$

And confirming the relative return state bounds, it can be shown that the slope is steeper, the higher is τ:

$$\frac{\partial^2 E_1}{\partial r_{l,1}\partial\chi(\tau)} = \frac{\rho_l}{\gamma\sigma_l^2} > 0 \tag{B.55}$$

This implies that under a stress-test constraint, the bank starts to retain equity only in relatively higher states, but once started, it reaches full retainment earlier. Naturally, there exists a threshold $\tilde{r}$ for which the two equity functions intersect.

3.2. Turning to the loans, one can show that $L_1^{*,s} < L_1^{*,e}$. Her,e we first start with the loan rates implying $E_1 < E_0$. Then:

$$L_1^{*,s} < L_1^{*,e} \tag{B.56}$$

$$-\chi(\tau)\left(\frac{1}{\beta} - 1 - r_d\right) < \chi\left(\frac{1}{\beta} - 1 - r_d\right) \tag{B.57}$$

$$\chi < \chi(\tau) \tag{B.58}$$

$$\tilde{\tau} < \tau \tag{B.59}$$

Now, we consider the high return states inducing $E_1^* = E_0$:

$$L_1^{*,s} < L_1^{*,e} \tag{B.60}$$

$$\frac{E_0}{\chi(\tau \geq \tau)} < \frac{E_0}{\chi} \tag{B.61}$$

$$\chi < \chi(\tau) \tag{B.62}$$

$$\tilde{\tau} < \tau \tag{B.63}$$

We omit the proof for the variance of lending here due to its complexity here, and discuss it in detail during the supervisory problem. We would nevertheless like to highlight here, that lending L_1^* follows a rectified normal distribution with a lower and an upper bound. By increasing τ (above $\tilde{\tau}$), we bring the bounds closer together, thus reducing the variance of the overall distribution.

Appendix C. The Optimal Tightness τ

In this section, we derive the optimal supervisory choice under two different objective functions. To maintain tractability, we will assume that the realization of return states above $r_{l,1}^{f,s}$ are very low probability events for large banks with sufficient equity stocks. Thus, loan levels are fully characterized. Let us denote the optimal lending in the absence of feasibility constraints with L_1^x, where:

$$L_1^x = \frac{1}{\gamma\sigma_l^2}\left[\mu_l + \rho_l r_{l,1} - r_d - \chi(\tau)\left(\frac{1}{\beta} - 1 - r_d\right)\right] \tag{C.1}$$

$$L_1^x \sim N(\mu_x, \sigma_x^2) \tag{C.2}$$

$$\mu_x = \frac{1}{\gamma\sigma_l^2}\left[\mu_l + \rho_l(\mu_l + \rho_l r_{l,0}) - r_d - \chi(\tau)(1/\beta - 1 - r_d)\right] \tag{C.3}$$

$$\sigma_x^2 = \left(\frac{\rho_l}{\gamma\sigma_l}\right)^2 \tag{C.4}$$

The optimal bank lending L_1^* thus takes the following step-function.

$$L_1^* = \begin{cases} 0 & L_1^x < 0 \\ L_1^x & 0 \geq L_1^x \geq \frac{E_0}{\chi(\tau)} \\ \frac{E_0}{\chi(\tau)} & L_1^x > \frac{E_0}{\chi(\tau)} \end{cases} \tag{C.5}$$

Appendix D. Additional Proofs

D.1. Proofs for Voluntary Violation

Voluntary violation of the stress-test constraint implies a ban on dividends and, thus, the following equalities:

$$d_1 = 0 \tag{D.1}$$

$$E_1 = E_0 \tag{D.2}$$

$$D_1 = L_1 - E_0 \tag{D.3}$$

With this, the optimization problem reduces to:

$$\max_{L_1} \quad (\mu_l + \rho_l r_{l,1})L_1 - r_d(L_1 - E_0) + E_0 - \frac{\gamma}{2}\sigma_l^2 L_1^2 \tag{D.4}$$

$$s.t.$$

$$L_1 \in \left[E_0, \frac{E_0}{\chi}\right] \tag{D.5}$$

Here note that the upper feasibility limit is now determined by χ and not anymore $\chi(\tau)$.

Ignoring the two feasibility constraints for now, the FOC and the consequent optimal lending level are:

$$\mu_l + \rho_l - r_d - \gamma\sigma_l^2 L_1 = 0 \tag{D.6}$$

$$L_1^{*V} = \frac{\mu_l + \rho_l r_{l,1} - r_d}{\gamma\sigma_l^2} \tag{D.7}$$

Recall that L_1^{*V} is bounded above by the minimum asset-to-equity ratio constraint which allows us to derive a threshold $\overline{r_l^V}$. Similarly, in this business model L_1 can never be below E_0, allowing us a lower threshold $\underline{r_l^V}$

$$\overline{r_l^V} = \frac{1}{\rho_l}\left[\frac{\gamma\sigma_l^2}{\chi}E_0 + r_d - \mu_l\right] \tag{D.8}$$

$$\underline{r_l^V} = \frac{1}{\rho_l}\left[\gamma\sigma_l^2 E_0 + r_d - \mu_l\right] \tag{D.9}$$

With this in mind, it remains to be shown when the total utility exceeds the one of complying to the stress-test constraint. The resulting total utility from violation is:

$$r_{l,1} < \underline{r_l^V}: \quad U^V(d_1, d_2) = \beta(\mu_l + \rho_l r_{l,1} + 1 - \gamma\sigma_l^2 E_0)E_0 \tag{D.10}$$

$$r_{l,1} \in [\underline{r_l^V}, \overline{r_l^V}]: \quad U^V(d_1, d_2) = \beta\left[\left(\mu_l + \rho_l r_{l,1} - r_d\right)L_1^{*V} - \frac{\gamma\sigma_l^2}{2}\left(L_1^{*V}\right)^2 + \left(1 + r_d\right)E_0\right] \tag{D.11}$$

$$\text{where} \quad L_1^{*V} = \frac{\mu_l + \rho_l r_{l,1} - r_d}{\gamma\sigma_l^2} \tag{D.12}$$

$$r_{l,1} > \overline{r_l^V}: \quad U^V(d_1, d_2) = \beta\left[\left(\mu_l + \rho_l r_{l,1} - r_d\right)\frac{E_0}{\chi} - \frac{\gamma\sigma_l^2}{2}\frac{E_0^2}{\chi^2} + E_0\left(1 + r_d\right)\right] \tag{D.13}$$

This, we have to compare to the following aggregate utilities from complying:

$$r_{l,1} < \underline{r_l}: \quad U(d_1, d_2) = E_0 \tag{D.14}$$

$$r_{l,1} \in \left[\underline{r_l}, \overline{r_l}\right]: \quad U(d_1, d_2) = E_0 - E_1^* + \beta\left[\left(\mu_l + \rho_l r_{l,1} - r_d\right)L_1^* - \frac{\gamma\sigma_l^2}{2}\left(L_1^*\right)^2 + E_1^*(1 + r_d)\right] \tag{D.15}$$

$$\text{where} \quad L_1^* = \frac{E_1^*}{\chi(\tau)} = \frac{\mu_l + \rho_l r_{l,1} - r_d - \chi(\tau)(1 - 1/\beta + r_d)}{\gamma\sigma_l^2} \tag{D.16}$$

$$r_{l,1} > \overline{r_l}: \quad U(d_1, d_2) = \beta\left[\left(\mu_l + \rho_l r_{l,1} - r_d\right)\frac{E_0}{\chi(\tau)} - \frac{\gamma\sigma_l^2}{2}\left(\frac{E_0}{\chi(\tau)}\right)^2 + E_0(1 + r_d)\right] \tag{D.17}$$

To derive when violation would be optimal, one must compare the appropriate utilities given the return state $r_{l,1}$. A challenge here is that $\underline{r_l^V} \lesseqgtr \underline{r_l}$ and $\overline{r_l^V} \lesseqgtr \overline{r_l}$, depending on E_0:

$$\underline{r_l^V} \lesseqgtr \underline{r_l} \tag{D.18}$$

$$\frac{1}{\rho_l}\left[\gamma\sigma_l^2 E_0 + r_d - \mu_l\right] \lesseqgtr \frac{1}{\rho_l}\left[\chi(\tau)\left(\frac{1}{\beta} - 1 - r_d\right) + r_d - \mu_l\right] \tag{D.19}$$

$$E_0 \lesseqgtr \frac{\chi(\tau)}{\gamma\sigma_l^2}\left(\frac{1}{\beta} - 1 - r_d\right) \tag{D.20}$$

$$\overline{r_l^V} \lesseqgtr \overline{r_l} \tag{D.21}$$

$$\frac{1}{\rho_l}\left[\frac{\gamma\sigma_l^2}{\chi}E_0 + r_d - \mu_l\right] \lesseqgtr \frac{1}{\rho_l}\left[\frac{\gamma\sigma_l^2}{\chi(\tau)}E_0 + \chi(\tau)\left(\frac{1}{\beta} - 1 - r_d\right) + r_d - \mu_l\right] \tag{D.22}$$

$$E_0 \lesseqgtr \frac{\chi\chi(\tau)^2}{(\chi(\tau) - \chi)\gamma\sigma_l^2}\left(\frac{1}{\beta} - 1 - r_d\right) \tag{D.23}$$

Without further restrictions on E_0, a closed-form proof is a cumbersome comparison of all possible combinations for the different functional forms that the utilities may take. As this provides little additional insight without restricting the parameter space, we refrain from doing so. Instead, we show when voluntary violation is optimal for the above calibrated parameters and several different values of E_0. Please refer to the main text for results.

D.2. Covid-19 Dividend Ban

Sketch of proof for Proposition 2.

1. A ban on bank dividend payments implies the following equalities:

$$d_1 = 0 \tag{D.24}$$

$$E_1 = E_0 \tag{D.25}$$

$$D_1 = L_1 - E_0 \tag{D.26}$$

2. As the stress-test constraint is still binding, the optimization problem reduces to:

$$\max_{L_1} \quad (\mu_l + \rho_l r_{l,1})L_1 - r_d(L_1 - E_0) + E_0 - \frac{\gamma}{2}\sigma_l^2 L_1^2 \tag{D.27}$$

$$s.t.$$

$$L_1 \in \left[E_0, \frac{E_0}{\chi(\tau)}\right] \tag{D.28}$$

3. Temporarily ignoring the two feasibility constraints, taking the FOC and equating it to zero yields the following optimal lending level:

$$\mu_l + \rho_l - r_d - \gamma\sigma_l^2 L_1 = 0 \tag{D.29}$$

$$L_1^{*B} = \frac{\mu_l + \rho_l r_{l,1} - r_d}{\gamma\sigma_l^2}. \tag{D.30}$$

4. Now, we turn to the upper feasibility limit on L_1^{*B} determined by the stress-test-implied minimum asset-to-equity ratio constraint. This allows us to derive a threshold $\overline{r_l^B}$:

$$L_1^{*B} \geq \frac{E_0}{\chi(\tau)} \tag{D.31}$$

$$r_{l,1} \geq \frac{1}{\rho_l}\left[\frac{\gamma\sigma_l^2}{\chi(\tau)}E_0 + r_d - \mu_l\right] = \overline{r_l^B} \tag{D.32}$$

Similarly, in this business model L_1 can never be lower than E_0, allowing us to define the lower threshold $\underline{r_l^B}$

$$L_1^{*B} \leq E_0 \tag{D.33}$$

$$r_{l,1} \leq \frac{1}{\rho_l}\left[\gamma\sigma_l^2 E_0 + r_d - \mu_l\right] = \underline{r_l^B} \tag{D.34}$$

5. Then, the total utility under the Covid-19 dividend ban, denoted with $U^B(d_1, d_2)$, becomes:

$$r_{l,1} < \underline{r_l}: \quad U^B(d_1, d_2) = \beta(\mu_l + \rho_l r_{l,1} + 1 - \gamma\sigma_l^2 E_0)E_0 \tag{D.35}$$

$$r_{l,1} \in [\underline{r_l}, \overline{r_l}]: \quad U^B(d_1, d_2) = \beta\left[\left(\mu_l + \rho_l r_{l,1} - r_d\right)L_1^{*B} - \frac{\gamma\sigma_l^2}{2}\left(L_1^{*B}\right)^2 + \left(1 + r_d\right)E_0\right] \tag{D.36}$$

$$\text{where} \quad L_1^{*B} = \frac{\mu_l + \rho_l r_{l,1} - r_d}{\gamma\sigma_l^2} \tag{D.37}$$

$$r_{l,1} > \overline{r_l}: \quad U^B(d_1, d_2) = \beta\left[\left(\mu_l + \rho_l r_{l,1} - r_d\right)\frac{E_0}{\chi} - \frac{\gamma\sigma_l^2}{2}\frac{E_0^2}{\chi(\tau)^2} + E_0\left(1 + r_d\right)\right] \tag{D.38}$$

6. We are left with showing that $L_1^* < L_1^{*,B}$:

6.1. Assume a realized $r_{l,1}$ in the range $(-\infty,\ \min\{\underline{r_l};, \underline{r_l^B}\}]$. Then:

$$L_1^* < L_1^{*B} \tag{D.39}$$

$$0 < E_0 \tag{D.40}$$

6.2. Assume a realized return in the range $(\underline{r_l},\ \underline{r_l^B}]$. Then:

$$
\begin{aligned}
L_1^* &< L_1^{*B} && \text{(D.41)}\\
\frac{\mu_l + \rho_l r_{l,1} - r_d - \chi(\tau)(1/\beta - 1 - r_d)}{\gamma\sigma_l^2} &< E_0 && \text{(D.42)}\\
r_{l,1} &< \frac{1}{\rho_l}\left(\gamma\sigma_l E_0 - \mu_l + r_d + \chi(\tau)(1/\beta - 1 - r_d\right) && \text{(D.43)}\\
&< \underline{r_l}^B + \frac{1}{\rho_l}\chi(\tau)(1/\beta - 1 - r_d) && \text{(D.44)}
\end{aligned}
$$

Which holds true by assumption.

6.3. Assume a realized return $r_{l,1}$ in the range $(\underline{r_l}^B,\ \underline{r_l}]$. Then:

$$
\begin{aligned}
L_1^* &< L_1^{*B} && \text{(D.45)}\\
0 &< \frac{\mu_l + \rho_l r_{l,1} - r_d}{\gamma\sigma_l^2} && \text{(D.46)}\\
\underline{r_l}^B - \frac{\gamma\sigma_l^2 E_0}{\rho_l} &< r_{l,1} && \text{(D.47)}
\end{aligned}
$$

Which holds true by assumptions.

6.4. Assume a realized $r_{l,1}$ in the range $\left(\max\{\underline{r_l}, \underline{r_l}^B\},\ \min\{\overline{r_l}, \overline{r_l^B}\}\right]$. Then:

$$
\begin{aligned}
L_1^* &< L_1^{*B} && \text{(D.48)}\\
\frac{\mu_l + \rho_l r_{l,1} - r_d - \chi(\tau)(1/\beta - 1 - r_d)}{\gamma\sigma_l^2} &< \frac{\mu_l + \rho_l r_{l,1} - r_d}{\gamma\sigma_l^2} && \text{(D.49)}\\
-\chi(\tau)(1/\beta - 1 - r_d) &< 0 && \text{(D.50)}
\end{aligned}
$$

Which holds true by parameter assumption.

6.5. Assume a realized $r_{l,1}$ in the range $(\overline{r_l},\ \overline{r_l^B})$. Then:

$$
\begin{aligned}
L_1^* &< L_1^{*B} && \text{(D.51)}\\
\frac{E_0}{\chi(\tau)} &< \frac{\mu_l + \rho_l r_{l,1} - r_d}{\gamma\sigma_l^2} && \text{(D.52)}\\
\overline{r_l} - \frac{1}{\rho_l}\chi(\tau)(1/\beta - 1 - r_d) &< r_{l,1} && \text{(D.53)}
\end{aligned}
$$

Which holds true by assumption.

6.6. Assume a realized $r_{l,1}$ in the range $(\overline{r_l^B},\ \overline{r_l})$. Then:

$$
\begin{aligned}
L_1^* &< L_1^{*B} && \text{(D.54)}\\
\frac{\mu_l + \rho_l r_{l,1} - r_d - \chi(\tau)(1/\beta - 1 - r_d)}{\gamma\sigma_l^2} &< \frac{E_1}{\chi(\tau)} && \text{(D.55)}
\end{aligned}
$$

$$r_{l,1} < \overline{r_l} \tag{D.56}$$

This holds true by assumption.

6.7. Finally, assume a realized return state $r_{l,1} \in [\max\{\overline{r_l},\ \overline{r_l^B}\},\ +\infty$. Then:

$$L_1^* = L_1^{*B} \tag{D.57}$$

$$\frac{E_1}{\chi(\tau}) = \frac{E_1}{\chi(\tau)} \tag{D.58}$$

D.3. Proof for CCyB

Proof omitted due to its triviality. Please see the main-text for results.

D.4. Proof for a Dividend Prudential Target

The steady state of our model is characterized by the unconditional mean $\overline{\mu}_l$ and implies a dividend of:

$$d_1^{SS} = E_1^{SS} + \bar{\mu}\frac{E_1^{SS}}{\chi(\tau)} - r_d\left(\frac{E_1^{SS}}{\chi(\tau)} - E_1^{SS}\right) - E_1^{SS} \tag{D.59}$$

$$= \bar{\mu}\frac{E_1^{SS}}{\chi(\tau)} - r_d\left(\frac{E_1^{SS}}{\chi(\tau)} - E_1^{SS}\right) \tag{D.60}$$

$$= \left[\frac{\bar{\mu} - r_d}{\chi(\tau)} + r_d\right]\frac{\chi(\tau)}{\gamma\sigma_l^2}\left[\bar{\mu} - r_d - \chi(\tau)\left(\frac{1}{\beta} - 1 - r_d\right)\right]. \tag{D.61}$$

Given this, a state-dependent dividend prudential target is introduced:

$$d_1^T = \frac{r_{l,1}}{\overline{\mu}_l} d_1^{SS} \tag{D.62}$$

Any deviations from the target are punished with the following fine:

$$\frac{\kappa}{2}\left(d_1 - d_1^T\right)^2 \tag{D.63}$$

$$\frac{\kappa}{2}\left(E_0 - E_1 - \frac{r_{l,1}}{\overline{\mu}_l} d_1^{SS}\right)^2 \tag{D.64}$$

This results in the following revised optimization problem:

$$U(E_0 - E_1, d_2) = \max_{L_1, E_1} \quad E_0 - E_1 - \frac{\kappa}{2}\left(E_0 - E_1 - \frac{r_{l,1}}{\overline{\mu}_l} d_1^{SS}\right)^2$$

$$+\beta E_1(1+r_d)+\beta\left[L_1(\mu_l+\rho_l r_{l,1})-L_1 r_d-\frac{\gamma\sigma_l^2}{2}L_1^2\right] \tag{D.65}$$

$s.t.$

$$\lambda_1 : \qquad L_1 \in \left[E_1, \frac{E_1}{\chi(\tau)}\right] \tag{D.66}$$

$$\lambda_2 : \qquad E_1 \in \left[0, E_0\right] \tag{D.67}$$

1. We start by ignoring the feasibility constraints on L_1 and derive the optimal equity.

1.1. The FOC with respect to equity yields the following optimal equity levels:

$$\frac{\partial U(d_1,d_2)}{\partial E_1} = -1-\frac{\kappa}{2}\left(-2E_0+2\frac{r_{l,1}}{\overline{\mu}_l}d_1^{SS}+2E_1\right)+\beta(1+r_d)= \tag{D.68}$$

$$E_1 =\frac{1}{\kappa}\left(\beta(1+r_d)-1\right)+E_0-\frac{r_{l,1}}{\overline{\mu}_l}d_1^{SS} \tag{D.69}$$

1.2. The equity in equation (D.69) is the unconstrained equity level and decreases in $r_{l,1}$. Hence, we know that for low $r_{l,1}$ below a threshold r_l^*, the upper feasibility limit binds:

$$E_1 \geq E_0 \tag{D.70}$$

$$r_{l,1} \leq r_l^* = \frac{\overline{\mu}_l}{d_1^{SS}}\frac{1}{\kappa}(\beta(1+r_d)-1). \tag{D.71}$$

1.3. Similarly, the equity level is constrained below at zero:

$$E_1 \leq 0 \tag{D.72}$$

$$r_{l,1} \geq = r_l^{**} = \frac{\overline{\mu}_l}{d_1^{SS}}\left[\frac{1}{\kappa}(\beta(1+r_d)-1)+E_0\right]. \tag{D.73}$$

2. The above derived thresholds on equity ignore that the equity choice may relax feasbility constraints on lending. They are nevertheless necessary for a complete proof.

3. Next, assume that a feasible E_1 has been chosen and thus the bank is left with the optimal lending choice. Here, we can rely on results from the bank section and now for a given level E_1, the bank chooses:

$$L_1 =E_1 \qquad \forall r_{l,1} \leq r_l^l = \frac{1}{\rho_l}\left[\gamma\sigma_l^2 E_1 + r_d - \mu_l\right] \tag{D.74}$$

4. Notice that, unlike equity, lending increases in $r_{l,1}$. Hence, for low return states bank would lend out less than feasible and vice versa. Unconstrained, optimal lending is:

$$L_1^* = \frac{\mu_l+\rho_l r_{l,1}-r_d}{\gamma\sigma_l^2}. \tag{D.75}$$

5. Let us start with the upper feasibility limit. When is lending larger than optimal $E_1/\chi(\tau)$.

5.1. First, we assume that L_1 is already constrained below r_l^{**}:

$$L_1 \geq \frac{E_1^*}{\chi(\tau)}, \tag{D.76}$$

$$r_{l,1} \geq r_l^h = \frac{\bar{\mu}\chi(\tau)}{\chi(\tau)\rho_l\bar{\mu} + \gamma\sigma_l^2 d_1^{SS}}\left[\frac{\gamma\sigma_l^2}{\chi(\tau)}\left(\frac{1}{\kappa}\left(\beta(1+r_d)-1\right)+E_0\right)+r_d-\mu_l\right]. \tag{D.77}$$

5.2. Next, we verify that indeed $r_h^< r_l**$:

$$r_l^h \leq r_l^{**} \tag{D.78}$$

$$\frac{\bar{\mu}\chi(\tau)}{\chi(\tau)\rho_l\bar{\mu} + \gamma\sigma_l^2 d_1^{SS}}\left[\frac{\gamma\sigma_l^2}{\chi(\tau)}\left(\frac{1}{\kappa}\left(\beta(1+r_d)-1\right)+E_0\right)+r_d-\mu_l\right] < \frac{\overline{\mu}_l}{d_1^{SS}}\left[\frac{1}{\kappa}(\beta(1+r_d)-1)+E_0\right] \tag{D.79}$$

$$0 < \frac{\chi(\tau)\rho_l\overline{\mu}_l}{d_1^{SS}}\left[\frac{1}{\kappa}(\beta(1+r_d)-1)+E_0+\frac{\mu_l-r_d}{\chi(\tau)}\right]. \tag{D.80}$$

5.3. we can then conclude that for all levels above r_l^h retaining more equity relaxes the upper feasibility constraint on lending.

6. Taking this into account, we define an alternative optimization problem for high return states above r_l^h, where $L_1 = E_1/chi$,

6.1. Next, we derive the revised FOC wrt. E_1 that assumes $L_1 = E_1/\chi(\tau)$:

$$-1-\frac{\kappa}{2}\left(-2E_0+2\frac{r_{l,1}}{\overline{\mu}_l}d_1^{SS}+2E_1\right)+\beta(1+r_d)+\beta\frac{\mu_l+\rho_l r_{l,1}-r_d}{\chi(\tau)}-\beta\frac{\gamma\sigma_l^2}{\chi(\tau)^2}E_1=0, \tag{D.81}$$

$$\kappa E_1+\beta\frac{\gamma\sigma_l^2}{\chi(\tau)^2}E_1=-1+\kappa E_0-\kappa\frac{r_{l,1}}{\bar{\mu}}d_1^{SS}+\beta(1+r_d)+\beta\frac{\mu_l+\rho_l r_{l,1}-r_d}{\chi(\tau)}, \tag{D.82}$$

$$E_1=\frac{\chi(\tau)^2}{\chi(\tau)^2\kappa+\beta\gamma\sigma_l^2}\left[-1+\kappa E_0-\kappa\frac{r_{l,1}}{\bar{\mu}}d_1^{SS}+\beta(1+r_d)+\beta\frac{\mu_l+\rho_l r_{l,1}-r_d}{\chi(\tau)}\right] \tag{D.83}$$

The optimal equity level E_1 above r_l^h is strictly decreasing in $r_{l,1}$. Eventually, as $r_{l,1}$ increases it will meet the lower feasibility limit on E_1 of zero once again. The threshold return state r_l^{hh} is:

$$0=\frac{\chi(\tau)^2}{\chi(\tau)^2\kappa+\beta\gamma\sigma_l^2}\left[-1+\kappa E_0-\kappa\frac{r_{l,1}}{\bar{\mu}}d_1^{SS}+\beta(1+r_d)+\beta\frac{\mu_l+\rho_l r_{l,1}-r_d}{\chi(\tau)}\right], \tag{D.84}$$

$$\kappa\frac{r_{l,1}}{\bar{\mu}}d_1^{SS}-\frac{\beta\rho_l}{\chi(\tau)}r_{l,1}=\left[-1+\kappa E_0+\beta(1+r_d)+\beta\frac{\mu_l-r_d}{\chi(\tau)}\right], \tag{D.85}$$

$$r_l^{hh}=\frac{\bar{\mu}\chi(\tau)}{\kappa d_1^{SS}\chi(\tau)-\overline{\mu}_l\beta\rho_l}\left[-1+\kappa E_0+\beta(1+r_d)+\beta\frac{\mu_l-r_d}{\chi(\tau)}\right]. \tag{D.86}$$

7. Next, we turn to the lower feasibility limit on lending. Here we can distinguish two cases: L_1 intersects with E_1 below and above r_l^*. These two cases are determined by a threshold on E_0:

$$L_1^* \leq E_1 * \tag{D.87}$$

$$\frac{\mu_l + \rho_l r_{l,1} - r_d}{\gamma \sigma_l^2} \leq \frac{1}{\kappa}\big(\beta(1+r_d) - 1\big) + E_0 - \frac{r_{l,1}}{\overline{\mu}_l} d_1^{SS}, \tag{D.88}$$

$$r_{l,1} \leq r_l^l = \frac{\overline{\mu}_l}{\rho_l \overline{\mu}_l + \gamma \sigma_l^2 d_1^{SS}} \left[\frac{\gamma \sigma_l^2}{\kappa} (\beta(1+r_d) - 1) + \gamma \sigma_l^2 E_0 + r_d - \mu_l \right], \tag{D.89}$$

$$r_l^l \leq r_l^*, \tag{D.90}$$

$$E_0 \geq \frac{\rho_l \overline{\mu}_l}{\gamma \sigma_l^2 d_1^{SS}} \frac{1}{\kappa} (\beta(1+r_d) - 1) + \frac{\mu_l - r_d}{\gamma \sigma_l^2} = \overline{E_0}. \tag{D.91}$$

8. We first study the case, where $r_l^l \geq r_l^*$ as $E_0 \geq \overline{E_0}$. Here, any reduction in equity allows the bank to relax the lower feasibility limit.

8.1. Accounting for $E_1 = L_1$ in the optimization problem, we obtain the following FOC fir equity:

$$-1 - \frac{\kappa}{2}\left(-2E_0 + 2\frac{r_{l,1}}{\overline{\mu}_l} d_1^{SS} + 2E_1\right) + \beta(1+r_d) + \beta(\mu_l + \rho_l r_{l,1} - r_d) - \beta\gamma\sigma_l^2 E_1 = 0 \tag{D.92}$$

$$\kappa E_1 + \beta\gamma\sigma_l^2 E_1 = -1 + \kappa E_0 - \kappa \frac{r_{l,1}}{\overline{\mu}} d_1^{SS} + \beta(1 + \mu_l + \rho_l r_{l,1}) \tag{D.93}$$

$$E_1 = \frac{1}{\kappa + \beta\gamma\sigma_l^2}\left[-1 + \kappa E_0 - \kappa \frac{r_{l,1}}{\overline{\mu}} d_1^{SS} + \beta(1 + \mu_l + \rho_l r_{l,1})\right] \tag{D.94}$$

8.2. Mirroring this, for low $r_{l,1}$, the upper feasibility limit of E_1 not exceeding E_0 applies:

$$E_0 \geq \frac{1}{\kappa + \beta\gamma\sigma_l^2}\left[-1 + \kappa E_0 - \kappa \frac{r_{l,1}}{\overline{\mu}} d_1^{SS} + \beta(1 + \mu_l + \rho_l r_{l,1})\right] \tag{D.95}$$

$$r_{l,1} \leq = r_{l,1}^{ll} = \frac{\overline{\mu}_l}{\beta \rho_l \overline{\mu}_l - \kappa d_1^{SS}}\left[\beta\gamma\sigma_l^2 E_0 + 1 - \beta(1+\mu_l)\right] \tag{D.96}$$

9. Next, we study the case where $E_0 \leq \overline{E_0}$ and thus, $r_l^l \leq r_l^*$. Here, again the bank could relax the feasbility limit on L_1 by retaining more in equity states below r_l^l. That this is not optimal can easily be shown by the fact that:

$$r_l^l \leq r_l^{ll} \tag{D.97}$$

$$\frac{\overline{\mu}_l}{\rho_l \overline{\mu}_l + \gamma \sigma_l^2 d_1^{SS}} \left[\frac{\gamma \sigma_l^2}{\kappa} (\beta(1+r_d) - 1) + \gamma \sigma_l^2 E_0 + r_d - \mu_l \right] \leq \frac{\overline{\mu}_l}{\beta \rho_l \overline{\mu}_l - \kappa d_1^{SS}}\left[\beta\gamma\sigma_l^2 E_0 + 1 - \beta(1+\mu_l)\right] \tag{D.98}$$

$$E_0 \leq \frac{\rho_l \overline{\mu}_l}{\gamma \sigma_l^2 d_1^{SS}} \frac{1}{\kappa} (\beta(1 + r_d) - 1) + \frac{\mu_l - r_d}{\gamma \sigma_l^2} = \overline{E_0} \tag{D.99}$$

Because the above inequality (D.99) holds by assumption, we have that the bank never finds it optimal to pay out more equity to reduce lending.

10. For a given κ, assume that:

10.1.

$$E_0 \geq \overline{E_0} \tag{D.100}$$

Then, whenever we are in a very low return state $r_{l,1} \leq r_l^{ll}$, we have:

$$r_l^{ll} = \frac{\overline{\mu}_l}{\beta \rho_l \overline{\mu}_l - \kappa d_1^{SS}} \left[\beta \gamma \sigma_l^2 E_0 + 1 - \beta(1 + \mu_l) \right] \tag{D.101}$$

$$E_1 = E_0 = L_1 \tag{D.102}$$

For low return states, where $r_{l,1} \in (r_l^{ll}, r_l^l]$, whe have:

$$r_l^l = \frac{\overline{\mu}_l}{\rho_l \bar{\mu} + \gamma \sigma_l^2 d_1^{SS}} \left[\gamma \sigma_l^2 \left(\frac{1}{\kappa} \left(\beta(1 + r_d) - 1 \right) + E_0 \right) + r_d - \mu_l \right] \tag{D.103}$$

$$E_1 = \frac{1}{\kappa + \beta \gamma \sigma_l^2} \left[-1 + \kappa E_0 - \kappa \frac{r_{l,1}}{\bar{\mu}} d_1^{SS} + \beta(1 + \mu_l + \rho_l r_{l,1}) \right] \tag{D.104}$$

$$L_1 = E_1 \tag{D.105}$$

For intermediate return states $r_{l,1} \in (r_l^l, r_l^h]$, we have that :

$$r_l^h = \frac{\bar{\mu}\chi(\tau)}{\chi(\tau)\rho_l \bar{\mu} + \gamma \sigma_l^2 d_1^{SS}} \left[\frac{\gamma \sigma_l^2}{\chi(\tau)} \left(\frac{1}{\kappa} \left(\beta(1 + r_d) - 1 \right) + E_0 \right) + r_d - \mu_l \right] \tag{D.106}$$

$$E_1 = \frac{1}{\kappa} \left(\beta(1 + r_d) - 1 \right) + E_0 - \frac{r_{l,1}}{\overline{\mu}_l} d_1^{SS} \tag{D.107}$$

$$L_1 = \frac{\mu_l + \rho_l r_{l,1} - r_d}{\gamma \sigma_l^2} \tag{D.108}$$

For high return states, where $r_{l,1} \in (r_l^h, r_l^{hh}]$, we have that:

$$r_l^{hh} = \frac{\bar{\mu}\chi(\tau)}{\kappa d_1^{SS} \chi(\tau) - \overline{\mu}_l \beta \rho_l} \left[-1 + \kappa E_0 + \beta(1 + r_d) + \beta \frac{\mu_l - r_d}{\chi(\tau)} \right] \tag{D.109}$$

$$E_1 = \frac{\chi(\tau)^2}{\chi(\tau)^2 \kappa + \beta \gamma \sigma_l^2} \left[-1 + \kappa E_0 - \kappa \frac{r_{l,1}}{\bar{\mu}} d_1^{SS} + \beta(1 + r_d) + \beta \frac{\mu_l + \rho_l r_{l,1} - r_d}{\chi(\tau)} \right] \tag{D.110}$$

$$L_1 = \frac{E_1}{\chi(\tau)} \tag{D.111}$$

And finally, for very high return states, where $r_{l,1} > r_l^{hh}$, we have:

$$
\begin{aligned}
r_l^{hh} =& \frac{\bar{\mu}\chi(\tau)}{\kappa d_1^{SS}\chi(\tau) - \overline{\mu}_l \beta\rho_l}\left[-1 + \kappa E_0 + \beta(1 + r_d) + \beta\frac{\mu_l - r_d}{\chi(\tau)}\right] && \text{(D.112)}\\
E_1 =& L_1 = 0 && \text{(D.113)}
\end{aligned}
$$

10.2. If we have $E_0 \leq \overline{E_0}$, then the bank no longer retains less equity in low return states.

For very low return states $r_{l,1} \leq r_l^l$, the optimal lending is thus:

$$
\begin{aligned}
r_l^l =& \frac{\gamma\sigma_l^2 E_0 - \mu_l + r_d}{\rho_l} && \text{(D.114)}\\
L_1 =& E_0. && \text{(D.115)}
\end{aligned}
$$

For intermediate return states, $r_{l,1} \in [r_l^l, r_l^h]$, the lending choice is unrestricted and:

$$
L_1 = \frac{\mu_l + \rho_l r_{l,1} - r_d}{\gamma\sigma_l^2}. \tag{D.116}
$$

Similar to case 10.2, for high return states, where $r_{l,1} \in (r_l^h, r_l^{hh}]$, we have that:

$$
\begin{aligned}
r_l^{hh} =& \frac{\bar{\mu}\chi(\tau)}{\kappa d_1^{SS}\chi(\tau) - \overline{\mu}_l \beta\rho_l}\left[-1 + \kappa E_0 + \beta(1 + r_d) + \beta\frac{\mu_l - r_d}{\chi(\tau)}\right] && \text{(D.117)}\\
E_1 =& \frac{\chi(\tau)^2}{\chi(\tau)^2\kappa + \beta\gamma\sigma_l^2}\left[-1 + \kappa E_0 - \kappa\frac{r_{l,1}}{\bar{\mu}}d_1^{SS} + \beta(1 + r_d) + \beta\frac{\mu_l + \rho_l r_{l,1} - r_d}{\chi(\tau)}\right] && \text{(D.118)}\\
L_1 =& \frac{E_1}{\chi(\tau)} && \text{(D.119)}
\end{aligned}
$$

And again, for very high return states, where $r_{l,1} > r_l^{hh}$, we have:

$$
\begin{aligned}
r_l^{hh} =& \frac{\bar{\mu}\chi(\tau)}{\kappa d_1^{SS}\chi(\tau) - \overline{\mu}_l \beta\rho_l}\left[-1 + \kappa E_0 + \beta(1 + r_d) + \beta\frac{\mu_l - r_d}{\chi(\tau)}\right] && \text{(D.120)}\\
E_1 =& L_1 = 0, && \text{(D.121)}
\end{aligned}
$$

D.5. Alternative Welfare Function

As a sensitivity analysis we investigate how a supervisor would optimally set the severity of the stress test if he also takes into account the welfare of the bank's shareholders. To capture this trade-off, we assign the welfare weight $\phi \geq 0$ to the expected utility of the bank's shareholder. We, furthermore, assume that both the supervisor and the bank shareholder assign the same welfare weight γ to the expected variance of loans and dividends, respectively. Then, observing E_0 and $r_{l,0}$, the supervisor solves:

$$
\max_{\tau} \quad \mathbb{E}[L_1^* \mid r_{l,0}, E_0] - \frac{\gamma}{2}\mathbb{VAR}_0[L_1^* \mid r_{l,0}, E_0]
$$

$$
\begin{aligned}
&+ \phi\Big(\mathbb{E}[d_1^* \mid r_{l,0}, E_0] + \beta\mathbb{E}[d_2^* \mid r_{l,0}, E_0]\Big) && \text{(D.122)}\\
&- \phi\frac{\gamma}{2}\Big(\mathbb{VAR}[d_1^* \mid r_{l,0}, E_0] + \beta\mathbb{VAR}[d_2^* \mid r_{l,0}, E_0]\Big)
\end{aligned}
$$

s.t.

$$
\chi(\tau) \in [\chi, 1) \tag{D.123}
$$

where

$$
\begin{aligned}
r_{l,1} \le \underline{r_l}: \quad & L_1^* = 0 && \text{(D.124)}\\
r_{l,1} \in (\underline{r_l},\ \overline{r_l}): \quad & L_1^* = \frac{\mu_l + \rho_l r_{l,1} - r_d - \chi(\tau)(1/\beta - 1 - r_d)}{\gamma\sigma_l^2} && \text{(D.125)}\\
r_{l,1} \ge \overline{r_l}: \quad & L_1^* = \frac{E_0}{\chi(\tau)} && \text{(D.126)}
\end{aligned}
$$

As is the case for optimal loan levels L_1^*, the supervisor anticipates a rectified normally distributed d_1^* and d_2^* with lower and upper bounds. In period $t = 1$, dividends are set to $d_1^* = E_0$ for return states below $\underline{r_{l_1}}$ (no retainment); dividends are set to $d_1^* = 0$ for return states above $\overline{r_{l,1}}$ (full retainment; between $\underline{r_{l_1}}$ and $\overline{r_{l,1}}$ dividends are normally distributed with $N(\mu_{d_1}, \sigma_{d_1}^2)$:

$$
\begin{aligned}
d_1^x =& E_0 - \frac{\chi(\tau)}{\gamma\sigma_l^2}\Big(\mu_l + \rho_l r_{l,1} - r_d - \chi(\tau)(\frac{1}{\beta} - 1 - r_d)\Big) && \text{(D.127)}\\
d_1^x \sim& N(\mu_{d_1}, \sigma_{d,1}^2) && \text{(D.128)}\\
\sigma_{d,1}^2 =& \left(\frac{\chi(\tau)}{\gamma\sigma_l}\rho_l\right)^2 && \text{(D.129)}
\end{aligned}
$$

The optimal bank dividends d_1^* thus take the following step-function.

$$
d_1^* = \begin{cases} E_0 & d_1^x > E_0 \\ d_1^x & 0 \ge d_1^x \ge E_0 \\ 0 & d_1^x < 0 \end{cases} \tag{D.130}
$$

Consequently, in period $t = 2$, dividends are equal to $d_2^* = E_0\left(\frac{r_{l,2}-r_d}{\chi(\tau)} + 1 + r_d\right)$ if $r_{l,1} > \overline{r_{l,1}}$ with variance $(1+\rho_l^2)(\frac{E_0}{\chi(\tau)}\sigma_l)^2$; if $r_{l,1} \in (\underline{r_{l,1}}, \overline{r_{l,1}})$ dividends are normally distributed with $N(\mu_{d_2}, \sigma_{d_2}^2)$:

$$
\begin{aligned}
d_2^x =& E_1^*\left(\frac{r_{l,2} - r_d}{\chi(\tau)} + 1 + r_d\right) && \text{(D.131)}\\
=& \frac{1}{\gamma\sigma_l^2}\Big(\mu_l + \rho_l r_{l,1} - r_d - \chi(\tau)(\frac{1}{\beta} - 1 - r_d)\Big)\Big(r_{l,2} - r_d + \chi(\tau)(1 + r_d)\Big) && \text{(D.132)}\\
d_2^x \sim& N(\mu_{d_2}, \sigma_{d,2}^2) && \text{(D.133)}\\
\mu_{d,2} =& \frac{1}{\gamma\sigma_l^2}\Big(\mu_l(1 + \rho_l) + \rho_l^2 r_{l,0} - r_d - \chi(\tau)(\frac{1}{\beta} - 1 - r_d)\Big) && \text{(D.134)}
\end{aligned}
$$

$$\cdot\Big((1+\rho_l)\mu_l + \rho_l^2 r_{l,0} - r_d + \chi(\tau)(1+r_d)\Big) \tag{D.135}$$

$$\sigma_{d,2}^2 = \mathbb{E}[d_2^{*2}] - \mathbb{E}[d_2^*]^2 = \left(\frac{\rho_l}{\gamma\sigma_l}\right)^2 b^2 + (1+\rho_l^2)\sigma_l^2 a^2 + \frac{\rho_l^4}{\gamma^2} + 4ab\frac{\rho_l^2}{\gamma} \tag{D.136}$$

where

$$a = \frac{1}{\gamma\sigma_l^2}\left(\mu_l(1+\rho_l) + \rho_l^2 r_{l,0} - r_d - \chi(\tau)(\frac{1}{\beta} - 1 - r_d)\right) \tag{D.137}$$

$$b = \Big(\left(1+\rho_l\right)\mu_l + \rho_l^2 r_0 - r_d + \chi\left(\tau\right)\left(1+r_d\right)\Big) \tag{D.138}$$

Conditional on $r_{l,1} \in (\underline{r_{l,1}}, \overline{r_{l,1}})$, the optimal bank dividends d_2^* thus take the following step-function.

$$d_2^* = \begin{cases} d_2^x & d_2^x \geq E_0\left(\frac{r_{l,2}-r_d}{\chi(\tau)} + 1 + r_d\right) \\ E_0\left(\frac{r_{l,2}-r_d}{\chi(\tau)} + 1 + r_d\right) & d_2^x < E_0\left(\frac{r_{l,2}-r_d}{\chi(\tau)} + 1 + r_d\right) \end{cases} \tag{D.139}$$

Appendix E. Additional Figures

Figure E.1: Optimal Punishment Factor κ^* under a Dividend Prudential Target

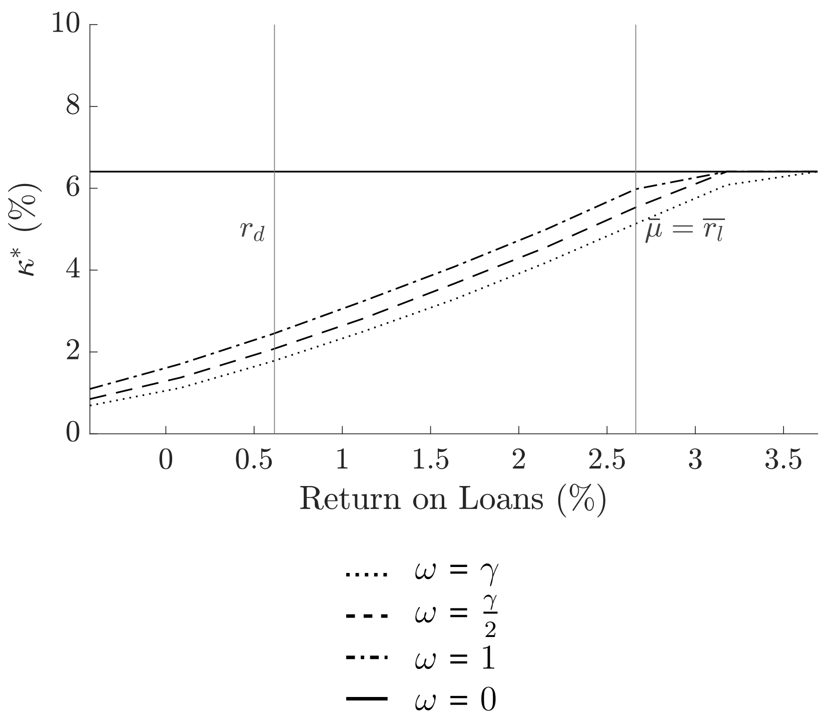


Figure E.2: Capital Buffers under Optimal Stress Tests + Macroprudential Policies

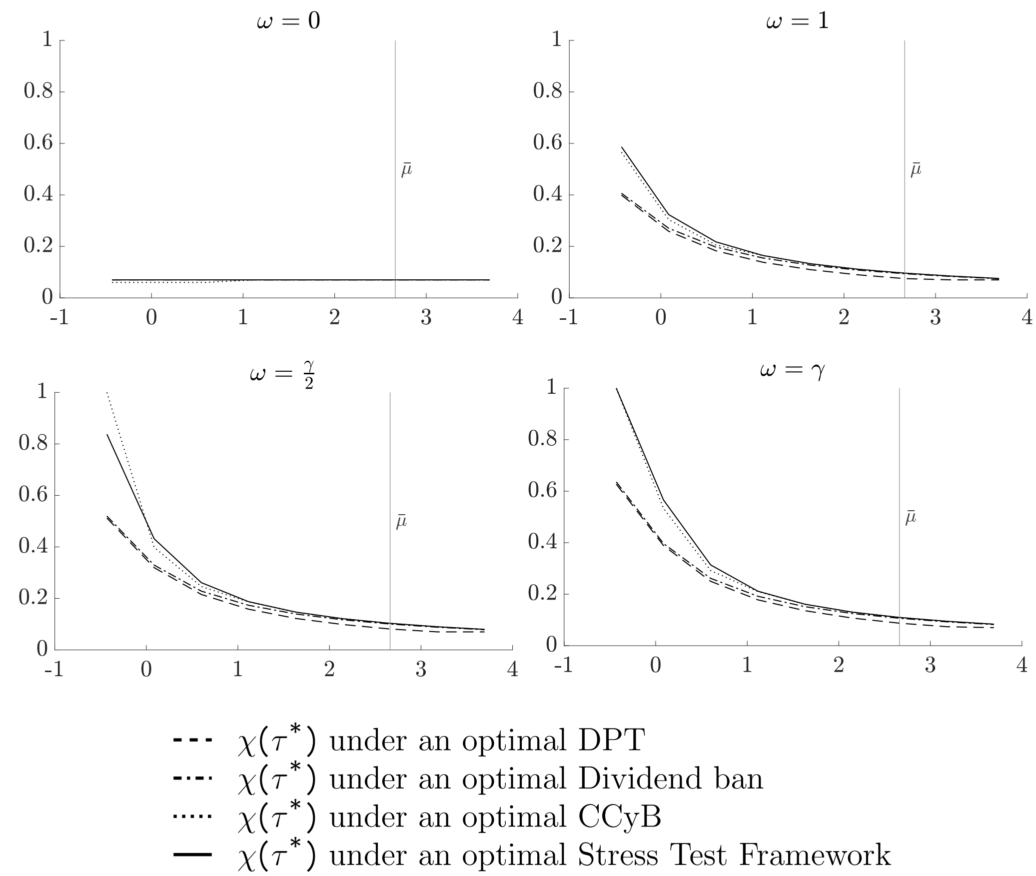


Figure E.3: Expected Lending under Optimal Stress Tests + Macroprudential Policies

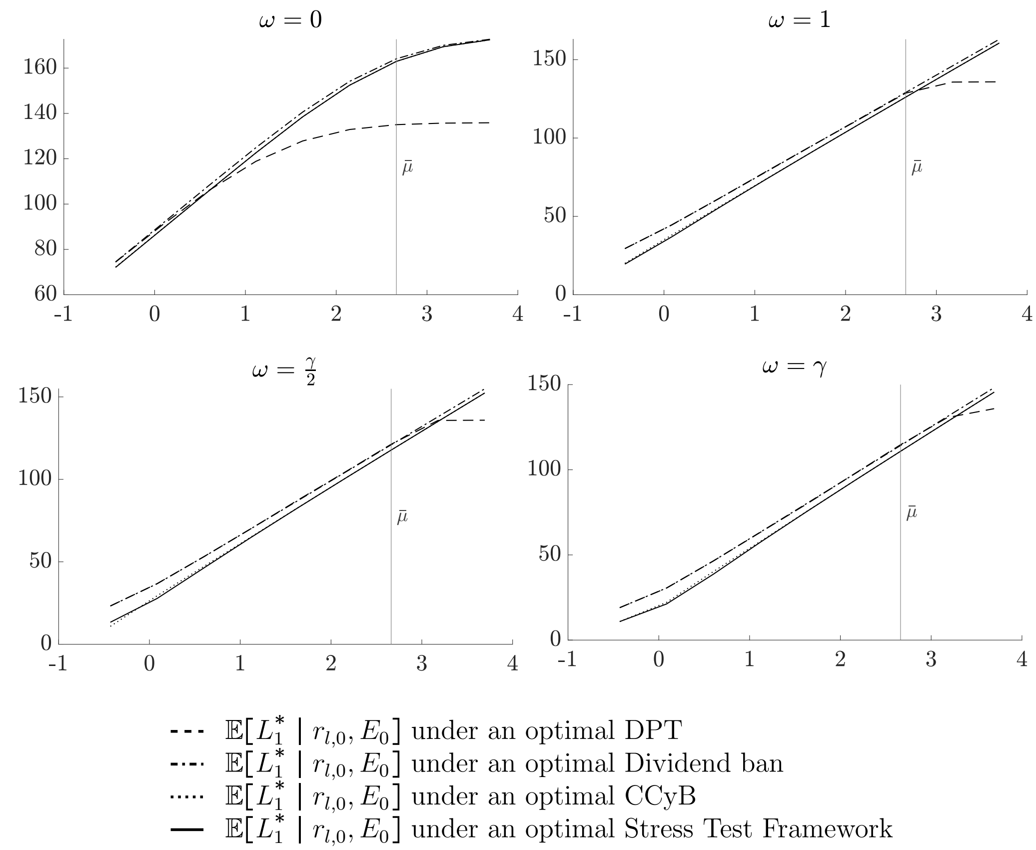
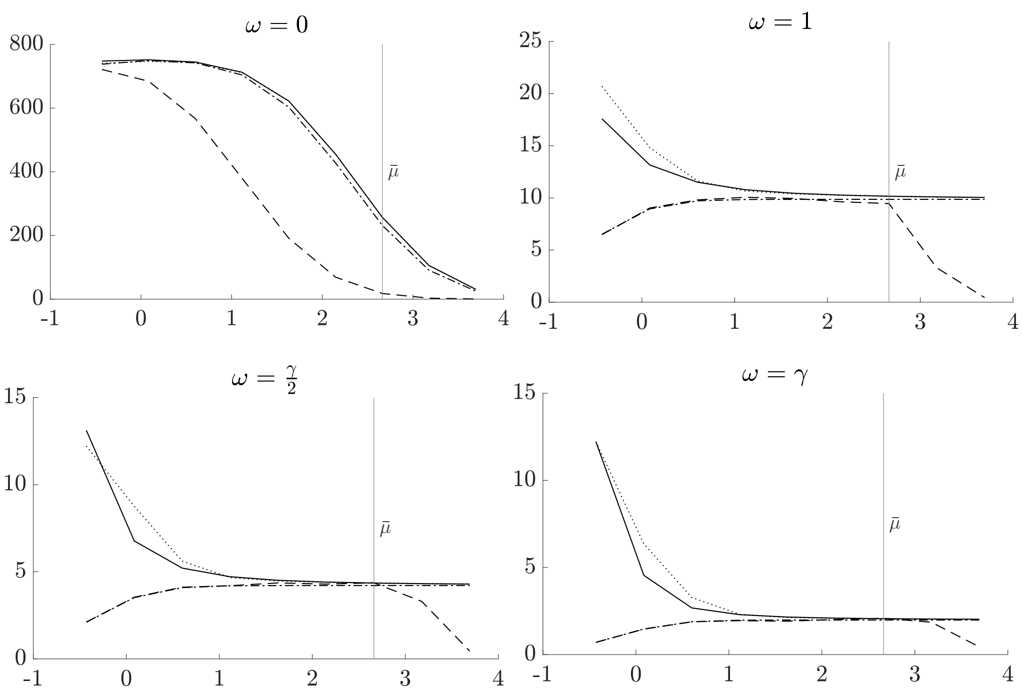


Figure E.4: Expected Variance of Lending under Optimal Stress Tests + Macroprudential Policies



- - - $\mathbb{VAR}[L_1^* \mid r_{l,0}, E_0]$ under an optimal DPT
-·-· $\mathbb{VAR}[L_1^* \mid r_{l,0}, E_0]$ under an optimal Dividend ban
····· $\mathbb{VAR}[L_1^* \mid r_{l,0}, E_0]$ under an optimal CCyB
— $\mathbb{VAR}[L_1^* \mid r_{l,0}, E_0]$ under an optimal Stress Test Framework