Current Policy Perspectives

FEDERAL RESERVE BANK OF BOSTON™

No. 17-6

The Equilibrium Real Policy Rate Through the Lens of Standard Growth Models

Daniel E. Sichel and J. Christina Wang

Abstract:
The long-run equilibrium real policy rate is a key concept in monetary economics and an important input into monetary policy decision-making. It has gained particular prominence lately as the Federal Reserve continues to normalize monetary policy. In this study, we assess the evolution, current level, and prospective values of this equilibrium rate within the framework of standard growth models. Our analysis considers as a baseline the single-sector Solow model, but it places more emphasis on the multi-sector neoclassical growth model, which better fits the data over the past three decades. We find that the long-run equilibrium policy rate has fallen between 0.3 and more than 1.6 percentage points from the 1973–2007 historical average, depending on the model and parameter values, mainly because of slower growth in total factor productivity (TFP) and the labor force. To the extent that the recent sluggish TFP growth persists, our estimates suggest a range of 0 percent to 1 percent for the equilibrium *real* rate in the current policy setting. But these estimates are subject to a substantial degree of uncertainty, as has been found in other studies. Policymakers thus need to interpret cautiously the guidance from policy rules that depend on the long-run equilibrium rate. This uncertainty also highlights the importance of the Federal Reserve's standard practice of constantly monitoring a wide range of indicators of inflation and real activity to gauge as accurately as possible the economy's reaction to policy.

Keywords: Natural rate of interest, productivity, growth models, monetary policy
JEL Classification: E43, E52, O33, O41

Daniel E. Sichel is a Professor of Economics at Wellesley College. J. Christina Wang is a senior economist and policy advisor in the research department of the Federal Reserve Bank of Boston. Their e-mail addresses are dsichel@wellesley.edu and christina.wang@bos.frb.org, respectively.

This paper presents preliminary analysis and results intended to stimulate discussion and critical comment. It is available on the website of the Federal Reserve Bank of Boston at http://www.bostonfed.org/economic/current-policy-perspectives/index.htm.

The authors thank José Fillat, Giovanni Olivei, Jonathan Parker, and Dongho Song for helpful discussions and comments.

This version: November 21, 2017

Introduction

The long-run equilibrium real policy rate is a key concept in monetary economics and an important input into monetary policy decision making. It is generally defined as the real interest rate that prevails when the economy is at full employment and inflation is at the central bank's target rate. It has gained particular prominence lately as the Federal Reserve continues to normalize the monetary policy stance by lifting the policy rate, which should in principle converge toward the long-run equilibrium policy rate, equal to the corresponding real rate plus the inflation target.

In this study, we assess the evolution, current level, and prospective values of this equilibrium rate within the framework of standard growth models. In other words, our exercise puts the role of total factor productivity (TFP) and, in turn, potential growth, front and center in determining the equilibrium real rate. This focus complements existing studies.

Our analysis considers as a baseline the single-sector Solow model, but it places more emphasis on the multi-sector neoclassical growth model, which better fits the data from the past three decades. Our key results include the following:

- Over this last cycle, the equilibrium real policy rate has fallen between 0.3 and more than 1.6 percentage points from the 1973–2007 historical average, depending on the model and parameter values.
- The key factors driving down the equilibrium real rate are the decline in growth rates of the labor force and TFP.
- Given the recent sluggish trend in TFP growth, these models suggest a range of 0 percent to 1 percent for the equilibrium *real* rate in the current policy setting (with a *nominal* rate in the range of 2 percent to 3 percent). If we consider probable improvements in the trend rate of TFP growth, our estimate for the equilibrium real rate climbs to almost 2 percent.
- A high degree of uncertainty surrounds these estimates. A key source is the uncertainty about future TFP growth rates, which have proven extremely difficult to forecast historically.

- Even though pronounced uncertainty is hardly unique—estimates of potential output and the natural rate of unemployment are also uncertain—the high uncertainty relative to the low average estimated level of the real equilibrium rate suggests that the guidance from simple policy rules that rely on the level of the equilibrium rate need to be regarded with greater caution during this tightening cycle.
- Outside the scope of the models we consider, global forces (such as greater demand for safe assets and a heightened sense of uncertainty) may also have helped push down the equilibrium risk-free real rates the world over, although the magnitude is hard to estimate.

In recent years, a growing number of studies have tackled the difficult task of estimating the equilibrium real policy rate. Most of those studies have concluded that the equilibrium real rate has fallen, albeit to varying degrees. Several of those studies, including Laubach and Williams (2016) and Kiley (2015), model this rate as an unobserved component and use filtering techniques to estimate it. Laubach and Williams' estimates are among the most widely followed, and as shown in the left panel of Figure 1, their estimates have fallen dramatically since the early 2000s and now stand at essentially zero.[1] For comparison, the right panel of Figure 1 shows the median and central tendency of Federal Open Market Committee (FOMC) participants' projections of the long-run real funds rate—as revealed in the Summary of Economic Projections (SEP). (This panel zooms in to the period since 2015 because the SEP started reporting the long-run funds rate in June 2015.[2]) These projections have been revised downward repeatedly since June 2015, and the median was down to 1 percent as of the SEP released on March 15, 2017. These downward revisions are consistent with the continuing downward revisions in the SEP estimate of the long-run growth rate of potential output, which generally imply a lower expected steady-state *real* rate, all else being equal.

As noted, this study investigates the evolution of the long-run equilibrium real policy rate within the framework of two standard growth models. First, we derive what these two growth models imply about the factors that determine the equilibrium real rate. We then generate empirical estimates of the equilibrium rate by calibrating variables, such as the growth rate of TFP, to

---

[1] Data were downloaded from www.frbsf.org/economic-research/files/Laubach_Williams_updated_estimates.xlsx on May 24, 2017, corresponding to "Estimates based on data available on April 21, 2107. Final data point: 2016Q4."

[2] The real policy rate from the SEP is imputed by subtracting the target inflation rate of 2 percent from the reported nominal funds rate.

historical averages. Our estimates suggest that the equilibrium rate has fallen in recent years because of slower growth of TFP and the labor force. Past performance of TFP growth then helps inform our projections of the equilibrium real rate relevant for this tightening cycle. Finally, we consider the implications of our results for monetary policy.

The Equilibrium Policy Rate in the Solow Model

The Solow (1957) growth model provides a simple representation of the economy that usefully highlights some key long-run relationships. Our approach is to use the model to generate an estimate of the return to capital and then to adjust this return to capital to obtain an estimate of the real policy rate, which is an overnight risk-free rate.

Specifically, we begin with the Cobb-Douglas production function:

$$Y_t = A_t K_t^a L^{1-a},$$

where Y_t is output, K_t is capital, L_t is labor input, A_t is technology or TFP, and a is the income share of capital. L_t and A_t are assumed to grow at exogenous rates n and g, respectively. K_t increases each period by the amount of investment net of depreciation on the prior period's capital stock. Finally, investment is assumed to be a constant share (s) of output. This assumption of an exogenous and constant saving rate is an important limitation of the Solow model. We relax it below in our analysis using the neoclassical growth model.

The model can be solved for its steady state, at which capital per effective worker is constant, and thus aggregate capital and output grow at the same rate of $n+g$.

In the standard Solow model, there is only one rate of return, which is the return on the only long-lived asset—productive capital, because the model neither explicitly accounts for risk nor considers assets of different maturities. This rate of return on capital equals the marginal product of capital (MPK) net of the depreciation rate (denoted as d), and the steady-state return to capital can thus be expressed as follows:

$$R_{SS} = MPK_{SS} - d = \frac{a\left[\frac{1}{1-a}g+n+d\right]}{s} - d\ . \qquad (1)$$

If we map R_{SS} from the Solow model to the return on capital in the data, then we can extract an estimate of the equilibrium real policy rate if we adjust R_{SS} by the spread between R_{SS} and the policy rate. Specifically, we set

$$R_{EQUIL\ POLICY\ RATE} = R_{SS} - SPR,$$

where the spread variable *(SPR)* is the spread between the average return on capital and the federal funds rate. This spread captures term and risk premiums as well as all other factors that affect the relationship between the Solow model's steady-state return to capital and the policy rate. This somewhat ad-hoc approach to estimating the equilibrium real policy rate highlights the single-rate limitation of the Solow model.

On the other hand, this expression for the equilibrium real policy rate highlights key factors influencing the rate in a relatively transparent manner. Faster TFP growth (g) implies a higher equilibrium rate: faster technological advances make capital more productive and, given a fixed saving rate, lead to a higher equilibrium rate. Faster labor force growth (n) implies a higher equilibrium rate: more labor per unit of capital raises the marginal product of capital and, given a fixed saving rate, generates a higher equilibrium rate. A lower saving rate (s) similarly leads to a higher equilibrium rate. Likewise, a higher capital share of income (a) implies that an additional dollar of capital generates more output, making capital more desirable and raising the equilibrium rate.

To implement this approach empirically, we assume that the spread variable (*SPR*) can be roughly proxied by the spread between the yield on BBB corporate bonds and the federal funds rate. While this adjustment is somewhat ad hoc, we argue it has the virtue of simplicity and transparency because it uses readily observable interest rates.

The Equilibrium Real Policy Rate in a Neoclassical Growth Model

Compared to the Solow model, the neoclassical growth model features risk-averse consumers who choose consumption, and hence saving, optimally. Risk aversion means consumers maximize utility by equalizing (the present value of) marginal utility in every period, which implies consumption smoothing over time, as summarized in the following first-order condition:

$$u'(c_t) = \mathrm{E}_t[(1+\rho)^{-1} R_{t+1} u'(c_{t+1})],$$

where ρ is the consumer's subjective discount rate, $u'(c_t)$ and $u'(c_{t+1})$ are the marginal utility of consuming today versus tomorrow, and R is the rate of return received on any investment.

This Euler equation implies a relationship between the equilibrium real overnight (policy) rate and consumption growth; it also implies that consumers require higher rates of return on risky assets—assets that pay less when consumption is low. Most macro models feature a representative consumer with a constant relative risk aversion (power) utility function $u(c) = c^{1-\sigma}/(1-\sigma)$. Such models imply the following equation for the risk-free overnight real (policy) rate of interest:[3]

$$r_t = \rho + \frac{1}{\psi}\mathrm{E}_t(\Delta c_{t+1}) - \frac{1}{2\psi^2}\mathrm{var}_t(\Delta c_{t+1}) - \eta_t, \quad (2)$$

where ψ is the intertemporal elasticity of substitution (IES), $\mathrm{E}_t(\Delta c_{t+1})$ is the expected growth rate of *per capita* consumption over the next period, and $\mathrm{var}_t(\Delta c_{t+1})$ is the expected variance of this consumption growth. The intuition for this relationship is that consumers desire to smooth consumption, so if they expect high growth leading to high consumption tomorrow, they will want to raise consumption (that is, reduce saving) today to smooth its path. This desired shift then leads to higher returns on assets to restore equilibrium, resulting in the positive relationship between the real policy rate and expected consumption growth. The second-to-last term in equation (2) is often called the precautionary saving term: expectations of more volatile consumption growth induces consumers to want to save more, driving down the real rate. In models solved with a linear approximation—such as Laubach and Williams (2016)—this last term is absent.

Equation (2) is written generally to allow for factors that can affect the real policy rate in the short run, as represented by the last term, $-\eta_t$. These are frictions that can prevent consumers from achieving their desired consumption in the short run. For example, if consumers face borrowing constraints, then the negative term $-\eta_t$ will be proportional to the shadow cost of the

[3] Equation (2) holds for the conditional expectation at any point in time. This relationship then implies a point-in-time "natural" rate in the absence of nominal frictions, such as those estimated in DSGE models. Note that equation (2) is an equilibrium relationship between endogenous variables—the interest rate and consumption growth—that are jointly determined.

borrowing constraint. One way to see the intuition is that for any given risk-free rate, expected consumption growth is faster due to lower current-period consumption caused by the borrowing constraint (which is assumed binding only in the current period). Another factor that can depress the equilibrium policy rate during certain periods is a liquidity/safety premium enjoyed by short-term risk-free assets. This would again introduce an additional term to the right-hand side of equation (2), and it would have qualitatively the same effect as the term due to the borrowing constraint. In the first few years following the financial crisis, both of these forces may have further depressed the equilibrium rate to a level below that which would prevail over the long run. Globally, these forces may yet persist for a while longer.

Over the long run, when the economy has reached the steady state so that aggregate output grows at the potential rate while inflation is at the central bank's target rate, the steady-state equilibrium real rate of policy, r_{ss}, can be expressed as:

$$r_{SS} = \rho + \frac{1}{\psi} \mathrm{E}\ (\Delta c_{t+1}) - \frac{1}{2\psi^2} \mathrm{var}\ (\Delta c_{t+1}), \tag{2'}$$

which results from taking unconditional expectations of Equation (2) and removing the term due to short-run forces.

A Single-Sector Neoclassical Growth Model

In the basic neoclassical model with a single good, consumption is equivalent to output, and so consumption and output have the same growth rate. Given a Cobb-Douglas production function as used in our implementation of the Solow growth model, the steady-state growth rate of aggregate consumption in the neoclassical model, denoted as g_C, is the sum of growth contributions from TFP, capital deepening, and the labor force as follows:

$$g_C = \frac{z}{1-\alpha} + n = \left(z + \frac{\alpha z}{1-\alpha} \right) + n, \tag{3}$$

where z is the overall TFP growth rate, while n is the labor force growth rate, and α is the income share of capital.[4] z clearly measures the growth contribution from TFP, while $\alpha z/(1-\alpha)$ measures the contribution from capital deepening, that is, the increasing amount of capital per worker, which is entirely determined by TFP in this model.

The growth rate of consumption per capita, denoted as Δc, is simply g_c net of population growth, denoted n':

$$\Delta c = g_C - n' = z/(1-\alpha) + (n - n') . \tag{4}$$

If population grows at the same rate as the labor force, then Δc is simply $z/(1-\alpha)$. This is an intuitive result: per capita output (equivalent to consumption in a one-sector neoclassical model) grows in the steady state solely because of TFP growth and the induced capital deepening. More generally, Δc also depends on labor force growth relative to population growth. If population is expected to grow faster than the labor force, as will be the case for many developed countries in the coming decades, then per capita consumption will grow at a rate slower than what would be implied by technology alone.

A Multi-Sector Neoclassical Growth Model

Our analysis in this section goes beyond the standard single-sector model to consider a multi-sector model in which some sectors grow faster than others. We use a multi-sector model primarily because the U.S. data since the early to mid-1980s are better described by a two- or three-sector model: data indicate that consumption has grown consistently more slowly than GDP, especially during the productivity boom years of the late 1990s and early 2000s, when the growth differential between GDP and consumption widened to 0.8 percentage point from 0.4 percentage point over the productivity-slowdown period of 1973–1996. The choice of a multi-sector model has direct bearing on the equilibrium rate estimation: the Euler equation above makes clear that in a multi-sector model, it is consumption growth, rather than output growth, that matters for the equilibrium real rate. One immediate implication is that because the decline in the growth rate of

[4] z, instead of g, is used to denote the TFP growth rate to enable a clearer expression for consumption growth in the multi-sector version of the model later.

potential consumption since the previous high-growth period is less than that in potential output growth, the implied decline in the equilibrium rate since the high-growth era is also smaller.

To derive the steady-state growth rate of aggregate consumption as a function of TFP by sector and labor force growth, we adopt the model in Fernald (2015) that features three sectors producing 1) consumption goods (that is, services and nondurables), 2) structures, and 3) equipment and durable goods, respectively, with the same Cobb-Douglas production function in all three sectors except for the growth rate of TFP. Moreover, this model considers land and inventory as inputs into composite capital services. In this model, the steady-state growth rate of the consumption sector (denoted as g_C) can be expressed as:

$$g_C = \underbrace{z_C}_{\substack{\text{Contribution of} \\ \text{consumption TFP}}} + \underbrace{\frac{\alpha\left(1-c_T\right)}{1-\alpha\left(1-c_T\right)} g_I}_{\substack{\text{Contribution of} \\ \text{reproducible capital}}} + \underbrace{\frac{\alpha c_T}{1-\alpha\left(1-c_T\right)} g_T}_{\text{Contribution of land}} + \underbrace{\frac{1-\alpha}{1-\alpha\left(1-c_T\right)} n}_{\text{Contribution of labor}} , \tag{5}$$

where z_C is the TFP growth rate for the consumption sector, α is the income share of all capital, g_I is the growth rate of reproducible capital (inclusive of inventories), while g_T is the (exogenous) growth rate of land. The term g_I can be further expressed as the weighted average growth rates of equipment capital, structures, and inventories, with weights equal to each category's share in total capital income.[5] The term c_T is the share of equipment in overall capital income. Equation (5) clearly has an analogous structure to (3): the steady-state growth of consumption, g_C, is the sum of contributions from the TFP growth of the consumption sector, increasing intensity of reproducible capital (which depends on the TFP growth of the other two sectors), land, and labor input.[6]

Intuitively, g_C also rises with the growth rate of labor input. During the expansion after the Great Recession, employment, in its recovery from the deep cuts during the downturn, grew much faster than population, giving g_C a significant boost over this period. On the other hand, the slower

---

[5] Specifically, $g_I = \frac{c_E + c_V v_D}{1-c_T} z_D + \frac{1-c_E-c_V-c_T}{1-c_T} z_S + \frac{c_V\left(1-v_D\right)}{1-c_T} z_C$, where z_C and z_S are the TFP growth rates for the consumption (that is, nondurable goods, which contribute to inventories) and structures sectors, respectively, while c_E and c_V are the shares of equipment and inventory in overall capital income, respectively. v_D is the share of durables in inventories. So g_I is the weighted average growth of its three components: equipment capital, structures and (nondurable) inventories, each weighted by its share in total capital income.

[6] Compared to the one-sector model, TFP of the consumption sector grows slower, whereas capital deepening is faster in the multi-sector model. The net effect is ambiguous, depending on the exact parameter values.

labor force growth due to population aging will mean slower aggregate consumption growth going forward. In terms of per capita consumption growth, the logic is the same in the multi-sector model as in the single-sector model. That is, it depends on the relative growth rate of the labor force versus the population. A slight wrinkle in this multi-sector model with non-reproducible capital (that is, land) is that even if labor force and population grow at the same rate (n), there will be a negative relationship between per capita consumption growth and population growth, as can be seen from Equation (5), since the coefficient for n is less than 1.[7] This relationship implies that if population and labor force growth slowed in tandem, it would in fact raise per capita consumption growth very slightly. Unfortunately, the aging of the population in all the developed countries means that labor force growth will slow noticeably more than population growth, causing per capita consumption to decelerate even more than aggregate consumption in the coming years.

Equation (2') makes precise the positive connection between expected long-run potential growth and the neutral real policy rate. Diminished expectations of long-run growth thus correspond to a lower long-run real rate, and the smaller the intertemporal elasticity of substitution (IES), the more the long-run rate has to fall. Moreover, once we go beyond the linear approximation, the equilibrium real rate also depends negatively on the expected volatility of consumption growth. Perceptions of more uncertain consumption growth following the Great Recession may have led to more precautionary savings, perhaps globally, thus lowering estimates of the equilibrium rate even over the medium to long run.

Moreover, to the extent households have raised their desired saving rate exogenously, such an increase can be mapped into this framework as a lower subjective time discount rate ρ in equation (2'). This would lower the neutral real rate one for one. A higher saving rate would be consistent with what Bernanke (2005) termed the "global savings glut," which is one factor often used to explain the low rates prevailing worldwide since the 2001 U.S. recession.

Estimates of the Equilibrium Real Rate in the Solow Model

[7] This is because land is not reproducible, and so faster population growth leads to a lower rate of overall capital deepening.

Historical results. To obtain numerical estimates over different periods, we use averaged actual historical values over each period for each variable and parameter in the equation for the equilibrium real rate.[8] Wherever possible, we use data from John Fernald's growth accounting decomposition, available on the San Francisco Federal Reserve Bank's website.[9]

Historical results from the Solow model are shown in Table 1.[10] These estimates imply that the return to capital stepped down 1 percentage point after 2007, coinciding with the slowdown in productivity and labor force growth that occurred during that period. The equilibrium policy rate fell more dramatically after 2007, reflecting a jump in the spread between the BBB yield and the federal funds rate during the financial crisis. To smooth through that effect and obtain an estimate of the rate for the current period, we focus on the 2015–2016 period in which the estimate of the equilibrium real policy rate is 0 percent, down 1.6 percentage points from its value before 2008. We regard this estimate as the Solow model's take on the decline in the equilibrium real rate since 2007.

These estimates and others reported in this paper are obtained using the TFP growth estimates from Fernald that were not adjusted for unmeasured input (such as labor effort and the capital workweek). Such time-varying resource utilization mostly averages out over a business cycle and thus has a limited impact on long averages for TFP growth.[11] Nevertheless, as robustness checks, we also calculate an alternative set of estimates using Fernald's utilization-adjusted TFP, which at the business-cycle and higher frequencies can be argued to be a more precise measure of

---

[8] For the saving rate, we use the economy-wide investment rate, calculated as the ratio of the sum of gross private domestic investment and government investment over GDP. Because domestic saving does not necessarily equal domestic investment in an open economy, this ratio focuses on the share of the economy's resources that are being invested to form capital.

[9] These data focus on the business sector, and output growth is an average of that for Gross Domestic Product and Gross Domestic Income. See Fernald (2014) for details.

[10] We choose to start our sample in 1973 because it is taken in many studies as the starting year of the productivity slowdown. More specifically, the fourth quarter of 1973 is chosen because our projection estimates later will divide the sample by business cycle, and 1973:Q4 was a cyclical peak.

[11] A number of studies, including Basu, Fernald and Kimball (2006), have demonstrated that the Solow residual is affected not only by technical change but also changes in unmeasured inputs, which is most relevant within a business cycle and especially at high frequency (such as in quarterly data). On the other hand, since the input utilization measure used to adjust the Solow residual has to be estimated, the adjusted TFP growth rates inevitably introduce some more measurement error. Moreover, as found in Basu et al. (2014), the relative price change between consumption and investment goods is more closely correlated with their relative TFP growth, not their relative utilization-adjusted TFP growth. Hence, the relative price change is a better proxy for the relative unadjusted TFP growth. For these reasons, we prefer the unadjusted TFP estimates for the exercise here.

the underlying pace of technical change. Those estimates based on utilization-adjusted TFP (not reported for brevity) tell a story that is largely similar to the one using the unadjusted figures.[12]

What factors accounted for the decline in the Solow return to capital? Table 2 provides an answer from the perspective of the Solow model. We specifically compare the most recent cycle—beginning just before the Great Recession—with the average over the prior period 1973:Q4–2007:Q4, which can be regarded as approximating the historical average. This table decomposes the total change between these two periods into the amount caused by changes in each parameter or variable in the formula for the Solow-model steady-state return to capital.[13] As shown, the equilibrium return to capital dropped 1 percentage point across these two periods. The main message of the table is that this decline reflected a slowdown in the average rate of TFP growth and a very sizable slowdown in the growth of the labor force. At the same time, the rise in the capital share of income across the periods and the decline in the saving rate exerted a positive partial effect on the model's estimate of the equilibrium rate across these periods, damping the negative effect caused by the slowdown in TFP and labor force growth.[14]

Projections of the real equilibrium policy rate. Using this framework to project the equilibrium policy rate from the Solow model, we consider three scenarios for possible future values of the rate. Because trend TFP growth is so essential to determining the equilibrium rate, our scenarios focus on a range of possible outcomes for trend TFP, with the scenarios broadly based on the range of historical trends in TFP growth.

Our "Baseline" scenario assumes that the current sluggish trend in TFP continues, and we set TFP growth in this scenario to be just 0.6 percent, which matches the average growth rate of TFP over 2011:Q1–2016:Q4. It must be noted though that this scenario implicitly assumes the

[12] As will be noted below, the only noticeable change that would be brought about by the use of utilization-adjusted TFP is that it would call for a swap in the two periods used to calibrate the Baseline versus the Better scenarios for making projections of TFP. This is because we use the average TFP growth over only the expansion phase of each business cycle, and utilization adjustment affects the average growth rate more noticeably when using data shorter than a full cycle.

[13] Each line of the decomposition reported in the table is calculated as the partial effect of each variable while keeping all the other variables at their average values across the 1973:Q4–2007:Q4 and 2008:Q1–2016:Q4 periods.

[14] In the golden-rule steady state, the saving rate (s) and the income share of capital (a) cancel and drop out of the expression for the equilibrium rate of return. Accordingly, the drop in the equilibrium rate across the periods would be larger for the golden-rule version because there would be no upward boost to the equilibrium real rate from the saving rate and the income share of capital.

pace of technological progress and diffusion will be noticeably below historical norms for the foreseeable future.

Our "Better" scenario considers the possibility that trend TFP growth improves by about 0.25 to 0.8 percentage point. This scenario represents an improvement to a better pace, but one that still is less than the long-run historical trend in TFP. (The pace in this scenario matches the average from 2004:Q1–2007:Q4, the period after the sizable step-down in TFP around 2004 but before the onset of the financial crisis.[15])

Finally, our "Optimistic" scenario assumes a much brighter outlook for technology trends and for TFP, almost 0.5 percentage point faster than our Better scenario. While the boom period of 1996:Q1–2003:Q4 provides a possible point of reference for the Optimistic scenario, we believe it is unlikely that TFP growth will return to such an elevated pace on a persistent basis. Accordingly, our optimistic scenario assumes that trend TFP growth revives halfway back to the rate during the boom period, reaching 1.2 percent. (This rate is thus the average of that during 1996:Q1–2003:Q4 and during the current sluggish period of 2011:Q1–2016:Q4.)

For all three scenarios, we set the projected growth of the labor force to the average of the Congressional Budget Office's (CBO) projected growth in the potential labor force over the future period 2017–2027, and we set the spread between the BBB corporate bond yield and the federal funds rate to its long-run average from 1973:Q4–2016:Q4.[16] The other variables and parameters in all three scenarios (the income share of capital, the depreciation rate, and the saving rate) are set to their average values from 2011:Q1–2016:Q4.

The equilibrium real policy rates generated under each scenario are reported in Table 3. In the Baseline scenario that captures the recent sluggish trend in TFP growth, the implied value of the equilibrium real policy rate is 0 percent. In the Better scenario, the modest improvement in trend TFP generates an estimate of the equilibrium policy rate of 0.5 percent. Finally, in our

[15] As is apparent from the scenario descriptions, the TFP growth was 0.2 percentage point slower during the period covering 2011:Q1–2016:Q4 than during 2004:Q4–2007:Q4. If we had used the utilization-adjusted TFP figures from Fernald rather than the unadjusted figures, this ranking reverses, with average growth slightly faster during 2011:Q1–2016:Q4 than during the earlier period.

[16] See Congressional Budget Office (2017) for labor force projections. Regarding the spread between the BBB yield and the federal funds rate, note that the long-run average during 1973:Q4–2016:Q4 is very close to the 2016:Q4 value.

Optimistic scenario, the more substantial improvement in trend TFP growth delivers an equilibrium real policy rate of 1.9 percent.

In sum, looking across the three scenarios, we can see that estimates of the equilibrium real policy rate based on the single-sector Solow model spans a wide range, merely because of different projections of the trend TFP growth rate. The band of uncertainty would be even greater if we allowed other parameters to vary across scenarios, such as the spread between risky and risk-free rates. Moreover, the Solow framework describes a closed economy, and uncertainty about foreign developments that affect U.S. rates would boost the band of uncertainty further.

Estimates of Long-Run Growth and the Equilibrium Real Rate in a Multi-Sector Model

In exogenous growth models, the economy's long-run growth rate is entirely pinned down by the rate of labor force growth and TFP growth, which determines the rate of capital deepening. The U.S. data since the early 1980s suggest that the pace of technological progress for producing equipment, software, and consumer durables (referred to as the investment sector for short) has grown at a noticeably faster pace than that for the rest of the economy, especially the sectors producing services and nondurable goods.[17] As shown in Figure 2, over the subperiods typically used in productivity studies, technological progress in the investment sector has been consistently faster than in other sectors, especially during the late 1990s to early 2000s.[18]

Because the share of relatively slow-growing services and nondurables in consumption is greater than the share in business-sector output, consumption thus grows at a slower rate than does business-sector output. Even when compared with GDP, consumption comprises more of those components characterized by slower TFP growth. The share of services, which exhibit the slowest TFP growth rate, has in fact risen from 52 percent of consumption in 1980 to now nearly 68 percent. The relatively slower consumption growth rate is arguably the most important feature

[17] At the same time, the implicit deflator of the investment sector has been falling faster relative to the deflator of the rest of economy so that nominal shares of the two sectors have been stationary.

[18] It is well documented that labor productivity growth slowed substantially starting in 1973. It then switched into a higher gear during 1996–2003, reflecting faster TFP growth (and greater capital deepening). TFP then decelerated noticeably starting around 2004, as documented in Fernald (2015). The pattern of average TFP growth is rather similar however one divides the period since 2004: Figure 2 reports the last subperiod including the Great Recession (from 2008) to match the timing used in the tables on historical decomposition (Tables 1, 2, and 4), while Figure A2 singles out the last expansion (from 2011) to match the timing used in the projection tables.

captured by the multi-sector growth model, as compared to a single-sector model, for our purpose of estimating the equilibrium real rate.[19]

Overall consumption or output growth, of course, also depends on labor input growth, as can be seen in equation (5).[20] It is instructive to compare realized output growth to potential growth as implied by the sectoral TFP growth according to the model along with potential labor force growth according to the CBO. Figure 3 carries out a qualitative comparison between actual growth (the stack of bars labeled "Data") and potential growth, depicting just the sum of the two components that matter for the difference—potential labor force growth and the contribution of capital deepening implied by TFP growth. Actual growth basically equals potential growth for the 1973:Q4–1995:Q4 period, as the above-potential capital deepening is offset by the below-potential labor growth.[21] Actual growth falls short of potential growth over the high-TFP-growth period of 1996:Q1–2003:Q4 because of the labor component. This shortfall in actual labor input growth becomes even more pronounced during the 2004–2010 period because of the deep recession.

The recent years stand out in contrast: labor surpasses its potential rate by enough to more than offset the below-potential pace of capital deepening, so that output in fact grew faster than the potential rate implied by the model. The rapid growth in employment since 2011 (relative to the growth of the potential labor force) is at least in part the inevitable reversal of the deep payroll cuts firms had to make during the Great Recession. In contrast, capital accumulation falls short of even the steady-state rate that would be implied by the weaker TFP growth since the mid-2000s, so that the capital-labor ratio has in fact clearly fallen well below trend over this expansion, resulting in a gap that is unprecedented for the post-World War II period. To the extent that factors that have restrained investment dissipate over time, it seems likely that firms will step up capital accumulation in the coming years to take advantage of the available technology, cushioning output

[19] The other important, perhaps subtler, difference brought about by a multi-sector model where the investment sector grows faster is that the pace of capital deepening is faster than what would be implied by a single-sector model for any given aggregate TFP growth rate. And this prediction is borne out by the U.S. data since the 1980s. But this factor is overwhelmed by the lower consumption TFP growth, so that on net the growth rate of consumption is lower than would be implied by a single-sector model.

[20] As noted previously, output growth can be written as the sum of labor input growth, capital deepening (multiplied by capital's income share) and TFP growth. Since multi-sector models with balanced growth assume sectors all have the same capital and labor ratio and differ only in their TFP growth, we can simply compare realized versus potential growth of labor input and capital deepening, which apply to all sectors.

[21] One plausible explanation is that firms were carrying out various intangible investments in order to take advantage of the information-technology revolution. This would be consistent with the lackluster TFP growth over those years followed by strong TFP performance in the subsequent period.

growth from the eventual convergence of labor input growth toward the slower rate of potential labor force growth.

We next examine what the real equilibrium rate would have been over historical subperiods given the average realized growth rates of TFP and labor input, and the implied steady-state contribution of capital deepening. For this exercise, we use the realized volatility of per capita consumption over each relevant subperiod to approximate the expected value. We again compare the most recent cycle with the historical average from 1973:Q1–2007:Q4. We first report the comparison according to the one-sector model, assuming, that is, all goods and services are produced by the same technology (Table 4). The overall contribution of 0.4 percentage point (0.3 of which is attributed to TFP itself and 0.1 to the associated slower capital deepening) of TFP growth to the slowdown in consumption growth is much smaller than the contribution from slower growth of labor input (smaller by a full percentage point). The slower growth of consumption per capita would imply a lower equilibrium real rate: depending on the parameter values for IES and ρ, the decline in the real rate ranges from 0.9 to 1.1 percentage points.[22] The lower the IES, the larger the decline in the equilibrium rate, since households are less willing to substitute across time so that a larger change (decline) in the equilibrium rate is needed to "induce" consumers to reduce consumption (that is, shift to a path of slower consumption growth).

Turning to the multi-sector model, Table 5 reports the growth rate of consumption per capita over this last cycle, which slowed by 0.6 percentage point compared to the historical average. Again, this is in large part because of the slower growth of labor input, but also because of the somewhat slower pace of TFP growth in the equipment sector. As in the one-sector model, this slowdown would imply a lower equilibrium real rate (for any given parameter values for IES and ρ), but this effect is slightly offset by the lower volatility of consumption growth. On net, estimates of the equilibrium real rate fall by about half a percentage point over this last cycle relative to the historical average. The decline in TFP growth in the multi-sector model is slightly smaller than in the one-sector model and thus leads to smaller reductions in consumption per capita, and in turn the equilibrium real policy rate.

[22] See the next subsection on the projections based on a multi-sector growth model, which is our focus, for more detailed discussion of the influence of the parameter value choices.

Projections of the Equilibrium Real Rate Based on TFP Growth Rates

We next make projections of expected consumption growth going forward, using the scenarios that parallel those for the projections from the Solow model. Table 5 summarizes our projections of consumption growth based on TFP growth rates, and what these imply about the steady-state equilibrium real rate. (Potential GDP growth is reported for comparison.) The first scenario, the Baseline, relies on the assumption that the average sectoral TFP growth rates over the current expansion (that is, since 2011:Q1) would prevail over the long run. At the same time, we compare these forecast values with the projections based on a somewhat better assumption of TFP performance, which is calibrated to the average rate over the previous expansion from 2004:Q1–2007:Q4. We also consider an Optimistic scenario featuring higher TFP growth rates, which are assumed to be halfway between the high-growth era of 1996:Q1–2003:Q4 and the most recent years. For the variance of per capita consumption growth, we use the relatively low volatility values from the current expansion as the baseline. To the extent that households' perception of consumption variability has risen because of the deep downturn, it would push down estimates of the real rate. In fact, if we adopt the variance over all the years since 2008, we obtain some estimates of a negative equilibrium real rate.

For parameter values, we take a range of 1.5 to 0.8[23] for IES estimates, as suggested by previous studies. The lower the assumed value for the IES, the lower the imputed long-run rate, as the influence of consumption's variability dominates the influence of the average growth. Note that this relationship between the IES and the equilibrium rate is conditional on the values of per capita consumption growth and variance used in the projection scenarios, because it is the net effect of two countervailing forces. On the one hand, the smaller the IES (meaning households really want to smooth consumption over time), the higher the interest rate needs to be to discourage consumers from wanting to borrow from the future given a positive expected consumption growth rate. On the other hand, a lower IES corresponds to a higher degree of risk aversion under the power utility function, implying a greater precautionary savings motive, pushing down the

[23] Finance studies, especially those that use a recursive utility function (such as the Epstein-Zin), tend to adopt values above 1 to better fit asset price data. See Bansal and Yaron (2004) for an example. Macro studies tend to estimate lower values of IES. Guvenen (2006) offers an intuitive explanation reconciling these findings. By way of comparison, the coefficient estimate from Laubach and Williams (2015) implies a value equivalent to an IES of about 0.77. Note that here we ignore the potential issue that our chosen values of the IES, which equals the inverse of the coefficient of risk aversion in power utility functions, may imply unreasonable values for the equity risk premium.

equilibrium rate, all else being equal. The net effect is a positive association between the IES and the equilibrium real rate in our chosen scenarios. The parameter most difficult to calibrate is perhaps the subjective discount rate ρ. But its effect is easy to gauge, since the long-run rate shifts one for one with respect to ρ. So we take ρ = 1 to be the baseline.

In sum, if TFP growth rates were the only difference between the Baseline and the Better scenarios (calibrated to the current and the previous expansion, respectively), then the scenarios' equilibrium real rates would differ only slightly, by no more than 0.3 percentage point (under a low IES). The estimates of the equilibrium real rate under the Optimistic scenario for TFP growth are 0.4 to 0.7 percentage point higher than in the Baseline scenario. A lower value of ρ would certainly pull down the equilibrium rates, but it would not change how the rates differ across these three scenarios. If the expected variance of consumption growth is assumed to be higher than the realized value over this expansion, for example by incorporating the volatility experienced during the Great Recession (so that the expected variance rises from 1.4 to 3.2), then it can lower the rate projections by a full percentage point or more. In short, under the fairly plausible parameter values chosen here, estimates of the equilibrium rate are around 1 percent or somewhat lower. That figure, around 1 percent, also happens to be the median value in the March 2017 SEP. At the same time, the margin of error surrounding this 1 percent estimate from the multi-sector model is probably skewed to the downside, since alternative (but still more or less plausible) parameter values tend to generate estimates that are lower by around 0.5 to 1 percentage point.[24] As a robustness check, we also estimate the equilibrium real rate using the utilization-adjusted TFP growth rates (not reported for brevity). The range of values for the projected equilibrium real rate remain largely comparable.[25]

Putting the Pieces Together

[24] Faster growth of utilization-adjusted TFP than unadjusted TFP means that the pace of unmeasured input growth, imputed based on hours per worker, has been slower during this expansion than during earlier periods.

[25] The only difference is that the steady-state growth of consumption per capita calibrated using the utilization-adjusted TFP growth is somewhat faster during the current expansion than during the expansion after 2001, despite the similar range of values. This is because the average rate of utilization does not average to zero, and hence the adjustment affects the average TFP growth when using data over only the expansion phase of a full business cycle.

Table 6 pulls together the projections reported above for the equilibrium real policy rate from both the Solow model and the multi-sector neoclassical model. As shown, the Solow model projection in the Baseline scenario (calibrated to the current expansion) is noticeably lower than those using the TFP growth in the Better scenario (calibrated to the previous expansion). In contrast, the difference between these scenarios is much smaller in the neoclassical model. This outcome in the neoclassical model mainly is the result of per capita consumption growth declining less than GDP growth, which can be distinguished in the multi-sector model. If one takes a more optimistic view about the prospects for future TFP growth, the estimated equilibrium policy real rates are somewhat higher, climbing to 1.3 percent in the neoclassical model and to almost 2 percent in the Solow model.

Implications for Monetary Policy

While standard growth models provide useful intuition about the influence of different factors on the equilibrium policy rate, they hardly reduce the uncertainty surrounding the rate's value. Our estimates suggest a plausible range for the real equilibrium policy rate in the current policy setting—given our baseline projection of trend TFP growth—from 0 percent to 1 percent, and alternative choices for calibrations easily could have generated a wider range. This range includes the Laubach-Williams estimate of about 0 percent as well as the value implied by the SEP of 1 percent, which are themselves surrounded by much uncertainty.[26] The absolute magnitude of the uncertainty may or may not have risen relative to the past, but it is certainly much higher relative to the low average level of the real rate estimates. This configuration highlights the elevated risk of the economy hitting the effective lower bound for nominal rates set by the FOMC.

The uncertainty surrounding the equilibrium real rate is hardly unique. Estimates of other key variables for policy—such as potential output growth and the NAIRU—are also subject to nontrivial uncertainty.[27] Nevertheless, the high uncertainty, especially relative to the low central

[26] Since the middle of 2014, market expectations of the policy rate, as measured by the market-implied forward short-term rates over the SEP forecast horizon, have consistently risen less than the SEP projections of the funds rate, sometimes markedly. This suggests that market participants' estimates of the real equilibrium rate may be lower than the median of the SEP. Alternatively, it could be because over the recent past, the term premium has turned more negative than the estimates using more distant past data.

[27] Although, according to Kiley (2015), macro data are comparatively more informative about potential growth and the natural rate of unemployment.

tendency of the real rate estimates, suggests the guidance from policy rules that rely heavily on equilibrium-rate estimates may be less useful now than in other periods. While such rules—which include some versions of the so-called Taylor Rule—are able to give clear directional signals at times when output is well above or below potential, in the current economic situation their guidance likely should be interpreted more cautiously.

In addition, the uncertainty also points to the importance of weighing the potential costs of errors in either direction in the estimated equilibrium rate. In the current policy setting, if the true equilibrium real policy rate is near the lower end of the estimated range, but policy is set in the belief that the equilibrium rate is higher and the policy rate is raised to beyond the equilibrium level, the economy could be pushed back to a period of weak growth and stuck near the effective lower bound for interest rates.[28] Unconventional policy tools (such as asset purchases and negative interest rates) could be used to counter such economic anemia, but further use of these tools raises a host of issues. On the other hand, if the true equilibrium rate is at the upper end of the range we estimated, but policymakers acted on the belief that the equilibrium rate is nearer to the lower end of the range and removed accommodation too slowly, then the economy could overheat. Correcting this overheating could engender a recession, since the historical record suggests that it is difficult for monetary policy to engineer a soft landing for the economy.

Given that the potential costs of errors in either direction in the current policy setting are likely to stem from hitting the effective lower bound because of the low equilibrium real rate, possible changes to the policy framework that can help combat this constraint may warrant more serious consideration. At the same time, the uncertainty about the true equilibrium rate also highlights the importance of the Federal Reserve's standard practice of monitoring in real time a wide range of indicators of inflation and real activity in order to gauge as accurately as possible the state of the economy, including the economy's response to policy actions.

Conclusion

[28] We should note a subtle distinction here: As highlighted in previous sections, a lower equilibrium rate is often associated with slower long-run growth of potential output. If that is the case, then, for any *given* path of actual output, slower trend growth would imply that the output gap had evolved to be *more positive* (or less negative) than otherwise. Such a difference in the path of the output gap also would have implications for the appropriate path of the policy rate over the tightening cycle.

This paper examines the long-run equilibrium interest rates through the lens of simple and standard growth models. While these models have some limitations for estimating equilibrium rates, they also provide transparent intuition about the factors affecting these rates. As in other studies, we find that estimates of the equilibrium rate have moved down considerably in recent years, with the declines largely driven by the slowdown in TFP growth and in labor force growth. Going forward, our point estimates, taken as a whole, suggest an equilibrium real policy rate in the neighborhood of 0 percent to 1 percent. Note that our estimates are based on TFP growth rates free of business-cycle influences (such as unmeasured input utilization), so any cyclical gain in measured TFP growth in the near term will have no bearing on our estimates of the equilibrium rate. That being said, factors outside of the long-run trend growth of technology and the labor force can exert a nontrivial influence on the real policy rate over an extended period, and we suspect that many of these factors are more likely than not to exert downward pressure on the real policy rate. On the other hand, the uncertainty band surrounding our estimates is wide, which primarily reflects uncertainty about future TFP growth rates. Past experience suggests that it is exceedingly difficult to predict future performance of TFP. There is some evidence that there exists medium-term (from 10 years to a few decades) fluctuations of TFP growth, and if an improvement in trend TFP growth were to materialize, the equilibrium real policy rate in the future would be higher, all else being equal.

References

Bansal, Ravi, and Amir Yaron. 2004. "Risks for the Long Run: A Potential Resolution of Asset Pricing Puzzles." *Journal of Finance* 59(4): 1481–1509.

Basu, Susanto, John Fernald, and Miles Kimball. 2006. "Are Technology Improvements Contractionary?" *American Economic Review* 96(5): 1418–1448.

Basu, Susanto, John Fernald, Jonas Fisher, and Miles Kimball 2014. *Sector-Specific Technical Change*. Unpublished manuscript.

Bernanke, Ben S. 2005. "The Global Saving Glut and the U.S. Current Account Deficit." Remarks by Governor Bernanke, Board of Governors of the Federal Reserve System. March 10.

Congressional Budget Office. 2017. *The Budget and Economic Outlook: 2017 to 2027*. January.

Fernald, John. 2014. "A Quarterly, Utilization-Adjusted Series on Total Factor Productivity." Working Paper No. 2012-19. San Francisco Federal Reserve Bank.

Fernald, John. 2015. "Productivity and Potential Output Before, During, and After the Great Recession." *NBER Macroeconomics Annual*.

Feroli, Michael. 2017. "Exploring the Economic Impact of Immigration Reform." *JPMorgan Economic Research Note*. March 3.

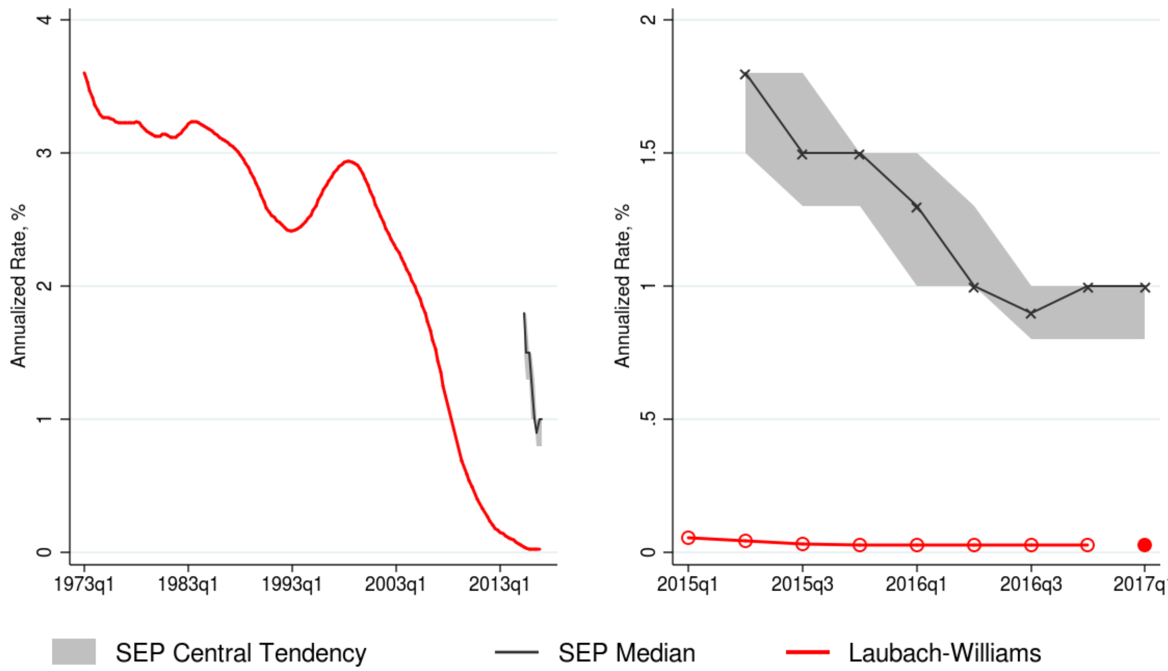
Guvenen, Fatih. 2006. "Reconciling Conflicting Evidence on the Elasticity of Intertemporal Substitution: A Macroeconomic Perspective. *Journal of Monetary Economics*. 53(7): 1451–1472.

Kiley, Michael. 2015. "What Can the Data Tell Us about the Equilibrium Real Interest Rate?" Finance and Economics Discussion Series 2015-77. Board of Governors of the Federal Reserve System.

Laubach, Thomas, and John C. Williams. 2016. "Measuring the Natural Rate of Interest Redux." *Business Economics*. 51(2): 57–67.

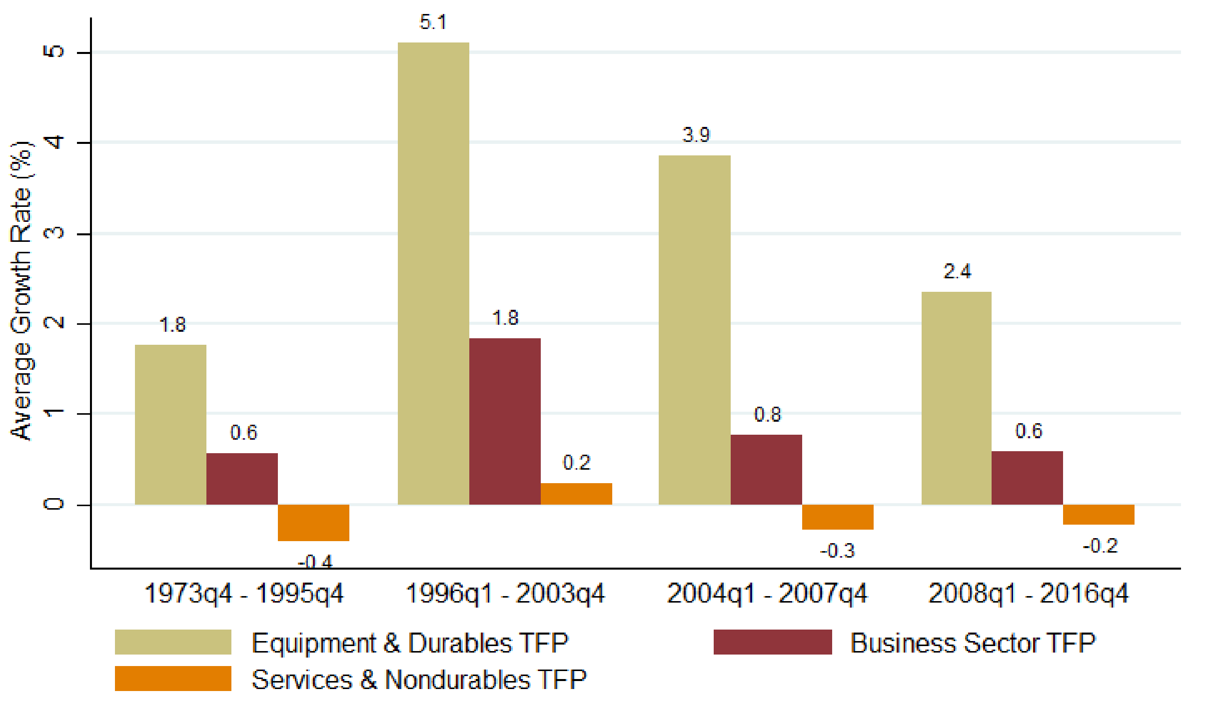
Solow, Robert. 1957. "Technical Change and the Aggregate Production Function." *Review of Economics and Statistics.* 39(3): 312–320.

Figure 1. Estimates of the Equilibrium Real Rate



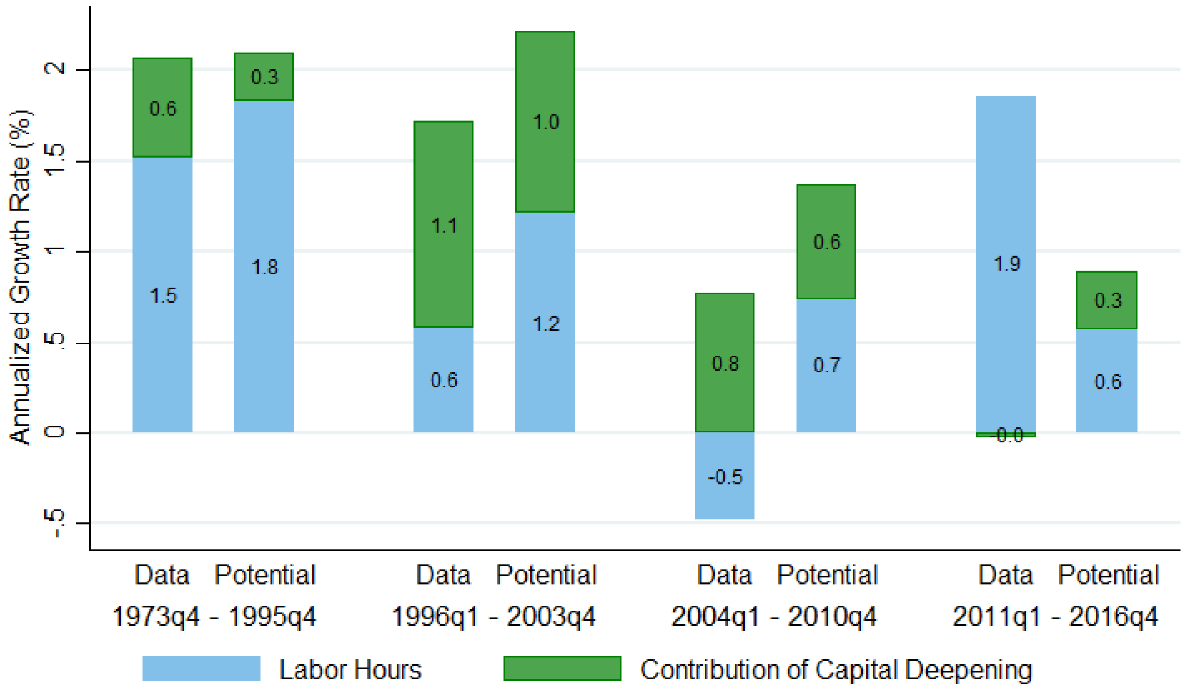
Note: The left panel shows the full time series of Laubach and Williams' estimates over our sample period to highlight the decline over time. The estimates are based on data available on April 21, 2017, and their final data point is 2016:Q4, the value of which is copied to 2017:Q1. The right panel zooms in to the period since 2015:Q1; the SEP started to include the long-term funds rate forecast in June 2015. The real rate is imputed by subtracting an inflation target of 2 percent from the nominal rate reported in the SEP.

Figure 2. Growth Rates of TFP by Sector in a Multi-Sector Growth Model



Notes: The "Equipment & Durables" sector includes software as well. "Services & Nondurables" refers to nondurables and services in personal consumption expenditures. These estimates are based on a more detailed five-sector model (Fernald, 2015) that considers land as an input. Business sector TFP grew at the same rate over the latest period as it did over the 1973:Q1–1995:Q4 period despite faster TFP growth in both the investment (Equipment & Durables) and the consumption (Services & Nondurables) sectors, because the residual portion of the business sector grew more slowly over recent years.

Figure 3. Actual versus Potential Rate of Labor Input and Contribution of Capital Deepening



Notes: Potential growth rate of labor hours equals the growth of potential labor force according to the CBO. Potential capital deepening is determined by the sectoral TFP growth rate according to a multi-sector growth model. The average rate for each subperiod is reported.

Table 1. Equilibrium Return to Capital and Policy Rate: Solow Model

	1973:Q4-2007:Q4	2008:Q1-2016:Q4	2015:Q1-2016:Q4
Return to capital	4.3	3.3	3.9
Spread: BBB - funds rate	2.7	4.9	3.9
Equilibrium policy rate	1.6	-1.6	0.0

Table 2. Sources of Change in Solow Equilibrium Return to Capital, 1973:Q4–2007:Q4 to 2008:Q1–2016:Q4

	Contributions	Average Parameter Value (%)	
		1973:Q4-2007:Q4	2008:Q1-2016:Q4
Total change in R_{EQUIL}	**-1.0**		
Contributions from:			
a (cap income share)	.8	32	38
g (TFP growth)	-.8	.88	.59
n (LF growth)	-1.6	1.7	.7
d (deprec rate)	.0	4	5
s (saving/invest rate)	.6	23	19

Table 3. Projections for the Real Equilibrium Policy Rate from the Solow Model with Alternative TFP Scenarios

	Baseline: Current TFP Trend	**Better: Prior Expansion TFP Trend**	**Optimistic: Stronger TFP Trend**
TFP growth set to average across:	**2011:Q1-2016:Q4**	**2004:Q1-2007:Q4**	**1996:Q1-2003:Q4 average and 2011:Q1-2016:Q4 average**
Equilibrium policy rate	0.0	0.5	1.9
For reference:			
Return to capital	3.1	3.6	5.1
Spread: BBB yield over fed funds rate	3.1	3.1	3.1
TFP Growth	.6	.8	1.2

Note: These projections use average parameter and variable values from 2011:Q1–2016:Q4, except for TFP growth, labor force growth, and the yield spread. For the Baseline and Better Scenarios, TFP growth is the average value for the periods shown. For the Optimistic Scenario, TFP growth is set to the average across the fast-TFP-growth era of 1996:Q1–2003:Q4 and the current expansion of 2011:Q1–2016:Q4. Labor force growth in all scenarios is the average value of trend labor force growth projected by the CBO for 2017–2027. The yield spread in all scenarios is the difference between the BBB corporate bond yield and the federal funds rate during the period 1973:Q4–2016:Q4.

Table 4. Change in Equilibrium Real Rate and Its Sources: One-Sector Neoclassical Growth Model

	1973:Q4 to 2007:Q4	2008:Q1 to 2016:Q4	Change
TFP of all sectors	0.9	0.6	-0.3
Steady-State capital deepening	0.4	0.3	-0.1
Actual Labor input	1.7	0.7	-1.0
Steady-State Consumption	3.0	1.6	-1.4
Steady-State Consumption/capita	1.9	0.8	-1.1
Variance of consumption/capita	1.9	1.6	-0.3
Implied equilibrium real rate:			
IES = 1.5, rho = 1	1.9	1.2	-0.7
IES = 1, rho = 1	2.0	1.0	-0.9
IES = 0.8, rho = 1	1.9	0.8	-1.1

Note: This table compares the aggregate and per capita consumption growth between this last cycle (since 2008:Q1, the onset of the Great Recession) with the historical average, based on the one-sector neoclassical growth model (that is, assuming all goods and services are produced by the same technology). Compared to the multi-sector model (whose estimates are presented in Table 5 below), the slowdown in aggregate TFP is greater, and in turn so is the rate of capital deepening (albeit by a small margin). Hence, the slowdown in growth is greater, resulting in larger falls in the equilibrium real policy rate.

Table 5. Change in Equilibrium Real Rate and Its Sources: Multi-Sector Neoclassical Growth Model

	1973:Q4 to 2007:Q4	2008:Q1 to 2016:Q4	Change
TFP of equipment & durables	2.8	2.4	-0.4
TFP of services & nondurables	-0.2	-0.2	0.0
Steady-State capital deepening	1.3	1.3	0.0
Actual Labor input	1.7	0.7	-1.0
Steady-State Consumption	2.2	1.3	-0.9
Steady-State Consumption/capita	1.2	0.5	-0.6
Variance of consumption/capita	1.9	1.6	-0.3
Implied equilibrium real rate:			
IES = 1.5, rho = 1	1.4	1.0	-0.4
IES = 1, rho = 1	1.2	0.7	-0.5
IES = 0.8, rho = 1	1.0	0.4	-0.6
For comparison: Actual Real Growth Rates			
GDP	3.0	2.0	-1.0
Consumption	2.8	1.7	-1.1
Consumption/capita	1.8	1.0	-0.7

Note: This table is the multi-sector counterpart to Table 4 above. It compares the aggregate and per capita consumption growth of this last cycle (since 2008:Q1, the onset of the Great Recession) to the historical average, and it shows that the slower growth rate of consumption and, in turn, the decline in the equilibrium real rate are caused by the slowdown in growth of both TFP and labor input.

Table 6. Growth Projections and the Implied Long-Run Equilibrium Real Rate with Alternative TFP Growth Scenarios

	Baseline: Current Expansion TFP Trend	**Better: Prior Expansion TFP Trend**	**Optimistic: Stronger TFP Trend**
TFP growth set to average over:	(2011:Q1-2016:Q4)	(2004:Q1-2007:Q4)	(1996:Q1-2003:Q4)*50% (2011:Q1-2016:Q4)*50%
TFP of equipment & durables	2.4	3.9	3.8
TFP of services & nondurables	-0.2	-0.3	0.0
Steady-state capital deepening	0.9	1.3	2.0
Potential labor input	0.7	0.7	0.7
Steady-state consumption	1.2	1.4	1.9
Steady-state consumption/capita	0.5	0.7	1.1
Variance of consumption/capita	1.4	1.4	2.1
Implied equilibrium real rate:			
IES = 1.5, rho = 1	1.0	1.1	1.3
IES = 1, rho = 1	0.8	1.0	1.1
IES = 0.8, rho = 1	0.5	0.8	0.7
For comparison:			
Actual consumption per capita	1.0	1.5	1.5
Steady-state GDP	1.6	1.7	2.5

Note: This table reports projections of the equilibrium real rate according to a multi-sector model under three scenarios for TFP trend growth, which are calibrated to the same three periods as in Table 3 above. For the Baseline and Better Scenarios, TFP growth is the average value for the periods shown. For the Optimistic Scenario, TFP growth is set to the average across the fast-TFP-growth era of 1996:Q1–2003:Q4 and the current expansion of 2011:Q1–2016:Q4. The variance of consumption per capita growth is assumed to take on the realized value over 2011:Q1–2016:Q4 for the first two scenarios, but take on the average of that period and the earlier period for the optimistic scenario. Labor force growth in all scenarios is the average value of trend labor force growth projected by the CBO for 2017–2027.

Table 7: Real Equilibrium Policy Rate Projections with Alternative TFP Growth Scenarios

	Baseline: Current TFP Trend	Better: Prior Expansion TFP Trend	Optimistic Scenario: Stronger TFP trend
Solow Model	0.0	0.5	1.9
Multi-Sector Neoclassical Model			
IES=1.5, rho=1	1.0	1.1	1.3
IES=1.0, rho=1	0.8	1.0	1.1
IES=0.8, rho=1	0.5	0.8	0.7

Note: Optimistic Scenario sets TFP growth equal to the average of the 1996:Q1–03:Q4 mean growth and the 2011:Q1–16:Q4 mean growth rate.