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Variable Annuities: Underlying Risks and Sensitivities

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Abstract and motivation

This paper presents a quantitative model designed to understand the sensitivity of variable annuity (VA) contracts to market and actuarial assumptions and how these sensitivities make them a potentially important source of risk to insurance companies during times of stress. VA contracts often include long dated guarantees of market performance that expose the insurer to multiple non-diversifiable risks. Our modeling framework employs a Monte Carlo simulation of asset returns and policyholder behavior to derive fair prices for variable annuities in a risk neutral framework and to estimate sensitivities of reserve requirements under a real-world probability measure. Simulated economic scenarios are applied to four hypothetical insurance company VA portfolios to assess the sensitivity of portfolio pricing and reserve levels to portfolio characteristics, modelling choices, and underlying economic assumptions. Additionally, a deterministic stress scenario, modeled on Japan beginning in the mid-90s, is used to estimate the potential impact of a severe, but plausible, economic environment on the four hypothetical portfolios. The main findings of this exercise are: (1) interactions between market risk modeling assumptions and policyholder behavior modeling assumptions can significantly impact the estimated costs of providing guarantees, (2) estimated VA prices and reserve requirements are sensitive to market price discontinuities and multiple shocks to asset prices, (3) VA prices are very sensitive to assumptions related to interest rates, asset returns, and policyholder behavior, and (4) a drawn-out period of low interest rates and asset under-performance, even if not accompanied by dramatic equity losses, is likely to result in significant losses in VA portfolios.

Keywords: insurance risk, market risk, variable annuities, derivative pricing, policyholder behavior
JEL Classification: C15, G12, G17, G22, G23

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A. Introduction

In the period leading up to the financial crisis of 2008, variable annuities (VAs) emerged as an important and high-growth offering for U.S. life insurance companies. During the 1998 to 2007 period, VA sales grew at a 6.9% annualized rate, reaching $182 Billion in 2007[2]. At the same time, the VA product was evolving from one that offered relatively straight-forward guaranteed death benefits to one that offered increasingly complex living-benefit guarantees. The rapid growth in these guarantees resulted in insurers assuming substantial, non-diversifiable exposures to equities and bond markets within their VA portfolios. After substantial losses and a contraction following the financial crisis of 2007-2008, the VA market has seen renewed growth in recent years driven by the aging of the baby boomers and strong recent performance of the equities markets. As of December 2015, the value of separate account assets associated with guaranteed variable annuities have grown to approximately $1.7 trillion.

This paper examines the sensitivity of the prices and reserves related to guarantees associated with VAs to market and actuarial assumptions and whether these guarantees may become an important source of risk to insurance companies during times of stress. We develop a quantitative model, calibrated to industry data, to examine the sensitivities of VA fair pricing and reserve requirements to various underlying risks. Our model expands upon existing literature by simultaneously incorporating, within a single Monte Carlo simulation framework, several essential aspects of financial markets and policyholder behavior that have previously been examined in isolation.

We apply our model to derive fair prices for various VA guarantees and to estimate reserve and capital requirements for firm-level proxy portfolios (derived from regulatory reported data) under simulated economic scenarios. Using these results, we evaluate the sensitivities of VA guarantee prices and reserve requirements to modeling choices related to macroeconomic scenario generation and policyholder behavior. It is important to note that our model does not incorporate a rigorous accounting of VA hedging strategy. For this reason, our estimates of absolute levels of prices and reserve requirements are unlikely to be representative of those observed in the market by firms that hedge a considerable amount of risk. However, while we do not draw conclusions about the levels of reserves associated with a portfolio of VAs, we can offer insight into how those levels are likely to be impacted by various modeling assumptions.

Additionally, a deterministic stress scenario, modeled on Japan beginning in the mid-90s, is used to estimate the potential impact of a severe, but plausible, economic environment on the four portfolios. The main findings of this exercise are: (1) VA guarantee prices and reserves are very sensitive to assumptions related to interest rates, asset returns, and policyholder behavior, (2) assumptions interact with each other to produce impacts that are highly non-linear and, (3) a drawn-out period of low interest rates and asset under-performance, even if not accompanied by dramatic equity losses, results in significant losses in VA portfolios.

The remainder of this paper will proceed in the following manner. Section B provides an overview of the structure of VA contracts and their features that we incorporate into our model. Section C provides a description of the types of risks inherent to insurance firms' VA portfolios that we attempt to assess within

[2] U.S. Individual Annuities Sales Survey, LIMRA

our model. In section D we provide a brief overview of the academic literature related to the pricing of VA contracts and guarantees and the impact of modeling choices related to financial market features and policyholder behavior. We present our modeling framework in section E. Section F presents results of sensitivity testing of prices and reserves related to various contract, financial market, and policyholder feature assumptions. The application of the model to four hypothetical VA portfolios derived from statutory filings at individual firms is presented in section G under a large number of simulated economic scenarios. Finally in section H we consider the impact of a deterministic economic scenario modeled after Japan in the 1994-2017 period and conclude in section I.

B. Overview of Variable Annuities

Before we present our model, we describe the characteristics and types of VAs offered. This will allow us to better highlight the features of VAs that we are modeling.

Variable annuities are long-term insurance contracts, used by individuals for saving and guaranteed income purposes. Initial premiums are paid up-front by the policyholder who then receives contractually guaranteed payments from the insurer in the future. The underlying assets of a VA (i.e. the premium paid by the policyholder less origination fees) are invested in mutual funds. The policyholder typically pays fees, in addition to the initial premium amount, related to optional guarantees embedded into the VA, administrative fees, and investment management fees. The policyholder may also be charged additional fees for exiting the contract before a pre-established holding period elapses.

Variable annuities have a lifecycle consisting of three phases: an accumulation phase, a withdrawal phase, and an insured phase. During the accumulation phase, the account value is allocated across, and its value varies with, investment options provided by the insurer. The policyholder determines the fund allocation across the investment options (subject to restrictions placed on the investments by contract).

In the withdrawal phase guaranteed cash flows are paid out to the policyholder. The withdrawal rates are generally established at the contract inception and are guaranteed by the insurer. The payments may be a lump sum or a recurring percentage of the benefit base. The benefits base, which is discussed in more detail below, is determined by the performance of the underlying investments during the accumulation phase and any additional guarantees that the customer may have purchased to reduce the investment risk associated with the underlying account during the accumulation period.

Cash flows during the withdrawal phase are first paid out of the underlying account value of the VA (and thus reduce the underlying account value). If the underlying account value is insufficient to support the guaranteed payments (generally owing to poor investment returns experienced during the accumulation phase), then the insurer is responsible for covering any shortfalls. If at some point the insurer becomes responsible for making guaranteed payments to a policyholder (i.e. the underlying account value is insufficient to support the payments), then the VA is said to have entered the insured phase.

Payment Guarantees:

Variable annuities are sold with various types of guarantees. These guarantees provide the policyholders with assurance that they will receive certain future cash flows. These cash flows may be either recurring or lump sum payments. There are four main types of guarantees (death, income, withdrawal, and accumulation) that insurers sell with VAs.

GMDB riders provide guaranteed certain lump-sum payment at the time of the policyholder's death, regardless of the account value. Guaranteed Minimum Income Benefit (GMIB) riders provides the account holder the right to convert the underlying account value into a fixed annuity that provides a lifetime fixed payment. The annuity payment amount will be equal to a percentage, established at the time of the initial VA sale, of the benefit base. The timing of the conversion is determined by the policyholder, assuming that an initial waiting period has elapsed.

Guaranteed Minimum Accumulation Benefit (GMAB) riders guarantee the policyholder a certain lump sum payment amount on a certain date, regardless of the investment performance of the underlying account.

GMWB and Guaranteed Lifetime Withdrawal Benefit (GLWB) riders provide assurance that a policyholder will be able to make withdrawals that are less than or equal to a fixed percentage of the benefits base for a fixed number of years. For example, the policyholder may be guaranteed the right to withdraw up to 5 percent of the benefit base annually for 20 years, even if the payment amount exceeds the account value at any point. In this example, the policyholder is guaranteed withdrawals that total, at a minimum, the original amount of the benefit base, regardless of the performance of the underlying account. The GLWB extends the withdrawal period to encompass the policyholder's lifetime, rather than a fixed interval.

Benefit Base Contract Provisions:

Benefit payments are calculated in reference to a "benefit base." The benefit base is an amount that is related to, but separate from, the underlying account value. Differences between the benefit base and the account value arise when the policyholder elects (in exchange for paying additional fees) to receive protection against poor performance of the investment funds selected for the underlying assets.

Return of principle (RoP) guarantees ensure that the policyholder will not incur any losses below the original account value (adjusted for fees and withdrawals) due to underperformance of the underlying funds that the account is invested in.

Roll-ups are a contract provisions that protect the policyholder from underperformance of the underlying account value relative to a baseline growth rate. When a roll-up provision is in place, the benefit base will be the greater of the actual account value (reflective of investment returns) and a value derived from applying a pre-specified minimum rate to the original account value. A roll-up guarantee, therefore, establishes a minimum growth rate for the benefit base, but does not limit upside potential for the policyholder.

A ratchet is a contract feature that establishes the benefit base as the maximum value of the underlying asset account value in a specified period. Ratchets are evaluated at pre-established intervals during the accumulation period (annually, quarterly, or monthly) and offer protection against short-term downward movement in the underlying account value. For example, an annual ratchet would set the benefit base to the highest between account values observed at prior year, the current account value, and the past values of the benefit base that have been set by the application of the ratchet in prior periods. In the case that the account values were to fall over the course of the upcoming year, the benefit base would remain at the "high-water" mark observed at the ratchet reset date or "anniversary".

The insurer offers these contract features to customers purchasing VAs with all types of cash flow guarantees, in exchange for an additional rider charge that is typically established as percentage of the account value. A particular policy may include multiple benefit base guarantees simultaneously (e.g. a roll-up and a ratchet), if the policyholder elects to purchase multiple guarantees.

A final contract feature of VAs is the right for the policyholder to withdraw more than the guaranteed withdrawal amount or to withdraw the value of the underlying account and terminate the policy. By withdrawing excess funds from the account, the policyholder relinquishes the guarantees that were included in the VA contract for the withdrawn amount. The embedded guarantees are primarily designed to protect the policyholder from downside moves in the account value. Thus, lapsing may be rational for the policyholder if the account value has increased significantly, and is therefore far above the guaranteed value. In this case, the policyholder may benefit from forfeiting the now relatively low-value (or a portion of it) guarantees and removing the underlying account value for other uses (e.g. purchasing a new VA that offers downside protection relative to the new, higher account value). Typically, surrender fees are waived if the policy has been in force for some prespecified period of time. If a policyholder surrenders, all guarantees associated with the contract are relinquished.

C. Main Underlying Risks

In this section, we discuss the main type of risks that are present for insurance companies with VA contract liabilities. The risks described here, are the exposures that our model attempts to quantify under various economic conditions. At the end of the section, we explain why we refrain from modelling the cost to hedge these contracts.

Market Risk:

Market risk is the risk of adverse price movement that impacts guaranteed payments or required reserve levels due to changes in market factors, and it includes interest rate risk, equity market risk, foreign exchange risk and credit risk.

When pricing and reserving for variable annuity products, insurers make assumptions about the performance of several capital market factors. Market risk exists to the extent that capital markets perform in different way than assumed in pricing and reserving calculations.

The main market risks that variable annuities involve are equity and interest rate risks. Declines in policyholder account values due to equity price declines or shifts in the interest rate environment increase the exposure of VA issuer to the risk that the account value may be insufficient to cover the level of guarantees promised by the contract which increases expected living benefit future claims. Further, declining interest rates increases the present value of the long-term income provided by the contract guarantees. A change in capital markets can result in increases or decreases in the required reserves and the capital that a VA issuer has to set aside to meet its commitments to policyholders and regulatory requirements. If market risk is managed inappropriately, it can lead to significant balance-sheet volatility and solvency risk for VA issuers. In the current framework, we focus on equity and interest rate risks as these are the main market risks.

Insurance risk:

Longevity risk reflects the risk that mortality assumptions are not as expected and policyholders live longer than expected resulting in increased duration of guarantee payments associated with living benefits. This risk impacts the living benefits (GMWB, GMIB, GMAB and GLWB).

Mortality assumptions are derived from actuarial studies. Such studies involve long periods of time and large population which creates a high degree of precision and confidence in their estimate.

However, longevity risk is still present as holders of these insurance products may not exactly match the overall population expectations, due to many different factors, resulting in different mortality experiences within a given VA portfolio population than the population used to derive the tables. Causal factors may include population health improvements, medical enhancements, and different demographic profile. There may also be adverse selection and moral hazard aspects related to insurance risk. Customers who buy individual annuities may tend to be those that are expected to live longer than average. On the other hand, customers with shorter life expectancy would find a VA based on average mortality to be unfavorably priced.

By contrast, living benefit contract holders might change their habits and live longer than previously expected by the insurer. Such changes in policyholder behavior after VA issuances is purchased is referred to as moral hazard.

To manage longevity risks, insurers design products with age restrictions on both receiving guaranteed benefits and on income commencement. Other tools available to insurers are risk pooling and product diversification where large pools of contracts are likely to behave in terms of mortality, on average as expected. A mix of guaranteed death and living benefit contracts is a natural hedge against mortality and longevity risk.

Policyholder Risk:

Policyholder risk is the risk that policyholder behavior in terms of benefit utilization and contract surrender doesn't align with insurer's expectation or past experience resulting in unexpectedly high or low utilization of benefits. Insurers make assumptions regarding the frequency and magnitude of contract lapse rates and benefit utilization. For example, insurers can assume a static lapse rate of 5% annually and 90% of benefit utilization (90% of the withdrawal in the case of guaranteed withdrawals). Assumptions made by VA issuers vary based on the types of guarantees offered, as well as internal and external policyholder behavior studies. Such studies[3], generally based on limited data, show that surrender rate and utilization are becoming increasingly dynamic. Several factors contribute to shaping this dynamic

[3] SOA/LIMRA, Variable Annuity Guaranteed Living Benefits Utilization, 2013

behavior, including policyholder characteristics (age, gender), contract size (retail vs institutional), and the moneyness of the guaranteed benefit. The dynamic nature of these assumptions is generally linked to the gap between the guaranteed benefit amount and the VA account value. For instance, a large difference between these two values where the guaranteed amount exceeds the account value can lead to low lapse rates. Rational holder's behavior suggests keeping risk with the insurer when market is declining (lower lapse rates) and seek better opportunities when market is rising (higher lapse rates).

As for insurance risk, product design can also help mitigate risks arising from policyholder behavior. Insurers can design VA contracts that penalize lapses within the first 5 or 7 years of contract life. Typically insurers will apply penalties on early surrenders during this window. Consequently, policyholders will modify their surrender behavior to avoid penalties. Lapsing penalties are effective at reducing the rate of lapses during the early years of contracts, but do not offset the dynamic lapsing effect with relation to contract moneyness. Pricing is another component of product design where VA issuers may adopt prudent assumptions with regards to policyholder behavior and in order to avoid unexpected changing behavior.

As will be shown later in this paper, failure to model moneyness-based dynamic lapsing poses a serious risk to VA providers; as dynamic lapsing exacerbates the impact on VA price (reserve or capital) or market risk shocks. This illustrates that market and policyholder risks are multiplicative, rather than additive. Furthermore, there is an increasing awareness, in policy behavior studies, of the moneyness-based utilization of VA benefits. Such changing behavior from static to dynamic utilization will likely to expose VA providers to further risks and increase the cost of guaranteed benefits, reserves and capital.

Hedge Risks:

Hedge risk is the risk that hedging strategies do not perform as intended, creating basis risk, or the risk that a hedge provider defaults on its obligation, creating counterparty risk. VA issuers rely on hedging strategies to neutralize (or reduce) residual risks that are inherent to the product design or emerging due to changes in underlying assumptions. Hedging strategies are also used to mitigate the required levels of reserve and capital. There are several aspects characterizing hedging strategies such as the risks that need to be hedged, which include sensitivity to changes in the underlying portfolio price (Delta), sensitivity to changes in interest rate (Rho) or sensitivity to guarantee value to changes in the implied volatility of the underlying asset (Vega). Hedging strategies also include cross Greek sensitivities and sensitivity to convexity risk (variation of Delta and Rho). Hedging strategies are based on a variety of instruments where equity risk is managed mainly via equity future contracts while interest rate risk is managed primarily using interest rate futures contracts. Volatility is generally hedged using equity put options and swaptions. Due to the long duration of VA contracts, hedging strategies require frequent rebalancing in order to maintain hedging efficiency at targeted levels. The cost of hedging is in general included in guarantee prices in cases where hedging is part of product design.

Continuous rebalancing associated with dynamic hedging results in high costs. A study by PIMCO[4] on the cost of delta hedge from December 2005 to June 2011 show hedge cost ranging from a few basis points (prior to 2008) and near 200 basis points during 2008 financial crisis.

[4] PIMCO, Viewpoints November 2011

In this paper, we do not model the cost of hedging VA guarantees. There are several difficulties inherent to modeling the impact of hedging. Simulating the impact of a hedging strategy on cash flows would require us to make assumptions about the hedge objective, the guarantees that firms are hedging, and firms' risk tolerances. In addition, we would need to assume whether firms were employing static hedges, dynamic hedges, or partial hedges. Given the wide variety of approaches and risk tolerances that could reasonably be adopted, the large number of assumptions that would need to be made, and the significant impact on results, we opt rather to focus on evaluating sensitivities in prices, and reserve requirements rather than attempting to establish these values net of the effect of hedging.

Given that our model quantifies VA price under risk neutral measure, we expect the cost of lifetime hedges to be equivalent to VA fair value produced by our model. A hedge cost index produced by Milliman[5] confirms this fact and shows a cost for hedging a hypothetical GMWB spanning between 100 and 172bps during September 2014 to September 2016 period. These values are equivalent to the 150bps VA price produced by our model for scenario 1.

D. Literature review

Our paper extends the existing literature in several dimensions: (a) our model is designed to price contracts that include various combinations of guarantees and features whereas the literature has largely focused on pricing individual guarantees, (b) our model includes several features of realistic financial markets that have not been incorporated in much of the prior work on VA pricing, (c) we incorporate several important features of policyholder behavior that have been shown to play an important role in accurately modeling contract cash flows and, (d) we apply our model to proxy portfolios developed using industry data. This allows us to evaluate the interaction between market risk factors, insurance factors, and various combinations of contract features simultaneously, rather than in isolation. In addition, we apply these results to evaluating the expected impact on industry-representative portfolios, rather than on individual VAs in isolation.

While our model is implemented using a Monte Carlo simulation framework, it is related to the early literature on VA pricing that focused primarily on deriving closed-form solutions for the pricing and valuation of guarantees. Closed-form pricing models are generally limited to pricing specific types of guarantees independently, as combinations of guarantees do not generally admit exact closed-form solutions. Aase and Persson (1994) applied tools typically used for pricing financial products, such as no-arbitrage pricing and martingale theory, to derive the optimal pricing for traditional life insurance products.

Milevsky and Salisbury (2001, 2002) were early examples of the application of closed-form pricing of guarantees associated with variable annuities. The authors employed option pricing theory to value Guaranteed Minimum Death Benefit (GMDB) and Guaranteed Minimum Withdrawal Benefit (GMWB) guarantees using a static approach that assumed policies did not lapse and withdrawals are made exactly as specified in riders. Milevsky and Salisbury (2006) expanded this framework to incorporate Guaranteed

[5] The Milliman Hedge Cost Index

Lifetime Withdrawal Benefit (GLWB) and dynamic withdrawal behavior. Their results generally show that guarantees were, at the time, underpriced in the market. However, approaches that rely on closed-form pricing solutions are limited because of the complexity of contracts that can be priced and must generally assume relatively simple models related to behavior of the underlying assets.

Several papers have provided analytical pricing formulas for specific individual guarantee types under stochastic interest rates and stochastic mortality. A relevant example is Krayzler et al (2012) for GMDB guarantees that incorporate various riders such as roll-ups and ratchets, though Krayzler and coauthors only model a single asset.

Our work extends the literature that discusses the pricing of various types of guarantees together. Sun (2006) studied the impact of combining multiple products while incorporating dynamic policyholder behavior related to lapsing, the “annuitization” decision, and withdrawal rates utilizing a set of pre-defined economic scenarios. Sun shows that incorporating policyholder behavior that is conditional on the economic scenario can have a dramatic effect on pricing. However, Sun’s approach does not result in prices that are consistent with market expectations related to asset behavior.

Bauer et al (2008) developed a framework that could consistently price contracts that include multiple guarantees using a stochastic economic scenario generator. Their model uses a Monte Carlo framework to simulate asset returns using Geometric Brownian Motion and imposes a deterministic mortality model while assuming that policyholders follow an optimal strategy related to surrender and withdrawal. While able to price multiple guarantees simultaneously, their approach relied on relatively simple models for asset returns and policyholder behavior.

Several authors have worked to expand this general framework to incorporate various riders, dynamic policyholder behavior, and multiple assets. Bacinello et al (2011) developed a dynamic approach to policyholder behavior. Their model incorporates stochastic interest rates as well as stochastic mortality into a Monte Carlo framework to price VAs with multiple guarantees simultaneously. The authors derive pricing under a solution for optimal withdrawal strategy for rational policyholder. Holz et al (2007) applied a framework similar to the one employed in Bauer (2006) to GLWBs while incorporating roll-ups and ratchets. Kling et al. (2010) further expand on this framework by applying stochastic rates and volatilities rather than log-normal returns. Chen et al. (2009) incorporates asset jumps and dynamic, but sub-optimal, policyholder behavior in order to extend research to better reflect the observed behavior of both asset returns and lapsing experiences.

While the literature has increasingly moved towards simultaneously modeling multiple guarantees, dynamic policyholder behavior, and realistic asset returns, our model contributes to the understanding of the risks inherent in VAs to firms by incorporating each of these modeling features and applying them to representative firm portfolios derived using data included in regulatory reporting. This allows us to examine the sensitivity of firms’ VA portfolios to various risk factors and to estimate the potential impact of various counterfactual economic scenarios on the performance of these portfolios.

E. Model framework

In order to illustrate our framework, we present in more detail a formal model for pricing a GMWB with T withdrawals. We introduce first the following notations. We denote as AV_t the value of assets backing the VA at time t and we call it the account value from now on, as P_t the insurer's payment to the policyholder at time t, as G_t the amount of guaranteed withdrawal accruing to the policyholder at time t, as α_g the guarantee fee paid by the policyholder, and as α_m the management fee (in % of AV_t) accruing to the manager of the assets backing the VA.

The GMWB's price is then a guarantee fee α_g such that the expected present value under the risk neutral measure of all future payments P_t made by the insurer net of the guarantee payments made by the policyholder is 0,

$$E\left(\sum_1^T PV\left[P_t \ - \alpha_g(1-\alpha_m)AV_{t-1})\right]\right) = 0 \qquad \text{Equation 1,}$$

where the guarantee payment at time t is the α_g% of the account value left after the payment of the management fee.

Formally, the insurer's payment P_t can be expressed as the payoff of an European option:

$$P_t = Max\left(G_t - \left(AV_{t-1} - \alpha_m AV_{t-1} - \alpha_g(1-\alpha_m)AV_{t-1}\right), 0\right)$$

$$= Max\left(G_t + \alpha_m AV_{t-1} + \alpha_g(1-\alpha_m)AV_{t-1} - AV_{t-1}, 0\right)$$

The reason is that the insurer's payment depends on the account value in each period t. If AV_{t-1} is sufficient to cover the guaranteed withdrawal (G_t), the management fee ($\alpha_m AV_{t-1}$) and the annual guarantee fee ($\alpha_g(1-\alpha_m)AV_{t-1}$)), then the insurer's payment is 0. Otherwise, in order to be able to honor the guaranteed withdrawal, the insurer has to pay the difference between the withdrawal and what is left of the account value after the fee payments. As we can see from the last equation, the chance that the insurer has to make a positive payment is higher, the more generous guarantee G_t and/or the lower account value AV_{t-1} is. The above option is assumed to be fully or partially exercised. Scenarios of partial option exercise (benefit utilization) are presented in the sensitivity analysis section.

In order to incorporate risks and contract features described in previous sections into the above framework as well as perform quantitative analysis, we make several assumptions. First, we model the path of the account value AV_t using a risk-neutral simulation framework that relies on a geometric Brownian motion with jump-diffusion processes for equities and bonds. Formally, we have

$$dS_t = \left(\mu_s - \lambda_s\, \upsilon_s \right) S_t \mathrm{dt} + \sigma_s\, S_t \mathrm{d}Z_{st} + \ S_t \mathrm{d}J_{st} \qquad \text{Equation 2}$$

for equities and

$$dB_t = \left(\mu_b - \lambda_b\, \upsilon_b \right) B_t \mathrm{dt} + \sigma_b\, B_t \mathrm{d}Z_{bt} + \ B_t \mathrm{d}J_{bt} \qquad \text{Equation 3}$$

for bonds, where S_t and B_t are values of equities and bonds indices at time t, μ_s and μ_b are drifts, σ_s and σ_b are volatilities, $\mathrm{d}Z_{st}$ and $\mathrm{d}Z_{bt}$ are correlated Brownian motion processes, and $\mathrm{d}J_{st}$ and $\mathrm{d}J_{bt}$ are stochastic jump processes. Drifts have the starting risk free rates of 0.7% as of June 2016. The bond portfolio is assumed to be actively managed and follow an index which supports the use of GBM process. The probability of having at least one jump is assumed to be 1% each month corresponding to one in eight years event. Historically, market price jumps occurred, on average, with one in seven years frequency. Our model framework assumes frequency of jumps to follow a Poisson distribution with intensity λ.

$$\Pr(\mathrm{X}=\mathrm{k})=\frac{\lambda^k e^{-\lambda}}{k!}$$ Equation 4

The jump size is assumed to follow a lognormal distribution calibrated to the 1% most severe monthly historical drops of equity and bond indices. Jump size is drawn from lognormal distributions (μ_{sj},σ_{sj}) and (μ_{bj},σ_{bj}) for equity and bond respectively. $\mu_s - \lambda_s\, v_s$ and $\mu_b - \lambda_b\, v_b$ are the jump-adjusted drifts for equities and bonds where:

$$v_s = \exp(\mu_{sj} + 0.5\, \sigma_{sj}{}^2) \quad \text{and} \quad v_b = \exp(\mu_{bj} + 0.5\, \sigma_{bj}{}^2)$$

In our baseline calculation we assume a typical 60%/40% distribution between equity and bonds, implying AV_t =0.6.S_t + 0.4 B_t. However, our framework allows for varying this asset allocation (such as the 80%/20% allocation presented in the sensitivity analysis section).Next we assume that interest rates evolve according to a Vasicek[6] model:

$$dr_t = \mathrm{a}(b - r_t)\mathrm{dt} + \sigma_r\, dZ_{rt}$$ Equation 5

where Z_{rt} is a Brownian motion process under the risk-neutral framework and is correlated with equity and bond processes (Z_{st} and Z_{bt}), b is the long-term mean level of risk free rate, a is the speed at which interest rate path reverts to level b, and σ_r is the instantaneous interest rate volatility. Calibration of the Vasicek model results in an estimated long-run equilibrium rate of 1.63%, volatility of 14.2% and a reversion speed of 0.99%.

We calibrate the equity process using monthly data on the S&P500 from 1999 to 2016, the bond process using Merrill Corporate Bond Total Return Indices from 1999 to 2016, and the interest rate process using 3-months treasury rates from 1982 to 2016. The calibration period spans back to the 80s in order to reflect life benefit's long guaranteed period (30 years or higher) and capture the levels of interest rates observed in the early 80s. Moreover, we allow for correlations between equity, bond and interest rate processes. We use the Cholesky transformation to generate correlated random processes with correlations calibrated to 1999-2016 data period.

The rest of the assumptions cover those specific for the insurance contracts. First, we model mortality/longevity risk in a deterministic fashion[7] based on the most recent life tables by age, gender and date of birth. However, we assessed the potential impact of longevity risk by conducting sensitivity

[6] *Vasicek, Oldrich (1977). "An Equilibrium Characterisation of the Term Structure". Journal of Financial Economics*
[7] We abstract from stochastic mortality since E Marceau, P-A Veilleux, GMWB Guarantee: Hedge Efficiency & Longevity Analysis, 2015 and a study led by Milliman demonstrated that it has limited impact on VA prices.

analyses on VA prices in response to changes in mortality rates and found limited impact. In addition, we abstract from any correlation between mortality/longevity and financial risks.

To assess the impact of policyholder behavior, we allow for static and dynamic lapsing of contracts. As mentioned in section B, VA contracts allow policyholders to withdraw a percentage of the account value in excess of guaranteed withdrawal. Under static lapsing, we assume that this percentage is fixed at 5%. Under dynamic lapsing we assume that this percentage is a function of the VA contract's moneyness (ITM_t) and other policyholder's characteristics. ITM_t is the in-the-money indicator at time t and is defined as the ratio of the account value over the guaranteed annual withdrawal,

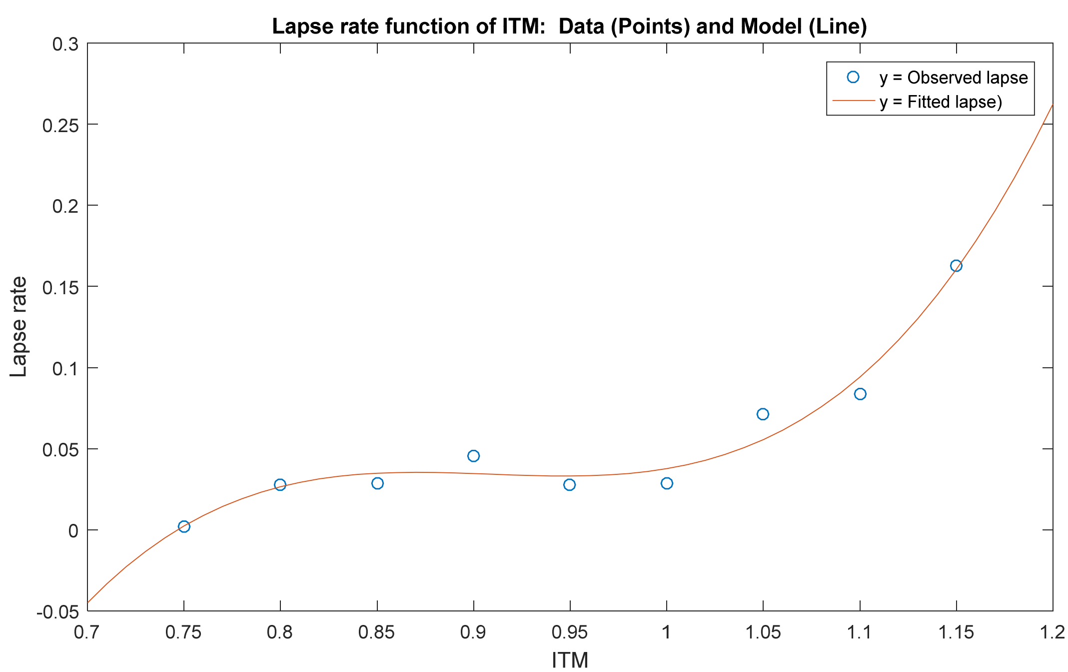
$$\mathrm{ITM_t} = \frac{\mathrm{AV_t}}{\mathrm{G_t}} \qquad \text{Equation 6}$$

A variable annuity contract is considered in the money if the present value of future guaranteed benefits (G_t) is below the account value (AV_t) of the underlying asset at time t ($\mathrm{ITM_t}$ =>1). To calibrate policyholder behavior we use data from Milliman's dynamic lapse study.[8] We then use a cubic polynomial function to regress lapse rate on the "in-the-moneyness" indicator ITM_t,

$$\mathbf{Lapse\ rate_t} = \boldsymbol{\alpha_0} + \boldsymbol{\alpha_1}\mathbf{ITM_t} + \boldsymbol{\alpha_2}\mathbf{ITM_t}^2 + \boldsymbol{\alpha_3}\mathbf{ITM_t}^3 \qquad \text{Equation 7}$$

Figure 1 shows calibration results of observed versus fitted lapse rates as a function of ITM.

Figure 1: Lapse Rate Calibration



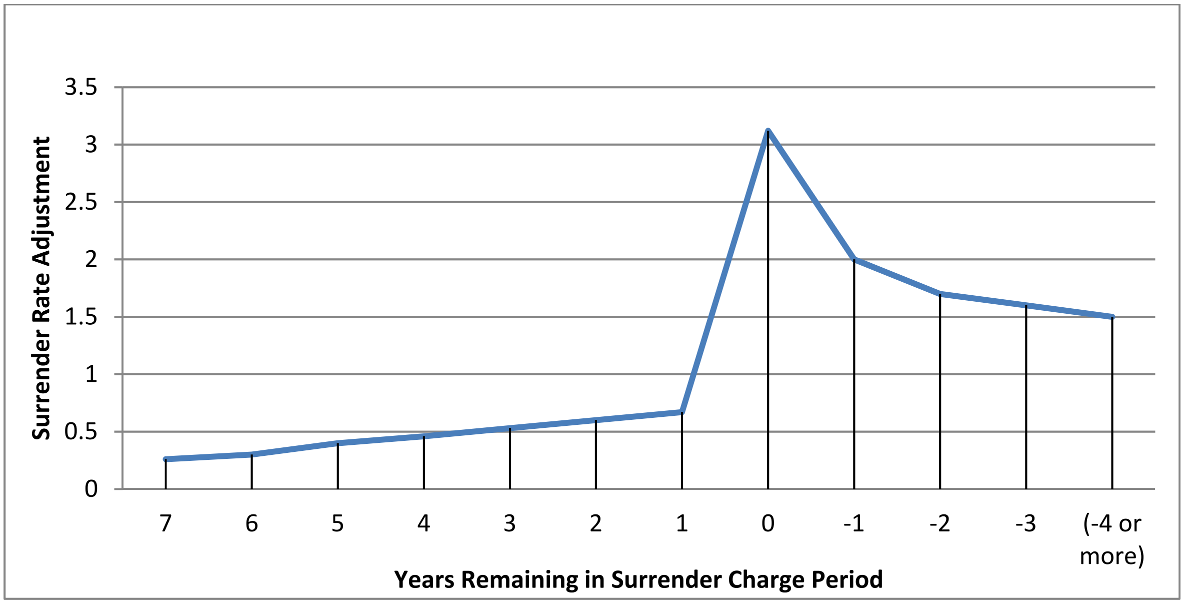
Second, we assume the following pattern of lapsing penalties for the first seven years of the contract: 7%, 6%, 5%, 4%, 3%, 2% and 1% of the account value. In addition to moneyness calibration, the framework

[8] Milliman, Variable annuity dynamic lapse study: A data mining approach, June 2011.

adjusts lapse rates to the levels observed before and after the surrender penalty period (9). Figure 2 shows the lapse rate adjustment reflecting policyholder behavior of relatively lower lapsing during the penalty period (first 7 years). To reflect this policyholder behavior before and after the penalty period, we adjust lapse rate from Equation 7 with the surrender rate adjustment (Equation 8).

$$Final\ Lapse\ Rate_t = Surrender\ Rate\ Adjustment_t \times Lapse\ Rate_t \qquad \text{Equation 8}$$

Figure 2: VA Surrender Experience by Years Remaining to Maturity



Finally, our model allows using ratchets and roll-ups as performance guarantees of the benefit base. The benefit base which is the sum of all guaranteed withdrawals (including the death benefit) is guaranteed to grow regardless of market performance. The Ratchet feature guarantees the greater of a return of benefit base or the highest "anniversary" account value adjusted for withdrawals. The "anniversary" decides the frequency of the ratchet (monthly, semiannually, annually, etc.). An annual ratchet will recalculate the base benefit once a year by taking the maximum of previous benefit base and the account value at the end of a chosen month of the year. The rollup feature guarantees a minimum return of the account value where the benefit base at t will be equal to the maximum of $A(t)$ and $(1 + .05) * A(t - 1)$ if we assume 5% rollup rate.

F. Model Results and Price sensitivity: Application to Standard GMWB Contract

In this section we first present the results from pricing of a VA contract with a GMWB using the framework described in the previous section. We price the GMWB using a set of parameters reflecting average contract features and financial conditions. Next, we analyze the sensitivity of the GMWB price to changes in these assumed parameters.

[9] Society Of Actuaries, What is the Market Price of Policyholder Behavior? May 2015.

1) Results

Table 1 summarizes all contract features, financial conditions and assumptions on policyholder behavior we use to price the GMWB in the baseline scenario 1. Parameters on contract features reflect average contract features described in the literature[10]. Financial inputs are obtained using calibration approaches described in the previous section.

Table 1: Model Inputs

	Model Input	Scenario 1	Source
	Policyholder age	55	Assumed contract feature
Insurance Inputs	Contract maturity	15	Assumed contract feature
	Accumulation period	5	Assumed contract feature
	Acquisition fee	5%	Assumed contract feature
	Management fee	1%	Contract feature/Assumed based on literature
	Fee base	Account value	Assumed contract feature
	Death guarantee	N	Assumed contract feature
	Roll-up guarantee	N	Assumed contract feature
	Guaranteed roll-up rate	2%	Contract feature/Assumed based on literature
	Ratchet at accumulation	N	Assumed contract feature
	Frequency of ratchet at accumulation	once a year	Assumed contract feature
	Ratchet at distribution	N	Assumed contract feature
	Frequency of ratchet at distribution	once a year	Assumed contract feature
	Static lapse rate	5%	Contract feature/Assumed based on literature
	Dynamic lapsing	N	Assumed contract feature
Financial Inputs	Equity vs bond weight	60/40	Assumed contract feature
	Equity volatility	13.4%	Calibrated
	Bond volatility	4.5%	Calibrated
	Correlation equity/bond	15%	Calibrated
	Correlation equity/rate	-10.40%	Calibrated
	Correlation bond/rate	-0.15%	Calibrated
	Starting short rate	0.70%	Observed
	Interest rate model	Vasicek (14.2%, 1.63%, 0.99%)	Calibrated
	Jump frequency-monthly	1%	Calibrated/Assumed
	Jump size equity	lognormal(-0.156,0.044)	Calibrated
	Jump size bond	lognormal(-0.062,0.031)	Calibrated

We assume constant correlation between stock and bond indices. This is a limitation to the model framework since correlation can vary over time and be market state dependent. US stock-bond yield correlations have been consistently positive since the late 90s, increased significantly during 2008 financial crisis and stayed at a high level for a long period thereafter. We perform, however, sensitivity on

[10] Variable Annuity Guaranteed Living Benefit Utilization, SOA/LIMRA, 2013

the assumed correlation by testing a constant negative correlation of -15% between stock and bond. This sensitivity test shows limited impact on guarantee price from change of correlation.

The fair price of the GMWB is a guarantee fee for which the contract's expected PV given by equation 1 under scenario 1 is equal to 0. To obtain the contract's expected PV we simulate 10000 times the path of market conditions and policyholder behavior based on a given realization of stochastic processes of equity, bond, interest rate and jumps over the duration of the contract. Figure 3 shows the distribution of contract's present values generated from 10000 simulated paths of scenario 1 for a guarantee fee set at 150bps. Here a guarantee fee of 150bps is chosen because it is the fee that, when charged, results in an expected PV of 0 for the guarantee.

Figure 3: Contract Present Value Distribution for Guarantee Fee of 150 bps.

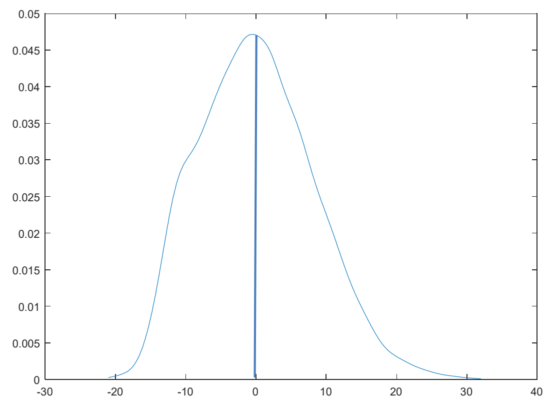


Figure 4: Typical GMWB Valuation: Average Path.

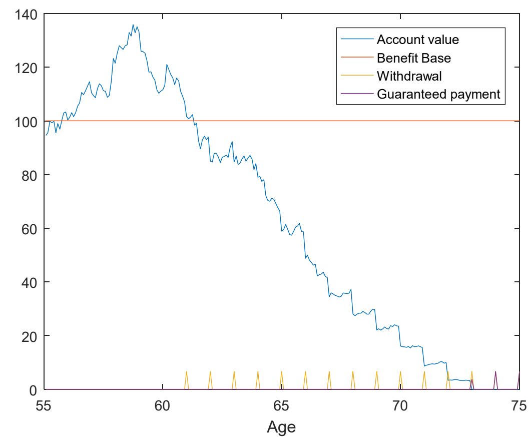


Figure 5: Typical GMWB Valuation: Good Path

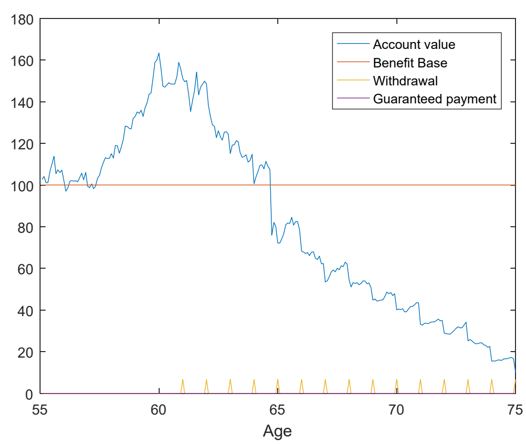


Figure 6: Typical GMWB Valuation: Bad Path

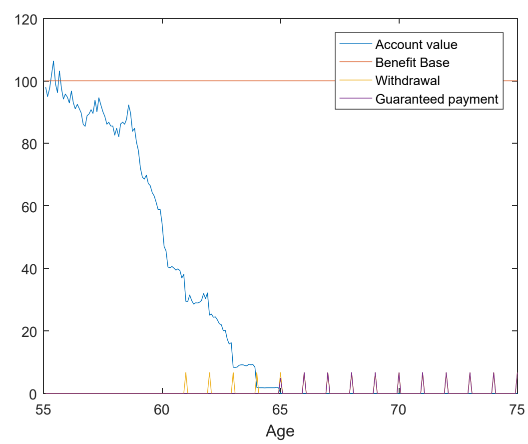


Figure 4-6 present three cases for patterns of payments to the policyholders that are covered by the account value (yellow) and the out-of-pocket payments that the insurer has to make in case the account value is not enough (purple). The patterns of payments differ across the pictures depending on the realization of the simulated paths of account values for benefit base equal to the initial value of the account (100). Because the annual withdrawal is calculated as a fraction of the benefit base and the withdrawal duration is 15 years, the annual withdrawal is around 6.7% of the benefit base.

Figure 4 depicts the case in which future market conditions and policyholder behavior are such that the contract's PV is zero. In this case the insurance company makes only three out-of-pocket payments at the end of contract's life at the age of 73 after 13 payments that were fully covered by the account value. Figure 5 presents the case where the performance of the underlying assets cover all guaranteed withdrawal benefits without a need of out-of-pocket payments. The contract's present value is positive and insurers make profit on this contract. In contrast, Figure 6 presents a poor performance of the underlying assets and early out-of-pocket guarantee payments. Benefit payments are paid by the insurer due to insufficient account value for covering the guaranteed amount for 12 out of 15 total guaranteed withdrawals.

2) Sensitivity Analysis

In this section we present sensitivity analysis of guarantee price to varying insurance and financial inputs. We use scenario 1 shown in Table 1 as a base case for the sensitivity analysis.

We first analyze the sensitivity of guarantee price to insurance factors. The results are in Figure 7. Incorporating dynamic lapsing behavior in the model has the largest impact on the price of a GMWB contract. Incorporating more realistic modeling of policyholder behavior in the form of dynamic lapsing (calibrated to observed lapsing behavior) results in break-even guarantee costs that are around 280bps higher than those calculated using an assumed static lapsing rate of 5%. Adding roll-ups as a contract feature has a significant impact with an incremental cost around 190 bps. Adding a death guarantee to a GMWB increases the guarantee cost by around 100bps. Except the sensitivity tests related to initial value based fee, longer maturity and lower utilization, other insurance features have increasing but limited impact on the guarantee price. Longer maturity lowers the likelihood of triggering withdrawal guarantee, because it decreases the maximum annual withdrawal $\left(\frac{1}{T}\right)$. Lower policyholder age (50 instead of 55) will increase contract exposure to market risk given that relatively more policyholders will survive until maturity and increases contract price by 30 bps. Assuming lower benefit utilization (from 100% to 70%) will decrease guarantee price by around 130 bps as it lowers the likelihood of out-of-pocket payments. Insurance companies generally assume that utilization is static and is lower than the guaranteed benefit. However a SOA/LIMRA recent study[11] shows that the assumption of static utilization is increasingly less realistic and that increasingly utilization is a function of contract's moneyness. We also believe, as shown with dynamic lapsing, that modeling utilization dynamically as a function of contract's moneyness will further increases the cost of the guarantee. Public data on levels of VA guarantee utilization is limited and doesn't allow further modelling.

Similarly, calculating guarantee fee as a percentage of initial account value as opposed to a percentage of current account value decreases guarantee price by up to 50 bps. Using initial account value as fee's base lowers issuer's inflows in market upturn but also lowers issuer's outflows (out-of-pocket payments) in market downturn. Initial account value based fees constitute a natural hedge against low returns on the underlying assets. The fee is flat over the duration of the contract. An increase in the management fee

[11] SOA/LIMRA, Variable Annuity Guaranteed Living Benefits Utilization, 2013

will result in an equal increase in the price given that the fee reduces the issuer's inflows in all market conditions by the amount of the fee.

Figure 7: Sensitivity of Guarantee Price to Insurance Factors (% of Account Value)

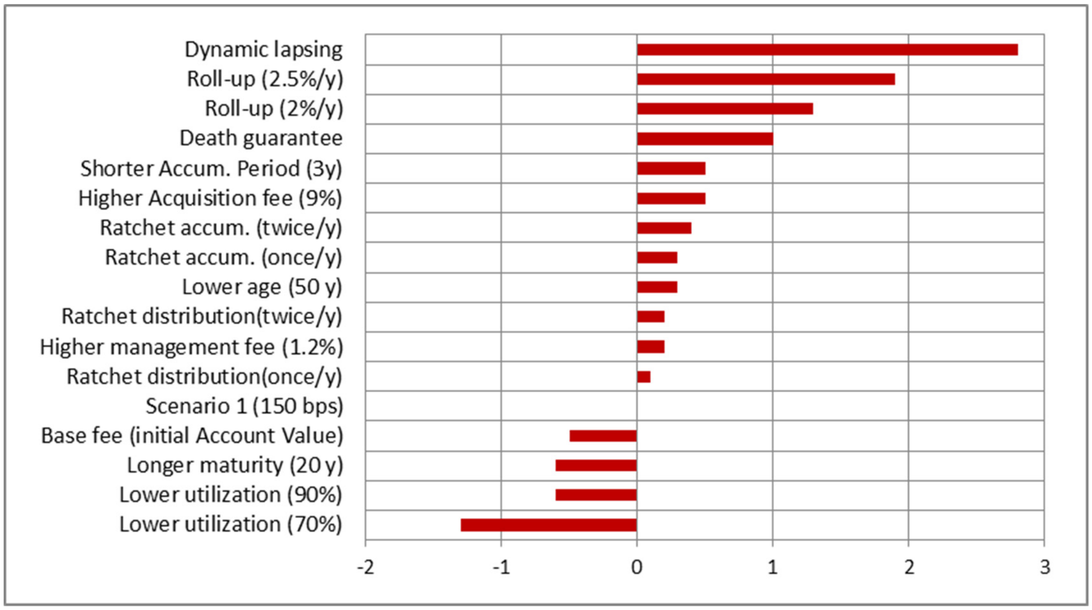


Figure 8 presents the results of the sensitivity analysis performed on financial factors. The inclusion of jumps in equity and bond prices is the most impactful factor on guarantee price, followed by changes to long-run interest rates, changes in equity volatilities and changes in portfolio composition (equity/bond). Incorporating negative correlation reflected in observed historical levels of correlation between equities and bonds, and higher bond volatility have intuitive, but limited (less than 10 bps), impacts on guarantee price. We also note that the sensitivity to financial factors is not symmetrical, meaning that the impact on price of increasing equity volatility is higher than that for decreasing volatility. The same observation applies to changes in the long-run rate assumption. In terms of allocation of the underlying assets (equity/bond), increasing equity allocation from 60% to 80% increases guarantee price by 40 bps due to higher equity volatility and jump size.

It is therefore important to highlight that the assumption of the long-run risk free rate and the inclusion of jumps in asset paths in the modeling framework have significant impact on the modeled guarantee price. More importantly, the reversion speed to long-run interest rate which is the speed at which spot rate converge to long-run risk free rate is the most impactful factor. As seen in Figure 8, an increase of the reversion speed from around 1% (base value) to 3% increases guarantee prices by 180 bps. An increase of rate volatility from 14% (base value) to 30% lowers guarantee prices by 30bps only.

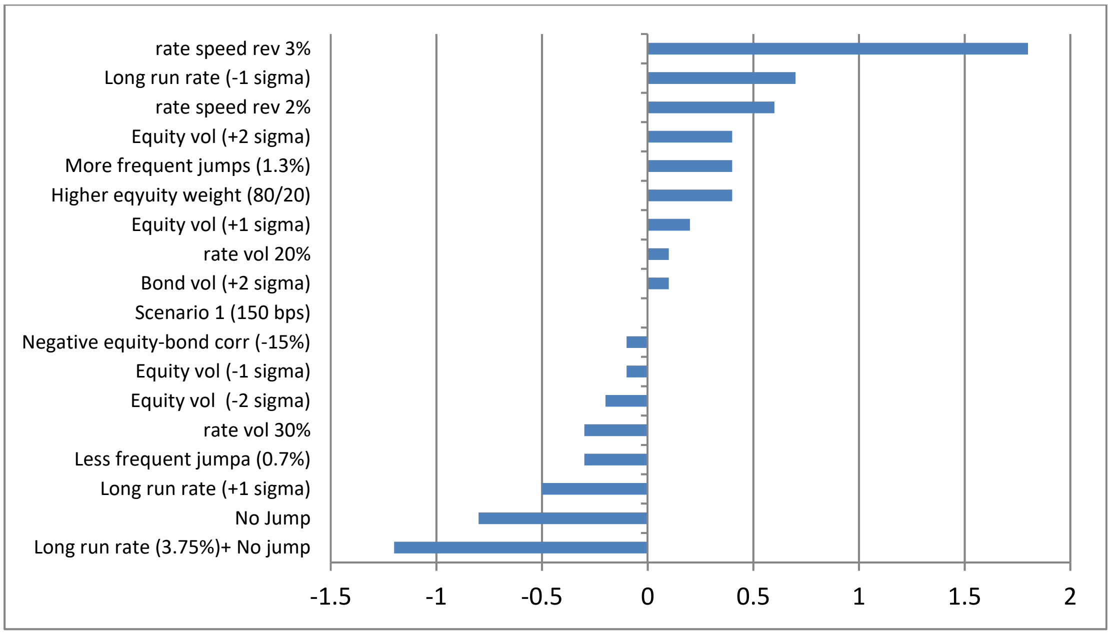
In the implementation of this framework, we assume price discontinuity (jumps) consistent with empirical evidence[12] and the lengthy literature on price discontinuities. We assume a jump frequency of 1 each 8-9

[12] Rama Cont and Peter Tankov, Financial Modelling With Jump Processes, 2004

years, which is calibrated to long-run data. The observed frequency of market price jumps since 1980 exhibits a slightly higher frequency of jumps with a jump occurring once approximately every seven years.

Moreover, we test a combination of no jump assumption and a relatively higher long-run risk free rate of 3.75%. The resulting guarantee price is 120bps lower than scenario1 (priced at 150bp). Note that in scenario 1 we assume price discontinuity and a long-run risk free rate of 1.6% (calibrated to market data spanning from the 80s).

Figure 8: Sensitivity of Guarantee Price to Market Factors (% of Account Value)



Guarantee price sensitivity under dynamic lapsing assumption:

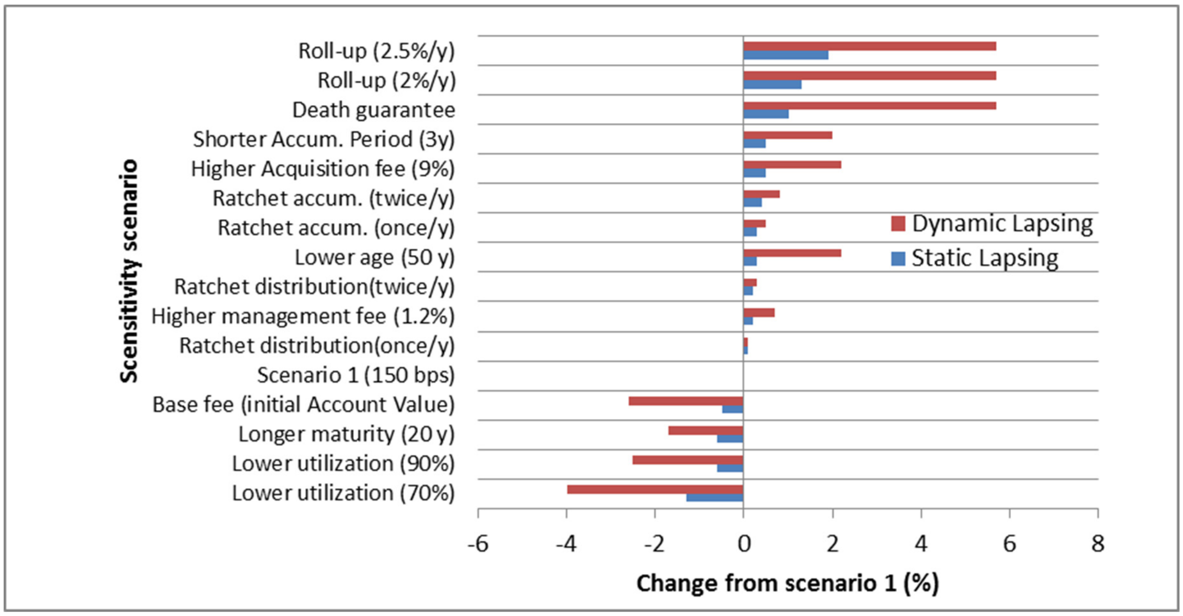
Since we found that dynamic lapsing has the most pronounced impact on the guarantee price, we compare the sensitivity of the guarantee price to the changes in the inputs between the cases of static and dynamic lapsing. In order to do so, we calculate differences in guarantee price before and after applying the shock (change in underlying input) for both static and dynamic lapsing scenario. For a single shock i the impact is as follow:

$$impact_S^i = price_S^i - price_{scenario\ 1} \qquad \text{Equation 9}$$

$$impact_D^i = price_D^i - price_{scenario\ 1+Dynamic\ lapsing} \qquad \text{Equation 10}$$

In terms of combined effect of dynamic lapsing and changes in insurance inputs, Figure 9 shows that GMWB prices are significantly more sensitive to changes in insurance features when dynamic lapsing is considered. The cost of "richer" contracts is higher when dynamic lapsing behavior is included in the model. For example, the cost of combined death guarantee and dynamic lapsing is considerably higher than the sum of death guarantee cost and dynamic lapsing cost. Dynamic lapsing has an aggravating effect when combined to other insurance features. The residual cost of a death guarantee or roll-up are less than 200bps under static lapsing condition but more than 500bps under dynamic lapsing. The dynamic lapsing condition also exacerbates the price reduction impact from lower benefit utilization where assuming benefit utilization of 70% instead of 100% reduces guarantee price by 4% which illustrates that guarantee price is very sensitive to policyholder behavior both in terms of level of benefit utilization and dynamic lapsing. This suggests that pricing variable annuity guarantees without accounting for the interaction effect of dynamic lapsing and other insurance features will likely lead to significant estimate bias and potentially to underestimating guarantee fees. Changes in other insurance features such as lower age of policyholder at issuance, higher acquisition fee and shorter accumulation period have also significant impact on guarantee price under dynamic lapsing compared to static lapsing.

Figure 9: Sensitivity of Guarantee Price to Insurance Factors Conditional on Dynamic Lapsing (% of Account Value)



Similar to insurance factors, sensitivities to financial factors (Figure 10) are also amplified when dynamic lapsing is included in the model. The sensitivities to long-run interest rate and jump parameters are significantly higher if dynamic lapsing is included in the model. Reducing equity volatility shock magnitude, changing the equity-bond correlation, and altering bond volatilities do not have significant interaction with dynamic lapsing behavior. However the most pronounced effect is from lower long term interest rates whereas the impact on guarantee price under static versus dynamic lapsing increases from 70bps to 430bps. The low interest rate environment and a more frequent market price discontinuity have an extreme impact on VA guarantee value when measured under dynamic policyholder behavior.

GMWB pricing depends mainly on assumptions of policyholder behavior, long-run interest rate environment, discontinuity of asset prices and to lesser extent on equity volatility and allocation of the underlying asset investments.

Figure 10: Sensitivity of Guarantee Price to Financial Factors Conditional on Dynamic Lapsing (% of Account Value)

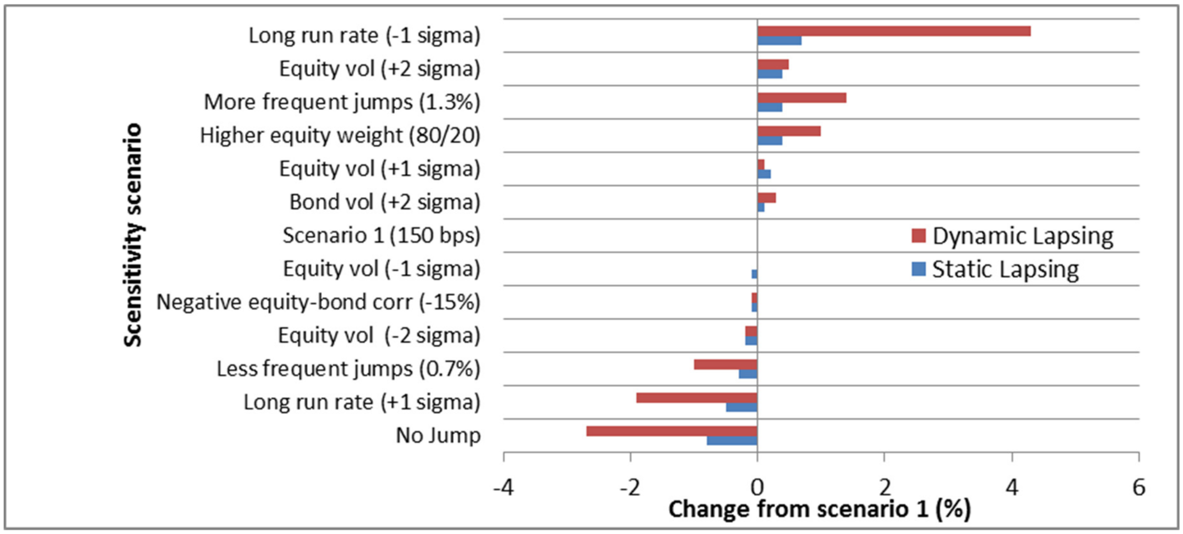
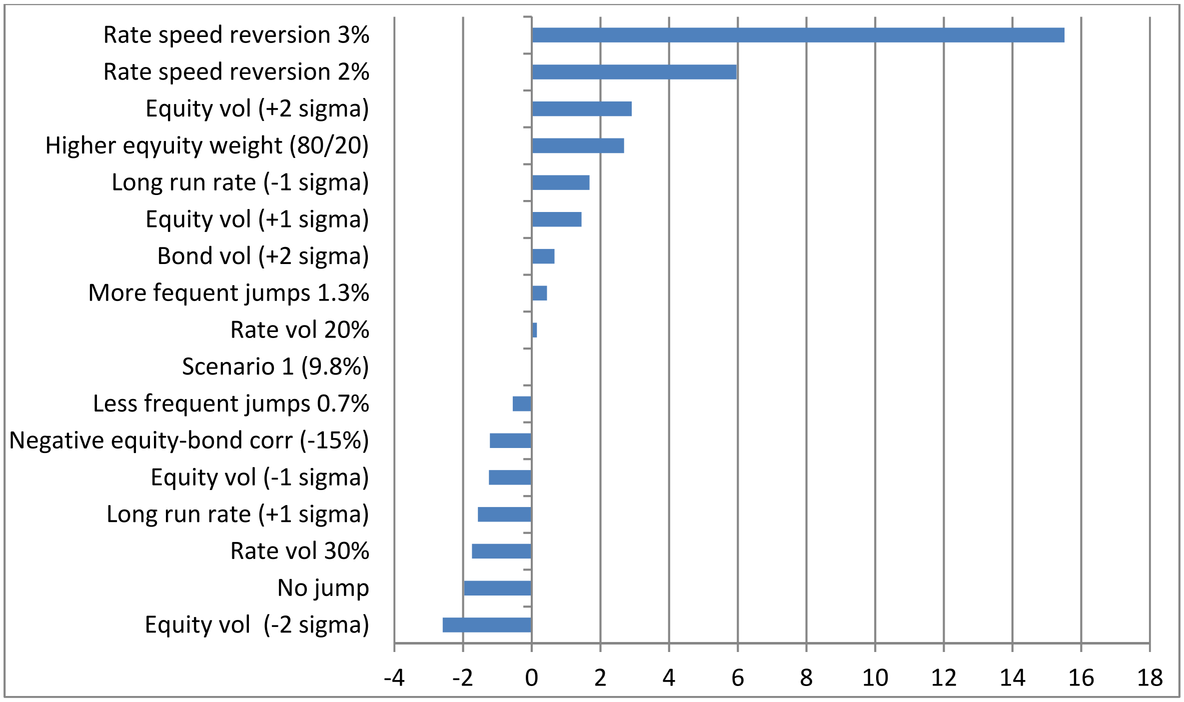


Figure 11 shows that VA reserves calculated as the 70th conditional tail expectation (CTE70) of guaratee's cash flow are most sensitive to rate reversion speed, long-run interest rate, increased volatility and portfolio composition (higher concentration i equity). These results are similar to the those observed for price sensitivity to market factors. Bond volatility and correlation have an intuitive, but limited, impact on reserves. Price discontinuity (jump) shows less impact on reserve as opposed to guarantee price. For example, an increase of rate reversion speed from 0.99% (base value) to 2% increases required reserves by 6% which is a 61% increase from its base value of 9.8%. We also note that while assuming no price discontinuity reduces gurantee price by around 50% it reduces guarantee reserve by only 20%.

Figure 11: Sensitivity of Guarantee Reserve (in %) to Market Factors



G. Application to four hypothetical VA portfolios

We now apply our framework to data that are representative of real-world insurance VA portfolios. In particular, using our Monte Carlo simulation framework, we estimate the model price and reserve requirements of four proxy portfolios created for four firms using regulatory reports.

1) Data

We created four stylized portfolios designed to reflect the risk exposure of four of the largest VA issuers by combining a number of publicly available data sources. The first of these are the 2015 annual statement general interrogatories (life interrogatories) for each firm. These data are filed by firms (at the legal entity level) with state insurance regulators and provide high-level data regarding the amount and types of VA liabilities held by the firm. Firms provide data grouped by contract types (i.e. death guarantee benefit type, living guarantee benefit type, and by rider type). For example, a particular line item in this data may include data on the value of separate account assets associated with contracts with a 5% rollup GMDB and a GMAB with 5% rollup and 1 year ratchet with one year remaining on the waiting period.

These data provide a reasonable, but not complete, source of data regarding the scope and nature of a firm's VA liabilities. Clearly, in order to facilitate presenting these data in regulatory reports, there is a fair degree of aggregation among contract features. For the purposes of reporting, firms aggregate some contracts with similar, but slightly different characteristics (e.g. contracts with 4%, 5% and 6% rollup rates) into a single line item which is designated as "5% roll-up"). Thus, we are working with data that has had some detail removed regarding contract features. This will naturally introduce some uncertainty into our approximation of firm-specific risk as some contracts are simulated using average values (for example the

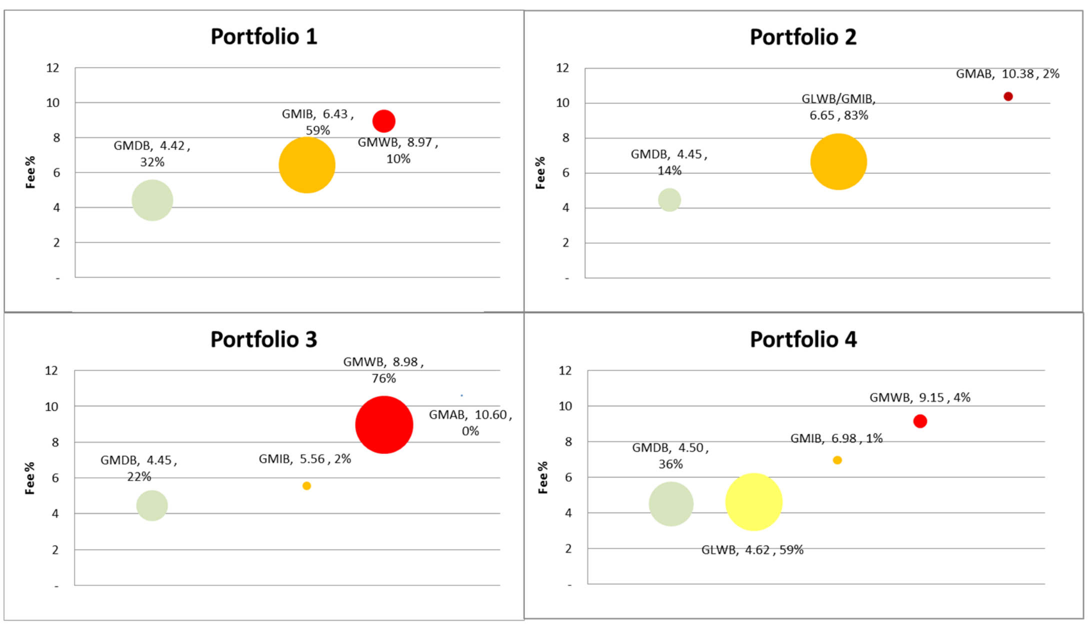
4% rollup contracts being simulated as 5% rollups based on their inclusion on this line item). Without access to confidential contract-level data, our data are the best source of data regarding the exposures within firms.

Additionally the general interrogatory data does not include some data that is required to generate results within our simulation framework. These data include: age and gender of policyholder, and GLWB and GMWB withdrawal rates. To address this, we supplement regulatory data with data from a joint study sponsored by SOA and LIMRA[13]. This study surveyed 20 firms regarding VA contracts and their policyholders. The data covered 4.7 million contracts in force in 2013. These data are analyzed to provide industry-representative data of policies and policyholders. For each VA type, the authors calculate the age distribution of policyholders and the portion of policyholders that are male and female, across all firms. Within a particular type of guarantee, the portion of contracts that include certain features or guarantees—in particular, the portion of contracts that were issued with each withdrawal rates—are provided.

These data are crossed with the firm-level data to create a proxy for contract-level data. For example, if $100 million in separate account assets are associated with GMAB guarantees and in the industry-level data 49% of GMAB policyholders are male, we assume that at the firm level, $49 million of separate account assets that are associated with GMAB contracts that are held by males. Using this process we are able to define VA and policyholder characteristics at a policy level for data that are not included in the general interrogatory. This process assumes independence between the observed characteristics of contracts and the industry-level proxies.

2) Proxy Portfolio Descriptions

Figure 12: Risk Profile by Hypothetical Portfolio



[13] Variable Annuity Guaranteed Living Benefits Utilization: 2013 Experience

The four selected firms that provide a basis for our proxy portfolios have large guaranteed VA portfolios on their balance sheets as measured by separate account assets as a portion of total assets. Products offered are primarily GMIB, GMWB and GLWB. GMDB and GMAB constitute smaller portions of the sample portfolios.

Figure 12 shows the size of the proxy portfolios across each type of offered guarantee and the average price as estimated by our model (i.e. the fair price result). For example, 59% of portfolio 1's VA is made up of GMIBs with an average guarantee price of 6.43%. Our results show that the modeled price of GMABs is higher than the other guarantee types followed by GMWB, GMIB/GLWB and GMDB.

Figure 18 to Figure 22 (in the appendix) provide the descriptions of the proxy VA portfolios in terms of guarantee type composition, and roll-up and ratchet features. More than 80% of offered contacts provide guaranteed roll-up (Figure 19). Ratchets which are intra-annual performance guarantees are offered in relatively lower scale except for firm 1 (Figure 20) where 60% of the VA portfolio are guaranteed intra-annual ratchets. Offered roll-rates are mostly between 0% and 5%. Here a 0% roll-up rate corresponds to a "return-of-principle" guarantee that protects against decreases in the value of the underlying assets while a 5% roll-up guarantees an annual increase in the value of the benefits base of 5% regardless of the returns on the underlying portfolio.

Based on these data, we constructed a sample of 1414 contracts representative of those held in the VA portfolios at the four companies and covering most of the observed contract features combinations. This sample is used to generate results for these firms and assess guarantee prices and reserves. In the following section, we discuss the guarantee price results and in section 4) we present model results on guarantee reserves.

3) Model Results- Application to Proxy Portfolios

For each contract in the proxy portfolios, we run 10000 simulations and solve for the "breakeven" guarantee fee. Simulations are run for multiple guarantee fees and the guarantee fee leading to an expected 0 present value is determined to be the fair price for the contract. Table 2 shows VA price results from the model summarized by firm and product. Our model results are presented as a range of values where the lower boundary is obtained when guarantee fee is modeled as a percentage of initial account value and the higher boundary represents contracts modeled with guarantee fee as a percentage of current account value. As shown in the sensitivity analysis section, the guarantee price of scenario 1 (GMWB contract) is 150bps and 430bps if priced under dynamic lapsing assumption.

Our model results show that GMABs are the most expensive product, followed by GMIBs and GMWBs (Table 2). In table 2, fees are presented as a range a lower bound when calculated using initial account value and an upper bound given by the fee calculated using current account value. GMLB with GMDB are the least expensive. Such results are intuitive knowing that GMABs have longer duration on average, and thus have greater exposure to market risk over the life of the contract, while GMWB and GMIB are more sensitive to combined effect of financial risks and policyholder's dynamic behavior.

Table 2: Fair Value Fees Calculated as a Percentage of Initial or current Account Value

Model Results: (Lower values are Fees as % of Initial account value, Upper values are Fees as a % of current value)	GMDB	GLWB	GMIB	GMWB	GMAB
Firm 1	3.2-5.7		3.2-9.6	2.9-10+	
Firm 2	3.2-5.7	3.7-9.6[14]			7.2-13.6
Firm 3	3.2-5.7		2.8-8.3	3-10+	7.4-13.8
Firm 4	3.2-5.8	2.3-6.9	3.8-11.2	3.3-10+	

Beyond the fact that our model contains a lot of simplifying assumptions in modelling the hypothetical portfolios, there are several factors that may explain the differences between our modeled fees and market fees:

- Reported fees may be specific to limited guarantees and reflect guarantee price instead of contract price that combines living and death benefits.
- Firms may price these guarantees using real world measures as shown in the 2015 NAIC QIS where 87% of firms assumed equity returns of more than 4% through VA contract duration. Such assumption significantly reduces the guarantee price.
- Firms may not incorporate dynamic lapsing and multiple shocks. Or may use static rates and lower than historically observed volatilities. They may also model these differently using internal data and assumptions.

As a result of these different modelling approaches and assumptions, our model outputs aren't not directly comparable to market reported fees. Results from our model aim primarily to quantify sensitivities to risk coverage, modelling approaches, inputs and assumption.

4) Estimating VA Reserves.

This section discusses reserve quantification for the 1414 contracts in our sample. Reserves are estimated by calculating the conditional tail expectation at 70th percentile (CTE70) of the P&L at the contract level and aggregated to the guarantee type and proxy portfolio levels. Reserves are calculated using real world valuation by assuming an equity return premium over risk free rate and discounting cash-flows using firms' funding cost proxy curves.

For equity returns, we adjust modelled stochastic risk free rates (OIS-based) by adding an equity return premium (ERP). A study by Modugno 2012[15] shows that historically ERP ranges from 2.67% to 7.62% depending on the chosen period and the underlying index. In this framework, we assume a 3.5% ERP such that asset drift in the geometric Brownian process is set to stochastic risk free rate plus 3.5% ERP. This

[14] Portfolio 2 is generated using a combined GLWB and GMIB portfolios because of the optionality of annuitization or guaranteed withdrawal as this is the reporting method for this firm.

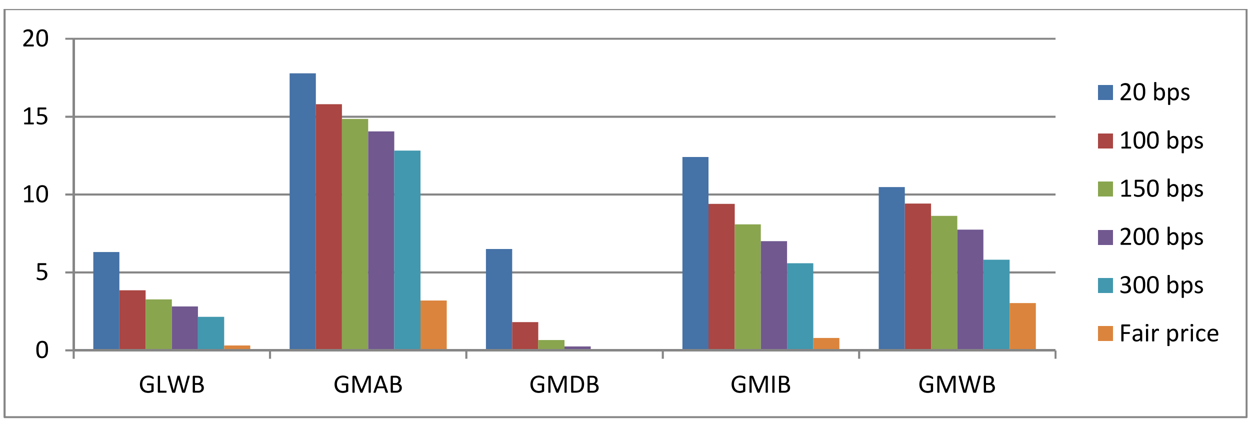
[15] Victor Modugno, Estimating Equity Risk Premium, SOA 2012

rate is observed to roughly correspond to the median ERP in the Modugno study. The discount curve is selected to match the funding cost for the four selected insurance firms. To that end, we use 20-year CMT (constant maturity treasury) to calibrate firms' stochastic funding cost. We also explored a two other yield curve tenors 0.5 to 20 years and 0.5 to 50 years HQM discounting curves. Furthermore, we noted that CMT is discussed in industry literature as an appropriate choice of a discounting curve for reserve deterministic scenario.[16] Vasicek interest rate models are applied to all three discounting curves.

Figure 13, Figure 23 and Figure 24 present modelled reserves as percentages of current value for the three different discounting scenarios (CMT20 and HQM50, HQM20) and for six assumed charged fees. In addition to the fair price produced by our model, values assumed for charged fees are 20, 100, 150, 200 and 300 bps. Fair price values are specific to each contract in the portfolio and vary buy contact type, features and "richness". Other assumed fees (20 to 300bps) are set flat for each contract in the portfolio. Note that if fair price is charged, the required reserves (from our model) are below 4% across the different guarantee types.

Reserves calculated at CTE70 reflect the shape of the guarantee P&L distribution. The probability of large losses is higher for GMABs which show relatively higher CTE70 compared to GMIBs and GMWBs. Due to lower withdrawal rates, GLWBs also show lower reserve requirements. As seen in the previous sections, GMDBs are relatively less expensive and require limited reserves if priced above 100bps.

Figure 13: Modeled Reserves (% Current AV) – Discount Curve calibrated to 20-year CMT yield curve.



Vasicek Model Parameters for 20-year CMT:

Initial rate	1.57%
Theta	3.12%
Kappa	14.56%
Eta	0.22%

The Vasicek model calibrated to the 20-year CMT shows different parameters compared to our risk neutral model where the starting rate is 7.57% (higher than 0.7%), the equilibrium rate is 3.12% (higher than 1.63%), similar rate volatility (14.56%) but lower speed reversion (0.22% vs 0.99%). The HQM20 is most optimistic scenario with an equilibrium rate of 6.53% and a reversion to equilibrium speed of 2%. Note

[16] NAIC VA Reserve and Capital Reform, Recommended Revisions to AG43 and C3P2, August 2016

that we showed in the sensitivity analysis section that these two parameters are the most impactful on guarantee price and reserve.

There are several features regarding reserving practices at firms that may result in reserve requirements that are significantly different than those represented here:

- As shown in figure 13, required reserves are conditional on contract pricing practices. To the extent that firms employ pricing that differs from our modeled fair value prices, calculated reserve requirements will differ as well.
- Firms may not incorporate dynamic lapsing and multiple shocks. Or may use static rates and lower than historically observed volatilities. They may also model these differently using internal data and assumptions.
- Firms determine reserves net of any impact of hedging, which we do not consider in our modeling.
- There is a standard scenario floor used to established reserves which may require firms to post reserves beyond those specified in AG 43.

Given the differences stated above, our model results are not directly comparable to firms' reported reserves. Our model results show that reserves are very sensitive and inversely proportional to VA price (the lower the fee, the higher the reserve).

H. Stressing VA portfolio

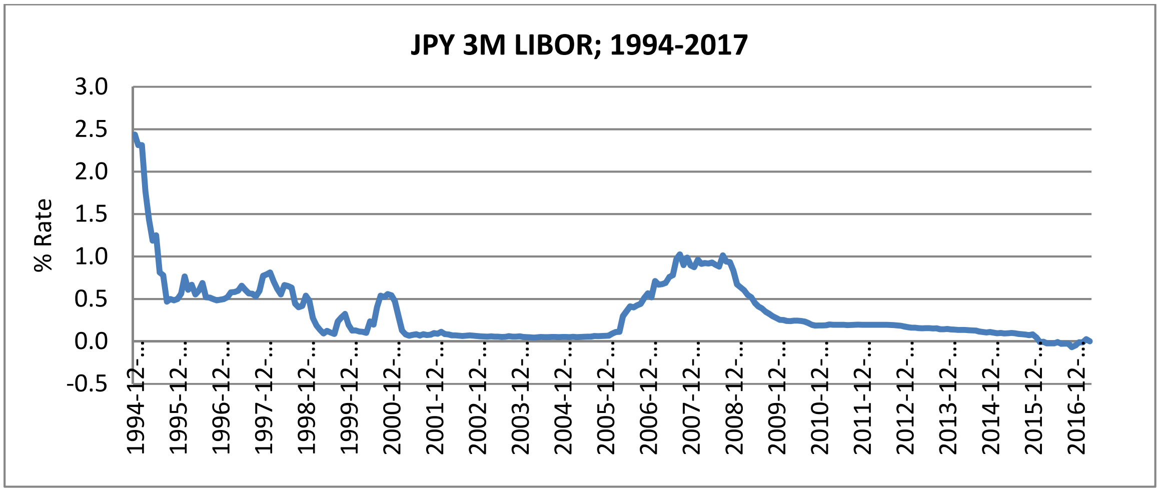
In addition to pricing and reserving for VAs, our model can be used to stress-test the portfolio of VAs with guarantees. In this section we provide a hypothetical example of applying a stress scenario to hypothetical VA portfolios.

As described above, variable annuities guarantees are long dated liabilities with present values depending on long-term market factor assumptions such as 20 to 30 years projections of interest rates and equity returns. Variable annuities are also sensitive to market discontinuity and to multiple (versus single) market shocks within contract lifetime. A meaningful stress test of these liabilities should assume a stressful, but plausible, macroeconomic scenario for the duration of VA contracts. As our model shows, variable annuities are most vulnerable to a macroeconomic scenario that includes a low interest rate environment and a protracted equity/bond decline.

As an example of a possible meaningful scenario, we propose the Japanese macroeconomic conditions observed between December 1994 and April 2017 (269 months). We use the Japanese 3-month LIBOR rate and the return on the NIKKEI 225 as proxies for risk free rates and equity returns. The VA portfolio composition as described in section G is used as the base for stressed cash flow projections. December 1994 is chosen as a starting date for the Japanese proxy scenario because of the comparability to US rates as of today. Note that the Japanese stock market was stable and with near 0% average return during the selected period (Figure 15). Beyond the 269 months form Japan scenario, rates and returns revert to stochastic values calibrated from US experience (same as used in calculating guarantee fair price).

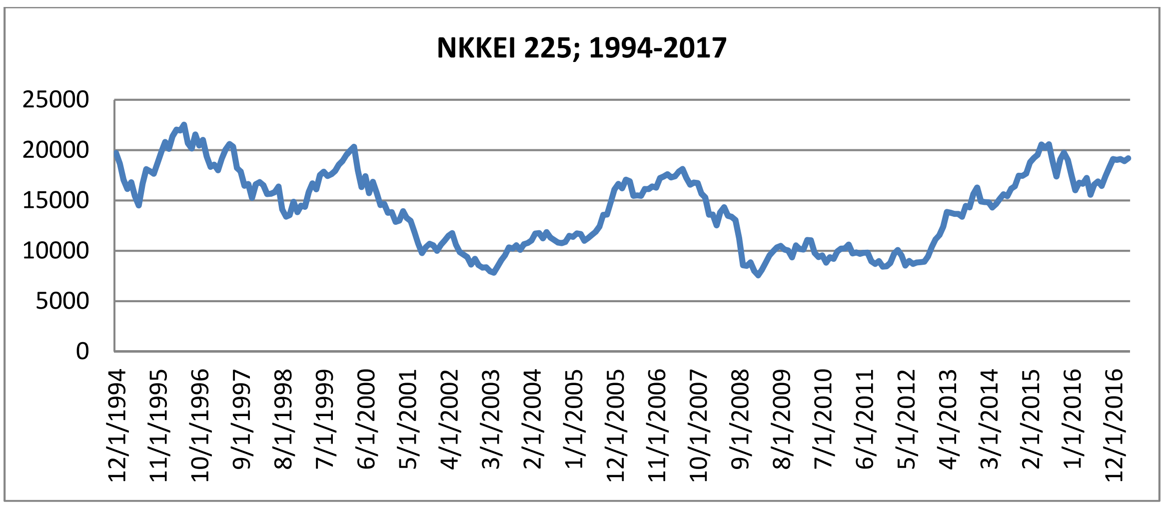
Within this deterministic scenario we keep insurance and policyholder behavior risks unstressed. However, the policyholder behavior in our model framework is tied to market performance via the lapsing versus moneyless relationship (see section 0).

Figure 14: Japan 3M LIBOR 1994-2017



In December 1994 JPY 3M LIBOR was about 2.3% (Figure 14) and dropped to near zero levels for a long period of time. Rates recovered during a brief period between August 2006 and August 2008. Returns on equity as proxied by NIKKEI 225 (Figure 15) do not appear to be stressful during this same period with an average return of 0.1%.

Figure 15: NIKKEI 225; 1994-2017



In order to project cash flows, we need to assume a value for the fees that are charged by insurance companies for VA contracts. First we assume that VA contacts are fairly priced by the four insurance companies using the risk neutral framework simulation outputs shown in prior section related to model price results. We also run the Japan scenario assuming two average guarantee prices of 150 and 100 bps. Note that 150 bps is the price we presented in the "standard scenario".

1) Japan Deterministic Scenario (1994-2017) under a fair price assumption:

Based on our model, Figure 16 shows the cumulative discounted gain/loss generated from the overall proxy portfolio made of the four individual proxy portfolios after applying the stress scenario. The cumulative gain will reach its maximum after 6 years from the beginning of the stress and then starts to decrease as the early losses begin offsetting previous gain. The portfolio loses all its previous gain at year 16 and shows 10% cumulative loss in year 24 with a maximum expected cumulative loss of 20%.

Such scenario applied to the overall VA industry ($1.7 trillion) could generate a cumulative loss of over $300 billion. Results from this stress analysis are sensitive to the assumed guarantee price. In Figure 16, we assume that VA guarantees are fair-value priced. Figure 25 and Figure 27 present stress results for guarantee prices of 150bps and 100bps respectively and show that pricing below fair value lead to earlier and more severe cumulative losses.

Note that all Japan scenarios do not assume replenishment of the portfolio as we attempt to estimate losses generated from the current portfolio. However, this analysis aims to produce stress losses (under different price assumptions) for different guarantee profiles that can be used to allocate stress losses to future portfolio acquisition or replacement.

Figure 16: Portfolio Cumulative Losses under Japan Scenario and Fair Price Assumption[17]

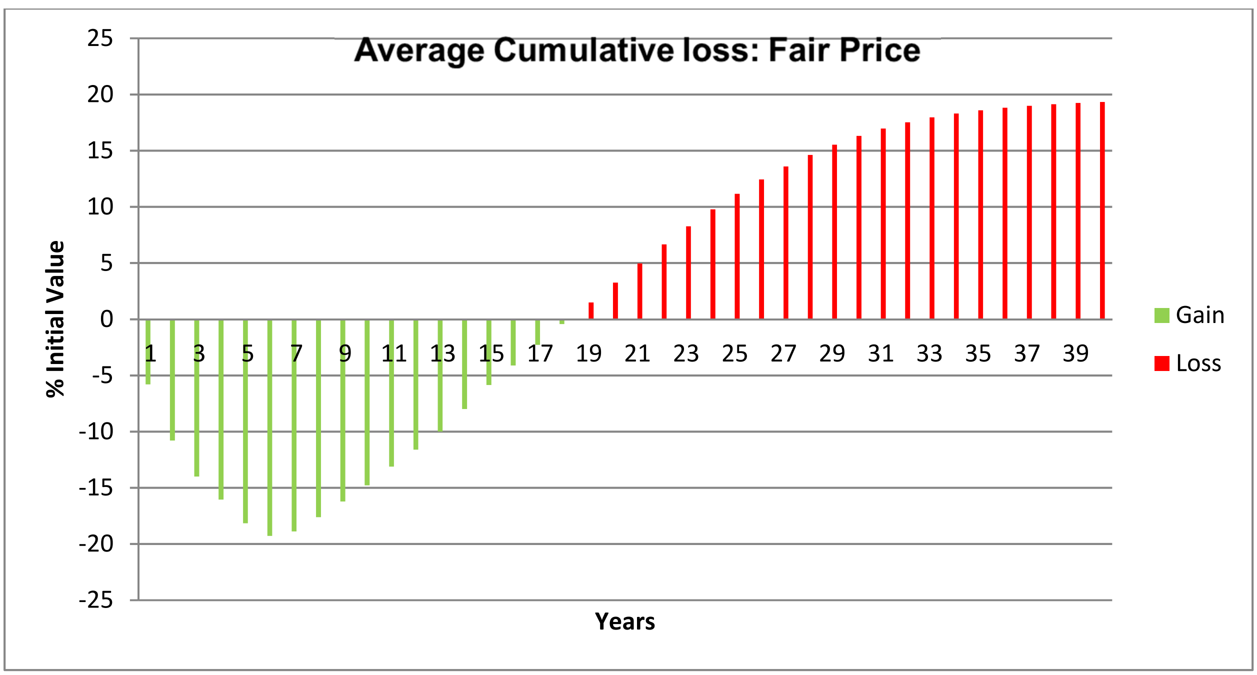
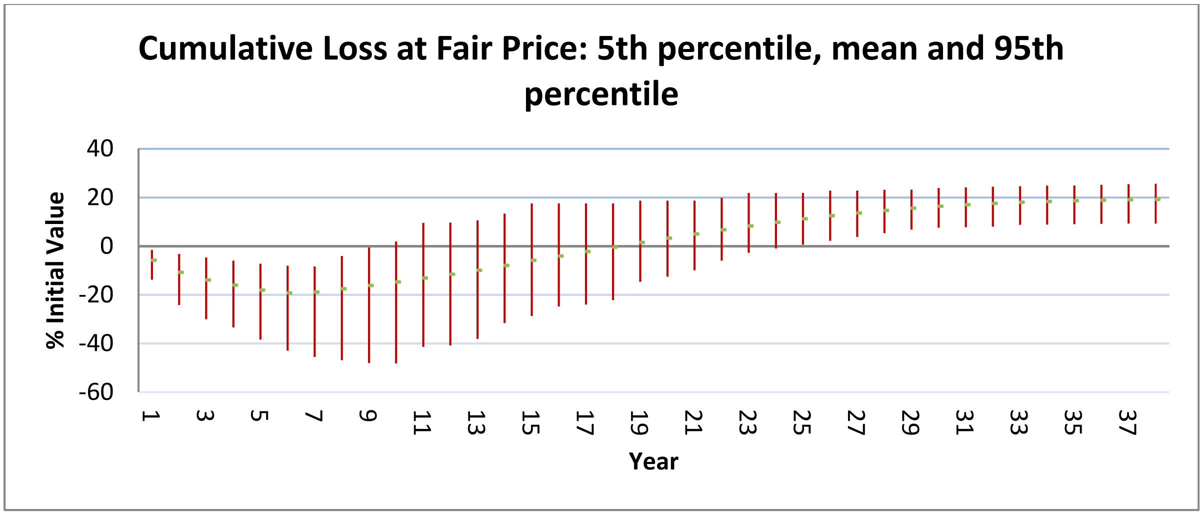


Figure 17 below shows the 5th percentile, the mean, and the 95th percentile of cumulative losses at the contract level. Contract losses below the 5th percentile do not manifest until year 25, while contracts with losses above 95th percentile are observed starting from year 9. This means that 5% of portfolio contracts

[17] The “sawtooth” observed in the gain area (green zone) is due to cash flow timing mismatch; death claims occur anytime within a year while withdrawals and fee payments are annual.

will start seeing losses before year 9. Contracts below the 5th percentile can be described as the least risky (i.e. less sensitive to Japan scenario) among all contracts from the four firms. Contract above the 95th percentile are the riskier (and more sensitive to Japan scenario) contract that likely to show losses the earliest. These least risky contracts will not report losses until year 24.

Figure 17: Cumulative Contract Level Losses under Japan Scenario and Fair Price Assumption



2) Japan Deterministic Scenario (1994-2017) under 150 and 100 bps average prices

Given that firms' specific guarantee prices are not public, we assume average prices across firms using prices reported by industry reports and literature. Therefore we run also the Japan stress scenario assuming VA guarantees priced at 150 bps and 100 bps (Figure 25 and Figure 27, respectively). Contracts priced at 150 bps and subjected to "Japan deterministic scenario", will incur losses starting from year 10 and cumulative losses will reach 10% of portfolio initial value by year 16. The overall cumulative loss for this case nears 30% of initial value. Assuming guarantee price at 100 bps, losses will be incurred starting from year 8 and will reach a cumulative loss of 5% of portfolio initial value by year 11. The overall cumulative losses for contracts priced at 100 bps under "Japan deterministic scenario" will exceed 30% of portfolio initial value. Note that these cumulative losses are portfolio losses and single contacts may incur more or less severe losses than average depending on contract features and guaranteed benefits.

In Figure 26 we present percentiles (5th, mean and 95th) of cumulative losses at contract level under the 150 bps price assumption. Contracts with 5% lower losses will not observe losses until year 23 while, 95th percentile contracts will observe losses starting from year 4.

In Figure 28 we present percentiles (5th, mean and 95th) of cumulative losses at contract level under the 100 bps price assumption. Contracts with 5% lower losses will not observe losses until year 21, while 95th percentile contracts will observe losses as early as year 3.

As shown in Figure 28, stress losses increase with lower guarantee prices and firms may observe losses in some underpriced contracts as early as the first or second year of the stress test horizon. Under 100bps price assumption, more than 5% of the four firms' contracts will see losses within the first three years of the stress horizon.

3) Japan Deterministic Scenario (1994-2017) under 150 bps average prices: Stress Losses by Guarantee Type

In this section we apply the "Japan deterministic Scenario" to four types of guarantees: GMWB, GMIB, GMAB and GMDB assuming they are priced at 150 bps.

From prior sections, we observed that for contracts from the four firms that we analyzed, GMAB, GMIB and GMWB are (on average) relatively riskier in terms of fair prices. Note that most living benefit contracts at these firms include death benefits which, when offered alone, have lower fair prices than living benefit guarantees. We also noted that GMABs are relatively longer duration contracts that typically include supplementary guarantees such as rollups and ratchets. Thus, GMAB contracts are both "rich" and face longer exposure to market risks. Figure 29 shows the 5^{th} percentile, the mean, and the 95^{th} percentile of losses among GMAB contracts with 95th riskier contracts observing losses as early as year 4 and a lifetime cumulative losses over 50% of initial value. Less risky contracts do not realize losses until year 19 which reflects the variety of "richness" of these contracts within the four analyzed companies. GMWBs (Figure 30) shows 5^{th}, mean and 95^{th} percentile of stressed losses under the Japan scenario and a 150 bps price assumption. The 5% riskier contracts observe losses just before year 4 and have a lifetime cumulative loss of more than 30%. Lifetime cumulative losses are reached at year 15 relatively earlier than other guarantee types. This is due to the fact that most GMWBs are offered for 15 years duration. GMIBs (Figure 31) which guarantee lifetime income exhibit the most losses under the Japan scenario and under the 150 bps assumption. Lifetime cumulative losses range from 20% to 60% (5^{th} and 95^{th} percentiles). As noted before, most of these contracts offer death benefit and guarantee lifetime income which is equivalent to a long series of put options exercised over a long period. GMDBs (Figure 32) are the least impacted by Japan scenario where no contract observes losses before year 11 and contract lifetime losses between 25% and 30%. Our results show that GMIB and GMAB are more vulnerable to prolonged low interest rate due to their longer duration compared to GMDB and GMWB where claims occur in general relatively earlier in contract life.

Our results from the Japan stress test corroborate conclusions reached by the Financial Services Agency of Japan (JFSA) in a study on the impact of low interest rate environment on the Japanese insurance sector[18]. The study found that life insurance bankruptcies were caused by a persistent "negative spread" (guaranteed interest rate minus investment yield) which resulted in 7 life insurers failing between 1997 and 2001 with negative equity ranging from 11% to 30% of total assets. These firms were guaranteeing returns above 5% prior to failing.

[18] Impact of interest rate environment on Japanese insurance sector, JFSA, Nov 2016

I. Conclusion

This paper develops a Monte Carlo model that is used to examine the sensitivities of VA fair pricing and reserve requirements to various underlying risks and applies this model to several proxy portfolios to examine the risks associated with a portfolio of VAs and the impact of a stress scenario on potential losses associated with these portfolios. We expand upon the existing literature by incorporating several essential aspects of financial markets and policyholder behavior simultaneously within a Monte Carlo simulation framework that allows us to examine the risks and sensitivities associated with portfolios designed to mimic those that currently exist within four large insurers.

Our examination of the pricing and reserving sensitivity of VA contracts to underlying risks confirms the results of previous research that features that incorporating elements such as asset return correlation and jump processes has a material impact on pricing. Similarly, VA prices can exhibit large sensitivities to extended periods of sub-par returns or to multiple periods of shocks and recovers. The findings have important implication for evaluating both the stochastic economic scenario generation framework and for evaluating the deterministic scenarios that are both used by insurers.

Our results illustrate the sensitivity of pricing and reserving calculations to underlying assumptions regarding the economic scenario generation and policyholder behavior modeling. In particular, the choice of discount curve has a large impact on fair-value pricing and reserve requirements, with the use of risk-adjusted discount curves with higher long-run equilibrium rate assumptions dramatically reduces the modeled reserve requirements conditional on a given price. Additionally, the incorporation of dynamic lapsing behavior has a large impact on results as well. Our findings indicate that realistic assumptions regarding these modeling features will be critical in developing reliable pricing and reserve estimates.

Finally, applying our model to a deterministic stress scenario, modeled on Japan beginning in the mid-90s, shows significant cumulative losses of up to 25% of the initial contract value. Cumulative losses vary by contract type and can reach more than 40% for some types of GMIB and GMAB contracts.

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K. Appendix

Figure 18: Firm Portfolio Composition by Guarantee

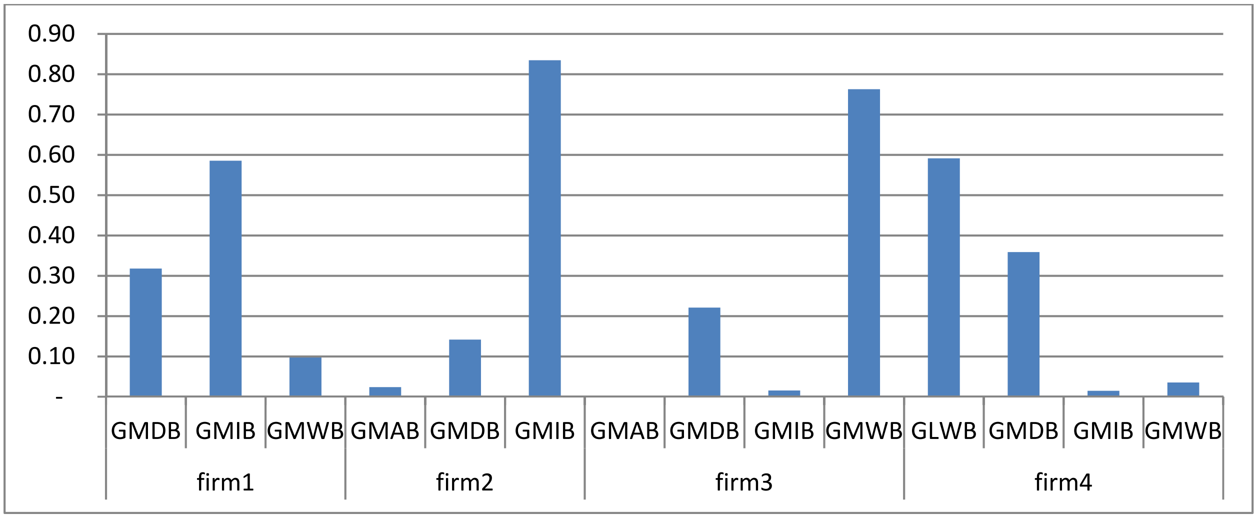


Figure 19: Proportion of VA contracts with Roll-Up by firm

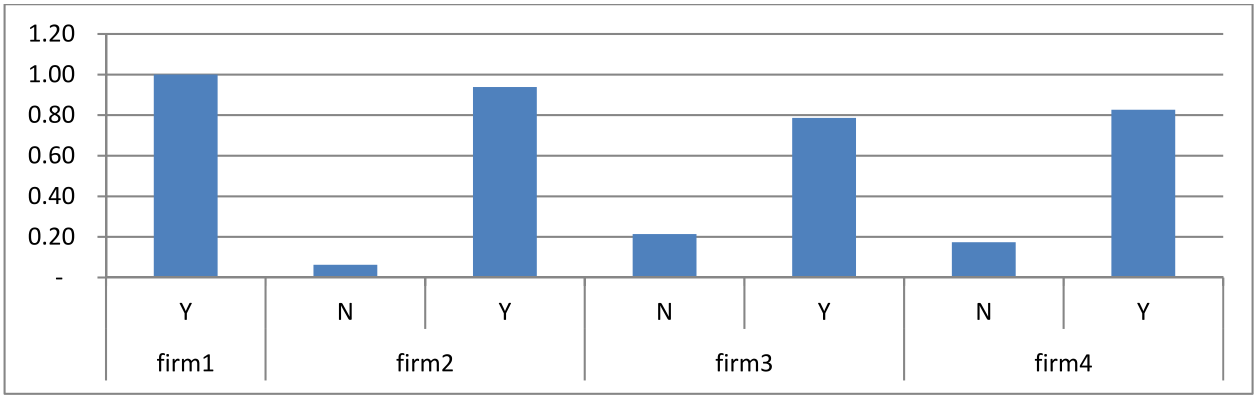


Figure 20: Proportion of VA contracts with Ratchet by firm

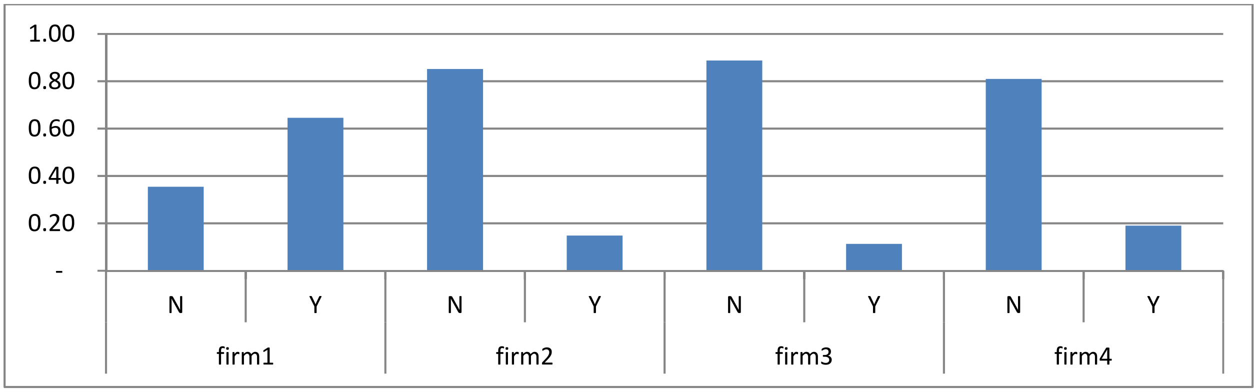


Figure 21: Proportion of VA contracts with Roll-Up and Ratchet by firm

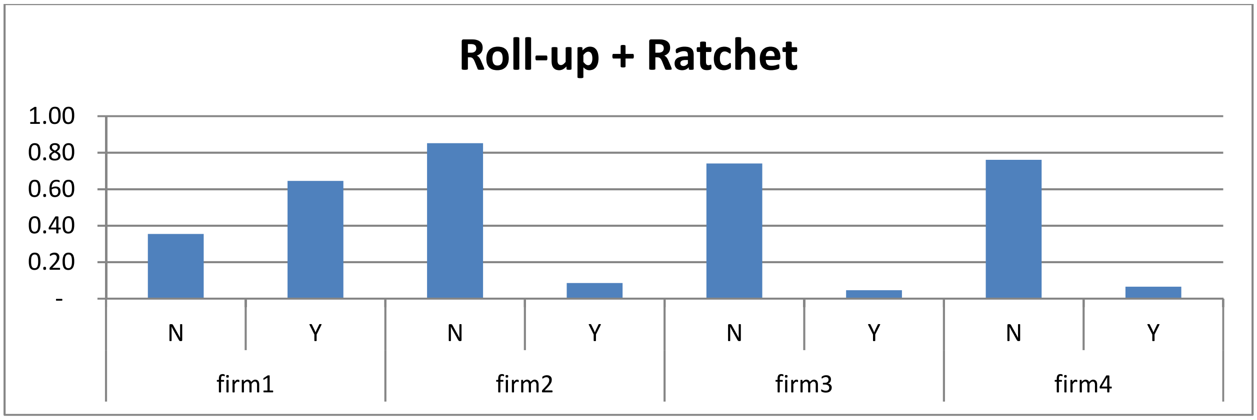


Figure 22: Roll-Up rate by firm

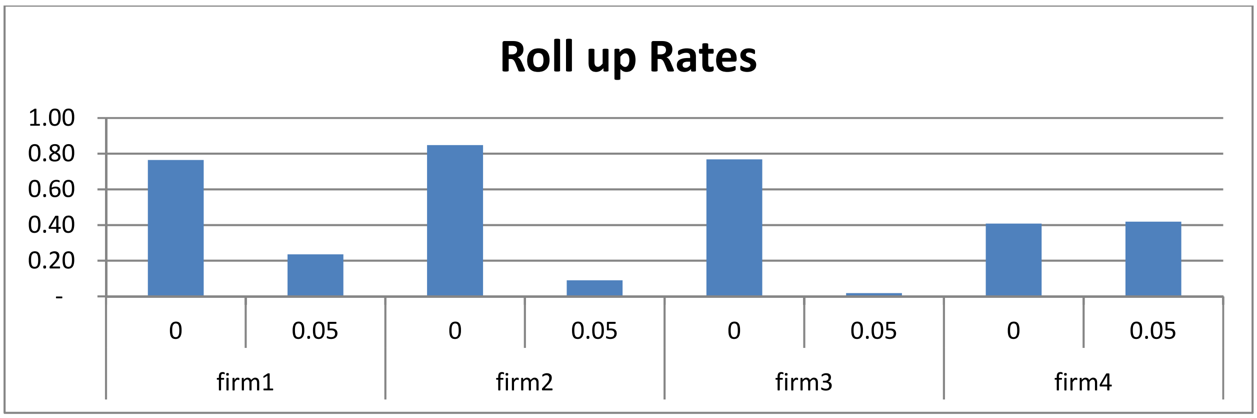
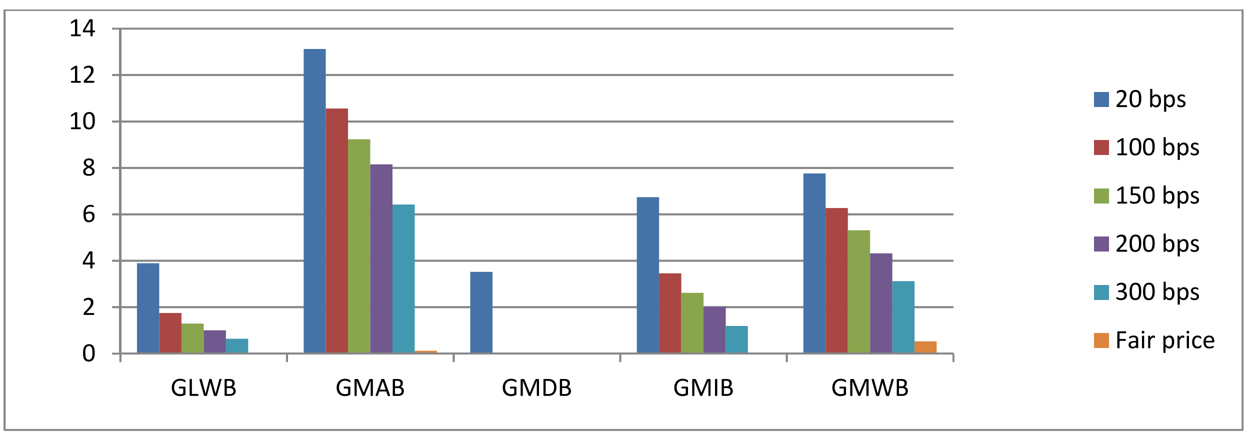


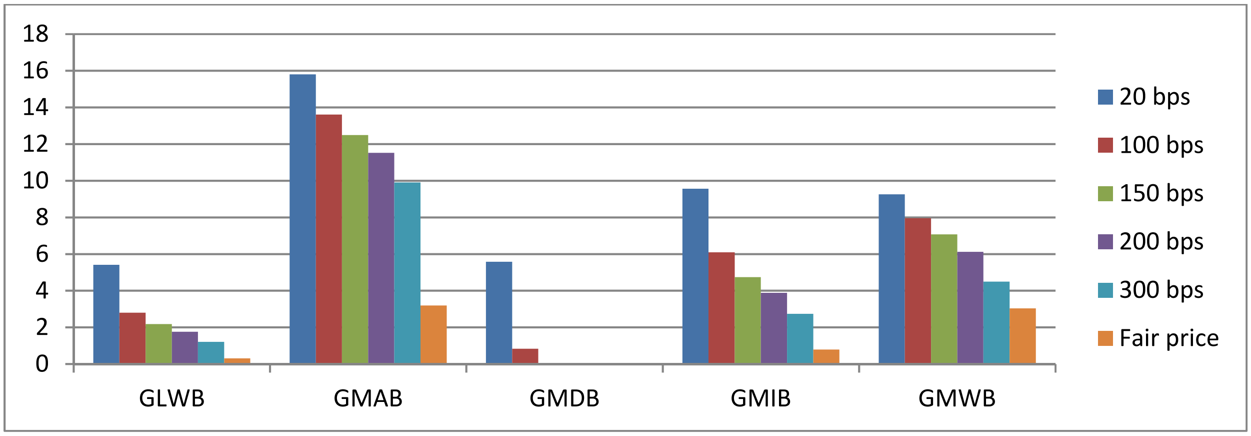
Figure 23: Modeled Reserves (% Current AV) – Discount Curve calibrated to 0.5-50 years HQM yield curve.



Vasicek Model Parameters for 0.5-50 years HQM yield curve:

Initial rate	0.60%
Theta	5.07%
Kappa	23.37%
Eta	1.00%

Figure 24: Modeled Reserves (% Current AV) – Discount Curve calibrated to 0.5-20 years HQM yield curve.



Vasicek Model Parameters for 0.5-20 years HQM yield curve:

Initial rate	0.60%
Theta	6.53%
Kappa	12.27%
Eta	2.09%

Figure 25: Portfolio Cumulative Losses under Japan Scenario and 150 bps Price

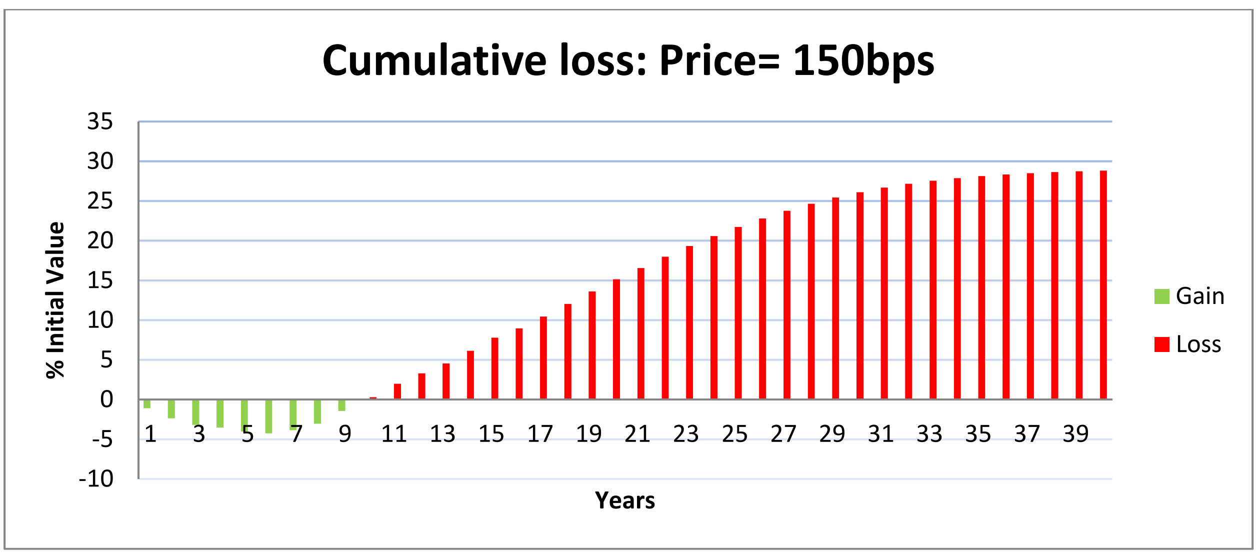


Figure 26: Cumulative Contract Level Losses under Japan Scenario and 150 bps Price

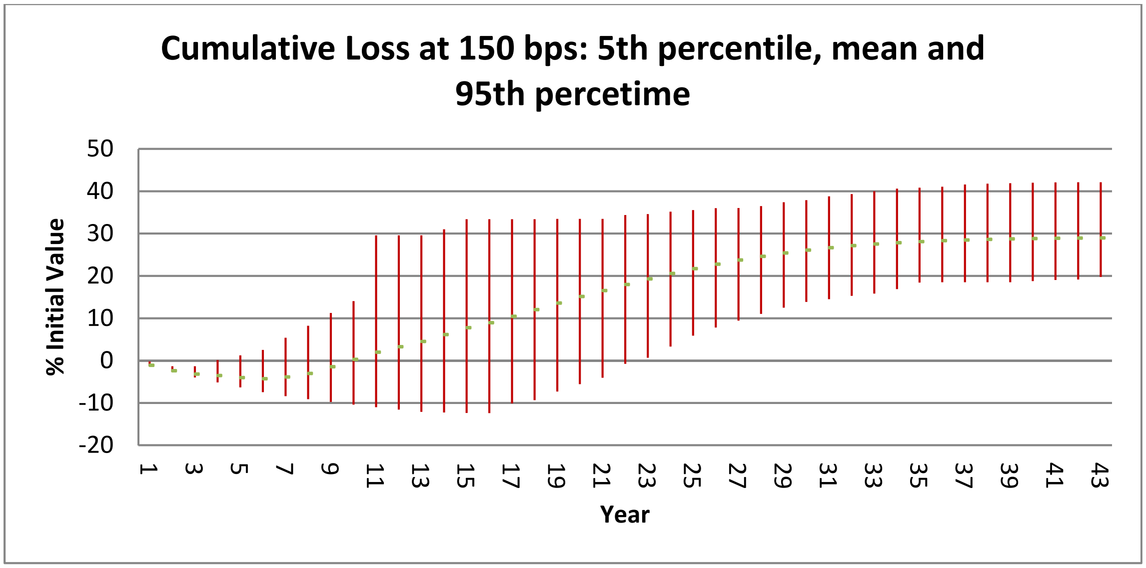


Figure 27: Portfolio Cumulative Losses under Japan Scenario and 100 bps Price

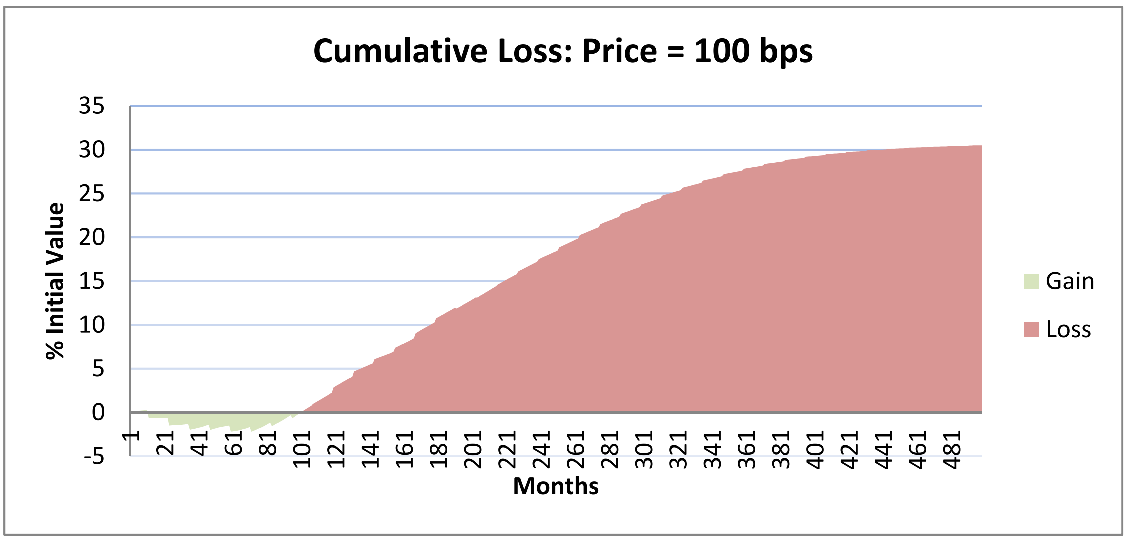


Figure 28: Cumulative Contract Level Losses under Japan Scenario and 100 bps Price

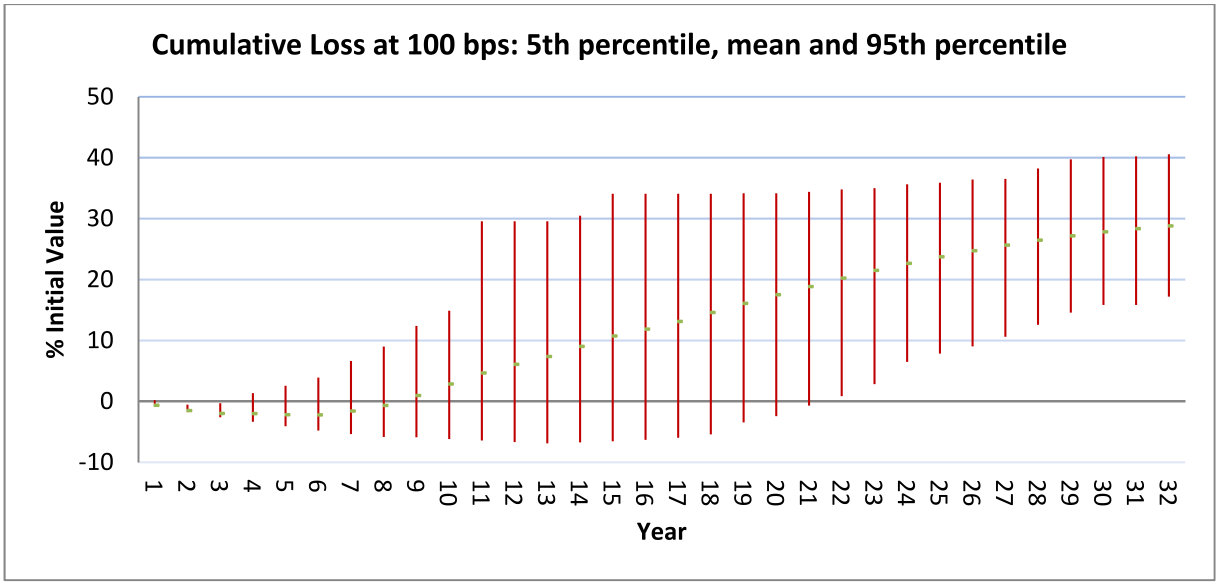


Figure 29: GMAB Contracts Cumulative Losses at 150 bps: 5th and 95th percentiles

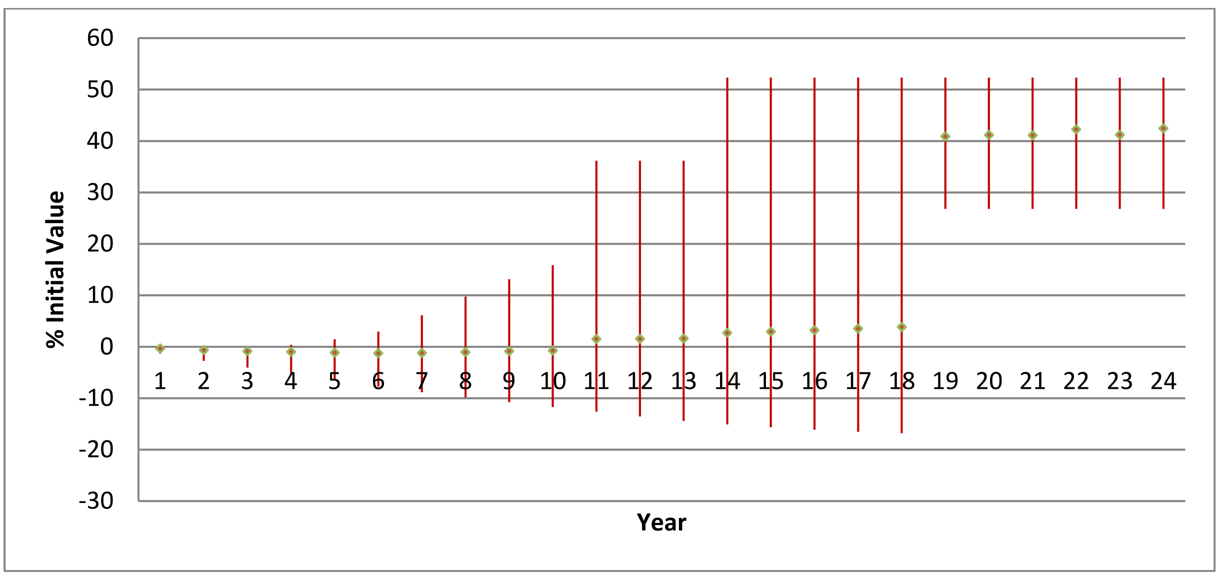


Figure 30: GMWB Contracts Cumulative Losses at 150 bps: 5th and 95th percentiles

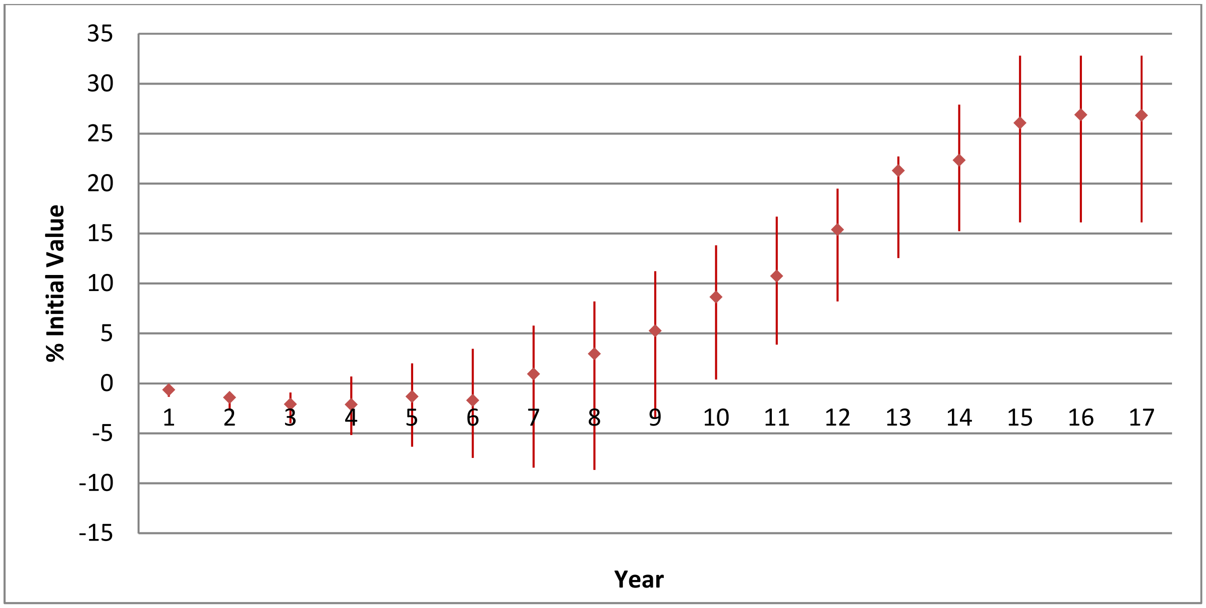


Figure 31: GMIB Contracts Cumulative Losses at 150 bps: 5th and 95th percentiles

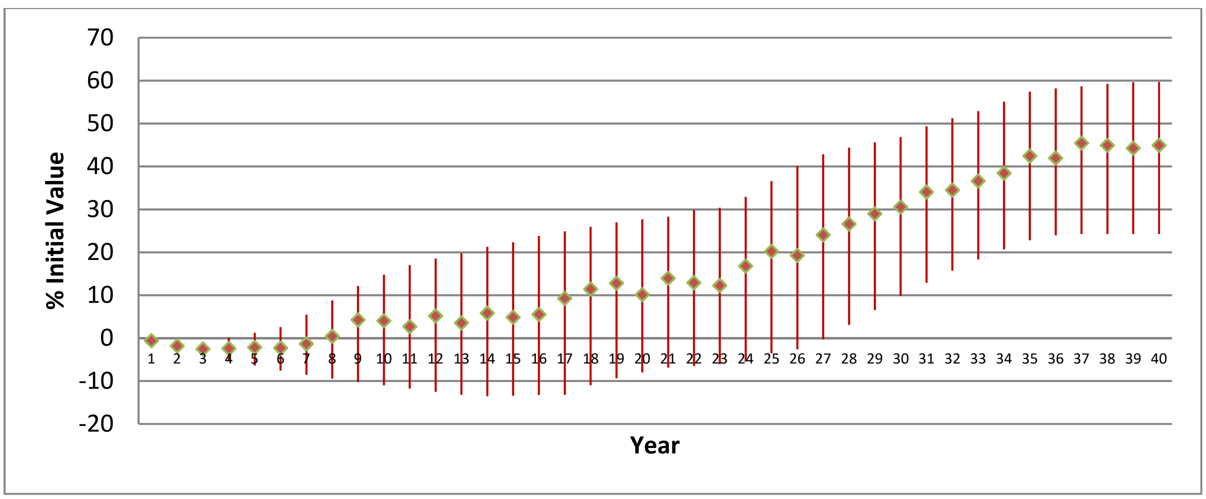


Figure 32: GMDB Contracts Cumulative Losses at 150 bps: 5th and 95th percentiles