Output Spillovers from U.S. Monetary Policy: The Role of International Trade and Financial Linkages

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Abstract:

We estimate that U.S. monetary policy has sizable spillover effects on global economic activity. In response to a surprise increase in the federal funds rate of 25 basis points, real output in our sample of 44 countries declines on average by 0.9% after three years. We find that international trade is a more important factor than international finance in explaining these spillovers. In particular, countries with a high share of exports and imports in output have 79% larger responses than countries with a low share, whereas we do not find significant heterogeneity depending on a country's financial openness. Bilateral trade linkages appear to be quantitatively important, as the network amplification effect accounts for 45% of the total spillover effect at the peak horizon. We conclude that trade networks could be an important ingredient of theoretical models focusing on the international effects of U.S. monetary policy shocks.

Keywords: financial linkages, international spillovers, monetary shocks, trade networks

JEL Classifications: E52, F42, F44, G15

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1 Introduction

The transmission of shocks in the global economy and the cross-country comovement of business cycles are among the central questions of international macroeconomics (e.g., Kose, Otrok, and Whiteman 2003; Lumsdaine and Prasad 2003; Imbs 2004; Baxter and Kouparitsas 2005; Aguiar and Gopinath 2007). In this context, international trade and financial linkages may be an important channel through which economic shocks propagate across countries.[1] For example, a contractionary U.S. monetary policy shock may reduce U.S. demand for foreign goods through the income effect, with a negative impact on foreign economies. At the same time, the supply of credit provided by U.S. banks may tighten, which would then reduce the liquidity they supply to foreign banks, thereby tightening international credit availability. In addition, a chain of indirect effects propagating through international trade and financial linkages could further amplify these spillovers. Therefore, some important empirical questions include (1) how strong these spillover effects are; (2) through which networks do these effects propagate across countries; and (3) whether the indirect spillover effects are quantitatively important.

   In this paper, we contribute to this debate by analyzing how U.S. monetary policy shocks affect global economic activity by transmission through international trade and financial networks. We study U.S. monetary shocks because they are often perceived as an important driver of international business cycles, due to the size of the U.S. economy and the dollar's role as a dominant currency (e.g., Goldberg and Tille 2008; Gopinath and Stein 2018). As U.S. monetary shocks likely affect both international trade and finance, they can help us evaluate the relative importance of the two propagation channels. We also take advantage of a long-standing literature dedicated to the identification and analysis of the real effects of monetary shocks in the United States (e.g., Romer and Romer 2004; Nakamura and Steinsson

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[1]As transmission channels for shocks, the previous literature has emphasized both trade (e.g., Frankel and Rose 1998; Eichengreen and Rose 1999; Glick and Rose 1999; Forbes 2004) and financial linkages (e.g., Kaminsky and Reinhart 2000; Van Wincoop and Yi 2000; Van Rijckeghem and Weder 2001).

2018).[2] Our paper extends this analysis to foreign economies.[3]

To estimate the spillover effects, besides U.S. monetary shocks, we employ quarterly data on real GDP per capita, bilateral trade flows, bilateral international banking claims, balance of payments, international investment positions (IIPs), and other variables. Our baseline sample covers 44 countries during the period 1995–2017.[4] We employ a local projection method (Jordà 2005) with instrumental variables. To this end, we instrument the U.S. policy rate with high-frequency monetary shocks. Because a significant portion of our sample includes the zero lower bound (ZLB) period, our benchmark policy rate is the Wu and Xia (2016) shadow rate.

We document three major results. First, U.S. monetary tightening reduces foreign output, with larger effects in countries that are relatively more open to international trade. Second, monetary shocks generate significant indirect effects propagating through the network of bilateral trade linkages. Third, a country's IIPs do not appear to be an important factor in explaining the heterogeneous responses, and the associated indirect effects are small. Overall, trade linkages are more potent than financial linkages in explaining the international spillover effects of monetary shocks on *real* economic variables.

We start by measuring the average spillover effect in the full sample. We estimate that, in response to a U.S. monetary policy tightening of 25 basis points, a country's real GDP per capita decreases on average by 0.9% over a three-year horizon, with a statistically significant decline between two and four years after the shock. The estimates are robust to using alternative measures of policy instruments and to excluding the ZLB period. For example, our estimates are qualitatively similar when we use the actual federal funds rate or the one-year Treasury yield instead of the shadow rate.

We then estimate the heterogeneous responses (β_j) of output in country j to a U.S.

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[2]See Ramey (2016) for a comprehensive survey of this literature.

[3]Papers studying the spillover effects from U.S. monetary policy often focus on financial effects (e.g., Bruno and Shin 2015; Miranda-Agrippino and Rey 2015; Forbes, Hjortsoe, and Nenova 2018; Bräuning and Ivashina, forthcoming). A notable exception is Iacoviello and Navarro (2019).

[4]The bilateral claims data are available for only 22 countries and start in 2005.

monetary shock. To understand the transmission mechanism behind these spillover effects, we regress β_j on the degree of country j's openness to trade, measured as the output share of total trade, and on country j's degree of openness to foreign finance, using gross IIPs normalized by output. We also control for an individual country's level of development, its exchange rate regime, and other country-specific characteristics. We find that trade openness is positively associated with the spillover effects, whereas we do not find evidence that these spillover effects are larger in countries with a high degree of financial openness compared to countries with a low degree. Nor do we find significant differences depending on net exports, suggesting that gross trade flows are more important for our analysis than net trade flows. Quantitatively, the spillover effect, at its peak, is 79% stronger for high-trade countries than for low-trade countries. These differential effects hold both in countries with a fixed exchange rate regime and in countries with a flexible rate, and also when we focus on a homogeneous group of countries, which share a common monetary policy, such as the euro area members.

To quantify the endogenous amplification through the international trade network, we estimate a spatial model, wherein output changes in one country can affect output in other countries. This approach enables us (1) to measure the spillover effect for a given network; (2) to decompose the total effect into the direct and indirect effects; and (3) to rank the different networks according to the sizes of these effects. We measure the trade network using bilateral gross flows, and we also consider alternative networks such as those based on financial linkages.

Our results suggest strong amplification effects through the trade network: in the baseline model, 45% of the peak total spillover effect of U.S. monetary policy on foreign output is attributed to an indirect effect. We also find that the share of the indirect effect (relative to the total effect) increases somewhat over time, thus helping to explain the delayed output response. We do not find, however, that U.S. monetary policy shocks propagate differently through the export network than through the import network, or that they propagate through the network of net trade flows. In contrast to gross trade flows, we estimate

small and statistically insignificant indirect effects when using financial networks. Hence, gross trade linkages rather than net trade or financial linkages are associated with a strong network amplification of spillover effects from U.S. monetary policy.

Related Literature. This paper relates to several strands of literature. First, our paper contributes to the literature on global spillover effects stemming from U.S. monetary policy. In particular, several recent studies investigate the spillover effects that U.S. monetary shocks have on foreign financial markets, including foreign exchange markets and international banking (e.g., Bruno and Shin 2015; Miranda-Agrippino and Rey 2015; Forbes, Hjortsoe, and Nenova 2018; Bräuning and Ivashina, forthcoming). However, relatively few papers focus on the response of macroeconomic real variables (e.g., Iacoviello and Navarro 2019; Kim 2001). We contribute to this literature, but we focus on estimating the real spillover channels and amplification effects operating through the international trade and financial networks using identified monetary shocks.

Second, our paper relates to the literature that studies more broadly the role of finance and trade in the international transmission of shocks. As transmission channels, the previous literature has emphasized both trade (e.g., Frankel and Rose 1998; Eichengreen and Rose 1999; Glick and Rose 1999; Forbes 2004) and finance (e.g., Kaminsky and Reinhart 2000; Van Wincoop and Yi 2000; Van Rijckeghem and Weder 2001; Caballero and Krishnamurthy 2004). While the nature of the shock likely matters for the relative importance of the two transmission channels, we find that an important aggregate demand shock (i.e., U.S. monetary policy shock) is strongly transmitted to output through international trade networks, with international financial linkages playing less of a role.

Third, our paper provides direct evidence on the importance of network amplification in the propagation of shocks to macroeconomic aggregates. Theoretically, the literature has only recently started to study the implication of network effects in macroeconomics (e.g., Acemoglu et al. 2012; Carvalho and Gabaix 2013; di Giovanni, Levchenko, and Mejean

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2014). Focusing on input–output matrices, recent empirical work supports the relevance of such network effects in aggregate responses (e.g., Barrot and Sauvagnat 2016; Ozdagli and Weber 2017). In an international context, we show that a shock emanating from a single country, amplified through indirect linkages, can have sizable effects on the global economy.

This paper proceeds as follows. Section 2 summarizes the data used. Section 3 presents our empirical specifications and estimates of output spillovers from U.S. monetary policy, including the heterogeneous effects across countries and their determinants. Section 4 then analyzes spatial networks based on trade and financial linkages, and shows that trade linkages are associated with significant amplification effects. Section 5 concludes.

2 Data

We use quarterly data for 44 countries during the period 1995–2017. To measure real economic activity, we collect real GDP data (in local currency), depending on availability, from the International Monetary Fund (IMF) and the Organization for Economic Cooperation and Development (OECD). We compute per-capita measures using annual population data from the Penn World Table database (version 9.1), which we interpolate to a quarterly frequency.

As a measure of the U.S. monetary policy rate, we splice the federal funds rate data prior to the ZLB period with Wu and Xia's (2016) estimates of the shadow rate during the ZLB period. For simplicity, we refer to this measure as the policy rate. For a robustness exercise, we also use the one-year Treasury yield (Gertler and Karadi 2015). We use three measures of monetary shocks, based on the high-frequency identification methods available from the recent literature: Gürkaynak, Sack, and Swanson (2005); Gertler and Karadi (2015); and Nakamura and Steinsson (2018). These data are publicly available at the authors' websites.

To study the transmission of monetary policy through the international economic network, we rely on bilateral trade flows (exports and imports) obtained from the United Nations' Comtrade database. To measure trade openness at the country level, we obtain data

on total exports and imports from the World Bank. To measure the network of international financial linkages, we collect quarterly data on bilateral banking claims from the Consolidated Banking Statistics Claims database compiled by the Bank for International Settlements (BIS). These data report, for example, the claims of all Italian banks on all Japanese counterparties. There are two main reasons to rely on the BIS data. First, these are the only data on international financial linkages that are consistently available for a relatively large number of countries and a relatively long period. Second, international banking flows (in contrast to investment fund flows, for example) comprise a major portion of financial linkages, especially for developing countries (e.g., Bräuning and Ivashina, forthcoming). In sum, banking flows strongly correlate with other types of financial flows. In contrast to the data on bilateral trade flows, the BIS data start in 2005 and are available for only 18 of the 44 countries in our full sample. To measure the overall degree of a country's financial openness, we used the gross and net IIP data from the IMF's International Financial Statistics (IFS).

In addition to these main variables, we use several other measures. We take current account balances from the IFS Balance of Payments and quarterly nominal and real exchange rates from the OECD. We also use Shambaugh's (2004) classification of exchange rate regimes, extended through the end of our sample period, and the BIS development classification.

The list of countries and key summary statistics are provided in Appendix Table A.1.

3 Spillovers from U.S. Monetary Policy

3.1 Average Effects

We first focus on estimating the semi-elasticities of output with respect to U.S. monetary policy. We employ the local projection method (Jordà 2005) with instrumental variables in a panel setup. We first estimate the average output response across all countries. For each

response horizon h between 0 and 20 quarters, we estimate the following specification:

$$y_{i,t+h} = \alpha_i^h + \beta^h\, r_t + \sum_{k=1}^{4} \gamma_k^h\, y_{i,t-k} + \sum_{k=1}^{4} \delta_k^h\, r_{t-k} + \sum_{k=1}^{4} \zeta_k^h\, \boldsymbol{s}_{t-k} + \varepsilon_{i,t+h}^h, \qquad (1)$$

where the response variable $y_{i,t+h}$ is the logarithm of real GDP per capita h quarters ahead, r_t is the U.S. policy rate instrumented with the vector of shocks $\boldsymbol{s}_t$, $\varepsilon_{i,t+h}^h$ is the error term, and $\left\{\alpha_i^h,\ \beta^h,\ \gamma_k^h,\ \delta_k^h,\ \zeta_k^h\right\}$ are estimated parameters. We include lagged output to control for the predetermined path, and the lagged policy rate and shocks to account for serial correlation. The sequence of coefficients $\{\hat{\beta}^h\}$ is the estimated average impulse-response function. Because our dependent variable is in logs, $100 \cdot \hat{\beta}^h$ can be interpreted as the percentage change in output per capita in response to a 1 percentage point (surprise) increase in the U.S. policy rate. We report standard errors two-way clustered at quarters and countries. This conservative approach allows for the arbitrary correlation of residuals both within a country and within a quarter (i.e., we account for serial correlation in local projections as well as for the contemporaneous comovement of errors across countries).

Because of the well-known problem of identifying macroeconomic shocks (Ramey 2016), we instrument the federal funds rate with several recently proposed monetary policy surprises that exploit changes in asset prices occurring around policy announcements made by the Federal Open Market Committee. We consider three different shock measures: the federal funds rate shock from Gürkaynak, Sack, and Swanson (2005); the policy surprise by Gertler and Karadi (2015); and the policy news shock from Nakamura and Steinsson (2018).[5] Those high-frequency shocks are arguably exogenous to economic conditions (both in the United States and abroad), allowing the identification of a causal relationship between monetary policy and economic activity. In our baseline specification, we use all three policy surprises as instruments, as suggested by various tests of instrument relevance and identifying restrictions; we discuss this choice and the robustness of our results later in this section.

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[5]We aggregate the shocks to match the quarterly frequency in our analysis.

Figure 1 presents the impulse response function of output to a tightening U.S. monetary shock. Output decreases significantly, with delayed effects materializing after 8 quarters and peaking at 12 quarters after the shock. At the three-year horizon, output falls by 0.9% in response to a 25 basis point surprise increase in the U.S. policy rate. The effects for horizons longer than 3.5 years are small and not statistically different from zero (see Table 1).[6]

These baseline estimates indicate a sizable output semi-elasticity. However, it is important to highlight that our estimates are identified by instrumenting the federal funds rate with the monetary policy shocks. For comparison, the standard deviation of the projected changes in the U.S. policy rate on the shocks is 21 basis points, or 45% of the standard deviation of the actual changes in the federal funds rate (47 basis points). Hence, a 25 basis point surprise increase in the federal funds rate is a rather large shock.

In our baseline specification, we use all three policy shocks as instruments, which enable us to exploit the different information about policy movements contained in the three measures. For example, the Gürkaynak, Sack, and Swanson (2005) shocks are based on movements in federal funds *futures* rates, while the Nakamura and Steinsson (2018) shocks exploit variation in longer-term yields, which could be more relevant in characterizing monetary policy during the ZLB period, when U.S. policymakers focused primarily on the longer end of the yield curve. The Gertler and Karadi (2015) shocks, in addition to high-frequency changes in asset prices, exploit information from the macroeconomic variables typically used in a VAR setup. Hence, each of these various shocks may have advantages over the others during certain periods in our sample, and thus using all three types may enrich the identification strategy when applied to heterogeneous samples.

In Table 1, we show more diagnostics taken from our baseline method using all three types of shocks. Overall, the statistical tests support the relevance and validity of our baseline specification. First, for all the horizons considered, we cannot reject the null hypothesis that

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[6]Appendix Figure A.1 shows the responses for other important macroeconomic variables. Consistent with U.S. monetary tightening, foreign currencies depreciate relative to the dollar, whereas foreign prices do not respond significantly. This evidence suggests that foreign central banks, on average, do not offset the U.S. shocks by implementing symmetric policies.

Figure 1: Average Output Response to a U.S. Monetary Policy Shock
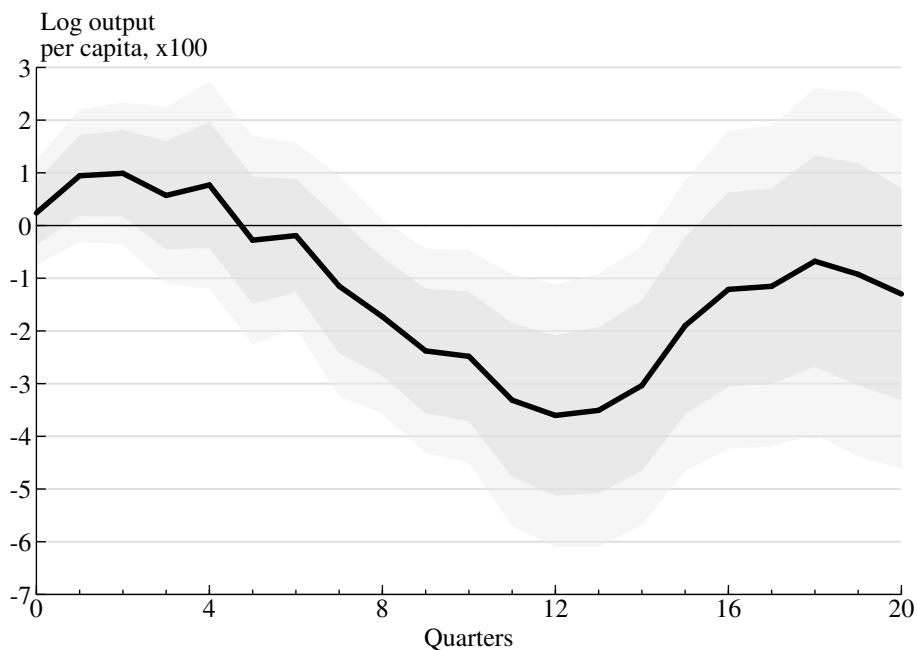
Source: All tables and figures are based on the authors' calculations using the data described in Section 2.
Note: This figure shows the responses of (log) real GDP per capita to a 1 percentage point monetary tightening. The estimates are based on quarterly data for 44 countries during the period 1995–2017. The responses are shown through 20 quarters after the shock. Shaded areas indicate 90% and 68% confidence intervals, based on standard errors two-way clustered at quarters and countries.

Table 1: Average Responses and Model Diagnostics

	(1) 8 qtrs	(2) 10 qtrs	(3) 12 qtrs	(4) 14 qtrs	(5) 16 qtrs
Log output p.c., x100	−1.73	−2.48**	−3.60**	−3.04*	−1.21
	(1.12)	(1.23)	(1.52)	(1.62)	(1.84)
Kleibergen–Paap LM statistic	9.734	9.734	9.734	9.734	9.734
p-value	0.021	0.021	0.021	0.021	0.021
Hansen J statistic	1.023	1.319	2.010	2.364	4.230
p-value	0.600	0.517	0.366	0.307	0.121

Note: This table shows the responses of (log) real GDP per capita to a 1 percentage point monetary tightening, as well as the IV diagnostics. The estimates are based on quarterly data for 44 countries during the period 1995–2017. Standard errors shown in parentheses are two-way clustered at quarters and countries. Significance at the 1%, 5%, and 10% level is denoted by ***, **, and *, respectively.

Figure 2: Alternative IV Strategies
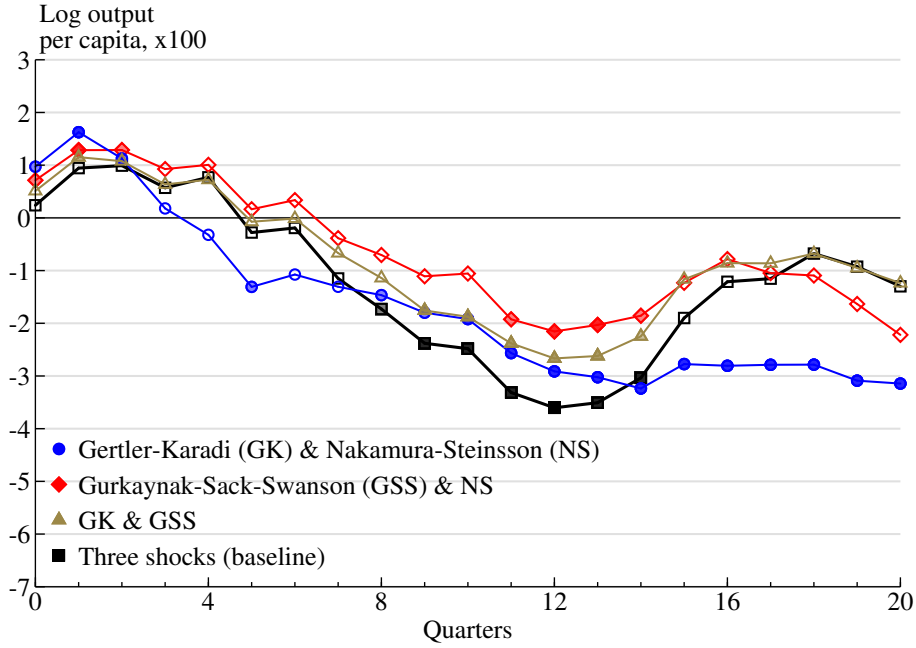
Note: This figure shows the responses of (log) real GDP per capita to a 1 percentage point monetary tightening, using alternative identification strategies. The estimates are based on quarterly data for 44 countries during the period 1995–2017. Solid and shaded symbols indicate significance at the 90% and 68% levels, respectively, based on standard errors two-way clustered at the country and quarter levels.

the shocks are uncorrelated with the error terms and therefore that it is correct to exclude them from the first-stage regression. Appendix Figure A.2 shows that using the policy rate in a simple OLS regression results in output responses of a different shape and magnitude. Thus, it is important to focus on the *exogenous* changes in the policy rate. Second, using the Kleinbergen–Paap LM test that returns a p-value of 0.02, we reject the null hypothesis that the monetary shocks are irrelevant instruments. The corresponding first-stage F-statistic is 22. In Figure 2, we present estimated impulse responses for different combinations of the instrumental variables. The overall pattern of the alternative output responses is similar to the baseline results, confirming that the spillover effect is delayed and varies between two and four years. Moreover, a comparison of the different instruments shows that our baseline strategy is supported by obtaining stronger first-stage diagnostics (Appendix Table A.2).

Appendix Figure A.3 shows that our baseline estimates are not sensitive to using indicators of the U.S. monetary policy stance other than the federal funds rate. In particular, we

address a potential concern that the federal funds rate may not be a good policy indicator during the ZLB period by excluding the ZLB period and, separately, by using the one-year Treasury rate (similar to Gertler and Karadi 2015). Thus, our results are not driven by the Great Recession of 2008–2009.[7]

3.2   Heterogeneous Spillover Effects

Next, we present heterogeneous responses across countries, depending on their openness to international trade and finance. We start by exploiting different subsamples of the data. Specifically, we group countries into high- and low-trade countries, according to their mean trade-to-GDP ratios during our sample period. Countries with a mean ratio above and below the median are classified as high- and low-trade countries, respectively.

Figure 3 presents the estimated average output responses for the two groups (Panel a) and the difference between them (Panel b). Panel (a) shows a stronger peak response for high-trade countries than for low-trade countries, especially pronounced at horizons between two and four years. Panel (b) shows that the difference between the two groups is significant, peaking at 2 percentage points at three years after the shock. In other words, the peak output response for high-trade countries is up to 79% larger than the peak output response for low-trade countries.

In Figure 4, we replicate these results for different subsamples of high- and low-trade countries. This figure shows that the differential responses hold for countries with a low degree of financial openness (low average IIP-to-GDP ratio). The results shown in Panels (a) and (b) confirm that the stronger output responses for high-trade countries are not driven by a positive correlation between openness to trade and openness to finance.[8] In Panels (c) and (d), we show similar results when we focus on countries with a floating exchange rate regime, according to the classification by Shambaugh (2004) extended through the end of our

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[7]We also do not find significant differences in size between the effects of monetary tightening and easing in our sample.
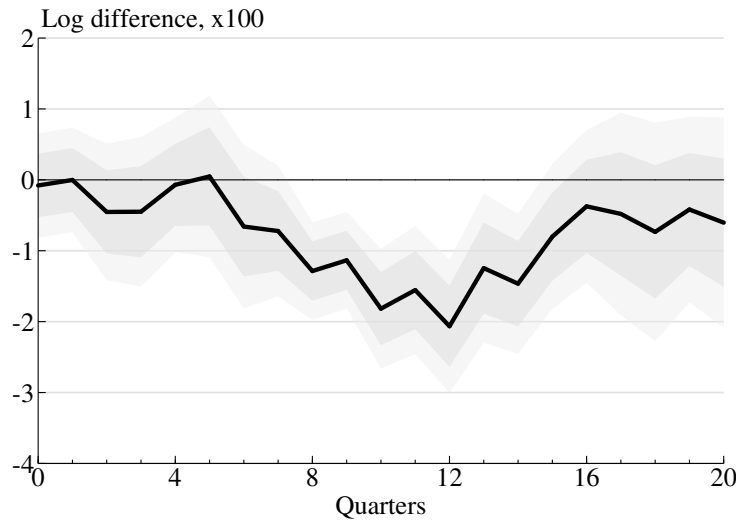
[8]The corresponding correlation coefficient is 0.4.

Figure 3: Differential Effects of U.S. Monetary Policy in High- and Low-Trade Countries

(a) Average Responses
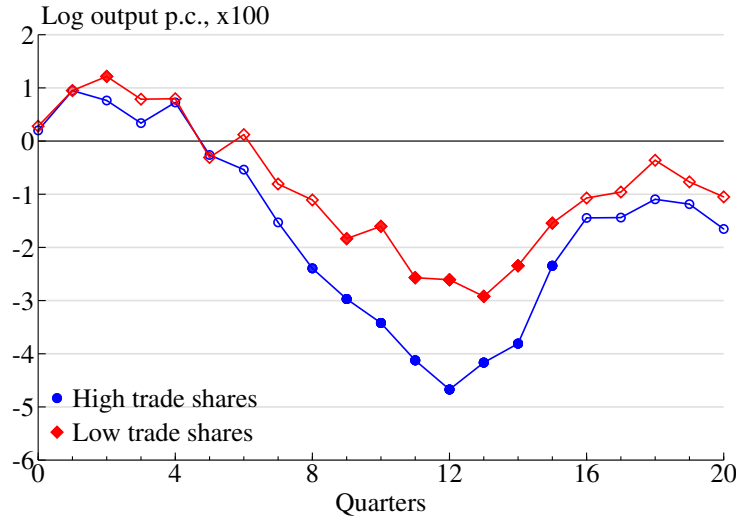
(b) Difference between High- and Low-Trade Samples
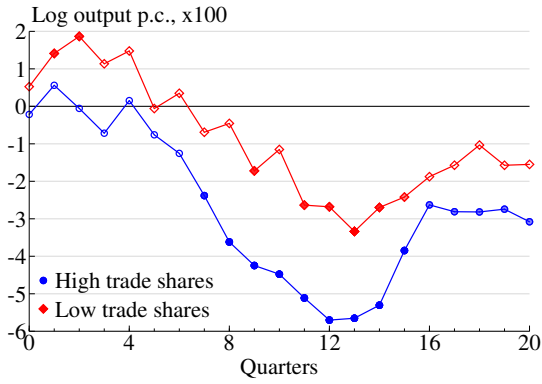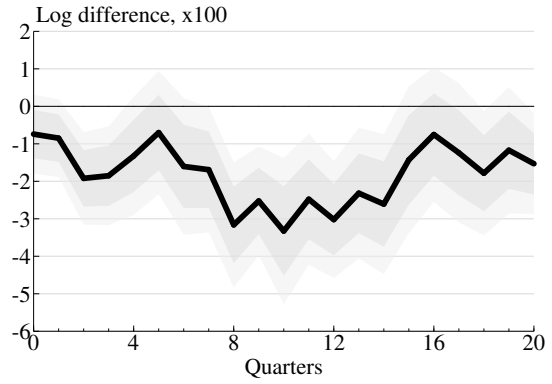

Note: Country groups are based on average trade-to-GDP ratios during our sample period from 1995 through 2017. High (low) trade countries are defined as having an average trade-to-GDP ratio above (below) the median. In Panel (a), solid and shaded symbols indicate significance at the 90% and 68% levels, respectively; in Panel (b), 90% and 68% confidence intervals are shaded.

Figure 4: High- and Low-Trade Countries: Robustness

(a) Low IIP: Response

(b) Low IIP: Difference

(c) Floating Exchange Rate: Response
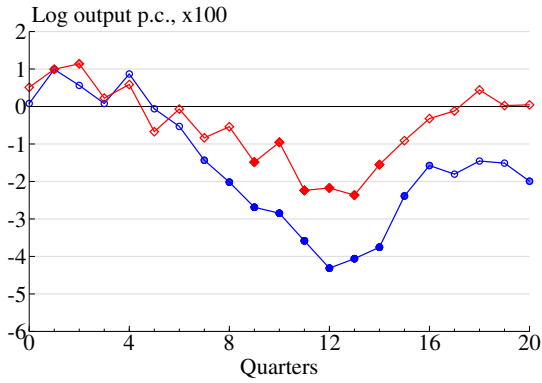
(d) Floating Exchange Rate: Difference
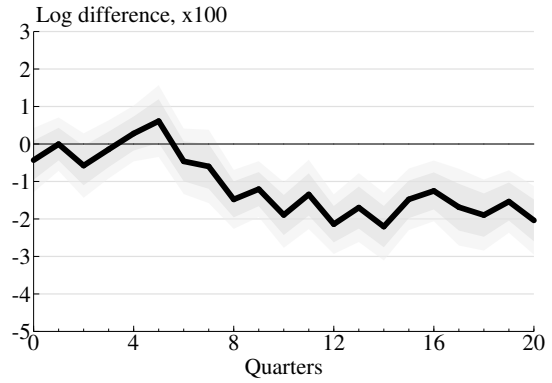
(e) Euro Area: Response
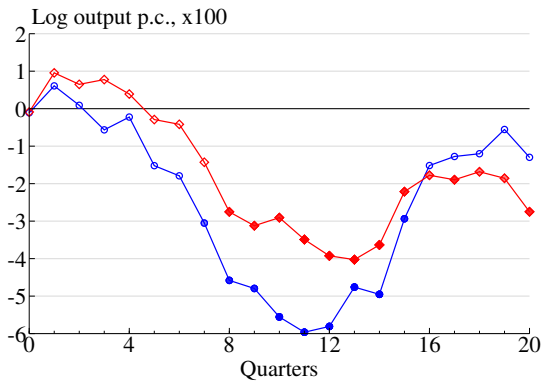
(f) Euro Area: Difference
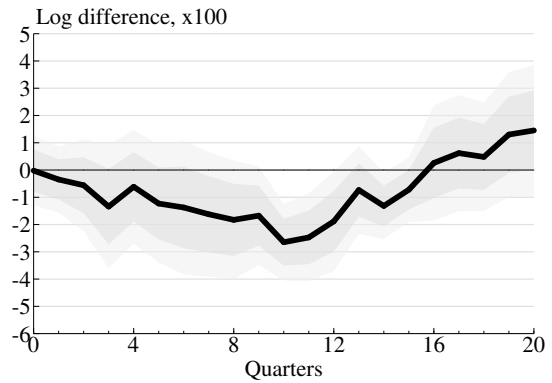
Note: Trade groups are based on the average trade-to-GDP ratios during our sample period from 1995 through 2017. High (low) trade countries are defined as having an average trade-to-GDP ratio above (below) the median. Low IIP countries are those with an average IIP-to-GDP ratio below the median. The exchange rate regimes are based on the Shambaugh (2004) classification extended through the end of our sample. In Panels (a), (c), and (e), the solid and shaded symbols indicate significance at the 90% and 68% levels, respectively; in Panels (b), (d), and (f), the 90% and 68% confidence intervals are shaded.

sample period. In Panels (e) and (f), we show that the differential also holds for countries that are part of the euro area. Hence, given the common monetary policy shared by these countries, we can rule out that the differential is potentially driven by the heterogeneous responses of foreign banks to the U.S. shock. In all the subsamples, we estimate differential effects that are close to our baseline result of 2 percentage points (at the peak).

We now estimate heterogeneous output responses and compare the roles that a country's openness to trade and finance plays in these responses. To do this, in Equation (1) we relax the assumption of a pooled response parameter $\beta_i^h = \beta^h$ for all countries i. Specifically, separately for each i, we estimate the following equation:

$$y_{i,t+h} = \alpha_i^h + \beta_i^h \, r_t + \sum_{k=1}^{4} \gamma_{i,k}^h \, y_{i,t-k} + \sum_{k=1}^{4} \delta_{i,k}^h \, r_{t-k} + \sum_{k=1}^{4} \zeta_{i,k}^h \, \boldsymbol{s}_{t-k} + \varepsilon_{i,t+h}^h, \qquad (2)$$

and then model β_i^h as

$$\hat{\beta}_i^h = b_0^h + b_1^h \, \text{Trade Openness}_i + b_2^h \, \text{Finance Openness}_i + b_3^h \, \text{Controls}_i + u_i^h, \qquad (3)$$

where *Trade Openness* is a country's average (over time) ratio of total trade (exports + imports) to GDP, in percentages, during our sample period; *Finance Openness* is a country's average total gross IIP (assets + liabilities) as a percentage of its GDP; and *Controls* include other country-specific characteristics (e.g., development indicator). To measure a country's openness, we also use net exports and net IIP, but we find that these variable have less importance than do the gross measures. We also include current accounts, since they contain income transfers that can be important for many developing countries.

We estimate the model in two steps. In the first step, we estimate the country-specific responses β_i^h in Equation (2). (As before, we use an instrumental variables approach.) Then, in the second step, we regress these responses on trade and financial openness as well as on other controls, as in Equation (3).[9] Our second-stage inference is based on heteroskedasticity-

[9]In an alternative approach, we estimate a panel regression wherein we interact the federal funds rate in

Figure 5: U.S. Monetary Spillovers, Trade, and Finance
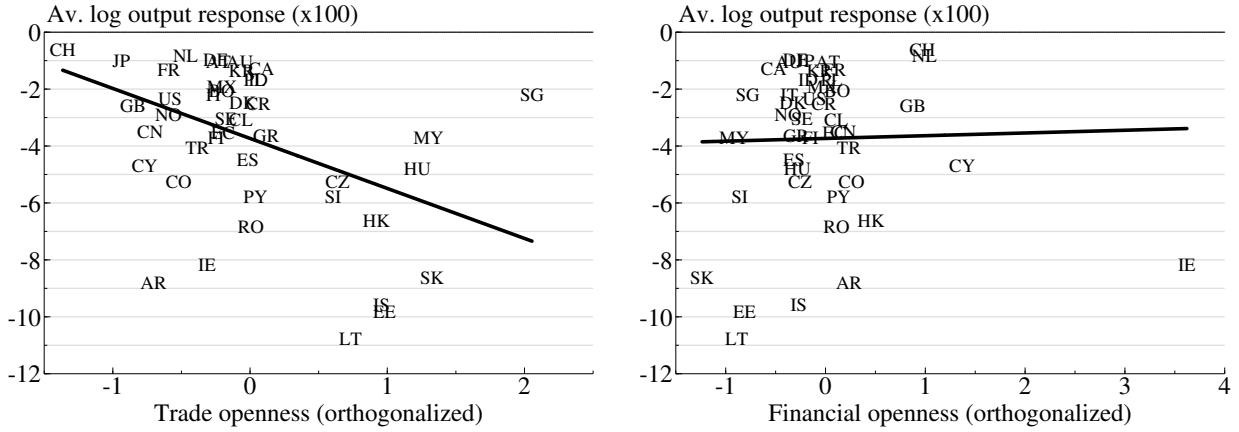
Note: This figure shows scatterplots for the spillover coefficient of a country after 12 quarters (vertical axis) against its average trade openness (horizontal axis, left panel) and financial openness (right panel), orthogonalized with respect to each other as well as to control variables. For visibility, the only country with a positive spillover coefficient (Israel) is dropped from the figure, but not from the analysis or the fitted lines. Appendix Figure A.4 shows that this has little effect on the slopes. The country codes are based on the ISO2 standard.

robust standard errors.

In Figure 5, we plot the output responses to a U.S. monetary shock in country i at a 12-quarter horizon (β_i^{12}) against its average degrees of trade openness (left panel) and financial openness (right panel), orthogonalized with respect to the control variables. We find a strong negative relationship between the spillover effects and trade openness, but no visible relationship between the spillover effects and financial openness. Note that Ireland has a large degree of financial openness and a large spillover coefficient, making the slope more negative.[10] However, even if financial openness can matter for some countries, it is generally not enough to make the slope negative in the full sample. Figure 6 shows that these patterns hold across relatively long periods. Consistent with our findings from the analysis of different subsamples, the effect of trade openness on the spillover effects is negative and significant, starting at the six-quarter horizon. The effect of financial openness, on the contrary, is close

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Equation (2) with the measures of trade and financial openness, as well as other covariates. This one-step procedure leads to results similar to those obtained from the two-step method.

[10]Appendix Figure A.4 shows that when we exclude Ireland from the sample, the slope becomes mildly positive.

Figure 6: Effects of Trade and Financial Openness on the Output Response

(a) Trade Openness

(b) Financial Openness
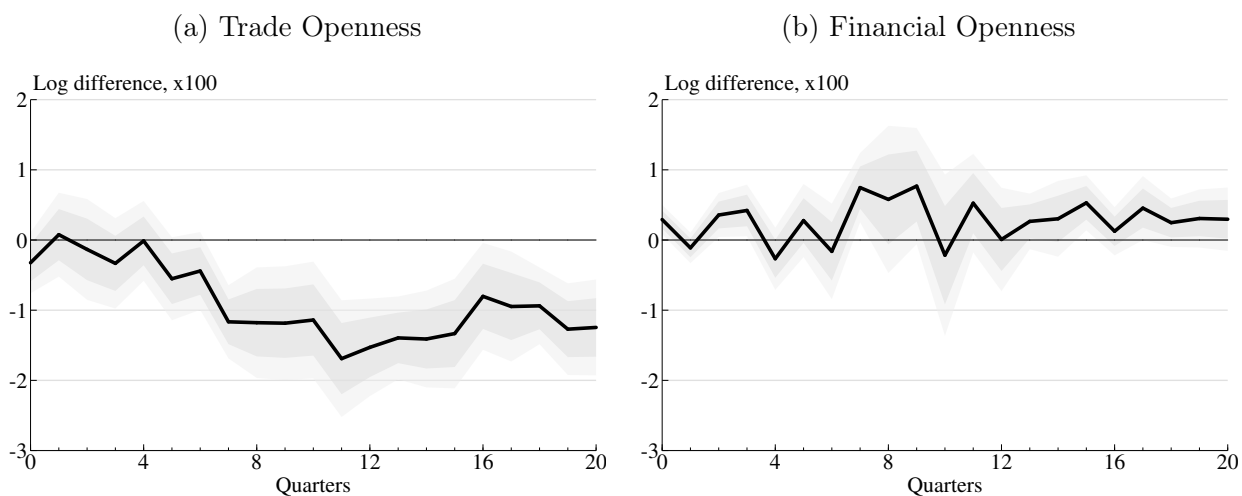
Note: Effects of a one-standard-deviation change in trade (Panel a) and finance (Panel b) on the output response to U.S. monetary policy. Estimates are orthogonalized with respect to financial openness (Panel a) or trade openness (Panel b) as well as control variables. The 90% and 68% confidence intervals are shaded.

Table 2: Effects of Trade and Finance Openness on Countries' Spillover Coefficients

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
	8 qtrs		10 qtrs		12 qtrs		14 qtrs		16 qtrs	
Total trade	−1.18**	−1.21**	−1.14**	−1.26**	−1.53***	−1.76***	−1.41***	−1.52***	−0.80*	−0.68
	(0.48)	(0.58)	(0.51)	(0.58)	(0.42)	(0.49)	(0.42)	(0.51)	(0.46)	(0.51)
Gross IIP	0.58	0.59	−0.22	−0.16	0.01	0.10	0.30	0.35	0.12	0.07
	(0.64)	(0.62)	(0.70)	(0.62)	(0.45)	(0.41)	(0.33)	(0.33)	(0.21)	(0.26)
Net Exports		0.02		−0.25		−0.05		−0.12		0.22
		(0.57)		(0.43)		(0.36)		(0.27)		(0.31)
Net IIP		0.15		0.07		0.73		0.23		−0.10
		(1.16)		(0.65)		(0.49)		(0.55)		(0.65)
Current account	1.42***	1.31	1.59***	1.72**	1.91***	1.48**	1.91***	1.84**	1.31***	1.23**
	(0.50)	(0.90)	(0.45)	(0.64)	(0.44)	(0.59)	(0.45)	(0.68)	(0.40)	(0.59)
Developing (indicator)	0.86	0.88	0.50	0.63	0.16	0.34	0.07	0.18	−0.40	−0.52
	(0.94)	(1.09)	(0.95)	(1.04)	(0.85)	(0.93)	(0.92)	(1.04)	(0.86)	(1.01)
Constant	−2.21***	−2.22***	−3.03***	−3.08***	−3.80***	−3.87***	−3.14***	−3.18***	−1.24***	−1.19***
	(0.51)	(0.58)	(0.46)	(0.49)	(0.45)	(0.45)	(0.42)	(0.45)	(0.37)	(0.41)

Note: The dependent variable is the estimated response $\hat{\beta}_i^h$ of the (log) real GDP per capita to U.S. monetary policy, obtained from Equation (3). All continuous independent variables are normalized by GDP and standardized to have zero mean and unit variance. Our sample includes quarterly data for 44 countries during the period 1995–2017. Heteroskedasticity-robust standard errors are in parentheses. Significance at the 1%, 5%, and 10% levels is denoted by ***, **, and *, respectively.

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to zero and, by and large, statistically insignificant.

Table 2 shows detailed regression results for output spillovers at key horizons. To make the coefficients comparable, we standardize all continuous regressors to have a zero mean and a unit variance, which also implies that the constant represents the average effect for developed countries. At a three-year horizon, a one-standard-deviation increase in total trade increases (in absolute value) the semi-elasticity of output responses by 1.5 (column 5), or about 40% relative to the mean response (regression constant). Moreover, the R-squared drops from 39% to 22% when we exclude trade from the regression. Interestingly, while gross trade flows are an important determinant of the spillover effects, the net trade flows (measured by net exports) are not. While a country's current account plays some role, it likely does so through income transfers rather than through the trade channel. We also do not find any significant differences between developed and developing countries.

4   Network Amplification and Indirect Effects

4.1   Measuring Network Effects

To what extent is the aggregate demand effect of U.S. monetary policy amplified through the international trade network? In other words, what share of the effect is driven by indirect output spillovers? To answer these questions, we employ a model in which not only U.S. monetary shocks may have heterogeneous direct effects, but also output comoves across countries, giving rise to an indirect effect. We estimate the network amplification effects by extending the model in Equation (2) to directly include international output spillovers:

$$y_{i,t+h} = \alpha_i^h + \beta_i^h \, r_t + \sum_{j \neq i} \eta_{i,j}^h \, y_{j,t+h} + \kappa \, \text{Controls} + \varepsilon_{i,t+h}^h, \qquad (4)$$

where $\eta_{i,j}^h$ is the theoretical effect of output in country j on output in country i. To associate the strength of these output spillovers with a network effect, we assume that the individual

output effects, $\eta_{i,j}^h$, are proportional to predetermined bilateral trade linkages, $w_{i,j}$, and the aggregate network effect, ρ^h. That is, we assume that $\eta_{i,j}^h = \rho^h w_{i,j}$ and focus on estimating the aggregate network effect (ρ^h) given the network structure. The size of the coefficients ρ^h determines the importance of the total spillover effect and the direct and indirect effects that make up this total effect.

To derive the expressions for direct and indirect effects, it is useful to switch to vector notation. We denote the vector of the individual countries' log output in quarter t as $\boldsymbol{y}_t = (y_{1,t}, \ldots, y_{i,t}, \ldots, y_{N,t})'$. Denote further the weighting matrix of elements $w_{i,j}$ as $\boldsymbol{W}$, setting $w_{i,i} = 0$. Thus, the model in Equation (4) can be written as

$$\boldsymbol{y}_{t+h} = \boldsymbol{\alpha}^h + \boldsymbol{\beta}^h r_t + \rho^h \boldsymbol{W} \boldsymbol{y}_{t+h} + \kappa \, \text{Controls} + \boldsymbol{\varepsilon}_{t+h}^h, \tag{5}$$

where the vector $\boldsymbol{\beta}^h$ collects the elements β_i^h, and the vector $\boldsymbol{\alpha}^h$ collects the country-specific intercepts, α_i^h. The residuals $\boldsymbol{\varepsilon}_{t+h}^h$ could be either i.i.d. Gaussian or correlated across countries, as in the spatial error specification. Our control variables are the same as in Equation (2).[11] Note that we simplified Equation (3) to assume that only α_i^h and β_i^h are heterogeneous parameters, while the slope coefficients on the controls are pooled across countries. For the identification and the conventional interpretation of the parameter ρ^h, we use a standard row normalization of $\boldsymbol{W}$. With this normalization, the spatial lag $\boldsymbol{W} \boldsymbol{y}_{t+h}$ contains the mean values of trading partners' log output growth weighted by the trade shares. (We discuss the weight matrix and the spatial lag in detail in Section 4.2.)

A crucial parameter summarizing the network amplification is ρ^h. If $\rho^h = 0$, the above model collapses to the standard linear model in Equation (2). Furthermore, solving for $\boldsymbol{y}_{t+h}$ and taking the derivative with respect to r_t—for simplicity, abstracting from the intercept

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[11]Our baseline specification is a spatial Durbin model, which includes a spatial lag, a spatial error, and spatial controls. In the appendix, we compare our results to estimates obtained from alternative spatial models.

and controls—we obtain

$$
\begin{aligned}
\frac{\partial \boldsymbol{y}_{t+h}}{\partial r_t} &= \left(\boldsymbol{I}_N - \rho^h \, \boldsymbol{W}\right)^{-1} \boldsymbol{\beta}^h \\
&= \boldsymbol{\beta}^h + \rho_h \, \boldsymbol{W} \boldsymbol{\beta}^h + \rho_h^2 \, \boldsymbol{W}^2 \boldsymbol{\beta}^h + \rho_h^3 \, \boldsymbol{W}^3 \boldsymbol{\beta}^h + \dots,
\end{aligned}
\tag{6}
$$

where $\boldsymbol{I}_N$ is the $N \times N$ identity matrix. Again, if $\rho^h = 0$, we recover the linear effects $\boldsymbol{\beta}^h$. If $\rho^h > 0$, the initial output responses induce endogenous amplification through an infinite chain of bilateral linkages.[12]

To highlight the role of network effects in the shock transmission, we can decompose the total effect in Equation (6) into a direct effect and an indirect effect. Denoting $\boldsymbol{J} \equiv (\boldsymbol{I}_N - \rho^h \, \boldsymbol{W})^{-1}$, the vectors of direct and indirect effects are

$$
\frac{\partial \boldsymbol{y}_{t+h}}{\partial r_t}^{\text{direct}} = \operatorname{diag}(\boldsymbol{J}) \, \boldsymbol{\beta}^h
\tag{7}
$$

$$
\frac{\partial \boldsymbol{y}_{t+h}}{\partial r_t}^{\text{indirect}} = \left(\boldsymbol{J} - \operatorname{diag}(\boldsymbol{J})\right) \boldsymbol{\beta}^h,
\tag{8}
$$

where $\operatorname{diag}(\boldsymbol{J})$ sets all off-diagonal elements of $\boldsymbol{J}$ to zero. We estimate the model parameters with maximum likelihood, and adjust standard errors to account for the instrumented U.S. policy rate using the Murphy and Topel (1985) procedure. The details are presented in Appendix B.

Special Case: $N = 3$

To provide intuition for the mechanics of the general model, we consider the case of $N = 3$ countries. The Jacobian matrix of this system is as follows:

$$
\frac{\partial \boldsymbol{y}_{t+h}}{\partial r_t} = \frac{1}{D}
\begin{pmatrix}
1 - \rho^2 \, w_{23}w_{32} & \rho \, w_{12} + \rho^2 \, w_{13}w_{32} & \rho \, w_{13} + \rho^2 \, w_{12}w_{23} \\
\rho \, w_{21} + \rho^2 \, w_{23}w_{31} & 1 - \rho^2 \, w_{13}w_{31} & \rho \, w_{23} + \rho^2 \, w_{21}w_{13} \\
\rho \, w_{31} + \rho^2 \, w_{32}w_{21} & \rho \, w_{32} + \rho^2 \, w_{31}w_{12} & 1 - \rho^2 \, w_{12}w_{21}
\end{pmatrix}
\begin{pmatrix}
\beta_1^h \\
\beta_2^h \\
\beta_3^h
\end{pmatrix},
\tag{9}
$$

---

[12]With standard normalizations, the model converges if $\left| \rho^h \right| < 1$.

Figure 7: Indirect Spillover Effects of External Monetary Policy on Output in Country 1

(a) Indirect Effect on y_1 from y_2 (b) Indirect Effect on y_1 from y_3
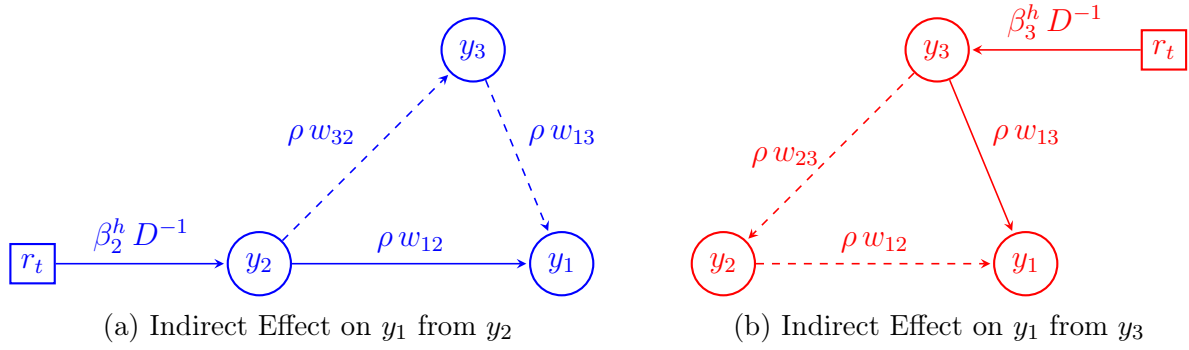
where

$$D \equiv \det(\boldsymbol{I}_3 - \rho\,\boldsymbol{W})$$

$$= 1 - \rho^2\,w_{12}w_{21} - \rho^2\,w_{13}w_{31} - \rho^2\,w_{23}w_{32} - \rho^3\,w_{12}w_{23}w_{31} - \rho^3\,w_{13}w_{32}w_{21}.$$

Each diagonal element of the Jacobian matrix is larger than the corresponding β_i^h. Thus, a unit shock to the U.S. policy rate induces amplification as a result of indirect output spillovers propagating through the international trade network. This mechanism contrasts with the one at work in the linear model. The off-diagonal elements in Equation (9) measure the indirect spillover effects (i.e., how a change in one country's output affects output in other countries).

Solving for individual countries, we can express the total effect on output in country 1 as follows:

$$\frac{\partial y_{1,t+h}}{\partial r_t} = \overbrace{\beta_1^h\,D^{-1}\left(1 - \rho^2\,w_{23}w_{32}\right)}^{\text{Direct Effect}}$$
$$+ \underbrace{\beta_2^h\,D^{-1}\left(\rho\,w_{12} + \rho^2\,w_{13}w_{32}\right)}_{\text{Indirect Effect from }y_2} + \underbrace{\beta_3^h\,D^{-1}\left(\rho\,w_{13} + \rho^2\,w_{12}w_{23}\right)}_{\text{Indirect Effect from }y_3}. \tag{10}$$

The total effect can be decomposed into the direct effect, due to the effect of r_t on output in country 1 (the top line of Equation 10), and the indirect effect (the bottom line), which

can be split further into the indirect effects emanating from the change in y_2 (in blue) and from the change in y_3 (in red).

Figure 7 demonstrates these indirect effects schematically. In Panel (a), we show the indirect effect on country 1 emanating from output in country 2. A unit change in the U.S. policy rate has an effect on output in country 2 of the size β_2^h. Through the bilateral linkage between countries 2 and 1, this effect is amplified by $\rho\, w_{12}$ and, through the multilateral linkage $2 \to 3 \to 1$, by $(\rho\, w_{32}) \cdot (\rho\, w_{13})$. Because the shock generates an infinite chain of responses, the sum is scaled up by $D^{-1} > 0$. To obtain the total indirect effect on country 1, one needs to consider the indirect effects emanating from all other countries—in our example, also from country 3 (Panel b). The share of the indirect effect in the total effect is a useful statistic summarizing both the strength of the network (ρ) and its structure ($\boldsymbol{W}$). Note that as $\rho \to 0$ (and so $D \to 1$), the direct effect approaches β_1^h and the indirect effect converges to zero.

4.2   The International Trade Network

In our baseline model, we use a weight matrix $\boldsymbol{W}$ based on bilateral gross trade flows (exports + imports) across 44 countries in our sample. We normalize each individual weight by the sum of the elements in each row, so that the total spillover effects depend on the relative sizes of the bilateral trade linkages but not on an individual country's overall exposure to international trade. We then analyze if the spillover effects measured through the lens of this network also depend on the country's degree of trade openness.[13] To abstract from the effects of output growth and spillovers on trade patterns, we fix the weights at their 1995 values (i.e., at the beginning of the sample period).

Figure 8 depicts the implied trade network. The size (and the shading) of the nodes correspond to the degree of network centrality. The larger and darker nodes represent countries that are important trading partners for other countries, and hence the shocks originating

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[13]We also consider alternative normalizations such as the largest-eigenvalue normalization, which accounts for overall trade openness, and reach qualitatively similar conclusions.

Figure 8: International Trade Network
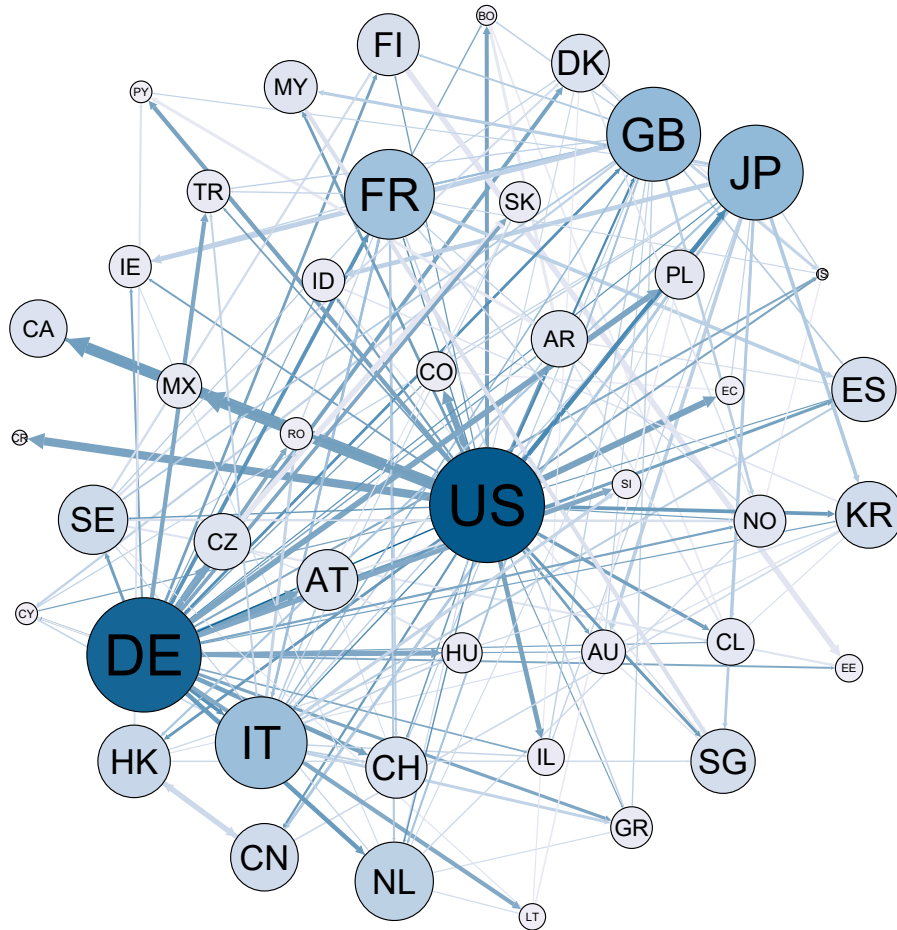
Note: The figure visualizes the total trade network for our baseline sample of 44 countries in 1995, as implied by our baseline weighting matrix $\boldsymbol{W}$. To increase visibility, only weights (total-trade shares) larger than 5% are shown. The country codes are based on the ISO2 standard.

in such countries, or propagating through them, are likely to be amplified. In contrast, the shocks originating in or reaching the countries represented by relatively small nodes are likely to die out.[14] As a measure of network centrality, we use the average share of a country in the total trade for every other country. Given our normalization, network centrality is computed as the average weight in a corresponding column of $\boldsymbol{W}$. Predictably, large open economies, such as the United States and Germany, are central to the international trade network. However, Sweden, a relatively small economy, has a centrality index comparable to China's, due to their relatively different degrees of openness. Note that this measure reflects not just a country's overall amount of trade but also the geographical diversification of its trading partners.

The size and shading of the *arrows* in Figure 8 correspond to bilateral trade shares: thicker and darker arrows represent larger trade shares. Naturally, countries with a high index of network centrality also have on average thicker arrows originating from them. For example, the United States (a large and dark node) is an important trading partner for many countries, in particular for Canada and Mexico, as indicated by the thick and dark arrow emanating from the United States to these two countries. Thus, through direct trade linkages, a U.S. demand shock is likely to affect many other countries, including Canada and Mexico.

Before discussing the results of the full spatial model, we illustrate the spatial correlation in GDP growth as implied by our baseline $\boldsymbol{W}$. Specifically, in Panel (a) of Figure 9, we plot the correlation between year-over-year GDP growth rates, $\boldsymbol{y}$, and the trade-weighted mean GDP growth of other countries, i.e., the spatial lag of GDP growth ($\boldsymbol{W}\,\boldsymbol{y}$).[15] The strong positive correlation means that a country's growth rate is related to growth of its trading partners as of 1995.

Because such positive correlation may partly be explained by the global business cycle

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[14]The node location is arbitrary and is chosen to enhance visibility.

[15]As before, our data include quarterly GDP growth rates for 44 countries for the period 1995–2017. To avoid cluttering up the figure, we present the bin scatterplots for 50 bins.

Figure 9: Spatial Correlation in Output Growth

(a) Raw   (b) Residualized
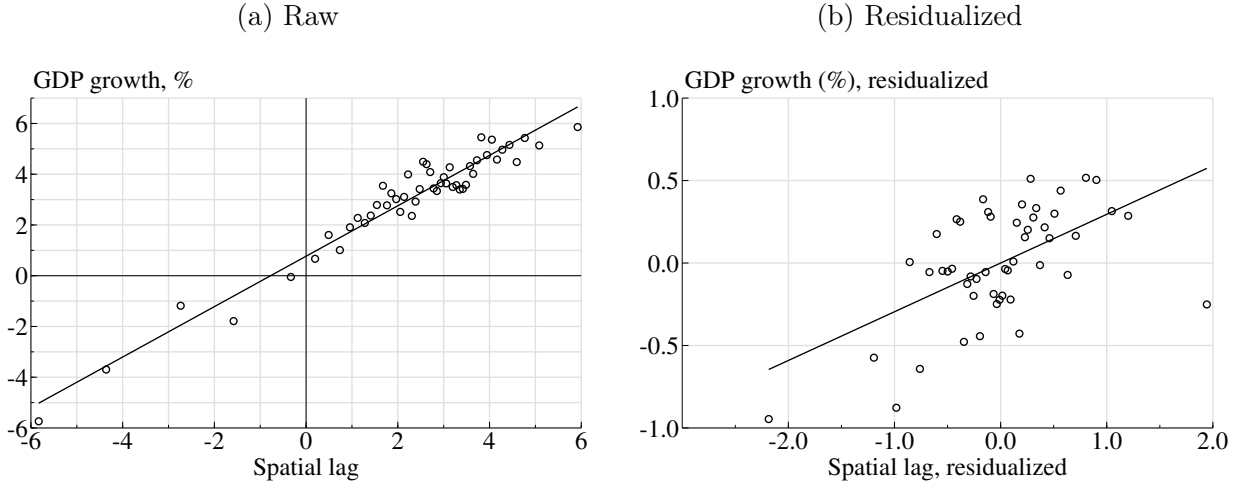
Note: This figure shows a bin scatterplot for 50 percentiles of GDP growth per capita against its spatial lag (Panel a). In Panel (b), we orthogonalize these variables with respect to country and time fixed effects as well as four lags of GDP growth.

(e.g., Kose, Otrok, and Whiteman 2003), we show in Panel (b) a positive association between growth rates and their spatial lags also when we partial out country and time fixed effects as well as four lags of GDP growth. Thus, the spatial correlation in GDP growth is not only driven by common time effects, or the tendency of high-growth countries to trade more often with other high-growth countries. Instead, we find a spatial correlation in the innovations of output growth relative to global and country-specific trends (as proxied with lagged GDP growth).

4.3 Spatial Model Estimates

In Table 3, we show the estimates of direct and indirect spillover effects obtained from our baseline spatial model with heterogeneous coefficients in Equation (5). The estimates of the spatial correlation parameter ρ are large and highly significant, supporting the relevance of endogenous feedback loops. While varying with the estimation horizon, the spatial-lag coefficient is close to 0.5 two to three years after the shock. The slope coefficient β, which due to nonlinear spillovers no longer measures the output response to a monetary shock, is

Table 3: Direct and Indirect Effects

	(1) 8 qtrs	(2) 10 qtrs	(3) 12 qtrs	(4) 14 qtrs	(5) 16 qtrs
Spatial lag, ρ	0.469***	0.489***	0.530***	0.584***	0.623***
	(0.066)	(0.061)	(0.058)	(0.053)	(0.049)
Slope, β	−1.02	−1.48*	−2.38***	−2.17**	−1.01
	(0.76)	(0.79)	(0.85)	(0.85)	(0.86)
Total effect	−1.63	−2.48	−4.49**	−4.41**	−1.62
	(1.42)	(1.53)	(1.75)	(2.02)	(2.27)
Direct effect	−1.04	−1.51*	−2.44***	−2.23**	−1.03
	(0.77)	(0.81)	(0.86)	(0.88)	(0.89)
Indirect effect	−0.59	−0.98	−2.05**	−2.18*	−0.59
	(0.66)	(0.74)	(0.94)	(1.18)	(1.38)

Note: This table shows the average total, direct and indirect effect of U.S. monetary policy on (log) real GDP per capita. The decomposition into direct and indirect effect is obtained using the total-trade bilateral linkages. Our sample includes 44 countries from 1995 through 2017. Significance at the 1%, 5%, and 10% level is denoted by ***, **, and *, respectively.

Figure 10: Share of Indirect Effects by Horizon
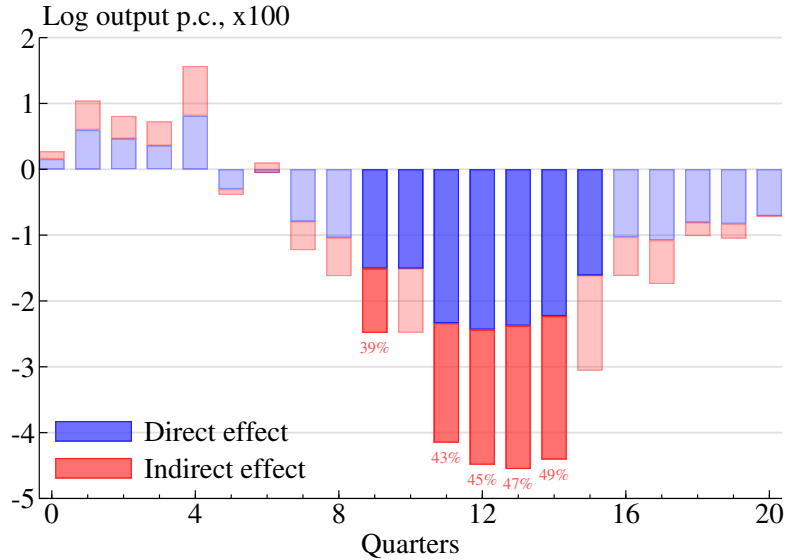
Note: The dependent variable is (log) real GDP per capita. Data are quarterly for 44 countries from 1995 through 2017. Decomposition into direct and indirect effect is obtained using the total-trade bilateral linkages. Solid bars represent significant effects at the 10% level. Percentage of indirect effects of significant responses are shown below the bars.

significantly negative at the horizons of 2.5 to 3.5 years.

We then use our estimates from the spatial model to compute the total, direct, and indirect spillover effects from U.S. monetary policy, using Equations (6)–(8). The average total effect after three years, when taking into account the trade linkages, increases in absolute value to −4.5. In comparison with the linear models, the size of the effect increases in absolute value by about 25%. Thus, ignoring spatial dependence leads to a substantial underestimation of the international spillover effects on output from U.S. monetary policy.

As Figure 10 shows, about half (43%–49%) of the total effect at the peak horizons can be attributed to the indirect effects.[16] In line with the estimates from our linear model, the responses at horizons shorter than two years or longer than four years are small and not significant.

Appendix Table A.3 shows our results when, as a measure of economic linkages, we use bilateral exports (Panel a) or bilateral imports (Panel b) instead of total trade. Similar to the baseline model, we find that about 45% of the total effect results from indirect spillovers when using export shares or import shares separately, and the estimates of the total effect are similar to the baseline results. Indeed, the correlation between these export and import weights is high.

We then compare our baseline results with those obtained from alternative specifications. In particular, we consider models wherein we set to zero the spatial error coefficient and/or the coefficients on the spatial controls, which are used in the baseline. When we remove both components, we obtain a basic spatial autoregressive (spatial lag) model. Setting the spatial error component to zero (Panel b of Appendix Table A.4) leads to an increase in the estimates of ρ to 0.65, and the share of indirect effects rises to 56%. The total effect, however, remains almost unchanged. Removing the spatial controls (Panel c) leads to similarly sized decreases in these two metrics (to 0.35 and 32%, respectively). While we observe some variation in the estimates across these various specifications, our conclusions about the role of spatial

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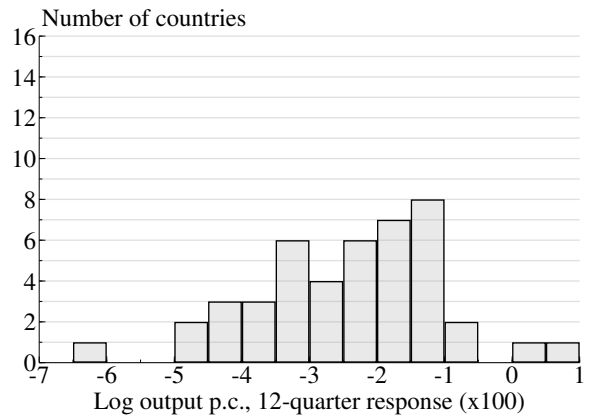[16]Appendix Figure A.5 shows that the share of the indirect effect is similar in the model with homogeneous β.

Figure 11: Heterogeneity in Spillover Effects across Countries
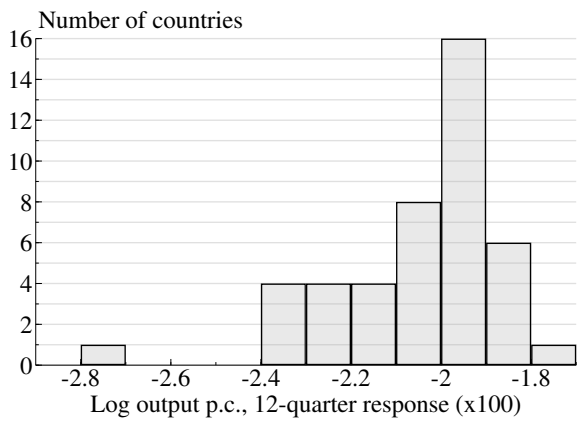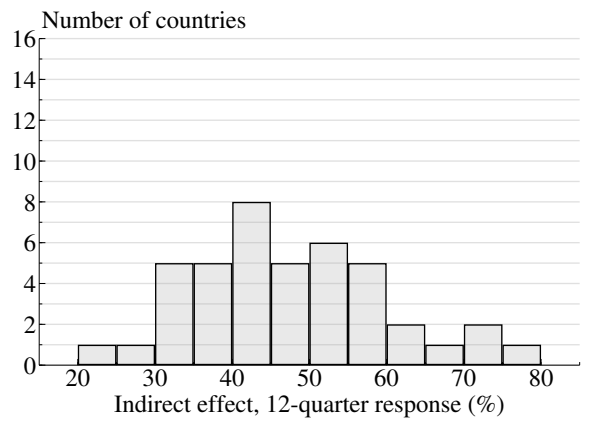
(a) Total Effect
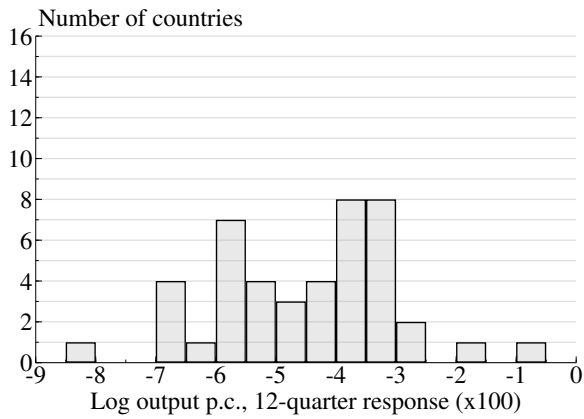
(b) Direct Effect

(c) Indirect Effect

(d) Share of Indirect Effect

Note: Decomposition of heterogeneous output responses after three years into direct and indirect effects is obtained using the total-trade bilateral linkages.

Table 4: Effects of Trade and Finance Openness on Direct and Indirect Effects

	(1)	(2)	(3)	(4)	(5)	(6)
	Direct effect		Indirect effect		Share of indirect effect, %	
Total trade	−0.62**	−0.60**	−0.06**	−0.06**	−4.56**	−4.33*
	(0.23)	(0.25)	(0.02)	(0.03)	(2.07)	(2.39)
Gross IIP	0.62	0.52	0.02	0.02	0.20	0.08
	(0.40)	(0.47)	(0.02)	(0.02)	(2.04)	(2.18)
Net Exports		−0.17		−0.17		−0.17
		(0.28)		(0.28)		(0.28)
Net IIP		−0.14		−0.14		−0.14
		(0.29)		(0.29)		(0.29)
Current account	0.10	0.30	−0.03	−0.08**	3.40*	3.68
	(0.19)	(0.37)	(0.02)	(0.03)	(1.80)	(3.20)
Developing (indicator)	−1.20***	−1.27***	−0.16***	−0.17***	−14.12***	−14.32***
	(0.41)	(0.42)	(0.04)	(0.04)	(3.53)	(3.81)
Constant	−1.84***	−1.82***	−1.95***	−1.95***	53.22***	53.27***
	(0.22)	(0.23)	(0.02)	(0.02)	(2.12)	(2.25)

Note: Heterogeneous direct and indirect effects of output spillovers after three years explained by country characteristics. All continuous independent variables are demeaned and have unit variance. The decomposition into direct and indirect effect is obtained using the total-trade bilateral linkages. Significance at the 1%, 5%, and 10% level is denoted by ***, **, and *, respectively.

spillovers and indirect effects in the international transmission of monetary policy still hold.

Similar to the linear case, we document significant cross-country heterogeneity in output spillovers, both in the size of the total effect and in the share of the indirect effect. The total effect, depicted in Panel (a) of Figure 11, has a significant mass of semi-elasticities in the interval between −7 and −3, with the largest negative effect above −8. As Panels (b) and (c) show, this result is due mostly to heterogeneity in the direct effects. The indirect effects are relatively homogeneous: For example, 34 of the 44 countries have an indirect effect in a narrow interval of −1.8 to −2.2. Consequently, the share of indirect effects (Panel d) varies significantly, between 20% and 80%. Importantly, for a majority of countries, the share of indirect effects are above 30% and hence play a nontrivial role in the spillover effects.

What can explain this heterogeneity? We explore the same factors from the linear model: namely, a country's openness to international trade and investment. In Table 4, we regress the direct, indirect, and relative indirect effects at the 12-quarter horizon on country char-

acteristics.[17] The results show that those countries that are relatively more open to trade exhibit larger direct and indirect effects than do countries with a low degree of trade openness. Given a stronger direct effect, we find that these countries have a smaller share of the indirect effects (relative to the total effects). Thus, U.S. monetary shocks affect countries with a relatively low openness to trade through indirect effects rather than through direct effects. This result is also consistent with a relative homogeneity in the size of the indirect effects. In contrast, we do not find that the direct and indirect effects depend on net exports or IIPs (whether gross or net). Thus, we again find that a country's degree of openness to trade is more important for the propagation of monetary shocks than is its degree of financial openness. Interestingly, for emerging markets, we estimate both a larger direct effect and a larger indirect effect, in contrast to the estimates obtained from the linear model. This evidence is consistent with the heterogeneous effects stemming from fiscal shocks (Miyamoto, Nguyen, and Sheremirov 2019).

International Trade and Financial Networks

We use international banking statistics data from the BIS to construct alternative spatial matrices that focus on financial linkages rather than on trade linkages. This approach helps us to assess whether shocks are propagated through financial networks in a similar way as through trade networks. Figure 12 visualizes the trade and financial linkages across a set of countries for which financial data are available.[18] While we observe certain similarities between the two networks (e.g., in both cases, the United States is an important partner for many countries), the two networks also exhibit important differences. For example, the United Kingdom plays a relatively more important role in the financial network than it does in the trade network. On the other hand, Germany has a larger centrality index in the trade network than in the financial network.

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[17]Because of one large positive indirect effect, we estimate the model with robust regressions.

[18]Note again that bilateral claims data are available for only 18 countries (for which we have also bilateral trade data) and start in 2005.

Figure 12: International Trade and Financial Networks (Constant Sample)

(a) Trade Network
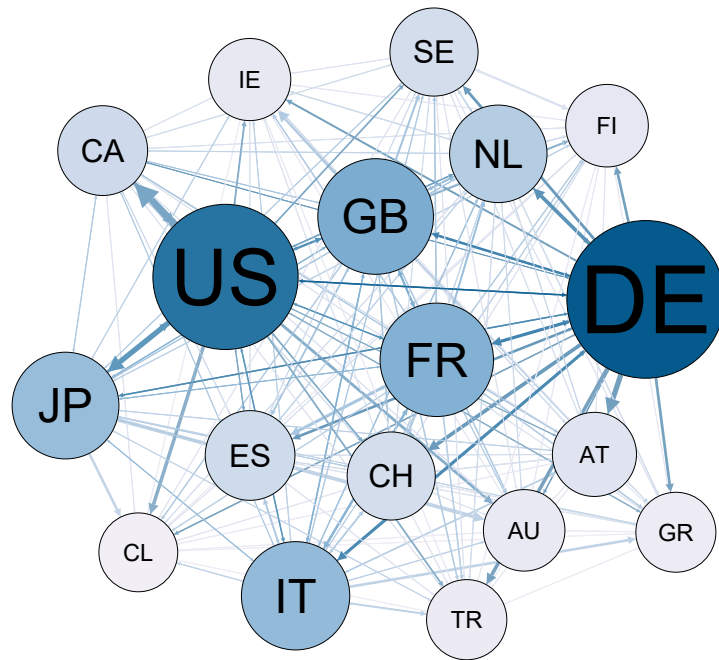
(b) Financial Network

Note: This figure shows the trade and financial linkages for the constant sample of 18 countries. The country codes are based on the ISO2 standard.

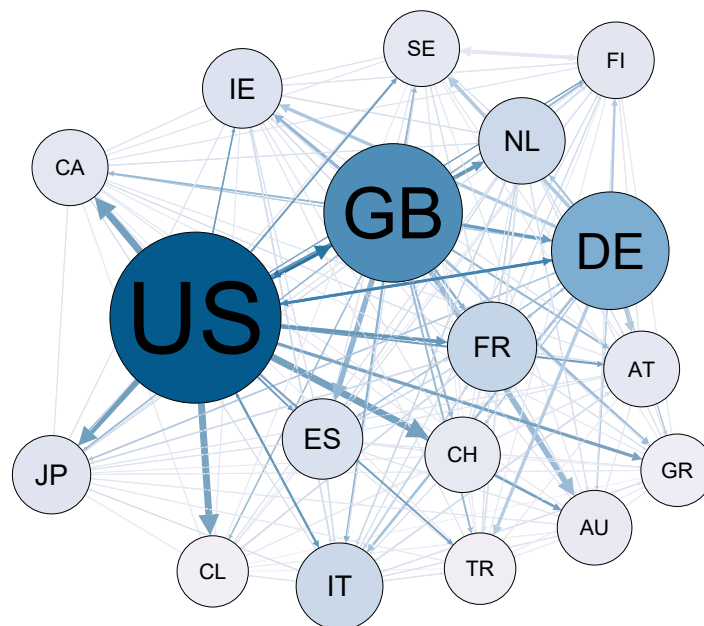Table 5: Trade and Financial Networks

	(a) Trade Linkages				
	(1) 8 qtrs	(2) 10 qtrs	(3) 12 qtrs	(4) 14 qtrs	(5) 16 qtrs
Spatial lag, ρ	0.222***	0.196**	0.200**	0.205**	0.227***
	(0.086)	(0.085)	(0.084)	(0.083)	(0.081)
Indirect effect	−0.42	−0.49	−0.69*	−0.55	−0.20
	(0.30)	(0.31)	(0.39)	(0.36)	(0.35)
% Indirect	21.4	18.7	19.0	19.2	19.2
	(b) Financial Linkages				
	(1) 8 qtrs	(2) 10 qtrs	(3) 12 qtrs	(4) 14 qtrs	(5) 16 qtrs
Spatial lag, ρ	0.188	0.132	0.073	0.030	0.005
	(0.130)	(0.127)	(0.141)	(0.141)	(0.139)
Indirect effect	−0.27	−0.25	−0.21	−0.06	−0.00
	(0.27)	(0.28)	(0.41)	(0.28)	(0.02)
% Indirect	18.1	12.7	7.0	2.8	0.3

Note: Panel (a) uses the trade network based on bilateral gross flows, and Panel (b) uses the financial network based on bilateral banking claims. Data are quarterly for 18 countries from 2005 through 2017. Significance at the 1%, 5%, and 10% level is denoted by ***, **, and *, respectively.

In Table 5, we compare estimates obtained from the spatial models using the trade and financial networks described above. In Panel (a), we re-estimate our baseline model based on the trade network for the sample with available IIP data, and find somewhat smaller average indirect effects than in the full sample: in this smaller sample, the indirect effects account for close to 20% of the total effects.[19] In Panel (b), using the financial network, we report small and insignificant estimates of the spatial lag parameter and of the indirect effect. Thus, we do not find evidence for the hypothesis that real output spillovers are transmitted through financial linkages rather than through trade linkages.

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[19]The drop in the spatial lag coefficient, ρ, is mostly driven by the smaller cross-section of countries and not by the shorter time series.

5 Conclusion

In this paper, we document three major results. First, U.S. monetary tightening shocks reduce foreign output, but with heterogeneous effects depending on the characteristics of an individual country. These spillovers are larger in countries that are relatively more open to trade. Second, monetary shocks generate significant indirect effects that propagate through the network of bilateral trade linkages. Third, a country's openness to foreign investment does not appear to be important in explaining these heterogeneous responses, and the associated indirect effects are small. Overall, trade linkages appear to be more potent than financial linkages in explaining international spillover effects of monetary shocks on real economic variables.

Based on these findings, we conjecture that both empirical and theoretical studies of international effects of shocks should incorporate measures of endogenous amplification through network effects. Abstracting from these indirect spillovers may result in mismeasurement of the effects and potentially yield both quantitatively and qualitatively different theoretical predictions. Policymakers, especially in large open economies, should consider these spillover effects—and the potential feedback loops—when designing optimal policy. To understand and to predict the effects of foreign shocks, small open economies could benefit from analyzing their trade linkages.

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A   Additional Results: Tables and Figures

Table A.1: Summary Statistics (in percentages)

Country	GDP Growth	Trade/GDP	IIP/GDP
Argentina	1.5	32.6	33.5
Australia	1.8	41.0	212.6
Austria	1.5	89.9	537.0
Bolivia	2.3	62.3	139.0
Canada	1.5	70.3	255.3
Chile	3.8	65.8	178.5
China	8.5	47.4	51.9
Colombia	2.6	36.3	78.2
Costa Rica	2.6	82.4	51.3
Cyprus	1.1	120.1	$1,044.5$
Czech Republic	2.6	109.9	149.4
Denmark	1.0	87.0	371.2
Ecuador	1.9	54.4	98.5
Estonia	4.8	139.3	222.6
Finland	2.1	73.9	419.7
France	1.2	53.3	420.2
Germany	1.2	67.0	333.7
Greece	0.5	51.2	233.7
Hong Kong	2.8	313.5	$1,437.7$
Hungary	2.4	132.4	312.6
Iceland	1.9	78.6	550.0
Indonesia	2.5	59.2	64.1
Ireland	3.0	162.3	$2,969.6$
Israel	1.6	70.7	167.6
Italy	0.3	49.6	222.1
Japan	-0.5	24.7	142.2
Korea, Republic of	3.8	76.4	100.0
Lithuania	5.3	109.6	111.9
Malaysia	2.6	189.5	149.3
Mexico	1.3	55.0	22.6
Netherlands	1.5	123.0	$1,153.9$
Norway	1.2	70.7	295.4
Paraguay	0.7	81.7	171.0
Poland	4.3	68.8	105.5
Romania	3.3	61.5	97.8
Singapore	3.0	370.8	$1,196.2$
Slovakia	4.2	138.1	138.8
Slovenia	2.2	114.4	177.9
Spain	3.6	54.5	262.1
Sweden	1.7	81.8	421.0
Switzerland	1.1	101.1	909.7
Turkey	5.0	47.2	81.5
United Kingdom	1.5	53.1	824.8
United States	1.7	25.5	217.0

Note: Country-level summary statistics for key variables in our baseline estimation sample.

Figure A.1: Responses for Exchange Rates and the Consumer Price Index

(a) Nominal Exchange Rate
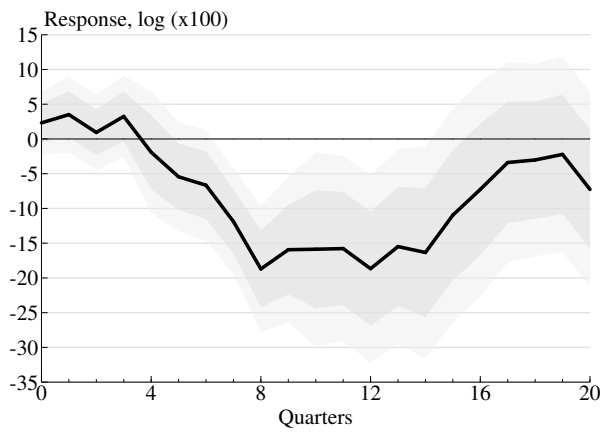
(b) Real Exchange Rate
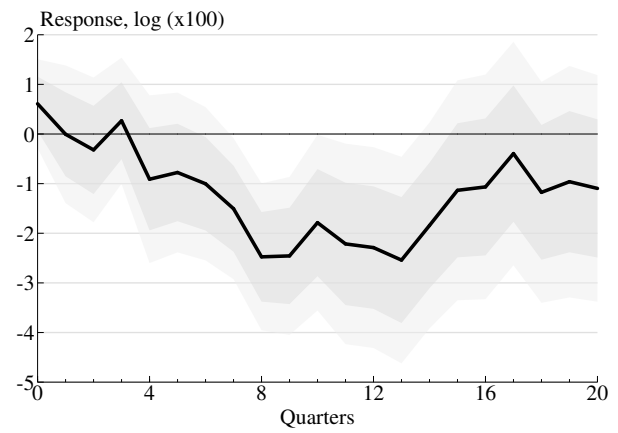
(c) Consumer Price Index
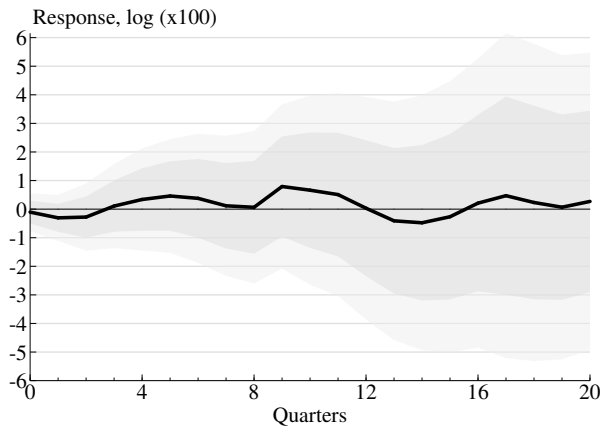
Note: This figure shows the responses of the logarithm of nominal and real exchange rates, expressed in U.S. dollars per the corresponding currency, and the logarithm of the consumer price index. The 90% and 68% confidence intervals are shaded.

Figure A.2: OLS and IV Estimates of Output Responses to U.S. Monetary Tightening

Note: The dependent variable is the logarithm of quarterly real GDP per capita for 44 countries from 1995 through 2017. Responses are shown for 0 through 20 quarters after a U.S. monetary tightening of 1 percentage point. Standard errors are two-way clustered at the country and time level. Shaded areas indicate 90% and 68% confidence intervals. Solid and shaded symbols indicate significance at the 90% and 68% levels, respectively.
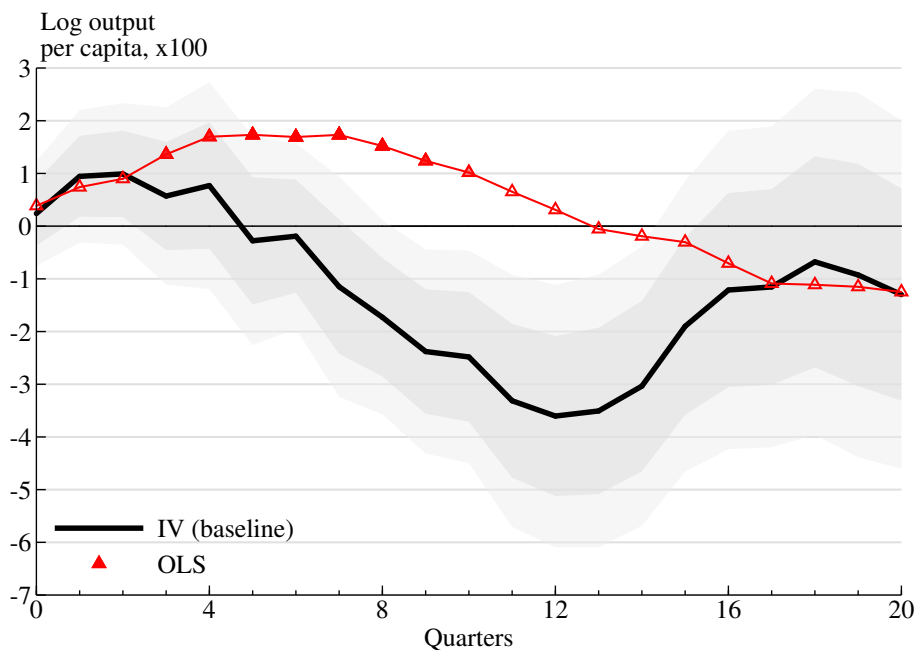
Table A.2: First-Stage Results for Different IV Strategies

	(1) Baseline	(2) NS/GSS	(3) NS/GK	(4) GSS/GK
Kleibergen–Paap LM statistic	9.734	7.619	7.077	10.308
p-value	0.021	0.022	0.029	0.006
Hansen J statistic	2.010	0.533	1.077	3.409
p-value	0.366	0.466	0.299	0.065

Note: First-stage diagnostics of instrument relevance and validity for selected alternative instruments of the federal funds rate. GSS is the federal funds rate shock from Gürkaynak, Sack, and Swanson (2005); GK is the policy surprise in Gertler and Karadi (2015); and NS refers to the policy-news shock from Nakamura and Steinsson (2018). Response horizon: 12 quarters.

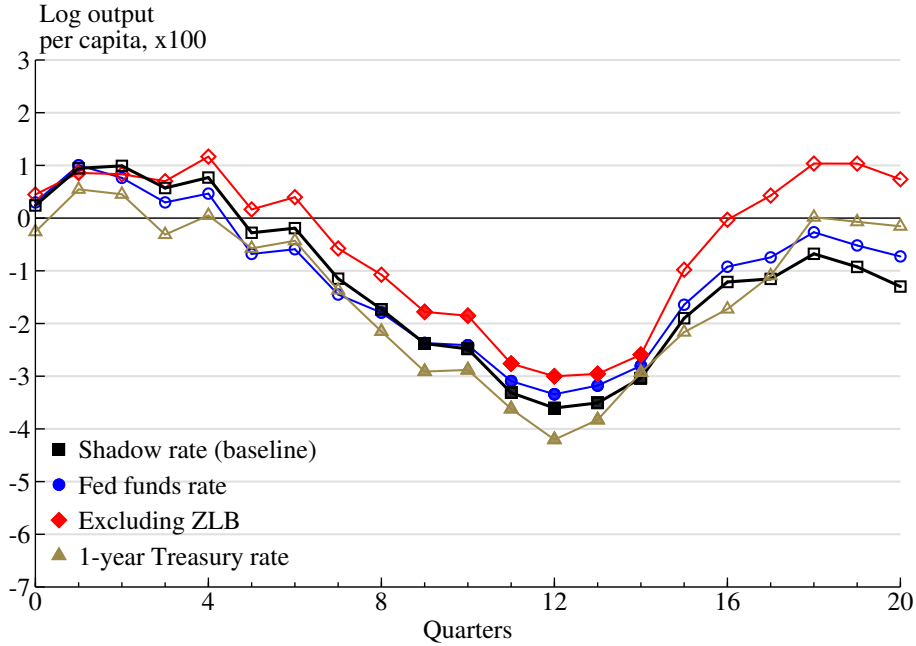# Figure A.3: Alternative U.S. Monetary Policy Measures

Note: The dependent variable is the logarithm of quarterly real GDP per capita for 44 countries from 1995 through 2017. Responses are shown for 0 through 20 quarters after a change in U.S. monetary policy. Standard errors are two-way clustered at the country and time level. Solid and shaded symbols indicate significance at the 90% and 68% levels, respectively.

Figure A.4: Country Characteristics Affecting U.S. Monetary Spillovers: The Role of Outliers
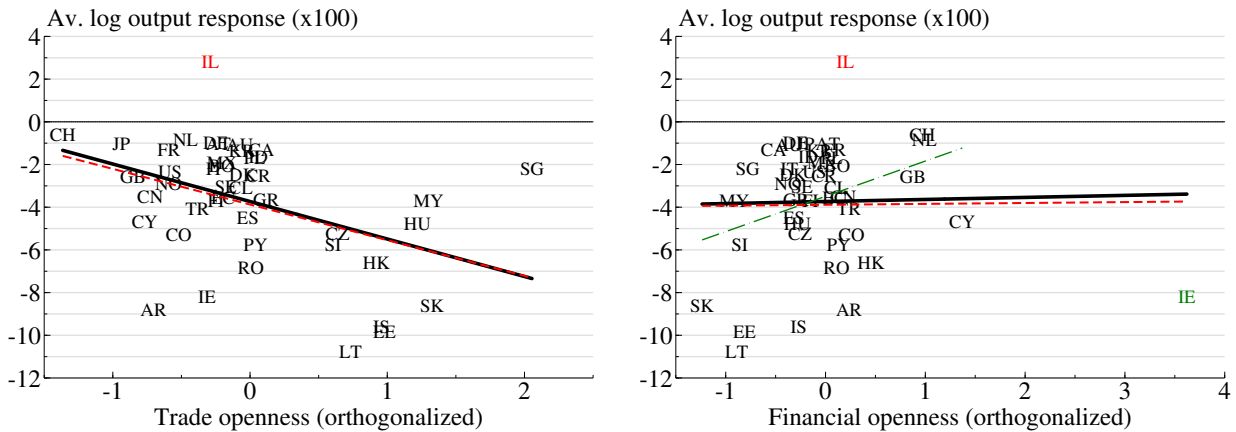
Note: This figure explores the role of outliers in Figure 5. The black solid lines show the fit in our baseline sample. The red dashed line excludes Israel from the sample, as this country has a positive spillover coefficient. Excluding Israel has a negligible effect on the slopes. The green dash-dot line in the right panel excludes Ireland, which has a disproportionately large degree of financial openness. Including Ireland makes the slope more negative—supporting some role for financial openness in the spillover effects—but not enough to make the overall slope negative.

Figure A.5: Share of Indirect Effects by Horizon: Pooled β
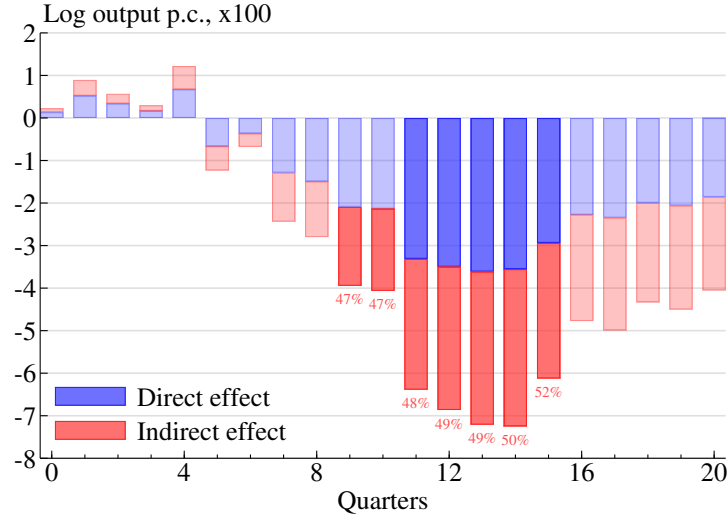
Note: The dependent variable is (log) real GDP per capita. Data are quarterly for 44 countries from 1995 through 2017. The decomposition into direct and indirect effects is obtained using the total-trade bilateral linkages. The solid bars represent significant effects at the 10% level. The percentage of indirect effects for significant output responses are shown below the bars.

Table A.3: The Role of Exports and Imports in Bilateral Trade Linkages

	(a) Export Linkages				
	(1) 8 qtrs	(2) 10 qtrs	(3) 12 qtrs	(4) 14 qtrs	(5) 16 qtrs
Spatial lag, ρ	0.462***	0.477***	0.520***	0.577***	0.621***
	(0.069)	(0.064)	(0.060)	(0.055)	(0.050)
Indirect effect	−0.60	−1.00	−2.04**	−2.18*	−0.69
	(0.63)	(0.70)	(0.90)	(1.13)	(1.34)
% Indirect	36.1	38.9	45.1	49.1	39.2
	(b) Import Linkages				
	(1) 8 qtrs	(2) 10 qtrs	(3) 12 qtrs	(4) 14 qtrs	(5) 16 qtrs
Spatial lag, ρ	0.468***	0.491***	0.530***	0.582***	0.617***
	(0.065)	(0.060)	(0.056)	(0.052)	(0.049)
Indirect effect	−0.58	−0.95	−2.03**	−2.14*	−0.53
	(0.67)	(0.76)	(0.96)	(1.20)	(1.40)
% Indirect	35.9	39.1	45.5	48.9	34.1

Note: This table presents estimates for the model results shown in Table 3, with linkages based separately on exports flows (Panel a) and imports flows (Panel b).

Table A.4: Alternative Spatial Models

	(a) Baseline model				
	(1) 8 qtrs	(2) 10 qtrs	(3) 12 qtrs	(4) 14 qtrs	(5) 16 qtrs
Spatial lag, ρ	0.469*** (0.065)	0.488*** (0.060)	0.529*** (0.053)	0.583*** (0.050)	0.621*** (0.049)
Indirect effect	−0.62 (0.64)	−1.00 (0.72)	−2.07** (0.89)	−2.21* (1.15)	−0.76 (1.36)
% Indirect	36.9	39.7	45.9	49.6	40.4
	(b) No spatial error				
	(1) 8 qtrs	(2) 10 qtrs	(3) 12 qtrs	(4) 14 qtrs	(5) 16 qtrs
Spatial lag, ρ	0.621*** (0.032)	0.625*** (0.033)	0.653*** (0.031)	0.686*** (0.029)	0.711*** (0.028)
Indirect effect	−0.78 (0.85)	−1.23 (0.90)	−2.50** (0.99)	−2.56** (1.25)	−0.78 (1.55)
% Indirect	48.0	50.1	56.2	58.1	44.2
	(c) No spatial controls				
	(1) 8 qtrs	(2) 10 qtrs	(3) 12 qtrs	(4) 14 qtrs	(5) 16 qtrs
Spatial lag, ρ	0.241*** (0.030)	0.308*** (0.031)	0.374*** (0.033)	0.460*** (0.033)	0.526*** (0.034)
Indirect effect	−0.32 (0.31)	−0.59 (0.41)	−1.34** (0.56)	−1.52* (0.79)	−0.45 (1.03)
% Indirect	19.1	24.8	32.2	38.4	29.8
	(d) Spatial lag only				
	(1) 8 qtrs	(2) 10 qtrs	(3) 12 qtrs	(4) 14 qtrs	(5) 16 qtrs
Spatial lag, ρ	0.311*** (0.023)	0.376*** (0.025)	0.454*** (0.026)	0.540*** (0.026)	0.606*** (0.025)
Indirect effect	−0.28 (0.24)	−0.56 (0.34)	−1.26** (0.50)	−1.30* (0.73)	−0.12 (1.00)
% Indirect	21.3	27.8	36.8	41.8	13.3

Note: This table presents estimates of alternative spatial specifications. The baseline model is in Panel (a). The weights are based on bilateral trade flows.

B Spatial Model: Estimation Details

Because we employ a two-stage instrumental variable approach, we face the problem of inconsistent standard errors from the second-stage estimation. In this appendix, we show how we compute the standard errors for the spatial models, following the general procedure of Murphy and Topel (1985). Dropping superscripts h to simplify notation, our second-stage model in Equation (5) can be written as

$$\boldsymbol{y}_{t+h} = \boldsymbol{\alpha} + \rho \boldsymbol{W} \boldsymbol{y}_{t+h} + \sum_{i=1}^{p} \boldsymbol{A_i} \boldsymbol{r_t} \beta_i + \boldsymbol{X_t} \boldsymbol{\beta_X} + \boldsymbol{\varepsilon}_t,$$

with $\boldsymbol{\varepsilon}_t = \lambda \boldsymbol{W} \boldsymbol{\varepsilon}_t + \boldsymbol{v}_t$ and $\boldsymbol{v}_t \sim N(0, \sigma_2^2 \boldsymbol{I})$. For appropriately chosen $N \times N$ matrices $\boldsymbol{A_i}$, $i = 1, ..., p$, we get $\boldsymbol{\beta_r} \boldsymbol{r_t} = \sum_{i=1}^{p} \boldsymbol{B} \boldsymbol{r_t}$, where $\boldsymbol{B} = \sum_{i=1}^{p} \boldsymbol{A_i} \beta_i$ and $\boldsymbol{\beta_r} = [\beta_1, ..., \beta_p]^\top$. For example, with $p = N$ and the elements of $\boldsymbol{A_i}$ being zero everywhere except for value 1 on the ith diagonal element, we obtain our baseline heterogeneous $\boldsymbol{\beta_r}$ model.[20]

Following Lee and Yu (2010), we difference out the fixed effects and focus on the parameter vector $\boldsymbol{\theta_2} = (\boldsymbol{\beta_r}^\top, \boldsymbol{\beta_X}^\top, \rho, \lambda, \sigma_2)^\top$. Using the definition $\tilde{\boldsymbol{x}}_t = \boldsymbol{x}_t - \frac{1}{T} \sum_{t=1}^{T} \boldsymbol{x}_t$, the log-likelihood of this model can be written as

$$\log L_2 = -(T-1)\left(\frac{N}{2}\log(2\pi\sigma_2^2) + \log|\boldsymbol{S}(\rho)| + \log|\boldsymbol{Q}(\lambda)|\right) - \frac{1}{2\sigma_2^2}\sum_{t=1}^{T}\tilde{\boldsymbol{v}}_t^\top(\xi)\tilde{\boldsymbol{v}}_t(\xi)$$

$$\boldsymbol{S}(\rho) = (\boldsymbol{I}_N - \rho\boldsymbol{W}) \qquad \boldsymbol{Q}(\lambda) = (\boldsymbol{I}_N - \lambda\boldsymbol{W}) \qquad \tilde{\boldsymbol{v}}_t(\boldsymbol{\xi}) = \boldsymbol{Q}(\lambda)[\boldsymbol{S}(\rho)\tilde{\boldsymbol{y}}_{t+h} - \tilde{\boldsymbol{X}}_t\boldsymbol{\beta_X} - \boldsymbol{B}\tilde{\boldsymbol{r}}_t].$$

The log-likelihood of the first-stage estimation, which is a linear regression of the federal funds rate on the monetary shocks and the controls of the second stage, is given by

$$\log L_1 = -\frac{N(T-1)}{2}\log(2\pi) - \frac{N(T-1)}{2}\log(\sigma_1^2) - \frac{1}{2\sigma_1^2}\sum_{i=1}^{NT}(\tilde{y}_i - \tilde{\boldsymbol{x}}_i\boldsymbol{\beta}_1)^2,$$

where the parameters of the first-stage model are collected in $\boldsymbol{\theta}_1 = (\boldsymbol{\beta}_1^\top, \sigma_1)$. The covariance

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[20]This representation is easier to work with. We obtain the pooled parameter case ($\beta_i = \beta_j$, $\forall i, j$), by setting $p = 1$ and restricting $\boldsymbol{A_1}$ to be an identity matrix.

matrix of the maximum likelihood estimator for $\boldsymbol{\theta_2}$ is

$$\boldsymbol{\Sigma} = \boldsymbol{R}_2^{-1} + \boldsymbol{R}_2^{-1}[\boldsymbol{R}_3^\top \boldsymbol{R}_1^{-1} \boldsymbol{R}_3 - \boldsymbol{R}_4^\top \boldsymbol{R}_1^{-1} \boldsymbol{R}_3 - \boldsymbol{R}_3^\top \boldsymbol{R}_1^{-1} \boldsymbol{R}_4]\boldsymbol{R}_2^{-1},$$

where

$$\boldsymbol{R}_1(\boldsymbol{\theta}_1) = -\mathbb{E}\left[\frac{\partial^2 L_1}{\partial \boldsymbol{\theta}_1 \partial \boldsymbol{\theta}_1^\top}\right] \qquad \boldsymbol{R}_2(\boldsymbol{\theta}_2) = -\mathbb{E}\left[\frac{\partial^2 L_2}{\partial \boldsymbol{\theta}_2 \partial \boldsymbol{\theta}_2^\top}\right]$$

$$\boldsymbol{R}_3(\boldsymbol{\theta}_1, \boldsymbol{\theta}_2) = \mathbb{E}\left[\frac{\partial L_2}{\partial \boldsymbol{\theta}_1}\left(\frac{\partial L_2}{\partial \boldsymbol{\theta}_2}\right)^\top\right] \qquad \boldsymbol{R}_4(\boldsymbol{\theta}_1, \boldsymbol{\theta}_2) = \mathbb{E}\left[\frac{\partial L_1}{\partial \boldsymbol{\theta}_1}\left(\frac{\partial L_2}{\partial \boldsymbol{\theta}_2}\right)^\top\right].$$

$\boldsymbol{R}_1^{-1}$ and $\boldsymbol{R}_2^{-1}$ are the Fisher information matrices from the first- and second-stage estimation, respectively, and these estimates are readily obtained from standard likelihood optimization procedures. To derive $\boldsymbol{R}_3$ and $\boldsymbol{R}_4$, we need to compute the gradients of the log likelihoods from each stage. For ease of notation, all matrices are converted to block-diagonal form using the Kronecker product with the $T \times T$ identity matrix, unless indicated otherwise by a subscript N; i.e., $X = I_T \otimes X_N$. Using this notation with $\boldsymbol{G} = \boldsymbol{W}\boldsymbol{S}^{-1}$ and $\boldsymbol{H} = \boldsymbol{W}\boldsymbol{Q}^{-1}$, we have

$$\frac{\partial L_2}{\partial \boldsymbol{\theta}_2} = \begin{bmatrix} \frac{1}{\sigma_2^2}\tilde{\boldsymbol{r}}^\top \boldsymbol{A}_1^\top \boldsymbol{Q}^\top \tilde{\boldsymbol{v}} \\ \vdots \\ \frac{1}{\sigma_2^2}\tilde{\boldsymbol{r}}^\top \boldsymbol{A}_i^\top \boldsymbol{Q}^\top \tilde{\boldsymbol{v}} \\ \vdots \\ \frac{1}{\sigma_2^2}\tilde{\boldsymbol{r}}^\top \boldsymbol{A}_p^\top \boldsymbol{Q}^\top \tilde{\boldsymbol{v}} \\ \frac{1}{\sigma_2^2}\tilde{\boldsymbol{X}}_2^\top \boldsymbol{Q}^\top \tilde{\boldsymbol{v}} \\ \frac{1}{\sigma_2^2}(\boldsymbol{Q}\boldsymbol{W}\tilde{\boldsymbol{y}})^\top \tilde{\boldsymbol{v}} - (T-1)\operatorname{tr}(\boldsymbol{G_N}) \\ \frac{1}{\sigma_2^2}(\boldsymbol{H}\tilde{\boldsymbol{v}})^\top \tilde{\boldsymbol{v}} - (T-1)\operatorname{tr}(\boldsymbol{H_N}) \\ \frac{1}{2\sigma_2^4}(\tilde{\boldsymbol{v}}^\top \tilde{\boldsymbol{v}} - N(T-1)\sigma_2^2) \end{bmatrix}$$

$$\frac{\partial L_2}{\partial \boldsymbol{\theta}_1} = \begin{bmatrix} \frac{1}{\sigma^2} \tilde{\boldsymbol{X}}_1^\top \boldsymbol{B}^\top \boldsymbol{Q}^\top \tilde{\boldsymbol{v}} \\ 0 \end{bmatrix} \qquad \frac{\partial L_1}{\partial \boldsymbol{\theta}_1} = \begin{bmatrix} \frac{\tilde{\boldsymbol{X}}_1^\top \tilde{\boldsymbol{\varepsilon}}_1}{\sigma_1^2} \\ -\frac{N(T-1)}{2\sigma_1^2} + \frac{1}{2\sigma^4} \tilde{\boldsymbol{\varepsilon}}_1^\top \tilde{\boldsymbol{\varepsilon}}_1 \end{bmatrix}.$$

Thus, the matrix $\boldsymbol{R}_3$ is given by the following $2 \times (4+p)$ blocks:

$$\boldsymbol{R}_3(1,j) = \mathbb{E}\left[\frac{1}{\sigma_2^4} \tilde{\boldsymbol{X}}_1^\top \boldsymbol{B}^\top \boldsymbol{Q}^\top \tilde{\boldsymbol{v}} \tilde{\boldsymbol{v}}^\top \boldsymbol{Q} \boldsymbol{A}_i \tilde{\boldsymbol{r}} \right] = \frac{T-1}{T\sigma_2^2} \tilde{\boldsymbol{X}}_1^\top \boldsymbol{B}^\top \boldsymbol{Q}^\top \boldsymbol{Q} \boldsymbol{A}_i \tilde{\boldsymbol{r}}, \ \forall j = 1, ..., p$$

$$\boldsymbol{R}_3(1,p+1) = \mathbb{E}\left[\frac{1}{\sigma_2^4} \tilde{\boldsymbol{X}}_1^\top \boldsymbol{B}^\top \boldsymbol{Q}^\top \tilde{\boldsymbol{v}} \tilde{\boldsymbol{v}}^\top \boldsymbol{Q} \tilde{\boldsymbol{X}}_2 \right] = \frac{T-1}{T\sigma_2^2} \tilde{\boldsymbol{X}}_1^\top \boldsymbol{B}^\top \boldsymbol{Q}^\top \boldsymbol{Q} \tilde{\boldsymbol{X}}_2$$

$$\boldsymbol{R}_3(1,p+2) = \mathbb{E}\left[\frac{1}{\sigma_2^4} \tilde{\boldsymbol{X}}_1^\top \boldsymbol{B}^\top \boldsymbol{Q}^\top \tilde{\boldsymbol{v}} \tilde{\boldsymbol{v}}^\top \boldsymbol{Q} \boldsymbol{W} \tilde{\boldsymbol{y}} \right] = \frac{T-1}{T\sigma_2^2} \tilde{\boldsymbol{X}}_1^\top \boldsymbol{B}^\top \boldsymbol{Q}^\top \boldsymbol{Q} \boldsymbol{W} \tilde{\boldsymbol{y}}$$

$$\boldsymbol{R}_3(1,p+3) = \mathbb{E}\left[\frac{1}{\sigma_2^4} \tilde{\boldsymbol{X}}_1^\top \boldsymbol{B}^\top \boldsymbol{Q}^\top \tilde{\boldsymbol{v}} \tilde{\boldsymbol{v}}^\top \boldsymbol{H} \tilde{\boldsymbol{v}} \right] = \boldsymbol{0}$$

$$\boldsymbol{R}_3(1,p+4) = \mathbb{E}\left[\frac{1}{\sigma_2^4} \tilde{\boldsymbol{X}}_1^\top \boldsymbol{B}^\top \boldsymbol{Q}^\top \tilde{\boldsymbol{v}} \tilde{\boldsymbol{v}}^\top \tilde{\boldsymbol{v}} \right] = \boldsymbol{0}$$

$$\boldsymbol{R}_3(2,j) = \boldsymbol{0}, \ \forall j = 1, ..., p+4.$$

To derive the blocks of the matrix $\boldsymbol{R}_4$, we use the projection matrix of the first-stage, $\boldsymbol{P} = \tilde{\boldsymbol{X}}_1 (\tilde{\boldsymbol{X}}_1^\top \tilde{\boldsymbol{X}}_1)^{-1} \tilde{\boldsymbol{X}}_1^\top$, as well as $\mathbb{E}[\tilde{\boldsymbol{\varepsilon}}_1^\top \tilde{\boldsymbol{\varepsilon}}_1 \tilde{\boldsymbol{\varepsilon}}_1^\top A \tilde{\boldsymbol{\varepsilon}}_1] = \frac{N(T-1)^2}{T}(NT+2)\operatorname{tr}(A)\sigma_1^4$, which follows from the normality and independence of the errors. Finally, we use the identity $\tilde{\boldsymbol{r}} = \tilde{\boldsymbol{X}}_1 \boldsymbol{\beta}_1 + \boldsymbol{P} \tilde{\boldsymbol{\varepsilon}}_1$ and the identity $\boldsymbol{P} \tilde{\boldsymbol{X}}_1 = \tilde{\boldsymbol{X}}_1$. Using these equations, we can derive the $2 \times (4+p)$ blocks of the matrix $\boldsymbol{R}_4$ as

$$\boldsymbol{R}_4(1,j) = \mathbb{E}\left[\frac{1}{\sigma_1^2 \sigma_2^2} \tilde{\boldsymbol{X}}_1^\top \tilde{\boldsymbol{\varepsilon}}_1 \tilde{\boldsymbol{v}}^\top \boldsymbol{Q} \boldsymbol{A}_i \tilde{\boldsymbol{r}} \right]$$
$$= \frac{T-1}{T\sigma_2^2} \tilde{\boldsymbol{X}}_1^\top \left(\boldsymbol{A}_i^\top \boldsymbol{Q}^\top \boldsymbol{Q} (\boldsymbol{S}\tilde{\boldsymbol{y}} - \boldsymbol{B}\tilde{\boldsymbol{X}}_1 \boldsymbol{\beta}_1 - \tilde{\boldsymbol{X}}_2 \boldsymbol{\beta}_X) - \boldsymbol{B}^\top \boldsymbol{Q}^\top \boldsymbol{Q} \boldsymbol{A}_i \tilde{\boldsymbol{X}}_1 \boldsymbol{\beta}_1 \right)$$

$$\boldsymbol{R}_4(1,p+1) = \mathbb{E}\left[\frac{1}{\sigma_1^2 \sigma_2^2} \tilde{\boldsymbol{X}}_1^\top \tilde{\boldsymbol{\varepsilon}}_1 \tilde{\boldsymbol{v}}^\top \boldsymbol{Q} \tilde{\boldsymbol{X}}_2 \right] = -\frac{T-1}{T\sigma_2^2} \tilde{\boldsymbol{X}}_1^\top \boldsymbol{B}^\top \boldsymbol{Q}^\top \boldsymbol{Q} \tilde{\boldsymbol{X}}_2$$

$$\boldsymbol{R}_4(1,p+2) = \mathbb{E}\left[\frac{1}{\sigma_1^2 \sigma_2^2} \tilde{\boldsymbol{X}}_1^\top \tilde{\boldsymbol{\varepsilon}}_1 \tilde{\boldsymbol{v}}^\top \boldsymbol{Q} \boldsymbol{W} \tilde{\boldsymbol{y}} \right] = -\frac{T-1}{T\sigma_2^2} \tilde{\boldsymbol{X}}_1^\top \boldsymbol{B}^\top \boldsymbol{Q}^\top \boldsymbol{Q} \boldsymbol{W} \tilde{\boldsymbol{y}}$$

$$\boldsymbol{R}_4(1,p+3) = \mathbb{E}\left[\frac{1}{\sigma_1^2 \sigma_2^2} \tilde{\boldsymbol{X}}_1^\top \tilde{\boldsymbol{\varepsilon}}_1 \tilde{\boldsymbol{v}}^\top \boldsymbol{H} \tilde{\boldsymbol{v}} \right]$$
$$= -2\frac{T-1}{T\sigma_2^2} \tilde{\boldsymbol{X}}_1^\top \boldsymbol{B}^\top \boldsymbol{Q}^\top \boldsymbol{W} (\boldsymbol{S}\tilde{\boldsymbol{y}} - \boldsymbol{B}\tilde{\boldsymbol{X}}_1 \boldsymbol{\beta}_1 - \tilde{\boldsymbol{X}}_2 \boldsymbol{\beta}_X)$$

$$\boldsymbol{R}_4(1,p+4) = \mathbb{E}\left[\frac{1}{2\sigma_1^2 \sigma_2^4} \tilde{\boldsymbol{X}}_1^\top \tilde{\boldsymbol{\varepsilon}}_1 \tilde{\boldsymbol{v}}^\top \tilde{\boldsymbol{v}} \right] = -\frac{T-1}{T\sigma_2^4} \tilde{\boldsymbol{X}}_1^\top \boldsymbol{B}^\top \boldsymbol{Q}^\top \boldsymbol{Q} (\boldsymbol{S}\tilde{\boldsymbol{y}} - \boldsymbol{B}\tilde{\boldsymbol{X}}_1 \boldsymbol{\beta}_1 - \tilde{\boldsymbol{X}}_2 \boldsymbol{\beta}_X)$$

$$\boldsymbol{R}_4(2,j) = \mathbb{E}\left[\frac{1}{2\sigma_1^4\sigma_2^2}\tilde{\boldsymbol{\varepsilon}}_1^\top\tilde{\boldsymbol{\varepsilon}}_1\tilde{\boldsymbol{v}}^\top\boldsymbol{Q}\boldsymbol{A}_i\tilde{\boldsymbol{r}}\right]$$

$$= \frac{N(T-1)}{2\sigma_1^2\sigma_2^2}(\boldsymbol{S}\tilde{\boldsymbol{y}} - \boldsymbol{B}\tilde{\boldsymbol{X}}_1\boldsymbol{\beta}_1 - \tilde{\boldsymbol{X}}_2\boldsymbol{\beta}_X)^\top\boldsymbol{Q}^\top\boldsymbol{Q}\boldsymbol{A}_i\tilde{\boldsymbol{X}}_1\boldsymbol{\beta}_1$$

$$- \frac{N(T-1)^2}{T}\frac{NT+2}{2\sigma_2^2}\operatorname{tr}(\boldsymbol{P}\boldsymbol{B}^\top\boldsymbol{Q}^\top\boldsymbol{Q}\boldsymbol{A}_i\boldsymbol{P})$$

$$\boldsymbol{R}_4(2,p+1) = \mathbb{E}\left[\frac{1}{2\sigma_1^4\sigma_2^2}\tilde{\boldsymbol{\varepsilon}}_1^\top\tilde{\boldsymbol{\varepsilon}}_1\tilde{\boldsymbol{v}}^\top\boldsymbol{Q}\tilde{\boldsymbol{X}}_2\right] = \frac{N(T-1)}{2\sigma_1^2\sigma_2^2}(\boldsymbol{S}\tilde{\boldsymbol{y}} - \boldsymbol{B}\tilde{\boldsymbol{X}}_1\boldsymbol{\beta}_1 - \tilde{\boldsymbol{X}}_2\boldsymbol{\beta}_X)^\top\boldsymbol{Q}^\top\boldsymbol{Q}\tilde{\boldsymbol{X}}_2$$

$$\boldsymbol{R}_4(2,p+2) = \mathbb{E}\left[\frac{1}{2\sigma_1^4\sigma_2^2}\tilde{\boldsymbol{\varepsilon}}_1^\top\tilde{\boldsymbol{\varepsilon}}_1\tilde{\boldsymbol{v}}^\top\boldsymbol{Q}\boldsymbol{W}\tilde{\boldsymbol{y}}\right] - C_1 = \frac{N(T-1)}{2\sigma_1^2\sigma_2^2}(\boldsymbol{S}\tilde{\boldsymbol{y}} - \boldsymbol{B}\tilde{\boldsymbol{X}}_1\boldsymbol{\beta}_1 - \tilde{\boldsymbol{X}}_2\boldsymbol{\beta}_X)^\top\boldsymbol{Q}^\top\boldsymbol{Q}\boldsymbol{W}\tilde{\boldsymbol{y}} - C_1$$

$$\text{where } C_1 = \frac{N(T-1)^2}{2\sigma_1^2}\operatorname{tr}(\boldsymbol{G}_N)$$

$$\boldsymbol{R}_4(2,p+3) = \mathbb{E}\left[\frac{1}{2\sigma_1^4\sigma_2^2}\tilde{\boldsymbol{\varepsilon}}_1^\top\tilde{\boldsymbol{\varepsilon}}_1\tilde{\boldsymbol{v}}^\top\boldsymbol{H}\tilde{\boldsymbol{v}}\right] - C_2$$

$$= \frac{N(T-1)}{2\sigma_1^2\sigma_2^2}(\boldsymbol{S}\tilde{\boldsymbol{y}} - \boldsymbol{B}\tilde{\boldsymbol{X}}_1\boldsymbol{\beta}_1 - \tilde{\boldsymbol{X}}_2\boldsymbol{\beta}_X)^\top\boldsymbol{Q}^\top\boldsymbol{W}(\boldsymbol{S}\tilde{\boldsymbol{y}} - \boldsymbol{B}\tilde{\boldsymbol{X}}_1\boldsymbol{\beta}_1 - \tilde{\boldsymbol{X}}_2\boldsymbol{\beta}_X)$$

$$+ \frac{N(T-1)^2}{T}\frac{NT+2}{2\sigma_2^2}\operatorname{tr}(\boldsymbol{P}\boldsymbol{B}^\top\boldsymbol{Q}^\top\boldsymbol{W}\boldsymbol{B}\boldsymbol{P}) - C_2, \quad \text{where } C_2 = \frac{N(T-1)^2}{2\sigma_1^2}\operatorname{tr}(\boldsymbol{H}_N)$$

$$\boldsymbol{R}_4(2,p+4) = \mathbb{E}\left[\frac{1}{4\sigma_1^4\sigma_2^4}\tilde{\boldsymbol{\varepsilon}}_1^\top\tilde{\boldsymbol{\varepsilon}}_1\tilde{\boldsymbol{v}}^\top\tilde{\boldsymbol{v}}\right] - C_3$$

$$= \frac{N(T-1)}{4\sigma_1^2\sigma_2^4}(\boldsymbol{S}\tilde{\boldsymbol{y}} - \boldsymbol{B}\tilde{\boldsymbol{X}}_1\boldsymbol{\beta}_1 - \tilde{\boldsymbol{X}}_2\boldsymbol{\beta}_X)^\top\boldsymbol{Q}^\top\boldsymbol{Q}(\boldsymbol{S}\tilde{\boldsymbol{y}} - \boldsymbol{B}\tilde{\boldsymbol{X}}_1\boldsymbol{\beta}_1 - \tilde{\boldsymbol{X}}_2\boldsymbol{\beta}_X)$$

$$+ \frac{N(T-1)^2}{T}\frac{NT+2}{4\sigma_2^4}\operatorname{tr}(\boldsymbol{P}\boldsymbol{B}^\top\boldsymbol{Q}^\top\boldsymbol{Q}\boldsymbol{B}\boldsymbol{P}) - C_3, \quad \text{where } C_3 = \frac{N^2(T-1)^2}{4\sigma_1^2\sigma_2^2}.$$