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Corporate Finance and the Transmission of Shocks to the Real Economy

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Abstract:
Credit availability from different sources varies greatly across firms and has firm-level effects on investment decisions and aggregate effects on output. We develop a theoretical framework in which firms decide endogenously at the extensive and intensive margins of different funding sources to study the role of firm choices on the transmission of credit supply shocks to the real economy. As in the data, firms can borrow from different banks, issue bonds, or raise equity through retained earnings to fund productive investment. Our model is calibrated to detailed firm- and loan-level data and reproduces stylized empirical facts: Larger, more productive firms rely on more banks and more sources of funding; smaller firms mostly rely on a small number of banks and internal funding. Our quantitative analysis shows that bank credit supply shocks lead to a sizable reduction in aggregate output, with substantial heterogeneity across firms, due to the lack of substitutability among alternative credit sources. Finally, we show that our insights have important implications for the validity of standard empirical methods used to identify credit supply effects (Khwaja and Mian 2008).

JEL Classifications: E32, E43, E50, G21, G32
Keywords: credit supply shocks, firm financing, bank-firm matching, shock transmission

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1 Introduction

Credit markets play a central role in the propagation of shocks to the real economy. Because firms finance the purchase of inputs and productive investments primarily by borrowing from debt investors, the cost and availability of credit affects output in standard macroeconomic models with financial sectors.[1] However, the importance of different financing sources varies greatly across the firm-size distribution, with small firms mostly relying on only one or a few banking relationships for external financing, while large firms have multiple established relationships with different banks and also have access to market finance (bonds). The transmission of shocks—such as credit supply or monetary policy shocks—to the economy thus depends crucially on frictions in different credit markets. For example, the pass-through of a policy rate change to firms' funding costs depends on the substitutability of bond and bank financing as well as the substitutability of loans across different banks. Additionally, idiosyncratic productivity shocks may have an aggregate impact, depending on how firms can lever up investment in more productive capital.

In this paper, we introduce a theoretical framework to study firms' corporate finance choices and their implications for the transmission of shocks to the real economy. Our model allows a firm to choose between self-financing (equity) or external debt funding through the bond market or bank loans. A key focus is on the formation of lending relationships with individual banks, which is at the core of a large literature that studies the transmission of shocks to firms through the banking sector. To this end, we depart from the existing literature that takes lending relationships as given and fully endogenize both the intensive and extensive margins of credit across a finite, countable set of banks.

More precisely, we adapt and extend the Ricardian trade model by Eaton and Kortum (2002) to a corporate finance setup, as in Herreño (2020). After observing a menu of expected interest rates, firms choose which banking relationships to build (extensive margin) and, conditional on forming a relationship, their magnitude (intensive margin). Firms can also obtain external finance through the bond market. Building a relationship with a bank or accessing the bond market is costly due to, for example, the need to obtain external credit ratings or provide regular audited financial reports. Conditional on established bank relationships and bond

[1] See Bernanke and Gertler (1989), Kiyotaki and Moore (1997), and Brunnermeier and Sannikov (2014).

market access, the interest rates that firms ultimately face are subject to correlated shocks, and the firm picks the option with the lowest financing cost (post shock) to fund its intermediate inputs and productivity-enhancing investment. Because the final good is produced using a continuum of intermediate inputs, the firm's cost of funding and the shares of expenditures funded with each source (banks, bonds, and self-financing) are characterized by the relative interest rates offered across the alternative funding sources. [2] The loan market equilibrium is characterized by monopolistic competition, in which banks know firms' full demand schedule (including the extensive margin) and set prices at a markup over the interest rates paid to depositors while taking the actions of other banks that compete for market shares of each borrower as given.

Our model design and calibration are informed by several key stylized facts about firms' corporate finance choices that we establish using a detailed supervisory data set containing information about borrowers from large banks. Specifically, we use the Federal Reserve Y-14Q data set, which contains facility-level information of virtually all commercial and industrial (C&I) loans originated by US bank holding companies, US intermediate holding companies of foreign bank organizations, and covered savings and loans holding companies, with $100 billion or more in total consolidated assets.[3] A key advantage of the Y-14Q data is that they contain detailed income and balance sheet information about the borrowing firms—irrespective of their size or ownership status—thereby not restricting the analysis to large, public firms for which this information is readily available through required filings with the Securities and Exchange Commission (SEC) and distributed to common data vendors such as Compustat. Hence, we use this data set as a novel window into the capital structure of firms across the entire firm-size distribution, including small and private firms, which account for a large share of the economy's output, wage bill, and employment.

Importantly, we show that the funding mix varies substantially along the firm-size distribution. More than 50 percent of firms in our data set (those with annual sales of less than $150

[2] We do not explicitly model other loan characteristics, such as maturity, collateral, or covenants, which, in turn, may induce imperfect substitutability across different banks. However, these characteristics enter into our model in two related ways. First, rates in our model can be interpreted as shadow rates, that is, already including the rate equivalent effect of other loan characteristics. Second, we assume that there is random within-firm variation in interest rates for each input, which can come from specific loan characteristics.

[3] Reports are filed by the holding companies, but, for simplicity, we refer to them as banks or Y-14 banks in this article. Banks are not required to report facilities with an outstanding balance of less than $1 million in the Y-14 C&I loan data (schedule H).

million), predominantly rely on bank loans from just one Y-14 bank as external financing and retained earnings as internal financing to fund their operations.[4] Larger firms tap into the bond markets, and the share of funding through bank loans decreases to close to 0 percent at the very large firms. While the overall share of bank funding decreases, we also observe that larger firms borrow from multiple banks and the number of banking relationships increases with firm size. Among firms with multiple banking relationships, firms borrow larger shares from banks that offer lower interest rates. Overall, we also document that the cost of debt funding (interest rate expenses relative to total debt) decreases with firm size. Our structural model replicates all these key stylized facts and rationalizes them through the costs of formation of credit relationships, paired with imperfect substitutability across funding sources. Firm productivity is the key source of heterogeneity in the model. Only firms with high productivity find it worthwhile to invest in multiple banking relationships or to access the bond market, but doing so reduces their total cost of funding, allowing them to invest and produce more.

The analysis of our calibrated model, which matches key empirical characteristics of corporate finance choices, reveals a sizable differential impact of bank credit supply shocks on economic activity depending on firm size. Intuitively, small firms are most affected by a bank shock that increases banks' funding cost by 1 percentage point. After the shock, these firms face substantially higher costs of funding investment (20 percent increase) and a resulting decline in output of as much as 4 percent. Part of this increase is due to the lack of substitutability (lock-in effect) of bank credit combined with the high cost of internal financing. However, the largest firms, many of which borrow from multiple banks in addition to the bond market, also face a meaningful reduction in output, more than half a percent. The differential impact of financial shocks on firms implies that there would be an increase in product market concentration following such shocks as large firms' sales shares increase. Finally, we show that credit market frictions have aggregate effects on investment and output. A decline in the substitutabilty of bank credit (from 10 to 6) increases the output response to a 1 percentage point funding cost shock to one bank in the economy by a sizable 4 percent (in absolute value).[5]

[4] We only observe banking relationships with Y-14 banks in our data, thereby potentially underestimating the true importance of bank loans as a source of firm financing.

[5] We estimate a reduction in output of 0.71 percent with an elasticity of 10, compared with a reduction of 0.74

Our insights have important implications for a large empirical literature that emerged in the past decade and estimates credit supply effects using loan-level data. The now-standard approach of this literature is to estimate supply effects using within-borrower estimation similar to Khwaja and Mian (2008). The key assumption behind this approach is that firms' credit demand is the same across all banks. This assumption can be violated if demand for credit is not perfectly substitutable across banks (Paravisini, Rappoport and Schnabl, 2015), if there is bank-firm sorting (Chang, Gomez and Hong, 2021), or if the effect of these shocks is heterogeneous across firms or loan types (Ivashina, Laeven and Moral-Benito, 2020). Our model incorporates all of these channels and allows for imperfect substitutability across funding sources, a comparative-advantage-based matching between firms and banks, and for these effects to be heterogeneous across firms. We develop a quantitative framework to interpret and evaluate empirical evidence on credit supply shocks. Specifically, a simulated version of the model allows us to estimate the true effect of a credit supply shock on borrowers from affected banks and compare the true effect with that obtained from a regression exploiting within-firm variation, as in Khwaja and Mian (2008). We find that these methods can result in substantial bias of the estimated credit supply effects even in the absence of correlated demand shocks due to bank-firm sorting at the extensive and intensive margins.

Our paper also contributes to the literature on heterogeneity in financial frictions in the transmission of shocks to the real economy.[6] Closest to our paper is Crouzet (2018), which studies the aggregate implications of corporate debt choices. The key channel in Crouzet (2018) is the flexibility of bank financing versus a lower cost of bond issuance. Our model allows for a reduced-form version of this channel by including an imperfect substitution between bank and bond financing, but our framework also encompasses within-bank financing substitution, bank market power, and specialization, and it explicitly incorporates firms' financing choices at the extensive margin. Therefore, our model features rich heterogeneity not only in the quantity, but also in the cost of borrowing across firms, as well as within and across funding sources. Our paper also contributes to the literature that takes into account bank market power in addressing macro-finance questions, in particular monetary policy transmission

percent with an elasticity of 8.

[6]See, for instance, Gomes (2001), Cooley and Quadrini (2001), Khan and Thomas (2013), Gilchrist, Sim and Zakrajšek (2014) al. (2014), Khan, Senga and Thomas (2014), Jeenas (2019), (Ottonello and Winberry, 2020), Bacchetta, Benhima and Poilly (2019), and Greenwald, Krainer and Paul (2021).

(Drechsler, Savov and Schnabl (2017) and Egan, Hortaçsu and Matvos (2017); Wang et al. (2020), and Xiao (2020)).

Finally, our paper joins the growing literature that uses the Y-14 supervisory data. Chodorow-Reich et al. (2021) focus on the heterogeneity of terms and usage of credit lines across the firm-size distribution. Greenwald, Krainer and Paul (2021) explore the role of credit lines in the transmission of aggregate shocks. Caglio, Darst and Kalemli-Özcan (2021) show that small and large firms have a different funding mix in terms of maturity and collateral, and thus their responses are different following a monetary policy shock. Our work contributes to this literature by focusing on the role of bank-firm relationships and substitutability at the intensive *and* extensive margins within and across different sources of finance.

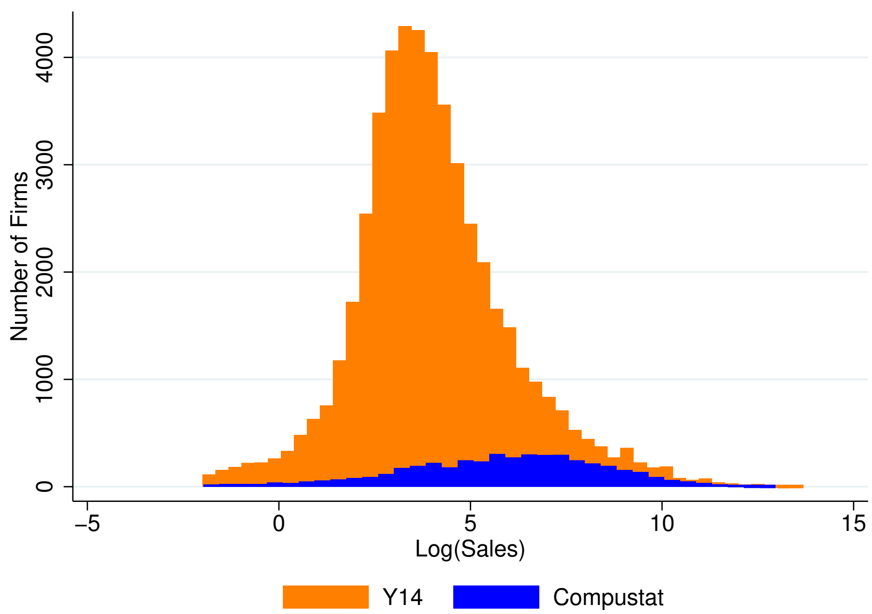
The remainder of this paper is organized as follows. Section 2 provides some basic empirical facts that motivate the model design and calibration. Section 3 introduces the model. Section 4 provides a quantitative analysis. Section 5 discusses the implications for reduced-form estimates of credit supply shocks. Section 6 concludes.

2 Motivational Evidence

In this section, we establish some basic empirical facts about firms' corporate finance structure that we will use to inform our economic model. Our insights draw on supervisory information for a large set of borrowers covering a wide range of the firm-size distribution, including small and medium-sized businesses, primarily private firms, as well as large, primarily publicly traded, firms. In contrast, previous studies of firms' financing decisions rely often on Compustat data that collect financial statements that public firms are required to file with the Securities and Exchange Commission (SEC). Due to the focus on public firms, the sample in Compustat omits an important segment of the firm-size distribution and thus cannot be used to study debt choices of smaller or even most mid-sized firms, for many of which bank funding is, as we will show, an important source of external funding.

We overcome this data limitation by exploiting firms' financial statement information recorded in supervisory bank data collected by the Federal Reserve. As of 2012, as part of the mandated Dodd-Frank Act Stress Tests (DFAST), the Federal Reserve collects detailed portfolio information in the FR Y-14Q report form. Specifically, banks with more than $100 billion in total consolidated assets are subject to DFAST and are required to report facility-level information on their balance sheet, along with borrower information, on a regular basis to the Federal Reserve.[7] In our analysis, we use the schedule H.1 containing quarterly data with detailed information on nearly all individual C&I loans held by the reporting banks. Importantly, the data set includes detailed balance sheet and income statement information for the borrowing firms, irrespective of borrower size or ownership status. We provide more details on the Y-14 data processing in Appendix A.

Figure 1: Firm-Size Distribution



Notes: Number of firms with sales information for 2019 in FR Y-14Q H.1 schedule and Compustat.

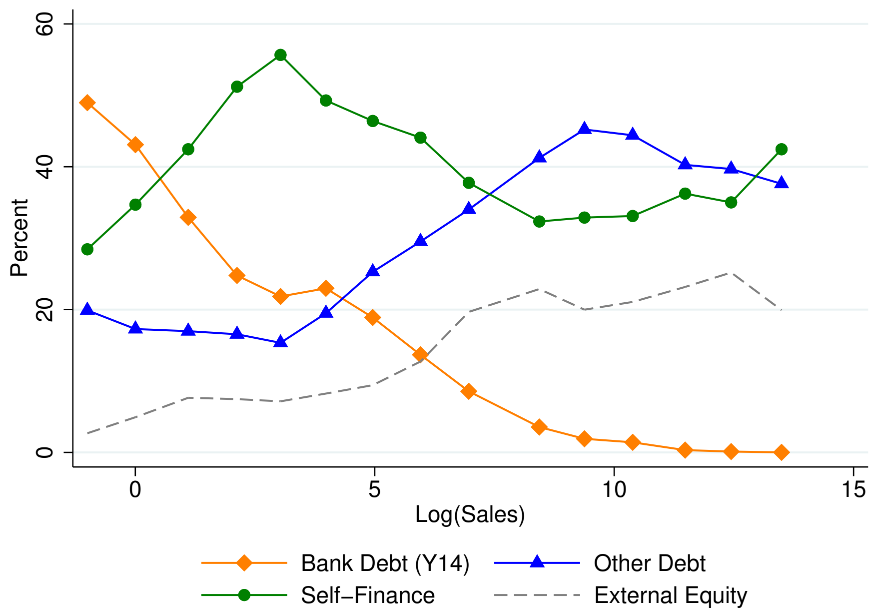
Figure 1 compares the size distribution of the firms in the Compustat data with that of firms in the Y-14 data, based on 2019 sales values. Two important takeaways stand out. First, the number of firms in the Y-14 data is substantially larger than the number in Compustat, due in large part to the presence of firms with low or medium sales values, which are largely absent from Compustat.[8] Second, even for the right tail of the distribution, the number of

[7] The reported data are confidential supervisory information, but the list of variables collected by the Federal Reserve is publicly available at https://www.federalreserve.gov/apps/reportforms/Default.aspx.

[8] Y-14 contains only loans of at least $1 million in value. This truncation implies that we likely miss some very small firms in the Y-14 data. Moreover, anecdotal evidence suggests that small firms tend to bank with small banks, potentially introducing another layer of sample selection. In fact, a comparison with the population of US firms as recorded by the US Census shows that very small firms are substantially underrepresented in both Compustat and Y-14.

firms covered in Y-14 is larger than the number of firms in Compustat. Overall, the Y-14 data are more suitable for studying corporate finance decisions for a range of firms along the size distribution.

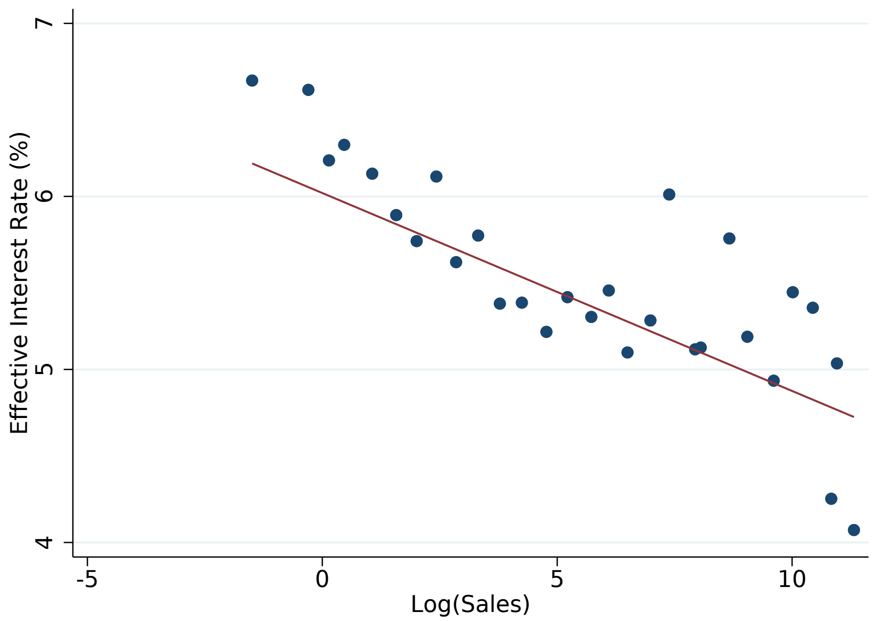
Figure 2: Firm Size and Funding Mix



Notes: The figure shows the average funding mix for different bins of the firm-size distribution. The figure is based on all firms with 2019 information in Y-14. Bank Debt includes only debt from Y-14 banks. Other Debt includes bonds and loans obtained outside of Y-14 banks. Self-Finance is the stock of retained earnings. External Equity is defined as total assets minus total liabilities minus retained earnings. Source: FR Y-14Q data, H.1 schedule with C&I lending, authors' calculations.

Figure 2 provides a high-level overview of the relationship between a firm's size and its funding mix, using 2019 data. On the horizontal axis, we plot bins of (the logarithm of) sales. On the vertical axis, we show the percentage of total funding for four key categories: (i) debt obtained from Y-14 banks, (ii) other debt, (iii), self-financing (retained earnings), and (iv) external equity. The figure shows that there is substantial heterogeneity in the funding mix across the firm-size distribution. For small firms, with annual log sales below 5, bank debt is the single most important external financing source. For very small firms, bank debt constitutes about half of all funding. Only for the very large firms does other debt play an important role. Note that we observe bank debt only from Y-14 banks in the Y-14 data set. Borrowing from other banks (outside Y-14) would be classified as "Other Debt." In this sense, we expect that the actual role of banks is even larger than suggested by the figure, while the role of nonbank debt (bonds, or nonbank intermediaries) is smaller. Finally, self-financing through retained earnings seems an important source of funding throughout the firm-size distribution. External equity funding plays a minor role, but it increases steadily in firm size to about 20 percent for the largest firms.

Figure 3: Firm Size and Effective Interest Rate

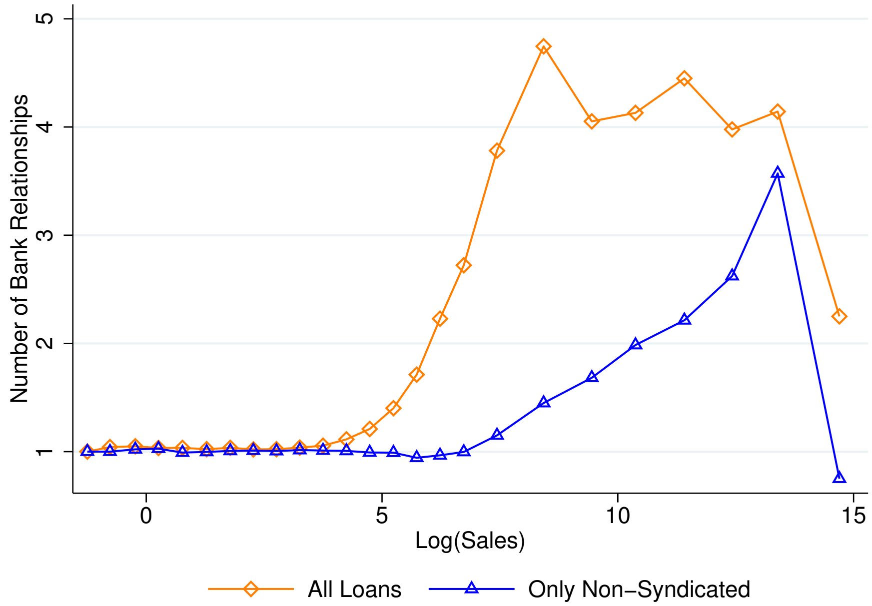


Notes: The figure shows a bin scatter plot of the average effective interest rate, defined as interest expenses relative to the stock of total debt in 2019, against the average logarithm of 2019 sales. Before computing the averages for each bin, we orthogonalize the data with respect to debt over assets and firm rating. Source: FR Y-14Q data, H.1 schedule with C&I lending, authors' calculations.

In addition to the funding mix, we find that the cost of funding varies with firm size. Figure 3 shows the effective interest rate paid on debt as the cost measure, computed as interest expenses over the stock of total debt, as a function of firm size. In order to focus on the relationship between the interest rate and firm size alone, we hold constant borrower risk, as measured by leverage and rating. The figure clearly shows a strong negative correlation between firm size and loan interest rates. Small firms pay higher interest rates on their debt compared with large firms of the same level of riskiness.

Figure 4 zooms in on bank funding and reports the number of unique banking relationships that we observe within the Y-14 data. Very strikingly, the figure shows that the majority of firms obtain credit from only one single bank. The number of relationships increases to about

Figure 4: Firm Size and Banking Relationships



Notes: The figure shows a bin scatter plot of the average number of bank relationships as observed in 2019 against the average logarithm of 2019 sales. The orange line shows the total number of relationships, and the blue line shows the number of relationships from nonsyndicated loans. Source: FR Y-14Q data, H.1 schedule with C&I lending, authors' calculations.

four for the largest firms in the sample, but even then, excluding syndicated loans, most firms borrow from only one or two banks. Note that, as mentioned above, we observe only banking relationships with Y-14 banks with committed loan balances of at least $1 million, potentially underestimating a firm's true number of relationships.[9]

3 Model of Firms' Corporate Finance Choices

In this section, we develop our theoretical model of firms' corporate finance choices. Our model is static and features a continuum of firms, a discrete number of banks, and a representative household. Firms are heterogeneous in their productivity and must finance their operating costs and investment with a mix of borrowing from multiple banks, external non-bank debt (primarily bonds), and self-finance. Different banks, as well as different sources of finance, are imperfect substitutes. Our model is set up in nests, such that the substitutability

[9]We may underestimate the importance of bank loans, especially for smaller firms, which tend to borrow from small banks that do not have to report in the Y-14 data.

between banks (bank nest) is different from the substitutability between bank and nonbank debt (external finance nest), and external finance has a different degree of substitutability with self-finance. Firms choose which sources of finance to use (including within the bank nest, that is, which banks to borrow from) both at the extensive and intensive margins. Due to the imperfect substitutability across banks and sources of finance, banks face a downward-sloping credit demand curve. Banks have heterogeneous costs and choose their offer rates to maximize profits. Finally, our model features a household that supplies labor inelastically and consumes a composite good of the varieties each firm produces. In this version of the model, we solve a partial equilibrium problem for firms and banks.

Representative Household. The only role of the representative household in the economy is to elastically supply an input (for example, labor) to a continuum of firms, indexed by j, and consume a composite good. The composite good is a constant elasticity of substitution (CES) aggregation of a continuum of intermediate goods, each produced by a firm j, with elasticity of substitution $\eta > 1$. The demand schedule for each variety is given by:

$$Y_j = Y P_j^{-\eta}, \tag{1}$$

where P_j is the price of the good produced by firm j, Y is aggregate demand, and the aggregate price index is the numeraire.

3.1 Firms

We focus now on the problem of a firm j. Economically, the problem of the firm consists of the following steps. First, a firm chooses which sources of finance to use at the extensive margin (subject to a fixed cost of using financing from a given source). Second, given the choices at the extensive margin, the firm pays a cost to reduce the rates of each source of finance (for instance, the firm invests in reducing monitoring costs from banks). Third, given the financing choices of a firm at the extensive and intensive margins, the firm chooses how much to invest to increase its productivity, sets prices, and decides how much to produce in a monopolistically competitive setting that results from the CES aggregate demand. For exposition purposes, we present and solve the problem of the firm backward in this section. We start with the production function and then move step by step to the firms' choices of sources of finance at the extensive margin.

Firms' Production Function. Firm j produces a quantity Y_j of its variety using a CES aggregation of input goods $y_j(\omega)$:

$$Y_j = \left(\int_0^1 y_j(\omega)^{\frac{\sigma-1}{\sigma}} d\omega \right)^{\frac{\sigma}{\sigma-1}}. \tag{2}$$

Each intermediate good $y_j(\omega)$ is produced with a linear production function of a single input, $l_j(\omega)$, a firm-specific productivity parameter a_j, and a firm-specific productivity shifter I_j that is:

$$y_j(\omega) = I_j a_j l_j(\omega). \tag{3}$$

Without loss of generality of the qualitative insights in this paper, we parameterize the productivity distribution across firms as a Pareto distribution:

$$a_j \sim \text{Pareto}(a_a, m_a, l_a),$$

where a_a is the exponent, m_a is the scale parameter, and l_a is the location parameter. The firm can invest in a productivity-enhancing technology, I_j, thereby shifting the initial productivity parameter. The investment cost, $c_I(I_j)$, is given by:

$$c_I(I_j) = \frac{1}{\psi+1} I^{\psi+1}, \;\; \psi \geq 0. \tag{4}$$

Given initial productivity and investment, the cost of producing $y_j(\omega)$ is:

$$c_j(\omega) = w_j l_j(\omega),$$

where w_j is the unit cost of the input $l_j(\omega)$. We do not close the labor markets in this version of the model and take w_j as given. In a general version of the model, the w_j's and aggregate demand Y are determined in equilibrium after solving the household problem.

Source of Finance for Each Variety. For a given intermediate good $y_j(\omega)$, firms must choose a source of finance. The firm can borrow from multiple banks, indexed by $b = 1,...,N_b$; nonbank external funding, denoted by nb; or self-finance (retained earnings), denoted by sf. Let $F \equiv \{1,...,N_b, nb, sf\}$ denote the set of potential sources of funding and f be a generic element of this set. Each firm faces a gross offer rate $R_{j,f}$ for each f. Let $\boldsymbol{R}_j$ denote the set of offer rates for firm $f$that is, $\boldsymbol{R}_j \equiv \{R_{j,f}\}_f$. The cost of finance for each variety ω, choosing a source of finance f, is given by:

$$R_{j,f}(\omega) = \epsilon_{j,f}(\omega)^{-1}\left[\lambda_f R_{j,f} + (1-\lambda^f)t_f\right], \tag{5}$$

where λ_f is the probability of repayment for source of finance f, t_f is collateral seized by the creditor in case of default, and $\epsilon_{j,f}(\omega)$ are shocks to the offered rates of different sources of funding for different varieties ω. These shocks are a reduced-form representation of factors through which financing an intermediate input with one source of finance is more advantageous. For instance, some projects are easier to monitor, and thus the shocks are a representation of heterogeneity in the bank-monitoring technology for each of the varieties. It is worth noting that although our model is set up with $\epsilon_{j,f}(\omega)$ varying at the firm-bank level, it is isomorphic to a model where the shocks across varieties occur at the bank level (that is, some banks are better at financing some varieties) and each firm in the model faces a different effect from the same bank shock. In this case, the shocks can be interpreted simply as some banks being better at financing some types of projects (longer-term, riskier, etc.).

Omitting the index ω to reduce notation, the vector $\boldsymbol{\epsilon}_j \equiv \{\epsilon_{j,f}\}_f$ is drawn from a nested Fréchet Distribution:

$$F_j(\boldsymbol{\epsilon}) = \exp\left\{-\zeta_{j,e}\left[\zeta_{j,B}\left(\sum_{b=1}^{N_b} T_{j,b}\epsilon_{j,b}^{-\theta}\right)^{\frac{\xi_E}{\theta}} + \zeta_{j,nb}\epsilon_{j,nb}^{-\xi_e}\right]^{\frac{\xi_S}{\xi_E}} - \zeta_{j,sf}\epsilon_{j,sf}^{-\xi_S}\right\}, \tag{6}$$

where $T_{j,b}, \zeta_{j,B}, \zeta_{j,nb}, \zeta_{j,e}$, and $\zeta_{j,sf}$ are firm-specific scale parameters that capture the intensity of the lending relationship of firm j with a given source of finance relative to other sources in the same nest—which ultimately are endogenous choices in our model, and θ, ξ_E, and ξ_S are shape parameters (constant across firms) that will capture correlation between shocks and will thus pin down the elasticity of substitution across sources of finance. If a scale parameter is zero, the firm has no relationship with that funding source and cannot access funding from it. For instance, if $T_{j,b} = 0$, the firm has no relationship with bank b and cannot access loan funding from bank b. If $\zeta_{nb} = 0$, the firm has no access to external nonbank (bond) funding. At the intensive margin, the scale parameters, T's and ζ's, capture the absolute advantage of a given source of funding, given that a higher $T_{j,b}$ implies a higher value of $\varepsilon_{j,b}$ (and thus a lower rate) is more likely.

Firms choose the source of finance with the lowest (post- shock) interest rate for each variety:

$$R_j(\omega) = \min_{f \in F} R_{j,f}(\omega). \tag{7}$$

As there is a continuum of varieties ω for each firm, the overall cost of funds to firm j is determined not by the realized cost of financing each variety $y_j(\omega)$, but rather the structural pricing among options. The structural pricing will be determined by the differential of surpluses between using a specific option and the best alternative. The shocks $\boldsymbol{\epsilon}_j$ in Eq. (6) are correlated within and across types of sources of funding, and the substitutability across different options will be given by how correlated shocks to different choices are for each given variety. This correlation, in turn, is governed by the three elasticity-of-substitution parameters, θ, ξ_E, and ξ_S.

The structure and functional forms of Eqs. (5), (6) and (7) are similar to the structure in Herreño (2020) and are an extension of the Ricardian model of international trade in Eaton and Kortum (2002) and the ensuing literature on corporate finance choices. In Eaton and Kortum (2002), the ϵ are productivity shocks from a country for a given good, and other countries choose which country to buy from according to a minimization rule, as in Eq. (7). In our setting, the ϵ are interest rate differences across varieties when financed from the same source of finance, and the firm chooses which sources of finance to use for each variety. The advantage of the nested Fréchet Distribution is that it allows for a parsimonious representation in which different sources of funding differ in their levels (absolute advantage) and substitutability (comparative advantage) at the firm level. From a modeling perspective, the Fréchet Distribution is a generalized extreme value distribution that has a closed-form solution for the minimum in Eq. (7).

In our model, we opt to focus on three sources of finance at the extensive margin (banks, nonbank external finance, and self-finance), but our theoretical framework can encompass any number of nests, with different degrees of substitution within and across nests. Therefore, our framework can be readily extended for a multitude of settings. For instance, one could consider a case where loans of different types have a different degree of substitutability across them (for empirical evidence of this channel, see Ivashina, Laeven and Moral-Benito (2020)).

Firms' Investment, Production, and Pricing Problem. The problem of the firm involves the choices of how much to produce across varieties, $y_j(\omega)$; output prices P_j; and how much to invest in increasing productivity, I_j, given a set of potential sources of finance with a correlation structure given by

$$\max_{I_j, P_j, y_j(\omega)} \; P_j \left[\int y_j(\omega)^{\frac{\sigma-1}{\sigma}} d\omega \right]^{\frac{\sigma}{\sigma-1}} - \frac{w_j}{I_j a_j} \left[\int R_j(\omega) y_j(\omega) d\omega \right], \tag{8}$$

where $R_j(\omega)$ is given by Eq. (7). The solution to the problem of the firm in Eq. (8) is given by the standard CES optimization for $y_j(\omega)$, a markup price condition for P_j, and a first-order condition for I_j:[10]

$$y_j(\omega) = \left(\frac{R_j(\omega)}{R_j} \right)^{-\sigma} Y_j, \;\; P_j = \frac{\eta}{\eta - 1} \frac{w_j}{a_j I_j} R_j, \text{ and } I_j = \Omega_{I,j} R_j^{-\gamma}, \tag{9}$$

where R_j is the rate index of the firm; that is,

$$R_j \equiv \left[\int R_j(\omega)^{1-\sigma} d\omega \right]^{\frac{1}{1-\sigma}},$$

and $\Omega_{j,I} \equiv \left\{ Y \left[\frac{\eta-1}{\eta}\right]^{\eta} \left[\frac{a_j}{w_j}\right]^{\eta-1} \right\}^{\frac{1}{\psi-\eta+2}}$. The value function from Eq. (8) can be written as a product of a firm-specific term, Ω_j (that depends on productivity a_j), and the effective rate of finance R_j; that is,

$$\Pi(\Omega_j, R_j) = \Omega_j R_j^{-\gamma}, \tag{10}$$

where $\Omega_j = \left[\frac{1}{\eta-1} - \frac{1}{\psi+1}\right] \Omega_{j,I}^{\psi+1}$ and $\gamma \equiv \frac{(\psi+2)(\eta-1)}{\psi-\eta+2}$.

The key to understanding the problem of the firm is the effective cost of finance, R_j. Our theory of R_j has two steps. First, we take the parameters in Eq. (6) as given and analyze what

[10] Derivations in Appendix B.

the shock distribution (Eq. (6)) and the choice across funding sources (Eq. (7)) imply for the firm's effective cost of funding, R_j. Second, we allow firms to optimally choose the parameters in Eq. (6) and thus provide a complete theory of the cost of finance heterogeneity across firms.

Firm's Effective Cost of Funding and Financing Shares. Conditional on a set of scale and shape parameters of the distribution in Eq. (6)—which we will endogenize below—we can derive the expected minimum interest rate of financing across all funding sources. In the main text, we focus on the results and leave the derivation to Appendix C. The firm's effective cost of funding, R_j, aggregates bank, bond, and self-financing rates into one index, similar to a CES price index given by

$$R_j = \tilde{\Gamma} \cdot \left[\zeta_{j,e} R_{j,e}^{-\xi_S} + \zeta_{j,sf} R_{j,sf}^{-\xi_S}\right]^{-\frac{1}{\xi_S}}, \tag{11}$$

where

$$\tilde{\Gamma} = \Gamma\left(\frac{1-\sigma}{\xi_S}+1\right)^{\frac{1}{1-\sigma}}, \quad R_{j,E} \quad = \left[\zeta_{j,B} R_{j,B}^{-\xi_E} + \zeta_{j,nb} R_{j,nb}^{-\xi_E}\right]^{-\frac{1}{\xi_E}}, \quad \text{and } R_{j,B} = \left[\sum_b^{N_b} T_{j,b} R_{j,b}^{-\theta}\right]^{-\frac{1}{\theta}}, \tag{12}$$

with $\Gamma(.)$ being the standard Gamma function. The share of total expenditures a firm finances with each different funding source can also be written in closed form. For a given bank b, for instance, the within-firm market share is given by

$$s_{j,b} = \underbrace{\frac{\zeta_{j,e} R_{j,E}^{-\xi_S}}{\zeta_{j,e} R_{j,E}^{-\xi_S} + \zeta_{j,sf} R_{j,sf}^{-\xi_S}}}_{\equiv s_{j,E}} \cdot \underbrace{\frac{\zeta_{j,B} R_{j,B}^{-\xi_E}}{\zeta_{j,B} R_{j,B}^{-\xi_E} + \zeta_{j,nb} R_{j,nb}^{-\xi_E}}}_{\equiv s_{j,B}} \cdot \underbrace{\frac{T_{j,b} R_{j,b}^{-\theta}}{\sum_k T_{j,k} R_k^{-\theta}}}_{\equiv \nu_{j,b}}, \tag{13}$$

where $s_{j,E}$ is the share of external funding (and $s_{j,sf} = 1 - s_{j,E}$ is the share of self-finance), $s_{j,B}$ is the share of banks within external funding sources (and $s_{j,nb} = 1 - s_{j,B}$ is the share of nonbank external funding), and $\nu_{j,b}$ is the share of bank b within banks.

A few comments are in order. First, note that R_j is the exact CES price index for the problem of the firm. Second, the term $\tilde{\Gamma}$ appears in R_j due to firms taking the minimum across different, but correlated, options. This term is a function of how much firms can substitute across varieties to produce the intermediate good (σ) and how much variability and correlation there are in the Fréchet Distribution (ξ_S). Third, from Eq. (13) it is clear that θ captures the elasticity of substitution of funding across banks, ξ_E captures the elasticity of substitution between banks and nonbank external finance, and ξ_S captures the elasticity of substitution between internal and external finance. These imperfect substitution patterns come from the correlation pattern in the Fréchet distribution: Two sources of finance that receive very similar shocks have similar rates from different varieties and thus are not substitutable from the perspective of the firm. Finally, note that this relative demand schedule depends on all interest rates, both across banks and across nonbank funding sources, to determine the total demand for a given source of finance.

Optimal Lending Relationships and Firm Entry. Let $\boldsymbol{T}_j \equiv \{T_{j,b}\}_b$ and $\boldsymbol{\zeta}_j \equiv \{\zeta_{j,B}, \zeta_{j,nb}, \zeta_{j,e}, \zeta_{j,sf}\}$ denote the vectors of relationship intensities for each source of finance. So far, we have focused on the problem of the firm conditional on a set of potential sources of finance at the extensive margin and relationships at the intensive margin, that is, exogenous $\boldsymbol{T}_j$ and $\boldsymbol{\zeta}_j$. We now endogenize this choice. The timing of the problem of the firm is as follows. The firm receives offer rates $\boldsymbol{R}_j$ and chooses $\boldsymbol{T}_j$ and $\boldsymbol{\zeta}_j$ to maximize its profits. Changes in $\boldsymbol{T}_j$ and $\boldsymbol{\zeta}_j$ change the effective rate of finance, R_j, and thus will affect profits through an increase in both investment and scale. Note that, conditional on $\boldsymbol{T}_j$ and $\boldsymbol{\zeta}_j$, and a vector of offer rates $\boldsymbol{R}_j$, the effective cost of finance is the same across firms; that is, we can write $R_j = \mathcal{R}(\boldsymbol{T}, \boldsymbol{\zeta}, \boldsymbol{R})$. The cost of finance variation in our model across firms comes from different choices of $\boldsymbol{T}$ and $\boldsymbol{\zeta}$, which have two complementary effects. First, different relationship intensities change the cost of finance for any given vector of offer rates. Second, the relationship intensities also change the offer rates through the profit maximization of banks (details to follow). The problem of the firm in the choice of optimal lending relationships is given by:

$$\max_{\boldsymbol{T},\boldsymbol{\zeta}} \; \Omega_j \mathcal{R}(\boldsymbol{T}, \boldsymbol{\zeta}, \boldsymbol{R})^{-\gamma} - c_{\mathcal{E}} \mathcal{E}(\boldsymbol{T}, \boldsymbol{\zeta}) \tag{14}$$

$$\text{s.t. } T_b \geq 0, \forall b, \text{ and } \zeta_f \geq 0, \forall f, \tag{15}$$

where the cost function $\mathcal{E}(.)$ has the following functional form:

$$\mathcal{E}(\boldsymbol{T},\boldsymbol{\zeta}) = \left\{ \zeta_E^{\rho} \left[\zeta_b^{\rho} \left(\sum_b T_b^{\rho} \right)^{\frac{\xi_E}{\theta}} + \zeta_{nb}^{\rho} \right]^{\frac{\xi_S}{\xi_E}} + \zeta_{sf}^{\rho} \right\}^{\xi_S^{-1}} + \sum_b FC_b[T_b > 0] + FC_{nb}[\zeta_{nb} > 0]. \tag{16}$$

The functional form of $\mathcal{E}(.)$ matches the correlation structure of the Fréchet effective cost of finance for the intensive margin investment in relationships. Therefore, investments at the intensive margin have degrees of substitutability that are similar to those of the sources of finance themselves, as well as an effect on the effective cost of finance that is comparable to the cost of investment itself. Alternative cost functions (for example, a linear cost function) miss this correlation structure. For instance, using the functional form in Eq. (16), increasing ζ_e and ζ_{sf} by a factor of α costs $c_E \alpha^{\frac{\rho}{\xi_S}}$ and has the effect of reducing the effective rate of finance by a factor of $\alpha^{\frac{1}{\xi_S}}$.

We solve the problem of the firm in two steps. First, conditional on which $\boldsymbol{T}$ and ζ are strictly positive, we determine their optimal values, that is, the optimum intensity of relationships. Second, we focus on which elements of $\boldsymbol{T}$ and $\boldsymbol{\zeta}$ are strictly positive to maximize the profit of the firm, taking into account the implied profit from our first step.

Note that the problem of the firm in Eq. (14) does not feature a unique solution due to the scale of $\boldsymbol{T}$ and $\boldsymbol{\zeta}$.[11] Therefore, for simplicity, we focus on the solution where we normalize the within-nest intensities scale such that $\sum_b T_b^{\rho} = 1$, $\zeta_b^{\rho} + \zeta_{nb}^{\rho} = 1$. In this case, for any $b, \tilde{b}$

$$\frac{T_{j,b}}{T_{j,\tilde{b}}} = \left[\frac{R_b}{R_{\tilde{b}}} \right]^{\frac{-\theta}{\rho-1}}, \quad \frac{\zeta_{j,B}}{\zeta_{j,nb}} = \left[\frac{R_B}{R_{nb}} \right]^{\frac{-\xi_E}{\rho-1}}, \text{ and } \frac{\zeta_{j,e}}{\zeta_{j,sf}} = \left[\frac{R_E}{R_{sf}} \right]^{\frac{-\xi_S}{\rho-1}} \tag{17}$$

and $\zeta_{sf,j}$ is pinned down by the first-order condition of Eq. (14) with respect to ζ_{sf}; that is

$$\zeta_{j,sf} = \left\{ \frac{\gamma}{c_{\mathcal{E}}} \Omega_j R_{sf}^{-\xi_S} \left[\left[\frac{R_E^{-\xi_S}}{R_{sf}^{-\xi_S}} \right]^{\frac{1}{\rho-1}} R_E^{-\xi_S} + R_{sf}^{-\xi_S} \right]^{\xi_S^{-1}-1} \right\}^{\frac{1}{\rho+1-\gamma/\xi_S}}. \tag{18}$$

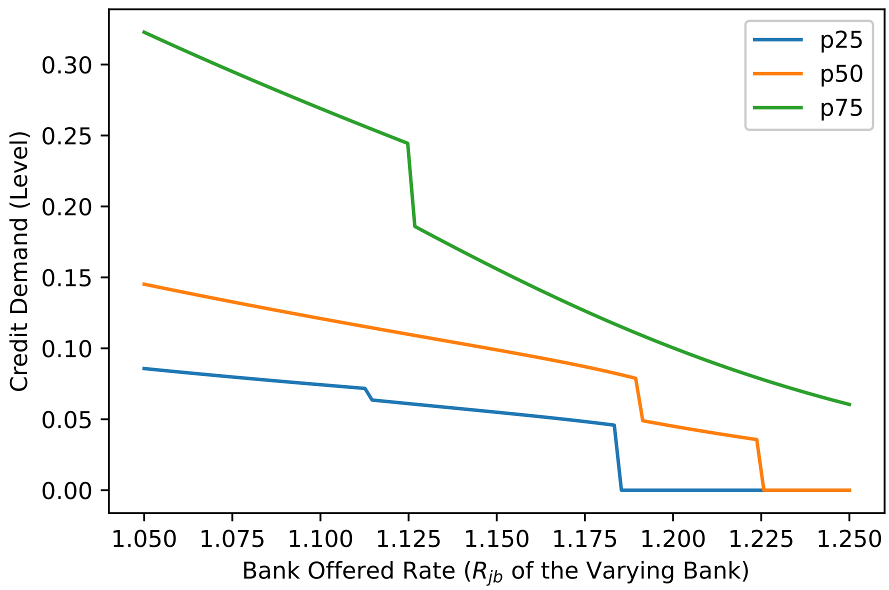
[11] For instance, let $\boldsymbol{T}_0$ be part of a solution to the problem of the firm. Now, let $\boldsymbol{T}_1$ be such that: $T_b^1 = \frac{T_b^0}{\sum_b T_b^0}$ $\zeta_b^1 = \zeta_b^0 \left(\sum_b T_b^0 \right)^{\frac{\xi_E}{\theta}}$. Then, $\boldsymbol{T}_1$ is also a solution.

We next focus on which elements of $\boldsymbol{T}$ and $\boldsymbol{\zeta}$ are strictly positive. We start with the understanding that there is a pecking order within banks; that is, the firm chooses to build relationships with the cheapest banks before moving to the more expensive ones. Mathematically, this result is stated in Appendix D. The existence of a pecking order greatly facilitates the computation of the corner solutions in terms of relationships with banks. Given a set of $\boldsymbol{\zeta}$, the firm chooses to establish a banking relationship with the cheapest bank (the one with the lower offer rate $R_{j,b}$) if and only if total profits net of the cost of establishing a relationship are larger than the profits without access to bank credit. Similarly, the firm adds new bank relationships to its existing set of banking relationships, adding banks with higher offer rates $R_{j,b}$ to the mix, as long as the marginal bank relationship compensates for the additional fixed cost and variable cost of creating this relationship. Beyond that, we determine if the firm uses a bank relationship or nonbank external funding by numerically checking each of the four cases individually.

Finally, let $\mathcal{V}(a_j)$ be the value function of the firm in Eq. (14). Firm entry is determined by the zero-profit entry condition, $\mathcal{V}(a_j) = 0$. This condition determines an a^* threshold below which firms do not enter.

The elements introduced in this section are novel to both the corporate finance literature and the international trade literature and constitute a key theoretical contribution of this paper. We fully endogenize the sources of finance that firms use, at both the intensive and extensive margins, thus providing a theory of the effective cost of finance determination across firms. Our theory of optimal relationships is Ricardian in nature; that is, it is based on the comparative advantages of different sources of finance. In the international trade context, this would be equivalent to countries investing in improving their productivity distribution and in reducing bilateral trade costs. This novel element is what allows us to speak directly to the observed heterogeneity in corporate finance choices across the firm-size distribution (Section 2), understand the transmission of shocks to the real economy (Section 4), and evaluate state-of-the-art empirical methods for estimating the effect of credit supply shocks (Section 5).

Figure 5: Bank-Firm Specific Credit Demand Functions



Notes: The figure shows the demand function for credit from bank b for three different firms j, with productivity in the 25th, 50th, and 75th percentiles. Source: authors' calculations.

3.2 Credit Demand Functions

Let $R_{j,b}$ be an offer rate of bank b to firm j and $\boldsymbol{R}_{-b}$ a vector of offers from all other banks $\tilde{b} \neq b$. Total demand for credit from bank b is the relative demand schedule multiplied by the total financing need, which is given by

$$\mathcal{D}_b(R_b, \boldsymbol{R}_{-b}) \equiv \gamma \Omega_j R_j^{-\gamma-1} s_{j,b}. \tag{19}$$

Combining the optimal intensities for different lending relationships with the optimal funding shares, we obtain a nonlinear and discontinuous demand schedule—a function that maps prices to demanded quantities—that encompasses both the extensive and intensive margins of credit. Previous work in the trade literature and finance literature has taken the intensities as given, implying that changes in prices lead to adjustment only at the intensive margin but neglect an important channel of adjustment in credit markets.

Figure 5 illustrates the key features of the credit demand function that is at the core of this paper.[12] For large enough offer rates, the firm decides not to establish a relationship with a given bank and hence demanded credit volumes are zero. For sufficiently low offer rates, the firm finds it profitable to incur the fixed cost of establishing a relationship. At this point, the credit demand exhibits a jump, as the firms will demand a quantity of loans that satisfies an interior solution with a strictly positive loan volume. As the offer rates decline further, the firm continuously increases its borrowing from the bank. However, for small enough rates, the credit demand function may exhibit another discontinuity, because the firm may find it profitable to discontinue some of the relationships it has with other banks, stop borrowing from them, and shift its demand to the bank with lower offer rates, leading to a discontinuous jump in the credit demand function.

3.3 Banks' Problem and Loan Market Equilibrium

In the loan market, banks compete by offering interest rates to firms. Banks have complete information about the credit demand curve of each firm, but they do not internalize the general equilibrium effects of their actions (prices, output).

Each bank b chooses the interest rate it charges firm j, $R_{j,b}$, taking the interest rates of all competitor banks as given and funding the loan with deposits. The problem of the bank, given offer rates $\boldsymbol{R}_{j,-b}$ of the other banks, is given by

$$\max_{R_{j,b}} \mathcal{D}\Big(R_{j,b}, \boldsymbol{R}_{j,-b}\Big)\Big[\lambda_b R_j^{min}\Big(R_{j,b}, \boldsymbol{R}_{j,-b}\Big) + (1-\lambda_b)t - R_{b,d},\Big] \tag{20}$$

where $1-\lambda_b$ is the probability of default, and $R_{b,d}$ is the rate the bank pays to its depositors, and

$$R_j^{min}\Big(R_{j,b}, \boldsymbol{R}_{j,-b}\Big) \equiv \mathbb{E}[\tilde{R}_{j,b}|\tilde{R}_{j,b} = \min_f \tilde{R}_{j,f}] = \frac{\hat{\Gamma}}{\tilde{\Gamma}} R_j, \tag{21}$$

where $\tilde{R}_{j,f} \equiv \varepsilon_f^{-1} R_{j,f}$ and $\hat{\Gamma} \equiv \Gamma\left(\frac{1}{\xi_S}+1\right)$. The only difference between R_j^{min} and the effective cost of finance R_j (which corresponds to the CES rate index for the firm) is the Gamma function term.

[12] While the figure is computed using our baseline parameterization discussed below, the qualitative insights hold more generally.

A loan market Nash equilibrium $\boldsymbol{R}_B^*$ is a solution to Eq. (20) for every bank $b = 1, \ldots, N_b$. Everything else being equal, banks with lower cost can offer lower rates for firms. Note that the bank's choice variable, the offer rate $R_{j,b}$, is not the interest rate that the firm pays when borrowing from bank b, and the bank internalizes this in its profit function. We show in Appendix C that conditional on being the minimum across all sources of finance, banks receive R_j times a factor, which we can normalize to 1 in our calibration. We discuss the difference between offer rates and the amount paid by firms in detail in Section 3.4.

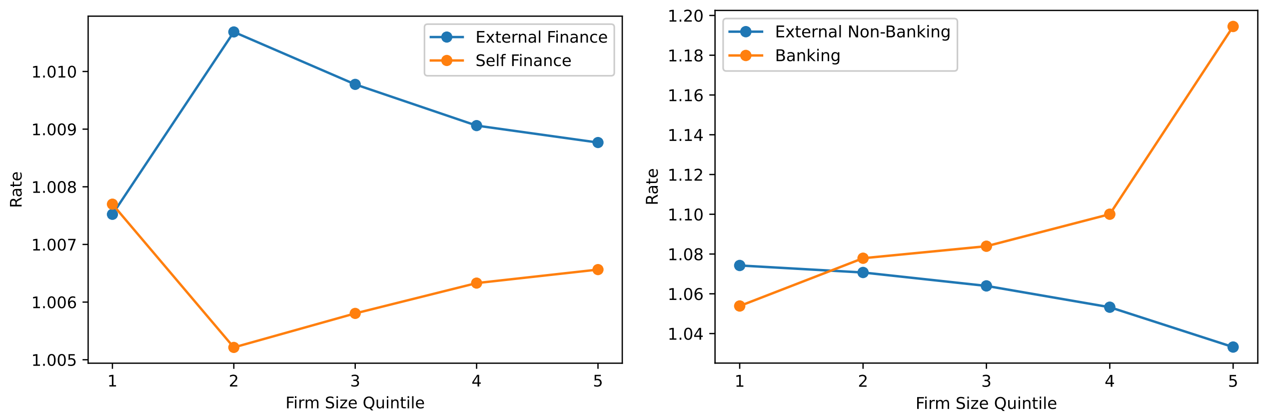
Numerical Solution. To compute the model solution, we adopt a best iterated response algorithm across banks. We solve the model separately for each firm j. We initialize the algorithm with a set of offer rates for each bank. For each bank, we vary the offer rate of this bank, taking what competitors did in the previous round as given, and construct the profit function as follows. We compute for each firm the profit and optimal relationships under four alternative extensive margin choices: with and without nonbank external finance and with and without bank funding (the firm always uses self-finance). We maximize the extensive margin choice over these four alternatives for the firm, which in turn gives a bank's profit at this offer rate. Each bank updates its offer rate based on maximizing profits. We iterate this process until we find a Nash equilibrium.

3.4 Offered versus Observed Rates

In this section, we explore the differences between offered and observed rates in our model, discuss how to infer offer rates from the data, and test one of the model predictions for observed rates.

Firms in our model are faced with offered rates $\boldsymbol{R}_j$ that are not the rates (or even the average or representative rates) a firm actually pays when borrowing from a given financial source. First, each offered rate $\boldsymbol{R}_j$ is subject to a multiplicative shock that does not necessarily have an average of 1. Second, firms choose the source of finance that offers the minimum rate. The interest rate that a firm ends up paying for a source of finance is *conditional* on that source of finance having the lowest offered rate across *all* sources of finance. Therefore, the rates we observe in the data are not offered rates or effective rates. The model-implied observed rate is R_j^{min}.

Figure 6: Implicit Offer Rates (Relative to the Effective Cost of Finance)



(a) Self-Finance ($\hat{R}_{sf}$) and External ($\hat{R}_E$) (b) Bank ($\hat{R}_B$) and Nonbank ($\hat{R}_{nb}$) debt

Note: Implied offer rates relative to the effective cost of finance of firm j, computed from Eq. (22). We compute the rates by firm j and plot the median within firm-size quintiles. The figure is based on all borrowers with 2019 information in Y-14. Shares are computed as in Figure 2. Bank Debt includes only debt from Y-14 banks. Other Debt includes bonds and loans obtained outside of Y-14 banks. Self-Finance is the stock of retained earnings. Source: FR Y-14Q data, H.1 schedule with C&I lending, authors' calculations.

To understand which offer rates would be consistent with the data and highlight how they differ from observed rates, we can invert our credit demand system in Eq. (13), up to a firm specific normalization on $\zeta_{j,sf}$, to obtain [13]

$$\hat{R}_{sf} = s_{SF}^{-\frac{\rho-1}{\xi_S \rho}}, \quad \hat{R}_E = s_E^{\frac{-\rho-1}{\xi_S \rho}}, \quad \hat{R}_{nb} = \hat{R}_E s_{nb}^{-\frac{\rho-1}{\xi_E \rho}} \text{ and } \hat{R}_B = \hat{R}_E s_B^{-\frac{\rho-1}{\xi_E \rho}}. \tag{22}$$

Figure 6 shows the median of the implicit offer rates (relative to R_j) across firms within firm-size quintiles.[14] Relative to their effective cost of finance, larger firms have higher implicit offer rates from banks and lower implicit rates from external nonbank debt. This is the counterpart in our model of the shares observed in Figure 2. Even though larger firms pay, on average, lower rates on their bank loans (Figure 3), larger firms have a comparative advantage in self-finance and thus use less bank funding relative to smaller firms. Our model features endogenous absolute and comparative advantages of different sources of finance across firms and can therefore speak directly to this evidence.

Finally, note that R_j^{obs} in Eq. (21) is not a function of the source of finance f. Given that all

[13]We normalize $\zeta_{j,sf}$ to have the results analogous to the external finance nest.

[14]We focus on the median (and not the mean) to avoid issues with firms that don't use a given source of finance and, therefore, have an infinite implicit offer rate.

sources of finance have the exact same *expected* interest rate, conditional on being chosen for a given project in our model, we shouldn't observe systematic within-firm differences in interest rates in new loan originations. To test this hypothesis, we run a set of regressions of loan-level interest rates on firm (or firm-time fixed effects) and loan characteristics, and focus on the unexplained component of within-firm variation in interest rates, controlling for loan characteristics. More specifically, we run

$$i_{l,f,t} = \alpha_{f,t} + \boldsymbol{\beta} X_{l,t} + \varepsilon_{l,f,t}, \tag{23}$$

where $i_{l,f,t}$ is the interest rate on loan l for firm f at time t, and $\alpha_{f,t}$ are firm-quarter fixed effects. Although our model does not take some loan characteristics into account (for example, maturity), we control for those characteristics, because ultimately the implicit rate R_j^{obs} is the one perceived by firms given a set of loan characteristics. For instance, a loan that requires more collateral will require a higher t_f in our model and thus a lower offer and observed rate, even in a within-firm analysis.

To test for systematic within-firm differences in interest rates, we adopt a version of the nonparametric permutation test used in Schoefer and Ziv (2020). Given that every firm borrows from a few sources, and because our theory applies only to expected rates, there will be natural variation in within-firm rates, as the law of large numbers does not apply to a handful of bank loans. Therefore, the idea is to understand if the within-firm residuals simply have some random variation (which would be consistent with our theory), or if they are systematically differently distributed.

The test proceeds as follows. We take the residuals at the firm level and randomly allocate them across different firms. We repeat this random allocation procedure and obtain the distribution of the within-firm standard deviation of residuals, which we then compare with the observed within-firm standard deviation of residuals. Table 1 describes the control variables in detail and shows our results. We find that within a given firm-quarter, the variation in interest rates is approximately random, which is consistent with our model.

Table 1: Loan-Level Within-Firm Variation in Interest Rates in Y-14

	(1)	(2)	(3)	(4)
Mean Within-Firm SD	0.318	0.324	0.330	0.329
Mean Within-Firm SD (Permutation)	0.330	0.308	0.452	0.483
Median Within-Firm SD	0.0750	0.167	0.173	0.165
Median Within-Firm SD (Permutation)	0.195	0.209	0.309	0.351
Percentile	0	1	0	0
Firm FE	Yes	Yes	Yes	Yes
Quarter FE	Yes	Yes	Yes	Yes
Firm-Quarter FE	Yes	Yes	No	No
Facility-Og Date FE	No	Yes	Yes	Yes
Securitized-Og Date FE	No	Yes	Yes	Yes
IR Index-Og Date FE	No	Yes	Yes	Yes
Maturity-Bin-Og Date FE	No	Yes	Yes	Yes
R-Squared	0.868	0.897	0.829	0.790
Adj. R-Squared	0.802	0.842	0.781	0.726
RMSE	0.646	0.577	0.679	0.729
Observations	73518	73518	73518	144526

Note: This table shows output for regressions of loan interest rates on firm-quarter fixed effects (both columns) and including loan controls (column 2). The interest rates are for all C&I loan originations recorded in Y-14Q from 2011Q3 through 2021Q2 and are expressed in percentage points. The sample in column (1) is fixed to be that which is generated by the specification in (2). Permutation test is the average within-firm standard deviation from the permuted data. Og Date FE stands for origination quarter fixed effect. Facility refers to facility type, which reflects whether the loan is categorized as a term loan, revolving credit facility, or "other." The IR (interest rate) Index is a categorical variable representing the type of interest rate index on the loan (undrawn, LIBOR, prime, Treasury Index, SOFR, fixed, mixed, other). Maturity bins categorically represent the number of years until the loan matures and range from 1 to 15. Loans that mature in 15 years or longer are top-coded to 15. The data have been trimmed at the top 99 percent of the left-hand-side variable to account for large outliers. The sample also excludes observations for which negative loan volumes were reported.

4 Quantitative Analysis

Table 2 shows the parameters that we use in the quantitative model analysis. These parameters were chosen to match the key empirical moments discussed in Section 2. Common parameters, such as the elasticity of substitution of output (η), are calibrated to standard values, and we focus here on the discussion of parameters of our novel modeling elements. It is noteworthy that we calibrate the fixed cost of accessing the bond market to be about three times larger than the fixed cost of establishing a banking relationship, as issuing public debt typically requires expensive external credit ratings and audits. The elasticity of substitution between external and internal financing (ξ_S), between bond and bank debt (ξ_E), and between debt from different banks θ is calibrated to 80, 20, and 10, respectively.[15] We set the offer rate of

[15] Although the elasticity parameters may seem high at first glance, they represent the change in market shares for two banks based on changes in the gross rates. For instance, a change of interest rates from 2 percent to 5 percent (a 150 percent increase), which should generate significant movement in market shares, corresponds to a

bond loans to 17 percent, the offer rate of self-financing to 19 percent (as discussed above, these are not the actual rates paid), and cap the implied R_j, given this model parameterization, at 2.5 percent, without having a material effect on our insights. Moreover, we calibrate the exponent of the Pareto distribution to 2. For computational reasons, we simulate the economy with 300 firms and four banks with heterogeneous deposit rates, although increasing the number of firms has little effect on the quantitative results in this section.

In Figures 7 and 8, we show targeted moments (auxiliary statistics) for both the Y-14 data and the simulated data under the baseline parameterization. Our focus is on the variation of key moments that summarize the funding mix across the firm-size distribution; that is, we are interested in the relative size of variables across different size groups. Therefore, we group firms into quintiles and report standardized values of within-quintile means (that is, the average across quintiles is 0 and the variance is 1).[16]

Figure 7 depicts the product-market concentration and effective cost of financing across quintiles. Panel 7a shows that, both in our model-simulated data and the Y-14 data, the implied sales distribution is highly skewed, with the top quintile of firms having an average sales share about two standard deviations larger than the average sales share across all firms. Panel 7b depicts the effective cost of financing. We measure the cost of financing for Y-14 firms as the 2019 interest rate expenses as a percentage of beginning-of-period stock of total debt. In both data sets, small firms pay substantially higher interest rates compared with large firms: The smallest quintile pays about 1.5 standard deviations more than the average firm, while the firms in the top quintile pay about 0.5 to 1.0 standard deviations less.[17]

Figure 8 depicts the corporate finance structure across the firm-size distribution, both for simulated and empirical data. Panel 8a shows the standardized share of bank funding. In both the simulated and empirical data, small firms have a substantially larger reliance on banks in their overall funding mix, and this share decreases as firms become larger. Note that in our model, we also find that the very smallest firms in the model rely exclusively on self-finance and do not establish any bank relationship, as it is too costly. For comparison with the firms in the Y-14 data, any of which have some bank debt, we remove those firms from the figure.

change of only 2.9 percent in terms of gross rates.

[16] The raw (nonstandardized) variables for simulated and empirical data are reported in Appendix E.

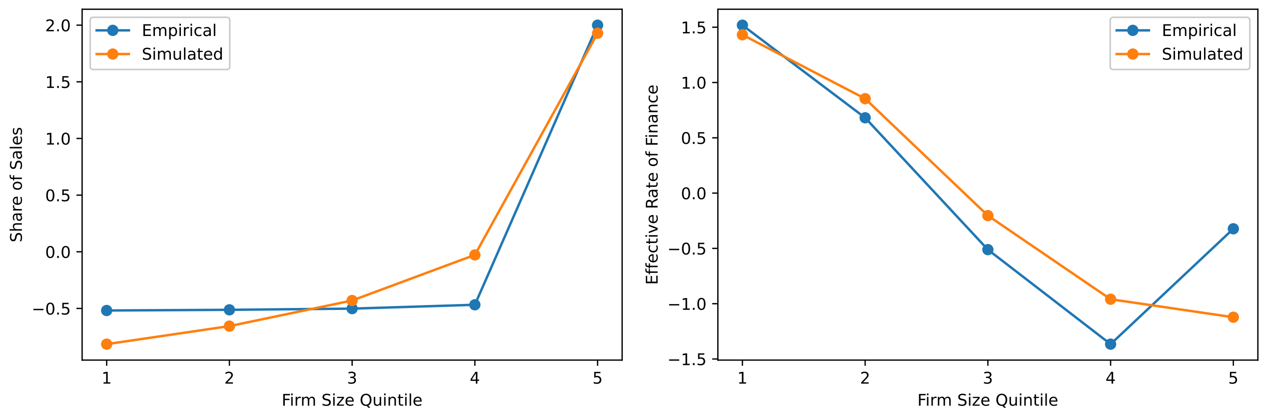
[17] As in Figure 3, we orthogonalize the Y14 data in Figure 7b with respect to rating. Note also that Figure 3 is based on bins of equal length in log(Sales) and not based on quantiles as Figure 7b.

Table 2: Model Parameterization

Parameter	Baseline	Note
		Number of Banks and Firms
N	300	Number of firms
N_B	4	Number of banks
		Production and Investment
η	1.6	Elasticity of Substitution - Intermediate Goods
ψ	0	Investment Cost Curvature
		Elasticity of Substitution: Sources of Finance
θ	10	Banks
ξ_E	20	Bank and Nonbank External Finance
ξ_S	80	External and Self-Finance
		Cost of Lending Relationships
c_ε	0.01	Scale
ρ	8	Curvature on $\boldsymbol{T}$ and $\boldsymbol{\zeta}$
FC_b	0.1	Fixed Cost of Bank Relationship
FC_{nb}	0.3	Fixed Cost of Nonbank External Finance
		Nonbank Offer Rates and Deposit Rates
R_{nb}	1.17	Offer Rate of Nonbank External Finance
R_{sf}	1.192	Offer Self-Finance Rate
$R_{b,d}$	$1.1 \cdot \langle 1.04, 1.02, 1.034, 1.0341 \rangle$	Deposit Rates
		Pareto Productivity Distribution
a_a	2	Shape
m_a	4	Scale
l_a	-0.81	Location

Note: All parameters are pooled across firms unless indicated otherwise. All interest rates are presented in decimals and expressed as gross rates, that is, 5 percent is expressed as 1.05. We set $w = Y = 1$ (partial equilibrium) and $\lambda_f = t_f = 1$ (no default). We don't need to define σ in our model: it affects only the scale of R_j by a factor that is constant across firms, and thus it simply scales profits. As we calibrate the relative cost scale, $c_\mathcal{E}$, we therefore do not need to consider calibrating σ.

Figure 7: Market Concentration and Funding Cost



(a) Sales Share

(b) Interest Rate

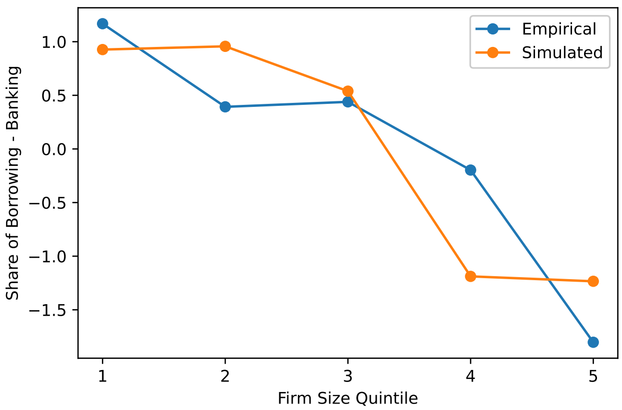
Note: The figure depicts the simulated and empirical market shares (Panel a) and the effective rate of finance (Panel B). Each series is standardized across quintiles to have zero mean and unit variance. The effective rate of finance for firms in Y-14 is computed as interest expenses as a percentage of stock of debt. Y-14 figures refer to 2019 values. Source: FR Y-14Q data, H.1 schedule with C&I lending, authors' calculations.

Panel 8b shows the standardized share of nonbank external funding (that is, bond funding). Larger, more productive firms find it profitable to pay the large fixed cost of accessing bond funding, and, as a result, the share of bond funding increases for larger firms. Indeed, both in simulated and empirical data, the largest quintile of firms has a share of bond funding that is more than one standard deviation larger than the average share across all firms. For these large firms, along with the increase in bond funding, the reliance on bank loans in the overall funding mix declines. The relationship between the share of self-financing and firm size declines somewhat in the simulated data, but the relationship between the self-financing share and size is less clear in the empirical data (Figure 8c).[18]

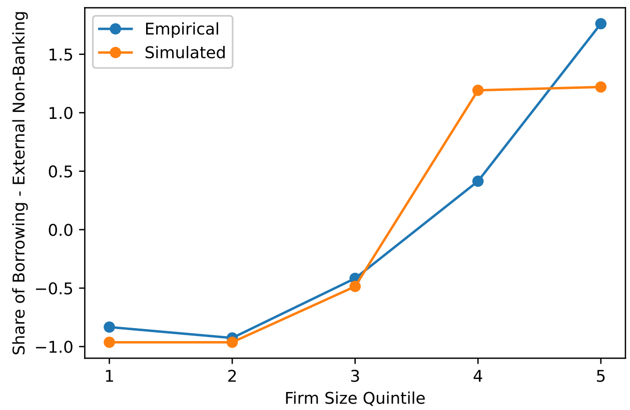
Moreover, while the overall dependence on bank funding decreases with firm size in favor of bond funding, we also find in Panel 8d that larger firms have more established relationships with different banks. This feature is also present in the Y-14 data. In our model, those large and productive firms find it worthwhile investing in multiple banking relationships at the extensive margin to decrease their overall cost of funding. Given that the large firms have higher overall demand to fund their investment, the reduction in interest rates brought about

[18]The levels of the self-financing share are shown in Appendix Figure A.2 and support the view that there is a less clear relationship between the self-financing share and firm size compared with the shares of banking and nonbanking and firm size.

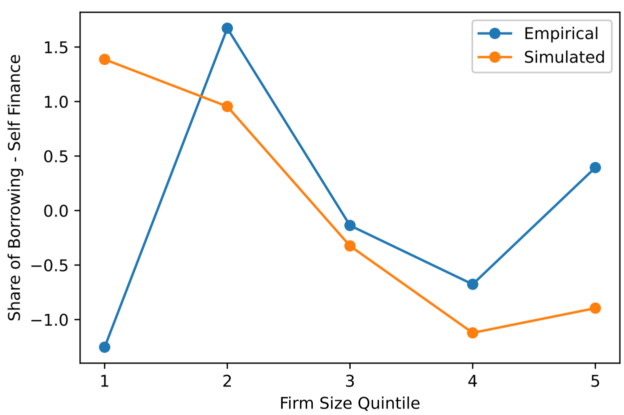
Figure 8: Corporate Finance Structure Across Firm-Size Distribution



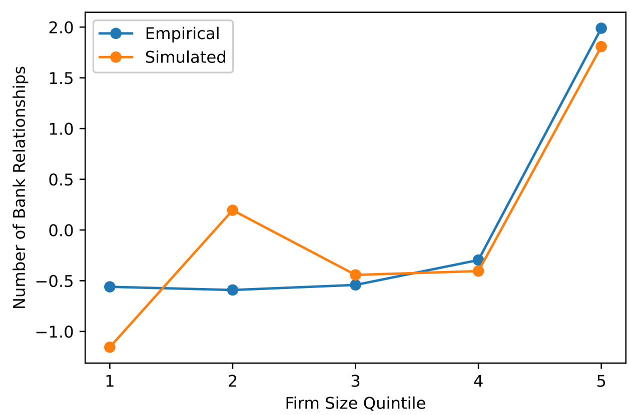
(a) Bank Share



(b) Nonbank Share



(c) Self-finance Share



(d) Number of Bank Relationships

Note: The figure depicts the simulated and nonbank funding share (Panel a), bank funding share (Panel b), self-financing share (Panel c), and the number of banking relationships (Panel d). Each series is standardized across quintiles to have zero mean and unit variance. Source: FR Y-14Q data, H.1 schedule with C&I lending, authors' calculations.

by establishing additional banking relationships outweighs the fixed cost of doing so.

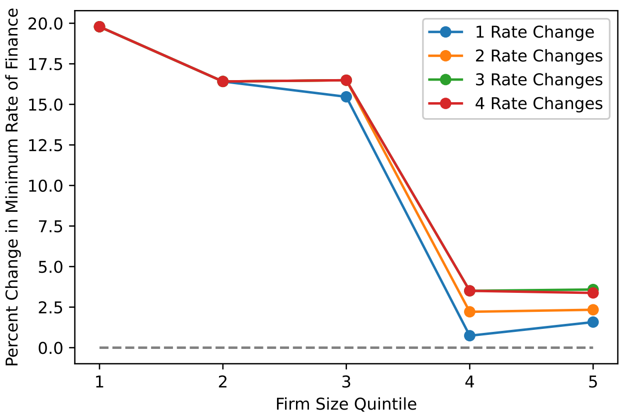
How does a shock to banks affect firms' financing, investment, and production decisions, and what are the macroeconomic implications? We use our model to answer these questions by focusing first on the differential impact of an increase in banks' financing costs on firms. Given the heterogeneity in financing across firms, our focus is first on the differential impact depending on firm size. Our key structural parameter that we vary is banks' deposit rate. We start by increasing only the funding cost of the bank with the lowest deposit rate, then add the shock to the second-most efficient bank, etc., until all banks in the economy face an increase in their deposit rate by 1 percentage point. The results are presented in Figure 9.

Figure 9a shows, by firm-size quintile, the average percentage change in firms' funding cost and output when different numbers of banks in the economy face a 1 percentage point increase in their deposit rate. Given the strong bank dependence of small firms, the response of the first quintile is strongest with an increase in funding cost of 20 percent. Part of the increase is driven by the higher cost of borrowing from banks that pass on their own increase in funding cost to their borrowers. However, another channel at work is that some firms (those with low productivity) stop borrowing from banks because the cost of establishing a relationship is too high. Instead, those firms resort to more costly (in terms of interest rate) self-financing, which doesn't require a fixed cost payment. The increase in firms' funding cost is less strong for larger firms that tap into the bond market or, in case of a shock to select banks only, use multiple banking relationships to satisfy their overall funding needs. These firms can partly offset the increase in borrowing cost from shocked banks by substituting away to other sources of finance.

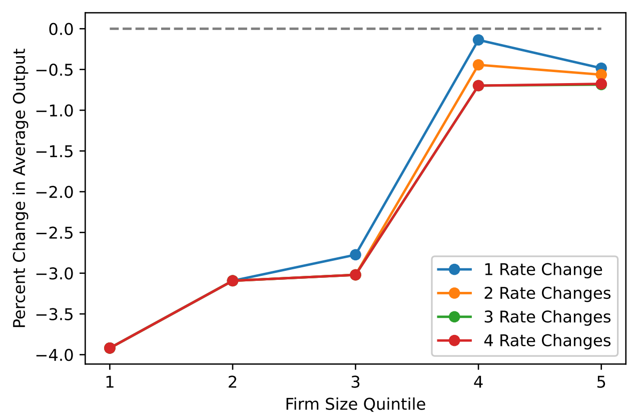
As a result of the transmission of bank shocks to firms' financing cost, firms reduce investment, output (production), and sales. Figure 9b shows that the decline in output is strongest for small firms, with a contraction of close to 4 percent. But even the largest firms reduce output by about three-quarters of a percent due to the bank shock indicating sizable transmission, even to the largest firms. This happens because large banks also tap into the bank loan market for funding, or, if they substitute to other sources, they need to pay more at the intensive margin.

Along with the output decline, Figure 9c shows a corresponding decline in sales value; that is, prices do not adjust in response to a drop in output to keep the sales value constant. A

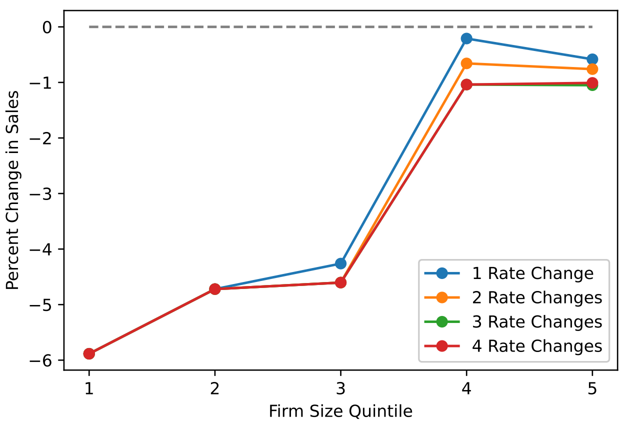
Figure 9: Impact of Bank Shock across Firm-Size Distribution



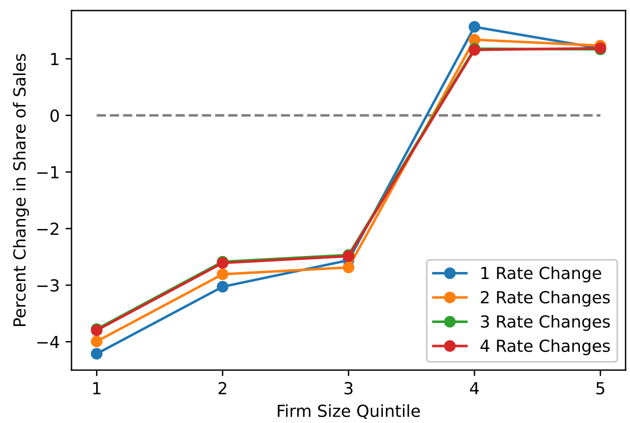
(a) Interest Rate



(b) Output



(c) Sales



(d) Market Share

Note: The figure depicts the effect of shocks to bank funding costs on firms' funding cost (Panel a), output (Panel b), sales (Panel c), and market shares (Panel d). We depict the effects for four different scenarios where we shock one, two, three, or four banks' funding costs (deposit rate) by 1 percentage point. When one bank is shocked, we shock the bank with the lowest deposit rate. When two banks are shocked, we shock the two banks with the two lowest deposit rates, etc. See text for more details. Source: authors' calculations.

further implication of the differentially stronger impact of bank shocks on small as compared with large firms is that, while total output and sales decrease, the product market also becomes more concentrated toward large firms. This result is shown in Figure 9d. The smallest firms experience a 4 percent decline in their market share (share of total sales), while the largest firms see an increase of their market share of about 1 percent.

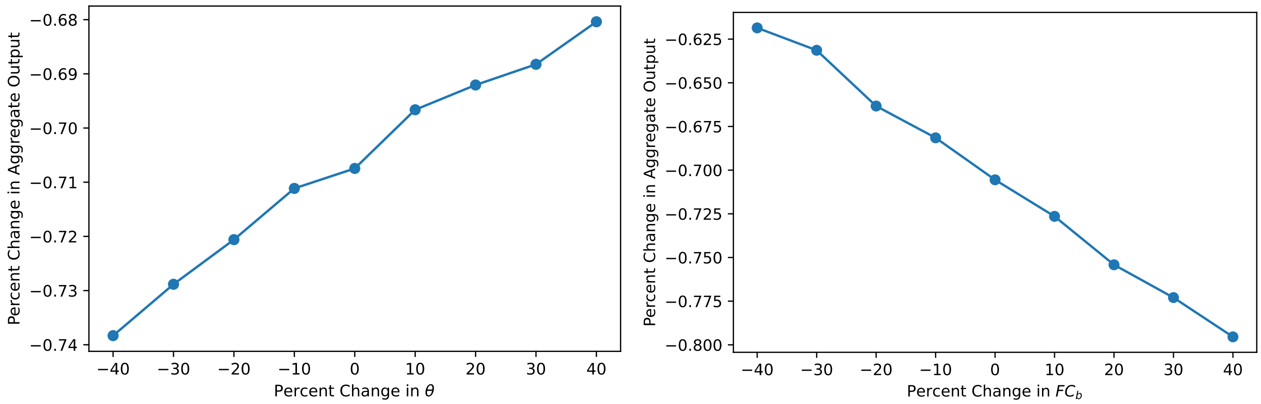
The analyses presented in Figure 9 are derived under a fixed sequence of banks that are shocked. In particular, as discussed above, we first shock the lowest-deposit-rate bank, then add a shock to the second-lowest-deposit-rate bank, etc. In Figure A.3, we present the results for alternative scenarios in which we permute the sequence of banks being shocked. As the figure shows, the qualitative insights remain the same, but the quantitative conclusions are somewhat smaller, because a shock to the lowest-deposit-rate bank always has the largest effect, given that firms borrow more from this bank, which can offer cheaper rates compared with competitor banks.

We can also use our calibrated model to study the impact of frictions in credit markets on aggregate output losses after a bank shock. To this end, we simulate the aggregate output response to a 1 percentage point increase in the deposit rate of the lowest-deposit-rate bank under different parameterizations of the model. In particular, we vary the elasticity of substitution of bank credit (θ) and the fixed cost of establishing a bank relationship (c_b).

Figure 10 shows that when the elasticity of substitution between credit from different banks decreases, the aggregate output falls more in response to a bank shock. In particular, a decline in substitutabilty of bank credit by 40 percent (from our baseline of 10 to 6), increases the output response to a 1 percentage point funding cost shock to one bank in the economy by a sizable 4 percent (in absolute values).[19] Similarly, when it becomes more costly to switch banks because of higher fixed costs of establishing banking relationships, the output response becomes stronger (larger in absolute values). These results highlight that frictions in the bank loan market have a sizable effect on the transmission of financial shocks to the macro economy.

[19] We estimate a reduction in output of 0.71 percent with an elasticity of 10, compared with a reduction of 0.74 percent with an elasticity of 8.

Figure 10: Credit Market Frictions and Aggregate Impact of Bank Shock



(a) Output Response vs. θ

(b) Output Response vs. FC_b

Note: The figure depicts how changes in the elasticity of substitution (θ, in Panel a) and the fixed cost of establishing bank relationships (FC_b, in Panel b) affect the response of aggregate output to a 1 percentage point increase in the deposit rate of one bank in the economy. Specifically, we shock the funding cost of the bank with the lowest deposit rate. See text for more details. Source: authors' calculations.

5 Implications for Within-Firm Regressions

A prominent empirical approach to studying the effect of bank shocks on credit was pioneered by Khwaja and Mian (2008), and it has become the state of the art in the empirical analysis of micro (loan-level) data. At a high level, this approach exploits the fact that some firms borrow from multiple banks, some of which are subject to a capital or liquidity shock while others are not. If the researcher now observes, for the same bank at the same time, the loan volume of shocked banks to decline by more than the loan volume of unaffected banks, the argument is that the differential reduction of loan volumes is due to a credit supply shift of the affected bank, as any bank or common time variation could be differenced out with a firm*time fixed effect.

Several studies, including one by Paravisini, Rappoport and Schnabl (2015), point out that under certain circumstances this approach does not isolate a supply shift. For example, if firm-bank-specific credit-demand shocks and bank-supply shocks are correlated, which may be a relevant scenario in the case of bank-firm specialization (sorting at the extensive margin), this approach breaks down. Paravisini, Rappoport and Schnabl (2015) discuss an example in which exporting firms tend to borrow from banks to fund their exports with international exposure such that an international shock can lead to a simultaneous credit supply reduction and a demand reduction for financing exports. Ivashina, Laeven and Moral-Benito (2020) find that borrowers source different types of loans from different lenders and, thus, the identifying assumption that loan demand is fixed across different lenders does not hold.

Our model can help us better understand when and why the reduced-form within-firm estimator breaks down and does not (fully) recover the supply effect. Taking a step back, clearly one source of bias of the within-firm estimator is related to the sample composition: The within-firm estimator requires firms to have at least two credit relationships before and after the shock. The estimator cannot be applied to firms with just one bank lender. Hence, if the credit-supply effect is heterogeneous across firms, the within-estimator based on a restricted sample of firms does not uncover the true average effect. This source of bias is intuitive and inherent to the approach but nevertheless important, as we will show. In addition to the bias of the average affect across all firms stemming from sample selection, the within-firm estimator of the average effect can also be biased for the set of firms that have multiple lenders in both periods (pre- and post-shock), due to nonrandom matching between firm types and shocked banks. This sort of bias is more interesting economically, and our model provides an intuitive way to understand it.

Banks with lower funding cost (deposit rates) will be able to offer lower rates than other banks to the same customer. As a result, firms with low productivity, which borrow externally from only one bank, will always borrow from the bank with the cheapest deposit rate (although the rate they pay on this debt will be higher than what more productive firms pay to the same bank). Hence, lower-productivity firms will satisfy a larger share of their funding needs from banks with lower deposit rates as compared with higher-productivity firms. In other words, there is bank-firm sorting at both the extensive and intensive margins of credit. This sorting leads to a bias in the within-firm estimator for the set of firms with multiple banking relationships, even in the absence of any correlated credit-demand shock, just by the mere fact that firms with low productivity have a different slope (and level) to their credit-demand curve compared with shocked banks. In particular, the credit-demand curve of lower-productivity firms is less elastic than that of higher-productivity firms.

To quantify the different biases of the within-firm estimator for data simulated from our calibrated model, we simulate two "periods" of our (static) model, a period $t = 0$, in which we simulate from the model under our baseline parameterization, and period t =1 , where we shock one bank's deposit rate by 1 percentage point but keep all the other parameters—including the deposit rates of the other banks—constant. This scenario resembles a credit-supply shock to one bank in the economy, and we can study how it affects the credit allocation and what a reduced-form within-firm estimate would recover in this specific scenario. The resulting simulated data report the lending volume of all firm-bank pairs for each of the two periods.

Table 3: Bias of Within-Firm Estimator of Credit Supply Effects in Simulated Data

	Dependent Variable:			
	Credit Growth (%) Full Sample (1)	Credit Growth (%) KM Sample (2)	Credit Growth (%) KM Sample (3)	Firm FE (4)
Shocked Bank (0/1)	-45.88***	-14.66*	-21.55***	
	(5.79)	(8.72)	(4.32)	
Productivity (a_j)				11.16***
				(2.90)
Constant	-17.41***	-17.41***	-14.15***	-32.67***
	(5.11)	(6.00)	(2.96)	(10.77)
Unit of Obs.	Bank-Firm Level	Bank-Firm Level	Bank-Firm Level	Firm Level
Firm FE	No	No	Yes	-
Observations	366	154	154	73
R-squared	0.15	0.02	0.87	0.17

Note: This table is based on data simulated from our model under the baseline parameterization. Columns (1) through (3) show output of a regression of the bank-firm level percentage change in credit after a bank credit-supply shock on a dummy variable that equals 1 if the bank lender is subject to a shock and 0 otherwise. Column (4) regresses the estimated firm fixed effects from column (3) on firm productivity. See text for details.

Table 3 presents the results of our analysis. Column 1 shows the true average (differential) credit-supply effect for all firms in the economy, pointing to a stronger contraction of 45.9 percentage points for credit from shocked banks versus for banks without a change in funding cost. Column (2) reports the same effect for the sample of firms that enter the within-firm estimation, that is, where the firm fixed effects are identified.[20] The true differential effect is about one-third the size for this set of firms compared with the average effect for the entire population of firms, with a differential reduction of about 14.7 percentage points. In Column

[20]Note that we are estimating the model in first differences. Hence, a firm fixed effect resembles a firm*time fixed effect in the levels model with two periods.

(3), we report the results of the within-firm estimator, for the same set of observations, with an estimated reduction of 21.6 percentage points—about 50 percent larger (in absolute values) than the true effect in Column (2), indicating a substantial bias in the effect. More important, this bias is not due to sample difference but a result of bank-firm sorting. In Column (4), we show the correlation of the FEs estimated in Column (3) with firm productivity, showing that lower-productivity firms have smaller fixed effects, meaning their average credit growth is smaller (more negative).

We conclude that bank-firm matching, at both the intensive and extensive margins, can lead to substantial biases of widely applied empirical methods for identifying credit-supply shocks.

6 Conclusion

In this paper, we explore the role of corporate debt choices in the transmission of shocks to the real economy. We use the Federal Reserve Y-14Q supervisory data set to establish various stylized facts about corporate debt choices across the firm-size distribution. This provides a novel window into the heterogeneity in capital structure, in particular of small and private firms, which account for a large share of the economy's output. First, most firms in our sample rely predominantly on bank loans from one bank and retained earnings to fund their operations. Second, larger firms tap into the bond market, and the share of funding through bank loans decreases to close to 0 percent at the very large firms. Third, although the overall share of bank funding decreases, the number of banking relationships increases with firm size. Fourth, among firms with multiple banking relationships, firms borrow larger shares from banks that offer lower interest rates. Finally, we also document that the cost of debt funding decreases with firm size.

We introduce a theoretical framework informed by the stylized facts we establish. Firms can borrow from several banks, issue bonds, or raise equity internally. Our model incorporates bank market power and allows for a rich pattern of (imperfect) substitutability within and across funding sources, at both the intensive and extensive margins. A key component of our analysis is that the formation of lending relationships with individual banks is endogenous. We calibrate our model to detailed firm- and loan-level data and match key moments of the corporate debt choices across the firm-size distribution.

The insights of our quantitative analysis highlight the differential impact of bank shocks to different segments of the firm-size distribution and the implications for real economic activity and market concentration. We also show that credit market frictions that prevent a perfect substitutability across funding sources are key for the transmission of financial shocks to aggregate output. Our results suggest that policies targeted at improving credit availability to firms, especially smaller ones, may be important for mitigating the adverse effect of bank shocks on output, especially when substitutability of bank credit may be low, such as during a crisis. Moreover, our model analysis, especially the role of discontinuities in firms' credit-demand functions related to the extensive margin of credit, provides important implications for a large literature that uses reduced-form regressions to study the transmission of bank shocks to firms.

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A Details on Y-14 Data Processing

In this section, we provide details on how we process the Y-14 data. As discussed in the main body of the text, the Y-14 data are at the facility level. Each reporting bank (see main text) reports any outstanding loan (facility) commitment larger than or equal to $1 million in value. This is the original unit of observation that is collected. Each facility comes with facility-specific information relating to the type of loan and its characteristics, such as interest rate, loan amount, maturity date, collateralization, type of facility (line of credit, term loan, etc.). Other information relates to the lender bank, most notably its RSSD, a unique identification for credit institutions. Most importantly, the reporting banks also provide information about the borrower of each facility, most notably a tax identification number (TIN) and financial statement data, in particular key items of the borrower's income statement and balance sheet statement. The TIN allows us to match borrowers across different facilities (both within the same bank and across different banks), such that we can compute the total debt from all Y-14 banks or the number of different lenders of a given borrower.

In most cases, the financial statement information of the borrowers is as of a date (in most cases end of quarter) that differs from the reporting date of the Y-14 bank. For example, bank A may report in 2019:Q3 an exposure to firm Z, but the financial information about the borrower may refer to a balance sheet and income statement as of 2018:Q4. As many of the firms are smaller in size, for most firms we have only annual financial statement information as of Q4 of each year (some of which is not financially audited). Because we want to focus on as large a segment of the firm-size distribution as possible, we focus our analysis on financial statement data as of Q4. In particular, we use financial statement data as of 2019:Q4, so that we don't contaminate our analysis with the extraordinary events related to the outbreak of COVID-19 in 2020. We then create from the original facility-level data a borrower-level data set by collapsing all information to the borrower level.

Unfortunately, not all fields are (meaningfully) populated in the original data and so, for our analysis, before collapsing the original Y-14 data to the borrower level, we process the raw data as follows. First, we clean the data for obvious data errors, such as reported (origination or maturity) dates being missing or in the past. We also drop any facility where the borrower has a missing TIN, as we need this to map firms across different facilities. We also drop negative loan values, negative assets, and negative total debt (including non-Y-14 debt).

Our focus is on the role of the corporate finance structure across firms of different sizes, measured by sales, for the transmission of shocks. Therefore, we group firms into quintiles depending on their 2019 annual sales.

B Optimal Production Choices and Investment

We solve the problem of the firm (Eq. 8) in three steps: (i) optimal choice across varieties, (ii) optimal prices, and (iii) optimal investment choices. Finally, we compute the total cost for each firm.

Step 1. Optimal Choice across Varieties. Taking the FOC with respect to $y_j(\omega)$ in Eq. (8) delivers $y_j(\omega) = \left(\frac{R_j(\omega)}{R_j}\right)^{-\sigma} Y_j$, where $R_j \equiv \int R_j(\omega)^{1-\sigma} d\omega$ and Y_j is total output for the firm.

Step 2. Optimal Pricing. Given the optimal choice of varieties, the pricing problem of the firm is given by

$$\max_{P_j} \; P_j Y_j - \frac{w_j}{I_j a_j} R_j Y_j = \max_{P_j} \; P_j^{1-\eta} - \frac{w_j}{I_j a_j} R_j P_j^{-\eta}$$

Taking the FOC

$$(1-\eta)P_j^{-\eta} + \eta \frac{w_j}{I_j a_j} R_j P_j^{-\eta-1} = 0 \Rightarrow P_j = \frac{\eta}{\eta-1} \frac{w_j}{I_j a_j} R_j$$

The total profit without taking into account investment, $\Pi_{-I}(R_j, I_j)$, is given by

$$\Pi_{-I}(R_j, I_j) = Y\tilde{\eta} \left[\frac{I_j a_j}{w_j R_j}, \right]^{\eta-1}, \text{ where } \tilde{\eta} \equiv (\eta-1)^{\eta-1}\eta^{-\eta}$$

Step 3. Optimal Investment. The optimal investment choice is the solution to

$$\max_{I_j} \; \Pi_{-I}(R_j, I_j) - R_j c_I(I_j) = \max_{I_j} \; Y\tilde{\eta} \left[\frac{I_j a_j}{w_j R_j} \right]^{\eta-1} - R_j c_I(I_j)$$

With smooth and convex investment costs, $c_I(I_j) = \frac{1}{\psi+1} I^{\psi+1}$, and $\psi > \eta - 2$ to guarantee an interior solution, the FOC on the investment problem in equation (B) is

$$Y\tilde{\eta} \left[\frac{a_j}{w_j R_j} \right]^{\eta-1} (\eta-1) I_j^{\eta-2} = R_j I_j^{\psi} \Rightarrow I_j^* = \underbrace{\left\{ Y \left[\frac{\eta-1}{\eta} \right]^{\eta} \left[\frac{a_j}{w_j} \right]^{\eta-1} \right\}^{\frac{1}{\psi-\eta+2}}}_{\equiv \Omega_I} R_j^{-\frac{\eta}{\psi-\eta+2}}.$$

The total profit with investment is given by

$$\Pi_j = Y\tilde{\eta}\left[\frac{I_j^* a_j}{w_j R_j}\right]^{\eta-1} - \frac{R_j}{\psi+1}\left(I_j^*\right)^{\psi+1} = \left[\frac{1}{\eta-1} - \frac{1}{\psi+1}\right] R_j I_j^{\psi+1} \quad = \Omega_j R_j^{-\gamma}, \;\; \gamma \equiv \frac{(\psi+2)(\eta-1)}{\psi-\eta+2} > 0$$

where the second to third equality come from the fact that from the FOC we know that

$$Y\tilde{\eta}(\eta-1)\left[\frac{a_j I_j}{w_j R_j}\right]^{\eta-1} = R_j I_j^{\psi+1}$$

Total Cost. Note from the problem of the firm that the total cost C_j is given by

$$C_j \equiv \int R_j(\omega)\left[c_j(\omega) + c_I(I_j)\right] d\omega = \gamma \Omega_j R_j^{-\gamma}$$

C Shares and Rates across Different Sources of Funding

In this section, we want to derive three objects: (i) the probability that the firm j chooses source of finance f for a given variety (which is also the share of varieties financed from this source, since there is a continuum of varieties ω), (ii) the expected rate the firm pays for financing when faced with offer rates $\boldsymbol{R}_j$, and (iii) the conditional distribution of interest rates from a source of finance when this source of finance is the minimum among all sources of finance. Together, these derivations provide details of Eq. (11)-(12) in the main text, as well as the rate R_j^{min} that enters into the problem of the bank in Eq. (20).

To derive these objects, we follow Lind and Ramondo (2018). Lind and Ramondo (2018) generalize the Eaton and Kortum (2002) for correlated Fréchet shocks, which is the case for the distribution in Eq. (6).

Notation. Before proceeding, let us introduce some notation. Let $\boldsymbol{x} \equiv \left\{\{x_b\}_b, x_{nb}, x_{sf}\right\}$ for any vector of variables $\boldsymbol{x}$ across all sources of finance. Let $\tilde{\boldsymbol{T}}$ be adjusted scale parameters T_b and ζ's as follows

$$\tilde{T}_b = \zeta_e^{\frac{\theta}{\xi_e}\frac{\xi_e}{\xi_s}} \zeta_b^{\frac{\theta}{\xi_e}} T_{j,b} R_{j,b}^{-\theta}, \;\; \tilde{T}_{nb} = \zeta_e^{\frac{\theta}{\xi_e}\frac{\xi_e}{\xi_s}} \zeta_{nb}^{\frac{\theta}{\xi_e}} R_{j,nb}^{-\theta}, \text{ and } \tilde{T}_{sf} = \zeta_{sf}^{\frac{\theta}{\xi_s}} R_{j,sf}^{-\theta} \tag{A.1}$$

and let $\tilde{R}_f \equiv \varepsilon_f^{-1} R_b$. Let $\mathcal{C}(\boldsymbol{x})$ be given by (A.2)

$$\mathcal{C}(\boldsymbol{x}) = \left\{ \left[\left(\sum_{b=1}^{N_B} x_b \right)^{\frac{\xi_e}{\theta}} + x_{nb}^{\frac{\xi_e}{\theta}} \right]^{\frac{\xi_s}{\xi_e}} + x_{sf}^{\frac{\xi_s}{\theta}} \right\}^{\frac{\theta}{\xi_s}} \tag{A.2}$$

This function has several important properties. First, it is homogeneous of degree one. Second, we can write

$$\mathbb{P}\left[\{\tilde{R}_f \geq r_f\}_f \right] = \exp\left\{ -\mathcal{C}\left(\{\tilde{T}_f r_f^{\theta}\}_f \right)^{\frac{\xi_s}{\theta}} \right\} \tag{A.3}$$

which implies that

$$\mathbb{P}\left[\boldsymbol{R} \geq r \right] = \exp\left\{ -\mathcal{C}\left(\tilde{\boldsymbol{T}} \right)^{\frac{\xi_s}{\theta}} r^{\xi} \right\} \tag{A.4}$$

which greatly simplifies out analysis. This function $\mathcal{C}(\boldsymbol{x})$ is similar to what Lind and Ramondo (2018) define as a correlation function and captures the dependency structure of our Fréchet shocks. Finally, let the function $\mathcal{C}(\boldsymbol{x})$ evaluated at $\boldsymbol{T}$ and the partial derivative of $\mathcal{C}(\boldsymbol{x})$ to argument f evaluated at $\boldsymbol{T}$ be given by

$$\tilde{\mathcal{C}} \equiv \mathcal{C}\left(\tilde{\boldsymbol{T}} \right), \quad \tilde{\mathcal{C}}_f \equiv \frac{\partial}{\partial x_f} \mathcal{C}\left(\tilde{\boldsymbol{T}} \right) \tag{A.5}$$

C.1 Probability of a Given Source of Finance Being Chosen

We focus here on the probability that a single bank is chosen, but the argument can be easily generalized for the self-finance and nonbank external finance sources. To compute the probability of a specific bank being chosen, we follow a two-step strategy. First, we consider the probability of a specific bank being chosen conditional on $\min_f \tilde{R}_f \geq r$. Second, we take $r \to 0$ to recover the probability of that bank being chosen. In the final step of the proof, we replace the $\tilde{C}$ and its derivative by the actual values to reach the equations in the main text. We omit the firm j index.

Step 1. Probability of a specific bank being chosen conditional on $\min_f \tilde{R}_f \geq r$. We can write this probability as

$$\mathbb{P}\left[\min_f \tilde{R}_f \geq r \text{ and } \tilde{R}_b = \min_f \tilde{R}_f\right] = \mathbb{P}\left[\tilde{R}_f \geq r \text{ and } \tilde{R}_b = \min_f \tilde{R}_f\right]$$
$$= \mathbb{P}\left[\tilde{R}_f \geq r \text{ and } \tilde{R}_b \leq \tilde{R}_f\right] \tag{A.6}$$

Writing the joint probability as the conditional times the marginal, we have that

$$\mathbb{P}\left[\min_f \tilde{R}_f \geq r \text{ and } \tilde{R}_b = \min_f \tilde{R}_f\right] = \int_0^r \mathbb{P}\left[\tilde{R}_{f,-b} \geq t \mid \tilde{R}_b = t\right] \frac{\partial}{\partial t} \mathbb{P}\left[\tilde{R}_b = t\right] dt$$
$$= \int_0^r \frac{\partial}{\partial t} \mathbb{P}\left[\tilde{R}_{f,-b} \geq z \text{ and } \tilde{R}_b \geq t\right]\Big|_{z=t} dt \tag{A.7}$$

For the distribution in (6)

$$\mathbb{P}\left[\min_f \tilde{R}_f \geq r \text{ and } \tilde{R}_b = \min_f \tilde{R}_f\right] = \int_0^r \frac{\partial}{\partial r_b} e^{-\mathcal{C}(\tilde{T}_f r_f^{\theta})^{\frac{\xi_s}{\theta}}}\Big|_{r_f = t} dt$$
$$= \int_0^r e^{-\tilde{\mathcal{C}}^{\frac{\xi_s}{\theta}} t^{\xi_s}} \frac{\xi}{\theta} \tilde{\mathcal{C}}^{\frac{\xi}{\theta}-1} t^{\xi-\theta} \tilde{\mathcal{C}}_b \tilde{T}_b \theta t^{\theta-1} dt$$
$$= \frac{\tilde{T}_b \tilde{\mathcal{C}}_b}{\tilde{\mathcal{C}}} \int_0^r e^{-\tilde{\mathcal{C}}^{\frac{\xi_s}{\theta}} t^{\xi_s}} \tilde{\mathcal{C}}^{\frac{\xi_s}{\theta}} \xi t^{-\xi-1} dt = \frac{\tilde{T}_b \tilde{\mathcal{C}}_b}{\tilde{\mathcal{C}}} e^{-\tilde{\mathcal{C}}^{\frac{\xi_s}{\theta}} r^{\xi_s}} \tag{A.8}$$

Therefore

$$\mathbb{P}\left[\min_f \tilde{R}_f \geq r \text{ and } \tilde{R}_b = \min_f \tilde{R}_f\right] = \frac{\tilde{T}_b \tilde{\mathcal{C}}_b}{\tilde{\mathcal{C}}} e^{-\tilde{\mathcal{C}}^{\frac{\xi_s}{\theta}} r^{\xi_s}} \tag{A.9}$$

Step 2. Taking $r \to 0$. With $r \to 0$, we have that from (A.8)

$$\mathbb{P}\left[\tilde{R}_b = \min_f \tilde{R}_f\right] = \frac{\tilde{T}_b \tilde{\mathcal{C}}_b}{\tilde{\mathcal{C}}} \tag{A.10}$$

Step 3. Replacing the $\mathcal{C}$ function. Given (A.10), we have that the market share of one specific bank across banks is given by:

$$\nu_{\hat{b}} \equiv \frac{\tilde{T}_{\hat{b}} \tilde{\mathcal{C}}_{\hat{b}}}{\sum_b \tilde{T}_b \tilde{\mathcal{C}}_b} = \frac{T_{\hat{b}} R_{\hat{b}}^{-\theta}}{\sum_b T_{j,b} R_{j,b}^{-\theta}} \tag{A.11}$$

To arrive at this result, note that all of other terms cancel out from the derivative (since we are taking the derivative of power functions). Similarly, we can compute the market share of all banks within external finance, s_B, as follows

$$s_B \equiv \frac{\frac{\theta}{\xi_s}\frac{\xi_s}{\xi_e}\frac{\xi_e}{\theta}\left[\left(\sum_{b=1}^{N_B}\tilde{T}_b\right)^{\frac{\xi_e}{\theta}-1}\left(\sum_{b=1}^{N_B}\tilde{T}_b\right)\right]}{\left[\left(\sum_{b=1}^{N_B}\tilde{T}_b\right)^{\frac{\xi_e}{\theta}}+\tilde{T}_{nb}^{\frac{\xi_e}{\theta}}\right]} = \frac{\left(\sum_{b=1}^{N_B}\tilde{T}_b\right)^{\frac{\xi_e}{\theta}}}{\left(\sum_{b=1}^{N_B}\tilde{T}_b\right)^{\frac{\xi_e}{\theta}}+\tilde{T}_{nb}^{\frac{\xi_e}{\theta}}} = \frac{\zeta_B R_B^{-\xi_e}}{\zeta_B R_B^{-\xi_e}+\zeta_{nb}R_{nb}^{-\xi_e}} \tag{A.12}$$

Finally, we can compute the market share of external finance as

$$s_e \equiv \frac{\sum_b \tilde{T}_b\tilde{C}_b+\tilde{T}_{nb}\tilde{C}_{nb}}{\tilde{C}} = \frac{\zeta_e\left[\zeta_b R_B^{-\xi_e}+\zeta_{nb}R_{nb}^{-\xi_e}\right]^{\frac{\xi_s}{\xi_e}}}{\zeta_e\left[\zeta_b R_B^{-\xi_e}+\zeta_{nb}R_{nb}^{-\xi_e}\right]^{\frac{\xi_s}{\xi_e}}+\zeta_{sf}R_{sf}^{-\xi_s}} = \frac{\zeta_e R_e^{-\xi_s}}{\zeta_e R_e^{-\xi_s}+\zeta_{sf}R_{sf}^{-\xi_s}} \tag{A.13}$$

Therefore, we have that the probability a variety is financed with bank b (and thus the share of varieties), is given by

$$\mathbb{P}\left[\tilde{R}_b = \min_f \tilde{R}_f\right] = s_E s_B \nu_b \equiv s_b \tag{A.14}$$

We denote this share by μ_b and refer to it as the market share of bank b across sources of finance for firm j.

C.2 The Rate Index

Let R_j be the rate index from the firms' optimization across varieties, that is

$$R_j^{1-\sigma} \equiv \int_0^\infty R_j(\omega)^{1-\sigma}d\omega = \int_0^\infty r^{1-\sigma}dG_j(r) \tag{A.15}$$

where $G_j(r)$ is the distribution of the minimum of the sources of funding across all sources; that is, $G_j(r) \equiv \mathbb{P}\left[\tilde{R}_f \le r\right]$. Using our notation, we have that $G_j(r) = 1-\exp\left\{-\tilde{C}^{\frac{\xi_s}{\theta}}r^{\xi_S}\right\}$. Therefore:

$$R_j^{1-\sigma} = \int_0^\infty r^{1-\sigma}dG_j(r) = \int_0^\infty r^{1-\sigma}\tilde{C}^{\frac{\xi_s}{\theta}}\xi r^{\xi-1}\exp\left[-\tilde{C}^{\frac{\xi_s}{\theta}}r^{\xi}\right]dr$$

Let $x = \tilde{C}^{\frac{\xi_s}{\theta}}r^{\xi_s}$, $dx = \tilde{C}^{\frac{\xi_s}{\theta}}\xi_s r^{\xi_s-1}$. With a substitution of variables from r to x

$$R_j^{1-\sigma} = \int_0^\infty \left(\frac{x}{\tilde{C}^{\frac{\xi_s}{\theta}}}\right)^{\frac{1-\sigma}{\xi_s}} e^{-x}dx = \tilde{C}^{-\frac{1-\sigma}{\theta}}\Gamma\left(\frac{1}{\xi_S}+1\right) \tag{A.16}$$

Replacing back $\tilde{C}$, we have that

$$R_j = \Gamma\left(\frac{1-\sigma}{\xi_S}+1\right)^{\frac{1}{1-\sigma}} \left\{\zeta_e\left[\zeta_b\left(\sum_{b=1}^{N_B} T_{j,b}R_{j,b}^{-\theta}\right)^{\frac{\xi_e}{\theta}} + \zeta_{nb}R_{j,nb}^{-\xi_e}\right]^{\frac{\xi_s}{\xi_e}} + \zeta_{sf}R_{j,sf}^{-\xi_s}\right\}^{\frac{1}{\xi_s}} \tag{A.17}$$

or, alternatively,

$$R_j = \Gamma\left(\frac{1-\sigma}{\xi_S}+1\right)^{\frac{1}{1-\sigma}} \left\{\zeta_e R_{j,e}^{-\xi_s} + \zeta_{sf}R_{j,sf}^{-\xi_s}\right\}^{-\frac{1}{\xi_s}} \tag{A.18}$$

C.3 Conditional Rate Distribution and Expectation

Whenever firm j chooses bank b, we have that the expected price this firm pays is given by

$$R_j^{min} \equiv \mathbb{E}[\tilde{R}_{j,b}|\tilde{R}_{j,b} = \min_f \tilde{R}_{j,f}] = \frac{\int_0^\infty r d\mathbb{P}\left[\min_f \tilde{R}_f \geq r \text{ and } \tilde{R}_b = \min_f \tilde{R}_f\right]}{\mathbb{P}\left[\tilde{R}_b = \min_f \tilde{R}_f\right]} \tag{A.19}$$

We have that $\mathbb{P}\left[\tilde{R}_b = \min_f \tilde{R}_f\right] = s_E s_B \nu_b = s_b$ from (A.14). To compute the numerator, we replace $\mathbb{P}\left[\min_f \tilde{R}_f \geq r \text{ and } \tilde{R}_b = \min_f \tilde{R}_f\right]$ from the (A.9) to get

$$\int_0^\infty r d\mathbb{P}\left[\min_f \tilde{R}_f \geq r \text{ and } \tilde{R}_b = \min_f \tilde{R}_f\right] = s_E s_B \nu_b \int_0^\infty r dG_j(r) = \frac{\hat{\Gamma}}{\tilde{\Gamma}} s_E s_B \nu_b R_j \tag{A.20}$$

where $\hat{\Gamma} \equiv \Gamma\left(\frac{1}{\xi_S}+1\right)$. Therefore, $\mathbb{E}[R_{j,b}|R_{j,b} = \min_f R_{j,f}] = \frac{\hat{\Gamma}}{\tilde{\Gamma}} R_j$.

D Optimal Lending Relationships

In this section we describe the solution to the problem in Eq. (14). For that, we first introduce some notation.

Notation. Let the outer derivative of the rate index with respect to changes in $\sum_b T_b R_b^{-\theta}$ be denoted by: $\partial_{o,b} R_j \equiv \frac{\partial R_j}{\partial\left(\sum_b T_b R_b^{-\theta}\right)}$. Note that

$$\partial_{o,b} R_j = \frac{-1}{\xi_s} \frac{\partial \ln\left(\zeta_e R_e^{-\xi_S} + \zeta_{sf} R_{sf}^{-\xi_S}\right)}{\partial\left(\sum_b T_b R_b^{-\theta}\right)} \cdot R_j = -\theta^{-1} R_j s_E s_B \left[\sum_b T_b R_b^{-\theta}\right]^{-1} \tag{A.21}$$

Step 1. Solution Conditional Extensive Margin Relationships. Let χ_f be the Lagrange multiplier on the nonnegativity constraints. The KKT conditions are

$$-\gamma \Omega(.,a) R_j (\boldsymbol{T})^{-\gamma-1} R_b^{-\theta} \partial_{o,b} R_j = \frac{\partial \mathcal{E}(\boldsymbol{T}, \boldsymbol{\zeta})}{\partial T_b} T_b^{\rho-1} + \chi_b \tag{A.22}$$

and similarly for the other sources of finance. From the KKT conditions it follows that any interior solution (with $\chi_b = 0$, $\chi_{nb} = 0$, $\chi_{sf} = 0$), together with our normalization where $\sum_b T_b^\rho = 1$, $\zeta_b^\rho + \zeta_{nb}^\rho = 1$, satisfies Eq. (17). And therefore:

$$T_b = \left[1 + \sum_{\tilde{b} \neq b} \left[\frac{R_{\tilde{b}}^{-\theta}}{R_b^{-\theta}}\right]^{\frac{\rho}{\rho-1}}\right]^{-\frac{1}{\rho}} \text{ and } \zeta_b = \left[1 + \left[\frac{R_{nb}^{-\xi_E}}{R_B^{-\xi_E}}\right]^{\frac{\rho}{\rho-1}}\right]^{-\frac{1}{\rho}} \tag{A.23}$$

and ζ_{sf} solves a FOC analogous to Eq. (A.22), which delivers Eq. (18) in the main text.

Step 2. Extensive Margin Relationships. We solve the problem of optimal relationships at the extensive margin numerically. We compute the optimal profits from Step 1 for four different cases: (1) the firm does not use external finance ($\zeta_{j,e} = 0$); (2) the firm uses only nonbank external finance ($\zeta_{j,b} = 0$); (3) the firm uses only bank external finance ($\zeta_{j,nb} = 0$); and (4) the firm uses all sources of finance. In cases 3 and 4, when firms use bank financing, we use the fact that firms have a pecking order within-banks, that is, firms choose to building relationships with the banks with lower offer rates first. Therefore, we add banks sequentially, starting from the banks with the lowest cost, until the fixed cost of the additional relationship is lower than the marginal change in profits from lower rates coming from the marginal bank in the market.

To see that firms have this pecking order within banks, that is, that If $R_b > R_{\hat{b}}$, then $T_b > 0 \Rightarrow T_{\hat{b}} > 0$, assume by contradiction that $T_{\hat{b}} = 0$ and $T_b > 0$. Let the objective function of the firm be denoted by $\Pi_T(\boldsymbol{T}) \equiv \Omega(.,a) R_j (\boldsymbol{T})^{-\gamma} - \sum_b c_T(T_b)$. Let $\boldsymbol{T}_0$ be the vector of T's with $T_{\hat{b}} = 0$ and $T_b > 0$ and let $\boldsymbol{T}_1$ be a vector of T's that is such that $T_{1,\hat{b}} = T_b$ and $T_{1,b} = 0$, and for every other bank is the same as T_0.

$$\Pi_T(\boldsymbol{T}_1) - \Pi_T(\boldsymbol{T}_0) = \Omega(.,a)\left[R_j(\boldsymbol{T}_1)^{-\gamma} - R_j(\boldsymbol{T}_0)^{-\gamma}\right] = \Omega(.,a)\int_0^1 R_j(t)^{-\gamma}\,dt$$

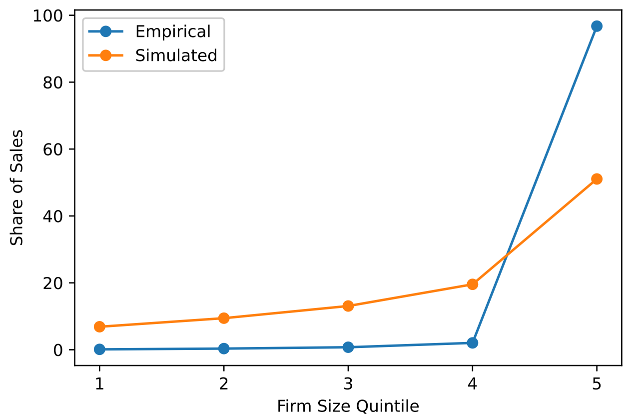
where $R_j(t)$ is the interest rate index evaluated at $T_{1,b} = (1-t)T_b$ and $T_{1,\hat{b}} = tT_b$ Note that

$$\frac{dR_j(t)}{dt} = \partial_o R_j \cdot [R_{\hat{b}}^{-\theta} - R_b^{-\theta}] > 0$$

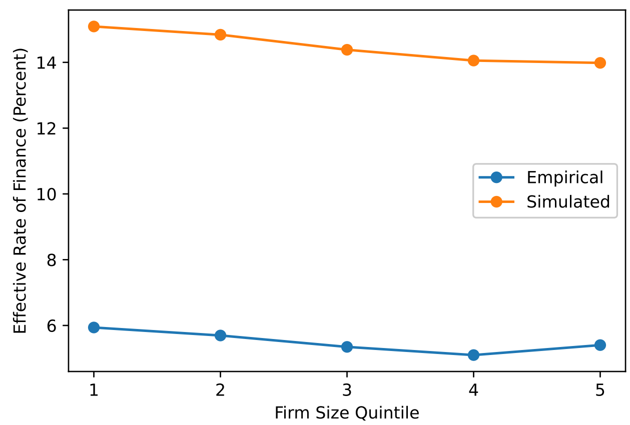
Therefore, $T_{\hat{b}} = 0$ and $T_b > 0$ can't be optimal since $\Pi_T(\boldsymbol{T}_1) > \Pi_T(\boldsymbol{T}_0)$.

E Additional Quantitative Analysis

Figure A.1: Simulated and Empirical Moments in Levels: Market Concentration and Funding Cost



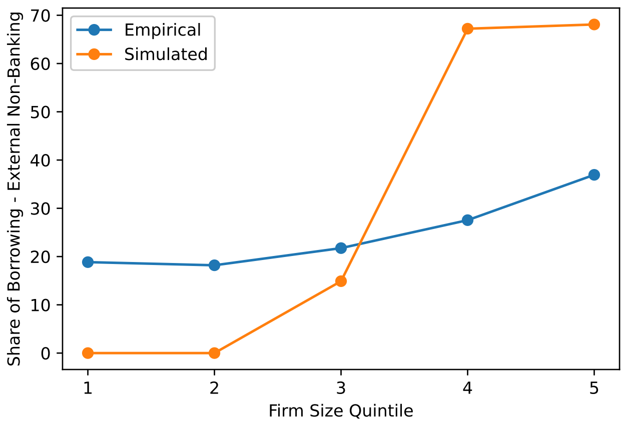
(a) Sales Share



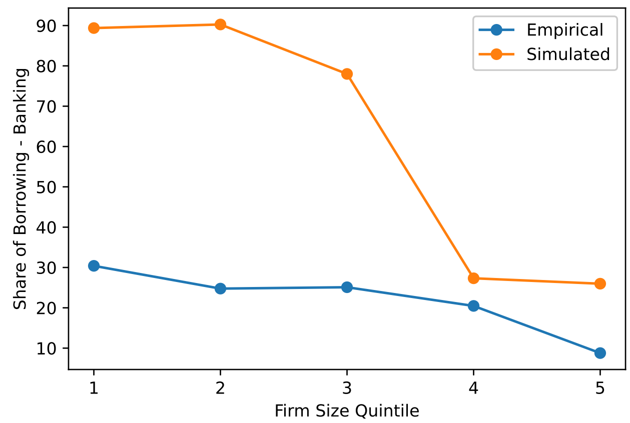
(b) Interest Rate

Note: The figure depicts the simulated and empirical market share (Panel a) and the effective rate of finance (Panel B). The effective rate of finance for firms in Y-14 is computed as interest expenses as a percentage of stock of debt. Y-14 figures refer to 2019 values. Source: FR Y-14Q data, H.1 schedule with C&I lending, authors' calculations.

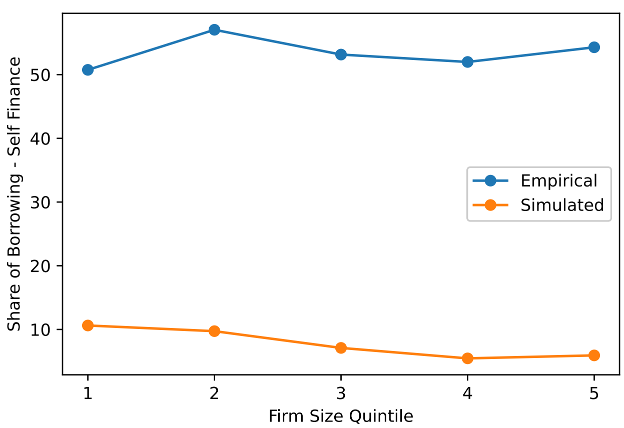
Figure A.2: Simulated and Empirical Moments in Levels: Corporate Finance Structure across Firm-Size Distribution



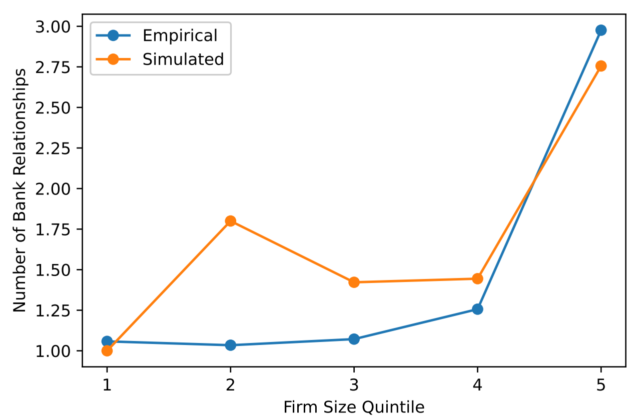
(a) Nonbank Share



(b) Bank Share



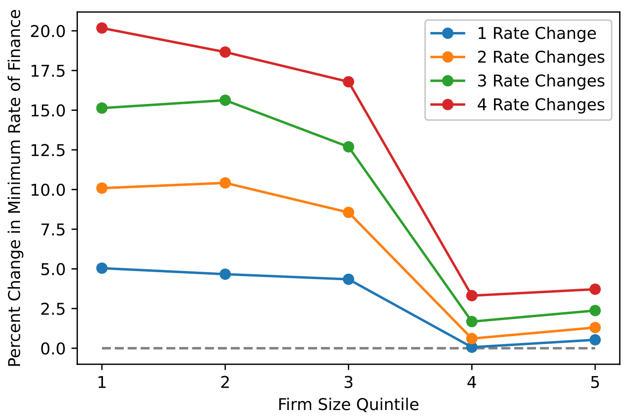
(c) Self-finance Share



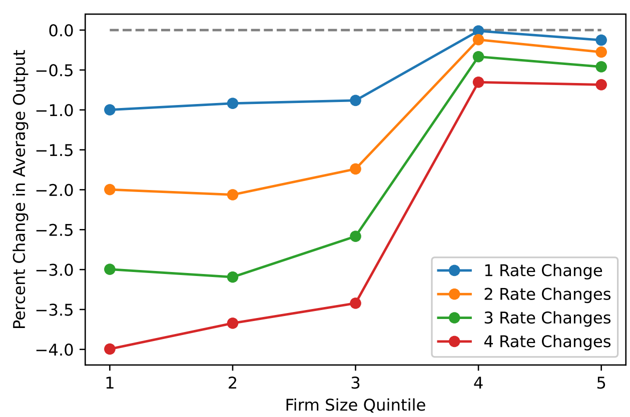
(d) Number of Bank Relationships

Note: The figure depicts the simulated and nonbank funding share (Panel a), bank funding share (Panel b), self-financing share (Panel c), and the number of banking relationships (Panel d). Source: FR Y-14Q data, H.1 schedule with C&I lending, authors' calculations.

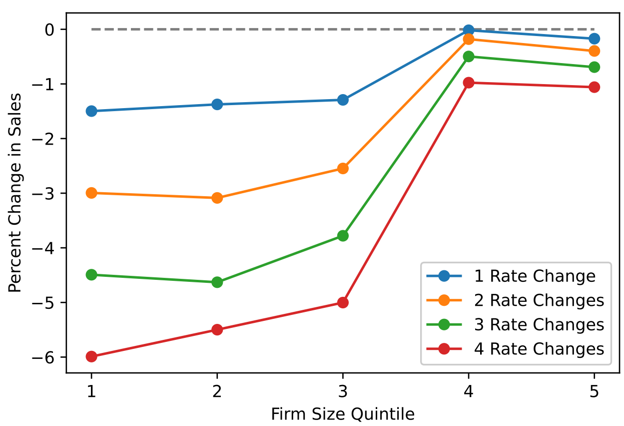
Figure A.3: Average Impact of Bank Shock across Firm-Size Distribution



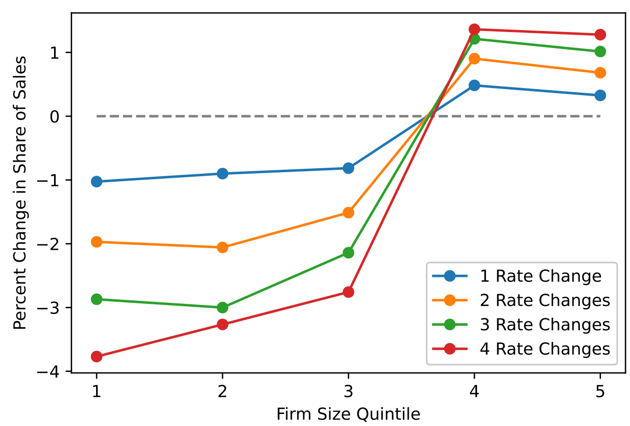
(a) Interest Rate



(b) Output



(c) Sales



(d) Market Share

Note: This figure is similar to Figure 9, but instead of shocking a fixed order of banks, we shock all combinations of one, two, three, or four banks and present the average effects across the different permutations for each number of banks being shocked. See text for further explanations.