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Downside Risk and Fund Flow

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Abstract

This paper examines the possibility that investor redemption is driven by return implied risk profile of the fund. We find that funds with asymmetric exposure to downside risk experience more than double the performance induced fund flow than those without. The magnitude of amplification increases significantly during market stress. These effects are both statistically and economically significant. From a financial stability perspective, this measure of downside risk exposure identifies funds that are more vulnerable to redemption in times of stress. Downside risk identification based on put-writing strategy yields more robust results on fund flow than alternative measures of return implied risk.

Keywords: Downside Risk, Mutual Fund, Fund Flows.

JEL Classification: G12, G20, G23

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1 Introduction

The US mutual fund industry is quite sizeable, with total asset under management of over \$17 trillion[1] dollars at the beginning of 2020. This number plummeted to just below \$15 trillion reaching its lowest point during the COVID shock. Massive outflow caused great concern that the industry has become a source of systemic risk. Compared to similar ETFs based on performance, size and age, recent research has shown that mutual funds sustained larger and more persistent outflows during this market episode (Falato, Goldstein, and Hortaçsu (2021)). Authors focused on corporate bond funds during the crisis, and found that such outflow is most severe for funds with illiquid assets and vulnerable to fire sales. However, outflow from mutual funds occurred across all fund types, which means similar fragility exists in all fund types giving rise to the forementioned systemic risk concern of the industry. On the other hand, not all mutual funds are made equal. Within the entire population of mutual funds, there is a wide range of cross-sectional variation in fund flows at all times, including but not limited to stress markets such as COVID shock.In times of stress, some funds are disproportionately more affected by adverse economic conditions than others. Their investors redeem in larger quantities causing funds to liquidate more positions. Fire sale creates excessive selling pressure on the underlying securities, pushing markets into a downward spiral. The outsized outflow they experience imposes larger risk to financial stability within the broader markets. Being able to identify these funds will serve as a first step to eventually arrive at appropriate policy that can effectively decrease investor redemption and reduce undesirable adverse amplification under stress. We propose a way to achieve such identification.

Taking a step back from the specific underlying assets and portfolios of a fund, we examine the possibility that investor behavior and redemption decisions are mostly driven by return implied risk profile of the fund. This hypothesis assumes rational investors who only care about risk adjusted returns in equilibrium, which is not too farfetched. Regardless of the

[1]Source: Investment Company Institute (ICI).

kind of risk investors choose to bear or implicitly end up bearing, required rate of return has to properly compensate for the size of the risk. The combination of traded products, fund strategy, and manager execution determines fund's risks and performances. Identification strategies in this paper do not distinguish amongst them. Risk profile is only implied by the performance outcome. When return pattern appears that a fund is generating its returns from taking extreme risks out on the left tail, investors should demand appropriate risk adjusted compensation for bearing this risk. If such investor required return cannot be met, investors should deem their stake in the fund too costly and exit by redeeming their shares. In other words, if a fund has asymmetric exposure to downside risk, where it exhibits characteristics consistent with allocating more losses to tail events, its investors will be more likely to withdraw in times of stress. These disproportionately large redemptions create excess selling pressure, and therefore contribute to adverse pressure on asset prices. In this paper, we focus on whether asymmetric exposure to downside risk indeed exacerbates flows.

Within the scope of this paper, asymmetric exposure to downside risk is defined specifically as nonlinear exposure to the broader underlying markets, with a heavy tail in downturns. Factor representation is through construction of returns to a range of out-of-money S&P 500 equity index options, as described by Jurek and Stafford (2015). Such short position in index put option magnifies negative skewness of underlying market shocks, and therefore continuously exposes option writer to deteriorating market conditions. The series of strategy returns, compared to Fama French factors or market portfolio, is nonlinear. Different downside exposure implied by the strategy is determined by moneyness of the options sold and amount of leverage used. The strategy was originally designed to replicate hedge fund return indices. Though hedge funds and mutual funds have significant structural differences, we believe that this notion of nonlinear exposure to downside risk embedded in the put-writing strategy is independent from its application to hedge funds. However, structural differences such as limitations on short selling and leverage for mutual funds might cause characteristics of downside exposure to change. We recalibrated strategy parameters to capture the shift in application.

We compare the put-writing characterization to alternative notion of downside risk stemming from applications in individual equities, where conditional beta is used to capture exposure to downside risk (Ang and Chen (2002), Ang, Chen, and Xing (2006), Bawa and Lindenberg (1977)). Portfolio return is treated as equity return to arrive at a similar concept of conditional beta. These two measures describe different notions of downside risk. Where put-writing strategy identifies whether return is skewed towards large but rare losses in market downturn, conditional beta measures portfolio's market exposure whenever market is performing below its mean. We find that put-writing strategy leads to better significance in estimation results, implying a more appropriate fit to fund's underlying risk profile, based on which investors are demanding returns.

This paper is also related to literature on the impact of fund performances on fund flows. Studies such as Ippolito (1992), Chevalier and Ellison (1997) and Sirri and Tufano (1998) find a positive relationship between fund flow and recent performances. Investors chase performance by allocating money to funds with higher lagged excess returns. Berk and Green (2004) derived a parsimonious model to reproduce the empirical results that fund flow rationally respond to past performances. This paper builds on these findings and explores cross sectional variation for additional amplification of the performance chasing phenomenon. Our results show that funds with asymmetric exposure to downside risk experience more than double the performance induced response in fund flows. For 1% in loss return, where funds without asymmetric exposure to downside risk will experience 9.2 bps performance induced outflow, their peers with an asymmetric exposure will experience a 20.2 bps outflow. The magnitude of amplification increases during market stress. Under stressed conditions, with 1% in loss return, funds without asymmetric exposure to downside risk will experience 5.7 bps performance induced outflow, and outflow their peers with an asymmetric exposure will experience increases to 14.8 bps. This effect continues to be statistically and economically significant when additional variation is introduced with a notion of the size of asymmetric exposure, which is measured by the size of unexplained return the put-writing strategy can explain away.

Results in this paper also persist for the broad category of fixed income funds, which can be interpreted as supporting evidence for regulatory interest in potential financial stability concerns in bond funds. Jin, Kacperczyk, Kahraman, and Suntheim (2022) discussed the fragility in bond funds and the efficacy of swing pricing to induce more financial resiliency. We show that the amplification results hold qualitatively when we reduce sample to fixed income funds. In further quantile regression analysis, we observe heterogenous absolute and relative impact of such asymmetric exposure to downside risk on fixed income funds. It is important to note that asymmetric exposure to downside risk is picking up another dimension of fragility, different from bond illiquidity as described in Goldstein, Jiang, and Ng (2017). The asymmetric allocation of risk is also applicable to equity funds, where illiquidity is not believed to vary vastly in the cross section. The same amplification results are nonetheless valid for a sample of only equity funds.

After establishing that put-writing strategy can be used to identify funds with potential for larger outflow during stress times, we examine the persistency of such identification and provide suggestions of its implementation for policy purposes. We find that such identification of asymmetric risk is heavily dependent on recent performances, and therefore does not persist through time. Policy implementation would require regulators to run identification criterion each time a new stress market presents itself to identify target funds. We view this time sensitive characteristic a strength of the identification strategy. By not yielding static fund population through time, it stands more chance to capture market relevant information and therefore provide more accurate selection of fragile mutual funds under different market stresses.

The rest of this paper is organized as follows. Section 2 describes the identification for asymmetric exposure to downside risk using put-writing strategy, and its classification outcome. Section 3 examines the empirical relationship between the presence of such asymmetric exposure and fund flow. Section 4 estimates persistence of the identification as a fund characteristics and outlines its policy implication. Section 5 concludes.

2 Classification of Downside Risk

The asymmetric exposure to downside risk defined in this paper is specific to the type that concentrates large losses in extremely adverse market conditions. Because of this negative skew in returns, implied risk profile should yield fund investors higher equilibrium risk compensation. In other words, when fund return implies that the fund has an asymmetric exposure to downside risk, its investors are bearing downside market risks that are not captured by linear replicating strategies. Unexplained excess return in common risk factor models is then merely fair compensation for bearing excess tail risks. Hence, we start with CAPM models, and select funds with positive significant alpha. We then regress excess returns of these funds on asymmetric factors, adopted from the put-writing strategy provided by Jurek and Stafford (2015) after calibrating relevant parameters to fit mutual funds. For any given mutual fund, if its positive significant alpha in linear factor models is explained away by the put-writing strategy, then it is said to have an asymmetric exposure to downside risk.

2.1 Put Writing Strategy

Let us briefly describe the construction of said put-writing strategy. It is a monthly rebalanced fully funded strategy holding short position in market index put and required margins. A put-writing strategy is completely characterized by $[Z, L]$, where Z measures moneyness of the option and L is leverage of the strategy. Each period at month end, form a simple portfolio consisting of a short position in a single S&P 500 index put $\mathcal{P}(K(Z), T)$ and equity capital, $\kappa_E(L)$, where $K(Z)$ is the option strike price, T is the option expiration date, and L is the leverage of the portfolio. Option is sold at bid price at time t, and strike is determined as follows:

$$K(Z) = S_t \cdot \exp(\sigma_{t+1} \cdot Z) \tag{1}$$

where S_t is level of S&P 500 index and σ_{t+1} is one-month VIX observed on date t. To resemble margin requirements, the strategy has to post capital κ_E[2] according to strategy defined leverage ratio L,

$$\kappa_E = \frac{e^{-r_{f,t+\tau}} \cdot K(Z) - \mathcal{P}_t^{bid}(K(Z), T)}{L} \tag{2}$$

where $e^{-r_{f,t+\tau}}$ is the risk free interest rate corresponding to the time to option expiration. On trade roll date $(t+1)$, option position is closed by repurchasing the index put at ask price, $\mathcal{P}_{t+1}^{ask}(K(Z), T)$. During this time period, strategy equity capital and premium from selling the put generate accrued interest of:

$$AI_{t+1} = \left(\kappa_E(L) + \mathcal{P}_t^{bid}(K(Z), T)\right) \cdot (e^{r_{f,t+1}} - 1) \tag{3}$$

Therefore, the monthly return of this put-writing strategy, to be used as nonlinear factors for downside risk classfication, is computed as:

$$r_{p,t+1} = \frac{\mathcal{P}_t^{bid}(K(Z), T) - \mathcal{P}_{t+1}^{ask}(K(Z), T) + AI_{t+1}}{\kappa_E(L)} \tag{4}$$

Repeat this process at $t+1$, until sample end. Options are selected at the closest strike to Equation (1) from below, with expiry closest to but after the end of the month.

Option prices and risk free rates are both obtained from Optionmetrics. Parameters are calibrated to $[Z = -2, L = 1]$ for mutual funds. This sits well with the fact that mutual funds can only trade with modest amount of leverage, if any. Moneyness at $Z = -2$ generates sufficient non-linearity with no leverage to fit general risk profiles of mutual funds. However,

[2] This is the maximum loss at expiry after leverage, making the put-writing strategy fully funded. Importance of satisfying margin requirement is argued by Santa-Clara and Saretto (2009).

classification results are fairly stable for values in a neighborhood of $[Z = -2, L = 1]$. For example, when option is further out of the money and leverage increases at $[Z = -2.5, L = 1.5]$, similar outcome on asymmetric downside exposure classification still holds.

We follow the setup where investors with CRRA preferences allocate to put-writing strategy optimally, and take calibration as given. Figure 1 demonstrates nonlinearity of the strategy. The upper panel plots the payoff profile of $[Z = -2, L = 1]$ and $[Z = -2.5, L = 1.5]$ as a function of market return. Market payoff is by construction the 45-degree line. The bottom panel plots the portfolio return skewness as a function of extent of downside exposure, as measured by allocation to the put-writing strategy. Skewness increases as investors allocate more to the put-writing strategy and increase their exposure to downside risk. This is not to say that mutual funds or their investors behave like these specialized hedge fund investors in actuality, but rather that their returns imply a similar risk profile where gains occur more often but the rare losses can be large. Mechanically, hedge funds have investment styles that map to the put-writing strategy in a much clearer manner. The channel through which mutual funds obtain such risk profile cannot be identified within the scope of this paper. Nor is it the focus. So long as returns imply similar risk profile, investors demand similar risk adjusted return regardless of reasons that give rise to such risks, whether it be intentional investment strategies or unintentional market outcomes.

2.2 Identification

We illustrate the differences between linear factor models and put-writing strategy, taking one specific fund as an example, in Figure 2. Instead of using a rolling window for identification as we would for later analysis, this illustration exercise uses a static window spanning the entire sample period. The upper-left panel shows the cumulative return based on the fitted values from two common factor models exclusive of the estimated intercept. Since alpha is unexplained return, it cannot be feasibly replicated. Therefore, excluding this term makes for a feasible linear replication. The upper-right panel shows return to the put-writing strategy.

Linear feasible portfolios miss most of the mean, which will be identified as alpha of the fund in regressions. However, put-writing strategy is able to track the return pattern better, not leaving significant gap between the actual and predicted returns. The fund will no longer have an alpha under this specification. The lower panels repeat the same comparison, but on drawdowns. While feasible linear replicating portfolio and put-writing portfolio both miss and underestimate quite a few sizeable drawdowns, linear replicating strategies exaggerate drawdowns incorrectly for 2003 and 2009. Put-writing strategy is able to track drawdowns more accurately than linear strategies, which means a better replication of the risk profile. Note that the replication of put-writing strategy on a specific mutual fund is not exact. Tracking errors are much more noticeable here than in Jurek and Stafford (2015). This is because the singular parameter combination of $[Z, L]$ needs to fit a range of mutual funds.

In following analysis, instead of using the entire sample period as window for identification, we use rolling window to classify whether a fund has asymmetric exposure to downside risk at any point in time. More formally, this time varying identification is defined as:

$$I_{i,t}^{P} = \begin{cases} 1 & \text{if } \alpha_{i,t}^{linear} \text{ is positive significant and } \alpha_{i,t}^{P} \text{ becomes insignificant} \\ 0 & \text{Otherwise} \end{cases} \tag{5}$$

where $\alpha_{i,t}^{linear}$ is the alpha of fund i in linear factor model during a rolling window ending at time t, and $\alpha_{i,t}^{P}$ is the alpha of said fund i in put-writing model during the same time period. The number of funds that have negative significant linear alpha to begin with, or end up with negative significant nonlinear alpha is minimal. Taking these funds out does not have significant impact on the results.

For main results here and after, we use a 24-month rolling window [3]. Identification results are shown in Figure 3. The left panels plot the count of funds identified to have asymmetric downside exposure, as defined by Equation (5) using $[Z = -2, L = 1]$ put-writing strategy, against total number of funds for each fund type on a rolling basis. From top to bottom,

[3] We repeat the same set of charts with 36-month rolling window. Similar observations still hold.

asset classes are equity, fixed income and allocation. The number of funds identified to have asymmetric exposure varies through time, and can represent a significant share of funds at various points in time. Timing where cluster of funds show asymmetric exposure is different across fund types, with the exceptions of two peaks in 2005 and 2010. Sample also spans asset classes alternative and convertibles. They are omitted due to limited number of funds and small aggregate size[4]. The right panels plot the aggregate asset under management of corresponding funds, along with total asset under management for the fund type. Total asset managed by these asymmetric strategies can be very sizeable compared to the aggregate size, but does not grow following a similar trend. Together with the left panels of fund count, the results suggest that identification is idiosyncratic to fund risk profile implied from recent returns. It is neither a static characteristic of the fund, nor an extension of sector trends. Similar patterns across different fund types also suggest that this is not a secondary characteristic solely driven by underlying assets. For example, fixed income funds can provide various extents of liquidity transformation. If other fund types do not have significant assets with asymmetric exposure through time, then it is possible that liquidity of the underlying is what gave rise to the nonlinearity. However, similar identification results prevail in equity funds where there is minimal liquidity transformation, if any at all. Therefore, underlying assets cannot be the sole or main reason that asymmetric exposure exists for any fund. Risk profile implied by fund returns is indeed a combination of assets traded and how they are traded. It does not distinguish or differentiate between the two channels. For mutual fund investors interested in risk adjusted returns, we assume here that they do not discriminate against where the risk originates from.

[4] Sample contains 7 convertibles funds with peak of $10b asset under management, and 8 alternative funds with peak of $22b asset under management. Identification results show similar time varying pattern, with non-trivial portion of asset under management exposed to downside risk

3 Regression Analysis

Now that we have specified what it means to have asymmetric exposure to downside risk, we want to examine whether this identification has an impact on fund flow. In our regression analysis, we find that funds with asymmetric exposure to downside risk experience disproportionately larger performance driven flow than those without. That is to say in times of stress, these funds will likely experience disproportionately more outflow and amplify adverse economic shocks. This effect is both statistically and economically significant. We also compare this put-writing identification of downside risk to other measures of risk implied by fund return, such as conditional beta and return volatility. We find that put-writing strategy based identification yields statistically significant results where the others do not.

3.1 Data

Mutual fund data from January 1996 to June 2020 is obtained from MorningStar. Data includes fund flow and montly returns for oldest shareclass, along with fund characteristics such as total net asset and type. Though fund flow can be computed from total net asset and returns, MorningStar reported monthly fund flow adjusts for distributions, reinvestments, and mergers. We remove funds with missing data to arrive at a balanced panel, to ensure a consistent notion of fund return and flow for regression analysis. We also remove small funds with less than \$5 million asset under management[5], to avoid extreme volatility and outliers. Summary statistics to compare the final sample to the entire universe of funds in MorningStar are reported in Table 1. Attrition from the full population is mainly due to requirement on non-missing data.

We construct fund flow as follows:

$$f_{i,t} = \frac{TNA_{i,t} - (1 + R_{i,t})TNA_{i,t-1}}{TNA_{i,t-1}} \tag{6}$$

[5]This filter removes less than 1% of the entire universe.

where $TNA_{i,t}$ is fund size for fund i at time t as measured by total net asset, and $R_{i,t}$ is return for the oldest share class of fund i at time t. We also construct alternative measure by normalizing MorningStar calculated fund flow with total net asset. Figure 4 shows cross sectional variation in fund flow through time under calculated and MorningStar fund flow measures. The top panel plots the top and bottom precentile of calculated fund flow as defined by Equation (6). The bottom panel plots the top and bottom percentile of normalized Morningstar fund flow. The two plots should be very similar by definition. Morningstar is included for comparison purposes to ensure that later results are not caused by measurement noise and adjustments. As expected, the patterns are almost the same, except for minor differences. Both plots show a significant variation between the top and botton percentiles at all times. It is precisely this large and persistent cross sectional variation that we try to explain.

3.2 Results

To examine whether funds with asymmetric exposure to downside risk will experience disproportionately larger performance-based flow than those without, we run the following specification:

$$f_{i,t} = \delta_0 + \delta_1 r_{i,t-1} + \delta_2 I^P_{i,t-1} + \delta_3 (r_{i,t-1} \times I^P_{i,t-1}) + \gamma_i + \gamma_t + \epsilon_{i,t} \tag{7}$$

where $r_{i,t}$ is return of fund i at time t, $I^P_{i,t}$ is the indicator function of whether fund is asymmetrically exposed to downside risk as defined by put-writing strategy at time t. This indicator function is backward looking based on a rolling 24-month window. We use lagged $I^P_{i,t-1}$ rather than $I^P_{i,t}$ to be consistent with investors chasing past returns and reflect that investors might not know current period performance of the fund at time of their redemption (or purchase) decision.

Regression results are shown as in Table 2. Column (1) reports simple regression on return and the cross term only, while column (2) adds fund and time fixed effects to the regression. Under full specification with fund characteristics, column (3) shows that if return has 9.2 bps impact on fund flow for funds that are not asymmetrically exposed to downside risk, funds that are identified to have asymmetric exposure will have an additional 11 bps of impact. This amplification is more than 100%, which is both statistically and economically significant. Comparing column (3) to previous columns, asymmetric exposure to downside risk is a time varying chracteristic orthogonal to other fund characteristics. This means that whether a fund has asymmetric exposure to downside risk is not inherited from any other time varying and constant fund characteristics. It is driven by return implied risk profile over the past period, which is a combination of market timing and fund strategy outcomes. Lastly, column (4) restricts the regression to crisis periods as defined by FRED[6]. Amplification is statistically significant, and slightly larger than non-crisis periods. For 1% loss return, funds that are not asymmetrically exposed to downside risk will experience 5.7 bps of performance induced outflow, but funds asymmetrically exposure will have an additional 9.1 bps of impact. This amplification is around 160%, which is an increase from the average impact of around 100%. Note that the coefficient for I_i^P is positive significant, which is consistent with funds allocating losses to extreme tail. Investors are more likely to enjoy higher return elsewhere outside the tail as a risk compensation. The coefficient is also on smaller magnitude compared to the return and interaction terms. During bad performance periods, investors have more incentive to run from the fund due to a concern of outsized losses than the benefit of better return associated with such asymmetric strategy.

We also introduce two other measures of return implied riskiness for a given fund, and compare them to $I_{i,t}^P$. First alternative is also a measure of downside risk, but in form of a conditional beta as defined in Bawa and Lindenberg (1977) and Ang et al. (2006). It measures fund's exposure to the market, conditional on market underperforming its in-period mean.

[6]This includes intervals from March 2001 to November 2001, from December 2007 to June 2009, and from February to March 2020.

Formally,

$$\beta^{-}_{i,t} = \frac{Cov(r_{i,t}, r_{m,t} | r_{m,t} < \mu_{m,t})}{Var(r_{m,t} | r_{m,t} < \mu_{m,t})} \tag{8}$$

where $r_{i,t}$ is fund i's excess return within the estimation window, $r_{m,t}$ is market's excess return, and $\mu_{m,t}$ is the average market excess return in the same window. A fund is said to have asymmetric exposure to downside risk if its conditional market exposure is larger than unconditional market exposure during the same period[7].

$$I^{\beta}_{i,t} = \begin{cases} 1 & \text{if } \beta^{-}_{i,t} > \beta_{i,t} \\ 0 & \text{Otherwise} \end{cases} \tag{9}$$

where $\beta^{-}_{i,t}$ is as defined by Equation (8), and $\beta_{i,t}$ is the corresponding unconditional beta. We rerun the same specification as Equation (7) with $I^{\beta}_{i,t}$ in place of $I^{P}_{i,t}$. Result for entire sample regression is shown in column (5) in Table 2, and crisis periods result is shown in column (6). In both cases, coefficient estimates of the interaction term are statistically insignificant. Performance of the $I^{\beta}_{i,t}$ identification differs from the put-writing strategy $I^{P}_{i,t}$ identification because conditional beta measures a different kind of downside risk. Intuitively, in a window of benign market conditions, put-writing portfolio return will pick up the absence of deteriorating market conditions. Conditional beta works a bit differently. Every point in time where market is below its in-period mean will be considered a down market, regardless of presence of stress in the underlying market. Mechanically, only a fraction of a rolling window is used for the estimation in Equation (8) by construction. This introduces a lot of noise[8], and therefore dampens its ability to identify cross sectional variation.

Second alternative of fund's implied riskiness is measured by return volatility, which is measured by sample standard deviation of returns within the window period. Cross term in Equation (7) is replaced by this measure of return risk. Full sample regression result is

[7] In unshown results, we have also defined $\beta^{-}_{i,t} = 1$ for top 20 percentile funds when ordered by $\beta^{-}_{i,t} - \beta_{i,t}$; and 0 otherwise. Qualitative results are the same, where regression results are statistically insignificant.

[8] For the same period, linear replicating regressions and put-writing strategy replicating regression have similar R^2 on average.

shown in column (7) in Table 2. Return volatility is negatively correlated with fund flow. When fund experiences increase in inflow during periods of positive return, an increase in return volatility will decrease such inflow. This is consistent with the higher probability of bad return next period when return fluctuates more. Conversely, when fund experiences increase in outflow during periods of negative return, an increase in return volatility will decrease such outflow. This is consistent with the higher probability of good return next period. Estimator as shown in column (7) is statistically significant, but less so than in column (3). Estimator becomes insignificant when regression is restricted to crisis periods, with results shown in column (8). Return volatility fails to yield significant result in times of stress, when amplification of adverse market shocks and therefore identification of more fragile funds are crucial.

To check robustness of the regression, we first replace constructed fund flow with Morningstar constructed fund flow, which accounts and adjusts for fund level events such as distribution. Results for the rerun are shown in Table 3. They are almost identical to the results from constructed fund flow regression in Table 2. Adjustments in fund flow for fund level events do not affect the results.

We then extend the rolling look-back window from 24 months to 36 months for estimations of $I^P_{i,t}$, $I^\beta_{i,t}$ and return volatility. Results for the rerun are shown in Table 4. When compared with Table 2, results hold qualitatively regardless of changes in specifications. Put-writing strategy yields better statistical significance than other alternative specifications of return implied risks. With a longer estimation window, results show that if return has 9.8 bps impact on fund flow for funds that are not asymmetrically exposed to downside risk, funds that are identified to have asymmetric exposure will have an additional 9.5 bps of impact. Economic significance decreases to under 100% from just above. Information on the far end of the rolling window might be outdated compared to more recent history, but it does not materially change the estimation results. Qualitative comparison between overall estimation and stressed period estimation is the same as in the case of shorter look-back window.

Now that we have established the presence of asymmetric exposure to downside risk matters to fund flow, we examine the impact of extent of such asymmetry. The extent to which a fund is exposed to downside risk is measured by the size of positive significant linear alpha that is explained away by the put-writing strategy. The larger the linear alpha that disappears, the more asymmetric said fund's exposure to downside risk is. This follows from the intuition that linear alpha is appropriate excess return for bearing downside risk, when it can be explained by put-writing portfolio. We run the following specification:

$$f_{i,t} = \delta_0 + \delta_1 r_{i,t-1} + \delta_2 (I^P_{i,t-1} \times \alpha^{linear}_{i,t-1}) + \delta_3 (r_{i,t-1} \times I^P_{i,t-1} \times \alpha^{linear}_{i,t-1}) + \gamma_i + \gamma_t + \epsilon_{i,t} \quad (10)$$

where $\alpha^{linear}_{i,t}$ is the alpha from linear regression run as first step to calculate $I^P_{i,t}$, and rest remains the same as earlier definitions. Regression results are reported in Table 5. Column (1) shows that a 1% increase in $\alpha^{linear}_{i,t}$ that was explained away has an additional impact of 2.2 bps to fund flow. The magnitude of this impact is similar in times of stress, as shown in column (2). Same regression run on MorningStar fund flow is reported in columns (3) and (4), which yields nearly identical results.

These results show that risk profile implied by fund return does matter, and this impact is robust. Funds with asymmetric exposure to downside risk experience disproportionally larger flow relative to fund performance. This makes them prone to larger outflow in times of stress, and potentially cause more fragility to the financial system. In fact, within the population of funds that have this asymmetric exposure, funds with larger asymmetry are more fragile than those with less.

Lastly, we restrict our sample to just fixed income funds and repeat the analysis as in Equation (7). Results are reported in Table 6. Similar to the entire sample, fixed income funds that are asymmetrically exposed to downside risk experience just under 100% increase in performance induced flow than their peers. While this is consistent with policy focus and concerns in bond fund fragility, the dimensional of fragility being picked up by I^P is different from bond illiquidity established in literature. We also run quantile regression analysis on the relationship, with results shown in Figure 5. Interestingly, both absolute and relative impacts of downside risk on performance induced flow are more prominent for the higher quantiles of flow. This suggests a larger impact during inflow rather than outflow periods. To further illustrate the difference between downside risk and bond illiquidity, we restrict the sample to equity funds. Regression results are reported in Table 7, and quantile regression results are shown in Figure 6. There are no qualitative differences between the two sets of results.

4 Policy Implication

In times of stress, financial stability policy would be very effective if it can precisely target funds that are particularly vulnerable and more likely to amplify adverse market shocks. Put-writing strategy based indicator provides a way to identify these funds. Funds with $I^P = 1$ have a 200% increase in outflow than those with $I^P = 0$, causing significantly more adverse pricing pressure on the market. How persistent is this identification? This question matters because if asymmetric exposure to downside risk is a persistent fund characteristics, then implementation of policy can be time invariant and therefore less costly. However, if such characteristics do not persist over time, then policy needs to be criterion based and evaluated every time a stress arises in the market. Regulators may well have to target different population of funds under these circumstances. The corrective action to take, based on specificities of different funds, very well may need to vary. This incurs excess implementation cost for a policy. It also means that preventative action too far into the future will not work.

Let us first define what it means for fund to have asymmetric exposure over a period of time. A fund is said to have asymmetric exposure to downside risk during the past N

months, if it had asymmetric exposure during any month within the period. Formally,

$$D_{i,t}^{-N} = \begin{cases} 1 & \text{if } \sum_{j=0}^{N-1} I_{i,t-j}^{P} > 0 \\ 0 & \text{Otherwise} \end{cases} \tag{11}$$

Similarly, a fund is said to have asymmetric exposure to downside risk during the future M months, if it has asymmetric exposure during any month within the period.

$$D_{i,t}^{M} = \begin{cases} 1 & \text{if } \sum_{j=1}^{M} I_{i,t+j}^{P} > 0 \\ 0 & \text{Otherwise} \end{cases} \tag{12}$$

Persistence can be estimated using probit regression:

$$P(D_{i,t}^{M} = 1 | D_{i,t}^{-N}) = \Phi(\beta_0 + \beta_1 D_{i,t}^{-N}) \tag{13}$$

Results are reported in Table 8 for $[N = 6, M = 1]$ and $[N = 6, M = 3]$. Columns (1) and (3) report results for Equation (13) without controls for additional fund characteristics. Columns (2) and (4) report results with additional controls. Estimators are statistically significant, implying strong persistence in asymmetric exposure in downside risk for funds. Funds with asymmetric exposure to downside risk within the past 6 months are very likely to persistently have asymmetric exposure to downside risk within the next month and quarter[9]. Time frame is kept relatively short (well under a year) for persistence regression because of the noise from outdated information on longer horizon will bring to the D^{-N} and D^{M} estimation. Extending reference time framework will mechanically increase the probability these variables become 1 by construct of Equations (11) and (12).

Recall from rolling identification results in Figure 3, very few funds are identified to have asymmetric exposure to downside risk for about half of the time. This prompts further

[9] In unreported results, we ran regressions for other combinations of $[N, M]$ where $N \in [3, 12]$ and $N \in [1, 3]$. Similar strongly significant probit results hold.

investigation into where strong statistical significance in the probit regression came from. To this end, we construct transition matrix for combinations of $[N, M]$, where outcome is denoted by combinations of $[D^{-N}, D^{M}]$. Results are shown in Figure 7. The left panels plot the transition matrix of funds that are identified to have asymmetric downside exposure over the past N months into the future M months. Series $[1, 1]$ is for the funds that remain asymmetrically exposed to downside risk, and series $[1, 0]$ is for funds that transition out of the asymmetric exposure. The probability that funds had asymmetric exposure will continue to have asymmetric exposure fluctuates a lot over time. It implies very low persistence amongst these $D^{-N} = 1$ funds. The right panels plot the transition matrix of funds that are not identified to have asymmetric downside exposure over the past N months into the future M months. Series $[0, 0]$ is for the funds that stay without asymmetric exposure to downside risk, and series $[0, 1]$ is for funds that become asymmetrically exposed to downside risk. Here we see high persistence amongst $D^{-N} = 0$ funds. If a fund does not have asymmetric exposure in the past, it will very likely remain without asymmetric exposure in the future. Persistence weakens when we extend the future period. Lower right panel has comparatively weaker persistent pattern relative to upper and mid right panels. This happens because extension of M makes presence of asymmetric exposure more likely, and therefore mechanically increase the probability of $[0, 1]$.

The breakdown of transition matrix shows very low persistence for funds to remain asymmetrically exposed to downside risk from one period to the next. This implies that static policy is infeasible as precaution to prevent more fragile funds from outsized outflow in times of stress. No combination of N and M will result in a meaningful rule, with which supervisor can predict the population of funds to target for price stabilization. Population of more vulnerable funds is very sensitive to risk profile implied by past returns during recent periods. This downside measure is dynamic with the market, and therefore has a lot of time variation. Criterion according to Equation (5) needs to be evaluated timely under current market conditions to correctly identify affected funds.

5 Conclusion

We define asymmetric exposure to downside for mutual funds in a very specific manner using $[Z = -2, L = 1]$ put-writing portfolio. Under this specification, funds with asymmetric exposure to downside risk will experience disproportionately larger outflow in times of stress, compared to funds without. For every 1% performance induced outflow, funds with asymmetric exposure experience an additional 1.20% impact. This effect is both statistically significant and economically large. Such outflow makes them more fragile from a financial stability perspective. Results also show that magnitude of amplification is positively correlated with the size of asymmetric exposure. Downside risk measured by conditional beta yields much weaker results.

However, the particular downside risk identification is very dependent on recent history, which makes it ill suited for static policy implantation that aims to reduce extreme redemption under fast deteriorating market conditions. Low persistency is a desirable feature of this identification, rather than a weakness. Though it requires implementation costs to select fund population and react accordingly under different stress scenarios, such market sensitive results yield nimble policies that are adaptive.

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Appendix

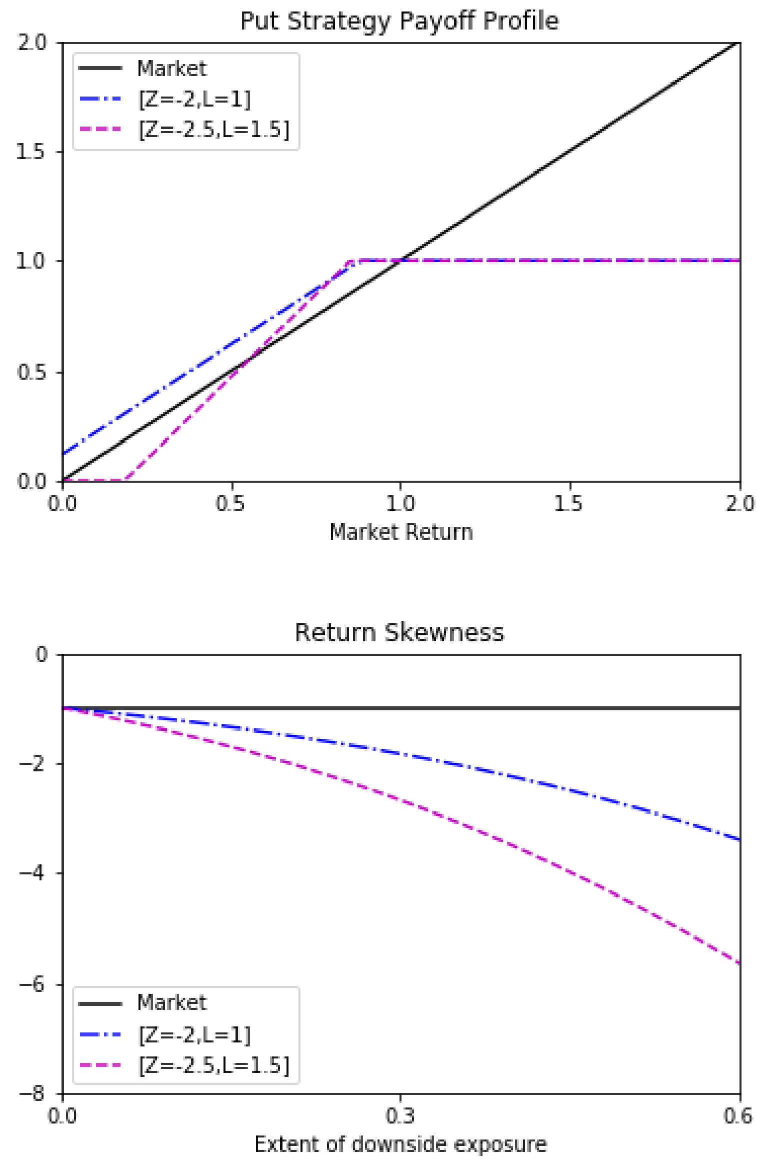


Figure 1: **Strategy payoff and return skewness.** The top panel plots of the payoff profile of the market and two put strategies ($[Z = -2, L = 1]$ and $[Z = -2.5, L = 1.5]$) as a function of the market realization. The bottom panel plots the return skewness as a function of extent of downside exposure, as measured by allocation to the put strategy. The underlying market distribution is assumed to follow $\mathcal{NIG}(0, 18\%^2, -1, 7)$.

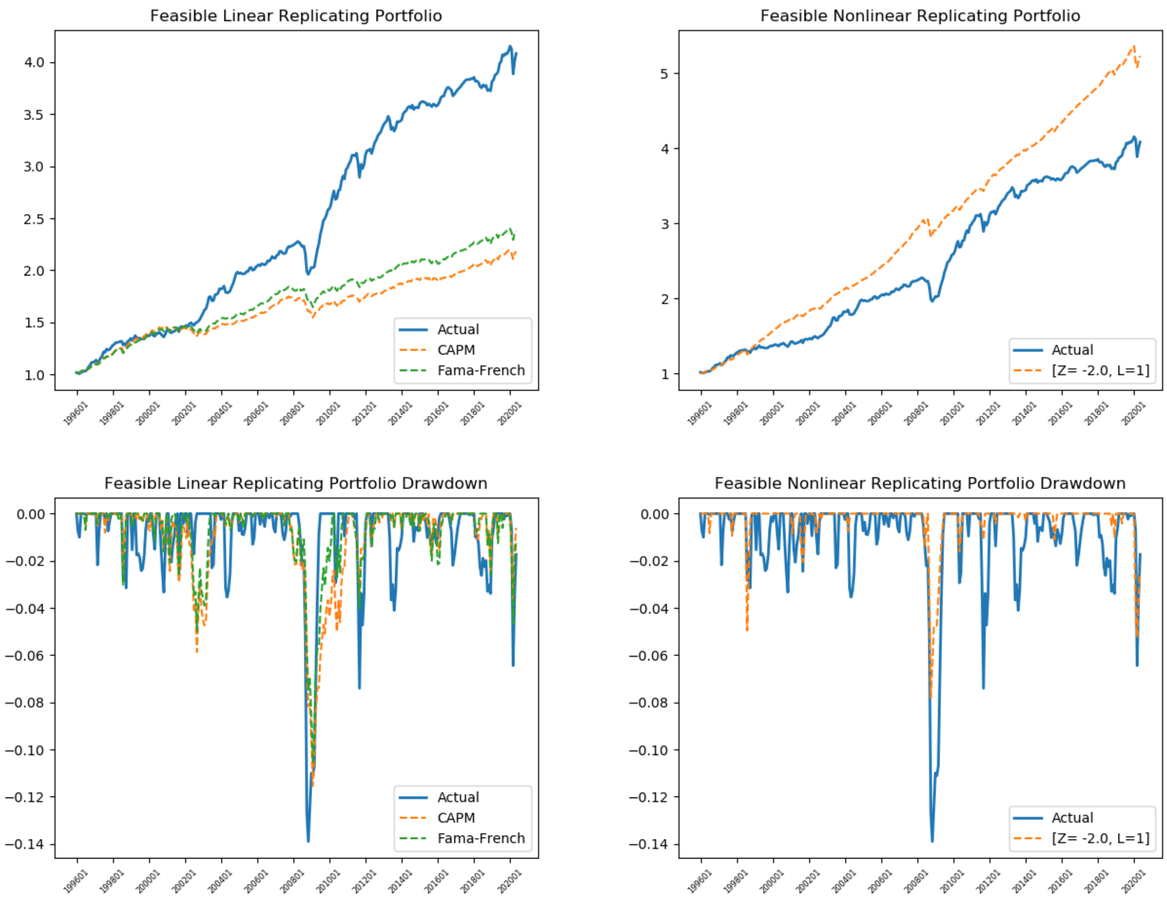


Figure 2: **Replicating the risks and returns of a mutual fund.** The top panels plot the cumulative value of $1 invested in the fund, along with common factor models (CAPM, Fama-French) on the left and $[Z = -2, L = 1]$ put-writing strategy on the right. The bottom panels plot the corresponding monthly drawdown series for the mutual fund and replicating strategies.

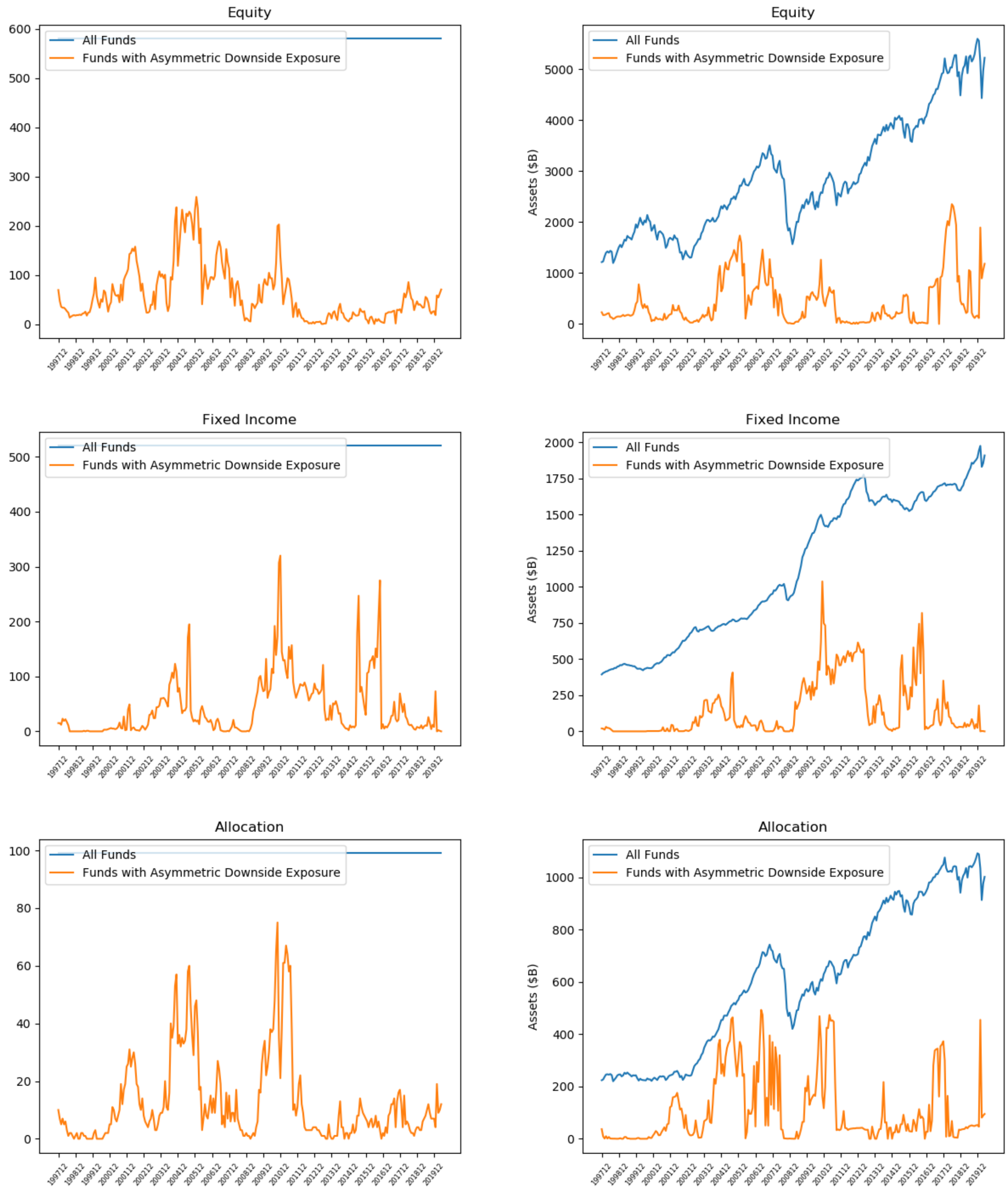


Figure 3: **Mutual funds with asymmetric downside exposure.** The left panels plot the count of funds identified to have asymmetric downside exposure (using $[Z = -2, L = 1]$ put-writing strategy) by industry on a rolling basis. The right panels plot the aggregate asset under management of corresponding funds, along with total asset under management for the fund type.

Table 1: **Summary Statistics**

	Universe	Sample
Size ($m)	200	977
	(1,070)	(2,680)
(min)	0	0.316
(max)	55,400	55,400
Inception Year	2002	1986
	(11.35)	(11.27)
(min)	1924	1924
(max)	2020	1995
Equity	7,622	580
Fixed Income	3,506	520
Allocation	2,319	99
Convertibles	39	7
Alternative	874	8

This table reports summary statistics on the sample relative to entire universe of mutual funds in MorningStar.

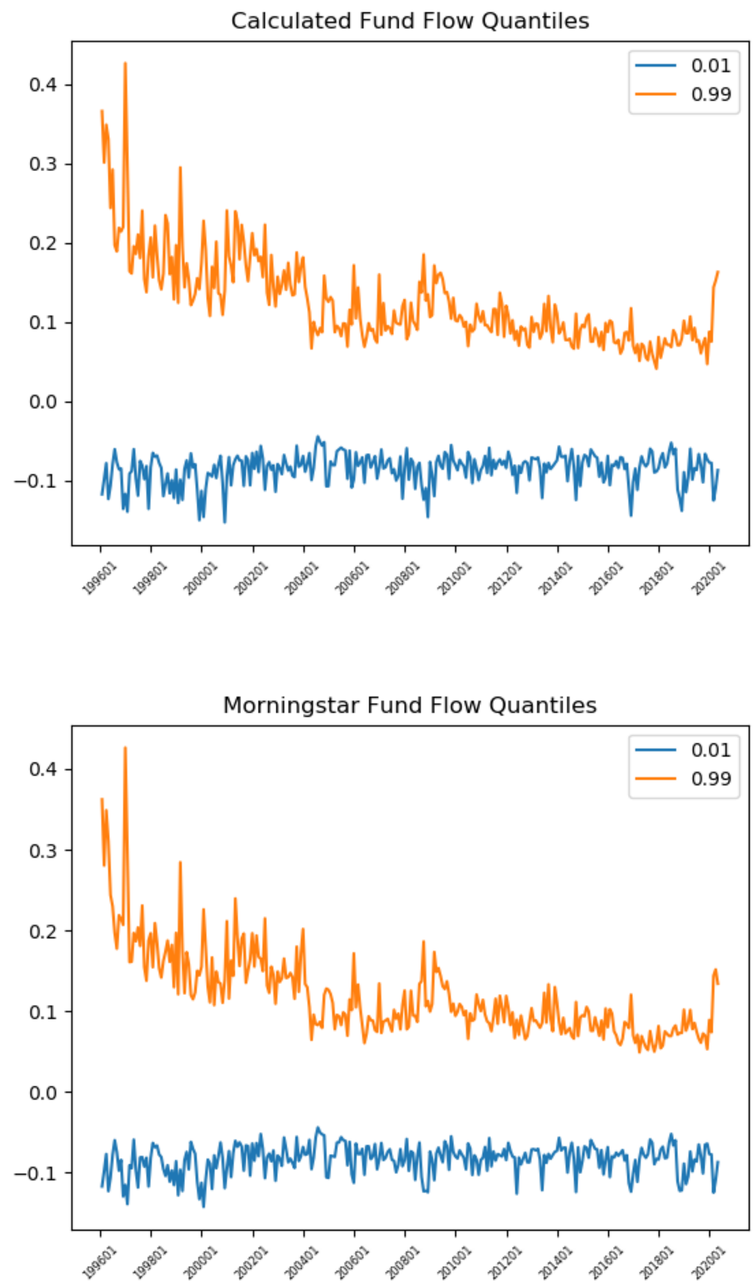


Figure 4: **Fund flows across mutual funds.** The top panel plots the top and bottom precentile of calculated fund flow as defined by Equation (6). The bottom panel plots the top and bottom percentile of fund flow as defined by Morningstar.

Table 2: **Fund Flow and Downside Risk Exposure**

	Put Writing Strategy				Conditional β		Return Risk	
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Return $\times I_i^P$	0.10***	0.11***	0.11***	0.091***				
	(0.02)	(0.018)	(0.018)	(0.021)				
I_i^P	0.0095***	0.0099***	0.010***	0.013***				
	(0.00093)	(0.00085)	(0.00085)	(0.0013)				
Return $\times I_i^\beta$					-0.005	0.0013		
					(0.008)	(0.011)		
I_i^β					-0.0006	-0.0002		
					(0.0004)	(0.0007)		
Return Volatility							-0.069**	-0.051
							(0.030)	(0.039)
Return	0.064***	0.092***	0.092***	0.057***	0.11***	0.075***	0.11***	0.075***
	(0.0083)	(0.011)	(0.011)	(0.013)	(0.013)	(0.015)	(0.012)	(0.015)
Fund Fixed Effect	N	Y	Y	Y	Y	Y	Y	Y
Time Fixed Effect	N	Y	Y	Y	Y	Y	Y	Y
Fund Characteristics	N	N	Y	Y	Y	Y	Y	Y
Observations	326,566	326,566	326,566	111,688	326,566	111,688	326,566	111,688
R-squared	0.013	0.054	0.055	0.084	0.048	0.076	0.048	0.076

This table reports coefficients from regression examining the impact of presence of asymmetric exposure to downside risk, as defined by $[Z = -2, L = 1]$ put-writing strategy in 24-month rolling window, on fund flow over time period from January 1996 to June 2020. The dependent variable is calculated fund flow. Fund characteristics included are fund size. Columns (5)-(6) is reporting on alternative downside risk measure in terms of conditional beta. Columns (7)-(8) is reporting on alternative return risk measure in terms of return volatility. Columns (4), (6) and (8) are restricted to stress periods identified by FRED. Standard errors are clustered at the fund date level, and reported in parenthesis. *, **, *** indicate statistical significance at the 10%, 5%, and 1% levels, respectively.

Table 3: **MorningStar Fund Flow and Downside Risk Exposure**

	Put Writing Strategy				Conditional β		Return Risk	
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Return $\times I_i^P$	0.10***	0.11***	0.11***	0.094***				
	(0.02)	(0.018)	(0.018)	(0.020)				
I_i^P	0.0097***	0.010***	0.010***	0.014***				
	(0.00091)	(0.00084)	(0.00084)	(0.0012)				
Return $\times I_i^\beta$					-0.006	-0.0011		
					(0.008)	(0.011)		
I_i^β					-0.0005	0.0002		
					(0.0004)	(0.0007)		
Return Volatility							-0.078**	-0.055
							(0.030)	(0.039)
Return	0.064***	0.093***	0.092***	0.057***	0.11***	0.074***	0.11***	0.072***
	(0.0082)	(0.011)	(0.011)	(0.013)	(0.052)	(0.015)	(0.012)	(0.015)
Fund Fixed Effect	N	Y	Y	Y	Y	Y	Y	Y
Time Fixed Effect	N	Y	Y	Y	Y	Y	Y	Y
Fund Characteristics	N	N	Y	Y	Y	Y	Y	Y
Observations	326,566	326,566	326,566	111,688	326,566	111,688	326,566	111,688
R-squared	0.015	0.059	0.060	0.095	0.048	0.085	0.053	0.085

This table reports robustness checks of the results in Table 2. The dependent variable is Morningstar fund flow, which adjusts for fund level events. Columns (5)-(6) is reporting on alternative downside risk measure in terms of conditional beta. Columns (7)-(8) is reporting on alternative return risk measure in terms of return volatility. Columns (4), (6) and (8) are restricted to stress periods identified by FRED. Standard errors are clustered at the fund date level, and reported in parenthesis. *, **, *** indicate statistical significance at the 10%, 5%, and 1% levels, respectively.

Table 4: **Downside Risk Exposure in Extended Window**

	Put Writing Strategy				Conditional β		Return Risk	
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Return $\times I_i^P$	0.093***	0.095***	0.095***	0.084***				
	(0.016)	(0.016)	(0.016)	(0.017)				
I_i^P	0.0082***	0.0086***	0.0088***	0.014***				
	(0.00074)	(0.00076)	(0.00077)	(0.0013)				
Return $\times I_i^\beta$					-0.006	-0.0082		
					(0.009)	(0.011)		
I_i^β					-0.00009	0.0003		
					(0.0004)	(0.0007)		
Return Volatility							-0.067**	-0.058
							(0.032)	(0.044)
Return	0.068***	0.099***	0.098***	0.062***	0.11***	0.081***	0.11***	0.075***
	(0.0081)	(0.012)	(0.012)	(0.014)	(0.013)	(0.016)	(0.012)	(0.015)
Fund Fixed Effect	N	Y	Y	Y	Y	Y	Y	Y
Time Fixed Effect	N	Y	Y	Y	Y	Y	Y	Y
Fund Characteristics	N	N	Y	Y	Y	Y	Y	Y
Observations	311,998	311,998	311,998	111,688	311,998	111,688	311,998	111,688
R-squared	0.013	0.053	0.054	0.084	0.048	0.076	0.048	0.076

This table reports robustness checks of the results in Table 2. The dependent variable is calculated fund flow. Independent variables are calculated based on a 36-month rolling window. Columns (5)-(6) is reporting on alternative downside risk measure in terms of conditional beta. Columns (7)-(8) is reporting on alternative return risk measure in terms of return volatility. Columns (4), (6) and (8) are restricted to stress periods identified by FRED. Standard errors are clustered at the fund date level, and reported in parenthesis. *, **, *** indicate statistical significance at the 10%, 5%, and 1% levels, respectively.

Table 5: **Fund Flow and Downside Risk Exposure Intensity**

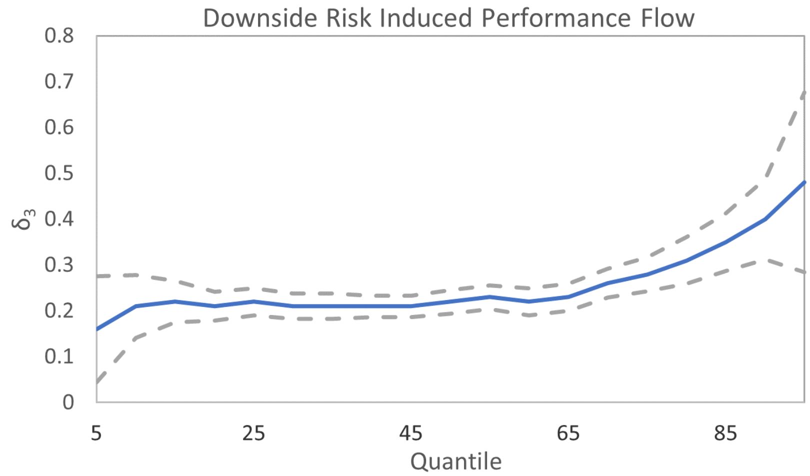
	Calculated Flow		MorningStar Flow	
	(1)	(2)	(3)	(4)
Return $\times I_i^P \times \alpha_i^{linear}$	3.319***	3.189***	3.329***	3.303***
	(1.035)	(1.027)	(1.036)	(1.022)
$I_i^P \times \alpha_i^{linear}$	1.103***	0.985***	1.115***	0.982***
	(0.097)	(0.121)	(0.096)	(0.119)
Return	0.093***	0.060***	0.093***	0.057***
	(0.011)	(0.014)	(0.011)	(0.014)
Firm Fixed Effect	Y	Y	Y	Y
Time Fixed Effect	Y	Y	Y	Y
Fund Characteristics	Y	Y	Y	Y
Observations	326,566	111,688	326,566	111,688
R-squared	0.052	0.074	0.058	0.085

This table reports coefficients from regression examining impact of the magnitude of asymmetric exposure to downside risk, as defined by $[Z = -2, L = 1]$ put-writing strategy in 24-month rolling window, on fund flow over time period from January 1996 to June 2020. The dependent variable in columns (1) and (2) is calculated fund flow. The dependent variable in columns (3) and (4) is Morningstar fund flow. Columns (2) and (4) are restricted to stress periods identified by FRED. Standard errors are clustered at the fund date level, and reported in parenthesis. *, **, *** indicate statistical significance at the 10%, 5%, and 1% levels, respectively.

Table 6: **Fund Flow and Downside Risk Exposure for Fixed Income Funds**

	(1)	(2)	(3)
Return $\times I_i^P$	0.25^{***}	0.11^{*}	0.11^{*}
	(0.091)	(0.059)	(0.059)
I_i^P	0.00064	0.0026^{***}	0.0028^{***}
	(0.0012)	(0.00081)	(0.00081)
Return	0.21^{***}	0.15^{***}	0.14^{***}
	(0.032)	(0.029)	(0.029)
Fund Fixed Effect	N	Y	Y
Time Fixed Effect	N	Y	Y
Fund Characteristics	N	N	Y
Observations	139,880	139,880	139,880
R-squared	0.009	0.060	0.062

This table reports similar regression results as in Table 2 when sample is restricted to the broad category of fixed income funds. The dependent variable is calculated fund flow. Independent variables are calculated based on a 24-month rolling window. Standard errors are clustered at the fund date level, and reported in parenthesis. *, **, *** indicate statistical significance at the 10%, 5%, and 1% levels, respectively.



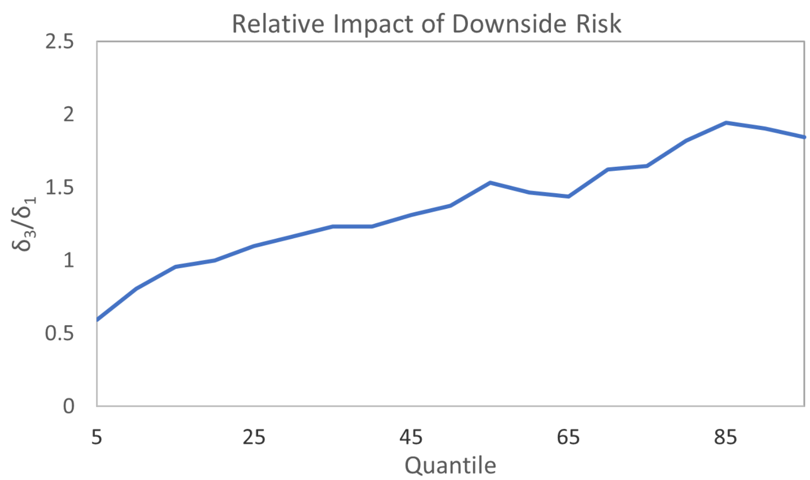
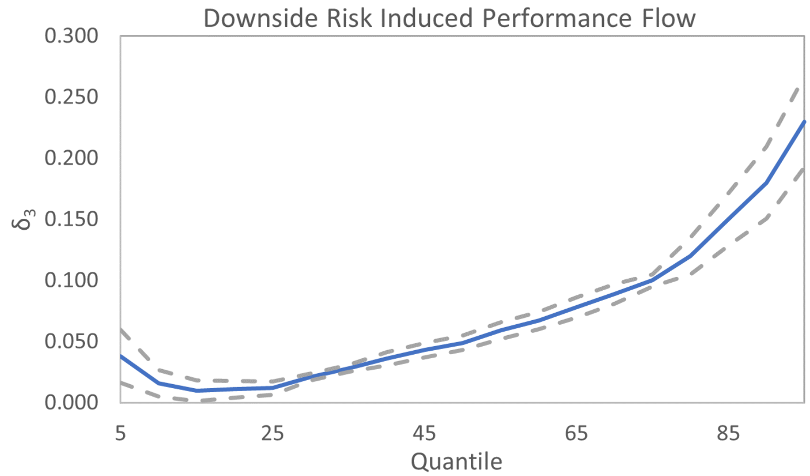


Figure 5: **Impact of downside exposure across quantiles of fixed income funds.** The top panel plots magnitude of estimator on the interaction term for different quantile regressions, with dotted lines being the 95% confidence interval. The bottom panel plots the magnitude of estimator on the interaction term relative to that on the return for different quantile regressions. Quantile regressions are run with robust standard errors.

Table 7: **Fund Flow and Downside Risk Exposure for Equity Funds**

	(1)	(2)	(3)
Return $\times I_i^P$	0.084***	0.084***	0.084***
	(0.018)	(0.017)	(0.017)
I_i^P	0.016***	0.015***	0.015***
	(0.001)	(0.001)	(0.001)
Return	0.055***	0.139***	0.138***
	(0.008)	(0.014)	(0.014)
Fund Fixed Effect	N	Y	Y
Time Fixed Effect	N	Y	Y
Fund Characteristics	N	N	Y
Observations	156,020	156,020	156,020
R-squared	0.024	0.065	0.066

This table reports similar regression results as in Table 2 when sample is restricted to the broad category of equity funds. The dependent variable is calculated fund flow. Independent variables are calculated based on a 24-month rolling window. Standard errors are clustered at the fund date level, and reported in parenthesis. *, **, *** indicate statistical significance at the 10%, 5%, and 1% levels, respectively.



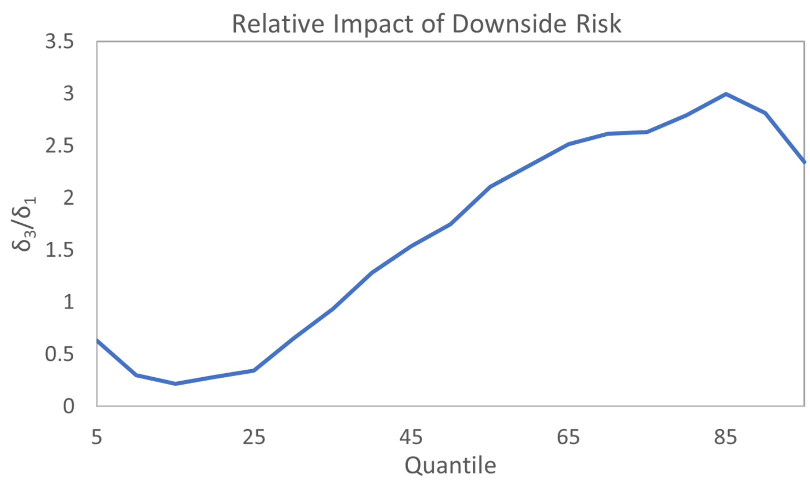


Figure 6: **Impact of downside exposure across quantiles of equity funds.** The top panel plots magnitude of estimator on the interaction term for different quantile regressions, with dotted lines being the 95% confidence interval. The bottom panel plots the magnitude of estimator on the interaction term relative to that on the return for different quantile regressions. Quantile regressions are run with robust standard errors.

Table 8: **Persistence of Downside Exposure**

	$[N=6, M=1]$		$[N=6, M=3]$	
	(1)	(2)	(3)	(4)
D_t^{-N}	1.9692***	1.9712***	1.7295***	1.7241***
	(0.008)	(0.008)	(0.007)	(0.007)
Fund Characteristics	N	Y	N	Y
Observations	320,496	320,496	318,068	318,068
Pseudo R-squared	0.3731	0.3855	0.2847	0.2961

This table reports coefficients from probit regression examining the likelihood of fund having asymmetric exposure in the future, conditional on having had asymmetric exposure in the past. Columns (1) and (2) are using 6 months for the lookback period, and 1 month for forward period. Columns (3) and (4) are using 6 months for the lookback period, and 3 months for forward period. Fund characteristics included are fund size, type, flow, and return. Standard errors are clustered at the fund date level, and reported in parenthesis. *, **, *** indicate statistical significance at the 10%, 5%, and 1% levels, respectively.

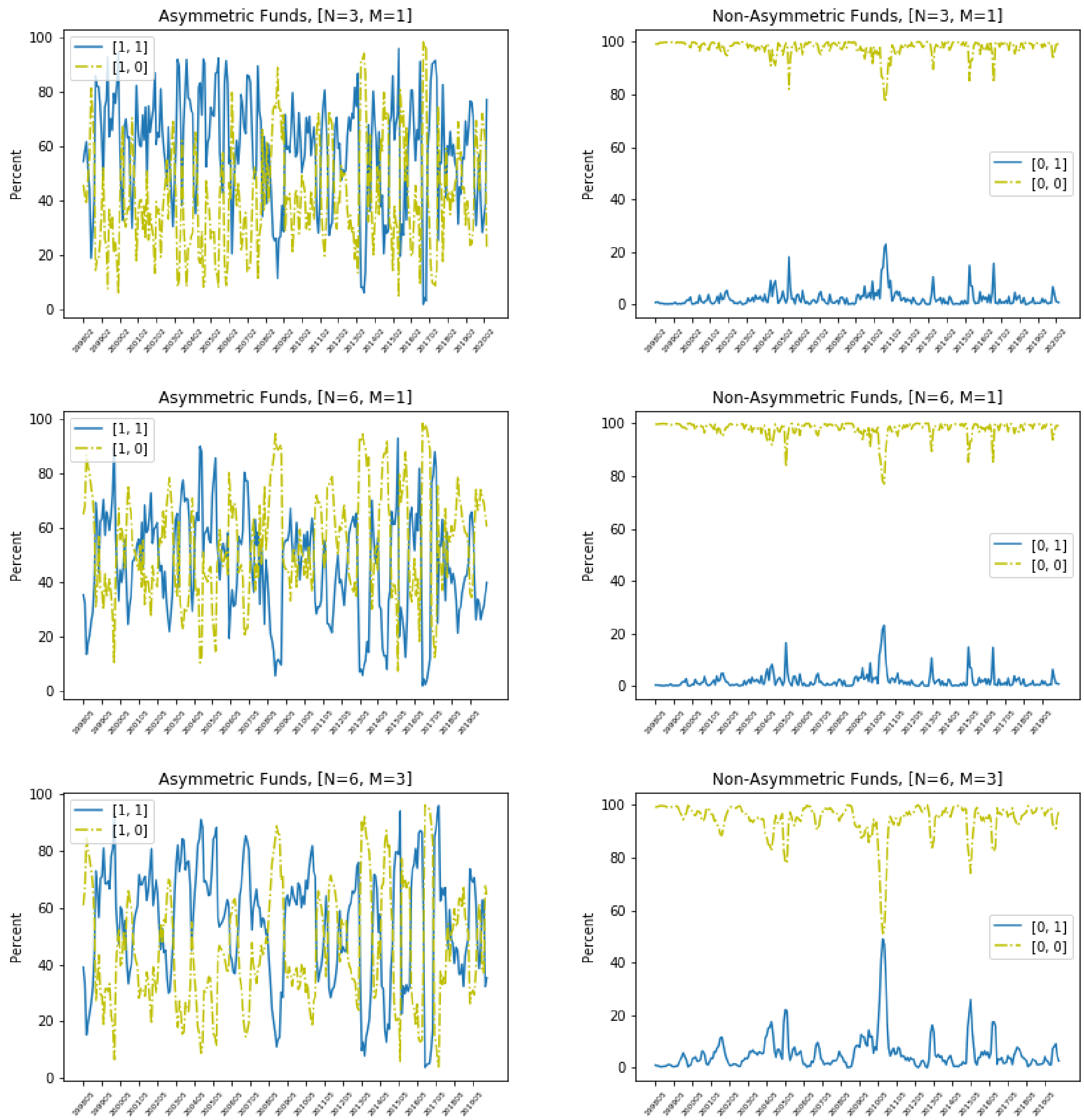


Figure 7: **Persistence of asymmetric downside exposure.** The left panels plot the transition matrix of funds that are identified to have asymmetric downside exposure over the past N months into the future M months. The right panels plot the transition matrix of funds that are not identified to have asymmetric downside exposure over the past N months into the future M months. The probability that funds with asymmetric exposure will continue to have asymmetric exposure fluctuates a lot over time. It implies very low persistence amongst these $D^{-N} = 1$ funds. The top panels are for $[N = 3, M = 1]$, middle panels are for $[N = 6, M = 1]$, and bottom panels are for $[N = 6, M = 3]$.