Working Papers

FEDERAL RESERVE BANK OF BOSTON™

No. 23-13

The Aggregate Effects of Sectoral Shocks in an Open Economy

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Abstract:
We study the aggregate effects of sectoral productivity shocks in a multisectoral New Keynesian open-economy model that allows for asymmetric input-output linkages, both within and between countries, as well as for heterogeneity in sectoral Calvo-type price stickiness. Asymmetries in the international production network play a key role in the model's ability to produce large domestic effects of foreign sectoral supply shocks and large differential effects of domestic shocks and global shocks. Larger trade openness and substitutability between domestic inputs and foreign inputs can also significantly amplify the effects of foreign and global sectoral shocks on domestic aggregates. In comparison, sectoral heterogeneity in price stickiness does not materially amplify the domestic responses to productivity shocks that originate abroad.

JEL Classifications: E12, E31, F41, F44
Keywords: International input-output linkages, open-economy New Keynesian model, sectoral shocks

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This version: October 2023 **https://doi.org/10.29412/res.wp.2023.13**

1 Introduction

Can sectoral shocks originating abroad significantly affect domestic aggregate fluctuations? In practice, they can. A recent example is the global shortage of semiconductors in 2022. This event, triggered by the disruptions in global supply chains due to the COVID-19 pandemic, engendered inflation spikes across consumption categories such as motor vehicles and electronics and materially contributed to an increase in aggregate inflation.

Several recent papers analyze the aggregate impact of sectoral shocks in a closed economy featuring nominal rigidities and production networks (see Afrouzi and Bhattarai, 2023; Andrade and Sheremirov, 2022; La'O and Tahbaz-Salehi, 2022; Pasten, Schoenle, and Weber, 2023; Rubbo, 2023, among others), with some studies aiming in particular to assess the consequences of the COVID-19 pandemic (Baqaee and Farhi, 2022; Guerrieri et al., 2022). Less progress, however, has been made in analyzing the aggregate impact of sectoral shocks in an open-economy setup.

This paper intends to start filling this gap by developing an open-economy model in which foreign sectoral shocks are propagated and amplified through the global supply chain and nominal rigidities. More precisely, our model combines the sectoral input-output linkages and heterogeneity in price rigidity of the closed-economy multisectoral models mentioned above with features of a one-sector open-economy New Keynesian model introduced in Galí and Monacelli (2005). In this setup, we focus on the effects of domestic, foreign, and global sectoral productivity shocks.

Specifically, we address the following question: Can the model generate large responses to foreign sectoral shocks and thereby create a wedge between the effects of domestic shocks and global shocks? We find that while a model with symmetric input-output linkages and heterogeneous price rigidity can produce large effects of domestic sectoral productivity shocks on inflation and consumption, the effects of foreign sectoral productivity shocks are nonetheless small. Hence, under symmetric linkages, global shocks operate mostly through their domestic component. The small domestic effect of foreign shocks can be explained by substitutability between domestic and foreign inputs and an exchange-rate adjustment.

By contrast, the effects of foreign and global shocks are found to be larger if the global input-output network is highly asymmetric and if the shocked foreign sector has a relatively large share of exports to the domestic economy. Moreover, while the relative degree of price stickiness in the shocked sector as well as aggregate price rigidity affect the responses to domestic and foreign shocks, the heterogeneity in price stickiness appears less important. This result holds for both symmetric and asymmetric production linkages.

The transmission mechanism of a foreign shock in our model is as follows. When a foreign production sector experiences a contractionary productivity shock, the relative price of the intermediate inputs produced in that sector rises. Because sectoral inputs are not perfectly substitutable, the price of the entire bundle of foreign inputs also increases. The sensitivity of the foreign aggregate price index depends on sectoral price stickiness and the cross-sector substitutability of inputs. The increase in the price of foreign inputs has two indirect effects on the domestic economy. First, as the foreign central bank raises its policy interest rate to combat cyclical inflation, the foreign currency appreciates and the domestic currency depreciates. Second, demand for domestic inputs relative to foreign inputs increases in both countries due to a decline in the relative price of domestic inputs. Both these channels have a stimulative effect on domestic production and consumption. In the baseline calibration, for both the model with symmetric linkages and the one with asymmetric linkages, these positive effects offset the negative direct effect of costlier foreign inputs. Thus, on balance, domestic consumption rises. Domestic inflation, however, also rises but significantly less so than in the case of a domestic sectoral shock of the same magnitude.

The transmission mechanism of a global shock is more nuanced. The important feature of the global shock is that it affects both the domestic and foreign economy simultaneously. The structure of the global production network is particularly important here. Under symmetric linkages, a global shock does not affect the price of domestic inputs relative to foreign inputs, nor does it affect the exchange rate. With asymmetric linkages, a shock to an import-intensive foreign sector leads to a deterioration of the terms of trade and an exchange-rate depreciation in the domestic economy. In practice, it implies that the differential between the effects of domestic shocks and global shocks is larger if international trade linkages are asymmetric.

We show that the degree of trade openness is important for our main result. Similarly to Galí and Monacelli (2005), we calibrate the foreign trade sector in the domestic economy to represent a small open economy such as Canada. In this context, we show two results. First, if the degree of trade openness is reduced to match the US imports share in output, the effects of foreign productivity shocks become small. Second, to generate a particularly large effect of the foreign component of a global shock, the model needs both a large degree of trade openness and high substitutability between domestic inputs and foreign inputs. The intuition for this result is simple: When the elasticity of substitution between domestic inputs and foreign inputs is large, a purely foreign shock has a limited effect on domestic production because it is relatively easy to substitute toward the cheaper inputs. But in the case of a global shock, such substitution is not possible because the domestic shocks and the foreign shocks are perfectly correlated.

The degree of price stickiness also plays a role in the transmission mechanism. But the important dimension is the level of price stickiness in the shocked sector (conditional on aggregate price stickiness) as opposed to heterogeneity in price stickiness per se. Because prices are more rigid in the shocked sector, the relative price of domestic inputs in terms of foreign inputs responds less to a shock. As a result, the differential effects of domestic shocks and global shocks decline in the degree of price stickiness in the shocked sector.

While an extension of closed-economy multisectoral models to an open-economy setting may seem straightforward, it is hindered by significant challenges. One such challenge arises because under a flexible exchange rate and no capital controls, the exchange rate strongly adjusts in response to foreign shocks, thereby limiting their domestic effects. Our results imply that asymmetric input-output linkages can limit this risk-sharing channel. Another challenge is that open-economy models often assume that domestic firms have a large share of foreign intermediate inputs, which aptly describes small open economies, such as Canada, but not large open economies, such as the United States or the euro area. While our model also requires that the foreign sector be large in order to generate significant domestic effects, asymmetries in the global production network mitigate this issue. The third challenge, which is outside the scope of this paper, is that building a quantitative model with realistic linkages covering a large number of countries—or at least one economy vis-à-vis the rest of the world—is not trivial. Therefore, this paper aims only to build a stylized model that clarifies a transmission mechanism of foreign and global shocks through international production networks. We leave fully quantitative analyses to future research.

Related Literature Our work is connected to the literature that, following seminal contributions by Gabaix (2011) and Acemoglu et al. (2012), studies the conditions under which idiosyncratic or sectoral shocks can account for aggregate fluctuations. While early work focuses on models with flexible prices, recent papers incorporate production networks in closed-economy multisectoral models with nominal rigidities and examine how these two key features affect the transmission of shocks to aggregate variables, including prices. While several works (Carvalho, 2006; Ghassibe, 2021, 2022; Nakamura and Steinsson, 2010; Pasten, Schoenle, and Weber, 2020) study the transmission of monetary-policy shocks, others such as Baqaee and Farhi (2019), Guerrieri et al. (2022), and Pasten, Schoenle, and Weber (2023) focus on sectoral shocks. Rubbo (2023) investigates both and also analyzes the optimal monetary policy and price index in such a setup. La'O and Tahbaz-Salehi (2022) do so in a model in which informational constraints, rather than sticky prices, generate relative price distortions. Afrouzi and Bhattarai (2023) introduce a sufficient-statistic approach to derive the quantitative impact of sectoral and aggregate shocks on aggregate variables in such a class of models.

Our paper is also related to the empirical literature assessing the contribution of sectoral shocks to aggregate fluctuations. Atalay (2017), Cesa-Bianchi and Ferrero (2021), and Foerster, Sarte, and Watson (2011) propose different methods for identifying domestic sectoral shocks and show that sectoral shocks contribute significantly to aggregate fluctuations. Boivin, Giannoni, and Mihov (2009) estimate a factor-augmented vector autoregression to show that domestic sectoral inflation rates respond slowly to aggregate shocks and rapidly to sectoral shocks. Carvalho, Lee, and Park (2021) show that an estimated multisectoral New Keynesian model with roundabout production and heterogeneous price stickiness can reproduce these patterns. Andrade and Sheremirov (2022) document that the cross-sectional distributions of sectoral inflation and consumption growth depart from the normal distribution, and that such an estimated multisectoral New Keynesian model with roundabout production cannot match these empirical facts. This suggests that sectoral shocks can have an even larger and more persistent impact on aggregate variables than this model would predict due to infrequent and large shocks. Andrade and Zachariadis (2016) emphasize the sizeable contribution of global sectoral shocks in international micro price dynamics. di Giovanni, Levchenko, and Mejean (2020) document how production networks transmit foreign shocks to domestic output through their impact on individual firms' value added. We contribute to this literature by investigating the transmission of local, foreign, and global shocks to the domestic economy.

Finally, we contribute to the literature analyzing the propagation of shocks in an open economy with nominal rigidities and a production network. Carvalho and Nechio (2011) introduce a quantitative multisector, two-country, sticky-price model to explain the persistence of real-exchange-rate responses to nominal shocks. While they consider roundabout production, we allow for a more general and potentially asymmetric input-output network. Comin, Johnson, and Jones (2023) also develop a multisectoral open-economy model, but they emphasize another mechanism that is based on occasionally binding production capacity constraints. They then analyze how shocks to these constraints, combined with demand shocks, can account for the recent inflation surge. Baqaee and Farhi (2022) analyze the welfare effects of trade shocks in a multicountry model with general input-output structure and potential nominal rigidities. We instead focus on the dynamic impact of sectoral productivity shocks on aggregate variables.

The paper proceeds as follows. We present our model in Section 2 and our parameterization strategy in Section 3. In Section 4, we study the effects of domestic, foreign, and global productivity shocks on inflation and consumption. In Section 4.1, we analyze the model with symmetric input-output linkages, and in Section 4.2, we introduce asymmetries in the global production network. This section pays considerable attention to heterogeneity in price stickiness and the relative rigidity of the shocked sector. Section 5 conducts sensitivity analyses to variations in consumer preferences and policy designs. Section 6 concludes.

2 Model

This section presents an open-economy New Keynesian model with input-output linkages that allows for heterogeneity in price rigidity and sector sizes both across sectors and between the domestic and foreign economies. The economy features supply and demand shocks at the global, country, and sectoral levels.

2.1 Households

The economy comprises a large number of infinitely lived households. The representative household's preferences are characterized by a utility function that is additively separable in consumption and labor. The representative household supplies labor to firms in each domestic sector and consumes a bundle of products produced in each domestic sector. The preferences are subject to an aggregate demand shock to the discount factor.

Specifically, the representative household chooses aggregate domestic consumption, C_t, and labor supply provided to each sector $k \in \{1, ..., K\}$, $L_{k,t}$, to maximize the following utility function:

$$\mathbb{E}_0 \left[\sum_{t=0}^{\infty} \beta^t \Gamma_t \left[\frac{C_t^{1-\sigma}}{1-\sigma} - \sum_{k=1}^{K} \chi_k \frac{L_{k,t}^{1+\varphi}}{1+\varphi} \right] \right], \tag{1}$$

where parameter β is the discount factor, σ is the inverse elasticity of intertemporal substitution, φ is the inverse Frisch elasticity of labor supply, and χ_k represents the relative disutility from working in sector k.[1] Γ_t, modeled as an AR(1) process in logarithms, represents an aggregate demand (preference) shock.

The representative household invests in state-contingent bonds, B_t, which are purchased at a discount in period $t-1$ and provide a unit payout in period t. The bond market is complete. The profits made by firm i in sector k, $\Pi_{k,t}(i)$, are distributed back to the household as a lump-sum payment. Thus, the budget constraint is as follows:

$$P_t C_t + \mathbb{E}_t[\Theta_{t,t+1} B_{t+1}] = B_t + \sum_{k=1}^{K} W_{k,t} L_{k,t} + \sum_{k=1}^{K} \int \Pi_{k,t}(i)\, \mathrm{d}i, \tag{2}$$

where P_t is the domestic aggregate consumer price index, $\Theta_{t,t+1}$ is the stochastic discount

[1] Our model allows for labor-market frictions across countries but not across sectors. Making the reallocation of labor across sectors costly would reinforce the effects of sectoral shocks in this model (see Ferrante, Graves, and Iacoviello, 2023).

factor, and $W_{k,t}$ is the nominal wage index in sector k.

Consumption within and across sectors is aggregated using the constant elasticity of substitution (CES) function, with the elasticity of substitution across sectors η and the elasticity of substitution across products within each sector θ. Denoting consumption of output produced by firm i in sector k by $C_{k,t}(i)$, aggregate and sectoral consumption bundles are as follows:

$$C_t = \left[\sum_{k=1}^{K} (\zeta_k D_{k,t})^{1/\eta} C_{k,t}^{(\eta-1)/\eta} \right]^{\eta/(\eta-1)} \tag{3}$$

$$C_{k,t} = \left[\left(\frac{1}{n_k} \right)^{1/\theta} \int C_{k,t}(i)^{(\theta-1)/\theta} \, \mathrm{d}i \right]^{\theta/(\theta-1)}, \tag{4}$$

where ζ_k is the consumption share of sector k, $D_{k,t}$ is a sectoral relative demand shock, and n_k is the size of sector k.

The first-order conditions of the household's dynamic optimization problem are as follows:

$$\Theta_{t,t+1} = \beta \left(\frac{\Gamma_{t+1}}{\Gamma_t} \right) \left(\frac{C_t}{C_{t+1}} \right)^{\sigma} \left(\frac{P_t}{P_{t+1}} \right) \tag{5}$$

$$\frac{W_{k,t}}{P_t} = \chi_k L_{k,t}^{\varphi} C_t^{\sigma} \tag{6}$$

$$C_{k,t} = \zeta_k D_{k,t} \left(\frac{P_{k,t}}{P_t} \right)^{-\eta} C_t \tag{7}$$

$$C_{k,t}(i) = \frac{1}{n_k} \left(\frac{P_{k,t}(i)}{P_{k,t}} \right)^{-\theta} C_{k,t}, \tag{8}$$

where $P_{k,t}$ is the price index in sector k, and $P_{k,t}(i)$ is the price of output produced by firm i in sector k. Here, Equation (5) is a stochastic Euler equation; Equation (6) is a standard intratemporal condition that equates the real wage to the ratio of the marginal utilities of labor and consumption, and Equations (7) and (8) characterize optimal demand. The aggregate and sectoral price indexes are computed as usual in the CES setup:

$$P_t = \left[\sum_{k=1}^{K} \zeta_k D_{k,t} P_{k,t}^{1-\eta} \right]^{\frac{1}{1-\eta}} \tag{9}$$

$$P_{k,t} = \left[\frac{1}{n_k} \int P_{k,t}(i)^{1-\theta} \, \mathrm{d}i \right]^{\frac{1}{1-\theta}}. \tag{10}$$

The same set of equations characterizes the foreign economy.

Denote the corresponding variables in the foreign economy with an asterisk, and define the real exchange rate, Q_t, as the price of foreign goods in terms of domestic goods. Then,

$$Q_t = \frac{\Gamma_t^*}{\Gamma_t}\left(\frac{C_t}{C_t^*}\right)^{\sigma}, \tag{11}$$

and thus the corresponding nominal exchange rate X_t (units of domestic currency per one unit of foreign currency) is as follows:

$$X_t = Q_t \frac{P_t}{P_t^*}. \tag{12}$$

2.2 Firms

Domestic firms combine domestic labor, $L_{k,t}(i)$, with intermediate inputs, $Z_{k,t}(i)$, which comprise intermediate goods produced domestically, $Z^{\mathrm{H}}_{k,t}(i)$, and intermediate goods produced abroad, $Z^{\mathrm{F}}_{k,t}(i)$. The technological process is represented by a Cobb–Douglas production function with the share of intermediate inputs δ:

$$Y_{k,t}(i) = \tilde{A}_t \tilde{A}_{k,t} A_t A_{k,t} L_{k,t}^{1-\delta}(i) Z_{k,t}^{\delta}(i), \tag{13}$$

where A_t and $A_{k,t}$ are *domestic* aggregate and sectoral productivity shocks, respectively, and $\tilde{A}_t$ and $\tilde{A}_{k,t}$ are global shocks.[2]

The intermediate inputs produced domestically and abroad are aggregated using the CES function:

$$Z_{k,t}(i) = \left[(1-\nu)^{1/\rho} Z^{\mathrm{H}}_{k,t}(i)^{(\rho-1)/\rho} + \nu^{1/\rho} Z^{\mathrm{F}}_{k,t}(i)^{(\rho-1)/\rho}\right]^{\rho/(\rho-1)}, \tag{14}$$

where ρ is the Armington (1969) elasticity of substitution between domestic and foreign goods, and ν pins down the share of imports in the intermediate goods used for domestic production.[3]

The aggregators for the home and foreign intermediate inputs are the following:

$$Z^{\mathrm{H}}_{k,t}(i) = \left[\sum_{\ell=1}^{K} (\omega^{\mathrm{H}}_{k,\ell} D_{\ell,t})^{1/\eta} Z^{\mathrm{H}}_{k,\ell,t}(i)^{(\eta-1)/\eta}\right]^{\frac{\eta}{\eta-1}} \tag{15}$$

$$Z^{\mathrm{F}}_{k,t}(i) = \left[\sum_{\ell=1}^{K^*} (\omega^{\mathrm{F}}_{k,\ell} D^*_{\ell,t})^{1/\eta} Z^{\mathrm{F}}_{k,\ell,t}(i)^{(\eta-1)/\eta}\right]^{\frac{\eta}{\eta-1}} \tag{16}$$

[2]That is, $\tilde{A}_t$ and $\tilde{A}_{k,t}$ take the same draws for both the domestic and foreign economies, whereas the foreign aggregate and sectoral shocks, A_t^* and $A_{k,t}^*$, need not be the same as the global or domestic shocks.

[3]While we generally adopt a symmetric parameterization, in our baseline, we assume that $\nu > \nu^*$. This choice reflects the notion that trade between a country and the rest of the world makes up a share of that country's output that is larger than its share of the output of all other countries combined.

$$Z^{\mathrm{H}}_{k,\ell,t}(i) = \left[\left(\frac{1}{n_\ell}\right)^{1/\theta} \int Z^{\mathrm{H}}_{k,\ell,t}(i,j)^{(\theta-1)/\theta}\,\mathrm{d}j\right]^{\theta/(\theta-1)} \tag{17}$$

$$Z^{\mathrm{F}}_{k,\ell,t}(i) = \left[\left(\frac{1}{n^*_\ell}\right)^{1/\theta} \int Z^{\mathrm{F}}_{k,\ell,t}(i,j)^{(\theta-1)/\theta}\,\mathrm{d}j\right]^{\theta/(\theta-1)}, \tag{18}$$

where the input-output linkages $\omega^{\mathrm{H}}_{k,\ell}$ represent the shares of domestic sector $\ell \in \{1,...,K\}$ output used in domestic sector $k \in \{1,...,K\}$ production, and linkages $\omega^{\mathrm{F}}_{k,\ell}$ are the shares of foreign sector $\ell \in \{1,...,K^*\}$ output used in domestic sector k production.[4]

Minimizing the cost of home inputs, we obtain the following conditions:

$$Z^{\mathrm{H}}_{k,t}(i) = (1-\nu)\left(\frac{P^{\mathrm{H}}_{k,t}}{P_{k,t}}\right)^{-\rho} Z_{k,t}(i) \tag{19}$$

$$Z^{\mathrm{H}}_{k,\ell,t}(i) = \omega^{\mathrm{H}}_{k,\ell} D_{\ell,t}\left(\frac{P^{\mathrm{H}}_{\ell,t}}{P^{\mathrm{H}}_{k,t}}\right)^{-\eta} Z^{\mathrm{H}}_{k,t}(i) \tag{20}$$

$$Z^{\mathrm{H}}_{k,\ell,t}(i,j) = \frac{1}{n_\ell}\left(\frac{P^{\mathrm{H}}_{\ell,t}(j)}{P^{\mathrm{H}}_{\ell,t}}\right)^{-\theta} Z^{\mathrm{H}}_{k,\ell,t}(i). \tag{21}$$

Similarly, for foreign inputs the cost minimization conditions are as follows:

$$Z^{\mathrm{F}}_{k,t}(i) = \nu\left(\frac{P^{\mathrm{F}}_{k,t}}{P_{k,t}}\right)^{-\rho} Z_{k,t}(i) \tag{22}$$

$$Z^{\mathrm{F}}_{k,\ell,t}(i) = \omega^{\mathrm{F}}_{k,\ell} D^*_{\ell,t}\left(\frac{P^{\mathrm{F}}_{\ell,t}}{P^{\mathrm{F}}_{k,t}}\right)^{-\eta} Z^{\mathrm{F}}_{k,t}(i) \tag{23}$$

$$Z^{\mathrm{F}}_{k,\ell,t}(i,j) = \frac{1}{n^*_\ell}\left(\frac{P^{\mathrm{F}}_{\ell,t}(j)}{P^{\mathrm{F}}_{\ell,t}}\right)^{-\theta} Z^{\mathrm{F}}_{k,\ell,t}(i). \tag{24}$$

Combining these equations, we obtain the optimal demand equations for home and foreign inputs:

$$Z^{\mathrm{H}}_{k,\ell,t}(i,j) = \frac{(1-\nu)\omega^{\mathrm{H}}_{k,\ell} D_{\ell,t}}{n_\ell}\left(\frac{P^{\mathrm{H}}_{\ell,t}(j)}{P^{\mathrm{H}}_{\ell,t}}\right)^{-\theta}\left(\frac{P^{\mathrm{H}}_{\ell,t}}{P^{\mathrm{H}}_{k,t}}\right)^{-\eta}\left(\frac{P^{\mathrm{H}}_{k,t}}{P_{k,t}}\right)^{-\rho} Z_{k,t}(i) \tag{25}$$

$$Z^{\mathrm{F}}_{k,\ell,t}(i,j) = \frac{\nu\omega^{\mathrm{F}}_{k,\ell} D^*_{\ell,t}}{n^*_\ell}\left(\frac{P^{\mathrm{F}}_{\ell,t}(j)}{P^{\mathrm{F}}_{\ell,t}}\right)^{-\theta}\left(\frac{P^{\mathrm{F}}_{\ell,t}}{P^{\mathrm{F}}_{k,t}}\right)^{-\eta}\left(\frac{P^{\mathrm{F}}_{k,t}}{P_{k,t}}\right)^{-\rho} Z_{k,t}(i). \tag{26}$$

Prices are sticky à la Calvo (1983). Each period, a domestic firm in sector k can adjust its price with probability $1-\alpha_k$, and a foreign firm can adjust with probability $1-\alpha^*_k$. The

[4]While the theory allows for asymmetric definitions of sectors, in practice we set $K = K^*$.

optimization problem is then as follows:

$$\max_{P^{\mathrm{H}}_{k,t}(i),P^{\mathrm{H}*}_{k,t}(i)} \mathbb{E}_t \sum_{s=0}^{\infty} \alpha_k^s \Theta_{t,t+s} \Pi_{k,t+s}(i), \tag{27}$$

where the profit function is

$$\begin{aligned}\Pi_{k,t+s}(i) = {} & P^{\mathrm{H}}_{k,t}(i) \frac{D_{k,t+s}}{n_k} \left(\frac{P^{\mathrm{H}}_{k,t}(i)}{P^{\mathrm{H}}_{k,t+s}} \right)^{-\theta} \left[\left(\frac{P^{\mathrm{H}}_{k,t+s}}{P_{t+s}} \right)^{-\eta} \zeta_k C_{t+s} + \sum_{\ell=1}^{K} (1-\nu) Z_{t+s} \omega^{\mathrm{H}}_{\ell,k} \left(\frac{P^{\mathrm{H}}_{k,t+s}}{P^{\mathrm{H}}_{\ell,t+s}} \right)^{-\eta} \left(\frac{P^{\mathrm{H}}_{\ell,t+s}}{P_{\ell,t+s}} \right)^{-\rho} \right] \\ & + X_{t+s} P^{\mathrm{H}*}_{k,t}(i) \sum_{\ell=1}^{K^*} \nu Z^*_{t+s} \frac{\omega^{\mathrm{H}*}_{\ell,k} D_{k,t+s}}{n_k} \left(\frac{P^{\mathrm{H}*}_{k,t}(i)}{P^{\mathrm{H}*}_{k,t+s}} \right)^{-\theta} \left(\frac{P^{\mathrm{H}*}_{k,t+s}}{P^{\mathrm{H}*}_{\ell,t+s}} \right)^{-\eta} \left(\frac{P^{\mathrm{H}*}_{\ell,t+s}}{P_{\ell,t+s}} \right)^{-\rho} \\ & - W_{k,t+s} L_{k,t+s}(i) - P^{\mathrm{H}}_{k,t+s} Z^{\mathrm{H}}_{k,t+s}(i) - P^{\mathrm{F}}_{k,t+s} Z^{\mathrm{F}}_{k,t+s}(i). \end{aligned} \tag{28}$$

Solving for the optimal reset prices, $\hat{P}$, we obtain the following equations:

$$\hat{P}^{\mathrm{H}}_{k,t} = \frac{\theta}{\theta-1} \frac{\mathbb{E}_t \sum_{s=0}^{\infty} \Theta_{t,t+s} \alpha_k^s \Lambda^{\mathrm{H}}_{k,t+s} \mathcal{M}_{k,t+s}}{\mathbb{E}_t \sum_{s=0}^{\infty} \Theta_{t,t+s} \alpha_k^s \Lambda^{\mathrm{H}}_{k,t+s}} \tag{29}$$

$$\hat{P}^{\mathrm{H}*}_{k,t} = \frac{\theta}{\theta-1} \frac{\mathbb{E}_t \sum_{s=0}^{\infty} \Theta_{t,t+s} \alpha_k^s \Lambda^{\mathrm{H}*}_{k,t+s} \mathcal{M}_{k,t+s}}{\mathbb{E}_t \sum_{s=0}^{\infty} \Theta_{t,t+s} \alpha_k^s X_{t+s} \Lambda^{\mathrm{H}*}_{k,t+s}}, \tag{30}$$

where

$$\begin{aligned}\Lambda^{\mathrm{H}}_{k,t} &= \frac{D_{k,t}}{n_k} \left(\frac{1}{P^{\mathrm{H}}_{k,t}} \right)^{-\theta} \left[\left(\frac{P^{\mathrm{H}}_{k,t}}{P_t} \right)^{-\eta} \zeta_k C_t + \sum_{\ell=1}^{K} (1-v) Z_t \omega^{\mathrm{H}}_{\ell,k} \left(\frac{P^{\mathrm{H}}_{k,t}}{P^{\mathrm{H}}_{\ell,t}} \right)^{-\eta} \left(\frac{P^{\mathrm{H}}_{\ell,t}}{P_{\ell,t}} \right)^{-\rho} \right] \\ \Lambda^{\mathrm{H}*}_{k,t} &= \sum_{\ell=1}^{K^*} \nu Z^*_t \frac{\omega^{\mathrm{H}*}_{\ell,k} D_{k,t}}{n_k} \left(\frac{1}{P^{\mathrm{H}*}_{k,t}} \right)^{-\theta} \left(\frac{P^{\mathrm{H}*}_{k,t}}{P^{\mathrm{H}*}_{\ell,t}} \right)^{-\eta} \left(\frac{P^{\mathrm{H}*}_{\ell,t}}{P_{\ell,t}} \right)^{-\rho}, \end{aligned}$$

and the marginal cost is given by

$$\mathcal{M}_{k,t} = \left(\frac{1}{1-\delta} \right) \left(\frac{\delta}{1-\delta} \right)^{-\delta} \frac{1}{\tilde{A}_t \tilde{A}_{k,t} A_t A_{k,t}} (W_{k,t})^{1-\delta} (P_{k,t})^{\delta}.$$

The price indexes for inputs are given by

$$P_{k,t} = \left[(1-\nu)(P^{\mathrm{H}}_{k,t})^{1-\rho} + \nu (P^{\mathrm{F}}_{k,t})^{1-\rho} \right]^{\frac{1}{1-\rho}} \tag{31}$$

$$P^{\mathrm{H}}_{k,t} = \left[\sum_{\ell=1}^{K} \omega^{\mathrm{H}}_{k,\ell} (P^{\mathrm{H}}_{\ell,t})^{1-\eta} \right]^{\frac{1}{1-\eta}} \tag{32}$$

$$P^{\mathrm{F}}_{k,t} = \left[\sum_{\ell=1}^{K^*} \omega^{\mathrm{F}}_{k,\ell} (P^{\mathrm{F}}_{\ell,t})^{1-\eta} \right]^{\frac{1}{1-\eta}}. \tag{33}$$

The equations for foreign firms are symmetric.

2.3 Policy

Monetary policy in both countries is conducted via the following rule:

$$\frac{I_t}{\bar{I}} = \left(\frac{I_{t-1}}{\bar{I}}\right)^{\rho_i} \left[\left(\frac{P_t}{P_{t-1}}\right)^{\phi_\pi} \left(\frac{Y_t}{\bar{Y}}\right)^{\phi_y}\right]^{(1-\rho_i)} \exp(\mu_t), \tag{34}$$

where I_t is the gross nominal interest rate, and μ_t represents a monetary shock. Steady-state values are marked with a bar. Parameters ρ_i, ϕ_π, and ϕ_y govern interest-rate smoothing, the policy response to inflation, and the response to the output gap, respectively.

2.4 Equilibrium

All markets clear. The goods market equilibrium in Equation (35) requires that the total production by firm i be either consumed as a final good or used as an intermediate input by domestic or foreign producers.

$$\begin{aligned}
Y_{k,t}(i) &= C_{k,t}(i) + \sum_{\ell=1}^{K} \int Z^{\mathrm{H}}_{\ell,k,t}(j,i)\,\mathrm{d}j + \sum_{\ell=1}^{K^*} \int Z^{\mathrm{H}*}_{\ell,k,t}(j,i)\,\mathrm{d}j \\
&= \underbrace{\frac{\zeta_k D_{k,t}}{n_k}\left(\frac{P_{k,t}(i)}{P_{k,t}}\right)^{-\theta}\left(\frac{P_{k,t}}{P_t}\right)^{-\eta} C_t}_{\text{Demand by Local Consumers}} \\
&\quad + \underbrace{\sum_{\ell=1}^{K} \int (1-\nu)\frac{\omega^{\mathrm{H}}_{\ell,k} D_{k,t}}{n_k}\left(\frac{P^{\mathrm{H}}_{k,t}(i)}{P^{\mathrm{H}}_{k,t}}\right)^{-\theta}\left(\frac{P^{\mathrm{H}}_{k,t}}{P^{\mathrm{H}}_{\ell,t}}\right)^{-\eta}\left(\frac{P^{\mathrm{H}}_{\ell,t}}{P_{\ell,t}}\right)^{-\rho} Z_{\ell,t}(j)\,\mathrm{d}j}_{\text{Input Demand by Home Firms}} \\
&\quad + \underbrace{\sum_{\ell=1}^{K^*} \int \nu X_t \frac{\omega^{\mathrm{H}*}_{\ell,k} D_{k,t}}{n_k}\left(\frac{P^{\mathrm{H}*}_{k,t}(i)}{P^{\mathrm{H}*}_{k,t}}\right)^{-\theta}\left(\frac{P^{\mathrm{H}*}_{k,t}}{P^{\mathrm{H}*}_{\ell,t}}\right)^{-\eta}\left(\frac{P^{\mathrm{H}*}_{\ell,t}}{P_{\ell,t}}\right)^{-\rho} Z^*_{\ell,t}(j)\,\mathrm{d}j}_{\text{Input Demand by Foreign Firms}}
\end{aligned} \tag{35}$$

The labor market equilibrium in Equation (36) requires that the amount of labor supplied to each sector equal the total labor demand in that sector. Finally, the condition that bonds are in zero supply, Equation (37), pins down the equilibrium interest rate in the financial markets.

$$L_{k,t} = \int L_{k,t}(i)\,\mathrm{d}i \tag{36}$$

$$B_t = 0 \tag{37}$$

The equilibrium conditions for the foreign economy are symmetric. The model is solved by log-linearization around a deterministic zero-inflation steady state and standard perturbation methods.

2.5 Shock Processes

All shocks follow an AR(1) process in logarithms, with parameters ρ and σ, each of which is enhanced by a superscript that describes the shocked variable, denoting persistence and volatility, respectively. Innovations ϵ, with a corresponding superscript, are each drawn from a standard normal distribution.

$$\log \Gamma_{t+1} = \rho^{\gamma} \log \Gamma_t + \sigma^{\gamma} \epsilon^{\gamma}_{t+1} \tag{38}$$
$$\log \tilde{A}_{t+1} = \rho^{\tilde{a}} \log \tilde{A}_t + \sigma^{\tilde{a}} \epsilon^{\tilde{a}}_{t+1} \tag{39}$$
$$\log \tilde{A}_{k,t+1} = \rho^{\tilde{a}}_k \log \tilde{A}_{k,t} + \sigma^{\tilde{a}}_k \epsilon^{\tilde{a}}_{k,t+1} \tag{40}$$
$$\log A_{t+1} = \rho^{\mathrm{a}} \log A_t + \sigma^{\mathrm{a}} \epsilon^{\mathrm{a}}_{t+1} \tag{41}$$
$$\log \tilde{A}_{k,t+1} = \rho^{\mathrm{a}}_k \log A_{k,t} + \sigma^{\mathrm{a}}_k \epsilon^{\mathrm{a}}_{k,t+1} \tag{42}$$
$$\log D_{k,t+1} = \rho^{\mathrm{d}}_k \log D_{k,t} + \sigma^{\mathrm{d}}_k \epsilon^{\mathrm{d}}_{k,t+1} \tag{43}$$

All foreign shocks are symmetric.

3 Parameterization

The model is parameterized as follows. Aggregate parameters for the utility, production, and policy functions are calibrated at a quarterly frequency using values commonly employed in other studies. Table 1 summarizes these parameters.

Panel A shows parameters in the utility function. The discount factor, β, is set to produce a steady-state nominal interest rate of 4 percent annually. The household utility function is logarithmic in consumption ($\sigma = 1$). The inverse Frisch elasticity of labor supply, φ, and the elasticity of substitution across sectors, η, each equal 2. The elasticity of substitution within sectors, θ, is set to 6 in order to produce a steady-state markup of 20 percent.

Panel B of Table 1 summarize production-function parameters. The share of intermediate inputs (δ), which determines the importance of input-output linkages, is set to 0.7, as in Carvalho, Lee, and Park (2021). The aggregation of domestic and foreign goods follows Galí and Monacelli (2005). In particular, the Armington elasticity of substitution between domestic and foreign goods, ρ, is set to 1, and the share of foreign inputs in the intermediate inputs bundle, ν, equals 0.4. We set the value for the share of imports in the foreign economy, ν^*, to 0.15, because from the perspective of a small open economy, the rest of the world is a

Table 1: Model Calibration

Parameter (1)	Value (2)	Description (3)	Target/Source (4)
		A. Utility	
β	0.99	Household discount factor	Annual interest rate = 4%
σ	1	Relative risk aversion	Log utility
φ	2	Inverse Frisch elasticity	Carvalho, Lee, and Park (2021)
η	2	Elasticity of substitution across sectors	Hobijn and Nechio (2019)
θ	6	Elasticity of substitution within sectors	Markup = 20%
		B. Production and Trade	
δ	0.7	Share of intermediate inputs	Carvalho, Lee, and Park (2021)
ρ	1	Armington elasticity	Galí and Monacelli (2005)
ν	0.4	Domestic openness	Galí and Monacelli (2005)
ν^*	0.15	Foreign openness	US imports / GDP
Mean α_i	0.75	Average Calvo rate	Average price spell = 1 year
SD α_i	0.12	Variability of Calvo rates	Andrade and Sheremirov (2022)
		C. Monetary Policy	
ϕ_π	1.25	Response to inflation	Pasten, Schoenle, and Weber (2020)
ϕ_y	0.33/4	Response to output gap	Pasten, Schoenle, and Weber (2020)
ρ_i	0.7	Policy smoothing	Pasten, Schoenle, and Weber (2020)

relatively closed economy.[5] Finally, the average degree of price stickiness reflects empirical evidence that firms adjust their prices, on average, once a year. The sectoral dispersion of price stickiness is based on the estimates in Andrade and Sheremirov (2022).

The policy rule parameters are shown in Panel C. The interest rate smoothing is set to 0.7; the response to inflation is 1.25; and the response to the output gap at an annual rate is 0.33.[6] Unless stated otherwise, all parameters in the foreign economy are the same as in the domestic economy.

To understand the features of the model with respect to the input-output linkages, we explore alternative configurations of these parameters that are not based on any particular data source. We consider an economy with three sectors, taking a parsimonious approach to represent heterogeneity.[7] In the baseline, the sectors have equal size. We start with a benchmark case of a symmetric production network wherein all input-output linkages are

[5] Galí and Monacelli (2005) choose the value of ν to represent the import share in GDP of a representative small open economy such as Canada. Our choice of ν^* corresponds approximately to the US imports share in GDP.

[6] We adjust the parameters in Pasten, Schoenle, and Weber (2020), who calibrate their model at a monthly frequency, to a quarterly frequency. For instance, the policy rate smoothing in that paper is 0.9, whereas we set it to 0.7. Both values correspond to a half-life measure of persistence equaling approximately two quarters.

[7] The three sectors can be thought of as above average, average, and below average in some specific sense, such as price rigidity. Future work could consider a quantitative model with the three sectors representing durables, nondurables, and services as well as a disaggregated model with a large number of sectors.

equal to one another. This case corresponds to a roundabout production as in Basu (1995) and Carvalho, Lee, and Park (2021). The perfectly symmetric case is represented by the following input-output matrices:[8]

$$\Omega^{\mathrm{H}} = \Omega^{\mathrm{F}} = \Omega^{\mathrm{H}*} = \Omega^{\mathrm{F}*} = \begin{bmatrix} \frac{1}{6} & \frac{1}{6} & \frac{1}{6} \\ \frac{1}{6} & \frac{1}{6} & \frac{1}{6} \\ \frac{1}{6} & \frac{1}{6} & \frac{1}{6} \end{bmatrix}. \tag{44}$$

We then consider asymmetries in the network to analyze the role of network structure in the propagation of sectoral shocks. For instance, adopting the convention that the shocked sector is sector 1, in the benchmark asymmetric case, all imported inputs come from a shocked sector:

$$\Omega^{\mathrm{F}} = \begin{bmatrix} \frac{1}{2} & 0 & 0 \\ \frac{1}{2} & 0 & 0 \\ \frac{1}{2} & 0 & 0 \end{bmatrix}. \tag{45}$$

We also consider alternative asymmetries in the input-output network.

4 Sectoral Supply Shocks in the Open Economy

In this section, we examine the model's aggregate dynamics following sectoral productivity shocks. We distinguish between domestic, foreign, and global shocks. In Section 4.1, we study the model with perfectly symmetric production linkages, both within and between countries, as described in Equation (44). Here, we focus on the role of heterogeneity in price stickiness. In Section 4.2, we study the dynamics of the model with skewed input-output linkages, as in Equation (45), focusing on the foreign sector.

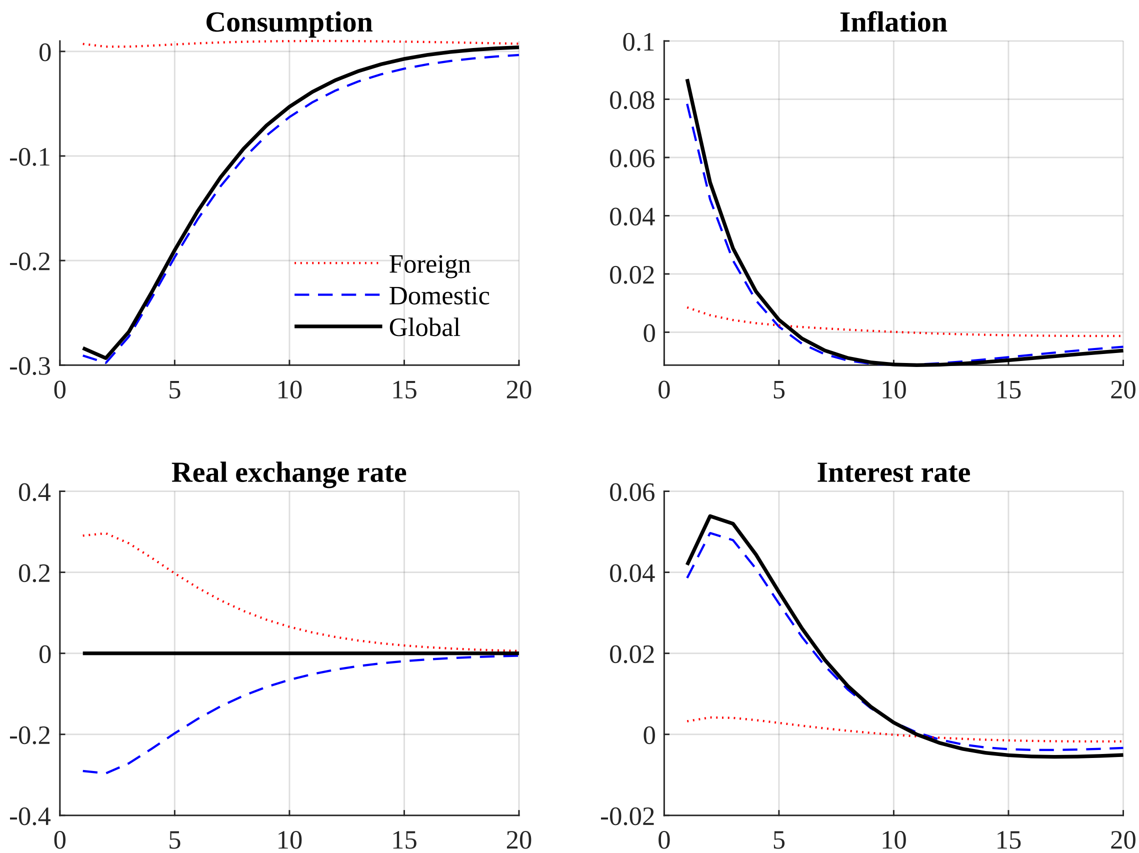
4.1 Symmetric Production Linkages

The four panels of Figure 1 show the impulse responses of consumption, inflation, the real exchange rate, and the nominal interest rate, respectively, to a contractionary domestic shock, a foreign shock, and a global sectoral productivity shock. Price rigidity is heterogeneous across sectors, with the shocked sector having average price rigidity. We start with the domestic shock (blue dashed lines).[9] Under the baseline calibration, a one standard deviation decline in domestic productivity leads, on impact, to a domestic consumption gap of 0.291 percent and domestic inflation that is 0.078 percentage point above its steady state (zero

[8] Here, matrix Ω^{H} collects individual linkages $\omega^{\mathrm{H}}_{k,\ell}$ and so on.

[9] For illustrative purposes, the persistence of each sectoral shock is set to 0.85, which corresponds to a half-life of about one year.

Figure 1: Aggregate Effects of Sectoral Shocks with Symmetric Production Networks



Notes: The figure shows responses to a one standard deviation contractionary sectoral productivity shock in the sector with average price rigidity. The input-output linkages are all equal to one another.
Source: All figures and tables in this paper are based on authors' calculations using a calibrated model.

inflation). At its peak response, two quarters out, the domestic central bank raises the nominal interest rate by a 0.050 percentage point. The real exchange rate appreciates by 0.290 percent in the home country.

The transmission mechanism of the domestic sectoral shock in the home country is standard. Consumption falls while inflation rises, which is typically the case for contractionary supply shocks. Because the central bank's policy rule is calibrated to be relatively more sensitive to inflation, monetary policy tightens. As a result, the domestic currency appreciates and the foreign currency depreciates. The role of the foreign sector is limited to dampening these effects quantitatively because domestic producers can substitute from domestic to foreign intermediate inputs, whose relative prices decline. This risk-sharing, however, is limited for two major reasons. First, domestic consumers derive utility from the final goods produced domestically; therefore, risk-sharing takes place through intermediate inputs only. Second, the bundle of intermediate inputs used in domestic production requires both domestic and foreign inputs, which are not perfectly substitutable. Hence, as the price of domestic inputs rises relative to the price of foreign inputs, domestic producers face an increase in marginal cost.

To understand the role of production networks and international risk-sharing in the model, we shut down the intermediate-inputs sector by setting $\delta = 0$. This results in an autarky, wherein foreign shocks have no effect on the domestic economy. Therefore, the effects of global shocks are the same as the effects of domestic shocks. Overall, the inflation responses are almost 70 percent larger than in the case of a domestic shock with production networks (Figure C.1 in the appendix). Accordingly, monetary-policy tightening is about 20 percent stronger at the peak and is substantially more persistent.[10]

For the same reasons that limit the dampening effects discussed earlier, the foreign sectoral productivity shocks have small effects on the domestic economy relative to the effects of domestic shocks of a similar magnitude (red dotted lines in Figure 1). Under the baseline calibration, foreign shocks have a somewhat larger effect on inflation than on consumption. But they are still too small for monetary policy to respond in a material way.

A notable and somewhat counterintuitive feature of the model is that a negative foreign sectoral productivity shock can have a positive effect on domestic consumption. This happens because an increase in the price of foreign inputs raises foreign demand for domestic inputs and encourages domestic firms to substitute from foreign to domestic inputs. As a result, domestic profits rise, stimulating domestic consumption of final goods. Whether domestic consumption increases or decreases in response to a foreign shock depends on the model's parameters. In the baseline, the increase in consumption stemming from a foreign shock is dwarfed by a decrease in consumption following a domestic shock of an equivalent magnitude. However, this small, positive consumption response to foreign shocks is robust across a variety of specifications considered. In Section 4.2.1, we show that domestic consumption declines in response to a negative foreign shock if the domestic economy is significantly more open to international trade than in the baseline.

A useful feature of our model is that it enables us to study global shocks. The main property of global shocks stems from the inability of domestic producers to substitute to or from foreign inputs, because the shock affects input prices in both countries simultaneously. Yet, under the baseline calibration, the foreign effects are so small that the effects of domestic and global shocks are similar to one another. For instance, the global shock leads to a 0.284 percent consumption gap and 0.087 percentage point increase in inflation on impact (black

[10] Although the responses of consumption appear to decline, they are not directly comparable. When $\delta = 0$, all output is consumed, whereas in the baseline, much of the output is used in production. When we consider the case of $\delta = 0.7$, $\Omega^{\mathrm{F}} = \Omega^{\mathrm{H}*} = \mathbf{0}_{3\times 3}$, with the elements of Ω^{H} and $\Omega^{\mathrm{F}*}$ all doubled, the consumption response increases at the peak by about one-third in absolute value. Note that this case is equivalent to a closed multisectoral economy with roundabout production.

solid line in Figure 1). That is, a global shock has an 11.5 percent larger effect on inflation and a 2.4 percent smaller effect on the consumption gap compared with the effects of a domestic shock only. Following the global shock, the nominal interest rate increases by 0.054 percentage point at the peak, an 8 percent larger response relative to the domestic shock. In the perfectly symmetric model, the global shock does not affect the real exchange rate or the terms of trade.

4.1.1 Heterogeneity in Price Rigidity

Next, we examine the role of heterogeneity in price rigidity. We consider the case in which the shocked sector has above-average price rigidity.[11] Overall, the responses of consumption, the real exchange rate, and the interest rate are similar to those in the baseline case (Figure 2). But the inflation response, on impact, is half of the baseline response. As expected, the characteristics of the shocked sector can play a role in the model's dynamics.

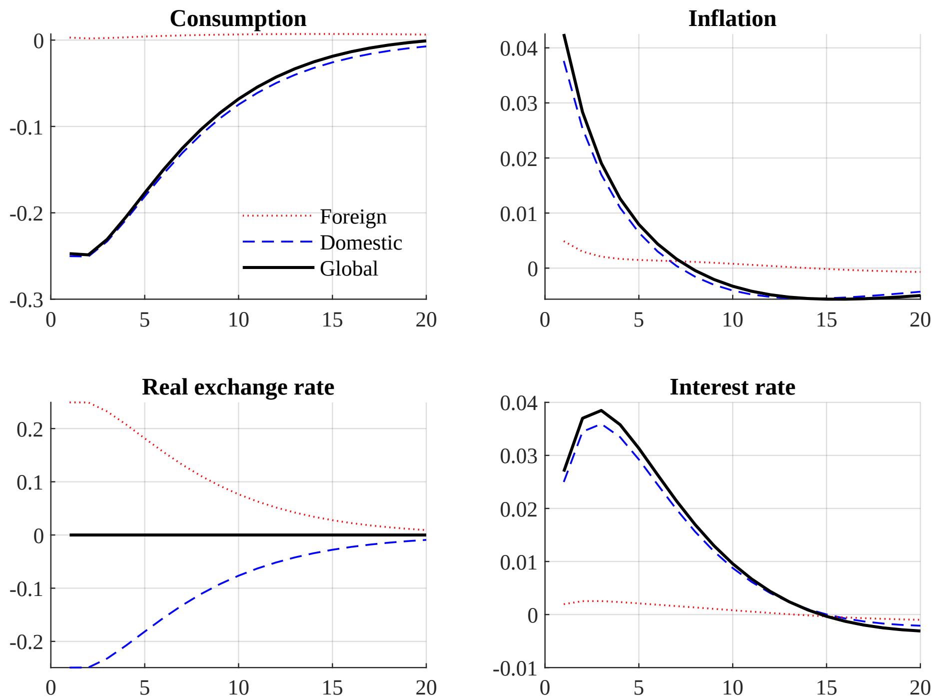
But overall heterogeneity in price stickiness plays only a minor role in this model. We provide additional evidence in the appendix. Figure C.3 considers the case in which price rigidity is homogeneous across sectors both in the domestic and foreign economies. The impulse responses are nearly indistinguishable from the baseline case for all four variables discussed. We also consider the case of asymmetries between countries. In Figure C.4, the domestic economy has heterogeneous price stickiness whereas the foreign economy does not. The shock is in the sector that is relatively rigid at home. In Figure C.5, the reverse is the case. That is, the domestic economy has homogeneous price stickiness, whereas the foreign economy features heterogeneous rigidity. In both cases, the heterogeneity has a very limited effect on the impulse responses.

4.2 Asymmetric Linkages

Figure 3 shows the dynamics of the model with asymmetric input-output linkages following sectoral shocks. As before, the sectors differ in price rigidity, and the shocked sector has the average price rigidity. The key difference from Figure 1 is that the shocked sector is solely responsible for all foreign inputs used in domestic production. The blue lines show that the responses to a domestic shock are nearly identical to those in the case of symmetric input-output linkages. This is expected because the asymmetries are introduced exclusively in the foreign sector.

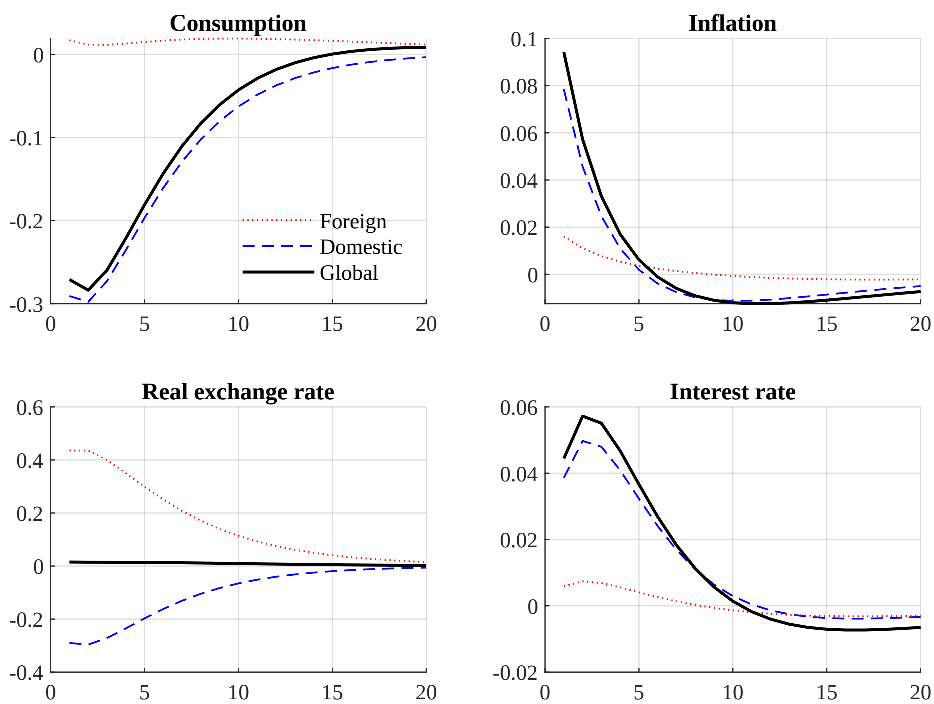
[11] Recall that such a sector is calibrated to have 17.5 percent higher price rigidity relative to the average sector. Appendix Figure C.2 shows a symmetric case wherein the shocked sector has 17.5 percent lower price rigidity.

Figure 2: Aggregate Effects of a Sectoral Shock to a Rigid Sector and Symmetric Linkages



Note: The shocked sector has above-average price rigidity.

Figure 3: Asymmetric Linkages: Shock to a High-Imports Sector and Average Price Rigidity



Notes: The shocked sector abroad supplies all foreign imports.

The responses to a foreign shock (red lines), however, become substantially larger in magnitude when asymmetries are introduced. For instance, the consumption gap is 0.017 percent and inflation rises by 0.016 percentage point on impact. Each response is about twice as large as in the preceding case. The real exchange rate depreciates 0.436 percent (a 50 percent increase), while the nominal interest rate increases 0.007 percentage point at its peak response (a 75 percent increase). Thus, the responses of all four variables in the asymmetric network are significantly larger than they are in the symmetric network.

We find significantly larger differences between the effects of a domestic shock and a global shock (black solid lines). For instance, the differential consumption response in the asymmetric model is 2.8 times the response in the symmetric model, while the corresponding differential for inflation is 1.9 times larger. We also compare the differential effects of global shocks and domestic shocks when the shocked sector has above-average price rigidity (Figure C.6 in the appendix).[12] In the model with asymmetric linkages, a shock to a high-rigidity sector reduces the consumption response 46.1 percent and reduces the inflation response 42.3 percent. The differential effect, however, is 3.6 times larger for consumption and 1.8 times larger for inflation than in the symmetric case with a shock to the rigid sector. Thus, asymmetries in the input-output structure of international trade play a far larger role in the propagation of foreign and global shocks compared with heterogeneity in price rigidity.[13]

4.2.1 Trade Openness

Next, we study how trade openness and substitutability between domestic inputs and foreign inputs affect the model's dynamics. We begin by considering the case where both countries have a relatively low degree of trade openness, $\nu = \nu^* = 0.15$. Here, we continue to maintain asymmetric production linkages.[14] This calibration may be appropriate for a large open economy, as opposed to a small open economy, which we consider in the baseline. With a small value of ν, the effects of foreign and global shocks on inflation are somewhat smaller than in the baseline case, but they are material nonetheless (Figure 4). The consumption responses are affected relatively less than inflation.

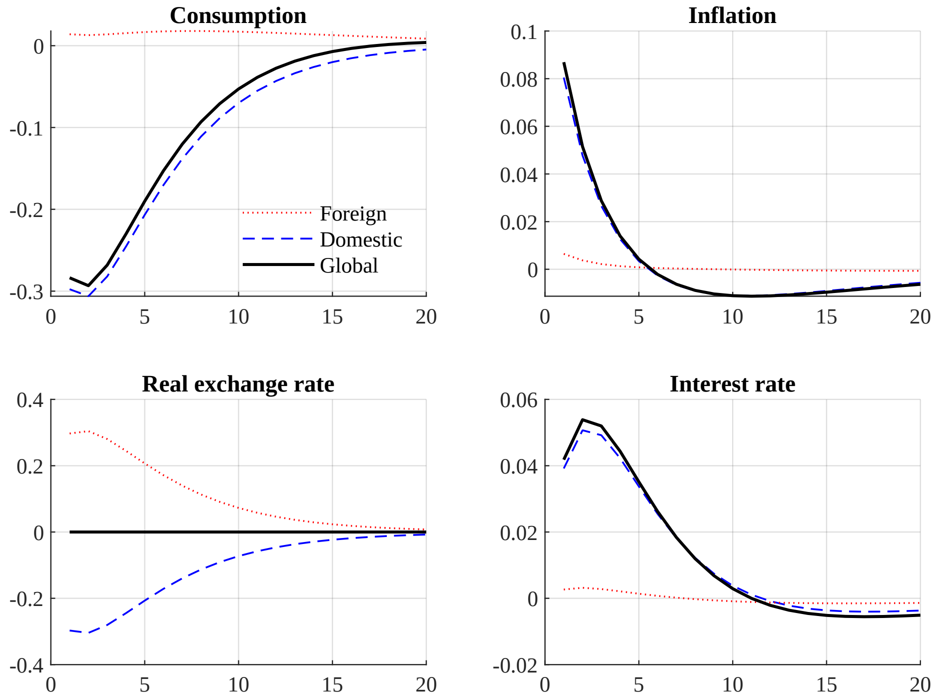
Alternatively, we set $\nu = 0.8$ and $\rho = 3$ to simulate a domestic economy that is very open and elastic with respect to foreign trade. We note that while we use a conservative parameterization of ρ in the baseline, the evidence on its value has not been settled. Feenstra et al. (2018), for example, contribute to the growing literature showing that for many

[12] Appendix Figure C.7 shows the impulse responses for the shock in a flexible price sector.

[13] We reach a similar conclusion when we examine the model with asymmetric linkages and no heterogeneity in price rigidity (Appendix Figure C.8).

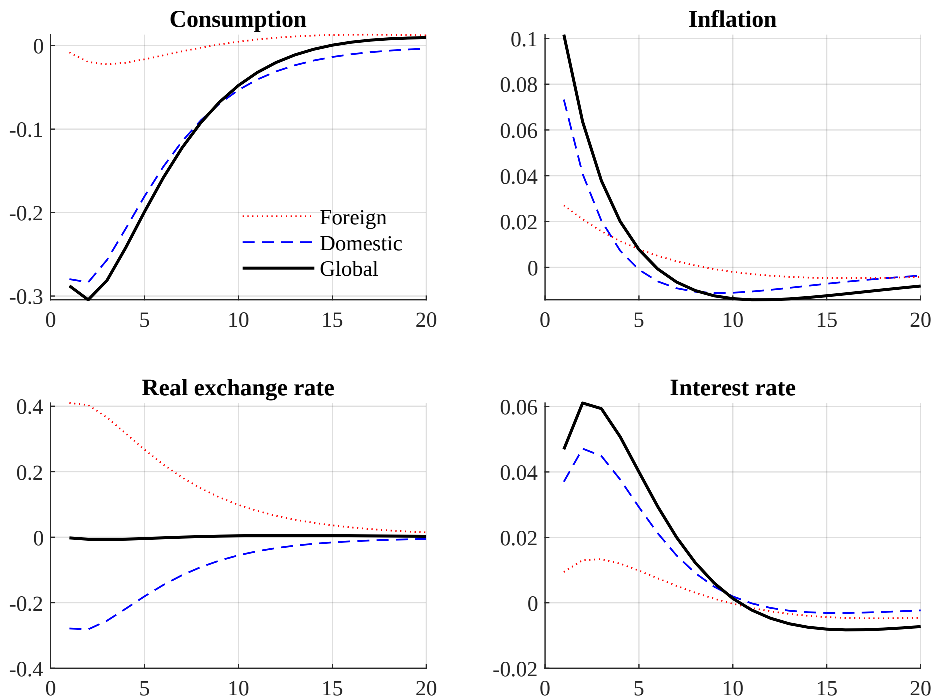
[14] Appendix Figure C.9 shows the impulse responses for the case of symmetric production linkages.

Figure 4: Aggregate Effects of a Sectoral Shock: Reduced Share of Imports



Notes: The import share parameter is reduced to $\nu = 0.15$. The production linkages are asymmetric.

Figure 5: Aggregate Effects of a Sectoral Shock: Increased Role of Imports



Notes: The import share parameter is increased to $\nu = 0.8$ and the Armington elasticity to $\rho = 3$. The linkages are asymmetric.

goods, the microelasticity of substitution between domestic goods and foreign goods can be substantially larger than the macroelasticity. Our alternative parameterization is in line with this evidence.

Figure 5 shows that with high values of ν and ρ as well as asymmetric input-output linkages, the differences between the effects of domestic shocks and global shocks on domestic inflation are especially large. In particular, the inflation response to a foreign sectoral productivity shock on impact is about twice the baseline response. In the appendix, we show that both parameters play an important role in generating this result (Figures C.11 and C.12). Moreover, in the model with an increased role of imports, negative foreign sectoral productivity shocks lead to a decline in domestic consumption. In this case, the share of foreign inputs in domestic production is so large that the substitution from foreign to domestic inputs is more than offset by the cost effects of foreign inputs' becoming relatively scarce.

Summary: This section's main results suggest that the effects of sectoral productivity shocks on inflation and consumption in an open-economy model with heterogeneous price stickiness and input-output linkages are nuanced. Whereas the literature shows that in the closed-economy case, price stickiness matters relatively more, in the open-economy case, the network structure and the degree of openness are relatively more important. In particular, highly open economies with asymmetric input-output foreign linkages are more susceptible to the effects of foreign sectoral shocks relative to open economies with asymmetric price rigidity but roundabout global production.[15]

5 Sensitivity to Consumer Preferences and Policy Designs

In this section, we examine the sensitivity of our baseline estimates to the model's parameters. We focus on the differential effects of a global sectoral supply shock relative to a domestic supply shock. The estimates are presented in Table 2. The table shows the responses to shocks on impact, which correspond to the peak inflation effects.[16] In columns (1) through (4), the input-output linkages are symmetric (roundabout production), while in columns (5) through (8), the linkages are skewed so that all foreign inputs are imported from the shocked sector. In all these cases, price rigidity is heterogeneous across sectors. In columns (1), (2), (5), and (6), we consider a shock to the sector with the average price rigidity, while in the

[15] We note that the effects of different model features are analyzed using stylized shocks. The impulse responses can be substantially larger in practice. For instance, Figures C.13 and C.14 in the appendix show much larger differentials when the persistence of sectoral shocks is increased to 0.95, a common value used to simulate aggregate productivity shocks.

[16] The peak consumption effects typically take place one quarter after impact, but the consumption responses on impact are comparable to those at the peak.

Table 2: The Differential Effects of Global and Domestic Sectoral Shocks

Parameters	Symmetric Linkages				Asymmetric Linkages			
	Average Rigidity		High Rigidity		Average Rigidity		High Rigidity	
	C	π	C	π	C	π	C	π
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Baseline	0.717	0.851	0.294	0.489	1.986	1.575	1.070	0.898
A. Preferences								
$\beta = 0.98$	0.732	0.861	0.321	0.503	2.010	1.594	1.118	0.923
$\sigma = 2$	0.839	0.935	0.528	0.575	1.778	1.677	1.043	0.920
$\varphi = 4$	0.243	0.959	0.040	0.607	0.996	1.675	0.511	1.004
$\eta = \theta = 6$	0.684	0.283	0.467	0.061	1.719	0.727	1.164	0.331
B. Policy								
$\phi_\pi = 2.5$	−1.189	0.773	−0.868	0.398	−1.273	1.377	−0.919	0.767
$\phi_y = 0.66/4$	0.690	0.798	0.005	0.412	2.457	1.578	1.162	0.911
$\rho_i = 0.85$	0.773	0.742	0.350	0.413	2.100	1.442	1.231	0.816
$\rho_i = 0.55$	0.644	0.924	0.233	0.541	1.854	1.663	0.934	0.946

Notes: This table shows the differential effects of a global sectoral supply shock relative to a domestic sectoral shock (times 100) on consumption and inflation. The model with symmetric linkages and average price rigidity (the first two columns) corresponds to the impulse responses shown in Figure 1. The model with a shock to the rigid sector (the next two columns) corresponds to Figure 2. In column (5) and (6), the input-output linkages are asymmetric and the shocked sector has the average price rigidity, as in Figure 3. The last two columns consider a shock to the high rigidity sector in the model with asymmetric production linkages.

rest of the table, the shocked sector features above-average price stickiness.

The first row recaps our previous discussions by showing the differences across the four models for the baseline values of preference and policy parameters. In Panel A, we examine the role of the preference parameters. The discount factor, β, and the intertemporal elasticity of substitution, $1/\sigma$, have a rather small effect on the responses. For instance, in the benchmark model with roundabout production and a shock to the average rigidity sector, reducing β from 0.99 in the baseline to 0.98, which corresponds to increasing the nominal interest rate from 4 percent to 8 percent in the steady state, raises the consumption differential 2.1 percent and the inflation differential only 1.2 percent. Varying σ from 1 in the baseline to 2 raises the consumption differential 17.0 percent and raises the inflation differential 9.9 percent. These numbers indicate a low sensitivity.

By contrast, the Frisch elasticity of labor supply, $1/\varphi$, is quantitatively important for the consumption differential but not for the inflation differential, while the opposite is true for the elasticity of substitution across sectors, η. Increasing φ from 2 in the baseline to 4 reduces the consumption differential 66.1 percent, and increasing η from 2 in the baseline to 6 lowers the inflation differential 66.7 percent.[17] These two parameters play a similarly important role when the input-output linkages are asymmetric and when the shocked sector has high price rigidity.

In Panel B of Table 2, we examine the model's sensitivity to policy-rule parameters. Notably, the inflation differentials are fairly stable across alternative policy rules. That is, while the differences in the rules can materially affect the inflation responses to domestic and foreign shocks, the differences between these effects are robust across policy designs. The consumption differential, however, is sensitive to how monetary policy responds to inflation. Increasing ϕ_π from 1.25 in the baseline to 2.5 reverts the sign and amplifies the difference between the consumption responses to domestic shocks and the consumption responses to global shocks in absolute value.

Crucially, across all the calibration strategies considered, our key result remains the same: Asymmetries in global production linkages play an important role in the propagation of foreign shocks and can substantially amplify the differential effects of global productivity shocks relative to domestic productivity shocks. While the role of heterogeneity in price rigidity is limited, the relative price stickiness of the shocked sector has a quantitative effect on both the inflation and consumption differentials across a wide spectrum of model parameters.

6 Conclusion

This paper develops an open-economy model that allows for heterogeneity in input-output linkages and the degree of price rigidity within and between countries. It documents that, in the model, asymmetries in global production linkages are key to the propagation of global sectoral shocks and to generating the differential effects of domestic shocks and global shocks on consumption and inflation. By contrast, heterogeneity in price rigidity is less important than the structure of the global supply chain. This result is robust to alternative designs of the monetary-policy rule.

This paper lays out a first but important step toward a larger-scale open-economy model that has a realistic global production network at its core. Accounting for international supply linkages is key to a better understanding of the economic effects of shocks to particular geographical and/or sectoral nodes of the global production network.

Future work can extend our analysis in several important directions. First, for nuanced policy analyses, a fully quantitative version of this model would be desirable. Hence, one could consider a model with more sectors for which production linkages and price rigidity are measured empirically. Allowing for more asymmetries across countries (for instance,

[17] In this case, we consider a triple baseline value so that $\eta = \theta$, which corresponds to a one-sector model wherein heterogeneity comes entirely from differing price rigidity across firms.

in preferences and policy designs) could also prove useful. Second, one could consider a medium-scale model that introduces capital, the public sector, and frictions in the foreign sector. The exchange-rate regime likely plays an important role in the cross-country transmission of sectoral shocks as well as aggregate shocks. The scope of our paper is limited to floating exchange rates and does not allow for capital controls. Third, future research could assess how optimal policy should respond to sectoral shocks. For instance, national governments can affect international input-output linkages by means of industrial policy. Open-economy models with production linkages may provide fertile ground for studying the welfare implications of such policies.

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Appendix

A Derivations

A.1 Solution to Firms' Static Optimization Problem

The first problem is optimal demand for home inputs across countries:

$$
\begin{aligned}
P_{k,t} Z_{k,t} - P^{\mathrm{H}}_{k,t} Z^{\mathrm{H}}_{k,t}(i) = P_{k,t} &\left[(1-\nu)^{1/\rho} Z^{\mathrm{H}}_{k,t}(i)^{(\rho-1)/\rho} + \nu^{1/\rho} Z^{\mathrm{F}}_{k,t}(i)^{(\rho-1)/\rho} \right]^{\rho/(\rho-1)} \\
&- P^{\mathrm{H}}_{k,t} Z^{\mathrm{H}}_{k,t}(i).
\end{aligned} \tag{A.1}
$$

It results in the optimal demand for home inputs:

$$
Z^{\mathrm{H}}_{k,t}(i) = (1-\nu) \left(\frac{P^{\mathrm{H}}_{k,t}}{P_{k,t}} \right)^{-\rho} Z_{k,t}(i). \tag{A.2}
$$

The cost minimization problem across home sectors is as follows:

$$
\begin{aligned}
P^{\mathrm{H}}_{k,t} Z^{\mathrm{H}}_{k,t}(i) - P^{\mathrm{H}}_{\ell,t} Z^{\mathrm{H}}_{k,\ell,t}(i) = P^{\mathrm{H}}_{k,t} &\left[\sum_{\ell=1}^{K} (\omega^{\mathrm{H}}_{k,\ell} D_{\ell,t})^{1/\eta} Z^{\mathrm{H}}_{k,\ell,t}(i)^{(\eta-1)/\eta} \right]^{\frac{\eta}{\eta-1}} \\
&- P^{\mathrm{H}}_{\ell,t} Z^{\mathrm{H}}_{k,\ell,t}(i).
\end{aligned} \tag{A.3}
$$

A.2 Firms' Dynamic Optimization Problem

Using $Z_t = \sum_{\ell=1}^{K} \int Z_{\ell,t}(j)\,\mathrm{d}j$ and $Z^*_t = \sum_{\ell=1}^{K^*} \int Z_{\ell,t}(j)\,\mathrm{d}j$, we obtain:

$$
\begin{aligned}
Y_{k,t}(i) &= C_{k,t}(i) + \sum_{\ell=1}^{K} \int Z^{\mathrm{H}}_{\ell,k,t}(j,i)\,\mathrm{d}j + \sum_{\ell=1}^{K^*} \int Z^{\mathrm{H}*}_{\ell,k,t}(j,i)\,\mathrm{d}j \\
&= \frac{\zeta_k D_{k,t}}{n_k} \left(\frac{P_{k,t}(i)}{P_{k,t}} \right)^{-\theta} \left(\frac{P_{k,t}}{P_t} \right)^{-\eta} C_t \\
&\quad + Z_t (1-\nu) \frac{\omega^{\mathrm{H}}_{\ell,k} D_{k,t}}{n_k} \left(\frac{P^{\mathrm{H}}_{k,t}(i)}{P^{\mathrm{H}}_{k,t}} \right)^{-\theta} \left(\frac{P^{\mathrm{H}}_{k,t}}{P^{\mathrm{H}}_{\ell,t}} \right)^{-\eta} \left(\frac{P^{\mathrm{H}}_{\ell,t}}{P_{\ell,t}} \right)^{-\rho} \\
&\quad + Z^*_t \nu X_t \frac{\omega^{\mathrm{H}*}_{\ell,k} D_{k,t}}{n_k} \left(\frac{P^{\mathrm{H}*}_{k,t}(i)}{P^{\mathrm{H}*}_{k,t}} \right)^{-\theta} \left(\frac{P^{\mathrm{H}*}_{k,t}}{P^{\mathrm{H}*}_{\ell,t}} \right)^{-\eta} \left(\frac{P^{\mathrm{H}*}_{\ell,t}}{P^{*}_{\ell,t}} \right)^{-\rho}.
\end{aligned} \tag{A.4}
$$

In equilibrium, $P_{k,t} = P^{\mathrm{H}}_{k,t}$. Thus,

$$
\begin{aligned}
Y_{k,t}(i) = {} & \frac{D_{k,t}}{n_k}\left(\frac{P^{\mathrm{H}}_{k,t}(i)}{P^{\mathrm{H}}_{k,t}}\right)^{-\theta}\left[\zeta_k C_t\left(\frac{P^{\mathrm{H}}_{k,t}}{P_t}\right)^{-\eta} + (1-\nu)Z_t\omega^{\mathrm{H}}_{\ell,k}\left(\frac{P^{\mathrm{H}}_{k,t}}{P^{\mathrm{H}}_{\ell,t}}\right)^{-\eta}\left(\frac{P^{\mathrm{H}}_{\ell,t}}{P_{\ell,t}}\right)^{-\rho}\right] \\
& + Z^*_t\nu X_t\frac{\omega^{\mathrm{H}*}_{\ell,k}D_{k,t}}{n_k}\left(\frac{P^{\mathrm{H}*}_{k,t}(i)}{P^{\mathrm{H}*}_{k,t}}\right)^{-\theta}\left(\frac{P^{\mathrm{H}*}_{k,t}}{P^{\mathrm{H}*}_{\ell,t}}\right)^{-\eta}\left(\frac{P^{\mathrm{H}*}_{\ell,t}}{P^*_{\ell,t}}\right)^{-\rho}.
\end{aligned}
\tag{A.5}
$$

Nominal profits and marginal cost for firm i, sector k, home country are as follows:

$$
\begin{aligned}
\Pi_{k,t}(i) &= P_{k,t}(i)Y_{k,t}(i) - W_{k,t}L_{k,t}(i) - P^{\mathrm{H}}_{k,t}Z^{\mathrm{H}}_{k,t}(i) - P^{\mathrm{F}}_{k,t}Z^{\mathrm{F}}_{k,t}(i) \\
&= P_{k,t}(i)\tilde{A}_t\tilde{A}_{k,t}A_tA_{k,t}L^{1-\delta}_{k,t}(i)\left[(1-\nu)^{1/\rho}Z^{\mathrm{H}}_{k,t}(i)^{(\rho-1)/\rho} + \nu^{1/\rho}Z^{\mathrm{F}}_{k,t}(i)^{(\rho-1)/\rho}\right]^{\delta\rho/(\rho-1)} \\
&\quad - W_{k,t}L_{k,t}(i) - P^{\mathrm{H}}_{k,t}Z^{\mathrm{H}}_{k,t}(i) - P^{\mathrm{F}}_{k,t}Z^{\mathrm{F}}_{k,t}(i).
\end{aligned}
\tag{A.6}
$$

Cost minimization gives the following conditions:

$$
Z^{\mathrm{H}}_{k,t}(i) = (1-\nu)\left[\frac{\delta}{1-\delta}\frac{W_{k,t}}{P^{\mathrm{H}}_{k,t}}L_{k,t}(i)\right]^{\rho}Z_{k,t}(i)^{1-\rho} \tag{A.7}
$$

$$
Z^{\mathrm{F}}_{k,t}(i) = \nu\left[\frac{\delta}{1-\delta}\frac{W_{k,t}}{P^{\mathrm{F}}_{k,t}}L_{k,t}(i)\right]^{\rho}Z_{k,t}(i)^{1-\rho}. \tag{A.8}
$$

Hence, the profits function is as follows:

$$
\begin{aligned}
\Pi_{k,t}(i) = {} & P_{k,t}(i)Y_{k,t}(i) - W_{k,t}L_{k,t}(i) \\
& - (1-\nu)P^{\mathrm{H}}_{k,t}\left[\frac{\delta}{1-\delta}\frac{W_{k,t}}{P^{\mathrm{h}}_{k,t}}L_{k,t}(i)\right]^{\rho}Z_{k,t}(i)^{1-\rho} \\
& - \nu P^{\mathrm{F}}_{k,t}\left[\frac{\delta}{1-\delta}\frac{W_{k,t}}{\xi_tP^{\mathrm{F}}_{k,t}}L_{k,t}(i)\right]^{\rho}Z_{k,t}(i)^{1-\rho}.
\end{aligned}
\tag{A.9}
$$

We obtain the following expression for inputs:

$$
Z_{k,t}(i) = \left[(1-\nu)\left(\frac{\delta}{1-\delta}\frac{W_{k,t}}{P^{\mathrm{H}}_{k,t}}L_{k,t}(i)\right)^{\rho-1} + \nu\left(\frac{\delta}{1-\delta}\frac{W_{k,t}}{P^{\mathrm{F}}_{k,t}}L_{k,t}(i)\right)^{\rho-1}\right]^{\frac{1}{\rho-1}}. \tag{A.10}
$$

With the production function, the expression for L is the following:

$$
\begin{aligned}
L_{k,t}(i) &= \left[\frac{Y_{k,t}(i)}{\tilde{A}_t\tilde{A}_{k,t}A_tA_{k,t}Z_{k,t}(i)^{\delta}}\right]^{\frac{1}{1-\delta}} \\
&= \left[\frac{Y_{k,t}(i)}{\tilde{A}_t\tilde{A}_{k,t}A_tA_{k,t}\left[(1-\nu)\left(\frac{\delta}{1-\delta}\frac{W_{k,t}}{P^{\mathrm{H}}_{k,t}}L_{k,t}(i)\right)^{\rho-1}+\nu\left(\frac{\delta}{1-\delta}\frac{W_{k,t}}{P^{\mathrm{F}}_{k,t}}L_{k,t}(i)\right)^{\rho-1}\right]^{\frac{\delta}{(\rho-1)}}}\right]^{\frac{1}{1-\delta}} \\
&= \left(\frac{1-\delta}{\delta}\right)^{\delta}\frac{Y_{k,t}(i)}{\tilde{A}_t\tilde{A}_{k,t}A_tA_{k,t}}\left[(1-\nu)\left(\frac{W_{k,t}}{P^{\mathrm{H}}_{k,t}}\right)^{\rho-1}+\nu\left(\frac{W_{k,t}}{P^{\mathrm{F}}_{k,t}}\right)^{\rho-1}\right]^{-\frac{\delta}{\rho-1}}.
\end{aligned}
\tag{A.11}
$$

Plugging expression (A.11) into Equation (A.10), we obtain:

$$
\begin{aligned}
Z_{k,t}(i) = \Bigg[&(1-\nu)\left(\frac{\delta}{1-\delta}\frac{W_{k,t}}{P^{\mathrm{H}}_{k,t}}\left(\frac{\delta}{1-\delta}\right)^{-\delta}\frac{Y_{k,t}(i)}{\tilde{A}_t\tilde{A}_{k,t}A_tA_{k,t}}\left[(1-\nu)\left(\frac{W_{k,t}}{P^{\mathrm{H}}_{k,t}}\right)^{\rho-1}+\nu\left(\frac{W_{k,t}}{P^{\mathrm{F}}_{k,t}}\right)^{\rho-1}\right]^{\frac{-\delta}{\rho-1}}\right)^{\rho-1} \\
&+\nu\left(\frac{\delta}{1-\delta}\frac{W_{k,t}}{P^{\mathrm{F}}_{k,t}}\left(\frac{\delta}{1-\delta}\right)^{-\delta}\frac{Y_{k,t}(i)}{\tilde{A}_t\tilde{A}_{k,t}A_tA_{k,t}}\left[(1-\nu)\left(\frac{W_{k,t}}{P^{\mathrm{H}}_{k,t}}\right)^{\rho-1}+\nu\left(\frac{W_{k,t}}{P^{\mathrm{F}}_{k,t}}\right)^{\rho-1}\right]^{\frac{-\delta}{\rho-1}}\right)^{\rho-1}\Bigg]^{\frac{1}{\rho-1}}.
\end{aligned}
\tag{A.12}
$$

Then, the final expression for inputs is the following:

$$
Z_{k,t}(i) = \left[\left(\frac{\delta}{1-\delta}\right)^{1-\delta}\frac{Y_{k,t}(i)}{\tilde{A}_t\tilde{A}_{k,t}A_tA_{k,t}}\right]\left[(1-\nu)\left(\frac{W_{k,t}}{P^{\mathrm{H}}_{k,t}}\right)^{\rho-1}+\nu\left(\frac{W_{k,t}}{P^{\mathrm{F}}_{k,t}}\right)^{\rho-1}\right]^{\frac{1-\delta}{\rho-1}}. \tag{A.13}
$$

Home country firm i, sector k profits are as follows:

$$
\begin{aligned}
\Pi_{k,t}(i) = P_{k,t}(i)Y_{k,t}(i) &- W_{k,t}\left(\frac{\delta}{1-\delta}\right)^{-\delta}\frac{Y_{k,t}(i)}{\tilde{A}_t\tilde{A}_{k,t}A_tA_{k,t}}\left[(1-\nu)\left(\frac{W_{k,t}}{P^{\mathrm{H}}_{k,t}}\right)^{\rho-1}+\nu\left(\frac{W_{k,t}}{P^{\mathrm{F}}_{k,t}}\right)^{\rho-1}\right]^{\frac{-\delta}{\rho-1}} \\
&- P_{k,t}\left[\left(\frac{\delta}{1-\delta}\right)^{1-\delta}\frac{Y_{k,t}(i)}{\tilde{A}_t\tilde{A}_{k,t}A_tA_{k,t}}\right]\left[(1-\nu)\left(\frac{W_{k,t}}{P^{\mathrm{H}}_{k,t}}\right)^{\rho-1}+\nu\left(\frac{W_{k,t}}{P^{\mathrm{F}}_{k,t}}\right)^{\rho-1}\right]^{\frac{1-\delta}{\rho-1}}.
\end{aligned}
\tag{A.14}
$$

Dividing through by $P_{k,t}$ and using the expression that

$$
\frac{W_t}{P_{k,t}} = \left[(1-\nu)\left(\frac{W_{k,t}}{P^{\mathrm{H}}_{k,t}}\right)^{\rho-1}+\nu\left(\frac{W_{k,t}}{P^{\mathrm{F}}_{k,t}}\right)^{\rho-1}\right]^{\frac{1}{\rho-1}},
$$

we obtain:

$$
\begin{aligned}
\Pi_{k,t}(i) &= \frac{P_{k,t}(i)}{P_{k,t}}Y_{k,t}(i) - \left(\frac{\delta}{1-\delta}\right)^{-\delta}\frac{Y_{k,t}(i)}{\tilde{A}_t\tilde{A}_{k,t}A_tA_{k,t}}\left[(1-\nu)\left(\frac{W_{k,t}}{P^{\mathrm{H}}_{k,t}}\right)^{\rho-1}+\nu\left(\frac{W_{k,t}}{P^{\mathrm{F}}_{k,t}}\right)^{\rho-1}\right]^{\frac{1-\delta}{\rho-1}} \\
&\quad - \left[\left(\frac{\delta}{1-\delta}\right)^{1-\delta}\frac{Y_{k,t}(i)}{\tilde{A}_t\tilde{A}_{k,t}A_tA_{k,t}}\right]\left[(1-\nu)\left(\frac{W_{k,t}}{P^{\mathrm{H}}_{k,t}}\right)^{\rho-1}+\nu\left(\frac{W_{k,t}}{P^{\mathrm{H}}_{k,t}}\right)^{\rho-1}\right]^{\frac{1-\delta}{\rho-1}} \\
&= \frac{P_{k,t}(i)}{P_{k,t}}Y_{k,t}(i) - \left(\frac{\delta}{1-\delta}\right)^{-\delta}\frac{Y_{k,t}(i)}{\tilde{A}_t\tilde{A}_{k,t}A_tA_{k,t}}\left[(1-\nu)\left(\frac{W_{k,t}}{P^{\mathrm{H}}_{k,t}}\right)^{\rho-1}+\nu\left(\frac{W_{k,t}}{P^{\mathrm{H}}_{k,t}}\right)^{\rho-1}\right]^{\frac{1-\delta}{\rho-1}}\left[1+\left(\frac{\delta}{1-\delta}\right)\right] \\
&= \frac{P_{k,t}(i)}{P_{k,t}}Y_{k,t}(i) - \left(\frac{1}{1-\delta}\right)\left(\frac{\delta}{1-\delta}\right)^{-\delta}\frac{Y_{k,t}(i)}{\tilde{A}_t\tilde{A}_{k,t}A_tA_{k,t}}\left[(1-\nu)\left(\frac{W_{k,t}}{P^{\mathrm{H}}_{k,t}}\right)^{\rho-1}+\nu\left(\frac{W_{k,t}}{P^{\mathrm{F}}_{k,t}}\right)^{\rho-1}\right]^{\frac{1-\delta}{\rho-1}}.
\end{aligned}
\tag{A.15}
$$

The expression for marginal cost is as follows:

$$
\begin{aligned}
\mathcal{M}_{k,t} &= \left(\frac{1}{1-\delta}\right)\left(\frac{\delta}{1-\delta}\right)^{-\delta}\frac{1}{\tilde{A}_t\tilde{A}_{k,t}A_tA_{k,t}}\left[(1-\nu)\left(\frac{W_{k,t}}{P^{\mathrm{H}}_{k,t}}\right)^{\rho-1}+\nu\left(\frac{W_{k,t}}{X_tP^{\mathrm{H}*}_{k,t}}\right)^{\rho-1}\right]^{\frac{1-\delta}{\rho-1}}P_{k,t} \\
&= \left(\frac{1}{1-\delta}\right)\left(\frac{\delta}{1-\delta}\right)^{-\delta}\frac{1}{\tilde{A}_t\tilde{A}_{k,t}A_tA_{k,t}}W_{k,t}^{1-\delta}(P_{k,t})^{\delta}.
\end{aligned}
\tag{A.16}
$$

A.3 Flexible-Price Case

Firm i in sector k solves the following problem:

$$
\max_{P^{\mathrm{H}}_{k,t}(i),P^{\mathrm{H}*}_{,k,t}(i)} \mathbb{E}_t\left[P_{k,t}(i)Y_{k,t}(i)-\mathcal{M}_{k,t}Y_{k,t}(i)\right]. \tag{A.17}
$$

Here, the total output of firm i in sector k, per Equation (A.5), can be separated into the home demand and foreign demand. That is, firms can choose the respective prices at home and abroad (pricing to market). The equilibrium conditions are given as follows:

$$
\begin{aligned}
&(\theta-1)\frac{D_{k,t}}{n_k}\left(\frac{P^{\mathrm{H}}_{k,t}(i)}{P^{\mathrm{H}}_{k,t}}\right)^{-\theta}\left[\left(\frac{P^{\mathrm{H}}_{k,t}}{P_t}\right)^{-\eta}\zeta_kC_t+Z_t(1-\nu)\omega^{\mathrm{H}}_{\ell,k}\left(\frac{P^{\mathrm{H}}_{k,t}}{P^{\mathrm{H}}_{\ell,t}}\right)^{-\eta}\left(\frac{P^{\mathrm{H}}_{\ell,t}}{P_{\ell,t}}\right)^{-\rho}\right] \\
&= \mathcal{M}_{k,t}\theta\frac{D_{k,t}}{n_k}\left(\frac{P^{\mathrm{H}}_{k,t}(i)}{P^{\mathrm{H}}_{k,t}}\right)^{-\theta-1}\left[\left(\frac{P^{\mathrm{H}}_{k,t}}{P_t}\right)^{-\eta}\zeta_kC_t+Z_t(1-\nu)\omega^{\mathrm{H}}_{\ell,k}\left(\frac{P^{\mathrm{H}}_{k,t}}{P^{\mathrm{H}}_{\ell,t}}\right)^{-\eta}\left(\frac{P^{\mathrm{H}}_{\ell,t}}{P_{\ell,t}}\right)^{-\rho}\right]
\end{aligned}
\tag{A.18}
$$

and

$$
\begin{aligned}
&(\theta-1)X_tZ^*_t\nu\frac{\omega^{\mathrm{H}*}_{\ell,k}D_{k,t}}{n_k}\left(\frac{P^{\mathrm{H}*}_{k,t}(i)}{P^{\mathrm{H}*}_{k,t}}\right)^{-\theta}\left(\frac{P^{\mathrm{H}*}_{k,t}}{P^{\mathrm{H}*}_{\ell,t}}\right)^{-\eta}\left(\frac{P^{\mathrm{H}*}_{\ell,t}}{P^*_{\ell,t}}\right)^{-\rho} \\
&= \mathcal{M}_{k,t}\theta Z^*_t\nu\frac{\omega^{\mathrm{H}*}_{\ell,k}D_{k,t}}{n_k}\left(\frac{P^{\mathrm{H}*}_{k,t}(i)}{P^{\mathrm{H}*}_{k,t}}\right)^{-\theta-1}\left(\frac{P^{\mathrm{H}*}_{k,t}}{P^{\mathrm{H}*}_{\ell,t}}\right)^{-\eta}\left(\frac{P^{\mathrm{H}*}_{\ell,t}}{P^*_{\ell,t}}\right)^{-\rho}.
\end{aligned}
\tag{A.19}
$$

Thus, the first-order conditions are as follows:

$$
\frac{P^{\mathrm{H}}_{k,t}(i)}{P^{\mathrm{H}}_{k,t}} = \frac{\theta}{\theta-1}\mathcal{M}_{k,t} \tag{A.20}
$$

$$
\frac{P^{\mathrm{H}*}_{k,t}(i)}{P^{\mathrm{H}*}_{k,t}} = \frac{\theta}{\theta-1}\frac{\mathcal{M}_{k,t}}{X_t}. \tag{A.21}
$$

A.4 Sticky-Price Case

The profit function is given:

$$
\begin{aligned}
\Pi_{k,t}(i) = {} & P^{\mathrm{H}}_{k,t}(i)\frac{D_{k,t}}{n_k}\left(\frac{P^{\mathrm{H}}_{k,t}(i)}{P^{\mathrm{H}}_{k,t}}\right)^{-\theta}\left[\left(\frac{P^{\mathrm{H}}_{k,t}}{P_t}\right)^{-\eta}\zeta_k C_t + (1-\nu)Z_t\omega^{\mathrm{H}}_{\ell,k}\left(\frac{P^{\mathrm{H}}_{k,t}}{P^{\mathrm{H}}_{\ell,t}}\right)^{-\eta}\left(\frac{P^{\mathrm{H}}_{\ell,t}}{P_{\ell,t}}\right)^{-\rho}\right] \\
& + X_t P^{\mathrm{H}*}_{k,t}(i)\nu Z^*_t\frac{\omega^{\mathrm{H}*}_{\ell,k}D_{k,t}}{n_k}\left(\frac{P^{\mathrm{H}*}_{k,t}(i)}{P^{\mathrm{H}*}_{k,t}}\right)^{-\theta}\left(\frac{P^{\mathrm{H}*}_{k,t}}{P^{\mathrm{H}*}_{\ell,t}}\right)^{-\eta}\left(\frac{P^{\mathrm{H}*}_{\ell,t}}{P^*_{\ell,t}}\right)^{-\rho} \\
& - W_{k,t}L_{k,t}(i) - P^{\mathrm{H}}_{k,t}Z^{\mathrm{H}}_{k,t}(i) - P^{\mathrm{F}}_{k,t}Z^{\mathrm{F}}_{k,t}(i) \\
= {} & \frac{D_{k,t}}{n_k}\left(P^{\mathrm{H}}_{k,t}(i)\right)^{1-\theta}\left(\frac{1}{P^{\mathrm{H}}_{k,t}}\right)^{-\theta}\left[\left(\frac{P^{\mathrm{H}}_{k,t}}{P_t}\right)^{-\eta}\zeta_k C_t + (1-\nu)Z_t\omega^{\mathrm{H}}_{\ell,k}\left(\frac{P^{\mathrm{H}}_{k,t}}{P^{\mathrm{H}}_{\ell,t}}\right)^{-\eta}\left(\frac{P^{\mathrm{H}}_{\ell,t}}{P_{\ell,t}}\right)^{-\rho}\right] \\
& + X_t\nu Z^*_t\frac{\omega^{\mathrm{H}*}_{\ell,k}D_{k,t}}{n_k}\left(P^{\mathrm{H}*}_{k,t}(i)\right)^{1-\theta}\left(\frac{1}{P^{\mathrm{H}*}_{k,t}}\right)^{-\theta}\left(\frac{P^{\mathrm{H}*}_{k,t}}{P^{\mathrm{H}*}_{\ell,t}}\right)^{-\eta}\left(\frac{P^{\mathrm{H}*}_{\ell,t}}{P^*_{\ell,t}}\right)^{-\rho} \\
& - \mathcal{M}_{k,t}\frac{D_{k,t}}{n_k}\left(P^{\mathrm{H}}_{k,t}(i)\right)^{-\theta}\left(\frac{1}{P^{\mathrm{H}}_{k,t}}\right)^{-\theta}\left[\left(\frac{P^{\mathrm{H}}_{k,t}}{P_t}\right)^{-\eta}\zeta_k C_t + (1-\nu)Z_t\omega^{\mathrm{H}}_{\ell,k}\left(\frac{P^{\mathrm{H}}_{k,t}}{P^{\mathrm{H}}_{\ell,t}}\right)^{-\eta}\left(\frac{P^{\mathrm{H}}_{\ell,t}}{P_{\ell,t}}\right)^{-\rho}\right] \\
& - \mathcal{M}_{k,t}\nu Z^*_t\frac{\omega^{\mathrm{H}*}_{\ell,k}D_{k,t}}{n_k}\left(P^{\mathrm{H}*}_{k,t}(i)\right)^{-\theta}\left(\frac{1}{P^{\mathrm{H}*}_{k,t}}\right)^{-\theta}\left(\frac{P^{\mathrm{H}*}_{k,t}}{P^{\mathrm{H}*}_{\ell,t}}\right)^{-\eta}\left(\frac{P^{\mathrm{H}*}_{\ell,t}}{P^*_{\ell,t}}\right)^{-\rho}.
\end{aligned}
\tag{A.22}
$$

Taking the first-order condition with respect to $P^{\mathrm{H}}_{k,t}(i)$ and $P^{\mathrm{H}*}_{k,t}(i)$ and using hats to denote the optimal reset prices, we obtain the following equations:

$$
\begin{aligned}
0 = {} & \mathbb{E}_t\sum_{s=0}^{\infty}\alpha_k^s\Theta_{t,t+s}\frac{D_{k,t+s}}{n_k} \\
& \times\left(\frac{\hat{P}^{\mathrm{H}}_{k,t}}{P^{\mathrm{H}}_{k,t+s}}\right)^{-\theta}\left[\left(\frac{P^{\mathrm{H}}_{k,t+s}}{P_{t+s}}\right)^{-\eta}\zeta_k C_{t+s} + (1-\nu)Z_{t+s}\omega_{\ell,k}\left(\frac{P^{\mathrm{H}}_{k,t+s}}{P^{\mathrm{H}}_{\ell,t+s}}\right)^{-\eta}\left(\frac{P^{\mathrm{H}}_{\ell,t+s}}{P_{\ell,t+s}}\right)^{-\rho}\right] \\
& \times\left[\hat{P}^{\mathrm{H}}_{k,t} - \frac{\theta}{\theta-1}\mathcal{M}_{k,t+s}\right]
\end{aligned}
\tag{A.23}
$$

and

$$
\begin{aligned}
0 = \mathbb{E}_t\sum_{s=0}^{\infty}\alpha_k^s\Theta_{t,t+s}\nu Z^*_{t+s}\frac{\omega^{\mathrm{H}*}_{\ell,k}D^*_{k,t+s}}{n_k}&\left(\frac{\hat{P}^{\mathrm{H}*}_{k,t}}{P^{\mathrm{H}*}_{k,t+s}}\right)^{-\theta}\left(\frac{P^{\mathrm{H}*}_{k,t+s}}{P^{\mathrm{H}*}_{\ell,t+s}}\right)^{-\eta}\left(\frac{P^{\mathrm{H}*}_{\ell,t+s}}{P^*_{\ell,t+s}}\right)^{-\rho} \\
\times\left[\hat{P}^{\mathrm{H}*}_{k,t} - \frac{\theta}{\theta-1}\frac{\mathcal{M}_{k,t+s}}{X_t}\right]&.
\end{aligned}
\tag{A.24}
$$

Finally, we can write these conditions as follows:

$$
\hat{P}^{\mathrm{H}}_{k,t} = \frac{\theta}{\theta-1}\frac{\mathbb{E}_t\sum_{s=0}^{\infty}\Theta_{t,t+s}\alpha_k^s\Lambda^{\mathrm{H}}_{k,t+s}\mathcal{M}_{k,t+s}}{\mathbb{E}_t\sum_{s=0}^{\infty}\Theta_{t,t+s}\alpha_k^s\Lambda^{\mathrm{H}}_{k,t+s}} \tag{A.25}
$$

$$
\hat{P}^{\mathrm{H}*}_{k,t} = \frac{\theta}{\theta-1}\frac{\mathbb{E}_t\sum_{s=0}^{\infty}\Theta_{t,t+s}\alpha_k^s\Lambda^{\mathrm{H}*}_{k,t+s}\mathcal{M}_{k,t+s}}{\mathbb{E}_t\sum_{s=0}^{\infty}\Theta_{t,t+s}\alpha_k^s X_{t+s}\Lambda^{\mathrm{H}*}_{k,t+s}}, \tag{A.26}
$$

where

$$\Lambda^{\mathrm{H}}_{k,t} = \frac{D_{k,t}}{n_k}\left(\frac{1}{P^{\mathrm{H}}_{k,t}}\right)^{-\theta}\left[\left(\frac{P^{\mathrm{H}}_{k,t}}{P_t}\right)^{-\eta}\zeta_k C_t + (1-\nu)Z_t\omega^{\mathrm{H}}_{\ell,k}\left(\frac{P^{\mathrm{H}}_{k,t}}{P^{\mathrm{H}}_{\ell,t}}\right)^{-\eta}\left(\frac{P^{\mathrm{H}}_{\ell,t}}{P_{\ell,t}}\right)^{-\rho}\right]$$

$$\Lambda^{\mathrm{H}*}_{k,t} = \nu Z^*_t\frac{\omega^{\mathrm{H}*}_{\ell,k}D_{k,t}}{n_k}\left(P^{\mathrm{H}*}_{k,t}(i)\right)^{-\theta}\left(\frac{1}{P^{\mathrm{H}*}_{k,t}}\right)^{-\theta}\left(\frac{P^{\mathrm{H}*}_{k,t}}{P^{\mathrm{H}*}_{\ell,t}}\right)^{-\eta}\left(\frac{P^{\mathrm{H}*}_{\ell,t}}{P^*_{\ell,t}}\right)^{-\rho}.$$

A.5 Steady State

- Prices:

$$P = P^{\mathrm{H}} = P^{\mathrm{H}}_k \tag{A.27}$$

$$P^* = P^{\mathrm{F}} = P^{\mathrm{F}}_k \tag{A.28}$$

$$\begin{aligned} P_k &= \left[(1-\nu)(P^{\mathrm{H}})^{1-\rho} + \nu(P^{\mathrm{F}})^{1-\rho}\right]^{\frac{1}{1-\rho}} \\ &= \left[(1-\nu)P^{1-\rho} + \nu(XP^*)^{1-\rho}\right]^{\frac{1}{1-\rho}} \end{aligned} \tag{A.29}$$

- Household Budget Constraint:

$$C = \frac{WL}{P} + \frac{\Pi}{P} \tag{A.30}$$

- Firms' Profits:

$$\frac{\Pi}{P} = Y - \frac{W}{P}L - Z \tag{A.31}$$

- Production Function:

$$Y = L^{1-\delta}Z^{\delta} \tag{A.32}$$

- Optimal Home Input:

$$Z^{\mathrm{H}} = Z^{1-\rho}(1-\nu)\left[\frac{\delta}{1-\delta}\frac{W}{P}L\right]^{\rho} \tag{A.33}$$

- Optimal Foreign Input:

$$Z^{\mathrm{F}} = Z^{1-\rho}\nu\left[\frac{\delta}{1-\delta}\frac{W}{XP^*}L\right]^{\rho} \tag{A.34}$$

$$\frac{Z_H}{Z_F} = \frac{1-\nu}{\nu}\left(\frac{XP^*}{P}\right)^{\rho} \tag{A.35}$$

- Labor Supply:

$$\frac{W}{P} = L^{\varphi}C^{\sigma} \tag{A.36}$$

- Market Clearing:

$$Y = C + Z \tag{A.37}$$

- Aggregate Input:[18]

$$\begin{aligned} Z &= \left[(1-\nu)^{\frac{1}{\rho}}(Z^{\mathrm{H}})^{\frac{\rho-1}{\rho}} + \nu^{\frac{1}{\rho}}(Z^{\mathrm{F}})^{\frac{\rho-1}{\rho}}\right]^{\frac{\rho}{\rho-1}} = \left[(1-\nu)^{\frac{1}{\rho}}(Z^{\mathrm{H}})^{\frac{\rho-1}{\rho}} + \nu\left(\frac{Z^{\mathrm{H}}}{1-\nu}\right)^{\frac{\rho-1}{\rho}}\left(\frac{P}{XP^*}\right)^{\rho}\right]^{\frac{\rho}{\rho-1}} \\ &= \left(\frac{\delta}{1-\delta}\right)^{1-\delta} Y\left[(1-\nu)\left(\frac{W}{P}\right)^{\rho-1} + \nu\left(\frac{W}{XP^*}\right)^{\rho-1}\right]^{\frac{1-\delta}{\rho-1}} = \frac{\delta L}{1-\delta}\frac{W}{P_k} \end{aligned} \tag{A.38}$$

- Optimal Reset Prices:

$$\begin{aligned} 1 &= \frac{\theta}{1-\theta}\left(\frac{1}{1-\delta}\right)\left(\frac{\delta}{1-\delta}\right)^{-\delta}\left[\left(\frac{W}{P}\right)^{\rho-1} + \left(\frac{W}{XP^*}\right)^{\rho-1}\right]^{\frac{1-\delta}{\rho-1}} \\ &= \frac{\theta}{1-\theta}\left(\frac{1}{1-\delta}\right)\left(\frac{\delta}{1-\delta}\right)^{-\delta}\left(\frac{W}{P_k}\right)^{1-\delta} \end{aligned} \tag{A.39}$$

From the last equation, solve for the real wage:

$$\frac{W}{P_k} = \left(\frac{\theta}{1-\theta}\frac{1}{1-\delta}\left(\frac{\delta}{1-\delta}\right)^{-\delta}\right)^{\frac{1}{1-\delta}}. \tag{A.40}$$

Now, use the aggregate input expression in the profits and production function equation:

$$Y = L\left(\frac{\delta}{1-\delta}\right)^{\delta}\left(\frac{W}{P_k}\right)^{\delta} \tag{A.41}$$

$$\begin{aligned} \frac{\Pi}{P} &= Y - \frac{W}{P}L - Z \\ &= Y - \frac{W}{P}L - \frac{\delta L}{1-\delta}\frac{W}{P_k}. \end{aligned} \tag{A.42}$$

Using the output expression in the expression for output and profits, we get an expression for labor:

$$\begin{aligned} L &= \left(\frac{\delta}{1-\delta}\right)^{-\delta} Y\left[(1-\nu)\left(\frac{W}{P}\right)^{\rho-1} + \nu\left(\frac{W}{XP^*}\right)^{\rho-1}\right]^{\frac{-\delta}{\rho-1}} \\ &= \left(\frac{\delta}{1-\delta}\right)^{-\delta} Y\left(\frac{W}{P_k}\right)^{-\delta}. \end{aligned} \tag{A.43}$$

In the steady state, $\Gamma = \tilde{A} = \tilde{A}_k = A = A_k = D_k = 1$.

[18]Note that $\frac{W}{P_k} = \left[(1-\nu)\left(\frac{W}{P}\right)^{\rho-1} + \nu\left(\frac{W}{XP^*}\right)^{\rho-1}\right]^{\frac{1}{\rho-1}}$.

A.6 Inflation Rates

We log-linearize the model around the symmetric steady state and denote the log-deviations of aggregate variables from their steady states with lowercase letters (for example, the log-deviation of the price level P_t is denoted by p_t).

Consumer price inflation is as follows:

$$\pi_t \equiv p_t - p_{t-1}. \tag{A.44}$$

Under the law of one price, the sectoral price index is given:

$$\begin{aligned} p_{k,t} &\approx (1-\nu)p^{\mathrm{H}}_{k,t} + \nu p^{\mathrm{F}}_{k,t} \\ &\approx p^{\mathrm{H}}_{k,t} + \nu s_{k,t}, \end{aligned} \tag{A.45}$$

where $s_{k,t}$ is the input terms of trade.

Domestic inputs inflation is defined as $\pi^{\mathrm{H}}_{k,t} \equiv p^{\mathrm{H}}_{k,t} - p^{\mathrm{H}}_{k,t-1}$. As consumers derive utility only from local goods, $\pi_t = \pi^{\mathrm{H}}_{k,t}$:

$$\begin{aligned} \pi_{k,t} &= \pi^{\mathrm{H}}_{k,t} + \nu \Delta s_{k,t} \\ &= \pi_t + \nu \Delta s_{k,t}. \end{aligned} \tag{A.46}$$

Log-linearizing the foreign input price $P^{\mathrm{F}}_t = X_t P^*_t$ around a symmetric steady state gives us:

$$p^{\mathrm{F}}_t = x_t + p^*_t. \tag{A.47}$$

Thus, the input terms of trade can be expressed as follows:

$$s_{k,t} = x_t + p^*_t - p^{\mathrm{H}}_{k,t}. \tag{A.48}$$

A.7 Derivation of the Phillips Curve

The sectoral price levels follow a Calvo process:

$$p^{\mathrm{H}}_{k,t} = (1-\alpha_k)\hat{p}^{\mathrm{H}}_{k,t} + \alpha_k p^{\mathrm{H}}_{k,t-1} \tag{A.49}$$

$$p^{\mathrm{H}*}_{k,t} = (1-\alpha_k)\hat{p}^{\mathrm{H}*}_{k,t} + \alpha_k p^{\mathrm{H}*}_{k,t-1}. \tag{A.50}$$

Sectoral Phillips curves are standard:

$$\pi^{\mathrm{H}}_{k,t} = \beta\,\mathbb{E}_t\,\pi^{\mathrm{H}}_{k,t+1} + \frac{(1-\alpha_k)(1-\alpha_k\beta)}{\alpha_k}(m_{k,t} - p^{\mathrm{H}}_{k,t}) \tag{A.51}$$

$$\pi^{\mathrm{H}*}_{k,t} = \beta\,\mathbb{E}_t\,\pi^{\mathrm{H}*}_{k,t+1} + \frac{(1-\alpha_k)(1-\alpha_k\beta)}{\alpha_k}(m_{k,t} - p^{\mathrm{H}*}_{k,t}). \tag{A.52}$$

Optimal labor supply and cost minimization produce the following condition:

$$\begin{aligned}\frac{1}{n_k}\int z_{k,t}(i)\,\mathrm{d}i &= w_{k,t} + l_{k,t} - (1-\nu)p^{\mathrm{H}}_{k,t} - \nu p^{\mathrm{F}}_{k,t} \\ &= (1+\varphi)l_{k,t} + c_t + p_t - (1-\nu)p^{\mathrm{H}}_{k,t} - \nu p^{\mathrm{F}}_{k,t}.\end{aligned} \tag{A.53}$$

Next, we integrate both sides of the production function:

$$\begin{aligned}y_{k,t} &= \tilde{a}_t + \tilde{a}_{k,t} + a_t + a_{k,t} + (1-\delta)l_{k,t} + \delta\frac{1}{n_k}\int z_{k,t}(i)\,\mathrm{d}i \\ &= \tilde{a}_t + \tilde{a}_{k,t} + a_t + a_{k,t} + (1+\varphi\delta)l_{k,t} + \delta c_t + \delta p_t - \delta(1-\nu)p^{\mathrm{H}}_{k,t} - \delta\nu p^{\mathrm{F}}_{k,t}.\end{aligned} \tag{A.54}$$

Then, we solve for labor:

$$\begin{aligned}l_{k,t} &= \frac{1}{1+\varphi\delta}\Big(y_{k,t} - \tilde{a}_t - \tilde{a}_{k,t} - a_t - a_{k,t}\Big) - \frac{\delta}{1+\varphi\delta}\Big(p_t + c_t - (1-\nu)p^{\mathrm{H}}_{k,t} - \nu p^{\mathrm{F}}_{k,t}\Big) \\ &= \frac{1}{1+\varphi\delta}\Big(y_{k,t} - \tilde{a}_t - \tilde{a}_{k,t} - a_t - a_{k,t}\Big) - \frac{\delta}{1+\varphi\delta}\Big(c_t + p_t - p_{k,t}\Big).\end{aligned} \tag{A.55}$$

With the expression for marginal cost, we obtain:

$$\begin{aligned}m_{k,t} &= -\tilde{a}_t - \tilde{a}_{k,t} - a_t - a_{k,t} + (1-\delta)w_t - (1-\delta)(p^{\mathrm{H}}_{k,t} + p^{\mathrm{F}}_{k,t}) + p_{k,t} \\ &= -\tilde{a}_t - \tilde{a}_{k,t} - a_t - a_{k,t} + (1-\delta)w_t + \delta p_{k,t} \\ &= -\tilde{a}_t - \tilde{a}_{k,t} - a_t - a_{k,t} + (1-\delta)w_t + \delta((1-\nu)p^{\mathrm{H}}_{k,t} + \nu p^{\mathrm{F}}_{k,t})\end{aligned} \tag{A.56}$$

and

$$\begin{aligned}m_{k,t} &= (1-\delta)(\varphi l_{k,t} + c_t + p_t) - \tilde{a}_t - \tilde{a}_{k,t} - a_t - a_{k,t} + \delta p_{k,t} \\ &= \frac{(1-\delta)\varphi}{1+\varphi\delta}y_{k,t} - \frac{1+\varphi}{1+\delta\varphi}a_t - \frac{1+\varphi}{1+\delta\varphi}\tilde{a}_t - \frac{1+\varphi}{1+\delta\varphi}a_{k,t} - \frac{1+\varphi}{1+\delta\varphi}\tilde{a}_{k,t} \\ &\quad + \frac{1-\delta}{1+\delta\varphi}c_t + \frac{1-\delta}{1+\varphi\delta}p_t + \frac{\delta(1+\varphi)}{1+\varphi\delta}p_{k,t}.\end{aligned} \tag{A.57}$$

It follows from the consumer optimal demand that the following condition holds:

$$p_t = p^{\mathrm{H}}_{k,t} - \frac{c_t - c_{k,t} + d_{k,t}}{\eta}. \tag{A.58}$$

Hence,

$$\begin{aligned}\pi^{\mathrm{H}}_{k,t} = \beta\, \mathbb{E}_t\, \pi^{\mathrm{H}}_{k,t+1} &+ \frac{(1-\alpha_k)(1-\alpha_k\beta)}{\alpha_k} \\ &\times \Bigg\{ \frac{(1-\delta)\varphi}{1+\varphi\delta} y_{k,t} - \frac{1+\varphi}{1+\delta\varphi} a_t - \frac{1+\varphi}{1+\delta\varphi}\tilde{a}_t - \frac{1+\varphi}{1+\delta\varphi} a_{k,t} - \frac{1+\varphi}{1+\delta\varphi}\tilde{a}_{k,t} \\ &\quad + \frac{(1-\delta)(\eta-1)}{\eta(1+\delta\varphi)} c_t + \frac{1-\delta}{1+\varphi\delta}\left[\frac{c_{k,t}-d_{k,t}}{\eta}\right] + \frac{\delta(1+\varphi)}{1+\varphi\delta}\left(p_{k,t} - p^{\mathrm{H}}_{k,t}\right)\Bigg\}.\end{aligned} \tag{A.59}$$

Using the fact that

$$\begin{aligned} p^{\mathrm{H}}_{k,t} &= \frac{z^{\mathrm{H}}_{k,t}(i) + d_{k,t} - z^{\mathrm{H}}_{\ell,k,t}(i)}{\eta} + p^{\mathrm{I,H}}_{k,t} \\ &= \frac{-\rho(p^{\mathrm{I,H}}_{k,t} - p^{\mathrm{I}}_{k,t}) + z_{k,t}(i) + d_{k,t} - z^{\mathrm{H}}_{\ell,k,t}(i)}{\eta} + p^{\mathrm{I,H}}_{k,t}, \end{aligned} \tag{A.60}$$

we can write:

$$\begin{aligned} p^{\mathrm{I}}_{k,t} - p^{\mathrm{H}}_{k,t} &= p^{\mathrm{I}}_{k,t} + \frac{\rho(p^{\mathrm{I,H}}_{k,t} - p^{\mathrm{I}}_{k,t})}{\eta} - \frac{d_{k,t}}{\eta} + \frac{z^{\mathrm{H}}_{\ell,k,t}(i) - z_{k,t}(i)}{\eta} - p^{\mathrm{I,H}}_{k,t} \\ &= \frac{\eta-\rho}{\eta}(p^{\mathrm{I}}_{k,t} - p^{\mathrm{I,H}}_{k,t}) - \frac{d_{k,t}}{\eta} + \frac{z^{\mathrm{H}}_{\ell,k,t}(i) - z_{k,t}(i)}{\eta} \\ &= \frac{\eta-\rho}{\eta}\left(\frac{z^{\mathrm{H}}_{k,t}(i) - z_{k,t}(i)}{\rho}\right) - \frac{d_{k,t}}{\eta} + \frac{z^{\mathrm{H}}_{\ell,k,t}(i) - z_{k,t}(i)}{\eta}. \end{aligned} \tag{A.61}$$

Firms' output is given:

$$y_{k,t}(i) = (1-\psi)\zeta_k c_{k,t}(i) + \psi\left[\sum_{\ell=1}^{K} \omega^{\mathrm{H}}_{\ell,k} \int z^{\mathrm{H}}_{\ell,k,t}(j,i)\,\mathrm{d}j + \sum_{\ell=1}^{K^*} \omega^{\mathrm{H}*}_{\ell,k} \int z^{\mathrm{H}*}_{\ell,k,t}(j,i)\,\mathrm{d}j\right]. \tag{A.62}$$

A.8 Aggregates and Market Clearing Conditions

- Input Terms of Trade: $S_{k,t} = \frac{P^{\mathrm{H}}_{k,t}}{P^{\mathrm{F}}_{k,t}}$
- Law of One Price: $P^{\mathrm{F}}_t = X_t P^*_t$
- Input Real Exchange Rate: $Q_{k,t} = X_t \frac{P_{k,t}}{P^*_{k,t}}$

- Sectoral Output: $y_{k,t} = \frac{1}{n_k} \int y_{k,t}(i) \, \mathrm{d}i$
- Intersectoral Demand for Intermediate Inputs: $z_{\ell,k,t} = \frac{1}{n_k n_\ell} \int \int z_{k,\ell,t}(i,j) \, \mathrm{d}i \, \mathrm{d}j$
- Aggregate Output: $y_t = \sum_k n_k y_{k,t}$
- Aggregate Consumption: $c_t = \sum_k \zeta_k c_{k,t}$
- CPI:

$$p_t = \sum_{k=1}^{K} \zeta_k p_{k,t}^{\mathrm{H}} \tag{A.63}$$

- Sectoral Price Level, Domestic:

$$p_{k,t}^{\mathrm{H}} = \frac{1}{n_k} \int p_{k,t}^{\mathrm{H}}(i) \, \mathrm{d}i \tag{A.64}$$

- Sectoral Price Level, Foreign:

$$p_{k,t}^{\mathrm{F}} = \frac{1}{n_k^*} \int p_{k,t}^{\mathrm{F}}(j) \, \mathrm{d}j \tag{A.65}$$

- Aggregate Sectoral Price Level:

$$p_{k,t}(i) = \frac{1}{n_k} \int p_{k,t}^{\mathrm{H}}(i) \, \mathrm{d}i + \frac{1}{n_k^*} \int p_{k,t}^{\mathrm{F}}(j) \, \mathrm{d}j \tag{A.66}$$

- Aggregate Home Consumption:

$$c_t = \sum_{k=1}^{K} \zeta_k c_{k,t} \tag{A.67}$$

- Sectoral Consumption:

$$c_{k,t} = \frac{1}{n_k} \int c_{k,t}(i) \, \mathrm{d}i \tag{A.68}$$

- Inputs Used by Firm i, k:

$$z_{k,t}(i) = \sum_{\ell=1}^{K} \omega_{k,\ell}^{\mathrm{H}} z_{k,\ell,t}^{\mathrm{H}}(i) + \sum_{\ell=1}^{K^*} \omega_{k,\ell}^{\mathrm{F}} z_{k,\ell,t}^{\mathrm{F}}(i) \tag{A.69}$$

- Aggregate Intermediate Inputs:

$$z_t = \sum_{\ell=1}^{K} \int z_{\ell,t}^{\mathrm{H}}(j) \, \mathrm{d}j + \sum_{\ell=1}^{K^*} \int z_{\ell,t}^{\mathrm{F}}(j) \, \mathrm{d}j \tag{A.70}$$

- Sectoral Labor Input:

$$l_{k,t} = \frac{1}{n_k} \int l_{k,t}(i)\,\mathrm{d}i \tag{A.71}$$

- Aggregate Home Hours:

$$l_t = \sum_{k=1}^{K} n_k l_{k,t} \tag{A.72}$$

- Aggregate Home Wage:

$$w_t = \sum_{k=1}^{K} n_k w_{k,t} \tag{A.73}$$

- Firm i, k Total Output:

$$\begin{aligned} y_{k,t}(i) = {} & (1-\psi)\zeta_k c_{k,t} \\ & + \psi \Big[\sum_{\ell=1}^{K} \omega^{\mathrm{H}}_{\ell,k} \int (1-\nu) z^{\mathrm{H}}_{\ell,k,t}(j,i)\,\mathrm{d}j + \sum_{\ell=1}^{K^*} \omega^{\mathrm{H}*}_{\ell,k} \int \nu z^{\mathrm{H}*}_{\ell,k,t}(j,i)\,\mathrm{d}j \Big] \end{aligned} \tag{A.74}$$

- Sectoral Output:

$$\begin{aligned} n_{k,t} y_{k,t} = {} & (1-\psi)\zeta_k c_{k,t} \\ & + \psi \Big[\sum_{\ell=1}^{K} n_\ell \omega^{\mathrm{H}}_{\ell,k} (1-\nu) z^{\mathrm{H}}_{\ell,k,t} + \sum_{\ell=1}^{K^*} n^*_\ell \omega^{\mathrm{H}*}_{\ell,k} \nu z^{\mathrm{H}*}_{\ell,k,t} \Big] \end{aligned} \tag{A.75}$$

- Aggregate Gross Output:

$$y_t = (1-\psi)c_t + \psi z_t \tag{A.76}$$

A.9 Log-linear Equilibrium Conditions

$$c_{k,t} - c_t = -\eta(p^{\mathrm{H}}_{k,t} - p_t) + d_{k,t} \tag{A.77}$$

$$c_{k,t}(i) - c_{k,t} = -\theta(p^{\mathrm{H}}_{k,t}(i) - p^{\mathrm{H}}_{k,t}) \tag{A.78}$$

$$z^{\mathrm{H}}_{k,t}(i) = -\rho(p^{\mathrm{H}}_{k,t} - p_{k,t}) + z_{k,t}(i) \tag{A.79}$$

$$z^{\mathrm{H}}_{k,\ell,t}(i) = -\eta(p^{\mathrm{H}}_{\ell,t} - p^{\mathrm{H}}_{k,t}) + z^{\mathrm{H}}_{k,t}(i) + d_{\ell,t} \tag{A.80}$$

$$z^{\mathrm{H}}_{k,\ell,t}(i,j) = -\theta(p^{\mathrm{H}}_{\ell,t}(j) - p^{\mathrm{H}}_{\ell,t}) + z^{\mathrm{H}}_{k,\ell,t}(i) \tag{A.81}$$

$$z^{\mathrm{F}}_{k,t}(i) = -\rho(p^{\mathrm{F}}_{k,t} - p_{k,t}) + z_{k,t}(i) \tag{A.82}$$

$$z^{\mathrm{F}}_{k,\ell,t}(i) = -\eta(p^{\mathrm{F}}_{\ell,t} - p^{\mathrm{F}}_{k,t}) + z^{\mathrm{F}}_{k,t}(i) + d^*_{\ell,t} \tag{A.83}$$

$$z^{\mathrm{F}}_{k,\ell,t}(i,j) = -\theta(p^{\mathrm{F}}_{\ell,t}(j) - p^{\mathrm{F}}_{\ell,t}) + z^{\mathrm{F}}_{k,\ell,t}(i) \tag{A.84}$$

$$\begin{aligned} y_{k,t}(i) = {} & (1-\psi)\zeta_k c_{k,t} \\ & + \psi\Big[\sum_{\ell=1}^{K} \omega^{\mathrm{H}}_{\ell,k} \int z^{\mathrm{H}}_{\ell,k,t}(j,i)\,\mathrm{d}j + \sum_{\ell=1}^{K^*} \omega^{\mathrm{H}*}_{\ell,k} \int z^{\mathrm{H}*}_{\ell,k,t}(j,i)\,\mathrm{d}j\Big], \end{aligned} \tag{A.85}$$

where $\psi = \delta\frac{\theta}{1-\theta}$.

Euler Equation:

$$c_t = \mathbb{E}_t\, c_{t+1} - \frac{1}{\sigma}\Big[i_t - \big(\mathbb{E}_t(p_{t+1}) - p_t\big)\Big] + \frac{1}{\sigma}(\gamma_t - \mathbb{E}_t\,\gamma_{t+1}) \tag{A.86}$$

$$w_{k,t} - p_t = \varphi l_{k,t} + \sigma c_t \tag{A.87}$$

$$y_{k,t}(i) = \tilde{a}_t + \tilde{a}_{k,t} + a_t + a_{k,t} + (1-\delta)l_{k,t}(i) + \delta z_{k,t}(i) \tag{A.88}$$

Log-linearization of the Minimization/ Efficiency Condition:

$$z_{k,t}(i) = w_{k,t} + l_{k,t}(i) - (1-\nu)p^{\mathrm{H}}_{k,t} - \nu p^{\mathrm{F}}_{k,t} \tag{A.89}$$

Log-linearization of Marginal Cost:

$$\mathcal{M}_{k,t} = \frac{1}{1-\delta}\left(\frac{\delta}{1-\delta}\right)^{-\delta} \frac{1}{\tilde{A}_t\tilde{A}_{k,t}A_tA_{k,t}}\left[(1-\nu)\left(\frac{W_{k,t}}{P^{\mathrm{H}}_{k,t}}\right)^{\rho-1} + \nu\left(\frac{W_{k,t}}{P^{\mathrm{F}}_{k,t}}\right)^{\rho-1}\right]^{\frac{1-\delta}{\rho-1}} \tag{A.90}$$

$$\mathcal{M}_{k,t}^{\frac{1}{1-\delta}} = \left(\frac{1}{1-\delta}\Big(\frac{\delta}{1-\delta}\Big)^{-\delta} \frac{1}{\tilde{A}_t\tilde{A}_{k,t}A_tA_{k,t}}\right)^{\frac{1}{1-\delta}}\left[(1-\nu)\left(\frac{W_{k,t}}{P^{\mathrm{H}}_{k,t}}\right)^{\rho-1} + \nu\left(\frac{W_{k,t}}{P^{\mathrm{F}}_{k,t}}\right)^{\rho-1}\right]^{\frac{1}{\rho-1}} \tag{A.91}$$

$$\mathcal{M}_{k,t}^{\frac{1}{1-\delta}} = \left(\frac{1}{1-\delta}\Big(\frac{\delta}{1-\delta}\Big)^{-\delta} \frac{1}{\tilde{A}_t\tilde{A}_{k,t}A_tA_{k,t}}\right)^{\frac{1}{1-\delta}} W_{k,t}\left[(1-\nu)\left(P^{\mathrm{H}}_{k,t}\right)^{1-\rho} + \nu\left(P^{\mathrm{F}}_{k,t}\right)^{1-\rho}\right]^{\frac{1}{\rho-1}} \tag{A.92}$$

$$\mathcal{M}_{k,t}^{\frac{-1}{1-\delta}} = \left(\frac{1}{1-\delta}\Big(\frac{\delta}{1-\delta}\Big)^{-\delta} \frac{1}{\tilde{A}_t\tilde{A}_{k,t}A_tA_{k,t}}\right)^{\frac{-1}{1-\delta}} W_{k,t}^{-1}\left[(1-\nu)\left(P^{\mathrm{H}}_{k,t}\right)^{1-\rho} + \nu\left(P^{\mathrm{F}}_{k,t}\right)^{1-\rho}\right]^{\frac{1}{1-\rho}} \tag{A.93}$$

Now, we log-linearize around a symmetric steady state and add prices:

$$m_{k,t} = -\tilde{a}_t - \tilde{a}_{k,t} - a_t - a_{k,t} + (1-\delta)w_t - (1-\delta)\Big((1-\nu)p^{\mathrm{H}}_{k,t} + \nu p^{\mathrm{F}}_{k,t}\Big) + p_{k,t}, \tag{A.94}$$

or

$$m_{k,t} = -\tilde{a}_t - \tilde{a}_{k,t} - a_t - a_{k,t} + (1-\delta)w_t + \delta p_{k,t}, \tag{A.95}$$

where

$$p_{k,t} = (1-\nu)p^{\mathrm{H}}_{k,t} + \nu p^{\mathrm{F}}_{k,t}. \tag{A.96}$$

Optimal Sectoral Prices:

$$\hat{p}^{\mathrm{H}}_{k,t} = (1-\alpha_k\beta)m_{k,t} + \alpha_k\beta\,\mathbb{E}_t[\hat{p}^{\mathrm{H}}_{k,t-1}] \tag{A.97}$$

$$p^{\mathrm{H}}_{k,t} = (1-\alpha_k)\hat{p}^{\mathrm{H}}_{k,t} + \alpha_k p^{\mathrm{H}}_{k,t-1} \tag{A.98}$$

A.10 Log-linear System

1) IS Curve:

$$c_t = E_t c_{t+1} - \frac{1}{\sigma}\left(i_t - E_t\pi_{t+1} + \gamma_t - \mathbb{E}_t\,\gamma_{t+1}\right) \tag{A.99}$$

2) Aggregate Labor Supply:

$$w_t - p_t = \varphi l_t + \sigma c_t \tag{A.100}$$

3) Aggregate Resource Constraint:

$$(1-\psi)c_t + \psi z_t = \tilde{a}_t + a_t + \sum_{k=1}^{\max_{K,K^*}} \tilde{a}_{k,t} + \sum_{k=1}^{K} a_{k,t} + (1-\delta)l_t + \delta z_t \tag{A.101}$$

4) Cost Minimization:

$$w_t = z_t - l_t + p_{k,t} \tag{A.102}$$

B Summary of Equilibrium Conditions

Demand Shocks:

$$\sum_{k=1}^{K} \zeta_k D_{k,t} = 1 \tag{B.1}$$

Sector Shares:

$$n_k = \underbrace{(1-\psi)\zeta_k}_{\text{Consumption share}} + \underbrace{\psi\left[\sum_{\ell=1}^{K} n_\ell \omega_{\ell,k}^{\mathrm{H}} + \sum_{\ell=1}^{K^*} n_\ell^* \omega_{\ell,k}^{\mathrm{H}*}\right]}_{\substack{\text{Outdegree: Usage of sector k input} \\ \text{by other local firms and foreign firms}}} \tag{B.2}$$

$$n_k^* = \underbrace{(1-\psi)\zeta_k^*}_{\text{Consumption share}} + \underbrace{\psi\left[\sum_{\ell=1}^{K^*} n_\ell^* \omega_{\ell,k}^{\mathrm{F}*} + \sum_{\ell=1}^{K} n_\ell \omega_{\ell,k}^{\mathrm{F}}\right]}_{\substack{\text{Outdegree: Usage of sector k input} \\ \text{by other local firms and foreign firms}}} \quad , \tag{B.3}$$

where $\psi = \delta \frac{\theta}{1-\theta}$.

Aggregate Equilibrium:

$$y_t = (1-\psi)c_t + \psi z_t \tag{B.4}$$

$$y_t^* = (1-\psi)c_t^* + \psi z_t^* \tag{B.5}$$

$$y_t = \tilde{a} + \sum \tilde{a}_k n_k + a + \sum a_k n_k + (1-\delta)l_t + \delta z_t \tag{B.6}$$

$$y_t^* = \tilde{a} + \sum \tilde{a}_k n_k^* + a^* + \sum a_k^* n_k^* + (1-\delta)l_t^* + \delta z_t^* \tag{B.7}$$

$$w_t = \sigma c_t + \varphi l_t + p_t \tag{B.8}$$

$$w_t^* = \sigma c_t^* + \varphi l_t^* + p_t^* \tag{B.9}$$

$$w_t = z_t - l_t + avp_t \tag{B.10}$$

$$w_t^* = z_t^* - l_t^* + avp_t^* \tag{B.11}$$

$$c_t = c_{t+1} - \sigma^{-1}(i - p_{t+1} - p_t) + \gamma_t - \gamma_{t+1} + \tilde{\gamma}_t - \tilde{\gamma}_{t+1} \tag{B.12}$$

$$c_t^* = c_{t+1}^* - \sigma^{-1}(i^* - p_{t+1}^* - p_t^*) + \gamma_t^* - \gamma_{t+1}^* + \tilde{\gamma}_t - \tilde{\gamma}_{t+1} \tag{B.13}$$

$$i = \rho_i i_{t-1} + (1-\rho_i)(\phi_\pi \pi_t + \phi_y y_t) + \mu_t \tag{B.14}$$

$$i^* = \rho_i i_{t-1}^* + (1-\rho_i)(\phi_\pi \pi_t^* + \phi_y y_t^*) + \mu_t^* \tag{B.15}$$

$$x_t = q_t + p_t - p_t^* \tag{B.16}$$

$$q_t = \gamma_t^* - \gamma_t + c_t - c_t^* \tag{B.17}$$

Sectoral Equilibrium:

$$c_{k,t} = c_t - \eta(p^{\mathrm{H}}_{k,t} - p_t) + d_{k,t} \tag{B.18}$$
$$c^*_{k,t} = c^*_t - \eta(p^{\mathrm{F}*}_{k,t} - p^*_t) + d^*_{k,t} \tag{B.19}$$
$$w_{k,t} = \frac{\varphi}{1+\varphi\delta} y_{k,t} - \frac{\varphi}{1+\varphi\delta}\left(\tilde{a}_t + \tilde{a}_{k,t} + a_t + a_{k,t}\right) + \frac{1}{1+\varphi\delta} c_t + \frac{\varphi\delta}{1+\varphi\delta} p_{k,t} + p_t \tag{B.20}$$
$$w^*_{k,t} = \frac{\varphi}{1+\varphi\delta} y^*_{k,t} - \frac{\varphi}{1+\varphi\delta}\left(\tilde{a}_t + \tilde{a}_{k,t} + a^*_t + a^*_{k,t}\right) + \frac{1}{1+\varphi\delta} c^*_t + \frac{\varphi\delta}{1+\varphi\delta} p^*_{k,t} + p^*_t \tag{B.21}$$
$$m_{k,t} = (1-\delta) w_{k,t} + \delta p_{k,t} - \tilde{a}_t - \tilde{a}_{k,t} - a_t - a_{k,t} \tag{B.22}$$
$$m^*_{k,t} = (1-\delta) w^*_{k,t} + \delta p^*_{k,t} - \tilde{a}_t - \tilde{a}_{k,t} - a^*_t - a^*_{k,t} \tag{B.23}$$

Sectoral Phillips Curves:

$$\beta\,\mathbb{E}_t\, p^{\mathrm{H}}_{k,t+1} - (1-\beta) p^{\mathrm{H}}_{k,t} + p^{\mathrm{H}}_{k,t-1} = \kappa_k (p^{\mathrm{H}}_{k,t} - m_{k,t}) \tag{B.24}$$
$$\beta\,\mathbb{E}_t\, p^{\mathrm{H}*}_{k,t+1} - (1-\beta) p^{\mathrm{H}*}_{k,t} + p^{\mathrm{H}*}_{k,t-1} = \kappa_k (p^{\mathrm{H}*}_{k,t} - m_{k,t} + x_t) \tag{B.25}$$
$$\beta\,\mathbb{E}_t\, p^{\mathrm{F}}_{k,t+1} - (1-\beta) p^{\mathrm{F}}_{k,t} + p^{\mathrm{F}}_{k,t-1} = \kappa^*_k (p^{\mathrm{F}}_{k,t} - m^*_{k,t} - x_t) \tag{B.26}$$
$$\beta\,\mathbb{E}_t\, p^{\mathrm{F}*}_{k,t+1} - (1-\beta) p^{\mathrm{F}*}_{k,t} + p^{\mathrm{F}*}_{k,t-1} = \kappa^*_k (p^{\mathrm{F}*}_{k,t} - m^*_{k,t}), \tag{B.27}$$

where $\kappa_k \equiv (1-\alpha_k)(1-\alpha_k\beta)/\alpha_k$.[19]

Price Indexes:[20]

$$p^{\mathrm{I,H}}_{k,t} = \sum_{\ell=1}^{K} \omega^{\mathrm{H}}_{k,\ell} p^{\mathrm{H}}_{\ell,t} \tag{B.28}$$
$$p^{\mathrm{I,F}}_{k,t} = \sum_{\ell=1}^{K^*} \omega^{\mathrm{F}}_{k,\ell} p^{\mathrm{F}}_{\ell,t} \tag{B.29}$$
$$p_{k,t} = (1-\nu) p^{\mathrm{I,H}}_{k,t} + \nu p^{\mathrm{I,F}}_{k,t} \tag{B.30}$$
$$p^{\mathrm{I,H}*}_{k,t} = \sum_{\ell=1}^{K} \omega^{\mathrm{H}*}_{k,\ell} p^{\mathrm{H}*}_{\ell,t} \tag{B.31}$$
$$p^{\mathrm{I,F}*}_{k,t} = \sum_{\ell=1}^{K^*} \omega^{\mathrm{F}*}_{k,\ell} p^{\mathrm{F}*}_{\ell,t} \tag{B.32}$$
$$p^*_{k,t} = \nu^* p^{\mathrm{I,H}*}_{k,t} + (1-\nu^*) p^{\mathrm{I,F}*}_{k,t} \tag{B.33}$$

[19] In practice, we let α_k differ between countries.

[20] Here, $p^{\mathrm{H}}_{k,t}$ is the aggregate price of output in home sector k sold in the domestic market, $p^{\mathrm{F}}_{k,t}$ is the aggregate price of output in foreign sector k sold in the domestic market, $p^{\mathrm{I,H}}_{k,t}$ is the aggregate price of domestic inputs purchased by firms in home sector k, and $p^{\mathrm{I,F}}_{k,t}$ is the aggregate price of foreign inputs purchased by firms in home sector k.

$$p_t = \sum_{k=1}^{K} \zeta_k p^{\mathrm{H}}_{k,t} \tag{B.34}$$

$$p^*_t = \sum_{k=1}^{K^*} \zeta^*_k p^{\mathrm{F}*}_{k,t} \tag{B.35}$$

Average Prices:

$$avph = \sum n_k p^{\mathrm{I,H}}_{k,t} \tag{B.36}$$

$$avpf = \sum n^*_k p^{\mathrm{I,F}}_{k,t} \tag{B.37}$$

$$avp = (1-\nu) \times avph + \nu \times avpf \tag{B.38}$$

$$avph^* = \sum n_k p^{\mathrm{I,H}*}_{k,t} \tag{B.39}$$

$$avpf^* = \sum n^*_k p^{\mathrm{I,F}*}_{k,t} \tag{B.40}$$

$$avp^* = \nu^* \times avph^* + (1-\nu^*) \times avpf^* \tag{B.41}$$

Sectoral Output:

$$\begin{aligned} y_{k,t} = {} & y_t + d_{k,t} - (1-\psi)\eta(p^{\mathrm{H}}_{k,t} - p_t) \\ & + \psi\Bigg\{\sum_{\ell=1}^{K} n_\ell \omega^{\mathrm{H}}_{\ell,k}\Big[-\eta(p^{\mathrm{H}}_{k,t} - p^{\mathrm{I,H}}_{k,t}) - \rho(p^{\mathrm{I,H}}_{k,t} - p_{k,t})\Big] \\ & + \sum_{\ell=1}^{K^*} n^*_\ell \omega^{\mathrm{H}*}_{\ell,k}\Big[-\eta(p^{\mathrm{H}*}_{k,t} - p^{\mathrm{I,H}*}_{k,t}) - \rho(p^{\mathrm{I,H}*}_{k,t} - p^*_{k,t}) + x_t\Big]\Bigg\} \end{aligned} \tag{B.42}$$

$$\begin{aligned} y^*_{k,t} = {} & y^*_t + d^*_{k,t} - (1-\psi)\eta(p^{\mathrm{F}*}_{k,t} - p^*_t) \\ & + \psi\Bigg\{\sum_{\ell=1}^{K^*} n_\ell \omega^{\mathrm{F}*}_{\ell,k}\Big[-\eta(p^{\mathrm{F}*}_{k,t} - p^{\mathrm{I,F}*}_{k,t}) - \rho(p^{\mathrm{I,F}*}_{k,t} - p^*_{k,t})\Big] \\ & + \sum_{\ell=1}^{K} n_\ell \omega^{\mathrm{F}}_{\ell,k}\Big[-\eta(p^{\mathrm{F}}_{k,t} - p^{\mathrm{I,F}}_{k,t}) - \rho(p^{\mathrm{I,F}}_{k,t} - p_{k,t}) - x_t\Big]\Bigg\} \end{aligned} \tag{B.43}$$

Shocks:

$$\gamma_{t+1} = \rho^{\gamma}\gamma_t + \sigma^{\gamma}\epsilon^{\gamma}_{t+1} \tag{B.44}$$

$$\gamma^*_{t+1} = \rho^{\gamma}\gamma^*_t + \sigma^{\gamma}\epsilon^{\gamma*}_{t+1} \tag{B.45}$$

$$\tilde{a}_{t+1} = \rho^{\tilde{\mathrm{a}}}\tilde{a}_t + \sigma^{\tilde{\mathrm{a}}}\epsilon^{\tilde{\mathrm{a}}}_{t+1} \tag{B.46}$$

$$\tilde{a}_{k,t+1} = \rho^{\tilde{\mathrm{a}}}_k\tilde{a}_{k,t} + \sigma^{\tilde{\mathrm{a}}}_k\epsilon^{\tilde{\mathrm{a}}}_{k,t+1} \tag{B.47}$$

$$a_{t+1} = \rho^{\mathrm{a}}a_t + \sigma^{\mathrm{a}}\epsilon^{\mathrm{a}}_{t+1} \tag{B.48}$$

$$a^*_{t+1} = \rho^{\mathrm{a}}a^*_t + \sigma^{\mathrm{a}}\epsilon^{\mathrm{a}*}_{t+1} \tag{B.49}$$

$$a_{k,t+1} = \rho^{\mathrm{a}}_k a_{k,t} + \sigma^{\mathrm{a}}_k\epsilon^{\mathrm{a}}_{k,t+1} \tag{B.50}$$

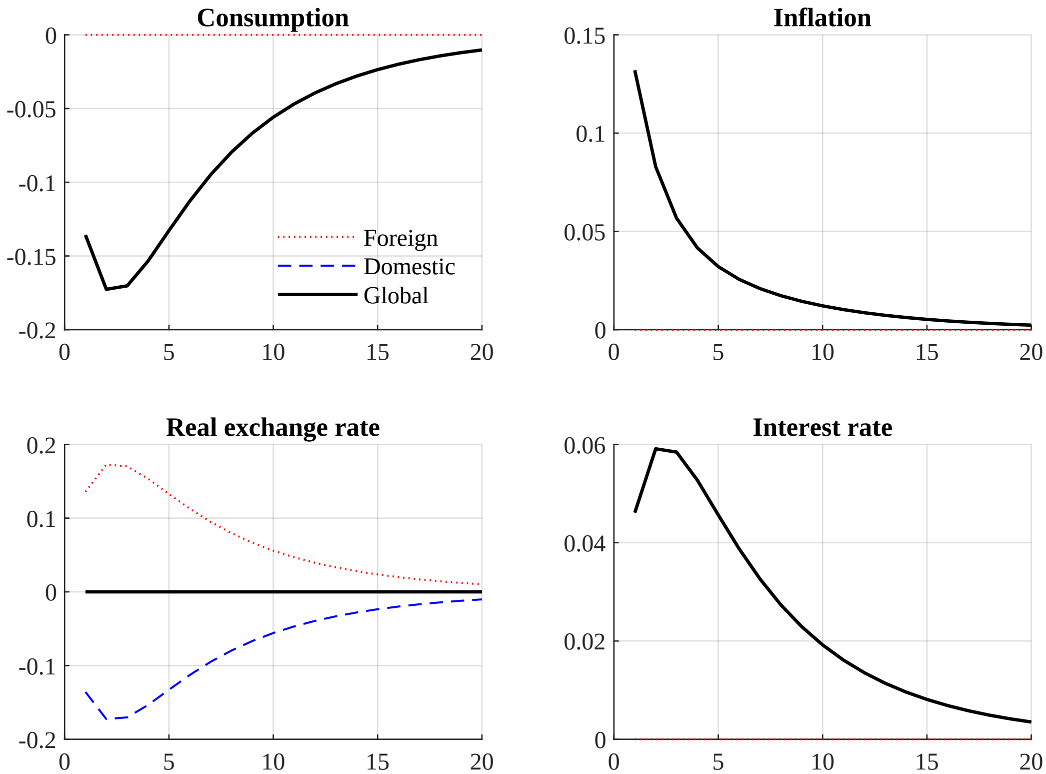
$$a^*_{k,t+1} = \rho^{\mathrm{a}}_k a^*_{k,t} + \sigma^{\mathrm{a}}_k\epsilon^{\mathrm{a}*}_{k,t+1} \tag{B.51}$$

$$d_{k,t+1} = \rho^{\mathrm{d}}_k d_{k,t} + \sigma^{\mathrm{d}}_k\epsilon^{\mathrm{d}}_{k,t+1} \tag{B.52}$$

$$d^*_{k,t+1} = \rho^{\mathrm{d}}_k d^*_{k,t} + \sigma^{\mathrm{d}}_k\epsilon^{\mathrm{d}*}_{k,t+1} \tag{B.53}$$

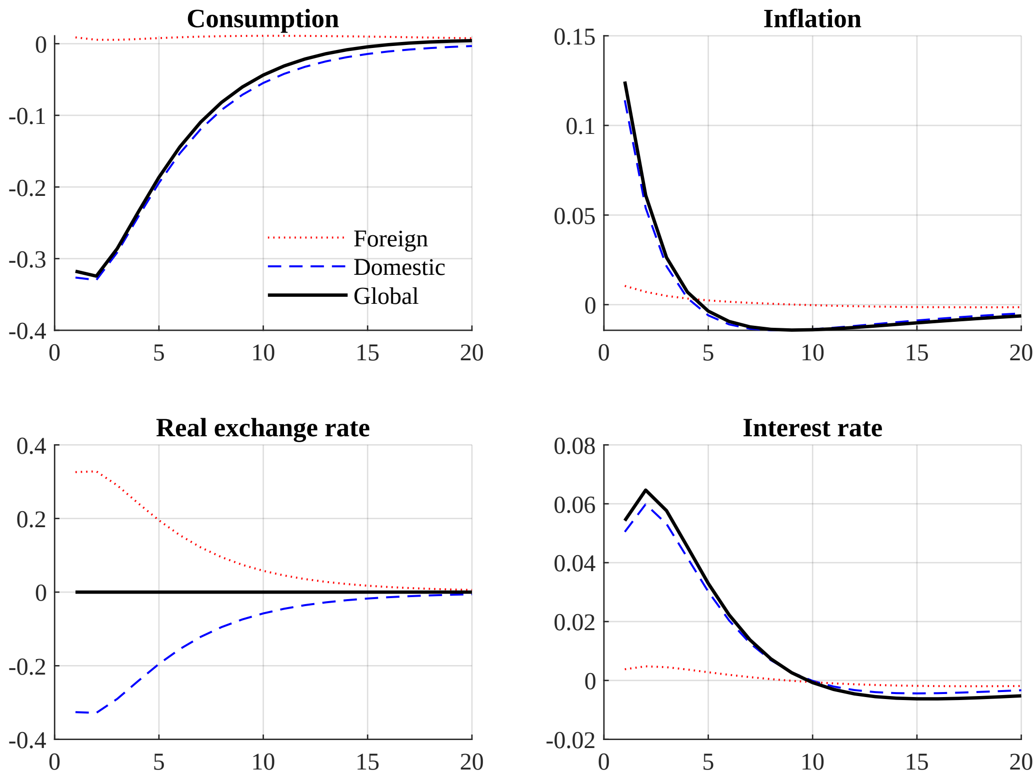
C Additional Results

Figure C.1: Aggregate Effects of a Sectoral Shock: No Intermediate Inputs



Notes: The share of intermediate inputs, both domestic and foreign, is zero.
Source: All figures in this appendix are based on authors' calculations using a calibrated model.

Figure C.2: Aggregate Effects of a Sectoral Shock to a Relatively Flexible Sector



Note: Price rigidity in the shocked sector is 17.5 percent lower than in the baseline.

Figure C.3: Aggregate Effects of a Sectoral Shock: All Sectors Have the Same Price Rigidity

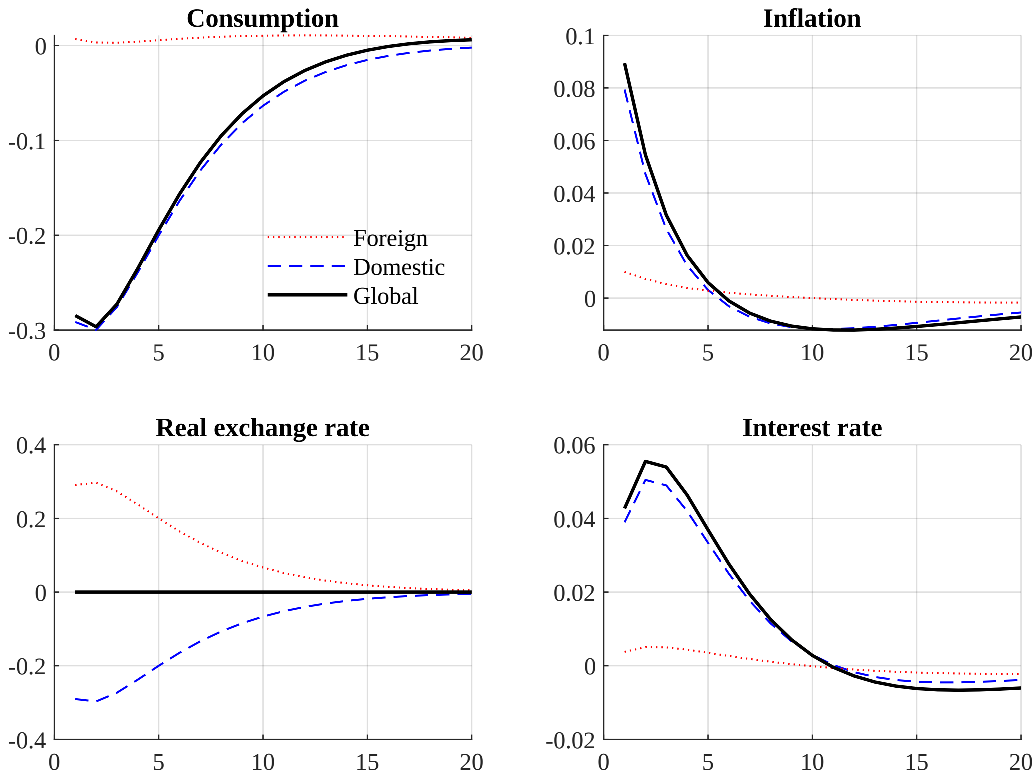
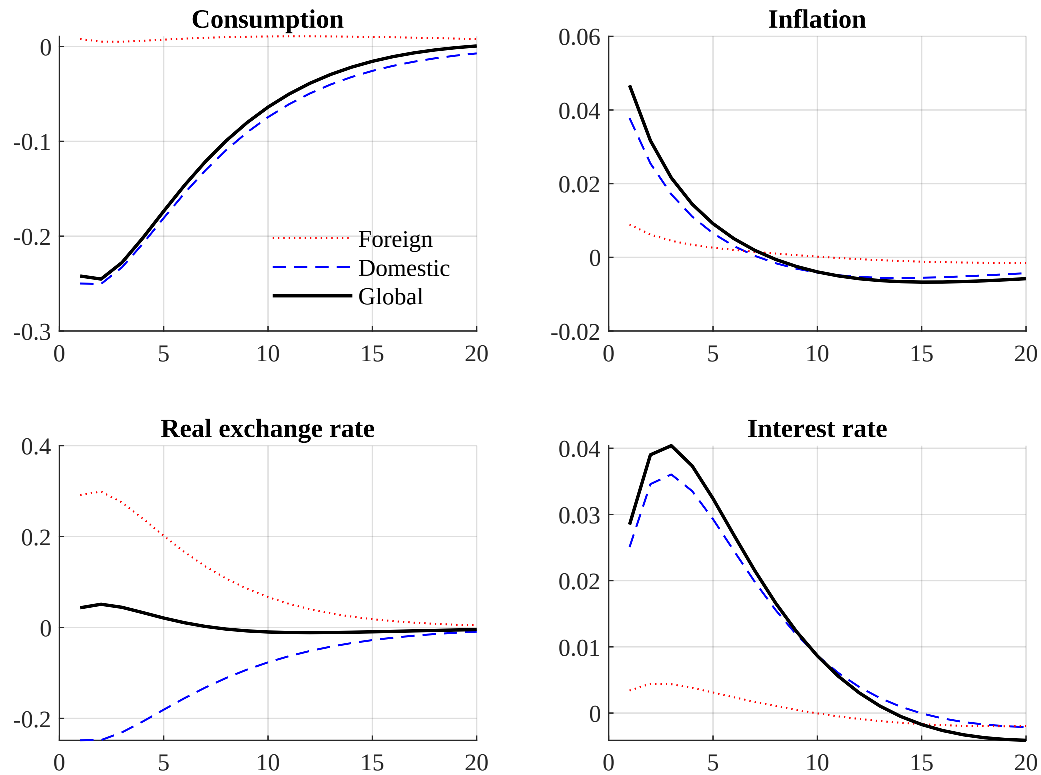
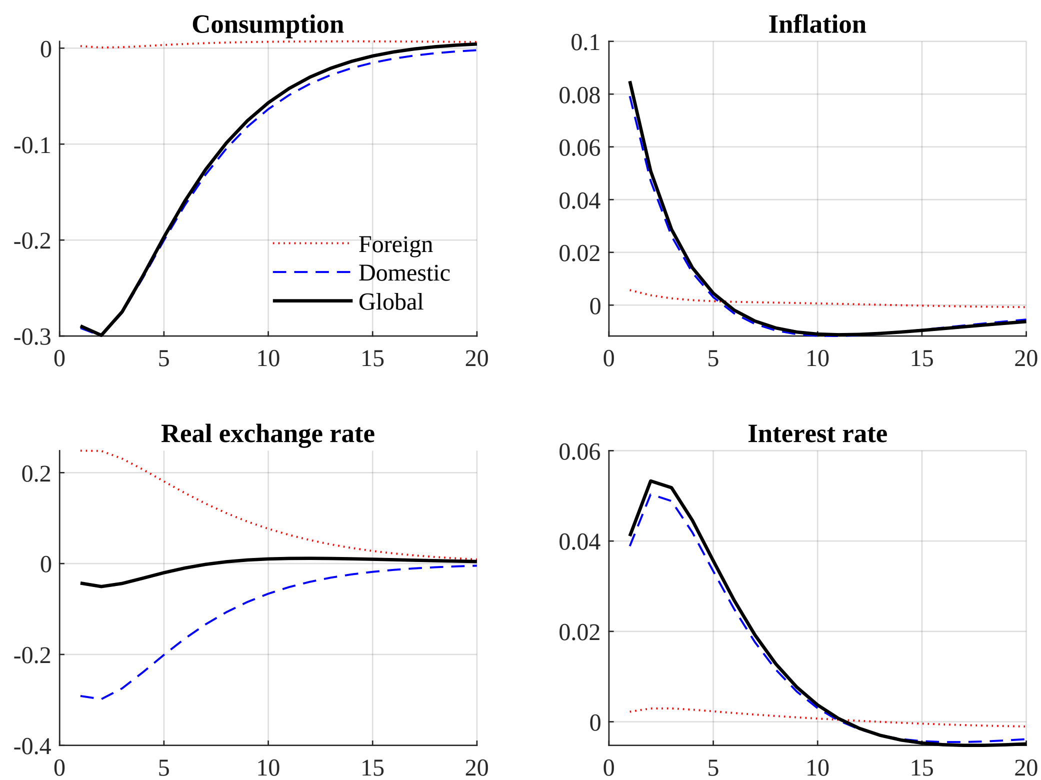


Figure C.4: Price Rigidity Is Heterogeneous at Home but Homogeneous Abroad



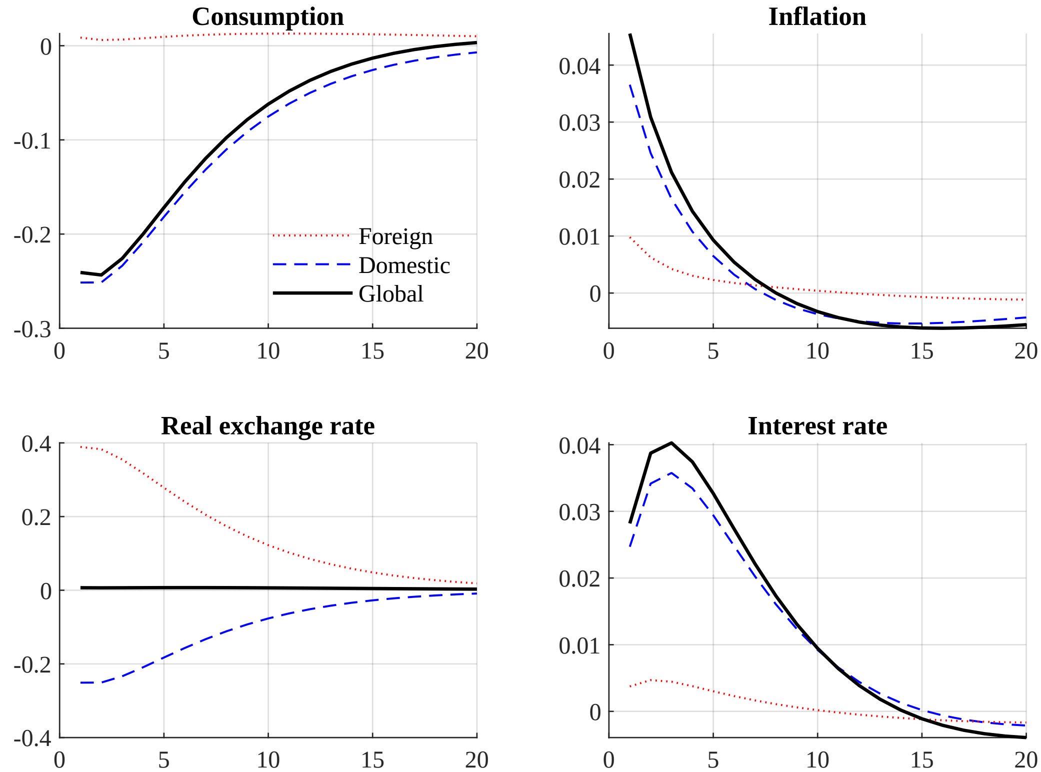
Note: In the economy with heterogeneity, price rigidity in the shocked sector is above average.

Figure C.5: Price Rigidity Is Homogeneous at Home but Heterogeneous Abroad



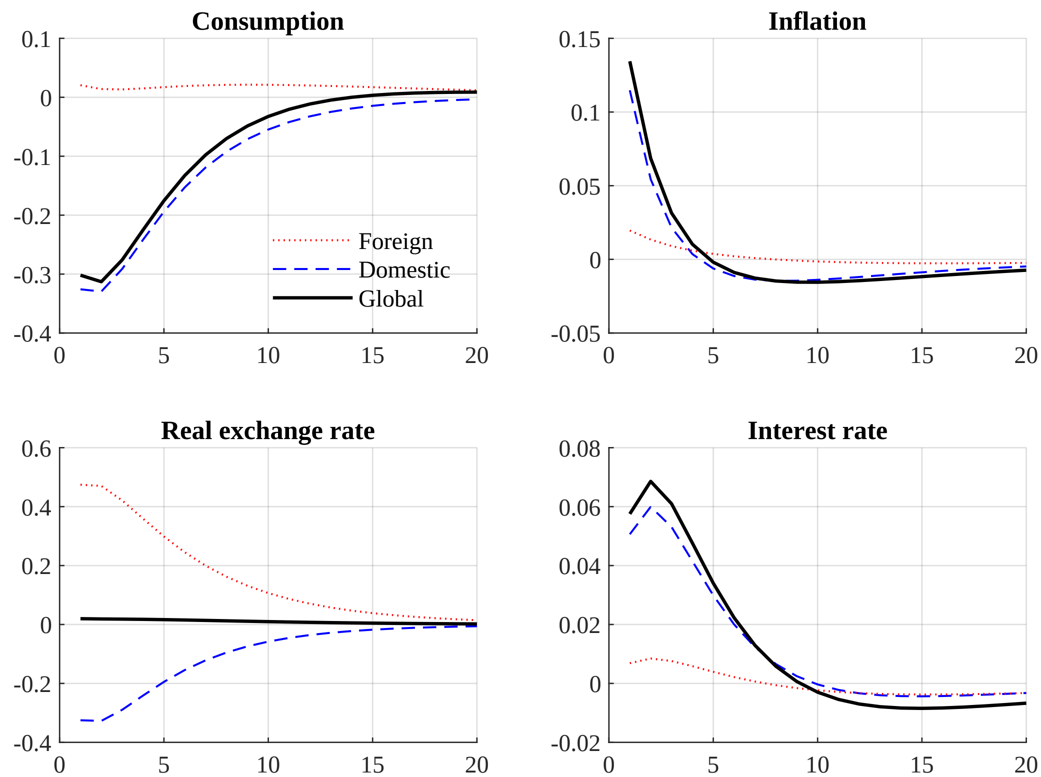
Note: In the economy with heterogeneity, price rigidity in the shocked sector is above average.

Figure C.6: Asymmetric Linkages: Shock to a High-Imports Sector and *Above-Average* Price Rigidity



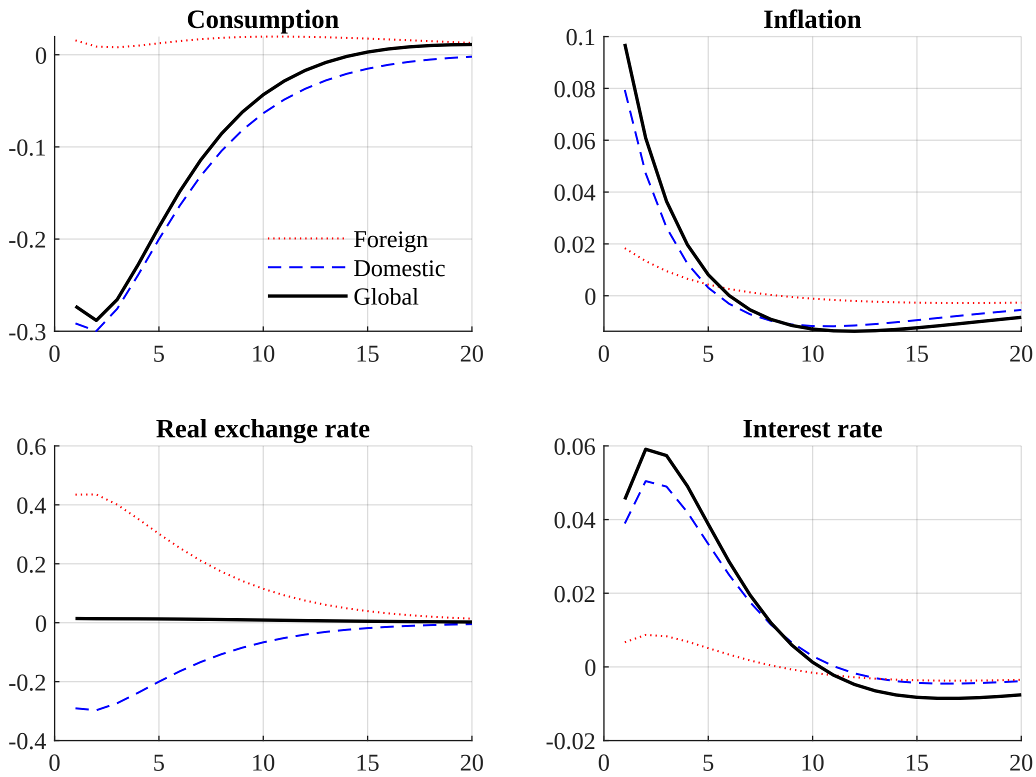
Notes: The shocked sector abroad supplies all foreign imports.

Figure C.7: Asymmetric Linkages: Shock to a High-Imports Sector and *Below-Average* Price Rigidity



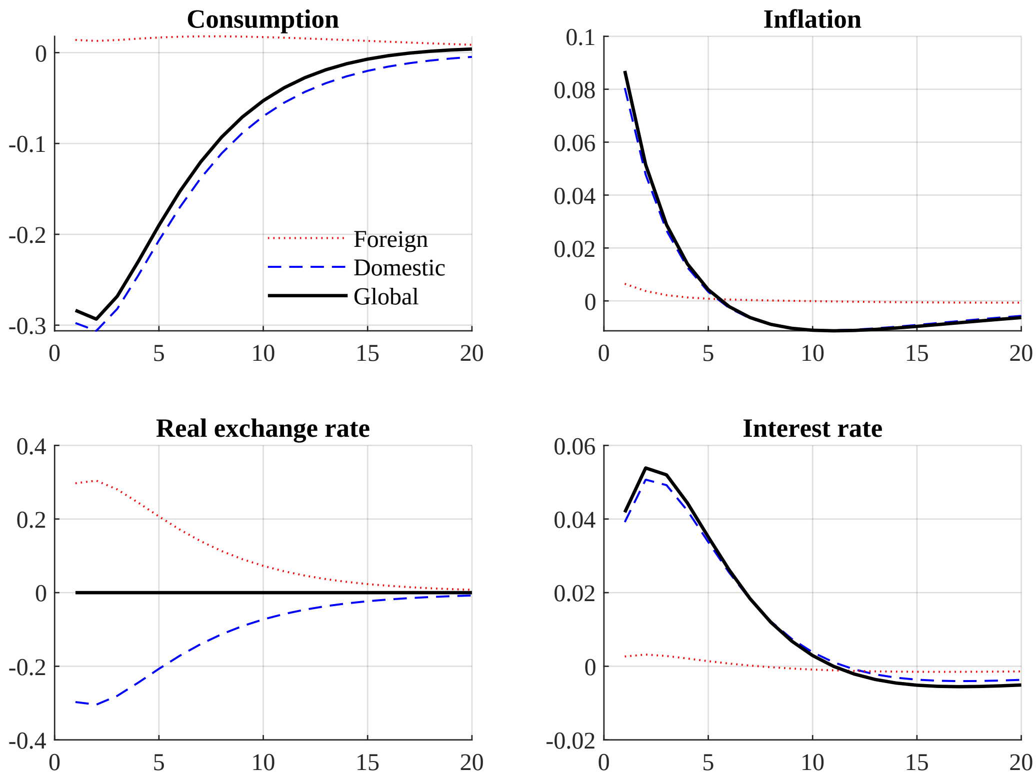
Notes: The shocked sector abroad supplies all foreign imports.

Figure C.8: Asymmetric Linkages: Shock to a High-Imports Sector and Homogeneous Price Rigidity



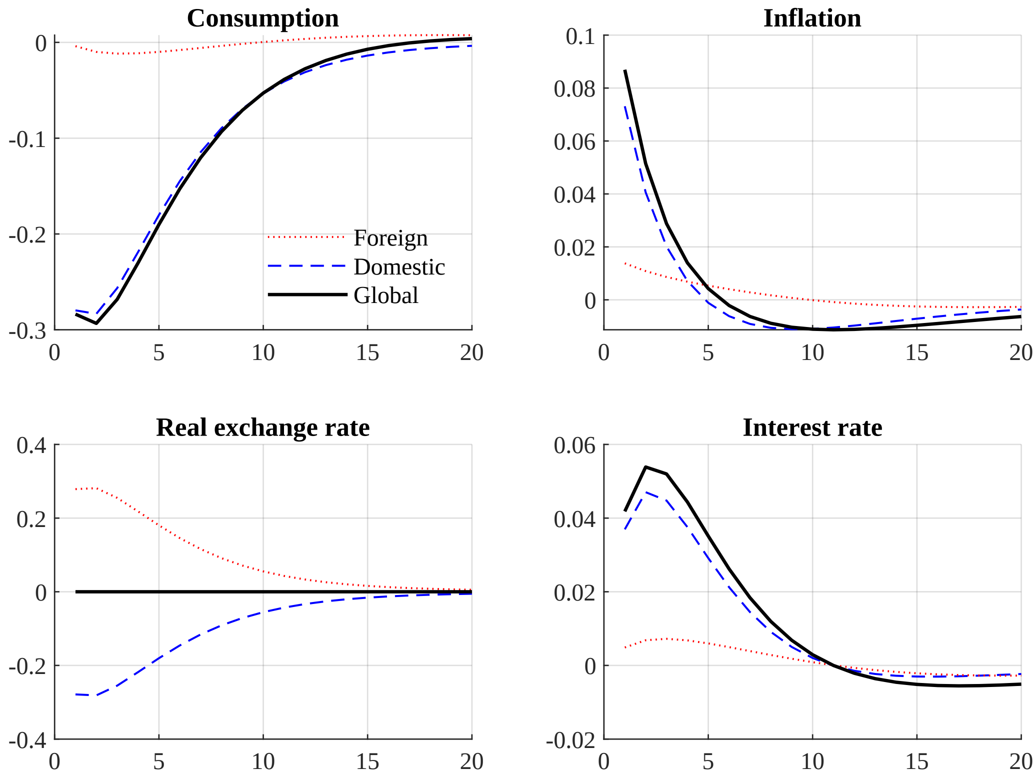
Notes: The shocked sector abroad supplies all foreign imports.

Figure C.9: Aggregate Effects of a Sectoral Shock: Reduced Share of Imports, Symmetric Linkages



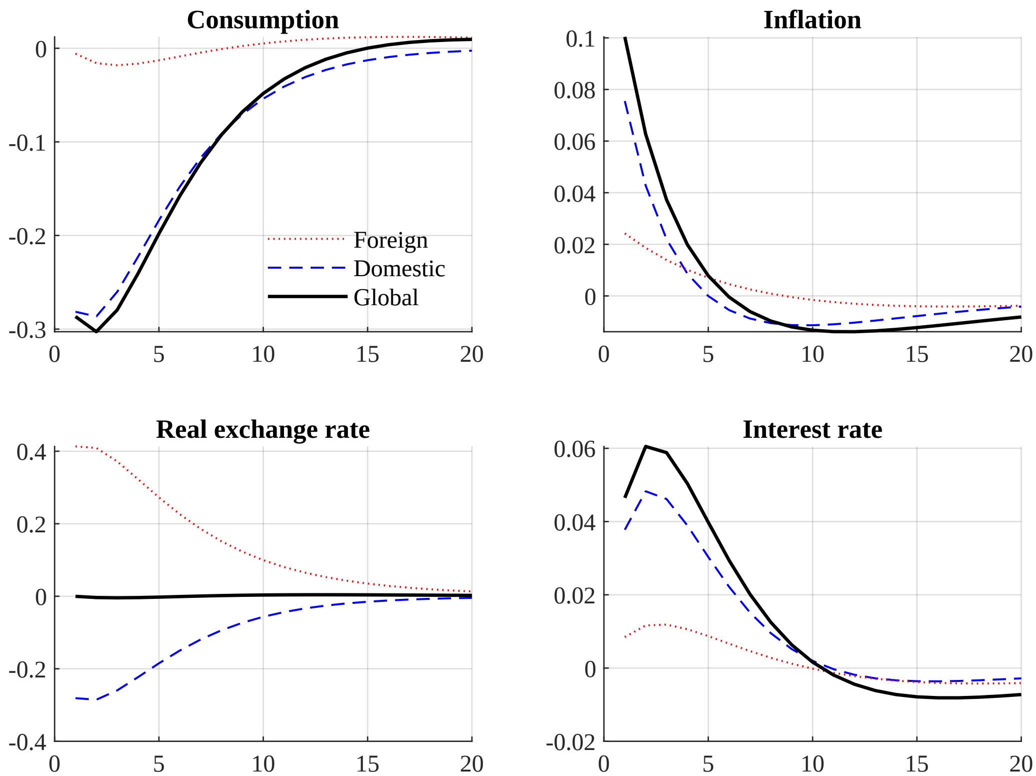
Notes: The import share parameter is reduced to $\nu = 0.15$. The production linkages are symmetric, as in Figure 1.

Figure C.10: Aggregate Effects of a Sectoral Shock: Increased Role of Imports, Symmetric Linkages



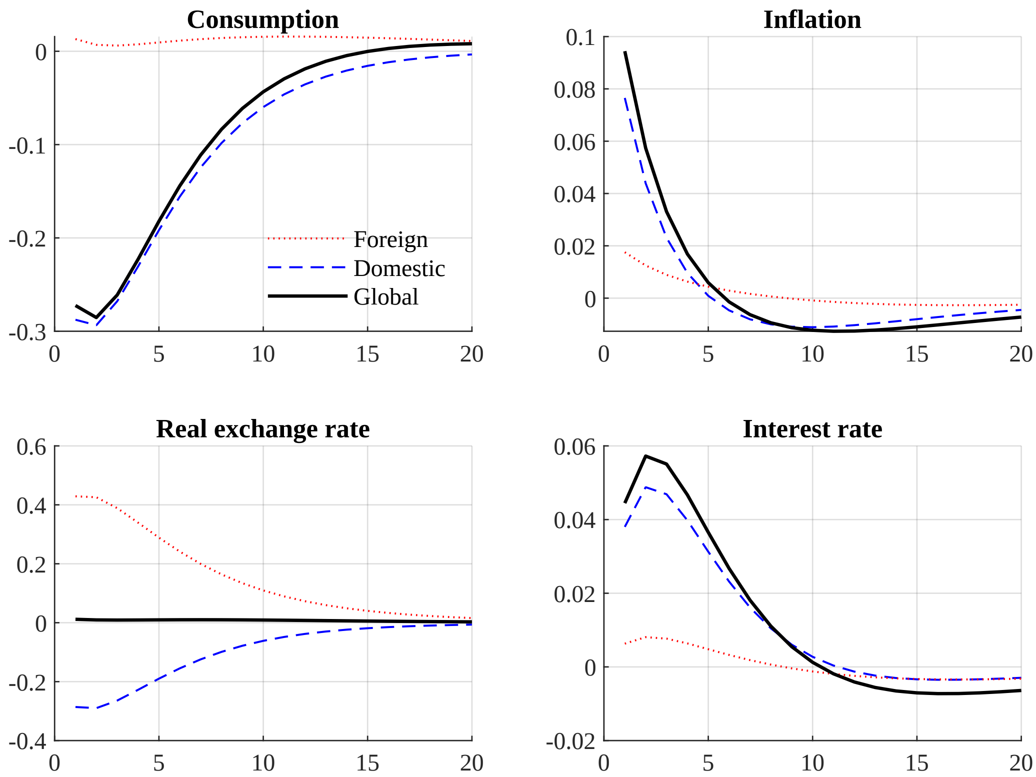
Notes: The import share parameter is increased to $\nu = 0.8$, and the Armington elasticity to $\rho = 3$. The linkages are symmetric, as in Figure 1.

Figure C.11: Aggregate Effects of a Sectoral Shock: Increased Imports Share Only, Asymmetric Linkages



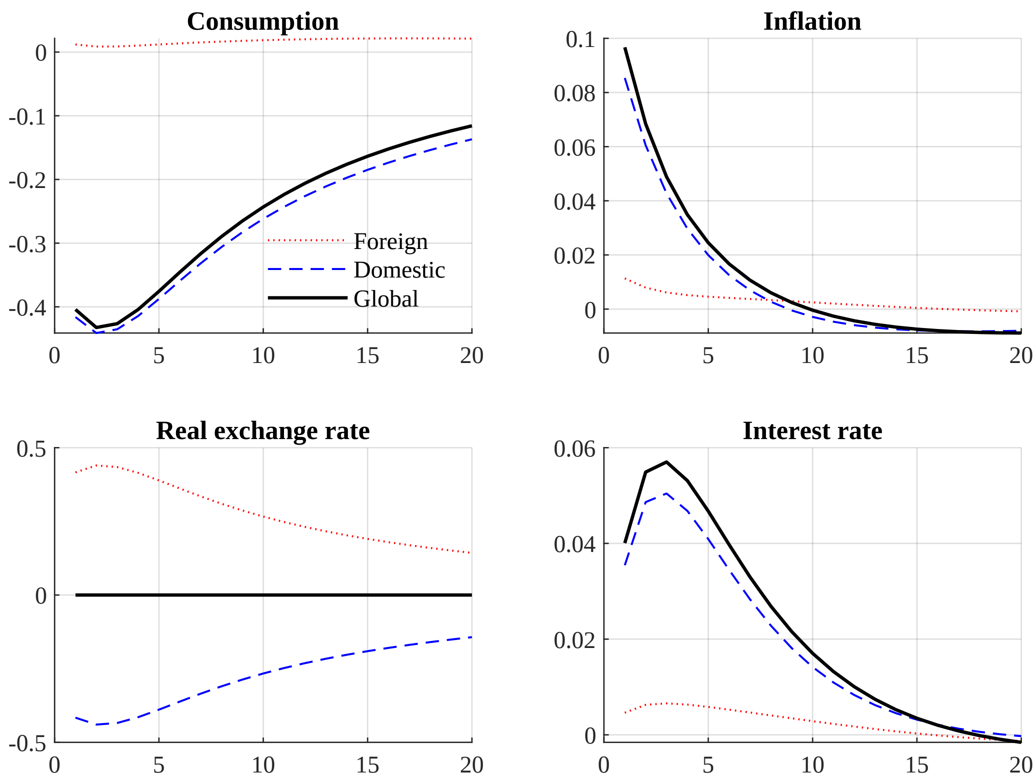
Notes: The import share parameter is increased to $\nu = 0.8$, but the Armington elasticity is unchanged, at $\rho = 1$.

Figure C.12: Aggregate Effects of a Sectoral Shock: Increased Armington Elasticity Only, Asymmetric Linkages



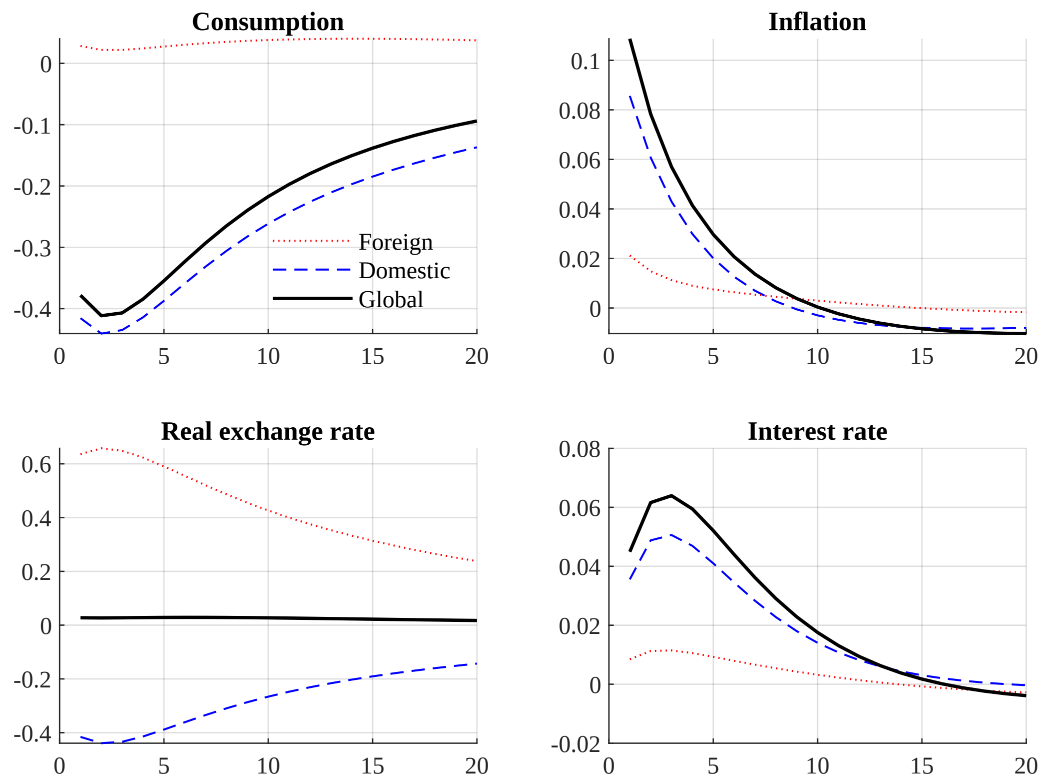
Notes: The import share parameter is unchanged, at $\nu = 0.4$, but the Armington elasticity is increased to $\rho = 3$.

Figure C.13: Symmetric Linkages and Persistent Shocks



Notes: The sectoral shock persistence increases from 0.85 to 0.95.

Figure C.14: Asymmetric Linkages and Persistent Shocks



Notes: The sectoral shock persistence increases from 0.85 to 0.95.