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Inflation in Disaggregated Small Open Economies

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Abstract:
This paper studies inflation in small open economies with production networks. I show that production networks alter the elasticity of the consumer price index (CPI) to changes in sectoral technology, factor prices, and import prices. Sectors can import and export directly but also indirectly through domestic intermediate inputs. Indirect exporting dampens the inflationary pressure from domestic forces, while indirect importing increases the inflation sensitivity to import price changes. Computing these CPI elasticities requires knowledge of the production network structure because these do not coincide with typical sufficient statistics used in the literature such as sectoral sales-to-GDP ratios, factor shares, or imported consumption shares. Using input-output tables, I provide empirical evidence that adjusting CPI elasticities for indirect exports and imports matters quantitatively for small open economies. I use the model to illustrate the importance of production networks during the COVID-19–related inflation in Chile and the United Kingdom.

JEL Classifications: E31, F41, D57, C67, F14, L16
Keywords: Inflation, small open economies, networks, input-output tables

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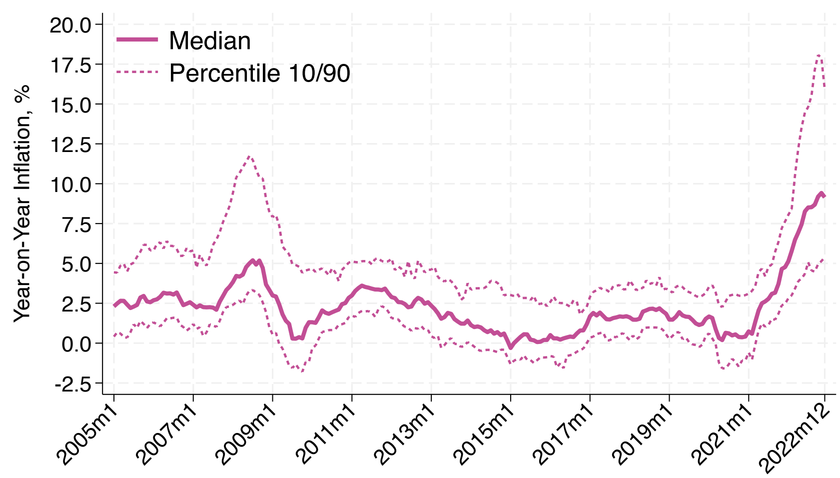
1 Introduction

In 2022, inflation reached 8 percent in the United States, its highest level in 40 years. The picture was similar on the other side of the Atlantic: Eurozone inflation was 8.4 percent, the highest since the region's formation. Explanations include shocks to commodity prices (Blanchard and Bernanke, 2023; Gagliardone and Gertler, 2023), sectoral demand changes (Ferrante et al., 2022), fiscal stimulus (Bianchi et al., 2023; di Giovanni et al., 2023b), and supply chain disruptions (di Giovanni et al., 2022, 2023a; Comin et al., 2023). As Figure 1 shows, high inflation was not restricted to these two economies: The median small open economy experienced an inflation rate of about 10 percent in 2022. However, inflation in this group of countries has not received much attention from researchers during the current episode. This paper attempts to fill this gap using both theory and data.

Figure 1. CPI Inflation in Small Open Economies



Note: The figure shows the median inflation rate (solid line) and the 10th and 90th percentiles (dashed lines). Small open economies are economies that represent less than 5 percent of world GDP and have a trade openness larger than 30 percent of GDP. See section 3.1 for more details. The plot shows an unbalanced panel of 46 small open economies over time. Source: Bank for International Settlements.

My starting point is the multisector and multifactor production network closed economy model in Baqaee and Farhi (2019b). It provides a useful benchmark for analyzing inflation during macroeconomic shocks such as the COVID-19 pandemic—that is, a combination of sectoral and aggregate shocks. Given my focus on small open economies, I augment this model to feature imports and exports at the sectoral level, adapting the production network model to the small open economy case. I use the model to study how the consumer price index (CPI) reacts to changes in sectoral technology, factor prices, and import prices, moving from the micro to the macro level.

I show that openness and production networks affect our understanding of inflation in small open economies via two distinct channels. On the one hand, exporting, either directly or indirectly through other economic producers, dampens the effect of sectoral technology shocks and factor price changes relative to a closed economy. On the other hand, direct importing gives rise to imported inflation, as the domestic consumer's basket now contains imported goods. On top of this channel, production networks imply that domestic goods are manufactured using imported inputs indirectly. As a result, production networks amplify imported inflation.[2] Uncovering these effects and quantifying their importance is possible only when both openness and network linkages are explicitly considered.

The key economic intuition is that opening up the economy is one way to break the link between a country's production and its domestic consumption. In an efficient closed economy—that is, an economy without distortions—with intersectoral linkages and domestic final consumption only, everything produced is consumed by the domestic consumer. *Network-adjusted domestic consumption*, by which I mean domestic consumption adjusted by domestic production network linkages, is thus equivalent to sales in the closed economy.[3] That domestic households consume everything produced, directly or indirectly, is one of the key reasons why the production network structure is irrelevant to first-order macroeconomic outcomes, such as real GDP or welfare, in closed economies. This irrelevance result is a useful benchmark because it allows us to use the ratios of sectoral sales to nominal GDP (the so-called "Domar weights") and factor payments to nominal GDP (factor shares) as sufficient statistics for the pass-through of sectoral technology changes or factor price changes into the CPI, respectively.[4] Increases in sectoral technology decrease consumer inflation by the Domar weight of the sector, while increases in factor prices increase inflation by the factor share.

I show that this irrelevance result no longer holds for consumer inflation in small open economies, without the need for second-order approximations, as in Baqaee and Farhi (2019b), or distortions, as in Baqaee and Farhi (2020) and Bigio and La'o (2020).[5] The

[2] This channel is distinct from inflation resulting from imported intermediate goods, as models can include intermediate goods without intersectoral linkages. See Svensson (2000) for an early analysis of imported inflation via intermediate goods.

[3] This definition is deliberately reminiscent of the network-adjusted labor share introduced in Baqaee (2015).

[4] As I show in the theory section, this can be thought of as a corollary of Hulten's theorem (Hulten, 1978) but for the CPI rather than for real GDP. Recall that for real GDP, Hulten's theorem states that in an efficient closed economy with inelastic factor supplies, the first-order effect of sectoral technology on real GDP is given by the Domar weights, and the first-order effect of changes in factor supply is given by its factor share.

[5] Strictly speaking, those papers I cite seek ways to break Hulten's theorem in closed economies, which refer to quantities, meaning the effect of sectoral technology changes or distortions on real GDP. However, because, as a corollary of Hulten's theorem, we can back out changes in the CPI, I reference them here.

reason is as follows. Consider first the impact of sectoral technology shocks on the CPI. In a small open economy, there are two final uses for goods produced within borders: domestic consumption or export. Unlike the closed economy case, sectoral sales do not map to the network-adjusted domestic consumption for two reasons: (1) direct exports and (2) indirect exports through domestic production network linkages. Instead, sectoral sales map to network-adjusted domestic consumption *and* network-adjusted exports. Because network-adjusted domestic consumption matters most for the CPI , one must subtract network-adjusted exports from sectoral sales. Hence, relative to consumers in a closed economy, consumers in a small open economy are slightly less exposed to changes in sectoral technology. Importantly, one must have knowledge of the domestic production network structure to compute these network-adjusted domestic consumption measures.

A similar intuition holds for how factor price changes affect the CPI. The relevant statistic here is the *network-adjusted domestic factor share*, that is, how much of each factor is embedded in goods consumed domestically after domestic production network linkages are taken into account (in the spirit of the domestic factor demand concept in Adao et al. (2022)). The total amount of a factor available in the economy can be "consumed" by domestic or foreign consumers,[6] with the production network potentially reshaping these patterns. While factor shares are sufficient statistics in a closed economy, in a small open economy with production networks, one must subtract from factor shares the fraction of each factor that is exported either directly or indirectly via production networks. This means that, relative to the situation in a closed economy, the domestic consumer is slightly less exposed to changes in factor prices.

The effect of import prices on the CPI, on the other hand, is stronger in a small open economy with production networks relative to a small open economy without production networks. The relevant statistics here are *network-adjusted import consumption shares*. Since the domestic consumer directly imports goods, the direct consumption share captures part of the exposure to import price changes. However, if there are intersectoral linkages across producers, domestic goods producers may end up importing intermediate inputs either directly, by buying from abroad, or indirectly, by buying from domestic sectors that buy from abroad or that buy from sectors that buy from abroad, and so on. This means that the imported content of domestically produced goods increases in the presence of production networks. To the extent that domestic goods increase their reliance on imported intermediate goods, so does the domestic consumer. Thus, domestic consumers'exposure to import prices must

[6]Here, consumers directly consume goods, not factors. Given that goods are ultimately made of factors of production, one can think of consumers implicitly consuming them. This notion can be found in the reduced factor demand system proposed by Adao et al. (2017).

account for both direct and indirect exposure, encapsulated in the *network-adjusted import consumption shares*.

Guided by the model, I turn to the data to measure the importance of these production network adjustments. Using data from the World Input-Output Tables (WIOT), I find that the adjustments matter quantitatively. I illustrate the adjustments by focusing on the three sources of variation I have considered so far: sectoral technology, factor prices, and import prices.

First, consider the electricity sector in the United Kingdom. The Domar weight of this sector is about 5.95 percent. When direct exports (but not indirect exports) are taken into account, the relevant ratio for the pass-through into CPI decreases to 5.90 percent, a negligible change. This is expected because the UK electricity sector does very little direct exporting to other countries. Yet, when indirect exporting is taken into account, the network-adjusted domestic consumption share decreases to 4.4 percent, a 25 percent decrease relative to the Domar weight benchmark. This is because other export-heavy sectors in the United Kingdom use electricity as a production input either directly or indirectly. Thus, Domar weights would overestimate the impact of a change in productivity in the UK electricity sector on the domestic CPI.

Second, consider the role of wage changes in moving the CPI. In a closed economy, the labor share is the relevant statistic for how wage changes pass through into the CPI. In the data, the labor share for the average small open economy is about 57 percent. However, the small open economy model with production networks suggests that one must subtract from the labor share the portion that is exported directly or indirectly. After accounting for network-adjusted exports, this average labor share decreases to 39 percent. This means that the same increase in domestic wages has a 32 percent weaker impact on inflation in a small open economy relative to a closed economy.

Finally, consider the role of import prices. In the data, the average small open economy exhibits a direct import consumption share of about 17 percent of its total expenditure. Yet, on average, the network-adjusted import consumption share is 30 percent. This implies that the impact of import prices on domestic inflation is (almost) twice what would be implied by a measure ignoring indirect linkages.

In the final section of the paper, I use the model to analyze the recent inflation in two small open economies: Chile and the United Kingdom. I chose these two countries because (1) they fit the small open economy definition, (2) they have experienced high inflation in recent years, and (3) they allow me to compute and contrast between emerging and developed markets. Using these countries, I show that network adjustments to exports and imports provide a quantitative improvement in matching data moments over both closed economy models and small open economy models without production networks.

From 2020 through 2022, the average annual inflation rate in Chile was 6.13 percent, with a standard deviation of 3.89. A quantitative closed economy model with production networks implies an average inflation rate of 0.98 percent, with a standard deviation of 9.69. A small open economy model without production networks delivers an average inflation rate of 1.45 percent, with a standard deviation of 6.88. Finally, the small open economy model with production networks delivers an average inflation rate of 2.41 percent, with a standard deviation of 6.67. Overall, the small open economy with production networks better matches the mean and the standard deviation.

For the United Kingdom, the average inflation rate was 3.69 percent over the 2020–2022 period, with a standard deviation of 3.11. The closed economy model with production networks implies an average inflation rate of 2.27 percent, with a standard deviation of 2.57. The small open economy model without network adjustments exhibits an average inflation rate of 2.72 percent, with a standard deviation of 2.64. The small open economy model with production networks shows an average inflation rate of 3.21 percent, with a standard deviation of 3.00. As in the Chilean case, the results involving production networks, coupled with openness, are closer to the data moments of UK inflation.

The measurement and application sections of this paper illustrate that the required network adjustments in a small open economy matter for understanding inflation not only as a theoretical curiosity, but also in practice.

Related Literature. This paper relates to several strands of the literature. The first one studies inflation in closed economies with production networks (Basu, 1995; La'o and Tahbaz-Salehi, 2022; Guerrieri et al., 2022; Baqaee and Farhi, 2022b; Luo and Villar, 2023; Afrouzi and Bhattarai, 2022; Ferrante et al., 2022; Rubbo, 2023; Minton and Wheaton, 2023; di Giovanni et al., 2023a,b; Lorenzoni and Werning, 2023).[7] These studies find consistently that the interaction between sectoral price/wage rigidities and production networks is key to understanding the behavior of inflation, which has implications for the conduct of monetary policy such as what inflation rate to target. This paper focuses on how introducing production networks in a small open economy model helps understand the pass-through of different shocks to inflation. Although there is no price rigidity in the model—and thus I cannot speak about the optimal conduct of monetary policy—I contribute to this literature by showing that the production network can have a first-order impact on inflation beyond its role in the sales share distribution, without the need for any distortions.

[7] There is also extensive literature on multisector models with sticky prices that do not necessarily feature a production network structure, so I omit them from the main text. For earlier contributions, see Woodford (2003) and the references therein.

Second, this paper relates to the literature on inflation in small open economies. In the second part of the 20th century, Latin America experienced episodes of high and persistent inflation, a term later coined "chronic inflation." In response, there was extensive literature during the 1990s on how to best control chronic inflation and the impact of different nominal and real policy rules in small open economies (see Calvo and Végh, 1995; Calvo et al., 1995; Calvo and Végh, 1999, especially the last one, for an overview of this earlier literature). Modern treatments that introduce New Keynesian features, such as sticky prices and monopolistic competition, into small open economy models include Gali and Monacelli (2005) and Faia and Monacelli (2008).[8] The literature has recently augmented these models to include multiple sectors (Matsumura, 2022) and intersectoral linkages (Qiu et al., 2024) and has applied these models to understand the recent inflationary episode in the United States (Comin and Johnson, 2020; Comin et al., 2023). Relative to this literature, I make four contributions. First, I analyze explicitly the role of production networks in inflation for small open economies. I show how production networks interact with international trade, affecting how domestic and foreign shocks ultimately affect CPI inflation both theoretically and quantitatively. Second, I show that this result holds without the need for any distortion or friction, further clarifing the key role of production networks beyond these forces. Third, using a first-order approximation allows me to consider unrestricted intersectoral linkages, without assuming any functional forms for production or utility. Fourth, my model also features multiple factors of production, while the previous models typically focus on only one factor: labor.

Finally, in focusing on the role of network-adjusted exports and imports, this paper connects inflation to the literature on indirect trade (Huneeus, 2018; Adao et al., 2022; Dhyne et al., 2021, 2023; Muñoz, 2023). This literature focuses on the firm-level real consequences of indirect trade, which is equivalent to my trade network adjustments. For example, Dhyne et al. (2021) use Belgian firm-to-firm-level transaction data and find that the relevant concept for a firm sale's exposure to international markets is total exports (network-adjusted exports), while its exposure in costs is total imports (network-adjusted import share). Importantly, the trade literature focuses on real outcomes, while this paper focuses on a nominal variable. Hence, I contribute to this literature by embedding indirect trade into a small open economy model to analyze how it matters for a nominal variable: inflation.

[8] There is also a large literature focusing on two or more countries. My work is not directly related to these models as I focus on small open economies. I refer the interested reader to Corsetti and Pesenti (2007) and Corsetti et al. (2010) for an overview of such models. Recent literature focusing on inflation using multi-country and multisector models include, for example, Auer et al. (2019) and Ho et al. (2022) during non–COVID-19 times and di Giovanni et al. (2022) and Andrade et al. (2023) during the pandemic.

2 A Small Open Economy Model with Production Networks

Environment. There is a set of domestically produced goods that I denote by N, with typical element i. These goods can be consumed domestically, used as intermediate inputs by other domestic sectors, and exported. I denote the imported goods set by M, with typical element m. These imported goods can be used as intermediate inputs to produce domestic goods or as final consumption. Finally, there is a set F of factors with typical element f.

Notation. I denote matrices and vectors using **bold text** (for example, $\boldsymbol{Y}$). I denote the transpose of a matrix as $\boldsymbol{Y}^T$. Unless otherwise noted, vectors are always column vectors. For example, the vector of Domar weights, defined later, is $\boldsymbol{\lambda} = (\lambda_1, \lambda_2, ..., \lambda_N)^T$. Log changes are expressed as $\mathrm{d}\log Y = \hat{Y}$.

Table 1 shows the different shares and matrices that are key for the analysis. I use a bar over a variable for shares based on *total expenditure*, while GDP-based measures do not contain a bar.

2.1 Households

There is a representative household that consumes domestically produced goods and foreign goods. It has an instantaneous utility function that I denote by $U(\boldsymbol{C}_D, \boldsymbol{C}_M)$, where $\boldsymbol{C}_D = \{C_i\}_{i\in N}$ denotes the vector of domestically produced goods consumption and $\boldsymbol{C}_M = \{C_m\}_{m\in M}$ is the vector of foreign goods consumption. These consumption vectors have associated vector prices $\boldsymbol{P}_D = \{P_i\}_{i\in N}$ and $\boldsymbol{P}_M = \{P_m\}_{m\in M}$. Unless otherwise stated, all prices are denominated in local currency. I assume the utility function $U(.)$ is homogeneous of degree one in its arguments. The representative consumer also owns all factors of production and supplies them inelastically ($\{\bar{L}_f\}_{f\in F}$) at the given factor prices ($\{W_f\}_{f\in F}$).

Given a vector of prices, for both domestically produced and foreign goods, the cost-minimization problem satisfies

$$PC = \min_{\boldsymbol{C}_D, \boldsymbol{C}_M} \sum_{i\in N} P_i C_i + \sum_{m\in M} P_m C_m \text{ subject to } U(\boldsymbol{C}_D, \boldsymbol{C}_M) \geq \bar{U}. \tag{2.1}$$

Solving this problem delivers a price index that is a function of good prices. I denote this price index by $P = P(\boldsymbol{P}_D, \boldsymbol{P}_M)$. As a reminder, up to a first-order approximation, changes

Table 1. Definitions

Name	Notation	Expression	Goods/Factors
GDP-based			
Domar Weight	λ_i	$\frac{P_i Q_i}{GDP}$	for $i \in N$
Consumption Share	b_i	$\frac{P_i C_i}{GDP}$	for $i \in N$
Imported Consumption Share	b_m	$\frac{P_m C_m}{GDP}$	for $m \in M$
Export Share	x_i	$\frac{P_i X_i}{GDP}$	for $i \in N$
Factor Shares	$\boldsymbol{\Lambda}$	$\Lambda_f = \frac{W_f L_f}{GDP}$	for $f \in F$
Expenditure-based			
Domar Weight	$\bar{\lambda}_i$	$\frac{P_i Q_i}{E}$	for $i \in N$
Consumption Share	$\bar{b}_i$	$\frac{P_i C_i}{E}$	for $i \in N$
Imported Consumption Share	$\bar{b}_m$	$\frac{P_m C_m}{E}$	for $m \in M$
Export Share	$\bar{x}_i$	$\frac{P_i X_i}{E}$	for $i \in N$
Factor Shares	$\bar{\boldsymbol{\Lambda}}$	$\bar{\Lambda}_f = \frac{W_f L_f}{E}$	for $f \in F$
Sector-level Shares			
Input-Output Matrix	$\boldsymbol{\Omega}$	$\Omega_{ij} = \frac{P_j M_{ij}}{P_i Q_i}$	$j \in N$
Leontief-Inverse Matrix	$\boldsymbol{\Psi}_D = (\boldsymbol{I} - \boldsymbol{\Omega})^{-1}$	$\boldsymbol{\Psi}_{ij} = \sum_{s=0}^{\infty} \boldsymbol{\Omega}_{ij}^s$	$i, j \in N$
Factor Spending Matrix	$\boldsymbol{A}$	$a_{if} = \frac{W_f L_{if}}{P_i^D Q_i}$	$i \in N; f \in F$
Intermediate Import Spending Matrix	$\boldsymbol{\Gamma}$	$\Gamma_{im} = \frac{P_m M_{im}}{P_i Q_i}$	$i \in N; m \in M$

in this price index satisfy

$$\widehat{P} = \bar{\boldsymbol{b}}_D^T \widehat{\boldsymbol{P}}_D + \bar{\boldsymbol{b}}_M^T \widehat{\boldsymbol{P}}_M, \tag{2.2}$$

where

$$\bar{\boldsymbol{b}}_D = \{\bar{b}_i\} = \frac{P_i C_i}{E}; \quad \bar{\boldsymbol{b}}_M = \{\bar{b}_m\} = \frac{P_m C_m}{E}; \quad E = \boldsymbol{P}_D^T \boldsymbol{C}_D + \boldsymbol{P}_M^T \boldsymbol{C}_M = PC,$$

are the expenditure share on domestically produced goods ($\bar{b}_i$), imported goods ($\bar{b}_m$), and total expenditure (E), respectively.

The consumer budget constraint reads

$$PC + T = \sum_{f \in F} W_f L_f + \sum_{i \in N} \Pi_i,$$

where T is an *exogenous* net transfer to the rest of the world, as in Baqaee and Farhi (2022b). In Appendix B, I provide a justification for having such a force in the current model by using a two-period model without changing the main results.

2.2 Sectors

There is a representative firm in each i sector that produces according to the following production function:

$$Q_i = Z_i F^i \left(\{L_{if}\}_{f \in F}, \{M_{ij}\}_{j \in N}, \{M_{im}\}_{m \in M} \right), \tag{2.3}$$

where Z_i is a sector-specific productivity, L_{if} is demand for factor f by firm i, M_{ij} represents intermediate input demand for good $j \in N$ by firm i, and M_{im} represents input demand for imported good $m \in M$. We can write cost-minimization firm i as

$$TC_i = \min_{\{L_{if}\}_{f=1}^{F}, \{M_{ij}\}_{j \in N}, \{M_{im}\}_{m \in M}} \sum_{f \in F} W_f L_{if} + \sum_{j \in N} P_j M_{ij} + \sum_{m \in M} P_m^M M_{im}^M$$
$$\text{subject to } Z_i F^i \left(\{L_{if}\}_{f \in F}, \{M_{ij}\}_{j \in N}, \{M_{im}\}_{m \in M} \right) \geq \bar{Q}_i.$$

This delivers a marginal cost function that only depends on prices and technology due to the constant returns to scale assumption. In particular,

$$MC_i = MC_i(Z_i, \boldsymbol{P}_D, \boldsymbol{P}_M, \boldsymbol{W}), \tag{2.4}$$

where $\boldsymbol{W} = \{W_f\}_{f \in F}$ is a vector of factor prices.

We can obtain conditional factor and intermediate input demand by applying Shephard's lemma to the optimized total cost, TC_i, such that

$$\frac{\partial MC_i}{\partial W_f} Q_i = L_{if} \quad \text{for each } f \in F, \tag{2.5}$$

$$\frac{\partial MC_i}{\partial P_j} Q_i = M_{ij} \text{ for each } j \in N, \tag{2.6}$$

$$\frac{\partial MC_i}{\partial P_m} Q_i = M_{im} \text{ for each } m \in M. \tag{2.7}$$

Due to constant returns to scale and perfectly competitive good and factor markets, each firm i makes zero profit:

$$P_i Q_i = \sum_{f \in F} W_f L_{if} + \sum_{j \in N} P_j M_{ij} + \sum_{m \in M} P_m M_{im} \quad \text{for all } i \in N. \tag{2.8}$$

2.3 Equilibrium

Market clearing conditions for good and factor markets satisfy

$$Q_i = C_i + X_i + \sum_{j \in N} M_{ji} \qquad \text{for each } i \in N. \tag{2.9}$$

Equation (2.9) is the good market clearing condition. I assume X_i is exogenous as in Adao et al. (2022) so that a price clearing the market always exists for each domestically produced good, even if it is exported.

Since this is a real model, nominal prices are indeterminate unless I supplement one additional equation. To do so, I impose the following:

$$PC \leq \mathcal{M} = E,$$

where $\mathcal{M}$ is the money supply that I take as exogenous in what follows. This is a cash-in-advance constraint used, for example, in La'o and Tahbaz-Salehi (2022) and Afrouzi and Bhattarai (2022).[9] One can think of this restriction as the small open economy's central bank effectively pinning down total nominal expenditure (E), providing an exogenous nominal anchor. The central bank, conditional on knowing C, which is determined by *real variables*, can implement any price level, P, that it desires, consistent with C. This model features the classic dichotomy, according to which real variables are determined independently of the

[9] It can be shown that this "constraint" is isomorphic to a model with money in the utility function that is separable from aggregate consumption. The cash-in-advance constraint thus serves no other purpose than pinning down nominal variables without affecting real allocations in this model.

nominal side.[10] Under these assumptions, one should interpret the results as highlighting the role of production networks in the CPI, conditional on an exogenous central bank monetary policy.

Similarly to Baqaee and Farhi (2019a), I define an equilibrium in this economy using a dual approach in which feasible and equilibrium allocations are found by taking as given factor prices $\boldsymbol{W}$ and a level of expenditure, E, as follows:

1. Given sequences $(\boldsymbol{W}, \boldsymbol{P}_D, \boldsymbol{P}_M, \boldsymbol{\Pi})$ and exogenous parameters (T), the household chooses $(\boldsymbol{C}_D, \boldsymbol{C}_M)$ to maximize its utility subject to its budget constraint.

2. Given $(\boldsymbol{W}, \boldsymbol{P}_D, \boldsymbol{P}_M)$ and production technologies, firms choose $(\boldsymbol{L}_i, \boldsymbol{M}_i)$ to minimize their cost of production.

3. Given $\boldsymbol{X}$, goods markets clear.

4. The cash-in-advance constraint holds with equality $PC = \mathcal{M} = E$.

2.4 Characterizing Changes in the Price Index

Having defined the environment, optimality, and equilibrium conditions, I can now study changes in the consumer price index, $\widehat{P}$. Inflation here consists of a log-linear approximation around the initial price-level equilibrium. The purpose of the model is to distill whether and how the production network may matter for inflation, which in the model is a cross-sectional statement rather than a dynamic statement. "Inflation" in this context can thus be understood in the space, rather than the time, dimension. This concept has been used, for example, to study inflation in the United States during the COVID-19 period (Baqaee and Farhi, 2022b; di Giovanni et al., 2022) and the role of sticky prices in production networks (La'o and Tahbaz-Salehi, 2022; Baqaee and Rubbo, 2023).

The following result characterizes how the CPI reacts to changes in exogenous variables.

Proposition 1. *Consider a perturbation $(\widehat{\boldsymbol{Z}}, \widehat{\boldsymbol{W}}, \widehat{\boldsymbol{P}}_M)$ around some initial equilibrium. Up to a first order, changes in the aggregate price index, $\widehat{P}$, satisfy*

$$\widehat{P} = -\left(\bar{\boldsymbol{\lambda}}^T - \widetilde{\boldsymbol{\lambda}}^T\right)\widehat{\boldsymbol{Z}} + \left(\bar{\boldsymbol{\Lambda}}^T - \widetilde{\boldsymbol{\Lambda}}^T\right)\widehat{\boldsymbol{W}} + (\bar{\boldsymbol{b}}_M^T + \widetilde{\boldsymbol{b}}_M^T)\widehat{\boldsymbol{P}}_M, \tag{2.10}$$

where

$$\widetilde{\boldsymbol{\lambda}}^T = \bar{\boldsymbol{x}}^T\boldsymbol{\Psi}_D; \qquad \widetilde{\boldsymbol{\Lambda}}^T = \bar{\boldsymbol{x}}^T\boldsymbol{\Psi}_D\boldsymbol{A}; \qquad \widetilde{\boldsymbol{b}}_M^T = \bar{\boldsymbol{b}}_D^T\boldsymbol{\Psi}_D\boldsymbol{\Gamma}$$

[10]The converse is not true since real shocks can affect nominal variables. See Végh (2013) chapter 5, especially footnote 11.

Proof. See Appendix A.1. ☐

The preceding expression highlights how opening up the economy to goods trade and introducing a production network structure alter the usual prediction of closed economy models. I now proceed with some illustrations that provide intuition for this expression.

Illustration 1: Closed economy. The following proposition characterizes CPI in a closed economy.

Proposition 2. *In a closed economy, equation (2.10) reduces to*

$$\widehat{P} = -\boldsymbol{\lambda}^T \widehat{\boldsymbol{Z}} + \boldsymbol{\Lambda}^T \widehat{\boldsymbol{W}}.$$

Proof. See Appendix A.2. ☐

Proposition 2 shows the exact form of changes in the CPI in a closed economy (see Baqaee and Farhi, 2022b). Intuitively, CPI changes comprise a weighted average of changes in productivity (weighted by the Domar weights, $\boldsymbol{\lambda}$) and factor prices (weighted by the factor shares, $\boldsymbol{\Lambda}$).

Equation (2.10) extends this for small open economies with production networks. There are four differences between the closed economy expression and equation (2.10). First, of course, inflation now depends on import price changes. Second, the Domar weights and factor shares in equation (2.10) are now based on expenditure rather than on nominal GDP. This distinction arises in small open economies that feature trade imbalances, in which the income from domestic production need not equal what they consume. Since domestic consumer spending matters most for the CPI, nominal expenditure is the relevant object for dividing sales and factor payments.

Third, the effect of sectoral productivity changes on the CPI is dampened relative to a closed economy or a small open economy without production networks. To see this, note that the relevant statistic for the effect of sectoral productivity changes on the CPI is $\bar{\boldsymbol{\lambda}}^T - \widetilde{\boldsymbol{\lambda}}^T$; thus, the Domar weight $\bar{\boldsymbol{\lambda}}^T$ is no longer the sufficient statistic for understanding how sectoral productivity changes affect the CPI. Importantly, the relevant elasticity requires adjusting the expenditure-based Domar weight $\bar{\boldsymbol{\lambda}}$ by subtracting $\widetilde{\boldsymbol{\lambda}}^T = \bar{\boldsymbol{x}}^T \boldsymbol{\Psi}_D$. This adjustment is based on the importance of the domestic consumer's exposure to changes in sectoral productivity.

To be precise, I write the price index as a function of domestic and imported goods prices; that is, $P = \mathcal{P}(\boldsymbol{P}_D, \boldsymbol{P}_M)$. Suppose there is a change in the productivity of sector k, $\widehat{Z}_k$, with no changes in factor or import prices. This shock impacts all domestic goods prices due to input-output linkages. Its propagation to the CPI is a tale of two elasticities. First, how exposed is the consumer to changes in domestic goods prices $\frac{\partial \log \mathcal{P}}{\partial \log P_i}$, for all i? According to the envelope theorem, this elasticity is simply the consumption share of the good at the initial equilibrium, $\bar{b}_i$. Second, the impact depends on how productivity passes through to each domestic good price, $\frac{\partial \log P_i}{\partial \log Z_k}$. This last term is simply given by $-\Psi_{ik}$, which measures the sensitivity of the price of good i to a change in productivity of sector k after taking into account all direct and indirect linkages via the production network. Collecting all these pieces, we can write

$$\widehat{P} = \sum_{i \in N} \underbrace{\frac{\partial \log \mathcal{P}}{\partial \log P_i}}_{=\bar{b}_i} \underbrace{\frac{\partial \log P_i}{\partial \log Z_k}}_{=-\Psi_{ik}} \widehat{Z}_k = -\bar{b}_D^T \boldsymbol{\Psi}_{(:,\,k)} \widehat{Z}_k,$$

where $\boldsymbol{\Psi}_{(:,\,k)}$ is the kth column of the Leontief-inverse matrix $\boldsymbol{\Psi}$. Note that, again, the reason why $\bar{\boldsymbol{b}}_D^T \boldsymbol{\Psi}$ is not equivalent to the vector of sales share is precisely because this is not the relevant exposure of the domestic consumer in the presence of input-output linkages and international trade.

Third, the effect of factor prices on the CPI is also dampened relative to the closed economy benchmark or a small open economy without production networks. Although the logic is similar to that of how productivity changes pass into the CPI, I analyze the factor price case in detail in the next example, as it also allows me to relate equation (2.10) to a well-known concept in the trade literature: the factor content of exports.

Illustration 2: Domestic factor demand and the factor content of exports. This example illustrates how exports' use of domestic factors lowers the sensitivity of prices to changes in domestic factor prices. Equation (2.10) highlights a tension between domestic factor demand and the factor content of exports, in the spirit of Adao et al. (2022). When an economy exports, some of its factors of production end up meeting foreign demand, which, everything else being equal, reduces *domestic factor demand*. These factors meet foreign demand because they are used to produce domestic goods that are exported. As a result, factor price changes put less pressure on the price index.[11] Moreover, this channel is in place whenever an economy exports to the rest of the world, even if there is no production network. To see this, notice that factor payments to a given factor f can be written as

$$W_f L_f = \sum_{i \in N} W_f L_{if} = \sum_{i \in N} a_{if} \lambda_i. \tag{2.11}$$

[11] Though the factor content of exports is already well known in the trade literature, I am unaware of previous work linking this precise notion to inflation.

Without intermediate inputs, the Domar weight of each sector, λ_i, is simply that sector's share in total final demand:

$$Q_i = C_i + X_i \Longrightarrow \lambda_i = b_i + x_i. \tag{2.12}$$

Combining equations (2.11) and (2.12) yields

$$W_f L_f - \underbrace{\sum_{i\in N} a_{if} x_i}_{\text{Factor Content of Exports}} = \underbrace{\sum_{i\in N} a_{if} b_i}_{\text{Domestic Factor Demand}}. \tag{2.13}$$

This equation shows the tension: A rise in exports—higher x_i—must be balanced by a fall in domestic factor demand on the right-hand side, conditional on aggregate payments to factor f being constant. This is one of the mechanisms through which exports cause domestic factor prices to put less pressure on domestic consumer prices.

Allowing for a production network and trade, sectors that do not export much directly (that is, have low x_i) could end up exporting indirectly via other producers. In the particular case I analyzed earlier, without a production network, $\boldsymbol{\Omega} = \mathbf{0}_{N\times N}$ and thus $\boldsymbol{\Psi_D} = \boldsymbol{I}$. This suggests that in the presence of intermediate input linkages, what matters for the price index changes is not only how much each sector exports directly, $\boldsymbol{x}^T$, but also how much it exports indirectly through intermediate input linkages, $\boldsymbol{x}^T\boldsymbol{\Psi}_D$ (see Dhyne et al., 2021). This mechanism also affects how much each factor ends up being exported and how much factor price changes are passed through to the CPI, since $\boldsymbol{x}^T\boldsymbol{\Psi}_D\boldsymbol{A}$ represents the factor content of exports when there are intermediate input linkages across sectors and -$\left(\bar{\boldsymbol{\lambda}}^T - \bar{\boldsymbol{x}}^T\boldsymbol{\Psi}_D\right)$ is the relevant Domar weight for the pass-through of sectoral technology shocks to inflation.

Illustration 3: Import price changes with intersectoral linkages and the network-adjusted import consumption share. This example illustrates that intersectoral linkages amplify the influence of import price changes on inflation. In the presence of intermediate input linkages and imported intermediate inputs, the *direct* import consumption shares $\bar{\boldsymbol{b}}_M^T$ are *not* a sufficient statistic for the effect of import prices on the CPI. To see this, fix factor prices and assume no productivity shocks, $\widehat{\boldsymbol{W}} = \mathbf{0}_F$ and $\widehat{\boldsymbol{Z}} = \mathbf{0}_N$. Then

$$\widehat{P} = \underbrace{\left(\bar{\boldsymbol{b}}_M^T + \bar{\boldsymbol{b}}_D^T\boldsymbol{\Psi}_D\boldsymbol{\Gamma}\right)}_{\text{Network-adjusted import consumption share}} \widehat{\boldsymbol{P}}_M.$$

As was the case for the factor content of exports, this equation shows the importance of *network-adjusted import consumption shares.* While domestic consumers purchase imports directly as final consumption ($\bar{\boldsymbol{b}}_M$), they also consume imports indirectly by purchasing domestically produced goods that directly or indirectly use imported intermediate inputs. This channel is captured by the second term on the right-hand side, $\bar{\boldsymbol{b}}_D^T \boldsymbol{\Psi}_D \boldsymbol{\Gamma}$, which captures the total import content of each domestically produced good when we account for intermediate input linkages. Intuitively, a rise in the price of import good m raises the marginal cost of a given producer h by Γ_{hm}. This rise in the marginal cost implies that P_h rises. This increase in P_h, through intermediate input linkages, raises the price of (say) good i by Ψ_{ih}, which denotes the exposure of sector i to changes in the price of sector h after taking into account intermediate input linkages. This increase in the price of good i, in turn, is passed through to the CPI via $\bar{b}_i$.

Additional Models. I provide two additional models in Appendix B and C. In Appendix B, I provide a detailed two-period model of a small open economy to show that the simplified model presented here shares the same intuition. Drawing on Baqaee and Farhi (2022b), who in turn build on Krugman (1998) and Eggertsson and Krugman (2012), the key idea is that a dynamic problem can be separated into two sub-periods: the present and the future. All action happens in the present, while the future can be taken as given. Shocks occur during the present and last only for that period, whereas in the "future," the economy returns to its initial no-shock equilibrium. Conditional on this interpretation, the model in this section is isomorphic to a multi-period model.

In Appendix C, I provide a three-sector (exportable, importable, and non-tradable) canonical small open economy dynamic model, as in Chapter 8 of Uribe and Schmitt-Grohé (2017). There, I embed a production network structure and show that the results also hold in that environment. In working out these additional models, I contribute to the literature by effectively embedding production networks into a small open economy setup and studying its consequences for inflation.

2.5 An Alternative Representation of Factor Markets: From Factor Prices to Factor Supplies

Factor price changes on the right-hand side of the equation (2.10) are exogenous and thus can be considered primitives in my exercise. However, typical neoclassical models treat factor prices as endogenous and factor supply as exogenous. Writing the problem by considering factor prices as given simplified the intuition for the main result of this paper. It also allowed me to differentiate the proximate causes of inflation between my model and a closed economy with or without production networks.[12] I now show that the same intuition holds if the ordering is reversed, treating factor prices as endogenous outcomes and factor supplies as exogenous objects.

2.5.1 Solving in terms of factor supply quantities

The key difference when solving for factor prices as endogenous objects is that one must introduce factor market clearing conditions as

$$\sum_{i \in N} L_{if} = \bar{L}_f \quad \forall f \in F,$$

where the left-hand side is factor demand, and the right-hand side is factor supply. In what follows, I assume that factor supplies, $\bar{L}_f$, are exogenous.

Recall that the expenditure-based share of factor f can be written as

$$\bar{\Lambda}_f = \frac{W_f \bar{L}_f}{\mathcal{M}},$$

where I have imposed the factor market clearing condition and the cash-in-advance constraint.

Thus, changes in factor prices can be written as

$$\widehat{W}_f = \widehat{\bar{\Lambda}}_f + \widehat{\mathcal{M}} - \widehat{\bar{L}}_f, \tag{2.14}$$

which in vector form is

$$\widehat{\boldsymbol{W}} = \widehat{\bar{\boldsymbol{\Lambda}}} + \mathbf{1}_F \widehat{\mathcal{M}} - \widehat{\bar{\boldsymbol{L}}}.$$

Intuitively, factor prices can increase because (1) demand is *reallocated* toward that factor, as captured by $\widehat{\bar{\boldsymbol{\Lambda}}}$; (2) aggregate demand is increasing ($\widehat{\mathcal{M}}$); or (3) there is a decrease in (inelastic) factor supply ($\widehat{\bar{\boldsymbol{L}}}$). As shown in the following proposition, this decomposition allows me to write changes in the price index as a function of sectoral and aggregate shocks and also changes in these expenditure-based factor shares.

Proposition 3. *Consider a perturbation* $(\widehat{\mathcal{M}}, \mathrm{d}T, \widehat{\boldsymbol{Z}}, \widehat{\boldsymbol{P}}_M, \widehat{\boldsymbol{X}}, \widehat{\bar{\boldsymbol{L}}})$ *around some initial equilib-*

[12] This is also the method that Baqaee and Farhi (2019a) use when studying aggregation in disaggregated economies via *aggregate cost functions* rather than aggregate production functions.

rium. Up to a first order, changes in the aggregate price index, $\widehat{P}$, satisfy

$$\widehat{P} = -\left(\bar{\boldsymbol{\lambda}}^T - \tilde{\boldsymbol{\lambda}}^T\right)\widehat{\boldsymbol{Z}} - \underbrace{\tilde{\boldsymbol{\Lambda}}^T\widehat{\bar{\boldsymbol{\Lambda}}} - \left(\bar{\boldsymbol{\Lambda}}^T - \tilde{\boldsymbol{\Lambda}}^T\right)\widehat{\bar{\boldsymbol{L}}} + \frac{\mathrm{d}T}{\mathcal{M}} + \left(1 - \tilde{\boldsymbol{\Lambda}}^T\mathbf{1}_F\right)\widehat{\mathcal{M}}}_{\textit{Factor Price Changes}}$$
$$+ \left(\bar{\boldsymbol{b}}_M^T + \tilde{\boldsymbol{b}}_M^T\right)\widehat{\boldsymbol{P}}_M. \tag{2.15}$$

Proof. See Appendix A.3. □

Proposition 3 is an *ex post* sufficient statistics result in the spirit of Baqaee and Farhi (2022a) since there is still one endogenous vector that needs to be solved for, namely $\widehat{\bar{\boldsymbol{\Lambda}}}$. Conditional on knowing this vector, one can compute the response of the CPI to changes in the other primitives. Note that in a closed economy, this term would be zero, since in this case $\tilde{\boldsymbol{\Lambda}} = \mathbf{0}_F$ and thus factor share reallocation would not have any first-order effect on inflation.

Note that the only difference relative to the model with exogenous factor price changes is that factor price changes are now being mapped to other exogenous objects $(\widehat{\mathcal{M}}, \mathrm{d}T, \widehat{\bar{\boldsymbol{L}}})$. As in Baqaee and Farhi (2022b), decreases in factor supply are inflationary because they increase factor prices conditional on factor demands. The impact on inflation of money supply changes, $\widehat{\mathcal{M}}$, is dampened relative to the closed economy because factor prices have less pass through to inflation. Increases in net transfers to the rest of the world, $\mathrm{d}T$, also increase CPI inflation because, conditional on money supply, they increase nominal GDP and thus increase factor prices. In this sense, factor price changes are sufficient statistics for how money supply and net transfer changes affect the CPI.

A few additional remarks regarding Proposition 3 are in order. First, note that sectoral export demands, $\boldsymbol{X}$, do not appear directly in this equation. This means that they can affect inflation only through its effect on $\widehat{\bar{\boldsymbol{\Lambda}}}$. Second, as I show in Appendix D, $\widehat{\bar{\boldsymbol{\Lambda}}}$ can be found by solving a linear system of equations. This system of equations depends on primitives, the production network structure, and the elasticities of substitution for producers and consumers. Thus, solving for $\widehat{\bar{\boldsymbol{\Lambda}}}$ requires one to take a stand on the values of elasticities of substitution of producers across different inputs and of consumers across different goods. Perhaps more importantly, through this endogenous vector, elasticities of substitution matter to a first order for CPI inflation in small open economies. Hence, even with a simple, sufficient statistics framework, elasticities of substitution matter to a first order for inflation in small open economies, a result that contrasts with the closed economy benchmark.

3 The Empirical Relevance of Adjustments of CPI Elasticity

This section examines the quantitative relevance of the proposed production network adjustments for inflation across small open economies. I start by describing the data sources and how I classify countries as small open economies. I then present three different results. First, I focus on the network-adjusted domestic consumption shares, which are the relevant elasticities for the pass-through of sectoral technology shocks to inflation. Second, I examine the adjustments to labor shares once I account for indirect exports. Finally, I compare direct and network-adjusted import consumption shares.

3.1 Data

Although network-adjusted shares require more information than sales and factor shares, they are still easily computable from available data. In this subsection, I briefly describe the necessary data to compute them.

Input-Output Tables. I calculate the objects $(\boldsymbol{\Omega}, \bar{\boldsymbol{x}}, \bar{\boldsymbol{b}}_D, \bar{\boldsymbol{b}}_M, \bar{\boldsymbol{\lambda}})$ using domestic input-output tables from the 2016 World Input-Output Database release, the latest available.

Penn World Tables (PWT). I use version PWT 10.01. This data set contains income, input, output, and productivity information between 1950 and 2019 for 183 countries. This database is freely available to download at https://www.rug.nl/ggdc/productivity/pwt/?lang=en.

Using this database, I construct two measures. First, I denote the share of world GDP accounted for by country c as α_c. Formally,

$$\alpha_c = \frac{nGDP_c}{nGDP_W}, \quad nGDP_W = \sum_{c \in C} nGDP_c.$$

I measure $nGDP_c$ using the series *cgdpo*, which corresponds to the output-side real GDP at current purchasing power parity (PPP) dollars (in 2017 US$ millions).

To measure trade openness, I use the series *csh_x* and *csh_m* in the PWT. The first corresponds to the ratio of merchandise exports over nominal GDP, while the second corresponds to imports over nominal GDP at current PPPs. I define the trade openness of country c as

$$\text{Openness}_c = \frac{\text{Exports}_c + \text{Imports}_c}{nGDP_c} = csh\_x_c - csh\_m_c,$$

where the last line follows since in the data $csh\_m_c = -\frac{\text{Imports}_c}{nGDP_c}$.

Classifying Small Open Economies. I apply two criteria to separate countries into small and non-small open economies according to the data. First, an economy is *small* if $\alpha_c \leq 0.05$. Second, an economy is *open* if $\text{Openness}_c \geq 0.3$. A country is a small open economy if it satisfies both conditions.

3.2 Results

In this subsection, I compare the network-adjusted objects with their closed economy and no-network-adjustment counterparts whenever possible. All cross-sectional plots are based on the year 2014 unless stated otherwise.

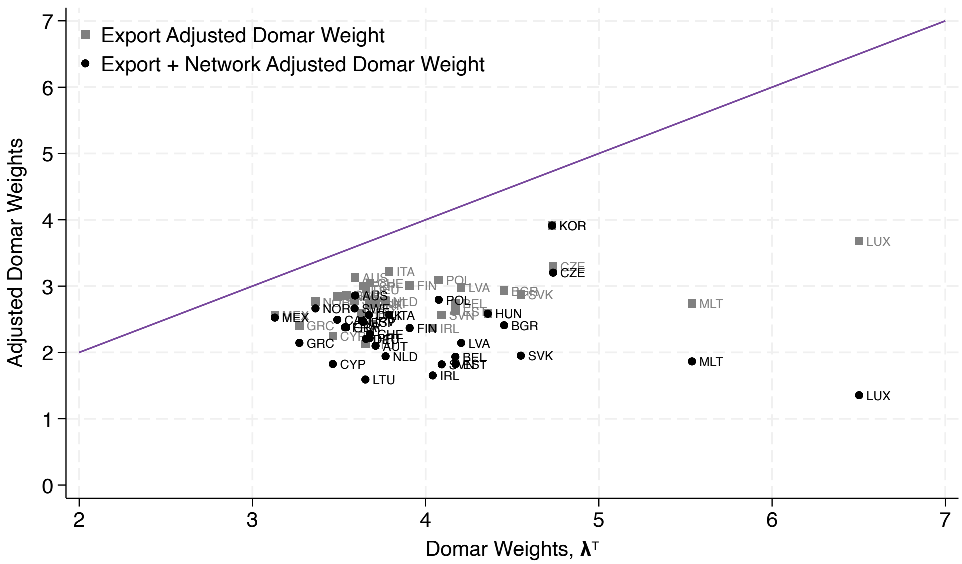
3.2.1 Network-adjusted domestic consumption shares

I start by showing results for network-adjusted domestic consumption shares $\bar{\boldsymbol{\lambda}} - \bar{\boldsymbol{x}}^T \boldsymbol{\Psi}_D$. Figure 2 shows three scenarios for the average sector in small open economies in panel (a) and for non-small open economies in panel (b). The x-axis shows the unadjusted Domar weights, while the y-axis shows the adjusted objects. Light squares in these figures refer to the export-adjusted weights, while dark points are export-network-adjusted weights. Thus, the dark points in these plots are the network-adjusted domestic consumption shares. As the figure shows, the adjustments are stronger for small open economies than non-small open economies. Moreover, the average Domar weight in small open economies is about 4 percent; it decreases to about 2.84 percent with the export adjustment and to about 2.31 percent with the network-adjusted exports. This is a non-negligible change, suggesting that the inflation impact of a given sized sectoral productivity shock in an average-sized sector will be dampened by about 50 percent for the average small open economy relative to the closed economy benchmark.

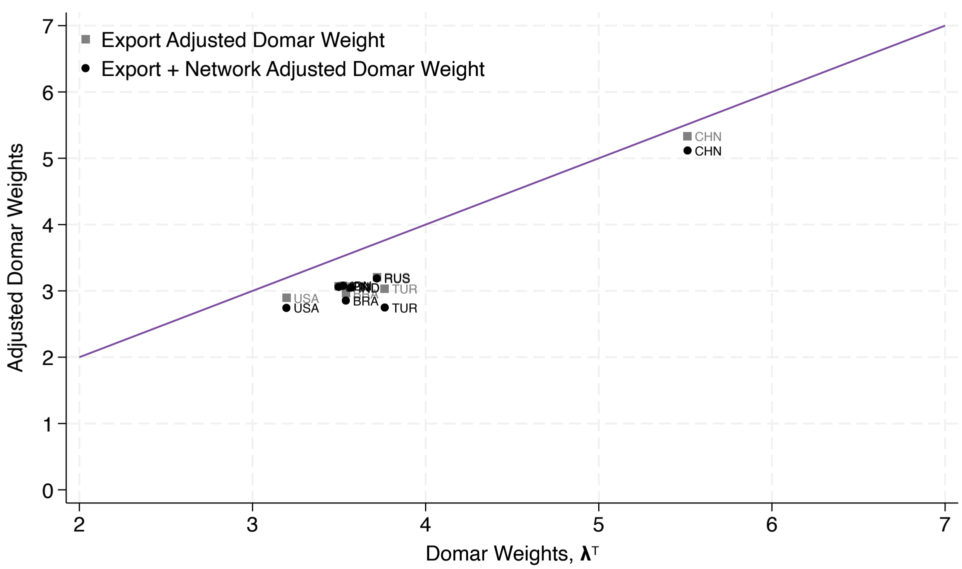
To provide a more concrete example, Figure 3 shows the three sectors for which network export adjustment is the largest in the United Kingdom: administrative support, legal and accounting, and electricity, gas, and water. The latter sector is illustrative. Its Domar weight is about 5.95 percent. This number decreases only to 5.9 percent when direct exports are subtracted. For all practical purposes, this means this sector is non-tradable. When indirect exports are taken into account, however, the network-adjusted consumption share decreases to 4.4 percent. This illustrates how indirect linkages are quantitatively relevant and provide information beyond the direct export share.

Figure 2. Export- and Network-Export-Adjusted Domar Weights

(a) Small Open Economies

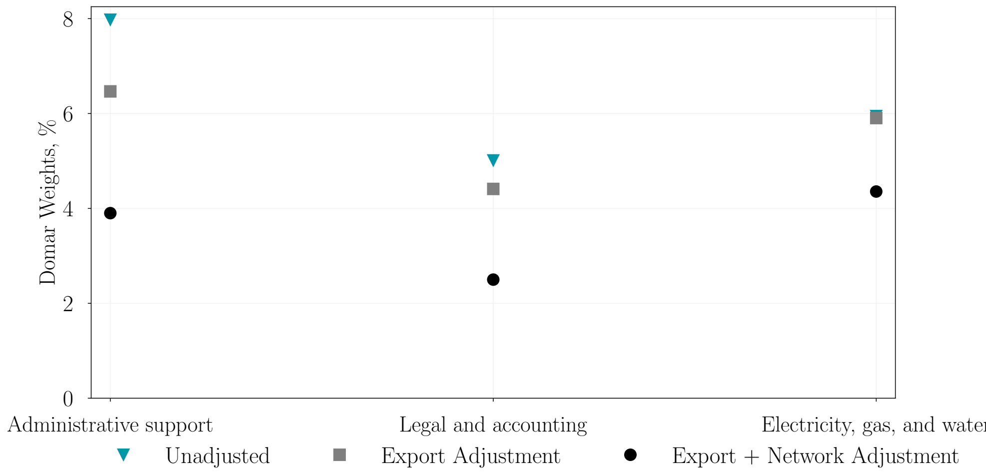


(b) Non-Small Open Economies



Note: This figure shows the average Domar weight for each country. The x-axis corresponds to the average Domar weight computed in the closed economy model, $\boldsymbol{\lambda}^T$. The gray squares subtract only for direct exports; that is, $\bar{\boldsymbol{\lambda}}^T - \bar{\boldsymbol{x}}^T$. The black circles further consider the production network structure, $\bar{\boldsymbol{\lambda}}^T - \bar{\boldsymbol{x}}^T \boldsymbol{\Psi}_D$. Panel (a) shows the results for small open economies, while Panel (b) shows the results for non-small open economies.

Figure 3. Three Sectors with Largest Adjustments: United Kingdom.



Note: This figure shows the three sectors with the largest network-export-adjusted share for the United Kingdom.

Regression framework. Which sectors and countries are most affected by network adjustments? To answer this question, I estimate the following cross-sectional regression:

$$y_{sc} = \alpha_s + \alpha_c + \varepsilon_{cs}, \tag{3.1}$$

where y_{sc} represents the difference between a measure for the small open economy with a production network relative to the small open economy without networks for a given country c and sector s. α_s is a sector-specific fixed effect, α_c is a country-specific fixed effect, and ε_{cs} is an error term. From this regression, I get estimates of sector- and country-specific fixed effects. Notice that these are identified up to a normalization, which in this case is that $\sum_{s \in S} \hat{\alpha}_s = 0$ and $\sum_{c \in C} \hat{\alpha}_c = 0$. All fixed effects are interpreted as deviations from the average fixed effects.

In Panel (a) of Figure 4, I show the country fixed-effects estimates when the left-hand side variable is the difference between the network-adjusted export share and the direct export share. Note that these country fixed-effects estimates show the average difference between these shares—as a fraction of aggregate expenditure—across sectors within a country. The countries with the largest adjustments are Luxembourg, Slovak Republic, Malta, and Latvia, while countries with the smaller adjustments are Korea, Hungary, and Mexico. These numbers indicate that the export sectors of the latter economies do not rely much on inputs from the domestic economy, not exporting much indirectly.[13]

Panel (b) does the same exercise for the sector fixed-effects estimates. These estimates show the average difference between shares across countries within a sector. Financial Ser-

[13] Remember, this does not mean that these countries do not export at all.

vices is the sector with the largest average production network adjustment. Note that the Electricity, Gas, and Water (EGSA) is the seventh sector with the largest difference, while Legal and Accounting is the sixth sector. Thus, the previously cited examples are not specific to the United Kingdom but have consistently large network adjustments across countries. Intuitively, these sectors are important suppliers for domestic sectors that export directly or indirectly.

These exercises suggest that accounting for the production network is important in computing inflation elasticities and that the adjustment varies substantially across countries and sectors.

3.2.2 Network-adjusted domestic factor demand

I now conduct a similar exercise to examine the importance of network adjustments for factor shares. First, I study how the aggregate labor share in different countries varies depending on the export and network export adjustment. I then consider how sector-specific labor shares vary when considering direct and indirect exports.

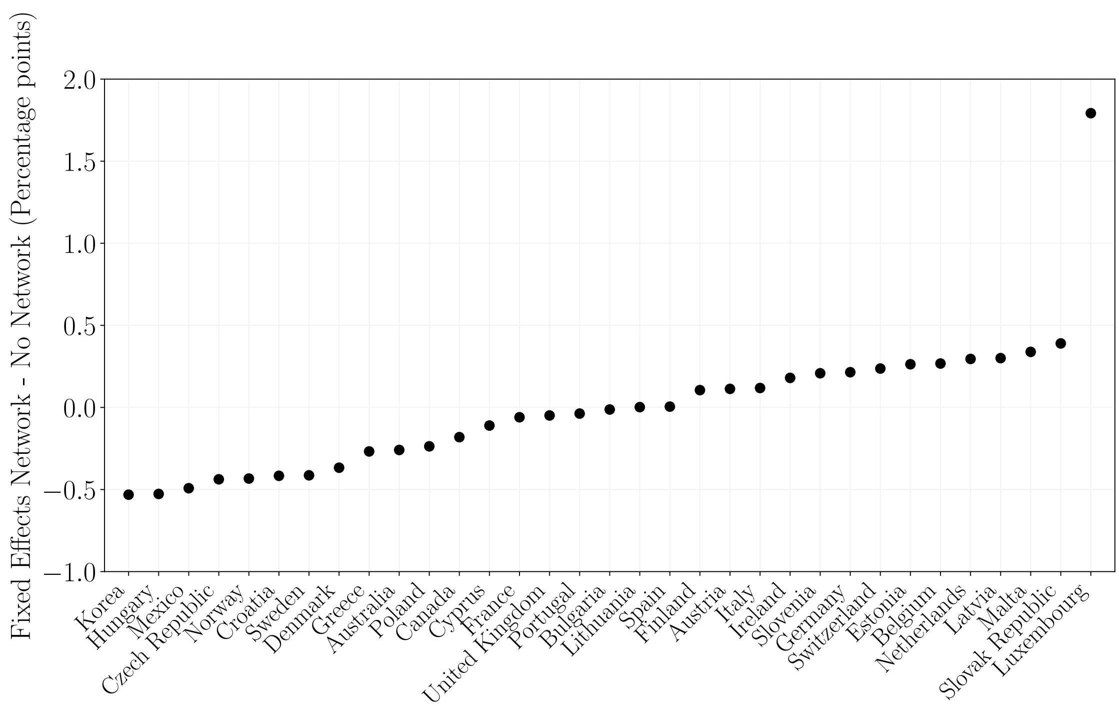
Labor share. Figure 5 shows the labor share for different economies on the x-axis and the network-export-adjusted labor share on the y-axis. Black diamonds show small open economies, and gray circles represent non-small open economies. As the figure illustrates, the adjustments are again significant for small open economies but not for non-small open economies. The average labor share across non-small open economies is 53 percent, while the network-export-adjusted labor share is 50 percent, a negligible change. By contrast, the average labor share in small open economies is about 57 percent, while the network-export-adjusted labor share is only 39 percent. This means the impact of a given wage increase will be 32 percent lower in a small open economy with production networks relative to an otherwise similar closed economy.

Sector-specific labor shares. I now conduct an exercise similar to that of the previous section. Here, I consider the dependent variable to be the difference between the network-adjusted labor content of exports relative to the non-network-adjusted labor content of exports.

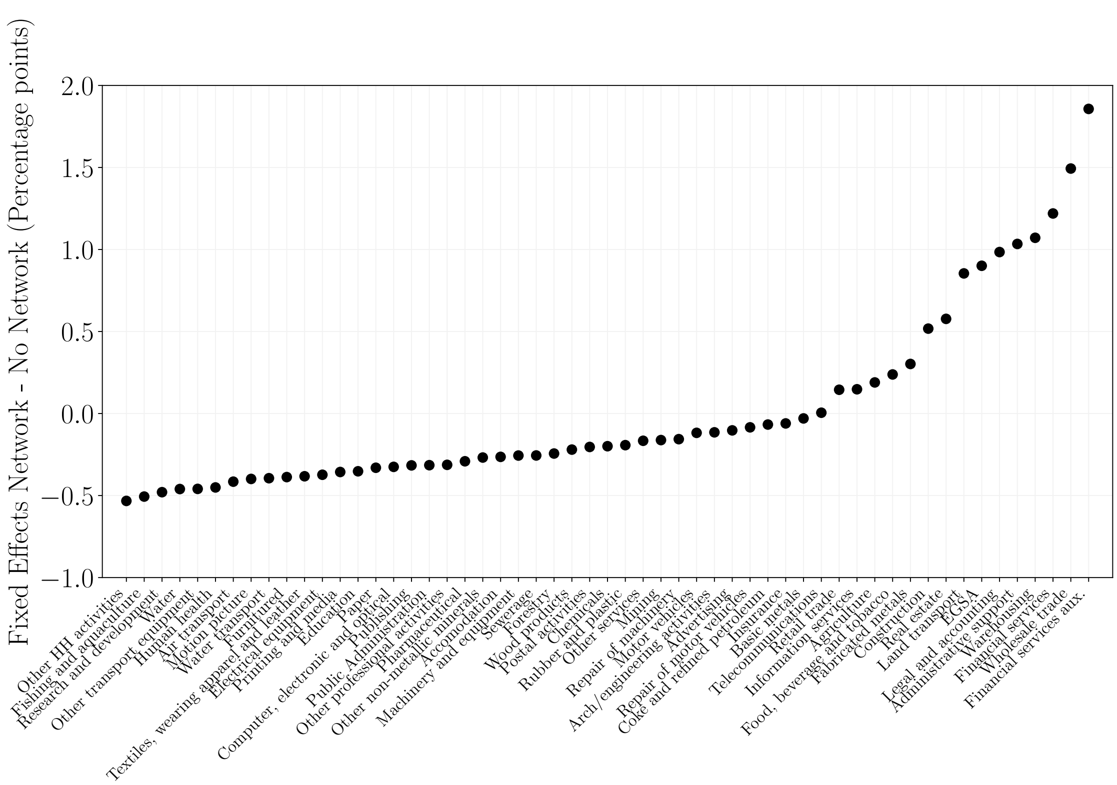
This exercise illustrates the heterogeneity across sectoral labor markets. Before, I considered the aggregate labor share. However, this aggregate labor share is a weighted average of what happens at the sectoral level. It can be a misleading statistic for addressing certain questions, especially in an environment such as COVID-19, where sectoral labor markets were affected differently.

Figure 4. Country and Sector Fixed Effects: Export-Network-Adjusted and Export-Adjusted

(a) Country Fixed Effects

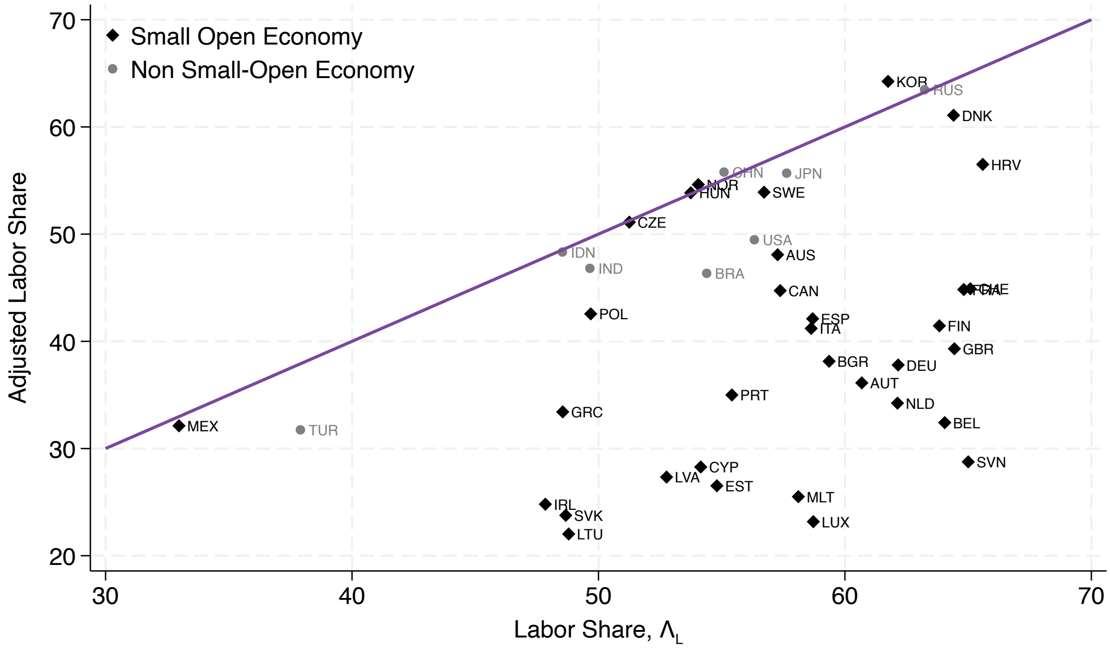


(b) Sector Fixed Effects



Note: This figure shows fixed effects from estimating equation (3.1), where the dependent variable is the difference between the network-adjusted export share and the direct export share. Panel (a) shows country fixed effects estimates, while Panel (b) shows sector fixed effects.

Figure 5. Labor Share Adjustments for Different Countries



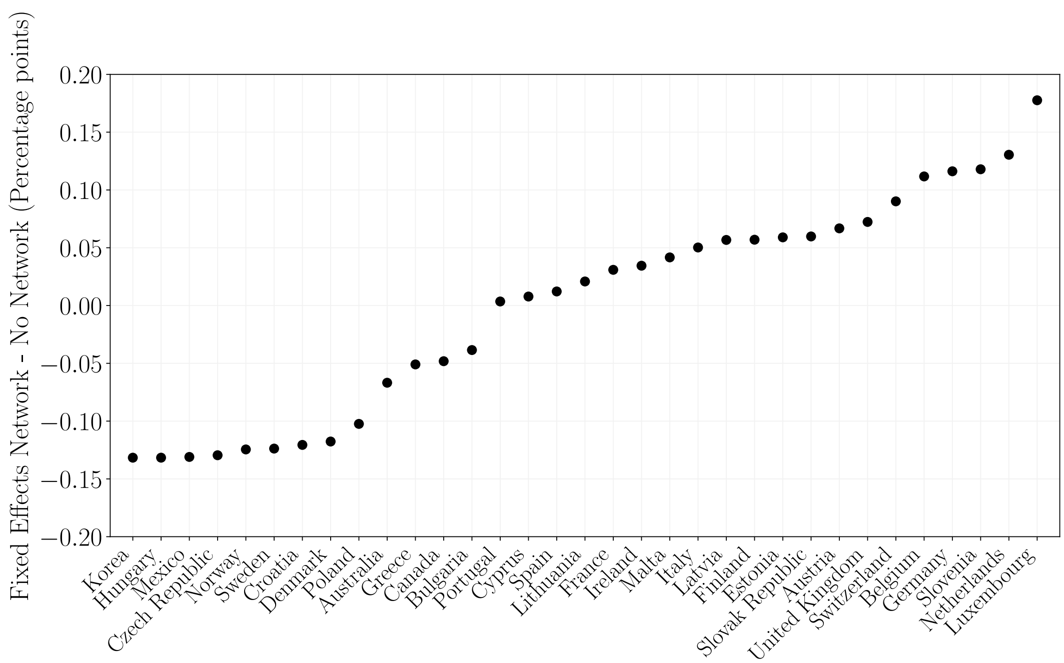
Note: This figure shows the average labor share on the x-axis and the network-export-adjusted labor share for small open economies in black diamonds and non-small open economies in gray circles.

Panel (a) of Figure 6 shows the results for the country fixed effects, while Panel (b) shows the same but for sector fixed effects. Apart from Luxembourg, the ranking differs from the network-adjusted domestic consumption share in Figure 4. Notably, countries where sector-specific labor shares adjusted the most due to the domestic production network are the Netherlands, Slovenia, and Germany, while the ranking at the bottom stays the same. This indicates that Germany exhibits an average production network adjustment of sector-specific labor shares 0.15 percentage points larger than the adjustment for the average country.

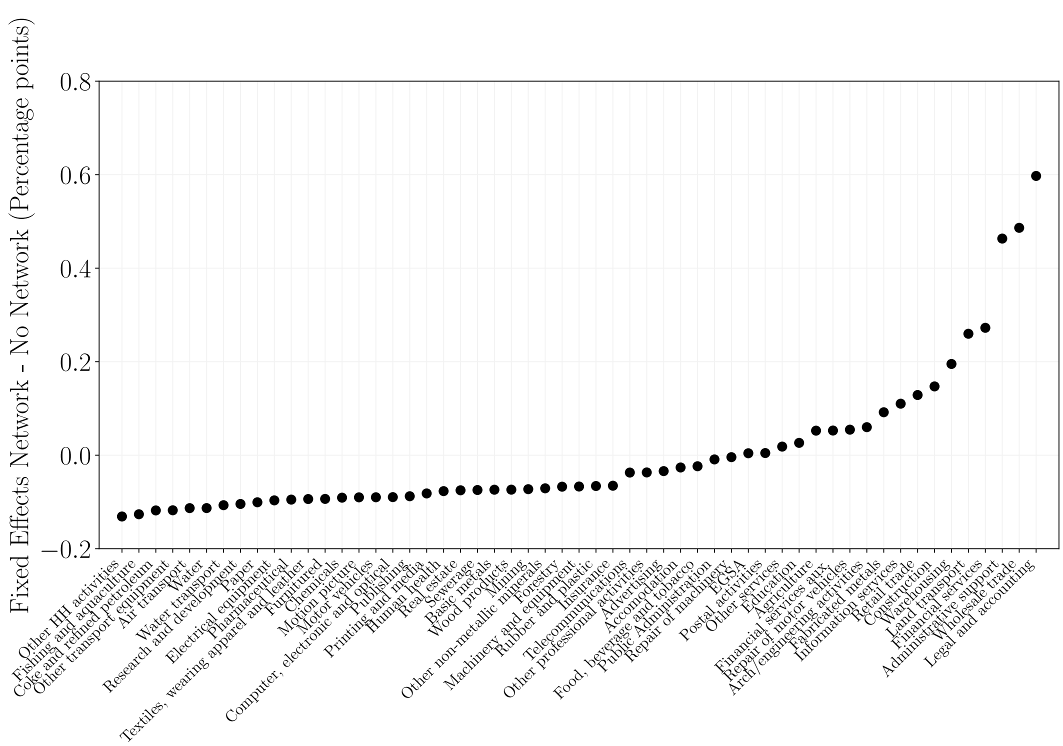
Turning to the sector fixed-effects results, the sectors with the largest production network adjustment are Legal and Accounting, Wholesale Trade, and Administrative Support. Legal and Accounting, for example, has an average adjustment 0.6 percentage points larger than the average sector. Since the 0.6 percentage point is an average across all countries, consider the Legal and Accounting sector in Germany as a concrete example. The share of this sector's labor on nominal GDP is about 2.6 percent of GDP. It decreases to 2.3 percent when exports are subtracted and to 0.8 percent when the domestic production network structure is taken into account. Thus, ignoring the production network adjustment would significantly overstate how much sectoral wage changes pass through into the CPI.

Figure 6. Country and Sector Fixed Effects: Export-Network-Adjusted Sector-Specific Factor Shares

(a) Country Fixed Effects

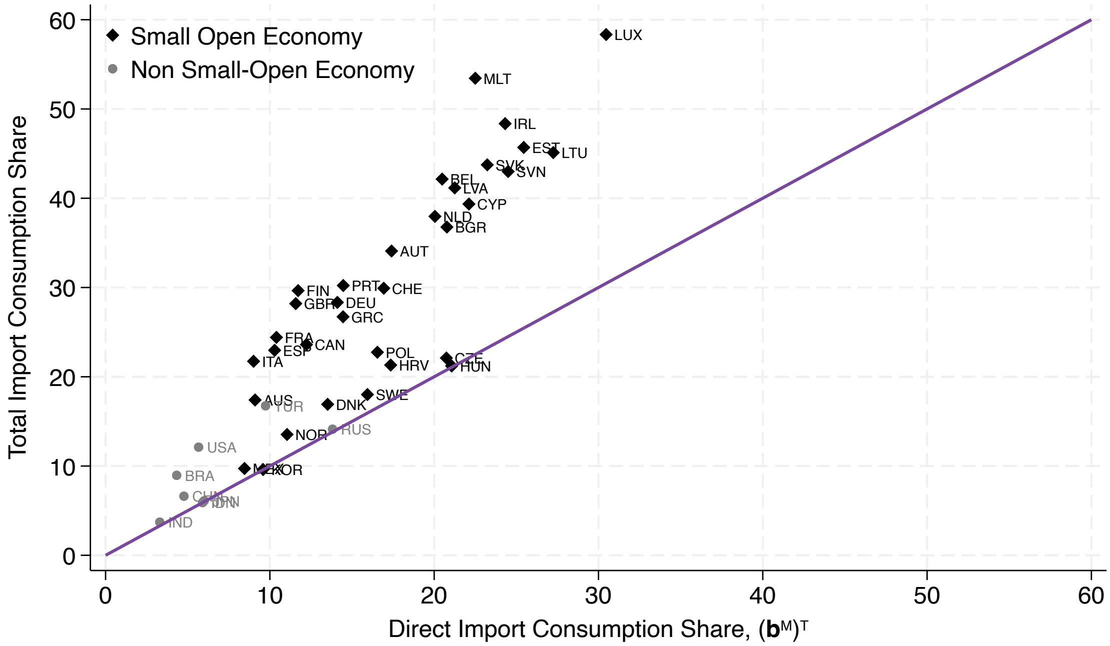


(b) Sector Fixed Effects



Note: This figure shows the fixed effects when the dependent variable is the difference between the network-adjusted sector-specific factor shares and the direct export share-adjusted sector-specific factor shares. Panel (a) shows this difference for the country fixed effects estimates, while Panel (b) does the same for sector fixed effects.

Figure 7. Direct and Network-Adjusted Import Consumption Shares



Note: This figure shows the direct import consumption share on the x-axis and the network-adjusted import consumption share on the y-axis. Small open economies are the black diamonds, and non-small open economies are the gray circles.

3.2.3 Network-adjusted import consumption shares

As a final empirical exercise, I consider import consumption shares. Figure 7 provides a scatterplot of these shares across economies. On the x-axis, I show the direct import consumption share, while on the y-axis, I show the network-adjusted import consumption share. The average direct import consumption share across non-small open economies is 6.7 percent; it increases to 9.3 percent when considering the production network structure. Despite the almost 3 percentage point increase, this change is small relative to the one I find for small open economies. The average direct import consumption share across small open economies is about 17 percent. This number increases to 30 percent when the production network structure is taken into account, representing a 13 percentage point increase. This suggests that the pass-through of import price changes into inflation (almost) doubles when intersectoral linkages are introduced.

4 The Evolution of Inflation in Chile and the United Kingdom during COVID-19

In this section, I use the model to study CPI inflation during the COVID-19 episode in Chile and the United Kingdom.

This empirical examination requires more data relative to the previous section. While the earlier section showed information on the CPI elasticities and compared these across countries and sectors using the WIOT alone, this section requires taking a stand on the processes $(\widehat{\boldsymbol{W}}_t, \widehat{\boldsymbol{P}}_{Mt}, \widehat{\boldsymbol{Z}}_t)$ that are not readily available for most countries worldwide. Therefore, I chose Chile and the United Kingdom, countries with all the necessary information to construct $(\widehat{\boldsymbol{W}}_t, \widehat{\boldsymbol{P}}_{Mt}, \widehat{\boldsymbol{Z}}_t)$ that also belong to the small open economy category.

This is an ex-post exercise using existing data to analyze the past behavior of inflation between 2020 and 2022. Yet, Proposition 1 is helpful for forecasting inflation and is thus a valuable tool for policymakers in small open economies. Provided that we have forecast information on the processes $(\widehat{\boldsymbol{W}}_t, \widehat{\boldsymbol{P}}_{Mt}, \widehat{\boldsymbol{Z}}_t)$, we can combine this information with input-output tables to estimate inflation. The accuracy of this exercise depends on the accuracy of elasticities and of the forecasted series. Throughout this section, I focus on the former, as it is the main point of this paper.

In what follows, I first describe the data. Then, I show how I map the model to the data. Finally, I discuss the results for Chile and the United Kingdom.

4.1 Data

4.1.1 Chile

Input-output tables. Since Chilean data are unavailable from the WIOT, I resort to Chilean National Accounts, using *Compilacion de Referencia* for 2013. The structure is similar to the WIOT's in that it includes information on input-output linkages, final uses, and factor payments. Moreover, it is highly disaggregated, containing information for up to 171 industries. I collapse these data to a 17-sector classification due to data availability on sectoral wages. This 17-sector classification is equivalent to Standard Industrial Classification 2 (SIC2).

Sectoral productivity. The ideal measure of productivity from the model is total factor productivity (TFP). However, TFP measures are hard to come by, especially at high frequencies and at the sectoral level. To circumvent this problem, I proxy sectoral TFP using sectoral labor productivity. I collect data on real GDP for the same 17 sectors and divide by total sectoral employment. Real GDP and sectoral employment data come from the Central Bank of Chile (CBCh) and are available quarterly from 1996 to 2022.

Sectoral wages. I source sectoral nominal wages from the Chilean National Institute of Statistics (INE) series *Indice de Remuneraciones Nominal*. This database is available monthly from January 2016 to December 2022. To be consistent with the productivity data, I collapse these data to a quarterly frequency.

Import prices. I use the import price index available from the CBCh quarterly from 2013 to 2022.

4.1.2 United Kingdom

Input-output tables. I source data from the WIOT domestic tables, as in the previous empirical section. I collapse these input-output tables into 20 industries to be consistent with the data on sectoral wages.

Sectoral productivity. I source data from the UK Office for National Statistics (ONS). I download quarterly estimates of labor productivity from the *Flash Productivity* report,[14] which contains information for up to 17 industries.

Sectoral wages. I source these data from the ONS. In particular, I use the data set `EARN03`, which contains monthly information on average weekly earnings for about 20 industries. This data set is available from 2000 to 2022.

Import prices. I use the import price index from the ONS (series `GD74`, data set `MM22`). This series is available at different frequencies. I use quarterly information from 2009 to 2022.

4.2 Mapping the Model to the Data

Since the model is static, all inherent inflation dynamics combine the dynamics of exogenous variables and their interaction with the CPI elasticities. First, I take all series and compute their level deviations from their value in 2018:Q4. Formally, the sources of variation I feed

[14] These data can be downloaded freely from the ONS website

in to construct implied inflation from the different models take the following form:

$$\hat{y}_t = y_t - y_{2018Q4},$$

where y_t represents (the log) of any time series, and y_{2018Q4} is its value in 2018:Q4. Notice that each vector now has a t subscript since they are all deviations from 2018:Q4 at each time t.

In the preceding equation, I call the deviation $\hat{y}_t$ a "shock." This differs from a structurally identified shock because I feed variation directly from the data, taking it as given. With this caveat in mind and throughout this section, I refer to these $\hat{y}_t$ simply as shocks.

Using this procedure, I construct counterparts to $\boldsymbol{\theta}_t = (\widehat{\boldsymbol{W}}_t, \widehat{\boldsymbol{P}}_{Mt}, \widehat{\boldsymbol{Z}}_t)$ in the model. I measure factor prices as sectoral wages. I assume segmented labor markets such that there are different wages across sectors to capture better the behavior of labor markets during the COVID-19 episode, as highlighted in the recent literature (Baqaee and Farhi, 2022b; di Giovanni et al., 2022, 2023a,b). Since I cannot observe sector-specific prices for other factors, such as capital or land, I assume that those factor prices do not change over the sample period.

CPI inflation in the data π_t, when t refers to a quarter, is

$$\pi_t = \log P_t - \log P_{t-4}.$$

Combining the model and shocks, I have $\widehat{P}_t^{\text{Model}}$ as

$$\widehat{P}_t^{\text{Model}} = -\sum_{i \in N} \mathcal{R}_i^{CPI,Z} \widehat{Z}_{it} + \sum_{f \in F} \mathcal{R}_f^{CPI,W} \widehat{W}_{ft} + \mathcal{R}_M^{CPI,M} \widehat{P}_{Mt}.$$

Note that here, $(\mathcal{R}_i^{CPI,Z}, \mathcal{R}_f^{CPI,W}, \mathcal{R}_M^{CPI,M})$ stand for the responses of the CPI to changes in sectoral technology, factor prices, and import price, respectively. These objects are *model-dependent* and thus will differ when considering the closed economy model, the small open economy model without production networks, and the small open economy with production networks.

Finally, inflation *from the model* is

$$\pi_t^{\text{Model}} = \widehat{P}_t^{\text{Model}} - \widehat{P}_{t-4}^{\text{Model}}.$$

Taking log differences relative to some initial point is the most transparent approach because it does not modify the data much, relative to other alternatives such as standard detrending procedures.

4.3 Results

In this subsection, I compare inflation implied by the models, π_t^{Model} with inflation in the data.

Figures 8 and 9 show inflation in the data and inflation implied by the model for Chile and the United Kingdom for 2020–2022, respectively. To highlight the distinct role of production networks and openness, I consider three models: a closed economy model (Closed, pink triangles), a small open economy without production networks (SOE–No Network, green $*$), and a small open economy with production networks (SOE–Network, orange circles). I plot the model's numbers using symbols rather than lines to emphasize the absence of dynamics within the model apart from those generated by the shocks I am adding.

Although the empirical exercise is fairly simple, it captures the data patterns well and more significantly for the small open economy with a production network in Chile and the United Kingdom.

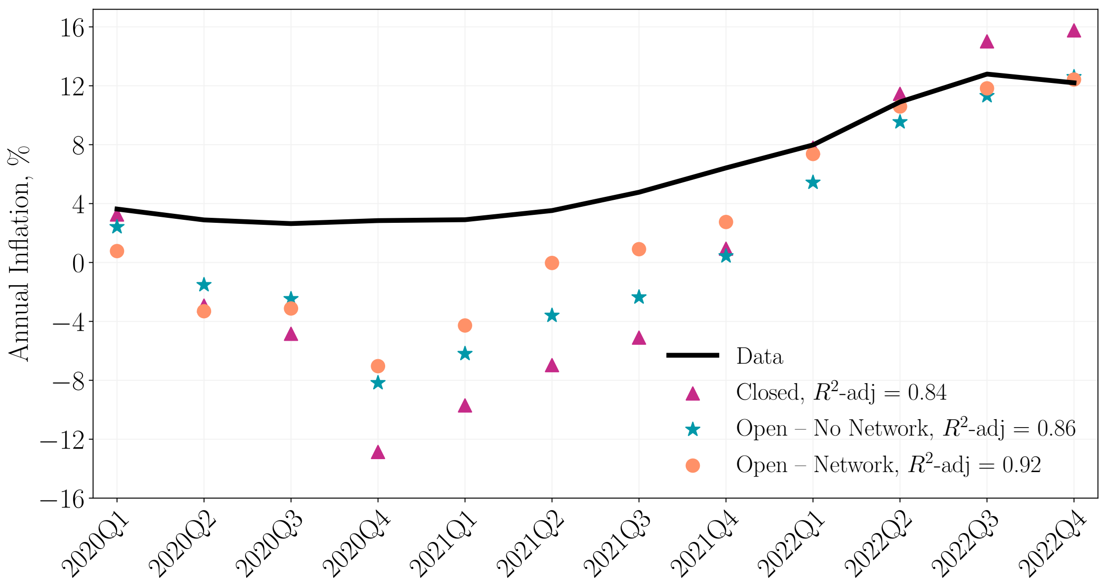
As noted, the model has no intrinsic dynamics: All the action over time comes from the dynamics in $\boldsymbol{\theta}_t$. It is more meaningful to compare the moments implied by the model and those in the data. Table 2 does precisely this and shows the first two moments of inflation in the data and the model. Panel (a) is for Chile, while Panel (b) shows the United Kingdom.

The average annual inflation in Chile between 2020 and 2022 was 6.13 percent, with a standard deviation of 3.89. The closed economy model delivers substantially lower mean inflation (0.98) and higher standard deviation (9.69) relative to the data. The sole introduction of a small open economy aspect, without production networks, quantitatively matters: it exhibits a larger mean relative to the closed economy benchmark (1.45) and a lower standard deviation (6.88). The small open economy with production networks more closely reflects the data, with an average inflation of 2.41 and a standard deviation of 6.67.

In the United Kingdom, the average inflation was 3.69 percent, almost half that of Chile during the same period, with a standard deviation of 3.11. The closed economy benchmark again generates too little average inflation (2.27 vs. 3.31) but now too low a standard deviation (2.57 vs. 3.11). As was the case for Chile, introducing the small open economy aspect quantitatively matters: Inflation is higher on average (2.72) and has a higher standard deviation (2.64). Considering production networks again improves the results: The model exhibits an even higher mean (3.21) and standard deviation (3.00).

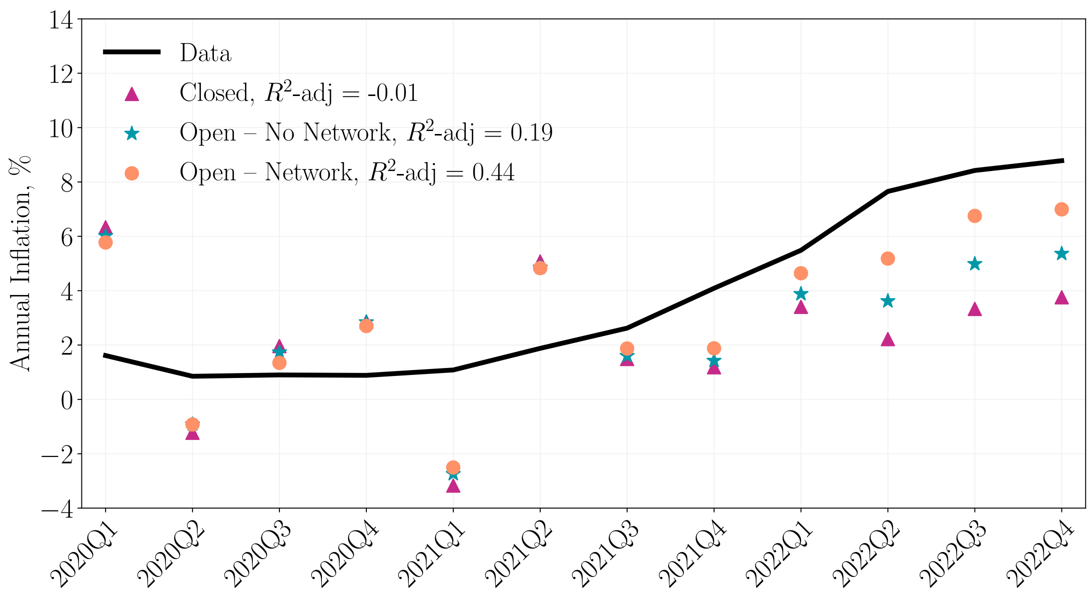
In summary, this exercise suggests that a small open economy model with production networks better matches inflation moments during 2020–2022 than a closed economy model and a small open economy model without production networks. To be fair, the small open economy model with production networks should indeed perform better because it adds a piece of realism missing from these other two models, namely, intersectoral linkages. The

Figure 8. Chile Inflation under Different Models



Note: This figure shows inflation in the data and inflation implied by the different models. The black line represents the data. The pink triangles correspond to the closed economy model. The green $*$ signify the small open economy model without production networks, and the orange circles correspond to the small open economy model with production networks.

Figure 9. UK Inflation under Different Models



Note: This figure shows inflation in the data and inflation implied by the different models. The black line represents the data. The pink triangles correspond to the closed economy model. The green $*$ signify the small open economy model without production networks, and the orange circles correspond to the small open economy model with production networks.

question is how much better?. The evidence here suggests that this is quantitatively relevant. Of course, the stylized model has many missing parts, but remarkably, such a stylized exercise matches these inflation data moments well.

Table 2. Average Inflation, 2020–2022

	Panel (a): Chile		Panel (b): United Kingdom	
	Mean	Std. Dev.	Mean	Std. Dev
Data	6.13	3.89	3.69	3.11
Model				
Closed	0.98	9.69	2.27	2.57
SOE no Network	1.45	6.88	2.72	2.64
SOE - Network	2.41	6.67	3.21	3.00

Note: This table shows the mean and standard deviation of inflation for the data and the different models. The Closed model uses the implied elasticities as if we consider the economies as closed. The SOE–No Network model considers the elasticities in a small open economy that does not feature any production network. Finally, the SOE–Network model accounts for network linkages.

5 Conclusion

I study inflation in small open economies with production networks. Theoretically and empirically, I show that production networks matter for the effect of sectoral technology shocks, factor prices, and import prices on CPI inflation. I argue that the interaction of trade and production networks matters because opening up the economy is one way to break the equivalence between what is produced within borders and what is consumed by the domestic consumer, for whom the CPI is relevant. With that relationship broken, the production network amplifies this discrepancy via indirect linkages: Non-exporters become indirect exporters, while non-importers become indirect importers. This ultimately affects the exposure of the domestic consumer to a different set of shocks. The production network thus has a first-order impact on inflation, with sales and factor shares no longer sufficient statistics for studying how changes in sectoral technology or factor prices pass through into inflation, as would be the case in a closed economy.

In a small open economy with production networks, indirect exporting dampens domestic shocks relative to an otherwise equivalent closed economy. Foreign shocks, such as import price shocks, are amplified relative to an otherwise equivalent small open economy without production networks. Which channels dominate at the aggregate level depends on the production network structure and is, in the end, a quantitative question. I show that the production network adjustments on both the export and import side matter quantitatively across a set of small open economies. I apply the small open economy model with production networks to the recent inflationary episode in Chile and the United Kingdom. Including production networks helps to better match the mean and variance of inflation in those countries during the 2020–2022 period.

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A Proofs

A.1 Proof of Proposition 1

By definition, changes in the aggregate price index satisfy

$$\widehat{P} = \bar{\boldsymbol{b}}_D^T \widehat{\boldsymbol{P}}_D + \bar{\boldsymbol{b}}_M^T \widehat{\boldsymbol{P}}_M. \tag{A.1}$$

By Shephard's lemma, the vector of domestic prices can be written as

$$\widehat{\boldsymbol{P}}_D = -\widehat{\boldsymbol{Z}} + \boldsymbol{A}\widehat{\boldsymbol{W}} + \boldsymbol{\Omega}\widehat{\boldsymbol{P}}_D + \boldsymbol{\Psi}_D \boldsymbol{\Gamma} \widehat{\boldsymbol{P}}_M,$$

where this result follows since in equilibrium, $MC_i = P_i$ for all $i = 1, 2, ..., N$.

Inverting this system yields

$$\widehat{\boldsymbol{P}}_D = -\boldsymbol{\Psi}_D \widehat{\boldsymbol{Z}} + \boldsymbol{\Psi}_D \boldsymbol{A}\widehat{\boldsymbol{W}} + \boldsymbol{\Psi}_D \boldsymbol{\Gamma} \widehat{\boldsymbol{P}}_M.$$

Using the definitions and equilibrium results

$$\bar{\boldsymbol{b}}_D^T \widehat{\boldsymbol{P}}_D = \bar{\boldsymbol{b}}_D^T \left[-\boldsymbol{\Psi}_D \widehat{\boldsymbol{Z}} + \boldsymbol{\Psi}_D \boldsymbol{A}\widehat{\boldsymbol{W}} + \boldsymbol{\Psi}_D \boldsymbol{\Gamma} \widehat{\boldsymbol{P}}_M \right]. \tag{A.2}$$

Thus, CPI changes can be written as

$$\widehat{P} = -\bar{\boldsymbol{b}}_D^T \boldsymbol{\Psi}_D \widehat{\boldsymbol{Z}} + \bar{\boldsymbol{b}}_D^T \boldsymbol{\Psi}_D \boldsymbol{A}\widehat{\boldsymbol{W}} + (\bar{\boldsymbol{b}}_M^T + \bar{\boldsymbol{b}}_D^T \boldsymbol{\Psi}_D \boldsymbol{\Gamma}) \widehat{\boldsymbol{P}}_M. \tag{A.3}$$

To derive the expression in the text, simply note that the goods market-clearing condition and the definition of factor shares can be written as

$$\begin{aligned}
\bar{\boldsymbol{\lambda}}^T &= (\bar{\boldsymbol{b}}_D^T + \bar{\boldsymbol{x}}^T)\boldsymbol{\Psi}_D \Longrightarrow \bar{\boldsymbol{b}}_D^T \boldsymbol{\Psi}_D = \bar{\boldsymbol{\lambda}}^T - \bar{\boldsymbol{x}}^T \boldsymbol{\Psi}_D \\
\bar{\boldsymbol{\Lambda}}^T &= \bar{\boldsymbol{\lambda}}^T \boldsymbol{A} \Longrightarrow \bar{\boldsymbol{b}}_D^T \boldsymbol{\Psi}_D \boldsymbol{A} = \bar{\boldsymbol{\Lambda}}^T - \bar{\boldsymbol{x}}^T \boldsymbol{\Psi}_D \boldsymbol{A}.
\end{aligned}$$

Replacing these expressions with changes in the CPI index yields the result.

A.2 Proof of Proposition 2

To prove this proposition, notice that Hulten's theorem in an efficient closed economy with inelastic factor supplies implies that changes in real GDP must satisfy

$$\widehat{Y} = \boldsymbol{\lambda}^T \widehat{\boldsymbol{Z}} + \boldsymbol{\Lambda}^T \widehat{\boldsymbol{L}}.$$

In turn, changes in factor supply must comply with$\widehat{\boldsymbol{L}} = -\widehat{\boldsymbol{W}} + \widehat{nGDP}\mathbf{1}_F + \widehat{\boldsymbol{\Lambda}}$, which upon replacing the preceding expression yields

$$\widehat{Y} = \boldsymbol{\lambda}^T \widehat{\boldsymbol{Z}} + \widehat{nGDP} - \boldsymbol{\Lambda}^T \widehat{\boldsymbol{W}}.$$

The definition of changes in nominal GDP has a quantity and a price component. The quantity component refers to $\widehat{Y}$ (real GDP). The price component is the GDP deflator, which is equivalent to the CPI in a closed economy[15]. This allows me to write changes in nominal GDP as

$$\widehat{nGDP} = \widehat{Y} + \widehat{CPI} \Longrightarrow \widehat{CPI} = \widehat{nGDP} - \widehat{Y}.$$

Combining the previous changes in real GDP as a function of productivity, factor prices, and nominal GDP yields

$$\widehat{CPI} = \widehat{nGDP} - (\boldsymbol{\lambda}^T \widehat{\boldsymbol{Z}} + \widehat{nGDP} - \boldsymbol{\Lambda}^T \widehat{\boldsymbol{W}}) = -\boldsymbol{\lambda}^T \widehat{\boldsymbol{Z}} + \boldsymbol{\Lambda}^T \widehat{\boldsymbol{W}},$$

which was the desired expression.

A.3 Proof of Proposition 3

As stated in the text,

$$\widehat{\boldsymbol{W}} = \widehat{\bar{\boldsymbol{\Lambda}}} + \mathbf{1}_F \widehat{\mathcal{M}} - \widehat{\bar{\boldsymbol{L}}}.$$

Multiplying the preceding expression by the weight on wages in the CPI from equation (2.10) yields

$$\left(\bar{\boldsymbol{\Lambda}}^T - \bar{\boldsymbol{x}}^T \boldsymbol{\Psi}_D \boldsymbol{A}\right) \widehat{\boldsymbol{W}} = \left(\bar{\boldsymbol{\Lambda}}^T - \bar{\boldsymbol{x}}^T \boldsymbol{\Psi}_D \boldsymbol{A}\right) (\widehat{\bar{\boldsymbol{\Lambda}}} + \mathbf{1}_F \widehat{\mathcal{M}} - \widehat{\bar{\boldsymbol{L}}}).$$

Now note that, in general, the budget constraint of a consumer in a small open economy can be written as

$$\mathcal{M} + T = GDP \Longrightarrow \widehat{nGDP} = \frac{\mathcal{M}}{nGDP} \widehat{\mathcal{M}} + \frac{\mathrm{d}T}{nGDP}.$$

By definition,

$$\Lambda_f = \bar{\Lambda}_f \frac{\mathcal{M}}{nGDP} \Longrightarrow \widehat{\bar{\Lambda}}_f = \widehat{\Lambda}_f - \widehat{\mathcal{M}} + \widehat{nGDP},$$

$$\bar{\boldsymbol{\Lambda}}^T \mathbf{1}_F = \sum_{f \in F} \bar{\Lambda}_f = \frac{nGDP}{\mathcal{M}} = \frac{\mathcal{M} + T}{\mathcal{M}} \Longrightarrow \bar{\boldsymbol{\Lambda}}^T \widehat{\bar{\boldsymbol{\Lambda}}} = \sum_{f \in F} \mathrm{d}\bar{\Lambda}_f = \mathrm{d}\left(1 + \frac{T}{\mathcal{M}}\right) = \frac{\mathrm{d}T}{\mathcal{M}} - \frac{T}{\mathcal{M}} \widehat{\mathcal{M}}$$

Then,

$$\begin{aligned}
\left(\bar{\boldsymbol{\Lambda}}^T - \bar{\boldsymbol{x}}^T \boldsymbol{\Psi}_D \boldsymbol{A}\right) \widehat{\boldsymbol{W}} &= \bar{\boldsymbol{\Lambda}}^T \widehat{\bar{\boldsymbol{\Lambda}}} - \bar{\boldsymbol{x}}^T \boldsymbol{\Psi}_D \boldsymbol{A} \widehat{\bar{\boldsymbol{\Lambda}}} + \left(\bar{\boldsymbol{\Lambda}}^T - \bar{\boldsymbol{x}}^T \boldsymbol{\Psi}_D \boldsymbol{A}\right) \mathbf{1}_F \widehat{\mathcal{M}} - \left(\bar{\boldsymbol{\Lambda}}^T - \bar{\boldsymbol{x}}^T \boldsymbol{\Psi}_D \boldsymbol{A}\right) \widehat{\bar{\boldsymbol{L}}} \\
&= \frac{\mathrm{d}T}{\mathcal{M}} - \frac{T}{\mathcal{M}} \widehat{\mathcal{M}} - \bar{\boldsymbol{x}}^T \boldsymbol{\Psi}_D \boldsymbol{A} \widehat{\bar{\boldsymbol{\Lambda}}} + \left(\bar{\boldsymbol{\Lambda}}^T - \bar{\boldsymbol{x}}^T \boldsymbol{\Psi}_D \boldsymbol{A}\right) \mathbf{1}_F \widehat{\mathcal{M}} - \left(\bar{\boldsymbol{\Lambda}}^T - \bar{\boldsymbol{x}}^T \boldsymbol{\Psi}_D \boldsymbol{A}\right) \widehat{\bar{\boldsymbol{L}}} \\
&= \frac{\mathrm{d}T}{\mathcal{M}} - \bar{\boldsymbol{x}}^T \boldsymbol{\Psi}_D \boldsymbol{A} \widehat{\bar{\boldsymbol{\Lambda}}} + \left(\left(\bar{\boldsymbol{\Lambda}}^T - \bar{\boldsymbol{x}}^T \boldsymbol{\Psi}_D \boldsymbol{A}\right) \mathbf{1}_F - \frac{T}{\mathcal{M}}\right) \widehat{\mathcal{M}} - \left(\bar{\boldsymbol{\Lambda}}^T - \bar{\boldsymbol{x}}^T \boldsymbol{\Psi}_D \boldsymbol{A}\right) \widehat{\bar{\boldsymbol{L}}} \\
&= \frac{\mathrm{d}T}{\mathcal{M}} - \bar{\boldsymbol{x}}^T \boldsymbol{\Psi}_D \boldsymbol{A} \widehat{\bar{\boldsymbol{\Lambda}}} + \left(\frac{\mathcal{M} + T}{\mathcal{M}} - \bar{\boldsymbol{x}}^T \boldsymbol{\Psi}_D \boldsymbol{A} \mathbf{1}_F - \frac{T}{\mathcal{M}}\right) \widehat{\mathcal{M}} - \left(\bar{\boldsymbol{\Lambda}}^T - \bar{\boldsymbol{x}}^T \boldsymbol{\Psi}_D \boldsymbol{A}\right) \widehat{\bar{\boldsymbol{L}}} \\
&= \frac{\mathrm{d}T}{\mathcal{M}} + \left(1 - \bar{\boldsymbol{x}}^T \boldsymbol{\Psi}_D \boldsymbol{A} \mathbf{1}_F\right) \widehat{\mathcal{M}} - \bar{\boldsymbol{x}}^T \boldsymbol{\Psi}_D \boldsymbol{A} \widehat{\bar{\boldsymbol{\Lambda}}} - \left(\bar{\boldsymbol{\Lambda}}^T - \bar{\boldsymbol{x}}^T \boldsymbol{\Psi}_D \boldsymbol{A}\right) \widehat{\bar{\boldsymbol{L}}}.
\end{aligned}$$

[15]This assumes that the CPI is interpreted broadly to include all final uses different from intermediate inputs, such as investment, government expenditure, and so on.

Placing this expression into equation (2.10) yields

$$\widehat{P} = -\left(\bar{\boldsymbol{\lambda}}^T - \tilde{\boldsymbol{\lambda}}^T\right)\widehat{\boldsymbol{Z}} - \underbrace{\tilde{\boldsymbol{\Lambda}}^T\widehat{\tilde{\boldsymbol{\Lambda}}} - \left(\bar{\boldsymbol{\Lambda}}^T - \tilde{\boldsymbol{\Lambda}}^T\right)\widehat{\tilde{\boldsymbol{L}}} + \frac{\mathrm{d}T}{\mathcal{M}} + \left(1 - \tilde{\boldsymbol{\Lambda}}^T\mathbf{1}_F\right)\widehat{\mathcal{M}}}_{\text{Factor price changes}}$$
$$+ \left(\bar{\boldsymbol{b}}_M^T + \bar{\boldsymbol{b}}_D^T\boldsymbol{\Psi}_D\boldsymbol{\Gamma}\right)\widehat{\boldsymbol{P}}_M, \tag{A.4}$$

which is the expression in the main text.

B Two-Period Model

In this section, I justify using the one-period model in the main text. The intuition from the main text is unchanged in this more complicated model.

Suppose there are two periods, 0 and 1. There is no uncertainty. The consumer has preferences over the consumption bundle in both periods according to some utility function $U(C)$. The consumer can also access an internationally traded bond that pays a nominal interest rate i_t^*. This nominal interest rate is *exogenous* from the perspective of the small open economy.

The consumer's budget constraint at times 0 and 1 read

$$P_0C_0 + \mathcal{E}_0B_0 = (1+i_{-1}^*)\mathcal{E}_0B_{-1} + nGDP_0,$$
$$P_1C_1 + \mathcal{E}_1B_1 = (1+i_0^*)\mathcal{E}_1B_0 + nGDP_1,$$

where P_t is the price index at time t, C_t is consumption at time t, B_t denotes asset holdings in foreign currency at time t, $\mathcal{E}_t$ is the nominal exchange rate at time t defined as local currency per unit of foreign currency, and $nGDP_t$ denotes nominal GDP at time t.

Combining the two budget constraints gives the intertemporal budget constraint

$$P_0C_0 + \frac{P_1C_1}{\frac{\mathcal{E}_1}{\mathcal{E}_0}(1+i_0^*)} = (1+i_{-1}^*)\mathcal{E}_0B_{-1} + nGDP_0 + \frac{nGDP_1}{\frac{\mathcal{E}_1}{\mathcal{E}_0}(1+i_0^*)}.$$

Under perfect mobility of capital flows, the *no-arbitrage condition*[16] is

$$(1+i_0) = (1+i_0^*)\frac{\mathcal{E}_1}{\mathcal{E}_0}. \tag{B.1}$$

With perfect foresight, there is no expectation regarding the future level of the exchange rate. I can also obtain this condition by adding a domestic bond in zero net supply. This does not change any of the following conclusions.

The no-arbitrage condition is important for the small open economy as it clearly illustrates that the central bank has two instruments for setting the nominal interest rate i_0: it can either choose i_0 directly and let the exchange rate $\mathcal{E}_0$ adjust, or it can pick $\mathcal{E}_0$ and let the nominal interest rate accommodate to comply with that rule.

I can thus rewrite the maximization problem as solving the following program:

$$\max_{C_0,C_1} \quad U(C_0) + \beta U(C_1) \quad \text{s.t} \quad P_0C_0 + \frac{P_1C_1}{(1+i_0)} = (1+i_{-1}^*)\mathcal{E}_0B_{-1} + nGDP_0 + \frac{nGDP_1}{(1+i_0)}. \tag{B.2}$$

[16] Under uncertainty, this is simply the uncovered interest parity (UIP) condition.

Letting λ be the multiplier on the intertemporal budget constraint yields

$$U'(C_0) = \lambda P_0$$
$$\beta U'(C_1) = \lambda \frac{P_1}{(1+i_0)}.$$

Combining both equations results in

$$\frac{U'(C_0)}{P_0} = \beta \frac{U'(C_1)}{P_1}(1+i_0).$$

Assume $U(C) = \log C$, as in Golosov and Lucas (2007) and Baqaee and Farhi (2022b), so that the intertemporal elasticity of substitution is unitary. Then,

$$\beta P_0 C_0 (1+i_0) = P_1 C_1.$$

As Baqaee and Farhi (2022b) argued (drawing on Krugman (1998) and Eggertsson and Krugman (2012)), this model assumes that anything happening at $t = 1$ is labeled as "future." The assumption here is isomorphic to an infinite horizon model, where an unexpected shock happens at $t = 0$ and the economy returns to the long-run equilibrium from $t = 1$ onward. For all practical purposes, this means I assume that any variables at $t = 1$ are exogenously given. Then, from the Euler equation, I derive

$$P_0 C_0 = \frac{P_1 C_1}{\beta(1+i_0)} = \frac{P_1 C_1}{\beta(1+i_0^*)} \frac{\mathcal{E}_0}{\mathcal{E}_1}.$$

Therefore, since $(\beta, i_0^*, P_1 C_1, \mathcal{E}_1)$ are exogenous, the nominal exchange rate $\mathcal{E}_0$ is determined via the no-arbitrage condition, equation (B.1). This equation provides a value for current expenditure in local currency, $P_0 C_0$.

For simplicity, suppose $B_{-1} = 0$. Replacing the Euler equation in the intertemporal budget constraint results in

$$P_0 C_0 + \frac{P_1 C_1}{(1+i_0)} = nGDP_0 + \frac{nGDP_1}{(1+i_0)}$$
$$P_0 C_0 + \frac{\beta P_0 C_0 (1+i_0)}{(1+i_0)} = nGDP_0 + \frac{nGDP_1}{(1+i_0)}$$
$$P_0 C_0 = \frac{1}{(1+\beta)} \left(nGDP_0 + \frac{nGDP_1}{(1+i_0)} \right)$$
$$P_0 C_0 = \frac{1}{(1+\beta)} \left(nGDP_0 + \frac{nGDP_1}{(1+i_0^*)} \frac{\mathcal{E}_0}{\mathcal{E}_1} \right).$$

Note that given $(\mathcal{E}_0, P_0 C_0, nGDP_1, i_0)$ from the Euler equation and no-arbitrage condition, the latter equation pins down $nGDP_0$.

B.1 Solving for Consumption at Time 0, C_0.

Before solving for consumption, I introduce the real exchange rate, $\mathcal{Q}_0$, as

$$\mathcal{Q}_0 = \frac{\mathcal{E}_0 P_0^F}{P_0},$$

where P_0^F is the rest of the world price index, which is exogenous from the perspective of the small open economy. Following Schmitt-Grohé et al. (2022), I write this foreign price index as $P_0^F = \mathcal{P}^F(\{P^*_{m,0}\}_{m\in M}, \{P^*_{j,0}\}_{j\in N^*})$, where M is the same set of goods imported by the small open economy and N^* is the set of all other goods consumed abroad by the foreign economy. Since I assume that all prices P^*_k for $k \in M \cup N^*$ are exogenous from the perspective of the small open economy, this allows me to write

$$\frac{P_0^F}{P^*_{m_0,0}} = \mathcal{P}^F(1, \{P^*_{m,0}/P^*_{0,0}\}_{m\in M\ 0}, \{P^*_{j,0}/P^*_{0,0}\}_{j\in N^*}).$$

Using this in the definition of the real exchange rate and the law of one price for imported goods $P_{m,0} = \mathcal{E}_0 P^*_{m,0}$ for all $m \in M$, then

$$\mathcal{Q}_0 = \frac{P_0^F/P^*_{m_0,0}}{P_0/P_{m_0,0}}.$$

Since the numerator is exogenous, the real exchange rate can be written as the relative price of CPI to one of the imported goods $P_{m_0,0}$. Let set $P_0^F/P^*_{m_0,0} = 1$, then

$$\mathcal{Q}_0 = \frac{P_{m_0,0}}{P_0}.$$

Note that consumption at time 0 is only a function of this real exchange rate, since from the Euler equation

$$C_0 = \frac{P_1 C_1}{\beta \mathcal{E}_1 (1+i_0^*)} \frac{\mathcal{E}_0}{P_0} = \frac{P_1 C_1}{\beta \mathcal{E}_1 (1+i_0^*)} \mathcal{Q}_0 = \frac{E_1}{\beta \mathcal{E}_1 (1+i_0^*)} \mathcal{Q}_0.$$

Changes in the real exchange rate represent changes in all prices relative to the imported good m_0.

$$\begin{aligned}
\widehat{P}_0 - \widehat{P}_{m_0,0} &= \sum_{i\in N} \frac{P_i C_i}{PC}(\widehat{P}_i - \widehat{P}_{m_0,0}) + \sum_{m\in M} \frac{P_m C_m}{PC}(\widehat{P}_m - \widehat{P}_{m_0,0}) = \bar{\boldsymbol{b}}_D^T(\widehat{\boldsymbol{P}}_D - \mathbf{1}_N \widehat{P}_{m_0,0}) + \bar{\boldsymbol{b}}_M^T(\widehat{\boldsymbol{P}}^*_M - \mathbf{1}_M \widehat{P}^*_{m_0,0}) \\
\widehat{P}_0 - \widehat{P}_{m_0,0} &= \bar{\boldsymbol{b}}_D^T(-\boldsymbol{\Psi}\widehat{\boldsymbol{Z}} + \boldsymbol{\Psi}\boldsymbol{A}(\widehat{\boldsymbol{W}} - \mathbf{1}_F \widehat{P}_{m_0,0}) + \boldsymbol{\Psi}\boldsymbol{\Gamma}(\widehat{\boldsymbol{P}}^*_M - \mathbf{1}_M \widehat{P}^*_{m_0,0})) + \bar{\boldsymbol{b}}_M^T(\widehat{\boldsymbol{P}}^*_M - \mathbf{1}_M \widehat{P}^*_{m_0,0}) \\
\widehat{P}_0 - \widehat{P}_{m_0,0} &= -\bar{\boldsymbol{b}}_D^T\boldsymbol{\Psi}\widehat{\boldsymbol{Z}} + \bar{\boldsymbol{b}}_D^T\boldsymbol{\Psi}\boldsymbol{A}(\widehat{\boldsymbol{W}} - \mathbf{1}_F \widehat{P}_{m_0,0}) + (\bar{\boldsymbol{b}}_D^T\boldsymbol{\Psi}\boldsymbol{\Gamma} + \bar{\boldsymbol{b}}_M^T)(\widehat{\boldsymbol{P}}^*_M - \mathbf{1}_M \widehat{P}^*_{m_0,0}) \\
\widehat{Q}_0 = -(\widehat{P}_0 - \widehat{P}_{m_0,0}) &= \bar{\boldsymbol{b}}_D^T\boldsymbol{\Psi}\widehat{\boldsymbol{Z}} - \bar{\boldsymbol{b}}_D^T\boldsymbol{\Psi}\boldsymbol{A}(\widehat{\boldsymbol{W}} - \mathbf{1}_F \widehat{P}_{m_0,0}) - (\bar{\boldsymbol{b}}_D^T\boldsymbol{\Psi}\boldsymbol{\Gamma} + \bar{\boldsymbol{b}}_M^T)(\widehat{\boldsymbol{P}}^*_M - \mathbf{1}_M \widehat{P}^*_{m_0,0}).
\end{aligned}$$

It follows that consumption changes satisfy

$$\begin{aligned}
\widehat{C}_0 &= \widehat{E}_1 - \widehat{\beta} - \widehat{\mathcal{E}}_1 - (\widehat{1+i_0^*}) + \widehat{Q}_0 \\
&= \widehat{E}_1 - \widehat{\beta} - \widehat{\mathcal{E}}_1 - (\widehat{1+i_0^*}) + \bar{\boldsymbol{b}}_D^T\boldsymbol{\Psi}\widehat{\boldsymbol{Z}} - \bar{\boldsymbol{b}}_D^T\boldsymbol{\Psi}\boldsymbol{A}(\widehat{\boldsymbol{W}} - \mathbf{1}_F \widehat{P}_{m_0,0}) - (\bar{\boldsymbol{b}}_D^T\boldsymbol{\Psi}\boldsymbol{\Gamma} + \bar{\boldsymbol{b}}_M^T)(\widehat{\boldsymbol{P}}^*_M - \mathbf{1}_M \widehat{P}^*_{m_0,0});
\end{aligned}$$

this illustrates how consumption at time 0 is not pinned down by real GDP, Y_0, as is the case in the closed economy model. Rather, it is pinned down by the real exchange rate, $\mathcal{Q}_0$, which in turn depends on factor prices in units of good m_0.

Since $P_0C_0 = E_0$ is given once I set either i_0 or $\mathcal{E}_0$, changes in the price index satisfy

$$
\begin{aligned}
\widehat{P}_0 &= \widehat{E}_0 - \widehat{C}_0 \\
&= \widehat{E}_0 - \widehat{E}_1 + \widehat{\beta} + \widehat{\mathcal{E}}_1 + \widehat{(1+i_0^*)} - \bar{\boldsymbol{b}}_D^T \boldsymbol{\Psi} \widehat{\boldsymbol{Z}} + \bar{\boldsymbol{b}}_D^T \boldsymbol{\Psi} \boldsymbol{A}(\widehat{\boldsymbol{W}} - \mathbf{1}_F \widehat{P}_{m_0,0}) + (\bar{\boldsymbol{b}}_D^T \boldsymbol{\Psi\Gamma} + \bar{\boldsymbol{b}}_M^T)(\widehat{\boldsymbol{P}}_M^* - \mathbf{1}_M \widehat{P}_{m_0,0}^*) \\
&= \widehat{\mathcal{E}}_1 + \widehat{(1+i_0^*)} - \widehat{(1+i_0)} - \bar{\boldsymbol{b}}_D^T \boldsymbol{\Psi} \widehat{\boldsymbol{Z}} + \bar{\boldsymbol{b}}_D^T \boldsymbol{\Psi} \boldsymbol{A}(\widehat{\boldsymbol{W}} - \mathbf{1}_F \widehat{P}_{m_0,0}) + (\bar{\boldsymbol{b}}_D^T \boldsymbol{\Psi\Gamma} + \bar{\boldsymbol{b}}_M^T)(\widehat{\boldsymbol{P}}_M^* - \mathbf{1}_M \widehat{P}_{m_0,0}^*).
\end{aligned}
$$

From real wages to aggregate demand and inelastic factor supply changes. To solve the model in terms of factor quantities, I define real wages, in relation to imported good m_0, as a function of factor supplies, factor shares, and the import price (denominated in foreign currency):

$$
\begin{aligned}
\widehat{W}_{f,0} - \widehat{P}_{m_0,0} &= \widehat{\bar{\Lambda}}_f - \widehat{\bar{L}}_{f,0} + \widehat{E}_0 - \widehat{P}_{m_0,0} \\
&= \widehat{\bar{\Lambda}}_f - \widehat{\bar{L}}_{f,0} + (\widehat{E}_0 - \widehat{\mathcal{E}}_0) - \widehat{P}_{m_0,0}^* \\
\widehat{\boldsymbol{W}} - \mathbf{1}_F \widehat{P}_{m_0,0} &= \widehat{\bar{\boldsymbol{\Lambda}}} - \widehat{\bar{\boldsymbol{L}}} + \mathbf{1}_F((\widehat{E}_0 - \widehat{\mathcal{E}}_0) - \widehat{P}_{m_0,0}^*),
\end{aligned}
$$

where the second line follows from the law of one price.

Note that expenditure denominated in foreign currency is exogenous from the Euler equation:

$$
\frac{E_0}{\mathcal{E}_0} = \frac{E_1}{\mathcal{E}_1} \frac{1}{\beta(1+i_0^*)}.
$$

Thus $(\widehat{E}_0 - \widehat{\mathcal{E}}_0)$ represents an *aggregate demand shifter*. It increases if the consumer becomes more impatient (β declines), future expenditure in local currency increases ($E_1 = P_1C_1$), the interest rate in foreign currency declines $(1+i_0^*)$, or the exchange rate in the future goes down, $\mathcal{E}_1$.

I set $\widehat{P}_{m0,0}^* = 0$ to simplify the exposition. Combining expenditure at time 0 in foreign currency with the expression for CPI changes, results in

$$
\widehat{P}_0 = \widehat{\mathcal{E}}_0 - \bar{\boldsymbol{b}}_D^T \boldsymbol{\Psi} \widehat{\boldsymbol{Z}} + \bar{\boldsymbol{b}}_D^T \boldsymbol{\Psi} \boldsymbol{A} \left(\widehat{\bar{\boldsymbol{\Lambda}}} - \widehat{\bar{\boldsymbol{L}}} + \mathbf{1}_F(\widehat{E}_0 - \widehat{\mathcal{E}}_0) \right) + (\bar{\boldsymbol{b}}_D^T \boldsymbol{\Psi\Gamma} + \bar{\boldsymbol{b}}_M^T) \widehat{\boldsymbol{P}}_M^*,
$$

which can be written as

$$
\begin{aligned}
\widehat{P}_0 = \; & \underbrace{\widehat{\mathcal{E}}_0}_{\text{Nominal Anchor}} + \underbrace{\bar{\boldsymbol{b}}_D^T \boldsymbol{\Psi} \boldsymbol{A} \mathbf{1}_F(\widehat{E}_0 - \widehat{\mathcal{E}}_0)}_{\text{Aggregate demand shifter}} - \underbrace{\bar{\boldsymbol{b}}_D^T \boldsymbol{\Psi} \widehat{\boldsymbol{Z}}}_{\text{Technology effects}} + \underbrace{\bar{\boldsymbol{b}}_D^T \boldsymbol{\Psi} \boldsymbol{A} \widehat{\bar{\boldsymbol{\Lambda}}}}_{\text{Factor share reallocation}} \\
& - \underbrace{\bar{\boldsymbol{b}}_D^T \boldsymbol{\Psi} \boldsymbol{A} \widehat{\bar{\boldsymbol{L}}}}_{\text{Factor supplies}} + \underbrace{(\bar{\boldsymbol{b}}_D^T \boldsymbol{\Psi\Gamma} + \bar{\boldsymbol{b}}_M^T) \widehat{\boldsymbol{P}}_M^*}_{\text{Import price channel}},
\end{aligned} \tag{B.3}
$$

where I use the no-arbitrage condition, which implies that changes in the nominal exchange rate at time 0 can be written as

$$
\widehat{(1+i_0)} = \widehat{(1+i_0^*)} + \widehat{\mathcal{E}}_1 - \widehat{\mathcal{E}}_0 \Longrightarrow \widehat{\mathcal{E}}_0 = \widehat{(1+i_0^*)} + \widehat{\mathcal{E}}_1 - \widehat{(1+i_0)}.
$$

This provides a nominal anchor at time 0.

B.2 Mapping the Net Transfer, T

We can use the previous two-period model to justify the exogenous net transfer in the static setup T. To see this, I write

$$T = nGDP_0 - P_0C_0 = (1+\beta)P_0C_0 - \frac{nGDP_1}{(1+i_0^*)}\frac{\mathcal{E}_0}{\mathcal{E}_1} - P_0C_0$$
$$T = \beta P_0C_0 - \frac{nGDP_1}{(1+i_0^*)}\frac{\mathcal{E}_0}{\mathcal{E}_1} = \beta P_0C_0 - \beta\frac{nGDP_1}{P_1C_1}P_0C_0 = \beta P_0C_0\left(1 - \frac{nGDP_1}{P_1C_1}\right);$$

this net transfer is positive or negative depending on whether nominal GDP in the future is higher or lower than future expenditure. This ultimately hinges on the difference between future consumption and income since if $nGDP_1/P_1C_1 > 1$, this ratio is negative, meaning $T < 0$. A negative net transfer means the economy receives resources at time 0 that do not come from their own production at time 0. In an intertemporal model, this comes from future resources. In a static model, this should come from the rest of the world. The converse also holds. Of course, if the steady state features no asset holdings, this equation collapses to $T = 0$. To see why, note the budget constraint satisfies

$$PC\left(1 + \frac{1}{(1+i)}\right) = \mathcal{E}i^*B + nGDP\left(1 + \frac{1}{(1+i)}\right).$$

If $B = 0$, then

$$PC = nGDP,$$

and therefore $T = 0$.

C A Small Open Economy Dynamic Model with Production Networks

In this appendix, I briefly explore how the intuition of the model in the main text extends to a small open economy *dynamic* environment.

Environment. The model is a variant of the canonical importable, exportable, and non-tradable model (MXN) as in Chapter 8 of Uribe and Schmitt-Grohé (2017), where I remove capital from the model.

Time is discrete, indexed by t, and runs forever. Households consume exportables, importables, and non-tradables. Non-tradable and exportable goods are produced using labor and intermediate inputs from other sectors. This intermediate input-output structure is one of the novelties of the model.

Financial markets are incomplete. The domestic household has access to two bonds. The first is a domestic bond denominated in local currency and pays a nominal interest rate i_t. The second is a foreign bond denominated in foreign currency and pays a nominal interest rate i_t^*. The small open economy takes this latter foreign interest rate as given.

Household. The household owns labor and consumes the three goods. Labor is supplied inelastically. The consumption aggregator is of the CES form

$$C = \left(b_N^{\frac{1}{\chi}} C_N^{\frac{\chi-1}{\chi}} + b_M^{\frac{1}{\chi}} C_M^{\frac{\chi-1}{\chi}} + (1 - b_N - b_M)^{\frac{1}{\chi}} C_X^{\frac{\chi-1}{\chi}} \right)^{\frac{\chi}{\chi-1}}, \tag{C.1}$$

where C_N is consumption of non-tradables, C_M is consumption of importables, and C_X is consumption of the exportable good. (b_N, b_M) are the expenditure shares on non-tradable and importables at the symmetric price steady state. In turn, $1 - b_N - b_M$ is the expenditure share on exportable goods. Finally, χ is the elasticity of substitution across the different goods.

I solve the household's problem in two steps. In the first step, I solve for the dynamic path of $\{C_t, B_t, B_t^*\}_{t=0}^{\infty}$. Conditional on knowing the path of C_t, I solve for (C_{Nt}, C_{Mt}, C_{Xt}) at each instant t. This allows me to simplify the exposition and is the method followed in, for example, Chapter 4 of Obstfeld and Rogoff (1996).[17]

The dynamic problem of the household is as follows. Taking as given paths of prices $\{P_t, W_t, \mathcal{E}_t\}$, interest rates $\{i_t, i_t^*\}$, and labor endowment $\{\bar{L}_t\}$, the consumer solves the following program:

$$\max_{\{C_t, B_t, B_t^*\}_{t=0}^{\infty}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \frac{C_t^{1-\sigma} - 1}{1 - \sigma}$$
$$\text{subject to } P_t C_t + \mathcal{E}_t B_t^* + B_t \leq W_t \bar{L}_t + (1 + i_{t-1}^*) \mathcal{E}_t B_{t-1}^* + (1 + i_{t-1}) B_{t-1},$$

where P_t is the consumer's price index, B_t^* is asset holdings of a foreign bond, B_t is asset holdings of a domestic bond, $\mathcal{E}_t$ is the nominal exchange rate defined as units of home currency per unit of foreign currency, W_t is the wage rate, $(1 + i_t)$ is the gross interest rate in the domestic bond, and $(1 + i_t^*)$ is the gross interest rate in the foreign bond. $\beta \in (0, 1)$ is a discount factor.

Letting $\beta^t \mu_t$ be the Lagrange multiplier on the flow budget constraint, results in the following first-order conditions:

$$C_t : C_t^{-\sigma} = \lambda_t P_t, \tag{C.2}$$
$$B_t : \lambda_t = \beta(1 + i_t) \mathbb{E}_t \lambda_{t+1}, \tag{C.3}$$
$$B_t^* : \lambda_t = \beta(1 + i_t^*) \mathbb{E}_t \lambda_{t+1} \frac{\mathcal{E}_{t+1}}{\mathcal{E}_t}, \tag{C.4}$$

plus the budget constraint.

Conditional on C, the intratemporal problem solves the following program (ignoring time subscripts):

$$\min_{C_N, C_M, C_X} P_N C_N + P_M C_M + P_X C_X \text{ subject to } C \geq \bar{C}. \tag{C.5}$$

That is, taking as given prices (P_N, P_M, P_X), a consumption aggregator function C, and a level of aggregate consumption $\bar{C}$, the household minimizes its expenditure to achieve $\bar{C}$.

The conditional demands that solve this problem are

$$C_N = b_N \left(\frac{P_N}{P} \right)^{-\chi} C; \quad C_M = b_M \left(\frac{P_M}{P} \right)^{-\chi} C; \quad C_X = (1 - b_N - b_M) \left(\frac{P_X}{P} \right)^{-\chi} C, \tag{C.6}$$

[17] Specifically, see Section 4.4.1. of Obstfeld and Rogoff (1996).

where the price index P satisfies

$$P = (b_N P_N^{1-\chi} + b_M P_M^{1-\chi} + (1 - b_N - b_M) P_X^{1-\chi})^{\frac{1}{1-\chi}}. \tag{C.7}$$

CPI inflation is thus defined as

$$\pi_t = \log P_t - \log P_{t-1}. \tag{C.8}$$

Production Side. There are two producing sectors: non-tradable (N) and exportable (X). I omit time indices whenever it causes no confusion.

Gross output of sector $i \in \{N, X\}$ is of the CES form

$$Q_i = Z_i \left(a_i^{\frac{1}{\sigma_i}} L_i^{\frac{\sigma_i - 1}{\sigma_i}} + (1 - a_i)^{\frac{1}{\sigma_i}} M_i^{\frac{\sigma_i - 1}{\sigma_i}} \right)^{\frac{\sigma_i}{\sigma_i - 1}}, \tag{C.9}$$

where L_i is labor demand and M_i is an intermediate input bundle. σ_i is the elasticity of substitution between labor and intermediate inputs. Z_i is a sector-specific productivity level. These productivity levels are exogenous. Finally, a_i represents the labor share in total costs (sales) at the symmetric price equilibrium.

The intermediate input bundle (M_i) aggregates non-tradable (M_{iN}) and tradable (M_{iT}) intermediate inputs according to another CES layer:

$$M_i = \left(\omega_i^{\frac{1}{\varepsilon_i}} M_{iN}^{\frac{\varepsilon_i - 1}{\varepsilon_i}} + (1 - \omega_i)^{\frac{1}{\varepsilon_i}} M_{iT}^{\frac{\varepsilon_i - 1}{\varepsilon_i}} \right)^{\frac{\varepsilon_i}{\varepsilon_i - 1}}, \tag{C.10}$$

where ε_i is the elasticity of substitution between non-tradable and tradable intermediate inputs. ω_i represents the expenditure share on non-tradable intermediates out of total intermediate input spending. Therefore, $1 - \omega_i$ is the expenditure share on tradable goods.

The tradable intermediate input bundle combines the exportable good and the imported input:

$$M_{iT} = \left(\omega_{iX}^{\frac{1}{\varepsilon_i^T}} M_{iX}^{\frac{\varepsilon_i^T - 1}{\varepsilon_i^T}} + (1 - \omega_{iX})^{\frac{1}{\varepsilon_i^T}} M_{iM}^{\frac{\varepsilon_i^T - 1}{\varepsilon_i^T}} \right)^{\frac{\varepsilon_i^T}{\varepsilon_i^T - 1}}, \tag{C.11}$$

where ε_i^T is the elasticity of substitution between importable and exportable goods. ω_{iX} is the expenditure share on exportable goods as a share of intermediate spending on tradable goods (both exportable and importable).

Cost minimization at each CES layer delivers the following conditional demands:

$$L_i = a_i \left(\frac{W}{MC_i} \right)^{-\sigma} Z_i^{\sigma_i - 1} Q_i; \quad M_i = (1 - a_i \left(\frac{P_i^I}{MC_i} \right)^{-\sigma} Z_i^{\sigma_i - 1} Q_i \tag{C.12}$$

$$M_{iN} = \omega_i \left(\frac{P_N}{P_i^I} \right)^{-\varepsilon_i} M_i; \quad M_{iT} = (1 - \omega_i) \left(\frac{P_i^T}{P_i^I} \right)^{-\varepsilon_i} M_i \tag{C.13}$$

$$M_{iX} = \omega_{iX} \left(\frac{P_X}{P_i^T} \right)^{-\varepsilon_i^T} M_{iT}; \quad M_{iM} = (1 - \omega_{iX}) \left(\frac{P_M}{P_i^T} \right)^{-\varepsilon_i^T} M_{iT}, \tag{C.14}$$

with marginal costs and price indices

$$MC_i = Z_i^{-1} \left(a_i W^{1-\sigma_i} + (1-a_i)(P_i^I)^{1-\sigma_i} \right)^{\frac{1}{1-\sigma_i}}, \tag{C.15}$$

$$P_i^I = \left(\omega_i P_N^{1-\varepsilon_i} + (1-\omega_i)(P_i^T)^{1-\varepsilon_i} \right)^{\frac{1}{1-\varepsilon_i}}, \tag{C.16}$$

$$P_i^T = \left(\omega_{iX} P_X^{1-\varepsilon_i^T} + (1-\omega_{iX})(P_M)^{1-\varepsilon_i^T} \right)^{\frac{1}{1-\varepsilon_i^T}}, \tag{C.17}$$

where MC_i stands for marginal cost, P_i^I is the price index of the intermediate input bundle, and P_i^T is the price index of the tradable intermediate input bundle.

Law of One Price. I assume that the law of one price holds for the exportable and importable good. This means

$$P_{Xt} = P_{Xt}^* \mathcal{E}_t \quad P_{Mt} = P_{Mt}^* \mathcal{E}_t. \tag{C.18}$$

Nominal Anchor. Households require local currency to buy the consumption bundles. I introduce this notion as a cash-in-advance constraint. I impose this as an additional aggregate equation

$$\mathcal{M}_t = P_t C_t, \tag{C.19}$$

where $\mathcal{M}_t$ is an *exogenous* money supply. Since prices are fully flexible and there are no market distortions, imposing this constraint does not affect relative prices and optimal allocations. It allows me to pin down the price level and, as a by-product, inflation. This is akin to a nominal GDP targeting rule, which in the open economy is instead an *expenditure* targeting rule.[18]

Exogenous Processes. In the model, there are six exogenous processes $(Z_{Nt}, Z_{Xt}, P_{Xt}^*, P_{Mt}^*, \mathcal{M}_t, \bar{L}_t)$. For the purposes of the exercise, I explore changes in (Z_{Nt}, P_{Mt}^*). I assume all processes follow AR(1) in logs. That is,

$$\log Z_{Nt} = \rho_{Z_N} \log Z_{Nt-1} + \nu_t^N \tag{C.20}$$

$$\log P_{Mt}^* = \rho_{P_M} \log P_{Mt-1}^* + \nu_t^{P_M}, \tag{C.21}$$

where $(\nu_t^N, \nu_t^{P_M})$ are disturbances.

Equilibrium. Since the domestic bond is traded only within the country, $B_t = 0$ in equilibrium. The non-tradable market-clearing condition and the labor market-clearing condition is

$$Q_{Nt} = C_{Nt} + M_{NNt} + M_{XNt} \tag{C.22}$$

$$\bar{L}_t = L_{Nt} + L_{Xt}. \tag{C.23}$$

[18] I can also specify a model with money in the utility function. As long as money and aggregate consumption preferences are separable, none of the results I present here change when using such a specification. This illustration closely follows the main text.

Combining these three conditions into the consumer's budget constraint, I write the law of motion for foreign assets as

$$B_t^* = (1 + i_{t-1}^*)B_{t-1}^* - \frac{1}{\mathcal{E}_t}\left(P_{Xt}(C_{Xt} + M_{XXt} + M_{NXt} - Q_{Xt}) + P_{Mt}(C_{Mt} + M_{NMt} + M_{XMt})\right) \quad \text{(C.24)}$$

I induce stationarity in this foreign asset position using a debt-elastic interest rate device, as in Schmitt-Grohé and Uribe (2003). This means

$$i_t^* = \bar{i}^* + \psi(e^{\bar{B}^* - B_t^*} - 1), \quad \text{(C.25)}$$

where $\bar{i}^*$ and $\bar{B}^*$ are steady-state values of the interest rate and foreign assets.

C.1 Calibration and Scenarios

A period is a year. To assess the role of the production network structure, I consider two different scenarios that vary the exposure and dependence of the exportable and non-tradable sectors to each other via input-output linkages.

In particular, I consider the following specifications:

1. Island: $(\omega_N, \omega_X, \omega_{NX}, \omega_{XX}) = (1, 0, 0, 0.5)$.
2. Intersectoral linkages: $(\omega_N, \omega_X, \omega_{NX}, \omega_{XX}) = (0, 1, 1, 0.5)$.

The first scenario ignores intersectoral linkages and treats both sectors as islands isolated from each other in the input-output structure. The non-tradable sector only buys intermediate input from the same sector, while the exportable sector buys from itself and the imported good. The idea of this scenario is to shut down intersectoral linkages. The second scenario assumes that the non-tradable sector only uses tradable intermediate inputs, while the exportable sector only uses non-tradable intermediate inputs. Importantly, these two scenarios change the intermediate expenditure share distribution while keeping the total intermediate expenditure share constant at $1 - a_i = 0.33$ in both sectors.

Table 3 shows the calibrated shares and parameters kept fixed in both scenarios that I borrow from the literature. Since the consumption share on tradables (exportables plus importables) is 0.3, I use the estimate expenditure share on importables (relative to tradable expenditure) from Table 8.2 in Uribe and Schmitt-Grohé (2017), which is $\chi_m = 0.898$ (in their notation). This implies that the consumption expenditure on importables as a share of total expenditure equals $b_M = 0.3 \times 0.898 = 0.27$. I set all elasticities in production and consumption to be Cobb-Douglas. I do this to highlight the first-order mechanisms, as it is well known that under low elasticities of substitution, the production network amplifies negative shocks on quantities and mitigates positive shocks Baqaee and Farhi (2019b); thus, production networks matter beyond sales shares to a second order. All remaining parameters are standard in the small open economy literature.

C.2 Results

I explore how inflation reacts to a 1 percent negative productivity shock in the non-tradable sector (ν_t^N) and a positive import price shock (ν_t^M). I solve the model using a first-order approximation around the non-stochastic steady state to be comparable with the model in the main text. Panel (a) of Figure 10 shows the response of inflation to a negative productivity shock in the non-tradable sector, while panel (b) of Figure 10 does the same for a positive import price shock. The solid

Table 3. Calibration

Parameters	Value	Description	Source
Shares			
$a_N = a_X$	0.66	Labor Share	Benigno et al. (2013)
b_N	0.70	Consumption share on non-tradables	Bianchi (2011)
b_X	0.03	Consumption share on exportable	Table 8.2 (Uribe and Schmitt-Grohé, 2017)
b_M	0.27	Consumption share on importable	Table 8.2 (Uribe and Schmitt-Grohé, 2017)
Elasticities			
χ	1	Elasticity of substitution in consumption	Cobb-Douglas specification
σ	2	Intertemporal elasticity of substitution	Table 8.2 (Uribe and Schmitt-Grohé, 2017)
$\sigma_N = \sigma_X$	1	Elasticity between value-added and intermediates	Cobb-Douglas specification
$\varepsilon_N = \varepsilon_X$	1	Elasticity across intermediates	Cobb-Douglas specification
$\varepsilon_N^T = \varepsilon_X^T$	1	Elasticity across tradable intermediates	Cobb-Douglas specification
Other Parameters			
$\rho_{Z_N} = \rho_{P_M}$	0.53	AR(1) non-tradable productivity and import price	Table 7.1 (Uribe and Schmitt-Grohé, 2017)
$\bar{B}^*$	0	Steady-state foreign assets position	Zero trade balance
ψ	0.000742	Interest rate sensitivity to foreign assets	Schmitt-Grohé and Uribe (2003)
$\bar{i}^*$	0.04	Steady-state foreign interest rate	Bianchi (2011)
β	$\frac{1}{(1+\bar{i}^*)} = 0.9615$	Discount factor	

purple line shows the implied impulse-response function for inflation in a model with intersectoral linkages. By contrast, the pink dashed line shows the impulse-response function for the island model.

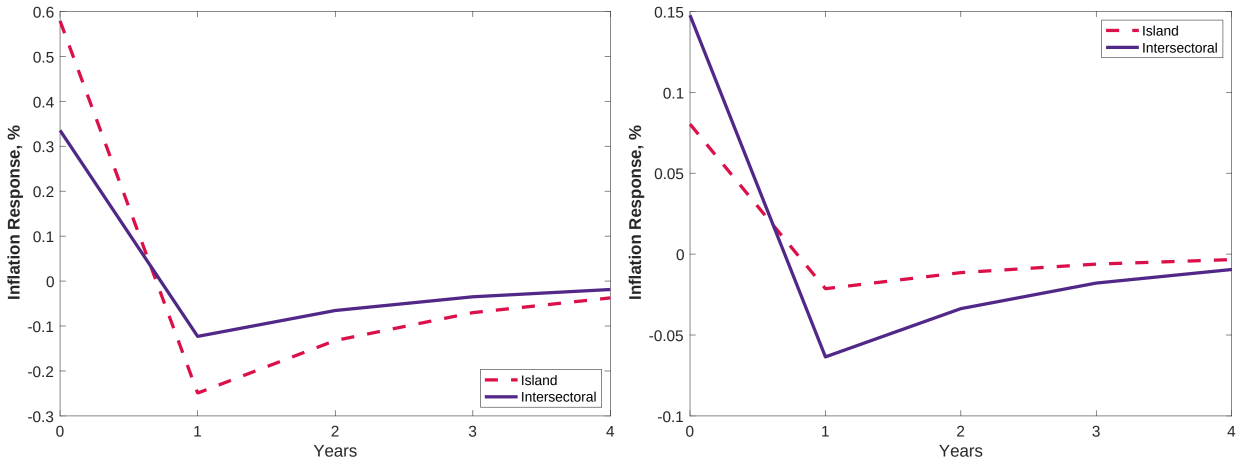
The dynamic model confirms the intuition of the static model. Panel (a) shows that a decrease in productivity generates inflation, which is lower in an economy with intersectoral linkages (solid line) due to indirect trade. A positive import price shock, on the other hand, increases inflation more so in a model with intersectoral linkages due to indirect trade. This highlights that the production network matters to a first order for the productivity pass-through to inflation. Importantly, Domar weights are no longer sufficient statistics for the productivity pass-through to inflation, as they are equal in both scenarios by construction.

The persistence in the dynamic responses is given by shocks' persistence, as the model has no inherent dynamics from the production network structure. Of course, the model does have dynamics from the foreign asset position and thus is richer relative to the static model in the main section. Moreover, a one-time shock at time 0 generates inflation in period 0 but requires deflation thereafter for the price level to recover its steady-state level in the long run.

Finally, inflation volatility due to productivity shocks in the non-tradable sector is lower under the intersectoral model than the island model. The opposite is true when one considers import price shocks. These inflation impulse responses are consistent with the results in Section 4, which show that the small open economy model with production networks decreases inflation volatility in Chile and increases it in the United Kingdom, as the production network adjustments put more weight on import prices and less on productivity shocks.

Figure 10. Inflation Impulse Responses

(a) Negative Productivity Shock in Non-Tradable Sector (b) Import Price Shock



Note: This figure shows the response of inflation to a 1 percent negative productivity shock (panel a) and a 1 percent positive import price shock (panel b). The solid purple line is for the model that considers intersectoral linkages, while the pink dashed line is the model that assumes an island production network structure.

D Shares

In this appendix I explicitly solve for $\widehat{\bar{\mathbf{\Lambda}}}$. Before doing so, I need to define several objects on the consumption and production sides.

D.1 Consumption

Let the consumer's share expenditure on good $i \in N \cup M$ be b_i and

$$\bar{b}_i = \frac{P_i C_i}{E}$$

Let the price elasticity of demand be $\varepsilon^C_{ik} = \frac{\partial \log C_i}{\partial \log P_k}$ and $\delta_{ik} = 1$ if $k = i$ and zero otherwise. This last element is usually called the Kronecker-delta. Log-differentiating the shares and using the homotheticity assumption, I have

$$\begin{aligned}
\widehat{\bar{b}}_i &= \sum_{k \in N \cup M} (\delta_{ik} + \varepsilon^C_{ik} - \bar{b}_i)\widehat{P}_k = \sum_{k \in N \cup M} \phi^C_{ik}\widehat{P}_k \\
\mathrm{d}\bar{b}_i &= \bar{b}_i \sum_{k \in N \cup M} \phi^C_{ik}\widehat{P}_k \qquad \text{for } i \in N \cup M
\end{aligned}$$

where $\phi^C_{ik} = (\delta_{ik} + \varepsilon^C_{ik} - \bar{b}_k)$ represents the elasticity of consumption share on good i, b_i, in response to a change in the price of good k, P_k.

Note that we must have

$$\sum_{i \in N \cup M} \mathrm{d}\bar{b}_i = 0 \Longrightarrow \sum_{i \in N \cup M} \bar{b}_i \sum_{k \in N \cup M} \phi^C_{ik}\widehat{P}_k = 0,$$

for any changes in prices. It thus follows that

$$\sum_{i \in N \cup M} \bar{b}_i \phi_{ik}^C = 0 \quad \text{for all } k \in N \cup M$$

For further reference, it proves useful to define changes in domestic expenditure shares as

$$\mathrm{d}\bar{\boldsymbol{b}}_D = diag(\bar{\boldsymbol{b}}_D)(\boldsymbol{\Phi}_D^C \widehat{\boldsymbol{P}}_D + \boldsymbol{\Phi}_M^C \widehat{\boldsymbol{P}}_M),$$

where $\boldsymbol{\Phi}_D^C$ is an $N \times N$ matrix with typical element ϕ_{ij}^C with $i, j \in N$, and $\boldsymbol{\Phi}_M^C$ is an $N \times M$ matrix with typical element ϕ_{im}^C, and $i \in N$, $m \in M$.

D.2 Production

On the production side, I must define an operator similar to the above but for each $i \in N$ producer. In addition, the decision of the representative firms also depends on factor prices $\widehat{W}_f$. With this in mind, define the expenditure share of producer i on input $j \in N \cup M$ as

$$\Omega_{ij} = \frac{P_j M_{ij}}{P_i Q_i}$$

Define

$$\phi_{jk}^i = \frac{\partial \log \Omega_{ij}}{\partial \log P_k} \quad \text{for } i = 1, ..., N; \quad k = 1, ..., N \cup M \cup F$$

Then

$$\begin{aligned}
\widehat{\Omega}_{ij} &= \widehat{P}_j + \sum_{k \in N \cup M} \varepsilon_{jk}^i \widehat{P}_k + \sum_{f \in F} \varepsilon_{jf}^i \widehat{W}_f - \sum_{k \in N \cup M} \Omega_{ik} \widehat{P}_k - \sum_{f \in F} a_{if} \widehat{W}_f \\
&= \sum_{k \in N \cup M} (\delta_{jk} + \varepsilon_{jk}^i - \Omega_{ik}) \widehat{P}_k + \sum_{f \in F} (\varepsilon_{jf}^i - a_{if}) \widehat{W}_f \\
\widehat{\Omega}_{ij} &= \sum_{k \in N \cup M} \phi_{jk}^i \widehat{P}_k + \sum_{f \in F} \phi_{jf}^i \widehat{W}_f \\
\mathrm{d}\Omega_{ij} &= \Omega_{ij} \left(\sum_{k \in N \cup M} \phi_{jk}^i \widehat{P}_k + \sum_{f \in F} \phi_{jf}^i \widehat{W}_f \right)
\end{aligned}$$

where

$$\begin{aligned}
\varepsilon_{jk}^i &= \frac{\partial \log M_{ij}}{\partial \log P_k} \\
\phi_{jk}^i &= \delta_{jk} + \varepsilon_{jk}^i - \Omega_{ik} \\
\phi_{jf}^i &= \varepsilon_{jf}^i - a_{if}
\end{aligned}$$

represents the elasticity of expenditure share on good j by producer i, Ω_{ij}, when there is a change in either good $k \in N \cup M$ or factors $f \in F$.

By a similar logic, I can write the change in expenditure share of producer i on factor f as

$$
\begin{aligned}
\mathrm{d}a_{if} &= a_{if}\left(\sum_{k\in N\cup M}(\varepsilon^i_{fk}-\Omega_{ik})\widehat{P}_k+\sum_{f'\in F}(\delta_{ff'}+\varepsilon^i_{ff'}-a_{if'})\widehat{W}_{f'}\right)\\
\mathrm{d}a_{if} &= a_{if}\left(\sum_{k\in N\cup M}\phi^i_{fk}\widehat{P}_k+\sum_{f'\in F}\phi^i_{ff'}\widehat{W}_{f'}\right)
\end{aligned}
$$

where again

$$
\begin{aligned}
\varepsilon^i_{fk} &= \frac{\partial \log L_{if}}{\partial \log P_k}\\
\varepsilon^i_{ff'} &= \frac{\partial \log L_{if}}{\partial \log W_{f'}}\\
\phi^i_{fk} &= \varepsilon^i_{fk}-\Omega_{ik}\\
\phi^i_{ff'} &= \delta_{ff'}+\varepsilon^i_{ff'}-a_{if'}
\end{aligned}
$$

The first two rows represent the demand elasticity of factor f relative to a change in other good prices (first row) or factors of producton (second row).

The last two rows represent the elasticity of expenditure share of producer i on factor f relative to either good or factor price changes. When these are positive, then expenditure changes increase after a change in other input prices, meaning that the producer substitutes away from those price increases towards factor f. If this term is negative, then the expenditure share in factor f declines with a change in other input prices: it means that it has to move resources away from factor f towards those goods that are seeing an increase in their price. This is the complementarity in the production case, and it arises with low elasticities of substitution (low ε's).

For imported intermediate, I can construct the same as

$$
\begin{aligned}
\mathrm{d}\Gamma_{im} &= \Gamma_{im}\left(\sum_{k\in N}(\varepsilon^i_{mk}-\Omega_{ik})\widehat{P}_k+\sum_{m'\in M}(\delta_{mm'}+\varepsilon^i_{mm'}-\Gamma_{im'})\widehat{P}_{m'}+\sum_{f\in F}(\varepsilon^i_{mf}-a_{if})\widehat{W}_f\right)\\
\mathrm{d}\Gamma_{im} &= \Gamma_{im}\left(\sum_{k\in N}\phi^i_{mk}\widehat{P}_k+\sum_{m'\in M}\phi^i_{mm'}\widehat{P}_{m'}+\sum_{f\in F}\phi^i_{mf}\widehat{W}_f\right)
\end{aligned}
$$

where again

$$
\begin{aligned}
\varepsilon^i_{mk} &= \frac{\partial \log M_{im}}{\partial \log P_k}\\
\varepsilon^i_{mm'} &= \frac{\partial \log M_{im}}{\partial \log P_{m'}}\\
\varepsilon^i_{mf} &= \frac{\partial \log M_{im}}{\partial \log W_f}\\
\phi^i_{mk} &= (\varepsilon^i_{mk}-\Omega_{ik})\\
\phi^i_{mm'} &= (\delta_{mm'}+\varepsilon^i_{mm'}-\Gamma_{im'})\\
\phi^i_{mf} &= (\varepsilon^i_{mf}-a_{if})
\end{aligned}
$$

D.3 Market clearing conditions and substitution patterns

Goods market clearing conditions. Recall that the market clearing conditions for the N domestic goods can be written as

$$Q_i = C_i + X_i + \sum_{j \in N} M_{ji}$$

In nominal terms and dividing by expenditure, I have

$$\frac{P_i Q_i}{E} = \frac{P_i C_i}{E} + \frac{P_i X_i}{E} + \sum_{j \in N} \frac{P_i M_{ji}}{P_j Q_j} \frac{P_j Q_j}{E}$$

Define *expenditure-based* ratios with a bar i.e. $\bar{\lambda}_i = \frac{P_i Q_i}{E}$. Then,

$$\bar{\lambda}_i = \bar{b}_i + \bar{x}_i + \sum_{j \in N} \Omega_{ji} \bar{\lambda}_j$$

Differentiating this expression, I have

$$\mathrm{d}\bar{\lambda}_i = \mathrm{d}\bar{b}_i + \mathrm{d}\bar{x}_i + \sum_{j \in N} (\mathrm{d}\Omega_{ji} \bar{\lambda}_j + \Omega_{ji} \mathrm{d}\bar{\lambda}_j)$$

Now, recall from shares and making some changes of indices

$$\mathrm{d}\Omega_{ji} = \Omega_{ji} \left(\sum_{k \in N \cup M} \phi^j_{ik} \widehat{P}_k + \sum_{f \in F} \phi^j_{if} \widehat{W}_f \right)$$

Then, the third term on the right-hand side can be written as

$$\begin{aligned}
\sum_{j \in N} \mathrm{d}\Omega_{ji} \bar{\lambda}_j &= \sum_{j \in N} \Omega_{ji} \bar{\lambda}_j \left(\sum_{k \in N \cup M} \phi^j_{ik} \widehat{P}_k + \sum_{f \in F} \phi^j_{if} \widehat{W}_f \right) \\
&= \sum_{j \in N} \Omega_{ji} \bar{\lambda}_j \sum_{k \in N \cup M} \phi^j_{ik} \widehat{P}_k + \sum_{j \in N} \Omega_{ji} \bar{\lambda}_j \sum_{f \in F} \phi^j_{if} \widehat{W}_f \\
&= \sum_{j \in N} \Omega_{ji} \bar{\lambda}_j \sum_{k \in N} \phi^j_{ik} \widehat{P}_k + \sum_{j \in N} \Omega_{ji} \bar{\lambda}_j \sum_{m \in M} \phi^j_{im} \widehat{P}_m + \sum_{j \in N} \Omega_{ji} \bar{\lambda}_j \sum_{f \in F} \phi^j_{if} \widehat{W}_f \\
&= \sum_{k \in N} \underbrace{\left[\sum_{j \in N} \Omega_{ji} \bar{\lambda}_j \phi^j_{ik} \right]}_{\equiv \phi_{ik}} \widehat{P}_k + \sum_{m \in M} \underbrace{\left[\sum_{j \in N} \Omega_{ji} \bar{\lambda}_j \phi^j_{im} \right]}_{\equiv \phi_{im}} \widehat{P}_m + \sum_{f \in F} \underbrace{\left[\sum_{j \in N} \Omega_{ji} \bar{\lambda}_j \phi^j_{if} \right]}_{\equiv \phi_{if}} \widehat{W}_f \\
&= \sum_{k \in N} \phi_{ik} \widehat{P}_k + \sum_{m \in M} \phi_{im} \widehat{P}_m + \sum_{f \in F} \phi_{if} \widehat{W}_f
\end{aligned}$$

A useful thing about writing this in this way, is that I can write this in matrix form

$$\mathrm{d}\boldsymbol{\Omega}^T \boldsymbol{\lambda} = \boldsymbol{\Phi}_D \widehat{\boldsymbol{P}}_D + \boldsymbol{\Phi}_M \widehat{\boldsymbol{P}}_M + \boldsymbol{\Phi}_F \widehat{\boldsymbol{W}}$$

where $\mathbf{\Phi}$ represents *direct substitution matrices*. A version of these substitution matrices appears in Baqaee and Farhi (2019b), although they consider both direct and indirect substitution. Each column represents the changing price and the rows represent where intermediate input demand is going. This only takes into account *first-round effects* and does not consider any input-output linkages beyond the direct exposure (or path of order 1). The next step is to recompute these matrices using the Leontief-inverse. To see this, note that the differentiated form of the market clearing condition write the problem as

$$
\begin{aligned}
\mathrm{d}\bar{\boldsymbol{\lambda}} &= \mathbf{\Psi}^T(\mathrm{d}\bar{\boldsymbol{b}}_D + \mathrm{d}\bar{\boldsymbol{x}} + \mathbf{\Phi}_D \widehat{\boldsymbol{P}}_D + \mathbf{\Phi}_M \widehat{\boldsymbol{P}}_M + \mathbf{\Phi}_F \widehat{\boldsymbol{W}}) \\
&= \mathbf{\Psi}^T \left(diag(\bar{\boldsymbol{b}}_D)(\mathbf{\Phi}_D^C \widehat{\boldsymbol{P}}_D + \mathbf{\Phi}_M^C \widehat{\boldsymbol{P}}_M) + \mathrm{d}\bar{\boldsymbol{x}} + \mathbf{\Phi}_D \widehat{\boldsymbol{P}}_D + \mathbf{\Phi}_M \widehat{\boldsymbol{P}}_M + \mathbf{\Phi}_F \widehat{\boldsymbol{W}} \right) \\
\mathrm{d}\bar{\boldsymbol{\lambda}} &= \mathbf{\Psi}^T \left(diag(\bar{\boldsymbol{b}}_D)\mathbf{\Phi}_D^C + \mathbf{\Phi}_D \right) \widehat{\boldsymbol{P}}_D + \left(diag(\bar{\boldsymbol{b}}_D)\mathbf{\Phi}_M^C + \mathbf{\Phi}_M \right) \widehat{\boldsymbol{P}}_M + \mathbf{\Psi}^T \mathrm{d}\bar{\boldsymbol{x}} + \mathbf{\Psi}^T \mathbf{\Phi}_F \widehat{\boldsymbol{W}}
\end{aligned}
$$

Factor shares changes. We need F more equations, which come from the factor market clearing conditions

$$
\bar{L}_f = \sum_{i \in N} L_{if}
$$

Write this in share form

$$
\bar{\Lambda}_f = \frac{W_f \bar{L}_f}{E} = \sum_{i \in N} \frac{W_f L_{if}}{P_i Q_i} \frac{P_i Q_i}{E} = \sum_{i \in N} a_{if} \bar{\lambda}_i
$$

In differential form,

$$
\mathrm{d}\Lambda_f = \sum_{i \in N} \mathrm{d}a_{if} \bar{\lambda}_i + \sum_{i \in N} a_{if} \mathrm{d}\bar{\lambda}_i
$$

Using the expression for changes in factor usage at the producer level, I have

$$
\begin{aligned}
\mathrm{d}\Lambda_f &= \sum_{i \in N} \left[a_{if} \left(\sum_{k \in N \cup M} \phi^i_{fk} \widehat{P}_k + \sum_{f' \in F} \phi^i_{ff'} \widehat{W}_{f'} \right) \right] \bar{\lambda}_i + \sum_{i \in N} a_{if} \mathrm{d}\bar{\lambda}_i \\
\mathrm{d}\Lambda_f &= \sum_{i \in N} \left[a_{if} \left(\sum_{k \in N} \phi^i_{fk} \widehat{P}_k + \sum_{m \in M} \phi^i_{fm} \widehat{P}_m + \sum_{f' \in F} \phi^i_{ff'} \widehat{W}_{f'} \right) \right] \bar{\lambda}_i + \sum_{i \in N} a_{if} \mathrm{d}\bar{\lambda}_i
\end{aligned}
$$

Taking each of the terms on the right-hand side, I can write

$$\sum_{i\in N} a_{if} \sum_{k\in N} \phi^i_{fk} \widehat{P}_k \bar{\lambda}_i = \sum_{k\in N} \underbrace{\left(\sum_{i\in N} a_{if} \phi^i_{fk} \bar{\lambda}_i \right)}_{\equiv \phi_{fk}} \widehat{P}_k = \sum_{k\in N} \phi_{fk} \widehat{P}_k$$

$$\sum_{i\in N} a_{if} \sum_{f'\in F} \phi^i_{ff'} \widehat{W}_{f'} \bar{\lambda}_i = \sum_{f'\in F} \underbrace{\left(\sum_{i\in N} a_{if} \bar{\lambda}_i \phi^i_{ff'} \right)}_{\equiv \phi_{ff'}} \widehat{W}_{f'} = \sum_{f'\in F} \phi_{ff'} \widehat{W}_{f'}$$

$$\sum_{i\in N} a_{if} \sum_{m\in M} \phi^i_{fm} \widehat{P}_m \bar{\lambda}_i = \sum_{m\in M} \underbrace{\left(\sum_{i\in N} a_{if} \bar{\lambda}_i \phi^i_{fm} \right)}_{\equiv \phi_{fm}} \widehat{P}_m = \sum_{m\in M} \phi_{fm} \widehat{P}_m$$

Replacing this into the differential form for the factor share

$$\mathrm{d}\Lambda_f = \sum_{k\in N} \phi_{fk} \widehat{P}_k + \sum_{f'\in F} \phi_{ff'} \widehat{W}_{f'} + \sum_{m\in M} \phi_{fm} \widehat{P}_m + \sum_{i\in N} a_{if} \mathrm{d}\bar{\lambda}_i$$

In matrix form,

$$\mathrm{d}\bar{\boldsymbol{\Lambda}} = \boldsymbol{\Phi}^F_D \widehat{\boldsymbol{P}}_D + \boldsymbol{\Phi}^F_F \widehat{\boldsymbol{W}} + \boldsymbol{\Phi}^F_M \widehat{\boldsymbol{P}}_M + \boldsymbol{A}^T \mathrm{d}\bar{\boldsymbol{\lambda}}$$

Export share changes. Since export demand is exogenous, I can write

$$\bar{x}_i = \frac{P_i X_i}{\mathcal{M}} \Longrightarrow \mathrm{d}\bar{x}_i = \bar{x}_i (\widehat{P}_i + \widehat{X}_i - \widehat{\mathcal{M}}),$$

which staking into a vector form

$$\mathrm{d}\bar{\boldsymbol{x}} = diag(\bar{\boldsymbol{x}})(\widehat{\boldsymbol{P}}_D + \widehat{\boldsymbol{X}} - \mathbf{1}_N \widehat{\mathcal{M}}) \tag{D.1}$$

D.4 Solving the system

$\mathrm{d}\bar{\boldsymbol{\Lambda}} = \boldsymbol{\Phi}^F_D \widehat{\boldsymbol{P}}_D + \boldsymbol{\Phi}^F_F \widehat{\boldsymbol{W}} + \boldsymbol{\Phi}^F_M \widehat{\boldsymbol{P}}_M + \boldsymbol{A}^T \mathrm{d}\bar{\boldsymbol{\lambda}}$ (F equations, $F+N+F+N$ unknowns)

$\mathrm{d}\bar{\boldsymbol{\lambda}} = \boldsymbol{\Psi}^T \left(diag(\bar{\boldsymbol{b}}_D)\boldsymbol{\Phi}^C_D + \boldsymbol{\Phi}_D \right) \widehat{\boldsymbol{P}}_D + \left(diag(\bar{\boldsymbol{b}}_D)\boldsymbol{\Phi}^C_M + \boldsymbol{\Phi}_M \right) \widehat{\boldsymbol{P}}_M + \boldsymbol{\Psi}^T \mathrm{d}\bar{\boldsymbol{x}} + \boldsymbol{\Psi}^T \boldsymbol{\Phi}_F \widehat{\boldsymbol{W}}$ (N equations, N add. unknowns)

$\widehat{\boldsymbol{P}}_D = -\boldsymbol{\Psi}\widehat{\boldsymbol{Z}} + \boldsymbol{\Psi}\boldsymbol{A}\widehat{\boldsymbol{W}} + \boldsymbol{\Psi}\boldsymbol{\Gamma}\widehat{\boldsymbol{P}}_M$ (N equations, no new unknowns)

$\mathrm{d}\bar{\boldsymbol{\Lambda}} = diag(\bar{\boldsymbol{\Lambda}}) \left(\widehat{\boldsymbol{W}} + \widehat{\boldsymbol{L}} - \mathbf{1}_F \widehat{\mathcal{M}} \right)$ (F equations, no new unknowns)

$\mathrm{d}\bar{\boldsymbol{x}} = diag(\bar{\boldsymbol{x}})(\widehat{\boldsymbol{P}}_D + \widehat{\boldsymbol{X}} - \mathbf{1}_N \widehat{\mathcal{M}})$ (N equations, no new unknowns)

This is a system of $2F + 3N$ unknowns: $(\mathrm{d}\bar{\boldsymbol{\Lambda}}, \widehat{\boldsymbol{W}}, \mathrm{d}\bar{\boldsymbol{\lambda}}, \widehat{\boldsymbol{P}}_D, \mathrm{d}\bar{\boldsymbol{x}})$ on the same number of equations, and thus it pins down all necessary objects.

Note that the distribution of factor shares ($\mathrm{d}\bar{\boldsymbol{\Lambda}}$) and Domar weights ($\mathrm{d}\bar{\boldsymbol{\lambda}}$) changes crucially depends on the substitution matrices ($\boldsymbol{\Phi}'s$ matrices). As a result, to the extent that substitution patterns are encapsulated in these matrices, they affect aggregate inflation in the small open economy via changing factor shares. It is in this sense that elasticities of substitution matters for inflation in this model, something that does not hold in the closed economy as these terms cancels out, to a first-order.