

Cyclical versus Secular: Decomposing the Recent Decline in U.S. Labor Force Participation

Michelle L. Barnes, Fabià Gumbau-Brisa, and Giovanni P. Olivei

Abstract:

Since the start of the Great Recession, one of the most striking developments in the U.S. labor market has been the pronounced decline in the labor force participation rate. The crucial issue in interpreting the decline in U.S. labor force participation is how much of the decline reflects cyclical factors and how much reflects more persistent developments such as the demographic effects of an aging population. We provide a decomposition of cyclical versus trend movements in the labor force participation rate, informed by the joint dynamics of this variable with the employment-to-population ratio. We find that since 2008 trend movements account for a significant portion of the decline in labor force participation. The cyclical response of the labor force participation rate over most of the Great Recession and ensuing recovery has been *smaller* than usual given the estimated cyclical behavior of the employment-to-population ratio. If the cyclical behavior of the labor force participation rate had followed historical norms, the unemployment rate over the period 2009–2011 would have been lower on average by roughly three-quarters of one percentage point. At this point, however, the unemployment rate should provide a fairly accurate signal of labor market conditions and further cyclical declines in labor force participation rates are unlikely to occur if the employment situation continues to improve.

JEL codes: E24, E52, E58, E63, E66

Michelle L. Barnes is a senior economist and policy advisor, Fabià Gumbau-Brisa is a senior economist, and Giovanni P. Olivei is a vice president and head of the macro-international group, all in the research department of the Federal Reserve Bank of Boston. Their e-mail addresses are michelle.barnes@bos.frb.org, fabia.gumbau-brisa@bos.frb.org, and giovanni.olivei@bos.frb.org, respectively.

The views expressed in this brief are the authors' and do not necessarily reflect the official position of the Federal Reserve Bank of Boston or the Federal Reserve System.

This brief, which may be revised, is available on the web site of the Federal Reserve Bank of Boston at <http://www.bostonfed.org/economic/ppb/index.htm>.

This version: July 2013

1. Introduction

Since the onset of the Great Recession, one of the most striking developments in the U.S. labor market has been the pronounced decline in the labor force participation rate. As of 2013:Q1 the drop amounted to 2-1/2 percentage points, which implies that if the participation rate were now at the same level as in December 2007, approximately 6.2 million additional people would be participating in the labor market. When interpreting the decline in the labor force participation rate, the crucial issue is how much of the decline reflects cyclical factors, namely the Great Recession and the slow ensuing recovery, and how much reflects more persistent developments such as the demographic effects of an aging population. Different studies have reached different conclusions on how to apportion the decline in the labor force participation rate between cyclical and long-term (trend) factors. For example, the U.S. Bureau of Labor Statistics (BLS) now interprets most of the decline in the labor force participation rate as reflecting long-term demographic and behavioral changes rather than cyclical developments.¹ Other studies, however, have reached the opposite conclusion: that most of the post-2007 decline in the rate of labor force participation results from cyclical factors.² According to this view, the depressed labor market conditions generated by Great Recession and the disappointing recovery that followed have led workers to temporarily leave the labor force. However, as the pace of economic activity improves, most of the workers who left the labor force are expected to return, and thus exert upward pressure on the unemployment rate.

Which view is correct has important implications for the conduct of monetary policy. If most of the decline in the rate of labor force participation reflects cyclical factors, then focusing on the unemployment rate as the relevant measure of labor market slack could lead to suboptimal monetary policy. This point is conveyed by Erceg and Levin (2013), who argue that the impact of a recession on the rate of labor force participation is nonlinear in the magnitude of the recession, with large recessions producing a decline in the rate of labor force participation that is larger in relation to the activity gap than is the case for small recessions. As a result, monetary policy rules devised for small recessions may not work well for large recessions; instead, in large recessions reliance on broader measures of slack that take into account the low rate of labor force participation could lead to better macroeconomic outcomes than reliance on the unemployment rate as the guide for policy. Specifically, focusing on the

¹ See Toossi (2012). It is interesting to note, however, that as of 2007 the BLS was projecting a much smaller trend decline in the labor force participation rate (see Toossi, 2007).

² See, for example, Erceg and Levin (2013), and Aaronson, Davis, and Hu (2012).

unemployment rate as the measure of labor market slack in a large recession could lead to a monetary policy response that is insufficiently accommodative, thus engendering too slow a recovery with only small improvements in the unemployment rate as many workers re-enter the labor market. Conversely, if large recessions do not generate an outside decline in labor force participation relative to small recessions, then even when the degree of resource underutilization is significant the unemployment rate may prove an accurate summary of labor market conditions for the conduct of policy.

In this brief, we provide a decomposition of cyclical versus trend movements in the labor force participation rate that is informed by the joint dynamics of this variable with the employment-to-population ratio. We assume that the two variables share a common trend, and illustrate how the cyclical movements in the two variables, while different in amplitude, are highly correlated. The main results of the analysis are as follows. Since 2008:Q1, trend movements have accounted for a significant portion of the decline in labor force participation. As of 2013:Q1, the rate of labor force participation is estimated to be three-quarters of one percentage point below trend. The cyclical response of the labor force participation rate over most of the Great Recession and ensuing recovery has been *smaller* than usual given the estimated cyclical behavior of the employment-to-population ratio. If the cyclical behavior of the labor force participation rate had followed historical norms, the unemployment rate over the period 2009–2011 would have been lower on average by roughly three-quarters of one percentage point. It is only recently that the rate of labor force participation has returned to display a cyclical response more in line with historical norms. Indeed, some of the more recent declines in the labor force participation rate can be interpreted as the participation rate adjusting to display a cyclical response that is more typical from a historical standpoint. The main implication of the analysis is that at this juncture the unemployment rate should provide a fairly accurate signal of labor market conditions, and that further cyclical declines in labor force participation rates are unlikely to occur if the employment situation continues to improve. In other words, we expect that declines in the unemployment rate going forward will be more tightly related to increases in the employment-to-population ratio than has been the case so far in this cycle.

The rest of the brief proceeds as follows. Section 2 illustrates the decomposition of the labor force participation rate and of the employment-to-population ratio into trend and cyclical movements. We use the same methodology as in Van Zandweghe (2012), although we make more explicit the interplay between the labor force participation rate and the employment-to-population ratio. The benefit of doing so is that it is then possible to assess the cyclical behavior of labor force participation

relative to the dynamics of the employment-to-population ratio, and to draw comparisons with earlier cycles. Section 3 shows that our findings are robust to changes in the estimation sample, in the definition of variables, and in the identification method for trend versus cyclical movements. Section 4 offers some concluding remarks.

2. Trend Estimates of the Labor Force Participation Rate and the Employment-to-Population Ratio

Estimates of the trend components for the labor force participation rate and the employment-to-population ratio are obtained via a Beveridge and Nelson (1981) trend-cycle decomposition in a multivariate setting. The Beveridge-Nelson trend for a variable is defined as the value to which the variable is expected to converge once transitory movements die out. Specifically, denoting by y_t a variable that is integrated of order one, so that $\Delta y_t \equiv y_t - y_{t-1}$ is stationary and (for simplicity only) with mean zero, the Beveridge-Nelson trend component of y_t is given by:

$$(1) \quad y_t^T = y_t + E_t \sum_{i=1}^{\infty} \Delta y_{t+i},$$

where the superscript T denotes the trend component of the series and E_t indicates expectations as of time t . Since Δy_t is stationary, the last term on the right-hand-side of (1) is stationary, too. Thus, the Beveridge-Nelson decomposition partitions y_t as the sum of a nonstationary component, identified as trend, and a stationary component, identified as cycle.³

This approach to identifying trend and cycle extends to a multivariate setting. Let \mathbf{y}_t denote a vector of n variables integrated of order one. With the same assumptions as in the univariate case, the Beveridge-Nelson trend component for the vector of variables \mathbf{y}_t can be written as in (1), with scalars replaced by vectors. Suppose, in addition, that $\Delta \mathbf{y}_t$ evolves as a vector autoregression of order p , which we write in first-order form as

$$(2) \quad \Delta \mathbf{Y}_t = \mathbf{A} \Delta \mathbf{Y}_{t-1} + \mathbf{u}_t,$$

³ In other words, $y_t = y_t^T + y_t^C$, where the cyclical component y_t^C is equal to $-E_t \sum_{i=1}^{\infty} \Delta y_{t+i}$.

where $\Delta \mathbf{Y}_t$ is a vector of size np , \mathbf{A} is a conforming square matrix of coefficients, and \mathbf{u}_t a conforming vector with n independent and uncorrelated shocks in non-zero positions. Then for a variable $y_{i,t}$ belonging to \mathbf{y}_t , the Beveridge-Nelson trend component is given by:

$$(3) \quad y_{i,t}^T = y_{i,t} + \mathbf{e}_i'(\mathbf{I} - \mathbf{A})^{-1} \mathbf{A} \Delta \mathbf{Y}_t,$$

where \mathbf{e}_i is a selection vector for $\Delta y_{i,t}$ in $\Delta \mathbf{Y}_t$.

In our setup, the vector of variables \mathbf{y}_t is given by the log of the labor force participation rate and the log of the employment-to-population ratio. These two series, for workers aged 16 years and over, are depicted in Figure 1. Grey-shaded areas represent NBER-dated recession periods. When applying the typical battery of unit root tests, the null hypothesis of a unit root in the two series is not rejected at standard confidence levels. The figure shows that lower-frequency movements in the two series are similar. We thus impose a cointegrating relationship between the log of the labor force participation rate and the log of the employment-to-population ratio. To estimate the level relationship between the labor force participation rate and the employment-to-population ratio, we use Stock and Watson's (1993) dynamic least squares procedure, which specifies an equation of the form:

$$(4) \quad \ln lp_t = \beta_0 + \beta_1 \ln lq_t + \sum_{l=-k}^k \lambda_l \Delta \ln lq_{t+l} + \varepsilon_t,$$

where lp denotes the labor force participation rate, lq the employment to population ratio, and Δ , as before, is the first-difference operator. This specification includes leads and lags of the first difference of the log of the employment-to-population ratio to eliminate the effects of the correlation between the regressor and the error term ε . Estimation of (4) on quarterly data over the period 1958:Q1 to 2007:Q4 using OLS gives the following point estimates:

$$(5) \quad \ln lp_t = .044328 + 1.00388 \ln lq_t, \\ (.077538) \quad (.018965)$$

where standard errors are in parentheses and we do not report estimates on the first differences.⁴ The hypothesis that the estimated residuals $\hat{\varepsilon}_t$ contain a unit root is rejected at standard confidence levels, lending some support to the view that the relationship in (5) is not spurious.

⁴ In estimating (4), we use eight leads and lags of the first difference of the log employment-to-population ratio.

The estimated trend deviation $\ln lp_t - 1.00388 \ln lq_t$ is then part of the information set when considering the law of motion for the first-difference of the log of the labor force participation rate and of the log of the employment-to-population ratio. In other words, the vector autoregression in (2) is augmented to include an “error-correction term” equal to the lagged value of the estimated trend deviation. The appendix shows how the expression in (3) for computing the Beveridge-Nelson trend is modified when the variables in the vector autoregression are cointegrated.

In our cointegrated vector autoregression, the log-difference of the labor force participation rate and the log-difference of the employment-to-population ratio are each regressed on four lags of each of the two variables and on the error-correction term. The sample period for the estimation is 1958:Q1 to 2007:Q4. The estimated vector autoregression is then used to compute in-sample and, for the most recent period, out-of-sample Beveridge-Nelson trends. The first two panels of Figure 2 plot the actual values and the estimated trends for the rate of labor force participation and the employment-to-population ratio, respectively, where the variables have been transformed back to levels. The figure shows that the labor force participation rate does respond to business cycle fluctuations. Indeed, the cyclical component of the rate of labor force participation and the employment-to-population ratio exhibit a relatively high degree of correlation, as the third panel in the figure illustrates. A regression of the estimated cyclical component of the labor force participation rate on the estimated cyclical component of the employment-to-population ratio over the period 1958:Q1 to 2007:Q4 yields the following estimates:

$$(6) \quad lp_t^C = -.017253 + .426244lq_t^C, \\ (.017033) \quad (.011810) \quad R^2 = .867.$$

In other words, a one percentage point cyclical increase (decline) in the employment-to-population ratio generates a cyclical increase (decline) in the labor force participation rate of approximately four-tenths of one percent.

For the period 2008:Q1 to present, which represents our out-of-sample exercise, the following considerations are in order. First, the estimated trend is negative, with the trend decline accounting for about 70 percent of the decline in the labor force participation rate over the period 2007:Q4 to 2013:Q1. As Figure 3 shows, for the most recent quarter of available data the actual level of labor force participation is estimated to be about 75 basis points below trend. The trend decline in the rate of labor force participation is mirrored by a decline in the trend component of the employment-to-population

ratio. As already shown, the employment-to-population ratio features more amplified cyclical fluctuations around trend than the labor force participation rate. Consequently, for the most recent quarter of available data the employment-to-population ratio is estimated to be roughly 175 basis points below trend. While this variable has shown little improvement in absolute terms since 2010, the estimated trend decline implies that in the ongoing recovery the gap in the employment-to-population ratio relative to trend has been narrowing. The estimated Beveridge-Nelson trends for the employment-to-population ratio and the labor force participation rate imply an estimate of the equilibrium rate of unemployment that currently stands at 6 percent. With an unemployment rate averaging 7.7 percent in 2013:Q1, the estimated distance from full employment thus remains relatively large.

An additional consideration for the most recent period is that the cyclical behavior of the labor force participation rate relative to the employment-to-population ratio has not been in line with the historical experience. This is shown in Figure 4, which plots the estimated cyclical component of the labor force participation rate together with the estimated cyclical component of the employment-to-population ratio. The figure also shows the predicted value of the cyclical component of labor force participation according to the historical estimated relationship reported in equation (6). From the figure it is apparent that given the cyclical behavior of the employment-to-population ratio, the predicted response of the cyclical component of the labor force participation rate would have called for a larger cyclical decline in the labor force participation rate than what we have estimated. Specifically, over the period 2009–2011, the labor force participation rate was 50 basis points higher on average than what would have been predicted by the behavior of the cyclical component of the employment-to-population ratio. It is only in the most recent quarters of available data that the relationship between the cyclical components of the labor force participation rate and the employment-to-population ratio appears to have reverted toward historical norms.

Figure 5 shows the actual behavior of the unemployment rate over the period 2008:Q1 to 2013:Q1 and the unemployment rate that would have prevailed had the labor force participation rate followed historical norms. According to this exercise, the unemployment rate would have been lower by roughly three-quarters of one percentage point on average over the period 2009–2011. Thus, any explanation about the recent unemployment rate dynamics, which featured a noticeable decline driven mainly by a drop in the labor force participation rate, should also take into consideration why the labor force participation rate did not decline as much as usual from a cyclical standpoint during the recession and the earlier stages of the recovery. The unprecedented expansion of unemployment insurance

benefits over the period 2009–2012 may have put upward pressure on the rate of labor force participation by reducing exit from the labor force. Farber and Valletta (2013) find that over this period a subset of unemployment insurance recipients remained nominally unemployed rather than exit the labor force. However, they estimate that extended unemployment benefits increased the unemployment rate by about four-tenths of one percentage point, which is roughly half the difference between the actual path of the unemployment rate and our predicted path with the labor force participation rate following historical norms—at least over the years 2009 to 2011. As a result, extended unemployment insurance benefits can only partly explain the unusual cyclical behavior of the labor force participation rate until very recently.

While our analysis is too aggregate and reduced-form to address the possible reasons for the behavior of the labor force participation rate in the recent recession and early stages of the recovery, our empirical exercise suggests that at present the labor force participation rate is, relative to trend, close to its predicted value as given by equation (6). Importantly, such a statement is conditional on having an adequate estimate of the cyclical component of the employment-to-population ratio. If this is indeed the case and the future behavior of labor force participation evolves roughly in accordance with the cyclical norm captured by the estimated relationship in (6), then future declines in the unemployment rate will have to rely on an improvement in the employment-to-population ratio.

Another implication of this exercise is that while over the period 2011:Q4 to 2013:Q1 the unemployment rate declined by one percentage point, the improvement would have been much smaller had the labor force participation rate followed the historical pattern predicted by the evolution of the employment-to-population ratio. As Figure 5 illustrates, with the cyclical behavior of the labor force participation rate at historical norms, the improvement in the unemployment rate over this period would have been less than one-quarter of one percentage point. In other words, recent progress in the labor market as measured by the unemployment rate can be interpreted as having been driven mainly by the labor force participation rate reverting to a level that is more in line with the typical relationship between the labor force participation rate and the employment-to-population ratio at this stage of the cycle.

3. Robustness

Estimation of the cointegrated vector autoregression in the previous section was restricted to a sample ending in 2007:Q4, in order to avoid allowing the recent dynamics of the labor force participation rate and the employment-to-population ratio to unduly influence the assessment of the

trends post-2007. The same results of the previous section hold if the estimation sample for the cointegrated vector autoregression stops much earlier, in 1987:Q4. In this case, the estimated cyclical components for the labor force participation rate and the employment-to-population ratio are depicted in Figure 6 over the (out-of-sample) period 1988 to present, and they are compared with the estimates obtained from the estimation sample ending in 2007. As the figure shows, the estimates are fairly close, implying that the results of the previous section are not especially sensitive to the sample period chosen when estimating the cointegrated vector autoregression. In particular, the notion that an important portion of the recent drop in the rate of labor force participation reflects a decline in the estimated Beveridge-Nelson trend still appears to be valid.

The analysis of the previous section has been conducted for people aged 16 years and over. The same qualitative results hold if we restrict the analysis to people in their prime working years (that is, ages 25 to 54). For this working-age category, the level relationship between the labor force participation rate and the employment-to-population ratio estimated over the period 1958:Q1 to 2007:Q4 by dynamic least squares yields

$$(5') \quad \ln lp_t^P = -.170911 + 1.05035 \ln lq_t^P, \\ (.046273) \quad (.010721)$$

where the superscript P indicates that the variable refers to workers in their prime working ages. As before, unit root tests typically reject the hypothesis that the estimated residuals from this regression contain a unit root. The estimated trend deviation $\ln lp_t^P + .170911 - 1.05035 \ln lq_t^P$ is used to estimate a cointegrated vector autoregression, where the log-difference of the labor force participation rate and the log-difference of the employment-to-population ratio for prime age workers are each regressed on four lags of each of the two variables and on the error-correction term. The sample period for the estimation is 1958:Q1 to 2007:Q4. The first two panels of Figure 7 show actual values and estimated Beveridge-Nelson trends for the prime age workers' labor force participation rate and employment-to-population ratio, respectively. The third panel in the figure plots the estimated cyclical component for the two variables. Again, the degree of comovement between the two series is high, with a regression of the estimated cyclical component of the labor force participation rate on the estimated cyclical component of the employment-to-population ratio over the period 1958:Q1 to 2007:Q4 yielding:

$$(6') \quad lp_t^{P,C} = -.007906 + .507446 lq_t^{P,C}, \\ (.026424) \quad (.015623) \quad R^2 = .841.$$

For the most recent period, the analysis carries the same qualitative implications discussed in the previous section. In particular, roughly 50 percent of the post-2007 decline in the labor force participation rate for the prime working age category is attributable to a trend decline (see the first panel of Figure 8). The actual level of labor force participation as of 2013:Q1 is estimated to be roughly 60 basis points below trend. For the employment-to-population ratio (the second panel in figure 8), the corresponding figure is somewhat less than two percentage points. Figure 9 illustrates that given the estimated cyclical behavior of the employment-to-population ratio, according to the historical relationship (6') the cyclical downturn in the labor force participation rate should have been larger than what we are estimating. As in the previous section, it is only recently that the relationship between the cyclical components of the labor force participation rate and the employment-to-population ratio appears to have reverted toward historical norms.

The results in the previous section also continue to hold if, rather than resorting to the Beveridge-Nelson trend-cycle decomposition, the transitory and permanent components of the labor force participation rate and the employment-to-population ratio are identified as in Blanchard and Quah (1989). The Blanchard and Quah decomposition, in contrast to the Beveridge-Nelson decomposition, does not require the permanent (trend) component to be restricted to a pure random walk.⁵ However, in Blanchard and Quah, innovations to the permanent and transitory components are assumed to be uncorrelated, while such a restriction is not imposed in the Beveridge-Nelson decomposition. Since the Blanchard and Quah identification method allows for both the transitory and the permanent component to have dynamic effects, the method is suited to computing impulse responses of the labor force participation rate and the employment-to-population ratio to transitory and permanent shocks. The impulse responses are depicted in Figure 10 and are estimated by means of the same vector autoregression with the error-correction term as in the previous section, over the period 1958:Q1 to 2007:Q4. With the Blanchard and Quah identification method, the permanent shock can affect the level of the labor force participation rate and the employment-to-population ratio permanently, and the error-correction term dictates the relationship between the responses of these two variables to the permanent shock in the long run. The other identified shock can have only transitory effects on the level of the labor force participation rate and the employment-to-population ratio. The impulse responses show that transitory shocks generate a positive relationship between the two variables. The response of labor force participation to the transitory shock is more muted, with the average response 5 to 12

⁵ See Quah (1988).

quarters after the shock amounting to roughly 39 percent of the average response of the employment-to-population ratio over the same period. This number is similar to the previous section's estimate reported in (6) when relating the Beveridge-Nelson cyclical component of the labor force participation rate to the cyclical component of the employment-to-population ratio.

As concerns the effect of permanent shocks, the Blanchard and Quah identification method for the (out-of-sample) period 2008:Q1 to 2013:Q1 shows a prevalence of negative disturbances, indicating that part of the decline in the labor force participation rate and the employment-to-population ratio over this period is nontransitory. In particular, it is possible to show that the effect of these shocks on the level of the labor force participation rate implies a permanent decline in labor force participation of 2-1/2 percent. This is about the same percentage decline as estimated in the previous section's Beveridge-Nelson trend labor force participation rate over the most recent 21 quarters.⁶

Overall, the results in this section provide—at least along the dimensions that we have considered—some corroborating evidence for our finding that a significant portion of the decline in the rate of labor force participation since 2008:Q1 reflects trend movements. Indirect evidence in support of the other finding that the cyclical response of the rate of labor force participation has been smaller than usual for most of the post-2007 period given the cyclical behavior of the employment-to-population ratio can also be gleaned from extending to 2013:Q1 the estimation sample for the vector autoregression with the error-correction term. The estimated responses to a temporary shock identified via the Blanchard and Quah method with this extended sample are shown in Figure 11, together with the original estimates for the same vector autoregression estimated up to 2007:Q4 (already reported in Figure 10). It is apparent that the inclusion of the most recent period reduces the magnitude of the estimated response of the rate of labor force participation relative to the response of the employment-to-population ratio.

4. Concluding Remarks

According to our empirical exercise, the cyclical behavior of the labor force participation rate now appears to be in line with historical norms. This was not the case for most of the Great Recession and subsequent recovery period, when the estimated cyclical response of the labor force participation rate was smaller than usual given the estimated cyclical behavior of the employment-to-population

⁶ In particular, the Beveridge-Nelson trend labor force participation rate was estimated at 65.9 in 2007:Q4, and at 64.2 in 2013:Q1. This represents a decline of 2.6 percent.

ratio. At this point, however, the discrepancy has largely vanished, and going forward the unemployment rate should convey an accurate signal of labor market conditions from a cyclical perspective. Further cyclical declines in the rate of labor force participation are unlikely to occur if the employment situation continues to improve. Our analysis is not supportive of the view that monetary policy under current circumstances should respond differently to labor market conditions than in the past, because the cyclical decline in labor force participation rates that has taken place in the post-2007 period has been abnormally large. If anything, the analysis shows that the opposite has been true.

These conclusions, however, need to be tempered by the fact that our analysis represents just one possible approach to estimating trend and cycle for the labor force participation rate. In addition, our empirical exercise is reduced-form and statistical in nature and therefore lacks the structure to frame the economic decision of whether to participate (or not) in the labor force. As such, the results of our analysis should be taken with caution. If anything the analysis, by reaching different conclusions than those reached by other studies, illustrates that there is a considerable degree of uncertainty when interpreting what portion of the decline in the rate of labor force participation is due to trend and what portion is due to cycle.

Figure 1.

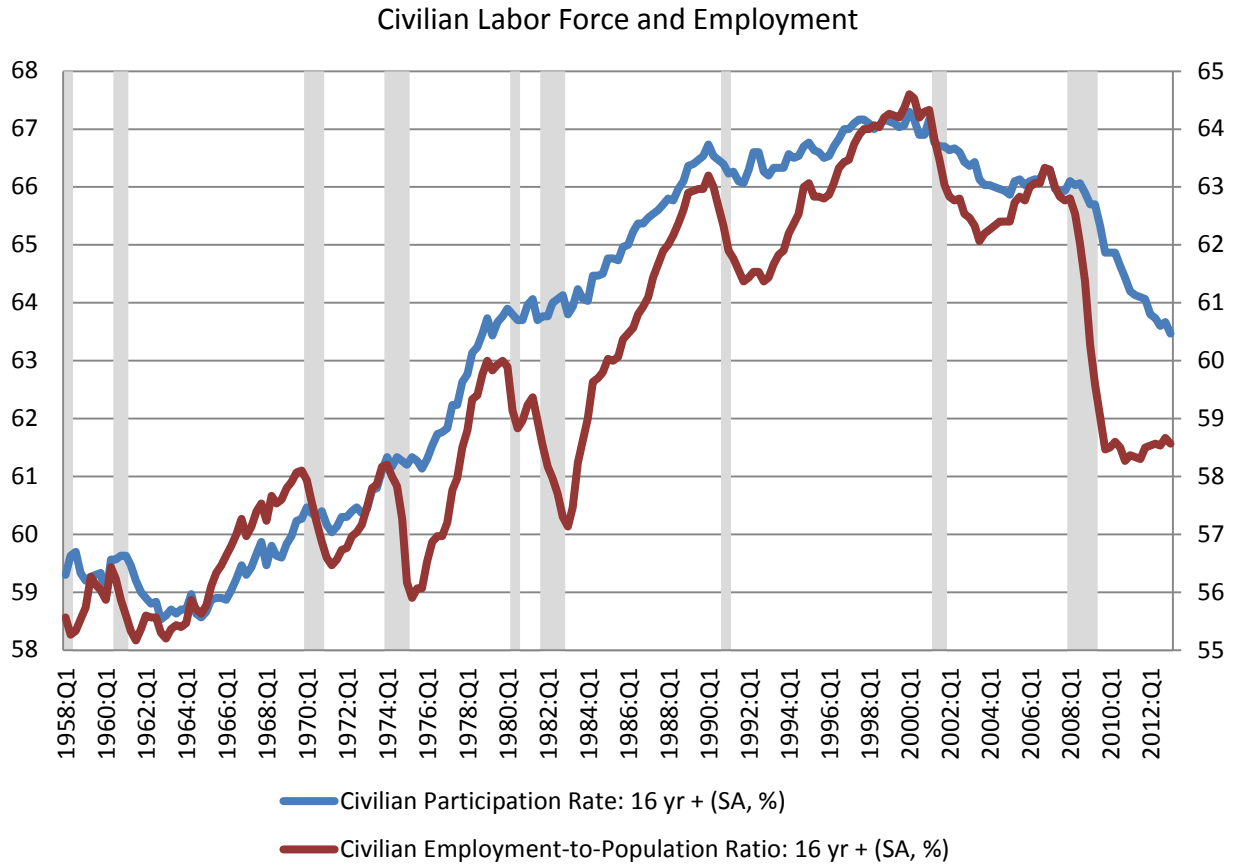


Figure 2.

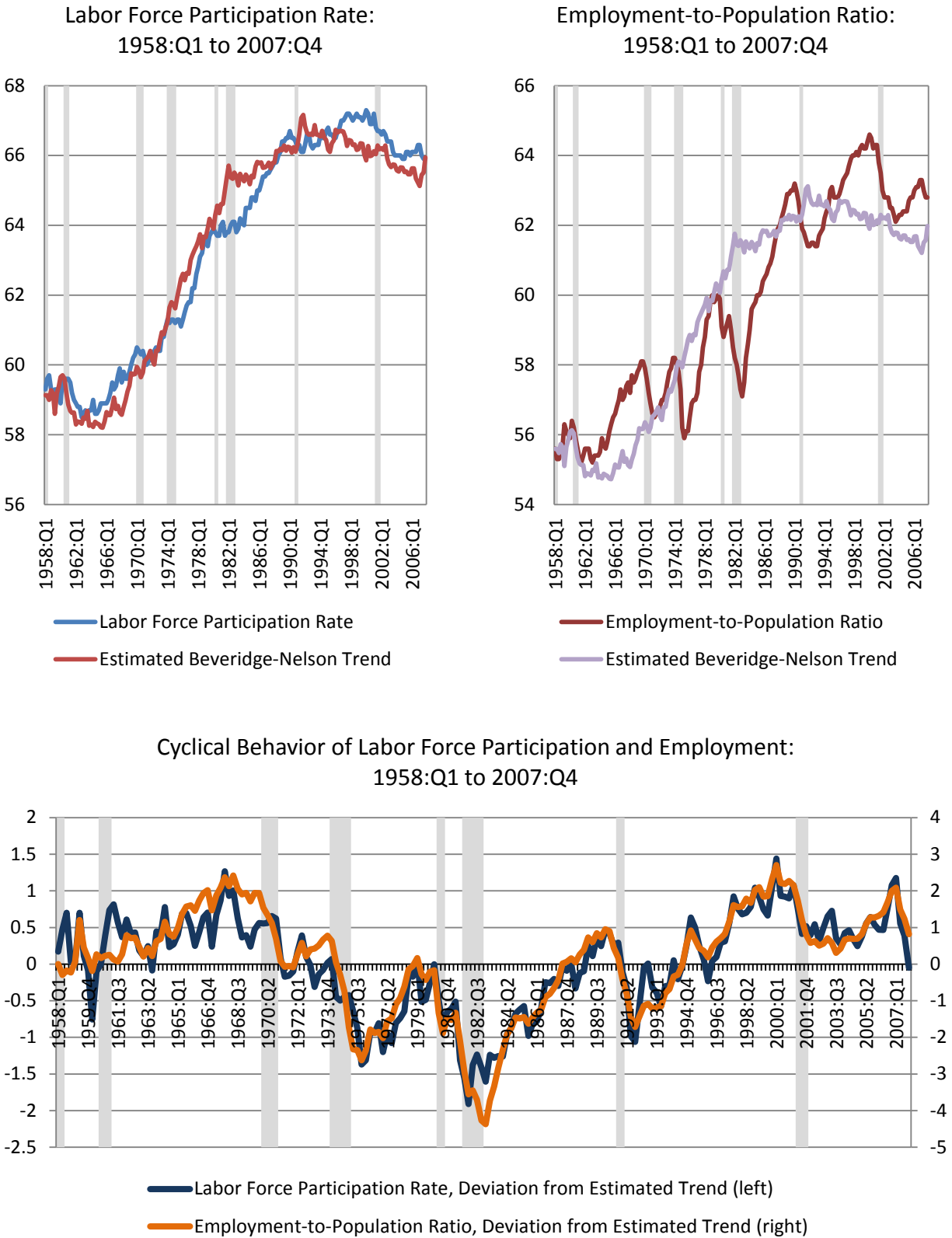


Figure 3.

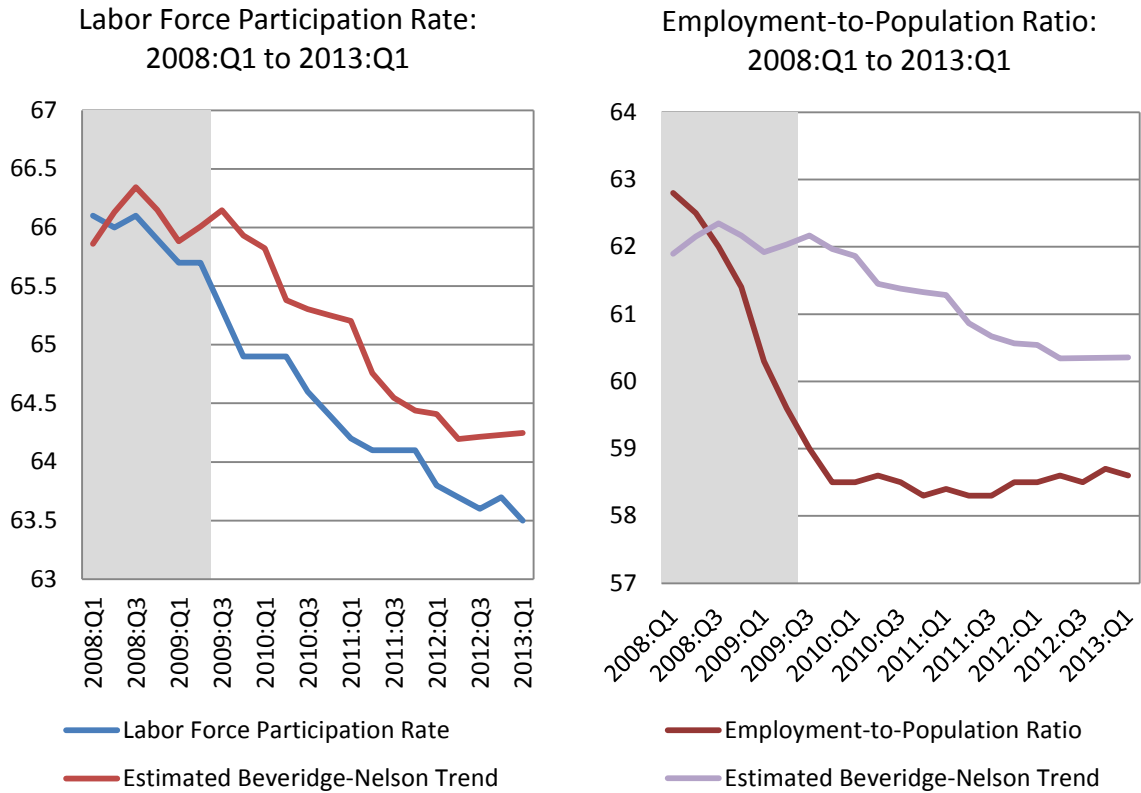


Figure 4.

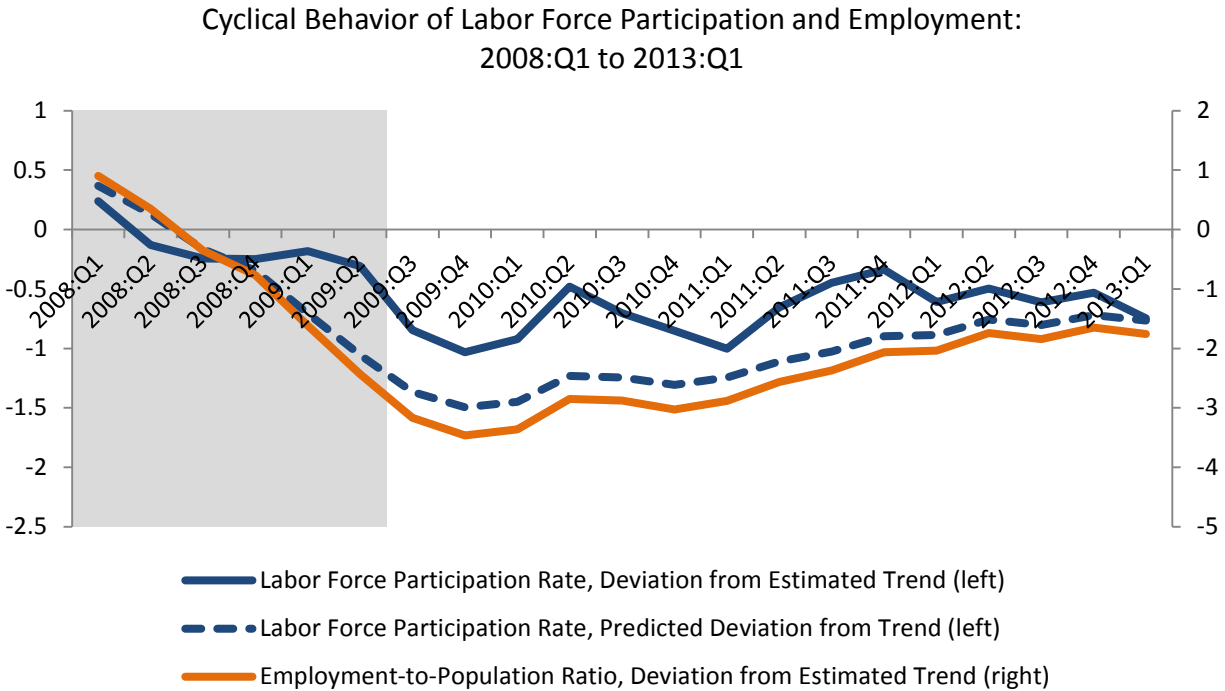


Figure 5.

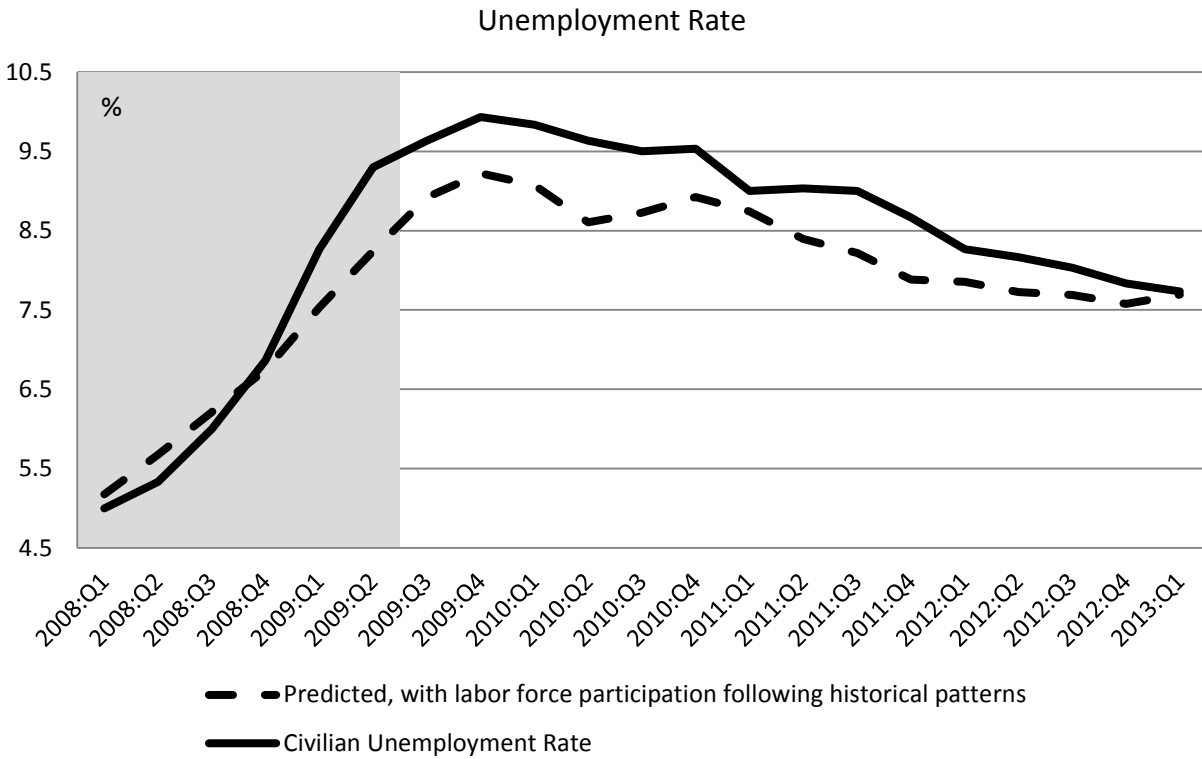


Figure 6.

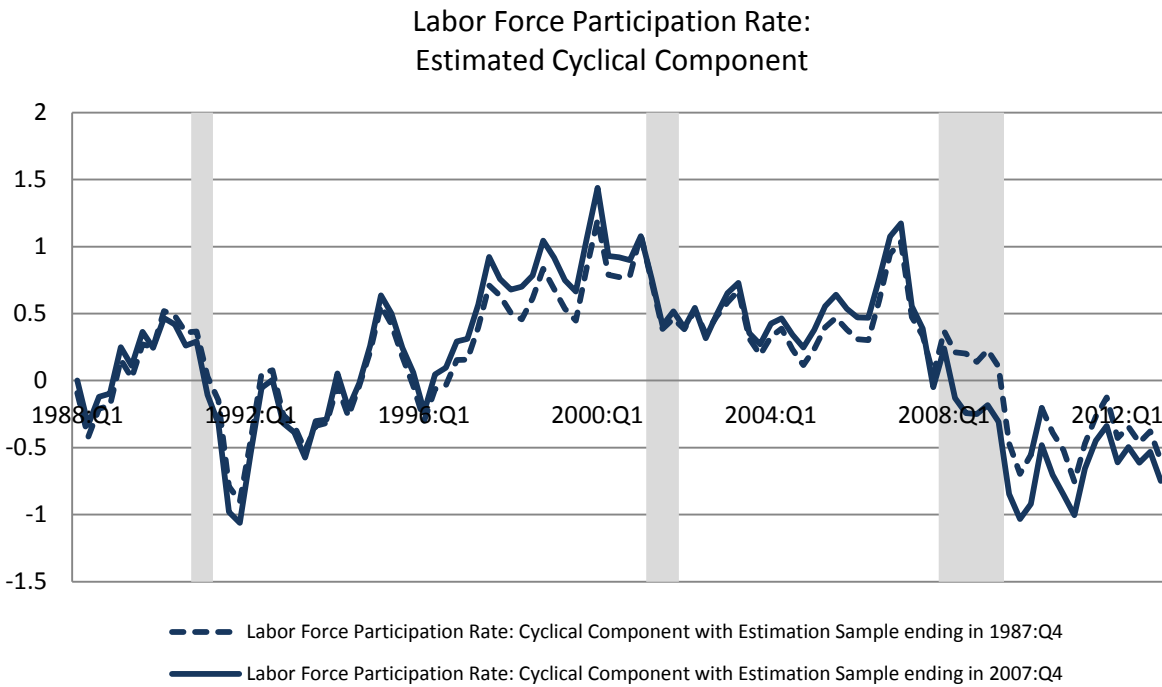


Figure 7.

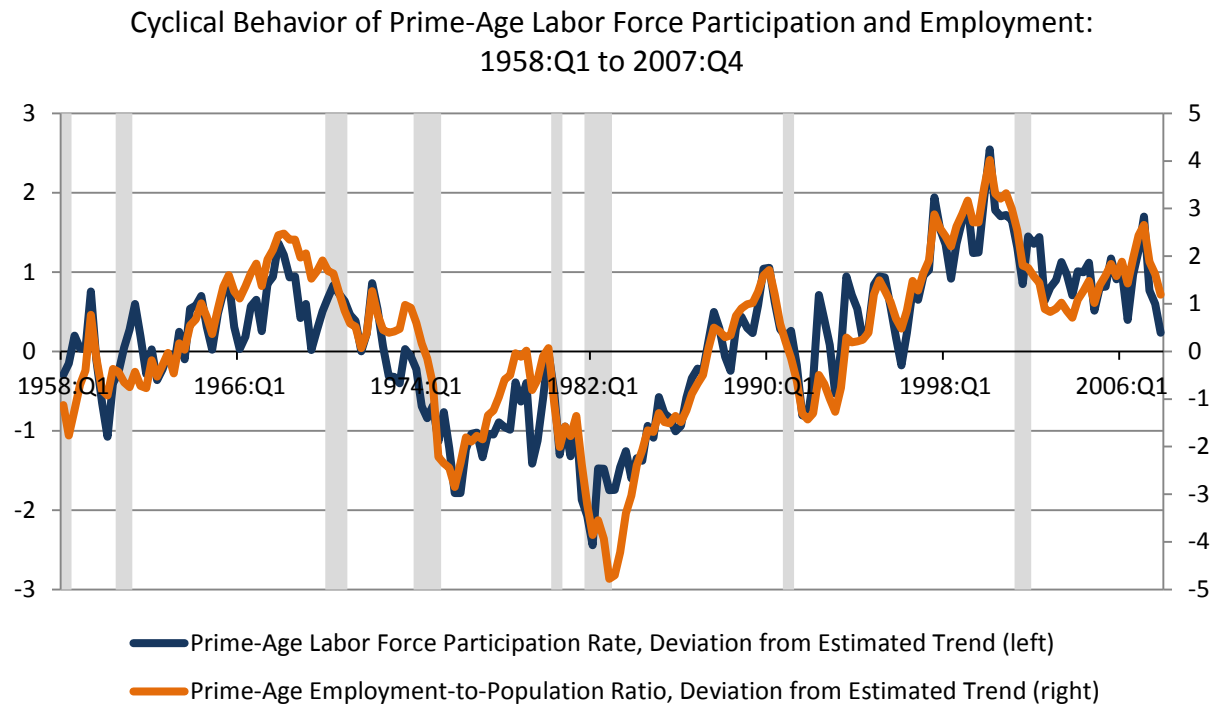
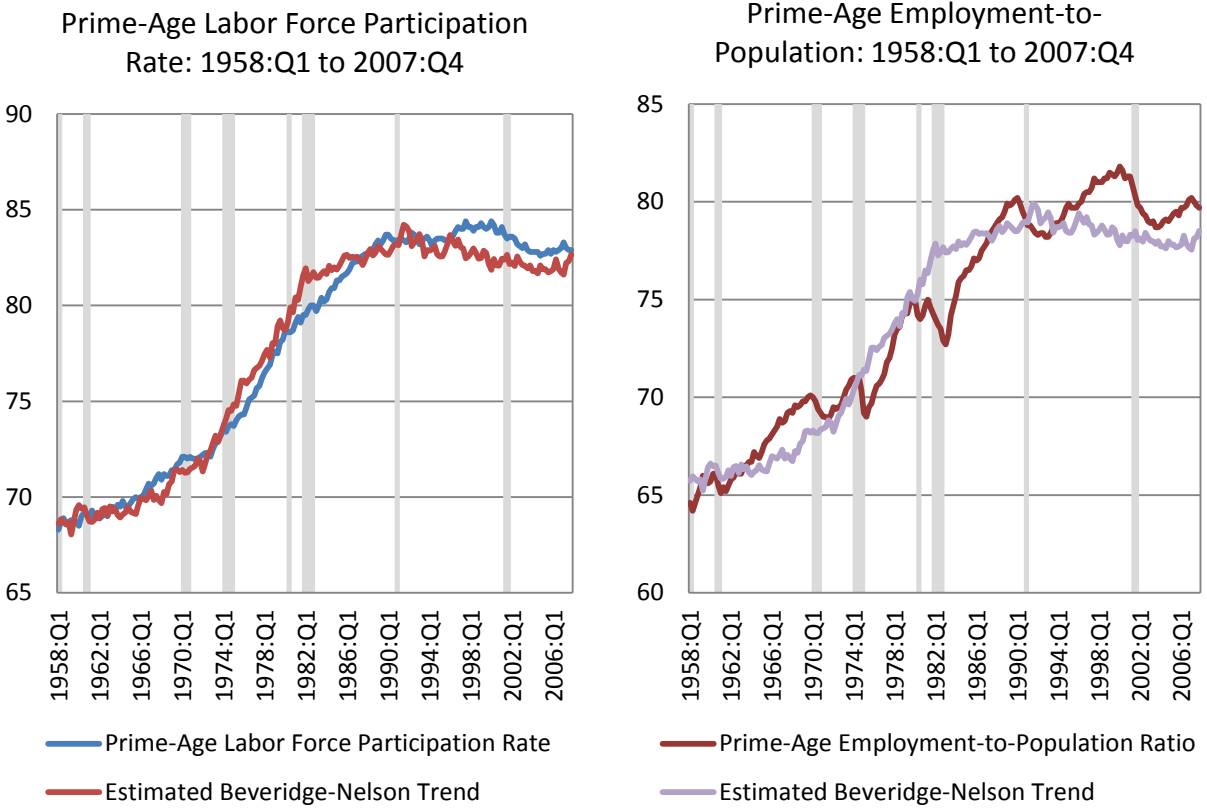


Figure 8.

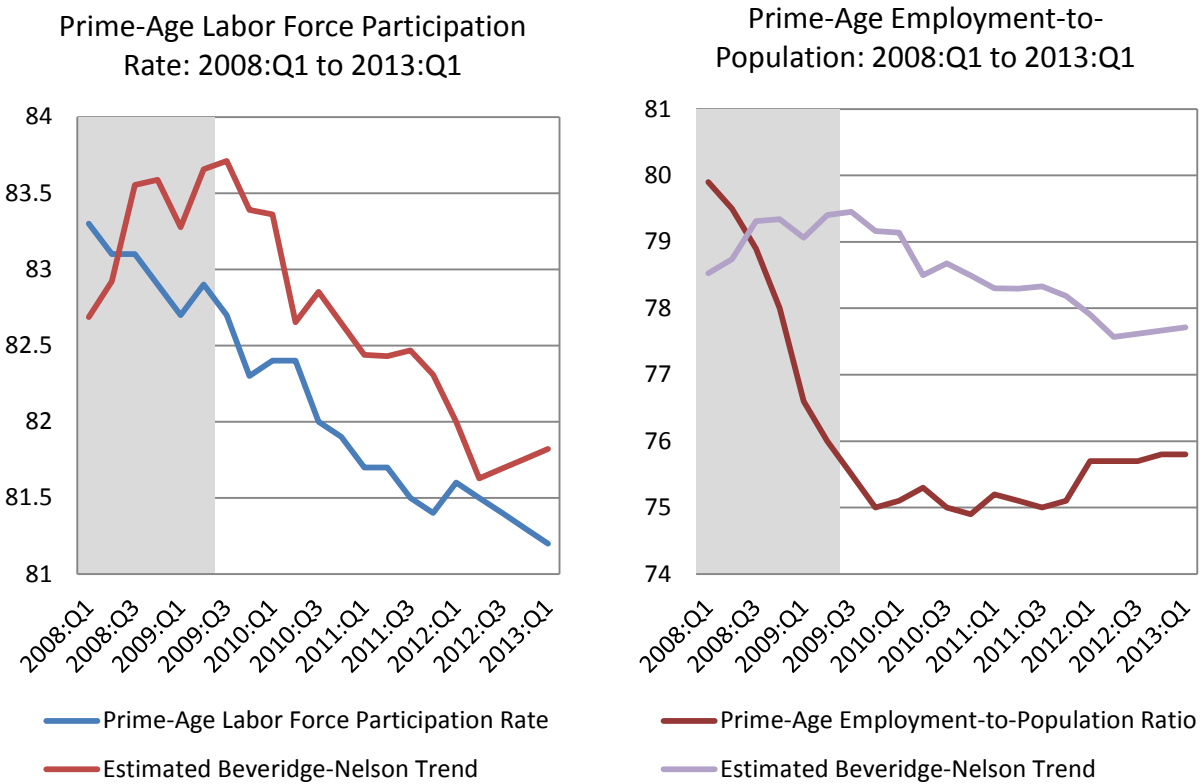


Figure 9.

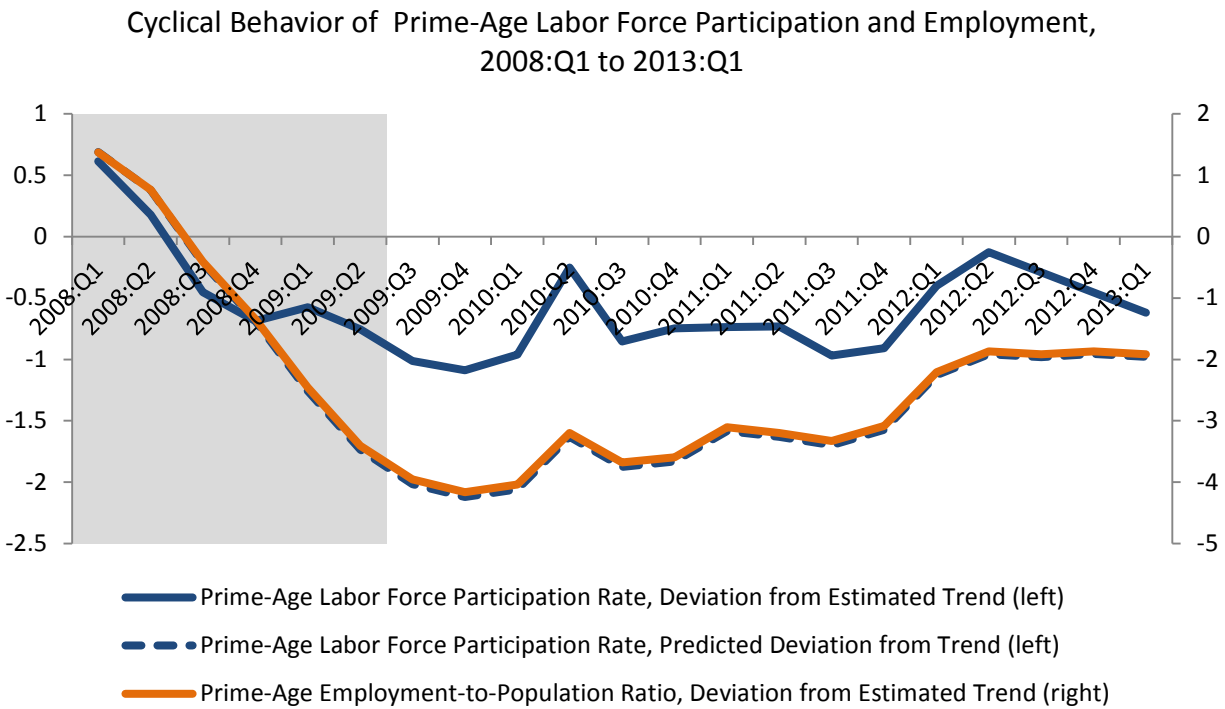


Figure 10.

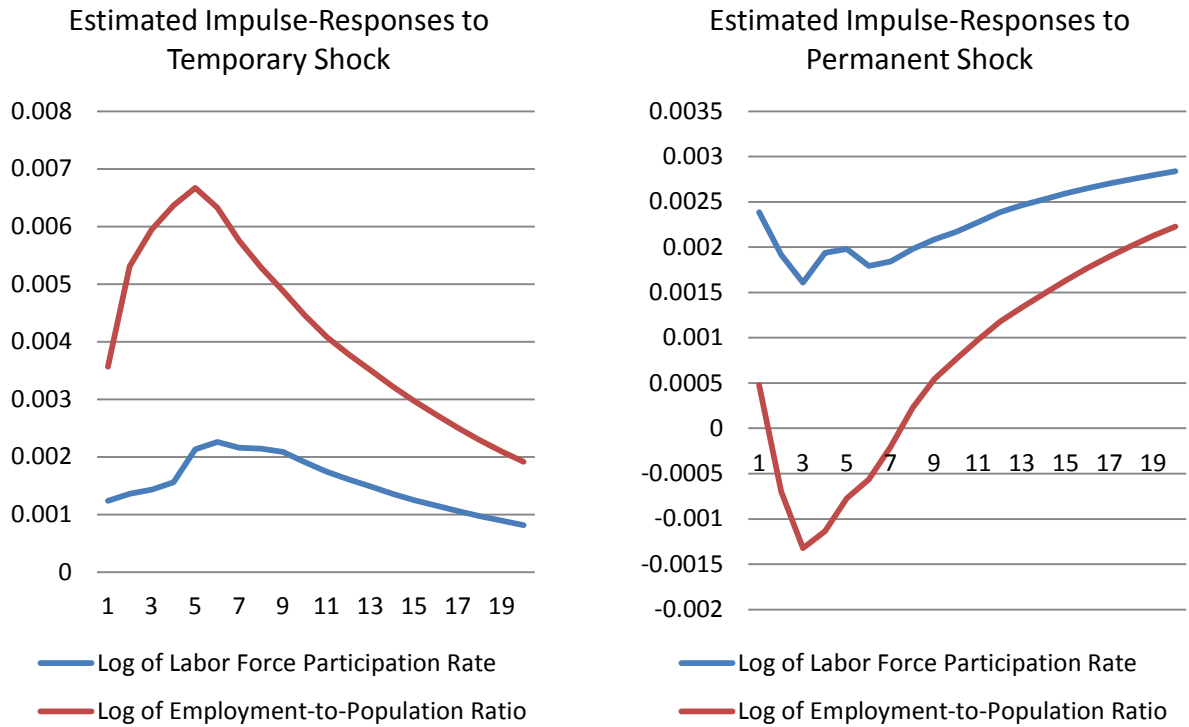
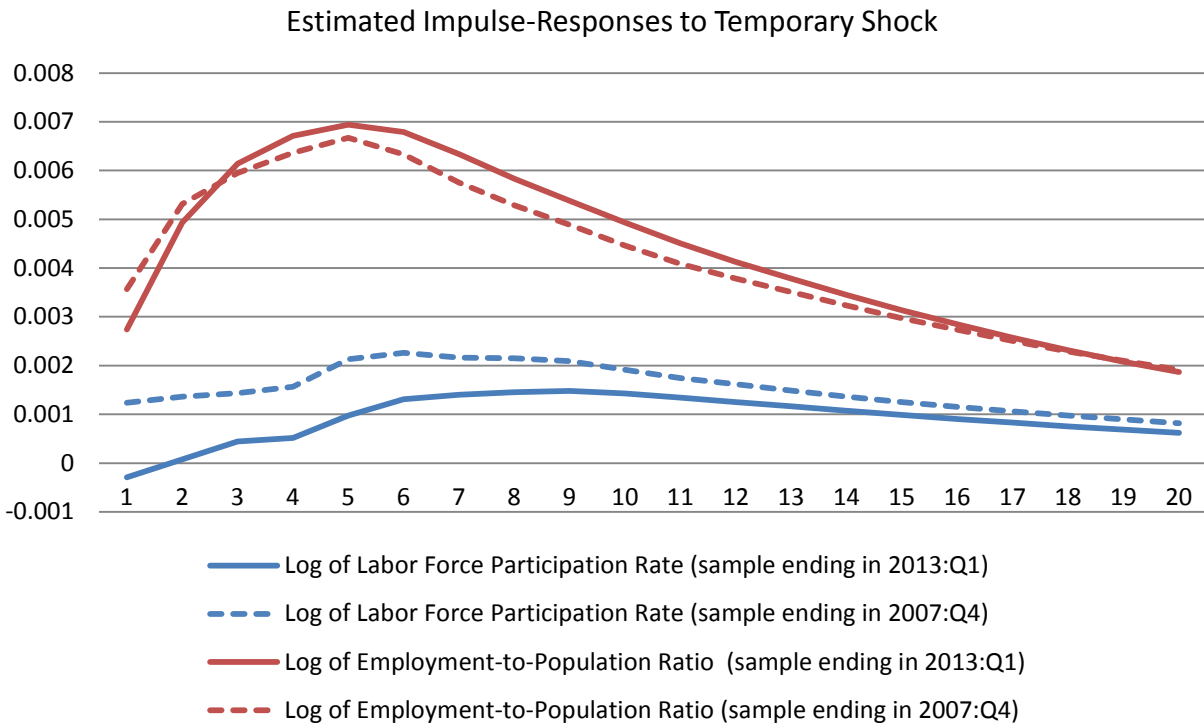


Figure 11.



Appendix

We illustrate here briefly how the computation of the Beveridge-Nelson decomposition in a multivariate setting summarized in Section 2 extends to the case with cointegrated variables. For more detail, see Ariño and Newbold (1998). Let the dynamic of the vector autoregression in first-order form be written as:

$$(2') \quad \Delta \mathbf{Y}_t = \mathbf{A} \Delta \mathbf{Y}_{t-1} + \gamma \beta' \mathbf{Y}_{t-1} + \mathbf{u}_t,$$

where γ and β are row vectors of size np , and $\beta' \mathbf{Y}_{t-1}$ describes the single (without loss of generality) cointegrating relationship among the series included in the information set.⁷ Pre-multiplying the above relationship by β' and rearranging terms yields

$$\beta' \mathbf{Y}_t = \beta' \mathbf{A} \Delta \mathbf{Y}_{t-1} + (1 + \beta' \gamma) \beta' \mathbf{Y}_{t-1} + \beta' \mathbf{u}_t,$$

so that the dynamics of the variables can be written as follows:

$$(2'') \quad \begin{bmatrix} \Delta \mathbf{Y}_t \\ \beta' \mathbf{Y}_t \end{bmatrix} = \begin{bmatrix} \mathbf{A} & \gamma \\ \beta' \mathbf{A} & 1 + \beta' \gamma \end{bmatrix} \begin{bmatrix} \Delta \mathbf{Y}_{t-1} \\ \beta' \mathbf{Y}_{t-1} \end{bmatrix} + \begin{bmatrix} \mathbf{u}_t \\ \beta' \mathbf{u}_t \end{bmatrix}.$$

The notation in the above expression can be collapsed to

$$(2''') \quad \mathbf{Z}_t = \mathbf{\Gamma} \mathbf{Z}_{t-1} + \mathbf{v}_t,$$

where the relationship between \mathbf{Z}_t , $\mathbf{\Gamma}$, \mathbf{v}_t , and the corresponding elements in (2'') is straightforward. Given the law of motion in (2'''), for a variable $y_{i,t}$ belonging to the vector autoregression's information set the Beveridge-Nelson trend component is given by:

$$(3') \quad y_{i,t}^T = y_{i,t} + \mathbf{e}_i' (\mathbf{I} - \mathbf{\Gamma})^{-1} \mathbf{\Gamma} \mathbf{Z}_t,$$

where \mathbf{e}_i is a selection vector for $\Delta y_{i,t}$ in \mathbf{Z}_t .

⁷ The potential nonzero elements in the vectors γ and β are positioned so as to select the time $t-1$ observation for each variable in \mathbf{Y}_{t-1} . All the other elements in γ and β are constrained to zero.

References

- Aaronson, Daniel, Jonathan Davis, and Luoia Hu. 2012. "Explaining the Decline in the U.S. Labor Force Participation Rate." *Chicago Fed Letter* Federal Reserve Bank of Chicago.
- Ariño, Miguel, and Paul Newbold. 1998. "Computation of the Beveridge-Nelson Decomposition for Multivariate Economic Time Series." *Economic Letters* 61: 37–42.
- Beveridge, Stephen, and Charles R. Nelson. 1981. "A New Approach to Decomposition of Economic Time Series into Permanent and Transitory Components with Particular Attention to Measurement of the Business Cycle." *Journal of Monetary Economics* 7: 151–174.
- Blanchard, Olivier J., and Danny Quah. 1989. "The Dynamic Effects of Aggregate Demand and Supply Disturbances." *American Economic Review* 79 (4): 655–673 (Sept.).
- Erceg, Christopher J., and Andrew T. Levin. 2013. "Labor Force Participation and Monetary Policy in the Wake of the Great Recession." mimeo.
- Farber, Henry S., and Robert G. Valletta. 2013. "Do Extended Unemployment Benefits Lengthen Unemployment Spells? Evidence from Recent Cycles in the U.S. Labor Market." NBER Working Paper 19048.
- Quah, Danny. 1988. "The Relative Importance of Permanent and Transitory Components: Identification and Some Theoretical Bounds." Department of Economics Working Paper 498, Massachusetts Institute of Technology.
- Stock, James H., and Mark W. Watson. 1993. "A Simple Estimator of Cointegrating Vectors in Higher Order Integrated Systems." *Econometrica* 61(4): 783–820 (July).
- Toossi, Mitra. 2012. "Labor Force Projections to 2020: A More Slowly Growing Workforce." *Monthly Labor Review* January: 43–64.
- Toossi, Mitra. 2007. "Labor Force Projections to 2016: More Workers in Their Golden Years," *Monthly Labor Review* November: 33–52.
- Van Zandweghe, Willem. 2012. "Interpreting the Recent Decline in Labor Force Participation," *Economic Review* Federal Reserve Bank of Kansas City. First Quarter: 5–34.