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Risk, Returns, and Multinational Production



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Risk, Returns, and Multinational Production*

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Abstract

This paper starts by unveiling a strong empirical regularity: multinational corporations exhibit higher stock market returns and earnings yields than non-multinational firms. Within non-multinationals, exporters exhibit higher earnings yields and returns than firms selling only in their domestic market. To explain this pattern, we develop a real option value model where firms are heterogeneous in productivity, and have to decide whether and how to sell in a foreign market where demand is risky. Firms selling abroad are exposed to risk: following a negative shock, they are reluctant to exit the foreign market because they would forgo the sunk cost that they paid to start investing abroad. Multinational firms are the most exposed due to the higher sunk costs they have to pay to enter. The model, calibrated to match aggregate U.S. export and foreign direct investment data, is able to replicate the observed cross-sectional differences in earnings yields and returns.

Keywords: Multinational firms, option value, cross-sectional returns

JEL Classification : F12, F23, G12

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1 Introduction

Multinational firms tend to exhibit higher stock market returns and earnings yields than non-multinational firms. Among non-multinationals, exporters tend to exhibit higher returns and earnings yields than firms selling only in their domestic market. Many studies in the new trade literature have documented features distinguishing firms that sell into foreign markets from firms that do not: exporters and multinational firms tend to be larger, more productive, to employ more workers, and sell more products than firms selling only domestically.¹ However, none of this literature has addressed the question of whether the international status of the firm matters for its investors. Similarly, in the financial literature, explanations of the cross section of returns overlooked the role of the international status of the firm.²

In this paper we attempt to fill this gap in the literature. We develop a real option value model where firms' heterogeneity, aggregate uncertainty and fixed and sunk costs of production provide the missing link between firms' international status and their stock market returns.

The fact that exporters and multinational firms give higher yields and returns than domestic firms does not constitute an anomaly *per se*. It indicates that these firms are riskier than firms that do not serve foreign markets: if this were not the case, rational agents would not hold shares of domestic firms in equilibrium. The concept of risk that we consider arises from the covariance of cash flows with the investors' intertemporal marginal rate of substitution. After documenting cross-sectional differences in returns and in their covariances with consumption growth, the purpose of our structural model is to identify a plausible channel that delivers differential exposure to risk of exporters and multinational corporations' cash flows.

The mechanism of the model is simple: suppose a firm decides to enter a foreign market where aggregate demand is subject to fluctuations, and entry involves a sunk cost. In "good times", when prospects of growth make entry profitable, a firm may decide to pay the sunk cost and enter. If – after entry – the shock reverses, the firm may experience losses due to the necessity of covering fixed operating costs. In this case, the firm will be reluctant to exit immediately because of the sunk cost it paid

¹See, among others, Bernard, Jensen, and Schott (2009).

²One notable exception is Fatemi (1984).

to enter, and may prefer to bear losses for a while, hoping for better times to come again. If sunk costs of establishing a foreign affiliate are larger than the sunk costs of starting to export, then the exposure to demand fluctuations and possible negative profits will be higher for multinational firms than for exporters, and will command a higher return in equilibrium.

The choice of whether to serve the foreign market and how (via export or foreign investment, henceforth FDI) is endogenous, and we model it following the literature on heterogeneous firms in international trade, namely the influential contribution by Helpman, Melitz, and Yeaple (2004). Exports are characterized by low sunk costs and high variable costs, due to the necessity of shipping goods every period, while FDI entails high sunk costs of setting up a plant and starting production abroad, but low variable costs, since there is no physical separation between production and sales. The model in Helpman, Melitz, and Yeaple (2004) is static, hence the value of a firm coincides with its profits and earnings yields are constant across firms. A dynamic but deterministic model, or a dynamic and stochastic model with idiosyncratic shocks share the same feature, with earnings-to-price ratios simply given by the discount rate. The same is true for the returns, which are given by the earning yields plus the expected change in the valuation of the firm (this last term being zero in the static framework). To generate heterogeneity in these variables across firms, we extend the basic Helpman, Melitz, and Yeaple (2004) framework to a dynamic and stochastic environment characterized by persistent shocks, using Dixit (1989) as a benchmark to model entry decision under uncertainty. In the model, firms choose whether to export or invest abroad based on their productivity and on prospects of growth of foreign demand. Larger sunk costs of foreign investment compared to export imply a larger inaction band for multinational firms compared to exporters, and multinationals may experience larger losses if the economy is hit by a negative shock.

How does this behavior generate heterogeneity in earnings yields and returns? Sunk costs of exports and FDI can be interpreted as the premia to be paid to exercise the option of entering the foreign market. The value of this option is an important component of the valuation of the firm. Hence profit flow and firm value are not proportional due to this extra component: the option value of entering/exiting the market, which differs across firms. To generate heterogeneity in stock market returns,

we nest the heterogeneous firms framework into an aggregate endowment economy, in the spirit of consumption-based asset pricing models à la Lucas (1978). Risk-averse consumers own shares of the firms, and discount future consumption streams with a stochastic discount factor dependent on the aggregate shocks. Firms' heterogeneity and endogenous status choices imply that different firms will differ in the covariance of their cash flows with the aggregate uncertainty, which affects consumers' marginal utility. As a result, the model endogenously determines cross-sectional differences in earnings-to-price ratios and returns, and provides a complementary explanation for the cross section of returns exploiting the production side from an international point of view.

The model can be parameterized to reproduce features of the data on trade and FDI dynamics and participation in export and FDI. With the calibrated model we simulate an artificial economy and compute the financial variables. We show that the model is able to generate the rankings of earnings yields and returns, which were not targeted in the calibration. While generally successful at replicating the ranking in financial variables, we show that the quantitative performance of the model is not completely satisfactory under our baseline CRRA specification of preferences. Adding recursive preferences and shocks to the expected growth rate of consumption significantly improves the quantitative fit.

There is a large body of literature that investigates cross-sectional differences in stock returns and earnings-to-price. Fama and French (1996) provide comprehensive evidence about returns differentials across portfolios formed according to particular characteristics (like size and book-to-market). In this paper we cut the data along an unexplored dimension, addressing the risk-return trade-offs of firms serving foreign markets. We focus on the cash flow dynamics of the firm and on how these are determined by endogenous decisions and exogenous risk. Multinational firms are exposed to foreign demand risk for longer due to the higher persistence in their status. This risk must be rewarded by a higher asset returns in equilibrium. Investors will be willing to hold these companies if the returns are high enough to compensate for the risk.

The existing finance literature that focuses on cross-sectional differences in earnings-to-price ratios and returns abstracts from the international organization of the firm.

There are numerous attempts to explain risk premium and cross sectional differences in expected returns that generalize the canonical power utility consumption-based model. These attempts entail different specifications of preferences, different specifications of the cash flow dynamics, or both.³ Our paper is aligned with the production-based models that link asset prices to firms' decisions, like Gomes, Kogan, and Zhang (2003), Gala (2012), and Gourio (2007). We contribute to the finance literature by endogenizing the exposure of firms' cash-flows to fundamental shocks. Exposure is directly linked to the decision of whether and how to serve the foreign market, which is ultimately driven by the interaction between productivity and cost structure.

This paper is also related to a strand of literature in corporate finance, studying the linkages between international activity and stock market variables. Our empirical evidence is consistent with the analysis in Denis, Denis, and Yost (2002), who find that multinational corporations trade at a discount, and with Baker, Foley, and Wurgler (2009), in linking empirically market valuations, returns, and FDI activity. Our approach departs from these contributions in explicitly acknowledging the endogeneity of the variables of interest and in using a structural model to understand the economic forces behind the correlations that we find in the data.

Our work is related to the literature on trade and FDI under uncertainty, mainly to Rob and Vettas (2003), Russ (2007), Ramondo and Rappoport (2010), and Ramondo, Rappoport, and Ruhl (2011). Rob and Vettas (2003) developed a model of trade and FDI with uncertain demand growth. In their framework FDI is irreversible, so it can generate excess capacity, but has lower marginal cost compared to export. The authors show that uncertainty implies existence of an interior solution where export and FDI coexist. Besides the different focus of the exercise, our work generalizes their model to one with many heterogeneous firms and a more general process for demand growth. Russ (2007) also formulates a problem of foreign investment under

³Yogo (2006) and Piazzesi, Schneider, and Tuzel (2007) are examples of non-separable goods in the utility function. Eichenbaum, Hansen, and Singleton (1988) find that non-separabilities between consumption and leisure decisions do not improve the CCAPM results. Campbell and Cochrane (1999) use internal habits specifications which impose non-separability of preferences over time. Epstein and Zin (1989) use recursive preferences to de-couple risk aversion from intertemporal elasticity of substitution and Bansal and Yaron (2004), and Hansen, Heaton, and Li (2008) add to recursive preferences persistent shocks to the endowment and cash flow dynamics to generate long run risk.

uncertainty to study the response of FDI to exchange rate fluctuations.⁴ Her model features firm heterogeneity, but does not allow trade as a way to serve foreign markets. Ramondo and Rappoport (2010) introduce idiosyncratic and aggregate shocks in a model where firms can locate plants both domestically and abroad. Multinational production allows firms to match domestic productivity and foreign shocks, and works as a mechanism for risk sharing. Ramondo, Rappoport, and Ruhl (2011) extend their setting to a model featuring also exports. Our framework allows for risk sharing and diversification in addition to the risk exposure driven by the combination of aggregate shocks and sunk costs. We allow for country-specific shocks with various correlation patterns.

This paper is also related to a growing body of literature on trade dynamics with sunk costs. Particularly, Alessandria and Choi (2007) and Irarrazabal and Oromolla (2009) model entry and exit into the export market in a world with idiosyncratic productivity shocks and sunk costs. Our model is closer to the framework in Irarrazabal and Oromolla (2009) for the use of the real option value analogy in solving the firm's optimization problem. While Irarrazabal and Oromolla (2009) concentrate their attention on the impact of idiosyncratic productivity shocks for firm dynamics, we model aggregate demand shocks that affect firms differently only through their endogenous choice of international status. Alessandria and Choi (2007) study the impact of firms' shocks and sunk costs on the business cycle. While the objective of our exercise is different from their paper, we follow their calibration methodology. Both papers analyze the decision to export, but do not consider the possibility of FDI sales. Roberts and Tybout (1997) and Das, Roberts, and Tybout (2007) empirically address the issue of market participation for export. Our model has similar predictions for both exports and FDI sales, and can be calibrated using information from trade and FDI data. In general, we contribute to the trade dynamics literature both empirically and theoretically: we document features of trade and FDI dynamics for large firms, and we incorporate in the model the mode of entry (*i.e.*, the decision between export and FDI sales).

While individual elements of our framework are found in other work, to our knowledge this paper is the first to propose a dynamic industry equilibrium model where

⁴Goldberg and Kolstad (1995) study the effect of exchange rate fluctuations on the location choices of multinational firms.

risk affects firms' international strategies and their financial variables in the stock market. The remainder of the paper is organized as follows. Section 2 presents empirical evidence establishing the ranking in financial variables and linking it to heterogeneity in risk exposure. Section 3 develops the model and characterizes the equilibrium. Section 4 brings the theory to the data: we calibrate the model using aggregate trade and FDI data and report quantitative results on earnings yields and returns predicted by the model. Section 5 concludes.

2 Empirical Evidence

In this section we document a novel empirical regularity linking firms international activities to stock market data. We find that multinational firms exhibit higher annual stock returns and earnings yields than exporters. In turn, exporters exhibit higher annual stock returns and earnings yields than firms selling only in their domestic market. To support the interpretation of cross-sectional differences in returns as differences in risk, we report evidence on the heterogeneous exposure of returns to consumption growth fluctuations.

2.1 The Data

The data used in this paper are from a sample of manufacturing firms that are publicly traded in the US stock market. Financial data is available from Compustat.⁵ Stock market data, like stock prices, dividends and returns, are obtained from the Center for Research in Security Prices (CRSP). We restrict our sample to firms incorporated in the U.S. whose headquarters are also located in the U.S.

In addition to the financial data, Compustat contains information about the geographical segments where the firms operate.⁶ The segments information allows us to classify the firms in three groups every fiscal year: multinationals, exporters, and domestic firms. Firms that report the existence of a foreign geographical segment as-

⁵Financial data refers to the data that firms report to the Securities and Exchange Commission (SEC) on the annual 10K files.

⁶Multinational and exporter dummies are constructed based on Compustat geographic and operating segments data. Appendix A contains a summary of data reporting criteria from the Financial Accounting Standards (FAS) Statement, and details about the construction of the sample.

Table 1: **Descriptive Statistics.** All statistics are averages across firm-year observations except the total number of firms, which accounts for the number of firms that belonged to a given group at least for one year.

	Domestic	Exporter	Multinational
Domestic sales (millions \$)	206.5	186.7	1589.9
Export sales (millions \$)	0	28.86	123.4
FDI sales (millions \$)	0	0	969.8
Number of employees (thousands)	1.539	1.716	10.95
Capital/labor ratio (millions \$ per worker)	0.153	0.0999	0.115
Market capitalization (millions \$)	192.0	117.9	1596.1
Book-to-market ratio	0.945	1.209	1.282
Total earnings (millions \$)	4.496	5.034	85.10
Annual earnings-to-price ratio	-0.0672	0.0296	0.0715
Annual returns (%)	4.147	9.156	9.443
Number of firms	2546	1798	3197

sociated with positive sales are classified as multinationals.⁷ Firms that do not report any foreign segment with positive sales but report positive exports are classified as exporters. All other firms are classified as domestic.⁸ The information contained in the segments data restricts our sample period to 1979-2009.

Table 1 shows firm-level descriptive statistics of the data by type of firm. Consistent with the empirical trade literature, multinational firms are overwhelmingly larger than exporters and domestic firms both in terms of sales and number of employees. They also have much higher earnings and market capitalization. In this sample, differences between exporters and domestic firms are more nuanced than what other papers report (see for example Bernard, Jensen, and Schott (2009) or Bernard et al. (2007)). This feature of our data is due to sample selection: Compustat is a sample of publicly listed, large firms. Many small (mostly domestic) firms are not included. As a result, since the sample includes the largest domestic firms only, domestic firms and exporters are less different than in a sample representative of the entire firm size

⁷Denis, Denis, and Yost (2002) also use this information from Compustat Segments to identify multinational firms in the data.

⁸Most multinational firms have also positive exports. For reasons that will become clear when we present our structural model, we believe that multinational firms that also export are exposed to at least the same risks affecting non-exporting multinational firms, and they are hence classified as multinationals.

distribution. Most importantly for our purposes, a pecking order in earnings-to-price ratios and stock returns appears in the summary statistics at the firm level.

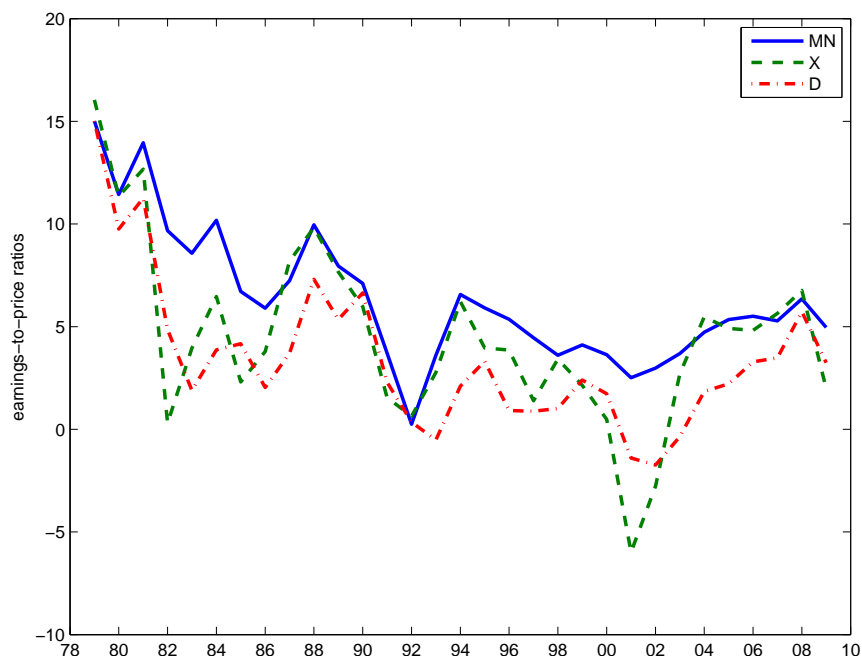


Figure 1: **Earnings-to-Price Ratios.** Portfolios formed yearly based on the international status of each firm. Data source: Compustat and CRSP, 1979-2009.

Annual earnings-to-price ratios are defined as annual earnings per share divided by the end-of-year price per share. Annual stock returns are defined as one-year capital gains plus dividend yields.⁹ Figure 1 presents more evidence on the ranking of earnings yields by status: a portfolio formed by multinational firms has earnings-to-price ratios on average above those of a portfolio of exporters, and in turn a portfolio formed by exporters has earnings-to-price ratios on average above those of a portfolio of firms selling only in their domestic market.¹⁰ The left panel of Table 2 reports sample means and standard deviations for earning-to price ratios. The ranking in

⁹Returns are defined as $R_{t+1} = \frac{p_{t+1} + d_t}{p_t}$ where p_t denotes the price of a share and d_t the dividends per share at time t . We identify firm-level returns with the returns of the firm's common equity.

¹⁰The portfolios are constructed as follows. For each firm i , determine its status S ($S = D, X, MN$) at the end of year $t - 1$, and collect data on earnings (e_t^i) and market capitalization (p_t^i) in year t . Portfolio earnings E_t^S and portfolio value P_t^S are constructed as equally weighted averages of individual values:

$$E_t^S = \frac{1}{N_t^S} \sum_{i \in S} e_t^i, \quad P_t^S = \frac{1}{N_t^S} \sum_{i \in S} p_t^i, \quad \forall S$$

Table 2: **Earnings Yields and Returns.** Summary statistics at the portfolio level.

	Earnings Yields		Returns	
	Mean (%)	Std. Dev.	Mean (%)	Std. Dev.
Domestic Firms	3.44	3.63	5.31	15.50
Exporters	4.47	4.36	8.91	14.70
Multinational Firms	6.33	3.29	9.84	13.20

earnings-to-price ratios at the portfolio level is consistent with the observation in Denis, Denis, and Yost (2002) that multinational corporations trade at a discount.

Figure 2 shows a similar, albeit noisier, pattern for the stock returns: a portfolio formed by multinational firms has returns on average above those of a portfolio of exporters, and in turn a portfolio formed by exporters has returns on average above those of a portfolio of firms selling only in their domestic market. The right panel of Table 2 reports sample means and standard deviation of returns. Not only mean returns are ordered across portfolios, but also the Sharpe ratios (expected returns divided by the standard deviation) are ordered, indicating that higher returns of exporters and multinational firms are not driven simply by higher volatility.

2.2 Firm-level Regressions

The data description above showed the ordering of the returns and earnings-to-price ratios in the raw data. However, simple averages across observations by type may hide other underlying characteristics not necessarily related to international status. To address this concern, we run firm level regressions of the financial variables of interest on a set of firm characteristics – financial and non-financial – that could be correlated with cross sectional differences in returns and earnings yields.

Table 3 displays the results of the following firm-level regressions:

$$Y_{it} = \alpha + \gamma_1 D_{it}^{MN} + \gamma_2 D_{it}^{EXP} + \gamma_3 X_{it} + \delta_{NAICSt} + \varepsilon_{it} \quad (1)$$

where N_t^S denotes the number of firms in status S at time t . Portfolio earnings yields are given by E_t^S/P_t^S . Data on returns are available at the monthly level: we annualize them for the corresponding firm fiscal year and then construct the returns of the portfolio by averaging over the firm-level yearly returns.

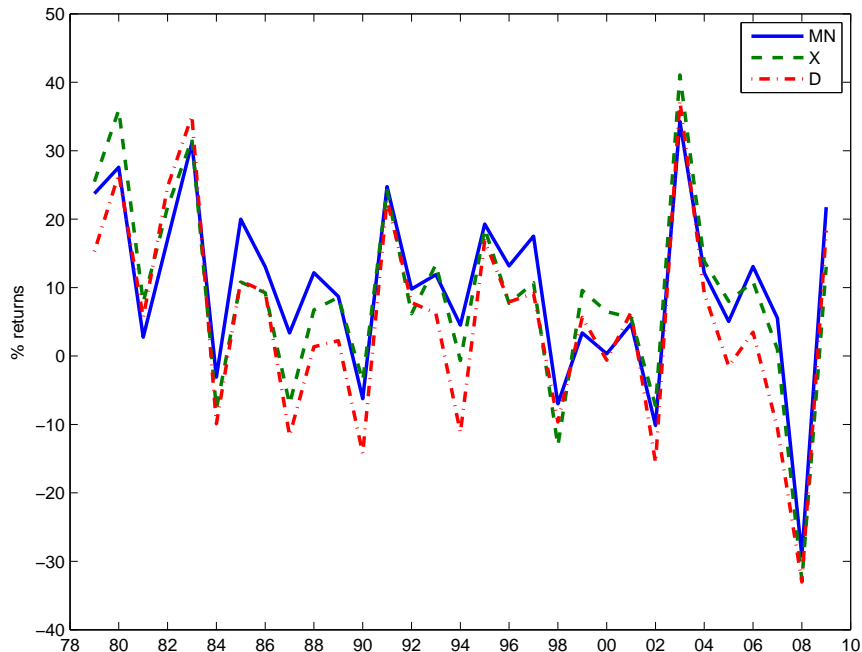


Figure 2: **Returns.** Portfolios formed yearly based on the status of each firm. Data source: Compustat and CRSP, 1979-2009.

where Y_{it} is the financial variable of interest for firm i at time t : earnings-to-price ratio in the left panel of the table, and returns in the right panel. D_{it}^{MN} and D_{it}^{EXP} are dummies assuming value 1 when firm i is a multinational or an exporter, respectively. X_{it} is a set of controls, including sales per employee (our measure of productivity), book-to-market ratio, leverage,¹¹ total revenues and market capitalization (measures of size). δ_{NAICS_t} are 4-digit industry-year fixed effects, and ε_{it} is an orthogonal error term.

The left panel of Table 3 shows the results for the earnings-to-price ratios. The coefficients associated with export and multinational status dummies are positive and significant. Moreover, the coefficient associated with multinational status is significantly larger than the one associated with export status, identifying a further difference between these two groups. We reject the null hypothesis that the coefficients of the two dummies are the same, confirming the difference in the earnings yields of multinationals *versus* exporters. The regression results in Table 3 include

¹¹Leverage is defined as the ratio of firm debt over firm book equity.

Table 3: **Earnings-to-Price and Returns Regressions.** Firm-level regressions of earnings-to-price ratios and stock returns on multinational and exporter dummies and other controls, with industry-year fixed effects. Standard errors are clustered by firm and status. (Top and bottom one percent of earnings-to-price sample excluded, top and bottom five percent of returns sample excluded. All dollar values are expressed in billions).

	Earnings-to-Price Ratios			Returns		
	(1)	(2)	(3)	(4)	(5)	(6)
MNE dummy	.137 (.006)***	.117 (.006)***	.137 (.006)***	.057 (.007)***	.058 (.007)***	.057 (.007)***
EXP dummy	.075 (.007)***	.075 (.006)***	.075 (.007)***	.048 (.008)***	.046 (.008)***	.049 (.008)***
revenue	.002 (.0007)***	.002 (.0006)***	.002 (.0007)***	.0007 (.0004)*	.0007 (.0004)	.0007 (.0004)
sales per emp.		.060 (.023)***	.061 (.024)**		.045 (.020)**	.044 (.020)**
book/market		.058 (.006)***			-.007 (.001)***	
leverage			.004 (.003)			.005 (.002)***
Prob > F :						
H_0 : MN=EXP	3.07e-31	2.06e-17	1.86e-30	.138	.065	.192
No of Obs.	51781	50969	50239	51639	50834	50104
adj. R^2	.153	.24	.158	.123	.125	.124

total revenues and sales per employee as controls related to size and profitability. Financial indicators like book-to-market ratio and leverage are meant to control for other potential sources of variation in the cross section of returns.¹²

The right panel of Table 3 reports the results of regression (1) with annual firm-level returns as the dependent variable. The coefficients on the multinational and exporter dummies are positive and significant, which confirms that firms selling in foreign markets tend to have higher returns than firms selling only domestically. The coefficient on the multinational dummy is higher than the one on the exporter dummy, indicating even larger excess returns for multinational firms. The ranking and significance of the coefficients are preserved across specifications.

Appendix B reports the results of regression (1) with a set of additional controls

¹²Book-to-market ratio and leverage enter the regressions only separately as they both give information on the relationship between a firm's own resources and its borrowed resources.

and robustness checks.

2.3 Asset Pricing Tests

After an exploration of earnings-to-price ratios and returns across the three groups of firms, it seems natural to explore the source of higher returns of firms selling in foreign markets. Higher average returns do not constitute a puzzle per se: they simply indicate that multinational firms and exporters are riskier than domestic firms. Research in finance has interpreted riskiness of a stock either as higher correlation with an aggregate “market” portfolio, or – in a more micro-founded perspective – as a higher correlation with the agents’ intertemporal marginal rate of substitution (IMRS).¹³ Consistently with the structural model that we develop in Section 3, we examine here the riskiness of three portfolios of firms formed by international status according to the micro-founded interpretation above. Reduced-form asset pricing tests (CAPM, Fama-French) are relegated to Appendix B.

If agents have CRRA preferences, the IMRS is a power function of aggregate consumption growth. According to the consumption CAPM model (CCAPM), the underlying source of risk driving the cross-section of returns is given by consumption growth: assets whose returns’ covariance with consumption growth is higher are riskier, and command a higher expected return in equilibrium:

$$E(R_{t+1}) = R^f - R^f \cdot \text{Cov} \left(\left(\frac{c_{t+1}}{c_t} \right)^{-\gamma} R_{t+1} \right) \quad (2)$$

where R_t denotes returns at time t , c_t denotes consumption at time t , R^f is the risk-free rate, and γ is the risk-aversion coefficient.

Figure 3 shows average realized quarterly excess returns for the three portfolios in the data plotted against predicted excess returns computed using (2) with an estimated risk-aversion parameter γ .¹⁴

¹³The finance literature refers to this term as intertemporal marginal rate of substitution, stochastic discount factor, or pricing kernel, equivalently.

¹⁴Excess returns are given by the difference between the returns and the risk-free rate. Predicted returns are computed as follows: we estimate the risk aversion parameter γ via GMM and construct the right-hand side of (2) using data on quarterly returns for the three portfolios from CRSP and data on quarterly consumption growth from NIPA, more precisely growth in personal consumption expenditures on non-durable goods.

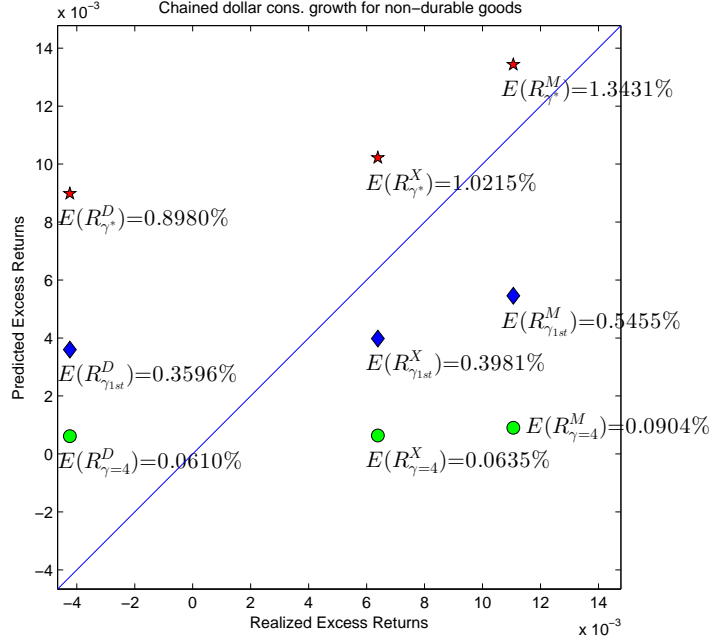


Figure 3: Realized and predicted quarterly excess returns of the three portfolios according to the CCAPM model.

The three stars in the picture show the results using the value of γ that we estimate using the efficient weighting matrix ($\gamma^* = 35$). The diamonds indicate the results obtained by estimating γ using the identity matrix as a weighting matrix ($\gamma_{1st} = 20$), while the circles correspond to the predicted returns computed with a “low” risk aversion parameter: $\gamma = 4$. There are two observations in order. First, the figure shows that realized and predicted returns display the same ranking, confirming the hypothesis that the higher returns of exporters and multinationals are in fact due to larger negative covariances with the agents’ IMRS. The CCAPM model captures the pecking order in the covariance of these portfolios’ returns with aggregate IMRS. Moreover, the model is fairly successful at matching the spread in the returns of exporters and multinationals. Returns of domestic firms are over-predicted, but are lower than the returns of the other two portfolios. Second, the spread in the predicted returns increases when γ increases, showing that multinational firms and exporters’ returns are more sensitive to risk-aversion. Table 4 reports the covariances of returns and consumption growth in the data – $\text{Cov} \left(\left(\frac{c_{t+1}}{c_t} \right)^{-\gamma} R_{t+1} \right)$ – for the three values of γ in the picture.

Table 4: **Covariances of Returns and Consumption Growth.** For the three groups of firms, from annualized quarterly results.

	$\gamma = 4$	$\gamma_{1st} = 20$	$\gamma^* = 35$
D	0.0006	0.0035	0.0089
X	0.0006	0.0039	0.0101
MN	0.0009	0.0054	0.0132

The covariance of realized returns with consumption growth explains (at least qualitatively) the cross-sectional variation in expected returns. This is direct empirical evidence of the fact that exporters and multinational corporations are riskier than domestic firms. Differences in the covariances are not enough to pin down quantitatively differences in returns, but the sensitivity of the results to the value of γ is indicative that the consumption based approach points to the right direction.

We take the results reported in Figure 3 as evidence that the ranking in the average returns we find in the data is driven by differences in the covariances of realized returns with the stochastic discount factor. The model we build in the next section is consistent with this finding. The goal of the model is to provide a structural explanation of the economic forces driving these differences in the covariances. In the model, consumers/investors have CRRA preferences and their IMRS determines the price of risk. Firms select endogenously their international status, which generates their risk exposure. Risk exposure and risk prices (IMRS) determine the risk premium of firms, which is the model-based equivalent of the excess returns we documented from the data.

The model is first specified with CRRA preferences to build the intuition behind the mechanism, and then augmented with recursive preferences to achieve a better quantitative fit.

3 Model

The model we develop in this section is designed to provide a structural explanation of the cross-section of earnings yields and returns by international status. At the aggregate level, the model is specified as an endowment economy, consistently with consumption-based asset pricing models. At the micro level, heterogeneous

firms' optimal choices determine how aggregate consumption is split into domestic goods, imported goods, and goods produced by affiliate plants of foreign multinationals. Firms' decisions endogenously determine profit flows. The agents' first-order conditions, firms' valuations and the covariance of their profits with the agents' intertemporal rate of substitution drive the returns.

3.1 Consumer Behavior and Aggregate Uncertainty

The economy is composed of two countries, Home and Foreign. Variables related to consumers and firms from the foreign country are marked with an asterisk (*). Both countries are populated by infinitely lived, risk-averse agents with preferences:

$$U = \int_0^{\infty} e^{-\vartheta t} \frac{Q(t)^{1-\gamma}}{1-\gamma} dt$$

where $\vartheta > 0$ is the subjective discount factor, and $\gamma > 1$ denotes risk aversion. $Q(t)$ is a constant elasticity of substitution (CES) aggregate of differentiated varieties:

$$Q(t) = \left(\int q_i(t)^{1-1/\eta} di \right)^{\eta/(\eta-1)}$$

where $\eta > 1$ denotes the elasticity of substitution across varieties.

Agents supply labor inelastically. Income is given by the wage plus the profit shares derived from ownership of firms incorporated in the country where the agents live.

In each country, aggregate consumption is hit by random shocks. Time is continuous, and Q and Q^* evolve according to geometric Brownian motions:

$$\frac{dQ}{Q} = \mu dt + \sigma dz \tag{3}$$

$$\frac{dQ^*}{Q^*} = \mu^* dt + \sigma^* dz^* \tag{4}$$

where $\mu, \mu^* \geq 0$, $\sigma, \sigma^* > 0$ and dz, dz^* are the increments of two standard Wiener processes with correlation $\rho \in [-1, 1]$. Since the model abstracts from growth considerations, we will later impose a zero drift: $\mu = \mu^* = 0$.

International markets are incomplete: consumers in the Home (Foreign) country

consume the stochastic endowment Q (Q^*), without any possibility of consumption smoothing. Moreover, consumers in the Home (Foreign) country own the firms incorporated in the Home (Foreign) country. As such, there is no possibility of international portfolio diversification because the model features perfect home bias in equity portfolios.¹⁵

In equilibrium, utility maximization implies that agents in each country discount future utility with stochastic discount factors described by the following geometric Brownian motions:

$$\frac{dM}{M} = -r dt - \gamma \sigma dz \quad (5)$$

$$\frac{dM^*}{M^*} = -r^* dt - \gamma \sigma^* dz^* \quad (6)$$

where $r = \vartheta + \gamma\mu - \gamma(\gamma + 1)\frac{\sigma^2}{2}$ ($r^* = \vartheta + \gamma\mu^* - \gamma(\gamma + 1)\frac{\sigma^{*2}}{2}$) is the risk-free rate and dz, dz^* are the increments of the Brownian motions ruling the evolution of Q and Q^* .¹⁶

This is a partial equilibrium model where labor is the only factor of production. As in Lucas (1978), we do not model how labor endowments produce the aggregate consumption levels Q, Q^* . We use the preferences to derive an expression for the stochastic discount factor and to find equilibrium goods and asset prices.¹⁷

¹⁵Tesar and Werner (1998) provide evidence of an extreme home bias in equity portfolios: about 90% of U.S. equity was invested in the U.S. stock market in the mid-1990s. Atkeson and Bayoumi (1993), Sorensen and Yosha (1998), and Crucini (1999) present evidence supporting the assumption of international market incompleteness.

¹⁶The stochastic discount factor is equal to the intertemporal marginal rate of substitution. The marginal utility of consumption is: $M = e^{-\vartheta t} Q(t)^{-\gamma}$. By applying Ito's Lemma to M and M^* one obtains (5) and (6).

¹⁷Alternatively, one could specify fully exogenous, country-specific productivity shocks and solve the model in general equilibrium. Shocks to aggregate productivity would imply equilibrium shock processes for Q and Q^* , making the behavior of the model qualitatively similar to our partial equilibrium one. Under such specification, we would need the full solution of the model to recover the process ruling the stochastic discount factor. For this reason we confine ourselves to a partial equilibrium analysis and model aggregate consumption as exogenous. It is well accepted that equilibrium consumption growth follows a random walk since Hall (1978). The unit root process is necessary to generate an option value component in the value of the firm, as will be made clearer below.

3.2 Technology and Firms Behavior

Each country is populated by a continuum of firms of total mass n (n^*), which operate under a monopolistically competitive market structure. Each firm produces a differentiated variety q_i taking the demand function as given. Firms produce with a linear technology defined by a firm-specific unit labor requirement a , which is a random draw from a distribution $G(a)$ ($G^*(a)$). Differentiated varieties are tradeable: a firm may sell its own variety only in its domestic market or both in the domestic and in the foreign market.

For simplicity we assume that there are no fixed costs associated with production for the domestic market, so every firm makes positive profits from domestic sales, and always sells in its domestic market.¹⁸ Besides producing for its domestic market, firms can produce also for the foreign country. Sales to the foreign market involve fixed operating costs, to be paid every period, and sunk costs of entry. If a firm decides to sell in the foreign market, it can do so either via exports or via foreign direct investment. We call multinationals those firms that decide to serve the foreign market through FDI sales.

We model the choice between trade and FDI along the lines of Helpman, Melitz, and Yeaple (2004): exports entail a relatively small sunk cost of entry, F_X , but a per-unit iceberg transportation cost τ to be paid every period.¹⁹ Instead, FDI is associated to a larger sunk cost, F_I ($F_I > F_X$), but there are no transportation costs to be covered every period, as both production and sales happen in the foreign market.²⁰

¹⁸We could have introduced positive fixed costs of domestic production, and modeled the initial decision of entry in the domestic market, like in Helpman, Melitz, and Yeaple (2004) and Irarrazabal and Opromolla (2009). This would have introduced additional complications in solving for firms' optimal dynamics, without any gains for our empirical analysis. Compustat includes only publicly listed firms, so when a firm enters or exits Compustat we do not have any information about whether the firm is in fact entering or exiting the market.

¹⁹ $\tau > 1$ units of good need to be shipped for one unit of good to arrive to the destination country.

²⁰The assumption that $F_I > F_X$ is key for our results. It seems intuitive to us that the costs of starting operations in a new production facility are higher than the costs of establishing an export channel. FDI entails a series of one-time costly activities, like acquiring licenses, dealing with local institutions (often in a foreign language), searching for qualified local labor, arrange relationships with suppliers, and so on. When the investment is greenfield, these costs are added to the cost of actually building a foreign plant. When the investment takes the form of a merger, the firm has to pay the initial acquisition cost. Clearly in both cases foreign plants can be sold, so part of the initial cost may not be sunk. However, all those activities related to starting production in a foreign

Both exports and FDI also entail fixed operating costs to be paid every period, that we denote with f_X and f_I for exports and FDI, respectively. After entering in the foreign market, a firm can exit at no cost. However, if it decides to re-enter, it will have to pay the sunk cost again.²¹ Sunk and fixed costs and stochastic demand imply that firms decide to enter when their expected profits are well above zero, and are reluctant to exit even in case of losses due to negative shocks. Dixit and Pindyck (1994) show that the “band of inaction”, or the set of realizations of the shocks such that a domestic firm is not willing to enter and an “international” firm is not willing to exit, is wider the larger the sunk cost of entry. Given our assumptions on the costs of export *versus* FDI, the band of inaction is wider for multinational firms than for exporters.

Notice also that the cost structure and the nature of uncertainty imply that if a firm decides to enter the foreign market, it will do so either as an exporter or as a multinational firm, but it will never adopt the two strategies at the same time.²²

Firms’ activities abroad are both subject to risk and to diversification potential. Correlation of the consumption processes in the two countries induces diversification. On the other hand, fixed costs of trade and FDI imply that fluctuations of Q and Q^* can induce negative profits, hence foreign activities are risky. In this sense, trade barriers limit the potential to diversify risk. In a frictionless world ($f_X = F_X = f_I = F_I = 0$), foreign activities would not be risky, and only the diversification channel would remain. Notice that this is true from the perspective of the firms, so different

country are.

²¹Roberts and Tybout (1997) report evidence on the fact that previous exporting experience matters as long as firms do not exit the foreign market. They find that the costs of entry for first-time exporters are not statistically different from the costs of entry for second-time exporters, *i.e.* firms that were once selling in the foreign market, exited, and decided to re-enter.

²²This feature of the model is the same as in Helpman, Melitz, and Yeaple (2004). Rob and Vettas (2003) obtain the existence of an equilibrium where firms can optimally choose to adopt simultaneously the two strategies because in their model firms choose the amount of the foreign investment, and given the structure of demand there may be the possibility of overinvestment. In their framework, FDI can be adopted to cover certain demand, while exports are used to serve the additional random excess demand without incurring the cost of a larger investment that could be underutilized. In the data we do observe a lot of firms that both export and have FDI sales (about 46% of the total). This fact can be rationalized within our framework by having multiproduct firms that choose different strategies for different product lines, or in a multi-country model where firms choose different strategies to enter different countries. Unfortunately, there is not enough information in the Compustat Segments data to check whether any of these is the case. Explaining the choice of firms to adopt both entry strategies would need a differently tailored framework, and is beyond the scope of this paper.

extents of risk/diversification have effects on firms' choices, but not on individuals' consumption levels.

For a given realization of (Q, Q^*) , in equilibrium, a firm with productivity $1/a$ must choose its optimal status S ($S \in \{D, X, I\}$, *i.e.* domestic, exporter, or multinational), the current selling price $p_S(a)$, and an updating rule (how to change the optimal price and status following changes in aggregate demand).

The CES aggregation over individual varieties implies that optimal prices are independent of (Q, Q^*) . From the firm's intratemporal first order condition: $p_S(a) = \frac{\eta}{\eta - 1} MC_S(a)$, where $MC_S(a)$ denotes the marginal cost of a firm with productivity $1/a$ in status S . The marginal cost of production varies with the status of the firm. Let w, w^* denote the wages in the Home and Foreign countries, respectively. For Home country firms, the marginal cost of domestic production is given by the labor requirement times the domestic wage, $MC_D = aw$. The marginal cost of exporting is augmented by the iceberg transportation cost: $MC_X = \tau aw$. When the firm serves the foreign market through FDI, firm-specific productivity is transferred to the foreign country and the firm employs foreign labor: $MC_I = aw^*$. The prices charged by firms from the Foreign country are determined in the same way.

Let $\pi_D(a; Q)$, $\pi_X(a; Q^*)$ and $\pi_I(a; Q^*)$ denote the maximal per-period profits from domestic sales, from exports and from FDI sales abroad, respectively, for a Home country firm with productivity $1/a$, given a realization of the aggregate quantity demanded (Q, Q^*) :

$$\pi_D(a; Q) = H(aw)^{1-\eta} P^\eta Q \tag{7}$$

$$\pi_X(a; Q^*) = H(\tau aw)^{1-\eta} P^{*\eta} Q^* - f_X \tag{8}$$

$$\pi_I(a; Q^*) = H(aw^*)^{1-\eta} P^{*\eta} Q^* - f_I \tag{9}$$

where $H \equiv \eta^{-\eta}(\eta - 1)^{\eta-1}$, and P (P^*) is the price index in the Home (Foreign) country, that firms take as given while solving their maximization problem.

3.3 Equilibrium

The state of the economy is described by the vector $\Sigma = (Q, Q^*, \Omega, \Omega^*)$, where $\Omega = (\omega_X, \omega_I)$ ($\Omega^* = (\omega_X^*, \omega_I^*)$) describes the distribution of firms from the Home (Foreign)

country into the three statuses.²³ Let $\mathcal{V}_S(a, Q, Q^*)$ denote the expected net present value of a Home country firm whose productivity is $1/a$, starting in status S ($S = D, X, I$) when the realization of aggregate demand is (Q, Q^*) , and following optimal policy. A firm's value coincides with the price at which its ownership is traded in the assets market.

Definition 1. *An equilibrium for this economy is defined by a set of value functions $(\mathcal{V}_S(a, Q, Q^*), \mathcal{V}_S^*(a, Q, Q^*))$ (for $S = D, X, I$), policy functions, price indexes (P, P^*) , and laws of motion for the distributions of firms into statuses (Ω, Ω^*) such that:*

- i. consumers maximize their lifetime utility subject to the budget constraint;*
- ii. firms' maximize their lifetime profits;*
- iii. goods and assets markets clear.*

3.3.1 Value Functions and Policy Functions

We solve the model along the lines of Dixit (1989). In the following, we omit the dependence of the value functions on Ω and Ω^* to ease the notation. Firms are active in their domestic market and make positive profits $\pi_D(a; Q)$ from domestic sales. Domestic activities are not directly affected by the realization of foreign demand Q^* . Similarly, the decision of whether to sell in the foreign market is not directly affected by the realization of domestic demand Q . For this reason, we can express the value function as:

$$\mathcal{V}_S(a, Q, Q^*) = \mathcal{S}(a, Q) + V_S(a, Q^*) \quad (10)$$

where $\mathcal{S}(a, Q)$ is the expected present discounted value of profits from domestic sales, which is independent on firm status, and $V_S(a, Q^*)$ is the expected present discounted value of profits from foreign sales for a firm in status S .

Over a generic time interval Δt , the two components of the value function for a

²³ $\omega_D = 1 - \omega_X - \omega_I$.

firm that is currently selling only in its domestic market can be expressed as:

$$\mathcal{S}(a, Q) = \pi_D(a, Q)M\Delta t + E[M\Delta t \cdot \mathcal{S}(a, Q')|Q] \quad (11)$$

$$V_D(a, Q^*) = \max \{E[M\Delta t \cdot V_D(a, Q^{*'})|Q^*] ; V_X(a, Q^*) - F_X ; V_I(a, Q^*) - F_I\}. \quad (12)$$

While (11) simply tracks the evolution of domestic profits, the right hand side of (12) expresses the firm's possible choices. If it sells only domestically, it gets the continuation value from not changing status, equal to the expected discounted value of the firm conditional on the current realization of foreign demand Q^* . If it decides to switch to exports (FDI) it gets the value of being an exporter, V_X (multinational, V_I) minus the sunk cost of entry F_X (F_I). Similarly, the present discounted value of profits from foreign sales for an exporter is:

$$V_X(a, Q^*) = \max \{\pi_X(a, Q^*)M\Delta t + E[M\Delta t \cdot V_X(a, Q^{*'})|Q^*] ; V_D(a, Q^*) ; V_I(a, Q^*) - F_I\} \quad (13)$$

and for a multinational:

$$V_I(a, Q^*) = \max \{\pi_I(a, Q^*)M\Delta t + E[M\Delta t \cdot V_I(a, Q^{*'})|Q^*] ; V_D(a, Q^*) ; V_X(a, Q^*) - F_X\}. \quad (14)$$

Notice that the continuation value of an exporter (a multinational) also includes the profit flow from sales in the foreign market. There are no costs of exiting the foreign market: if a firm decides to exit, its value is simply that of a domestic firm.

In Appendix C we show that the value functions $\mathcal{S}(a, Q)$, $V_S(a, Q^*)$ for $S \in \{D, X, I\}$ take the form:

$$\mathcal{S}(a, Q) = \frac{\pi_D(a, Q)}{r - \mu + \gamma\sigma^2} \quad (15)$$

$$V_D(a, Q^*) = A_D(a)Q^{*\alpha} + B_D(a)Q^{*\beta} \quad (16)$$

$$V_X(a, Q^*) = A_X(a)Q^{*\alpha} + B_X(a)Q^{*\beta} + \frac{H(\tau aw)^{1-\eta}P^{*\eta}Q^*}{r - \mu^* + \gamma\rho\sigma\sigma^*} - \frac{f_X}{r} \quad (17)$$

$$V_I(a, Q^*) = A_I(a)Q^{*\alpha} + B_I(a)Q^{*\beta} + \frac{H(aw^*)^{1-\eta}P^{*\eta}Q^*}{r - \mu^* + \gamma\rho\sigma\sigma^*} - \frac{f_I}{r} \quad (18)$$

where α and β are the roots of:²⁴

$$\frac{1}{2}\sigma^{*2}\xi^2 + (\mu^* - \frac{1}{2}\sigma^{*2} - \gamma\rho\sigma\sigma^*)\xi - r = 0.$$

$A_S(a)$ and $B_S(a)$ ($S \in \{D, X, I\}$) are firm-specific, time-varying parameters to be determined.²⁵

Since there are no fixed or sunk costs of domestic production, there is no option value associated with future profits from domestic sales. The value function $\mathcal{S}(a, Q)$ is simply equal to the discounted flow of domestic profits. Conversely, the option value of changing status (the term $A_S(a)Q^{*\alpha} + B_S(a)Q^{*\beta}$, for $S = D, X, I$) is a component of the expected present discounted value of foreign profits. In particular, the option value is the only component of the present discounted value of foreign profits for domestic firms. For exporters and multinationals, the value is given by the sum of the discounted foreign profit flow from never changing status plus the option value of changing status. The terms $\mu - \gamma\sigma^2$ and $\mu^* - \gamma\rho\sigma\sigma^*$ in the discount are the risk-adjusted drifts, result of taking expectations of the value function under the risk-neutral measure.

Equations (16)-(18) describe the value of foreign profits in the firms' continuation regions. We still need to solve for the updating rule, which consists of thresholds in the realizations of Q^* that induce firms to change status. Let $Q_{RS}^*(a)$ denote the quantity threshold at which a firm with productivity $1/a$ switches from status R to status S , for $R, S \in \{D, X, I\}$.²⁶ In order to find the six quantity thresholds $Q_{SR}^*(a)$ and the six value function parameters $A_S(a)$, $B_S(a)$, for $S \in \{D, X, I\}$, we impose

²⁴ $\alpha < 0$, $\beta > 1$.

²⁵The parameters $A_D(a)$ and $B_D(a)$ are time-varying because they also depend on the distribution of firms in the three statuses, which we are not making explicit in the value functions.

²⁶The quantity thresholds $Q_{RS}^*(a)$ also depend on the distribution of firm statuses Ω^* , which affects the equilibrium price index, and are hence time-varying.

the following value-matching and smooth-pasting conditions:

$$V_D(a, Q_{DX}^*(a)) = V_X(a, Q_{DX}^*(a)) - F_X \quad (19)$$

$$V_D(a, Q_{DI}^*(a)) = V_I(a, Q_{DI}^*(a)) - F_I \quad (20)$$

$$V_X(a, Q_{XD}^*(a)) = V_D(a, Q_{XD}^*(a)) \quad (21)$$

$$V_X(a, Q_{XI}^*(a)) = V_I(a, Q_{XI}^*(a)) - F_I \quad (22)$$

$$V_I(a, Q_{ID}^*(a)) = V_D(a, Q_{ID}^*(a)) \quad (23)$$

$$V_I(a, Q_{IX}^*(a)) = V_X(a, Q_{IX}^*(a)) - F_X \quad (24)$$

$$V'_R(a, Q_{RS}^*(a)) = V'_S(a, Q_{RS}^*(a)) , \text{ for } S, R \in \{D, X, I\} , S \neq R. \quad (25)$$

For each a , equations (19)-(25) define a system of twelve equations in twelve unknowns: the six quantity thresholds $Q_{SR}^*(a)$ and the six parameters $A_S(a)$, $B_S(a)$, for $S, R \in \{D, X, I\}$. To get an economically sensible solution, we follow Dixit (1989) and impose a series of restrictions on the parameters $A_S(a)$, $B_S(a)$, $S \in \{D, X, I\}$. Since $\alpha < 0$, $\beta > 1$, the terms in $Q^{*\alpha}$ ($Q^{*\beta}$) are large for low (high) realizations of Q^* . For low realizations of Q^* , entry is a remote possibility for a firm selling only in its domestic market, hence the value of the option of entering must be nearly worthless: $A_D(a) = 0$, $\forall a$. It must then be that $B_D(a) \geq 0$ to insure non-negativity of $V_D(a, Q^*)$.

Similarly, for high realizations of Q^* , the option of quitting FDI for another strategy is nearly worthless, hence $B_I(a) = 0$. Moreover, a multinational firm has expected value $\frac{H(aw^*)^{1-\eta} P^{*\eta} Q^*}{r-\mu^*+\gamma\rho\sigma\sigma^*} - \frac{f_I}{r}$ from the strategy of never changing status, hence the optimal strategy must yield a no lesser value: $A_I(a) \geq 0$.

Finally, an exporter has expected value $\frac{H(\tau aw)^{1-\eta} P^{*\eta} Q^*}{r-\mu^*+\gamma\rho\sigma\sigma^*} - \frac{f_X}{r}$ from the strategy of never changing status, hence its optimal strategy must yield a no lesser value for any realization of Q^* : $A_X(a), B_X(a) \geq 0$.

As a result of these restrictions, the value function of a domestic firm is increasing on its entire domain, indicating that as Q^* increases, the value of the option of entering the foreign market (either through trade or FDI) increases. The value functions of an exporter and of a multinational are U-shaped: for low levels of Q^* , the term with the negative exponent α dominates, and the value is high due to the option of leaving the market. Conversely, for high levels of Q^* , the value is high due to the profit stream that the firm derives from staying in the market and, for exporters, due to the

additional option value of becoming a multinational firm (the term with the positive exponent β).

The system of value-matching and smooth-pasting conditions includes among its variables the aggregate price index P^* . P^* is an endogenous variable and – as will be clearer in the next section – it depends on the realization of Q^* . For this reason, one should write each condition taking into account the equilibrium price at that specific realization of Q^* (*i.e.*, if $Q^* = Q_{DX}^*$, then $P^* = P^*(Q_{DX}^*)$). However, we appeal to the result developed in Leahy (1993) and Dixit and Pindyck (1994), Chapters 8-9, whereby a firm can ignore the effects of the actions of other firms when solving for the optimal thresholds triggering investment, and use the market equilibrium price P^* in all the value-matching and smooth-pasting equations. The intuition behind this result is as follows. The actions of other firms affect the problem of an individual firm via the price index in two ways: more firms entering the foreign market reduce the profit flows from foreign sales and also the option value of waiting to start selling abroad. It can be shown that these two effects exactly offset each other; hence, taking into account the effect of the actions of other firms on the price index is immaterial for the determination of the thresholds.²⁷

3.3.2 Price Indexes and Status Dynamics

In this section we discuss the computation of the price indexes and of the status distribution in the two countries.

²⁷Dixit and Pindyck (1994) and Leahy (1993) show this results for a perfectly competitive industry with free entry and CRS production technologies, where the shocks follow a general diffusion process. Leahy (1993) also shows that free entry is unnecessary to obtain the result. Our economy differs from the ones they study in that firms' technologies exhibit increasing returns to scale and the market structure is monopolistically competitive. However, we argue that the result still applies for the following reasons. The potential problem with increasing returns is that they may induce “too large” investment by the firms. This does not apply to our framework, where firms only decide whether to entry or not, and not the amount to invest. Imperfect competition in turn may invalidate the result if firms display some type of strategic behavior, which is clearly not the case for monopolistic competition with a continuum of firms.

The price indexes P and P^* are the solution of the following system:

$$P^{1-\eta} = n \int \left(\frac{\eta a w}{\eta - 1} \right)^{1-\eta} dG(a) + \dots$$

$$\dots n^* \left[\int_{\omega_X^*(Q)} \left(\frac{\eta \tau a w^*}{\eta - 1} \right)^{1-\eta} dG^*(a) + \int_{\omega_I^*(Q)} \left(\frac{\eta a w}{\eta - 1} \right)^{1-\eta} dG^*(a) \right] \quad (26)$$

$$(P^*)^{1-\eta} = n^* \int \left(\frac{\eta a w^*}{\eta - 1} \right)^{1-\eta} dG^*(a) + \dots$$

$$\dots n \left[\int_{\omega_X(Q^*)} \left(\frac{\eta \tau a w}{\eta - 1} \right)^{1-\eta} dG(a) + \int_{\omega_I(Q^*)} \left(\frac{\eta a w^*}{\eta - 1} \right)^{1-\eta} dG(a) \right]. \quad (27)$$

Each price index is an aggregate of prices of domestic sales, prices of imports, and prices of FDI sales of multinational firms from the other country. We denote by n (n^*) the exogenous mass of firms from the Home (Foreign) country, and by $\omega_X(Q^*)$, $\omega_I(Q^*)$ ($\omega_X^*(Q)$, $\omega_I^*(Q)$) the shares of these firms that export or have multinational sales when the realization of aggregate demand is (Q, Q^*) .

The law of motion of the status distribution is given by:

$$\omega'_D = \omega_D \cdot [1 - G(\max\{a_{DX}, a_{DI}\})] + \omega_X \cdot [1 - G(a_{XD})] + \dots$$

$$\dots \omega_I \cdot \{[1 - G(a_{ID})] \mathbf{1}_{a_{ID} \geq a_{IX}} + [G(a_{IX}) - G(a_{ID})] \mathbf{1}_{a_{ID} < a_{IX}}\} \quad (28)$$

$$\omega'_X = \omega_D \cdot \{[G(a_{DX}) - G(a_{DI})] \mathbf{1}_{a_{DX} \geq a_{DI}} + G(a_{DX}) \mathbf{1}_{a_{DX} < a_{DI}}\} + \dots$$

$$\dots \omega_X \cdot [G(a_{XD}) - G(a_{XI})] + \dots$$

$$\dots \omega_I \cdot \{[1 - G(a_{IX})] \mathbf{1}_{a_{ID} < a_{IX}} + [G(a_{ID}) - G(a_{IX})] \mathbf{1}_{a_{ID} \geq a_{IX}}\} \quad (29)$$

$$\omega'_I = \omega_D \cdot \{G(a_{DI}) \mathbf{1}_{a_{DX} \geq a_{DI}} + [G(a_{DI}) - G(a_{DX})] \mathbf{1}_{a_{DX} < a_{DI}}\} \dots$$

$$\dots \omega_X \cdot G(a_{XI}) + \omega_I \cdot G(\min\{a_{ID}, a_{IX}\}) \quad (30)$$

where $a_{RS}(Q^*)$, for $R, S \in \{D, X, I\}$ is the productivity threshold that induces a Home country firm to switch status from R to S when the realization of foreign demand is Q^* .²⁸ For a given initial distribution Ω_0 , equations (28)-(30) describe its evolution depending on the productivity thresholds ruling firms' allocation into statuses.

System (19)-(25) delivers firm-specific thresholds in Q^* : $Q_{RS}^*(a)$. To recover the

²⁸We omitted the dependence of ω_S and a_{RS} on Q^* to ease the notation.

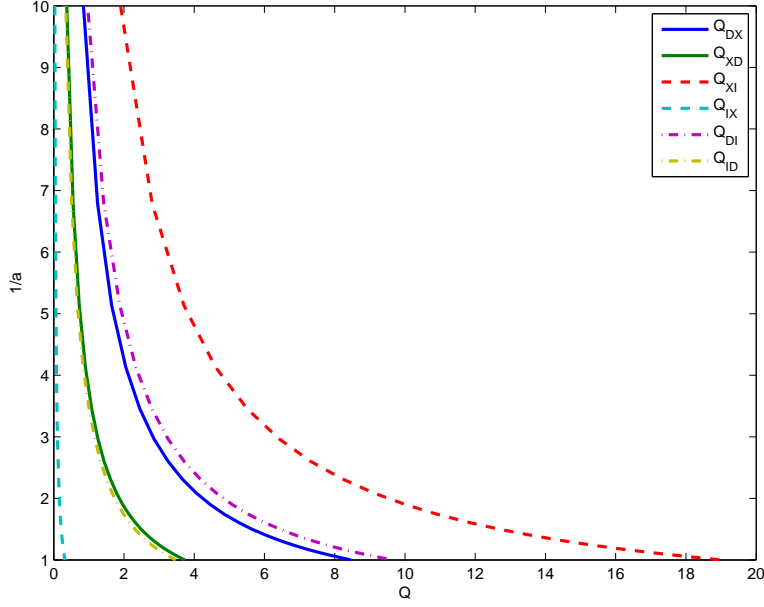


Figure 4: Quantity thresholds as functions of firm's productivity.

productivity thresholds driving selection we need to invert the $Q_{RS}^*(a)$ thresholds. Theorem 1 warrants this possibility.

Theorem 1.

$$\frac{\partial Q_{RS}^*(a)}{\partial a} > 0, \quad \text{for } R, S \in \{D, X, I\}, \quad \forall a. \quad (31)$$

Proof: See Appendix C.

Theorem 1 establishes that the six thresholds $Q_{RS}^*(a)$ are decreasing in firm productivity $1/a$, indicating that more productive firms need smaller positive shocks to demand to enter the foreign market, and larger negative shocks to exit. The one-to-one correspondence between productivities and quantity thresholds established by Theorem 1 implies that the functions $Q_{RS}^*(a)$ are invertible, hence for each realization of aggregate foreign demand Q^* we can compute six productivity thresholds $a_{RS}(Q^*)$, for $R, S \in \{D, X, I\}$, that determine the selection of heterogeneous firms into the three statuses and their likelihood of switching across statuses. This redefinition of the thresholds in terms of productivity is essential to compute the price indexes in (26)-(27). Figure 4 illustrates Theorem 1.

The equations describing aggregate prices depend – via the integration limits – on the distribution of firms into statuses, which in turn depends on the quantity

thresholds Q_{RS}^* . On the other hand, quantity thresholds themselves depend on aggregate prices (as evident from the value functions). In solving the firm's problem, we appeal to the equivalence result shown in Leahy (1993): when finding the quantity thresholds, each firm takes aggregate prices and the firms' distribution into statuses as given, and does not take into account the effect of its own entry and exit decisions on these variables. This result simplifies considerably the computation of the equilibrium, which we describe in Appendix E. Notice that the sets ω_S vary with the realization of Q^* , as firms may switch status, but only depend on the firms' status in the previous period, due to the Markov property of Brownian motions.

Since we abstract from the problem of entry in the domestic market, we take the mass of firms in the two countries as given. We normalize $n = n^* = 1$, and present the results in terms of shares of the total number of firms. The initial values of the processes ruling the evolution of the state, $Q(0)$ and $Q^*(0)$, and the initial status distribution are also taken as given.

3.4 Earnings-to-Price Ratios and Returns

The solution of the model delivers quasi-closed form solutions (up to multiplicative parameters) for the value functions $\mathcal{V}_S(a, Q, Q^*)$ ($S \in \{D, X, I\}$), and allows us to compute the earnings-to-price ratios and returns generated by the model.

Our earnings yields measure in the model is given by the ratio π_t/\mathcal{V}_t , where π_t represents per-period profits and \mathcal{V}_t is the market value of the firm. Let $ep_S(a, Q, Q^*)$ denote the earnings yields of a firm with productivity $1/a$ in status S when the realization of aggregate demand is (Q, Q^*) . Earnings yields in the model are given by:

$$ep_D(a, Q, Q^*) = \frac{\pi_D(a, Q)}{\mathcal{V}_D(a, Q, Q^*)} \quad (32)$$

$$ep_X(a, Q, Q^*) = \frac{\pi_D(a, Q) + \pi_X(a, Q^*)}{\mathcal{V}_X(a, Q, Q^*)} \quad (33)$$

$$ep_I(a, Q, Q^*) = \frac{\pi_D(a, Q) + \pi_I(a, Q^*)}{\mathcal{V}_I(a, Q, Q^*)}. \quad (34)$$

The empirical evidence presented in Section 2 suggests the following ordering in

aggregate earnings yields across groups:

$$\int_{\omega_D(Q^*)} ep_D(a, Q, Q^*)dG(a) < \int_{\omega_X(Q^*)} ep_X(a, Q, Q^*)dG(a) < \int_{\omega_I(Q^*)} ep_I(a, Q, Q^*)dG(a).$$

While is not possible to prove analytically that the model generates this ordering, the results of our numerical simulations confirm that the calibrated model is consistent with it.

Returns in the model are given by the earnings yields plus the expected change in the valuation of the firm:

$$ret_S(a, Q, Q^*) = ep_S(a, Q, Q^*) + \frac{E[d\mathcal{V}_S(a, Q, Q^*)]}{\mathcal{V}_S(a, Q, Q^*)}, \text{ for } S \in \{D, X, I\}. \quad (35)$$

Also in this case, the model does not have clear-cut analytical predictions for the ordering of $\frac{E[d\mathcal{V}_S(a, Q, Q^*)]}{\mathcal{V}_S(a, Q, Q^*)}$. The value of this object depends on the curvature of the value functions and it also critically depends on the calibration, since different firms exhibit different value functions and respond differently to the same realizations of the shocks.

To gain intuition on the main elements of the model at work – the presence of fixed and sunk costs (“hysteresis” channel) and the correlation of shocks across countries (diversification channel) – it is instructive to notice that returns can also be expressed as:

$$ret_S(a, Q, Q^*) = r + \frac{\gamma\sigma^2\mathcal{S}(a, Q) + \gamma\rho\sigma\sigma^*Q^*V'_S(a, Q^*)}{\mathcal{V}_S(a, Q, Q^*)} \quad (36)$$

and hence heterogeneity in returns across groups mostly depends on the elasticity of the value function with respect to Q^* . Holding revenues constant, larger fixed costs result in larger percentage changes in values with respect to changes in demand. Equation (36) also implies that in a frictionless economy (where $f_X = f_I = F_X = F_I$) the returns of domestic firms are higher than the returns of firms selling abroad. This makes perfect sense as without fixed costs only the diversification channel remains, and exporters and multinationals in fact diversify away the risk generated by country-specific shocks by selling in both markets. Similarly, with positive trade and FDI costs but imposing zero correlation between the consumption processes in the two

countries ($\rho = 0$), the returns of domestic firms are higher than the returns of firms selling abroad. In this case the diversification channel is strong enough to overcome the hysteresis channel. Appendix C shows the derivation of equation (36) together with these and other comparative statics results.

For the model to reproduce the ordering found in the data:

$$\int_{\omega_D(Q^*)} ret_D(a, Q, Q^*) dG(a) < \int_{\omega_X(Q^*)} ret_X(a, Q, Q^*) dG(a) < \int_{\omega_I(Q^*)} ret_I(a, Q, Q^*) dG(a)$$

we need the differences in $\frac{E[dV_S(a, Q, Q^*)]}{V_S(a, Q, Q^*)}$ not to overturn the ordering of the earnings yields, which is what our calibration delivers.

4 Empirical Results

The objective of this section is to evaluate the performance of the model in matching qualitatively and quantitatively features of the data on trade and FDI dynamics, and the pattern of earnings yields and returns across firms. We calibrate the parameters of the model to match the entry and exit pattern and the relative presence of the three types of firms in the data. We then use the calibrated version of the model to compute earnings yields and returns, which are not targeted moments in the calibration. We present both the calibration of the baseline CRRA model and of a richer version of it. The latter features shocks to the expected growth rate of consumption and recursive preferences to improve on the quantitative results of our baseline calibration.

4.1 Baseline Calibration

The calibration exercise is designed to match moments related to export and FDI activity and dynamics. We show that the model calibrated by targeting trade facts performs well also in matching non-targeted moments like earnings yields and returns.

We present a bilateral calibration exercise, that describes export and FDI activity between the U.S. and an aggregate set of trading partners. Due to data availability, we impose a series of symmetry assumptions. In particular, we assume that preferences and productivity distributions are identical in the U.S. and in the other countries,

and that the cost parameters τ , f_X , F_X , f_I , F_I are also the same across countries.²⁹

To calibrate the model, we need to choose a functional form for the cost distribution $G(a)$, and assign values to its parameters. We need to parameterize the Brownian motions and to choose values for parameters describing preferences and trade and FDI costs. We refer to the literature to assign parameters to the preferences and to the firms' productivity distributions. The parameters ruling the Brownian motions are chosen to match data on standard deviations and correlation of GDP growth between the U.S. and its major trading partners. We choose the remaining parameters to match data on export and FDI activity. We start describing the calibration with the parameters we adopt from the literature.

Several studies document that the tail of the empirical firm size distribution is well approximated by a Pareto distribution (see for example Luttmer (2007)). Since firm size (sales) is linked to the productivity distribution in the model, we assume that firms' productivities $1/a$ are distributed according to a Pareto law with location parameter b and shape parameter k .³⁰ We calibrate b together with the other parameters of the model, and choose k to match the coefficient of the empirical sales distribution: sales in the model are also Pareto-distributed with shape parameter $k/(\eta - 1)$. By regressing firm rank on firm size, Luttmer (2007) finds that $k/(\eta - 1) = 1.06$. We then choose k accordingly, given a value for η . There is little agreement in the literature on the value to attribute to the elasticity of substitution across differentiated varieties, η . Many papers that focus on long run macroeconomic predictions use a standard value of 2. Other papers that focus on matching data at business cycle frequencies choose much higher values. We set $\eta = 2.54$, equal to the median value in Broda and Weinstein (2006) SITC 3-digit estimates.³¹ This choice implies $k = 1.6324$. We set

²⁹Compustat records data of firms with activities in the U.S., among which there are both U.S.-based firms and foreign firms. However, only data of foreign firms with activities in the U.S. are reported (in other words, we have no data about foreign firms with activities only in their domestic market), which implies that we cannot construct shares of foreign firms in each status or their dynamic behavior.

³⁰The Pareto distribution is also a convenient choice for computational reasons, since it allows to solve explicitly for the aggregate prices P , P^* as functions of the productivity thresholds a_{RS} and of the other parameters of the model.

³¹Broda and Weinstein (2006) estimate sectoral elasticities of substitution from price and volume data on U.S. consumption of imported goods. By using data at the 10-digit Harmonized System, they estimate how much demand shifts between 10-digit varieties when relative prices vary, within each 3-digit SITC sector.

the subjective discount factor ϑ to 0.04, so that the risk-free rates in the two countries are about 3.5%. Finally, we set the risk aversion parameter to $\gamma = 4$, a low value among those proposed by the literature.³² We abstract from labor cost and market size differences, and set both wages to $w = w^* = 1$.

We impose that the drifts of the Brownian motions ruling the evolution of Q, Q^* have value $\mu = \mu^* = 0$. The need to impose zero expected demand growth arises from the fact that we abstract from firms' productivity growth.³³ We compute $\sigma = 0.022$ as the standard deviation of GDP growth in the U.S. over the sample period ($t = 1979-2009$), and $\sigma^* = 0.025$ as the average standard deviation of GDP growth in a set of main trading partners.³⁴ The correlation coefficient between the two Brownian motions, ρ , is a key parameter in this exercise. To select a value for ρ , we computed correlations in GDP growth rates between the U.S. and the same set of countries included in the calculation of σ^* , and took the median value: $\rho = 0.45$.³⁵

It remains to calibrate the lower bound of the productivity distribution b , the variable trade cost τ , fixed operating costs f_X, f_I , sunk costs F_X, F_I , and the initial value of the aggregate demand levels, $Q(0) = Q^*(0)$. We follow the methodology of the calibration in Alessandria and Choi (2007), and select values for these parameters to match a set of moments related to trade and FDI dynamics. We target data on firms' persistence in the same status and on the shares of the three types of firms in

³²Assigning a value to γ is a difficult choice to make in this setting. In their seminal contribution, Mehra and Prescott (1985) report evidence from several micro and macro studies suggesting a value of γ between 1 and 4. They also show that a model with CRRA preferences and such a low value of γ can match the risk-free rate, but generates returns that are too low compared with the data (the equity premium puzzle). Hansen and Singleton (1982) estimate the value of γ that matches returns, obtaining a value of $\gamma \approx 60$. This high value is implausible based on the empirical evidence, and generates too high a risk-free rate. Our model features CRRA preferences for the differentiated good, and is hence subject to the same problem: we are not able to match quantitatively both the risk-free rate and the returns. Moreover, the solution of the model imposes natural bounds on γ : a too high value of γ may bring the risk-free rate and/or the discount terms on profits flows to be negative.

³³If $\mu > 0$, $E(dQ/Q)$ would be increasing over time and for $t \rightarrow \infty$ all firms would become multinationals. By setting $\mu = \mu^* = 0$, we are implicitly assuming that Q, Q^* and b grow at a rate such that the distribution of firms in the three groups does not degenerate over time.

³⁴Data source: OECD Statistics. We use from 28 countries: 26 OECD countries plus China and South Africa (all the countries for which data are available for the entire sample period). This is a representative set to think about U.S. foreign sales, as it accounts for approximately 75% of total U.S. exports.

³⁵GDP growth correlations range from 0.177 between the U.S. and Portugal and 0.858 between the U.S. and Canada. For this reason, the choice of $\rho = 0.45$ is conservative: if one had to construct an export-weighted measure of correlation, for example, the value of ρ would increase to 0.81.

the data.

We compute the persistence moments from Compustat data. Every year on average 92.49% of domestic firms remain domestic the following year, while 3.63% of them become exporters, and the remaining become multinationals. 87.04% of exporters continue exporting the following year, while 6.95% of them become multinationals, and the remaining exit the foreign market to sell domestically only. Multinational firms exhibit even higher persistence, with 98.02% of them continuing being multinationals the following year, 1.1% of them exiting the market and only 0.88% of them becoming exporters the following year. Domestic firms' and exporters' persistence moments are close to the ones reported in Alessandria and Choi (2007), but we are unaware of other papers computing moments related to persistence in multinational activity. Next, we look at the average share of firms in each status. Bernard et al. (2007) report that the average share of manufacturing firms that export is about 18%, while Bernard and Jensen (2007) report that multinational firms represent only 1% of manufacturing firms.

Table 5: **Baseline Calibration: Summary.**

Parameter	Definition	Value
τ	iceberg export cost	1.3
f_X	fixed export cost	0.004
f_I	fixed FDI cost	0.08
F_X	sunk export cost	0.1
F_I	sunk FDI cost	0.2
b	lower bound of the productivity distribution	1
$Q(0), Q^*(0)$	initial demand	1

We simulate the model economy to obtain an acceptable match of these moments. The resulting calibrated parameters are reported in Table 5. The calibrated iceberg cost is 30%, consistent with a medium-range estimate in Eaton and Kortum (2002). Fixed costs of export and FDI are equal to 0.004 and 0.08, respectively. These fixed operating costs imply that an exporter must spend, on average, 5.41% of its revenues in operating fixed costs, according to the simulated model. Coincidentally, multinationals also spend about 5.18% of their revenues in operating fixed costs. Sunk costs of export and FDI are higher, equal to 0.1 and 0.2, respectively. The results of

the calibration show that a domestic firm must spend on average 10 times its per-period revenue to enter the foreign market as an exporter, and about 20 times its per-period revenue to start FDI operations there. Aggregate demand parameters are set at $Q = Q^*(0) = 1$, and the lower bound of the productivity distribution to $b = 1$.

Table 6 jointly displays the moments computed from the data and the moments generated by the calibrated model. Overall, the model performs quite well in matching the moments from the data, although it generates a bit more persistence and more exporters than what we observe in the data. The calibration error, computed as the sum of squared differences between the model-generated moments and the moments computed from the data, is equal to 0.044.

Table 6: **Moments.** Comparison of the moments, model *versus* data.

	Data	Model		Data	Model
$D \rightarrow D$ (%)	92.49	98.59	X (%)	18	32.82
$D \rightarrow X$ (%)	3.63	1.39	I (%)	1	2.52
$X \rightarrow X$ (%)	87.04	99.89			
$X \rightarrow I$ (%)	6.95	0.11			
$I \rightarrow I$ (%)	98.02	99.28			
$I \rightarrow X$ (%)	0.88	0.72			

4.2 Results: Earnings Yields and Returns

With the calibrated version of the model, we compute average earnings yields and average returns across the three groups of firms. In our calculations, we follow the construction of the portfolios we used in the data analysis presented in Section 2. We generate an artificial dataset of 100 firms with productivities drawn from a Pareto distribution with parameters $(b, k) = (8, 1.6324)$, and we simulate a 30-period economy 100 times.³⁶ In each simulation, we initialize the firms' distribution into status by assuming that all firms start domestic, and we generate a sample process for (Q, Q^*) .

³⁶The small number of Monte Carlo simulations is justified by the fact that for $\mu = \mu^* = 0$ there are small differences across simulations. The entire computation of the model for a given parametrization takes about 3 hours on a cluster of 10 CPUs. Details about the algorithm used are provided in Appendix E.

Given the process of the shocks, we simulate the economy, recording the distributions of firms into status in each period. For each firm and period, we compute earnings, equilibrium value (our model-based measure of market capitalization), and changes in the equilibrium value of the firm. For each year we create three portfolios of domestic firms, exporters, and multinationals, and we compute portfolio earnings, prices, value changes, earnings yields (earnings-to-price ratios), and returns (earnings-to-price ratios plus average percentage changes in value). For each simulation, we compute the mean of earning yields and returns over time.³⁷ We repeat this process for the 100 Monte Carlo simulations, and we average the results across simulations.

Table 7: **Earnings Yields and Returns - CRRA Preferences.** Summary statistics of earnings yields and returns computed from simulated data, and comparison with real data. All values are in percentage terms.

	Earnings Yields		Returns	
	Mean (data)	Mean (model)	Mean (data)	Mean (model)
DOM	3.44	2.04	5.31	2.05
EXP	4.47	2.06	8.91	2.26
MN	6.33	3.65	9.84	4.52

The left panel of Table 7 reports the results for earnings yields. The model generates average earnings yields of 3.65% for multinational firms, 2.06% for exporters, and 2.04% for firms selling only domestically. The right panel of Table 7 reports the results for the returns. The model generates average returns of 4.52% for multinational firms, 2.26% for exporters, and 2.05% for firms selling only domestically. Model-generated yields and returns are consistent with the ordering we found in the data, but present smaller differentials and are lower in levels.

While the model succeeds in matching the pecking order of earnings yields and returns across group, it fails in matching their magnitudes. As we mentioned previously, this comes at no surprise in light of the well known inability of models based on CRRA preferences to match quantitatively stock market returns for reasonable values of the parameter ruling risk aversion. To overcome this limitation, in the next

³⁷When averaging over time, we discard the first period of each simulation to reduce the importance of the fact that we initialize the firms' distribution into status at $\Omega_0 = \Omega_0^* = (0, 0)$.

section we propose a variation of the model that has the potential to improve the quantitative fit of the calibration.

4.3 Long Run Risk Calibration

Faced with the quantitative failure of CRRA preferences and time-separability of the utility function to explain the cross-section of returns, the finance literature has proposed more sophisticated specifications of preferences and of fundamental risk drivers' dynamics to improve the fit of the consumption-based models.

In this section we adopt one of these richer specifications and re-calibrate and re-compute the model accordingly. Particularly, to be as close as possible to the framework described in Section 3, we stick to a one-good framework and introduce shocks to the long run component of consumption growth (the expected growth rate), like in Hansen, Heaton, and Li (2008). The shock process is now described by:

$$\frac{dQ}{Q} = Xdt + \sigma dz \quad (37)$$

$$\frac{dQ^*}{Q^*} = X^*dt + \sigma^* dz^*. \quad (38)$$

The expected growth rates X , X^* are themselves random, and evolve according to the following mean-reverting processes:

$$dX_t = \kappa(\bar{X} - X_t)dt + \sigma_X dz_X \quad (39)$$

$$dX_t^* = \kappa^*(\bar{X}^* - X_t^*)dt + \sigma_X^* dz_X^* \quad (40)$$

where $\kappa, \kappa^*, \sigma_X, \sigma_X^* > 0$ and dz_X, dz_X^* are the increments of two standard Wiener processes with correlation $\rho_X \in [-1, 1]$. We assume zero correlation between consumption levels and expected growth rates.

To take full advantage of the shocks to the expected growth rate of consumption, agents have recursive preferences à la Epstein and Duffie (1992) with unit intertemporal elasticity of substitution like in Hansen, Heaton, and Li (2008). Preferences are

described by the following normalized aggregator:

$$f(Q, J) = \vartheta(1 - \gamma)J \left[\log(Q) - \frac{\log[(1 - \gamma)J]}{1 - \gamma} \right] \quad (41)$$

where J is the continuation utility associated with future consumption.

This modified specification helps us to match quantitatively the cross-section of returns. The intuition is the following: shocks to the long run component of growth are more persistent than i.i.d. shocks, and this higher persistence is more highly priced by the agents' intertemporal marginal rate of substitution via the continuation value component of recursive preferences.

We relegate to Appendix D the solution of the model under this specification of preferences and shocks, and we report here the calibration and the results.

4.4 Results: Earnings Yields and Returns Revisited

The values of the common parameters that we take from the literature are unchanged with respect to the baseline calibration. The steady state growth rates of consumption, \bar{X} and \bar{X}^* , are set equal to zero, consistently with $\mu = \mu^* = 0$ in the CRRA case. We assume that the shocks to the long run component of consumption growth follow the same process in the two countries, and we follow the calibration in Bansal and Yaron (2004) (appropriately converted to a continuous time setting) in choosing $\kappa = \kappa^* = 0.021$ and $\sigma_X = \sigma_X^* = 0.0012$. To choose a value for the correlation coefficient ρ_X , we show in Appendix D that the model reduces to the CRRA case for $\rho_X = 0$ and $\rho_X = 1$. We then choose an intermediate value $\rho_X = 0.5$.

With this modified version of the model, we need to re-calibrate the trade and FDI cost parameters, the lower bound of the Pareto distribution and the initial values of demand to match the persistence moments and the shares of firms in each group. Table 8 summarizes the newly calibrated parameters.

The calibrated iceberg cost is 1.36%, consistent with a medium-range estimate in Eaton and Kortum (2002). Fixed costs of export and FDI equal to 0.0006 and 0.008, respectively, indicate that an exporter must spend on average 5.96% of its per-period revenue in operating costs, and that a multinational firms must spend on average 6.07% of its per-period revenue in operating costs. Sunk costs of export and FDI are

Table 8: **Long Run Risk Calibration: Summary.**

Parameter	Definition	Value
τ	iceberg export cost	1.36
f_X	fixed export cost	0.0006
f_I	fixed FDI cost	0.008
F_X	sunk export cost	0.002
F_I	sunk FDI cost	0.01
b	lower bound of the productivity distribution	8
$Q(0), Q^*(0)$	initial demand	0.8

higher, equal to 0.002 and 0.01, respectively. They indicate that a domestic firm must spend on average 1.35 times its per-period revenue to enter the foreign market as an exporter, and about 6.77 times its per-period revenue to start FDI operations there. Aggregate demand parameters are set at $Q = Q^*(0) = 0.8$, and the lower bound of the productivity distribution to $b = 8$. Table 9 displays jointly the moments computed from the data and the moments generated by the calibrated model. Overall, the model performs quite well in matching the moments from the data: the calibration error, computed as the sum of squared differences between the model-generated moments and the moments computed from the data, is equal to 0.0195. The model generates a bit more persistence than what we observe in the data, while the shares of exporters and multinational are well matched.

Table 9: **Moments.** Comparison of the moments, long run risk model *versus* data.

	Data	Model		Data	Model
$D \rightarrow D$ (%)	92.49	98.92	X (%)	18	17.71
$D \rightarrow X$ (%)	3.63	1.06	I (%)	1	3.33
$X \rightarrow X$ (%)	87.04	97.86			
$X \rightarrow I$ (%)	6.95	2.14			
$I \rightarrow I$ (%)	98.02	97.84			
$I \rightarrow X$ (%)	0.88	2.16			

Table 10 reports the results of the calibration of the model with recursive preferences and shocks to the long run component of consumption growth. The model

generates the ranking in the financial variables that we observe in the data. The magnitudes of the earnings-to-price ratios are underpredicted (at about half of the values of the data), while it is noticeable how the addition of recursive preferences and shocks to the long run component of consumption improves on the quantitative fit of the returns. The model predicts very well the returns of domestic firms and exporters, and slightly over-predicts the returns of multinational corporations. Agents give a lot of weight to the higher persistence of the shocks via the continuation value component of recursive preferences.

Table 10: **Earnings Yields and Returns - Recursive Preferences and Long Run Risk Shocks.** Summary statistics of earnings yields and returns computed from simulated data, and comparison with real data. All values are in percentage terms.

	Earnings Yields		Returns	
	Mean (data)	Mean (model)	Mean (data)	Mean (model)
DOM	3.44	1.74	5.31	5.84
EXP	4.47	2.26	8.91	7.63
MN	6.33	3.49	9.84	11.45

5 Conclusions

This paper started by presenting a novel fact distinguishing multinational firms from exporters and from firms selling only in their domestic market. Multinational corporations tend to exhibit higher stock market returns and earnings yields than non-multinational firms. Within non-multinationals, exporters tend to have higher stock market returns and earnings yields than firms selling only in their domestic market. To explain this fact, we developed a real option value model where firms' heterogeneity, aggregate uncertainty and sunk costs provide a link between firms' choice of international status, risk exposure, and financial variables. We endogenize the exposure of these firms to sources of systematic risk.

The model is based on a very simple mechanism: firms decide to enter a risky foreign market when prospects of growth make entry profitable, and entry involves

a sunk cost. If after entry firms are subject to negative shocks, they will be reluctant to exit immediately because of the sunk cost they paid to enter, and may prefer to bear losses for a while. These losses, generated by the existence of fixed operating costs, are perceived as a risk by the firms' stockholders. Moreover, if the sunk costs of establishing a foreign affiliate are larger than the sunk costs of starting to export, the exposure to fluctuations and possible losses are higher for multinational firms than for exporters, commanding a higher return in equilibrium. While being consistent with facts about export and FDI participation and dynamics, the model endogenously determines cross-sectional differences in financial variables and provides a complementary explanation for the cross section of returns exploiting the international dimension of the data.

The model can be parameterized to reproduce features of the data on trade and FDI dynamics and participation in export and FDI. With the calibrated model we simulate an artificial economy and compute the financial variables. We show that the model is consistent, both qualitatively and quantitatively, with the observed rankings of earnings yields and returns.

We see this paper as the first step in a novel research agenda linking trade and FDI activities to asset pricing. The structural framework that we developed can be used to analyze the responses of heterogeneous agents (firms and investors) to different types of shocks: idiosyncratic, firm-specific, country-specific or aggregate, both in terms of real and financial variables. We think this is a promising avenue for research in finance and international trade that we plan to pursue in future work.

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Appendix of “Risk, Returns, and Multinational Production”, by José L. Fillat and Stefania Garetto

A Accounting Standards and Data Selection

The empirical analysis contained in this paper is based on annual, firm-level data. We limit the present study to the universe of publicly traded, US-based manufacturing firms included in the Standard & Poors Compustat Segments Database.¹ We use data from both the Center for Research in Security Prices (CRSP) and Compustat, obtained via CRSPs Securities Information Filtering Tool (CRSPSift). Our sample starts in 1979 and ends in 2009.

CRSP collects data on stock prices, earnings per share, numbers of shares outstanding, and returns, among other variables.² Compustat data is comprised of key components from annual regulatory filings and provides the link to Compustat Segments, which contains information on firms’ foreign operations. Segments data categorize a firm’s operations along a particular business division and report sales, assets, and other information. The four segment classifications are business, geographic, operating, and state. Multinational and export status dummies are constructed based on Compustat geographic and operating segments data.

The Financial Accounting Standards Board (FASB), in its Statement No. 131, sets the standards for the way in which public businesses report information about operating segments in their annual financial statements. Operating segments are defined by the FASB as “*components of an enterprise about which separate financial information is available that is evaluated regularly by the chief operating decision maker in deciding how to allocate resources and in assessing performance*”. The Statement of Financial Accounting Standards (FAS) No. 131 determines that firms should report data about revenues derived from the firm’s *products or services, countries* in which they earn revenues and hold assets, and about major *customers* regardless of whether that information is used in making operating decisions. However, the statement does not require firms to disclose the information on all the different segment types if it is

¹The NAICS codes for manufacturing firms contain the 2-digit prefix 31, 32, and 33.

²We only consider ordinary common shares that are the primary security of each firm in CRSP.

not prepared for internal use and reporting would be impracticable. Therefore, the firms decide how to report the data, disaggregated in several different ways: either by product, geography, legal entity, or by customer, but they do not necessarily have to report all of them. This method is referred to as the management approach. The statement establishes a minimum threshold to report separately information about an operating segment: either revenues of the segment are 10% or more of the combined revenue of all operating segments, or profits or losses are 10% or more of the combined reported profit or losses, or its assets are 10% or more of the combined assets of all operating segments. Hence, if a given firm considers best practice to aggregate the information upstream to the management level by customer, it may elect not to disclose geographical segments information.

According to the FAS 131, when a firm reports the existence of a geographical segment, it must report revenues and holdings of long-lived assets held in foreign countries. The FAS is not explicit in defining an ownership threshold for reporting, but the existence of accounting standards for the segments themselves leads us to think that the parent (U.S.-based) firm must have a control stake in the foreign entity. Moreover, one of the Financial Accounting Standards Board (FASB)'s roles is to *“require significant disclosures about the separate operating segments of an entity’s business so that investors can evaluate the differing risks in the diverse operations”*. Moreover, this information may or may not be disaggregated by individual foreign countries.

Clearly, the relevant segment for our classification of firms by status is the geographic segment. Faced with the potential measurement problems associated with the loose reporting requirements of Compustat Segments, we had two options to select our dataset: 1) include in the dataset only those firms that reported the existence of operating segments and drop all the others, or 2) include all firms in Compustat and impute as Domestic the status for those firms that did not report the existence of operating segments. The data analysis reported in Section 2 corresponds to the first selection criterium, which we prefer, because it generates a cleaner, albeit smaller, dataset.³ 96% of the firms that reported the existence of operating segments also report the existence of a geographic segment. For firms that report a geographic

³For robustness, we run all the regressions also using the dataset constructed with the second selection criterium, and the results on the ranking of earnings yields and returns are unchanged.

segment in a given year, all non-geographic segment observations were excluded. For the remaining firms, we aggregate data from the business segment and assume the firm's operations are entirely domestic. For these firms, all non-business segments are excluded from the sample. There are three types of geographic segments: (1) total, (2) domestic, and (3) non-domestic. A firm is classified as domestic in a given year if there is a missing or zero value for exports. A firm is classified as an exporter in a given year if the value for exports is non-missing and greater than zero. This includes firms that reported exports as insignificant or firms that reported exports in other data items.⁴ A firm is classified as multinational in a given year if its segments have a maximum Segment Geographic Type of 3 for a given year and if foreign sales are non-missing and greater than zero. Due to reporting errors, misclassifications, and multiple classifications, a few notes are required.

As is typical when a data point is unreliable, unreported, or otherwise a break from the traditional definition, the provider will report codes in place of an interpretable value. Compustat employs a similar methodology. In these instances we assume the segment to be of negligible importance and consider associated sales and exports to be null. As mentioned above our implementation of segment data relies entirely on the classification provided in the data. However, in a few instances sales for the non-domestic segments indicate the market of operation as the United States. In these cases we assume the reported classification was in error and re-classify the segment as domestic.

The data is aggregated by firm-year. For many firms this aggregation requires combining multiple segments and may result in competing classifications for a firm in a particular year. In these instances we classify the firm by the most "globally engaged" reported segment (for example domestic firms with exports are classified as exporters, while exporters with foreign sales are considered multinationals).

Examining a firm's international classification over time reveals what we believe to be reporting errors. These cases are characterized by a one-year "downward" status change, which results in a return to the original status. We believe this transient status change is a result of inaccurate segment reporting. As such we re-classify the observation to ensure continuity in the series. However, the opposite is not true: when

⁴Compustat specifically provides code values for "*value reported in another data item*".

a firm enters into an international market only to exit the following year, that firm retains the reported classification. The logic for this is evident: it is far easier to omit classification in a given year than to report segment details that were nonexistent.

Another dimension of selecting the data involves which criteria to use to establish the unit of observation and to eliminate outliers. The data span 31 years, from 1979 to 2009, but many firms have observations only for a part of this time interval.⁵ We drop firms that are active for less than one year and duplicate observations after a merge. The firms' classification is based on the fiscal year of each firm, as annual reports correspond to fiscal years and not annual years. As a result, annual returns are the result of compounding firm level monthly returns based on their fiscal year period. We disregard years for which we do not have 12 months of returns.

B Empirical Analysis: Robustness

This section complements the analysis of Section 2 by providing robustness checks of the firm-level regressions and by performing additional reduced-form portfolio-level asset pricing tests.

B.1 Firm-Level Regressions

Tables B.1 and B.2 report the results of regression (1) run with additional controls. We include in the analysis the capital/labor ratio, market capitalization, and the market *beta* of the individual firm, to make sure that differences in earnings yields and returns are not driven by systematic differences in these variables across groups.

The market *beta* of the primary security of firm i captures the comovement of the firm returns with the aggregate market returns.⁶ The purpose of adding the market *betas* is to control for each firm's individual exposure to aggregate market risk and to highlight the contribution of the international status to the magnitude of earnings yields once market risk is accounted for. Any cross-sectional differences in returns generated by exposure to aggregate risk is captured by cross sectional differences

⁵All variables have been deflated to constant 1984 dollars.

⁶The market *betas* have been computed by running a regression of individual security returns on the market aggregate returns (NYSE, AMEX, and Nasdaq) for the entire sample period.

Table B.1: **Robustness: Earnings-to-Price Ratios.** Firm-level regressions of earnings-to-price ratios on multinational and exporter dummies and other controls, with industry-year fixed effects. Standard errors are clustered by firm and status. (Top and bottom one percent of earnings-to-price sample excluded, top and bottom five percent of returns sample excluded. All dollar values are expressed in billions).

	(1)	(2)	(3)	(4)
MNE dummy	.135 (.006)***	.134 (.006)***	.115 (.006)***	.135 (.006)***
EXP dummy	.073 (.007)***	.074 (.007)***	.075 (.006)***	.076 (.007)***
revenue	.002 (.0009)*	.002 (.0009)*	.0009 (.0008)	.001 (.0009)
sales per emp.	.065 (.024)***	.063 (.024)***	.059 (.023)**	.063 (.024)***
K/L	21.787 (22.602)	21.102 (22.317)	.739 (12.905)	20.162 (21.802)
market cap.	.987 (.474)**	.929 (.474)*	1.794 (.484)***	1.026 (.470)**
β		-.269 (.140)*	-.073 (.143)	-.278 (.141)**
book/market			.056 (.006)***	
leverage				.004 (.003)
Prob > F :				
H_0 : MN=EXP	4.19e-31	1.80e-29	8.89e-16	5.78e-28
Obs.	50524	49658	49658	48851
adj. R^2	.156	.154	.231	.155

in their market β s. Hence, the significant coefficients on the multinational and exporter dummies identify a separate source of higher returns.

Table B.1 reports the results for the earnings-to-price ratios, and Table B.2 reports the results for the returns. The sign and significance of the coefficients on the status dummies are robust to the additional controls introduced in these specifications.

By looking at these regressions one could argue that differences among multinationals, exporters and domestic firms are not necessarily driven by intrinsic differences related to the status itself, but by unobservable firm characteristics. Clustering the standard errors by firm and status alleviates this concern. A more conservative specification of these regressions would include firm fixed effects. The problem of the firm fixed effect specification is that it identifies variation across groups by using only the

Table B.2: **Robustness: Returns.** Firm-level regressions of returns on multinational and exporter dummies and other controls, with industry-year fixed effects. Standard errors are clustered by firm and status. (Top and bottom one percent of earnings-to-price sample excluded, top and bottom five percent of returns sample excluded. All dollar values are expressed in billions).

	(1)	(2)	(3)	(4)
MNE dummy	.053 (.007)***	.053 (.007)***	.056 (.007)***	.054 (.007)***
EXP dummy	.046 (.008)***	.046 (.008)***	.046 (.008)***	.047 (.008)***
revenue	-.002 (.0007)***	-.002 (.0007)***	-.002 (.0007)***	-.002 (.0007)***
sales per emp.	.048 (.018)***	.048 (.018)***	.049 (.018)***	.048 (.018)***
K/L	19.341 (10.726)*	19.099 (10.595)*	22.027 (12.531)*	19.071 (10.519)*
market cap.	4.083 (.854)***	4.134 (.851)***	4.009 (.849)***	4.136 (.887)***
β		.581 (.205)***	.553 (.205)***	.582 (.205)***
book/market			-.008 (.002)***	
leverage				.005 (.002)**
Prob > F :				
H_0	.216	.258	.115	.317
Obs.	50390	49658	49658	48851
adj. R^2	.126	.127	.128	.126

information of the firms that change status at least once during the sample period. In our sample, on average every year 94% of firms do not change status, so the fixed effects specification loses a lot of information contained in the sample. This problem is particularly acute for multinational firms, which tend to change status even less than other firms (only 2% of MNEs change status every year). As a result, the fixed effect regressions show positive and significant coefficients on the export status dummy, but non-significant coefficients on the multinational status dummy.

Alternatively, one could argue that the results of our baseline regressions are driven by changes over time and correlations across years in the sample. The year fixed effects address this concern. For robustness, we also run regression (1) with a cross-sectional specification where every firm-level variable is calculated as an average over

the sample period. This specification suffers of the opposite problem of the firm fixed effect specification: since we have to define the status dummies over the entire sample period, we lose all the information coming from firms that switch status at least once in the sample. However, the results are robust to the cross-sectional specification: the status dummies are positive and significant, and the multinationality dummy is significantly higher than the export dummy.

B.2 Portfolio Regressions: CAPM and Fama-French

Empirical research in finance has attempted to explain the cross-section of returns by looking at how stock returns co-vary with aggregate risk, represented by risk factors. While in the body of the paper we privilege the micro-founded interpretation of risk as covariance of returns with the agents' IMRS, here we follow the reduced form literature and present CAPM and Fama-French asset pricing tests.

The CAPM model explains higher returns of certain assets as being generated by a larger covariance with systematic risk, represented by the returns on the aggregate market portfolio.⁷ To examine the returns of firms with different international status from a CAPM point of view, we run one time-series CAPM regression for each of the three portfolios of firms characterized by the same international status.⁸ The results are displayed in Table B.3. The risk to which multinationals and exporters are exposed, and the corresponding higher returns that they provide to investors, are not fully explained by higher *betas*: the portfolio formed by multinational corporations exhibits a slightly lower *beta* than the one of domestic firms, and the exporters portfolio exhibits the lowest.

If the exposure to the returns on the market portfolio does not explain the higher reward that multinationals' and exporters' stocks provide, it must be reflected in the pricing errors of the model. In fact, the *alpha* of the portfolio of multinational firms is significantly higher than the one of the exporters portfolio, which in turn is higher than the *alpha* of the portfolio of domestic firms.

Fama and French (1993) introduced a multifactor extension to the original CAPM.

⁷The return on the market is the value-weighted return on all NYSE, AMEX, and NASDAQ stocks minus the one-month Treasury bill rate.

⁸Every year portfolios are formed by equally-weighting the returns of firms belonging to each of the three categories.

Table B.3: **Portfolio Regressions: CAPM.** Time-series coefficient estimates of CAPM regressions for the three equally-weighted portfolios based on international status. The α coefficients capture the pricing errors.

	DOM	EXP	MN
R^{mkt}	.640 (.105)***	.580 (.104)***	.608 (.077)***
α	-.051 (.020)**	-.011 (.020)	-.003 (.015)
No of Obs.	31	31	31
adj. R^2	.548	.501	.671

Fama and French (1993) argue that a unique source of risk is not able to explain the cross section of returns. Instead, a three-factor model explains a higher portion of the variation in expected returns. Higher returns must be explained by higher exposure to either of these three factors: market excess returns, high-minus-low book-to-market, or small-minus-big portfolio, as these characteristics seem to provide independent information about average returns.⁹ Each of the three factors is assumed to mimic a macroeconomic aggregate risk. Therefore, any asset is represented as a linear combination of the three Fama-French factors. These regressions, though, are not informative about why exposures are different across portfolios. The purpose of our model is to endogenize these exposures.

Table B.4 shows the results of running one time-series Fama-French regression for each of the three portfolios of firms characterized by the same international status. The risk to which multinationals and exporters are exposed, and the corresponding higher returns that they provide to investors, are not explained by the three existing Fama-French factors. On the contrary, we find that the market *betas* of exporters and multinationals are lower than the ones of domestic firms. Multinationals and exporters' exposure to the SMB factor, related to size, and to the HML factor, related

⁹The small-minus-big (SMB) and high-minus-low (HML) factors are constructed upon 6 portfolios formed on size and book-to-market. The portfolios are the intersection of 2 portfolios formed on size (small and big) and 3 portfolios formed on book equity to market equity (from higher to lower: value, neutral, and growth.) This generates 6 portfolios: small-value, small-neutral, small-growth, big-value, big-neutral, and big-growth. SMB is the average returns on the three small portfolios minus the average returns on the three big portfolios. HML is the average return on the two value portfolios minus the average return on the two growth portfolios. For more details see Fama and French (1993).

Table B.4: **Portfolio Regressions: Fama-French.** Time-series coefficient estimates of Fama-French regressions for the three equally-weighted portfolios based on international status. The α coefficients capture the pricing errors.

	DOM	EXP	MN
R^{mkt}	.668 (.065)***	.578 (.075)***	.634 (.057)***
R^{SMB}	.625 (.093)***	.616 (.108)***	.402 (.082)***
R^{HML}	.250 (.074)***	.156 (.086)*	.187 (.065)***
α	-.079 (.013)***	-.032 (.015)**	-.023 (.011)**
Prob $> F$:			
$H_0: \alpha_D = \alpha_X = \alpha_I$	1.905e-06		
No of Obs.	31	31	31
adj. R^2	.854	.78	.851

to the value premium, is also lower than the exposure of domestic firms to the same factors. Particularly, the ranking of the coefficients on the SMB factors is opposite to the peking order in the returns that we observe in the data: higher returns of exporters and multinational corporations are not explained by higher exposure to the size factor which would command higher expected returns, all else equal. Differences in returns are not fully explained by the three factors, and are hence reflected in the pricing errors. Also in this specification, the *alpha* of the portfolio of multinational firms is significantly higher than the one of the exporters' portfolio, which in turn is higher than the *alpha* of the portfolio of domestic firms. The GRS test on the null hypothesis that the *alphas* are jointly equal to zero strongly rejects the hypothesis.

In Table B.5 we enlarge the set of factors by considering the excess returns on an international market portfolio that serves as a market benchmark for firms with foreign operations. Data on the excess returns on this global market portfolio are obtained from Kenneth French's data library on international indexes.¹⁰ The coefficients on the international market portfolio are not significant. Based on this evidence, the model we specify in Section 3 explains differences in returns across firms with different exposure to domestic and foreign shocks.

¹⁰http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/Data_Library/int_index_port_formed.html.

Table B.5: **Portfolio Regressions: 4-Factors Fama-French.** Time-series coefficient estimates of Fama-French regressions augmented with a global market portfolio. Three equally-weighted portfolios based on international status. The α coefficients capture the pricing errors.

	DOM	EXP	MN
R^{mkt}	.670 (.085)***	.567 (.098)***	.571 (.071)***
R^{SMB}	.625 (.096)***	.618 (.111)***	.416 (.081)***
R^{HML}	.250 (.076)***	.156 (.088)*	.184 (.064)***
R^{INT}	-.002 (.059)	.012 (.069)	.070 (.050)
α	-.081 (.064)	-.019 (.074)	.051 (.054)
No of Obs.	31	31	31
adj. R^2	.849	.772	.856

C Derivations and Proofs

In this section we present the details of the derivation of the value functions in Section 3, and some comparative statics properties.

C.1 Value Function of a Domestic Firm

In the continuation region, the value of a firm selling only in its domestic market is:

$$\pi_D(a, Q)M\Delta t + E[M\Delta t \cdot \mathcal{S}(a, Q')|Q] - \mathcal{S}(a, Q) + E[M\Delta t \cdot V_D(a, Q^*)|Q^*] - V_D(a, Q^*) = 0.$$

For $\Delta t \rightarrow 0$:

$$\pi_D(a, Q)Mdt + E[d(M \cdot \mathcal{S}(a, Q))] + E[d(M \cdot V_D(a, Q^*))] = 0.$$

The terms in the expectations can be written as:

$$\begin{aligned}
E[d(M \cdot \mathcal{S})] &= E[dM \cdot \mathcal{S} + M \cdot d\mathcal{S} + dM \cdot d\mathcal{S}] \\
&= M \cdot \mathcal{S} \cdot E \left[\frac{dM}{M} + \frac{d\mathcal{S}}{\mathcal{S}} + \frac{dM}{M} \cdot \frac{d\mathcal{S}}{\mathcal{S}} \right] \\
&= M \cdot \mathcal{S} \left[-r dt + E \left(\frac{d\mathcal{S}}{d\mathcal{S}} \right) + E \left(\frac{dM}{M} \cdot \frac{d\mathcal{S}}{\mathcal{S}} \right) \right] \\
&= M dt \left[-r \mathcal{S} + E \left(\frac{d\mathcal{S}}{dt} \right) + E \left(\frac{dM}{M} \cdot \frac{d\mathcal{S}}{dt} \right) \right] \tag{C.1}
\end{aligned}$$

$$\begin{aligned}
E[d(M \cdot V_D)] &= E[dM \cdot V_D + M \cdot dV_D + dM \cdot dV_D] \\
&= M \cdot V_D \cdot E \left[\frac{dM}{M} + \frac{dV_D}{V_D} + \frac{dM}{M} \cdot \frac{dV_D}{V_D} \right] \\
&= M \cdot V_D \left[-r dt + E \left(\frac{dV_D}{V_D} \right) + E \left(\frac{dM}{M} \cdot \frac{dV_D}{V_D} \right) \right] \\
&= M dt \left[-r V_D + E \left(\frac{dV_D}{dt} \right) + E \left(\frac{dM}{M} \cdot \frac{dV_D}{dt} \right) \right] \tag{C.2}
\end{aligned}$$

where the dependence of \mathcal{S} on (a, Q) and the dependence of V_D on (a, Q^*) have been suppressed to ease the notation. Plugging (C.1) and (C.2) into the no-arbitrage condition:

$$\pi_D - r\mathcal{S} + E \left(\frac{d\mathcal{S}}{dt} \right) + E \left(\frac{dM}{M} \cdot \frac{d\mathcal{S}}{dt} \right) - rV_D + E \left(\frac{dV_D}{dt} \right) + E \left(\frac{dM}{M} \cdot \frac{dV_D}{dt} \right) = 0. \tag{C.3}$$

By applying Ito's Lemma and using the expressions for the Brownian motions ruling Q and Q^* , we can derive expressions for some of the terms in (C.3):

$$\begin{aligned}
d\mathcal{S} &= \mathcal{S}' dQ + \frac{1}{2} \sigma^2 Q^2 \mathcal{S}'' dt = \mathcal{S}' [\mu Q dt + \sigma Q dz] + \frac{1}{2} \sigma^2 Q^2 \mathcal{S}'' dt \\
E[d\mathcal{S}] &= \mu Q \mathcal{S}' dt + \frac{1}{2} \sigma^2 Q^2 \mathcal{S}'' dt \\
dV_D &= V_D' dQ^* + \frac{1}{2} \sigma^{*2} Q^{*2} V_D'' dt = V_D' [\mu^* Q^* dt + \sigma^* Q^* dz^*] + \frac{1}{2} \sigma^{*2} Q^{*2} V_D'' dt \\
E[dV_D] &= \mu^* Q^* V_D' dt + \frac{1}{2} \sigma^{*2} Q^{*2} V_D'' dt .
\end{aligned}$$

Using these results and equation (5) (the expression for the evolution of M), we

can rewrite (C.3) as:

$$\begin{aligned}
& \pi_D dt - r\mathcal{S}dt + \mu Q\mathcal{S}'dt + \frac{1}{2}\sigma^2 Q^2 \mathcal{S}''dt + \dots \\
& E \left[(-rdt - \gamma\sigma dz) \cdot \left(\mu Q\mathcal{S}'dt + \sigma Q\mathcal{S}'dz + \frac{1}{2}\sigma^2 Q^2 \mathcal{S}''dt \right) \right] - \dots \\
& \dots rV_D dt + \mu^* Q^* V_D' dt + \frac{1}{2}\sigma^{*2} (Q^*)^2 V_D'' dt + \dots \\
& \dots E \left[(-rdt - \gamma\sigma dz) \cdot \left(\mu^* Q^* V_D' dt + \sigma^* Q^* V_D' dz^* + \frac{1}{2}\sigma^{*2} Q^{*2} V_D'' dt \right) \right] = 0. \quad (\text{C.4})
\end{aligned}$$

The terms in expectation can be reduced to:

$$\begin{aligned}
& E \left[(-rdt - \gamma\sigma dz) \cdot \left(\mu Q\mathcal{S}'dt + \sigma Q\mathcal{S}'dz + \frac{1}{2}\sigma^2 Q^2 \mathcal{S}''dt \right) \right] = -\gamma\sigma^2 Q\mathcal{S}'dt \\
& E \left[(-rdt - \gamma\sigma dz) \cdot \left(\mu^* Q^* V_D' dt + \sigma^* Q^* V_D' dz^* + \frac{1}{2}\sigma^{*2} Q^{*2} V_D'' dt \right) \right] = -\gamma\rho\sigma\sigma^* Q^* V_D' dt.
\end{aligned}$$

So we can rewrite (C.4) as:

$$\pi_D - r\mathcal{S} + (\mu - \gamma\sigma^2)Q\mathcal{S}' + \frac{1}{2}\sigma^2 Q^2 \mathcal{S}'' - rV_D + (\mu^* - \gamma\rho\sigma\sigma^*)Q^* V_D' + \frac{1}{2}\sigma^{*2} Q^{*2} V_D'' = 0. \quad (\text{C.5})$$

One possible solution of this equation is:

$$\pi_D(a, Q) - r\mathcal{S}(a, Q) + (\mu - \gamma\sigma^2)Q\mathcal{S}'(a, Q) + \frac{1}{2}\sigma^2 Q^2 \mathcal{S}''(a, Q) = 0 \quad (\text{C.6})$$

$$-rV_D(a, Q^*) + (\mu^* - \gamma\rho\sigma\sigma^*)Q^* V_D'(a, Q^*) + \frac{1}{2}\sigma^{*2} Q^{*2} V_D''(a, Q^*) = 0. \quad (\text{C.7})$$

We guess that the solution of (C.6) takes the form: $\mathcal{S}(a, Q) = Q^\chi + c_S Q$. By substituting this expression into (C.6), we find that χ is the root of:

$$\frac{1}{2}\sigma^2 \chi^2 + (\mu - \gamma\sigma^2 - \frac{1}{2}\sigma^2)\chi - r = 0,$$

while the linear parameter c_S is given by:

$$c_S = \frac{H(aw)^{1-\eta} P^\eta}{r - \mu + \gamma\sigma^2}.$$

Hence the value function describing the expected present discounted value of domestic

profits takes the form:

$$\mathcal{S}(a, Q) = A_S(a)Q^{\alpha_s} + B_S(a)Q^{\beta_s} + \frac{H(aw)^{1-\eta}P^\eta}{r - \mu + \gamma\sigma^2}$$

where α_s and β_s are the negative and positive value of χ , respectively, and $A_S(a)$ and $B_S(a)$ are firm-specific parameters to be determined. Since there is no option value associated with domestic profits, we can impose: $A_S(a) = B_S(a) = 0$, so that the solution is simply given by the value of profits discounted with the risk-adjusted measure:

$$\mathcal{S}(a, Q) = \frac{H(aw)^{1-\eta}P^\eta Q}{r - \mu + \gamma\sigma^2}. \quad (\text{C.8})$$

Similarly, we guess that the solution of (C.7) takes the form: $V_D(a, Q^*) = Q^{*\xi}$. By substituting this expression into (C.7), we find that ξ is the root of:

$$\frac{1}{2}\sigma^{*2}\xi^2 + (\mu^* - \gamma\rho\sigma\sigma^* - \frac{1}{2}\sigma^{*2})\xi - r = 0. \quad (\text{C.9})$$

Hence the value function describing the expected present discounted value of foreign profits of a domestic firm takes the form:

$$V_D(a, Q^*) = A_D(a)Q^{*\alpha} + B_D(a)Q^{*\beta} \quad (\text{C.10})$$

where α and β are the negative and positive value of ξ , respectively, and $A_D(a)$ and $B_D(a)$ are firm-specific parameters to be determined.

C.2 Value Function of an Exporter

The value of domestic profits is independent of status, hence the value of domestic profits for an exporter is also given by (C.8). The value of foreign profits of an exporter solves:

$$\pi_X(a, Q^*) - rV_X(a, Q^*) + (\mu^* - \gamma\rho\sigma\sigma^*)Q^*V'_X(a, Q^*) + \frac{1}{2}\sigma^{*2}Q^{*2}V''_X(a, Q^*) = 0.. \quad (\text{C.11})$$

We guess that the solution of (C.11) takes the form: $V_X(a, Q^*) = Q^{*\xi} + cQ^* + d$.

By substituting this expression into (C.11), we find that ξ is the root of (C.9), while the parameters c and d are given by:

$$c = \frac{H(\tau aw)^{1-\eta} P^{*\eta}}{r - \mu^* + \gamma \rho \sigma \sigma^*} \quad (\text{C.12})$$

$$d = -\frac{f_X}{r}. \quad (\text{C.13})$$

Hence the value function describing the expected present discounted value of foreign profits of an exporter takes the form:

$$V_X(a, Q^*) = A_X(a)Q^{*\alpha} + B_X(a)Q^{*\beta} + \frac{H(\tau aw)^{1-\eta} P^{*\eta} Q^*}{r - \mu^* + \gamma \rho \sigma \sigma^*} - \frac{f_X}{r} \quad (\text{C.14})$$

where α and β are the negative and positive value of ξ , respectively, and $A_X(a)$ and $B_X(a)$ are firm-specific parameters to be determined.

C.3 Value Function of a Multinational

Also for a multinational the value of domestic profits is independent of status and is given by (C.8). The value of foreign profits of a multinational solves:

$$\pi_I(a, Q^*) - rV_I(a, Q^*) + (\mu^* - \gamma \rho \sigma \sigma^*)Q^*V_I'(a, Q^*) + \frac{1}{2}\sigma^{*2}Q^{*2}V_I''(a, Q^*) = 0.. \quad (\text{C.15})$$

Notice that the functional form of (C.15) is identical to the one of (C.11), hence: $V_I(a, Q^*) = Q^{*\xi} + c'Q^* + d'$, where ξ is given by (C.9), and:

$$c' = \frac{H(aw^*)^{1-\eta} P^{*\eta}}{r - \mu^* + \gamma \rho \sigma \sigma^*} \quad (\text{C.16})$$

$$d' = -\frac{f_I}{r}. \quad (\text{C.17})$$

Hence the value function describing the expected present discounted value of foreign profits of a multinational takes the form:

$$V_I(a, Q^*) = A_I(a)Q^{*\alpha} + B_I(a)Q^{*\beta} + \frac{H(aw^*)^{1-\eta} P^{*\eta} Q^*}{r - \mu^* + \gamma \rho \sigma \sigma^*} - \frac{f_I}{r} \quad (\text{C.18})$$

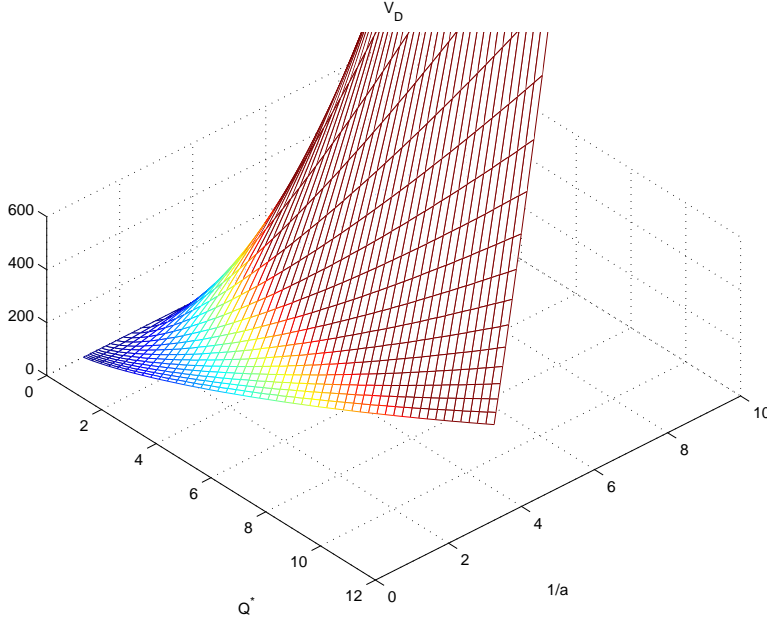


Figure C.1: Value of foreign profits of a domestic firm.

where α and β are the negative and positive value of ξ , respectively, and $A_I(a)$ and $B_I(a)$ are firm-specific parameters to be determined.

C.4 Comparative Statics: Value and Productivity

We show here qualitative properties of the value functions that are key to the solution of the model. Both the quantity thresholds and the parameters of the value functions depend on the productivity level $1/a$. Figure C.1 shows the value of foreign profits of a domestic firm as a function of the aggregate quantity demanded in the foreign market Q^* and of productivity $1/a$. V_D is increasing in Q^* , as the option value of entering the foreign market is increasing in the quantity demanded. V_D is also increasing in firm's productivity, as more productive firms can get higher profits from entering the foreign market.

Figure C.2 shows the value of foreign profits of an exporter and of a multinational firm as functions of Q^* and $1/a$. V_X and V_I are U-shaped functions of Q^* , indicating the high option value of exiting for low realizations of Q^* and the high option value of not changing status for high realizations of Q^* . For $Q^* \rightarrow \infty$, the value function of an exporter is steeper than the one of a multinational, because the exporter gets

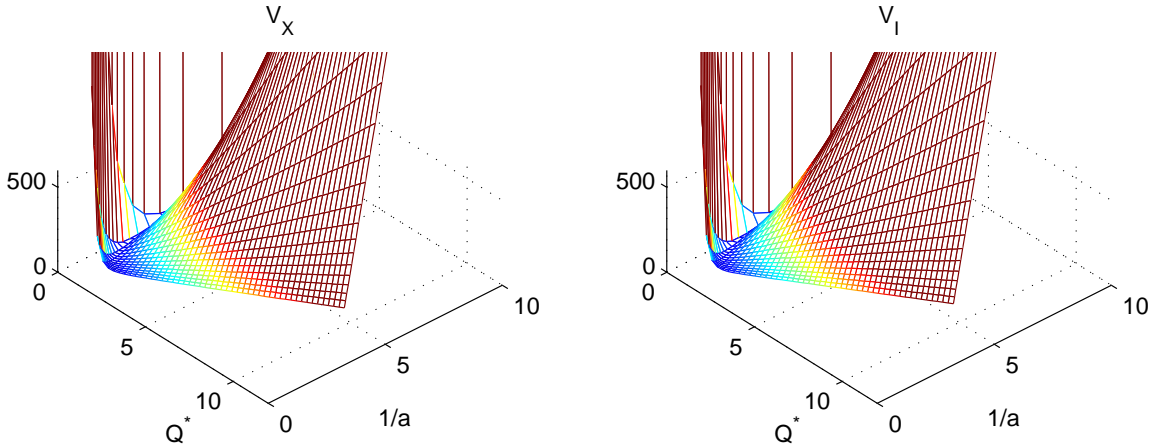


Figure C.2: Value of foreign profits of an exporter and of a multinational firm.

high value both from staying in the market as an exporter and from the option value of becoming a multinational. The behavior of the value functions for $Q^* \rightarrow 0$ does not vary much across the productivity dimension: when Q^* is low, the value is high as firms of all productivity levels associate a high value to the option of exiting. Conversely, the behavior of the value functions when Q^* is “large” varies with individual productivity: the value function is steeper for higher productivity firms, indicating that more productive firms obtain higher returns from staying in the foreign market when the realized aggregate demand is high.

From Figure C.2, the qualitative behavior of V_X and V_I appears very similar. Figure C.3 plots the difference between the value of foreign profits of firms serving the foreign market and of firms selling only domestically, $V_X - V_D$ and $V_I - V_D$. For each productivity level $1/a$, each plot has two stationary points, a local maximum and a local minimum. The value matching and smooth pasting conditions imply that the local maxima correspond to the “entry” thresholds (Q_{DX}^* and Q_{DI}^* in the left and right plot respectively), while the local minima correspond to the “exit” thresholds (Q_{XD}^* and Q_{ID}^*). Consistently with Theorem 1, both entry and exit thresholds are decreasing in $1/a$, indicating that more productive firms enter the foreign market for lower realizations of aggregate demand Q^* with respect to less productive firms. Similarly, more productive firms need larger negative shocks to demand to be induced to exit the foreign market with respect to less productive firms. Notice that for $Q^* \rightarrow 0$, $V_X - V_D$ and $V_I - V_D$ tend to infinity, because the option value of exiting

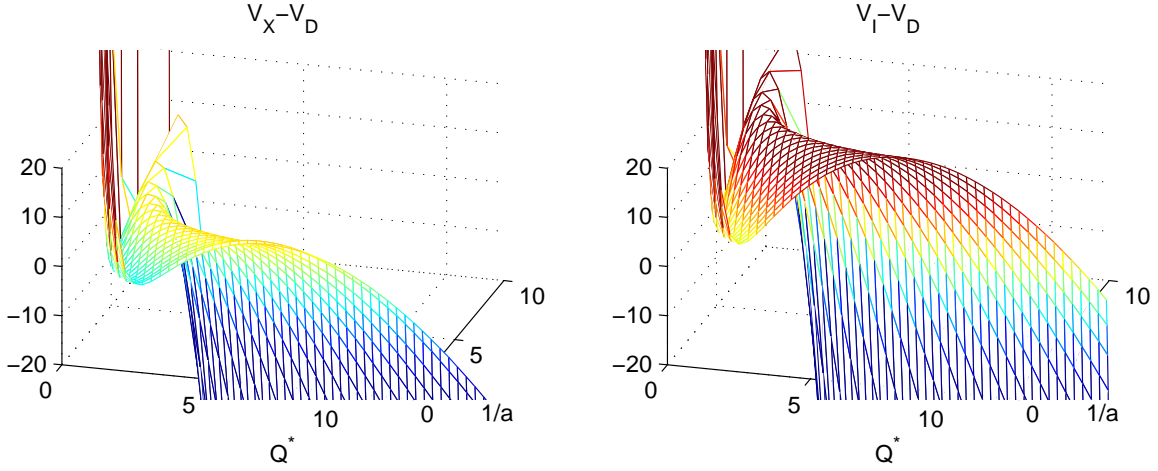


Figure C.3: Difference between the value of foreign profits of exporters and multinationals and that of domestic firms.

the foreign market is extremely high for very low realizations of Q^* (and irrespective of firm's productivity). Conversely, for $Q^* \rightarrow \infty$, $V_X - V_D$ and $V_I - V_D$ tend to negative infinity, because the domestic firms' option value of entering the foreign market is extremely high compared to the flow profits of staying for firms that are already serving that market. The difference between the value of foreign profits of a multinational firm and of an exporter displays similar properties.

C.5 Proof of Theorem 1: Thresholds and Productivity

$$\frac{\partial Q_{RS}^*(a)}{\partial a} > 0, \quad \text{for } R, S \in \{D, X, I\}, \forall a.$$

Proof: The proof closely follows Appendix B of Dixit (1989). We show the result for Q_{DX} only; the proof for the other thresholds follows the same steps.

The value-matching conditions for Q_{DX}^* , Q_{XD}^* are:

$$A_X Q_{DX}^{*\alpha} + (B_X - B_D) Q_{DX}^{*\beta} + \frac{H(\tau a w)^{1-\eta} P^{*\eta} Q_{DX}^*}{r - \mu^* + \gamma \rho \sigma \sigma^*} - \frac{f_X}{r} = F_X$$

$$A_X Q_{XD}^{*\alpha} + (B_X - B_D) Q_{XD}^{*\beta} + \frac{H(\tau a w)^{1-\eta} P^{*\eta} Q_{XD}^*}{r - \mu^* + \gamma \rho \sigma \sigma^*} - \frac{f_X}{r} = 0.$$

Differentiating and using the smooth-pasting conditions:

$$Q_{DX}^*{}^\alpha dA_X + Q_{DX}^*{}^\beta d(B_X - B_D) + \frac{(1-\eta)H(\tau w)^{1-\eta}a^{-\eta}P^{*\eta}Q_{DX}^*}{r - \mu^* + \gamma\rho\sigma\sigma^*} da = 0 \quad (\text{C.19})$$

$$Q_{XD}^*{}^\alpha dA_X + Q_{XD}^*{}^\beta d(B_X - B_D) + \frac{(1-\eta)H(\tau w)^{1-\eta}a^{-\eta}P^{*\eta}Q_{XD}^*}{r - \mu^* + \gamma\rho\sigma\sigma^*} da = 0. \quad (\text{C.20})$$

Dividing (C.19) by Q_{DX}^* and (C.20) by Q_{XD}^* and combining them:

$$dA_X = \left(\frac{Q_{DX}^*{}^{\beta-1} - Q_{XD}^*{}^{\beta-1}}{Q_{XD}^*{}^{\alpha-1} - Q_{DX}^*{}^{\alpha-1}} \right) d(B_X - B_D). \quad (\text{C.21})$$

Plugging (C.21) into (C.19):

$$d(B_X - B_D) = \left(\frac{Q_{XD}^*{}^{\alpha-1} - Q_{DX}^*{}^{\alpha-1}}{Q_{DX}^*{}^{\beta-1}Q_{XD}^*{}^{\alpha-1} - Q_{DX}^*{}^{\alpha-1}Q_{XD}^*{}^{\beta-1}} \right) \cdot \left(-\frac{(1-\eta)H(\tau w)^{1-\eta}a^{-\eta}P^{*\eta}}{r - \mu^* + \gamma\rho\sigma\sigma^*} da \right) \quad (\text{C.22})$$

and plugging (C.22) into (C.21):

$$dA_X = \left(\frac{Q_{DX}^*{}^{\beta-1} - Q_{XD}^*{}^{\beta-1}}{Q_{DX}^*{}^{\beta-1}Q_{XD}^*{}^{\alpha-1} - Q_{DX}^*{}^{\alpha-1}Q_{XD}^*{}^{\beta-1}} \right) \cdot \left(-\frac{(1-\eta)H(\tau w)^{1-\eta}a^{-\eta}P^{*\eta}}{r - \mu^* + \gamma\rho\sigma\sigma^*} da \right). \quad (\text{C.23})$$

The smooth-pasting condition for Q_{DX}^* is:

$$\alpha A_X Q_{DX}^*{}^{\alpha-1} + \beta(B_X - B_D) Q_{DX}^*{}^{\beta-1} + \frac{H(\tau aw)^{1-\eta}P^{*\eta}}{r - \mu^* + \gamma\rho\sigma\sigma^*} = 0.$$

Let $G_{DX}(\cdot) = V_X(\cdot) - V_D(\cdot)$. Differentiating the condition above:

$$G_{DX}''(\cdot)dQ_{DX}^* + \alpha Q_{DX}^*{}^{\alpha-1} dA_X + \beta Q_{DX}^*{}^{\beta-1} d(B_X - B_D) + \frac{(1-\eta)H(\tau w)^{1-\eta}a^{-\eta}P^{*\eta}}{r - \mu^* + \gamma\rho\sigma\sigma^*} da = 0. \quad (\text{C.24})$$

Let $\Delta \equiv Q_{DX}^*{}^{\beta-1}Q_{XD}^*{}^{\alpha-1} - Q_{DX}^*{}^{\alpha-1}Q_{XD}^*{}^{\beta-1}$. Substituting in the expressions for dA_X and $d(B_X - B_D)$, equation (C.24) can be rewritten as:

$$\underbrace{-G_{DX}''(\cdot)dQ_{DX}^*}_{>0} = \underbrace{\frac{(\eta-1)H(\tau w)^{1-\eta}a^{-\eta}P^{*\eta}}{\Delta[r - \mu^* + \gamma\rho\sigma\sigma^*]}}_{>0} da \cdot \dots \\ \dots \cdot \left[\alpha \left(Q_{DX}^*{}^{\alpha+\beta-2} - Q_{DX}^*{}^{\alpha-1}Q_{XD}^*{}^{\beta-1} \right) + \beta \left(Q_{DX}^*{}^{\alpha+\beta-2} - Q_{DX}^*{}^{\beta-1}Q_{XD}^*{}^{\alpha-1} \right) + \Delta \right]$$

In order to show that $\frac{\partial Q_{DX}^*(a)}{\partial a} > 0$, we must show that the last term of the expression above is positive:

$$\begin{aligned} & \left[\alpha \left(Q_{DX}^*{}^{\alpha+\beta-2} - Q_{DX}^*{}^{\alpha-1} Q_{XD}^*{}^{\beta-1} \right) + \beta \left(Q_{DX}^*{}^{\alpha+\beta-2} - Q_{DX}^*{}^{\beta-1} Q_{XD}^*{}^{\alpha-1} \right) + \Delta \right] = \dots \\ & \frac{1}{Q_{DX}^*{}^{\alpha+\beta-2}} \left[\alpha \left(1 - \left(\frac{Q_{XD}^*}{Q_{DX}^*} \right)^{\beta-1} \right) + \beta \left(\left(\frac{Q_{XD}^*}{Q_{DX}^*} \right)^{\alpha-1} - 1 \right) - \left(\frac{Q_{XD}^*}{Q_{DX}^*} \right)^{\alpha-1} + \left(\frac{Q_{XD}^*}{Q_{DX}^*} \right)^{\beta-1} \right] = \dots \\ & \underbrace{\frac{1}{Q_{DX}^*{}^{\alpha+\beta-2}}}_{>0} \left[\alpha \left(1 - \left(\frac{Q_{DX}^*}{Q_{XD}^*} \right)^{1-\beta} \right) + \beta \left(\left(\frac{Q_{DX}^*}{Q_{XD}^*} \right)^{1-\alpha} - 1 \right) - \left(\frac{Q_{DX}^*}{Q_{XD}^*} \right)^{1-\alpha} + \left(\frac{Q_{DX}^*}{Q_{XD}^*} \right)^{1-\beta} \right]. \end{aligned}$$

Let $z \equiv \frac{Q_{DX}^*}{Q_{XD}^*} > 1$ and let $\phi(z) \equiv [\alpha(1 - z^{1-\beta}) + \beta(z^{1-\alpha} - 1) - z^{1-\alpha} + z^{1-\beta}]$. Then $\phi(1) = 0$ and $\phi'(z) = (1 - \alpha)(\beta - 1)(z^{-\alpha} - z^{-\beta}) > 0$, which proves the result. \square

C.6 Returns

In this section we show how to derive the expression for the returns in equation (36) and some comparative statics properties. Starting from Ito's Lemma applied to the value of domestic profits and to the value of foreign profits (WLOG, for an exporter):

$$\begin{aligned} dS &= \left[\mu Q S' + \frac{1}{2} \sigma^2 Q^2 S'' \right] dt + \sigma Q S' dz \\ dV_X &= \left[\mu^* Q^* V_X' + \frac{1}{2} \sigma^{*2} Q^{*2} V_X'' \right] dt + \sigma^* Q^* V_X' dz^* \end{aligned}$$

which implies:

$$\begin{aligned} E[dS] &= \mu Q S' + \frac{1}{2} \sigma^2 Q^2 S'' \\ E[dV_X] &= \mu^* Q^* V_X' + \frac{1}{2} \sigma^{*2} Q^{*2} V_X''. \end{aligned}$$

The no-arbitrage condition derived from the Bellman equation is:

$$\pi_D - rS + (\mu - \gamma\sigma^2)QS' + \frac{1}{2}\sigma^2Q^2S'' + \pi_X - rV_X + (\mu^* - \gamma\rho\sigma\sigma^*)Q^*V_X' + \frac{1}{2}\sigma^{*2}Q^{*2}V_X'' = 0.$$

Adding and subtracting the term $\mu Q\mathcal{S}' + \mu^* Q^* V_X'$:

$$\begin{aligned}
& \pi_D - r\mathcal{S} + (\mu - \gamma\sigma^2)Q\mathcal{S}' - \mu Q\mathcal{S}' + \mu Q\mathcal{S}' + \frac{1}{2}\sigma^2 Q^2 \mathcal{S}'' + \dots \\
& + \pi_X - rV_X + (\mu^* - \gamma\rho\sigma\sigma^*)Q^* V_X' - \mu^* Q^* V_X' + \mu^* Q^* V_X' + \frac{1}{2}\sigma^{*2} Q^{*2} V_X'' = 0 \\
\pi_D - r\mathcal{S} - \gamma\sigma^2 Q\mathcal{S}' + \underbrace{\mu Q\mathcal{S}' + \frac{1}{2}\sigma^2 Q^2 \mathcal{S}''}_{E(d\mathcal{S})} + \pi_X - rV_X - \gamma\rho\sigma\sigma^* Q^* V_X' + \underbrace{\mu^* Q^* V_X' + \frac{1}{2}\sigma^{*2} Q^{*2} V_X''}_{E(dV_X)} = 0
\end{aligned}$$

where $E(d\mathcal{S}) + E(dV_X) = E[d(\mathcal{S} + V_X)] = E(d\mathcal{V}_X)$. Hence:

$$\begin{aligned}
E(d\mathcal{V}_X) + \pi_D + \pi_X &= r\mathcal{V}_X + \gamma\sigma^2 Q\mathcal{S}' + \gamma\rho\sigma\sigma^* Q^* V_X' \\
ret_X \equiv \frac{E(d\mathcal{V}_X) + \pi_D + \pi_X}{\mathcal{V}_X} &= r + \frac{\gamma\sigma^2 \mathcal{S} + \gamma\rho\sigma\sigma^* Q^* V_X'}{\mathcal{V}_X}
\end{aligned}$$

since \mathcal{S} is linear in Q .

This expression is useful to present some comparative statics results.

1. $F_X = F_I = f_X = f_I = 0$.

If sunk costs are zero, there is no option value. Then the value of foreign sales is just the present discounted value of the profit flow. With zero fixed costs:

$$\begin{aligned}
V_D &= 0 \\
V_X &= \frac{H(\tau aw)^{1-\eta} P^{*\eta} Q^*}{r - \mu^* + \gamma\rho\sigma\sigma^*} \\
V_I &= \frac{H(aw^*)^{1-\eta} P^{*\eta} Q^*}{r - \mu^* + \gamma\rho\sigma\sigma^*}.
\end{aligned}$$

From equation (36):

$$\begin{aligned}
ret_D &= r + \frac{\gamma\sigma^2 \mathcal{S}}{\mathcal{S}} = r + \gamma\sigma^2 \\
ret_X &= r + \frac{\gamma\sigma^2 \mathcal{S} + \gamma\rho\sigma\sigma^* V_X}{\mathcal{S} + V_X} \\
ret_I &= r + \frac{\gamma\sigma^2 \mathcal{S} + \gamma\rho\sigma\sigma^* V_I}{\mathcal{S} + V_I}.
\end{aligned}$$

If (as reasonable values for the parameters suggest) $\gamma\rho\sigma\sigma^* < 1$, then $ret_D >$

ret_X, ret_I . This holds for any value of ρ . Imposing $\rho = 0$ does not affect ret_D but lowers ret_X and ret_I (so with zero trade costs, ret_X and ret_I are increasing in ρ).

By comparing these equations with the general formula (36), one can conclude that:

- (a) the risk premium of domestic firms ($\gamma\sigma^2$) is identical to the one resulting from the simple CCAPM model;
- (b) if $\rho\sigma^*\beta < 1$, ret_D is higher in the case of no fixed costs than in the case with fixed costs;
- (c) ret_X and ret_I are higher in the case of positive fixed costs if the direct effect that fixed costs have on the profit flows is larger than the indirect effect they have on the option values;
- (d) similarly, in the case of positive fixed costs, ret_X and ret_I are higher than ret_D if the direct effect that fixed costs have on the profit flows is larger than the indirect effect they have on the option values.

2. $\rho = 0$.

From equation (36):

$$\begin{aligned} ret_D &= r + \frac{\gamma\sigma^2\mathcal{S}}{\mathcal{V}_D} = r + \frac{\gamma\sigma^2\mathcal{S}}{\mathcal{S} + V_D} \\ ret_X &= r + \frac{\gamma\sigma^2\mathcal{S}}{\mathcal{V}_X} = r + \frac{\gamma\sigma^2\mathcal{S}}{\mathcal{S} + V_X} \\ ret_I &= r + \frac{\gamma\sigma^2\mathcal{S}}{\mathcal{V}_I} = r + \frac{\gamma\sigma^2\mathcal{S}}{\mathcal{S} + V_I}. \end{aligned}$$

If $V_X, V_I > V_D$ (as it must be considering firms' selection), then $ret_D > ret_X, ret_I$.

3. Changes in ρ and γ .

To look at changes in ρ and γ we rewrite equation (36) as follows:

$$\begin{aligned}
ret_D &= r + \frac{\left[\frac{r-\mu}{\gamma\sigma^2 rev_D} + \frac{1}{rev_D}\right]^{-1} + \gamma\rho\sigma\sigma^*\beta BQ^{*\beta}}{\frac{rev_D}{r-\mu-\gamma\sigma^2} + BQ^{*\beta}} \\
ret_X &= r + \frac{\left[\frac{r-\mu}{\gamma\sigma^2 rev_D} + \frac{1}{rev_D}\right]^{-1} + \gamma\rho\sigma\sigma^*[\alpha AQ^{*\alpha} + \beta BQ^{*\beta}] + \left[\frac{r-\mu^*}{\gamma\rho\sigma\sigma^* rev_X} + \frac{1}{rev_X}\right]^{-1}}{\frac{rev_D}{r-\mu-\gamma\sigma^2} + AQ^{*\alpha} + BQ^{*\beta} + \frac{rev_X}{r-\mu-\gamma\rho\sigma\sigma^* - \frac{f_X}{r}}} \\
ret_I &= r + \frac{\left[\frac{r-\mu}{\gamma\sigma^2 rev_D} + \frac{1}{rev_D}\right]^{-1} + \gamma\rho\sigma\sigma^*\alpha AQ^{*\alpha} + \left[\frac{r-\mu^*}{\gamma\rho\sigma\sigma^* rev_I} + \frac{1}{rev_I}\right]^{-1}}{\frac{rev_D}{r-\mu-\gamma\sigma^2} + AQ^{*\alpha} + \frac{rev_I}{r-\mu-\gamma\rho\sigma\sigma^* - \frac{f_I}{r}}}
\end{aligned}$$

When ρ increases, the direct effect on the discount terms is that the numerator increases and the denominator decreases, so returns are increasing in ρ . However, an increase in ρ also affects the option value terms A and B : when ρ increases, the option value of entry (exit) decreases (increases), so B decreases (A increases). For the returns to be increasing in ρ , the direct effect through the discount term must be stronger than the effect on the option value terms.

Similarly, when γ increases, the direct effect on the discount terms is that the numerator increases and the denominator decreases, so returns are increasing in γ . However, an increase in γ also affects the option value terms A and B : when γ increases, the option value of entry (exit) decreases (increases), so B decreases (A increases). For the returns to be increasing in γ , the direct effect through the discount term must be stronger than the effect on the option value terms.

D Long Run Risk Model: Derivation and Proofs

D.1 Derivation of the Consumer's Value Function

The Bellman equation for a consumer with preferences described by (41) is:

$$\mathcal{DJ}(Q, X, t) + f(Q, J) = 0 \tag{D.1}$$

where $\mathcal{D}\mathcal{J}$ is the differential operator applied to J . The Bellman equation can be written as:

$$J_Q Q X + J_X \kappa (\bar{X} - X) + \frac{1}{2} J_{QQ} Q^2 \sigma^2 + \frac{1}{2} J_{XX} \sigma_X^2 + f(Q, J) = 0. \quad (\text{D.2})$$

Notice that we do not include the second mixed derivative because the correlation between Q and X is assumed to be zero.

We guess that the value function that solves (D.1) takes the following form:

$$J(Q, X) = \frac{1}{1 - \gamma} \cdot \exp\{u_0 \log(Q) + u_1 X + u_2\} \quad (\text{D.3})$$

where u_0 , u_1 and u_2 are parameters to be determined.

Plugging the derivatives of J , (41) and (D.3) into (D.2) and collecting common terms:

$$\begin{aligned} u_0 &= 1 - \gamma \\ u_1 &= \frac{1 - \gamma}{\kappa + \vartheta} \\ u_2 &= \frac{1 - \gamma}{\vartheta} \left[\frac{\kappa \bar{X}}{\kappa + \vartheta} - \frac{1}{2} \gamma \sigma^2 + \frac{1}{2} \frac{1 - \gamma}{(\kappa + \vartheta)^2} \sigma_X^2 \right]. \end{aligned}$$

D.2 Derivation of the Stochastic Discount Factor

Following Epstein and Duffie (1992), the stochastic discount factor M is given by:

$$\frac{dM}{M} = \frac{df_Q}{f_Q} + f_J dt. \quad (\text{D.4})$$

From (41):

$$\begin{aligned} f_Q &= \frac{\vartheta(1 - \gamma)J}{Q} \\ f_J &= -\vartheta(u_1 X_t + u_2 + 1). \end{aligned}$$

By applying Ito's lemma to f_Q :

$$\frac{df_Q}{f_Q} = \left[u_1 \kappa \bar{X} - (u_1 \kappa + \gamma) X_t + \frac{\sigma^2}{2} \gamma (1 + \gamma) + \frac{\sigma_X^2}{2} u_1^2 \right] dt - \gamma \sigma dz + \sigma_X u_1 dz_X.$$

Hence the stochastic discount factor can be written as:

$$\frac{dM}{M} = -r_t dt - \gamma \sigma dz + \left(\frac{1-\gamma}{\kappa + \vartheta} \right) \sigma_X dz_X \quad (\text{D.5})$$

where r_t is the risk-free rate:

$$r_t = X_t + \vartheta - \gamma \sigma^2. \quad (\text{D.6})$$

D.3 Derivation of the Firm's Value Function

We start by solving for the value function of a firm that is currently selling only in its domestic market. The firm's problem is unchanged with respect to the baseline model, so the no-arbitrage condition is still given by equation (C.3):

$$\pi_D - r_t \mathcal{S} + E \left(\frac{d\mathcal{S}}{dt} \right) + E \left(\frac{dM}{M} \cdot \frac{d\mathcal{S}}{dt} \right) - r_t V_D + E \left(\frac{dV_D}{dt} \right) + E \left(\frac{dM}{M} \cdot \frac{dV_D}{dt} \right) = 0.$$

Notice that the only difference in this condition compared to the baseline model is that the risk-free rate is now time-dependent.

By following the same methodology as for the baseline model, one can show that a possible solution is given by:

$$\begin{aligned} 0 = & \pi_D - r_t \mathcal{S} + \mathcal{S}_Q X Q + \mathcal{S}_X \kappa (\bar{X} -) + \frac{1}{2} (\mathcal{S}_{QQ} \sigma^2 Q^2 + \mathcal{S}_{XX} \sigma_X^2) + \dots \\ & - \gamma \sigma^2 \mathcal{S}_Q Q + \frac{1-\gamma}{\kappa + \vartheta} \sigma_X^2 \mathcal{S}_X \end{aligned} \quad (\text{D.7})$$

$$\begin{aligned} 0 = & -r_t V^D + V_{Q^*} X^* Q^* + V_{X^*}^D \kappa^* (\bar{X}^* - X^*) + V_X^D \kappa (\bar{X} - X) + \dots \\ & \dots + \frac{1}{2} (V_{QQ}^D \sigma^{*2} Q^{*2} + V_{XX}^D \sigma_X^2 + V_{X^* X^*}^D \sigma_{X^*}^2) + V_{XX^*}^D \rho_X \sigma_X \sigma_{X^*} + \dots \\ & \dots - \gamma \sigma \sigma^* \rho_Q Q^* V_{Q^*} + \frac{1-\gamma}{\kappa + \vartheta} \rho_X \sigma_X \sigma_{X^*} V_{X^*} + \frac{1-\gamma}{\kappa + \vartheta} \sigma_X^2 V_X. \end{aligned} \quad (\text{D.8})$$

D.3.1 Value of Domestic profits

We guess that the solution of (D.7) takes the form:

$$\mathcal{S} = e^{A_s X_t} Q^{\tilde{x}} + C_s Q \quad (\text{D.9})$$

By substituting this expression into (D.7) we find that:

$$\begin{aligned} C_s &= \frac{H(aw)^{1-\eta} P^\eta}{\vartheta} \\ A_s &= \frac{\tilde{\chi} - 1}{\kappa} \end{aligned}$$

while $\tilde{\chi}$ is given by the solution of the quadratic equation:

$$\begin{aligned} \frac{1}{2} \left(\sigma^2 + \frac{\sigma_X^2}{\kappa^2} \right) \tilde{\chi}^2 + \left[\bar{X} - \left(\frac{1}{2} + \gamma \right) \sigma^2 - \frac{\sigma_X^2}{2\kappa^2} + \frac{(1-\gamma)\sigma_X^2}{(\kappa+\vartheta)\kappa} \right] \tilde{\chi} + \dots \\ \left(\gamma\sigma_Q^2 - \vartheta - \bar{X} + \frac{\sigma_X^2}{2\kappa^2} - \frac{(1-\gamma)\sigma_X^2}{(\kappa+\vartheta)\kappa} \right) = 0. \end{aligned}$$

Since the option value of domestic profits is zero:

$$\mathcal{S}(a, Q) = \frac{H(aw)^{1-\eta} P^\eta Q}{\vartheta}. \quad (\text{D.10})$$

D.3.2 Value of Foreign Profits: Domestic Firms

We guess that the solution of (D.8) takes the form:

$$V^D = e^{AX+BX^*} Q^{*\tilde{\xi}} \quad (\text{D.11})$$

By substituting this expression into (D.8) we find that:

$$A = -\frac{1}{\kappa} \quad (\text{D.12})$$

$$B = \frac{\tilde{\xi}}{\kappa^*} \quad (\text{D.13})$$

and $\tilde{\xi}$ is the solution of the following quadratic equation:

$$\begin{aligned} \frac{1}{2} \left(\sigma^{*2} + \frac{\sigma_{X^*}^2}{\kappa^{*2}} \right) \tilde{\xi}^2 + \left(\bar{X}^* - \frac{1}{2} \sigma^{*2} - \frac{\rho_X \sigma_X \sigma_{X^*}}{\kappa \kappa^*} - \gamma \rho \sigma \sigma^* + \frac{(1-\gamma)\rho_X \sigma_X \sigma_{X^*}}{(\kappa+\vartheta)\kappa^*} \right) \tilde{\xi} + \dots \\ \dots \left(\gamma\sigma^2 - \vartheta - \bar{X} + \frac{\sigma_X^2}{2\kappa^2} - \frac{(1-\gamma)\sigma_X^2}{(\kappa+\vartheta)\kappa} \right) = 0. \end{aligned} \quad (\text{D.14})$$

So the general solution of (D.8) is given by:

$$V^D = \exp \left\{ \frac{\tilde{\xi}}{\kappa^*} X^* - \frac{1}{\kappa} X \right\} Q^{*\alpha} \quad (\text{D.15})$$

and the value of foreign profits for a domestic firm is:

$$V^D(a, Q^*, X^*, X) = B_D^l(a) \exp \left\{ \frac{\beta}{\kappa^*} X^* - \frac{1}{\kappa} X \right\} Q^{*\beta} \quad (\text{D.16})$$

where $\beta > 1$ is the positive root of $\tilde{\xi}$ and $B_D^l(a)$ is a positive parameter to be determined.

D.3.3 Value of Foreign Profits: Exporters and Multinational Firms

The analogous of (D.8) for exporters is:

$$\begin{aligned} H(\tau aw)^{1-\eta} P^{*\eta} Q^* - f_X &= -r_t V^D + V_{Q^*} X^* Q^* + V_{X^*}^D \kappa^* (\bar{X}^* - X^*) + V_X^D \kappa (\bar{X} - X) + \dots \\ &\dots + \frac{1}{2} (V_{Q^* Q^*}^D \sigma^{*2} Q^{*2} + V_{X^* X^*}^D \sigma_{X^*}^2 + V_{X^* X^*}^D \rho_X \sigma_X \sigma_{X^*}) + V_{X^* X^*}^D \rho_X \sigma_X \sigma_{X^*} + \dots \\ &\dots - \gamma \sigma \sigma^* \rho Q^* V_{Q^*} + \frac{1-\gamma}{\kappa + \vartheta} \rho_X \sigma_X \sigma_{X^*} V_{X^*} + \frac{1-\gamma}{\kappa + \vartheta} \sigma_X^2 V_X. \end{aligned} \quad (\text{D.17})$$

Equation (D.17) does not admit a closed-form solution. We approximate the solution with the following function, which is an exact solution in the option value term, and an approximation in the profit flow term, correct in the long run (for $X = \bar{X}$, $X^* = \bar{X}^*$).

$$V^X = Q^{*\tilde{\xi}} \exp\{AX + BX^*\} + CQ^* + D \quad (\text{D.18})$$

where $\tilde{\xi}$, A and B are given by (D.14), (D.12), and (D.13), respectively, and:

$$C = \frac{H(\tau aw)^{1-\eta} P^{*\eta}}{\vartheta - \gamma \sigma^2 + \gamma \rho \sigma \sigma^2} \quad (\text{D.19})$$

$$D = -\frac{f_X}{\vartheta - \gamma \sigma^2}. \quad (\text{D.20})$$

Hence the value of foreign profits for an exporter can be approximated as:

$$\begin{aligned}
V^X(a, Q^*, X^*, X) &= A_X^l(a) \exp \left\{ \frac{\alpha}{\kappa^*} X^* - \frac{1}{\kappa} X \right\} Q^{*\alpha} + B_X^l(a) \exp \left\{ \frac{\beta}{\kappa^*} X^* - \frac{1}{\kappa} X \right\} Q^{*\beta} + \dots \\
&\dots \frac{H(\tau aw)^{1-\eta} P^{*\eta} Q^*}{\vartheta - \gamma\sigma^2 + \gamma\rho\sigma\sigma^2} - \frac{f_X}{\vartheta - \gamma\sigma^2}
\end{aligned} \tag{D.21}$$

where $\alpha < 0$ and $\beta > 1$ are the roots of $\tilde{\xi}$, and $A_X^l(a)$, $B_X^l(a)$ are positive parameters to be determined.

Following an identical procedure one can show that the value of foreign profits for a multinational can be approximated as:

$$V^I(a, Q^*, X^*, X) = A_I^l(a) \exp \left\{ \frac{\alpha}{\kappa^*} X^* - \frac{1}{\kappa} X \right\} Q^{*\alpha} + \frac{H(aw^*)^{1-\eta} P^{*\eta} Q^*}{\vartheta - \gamma\sigma^2 + \gamma\rho\sigma\sigma^2} - \frac{f_I}{\vartheta - \gamma\sigma^2} \tag{D.22}$$

where $A_I^l(a)$ is a positive parameter to be determined.

Notice the role of the parameter ρ_X : when $\rho_X = 1$, $X = X^*$ and the exponential term in the option value is constant. In this case dV (and hence expected returns) depends only on the correlation between Q and Q^* like in the baseline model. Alternatively when $\rho_X = 0$, X and X^* are independent and hence also in this case dV depend only on the correlation between Q and Q^* .

D.4 Returns

In this section we show how to derive an expression for the returns (analogous to equation (36)) for the model with long run risk. Starting from Ito's Lemma applied to the value of domestic profits and to the value of foreign profits (WLOG, for an exporter) and taking expected values, we obtain:

$$\begin{aligned}
E[dS] &= XQ\mathcal{S}'_Q + \kappa(\bar{X} - X)\mathcal{S}'_X + \frac{1}{2}(\sigma^2 Q^2 \mathcal{S}''_{QQ} + \sigma_X^2 \mathcal{S}''_{XX}) \\
E[dV] &= X^*Q^*V'_{Q^*} + \kappa^*(\bar{X}^* - X^*)V'_{X^*} + \kappa(\bar{X} - X)V'_X + \dots \\
&\quad + \frac{1}{2}(\sigma^{*2} Q^{*2} V''_{Q^*Q^*} + \sigma_X^{*2} V''_{X^*X^*} + \sigma_X^2 V''_{XX}) + \rho_X \sigma_X \sigma_X^* V''_{XX^*}.
\end{aligned}$$

The no-arbitrage condition derived from the Bellman equation is:

$$\begin{aligned}
\pi_D - r\mathcal{S} + XQ\mathcal{S}'_Q + \kappa(\bar{X} - X)\mathcal{S}'_X + \frac{1}{2}(\sigma^2 Q^2 \mathcal{S}''_{QQ} + \sigma_X^2 \mathcal{S}''_{XX}) - \gamma\sigma^2 Q\mathcal{S}'_Q + \frac{1-\gamma}{\kappa+\vartheta}\sigma_X^2 \mathcal{S}'_X + \dots \\
+ \pi_X - rV + X^*Q^*V'_{Q^*} + \kappa^*(\bar{X}^* - X^*)V'_{X^*} + \kappa(\bar{X} - X)V'_X + \dots \\
+ \frac{1}{2}(\sigma^{*2} Q^{*2} V''_{Q^*Q^*} + \sigma_X^{*2} V''_{X^*X^*} + \sigma_X^2 V''_{XX}) + \dots \\
+ \rho_X \sigma_X \sigma_X^* V''_{XX^*} - \gamma\rho\sigma\sigma^* Q^* V'_{Q^*} + \frac{1-\gamma}{\kappa+\vartheta} (\rho_X \sigma_X \sigma_X^* V'_{X^*} + \sigma_X^2 V'_X) = 0.
\end{aligned}$$

Substituting in the expressions for $E[d\mathcal{S}]$ and $E[dV]$, we can write the no-arbitrage condition as:

$$\begin{aligned}
\pi_D - r\mathcal{S} + E[d\mathcal{S}] - \gamma\sigma^2 Q\mathcal{S}'_Q + \frac{1-\gamma}{\kappa+\vartheta}\sigma_X^2 \mathcal{S}'_X + \pi_X - rV + E[dV] - \gamma\rho\sigma\sigma^* Q^* V'_{Q^*} + \dots \\
+ \frac{1-\gamma}{\kappa+\vartheta} (\rho_X \sigma_X \sigma_X^* V'_{X^*} + \sigma_X^2 V'_X) = 0
\end{aligned}$$

and noticing that $E[d\mathcal{S}] + E[dV] = E[d(\mathcal{S} + V)]$:

$$\begin{aligned}
\pi_D + \pi_X + E[d(\mathcal{S} + V)] &= r(\mathcal{S} + V) + \gamma\sigma^2 Q\mathcal{S}'_Q + \gamma\rho\sigma\sigma^* Q^* V'_{Q^*} + \dots \\
&\quad + \frac{\gamma-1}{\kappa+\vartheta} (\sigma_X^2 \mathcal{S}'_X + \rho_X \sigma_X \sigma_X^* V'_{X^*} + \sigma_X^2 V'_X) \\
\frac{\pi_D + \pi_X + E[d(\mathcal{V})]}{\mathcal{V}} &= r + \frac{\gamma\sigma^2 Q\mathcal{S}'_Q + \gamma\rho\sigma\sigma^* Q^* V'_{Q^*} + \frac{\gamma-1}{\kappa+\vartheta} (\sigma_X^2 \mathcal{S}'_X + \rho_X \sigma_X \sigma_X^* V'_{X^*} + \sigma_X^2 V'_X)}{\mathcal{V}} \\
\frac{\pi_D + \pi_X + E[d(\mathcal{V})]}{\mathcal{V}} &= r + \frac{\gamma\sigma^2 \mathcal{S} + \gamma\rho\sigma\sigma^* Q^* V'_{Q^*} + \frac{\gamma-1}{\kappa+\vartheta} (\rho_X \sigma_X \sigma_X^* V'_{X^*} + \sigma_X^2 V'_X)}{\mathcal{V}}
\end{aligned}$$

since \mathcal{S} is linear in Q and does not depend on X .

E Computation Algorithm

Since the model features aggregate shocks and non-stationary dynamics, we simulate the economy and compute the variables of interest for a large number of simulations. We then average the results across simulations to obtain the model-generated moments. Each simulation proceeds as follows.

1. Define the exogenous parameters of the model (trade and FDI costs, preference parameters, parameters entering the shock processes). We simulate the economy

- for 100 firms and 30 periods.
2. Simulate the shocks $Q(t)$, $Q^*(t)$ by discretizing the Brownian motions in equations (3)-(4) (equations (37)-(38) in the long run risk case).
 3. Draw the productivities of 100 firms in each country from Pareto distributions with parameters (b, k) .
 4. Initialize the firm distribution into statuses at $\Omega_0 = \Omega_0^* = (0, 0)$ (all firms start by selling in their domestic market only). Compute P_0, P_0^* from (26)-(27).
 5. For $t = 1, \dots, 30$:
 - (a) For each firm in each country, solve system (19)-(25) to find the quantity thresholds and the parameters of the value functions. More details about the solution of this system are contained in the next subsection.
 - (b) Interpolate the solution of system (19)-(25) on a finer grid (10000 points) and invert each threshold function $Q_{RS}^*(a)$ to obtain the thresholds productivity levels $x_{RS}(Q^*)$. $x_{RS}(Q^*)$ is the level of productivity that induces a firm to switch from status R to status S when the realization of the shock in the foreign country is Q^* ($R, S, \in \{D, X, I\}$).
 - (c) Establish firm status at the end of the period by comparing the productivities of the simulated set of firms with the productivity thresholds $x_{RS}(Q^*)$. Compute the new distributions of firms into statuses Ω_t, Ω_t^* . Compute P_t, P_t^* from (26)-(27).
 - (d) Compute the profit and the value of each firm using equations (7)-(9) and (15)-(18), respectively.

We repeat this procedure 100 times. The results of the simulations are aggregated as follows.

I. Share of firms in each status.

The distribution Ω_t gives the share of firms in each status for every year and simulation. The moments in the right panel of Table 6 are obtained by averaging the shares across years (starting at $t = 2$ not to bias the results with the initialized degenerate distribution) and across simulations.

II. Persistence and changes of status.

We construct the moments in the left panel of Table 6 by tracking the status of each firm over time. For each simulation and year, we compute the share of firms in each status that i) remain in the same status in the following year, and ii) change status (by status) in the following year. The final moments are obtained by averaging these shares across years and across simulations.

III. Earnings yields.

For each simulation and year, and consistently with the construction of earnings yields in the data that we documented in Section 2, we sum the profits of firms in each group to obtain portfolio profits. Similarly, we sum the values within group to obtain portfolio values. Earnings yields are given by the ratio of portfolio profit and portfolio value, then averaged across years and across simulations.

IV. Returns.

For each simulation and year, we compute changes in the total value of each portfolio (dV). Portfolio returns are constructed as portfolio earnings yields plus percentage changes in the value of the portfolio ($\frac{\pi}{V} + \frac{dV}{V}$), then averaged across years and across simulations.

E.1 Solving the System of Value-Matching and Smooth-Pasting Conditions

System (19)-(25) is a non-linear system of 12 equations in 10 unknowns: the six thresholds inducing a firm to change status, for every status pair, and the four parameters entering the value functions (since we impose $A_D(a) = B_I(a) = 0$). We discipline the numerical solution of this system by a) splitting it in two perfectly identified, smaller systems, and b) choosing a “good” initial condition.

(a) Splitting the system into two perfectly identified subsystems.

To gain intuition about how to “separate” the system in two, consider the following example of two firms, call them y and z . Firm y is less productive than firm z , and following a positive shock to the quantity demanded abroad, it decides to start exporting. With the same realization of the shock, firm z decides to start

FDI. As a result, the entry threshold Q_{DI}^* is irrelevant for firm y , and the entry threshold Q_{DX}^* is irrelevant for firm z . In other words, the fact that firms are heterogeneous and the shock process is continuous imply that there are firms that move directly from domestic sales only to FDI and viceversa, while there are firms that gradually move from domestic sales only to exports and then (eventually) to FDI.

According to this reasoning, we split system (19)-(25) in two subsystems:

$$V_D(a, Q_{DX}^*(a)) = V_X(a, Q_{DX}^*(a)) - F_X \quad (\text{E.1})$$

$$V_X(a, Q_{XD}^*(a)) = V_D(a, Q_{XD}^*(a)) \quad (\text{E.2})$$

$$V_X(a, Q_{XI}^*(a)) = V_I(a, Q_{XI}^*(a)) - F_I \quad (\text{E.3})$$

$$V_I(a, Q_{IX}^*(a)) = V_X(a, Q_{IX}^*(a)) - F_X \quad (\text{E.4})$$

$$V'_D(a, Q_{DX}^*(a)) = V'_X(a, Q_{DX}^*(a)) \quad (\text{E.5})$$

$$V'_X(a, Q_{XD}^*(a)) = V'_D(a, Q_{XD}^*(a)) \quad (\text{E.6})$$

$$V'_X(a, Q_{XI}^*(a)) = V'_I(a, Q_{XI}^*(a)) \quad (\text{E.7})$$

$$V'_I(a, Q_{IX}^*(a)) = V'_X(a, Q_{IX}^*(a)) \quad (\text{E.8})$$

and:

$$V_D(a, Q_{DI}^*(a)) = V_I(a, Q_{DI}^*(a)) - F_I \quad (\text{E.9})$$

$$V_I(a, Q_{ID}^*(a)) = V_D(a, Q_{ID}^*(a)) \quad (\text{E.10})$$

$$V'_D(a, Q_{DI}^*(a)) = V'_I(a, Q_{DI}^*(a)) \quad (\text{E.11})$$

$$V'_I(a, Q_{ID}^*(a)) = V'_D(a, Q_{ID}^*(a)). \quad (\text{E.12})$$

System (E.1)-(E.8) is a system of eight equations in eight unknowns, that can be solved for the four quantity thresholds $Q_{DX}^*(a)$, $Q_{XD}^*(a)$, $Q_{XI}^*(a)$, $Q_{IX}^*(a)$, and the parameters of the value functions $B_D(a)$, $A_X(a)$, $B_X(a)$, and $A_I(a)$. Similarly, system (E.9)-(E.12) is a system of four equations in four unknowns, that can be solved for the two quantity thresholds $Q_{DI}^*(a)$, $Q_{ID}^*(a)$, and the parameters of the value functions $B_D(a)$ and $A_I(a)$. The two systems uniquely identify the six thresholds and the parameters $A_X(a)$, $B_X(a)$. To identify the remaining

parameters $B_D(a)$, $A_I(a)$, we determine if the firm moves directly from domestic sales only to FDI or gradually moves from domestic sales only to exports and then (eventually) to FDI. If $Q_{DX}^*(a) < Q_{DI}^*(a)$, then $B_D(a)$ is identified by system (E.1)-(E.8). Conversely, if $Q_{DX}^*(a) > Q_{DI}^*(a)$, $B_D(a)$ is identified by system (E.9)-(E.12). Similarly, if $Q_{ID}^*(a) > Q_{IX}^*(a)$, then $A_I(a)$ is identified by system (E.9)-(E.12). Conversely, if $Q_{ID}^*(a) < Q_{IX}^*(a)$, $A_I(a)$ is identified by system (E.1)-(E.8).

(b) **Choosing the initial condition.**

It is possible to show analytically (see Dixit (1989)) that the “entry” thresholds $Q_{DX}^*(a)$, $Q_{DI}^*(a)$, $Q_{XI}^*(a)$ are higher than the corresponding thresholds under certainty, while the “exit” thresholds $Q_{XD}^*(a)$, $Q_{ID}^*(a)$, $Q_{IX}^*(a)$ are lower than the corresponding thresholds under certainty. Accordingly, we solve the system for the absolute value of the differences between the equilibrium thresholds and the thresholds under certainty, with a vector of zeros as the initial condition.