

# Capital and Risk: New Evidence on Implications of Large Operational Losses<sup>\*</sup>

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## Abstract

Operational risk is currently receiving significant media attention, as financial scandals have appeared regularly and multiple events have exceeded one billion dollars in total impact. Regulators have also been devoting attention to this risk, and are finalizing proposals that would require banks to hold capital for potential operational losses. This paper uses newly available loss data to model operational risk at internationally active banks. Our results suggest that the amount of capital held for operational risk will often exceed capital held for market risk, and that the largest banks could choose to allocate several billion dollars in capital to operational risk. In addition to capital allocation decisions, our findings should have a direct impact on the compensation and investment models used by large firms, as well as on the optimal allocation of risk management resources.

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## Introduction

Financial institutions have experienced more than 100 operational loss events exceeding \$100 million over the past decade. Examples include the \$691 million rogue trading loss at Allfirst Financial, the \$484 million settlement due to misleading sales practices at Household Finance, and the estimated \$140 million loss stemming from the 9/11 attack at the Bank of New York. Recent settlements related to questionable business practices have further heightened interest in the management of operational risk at financial institutions. However, the absence of reliable internal operational loss data has impeded banks' progress in measuring and managing operational risk. Without such data, most firms have not quantified operational risk. This paper utilizes new databases that catalogue the industry's operational loss experience, and provides economic capital estimates for operational risk. These estimates are consistent with the 2-7 billion dollars in capital some large internationally active banks are allocating for operational risk.<sup>1</sup>

Current efforts to quantify operational risk follow more than a decade of immense change in banks' risk management practices. During this time, new quantification techniques and computational resources have greatly enhanced financial institutions' ability to measure, monitor, and manage risk. The new techniques were first applied to market risk, and most banks are now using some form of value at risk (VaR) model in which high frequency securities data are used to estimate the institution's market risk exposure. While the models used by financial institutions vary, there is substantial consistency in application since most models are based on the same set of market data. Credit risk management has also been greatly enhanced as banks have utilized risk models that calculate probabilities of default, loss given default, and exposure at default for credits.

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<sup>1</sup> In their 2001 Annual Reports, Deutsche Bank and JPMorgan Chase disclosed economic capital of 2.5 billion euros and 6.8 billion dollars for operational risk, respectively. It is our experience that such figures are representative of the amount of capital some other large internationally active banks are holding for operational risk.

Models can vary by financial institution, but tend to show similar qualitative responses to an economic downturn.

Many banks do not yet have an internal database of historical operational loss events. Those databases that exist are populated mostly by high frequency low severity events, and by a few large losses. As a result, relatively little modeling of operational risk has occurred, and banks have tended to allocate operational risk capital via a top down approach. A survey by the Basel committee's Risk Management Group found that on average, banks had allocated approximately 15 percent of their capital for operational risk. Many banks took this top down number and allocated the capital to operations based on scale factors. However, without operational loss data, banks could not verify that the scale factors were in fact correlated with operational losses (i.e., whether an increase in scale resulted in a proportional increase in operational risk).

With the impetus of a new Basel proposal to require capital for operational risk, banks and vendors have begun collecting more reliable data on operational losses. Several vendors have begun collecting data from public sources, and have constructed databases spanning a large cross-section of banks over several decades. The current paper focuses on these "external" loss data. In doing so, we are following the lead of early market and credit risk models, which also assumed that historical data are representative of the risks that large financial institutions face as a result of undertaking their business activities.

Publicly available operational loss data pose unique modeling challenges, the most significant being that not all losses are publicly reported. If the probability that an operational loss is reported increases as the loss amount increases, there will be a disproportionate number of very large losses relative to smaller losses appearing in the external databases. Failure to account for

this issue could result in sample selection bias. This paper addresses the problem of sample selection bias by using an econometric model in which the truncation point for each loss (i.e., the dollar value below which the loss is not reported) is modeled as an unobserved random variable. To model the underlying loss distribution, we rely on a result from extreme value theory which suggests that the logarithm of losses above a high threshold should have an exponential distribution. Using these procedures, we are able to extract the underlying loss distribution from the publicly available data. Our estimates are consistent with the amount of operational risk capital held by several large institutions.

Our main conclusions are as follows. First, while some have questioned the need for explicit capital requirements for operational risk, our estimates indicate that operational losses are an important source of risk for banks. In fact, the capital charge for operational risk will often exceed the charge for market risk.<sup>2</sup> Second, we find that reporting bias in external data is significant, and that properly accounting for this bias significantly reduces the estimated operational risk capital requirement. Third, the distribution of observed losses varies significantly by business line, but it is not clear whether this is driven by cross-business line variation in the underlying loss distribution or by cross-business line variation in the sample selection process. Finally, supplementing internal data with external data on extremely large rare events could significantly improve banks' models of operational risk.

The next section provides an overview of operational risk and its relationship to market and credit risk. The third section provides descriptive statistics for the operational losses reported in

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<sup>2</sup> Hirtle (2003) reports the market risk capital requirement for top U.S. banking organizations (by asset size) as of December 2001. The top three banks reported market risk capital between 1.9 and 2.5 billion dollars. These figures are of the same order of magnitude as the operational risk capital estimates reported in section six. The other 16 banks in Hirtle's sample reported market risk capital between 1 and 370 million dollars. Our results suggest that operational risk capital for most of these banks could significantly exceed this range.

external databases. The fourth section outlines the estimation techniques used to correct for reporting bias and to extract the underlying distribution of operational losses. Section five presents our main results. Section six considers the implications of our findings for capital allocation, and the final section provides some conclusions and implications for risk management.

## **II. Overview of operational risk**

The emergence of large diversified internationally active banks has created a need for more sophisticated risk management. Large institutions are increasingly relying on sophisticated statistical models to evaluate risk and allocate capital across the organization. These models are often major inputs into RAROC models (James (1996)), investment decisions, and the determination of compensation and bonuses. Thus, line managers have a strong incentive to verify the accuracy of the risk and capital attributed to their area.

As risk management becomes more sophisticated, bank supervisors are encouraging banks to develop their risk models, both by directly rating risk management through the supervisory process and by incorporating the models into the regulatory process. Market risk was the first area to migrate risk management practices directly into the regulatory process. The 1996 Market Risk Amendment to the Basel Accord used value at risk (VaR) models to determine market risk capital for large complex banks, and most large financial institutions now use such models to measure and manage their market risk exposures. Duffie and Pan (1997) provide an overview of value at risk modeling techniques, and numerous other papers evaluate the empirical performance of various VaR models (e.g., Hendricks (1996), Pritsker (1997)). However, research concerning how well these models perform in practice has been limited by the proprietary nature of both the models and the underlying trading book data. Berkowitz and O'Brien (2002) were able to obtain value at risk

forecasts employed by commercial banks, but concluded value at risk models were not particularly accurate measures of portfolio risk.

Regulators are now increasing the emphasis given to credit risk modeling. The new Basel proposal is much more risk sensitive, and requires significant detail in calculating the banks' credit risk exposure. Concerns that greater risk sensitivity will also make capital more procyclical have drawn academic attention to this area (Catarineu-Rabell, Jackson, and Tsomocos (2002), Jordan, Peek, and Rosengren (2003)). But while this proposal is intended to capture the best practices at major commercial banks, academic research has again been handicapped by the proprietary nature of the data and models. As a result, most studies of credit risk have examined similar market instruments, which are imperfect proxies for a bank's actual credit risk exposure (Carey (1998)).

Similarly, while analytical work by financial institutions in the area of operational risk has significantly advanced at some financial institutions, academic research has largely ignored the topic because of a lack of data. Not only was such data proprietary, but in addition, no consistent definition of an operational loss was applied within or across financial institutions. As a result, internal and external databases were difficult to develop. However, the Risk Management Group (RMG) of the Basel Committee and industry representatives have recently developed a standardized definition of operational risk. Their definition is, "The risk of loss resulting from inadequate or failed internal processes, people and systems or from external events."

The RMG has also provided eight standardized business lines and seven loss types as a general means of classifying operational events. The eight business lines are: Corporate Finance; Trading and Sales; Retail Banking; Payment and Settlement; Agency Services; Commercial Banking; Asset Management; and Retail Brokerage. The seven loss types are: Internal Fraud; External Fraud; Employment Practices and Workplace Safety; Clients, Products and Business

Practices; Damage to Physical Assets; Business Disruption and System Failure; and Execution, Delivery and Process Management. For additional discussion of operational risk and its treatment under the new Basel proposal, readers can refer to Netter and Poulsen (2003).

Adoption of the Basel standards has helped to enhance the consistency of data collection both within and between banking organizations. Such consistency will make it possible for banks to compare their loss experience across business lines, and for supervisors to compare loss experience across banks. The external data vendors are also applying the same standards, which will enable banks to compare their loss experience with that of their peers, and to apply findings from external data to modeling their own risk exposures.

### **III. Data**

This paper analyzes operational loss data provided by two vendors, OpRisk Analytics and OpVantage.<sup>3</sup> Both vendors gather information on operational losses exceeding \$1 million. These vendors collect data from public sources such as news reports, court filings, and SEC filings. As well as classifying losses by Basel business line and causal type, the databases also include descriptive information concerning each loss. We applied several filters to the raw data to ensure that losses included in our analysis occurred in the financial services sector, and that the reported

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<sup>3</sup> The specific data products we use are OpRisk Analytics' OpRisk Global Data and OpVantage's OpVar database. OpRisk Analytics has recently been acquired by the SAS Institute, Inc. OpVantage, a division of Fitch Risk Management, has recently acquired the IC2 database of operational loss events, and has renamed it OpVantage First. OpVantage is currently merging their two databases, and will offer a single integrated product by the end of 2003.

loss amounts were reliable.<sup>4</sup> In addition, all dollar amounts used in the analysis are expressed in real terms using the 2002 level of the Consumer Price Index.

Panel a of Table I reports descriptive statistics for losses that occurred in the United States. The table reports the distribution of losses across business lines, as well as the size of the loss in each business line at the 50<sup>th</sup>, 75<sup>th</sup>, and 95<sup>th</sup> percentiles. The two databases are remarkably similar at the aggregate level. In fact, the 50<sup>th</sup> percentile loss (\$6 million) and the 75<sup>th</sup> percentile loss (\$17 million) are the same in both databases. Even the 95<sup>th</sup> percentile losses are very close, equaling \$88 million in the OpRisk Analytics database and \$93 million in the OpVantage database.

The two databases are also quite similar at the business line level. The business line with the most observations in both databases is retail banking, which accounts for 38% of all OpRisk losses and 39% of all OpVantage losses. While retail banking has the largest number of losses, these losses tend to be smaller than in other business lines. In contrast, trading and sales has fewer observations, but has the largest loss by business line at the 95<sup>th</sup> percentile. These results suggest that it will be important to capture both frequency and severity in determining appropriate capital by business line.

Panel b of Table I reports descriptive statistics for losses that occurred outside the United States. The most striking result is that non-U.S. losses are significantly larger than U.S. losses. At both the aggregate and business line level, the reported percentiles for the non-U.S. losses are approximately double the equivalent percentiles for U.S. losses. Another result is that the two databases are less similar with respect to non-U.S. losses than with respect to U.S. losses. In retail

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<sup>4</sup> OpRisk Analytics segregates loss events of different types into different data sets. Our analysis focused on the primary data set, which contains events with a finalized loss amount. We also used some events from the supplementary data sets, but excluded all non-monetary operational losses, business risk losses, estimated loss amounts, unsettled litigation, and losses classified in the “Insurance” business line. OpVantage’s website



banking, for example, the 95<sup>th</sup> percentile loss is \$272 million for the OpVantage data but only \$101 million for the OpRisk Analytics data. We conclude that data collection processes may differ for U.S. versus non-U.S. losses, and that the underlying loss distributions may also differ. We thus restrict our attention to losses that occurred in the U.S. This is not much of a concession, as more than two thirds of reported losses occurred in the U.S., and our primary interest at this time is in U.S.-based banks.

Table II provides the same descriptive statistics classified by loss event type. The event types with the most losses are Internal Fraud and Clients, Products, and Business Practices. Those with the fewest losses are Damage to Physical Assets and Business Disruption and System Failures. The infrequency of these loss types may be an accurate reflection of their rarity. However, it could be that these types of losses are rarely disclosed, that loss amounts are not disclosed even if the event is disclosed, or that they are often misclassified under different loss types. It is also worth noting that while the losses at the 95<sup>th</sup> percentile are quite similar across the two databases overall, for particular event types such as internal fraud, the losses at the 95<sup>th</sup> percentile are quite different. Since the databases draw from similar news sources, this discrepancy could reflect the difficulty in classifying losses by event type.

#### **IV. Methodology**

Measuring operational risk from publicly available data poses several challenges, the most significant being that not all losses are publicly reported. One would also expect a positive relationship to exist between the loss amount and the probability that the loss is reported. If this relationship does exist, then the data are not a random sample from the population of all operational

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(www.opvantage.com) states that the raw data exclude “events such as rumors, estimates of lost income, pending litigations and unsettled disputes.”

losses, but instead are a biased sample containing a disproportionate number of very large losses. Standard statistical inferences based on such samples can yield biased parameter estimates. In the present case, the disproportionate number of large losses could lead to an estimate that overstates a bank's exposure to operational risk.

Another way of describing this sampling problem is to say that an operational loss is publicly reported only if it exceeds some unobserved truncation point.<sup>5</sup> Because the truncation point is unobserved, it is a random variable to the econometrician, and the resulting statistical framework is known as a random – or stochastic – truncation model. Techniques for analyzing randomly truncated data are reviewed in Amemiya (1984), Greene (1997), Maddala (1983), and many other sources. This section provides a brief overview of these techniques as they may be applied to operational loss data.

In related work, Baud et al. (2002) propose using a random truncation framework to model operational loss data, and provide initial empirical results suggesting the feasibility of the approach. To our knowledge, however, the current paper is the first to apply such techniques to the new databases of publicly disclosed operational losses, and is also the first to consider the implications of these data for the setting of operational risk capital.

Let  $x$  and  $y$  be random variables whose joint distribution is denoted  $j(x,y)$ . The variable  $x$  is randomly truncated if it is observed only when it exceeds the unobserved truncation point  $y$ . If  $x$  and  $y$  are statistically independent, then the joint density  $j(x,y)$  equals the product of the marginal densities  $f(x)$  and  $g(y)$ . Conditional on  $x$  being observed, this joint density can be written as:

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<sup>5</sup> We use the terms “threshold” and “truncation point” to describe two distinct features of the data collection process. The term “threshold” refers to the \$1 million level below which losses are not reported in the external databases. The threshold is a known constant, and is the same (in nominal terms) for each observation. The term “truncation point” refers to the unobserved, observation-specific random variable that determines whether a loss event is publicly reported and included in the external databases.

$$\begin{aligned}
j(x,y|x > y) &= f(x)g(y) / \text{Prob}(x > y) \\
&= f(x)g(y) / \int_{\mathbb{R}} \int_{-\infty, x} f(x)g(y) dy dx \\
&= f(x)g(y) / \int_{\mathbb{R}} f(x) G(x) dx,
\end{aligned} \tag{1}$$

where  $G(\cdot)$  denotes the cumulative distribution function of  $y$ . Integrating out the unobserved variable  $y$  yields the marginal with respect to  $x$ :

$$f(x | x > y) = f(x) G(x) / \int_{\mathbb{R}} f(x) G(x) dx \tag{2}$$

The above expression denotes the distribution of the observed values of  $x$ , and will form the basis for our estimation techniques.

As discussed previously, our data consist of a series of operational losses exceeding one million dollars in nominal value. Extreme value theory suggests that the distribution of losses exceeding such a high threshold can be approximated by a Generalized Pareto Distribution. To be more precise, let  $X$  denote a vector of operational loss amounts and set  $x = X - u$ , where  $u$  denotes a threshold value. The Pickands-Balkema-de Haan Theorem (page 158 of Embrechts et al. (1997)) implies that the limiting distribution of  $x$  as  $u$  tends to infinity is given by:

$$\text{GPD}_{\xi,b}(x) = \begin{cases} 1 - (1 + \xi x/b)^{-1/\xi} & \xi > 0, \\ 1 - \exp(-x/b) & \xi = 0. \end{cases} \tag{3}$$

Which of the two cases holds depends on the underlying loss distribution. If it belongs to a heavy-tailed class of distributions (e.g., Burr, Cauchy, Loggamma, Pareto), then convergence is to the GPD with  $\xi > 0$ . If it belongs to light-tailed class (e.g., Gamma, Lognormal, Normal, Weibull), then convergence is to the exponential distribution ( $\xi = 0$ ). Furthermore, it can be shown that if the

underlying loss distribution belongs to either the heavy-tailed or the light-tailed class, then the distribution of log losses belongs to the light-tailed class of distributions.<sup>6</sup>

That the exponential distribution has only one parameter makes it attractive for the current application. We thus model the natural logarithm of operational losses, and set  $f(x)$  in equation (2) as:

$$f(x) = \exp(-x/b)/b, \tag{4}$$

where  $x$  denotes the log of the reported loss amount  $X$  minus the log of the \$1 million threshold. The above method for modeling the distribution of large losses is referred to as the “Peaks Over Threshold” approach, and is discussed at greater length in Embrechts et al. (1997).

To model the distribution of the truncation point  $y$ , we assume that whether or not a loss is captured in public disclosures depends on many random factors. These factors include: the location of the company and loss event; the type of loss; the business line involved; whether there are any legal proceedings related to the event; as well as the personal idiosyncrasies of the executives, reporters, and other individuals involved in the disclosure decision. In this case, a central limit argument suggests that  $y$  should be normally distributed. In practice, however, we find that the normality assumption results in frequent non-convergence of the numerical maximum likelihood iterations. Alternatively, we assume that the truncation point has a logistic distribution, so that

$$G(x) = 1 / [1 + \exp(-(x - \tau) / \beta)]. \tag{5}$$

The Logistic distribution closely approximates the Normal distribution, but as noted in Greene (1997) its fatter tails can make it more suitable than the Normal for certain applications. The

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<sup>6</sup> See results 3.3.33 and 3.3.34 from Embrechts et al. (1997). In results not reported, we also estimated the distribution of excess losses (rather than excess log losses) using a GPD distribution (rather than an exponential distribution). The resulting parameter estimates were very close to those reported in Table 3.

logistic distribution seems more suitable for the current application as well, in that convergence issues are quite rare under this assumption. The logistic distribution has two parameters: the location parameter  $\tau$  that indicates the (log) loss amount with a 50% chance of being reported; and a scale parameter  $\beta$  that regulates how quickly the probability of reporting increases (decreases) as the loss amount increases (decreases).

The data consist of  $\{x_i, z_i\}_{i=1 \text{ to } n}$ , where  $x_i$  denotes the natural logarithm of the reported loss amount minus the natural logarithm of the \$1 million threshold value, and  $z_i$  denotes the log of the inflation adjustment factor for the year in which event  $i$  occurred.<sup>7</sup> The likelihood equation is as follows:

$$L(b, \beta, \tau | \mathbf{X}, \mathbf{z}) = \prod_{i=1 \text{ to } n} [f(x_i | b) G(x_i | \beta, \tau) / \int_{z(i), \infty} f(x | b) G(x | \beta, \tau) dx] \quad (6)$$

Likelihood estimation based on (6) underlies the results presented in the following section.

## V. Empirical Results

### A. Modeling losses at the bank-wide level.

The first and simplest model of the loss severity distribution restricts each parameter to be identical across business lines. The results for this model are presented in Panel a of Table III. The estimate for the exponential parameter  $b$  is remarkably consistent across the two databases, with a value of 0.64 for the OpRisk data and 0.66 for the OpVantage data. The parameter  $b$  provides a measure of tail thickness for the loss severity distribution, and the estimated parameter values indicate that the tails of these loss distributions are indeed quite thick. The 99<sup>th</sup> and 99.9<sup>th</sup> percentiles of the estimated loss distribution exceed \$20 million and \$90 million, respectively. However, the 99<sup>th</sup> percentile of the raw data exceeds \$300 million. Thus, our method of correcting

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<sup>7</sup> That is,  $z_i$  equals the log of the CPI for 2002 minus the log of the CPI for the year in which event  $i$  occurred. Because the \$1 million threshold is expressed in nominal terms,  $z_i$  arises as the minimum possible value for  $x$  in the denominator of equation 6.

for reporting bias leads to a dramatic reduction in the estimated probability of large losses. Basing value at risk calculations on the raw severity distribution of the external data could substantially overestimate the capital requirement for operational risk.

The parameter  $\tau$  indicates the size of loss that has a 50 percent chance of being reported. The estimates for OpRisk Analytics and OpVantage are consistent, and imply that an \$86 million loss has a 50% chance of being reported in each database. There are several reasons that a particular loss may not appear in a database. First, the bank where the loss occurred may not have disclosed the loss – either because it did not deem the amount to be material or because the loss involved a confidential legal settlement. Second, the press may have decided that the loss was not noteworthy enough to justify a news article. Third, the external vendor’s data collection process may not have located the news story reporting the loss. Finally, the reported loss amount may have been a rough estimate, in which case the loss would have been discarded in the data filtering process discussed in footnote 4.

The scale parameter  $\beta$  indicates how quickly the reporting probability increases (decreases) as the loss size increases (decreases). Both databases imply  $\beta$  estimates of about 0.8, which means that a \$500 million loss has a 90% probability of being reported, while a \$15 million loss has only a 10% probability of being reported.

To evaluate how well the model fits the observed loss data, we calculate Quantile-Quantile plots for both the OpRisk Analytics and OpVantage databases. These plots, which are reported in Figure 1, compare the predicted quantiles of the fitted loss distributions with the actual quantiles of the empirical loss distributions. Overall, the logit-exponential model fits both data sets quite well, which suggests that log losses follow an exponential distribution. Both plots do show some deterioration in fit towards the tail of the loss distribution. One possible reason is that \$1 million is

too low a threshold for the Peaks Over Threshold approach, so that the GPD approximation does not fully capture the tail behavior of losses. We will revisit the threshold selection issue later in the paper.

### *B. Modeling business line effects.*

Our second model allows the parameters  $b$ ,  $\beta$ , and  $\tau$  to each vary by business line. The results presented in Panel b of Table III suggest cross-business line variation in all three parameters. For example, the  $b$  estimates from the OpVantage data imply that the 99<sup>th</sup> percentile of the underlying loss distribution is \$59 million for Trading and Sales, and \$10 million for Commercial Banking. The  $\beta$  and  $\tau$  estimates from the OpRisk Analytics data imply that the probability of a \$169 million loss being reported is 96 percent if the loss occurs in Retail Banking, but only 50 percent if the loss occurs in Trading and Sales.

In addition, the two databases can imply quite different parameter values for the same business line. Such cross-database variation is not a problem for  $\beta$  and  $\tau$ , as differences in the two databases' sampling techniques could plausibly lead to cross-database variation in these parameters. However, cross-database variation in  $b$  is potentially troubling, as both OpRisk Analytics and OpVantage should be sampling from the same underlying loss distribution. It is thus important to determine whether the cross-business line parameter variation is statistically significant. To do so, we note that model 1 is a restricted version of model 2, and perform a likelihood ratio test in the usual manner.<sup>8</sup> We find that the test statistic exceeds the 1% critical value, which indicates that the cross-business line variation in the observed loss distribution is statistically significant.

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<sup>8</sup> The maximized values of the log likelihood function have been omitted at the data vendors' request.

We now consider whether cross-business line variation in the observed loss distribution derives from variation in the underlying loss distribution or from variation in the distribution of the unobserved truncation variable (or both). To do so, we first estimate model 3, in which  $b$  is held constant across business lines but  $\beta$  and  $\tau$  remain unrestricted. The results are presented in Panel b of Table III. Because model 3 is a restricted version of model 2, we can again use a likelihood ratio test to evaluate the hypothesis that  $b$  is constant across business lines. For both databases, the test statistics are less than the 5% critical value. Thus, we cannot reject the null that  $b$  is constant across business lines. Next, we estimate model 4, a restricted version of model 2 in which  $\beta$  and  $\tau$  are held constant across business lines (but  $b$  is unrestricted). The results are presented in Panel c of Table III. Here also, the likelihood test statistics for both databases are less than the 5% critical value. Thus, we cannot reject the null hypothesis that  $\beta$  and  $\tau$  are constant across business lines.

To summarize, we have found that while there is statistically significant cross-business line variation in the observed loss distribution, we cannot definitively attribute this finding to either variation in the severity parameter  $b$  or variation in the truncation parameters  $\beta$  and  $\tau$ .<sup>9</sup> However, the results for models 3 and 4 do resolve the issue of cross-database parameter variation. Panels b and c of Table III show that the parameter estimates for each of these models are quite consistent across the two external databases, which suggests that the cross-database variation in the results for model 2 may be attributed to estimation error.

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<sup>9</sup> The limited number of observations for most event types precludes a thorough analysis of how operational loss severity varies by event type. In results not reported, we considered the two event types with the most observations (Internal Fraud and Clients, Products and Business Practices), and found statistically significant variation in the observed loss distribution across Basel event types. As was the case with the cross-business line results presented in Table III, we could not definitively attribute this finding to either variation in the severity parameter  $b$  or variation in the truncation parameters  $\beta$  and  $\tau$ .



### *C. Robustness.*

The results presented in Table III were derived using a Peaks Over Threshold methodology, in which losses exceeding a high threshold are assumed to follow a Generalized Pareto Distribution. It is well known that this methodology can be highly sensitive to the choice of threshold. The GPD is only an approximation, and parameter estimates resulting from one threshold could differ substantially from those resulting from another even higher threshold (for which the GPD is a better approximation).

To check the sensitivity of our results to threshold selection, we re-estimate the models using various thresholds between \$2 million and \$10 million. The results for Model 1 are reported in Table IV.<sup>10</sup> Although the point estimates for  $b$  do vary somewhat according to the threshold, this variation does not appear statistically significant. Furthermore, there is no trend in the estimates as the threshold increases. Thus, the results suggest that the parameter estimates based on the original \$1 million threshold are robust with respect to threshold selection.

In section six, we will consider the implications of our findings for the level of capital that large internationally active banks might be expected to hold. To do so, we assume that the loss data underlying our results are representative of the risks to which these institutions are currently exposed. In results not reported, we performed a preliminary validation of these assumptions. The external databases report several measures of bank size (e.g., assets, revenue, number of employees). For each of these measures, we split the sample into banks below the median size and banks above the median size. We found no statistically significant relationship between the size of a bank and the value of the tail thickness parameter  $b$ . We also found no evidence of any significant time trend in  $b$ .

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<sup>10</sup> For the sake of compactness, we present these results only for model 1. The results for models 2 to 4 also suggest that parameter estimates are robust with respect to threshold selection.

We conclude this section with a summary of the more notable empirical results. Overall, the results based on U.S. data indicate that the logit-GPD model provides a good estimate of the severity of the loss data in external databases. In addition, the estimated loss severity is quite similar for the two databases examined. The observed loss distribution does vary by business line, but it is not clear whether this variation is a feature of the underlying losses or of the unobserved truncation variable. Finally, correcting the external data for reporting biases is important, and significantly decreases the estimated loss severity.

## **VI. Simulation results**

This section considers the implications of our findings for regulatory capital at large internationally active banks. The empirical work in the previous sections focuses on the severity of large operational losses. However, capital requirements also depend on how frequently these losses are likely to occur. We base our capital analysis on the simple and commonly-made assumption that the frequency of large losses follows a Poisson distribution. The Poisson assumption implies that the probability of a loss occurring does not depend on the time elapsed since the last loss, so that loss events tend to be evenly spaced over time. Some have suggested that the frequency of large operational losses does vary over time (Danielsson et al. (2001), Embrechts, Kaufmann, and Samorodnitsky (2002)). In this case, the Poisson assumption is not technically correct. However, we believe that time-variation in the loss arrival rate should result in a fatter-tailed aggregate loss distribution, so that our current capital estimates would be conservative. A rigorous investigation of this issue is left to future research.

To calibrate the Poisson parameter, we first turn to the results of the 2002 Loss Data Collection Exercise (LDCE), as publicly reported in a recent paper by the Risk Management Group (2003). Table 6 of that paper reports that the 89 banks participating in the LDCE together reported

712 operational losses exceeding one million Euros. Table 1 suggests that over all loss amounts, five banks accounted for 30% of the losses.<sup>11</sup> If one assumes that all banks share the same loss severity distribution, then the top five banks should have a total of 214 losses exceeding \$1 million, or 43 large losses per bank. It is also worth noting that only a minority of participating institutions supplied fully comprehensive data, so that the actual number of large losses at these five banks could well exceed 43.

The above calculation depends on a homogeneity assumption that may or may not hold in the underlying data. However, our informal discussions with banks confirm that a typical large internationally active bank experiences an average of 50 to 80 losses above \$1 million per year. Smaller banks and banks specializing in less risky business lines could encounter significantly fewer losses in excess of \$1 million, and extremely large banks weighted towards more risky business lines could encounter more large losses. The frequency of large losses may also depend on the control environment at an individual institution. We thus consider a wide range of values for the Poisson parameter  $\lambda$  of between 30 and 100 losses in excess of \$1 million per year.

We assume that the severity of losses exceeding \$1 million follows the log-exponential distribution that was estimated in previous sections, with values for the  $b$  parameter of 0.55, 0.65, and 0.75. These values provide a reasonable range around our estimates based on bank-wide losses.<sup>12</sup> This range also captures the possibility that the severity of large losses at a particular bank could depend on that bank's control environment.

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<sup>11</sup> Banks reported a total of 47,269 losses to the LDCE. One can infer from Table 1 of Risk Management Group (2003) that banks reporting 2,000 or fewer events account for no more than 33,450 of these losses.

<sup>12</sup> The results for model 4 suggest that the loss severity distribution may vary by business line. However, our current focus is on bank-wide risk capital at a large bank with a typical mix of business lines. Thus, we focus on those  $b$  estimates that apply to all business lines (models 1 and 3). Capital estimates for a bank specializing in one or two business lines could differ from those reported in Table 5.

Using the above range of frequency and severity assumptions, we simulate one million years' experience of losses exceeding \$1 million. The results are presented in Table V. Panel a presents the results for the 99.9 percent confidence level used by many banks. The capital for a bank with 60 events a year would range from \$600 million for a  $b$  estimate of 0.55 to \$4.0 billion for a  $b$  estimate of 0.75. For a bank with 80 events a year, the range increases to \$700 million to \$4.9 billion. For a very large bank with a poor control environment, 100 loss events with a  $b$  of 0.75 would result in capital of \$6 billion. Additional capital would be required to cover losses below \$1 million. Panel b reports the effect of increasing the soundness standard to the 99.97 percent confidence level favored by some banks. For the previous example of a large bank with a poor control environment, capital more than doubles to \$14.4 billion.

## **VII. Conclusion**

Despite the many headline-grabbing operational losses that have occurred over the past decade, banks have made limited progress in quantifying their exposure to operational risk. Instead, operational risk management has emphasized qualitative approaches, such as enhancing the firm's control environment and monitoring so-called "key risk" indicators. This qualitative emphasis has been driven in part by the lack of reliable operational loss data, and in part by the belief that the idiosyncrasies of large operational losses would make modeling operational risk exceedingly difficult even with reliable data.

This paper analyzes recently available databases of publicly disclosed operational losses. We find that because large losses are more often disclosed than small losses, these databases have a significant and unavoidable reporting bias. After correcting for this bias, we obtain robust and realistic estimates of operational risk. Our estimates are consistent with the level of capital some large financial institutions are currently allocating for operational risk, falling in the range of \$2-\$7

billion. Furthermore, we find that the operational losses reported by major banks display a surprising degree of statistical regularity, and that large losses are well modeled by the same Pareto-type distribution seen in phenomena as disparate as city sizes, income distributions, and insurance claim amounts.<sup>13</sup> This regularity suggests that while the details and chronologies of the loss events may be idiosyncratic, the loss amounts themselves can be modeled and quantified in a meaningful manner.

Some have suggested that even if it is possible to measure operational risk, the benefits of doing so may be outweighed by costs arising from the necessary investments in personnel and data infrastructure. However, we find that operational risk will often exceed market risk, an area in which banks have already made sizeable investments. As institutions increasingly rely on sophisticated risk models to aid them during strategic and business decision-making processes, it will become increasingly important for them to incorporate appropriately the quantification of operational risk. The failure to do so could seriously distort RAROC models, compensation models, economic capital models, and investment models frequently used by financial institutions.

The extent to which operational risk should inform strategic decisions (e.g., which business lines to grow) ultimately depends on how much this risk can vary within a firm. Our analysis of whether the underlying loss distribution varies across business lines was inconclusive, but it is possible that the cross-business line results will become clearer as the vendors expand and improve their databases. The amount of intra-firm variation in operational risk also depends on the answers to two open questions. First, to what extent does the frequency distribution of large losses vary across business lines? Second, are there dimensions other than business line that might be

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<sup>13</sup> Vilfredo Pareto introduced the Pareto distribution to explain income distribution at the end of the nineteenth century. For discussions of how Pareto-like distributions apply to city sizes and insurance claim amounts, refer to Gabaix (1999) and Embrechts et al. (1997).

associated with intra-firm variation in operational risk? The magnitude of our estimates suggests that if the level of operational risk does vary within a firm, the impact on economic capital could be large. Thus, answering these two questions is an important area for future research.

Another open question is how a bank's risk control environment affects the severity distribution of large operational losses. Because the external databases include losses experienced by a wide range of financial institutions, our severity estimates should apply to the "typical" large internationally active bank. Banks with better than average risk controls may have a thinner-tailed severity distribution, which would reduce the likelihood of very large losses. Similarly, banks with worse than average controls may have an increased exposure to large losses. Thus, a risk manager would need to adjust for the control environment at her institution before applying our results and techniques. Determining how to make such adjustments is another important area for future research.

Although much remains to be done, our findings do have several implications that are of immediate practical relevance. First, our analysis indicates that reporting biases in external data are significant, and can vary by both business line and loss type. Failure to account for these biases will tend to overstate the amount of operational risk that a bank faces, and could also distort the relative riskiness of various business lines. More generally, the analysis indicates that external data can be an important supplement to banks' internal data. While many banks should have adequate data for modeling high frequency low severity operational losses, few will have sufficient internal data to estimate the tail properties of the very largest losses. Our results show that using external data is a feasible solution for understanding the distribution of these large losses.

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Table I

## Descriptive Statistics by Basel Business Line

This table presents descriptive statistics for the loss data provided by OpRisk Analytics and OpVantage. The losses are expressed in 2002 dollars, and have been classified according to the RMG-defined business lines. A "-" indicates that there were too few observations for the quantile to be reported.

*Panel a. Losses that occurred in the U.S.*

Business Line	OpRisk Analytics				OpVantage			
	% of All Losses <sup>a</sup>	Percentiles (\$M)			% of All Losses <sup>a</sup>	Percentiles (\$M)		
		50%	75%	95%		50%	75%	95%
Corporate Finance	6%	6	23	-	4%	8	23	-
Trading & Sales	9%	10	44	334	9%	10	27	265
Retail Banking	38%	5	11	52	39%	5	12	60
Commercial Banking	21%	7	24	104	16%	8	28	123
Payment & Settlement	1%	4	11	-	1%	4	11	-
Agency Services	2%	22	110	-	3%	9	28	-
Asset Management	5%	8	20	-	6%	8	22	165
Retail Brokerage	17%	4	12	57	22%	4	13	67
Total	100%	6	17	88	100%	6	17	93

*Panel b. Losses that occurred outside the U.S.*

Corporate Finance	2%	13	-	-	3%	12	27	-
Trading & Sales	9%	30	125	-	12%	25	66	-
Retail Banking	41%	6	27	101	44%	9	29	272
Commercial Banking	30%	15	42	437	21%	35	91	323
Payment & Settlement	1%	5	-	-	1%	13	-	-
Agency Services	2%	45	-	-	3%	20	77	-
Asset Management	3%	5	47	-	5%	7	23	-
Retail Brokerage	12%	10	42	-	11%	8	34	-
Total	100%	10	36	221	100%	13	46	288

<sup>a</sup> The overall number of observations in each data set has been omitted at the data vendors' request.

Table II  
Descriptive Statistics by Basel Event Type

This table presents descriptive statistics for the loss data provided by OpRisk Analytics and OpVantage. The losses are expressed in 2002 dollars, and have been classified according to the RMG-defined event types. The following abbreviations are used: EPWS denotes Employment Practices and Workplace Safety; CPBP denotes Clients, Products and Business Practices; BDSF denotes Business Disruption and System Failures; and EDPM denotes Execution, Delivery and Process Management. A "-" indicates that there were too few observations for the quantile to be reported.

*Panel a. Losses that occurred in the U.S.*

Event Type	OpRisk Analytics				OpVantage			
	% of All Losses <sup>a</sup>	Percentiles (\$M)			% of All Losses <sup>a</sup>	Percentiles (\$M)		
		50%	75%	95%		50%	75%	95%
Internal Fraud	23.0%	4	10	42	27.0%	6	16	110
External Fraud	16.5%	5	17	93	16.6%	4	12	70
EPWS	3.0%	4	14	-	3.3%	5	11	-
CPBP	55.5%	7	20	95	48.1%	7	20	99
Damage Phys. Assets	0.4%	18	-	-	0.3%	20	-	-
BDSF	0.2%	36	-	-	0.4%	10	-	-
EDPM	1.3%	9	27	-	4.2%	4	11	-
Total	100.0%	6	17	88	100.0%	6	17	93

*Panel b. Losses that occurred outside the U.S.*

Internal Fraud	48.5%	9	35	259	42.9%	15	62	381
External Fraud	15.3%	7	27	-	21.6%	10	28	136
EPWS	0.8%	7	-	-	1.6%	2	7	-
CPBP	32.6%	14	51	374	28.6%	13	51	359
Damage Phys. Assets	0.0%	-	-	-	0.3%	163	-	-
BDSF	0.8%	7	-	-	0.5%	3	-	-
EDPM	1.9%	29	-	-	4.6%	5	19	-
Total	100.0%	10	36	221	100.0%	13	46	288

<sup>a</sup> The overall number of observations in each data set has been omitted at the data vendors' request.

Table III

Estimation results by Basel Business Line - US Losses Only.

Model 1 restricts each parameter to be equal across business lines, Model 2 places no parameter restrictions across business lines, Model 3 restricts  $b$  to be equal across business lines, and Model 4 restricts  $\beta$  and  $\tau$  to be equal across business lines. Parentheses denote standard errors for parameter estimates. Losses in the Agency Services, Asset Management, and Payment and Settlement business lines have been grouped together for estimation purposes. Results for these business lines are reported in the column labeled "AS, AM, PS." The following notation has been used: "Emp. 99th Percentile" denotes the empirical 99th percentile for the relevant business line; "Est. 99th Percentile" denotes the estimated 99th percentile for the relevant business line;  $\exp(\tau)$  is equal to the dollar loss amount that has a 50 percent chance of being reported.

Panel a. Models 1 and 2.

	Model 1	Model 2					
	All BL's	Corporate Finance	Trading & Sales	Retail Banking	Cmcl. Banking	AS, AM, PS	Retail Brok.
OpRisk Analytics Estimates							
Exponential: $b$	0.64 (0.08)	0.72 (0.31)	0.68 (0.20)	0.73 (0.20)	0.52 (0.16)	0.61 (0.63)	0.78 (0.22)
Scale: $\beta$	0.78 (0.10)	0.76 (0.26)	0.76 (0.22)	0.75 (0.09)	0.59 (0.20)	0.66 (0.59)	1.05 (0.30)
Threshold: $\tau$	4.45 (0.36)	3.98 (1.27)	5.13 (0.77)	2.81 (1.23)	4.78 (0.53)	4.67 (2.40)	3.68 (1.41)
Emp. 99th Percentile, \$M	302	278	709	186	283	297	261
Est. 99th Percentile, \$M	19	27	23	28	11	17	36
$\exp(\tau)$ , \$M	86	54	169	17	119	107	40
OpVantage Estimates							
Exponential: $b$	0.66 (0.07)	0.84 (0.25)	0.89 (0.23)	0.62 (0.11)	0.49 (0.13)	0.79 (0.27)	0.58 (0.13)
Scale: $\beta$	0.82 (0.09)	0.60 (0.11)	0.91 (0.17)	0.83 (0.17)	0.54 (0.14)	0.86 (0.21)	0.76 (0.20)
Threshold: $\tau$	4.46 (0.32)	2.64 (0.83)	4.14 (1.54)	4.27 (0.57)	4.82 (0.40)	4.16 (1.33)	4.43 (0.54)
Emp. 99th Percentile, \$M	388	326	591	203	332	400	188
Est. 99th Percentile, \$M	21	48	59	18	10	39	15
$\exp(\tau)$ , \$M	86	14	63	72	124	64	84

Table III (continued)

## Panel b. Model 3.

	All BL's	Corporate Finance	Trading & Sales	Retail Banking	Cmcl. Banking	AS, AM, PS	Retail Brok.
OpRisk Analytics Estimates							
Exponential: $b$	0.62 (0.08)						
Scale: $\beta$		0.67 (0.10)	0.69 (0.10)	0.77 (0.10)	0.71 (0.10)	0.67 (0.09)	0.83 (0.14)
Threshold: $\tau$		4.31 (0.65)	5.29 (0.63)	3.75 (0.48)	4.50 (0.43)	4.64 (0.61)	4.46 (0.82)
$\exp(\tau)$ , \$M		74	198	43	90	104	86
OpVantage Estimates							
Exponential: $b$	0.64 (0.07)						
Scale: $\beta$		0.54 (0.07)	0.71 (0.08)	0.85 (0.11)	0.68 0.08	0.72 (0.08)	0.83 (0.11)
Threshold: $\tau$		3.24 (0.46)	5.23 (0.64)	4.22 (0.47)	4.48 (0.35)	4.86 (0.58)	4.28 (0.49)
$\exp(\tau)$ , \$M		26	187	68	88	129	72
Panel c. Model 4.							
OpRisk Analytics Estimates							
Exponential: $b$		0.68 (0.09)	0.74 (0.11)	0.57 (0.06)	0.65 (0.08)	0.70 (0.10)	0.58 (0.07)
Scale: $\beta$	0.74 (0.09)						
Threshold: $\tau$	4.36 (0.33)						
$\exp(\tau)$ , \$M	78						
OpVantage Estimates							
Exponential: $b$		0.72 (0.08)	0.76 (0.09)	0.59 (0.05)	0.72 (0.08)	0.72 (0.08)	0.60 (0.05)
Scale: $\beta$	0.78 (0.07)						
Threshold: $\tau$	4.31 (0.28)						
$\exp(\tau)$ , \$M	74						

Table IV

Estimation results for various POT threshold values

The following table presents parameter estimates for Model 1 as the threshold  $u$  is increased from \$1 million to \$10 million. For example, a \$10 million threshold indicates that only losses exceeding \$10 million were included in the analysis.

Parentheses denote standard errors.

	Threshold value $u$ .				
	\$1M	\$2M	\$3M	\$5M	\$10M
OpRisk Analytics Estimates					
Exponential: $b$	0.64 (0.08)	0.69 (0.10)	0.66 (0.10)	0.64 (0.11)	0.70 (0.12)
Scale: $\beta$	0.78 (0.10)	0.81 (0.09)	0.81 (0.11)	0.82 (0.13)	0.73 (0.12)
Threshold: $\tau$	4.45 (0.36)	4.10 (0.50)	4.30 (0.56)	4.55 (0.71)	3.86 (0.83)
OpVantage Estimates					
Exponential: $b$	0.66 (0.07)	0.77 (0.08)	0.80 (0.09)	0.70 (0.11)	0.68 (0.18)
Scale: $\beta$	0.82 (0.09)	0.85 (0.06)	0.84 (0.06)	0.89 (0.10)	0.91 (0.12)
Threshold: $\tau$	4.46 (0.32)	3.69 (0.45)	3.46 (0.56)	4.34 (0.81)	4.52 (1.56)

Table V

Percentiles (in \$ Billions) of simulated aggregate loss distributions.

This table reports the 99.9th and 99.97th percentiles from simulated aggregate loss distributions. We assume for these simulations that the severity of operational losses exceeding \$1 million follows a log-exponential distribution with mean  $b$ , and that the frequency of these losses follows a Poisson distribution with mean  $\lambda$ . The percentiles are reported in billions of dollars, and can be interpreted as estimates of economic capital under the aforementioned distributional assumptions.

*Panel a. Capital estimates (in \$Billions) at the 99.9 percent confidence level. <sup>a</sup>*

Exponential parameter, $b$ .	Average Number of Losses Exceeding \$1 Million per Year							
	30	40	50	60	70	80	90	100
0.55	0.4	0.4	0.5	0.6	0.6	0.7	0.7	0.8
0.65	0.9	1.1	1.3	1.4	1.6	1.8	1.9	2.1
0.75	2.4	3.1	3.6	4.0	4.5	4.9	5.3	6.0

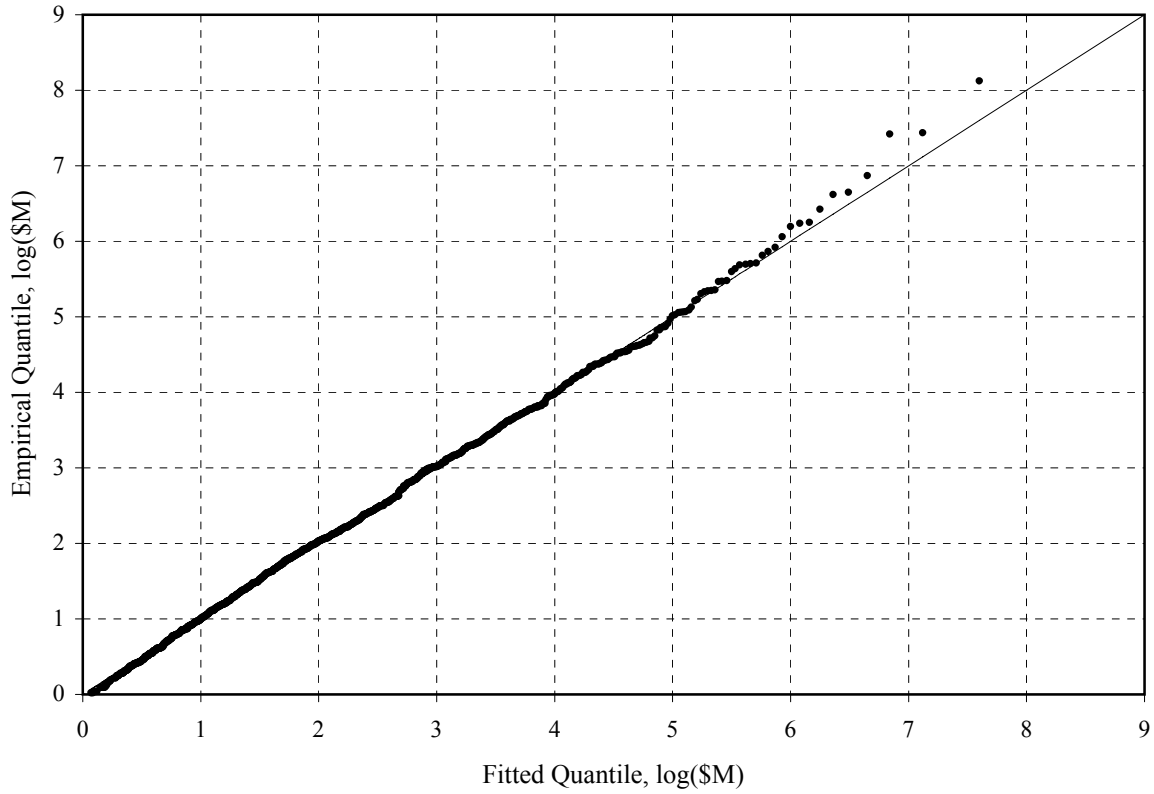
*Panel b. Capital estimates (in \$Billions) at the 99.97 percent confidence level. <sup>a</sup>*

0.55	0.6	0.8	0.8	1.0	1.1	1.2	1.2	1.3
0.65	1.8	2.2	2.5	2.7	3.2	3.5	3.8	4.0
0.75	5.8	7.3	7.7	10.0	10.6	12.0	12.7	14.4

<sup>a</sup> The above figures estimate the capital required to cover operational losses exceeding \$1 Million. Additional capital is required to cover losses below \$1 Million.

Figure 1. Quantile-Quantile plots for Model 1.

Panel a. *OpRisk Analytics data, all business lines, U.S. losses only.*



Panel b. *OpVantage data, all business lines, U.S. losses only.*

