

# Input and Output Inventories in General Equilibrium

Matteo Iacoviello, Fabio Schiantarelli, and Scott Schuh

**Abstract:**

We build and estimate a two-sector (goods and services) dynamic stochastic general equilibrium model with two types of inventories: materials (input) inventories facilitate the production of finished goods, while finished goods (output) inventories yield utility services. The model is estimated using Bayesian methods. The estimated model replicates the volatility and cyclicity of inventory investment and inventory-to-target ratios. Although inventories are an important element of the model's propagation mechanism, shocks to inventory efficiency or management are not an important source of business cycles. When the model is estimated over two subperiods (pre- and post-1984), changes in the volatility of inventory shocks, or in structural parameters associated with inventories play a minor role in reducing the volatility of output.

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Matteo Iacoviello is Professor of Economics at Boston College. His e-mail address is [iacoviel@bc.edu](mailto:iacoviel@bc.edu). Fabio Schiantarelli is Professor of Economics at Boston College. His e-mail address is [schianta@bc.edu](mailto:schianta@bc.edu). Scott Schuh is a Senior Economist and Policy Advisor at the Federal Reserve Bank of Boston. His e-mail address is [scott.schuh@bos.frb.org](mailto:scott.schuh@bos.frb.org).

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## 1. Introduction

Macroeconomists recognize that inventories play an important role in business cycle fluctuations, but constructing macroeconomic models that explain this role successfully has been an elusive task.<sup>1</sup> Early real business cycle (RBC) models, such as Kydland and Prescott (1982), treated inventories as a factor of production. However, Christiano (1988) showed that RBC models with aggregate inventories cannot explain the volatility and procyclicality of inventory investment without including a more complex information structure and restrictions on the timing of agents' decisions. Moreover, Christiano and Fitzgerald (1989) concluded that “the study of aggregate phenomena can safely abstract from inventory speculation.” Nevertheless, the recent empirical literature continues to affirm the conventional view of inventories as propagating business cycle fluctuations. For example, McConnell and Perez-Quiros (2000), among others, argue that structural changes in inventory behavior are an important reason for the decline in the volatility of U.S. GDP since the early 1980s.<sup>2</sup>

We re-examine the role of inventories in business cycle fluctuations by developing and estimating a dynamic stochastic general equilibrium (DSGE) model rich enough to explain essential elements of inventory behavior. To confront the data, the model requires four extensions over existing models with inventories: 1) two sectors, differentiated by whether they hold inventories; 2) a disaggregation of inventories into two theoretically and empirically distinct types, input and output inventories; 3) several modern DSGE features, which have been shown to be necessary to fit the data; and 4) multiple shocks, which provide a diverse array of economically interpretable sources of stochastic variation. Because these extensions increase the complexity of the model, we abstract from other potentially important features — variable markups, nominal rigidities, intermediate goods with input-output relationships, and nonconvexities.<sup>3</sup>

Studying inventories in a general equilibrium framework motivates a natural sectoral decomposition. Because inventories are goods mostly held by the firms that produce goods, our model contains a goods-producing sector that holds inventories and a service-producing sector that does not hold inventories. This inventory-based sector decomposition yields a broader goods sector than in prior

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<sup>1</sup>See Blinder and Maccini (1991) and Ramey and West (1999) for surveys. Ramey (1989) and Humphreys, Maccini, and Schuh (2001) study the importance of inventory investment in the decline of GDP during recessions.

<sup>2</sup>See also Blanchard and Simon (2000); Kahn, McConnell and Perez-Quiros (2002); Kahn and McConnell (2003); Irvine and Schuh (2005a); and Herrera and Pesevento (2005).

<sup>3</sup>Papers that incorporate variable markups and/or sticky prices include Bils and Kahn (2000); Hornstein and Sarte (2001); Boileau and Letendre (2004); Coen-Pirani (2004); Jung and Yun (2005); and Chang, Hornstein, and Sarte (2006). General equilibrium models with intermediate goods and input-output relationships include Huang and Liu (2001) and Wen (2005a). Models with nonconvexities include Fisher and Hornstein (2000) and Khan and Thomas (2007), which incorporate (S,s) policies for retail inventories and intermediate goods inventories respectively.

studies that distinguished goods from services because the model includes industries that distribute goods (wholesale and retail trade plus utilities).<sup>4</sup>

Our model disaggregates inventories into input (materials and work-in-progress) and output (finished goods) stocks, as suggested by the stage-of-fabrication approach employed in Humphreys, Maccini and Schuh (2001).<sup>5</sup> Wen (2005a) offers an alternative, purely theoretical analysis of input and output inventories in general equilibrium; Khan and Thomas (2007) develop a calibrated general equilibrium model with only inventories of intermediate goods, which are held by final goods producers and ordered with (S,s) policies. An advantage of these two studies is that they incorporate the interaction between firms in a supply-chain, which we do not. The advantages of our approach are: 1) we incorporate a utility benefit of holding output inventories; 2) we model the entire economy, including the services sector; and 3) we fit the model to the data.

Our estimated model motivates holdings of the two inventory stocks differently. As in earlier models, input inventories enter as a factor of production, but only in the goods-producing sector. Output inventories, however, pose a different specification challenge. Most of the inventory literature deals with partial-equilibrium analyses of the inventory-holding problem. Typically, a firm is assumed to hold output inventories either to avoid lost sales when stockouts occur (Kahn 1987) or to “facilitate” sales (Bils and Kahn 2000). In a general equilibrium framework, however, one must confront the value of output inventories to consumers more explicitly. Like Kahn and McConnell (2003), we assume that output inventories enter the consumers’ utility function directly; hence the utility services provided by output inventories are a proxy for shopping time, variety, or other consumer benefits associated with the underlying retailing service.

Empirically, the data strongly suggest disaggregating aggregate inventories. We define output inventories ( $F$ ) as stocks held by retailers for final sale; all other stocks are input inventories ( $M$ ). By these definitions, input inventories empirically are more volatile and procyclical than output inventories. Perhaps more importantly, as implied by the model, the ratios of each inventory type to its steady-state target exhibit very different cyclical behavior. Relative to output of goods, input inventories ( $M/Y_g$ ) are very countercyclical. However, we find that relative to the consumption of

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<sup>4</sup>Marquis and Trehan (2005a) define goods producers as manufacturing firms, while Lee and Wolpin (2006) use the broader NIPA definition (agriculture, mining, construction, and manufacturing). For multi-sector models based on consumption and investment, see Kimball (1994); Greenwood, Hercowitz, and Krusell (1997, 2000); Whelan (2003); and Marquis and Trehan (2005b).

<sup>5</sup>The importance of stage-of-fabrication inventories dates back to Lovell (1961) and Feldstein and Auerbach (1976). More recent models include Husted and Kollintzas (1987), Bivin (1988, 1993), Ramey (1989), and Rossana (1990). Cooper and Haltiwanger (1990) and Maccini and Pagan (2007) examine the linkages between firms created through inventories playing different input and output roles in production.

goods, output inventories ( $F/C_g$ ) are essentially acyclical.

Our setup also includes several important features now standard in estimated DSGE models, such as adjustment costs on all capital stocks (including inventories) and variable utilization of capital. We also allow for non-zero inventory depreciation (or, equivalently, an inventory holding cost that is proportional to the total stock). This addition is a relatively novel feature in the inventory literature, except in models of inventories with highly perishable goods (Pindyck 1994). We allow nonzero depreciation because it is theoretically plausible and essential to fit the data. The model incorporates six shocks. It is relatively standard to include two (correlated) sector-specific technology shocks and one demand-type shock to the discount rate. A fourth shock, necessary for the two-sector model, captures shifts in preferences between goods and services. Lastly, we introduce two inventory-specific shocks that create roles for unobserved changes in inventory technologies or preferences to influence the model.

We estimate the model using Bayesian likelihood methods. The estimated model fits the data well. Parameter estimates are consistent with the theory and are relatively precise. The estimated model replicates the volatility and procyclicality of inventory investment, and the qualitative differences in the observed cyclicity of the two inventory-target ratios. In particular, the model captures the countercyclicality of the input inventory ratio and the relatively acyclicity of the output inventory ratio. We also find that inventory shocks do explain some of the variation in investment and consumption, but little of the variation in aggregate output. Altogether, the results are consistent with the conventional view that inventories are an important part of the propagation mechanism, but in and of themselves are not an important source of macroeconomic fluctuations.

Our model and findings are related to Khan and Thomas (2007), who find that a calibrated equilibrium model with fixed delivery costs and driven by a single technology shock is successful in reproducing the cyclical properties exhibited by total inventories. We also match the cyclical properties of inventories, but our model distinguishes inventories by stage of fabrication and does not require fixed costs. The multiplicity of shocks in our model and our inclusion of a services sector (that does not hold inventories) provide additional richness in assessing the role inventories play in business cycle fluctuations, and allows us to better capture their volatility. Our use of Bayesian estimation (rather than calibration) constitutes another important difference.

The econometric results obtained with our model shed light on inventory behavior in general equilibrium. Consistent with Christiano (1988), we find that the elasticity of substitution between input inventories and fixed capital in the production function is much smaller than unity. In contrast,

the elasticity of substitution between consumption and output inventories in the utility function is closer to unity. Adjustment costs on fixed capital are large, while adjustment costs on inventory stocks are small and relatively insignificant. However, estimated depreciation rates for inventories are non-negligible, especially for output inventories, which decay at a rate estimated at about 8 percent per quarter. The magnitude of this depreciation rate and its importance in fitting the data motivate further research on understanding inventory depreciation.

Finally, we provide the first data-consistent, structural decomposition of the Great Moderation using an estimated DSGE model that incorporates independent roles for input and output inventories. By estimating the model over the sub-periods 1960–1983 and 1984–2004, we account for the notable changes in the steady-state values of the inventory-to-target ratios and for the relatively greater importance of the services sector in the U.S. economy since 1984. We find that most of the decline in aggregate output volatility is attributable to the lower volatility of shocks, which occurred primarily in the goods-sector technology shock.<sup>6</sup> The volatility of the input-inventory technology shock also declined, but this decline only accounts for a very small reduction in the volatility of aggregate output or goods output. We also find that structural changes in the parameters account for a small fraction of the reduction in aggregate output volatility. Among these changes, an increase in the cost of adjusting fixed capital (especially in the goods sector) and an increase in the cost of varying capital utilization (especially in the services sector) are the main contributors to structural change. The reduced ratio of input inventories to goods output observed in the data is associated with a decrease in GDP (and goods-sector) output volatility, but the size of the decrease is small.<sup>7</sup>

Section 2 describes the model and relates it to the literature. Section 3 discusses the choices present in applying the model to data. Specifically, we explain how we disaggregate the economy into goods and services sectors and how we disaggregate inventories by stages of fabrication. Section 4 presents the estimation methodology. Section 5 presents the estimation results. Section 6 concludes by summarizing our results and discussing the implications for future research.

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<sup>6</sup>This result is consistent with other aggregate analyses of the Great Moderation. See the VAR-based analyses of Blanchard and Simon (2001); Stock and Watson (2003); and Ahmed, Levin, and Wilson (2004). See also Khan and Thomas (2007) and Maccini and Pagan (2007) for analyses based on structural models with inventories. Arias, Hansen, and Ohanian (2006) use a calibrated RBC model without inventories, and Leduc and Sill (2006) use an equilibrium model to assess the quantitative importance of monetary policy.

<sup>7</sup>Ramey and Vine (2006), studying the automobile industry, also do not find much evidence of structural change related to inventories. They emphasize structural change in the persistence of exogenous sales and, to a much lesser extent, in the costs of adjusting employment.

## 2. The Model

### 2.1. Motivating Inventories

To motivate why input inventories are held, we follow the most of the literature and treat them as a factor of production.<sup>8</sup> This motivation for inventory holding assumes that the stock of input inventories on hand facilitates value-added production beyond the usage of materials and intermediate goods.<sup>9</sup> Although this motive does not explicitly identify the underlying mechanism, maintaining a stock of input inventories may facilitate production by minimizing resource costs of procuring input materials, by guarding against stockouts of input materials that reduce productivity, by allowing batch production, or by generally enabling better organization and efficiency of the production process. As a factor of production, one can think of inventory stocks as a type of capital. Viewed as a factor of production, inventories have associated costs of adjustment (but likely less than those associated with fixed capital), are subject to depreciation, and are likely to incur holding costs. If the costs of holding inventories are proportional to the stock, then the inventory depreciation rate will include physical wastage and the resource cost of holding inventories.

To motivate why output inventories are held, we take an approach parallel to that used for input inventories by entering the stock of output inventories into the utility function directly, as do Kahn and McConnell (2003). Inventories of finished goods that are readily available for consumption are assumed to provide convenience services to the consumer. One way output inventories may yield utility is by economizing on consumers' shopping time. For example, the idea we are trying to capture in the model is that a large stock of wine bottles at a local wine store increases utility beyond that derived from actually consuming the wine. This convenience argument is analogous to the general logic for including money in the utility function, but the argument for including inventories is even more compelling.<sup>10</sup> In other words, entering output inventories in the utility function is analogous to including the stock of consumer durables in the utility function, as this yields a service flow proportional to the stock.<sup>11</sup> However, the nature of the service flow from output inventories is different (yielding convenience, less shopping time, and greater variety) and it pertains

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<sup>8</sup>In addition to the early RBC models, see also Ramey (1989) and Feroli (2002).

<sup>9</sup>Humphreys, Maccini, and Schuh (2001) and Maccini and Pagan (2007) argue that it is important to model the delivery and usage of input materials in gross production together with the holding of input inventories. However, absent input-output (supply-chain) relationships among firms, a representative-firm approach in a general equilibrium model cannot admit deliveries of raw materials produced by an upstream supplier in an internally, model-consistent fashion. Thus, we make the simplifying assumption that the stock of inventories enters the production function.

<sup>10</sup>See, for instance, McCallum and Goodfriend (1987).

<sup>11</sup>See Baxter (1996) and Wen (2005b).

to nondurable goods as well.

Our representative-agent approach to output inventories abstracts from the decentralized problem of inventory holding by retailers (or by final good producers) that is common in partial-equilibrium analyses of inventories. To address this issue properly, one should model explicitly the relationship between individual consumers and retailers (or final good producers) in an imperfectly competitive setting. We leave this important task for future research in the context of a model that also allows for input-output (supply-chain) relationships, which are equally important to the decentralized problem. In this paper, we assume that households maintain community storehouses with output inventories from which they withdraw output inventories in such a way there are no stockout problems. More importantly, we assume that individual utility depends on the stock of privately-held inventories, rather than the aggregate inventory stock, so as to rule out any externalities or free-riding problems that may arise with the aggregate stock.

## 2.2. Preferences

The household chooses consumption of goods  $C_g$ , services  $C_s$ , output inventories  $F$ , and hours in the goods sector  $L_g$  and services sector  $L_s$  to maximize the following objective:

$$E_0 \sum_{t=0}^{\infty} \beta^t \left( \varepsilon_{\beta t} \left( \log \left( \gamma \varepsilon_{\gamma t} X_t^{-\phi} + (1 - \gamma \varepsilon_{\gamma t}) C_{st}^{-\phi} \right)^{-1/\phi} - \tau (L_{gt} + L_{st}) \right) \right),$$

where  $X_t$  is a CES bundle of goods and output inventories, and is defined as

$$X_t = \left( \alpha \varepsilon_{Ft} C_{gt}^{-\mu} + (1 - \alpha \varepsilon_{Ft}) F_{t-1}^{-\mu} \right)^{-1/\mu}, \quad (1)$$

where

$$0 < \gamma < 1, 0 < \alpha < 1 \text{ and } \mu \geq -1.$$

In this formulation,  $1 + \mu$  is the inverse elasticity of substitution between the consumption of final goods and output inventories. Similarly,  $1 + \phi$  is the inverse elasticity of substitution between services and the bundle of goods (consumption-output inventories). Utility is linear in leisure, following Hansen (1985) and Rogerson (1988), which both assume that the economy is populated by a large number of identical households that efficiently allocates individual members either to full-time work in the labor market or not at all.

We allow for three disturbances to impact the intertemporal and intratemporal margins of the

household. The shock  $\varepsilon_{\beta t}$  affects the preference for goods, services, and leisure today versus tomorrow. The preference shock  $\varepsilon_{\gamma t}$  affects the relative preference between goods and services.<sup>12</sup> Finally, the shock  $\varepsilon_{Ft}$  affects the relative preference between the consumption of goods and output inventories: this shock may also capture the reduced-form impact on utility due to changes in output inventory “technology” occurring in the storage of finished goods at retailers. Such technology may include the emergence of megastores like Walmart, Internet shopping, and other key U.S. retail developments, especially since the early 1980s.

### 2.3. Sectoral technologies

Following Christiano (1988), production in the goods sector is modeled as a Cobb-Douglas function in labor  $L_{gt}$ , and a CES aggregate of services from fixed capital and input inventories,

$$Y_{gt} = (A_{gt}L_{gt})^{1-\theta_g} \left( \sigma (z_{gt}K_{gt-1})^{-\nu} + (1 - \sigma) (\varepsilon_{Mt}M_{t-1})^{-\nu} \right)^{-\theta_g/\nu} , \quad (2)$$

where

$$0 < \sigma < 1 \text{ and } \nu \geq -1 .$$

In equation (2),  $K_{gt-1}$  is the end-of-period  $t - 1$  capital in the goods sector (plant, equipment, and structures),  $z_{gt}$  is the time-varying utilization rate of  $K_{gt-1}$ , and  $M_{t-1}$  is the end-of-period  $t - 1$  stock of input inventories. In this formulation,  $1 + \nu$  measures the inverse elasticity of substitution between fixed capital and input inventories. If  $\nu > 0$ , then fixed capital and input inventories will be defined here as complements; if  $-1 \leq \nu \leq 0$  then input inventories and capital are substitutes.

We allow for two disturbances in the goods sector technology:  $A_{gt}$  is a technology shock, while  $\varepsilon_{Mt}$  is a shock that affects the productive efficiency of input inventories, so that  $\varepsilon_{Mt}M_{t-1}$  is input inventories in efficiency units. This shock captures, in a reduced-form way, the impact on production efficiency of changes in input inventory technology. Such technological changes may include new methods of inventory management like just-in-time production, which are characterized by elaborate supply and distribution chains. Irvine and Schuh (2005a) offer evidence that such supply-chain management may have changed in the early 1980s, but the exact mechanisms underlying these new inventory management methods have not been incorporated well in existing macroeconomic models. Information and computing technology may also play an important and related role in these

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<sup>12</sup>For the model to admit a solution, a necessary condition is that  $\gamma\varepsilon_{\gamma t}$  never exceeds unity for each possible realization of  $\varepsilon_{\gamma t}$ . Even though we assume that  $\log \varepsilon_{\gamma t}$  has an unbounded support, empirically its standard deviation turns out to be rather small, so that this condition is always satisfied in practice.

new inventory management techniques, as argued by Kahn, McConnell, and Perez-Quiros (2002). We introduce  $\varepsilon_{Mt}$  as a shorthand term to capture variations in input-inventory management.<sup>13</sup> Obviously, evolution in input-inventory management techniques can also be reflected in the weight of input inventories in the CES aggregate,  $1-\sigma$ , in the parameter governing the elasticity of substitution,  $\nu$ , or, more generally, in the ratio between the stock of input inventories and goods output.

Production in the services sector is modeled by a Cobb-Douglas production function only for labor  $L_{st}$  and capital services:

$$Y_{st} = (A_{st}L_{st})^{1-\theta_s} (z_{st}K_{st-1})^{\theta_s} , \quad (3)$$

where  $K_{st-1}$  is the end-of-period  $t-1$  capital in the service sector, and  $z_{st}$  is the time-varying utilization rate of  $K_{st-1}$ . The empirical fact that service-producing firms do not hold inventories motivates our model's different specification of the services-production technology. We also allow for one general technology disturbance,  $A_{st}$ , in the services sector.

## 2.4. Aggregate Economy

Output from the goods sector provides consumption goods, new fixed investment in both sectors, and investment in output and input inventories. Output from the services sector provides services. Thus, the resource constraints for the goods and service sectors of the economy are, respectively,

$$Y_{gt} = C_{gt} + K_{gt} - (1 - \delta_{Kg}(z_{gt})) K_{gt-1} + K_{st} - (1 - \delta_{Ks}(z_{st})) K_{st-1} + F_t - (1 - \delta_F) F_{t-1} \quad (4)$$

$$+ M_t - (1 - \delta_M) M_{t-1} + \xi_{Kg}(K_{gt}, K_{gt-1}) + \xi_{Ks}(K_{st}, K_{st-1}) + \xi_F(F_t, F_{t-1}) + \xi_M(M_t, M_{t-1})$$

and

$$Y_{st} = C_{st} . \quad (5)$$

The capital depreciation rates in both sectors,  $\delta_{Kg}(z_{gt})$  and  $\delta_{Ks}(z_{st})$ , are increasing functions of the respective utilization rates. The inventory depreciation rates,  $\delta_F$  and  $\delta_M$ , are fixed and possibly capture inventory holding costs as well. We also allow for standard, convex adjustment costs in  $K_{gt}$ ,  $K_{st}$ ,  $F_t$  and  $M_t$ ,  $\xi_{Kg}(K_{gt}, K_{gt-1})$ ,  $\xi_{Ks}(K_{st}, K_{st-1})$ ,  $\xi_F(F_t, F_{t-1})$ , and  $\xi_M(M_t, M_{t-1})$ .

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<sup>13</sup>The information structure in Christiano (1988), as well as in Kahn and McConnell (2003), is more complex. For instance, Christiano (1988) assumes that hours worked and capital-investment decisions are made on the basis of noisy information from the shocks, while inventory and consumption decisions are made after the shock is revealed fully. This difference is not central to the present analysis.

Gross aggregate output (GDP) is a weighted sum of output from the two sectors. The relative size of each sector is determined, on average, by the preference weight  $\gamma$ , which relates the goods bundle to the services component in the utility function.

## 2.5. Optimality Conditions

Because the two welfare theorems apply, we solve the model as a social planner's problem. The first-order conditions for the planner's problem are standard and summarized here:

$$\frac{\partial u_t}{\partial C_{gt}} = \lambda_t \quad (6)$$

$$\frac{\partial u_t}{\partial Y_{st}} = \omega_t \quad (7)$$

$$-\frac{\partial u_t}{\partial L_{gt}} = \lambda_t \frac{\partial Y_{gt}}{\partial L_{gt}} \quad (8)$$

$$-\frac{\partial u_t}{\partial L_{st}} = \omega_t \frac{\partial Y_{st}}{\partial L_{st}} \quad (9)$$

$$\lambda_t \left( 1 + \frac{\partial \xi_{Kst}}{\partial K_{st}} \right) = \beta E_t \left( \lambda_{t+1} \left( 1 - \delta_{Kst+1} - \frac{\partial \xi_{Kst+1}}{\partial K_{st}} \right) + \omega_{t+1} \frac{\partial Y_{st+1}}{\partial K_{st}} \right) \quad (10)$$

$$\lambda_t \left( 1 + \frac{\partial \xi_{Kgt}}{\partial K_{gt}} \right) = \beta E_t \left( \lambda_{t+1} \left( 1 - \delta_{Kgt+1} - \frac{\partial \xi_{Kgt+1}}{\partial K_{gt}} + \frac{\partial Y_{gt+1}}{\partial K_{gt}} \right) \right) \quad (11)$$

$$\lambda_t \left( 1 + \frac{\partial \xi_{Ft}}{\partial F_t} \right) = \beta E_t \left( \frac{\partial u_{t+1}}{\partial F_t} + \lambda_{t+1} \left( 1 - \delta_F - \frac{\partial \xi_{Ft+1}}{\partial F_t} \right) \right) \quad (12)$$

$$\lambda_t \left( 1 + \frac{\partial \xi_{Mt}}{\partial M_t} \right) = \beta E_t \left( \lambda_{t+1} \left( 1 - \delta_M - \frac{\partial \xi_{Mt+1}}{\partial M_t} + \frac{\partial Y_{gt+1}}{\partial M_t} \right) \right) \quad (13)$$

$$\frac{\partial \delta_{Kgt}}{\partial z_{Kgt}} K_{gt-1} = \frac{\partial Y_{gt}}{\partial z_{Kgt}} \quad (14)$$

$$\frac{\partial \delta_{Kst}}{\partial z_{Kst}} K_{st-1} = \frac{\partial Y_{st}}{\partial z_{Kst}}. \quad (15)$$

The first two conditions equate the marginal utility of consumption of goods and services to their respective shadow costs,  $\lambda_t$  and  $\omega_t$ . Equations 8 and 9 are the optimality conditions between the consumption of goods and services versus leisure. Equations 10 to 13 are the intertemporal optimality conditions that govern the choices of  $K_{st}$ ,  $K_{gt}$ ,  $F_t$ , and  $M_t$ . The last two equations set the marginal benefit from capital utilization equal to its marginal cost.

## 2.6. Driving Processes

The model is closed by assumptions about the stochastic behavior of the preference and technology shocks. Shocks  $\varepsilon_{\beta t}$ ,  $\varepsilon_{\gamma t}$ ,  $\varepsilon_{Ft}$ ,  $\varepsilon_{Mt}$ ,  $A_{gt}$ , and  $A_{st}$  follow AR(1) stationary processes in logs:

$$\ln(\varepsilon_{\gamma t}) = \rho_{\gamma} \ln(\varepsilon_{\gamma t-1}) + (1 - \rho_{\gamma}^2)^{1/2} u_{\gamma t} \quad (16)$$

$$\ln(\varepsilon_{\beta t}) = \rho_{\beta} \ln(\varepsilon_{\beta t-1}) + (1 - \rho_{\beta}^2)^{1/2} u_{\beta t} \quad (17)$$

$$\ln(\varepsilon_{Ft}) = \rho_F \ln(\varepsilon_{Ft-1}) + (1 - \rho_F^2)^{1/2} u_{Ft} \quad (18)$$

$$\ln(\varepsilon_{Mt}) = \rho_M \ln(\varepsilon_{Mt-1}) + (1 - \rho_M^2)^{1/2} u_{Mt} \quad (19)$$

$$\ln(A_{gt}) = \rho_g \ln(A_{gt-1}) + (1 - \rho_g^2)^{1/2} u_{gt} \quad (20)$$

$$\ln(A_{st}) = \rho_s \ln(A_{st-1}) + (1 - \rho_s^2)^{1/2} u_{st}. \quad (21)$$

The innovations  $u_{\beta t}$ ,  $u_{\gamma t}$ ,  $u_{Ft}$ ,  $u_{Mt}$ ,  $u_{gt}$ , and  $u_{st}$  are serially uncorrelated with zero means and standard deviations given by  $\sigma_{\beta t}$ ,  $\sigma_{\gamma t}$ ,  $\sigma_{Ft}$ ,  $\sigma_{Mt}$ ,  $\sigma_{gt}$ , and  $\sigma_{st}$ . In addition, we allow for mutual correlation between the two technology innovations,  $u_{gt}$  and  $u_{st}$ .

## 2.7. Functional-form Assumptions

Adjustment costs are quadratic and given by the expression

$$\xi_{\Xi t} = \frac{\psi_{\Xi}}{2\delta_{\Xi}} \left( \frac{\Xi_t - \Xi_{t-1}}{\Xi_{t-1}} \right)^2 \Xi_{t-1} \quad (22)$$

for  $\Xi_t = (K_{gt}, K_{st}, M_t, F_t)$ . In this formulation, because the marginal adjustment cost is zero in steady state, it is straightforward to show that the elasticity of capital (investment) with respect to its shadow price is  $\delta_{\Xi}/\psi_{\Xi}$  ( $1/\psi_{\Xi}$ ). For the utilization function, we choose a parameterization such that the marginal cost of utilization equals the marginal product of capital in steady state.<sup>14</sup> The time  $t$  depreciation rate of  $K_{it}$ , defined as  $\delta_{Kit}$  (with  $i = g, s$ ), is given by

$$\delta_{Kit} = \delta_{Ki} + b_{Ki} \zeta_{Ki} z_{Kit}^2 / 2 + b_{Ki} (1 - \zeta_{Ki}) z_{Kit} + b_{Ki} (\zeta_{Ki} / 2 - 1) . \quad (23)$$

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<sup>14</sup>This way, steady-state depreciation is independent of the curvature of the function. See Christiano (2004).

In this formulation,  $\zeta_{Ki} > 0$  determines the curvature of the capital-utilization function, where  $b_{Ki} = 1/\beta - (1 - \delta_{Ki})$  is a normalization that guarantees that utilization is unity in the steady state.

## 2.8. The Steady State

In the absence of shocks, the model's equations imply that the variables will converge to a constant in the steady state. The first-order conditions for fixed capital in the goods sector and input inventories imply that in the steady state, the capital-to-output ratio in the goods sector,  $k_g = K_g/Y_g$ , and the input-inventories-to-output ratio,  $m = M_g/Y_g$ , can be written, respectively, as

$$k_g = \frac{\theta_g \sigma \beta}{1 - \beta(1 - \delta_{Kg})} \frac{1}{\sigma + (1 - \sigma) \left( \frac{\sigma}{1 - \sigma} \frac{1 - \beta(1 - \delta_M)}{1 - \beta(1 - \delta_{Kg})} \right)^{\frac{\nu}{1 + \nu}}} \quad (24)$$

$$m = \frac{\theta_g \beta (1 - \sigma)}{1 - \beta(1 - \delta_M)} \frac{1}{(1 - \sigma) + \sigma \left( \frac{\sigma}{1 - \sigma} \frac{1 - \beta(1 - \delta_M)}{1 - \beta(1 - \delta_{Kg})} \right)^{-\frac{\nu}{1 + \nu}}} . \quad (25)$$

These conditions state that the capital-to-output ratio and the input-inventory-to-output ratio are increasing in their relative weights in production,  $\sigma$  and  $1 - \sigma$ , respectively. At the same time, the different factor intensities depend on the degree of substitutability. When the ratios in large parentheses are larger than one, which is true empirically because input inventories are much smaller than the capital stock, then capital is decreasing in  $\nu$  and input inventories are increasing in  $\nu$ .

The optimality conditions for goods consumption and output inventories imply that in the steady state:

$$c_g = \left( \frac{\alpha}{1 - \alpha} \frac{1 - \beta(1 - \delta_F)}{\beta} \right)^{\frac{1}{1 + \mu}} f , \quad (26)$$

where  $c_g = C_g/Y_g$  and  $f = F/Y_g$ . The ratio of consumption to output inventories is increasing in  $\alpha$ , while it is decreasing (increasing) in  $\mu$  when the term in parentheses is larger (smaller) than one. Using the linear homogeneity of the CES aggregators and the first-order conditions for  $K_s$ ,  $C_g$ , and  $C_s$ , we derive the following expression for  $k_s$ :

$$k_s = \left( \frac{\lambda}{\omega} \right)^{\frac{-\phi}{1 + \phi}} \left( \frac{1 - \gamma}{\alpha \gamma} \right)^{\frac{1}{1 + \phi}} \frac{\theta_s \beta}{1 - \beta(1 - \delta_{Ks})} \left( \alpha + (1 - \alpha) \left( \frac{c_g}{f} \right)^\mu \right)^{\frac{\mu - \phi}{\mu(1 + \phi)}} c_g. \quad (27)$$

This equation says that capital in the services sector is higher when the relative price of goods in terms of services ( $\lambda/\omega$ ) is low, when the weight to services in utility,  $1 - \gamma$ , is high, or when the production function for services is capital intensive ( $\theta_s$  high). Using the first-order conditions for

labor and the linear homogeneity of the production functions, the relative price of goods is:

$$\frac{\lambda}{\omega} = \left( \frac{(1 - \theta_s) \left( \frac{\theta_s \beta}{1 - \beta(1 - \delta_{K_s})} \right)^{\frac{\theta_s}{1 - \theta_s}}}{(1 - \theta_g) (\sigma k_g^{-\nu} + (1 - \sigma) m_g^{-\nu})^{\frac{-\theta_g}{\nu(1 - \theta_g)}}} \right)^{1 - \theta_s} . \quad (28)$$

Finally, the market-clearing condition for the goods sector is

$$c_g + \delta_F f + \delta_{K_g} k_g + \delta_{K_s} k_s + \delta_M m = 1 . \quad (29)$$

For given parameter values, equations 24 to 29 above can be jointly solved for  $f$ ,  $c_g$ ,  $k_s$ ,  $k_g$ ,  $m$  and  $\lambda/\omega$ . The first-order conditions for  $L_g$ ,  $L_s$ ,  $C_g$ , and  $Y_s$ , together with the production functions, can be solved for  $L_g$ ,  $L_s$ ,  $Y_g$ ,  $Y_s$ ,  $\omega$ , and  $\lambda$ . (Details for all the derivations are given in the technical appendix.) The model's optimality conditions, together with the market-clearing conditions and the laws of motion for the shocks, can be used to obtain a linear approximation around the steady state for the decision rules of the model variables, given the initial conditions and the realizations of the shocks. Given the model's structural parameters, for a given set of values, the solution takes the form of a state-space econometric model that links the behavior of the endogenous variables to a vector of partially unobservable state variables that includes the six autoregressive shocks. In our econometric application, we use observed deviations from the steady states of 1) the output of goods and services, 2) the stock of input inventories and output inventories, 3) the relative price of goods, and 4) total fixed investment to estimate the model's parameters and the properties of the shocks. Before describing the procedure for constructing deviations from the steady state, an important task is to map the model variables into their data counterparts.

### 3. Data

#### 3.1. Sector and Inventory Definitions

To obtain model-consistent sectors, we divide the economy according to the inventory-holding behavior of industries. The goods sector in the model includes all seven industries that hold empirically measured inventories: agriculture, mining, utilities, construction, manufacturing, and wholesale and retail trade. All other private industries that do not hold inventories (as measured by statistical agencies) are classified as services in the model. Table 1 shows our sectoral classification, along with output shares in the year 2000. These model-consistent (inventory-based) sectors are different

from the now-conventional view of the U.S. economy as dominated by the service sector. The model’s goods sector is about twice as large as the service sector (59 percent versus 28 percent), and accounts for about two-thirds of private-sector output. However, because the model-consistent goods sector accounts for a much larger portion of international trade, the sectors become much closer in size when we abstract from net exports in the next subsection.

Table 1 also illustrates how the model-consistent sector definitions differ from standard national income and product account (NIPA) definitions, and why the goods sector is relatively more important in our data. We include the NIPA structures sector (which is essentially the construction industry) and roughly one-quarter of the NIPA services sector (utilities and wholesale and retail trade industries) in the model’s goods sector because these industries hold inventories. Including construction in the model’s goods sector is not a controversial choice because, in the NIPA two-sector classification scheme, the “goods-producing” definition does so. However, moving a sizable portion of the NIPA service sector to the goods sector warrants some justification. We include utilities and the trade industries as part of the goods sector because the “services” these industries provide involve making finished goods available to the consumer. We view this distribution of goods as reasonably classified as part of the overall production process of a representative goods-producing firm. Yet we recognize that separate treatment of the production and distribution of goods would be preferable in future research that incorporates stages of processing in the goods sector.<sup>15</sup>

We divide all NIPA inventories into input ( $M$ ) and output ( $F$ ) stocks, following the stage-of-fabrication perspective used by Humphreys, Maccini, and Schuh (2001) for manufacturing stocks. Table 2 shows the inventory definitions by industry and type, along with inventory shares in year 2000. The industries are ordered approximately according to their stage of processing, with the industries producing raw materials listed first and the industries distributing finished goods listed last.<sup>16</sup> We assume that, to a first approximation, most of the output (sales plus output inventories) of each industry becomes an input to the next industry situated along this supply and distribution chain. Prior research has focused mainly on the manufacturing sector, and thus generally has not grappled with the task of classifying total inventories in general equilibrium. However we defined output inventories as retail inventories because these stocks comprise the most finished set of goods in the supply and distribution chain. Under this empirical definition of the model’s inventories, output

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<sup>15</sup>The reclassification of utilities as goods production is consistent with already including other energy production in the goods sector. Petroleum refining is included in manufacturing, as part of the standard NIPA goods sector.

<sup>16</sup>According to the U.S. Census of Construction, inventories in the construction industry are materials and do not include unsold finished buildings.

(retail) inventories account for about one-fourth of all stocks while input (the remaining) inventories account for about three-fourths. By comparison to earlier research focusing on the manufacturing sector only, the analogous definitions of input and output inventories leaves finished goods (output) inventories accounting for about one-third (11.1 percent out of 31.1 percent) of all manufacturing stocks. Also, our model’s definition of input inventories is heavily oriented toward work-in-process (54 percent), whereas work-in-process inventories account for less than one-third of all manufacturing stocks (8.9 percent out of 31.1 percent).<sup>17</sup>

### 3.2. Data Construction

Based on the sector and inventory definitions, we use standard NIPA data and identities to construct the following model-consistent data for use in our econometric work:<sup>18</sup>

$$Y_g = C_g + I_g + I_s + \Delta F + \Delta M$$

$$Y_s = C_s$$

$$Y = Y_g + Y_s.$$

Because our model’s inventory-based sector definitions differ from the NIPA definitions, the consumption, investment, and inventory data require three types of adjustments to obtain model-consistent variables. First, consumption of energy services is partly consumption of goods output produced by the utilities industry because – even though the industry provides a service by distributing energy to consumers – the energy itself is essentially a good (gas, oil, electricity, and the like). Therefore, we redefine this output as goods consumption. Second, we use non-NIPA investment-by-industry data to obtain measures of investment in each sector. Finally, we splice inventory data from two industrial classification schemes – the old SIC system and the newer NAICS system – to obtain consistent time-series data for our sample. See the technical appendix for full details of the

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<sup>17</sup>The definition of stage-of-fabrication inventory stocks in general equilibrium is not well guided by theory or data. One could make a reasonable theoretical case for classifying manufacturing finished goods and wholesale stocks as output inventories too. However, no strong empirical justification exists for any particular alternative definition. For instance, wholesales inventories include construction material supplies, and manufacturing-output inventories contain goods that do not enter the consumer’s utility function. Moreover, each industry’s inventory investment exhibits different cyclical and trend characteristics, and the correlation of inventory investment between industries is low.

<sup>18</sup>For simplicity, we suppress the details of chain-weighting from equations included in the text. When constructing the actual real chain-weighted data, we use the Tornquist index that weights growth rates of subcategories by their nominal shares, as recommended by Whelan (2002). The Tornquist index is a good approximation to the BEA’s Fisher ideal chain index.

data adjustments and construction.<sup>19</sup>

### 3.3. Output and Investment

Figure 1 depicts the model-consistent output and investment data by sector. The left column contains output shares ( $Y_g/Y$  and  $Y_s/Y$ ) and the right column contains investment-to-output ratios ( $I_g/Y$  and  $I_s/Y$ ), plotted both in nominal and in real terms. Clearly, nominal output has been shifting away from the goods sector toward services, though the respective shares have stabilized in recent years. In contrast, real output shares have been relatively stable over the full sample, with each sector accounting for roughly half of real aggregate output, though the portion attributed to goods is slightly larger (roughly 50–55 percent). In nominal terms, gross investment relative to aggregate income has been rising in the services sector and falling in the goods sector. In real terms, investment-to-income in the goods sector is slightly decreasing but roughly stable, though in the services sector investment-to-income is clearly rising.

These nominal and real output trends differ from those previously noted in the two-sector literature. Sectoral decompositions based on consumption versus investment (for examples, see Greenwood, Hercowitz, and Krusell 1997, 2000; Marquis and Trehan 2005b), or based on durable goods versus nondurable goods and services (for example, Whelan 2003), tend to find that nominal shares are roughly stationary. In contrast to our results, these earlier studies find that real shares exhibit nonstationary trends because technological change is faster in the investment and durable goods sectors. Marquis and Trehan (2005a) also find nonstationary trends in real shares for the manufacturing sector only. However, in our model the inventory-holding goods sector contains durable goods, nondurable goods, and several services industries. Because these industries exhibit heterogeneous trends, combining them in our model produces substantially different aggregate real and nominal sectoral trends.

Although these trend differentials are important, we abstract from them for at least two reasons. First, our focus is on the relationship between inventory investment and business cycle fluctuations – hence on deviations from these trends. Second, to address the issue of differences in the rates of sector-specific technical change, it may well be necessary to divide the inventory-holding component

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<sup>19</sup>Despite these adjustments, our efforts to construct private-sector output for a model that excludes the foreign and government sector yield an imperfect approximation. Excluding international trade causes exports to be excluded from sector output, but imports are implicitly attributed to domestic sector output. One does not want to omit imports of capital goods, however, because investment goods purchased from abroad and installed domestically should be included in the capital accumulation equations. Similarly, some inventory holdings probably are associated with international trade. Finally, government spending probably has at least an indirect influence on actual consumption and investment.

of the economy into more than just two sectors. Following the common procedure in the inventory literature, we detrend all data used in the econometric work.<sup>20</sup>

Regarding business cycle properties, goods output is much more variable than services output. Output fluctuations in the inventory-based goods sector account for 76 percent of the variance of real output. By comparison, the growth rate of goods output in the narrower, more volatile three-sector NIPA definition (which includes only the agriculture, mining, and manufacturing industries) accounts for 89 percent of the variance of real GDP growth (Irvine and Schuh 2005b).

### 3.4. Input and Output Inventories

Figure 2 plots data on input and output inventories that portray the central inventory facts we wish to explain with our model. The left panels of Figure 2 plot actual data while the panels in the right column plot detrended data; all data are given in real chain-weighted terms (year-2000 dollars).

The upper left panel of Figure 2 shows that the actual inventory-target ratios, defined by  $F/C_g$  and  $M/Y_g$ , exhibit opposing trends.<sup>21</sup> Input inventories have been declining relative to their target ( $M/Y_g$ ), with a notably faster rate of decline since the early 1980s. This reduction in input inventories probably resulted at least partly from improved inventory management techniques and better information flows. Because this trend break occurs at about the same time that aggregate output volatility declined, a connection between these two events is a natural hypothesis to investigate, which we do later by allowing steady-state parameters to change between sample periods.

In contrast, output (retail) inventories have been rising relative to their target ( $F/C_g$ ). Little attention has been devoted to explaining this phenomenon and its implications for the aggregate economy. However, by separating inventories into input and output components, we highlight the need to understand the economic factors behind the trend increase in output inventories. Also, the output inventory-target ratio leveled off in the 1990s, much later than the break for the input

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<sup>20</sup>A trend is removed from the variables in logs, using the band-pass filter of Baxter and King (1999) that isolates frequencies between 3 and 32 quarters. Linear quadratic detrending and first-differencing are also common in the literature, but these techniques tend to yield similar cyclical properties in the detrended data. However, Wen (2005c) shows that the cyclical properties of detrended inventory investment are sensitive to the cyclical frequency. Business cycle frequencies like ours yield procyclical inventory investment, whereas higher frequencies (2–3 quarters) yield countercyclical inventory investment. After detrending, the data are restored to natural units by adding the detrended log residual to the mean of the log data, then taking the anti-log of this sum. Working with natural units is important for ratios, which are central to steady-state calculations. All detrended data used in estimation are divided by steady-state (average) goods output, but this scaling is not crucial.

<sup>21</sup>Although  $M/Y$  is consistent with traditional practice in the inventory literature, such as Lovell (1961) and Feldstein and Auerbach (1976),  $F/C$  differs from the traditional inventory-to-sales ratio specified by microeconomic models of the firm. In the two-sector general equilibrium model, the “sales” measure most analogous to that used in the inventory literature is final goods sales,  $S_g = C_g + I$ . Empirically, however, the choice of the scale variable for inventories does not alter the qualitative properties of inventory-target ratios.

inventory-target ratio, a fact that also warrants further investigation.

Distinctly different trends in inventory-target ratios provide one motivation for disaggregation of inventories in general equilibrium. Secular trends in these ratios have been analyzed by Kahn and McConnell (2002, 2003, and 2005), Ramey and Vine (2004), and Irvine (2005), but this research has not produced a consensus explanation for the kinds of low-frequency movements seen in Figure 2. Because our primary focus is on the role of inventories in business cycle fluctuations, we abstract from the secular trends in inventory-target ratios by detrending the data.

The key fact to emphasize regarding inventory-target ratios is the substantial differences in their cyclical properties (right panels, Figure 2). On average, the output-inventory ratio is roughly acyclical (the correlation with goods output is .10), as can be seen by the lack of consistent movement during recessions (shaded regions). Although the output-inventory ratio shot up during the 1973–1975 recession, it did not do so during previous or subsequent recessions. In contrast, the input-inventory ratio is very countercyclical (the correlation with goods output is  $-0.89$ ), as can be seen by its consistent increase during recessions. Thus, the existence of countercyclical inventory-target ratios for manufacturing output inventories, as emphasized by Bils and Kahn (2000), is not evident for all inventories. This result suggests that successful theories of aggregate inventory behavior must be comprehensive enough to explain heterogenous behavior among different types of stocks.

Another key fact seen in Figure 2 (lower left panel) is that input-inventory investment is nearly four times more variable than output-inventory investment (the ratio of variances is 3.8), when both investment series are scaled by goods output. This relative volatility is comparable to the analogous variance ratio observed within manufacturing (Blinder and Maccini 1991). However, the relative volatility of the two types of inventory investment has declined dramatically, from a ratio of 4.6 in the early sample (1960–1983) to a ratio of 2.5 since then. The volatility of input-inventory investment fell while the volatility of output-inventory investment remained about constant. Both types of inventory investment are procyclical over the full sample period, but input-inventory investment is more procyclical than output-inventory investment (the correlation with goods output is .64 for input inventories and .42 for output inventories). Their cross-correlation is only .26. The procyclicality of output-inventory investment decreased from .47 in the early sample (1960–1983) to .28 since then, but the cyclical correlation of input-inventory investment has remained relatively stable.

In sum, the distinctly different cyclical properties of input- and output-inventory investment provide additional motivation for disaggregating inventories. Thus, theoretical models that allow different inventory-target adjustments and volatility across stocks are likely to have an advantage in

explaining and understanding aggregate inventory behavior.

## 4. Model Estimation

### 4.1. Overview

We use observations on the following variables: (1) output from the goods sector; (2) output from the service sector; (3) the stock of input inventories; (4) the stock of output inventories; (5) the relative price of goods to services; and (6) total fixed investment. We estimate the model for the full sample from 1960:1–2004:4. We also estimate the model for the two sub-periods: 1960:1–1983:4 and 1984:1–2004:4. The breakpoint corresponds to point estimates of when the Great Moderation began, as indicated in McConnell and Perez-Quiros (2000). We plot our series in Figure 3.

We use Bayesian techniques to estimate the structural parameters.<sup>22</sup> For given values of the parameters, the solution to our linearized model takes the form of a state-space econometric model, and the Kalman filter enables to evaluate the likelihood of the observable variables as follows:

$$L\left(\{x_t\}_{t=1}^T \mid \Upsilon\right) ,$$

where  $\Upsilon$  is the vector collecting all the model parameters and  $x_t$  is the vector of observable variables. We combine the information observed in the data with prior information on the model parameters to construct the posterior density function:

$$p\left(\Upsilon \mid \{x_t\}_{t=1}^T\right) \propto L\left(\{x_t\}_{t=1}^T \mid \Upsilon\right) \Pi(\Upsilon) .$$

Specifically, we first calculate the posterior mode of the parameters using a numerical optimization procedure. Then we generate 250,000 draws from the posterior mode using the Metropolis-Hastings algorithm to obtain the posterior distribution. The mean of the posterior distribution is used to compute impulse response functions, variance decompositions, and moments of the estimated model.

### 4.2. Prior Distributions

We keep some parameters fixed during our estimation exercise. More specifically, we set the quarterly discount factor at 0.99, implying an annual interest rate of 4 percent. We also calibrate the depreciation rates for fixed capital, which we set at  $\delta_{Kg} = \delta_{Ks} = 0.02$  – a conventional choice in the

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<sup>22</sup>For the solution and the estimation of the model, we use the Dynare toolkit developed by Michel Juillard.

literature.<sup>23</sup> Once these values are set, 29 remaining parameters need to be estimated. We partition these into three groups:

1. The autocorrelation parameters  $(\rho_g, \rho_s, \rho_B, \rho_\gamma, \rho_F, \rho_M)$ , standard deviations of the innovation disturbances  $(\sigma_g, \sigma_s, \sigma_B, \sigma_\gamma, \sigma_F, \sigma_M)$ , and the correlation coefficient between the innovations in the goods-sector technology and the services-sector technology  $(\sigma_{g,s})$ .
2. The adjustment cost parameters  $(\psi_{Kg}, \psi_{Ks}, \psi_F, \text{ and } \psi_M)$ , and the parameters characterizing the curvature of the utilization functions for fixed capital  $(\zeta_{Kg}, \zeta_{Ks})$ .
3. The inventory depreciation rates  $(\delta_M \text{ and } \delta_F)$ , the elasticities of substitution  $(\nu, \phi, \mu)$ , the labor shares  $(\theta_g, \theta_s)$ , the weight of services in utility  $(\gamma)$ , the weight of input inventories in the CES capital aggregator  $(\sigma)$ , and the weight  $(\alpha)$  on consumption in the goods-bundle aggregator. This third group of parameters affects not only the model's dynamics, but also the steady-state values of fixed capital and input- and output-inventory stocks relative to output, as well as the relative size of the services versus the goods sector. For our sample (and for the two sub-samples), the average values of these ratios  $(F/Y_g, M/Y_g, K_g/Y_g, K_s/Y_g, (\omega/\lambda)Y_s/Y_g)$  are reported in Table 3. Observe that, for each combination of  $\delta_M, \delta_F, \nu, \phi, \mu$ , it is possible to determine a unique set of values for  $\theta_g, \theta_s, \gamma, \sigma$ , and  $\alpha$  that are consistent with these five ratios listed in Table 3 (see the technical appendix for details).<sup>24</sup> Accordingly, in the estimation of the model, for each value of the  $\delta_M, \delta_F, \nu, \phi$  and  $\mu$  parameters, we set the  $\theta_g, \theta_s, \gamma, \sigma$ , and  $\alpha$  parameters to the values that match the ratios.<sup>25</sup> Intuitively, we let the likelihood function use information on the behavior around the steady state of our observable variables to determine values for the depreciation rates,  $\delta_F$  and  $\delta_M$ , and the elasticity of substitution in the CES aggregates in the production and utility functions,  $\nu, \phi$ , and  $\mu$ . Doing this does not compromise the model's ability to be consistent with the first moments of the data. This procedure also allows us to account for the changes in the ratios over the entire sample period. In fact, when we estimate the model separately over the dates 1960:1–1983:4 and 1984:1–2004:4, we use the average values of the relevant ratios in each period.

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<sup>23</sup>In the data, the service sector has a higher proportion of structures in its total capital stock than the goods sector does. Because structures generally have lower depreciation rates than equipment, we also estimated a model with a smaller depreciation rate of capital in the service sector for robustness. The econometric results from the model with heterogeneous sectoral depreciation rates of capital are very similar the results from our baseline model with homogeneous depreciation rates reported below.

<sup>24</sup>The formulas for these parameters are in the technical appendix.

<sup>25</sup>Christiano (1988) follows the same strategy: in his model, which includes inventories in the production function, he chooses  $\sigma$  (in our notation) to maximize the likelihood function and  $\nu$  (our notation) to match the steady-state rental rate of inventories in the data.

Our prior distributions of the parameters are summarized in the first three columns of Table 4. For the parameters measuring adjustment costs  $\psi$ , we specify a beta prior over  $\frac{\psi}{1+\psi}$ , with a mean equal to 0.5 and a large standard deviation: this value corresponds to a prior mean of unity for the elasticity of investment to its shadow price. For the curvature of the utilization function, we choose a beta prior over  $\frac{\zeta}{1+\zeta}$  with the mean equal to 0.5. For the elasticity of substitution between services and the goods bundle, between consumption and inventories, and between input inventories and capital, we select priors centered around two-thirds (that is,  $1 + \mu = 1.5$ ). In other words, our prior goes slightly in favor of complementarity between these variables.

The existing literature and the national income and product accounts (NIPA) offer little guidance in choosing the inventory depreciation rates,  $\delta_F$  and  $\delta_M$ . An assumption in line with the procedures used in the NIPA would be that inventories do not depreciate. Yet, in addition to incurring holding costs, inventories are subject to various forms of “shrinkage,” such as breakage, wear and tear, perishability, and obsolescence, so the depreciation parameter may well be larger than the rate set for fixed capital. For instance, on a quarterly basis, Ramey (1989) reports inventory holding and storage costs of 4 percent per quarter, while Khan and Thomas (2007) set these costs at 3 percent per quarter. To limit the chances of obtaining an implausibly high estimate, we choose a conservative, tight prior mean for the depreciation rates equal to 0.02 per quarter.

The autoregressive coefficients of the exogenous shocks have beta prior distributions, as in Smets and Wouters (2003), centered at 0.75. The unconditional standard deviations of the shocks are assigned a diffuse inverse gamma distribution prior, which guarantees positive variance with a large domain. The correlation between  $u_{gt}$  and  $u_{st}$  is assumed to be normal and is centered around 0.50. The choices of the mean of the prior distribution for the standard deviation of the technology and preference shocks are in the ballpark of the previous findings in the literature.<sup>26</sup> Preliminary estimation attempts also suggested a higher standard deviation for the input-inventory shock.

## 5. Estimation Results

### 5.1. Full Sample

**Parameter estimates.** We begin by discussing the estimates over the full sample, 1960–2004. Table 4 reports the mean and the 5th and 95th percentiles of the posterior distribution of the

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<sup>26</sup>See, for instance, Ireland (2004) and Smets and Wouters (2003).

parameters obtained through the Metropolis-Hastings algorithm.<sup>27</sup>

Consistent with the autoregressive priors, all shocks are estimated to be quite persistent, with the autoregressive parameters ranging from .82 to .94. The unconditional standard deviation of the shocks ranges from .0032 (for the output-inventory shock) to .0944 (for the input-inventory shock): the quantitative relevance of each shock will be discussed below in the variance-decomposition exercise. Perhaps unsurprisingly, we find that the standard deviation of the technology shock in the goods sector (.0175) is higher than for that found in the services sector (.0142).

The elasticity of substitution between  $M$  and  $K$  (the inverse of  $1 + \nu$ ) equals 0.30. The elasticity of substitution between  $F$  and  $C_g$  (the inverse of  $1 + \mu$ ) equals 0.94, and it is not significantly different from unity. Similarly, the elasticity of substitution between services and the CES aggregator for consumption of goods and output inventories (the inverse of  $1 + \phi$ ) is close to one.

The estimates of the inventory adjustment-cost parameters,  $\psi_F$  and  $\psi_M$ , are close to zero, indicating very small costs of adjusting input and output inventories, while the bigger values of  $\psi_{Kg}$  and  $\psi_{Ks}$  indicate larger adjustment costs for fixed capital. At the posterior mean, the estimated values imply an elasticity to the user cost of investment equal to 1.12 in the goods sector, and equal to 2.96 in the service sector.<sup>28</sup> These different elasticities confirm that input inventories and fixed capital are indeed distinguished by having different degrees of adjustment costs.

Another important difference between inventories and fixed capital emerges from the estimates of depreciation rates for input and output inventories. The depreciation rate for  $M$  is 2.0 percent, about the same as capital, but the depreciation rate for  $F$  is 7.8 percent, much larger. Finally, estimates of the convexity of the utilization function suggest that the marginal cost of capital utilization (in terms of increased depreciation) is more sensitive to changes in the utilization rate in the goods sector than in the services sector. As a result, variable capital utilization is more important in the services sector than in the goods sector.

As we mentioned in the previous section, we do not directly estimate the parameters measuring the relative shares of consumption in utility and of capital, and inventories in production. Rather, given the estimated parameters, we calculate the values of  $\sigma$ ,  $\alpha$ ,  $\theta_g$ ,  $\theta_s$ , and  $\gamma$  (see Table 5) that are consistent with the sample means of  $F/Y_g$ ,  $M/Y_g$ ,  $K_g/Y_g$ ,  $K_s/Y_g$ , and  $(\omega/\lambda)Y_s/Y_g$  (see Table 3).

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<sup>27</sup>As is well known (see, for instance, Canova 2007), an important issue concerns the convergence of the simulated draws from the posterior distribution of the parameters. We fine tune our estimation algorithm in order to obtain acceptance rates between 30 and 40 percent, and we check for convergence using the cumulative sum of the draws statistics. Although convergence typically obtains within 50,000 iterations, we set the number of draws to 250,000 and calculate the statistics of our estimated model based on the last 75 percent of the draws.

<sup>28</sup>One can interpret  $\psi$  as the inverse elasticity of each type of investment to its shadow price. Our numbers are in line with microeconomic findings based on estimates of investment equations: see, for instance, Chirinko (1993).

**Impulse responses and variance decompositions.** Figure 4 presents the model impulse responses to the estimated shocks. In Table 6, we report asymptotic variance decompositions. Both in Figure 4 and in Table 6, we choose an orthogonalization scheme that orders the goods technology before the services technology shock. As a result, any variation in the responses due to the correlation between the goods and the services shock is attributed to the goods technology disturbance.

The first row plots the responses to a positive technology shock of one standard deviation.<sup>29</sup> This shock leads to an increase in  $A_g$  of .87 percent, and to an increase in  $A_s$  (given our ordering scheme) of .37 percent. This disturbance is fundamental in generating comovement of quantities in our model, and accounts for a large fraction of the fluctuations in economic activity. In response to the positive technology shock, consumption, business investment, and both types of inventory investment all rise. The goods shock spills over to the services sector (over and above the effect caused by the correlation of the shocks) because it facilitates the production of fixed capital that is then used in the service sector. The goods technology shock also accounts for a non-negligible fraction of the fluctuations in both types of inventory investment – around 12 percent for output inventories and 16 percent for input inventories. These responses of output and input inventory investment are, as a proportion of the respective stocks, larger than the one for fixed investment, relative to the fixed capital stock. For instance, the impact response of input inventories relative to business investment is half as big, when both variables are scaled by goods output. However, since the steady state stock of business capital is about ten times larger than the stock of input inventories, the response of input inventories is five times larger than that of business investment, when both variables are scaled by their *own* steady state stock. This is not surprising, since fixed capital is more costly to adjust. In this sense, inventories are an important part of the propagation mechanism, even if inventory investment counts for a small fraction of steady state output.

The second row shows the responses to a discount factor shock: this shock moves consumption and investment in opposite directions, and creates negative comovement between each sector’s output. It also contributes to fluctuations of input inventories – 16 percent of the total variance.

The third row shows responses to a shock that shifts preferences away from final inventories towards goods consumption. The mechanics of this disturbance have the classic implications of a demand shock. Consumption of goods increases; inventories of finished goods fall. Following the increase in demand, with a modest lag, the output of the goods sector increases while output of the

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<sup>29</sup>To facilitate comparison across all investment categories, we scale the response of inventory investment and business investment by steady-state goods output (rather than by their own steady state values). This way, the vertical axis measures the percent growth contribution of each investment category to the response of goods output.

services sector is only marginally affected (because the estimated elasticity of substitution implies an approximate separability in utility between goods and services). This preference-based shock accounts for a large share (about 80 percent) of the fluctuations in output-inventory investment.

The fourth row shows the response to a shock that shifts preferences away from services and towards goods. While this shock, which basically reflects shifts in the composition of demand, accounts only for a small fraction of GDP fluctuations, it accounts for a quarter of the variance of output in the services sector. It also accounts for about half of the total variance of sectoral hours, because the shock causes a reallocation of labor from one sector to the other (something not reported in Figure 4 or Table 6).

The fifth row plots the response to a positive shock to the efficiency of input inventories. This shock captures a large fraction (about 67 percent) of the variation in input-inventory investment. More efficient management of input inventories reduces their usage, increases the demand for fixed capital, and raises consumption (immediately) and output (with a slight delay). The positive input-inventory shock accounts for 6 percent of the variance of investment in fixed capital, and for about 2.5 percent of the variance of goods output. The rest of the variance of input inventories is explained by the general preference shock and by the general technology shock that occurred in the goods sector.

The last row plots responses to a positive technology shock in the services sector. While it is obviously important in explaining output of services, the effects of the shock in this sector are only marginally transmitted to the rest of the economy because the services sector does not produce capital.<sup>30</sup>

The literature has often looked at the cyclical properties of the inventory-target ratios, so Figure 5 reports the impulse responses of total output (model-based GDP) and the inventory-target ratios to the three disturbances — a goods technology shock, an output-inventory shock, and an input-inventory shock — that cause most of the variation in GDP and inventories. Following the goods technology shock, the input-inventory target ratio is strongly countercyclical, as is observed in the data. Input inventory investment rises but, because business capital is costly to adjust, the stock of input inventories — which is complementary to business capital — does not rise substantially, so that its ratio to GDP falls. The output inventory-target ratio is virtually acyclical (as in the data), since

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<sup>30</sup>The logic of this result can be interpreted using an analogy to the consumption-technology neutrality result described in Kimball (1994). With separable preferences over goods and services (as implied by our estimated model), technology shocks that only affect the consumption-producing sector (in our model, the service sector) also have no impact on employment or capital accumulation.

the household prefers to maintain a relatively constant balance of output inventories to consumption. The second and third row of Figure 5 show the aggregate responses to the inventory-specific shocks. While these shocks are central to reproducing the volatility of inventory investment observed in the data, these shocks mostly affect the inventory-target ratios through their effects on the numerators, and do not have large effects on output or consumption. In other words, inventory-specific shocks help fit the volatility of inventory investment, but they do not influence the cyclical properties of the inventory-target ratios, which are mostly driven by the aggregate productivity shocks.

**A comparison between the model and the data.** Figure 6 offers a visual check of the model’s ability to reproduce key features of the data. We compare the empirical impulse responses and the model responses, which were obtained from the model’s reduced form by ordering and orthogonalizing the model shocks, as was done in the VAR. In the first two columns of Table 9, by contrast, we focus on some unconditional correlations in the data, and compare these with those generated by our estimated model.<sup>31</sup>

The message that emerges both from Figure 6 and Table 9 is that our model accounts reasonably well for the variance of the key model variables, as well as for how these variables respond to various shocks and for many of the correlations among the variables in the data. In particular, the model simultaneously accounts for the volatility and procyclicality of inventory investment.<sup>32</sup> More specifically, it successfully mimics the greater volatility of input-inventory investment and its higher degree of procyclicality as compared to output-inventory investment. This result is true whether we look at the correlation between inventory investment and goods output, or the connection between changes in inventory investment and the change in GDP. Moreover, the model can reproduce the countercyclicality of the input-inventory target ratio, although not its magnitude, and the relative acyclicality of the output-inventory target ratio. Finally, the model successfully reproduces the relative volatilities of all types of investment.

To better gain insights into how our model achieves these results, it is useful to think of a reference model that treats all types of capital symmetrically with respect to adjustment costs, and assumes a zero depreciation rate on inventories. Recall that Christiano’s (1988) RBC model, which

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<sup>31</sup>The impulse responses are based on a 6-variable VAR with a constant and two lags and are based on the ordering shown in Figure 6.

<sup>32</sup>In Christiano (1988), it was necessary to rely on a more complex information structure in order to account for these two features of the data. He assumes that, at the time labor hours and capital decisions are made, firms observe the shocks with noise. Inventory and consumption decisions are, instead, made with full knowledge of the shocks. When there is no signal-extraction problem, his model can generate enough inventory-investment variability, but at the cost of a negative correlation between the change in inventory investment and output growth.

contains aggregate inventories as a factor of production, has no adjustment costs and assumes a zero depreciation rate for aggregate inventories. When we impose these two assumptions, our model’s ability to explain the volatility of inventories and fixed investment worsens considerably. Figure 7 demonstrates this result by plotting impulse responses to a positive technology shock in the goods sector from two restricted versions of our model – one with zero inventory depreciation and the other with no adjustment costs for  $K$ ,  $M$ , and  $F$ . With estimated (non-zero) depreciation rates but zero adjustment costs (starred lines), the responses of GDP and capital investment are much larger, and the response of output-inventory investment much smaller, than the respective responses from the unrestricted model (that, recall, match their empirical counterparts). With estimated (non-zero) adjustment costs but zero inventory depreciation rates (circled lines), the responses of GDP and capital investment are essentially the same as in the unrestricted model, but the responses of both types of inventory investment are too small – inventory investment is essentially flat in this counterfactual simulation.

This counterfactual exercise shows that positive depreciation rates and heterogeneous adjustment costs (large for fixed capital, small for inventories) are essential features of the model to fit the relative volatilities of all forms of investment. Absent depreciation, inventory investment would be excessively smooth and persistent. Due to standard consumption smoothing reasons, output inventories would be smooth too. Input inventories would not be very volatile because capital, in the absence of adjustment costs, would respond more quickly to general productivity shocks, since these shocks have a larger effect on the marginal return to fixed capital. This response occurs because a productivity shock has the same effect, percentage-wise, on the marginal return to fixed capital and inventories. When the depreciation rate on inventories is much smaller (equal to zero), and fixed capital must be compensated for the higher depreciation rate with a higher return, the absolute effect of a shock on the marginal return to capital is much greater in absolute value. As a result, capital would be more responsive to productivity shocks than would input inventories if inventory depreciation were zero.

## 5.2. Sub-samples

**Parameter estimates.** We re-estimated the model with the same priors over the sub-periods 1960:1–1983:4 and 1984:1–2004:4. In this exercise, we allow  $\sigma$ ,  $\alpha$ ,  $\theta_g$ ,  $\theta_s$ , and  $\gamma$  to differ across sub-samples to match the different sample means for the share of services in the economy and for the investment and the inventory ratios relative to goods output (reported in Table 3). This exercise allows us to investigate what lies at the root of the decline in output volatility since 1984, and what

role, if any, inventories may have played in this regard.

Table 7 reports the results of the sub-sample estimation. With few exceptions, the full-sample parameter estimates lie between those for the two sub-samples. Regarding the *structure* of the economy, three results are worth emphasizing. First, the depreciation rate for output inventories,  $F$ , is smaller in the second part of the sample, as it goes from 6 to 4 percent. Second, the utilization function for capital in the service sector becomes more convex ( $\zeta_{K_s}$  rises). Third, fixed capital becomes more costly to adjust in the latter sub-period.

It is difficult to provide exhaustive explanations for the changes in these “deep” parameters of the model. Potential reasons for the lower estimate of the depreciation rate  $\delta_F$  might be a change in the inventory mix or better inventory management in general. It is not clear how to interpret the higher adjustment costs for fixed capital, although these might reflect: 1) the increased weight of innovative investment in the second sub-period, and the greater associated costs in terms of learning and disruption; or 2) higher sector (or firm) specificity of capital goods.

We also find important changes in the parameters characterizing the stochastic processes for technology and preferences. The most important result is that the standard deviation of the technology shock in the goods sector and of the input-inventory shock experience the largest fall. The decrease in the importance of the input-inventory shock is consistent with the idea that new methods of inventory management adopted since the early 1980s have made it easier to control the level of input inventories in efficiency units. The reduced volatility of the technology shock in the goods sector is consistent with the idea that the decline in GDP volatility is due to a change in the nature of the shocks, particularly those affecting technology (Stock and Watson 2003). We also find that the correlation between the technology shocks in the two sectors decreases substantially between the two sub-periods, from 75 percent to 51 percent. Interestingly, however, unlike for technology (supply) shocks, we find that the variances of innovations coming from the preferences (demand) side have all increased across the sub-periods. This result is true for the output-inventory shock, for the discount-factor shock, and for the shock to preferences for goods versus services.

**Correlations and variance decompositions.** The last four columns of Table 9 show that, across the two sub-periods, the model can reproduce the volatility decline in most macroeconomic aggregates. There are no sizeable changes in the volatility of model’s input or output inventory investment variables, a result that is generally consistent with the data. The model accurately reproduces the reduced procyclicality of output inventory investment after 1983.

Table 8 shows how, in the second sub-period, inventory movements depend more on their own innovations (in relative terms). As for the other variables, a larger chunk of the volatility in economic activity (as summarized by the GDP measure) appears to be due to demand-preference shocks: in the second part of the sample, the share of GDP variance that can be accounted for by non-technological shocks rises from 13 percent to 31 percent. In the goods sector, technology shocks play a smaller role in accounting for the variance of input- and output-inventory investment. Finally, the fraction of variance in fixed investment that is explained by the input-inventory shock declines from 12 percent to 7 percent in the second sub-period.

### 5.3. The Role of Inventories in the Great Moderation

Prompted by the preceding results, a natural question is to what extent the reduced volatility of economic activity in the post-1984 sample is due to a reduction in the volatility of the shocks — often called the “good luck” hypothesis — or to a change in the economy’s structure?

We begin by observing that our estimated model successfully reproduces the reduction in volatility across the two sub-periods. In our data, the standard deviation of detrended GDP drops by .82 percentage points between the 1960–1983 and 1984–2004 sub-periods (from 2.16 percentage points to 1.35). As shown by the last four columns of Table 9, our sub-sample estimates match the volatility decline almost perfectly, showing a reduction in the standard deviation of GDP of .86 percentage points. With this in mind, we ask what features of the model contribute to the reduction in volatility? To answer this question, we partition the elements that can independently affect the implied volatility of the model variables into the following three sets:

1. Parameters that are determined by using the steady state of the model. Recall that when we estimate the model across sub-samples, we choose values of  $\alpha$ ,  $\gamma$ ,  $\sigma$ ,  $\theta_g$ , and  $\theta_s$  for each sub-sample that exactly match the average values of the ratios of input inventories to output and capital investment to output, plus the share of services in GDP.
2. Parameters that measure the unconditional standard deviations of the shocks.
3. Parameters that affect the dynamics of the model around the steady state, and which are estimated without using information on the steady-state ratios. This parameter set includes the autocorrelation parameters of the shocks, the inventory depreciation rates, the elasticities of substitution, the adjustment costs, and the parameters associated with capital utilization.

Table 10 breaks down how the three sets of parameters above contribute to the reduction in volatility captured by the model. Using the estimates obtained from the 1960–1983 sample as a reference point, we change one estimated parameter at a time, setting it to the value estimated for the 1984–2004 sample. This way, we can measure each parameter’s contribution to the change in volatility.<sup>33</sup> The main result is that most of the reduction in GDP volatility is attributable to the reduction in the volatility of the underlying shocks — especially of the technology shock in the goods sector. By themselves, smaller shocks can explain a reduction in GDP volatility of .70 percentage points (as measured by the standard deviation), compared to an estimated total decline of .82 percentage points. Most of the remainder (.12 percentage points) is attributable to larger capital-adjustment costs and capacity-utilization costs, as well as the increased importance (share) of services in the U.S. economy. Interestingly, while the rise of the service-based economy explains a non-negligible part of the reduced volatility, the net effect of changes in the steady-state parameters is smaller. In particular, the larger ratio of aggregate capital to goods output offsets the effects of the larger share of the services sector; this result occurs because a larger investment share renders output inherently more volatile.

What about the role inventories may have played in the Great Moderation? The smaller volatility of input-inventory shocks accounts for about .01 or .02 percentage points of the total reduction in volatility, depending on whether we look at GDP or just output of the goods sector. The reduced ratio of input inventories to goods output accounts for approximately .03 percentage points (.08) of the decrease in the volatility of GDP (output of the goods sector).

To summarize, our estimated model suggests that reductions in the volatility of the model’s structural innovations account for most of the reduction in GDP volatility — a result generally consistent with the “good luck” hypothesis. Structural changes in the model’s parameters have contributed to the reduction in GDP volatility by a smaller amount, working primarily through parameter changes that reduced the volatility of fixed investment. We are unable to find a significant role for inventory investment in the Great Moderation — neither reduced volatility of either inventory shock, nor changes structural parameters associated with inventories, appear to have played much of a role in the decline in GDP.<sup>34</sup>

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<sup>33</sup>Of course, because the model decision rules are nonlinear in the model structural parameters, it is possible that this exercise does not capture appropriately the actual contribution of a variable to the reduction in volatility.

<sup>34</sup>This conclusion is consistent with Khan and Thomas (2007), who consider how aggregate volatility changes in a general equilibrium model following a decrease in fixed ordering costs.

## 6. Conclusions

The most important lesson of this paper is that an estimated DSGE model can incorporate inventories and fit the data reasonably well with plausible and interesting estimates of structural parameters that help characterize the role of input and output inventories. Each type of inventory investment plays a logically different role in the model and exhibits different degrees of volatility and procyclicality. The model can replicate the observed volatility and cyclicity of both input and output inventory investment, and particularly the fact that input-inventory investment is more volatile and procyclical than output-inventory investment. Moreover, the model also can reproduce the countercyclicality of the input-inventory target ratio, and the relative acyclicity of the output-inventory target ratio. This finding represents a step forward relative to previous attempts to model inventories in DSGE models, especially given our model's ability to fit the data. Thus, our model provides a new, more expansive, and data-consistent framework for analyzing the cyclical properties of inventories.

When estimated across two sub-periods, 1960–1983 and 1984–2004, the model captures the volatility reduction observed in aggregate variables, as well as the decline in procyclicality of output-inventory investment. However, the model suggests that the bulk of the Great Moderation is explained primarily by a reduction in the volatility of the technology shock in the goods sector. The reduction in the volatility of inventory shocks accounts for only a small portion of the decrease in output volatility. Nevertheless, the model's framework identifies several dimensions along which the economy's structure changed in an economically important manner, and contributed to the reduction in GDP volatility. Some of these structural changes are related to inventory behavior and influence the propagation role inventories play in the macroeconomy, but, at best, they have only played a minor role in accounting for the reduced volatility of output.

These conclusions are based on an estimated two-sector general equilibrium model that includes novel features such as the distinction between the goods-producing and the services-producing sectors according to their inventory-holding behavior, and the distinction between input and output inventories. Non-zero inventory depreciation, which in the model provides an incentive to adjust inventories more in response to shocks, is another novel feature that is empirically important.

Despite the additional complexity, our model precludes an examination of certain aspects of inventory behavior that may be important to understanding business cycle fluctuations. First, we eschewed a richer examination of the stage-of-fabrication structure within the goods sector. For example, simplifying inventories into only two types abstracts from the supply and distribution chains that now pervade the actual input-output structure of the goods sector and probably play a

vital role in the propagation of shocks. A second issue is that the model is silent on how markup variations and nominal features matter for inventory behavior and business cycles. Some inventory research examines how markup variation or interest rate policies influence inventory behavior.<sup>35</sup> However, this work with nominal rigidities generally has not incorporated the stage-of-fabrication inventory distinction in a general equilibrium setting that we have advanced here. Third, we have sidestepped the micro-founded motivation for firms' holding of finished goods (output inventories). By focusing on the value of output inventories to households through utility and concentrating on the social planner's solution, we have not taken up a more detailed examination of the determinants of a firm's decision to hold output inventories in a market environment. We plan to address these issues in future work, and we hope that others will too.

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<sup>35</sup>See footnote 3 for detailed references on this issue and on supply and distribution chains.

## Appendix

### A: Modeling Details

#### The Model Equations

Below, we summarize the model equations:

$$Y_{gt} = C_{gt} + K_{gt} - \left(1 - \delta_{K_t^g}\right) K_{-1}^g + K_{st} - \left(1 - \delta_{K_t^s}\right) K_{-1}^s + F_t - \left(1 - \delta_F\right) F_{-1} + M_t - \left(1 - \delta_M\right) M_{-1} + AC \quad (1)$$

$$\varepsilon_{\beta t} \left(1 - \gamma \varepsilon_{\gamma t}\right) \frac{W_t^\phi}{Y_{st}^{1+\phi}} = \omega_t \quad (2)$$

$$\varepsilon_{\beta t} \varepsilon_{\gamma t} \varepsilon_{Ft} \gamma \alpha \frac{W_t^\phi X_t^{\mu-\phi}}{C_{gt}^{1+\mu}} = \lambda_t \quad (3)$$

$$\tau \varepsilon_{\beta t} = \lambda_t (1 - \theta_g) \frac{Y_{gt}}{L_{gt}} \quad (4)$$

$$\tau \varepsilon_{\beta t} = \omega_t (1 - \theta_s) \frac{Y_{st}}{L_{st}} \quad (5)$$

$$\lambda_t \left(1 + \frac{\psi_{K_g}}{\delta_{K_g}} \frac{\Delta K_{gt}}{K_{t-1}}\right) = \beta \lambda_{t+1} \left(1 + \frac{\psi_{K_g}}{2\delta_{K_g}} \frac{K_{gt+1}^2 - K_{gt}^2}{K_{gt}^2} - \delta_{K_{gt+1}} + \theta_g \sigma \frac{Y_{gt+1} H_{t+1}^\nu}{z_{K_{gt+1}}^\nu K_{gt}^{1+\nu}}\right) \quad (6)$$

$$\lambda_t \left(1 + \frac{\psi_{K_s}}{\delta_{K_s}} \frac{\Delta K_{st}}{K_{st-1}}\right) = \beta \left(\lambda_{t+1} \left(1 + \frac{\psi_{K_s}}{2\delta_{K_s}} \frac{K_{st+1}^2 - K_{st}^2}{K_{st}^2} - \delta_{K_{st+1}}\right) + \omega_{t+1} \theta_s \frac{Y_{st+1}}{K_{st}}\right) \quad (7)$$

$$\lambda_t \left(1 + \frac{\psi_M}{\delta_M} \frac{\Delta M_t}{M_{t-1}}\right) = \beta \lambda_{t+1} \left(1 + \frac{\psi_M}{2\delta_M} \frac{M_{t+1}^2 - M_t^2}{M_t^2} - \delta_M + \theta_g (1 - \sigma) \frac{Y_{gt+1} H_{t+1}^\nu}{\varepsilon_{M_{t+1}}^\nu M_t^{1+\nu}}\right) \quad (8)$$

$$\lambda_t \left(1 + \frac{\psi_F}{\delta_F} \frac{\Delta F_t}{F_{t-1}}\right) = \beta \left(\varepsilon_{\beta t+1} \varepsilon_{\gamma t+1} \gamma (1 - \alpha \varepsilon_{Ft+1}) \frac{W_{t+1}^\phi X_{t+1}^{\mu-\phi}}{F_t^{1+\mu}} + \lambda_{t+1} \left(1 + \frac{\psi_F}{2\delta_F} \frac{F_{t+1}^2 - F_t^2}{F_t^2} - \delta_F\right)\right) \quad (9)$$

$$\theta_g \frac{\sigma Y_{gt} H_t^\nu}{z_{K_{gt}}^{1+\nu} K_{gt}^\nu} = b_{K_g} (\zeta_{K_g} z_{K_{gt}} + 1 - \zeta_{K_g}) K_{gt-1} \quad (10)$$

$$\omega_t \theta_s \frac{Y_{st}}{z_{K_{st}}} = b_{K_s} (\zeta_{K_s} z_{K_{st}} + 1 - \zeta_{K_s}) K_{st-1} \lambda_t \quad (11)$$

$$\delta_{K_{gt}} = \delta_{K_g} + b_{K_g} \zeta_{K_g} z_{K_{gt}}^2 / 2 + b_{K_g} (1 - \zeta_{K_g}) z_{K_{gt}} + b_{K_g} (\zeta_{K_g} / 2 - 1) \quad (12)$$

$$\delta_{K_{st}} = \delta_{K_s} + b_{K_s} \zeta_{K_s} z_{K_{st}}^2 / 2 + b_{K_s} (1 - \zeta_{K_s}) z_{K_{st}} + b_{K_s} (\zeta_{K_s} / 2 - 1) \quad (13)$$

$$\xi_{K_{gt}} = \frac{\psi_{K_g}}{2\delta_{K_g}} \left(\frac{K_{gt} - K_{gt-1}}{K_{gt-1}}\right)^2 K_{gt-1} \quad (14)$$

$$\xi_{K_{st}} = \frac{\psi_{K_s}}{2\delta_{K_s}} \left(\frac{K_{st} - K_{st-1}}{K_{st-1}}\right)^2 K_{st-1} \quad (15)$$

$$\xi_{M_t} = \frac{\psi_M}{2\delta_M} \left(\frac{M_t - M_{t-1}}{M_{t-1}}\right)^2 M_{t-1} \quad (16)$$

$$\xi_{F_t} = \frac{\psi_F}{2\delta_F} \left(\frac{F_t - F_{t-1}}{F_{t-1}}\right)^2 F_{t-1} \quad (17)$$

$$Y_{gt} = (A_{gt} L_{gt})^{1-\theta_g} H_t^{\theta_g} \quad (18)$$

$$H_t = (\sigma (z_{gt} K_{gt-1})^{-\nu} + (1 - \sigma) (\varepsilon_{Mt} M_{t-1})^{-\nu})^{-1/\nu} \quad (19)$$

$$X_t = (\alpha \varepsilon_{Ft}^{-\mu} C_t^{-\mu} + (1 - \alpha) F_{t-1}^{-\mu})^{-1/\mu} \quad (20)$$

$$W_t = (\gamma \varepsilon_{\gamma t} X_t^{-\phi} + (1 - \gamma \varepsilon_{\gamma t}) \varepsilon_{Ft}^{-\phi} Y_{st}^{-\phi})^{-1/\phi} \quad (21)$$

$$Y_{st} = (A_{st}L_{st})^{1-\theta_s} (z_{st}K_{st-1})^{\theta_s}. \quad (22)$$

The stochastic processes for the shocks are described in Section 2.6 of the main text. In the first equation, the term  $AC$  denotes the total adjustment costs.

## The Steady State

After some algebraic manipulations, we can show that the main ratios that describe the steady state of our model are described by the following equations:

$$c_g = \left( \frac{\alpha}{1-\alpha} \frac{1-\beta(1-\delta_F)}{\beta} \right)^{\frac{1}{1+\mu}} f \quad (23)$$

$$k_s = \left( \frac{\omega}{\lambda} \right)^{\frac{\phi}{1+\phi}} \left( \frac{1-\gamma}{\alpha\gamma} \right)^{\frac{1}{1+\phi}} \frac{\theta_s\beta}{1-\beta(1-\delta_{K_s})} \left( \alpha + (1-\alpha) \left( \frac{c_g}{f} \right)^\mu \right)^{\frac{\mu-\phi}{\mu(1+\phi)}} c_g \quad (24)$$

$$k_g = \frac{\theta_g\sigma\beta}{1-\beta(1-\delta_{K_g})} \frac{1}{\sigma + (1-\sigma) \left( \frac{\sigma}{1-\sigma} \frac{1-\beta(1-\delta_M)}{1-\beta(1-\delta_{K_g})} \right)^{\frac{\nu}{1+\nu}}} \quad (25)$$

$$m = \frac{\theta_g\beta(1-\sigma)}{1-\beta(1-\delta_M)} \frac{1}{(1-\sigma) + \sigma \left( \frac{\sigma}{1-\sigma} \frac{1-\beta(1-\delta_M)}{1-\beta(1-\delta_{K_g})} \right)^{-\frac{\nu}{1+\nu}}} \quad (26)$$

$$\frac{\lambda}{\omega} = \left( \frac{(1-\theta_s) \left( \frac{\theta_s\beta}{1-\beta(1-\delta_{K_s})} \right)^{\frac{\theta_s}{1-\theta_s}}}{(1-\theta_g) (\sigma k_g^{-\nu} + (1-\sigma) m_g^{-\nu})^{\frac{-\theta_g}{\nu(1-\theta_g)}}} \right)^{1-\theta_s} \quad (27)$$

$$c_g + \delta_F f + \delta_{K_g} k_g + \delta_{K_s} k_s + \delta_M m = 1. \quad (28)$$

Given specific values for the model parameters, equations 23 to ?? can be solved for  $f$ ,  $c_g$ ,  $k_s$ ,  $k_g$ ,  $m$ , and the  $\omega/\lambda$  ratio.

## Matching Steady-State Ratios Through Choices of $\alpha$ , $\theta_g$ , $\theta_s$ , $\sigma$ , and $\gamma$

For each estimated value of  $\nu$ ,  $\phi$ ,  $\mu$ ,  $\delta_F$ , and  $\delta_M$ , and given calibrated values for  $\beta$ ,  $\delta_{K_g}$ , and  $\delta_{K_s}$ , our estimation procedure aims to exactly match the following steady-state ratios that we take to be the average values obtained from the data (denoted with a bar):

$$\begin{aligned} \bar{f} &= \text{output inventories over goods output} \\ \bar{m} &= \text{input inventories over goods output} \\ \bar{k}_g &= \text{capital stock in goods industries over goods output} \\ \bar{k}_s &= \text{capital stock in service industries over goods output} \\ \bar{y}'_s &= \text{services output over goods output,} \end{aligned}$$

where  $y'_s = \frac{\omega Y_s}{\lambda Y_g} = \frac{\omega}{\lambda} y_s$  measures services output in units of goods output.

Given  $\beta$ ,  $\delta_{K_s}$ ,  $\delta_{K_g}$ ,  $\phi$ ,  $\mu$ ,  $\nu$ ,  $\delta_F$ , and  $\delta_M$ , simple algebraic manipulation shows that there is a unique set of values

for  $\alpha, \theta_g, \theta_s, \sigma, \gamma$  that satisfies the five ratios above, obtained as follows. Given the  $(\overline{k_g}/\overline{m})$  ratios, we obtain

$$\sigma = \frac{(\overline{k_g}/\overline{m})^{1+\nu} \frac{1-\beta(1-\delta_{K_g})}{1-\beta(1-\delta_M)}}{1 + (\overline{k_g}/\overline{m})^{1+\nu} \frac{1-\beta(1-\delta_{K_g})}{1-\beta(1-\delta_M)}}. \quad (29)$$

From the  $\overline{c_g}/\overline{f}$  ratio, we derive

$$\alpha = \frac{(\overline{c_g}/\overline{f})^{1+\mu} \frac{\beta}{1-\beta(1-\delta_F)}}{1 + (\overline{c_g}/\overline{f})^{1+\mu} \frac{\beta}{1-\beta(1-\delta_F)}}. \quad (30)$$

The formula for  $k_g/y_g$  can be used to derive

$$\theta_g = \frac{(\sigma + (1-\sigma)(\overline{k_g}/\overline{m})^\nu)(1-\beta(1-\delta_{K_g}))}{\sigma\beta} \overline{k_g}. \quad (31)$$

Finally, we need to choose  $\gamma$  and  $\theta_s$  to match the values of  $k_s$  and  $y'_s$ . From the formula for  $k_s/y'_s$ , we derive an expression for  $\theta_s$ :

$$\theta_s = \frac{1-\beta(1-\delta_{K_s})}{\beta} \frac{\overline{k_s}}{\overline{y'_s}}. \quad (32)$$

Last, we need to obtain  $\gamma$ . Using the expressions for  $k_s/f$  and  $k_s/y'_s$  above, we obtain

$$\gamma = \frac{(\overline{c_g}/\overline{y'_s})^{1+\phi} (\omega/\lambda)^\phi c_g^{\mu-\phi} (\alpha \overline{c_g}^{-\mu} + (1-\alpha) \overline{f}^{-\mu})^{\frac{\mu-\phi}{\mu}}}{\alpha + (\overline{c_g}/\overline{y'_s})^{1+\phi} (\omega/\lambda)^\phi c_g^{\mu-\phi} (\alpha \overline{c_g}^{-\mu} + (1-\alpha) \overline{f}^{-\mu})^{\frac{\mu-\phi}{\mu}}}, \quad (33)$$

where  $\omega/\lambda$  is given by the equation ?? above.

To summarize the material thus far: (1) for given observed values in the data of  $k_g, k_s, m, c_g, f,$  and  $y'_s$ ; and (2) for any possible combination of  $(\beta, \delta_{K_s}, \delta_{K_g}, \phi, \mu, \nu, \delta_F,$  and  $\delta_M)$ , the values of  $\alpha, \theta_g, \theta_s, \sigma,$  and  $\gamma$  that satisfy equations 29 to 33 are consistent with the the steady-state values of the ratios  $k_g, k_s, m, c_g, f,$  and  $y'_s$ .

## Calculating Steady-state Hours, Output, and Prices

The optimal labor supply schedules satisfy

$$\tau = \lambda(1-\theta_g) \frac{Y_g}{L_g} \quad (34)$$

$$\tau = \omega(1-\theta_s) \frac{Y_s}{L_s}. \quad (35)$$

A closed-form solution can be calculated only when  $\phi = 0$ . Otherwise, we need to use the following expressions. From the first-order conditions for  $C_g$  and  $Y_s$ , after some algebraic manipulations, we obtain the following formula:

$$\lambda Y_g = \frac{\gamma\alpha}{c_g} \left( \gamma \left( \alpha + (1-\alpha) \left( \frac{c_g}{f} \right)^\mu \right) + (1-\gamma) \left( \alpha + (1-\alpha) \left( \frac{c_g}{f} \right)^\mu \right)^{\frac{\mu-\phi}{\mu(1+\phi)}} \left( \frac{\omega}{\lambda} \frac{\alpha\gamma}{1-\gamma} \right)^{\frac{\phi}{1+\phi}} \right)^{-1}. \quad (36)$$

By the same token, we find that:

$$\omega Y_s = (1-\gamma) \left( \gamma \left( \alpha + (1-\alpha) \left( \frac{c_g}{f} \right)^\mu \right)^{\frac{1+\mu}{\mu} \frac{\phi}{1+\phi}} \left( \frac{\omega}{\lambda} \frac{\alpha\gamma}{1-\gamma} \right)^{\frac{-\phi}{1+\phi}} + (1-\gamma) \right)^{-1}. \quad (37)$$

From the production functions, we know that:

$$Y_g = L_g (\sigma k_g^{-\nu} + (1 - \sigma) m_g^{-\nu})^{-\frac{\theta_g}{\nu(1-\theta_g)}} \quad (38)$$

$$Y_s = L_s \left( \frac{K_s}{Y_s} \right)^{\frac{\theta_s}{1-\theta_s}} . \quad (39)$$

Equations 34 through 39 can be then be solved for  $L_g$ ,  $L_s$ ,  $Y_g$ ,  $Y_s$ ,  $\omega$ , and  $\lambda$  using a non-linear equation algorithm.

## B: Data

Most of our data come from the national income and product accounts (NIPA) produced by the U.S. Commerce Department, Bureau of Economic Analysis (BEA), and obtained from Haver Analytics, Inc. All NIPA data are quarterly. The real data are expressed in chain-weighted terms (year-2000 dollars). Table B.1 lists the variable names, Haver mnemonics, and variable descriptions. Our model and data exclude net exports and government spending.

Although the formulas in this appendix suppress the notational details associated with the proper manipulation of chain-weighted real data, we use the appropriate Tornquist approximation for chain-weighted data in constructing the actual data, as recommended by Whelan (2002).

The NIPA data classify output by three sectors called goods ( $g$ ), structures ( $t$ ), and services ( $s$ ):

$$Y = Y^g + Y^t + Y^s .$$

In contrast, inventory investment,  $\Delta V$ , is classified by industry (goods inventories include the agriculture, mining, and manufacturing industries; structures inventories include the construction industry; and the services sector includes utilities and trade). Thus, the NIPA output and inventory data do not correspond to the inventory-based sectors of our model definitions of goods and services.

To obtain model-consistent data, we can think of the task as one that condenses the three NIPA sectors into two by redefining and combining the NIPA sector variables as follows. First, express the components of aggregate output as

$$Y = (C^g + I^g + \Delta V^g) + (I^t + \Delta V^t) + (C^{sg} + C^{ss} + I^s + \Delta V^s) .$$

There is no household consumption of structures ( $C^t$ ) because the construction of structures is pure physical capital, in which we assume each sector invests. Household consumption of services,  $C^s = C^{sg} + C^{ss}$ , includes two components, distinguished by a second superscript indicating the appropriate model sector to which the services consumption data should belong. Thus,  $C^{sg}$  represents the consumption of services from industries that distribute goods (utilities and trade) that we wish to redefine as goods consumption. Also,  $C^{ss}$  includes the service flow from housing.

Given these definitions, we can then write model-consistent goods output as

$$Y^g = (C^g + C^{sg}) + (I^g + I^t + I^s) + (\Delta V^g + \Delta V^t + \Delta V^s) ,$$

Table B.1: Variable Names and Data Definitions

Variable	Mnemonic	Description
$C$	C	Consumption
$C^{gn}$	CN	Consumption of nondurable goods
$C^{gd}$	CD	Consumption of durable goods
$C^s$	CS	Consumption of services
$C^{se}$	CSE	Consumption of energy services
$I$	F	Fixed investment (including residential)
$V^{ga}$	SF	Farm inventories
$V^{gm}$	SNM	Manufacturing inventories
$V^{sw}$	SNW	Wholesale trade inventories
$V^{sr}$	SNR	Retail trade inventories
$V_{SIC}^o$	SNO2	Other inventories, SIC (fixed-weight \$1996)
$V^{MUC}$	SNB	Mining, utilities, and construction inventories
$V_{NAICS}^o$	SNT	Other inventories, NAICS
$V^{CW}$	RES513	Inventory chain-weighted residual
$P$	JC	Consumption chain-weighted price index
$P^s$	JCS	Consumption of services chain-weighted price index
$P^{se}$	JCSE	Consumption of energy services chain-weighted price index

Note: These Haver mnemonics are for the nominal data; the real data have an ‘H’ added at the end and, unless otherwise noted, are in real chain-weighted terms (year-2000 dollars).

and model-consistent services output as

$$Y^s = C^{ss}.$$

The remainder of this appendix explains how each of the relevant variables is defined and constructed.

## Consumption

NIPA consumption data are classified by the type of good consumed by households,

$$C = C^{gn} + C^{gd} + C^s.$$

In this equation, goods consumption includes nondurables ( $gn$ ) plus durables ( $gd$ ); consumption of services ( $s$ ) includes the service flow obtained from housing. Theoretically, it would be preferable to construct the service flow from other consumer durable goods besides housing, rather than use actual expenditures, but this is not done in the NIPA data (except for automobile leasing, which is implicitly a service yield). Because we are ultimately trying to explain the volatility, and the change in volatility, of actual GDP data, we use the raw NIPA data instead.

Using the NIPA consumption data to construct model-consistent consumption data, we must reclassify the portion of consumption data pertaining to NIPA services obtained from the trade and utilities industries that our model defines as goods-producing ( $C^{sg}$ ) into consumption of goods ( $C^g$ ). Because the NIPA do not treat energy consumption (such as electricity) as a good, we must define household energy ( $e$ ) consumption services as model-consistent goods consumption:  $C^{sg} = C^{sge}$ . Because energy is output consumed as a good obtained from the utilities industry, which holds inventories, we assume that all types of energy are measurable goods distributed to consumers. In this regard, electric and natural gas utility firms are similar to firms specializing in wholesale and retail trade, which distribute

finished goods from goods producers to consumers. Thus, the model-consistent data (denoted by a double tilde) for services consumption are

$$\widetilde{C}^s = C^s - C^{sg} ,$$

and the model-consistent data for goods consumption are

$$\widetilde{C}^g = C^{gn} + C^{gd} + C^{sg} .$$

Because the underlying NIPA data are based on the type of good consumed,  $C^{gn}$  and  $C^{gd}$  already contain the output from the retail-trade industry and any output from the wholesale-trade industry that is categorized as consumption (meaning a final sale to consumers, as opposed to an intermediate input into retail trade or back into manufacturing).<sup>36</sup>

## Investment

Because capital is a good, it is logical to define investment as output from the goods sector. However, because our model has two distinct sectors that each accumulates sector-specific capital (and thus has two capital accumulation equations), the model requires that we construct investment data classified by the type of firm or industry (sector) in which the capital is installed. Although the NIPA data do not classify investment by the sector in which it is installed, the BEA provides other annual data on investment by industry that do, and we use these latter data to divide total investment into sector-specific investment components.<sup>37</sup>

We define the goods sector to include the seven inventory-holding industries: agriculture, mining, utilities, construction, manufacturing, wholesale trade, and retail trade. Using the BEA data on investment by industry (denoted by a double hat) for these seven specific industries, we define the share ( $s$ ) of goods investment as

$$\widehat{s} = \left( \widehat{I}^g / \widehat{I} \right) .$$

These annual-share data are interpolated to obtain a quarterly frequency. Then goods-sector investment data are described as

$$\widetilde{I}^g = \widehat{s} I ,$$

where  $I$  is total fixed investment. The services-sector investment data are

$$\widetilde{I}^s = \left( 1 - \widehat{s} \right) I = I - \widetilde{I}^g$$

and applied to the actual quarterly data on total fixed investment.

## Inventories

According to the NIPA definitions, each output sector is associated with at least one inventory-holding industry,

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<sup>36</sup>One way to think of the different types of “goods” is in terms of their depreciation rates:  $0 < \delta^{sh} < \delta^d < \delta^n < \delta^{so} = 1$ , where superscript  $sh$  denotes housing services and  $so$  denotes other services (that is, not a flow from a durable stock).

<sup>37</sup>These data can be obtained from <http://bea.gov/bea/dn/FA2004/Index.asp>.

$$\begin{aligned}
V^g &= V^{ga} + V^{gn} + V^{gm} \\
V^t &= V^{tc} \\
V^s &= V^{su} + V^{sw} + V^{sr} ,
\end{aligned}$$

with industries defined as agriculture ( $a$ ), mining ( $n$ ), manufacturing ( $m$ ), construction ( $c$ ), utilities ( $u$ ), wholesale trade ( $w$ ), and retail trade ( $r$ ). Thus, to construct model-consistent inventories, we redefine the goods sector as the holder of all inventories:

$$\widetilde{V}^g = V^g + V^t + V^s .$$

As discussed in the main text, by assumption the services sector holds no inventories ( $\widetilde{V}^s = 0$ ).<sup>38</sup>

We further divide total inventories into two types,

$$\widetilde{V}^g = M + F ,$$

where  $M$  denotes input and  $F$  denotes output. Economic theory provides no clear categorical definition of input and output inventories in general equilibrium. We view goods as being produced and distributed along a supply and distribution chain, so for our model one logical definition of output inventories is the last link in the chain, which is the retail industry:

$$\widetilde{M} = V^{sr} .$$

In this case, input inventories are expressed as

$$\widetilde{F} = V^{ga} + V^{gn} + V^{gm} + V^{tc} + V^{su} + V^{sw} .$$

In general, all non-retail inventory stocks can be considered inputs into production along the supply chain. According to the Census of Construction,  $V^{tc}$  (inventories in the construction industry of the structures sector) are raw materials and do not include unsold finished structures, thus they can be viewed as input inventories. In actuality, some fraction of the remaining stocks may be sold directly to consumers, and hence should be classified as output inventories, but we assume this fraction is small.

To obtain a long time series of inventory data, we combine non-farm stocks constructed under two different industry classifications: SIC (1947–1997) and NAICS (1987–present).<sup>39</sup> At this high level of industry definition, the manufacturing, wholesale, and retail inventory data are generally consistent across industry classification schemes, so we splice these data series without further manipulation. The inventories for all remaining industries (\*), however, are defined as follows:

$$\begin{aligned}
V_{SIC}^* &= V_{SIC}^o \\
V_{NAICS}^* &= V^{MUC} + V_{NAICS}^o + V^{CW} ,
\end{aligned}$$

where  $o$  denotes “other” industries in each classification system; MUC denotes mining, utilities, and construction. CW

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<sup>38</sup>The NIPA make this same assumption, setting output equal to final sales in the structures and services sectors and classifying all inventory investment in the goods sector.

<sup>39</sup>Farm, or agricultural, inventory stocks on a consistent industry classification are already available for the full sample period (1947–present).

denotes the chain-weighted residual for real data (real data on an SIC basis are expressed in fixed-weight 1996 dollars, and thus have no residual). In splicing the data, we use the SIC stocks through 1997, and then use the growth rates of the NAICS from 1997 on to extend the SIC data.

## Consumption Prices

The prices of goods and services consumption are constructed analogously to the respective quantities of consumption. Let  $w^{se}$  be the nominal expenditure weight for energy services, and  $w^{\bar{s}} = (1 - w^{se})$  be the nominal expenditure weight for model-consistent (non-energy) services. Then, having calculated the appropriate Tornquist index on the data, the model-consistent price of services consumption is

$$\widetilde{\widetilde{P}}^s = (1/w^{\bar{s}}) [P^s - w^{se} P^{se}] .$$

Likewise, let  $w^{\bar{s}}$  be the nominal expenditure weight for model-consistent services, and  $w^g = (1 - w^{\bar{s}})$  be the nominal expenditure weight for model-consistent goods. Then the model-consistent price of goods consumption is

$$\widetilde{\widetilde{P}}^g = (1/w^g) \left[ P - w^{\bar{s}} \widetilde{\widetilde{P}}^s \right] .$$

The ratio of the consumption prices,

$$\frac{\widetilde{\widetilde{P}}^g}{\widetilde{\widetilde{P}}^s} = \frac{\lambda}{\omega} ,$$

equals the ratio of Lagrange multipliers from the model's first-order conditions.

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Table 1: Sector Definitions and Output Shares

Model Sectors	NIPA Sectors and Industries (NAICS)		2000 GDP Share (in percent)	
	Sector	Industry		
Goods (59.3%) [67.6% of private sector]	Goods (35.1%)	Agriculture	1.0	
		Mining	1.2	
		Manufacturing	14.5	
	Structures (9.6%)	Construction	4.4	
	Services (28.4%) [32.4% of private sector]	Services (55.3%)	Utilities	1.9
			Wholesale Trade	6.0
Retail Trade			6.7	
Transportation			3.1	
Information			4.7	
FIREL			19.7	
Services			11.6	
Education & Health	6.9			
Leisure	3.6			
Public (excluded) (12.3%)		Government	12.3	

Notes: FIREL denotes Finance, Insurance, Real Estate, and Leasing.

Table 2: Inventory Stock Definitions and Shares

Model Inventories		NIPA Inventories (NAICS)	2000 Share (in percent)
Input & Output	Stage-of-Fabrication	Industry	
Input (73.4%)	Raw Materials (18.9%)	Agriculture	8.6
		Mining, utilities, construction (MUC)	2.9
		Mining	n.a.
		Utilities	n.a.
		Construction	n.a.
		Other	7.4
	Work-in-process (54.5%)	Manufacturing	31.1
		Materials and supplies	11.0
		Work-in-process	8.9
		Finished goods	11.1
Output (26.6%)	Finished goods (26.6%)	Wholesale trade	23.4
		Retail trade	26.6

Table 3: Target Steady-State Ratios of the Model

	Full sample	1960–1983	1984–2004
$F/Y_g$	0.32	0.29	0.34
$M/Y_g$	1.12	1.34	1.01
$K_g/Y_g$	6.89	7.73	6.54
$K_s/Y_g$	8.36	6.74	9.32
$Y'_s/Y_g$	0.75	0.53	0.81

Notes: Output is expressed in quarterly units. The last row is the ratio of nominal output of services over nominal output of the goods sector. The capital output ratios are calculated from the investment-to-output ratios, assuming depreciation rates of  $\delta_{Kg} = .02$  and  $\delta_{Ks} = .02$ .

Table 4: Prior Distributions and Parameter Estimates, Full Sample

	Prior			Full Sample		
	Mean	Distribution	St.dev.	Mean	5%	95%
$\delta_F$	0.020	beta	0.01	0.0784	0.0505	0.1088
$\delta_M$	0.020	beta	0.01	0.0204	0.0114	0.0316
$1 + \mu$	1.500	norm	0.5	1.0691	0.6290	1.5285
$1 + \nu$	1.500	norm	0.5	3.3277	2.8768	3.8174
$1 + \phi$	1.500	norm	0.5	1.0525	1.0290	1.0915
$\psi_F / (1 + \psi_F)$	0.500	beta	0.2	0.0242	0.0143	0.0367
$\psi_{Kg} / (1 + \psi_{Kg})$	0.500	beta	0.2	0.4722	0.2422	0.8103
$\psi_{Ks} / (1 + \psi_{Ks})$	0.500	beta	0.2	0.2523	0.1608	0.4180
$\psi_M / (1 + \psi_M)$	0.500	beta	0.2	0.0183	0.0101	0.0289
$\zeta_{Kg} / (1 + \zeta_{Kg})$	0.500	beta	0.2	0.9133	0.8130	0.9785
$\zeta_{Ks} / (1 + \zeta_{Ks})$	0.500	beta	0.2	0.5586	0.3446	0.7653
$\rho_g$	0.750	beta	0.1	0.8674	0.8287	0.9028
$\rho_B$	0.750	beta	0.1	0.8839	0.8428	0.9203
$\rho_F$	0.750	beta	0.1	0.8838	0.7803	0.9576
$\rho_\gamma$	0.750	beta	0.1	0.8173	0.7556	0.8761
$\rho_M$	0.750	beta	0.1	0.9381	0.9104	0.9619
$\rho_s$	0.750	beta	0.1	0.9343	0.9021	0.9611
$\sigma_g$	0.02	invg	Inf	0.0175	0.0149	0.0205
$\sigma_B$	0.02	invg	Inf	0.0187	0.0142	0.0250
$\sigma_F$	0.01	invg	Inf	0.0032	0.0025	0.0039
$\sigma_\gamma$	0.01	invg	Inf	0.0056	0.0047	0.0066
$\sigma_M$	0.10	invg	Inf	0.0944	0.0740	0.1193
$\sigma_s$	0.01	invg	Inf	0.0142	0.0114	0.0178
$\sigma_{g,s}$	0.50	norm	0.25	0.7115	0.6352	0.7771

Table 5: Values of the Share Parameters Implied by the Estimation Results

	Full sample	1960–1983	1984–2004
$\alpha$	0.9625	0.9716	0.9776
$\gamma$	0.4939	0.5562	0.4837
$\sigma$	0.9976	0.9958	0.9969
$\theta_g$	0.2305	0.2602	0.2217
$\theta_s$	0.3191	0.3653	0.3354

Table 6: Variance Decompositions of the Model, Full Sample

	Full Sample					
	$\sigma_g$	$\sigma_\beta$	$\sigma_F$	$\sigma_\gamma$	$\sigma_M$	$\sigma_s$
$\widehat{Y}_g$	78.9	16.1	1.5	1.0	2.5	0.0
$\widehat{Y}_s$	58.3	4.0	0.0	11.5	0.4	25.7
$\widetilde{I}$	51.1	42.1	0.1	0.7	6.0	0.0
$\widetilde{\Delta F}$	11.9	4.2	81.7	0.2	2.1	0.0
$\widetilde{\Delta M}$	16.5	16.0	0.2	0.0	67.3	0.0
$\widehat{C}_g$	77.5	3.1	4.9	9.8	4.7	0.0
$\widehat{GDP}$	84.3	11.3	0.4	0.1	1.2	2.8

Notes: For each variable, the columns indicate the fractions of the total variance explained by each shock. Variables with a hat are scaled by their steady state value. Variables with a tilde are scaled by steady state goods output.

Table 7: Parameter Estimates, Subsamples

	Prior			1960:1–1983:4			1984:1–2004:4		
				Mean	5%	95%	Mean	5%	95%
$\delta_F$	0.020	beta	0.01	0.0604	0.0351	0.0883	0.0426	0.0207	0.0688
$\delta_M$	0.020	beta	0.01	0.0187	0.0105	0.0285	0.0224	0.0124	0.0362
$1 + \mu$	1.500	norm	0.5	0.9783	0.5163	1.4586	1.1940	0.5364	1.8500
$1 + \nu$	1.500	norm	0.5	3.0918	2.6475	3.5884	3.1426	2.8101	3.6231
$1 + \phi$	1.500	norm	0.5	0.9626	0.9309	0.9855	1.1809	1.0910	1.2863
$\psi_F / (1 + \psi_F)$	0.500	beta	0.2	0.0214	0.0113	0.0347	0.0397	0.0214	0.0610
$\psi_{Kg} / (1 + \psi_{Kg})$	0.500	beta	0.2	0.4199	0.2149	0.8257	0.4956	0.2852	0.8160
$\psi_{Ks} / (1 + \psi_{Ks})$	0.500	beta	0.2	0.3335	0.1530	0.5936	0.3631	0.2165	0.5396
$\psi_M / (1 + \psi_M)$	0.500	beta	0.2	0.0278	0.0145	0.0440	0.0161	0.0073	0.0278
$\zeta_{Kg} / (1 + \zeta_{Kg})$	0.500	beta	0.2	0.8734	0.7396	0.9653	0.8680	0.7153	0.9673
$\zeta_{Ks} / (1 + \zeta_{Ks})$	0.500	beta	0.2	0.3047	0.1168	0.5120	0.8002	0.5960	0.9463
$\rho_g$	0.750	beta	0.1	0.8496	0.8035	0.8943	0.9034	0.8551	0.9461
$\rho_B$	0.750	beta	0.1	0.8780	0.8270	0.9210	0.9004	0.8577	0.9379
$\rho_F$	0.750	beta	0.1	0.9244	0.8306	0.9774	0.9121	0.8439	0.9655
$\rho_\gamma$	0.750	beta	0.1	0.8314	0.7505	0.9059	0.8181	0.7315	0.8974
$\rho_M$	0.750	beta	0.1	0.9547	0.9337	0.9714	0.9707	0.9551	0.9835
$\rho_s$	0.750	beta	0.1	0.9341	0.8956	0.9677	0.9460	0.9123	0.9725
$\sigma_g$	0.02	invg	Inf	0.0224	0.0188	0.0269	0.0142	0.0111	0.0183
$\sigma_B$	0.02	invg	Inf	0.0207	0.0150	0.0279	0.0235	0.0157	0.0351
$\sigma_F$	0.01	invg	Inf	0.0035	0.0026	0.0046	0.0047	0.0033	0.0065
$\sigma_\gamma$	0.01	invg	Inf	0.0053	0.0042	0.0069	0.0056	0.0044	0.0074
$\sigma_M$	0.10	invg	Inf	0.1340	0.1016	0.1769	0.1071	0.0816	0.1418
$\sigma_s$	0.01	invg	Inf	0.0159	0.0123	0.0215	0.0145	0.0111	0.0193
$\sigma_{g,s}$	0.50	norm	0.25	0.7565	0.6592	0.8345	0.5144	0.3572	0.6497

Table 8: Variance Decompositions, Subsamples

1960–1983						
	$\sigma_g$	$\sigma_\beta$	$\sigma_F$	$\sigma_\gamma$	$\sigma_M$	$\sigma_s$
$\widehat{Y}_g$	82.0	11.3	1.4	0.5	4.8	0.0
$\widehat{Y}_s$	62.1	2.4	0.0	15.9	1.1	18.6
$\widetilde{I}$	55.9	30.8	0.1	1.0	12.2	0.0
$\widetilde{\Delta F}$	14.1	5.1	79.1	0.1	1.7	0.0
$\widetilde{\Delta M}$	20.3	14.6	0.2	0.0	64.9	0.0
$\widehat{C}_g$	77.0	3.3	4.2	6.8	8.7	0.0
$\widehat{GDP}$	87.3	8.3	0.4	0.2	2.6	1.2
1984–2004						
	$\sigma_g$	$\sigma_\beta$	$\sigma_F$	$\sigma_\gamma$	$\sigma_M$	$\sigma_s$
$\widehat{Y}_g$	62.7	25.8	4.0	1.7	5.8	0.0
$\widehat{Y}_s$	38.6	7.4	0.1	9.3	0.7	43.9
$\widetilde{I}$	32.0	60.6	0.1	0.3	7.0	0.0
$\widetilde{\Delta F}$	2.3	3.4	93.9	0.0	0.4	0.0
$\widetilde{\Delta M}$	9.4	24.5	0.2	0.0	65.9	0.0
$\widehat{C}_g$	68.9	3.6	12.2	9.4	5.7	0.2
$\widehat{GDP}$	69.3	20.7	1.2	0.0	2.7	6.1

Table 9: Properties of the Estimated Model: Correlations and Standard Deviations

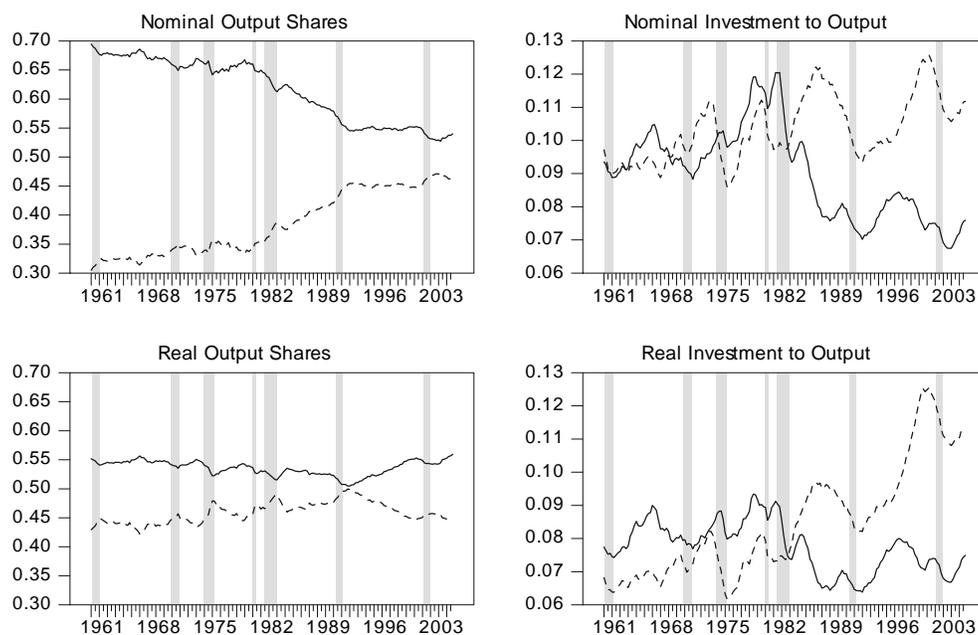
Standard deviation	Full sample		1960–1983		1984–2004	
	Model	Data	Model	Data	Model	Data
$\widehat{GDP}$	0.0167	0.0183	0.0216	0.0216	0.0134	0.0135
$\widehat{Y}_g$	0.0228	0.0297	0.0281	0.0349	0.0186	0.0223
$\widehat{Y}_s$	0.0132	0.0067	0.0156	0.0076	0.0117	0.0056
$\widetilde{I}$	0.0116	0.0139	0.0135	0.0161	0.0092	0.0108
$\widetilde{\Delta F}$	0.0045	0.0037	0.0051	0.0039	0.0055	0.0032
$\widetilde{\Delta M}$	0.0076	0.0068	0.009	0.0068	0.0068	0.0066
Correlations						
$\widehat{Y}_g, \widetilde{\Delta F}$	0.3183	0.4175	0.3396	0.4698	0.2627	0.2798
$\widehat{Y}_g, \widetilde{\Delta M}$	0.4517	0.6404	0.427	0.6574	0.4122	0.6199
$\widehat{\Delta GDP}, \widetilde{\Delta^2 F}$	0.4441	0.4163	0.5017	0.4661	0.3408	0.3702
$\widehat{\Delta GDP}, \widetilde{\Delta^2 M}$	0.5557	0.5324	0.5669	0.5544	0.6016	0.4546
$\widehat{Y}_g, \widehat{F}_g/C_g$	0.1133	0.104	0.0736	0.1067	-0.0328	0.0348
$\widehat{Y}_g, \widehat{M}_g/Y_g$	-0.3015	-0.8923	-0.3465	-0.9205	-0.2298	-0.8049
$\widehat{Y}_g, \widehat{Y}_s$	0.5573	0.6849	0.6215	0.7566	0.4675	0.5066

Table 10: Accounting for the Decline in Volatility

Parameter	Value		Contribution to change ( $\times 100$ )			
	1960-1983	1984-2004	$\sigma(\widehat{GDP})$	$\sigma(\widehat{Y}_g)$	$\sigma(\widehat{Y}_s)$	$\sigma(\tilde{I})$
$\delta_F$	0.0604	0.0426	0.00	0.00	0.00	0.00
$\delta_M$	0.0187	0.0224	0.00	0.02	0.00	0.02
$1 + \mu$	0.9783	1.194	0.00	0.00	0.00	0.00
$1 + \nu$	3.0918	3.1426	0.00	0.00	0.00	0.00
$1 + \phi$	0.9626	1.1809	0.02	-0.04	0.04	-0.12
$\psi_F / (1 + \psi_F)$	0.0214	0.0397	-0.01	-0.02	0.00	0.00
$\psi_{Kg} / (1 + \psi_{Kg})$	0.4199	0.4956	-0.06	-0.09	0.00	-0.11
$\psi_{Ks} / (1 + \psi_{Ks})$	0.3335	0.3631	-0.02	-0.03	-0.01	-0.05
$\psi_M / (1 + \psi_M)$	0.0278	0.0161	0.04	0.07	0.00	0.00
$\zeta_{Kg} / (1 + \zeta_{Kg})$	0.8734	0.868	0.01	0.01	0.00	0.01
$\zeta_{Ks} / (1 + \zeta_{Ks})$	0.3047	0.8002	-0.12	-0.09	-0.16	-0.12
<b>All estimated parameters</b>			<b>-0.13</b>	<b>-0.17</b>	<b>-0.14</b>	<b>-0.36</b>
$F/Y_g$	0.2881	0.3397	0.01	0.02	0.00	0.00
$M/Y_g$	1.3396	1.0148	-0.03	-0.08	0.00	-0.03
$(K_g + K_s)/Y_g$	14.47	15.86	0.17	0.25	-0.02	0.22
$Y'_s/Y_g$	0.5331	0.8061	-0.15	0.00	-0.03	0.00
<b>All steady state parameters</b>			<b>-0.03</b>	<b>0.20</b>	<b>-0.06</b>	<b>0.20</b>
$\sigma_F$	0.0035	0.0047	0.00	0.02	0.00	0.00
$\sigma_M$	0.1340	0.1071	-0.01	-0.02	0.00	-0.03
$\sigma_g$	0.0224	0.0142	-0.59	-0.81	-0.13	-0.25
<b>All shocks</b>			<b>-0.70</b>	<b>-0.77</b>	<b>-0.27</b>	<b>-0.16</b>
<b>All parameters and shocks</b>			<b>-0.82</b>	<b>-0.94</b>	<b>-0.39</b>	<b>-0.42</b>

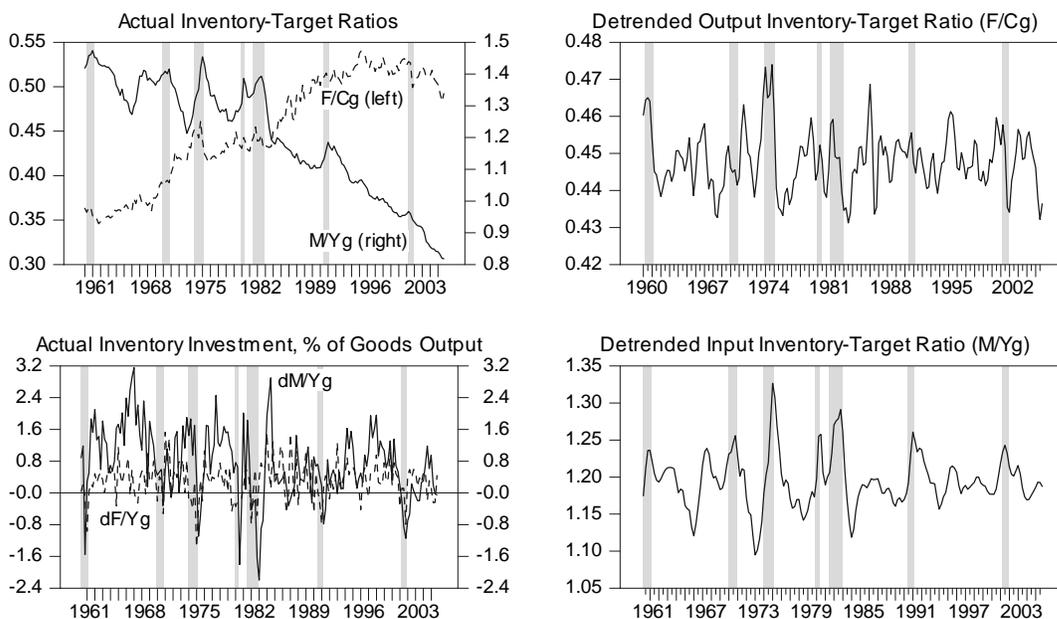
Notes: Columns 2 and 3 indicate the estimated value of the parameter in the first column in each sub-sample. In the last four columns, we take the period 1960–1983 as the baseline period and change each parameter to its 1984–2004 value to account for its contribution to reducing volatility. The columns indicate, for each variable, the change in the standard deviation (times 100) due to the change in that parameter. Two important caveats are that: (1) standard deviations are not additive; (2) the effects of each model parameter are not independent from the values of other model parameters. For this reason, the values in each column do not add up to the last value listed in the column.

Figure 1  
Output and Investment Data, by Sector.



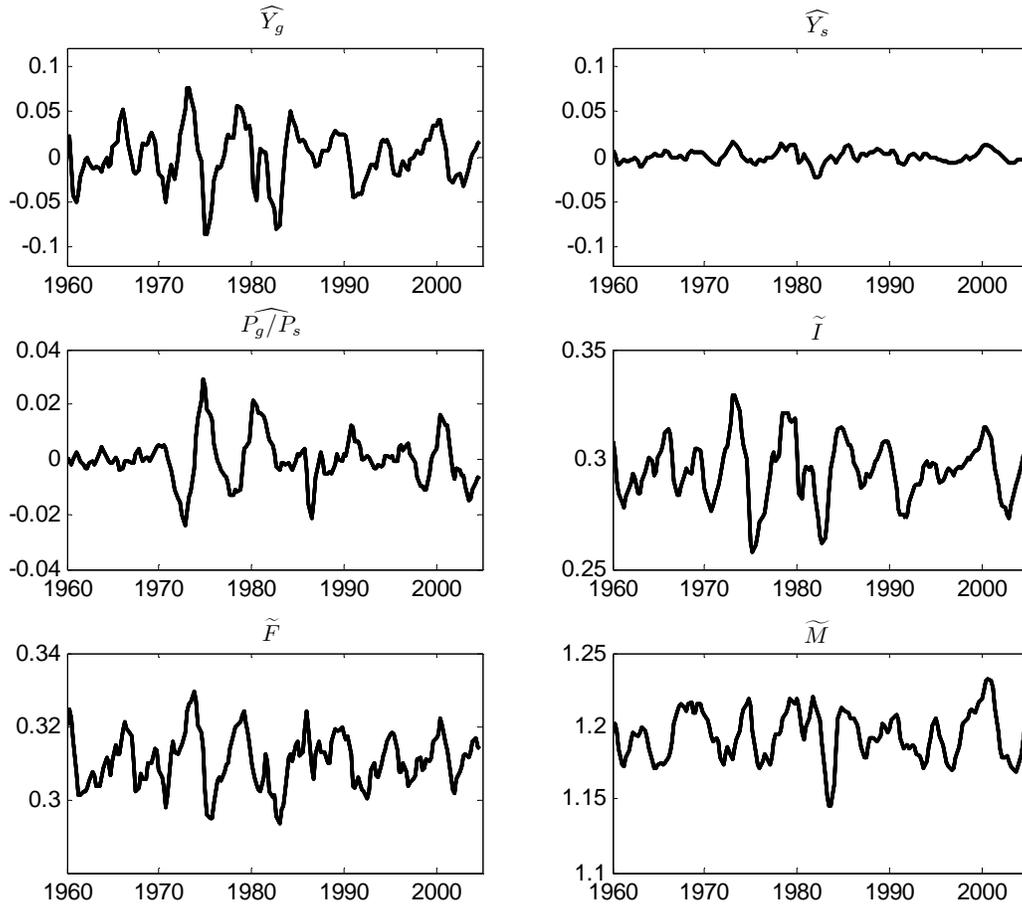
Notes: Solid lines: Goods sector; Dashed lines: Services sector. Shaded regions indicate NBER recession dates.

Figure 2  
Input and Output Inventory Data



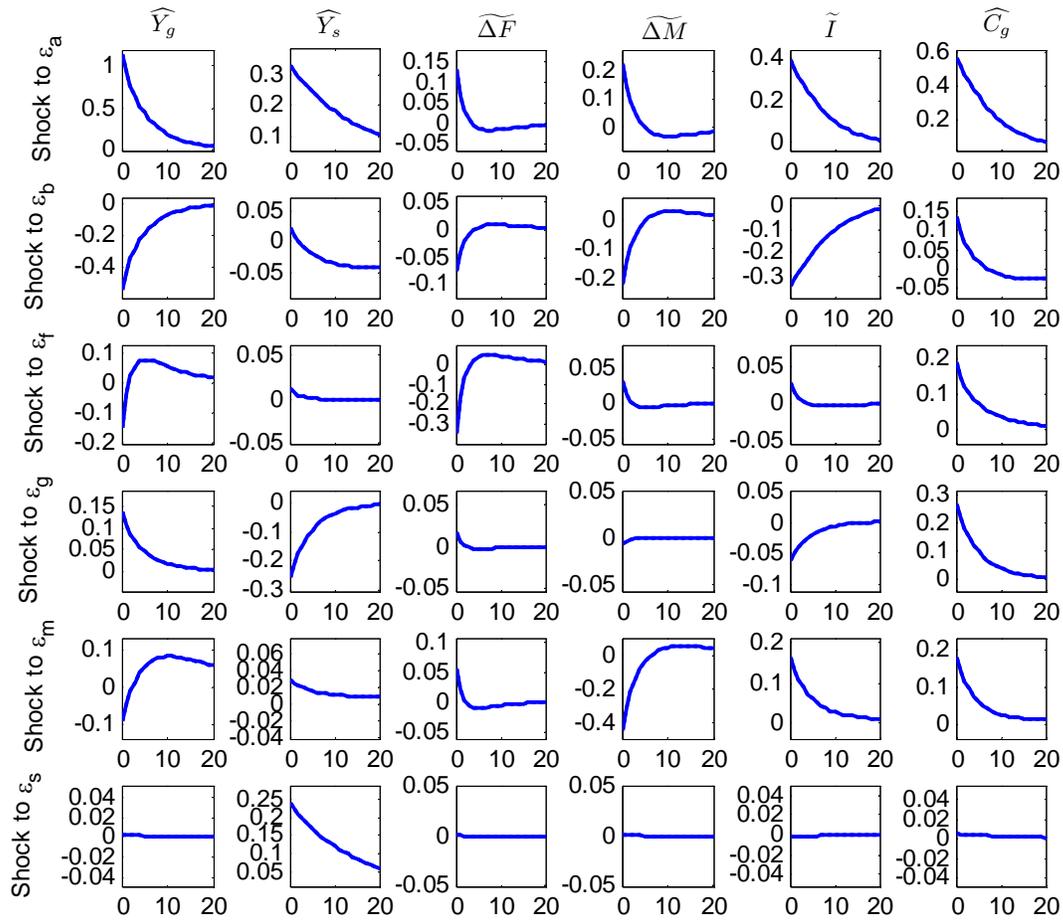
Notes: Shaded regions indicate NBER recession dates.

Figure 3  
Variables Used in Estimation



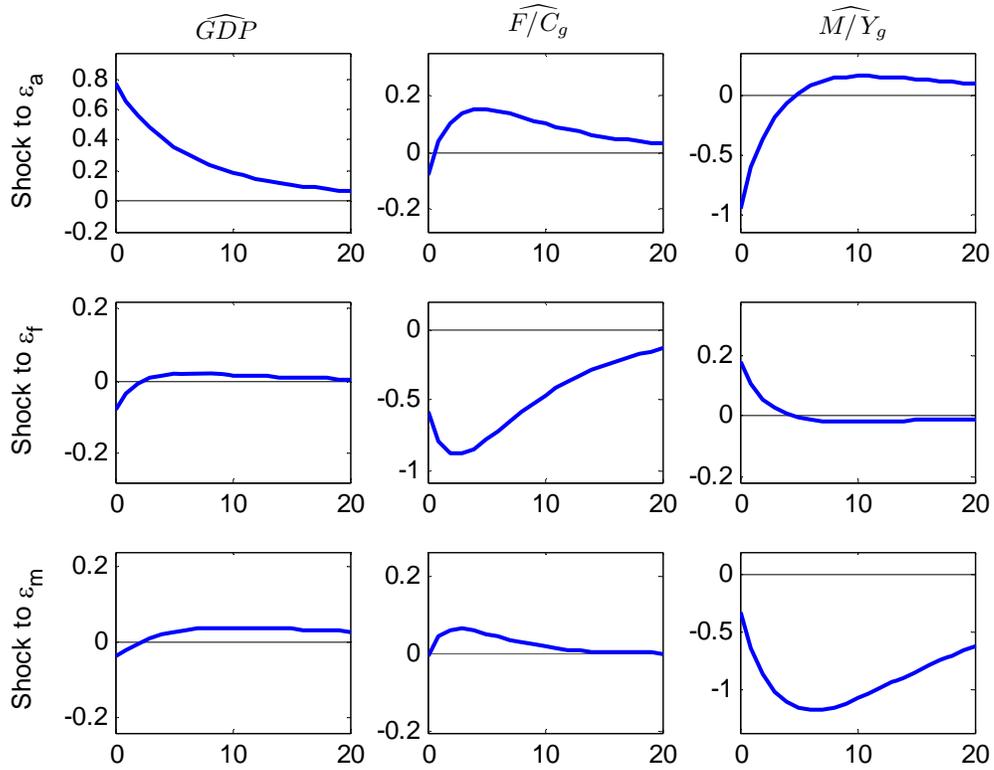
Notes: Variables with a hat are scaled by their steady-state values. Variables with a tilde are scaled by steady-state output in the goods sector.

Figure 4  
 Impulse Responses of the Estimated Model  
 Sectoral output, inventory and fixed investment (scaled by goods output), consumption.



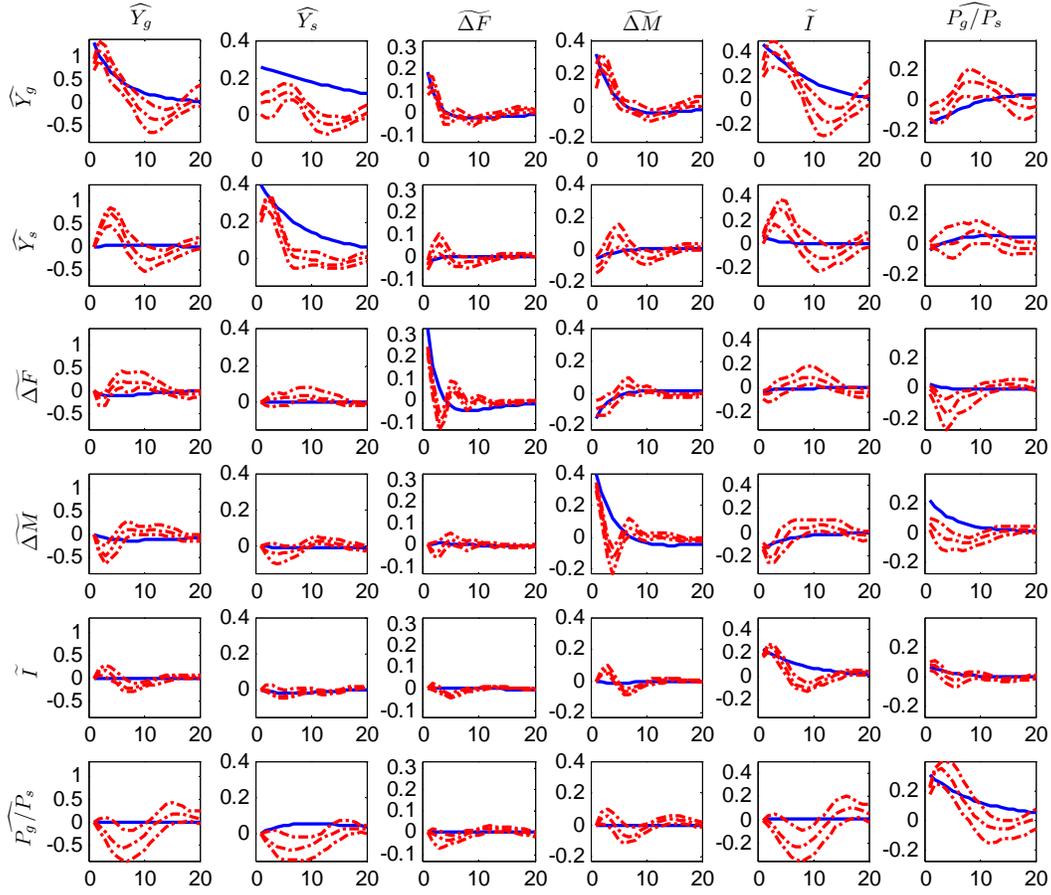
Notes: Each row shows the impulse responses to an estimated one-standard-deviation shock. Ordinate: Time horizon. Coordinate: Deviation from baseline, multiplied by one hundred. Variable with a hat are scaled by their steady state value. Variables with a tilde are scaled by steady state output in the goods sector.

Figure 5  
 Impulse Responses of the Estimated Model to Selected Shocks  
 GDP and inventory-to-target ratios



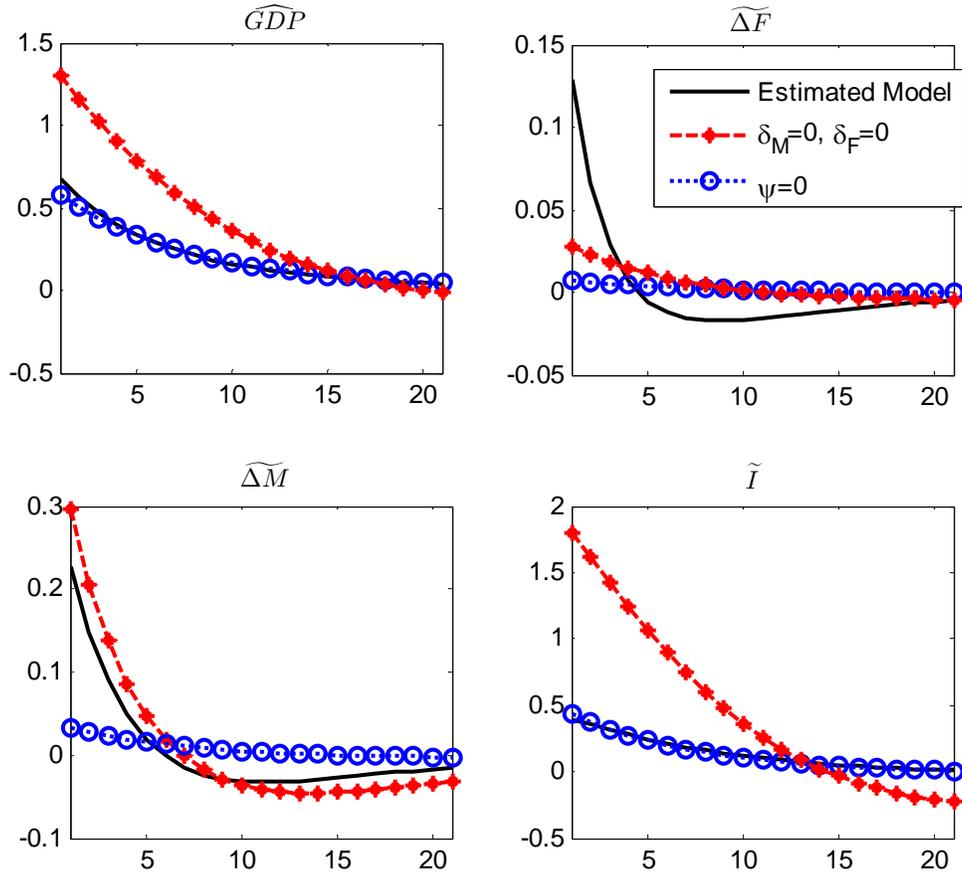
Notes: Each row shows the impulse responses to an estimated one-standard-deviation shock. Ordinate: Time horizon. Coordinate: Deviation from baseline, multiplied by one hundred.

Figure 6  
 Orthogonalized Impulse Responses of the Estimated Model, Comparison with VAR.



Notes: VAR based on actual data (dashed lines, with two lags and 95% bootstrapped confidence bands) and model. Each row represents one shock. Both sets of impulse responses have been orthogonalized in the same order. Shocks are one standard deviation. Ordinate: Time horizon. Coordinate: Deviation from baseline, multiplied by one hundred. Variables with a hat are scaled by their steady-state values. Variables with a tilde are scaled by steady-state output in the goods sector.

Figure 7  
 Impulse Responses to a Favorable Technology Shock in the Goods Sector



Notes: Responses to an estimated one-standard-deviation technology shock in the goods sector. Ordinate: Horizon in quarters. Coordinate: Deviation from baseline, multiplied by one hundred. Output is scaled by its steady-state value. Inventory investment and fixed investment are scaled by steady-state output in the goods sector, so that their impulse responses measure the growth contribution to goods output.