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Affective Decision Making: A Theory of Optimism Bias

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Abstract:

Optimism bias is inconsistent with the independence of decision weights and payoffs found in models of choice under risk, such as expected utility theory and prospect theory. Hence, to explain the evidence suggesting that agents are optimistically biased, we propose an alternative model of risky choice, affective decision making, where decision weights—which we label affective or perceived risk—are endogenized.

Affective decision making (ADM) is a strategic model of choice under risk where we posit two cognitive processes—the "rational" and the "emotional" process. The two processes interact in a simultaneous-move intrapersonal potential game, and observed choice is the result of a pure Nash equilibrium strategy in this game. We show that regular ADM potential games have an odd number of locally unique pure strategy Nash equilibria, and demonstrate this finding for ADM in insurance markets. We prove that ADM potential games are refutable by axiomatizing the ADM potential maximizers.

JEL Classifications: D01, D81, G22

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1 Introduction

Many of our everyday decisions such as working on a project, getting a flu shot, or buying insurance require an estimate of probabilities of future events: the probability of a project's success, of falling sick, or of being involved in an accident. In assessing these probabilities, decision makers tend to be optimistically biased, where optimism bias is defined as the tendency to overestimate the likelihood of favorable future outcomes and to underestimate the likelihood of unfavorable future outcomes (Irwin 1953; Weinstein 1980; Slovic, Fischoff, and Lichtenstein 1982; Slovic 2000). A young woman drinking at a bar thinking it will be safe for her to drive home is an example; an entrepreneur who starts a new business, confident that she is going to succeed where others have failed is another. Indeed, one can argue that although statistics for these events are well-documented, each of these persons has private information concerning her individual tolerance for alcohol and entrepreneurial ability, respectively. Hence, each woman may have good reasons to believe that overall empirical frequencies do not apply to her. The common feature in these examples is that decision makers have some freedom in choosing their probabilistic beliefs, and they are often optimistic—they appear to choose beliefs that are biased towards favorable outcomes.

Optimism bias is not merely a hypothetical bias—it translates into both microeconomic and macroeconomic activity. For example, optimism bias influences high-stakes decisions, such as startup investment, investment behavior, and merger decisions. It was found that 68 percent of startup entrepreneurs believe their company is more likely to succeed than similar companies, while in reality only 50 percent of startup companies survive beyond three years of activity (Baker, Ruback, and Wurgler 2006 and references therein). Malmendier and Tate (2005) find that CEOs who are optimistic regarding their firm's future performance are more sensitive to investment cash flow, which leads to distortions in investment decisions. Malmendier and Tate (2008) find that the optimistic CEOs are 65 percent more likely to complete mergers, are more likely to overpay for those target companies, and are more likely to undertake value-destroying mergers. On the macroeconomic level, Robert Shiller in his book *Irrational Exuberance* (2000; 2005), now a contemporary classic, defines irrational exuberance as "wishful thinking on the part of investors that blinds us to the truth of our situation," and makes the case that irrational exuberance contributes to generating bubbles in financial markets. Shiller points out several psychological and cultural factors that affect individuals' beliefs and consequently the investment behavior that leads to real macro-level effects. Many of these factors can be summarized as optimistically biased beliefs.

Optimism bias is inconsistent with the independence of decision weights and payoffs found in models of choice under risk, such as expected utility, subjective expected utility, and prospect theory. Hence, to explain the evidence suggesting that agents are optimistically biased, we suggest an alternative model of risky choice where decision weights—which we label affective or perceived risk—are endogenized. More specifically, we consider two systems of reasoning, which we label the *rational process* and the *emotional process*. The rational process decides on an action, while the emotional process forms a perception of risk and in doing so is optimistically biased. The two processes interact to yield a decision. This interaction is modeled as a simultaneous-move *intrapersonal* potential game, and consistency between the two processes, which represents choice, is the equilibrium outcome realized as a pure strategy Nash equilibrium of the game.

This novel formulation of optimism bias, by employing a simultaneous choice of action and beliefs and where tradeoff is done by a game, is the first contribution of our paper. This formulation may be viewed as a model of the specialization and integration of brain activity considered in recent neuroscience studies that identify distinct brain modules that specialize in different activities. For instance, the amygdala is associated with emotions, while the prefrontal cortex is associated with higher-level deliberate thinking (e.g., Reisberg 2001). Our model is also consistent with the psychology literature that draws a distinction between analytical and intuitive, or deliberate and emotional, processing (Chaiken and Trope 1999). However, in both neuroscience and psychology, behavior is thought to be a result of the different systems interacting (for example, Sacks 1985; Damasio 1994; Epstein 1994; LeDoux 2000; Gray, Braver, and Raichle 2002, Camerer, Loewenstein, and Prelec 2004; Pessoa 2008). For example, Gray, Braver, and Raichle (2002) conclude that "at some point of processing, functional specialization is lost, and emotion and cognition conjointly and equally contribute to the control of thought and behavior." More recently, Pessoa (2008) argues that "emotions and cognition not only strongly interact in the brain, but [that] they are often integrated so that they jointly contribute to behavior," a point also made in the specific context of expectation formation.

Although the evidence on modular brain and the dual-processes theory cannot typically be pinned down to the formation of beliefs, given that beliefs formation is partly affected by the beliefs we would like to have—that is, by affective considerations—decision making under risk maps naturally to the interplay between the two cognitive processes, as proposed by Kahneman (2003). That is, decision making under risk can be modeled as a *deliberate* process that chooses an optimal action, and an *emotional* cognitive process that forms risk perception.

Formally, the rational process coincides with the expected utility model, where for a given risk perception (meaning the affective probability distribution), the rational process chooses an action to maximize expected utility. The emotional process forms a risk perception by selecting an optimal risk perception that balances two contradictory impulses: (1) affective motivation and (2) a taste for accuracy. This is a definition of motivated reasoning, a psychological mechanism where emotional goals motivate agent's beliefs (see Kunda 1990), and is a source of psychological biases, such as optimism bias. Affective motivation is the desire to hold a favorable personal risk perception—optimism—and in the model it is captured by the expected utility term. The desire for accuracy is modeled as a mental cost the agent incurs for holding beliefs in lieu of her base rate probabilities given her desire for favorable risk beliefs. The base rate probabilities are the beliefs that minimize the mental cost function of the emotional process, i.e., the risk perception that is easiest and least costly to justify—in many instances, one can think of the baseline probabilities as the empirical, relative frequencies of the states of nature.

As an application of affective decision making we present an example of the demand for insurance in a world with a bad state and a good state. The relevant probability distribution in insurance markets is personal risk; hence, the demand for insurance may depend on optimism bias. Affective choice in insurance markets is defined as the insurance level and risk perception that constitute a pure strategy Nash equilibrium of the affective decision making (ADM) intrapersonal potential game.

The systematic departure of the ADM model from the expected utility model shows, consistent with consumer research (Keller and Block 1996), that campaigns intended to educate consumers on the magnitude of the potential loss they face in the unfavorable state may have the unintended consequence that consumers purchase less, rather than more, insurance. Hence, the ADM model suggests that the failure of the expected utility model to explain some datasets may be due to systematic affective biases. Furthermore, the ADM intrapersonal game is a potential game—where a (potential) function of a penalized subjective expected utility (SEU) form defines the best response dynamic of the game. This property has the natural interpretation of the utility function of the composite agent, or the integration of the two systems, and allows for the second contribution of this paper, discussed next.

Deviations from the basic rational choice models often raise the concern that the theory lacks the discipline imposed by a clear paradigm, and, as a result, "anything goes." This concern arises in the ADM model since we allow agents to choose both actions and beliefs. To investigate this concern we present an axiomatic characterization of ADM potential maximizers which subsumes the class of ADM models with a unique equilibrium. This shows that choosing both actions and beliefs in the ADM model does not per se imply that anything goes.

The remainder of the paper is organized as follows: In section 2 we discuss the related literature, and in section 3 we present the demand for insurance in a world with two states of nature. Section 4 presents an analysis of ADM potential games in a world with K-states of nature using a formal definition of optimistic preferences in the general case. In section 5 we present an axiomatic foundation of the ADM potential maximizers, and in the final section of the paper we discuss our results and the relationship between the axiomatized class of optimistic preferences and ambiguity attitudes. All proofs are in the Appendix.

2 Related Literature

Recent literature in economic theory recognizes the possibility that agents might choose their beliefs in a self-serving or optimistic way; see Akerlof and Dickens (1982), Bodner and

Prelec (2001), Bénabou and Tirole (2002), Yariv (2002), Caplin and Leahy (2004), Brunnermeier and Parker (2005), and Koszégi (2006). Likewise, the dual-processes hypothesis was recently recognized in economic modeling, specifically in models of self-control and addiction such as Thaler and Shefrin (1981), Bernheim and Rangel (2004), Loewenstein and O'Donoghue (2004), Benhabib and Bisin (2005), Fudenberg and Levine (2006), and Brocas and Carrillo (2008). Existing models are restricted in the sense that the choice of beliefs and the choice of action are not made in tandem and in that the models assume that an agent chooses beliefs strategically to resolve a tradeoff between a standard instrumental payoff and some notion of a psychologically based belief utility[1]. The existing models of dual processes are restricted in that the two systems, or decision modes, are conceived as mutually exclusive.

Although there are cases where a descriptive model seems to require mutually exclusive systems, as in the case of self-control and addiction, there are other cases where a descriptive model seems to require several different processes that together determine observed choice. We provide such a formulation: in this paper one process chooses action while the other forms perceptions, and both are necessary for decision making.

As mentioned, the ADM intrapersonal game is a potential game with a potential function defined as a penalized SEU model. This characterization allows us to axiomatize the set of ADM potential maximizers, which includes the class of unique equilibrium ADM models. More importantly the axiomatic foundation suggests an alternative interpretation of the ADM model as one that describes ambiguity-seeking behavior, an additional difference between this paper and the existing literature. In particular, the model in the literature that comes closest to optimistic preferences as described in the ADM intrapersonal game is the optimal expectations model of Brunnermeier and Parker (2005). The optimal expectations model considers an agent who chooses both beliefs and actions in a dynamic setting where beliefs are chosen in period one for all future periods, thus trading off greater anticipated utility against the cost of making poor decisions due to holding optimistic beliefs. Hence, optimal expectations are optimistic beliefs not constrained by reality. In contrast, ADM is a static model where beliefs and actions mutually determine observed choice, and where beliefs trade off greater anticipated utility against the mental cost of distorting beliefs—costs that are a function of reality. This captures the fact people do not hold arbitrary beliefs even in the absence of an action they can take based on their beliefs, in contrast to the optimal expectation model's prediction. Moreover, the ADM model's simultaneous framework, where cost is based solely on beliefs, is a parsimonious model consistent with cognitive dissonance, with differences between report and choice tasks, with integration of processes in the brain, and has a utility function for the composite agent that will allow future welfare analysis. This formulation allows further, as mentioned above and presented in section 5, axiomatic characterization of the ADM potential maximizers which then lead to an alternative interpretation of the ADM model as a model of ambiguity-seeking behavior. Both the model's axiomatic characteri-

[1] The axiomatic foundation for this is provided by Caplin and Leahy (2001) and Yariv (2001).

zation and its alternative interpretation as an ambiguity-seeking model are not shared by Brunnermeier and Parker (2005) or the other dual-process models.

3 The ADM Model of the Demand for Insurance

In this section we present a model of affective choice in insurance markets where probability perceptions are endogenous. We use a simple example of an agent facing two possible future states of the world; in section 4 we show that analogous results of this two-states case hold for K states of the world.

Consider an agent facing two states of the world, Bad and Good with associated wealth levels w_B and w_G , where $w_B < w_G$. The agent has a strictly increasing, strictly concave smooth utility function of wealth, $u(w)$, with $\lim_{w\to-\infty} Du(w) = \infty$, $\lim_{w\to\infty} Du(w) = 0$.[2] Risk perception is defined as the perceived probability $p \in [0, 1]$ of the Bad state occurring. For simplicity we allow the agent to purchase or sell insurance $I \in R$ at the fixed insurance premium rate, $\gamma \in (0, 1)$. The intuition and results for the case where the agent can only buy insurance are easily derived from this analysis.

The *rational process* chooses an optimal insurance level (I^*) to maximize expected utility given a perceived risk p. Specifically, the rational process maximizes the following objective function:

$$\underset{I}{max} \left\{ pu(w_B + (1-\gamma)I) + (1-p)u(w_G - \gamma I) \right\}.$$

The *emotional process* chooses an optimal risk perception (p^*) given an insurance level I, to balance the agent's affective motivation and taste for accuracy. Specifically, the emotional process maximizes the following objective function:

$$\underset{p}{max} \left\{ pu(w_B + (1-\gamma)I) + (1-p)u(w_G - \gamma I) - J^*(p; p_0) \right\}.$$

Affective motivation is captured by the expected utility term—the agent would like to assign the highest possible weight to her preferred state of the world. Taste for accuracy is modeled by introducing a mental cost function $J^*(p; p_0)$ that is nonnegative, strictly convex, and reaches a minimum at $p = p_0$, where p_0 is the baseline probability. $J^*(p; p_0)$ is essentially smooth—smooth on $(0, 1)$ with $lim_{p\to0}|DJ^*(p; p_0)| = lim_{p\to1}|DJ^*(p; p_0)| = +\infty$—and it's limit $lim_{p\to0} J^*(p; p_0) = lim_{p\to1} J^*(p; p_0) = +\infty$. See Figure 1.

[2] All qualitative results remain the same for the case of $\lim_{w\to0} Du(w) = \infty$, $\lim_{w\to\infty} Du(w) = 0$.

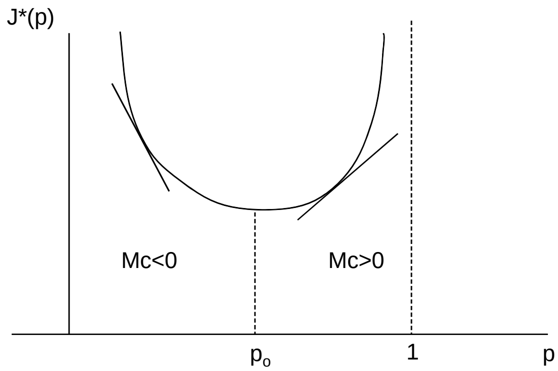

Figure 1

Why this U-shape? The psychology literature argues that individuals tend to use mental strategies such as bias search through memory to find justification for their desired beliefs (Kunda 1990). As the desired beliefs are further distanced from some baseline odds p_0—the odds that immediately come to mind and are easiest to justify (these could be the empirical relative frequency of the state of nature or available statistics like mortality tables)—the search costs are likely to increase. That is, it would be increasingly more difficult for the agent to come up with justifications and find anecdotes to support the optimistic view. This is exactly what the the mental cost captures. In addition, the behavior at the extreme is a formal description of a well-known phenomenon, namely, that decision makers assign a special quality to certain situations: getting $100 for certain is qualitatively different from a lottery with a 99 percent chance to win $100 (Kahneman and Tversky 1979). In the current simple settings, certainty corresponds to the two extreme beliefs $p \in \{0, 1\}$, and the behavior at the extremes captures the dramatic difference between certain, "safe", and risky events. Hence, the psychology literature is consistent with a mental cost $J^*(p)$ that is strictly convex, and essentially smooth on the interior of the probability simplex Δ. Moreover, since in situations that are inherently risky people do not hold beliefs that reflect certainty, we demand that in the limit, as the probability approaches zero or one, the cost function approaches infinity faster than the utility function does. This requirement implies that the insurance level desired by the rational process as the probability approaches a boundary is not sufficient for the emotional process to support these beliefs; hence, one would never hold beliefs $p \in \{0, 1\}$.

The fact that the mental cost is a function solely of probability is appealing as well. The mental cost formally reflects the psychological description of reasoning and gives rise to a special and important property of the intrapersonal game—our model of interaction between the rational and emotional processes. More specifically, the intrapersonal game is a potential game, and as such, this type of mental cost is consistent with *integration* of the two cognitive processes, a property supported by findings in psychology and recent research in neuroscience. Moreover, section 4 shows that, surprisingly, the psychological description of the mental cost function is in fact necessary and sufficient for optimistic preferences, where optimistic preferences are such that the assigned probabilities are skewed towards the good state and away from the bad state.

We now consider the interaction of the two processes in decision making. We model this interaction as an intrapersonal simultaneous-move game, a choice that reflects a recent view in cognitive neuroscience which holds that both processes mutually determine the performance of the task at hand (Damasio 1994).

Definition 1 *An intrapersonal game is a simultaneous-move game with two players, namely, the rational and the emotional processes. The strategy of the rational process is an insurance level,* $I \in R$, *and the strategy of the emotional process is a risk perception,* $p \in (0, 1)$. *The payoff function for the rational process is* $g : (0, 1) \times R \rightarrow R$ *is* $g(p, I) \equiv pu(w_B + (1 - \gamma)I) + (1 - p)u(w_G - \gamma I)$. *The payoff function for the emotional process is* $\Theta : (0, 1) \times R \rightarrow \overline{R}$ *is* $g(p, I) - J^*(p)$, *where* $J^*(\cdot)$ *is the mental cost function of holding belief* p, *which reaches a minimum at* p_0.

Proposition 2 below indicates that the intrapersonal game described in definition 1 is a potential game. Potential games are a class of strategic games introduced by Monderer and Shapley (1996), where all players have a common goal and therefore the game can be represented with one global common payoff function. This global payoff function is called the potential function of the game, and is used by each player to determine her best response. In the case of individual choice, since the players are decision processes and the game is a model of decision *making*, the potential function can be intuitively interpreted as a utility function of the composite agent. Below is the formal proposition:

Proposition 2 *The intrapersonal game is a potential game in which the emotional process's objective function is the potential function of the game. Because the potential function is strictly concave in each variable (risk perception and insurance), its critical points are the pure strategy Nash equilibria of the game.*

It is straightforward to show that the emotional process's objective function is the potential function of the game, as its first-order conditions with respect to I and p are the same as those of the rational process and emotional process, respectively. That is the potential function $\Theta(I, p) = < U(I), p > - J^*(p)$ captures the best-response dynamics of the intrapersonal game, and this potential game is therefore a model of affective decision making.

The equilibrium notion in the potential game is pure strategy Nash equilibrium, which is a natural candidate for choice, as it reflects a mutually determined choice and consistency between the rational and emotional processes. Excluding the case of tangency between the best responses of the two processes and given that the potential function is biconcave we have the following existence theorem (see Figure 2 for illustration).

Proposition 3 *The ADM intrapersonal game has an odd number of pure strategy Nash equilibria. The set of Nash equilibria is a chain in R^2 under the standard partial order on points in the plane.*

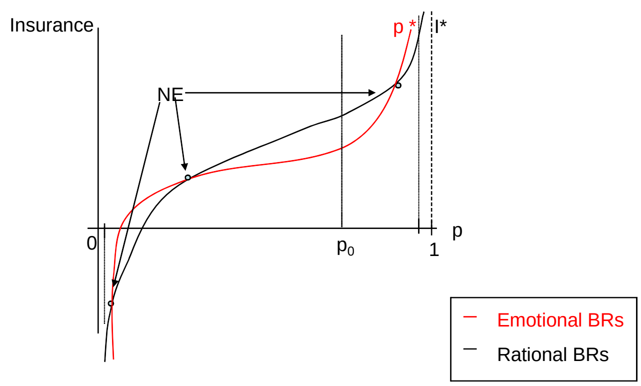

Figure 2

Note that with the ADM model one captures differences in report and choice tasks, as documented in different studies (in the context of normal form games, see Costa-Gomes and Weizsäcker 2008). In the insurance context, when asked to report the probability of, say, an accident with no subsequent action to take, the agent activates only the emotional process and tends to report low chances of an accident occuring. However, when asked to choose an action both processes are activated and together determine choice—hence the chosen action will generally be inconsistent with the reported beliefs.

Although the ADM model generally has multiple equilibria, which we believe is realistic given that it captures framing and attentional effects as will be discussed below, the case of a unique equilibrium ADM model is attractive due to its predictive power. The sufficient condition, due to Neyman (1997), for the uniqueness of pure strategy Nash equilibrium in a simultaneous-move potential game is strict concavity of the potential function and compact strategy sets. Due to the properties of the problem, one can always define a restricted game with compact strategy sets that will admit the same set of Nash equilibria as the original game. The potential is strictly concave if the Hessian of the potential be negative definite, which in this simple example reduces to the following condition:

Proposition 4 *A sufficient condition for a unique pure strategy Nash equilibrium for the intrapersonal game is:*

$$\frac{\partial^2 J^*(p;p_0)}{\partial p^2} > -\frac{[Du(w_B+(1-\gamma)I)(1-\gamma)+Du(w_G-\gamma I)\gamma]^2}{[pD^2u(w_B+(1-\gamma)I)(1-\gamma)^2+(1-p)D^2u(w_G-\gamma I)\gamma^2]}.$$

This condition is simply a statement regarding the relative slope of the two processes' best responses. Note that $\frac{\partial^2 J^*(p;p_0)}{\partial p^2}$ is the rate at which the marginal mental costs change with respect to perceived probabilities p, while $[Du(w_B + (1-\gamma)I)(1-\gamma) + Du(w_G - \gamma I)\gamma]$ is the rate at which the marginal benefits of distorting beliefs change with respect to insurance level I. The above condition therefore states that the ratio of change in the marginal mental costs with respect to perceived risk to the change in the marginal mental benefit with respect to insurance is always greater than a similar ratio defined on marginal expected utility. In this case, the emotional process's best response is everywhere steeper—to support a given change in perceived probability the emotional process demands a greater change in insurance level relative to the rational process's best response—and the intrapersonal game admits a unique equilibrium. One implication of this condition is that for large mental costs the equilibrium is unique (think of $\lambda > 0$, $\widehat{J}^*(\cdot) = \lambda J^*(\cdot)$), and for very large mental costs the ADM model reduces to the expected utility model.[3]

However, unless the mental costs are very large, even with a unique ADM model risk perceptions are endogenous and the model systematically departs from the expected utility model. This suggests that the failure of the expected utility model to explain some datasets may be due to systematic affective biases. How exactly does affective choice in insurance markets differ from the demand for insurance in the expected utility model? Proposition 5 shows that the expected utility outcome in the case of an actuarially fair insurance market (full insurance) falls within the choice set of the ADM agent. However, if the insurance market is not actuarially fair, then this is no longer the case.

Proposition 5 *If $\gamma = p_0$, there exists at least one Nash equilibrium (p^*, I^*) with $p^* = p_0 = \gamma$, and $I^* =$ full insurance.*

If $\gamma > p_0$, there exists at least one Nash equilibrium (p^*, I^*) with $p^* < p_0$ and $I^* < I^*(p_0)$.

If $\gamma < p_0$, there exists at least one Nash equilibrium (p^*, I^*) with $p_0 < p^*$ and $I^* > I^*(p_0)$.

To understand the intuition behind these results, consider a standard myopic adjustment process where the processes alternate moves. If $\gamma > p_0$, at p_0 the rational process, similar to the expected utility model, prescribes buying less than full insurance. The emotional process, in turn, leads the decision maker to believe "this is not going to happen to me" and to determine that she is at a lower risk. This effect causes a further reduction in the insurance purchase, with a result that the agent is not fully insured and is insured at a lower level than the expected utility model would predict. This proposition gives both an intuitive understanding of the effect of the emotional process in the ADM model, and intuitively shows existence of pure-strategy Nash equilibrium, affective choice. In the case of a unique ADM model, proposition 5 fully characterizes affective choice; in the case of multiple equilibria it points out only one equilibrium out of many possible. However, the effect of the two processes in enhancing each others' initial tendency is true whether one considers the unique ADM models or the entire class of ADM models.

[3] As $J^* \to \infty$, $p^* \to p_0$ for all values of I. As a result, the ADM model converges to the expected utility model.

Considering such an adjustment process, the ADM model is consistent with two widely discussed phenomena: cognitive dissonance and attention effects. Cognitive dissonance is when an individual holds two contradictory beliefs at the same time. Hence if one thinks of the adjustment process as a process of reaching a decision, in this process the agent suffers cognitive dissonance and her choice represents a resolution of it. As for attention effects—if one's attention is manipulated to first think of an action, or first think of risk beliefs, generally he or she will end up making different choices. In particular, according to our general ADM model, thinking first of the probabilities of adverse events occuring leads to greater optimism and lower insurance levels than if the agent's attention is given to thinking of insurance first.

Note that proposition 5 also implies that, from the viewpoint of an outside observer, both optimism and pessimism (relative to p_0) are possible. This is due to the characteristics of many insurance contracts reflected in the model: if an agent purchases an amount that is in excess of full insurance, then the "bad" state becomes the "good" state, and vice versa. Consequently, if there is no effective action, meaning that one cannot change the bad state to a good state, we would observe optimism and less-than-optimal level of insurance.

Another example of the difference between affective choice and the demand for insurance in the expected utility model is as follows: In the expected utility model, if people realize that they face a higher potential loss due to educational campaigns aimed at raising awareness of the possible catastrophe—much like warning labels such as "Smoking Kills" on cigarette packages, campaigns against speeding that show vivid pictures of people severely injured or killed in car accidents, and flood warnings "Like Never Before"—then they will purchase more insurance.[4] Yet, in the ADM model, if an agent realizes that she faces a higher possible loss, then she might purchase less insurance. The increased size of the potential loss affects both the emotional and the rational processes in different ways: the rational process prescribes more insurance, while the emotional process precribes a lower risk belief to every insurance level (due to greater incentives to live in denial). If the emotional effect is stronger the agent will buy less insurance than previously. That is, if the potential loss is great, agents might prefer to remain in denial and ignore the possible catastrophic outcomes altogether, which will lead them to take fewer precautions such as buying insurance. This result is consistent with consumer research showing that in educating people on the health hazards of smoking, a high fear arousal leads to agents discounting the potential threat (Keller and Block 1996; see also Ringold 2002 and references therein). Proposition 6 and Figure 3 below summarize the conditions for educational

[4] Indeed, there may be educational campaigns aimed at increasing the baseline probability of an event occurring, or changing the mental costs directly. If one analyzes this kind of educational campaign, it is still possible that the campaign will back fire. It depends on the specific assumptions made as to how the marginal mental costs change with the campaign, and the type of equilibria studied.

The approach we take is because many educational campaigns such as those for smoking, alcohol consumption, speeding, and natural disasters stress the adverse outcome and not the probability of getting into a bad state as described above. That is, many "educational" ads do not add information on the probability of suffering from the adverse consequence, but rather draw the public's attention to the possible dramatic consequences.

campaigns to produce the counterintuitive affective result.

Proposition 6 *An educational campaign results in more optimistic beliefs if*

$$\frac{u'(w_B + (1-\gamma)I)}{r(w_B + (1-\gamma)I)} > \frac{u'(w_G - \gamma I)}{r(w_G - \gamma I)},$$

and this will result in an agent purchasing less insurance if

$$\frac{u'(w_B + (1-\gamma)I)}{r(w_B + (1-\gamma)I)} > \frac{\partial^2 J^*(p; p_0)}{\partial p^2} p(1-p),$$

where $r(\cdot)$ is the absolute risk aversion property of the utility function $u(\cdot)$

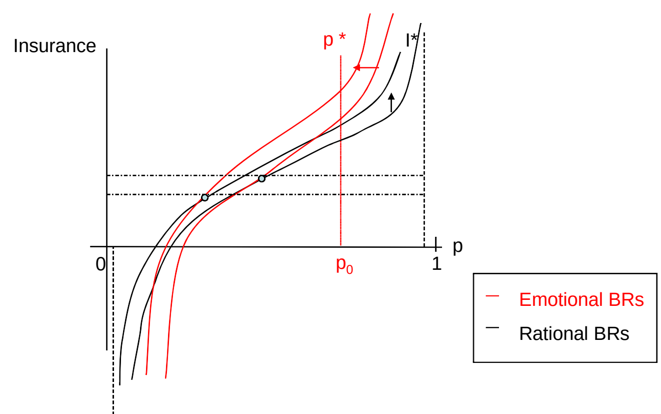

Figure 3

In Proposition 6, if the utility function $u(\cdot)$ exhibits constant or increasing absolute risk aversion, educational campaigns will lead to more optimistic view if and only if the agent initially buys less than full insurance coverage. This will translate into a lower insurance purchase depending on the curvature of the utility function at the bad state relative to the cost function. Taking the cost function discussed in the next section, $\sum_{j=1}^{j=K}[p_j \lg(p_j/p_{0j})]$, a backfire effect will occur if the marginal utility at the bad state is greater than the coefficient of absolute risk aversion at that point. Hence, for such utility functions, educational campaigns can divide the insurance market into a set of agents who purchase more insurance—the intended consequence—and a set of agents who purchase less insurance—the unintended consequence. These conditions are true for any equilibrium, even in the case of multiplicity of equilibria.

4 ADM Potential Games

In this section we present a state-preference model of affective decision making where an agent maximizes her utility subject to a budget constraint and is facing K possible states of nature; this section therefore extends the analysis in section 3. More specifically, consider an agent who faces K possible states of nature and has a utility function u over outcomes. The rational process chooses an action z that maps states into outcomes. Preferences over acts z are represented by a convex function J of state-utility vectors $U(z)$, where $U(z)$ is the vector of utilities of outcomes for the act z. That is, preferences over acts z are represented by a composite utility function $J(U(z))$. The emotional process chooses a belief p and the ADM potential game is $\Theta(z,p) = <U(z),p> -J^*(p)$, where $J^*(p)$ is a convex function of Legendre type. That is, $J^*(p)$ is a strictly convex, essentially smooth function on the interior of the probability simplex Δ[5].

Before proceeding to the results and extending the analysis of the two-states case into K-states, we pause to define optimism bias in the general case and discuss the relationship of optimism bias to the assumptions made on the affective cost function. We use Theorem 26.5 in Rockafellar (1970) showing that a closed convex function f is of a Legendre type on the interior of the effective domain of f denoted Ω if and only if the Legendre conjugate f^* is of Legendre type on the interior of the effective domain of f^*, denoted Ω^*. Moreover, the gradient mapping ∇f is a one-to-one map from the open convex set Ω onto the open convex set Ω^*, where the gradient map is continuous in both directions and $\nabla f^* = (\nabla f)^{-1}$.

Now, if $J^*(p)$ is a convex function of Legendre type, then it follows from the envelope theorem that the affective probabilities chosen by the emotional process for the act z are

$$\nabla_{U(z)} J(U(z)) = \arg\max_{p\in\Delta} \left\{ < U(z), p > -J^*(p) \right\}.$$

Moreover, for all state-utility vectors $U(z)$ and $U(y)$:

$$\left[\nabla_{U(z)} J(U(z)) - \nabla_{U(y)} J(U(y))\right] \cdot [U(z) - U(y)] > 0.$$

That is, the affective probabilities, $\nabla_{U(z)} J(U(z))$, are a strictly increasing monotone map of the state-utility vector $U(w)$. This is our definition of optimism bias. The definition of optimism bias in the K-state world subsumes the intuitive definition of optimism bias in the two-state case, where the assigned probability is skewed towards the more favorable outcomes of an act z.

Remarkably, the above exercise shows that the assumed shape of the mental cost function in the demand for insurance example (strictly convex and essentially smooth), reflecting the psychological characterization of affective costs, is both necessary and suf-

[5] The notion of a strictly convex, essentially smooth function is due to Rockafellar (1970)—see chapter 26. If Ω = the interior of the effective domain of a proper, extended real-valued convex function f : $R^K \longrightarrow R \cup \{+\infty\}$, then f is essentially smooth if: (i) Ω is not empty, (ii) f is differentiable throughout Ω, (iii) $\lim_{i\longrightarrow\infty} \|\nabla f(x_i)\| = +\infty$ whenever $x_1, x_2,...$ is a sequence in Ω converging to a boundary point x of Ω. He defines the class of strictly convex and essentially smooth functions to be of Legendre type.

ficient for optimism bias.

The next question is whether the assumed shape of the affective cost function is "typical"? That is, in some precise sense are most closed proper convex functions on R^K strictly convex and essentially smooth on the interior of their effective domains? Surprisingly, the answer is yes! This result is an immediate consequence of Howe's (1982) theorem that in the uniform topology, the family of strictly convex and differentiable functions on a compact, convex subset A of R^K is a residual subset of the family of convex functions on A. Any open and bounded convex subset of R^K can be exhausted by a countable family of compact, convex subsets. Since a countable intersection of residual sets is also residual, we see that in the topology of uniform convergence on compact sets (or topology of compact convergence) the family of strictly convex and differentiable functions on an open and bounded convex subsets, B of R^K is a residual subset in the family of convex functions on B. If $J^*(\pi)$ is strictly convex and differentiable, then it follows from the strict monotonicity of the gradient map that the gradient map is one-to-one. By Corollary 26.3.1. in Rockafellar—that is, if $J^*(\pi)$ is a closed proper, convex function on R^K, then the subgradient correspondence reduces to a one-to-one gradient map on the interior of the effective domain of $J^*(\pi)$ if and only if $J^*(\pi)$ is strictly convex and essentially smooth—these functions are strictly convex and essentially smooth. In other words, Legendre convex functions are generic[6].

Now we extend the existence and local uniqueness of pure strategy Nash equilibria results in the two-state ADM potential game to ADM potential games with K states of the world. To do this, we define regular potential games as potential games where the potential function $\Theta(z,p)$ is a Morse function. In other words, 0 is a regular value of $\nabla_{(z,p)}[\Theta(z,p)]$, where z is an action and p is a belief—the Hessian of the potential function $\Theta(z,p)$ evaluated at a critical point of the potential is non-singular. For a discussion of Morse functions, see chapter 1 in Guillemin and Pollack (1974) where they show that "most" smooth functions are Morse functions.

We show that essentially smooth and strictly biconcave regular potential games have an odd number of locally unique pure strategy Nash equilibria. In the proof we use the homotopy principle, which implies an algorithmic interpretation and allows for the computation of a pure strategy Nash equilibrium. Moreover, we show that if the potential function $\Theta(z,p)$ is essentially smooth and strictly biconcave, then the set of pure strategy Nash equilibria is non-empty, where 0 need not be a regular value of $\nabla_{(z,p)}[\Theta(z,p)]$.

Proposition 7 *If $\Theta(z,p)$ is the potential function for a regular potential game G, where the strategy sets Z and Π are the interiors of non-empty, convex, compact subsets of R^K, and the potential function is essentially smooth and strictly biconcave, then G has an odd number of locally unique, pure strategy Nash equilibria.*

Corollary 8 *If $\Theta(z,p) = < U(z), p > -J^*(p)$ is the potential function for a regular ADM potential game G, where Z is the interior of the budget hyperplane and Π is the*

[6]This theorem is due to Roger Howe - personal communication (March 2010).

interior of the probability simplex, and the potential function is essentially smooth and strictly biconcave, then G has an odd number of locally unique, pure strategy Nash equilibria.

Proposition 9 *If $\Theta(z,p)$ is the potential function for a potential game G, where the strategy sets Z and Π are the interiors of non-empty, convex, compact subsets of R^K, and the potential function is essentially smooth and strictly biconcave, then the set of pure strategy Nash equilibria is non-empty.*

Corollary 10 *If $\Theta(z,p) = < U(z), p > -J^*(p)$ is the potential function for an ADM potential game G, where Z is the interior of the budget hyperplane and Π is the interior of the probability simplex, and the potential function is essentially smooth and strictly biconcave, then the set of pure strategy Nash equilibria is non-empty.*

The conditions for uniqueness, given below, extend Neyman's (1997) theorem on the uniqueness of pure strategy Nash equilibrium for potential games with compact strategy sets to potential functions with bounded, open strategy sets.

Proposition 11 *If $\Theta(z,p) = < U(z), p > -J^*(p)$ is the potential function for a regular ADM intrapersonal game G, where the strategy set Z is the interior of the budget hyperplane and strategy set Π is the interior of the probability simplex, is essentially smooth and strictly concave, then G has a unique pure strategy Nash equilibria.*

In the insurance example, the affective cost function depends on baseline probabilities p_0. In the general K-states framework the analogous concept is Bregman divergence—a generalization of relative entropy, a concept in information theory used to measure the "directed distance" from a fixed probability distribution p_0 to other probability distributions p; see Banerjee et al. (2005) for a general discussion of Bregman divergences. Importantly, every convex function of Legendre type J^* on the interior of Δ and "prior" probability distribution p_0 in the interior of Δ defines a Bregman divergence $D(p_0, p)$ of Legendre type, where

$$D(p_0, p) \equiv J^*(p) - J^*(p_0) - \nabla J^*(p_0) \cdot (p - p_0).$$

For our purposes, notice that (i) for all p_0 and p in the interior of Δ, $D(p_0, p) \geq 0$ and (ii) $D(p_0, p_0) = 0$. Hence p_0 is the minimum of $D(p_0, p)$ on Δ or the baseline probabilities in the insurance example. That is, $D(p_0, p)$ is the "directed distance" from p_0 to $p \in \Delta$.

If $g^*(p) = -f^*(p)$, where $f^*(p)$ is a Bregman divergence, then we define $g^*(p)$ as a dual Bregman divergence. Relative entropy, $J^*(p) \equiv \sum_{j=1}^{j=K} [p_j \lg(p_j/p_{0j})]$, is a Bregman divergence of Legendre type and is of special interest as $H^*(p) = - J^*(p)$—its dual Bregman divergence of Legendre type—is the affective cost function in the multiplier preferences model (Hansen and Sargent 2000). That is, the structure of the cost function we suggest is consistent with a well-known cost function used in the literature.

Given this, here is an example of an essentially smooth, strictly biconcave *ADM* intrapersonal potential game:

Let

$$u(w) = w^{\delta}, where\ \delta \in (0,1)$$

and

$$J^{*}(p) \equiv \sum_{j=1}^{j=K} [p_j \lg(p_j/p_{0j})],$$

then

$$\Theta(z,p) = \sum_{j=1}^{j=K} z_j^{\delta} p_j - \sum_{j=1}^{j=K} [p_j \lg(p_j/p_{0j})]$$

The general setup of ADM potential games can be applied to insurance markets in a straightforward manner: instead of the two states considered in section 3 now the agent faces K possible future states of the world with an associated wealth-level vector w (where w_i is the wealth level in state $i \in K$). The different acts available to the agent are equivalent to different insurance vectors I, and the rational process chooses a utility vector $U(I)$ to maximize expected utility given a perceived risk p. Specifically, the rational process maximizes $< U(I), p >$, while the emotional process chooses an optimal risk perception (p^*) given an insurance level I, to maximizes $\Theta(I,p) = \{< U(I), p > -J^{*}(p)\}$. In the general case, we require that the mental cost function $J^{*}(p)$ be a strictly convex and essentially smooth function on the interior of the probability simplex Δ, meaning it is a convex function of the Legendre type. Given this set up, all results of the general ADM potential games immediately follow.

5 Axioms for Optimistic Preferences

In this section we show that attitudes towards optimism reduce to the convexity of the utility representation of preferences over acts, and use this property to derive axioms for optimistic preferences. We show that preferences over acts, z, are optimistic if and only if there exists a continuous utility function u over outcomes and a continuous convex function J over state-utility vectors $U(z)$, that is, the utility vector for the act z. The ADM model we present below is an example of optimistic preferences over acts.

We use the Legendre-Fenchel conjugate of a continuous convex function $J(U(z))$ to represent optimistic preferences as ADM potential games. That is, the Legendre-Fenchel conjugate is:

$$J^{*}(p) \equiv max_{U(z)\in R_{+}^{K}} \{< U(z), p > -J(U(z))\}.$$

It follows from the biconjugate theorem that

$$J(U(z)) = max_{p\in\Delta} \{< U(z), p > -J^{*}(p)\}.$$

That is, the double conjugate of J, $(J^{*})^{*} = J$. This equivalence is the same as the dual relationship between the cost and profit functions of a price-taking, profit-maximizing firm producing a single good.

Since we assume $J^*(p)$ is a convex function of Legendre type, that is, $J^*(p)$ is a strictly convex, essentially smooth function on the interior of the probability simplex Δ, its Legendre conjugate and biconjugate are well defined—see Theorem 26.5 in Rockafellar (1970) as previously stated in section 4. The potential function for the associated ADM potential game is

$$\Theta(z,p) \equiv < U(z), p > -J^*(p).$$

Next, we derive the set of axioms characterizing optimistic preferences, and show that these axioms also characterize the ADM potential maximizers. That is,

$$\arg\max_{z\in Z} J(U(z)) = \arg\max_{z\in Z, p\in\Delta} \Theta(z,p).$$

The axiomatic characterization of optimistic preferences amends the axiomatic characterization of variational preferences in Maccheroni, Marinacci and Rustichini (2006) $[MMR]$, where: S is the set of states of the world; Σ is an algebra of subsets of S, the set of events; and X, the set of consequences, is a convex subset of some vector space. F is the set of (simple) acts, that is, finite-valued Σ-measurable functions $f : S \longrightarrow X$. $B(\Sigma)$ is the set of all bounded Σ -measurable functions, and endowed with the sup-norm it is an AM-space with unit, the constant function 1. $B_o(\Sigma)$ the set of Σ-measurable simple functions is norm dense in $B(\Sigma)$. The norm dual of $B(\Sigma)$ is $ba(\Sigma)$, finitely additive signed measures of bounded variation on Σ (see Aliprantis and Border 1999 for further discussion). Below we present the axioms:

$A.1$ (Weak Order): If f, g, $h \in F$, (a) either $g \succsim f$ or $f \succsim g$, and (b) $f \succsim g$ and $g \succsim h \Longrightarrow f \succsim h$.

$A.2$ (Weak Certainty Independence): If f, $g \in F$, x, $y \in X$ and $\alpha \in (0,1)$, then
$\alpha f + (1-\alpha)x \succsim \alpha g + (1-\alpha)x \Longrightarrow \alpha f + (1-\alpha)y \succsim \alpha g + (1-\alpha)y$.

$A.3$ (Continuity): If f, g, $h \in F$, the sets$\{\alpha \in [0,1] : \alpha f + (1-\alpha)g \succsim h\}$ and
$\{\alpha \in [0,1] : h \succsim \alpha f + (1-\alpha)g\}$ are closed

$A.4$ (Monotonicity): If f, $g \in F$ and $f(s) \succsim g(s)$ for all $s \in S$,the set of states, then $f \succsim g$.

$A.\widehat{5}$ (Quasi-Convexity): If f, $g \in F$ and $\alpha \in (0,1)$, then
$f \sim g \Longrightarrow \alpha f + (1-\alpha)g \precsim f$.

$A.6$ (Nondegeneracy): $f \succ g$ for some f, $g \in F$.

These axioms where $A.\widehat{5}$ is replaced by $A.5$ (quasi-concavity) are due to MMR (2006).

Theorem 12 *Let $\succsim$ be a binary order on F. The following conditions are equivalent:*

(1) The relation $\succsim$ satisfies axioms $A.1 - A.6$

(2) There exists a nonconstant function $u : X \longrightarrow R$, unique up to a positive affine transformation, and a continuous, convex function $J^* : \Delta \longrightarrow [0,\infty]$ where for all f, g

$\in F,\ f \succsim g \iff W(f) \geq W(g)$, *where* $W(h) = J(U(h)) = \max_{p\in\Delta}\{\langle U(h), p\rangle - J^*(p)\}$ *is a convex function of* $U(h)$ *by the biconjugate theorem.*

In the standard approach, as in the expected utility theory and prospect theory, the agent maximizes over actions not over both action and beliefs. In our setup, $\arg\max_{z\in Z} J(U(z)) = \arg\max_{z\in Z, p\in\Delta} \Theta(z,p)$, are the potential maximizers, a subset of the set of pure strategy Nash equilibria of the ADM potential game. If the ADM model has a unique pure strategy Nash equilibrium, then maximizing the composite utility function $J(U(z))$ over actions and maximizing the potential $\Theta(z,p)$ over actions and beliefs rationalize the same observed choices. Hence, these models are refutable. That is, not every dataset can be rationalized with an ADM potential game.

6 Discussion

The set of axioms we present, and therefore the ADM model, is closely related to existing models in the literature: if axiom $\widehat{5}$ — ambiguity-seeking — is replaced by MMR's axiom 5—ambiguity aversion — then we get MMR's variational preferences model. If we replace axiom $\widehat{5}$ — ambiguity-seeking — with axiom $\widetilde{5}$ of ambiguity-neutrality we get the subjective expected utility model (both results are proved in MMR 2006). This relationship suggests that optimism bias and pessimism bias are equivalent to ambiguity-seeking and to ambiguity-aversion, respectively. That is, the ADM model has an alternative interpretation of ambiguity-seeking behavior and variational preference has an alternative interpretation of pessimism bias, where instead of a game against nature the game is against the emotional self that assigns a higher probability to the least favorable outcome.

We view these two interpretations (optimism bias and ambiguity-seeking) as two concepts that apply to different parallel choice situations: in risky situations, when the decision maker does have information, the appropriate interpretation of preference should be in terms of optimism or pessimism. In these situations, although decision makers have information they also have some freedom in choosing their probabilistic beliefs, as we argue in this paper. Hence, one can view such risky situations as instances of *endogenous ambiguity*, ambiguity that is generated by the individual and is created in a skewed manner. If the individual is optimistic then the generated endogenous ambiguity would be favorable to her; therefore being optimistic is being ambiguity-seeking in this case. In other words, the individual prefers the situation where she "does not know" the exact probability distribution over states of the world. If the individual is pessimistic, then the generated endogenous ambiguity would be unfavorable to her; therefore being pessimistic is being ambiguity-averse. In that sense, attitudes toward ambiguity are equivalent to holding optimistic or pessimistic attitudes. In contrast, uncertain or ambiguous situations are cases where the decision maker has no information regarding the probability distribution over future states of the world. Hence one can view such ambiguous situations as instances of *exogenous ambiguity*, meaning ambiguity that is imposed on the individual.

Using this distinction between endogenous and exogenous ambiguity, the existing evidence can be summarized as follows: evidence on ambiguity aversion is found in situations with no information—exogenous ambiguity—while optimism bias is found in situations where the decision maker has some information and freedom to choose her beliefs; these are instances of endogenous ambiguity. That is, we would expect to find ambiguity-seeking a là ADM in endogenously ambiguous situations, while we would expect ambiguity-aversion a là MMR in exogenously ambiguous situations. In fact, in their study of ambiguity Heath and Tversky (1991) state that: "We investigate the relation between judgments of probability and preferences between bets. A series of experiments provides support ... that people prefer betting on their own judgment over an equiprobable chance event when they consider themselves knowledgeable, but not otherwise. They even pay a significant premium to bet on their judgments. These data cannot be explained by aversion to ambiguity, because judgmental probabilities are more ambiguous than chance events."

This distinction between self-generated or imposed ambiguity helps understand the relationship between the game against nature in variational preferences and the game against the emotional process in the ADM model: in both the variational preferences model and the ADM model, nature and the emotional process, respectively, are formal descriptions of uncertainty. In MMR this is a formal description of an imposed uncertainty, while in the ADM model it is a formal description of generated uncertainty. Even if one considers the unique equilibrium ADM case, the emotional self still generates ambiguity in the sense that it allows the agent to consider several possible probability distributions. The attitude towards ambiguity is either negative —when uncertainty is imposed on the agent—or positive—when uncertainty is generated by the agent.

7 Appendix: Proofs

Proof of Proposition 2. Denote the rational process's payoff function as (R) and the emotional process's payoff function as (E). A necessary and sufficient condition for the intrapersonal game to have a potential function (Monderer and Shapley 1996) is $\frac{\partial^2 R}{\partial p \partial I} = \frac{\partial^2 E}{\partial p \partial I}$. This condition clearly is satisfied in the ADM model. The potential function $\Theta(p, I)$ is a function such that (Monderer and Shapley, 1996): $\frac{\partial \Theta}{\partial p} = \frac{\partial E}{\partial p}$, $\frac{\partial \Theta}{\partial I} = \frac{\partial R}{\partial I}$. Because $\frac{\partial E}{\partial I} = \frac{\partial R}{\partial I}$, (E) can serve as a potential function. The critical points of the potential function are $\frac{\partial \Theta}{\partial p} = \frac{\partial E}{\partial p} = 0$, $\frac{\partial \Theta}{\partial I} = \frac{\partial R}{\partial I} = 0$. The potential function is strictly concave in each variable, so at each critical point, each process is maximizing its objective function given the strategy of the other process. Therefore, the critical points of the potential function are the pure strategy Nash equilibria of the intrapersonal game, and all pure strategy Nash equilibria are critical points of the potential function. ■

Proof of Proposition 3. Given the assumption on the mental cost function, we know the relationship between the emotional process and the rational process best response at the extreme beliefs $\{0, 1\}$. We know that as $p \to 0$, the rational process best response would be "higher", since the optimal insurance given such a belief is greater than the required insurance level to support these beliefs (which would be infinite). The relationship exactly reverses when $p \to 1$. Hence, there exists a pure strategy Nash equilibria; since the best responses are monotonically increasing, it follows that there exists an odd number of Nash equilibria. ■

Proof of Proposition 4. Given the geometry of the problem, the Nash equilibria are not at the boundary; hence, one can always find values $0 < \underline{p} < \overline{p} < 1$ such that the set of the game's Nash equilibria $(p^{'}, I^{'})$ satisfy $p^{'} \in [\underline{p}, \overline{p}]$, and $I^{'} \in [I^*(\underline{p}), I^*(\overline{p})]$, where $I^*()$ represents the rational process's best response. One can therefore restrict attention to an intrapersonal game where the players' strategy spaces are compact. Neyman (1997) proved that a potential game with a strictly concave, smooth potential function, and one in which all players have compact, convex strategy sets, has a unique pure strategy Nash equilibrium. That is, the Hessian of the potential function is negative definite, which in the insurance example reduced to the stated condition. ■

Proof of Proposition 5. Consider the case in which $\gamma = p_0$. At full insurance, there is no mental gain for holding beliefs $p \neq p_0$ but holding these beliefs exacts a mental cost. Therefore, at the full insurance level, the mental process's best response is $p = p_0$. Given that $\gamma = p_0 = p$, the rational process's best response is full insurance. Consequently, for this case full insurance and $p = p_0$ is a Nash equilibrium. Next, consider the case $\gamma > p_0$; because the insurance premium is higher than p_0, $I^*(p = p_0) < z$. Also, $p^* = p_0$ only at the full insurance level, where $I = z$. Therefore, at $p = p_0$ the mental process's best response falls above the rational process's best response. This relationship is reversed at the limit $p \to 0$, and both the mental and the rational best responses increase. Therefore, there exists a Nash equilibrium with $p < p_0$ and less insurance than predicted by the expected utility model. A similar argument can be used to prove the result when $\gamma < p_0$.

■

Proof of Proposition 6. Define $\tilde{I}(p;p_0)$ as the inverse function p^{*-1}. Define $\Pi(p;p_0) = I^*(p) - \tilde{I}(p;p_0)$; a solution $\Pi(p;p_0) = 0$ is a Nash equilibrium, and $\frac{\partial \Pi}{\partial z} < 0$, where z is the loss size represents the unintended consequence of greater optimism due to such campaigns.

$$\frac{\partial \Pi}{\partial z} < 0 \Leftrightarrow \frac{\frac{\partial \tilde{I}}{\partial z}}{\frac{\partial I^*}{\partial z}} > 1$$

.

$$\frac{\partial I^*}{\partial z} = \frac{\left[u^{''}(w_G - z + (1-\gamma)I^*)\right]\left[u^{'}(w_G - \gamma I^*)\right]^2}{[u^{'}(w_G - \gamma I^*)]\left[\begin{array}{c} u^{''}(w_G - z + (1-\gamma)I^*)u^{'}(w_G - \gamma I^*)(1-\gamma) \\ +u^{'}(w_G - z + (1-\gamma)I^*)u^{''}(w_G - \gamma I^*)\gamma \end{array}\right]};$$

$$\frac{\partial \tilde{I}}{\partial z} = \frac{\left[u^{'}(w_G - z + (1-\gamma)\tilde{I})\right]}{\left[u^{'}(w_G - z + (1-\gamma)\tilde{I})(1-\gamma) + u^{'}(w_G - \gamma \tilde{I})\gamma\right]} \Rightarrow$$

$$\frac{\partial \Pi}{\partial z} < 0 \Leftrightarrow \frac{r(w_G - \gamma I)}{u^{'}(w_G - \gamma I)} > \frac{r(w_B + (1-\gamma)I)}{u^{'}(w_B + (1-\gamma)I)}, \text{where } r(x) = -\frac{u^{''}(x)}{u^{'}(x)}.$$

Define $\hat{p}(I)$ as the inverse function I^{*-1}. Define $\hat{\Pi}(I) = p^*(I) - \hat{p}(I)$; a solution $\hat{\Pi}(I) = 0$ is a Nash equilibrium, and $\frac{\partial \hat{\Pi}}{\partial z} < 0$, where z is the loss size represents the unintended consequence of obtaining less insurance due to such campaigns.

$$\frac{\partial \hat{\Pi}}{\partial z} < 0 \Leftrightarrow \frac{\frac{\partial \hat{p}}{\partial z}}{\frac{\partial p^*}{\partial z}} > 1$$

.

$$\frac{\partial p^*}{\partial z} = -\frac{u^{'}(w_B + (1-\gamma)I)}{J^{''}(p)};$$

$$\frac{\partial \hat{p}}{\partial z} = -\frac{pu^{''}(w_B + (1-\gamma)I)(1-\gamma)(-1)}{u^{'}(w_B + (1-\gamma)I)(1-\gamma) + u^{'}(w_G - \gamma I)\gamma} \Rightarrow$$

$$\frac{\partial \hat{\Pi}}{\partial z} < 0 \Leftrightarrow \frac{u^{'}(w_B + (1-\gamma)I)}{r(w_B + (1-\gamma)I)} > p(1-p)J^{''}(p), \text{where } r(x) = -\frac{u^{''}(x)}{u^{'}(x)}.$$

Note that when $\frac{\partial \Pi}{\partial z} < 0$ or $\frac{\partial \hat{\Pi}}{\partial z} < 0$ the result would be greater optimism.and less insurance respectively, even at an unstable equilibrium (under the usual Tatoumount adjustment process) This is because as a result of the change, a person whose choice was according to an unstable Nash equilibrium would converge now to a stable, lower, Nash equilibrium.

■

The proofs of propositions 7 and 9 use the homotopy principle–see chapters $1, 3$, and 22 in Garcia and Zangwill (1981). The homotopy principle admits an algorithmic interpretation that can be used to compute a pure strategy Nash equilibrium of the potential game—see chapter 2 in Garcia and Zangwill.

Proof of Proposition 7. Consider the following homotopy: $H(t, \theta, \theta_0) = (1-t)(\theta -$

$\theta_0) + t\nabla_\theta[P(\theta)]$, where $\theta = (z, \pi)$, $\theta_0 = (z_0, \pi_0)$ and $t \in [0, 1]$. Zero (0) is a regular value of $H(0, \theta, \theta_0)$, since $[\partial H(0, \theta, \theta_0)/\partial\theta] = I_{2K}$, the identity matrix on $R^K \times R^K$. Zero (0) is also a regular value of $H(1, \theta, \theta_0)$, since $[\partial H(1, \theta, \theta_0)/\partial\theta] = \nabla_\theta[P(\theta)]$ and $P(\theta)$ is a Morse function. For $t \in (0, 1)$, $[\partial H(t, \theta, \theta_0)/\partial\theta_0] = -\ I_{2K}$. Hence 0 is a regular value of $H(t, \theta, \theta_0)$ for all $t \in (0, 1)$ by the transversality theorem (parametric Sard's theorem). That is, 0 is a regular value of $H(t, \theta, \widehat{\theta_0})$ for almost all $\widehat{\theta_0} \in \Theta$, where $\Theta \equiv K \times \Pi$—see chapter 2 in Guillemin and Pollack for a proof of the transversality theorem. The assumption that $P(\theta)$ is essentially smooth, that is, $\|\nabla_\theta[P(\theta_n)]\| \to \infty$ as $\theta_n \to bdry(\Theta)$, implies that the homotopy is boundary-free. Hence by the homotopy principle, $\nabla_\theta[P(\theta)]$ has an odd number of regular points—see the proof of Theorem 3.2.3 in Garcia and Zangwill. Since $P(\theta)$ is strictly biconcave, it follows that $P(\theta)$ has an odd number of locally unique, pure strategy Nash equilibria. ■

Proof of Corollary 8. Proof is immediate. ■

Proof of Proposition 9. If the set of pure strategy Nash equilibria is empty, then the set of critical points is empty and 0 is a regular value, contradicting proposition 7. Hence there exists at least one singular critical point. That is, there exists at least one pure strategy Nash equilibrium. ■

Proof of Corollary 10. Proof is immediate. ■

Proof of Theorem 11. Axioms 1-4 are used in MMR to derive a nonconstant utility function, u, unique up to a positive affine transformation, over the space of consequences, X. The utility function u is extended to the space of simple acts, F, using certainty equivalents. That is, $U(f) = u(x_f) \in B_o(\Sigma)$ for each $f \in F$, where x_f is the certainty equivalent of f. This is lemma 28 in MMR, where $I(f) = U(f)$ is a niveloid on $\Phi = \{\varphi : \varphi = u(f) \text{ for some } f \in F\}$. Niveloids are functionals on function spaces that are monotone: $\varphi \leq \eta \Longrightarrow I(\varphi) \leq I(\eta)$ and vertically invariant: $I(\varphi + r) = I(\varphi) + r$ for all φ and $r \in R$—see Dolecki and Greco (1995) for additional discussion. Φ is a convex subset of $B(M)$ and by axiom 5 in Schmeidlers (1989), I is quasi-concave on Φ. We also assume axioms1-4, so lemma 28 in MMR holds for the niveloid J in the ADM representation theorem. By axiom $\widehat{5}$, J is quasi-convex on Φ. MMR (2006) show in lemma 25 that I is concave if and only if I is quasi-concave. Hence J is convex if and only if J is quasi-convex, since J is convex(quasi-convex) if and only if $-J$ is concave(quasi-concave). MMR extend I to a concave niveloid $\widehat{I}$ on all of $B(\Sigma)$—see lemma 25 in MMR. Epstein, Marinacci, and Seo [EMS] (2007) show in lemma $A.5$ that niveloids are Lipschitz continuous on any convex cone of an AM-space with unit and concave(convex) if and only if they are quasi-concave(convex). Hence, since $B(\Sigma)$ is a convex cone in an AM-space with unit, $\widehat{I}$ is Lipshitz continuous. It follows from the theorem of the biconjugate for continuous, concave functionals that $I(\varphi) = \inf_{p \in ba(\Sigma)} \left\{\int \varphi dp - \widehat{I}^*(p)\right\}$, where $\widehat{I}^*(p) = \inf_{\varphi \in B_o(\Sigma)} \left\{\int \varphi dp - \widehat{I}(\varphi)\right\}$ is the concave conjugate of $\widehat{I}(\varphi)$—see Rockafellar (1970, p.308), for finite state spaces. On pg. 1476 MMR show that we can restrict attention to Δ, the family of positive, finitely additive measures of bounded variation in $ba(\Sigma)$. Hence $I(\varphi) = \min_{p \in \Delta} \left\{\int \varphi dp - \widehat{I}^*(p)\right\} = \min_{p \in \Delta} \left\{\int u(f) dp + c(p)\right\}$, where

$\varphi = u(f)$ and

$J^*(p) = -\widehat{I}^*(p)$. $J^*(p)$ is convex since $\widehat{I}\,^*(p)$ is concave.

Extending -J to -$\widehat{J}$ on $B(\Sigma)$, using lemma 25 in MMR, it follows from the theorem of the biconjugate for continuous, convex functionals that

$$J(\varphi) = \max_{p \in ba(\Sigma)} \left\{ \int \varphi dp - \widehat{J}^*(p) \right\},$$

where

$$\widehat{J}^*(p) = \max_{\varphi \in B_o(\Sigma)} \left\{ \int \varphi dp - \widehat{J}(\varphi) \right\}$$

is the convex, conjugate of $\widehat{J}(\varphi)$—see Rockafellar (1970, p.104), for finite state spaces and Zălinescu (2002, p.77), for infinite state spaces. Again it follows from MMR that

$$J(\varphi) = \max_{p \in \Delta} \left\{ \int \varphi dp - \widehat{J}^*(p) \right\} = \max_{p \in \Delta} \left\{ \int u(f) dp - J^*(p) \right\} = W(f),$$

where $\varphi = u(f)$ and $J^*(p) = \widehat{J}^*(p)$. $J^*(p)$ is convex since $\widehat{J}\,^*(p)$ is convex.

$$f \succsim g \iff J(u(f)) \geq J(u(g)) \iff W(f) \geq W(g).$$

Hence $\arg\max_{f \in F, p \in \Delta} \left\{ \int u(f) dp - c(p) \right\} \subseteq$ **set** of pure strategy Nash equilibria of the ADM intrapersonal game, where $u(\cdot)$ is the Bernoulli utility function of the rational process and $\widehat{J}^*(\cdot)$ is the cost function of the emotional process. It follows that the axioms for ambiguity-seeking preferences also characterize the ADM intrapersonal games with a unique pure strategy Nash equilibrium: $\arg\max_{f \in F, p \in \Delta} \left\{ \int u(f) dp - J^*(p) \right\}$. ■

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