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Occupation-Level Income Shocks and Asset Returns: Their Covariance and Implications for Portfolio Choice

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Abstract:

This paper develops and applies a simple graphical approach to portfolio selection that accounts for covariance between asset returns and an investor's labor income. Our graphical approach easily handles income shocks that are partly hedgeable, multiple risky assets, multiple risky assets, many periods, and life cycle considerations.

We apply the approach to occupation-level components of individual income innovations estimated from repeated cross sections of the Current Population Survey. We characterize several properties of these innovations, including their covariance with aggregate equity returns, long-term bond returns, and returns on several other assets. Aggregate equity returns are uncorrelated with the occupation-level income innovations, but a portfolio formed on firm size is significantly correlated with income innovations for several occupations, and so are selected industry-level equity portfolios.

An application of the theory to the empirical results shows (a) large predicted levels of risky asset holdings compared to observed levels, (b) considerable variation in optimal portfolio allocations over the life cycle, and (c) large departures from the two-fund separation principle.

JEL Classifications: G11, D91, D52, J30

Key words: life cycle portfolio choice, risky labor income, graphical approach, occupation-level income shocks

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1 Introduction

This paper develops and applies a simple graphical approach to portfolio selection that accounts for covariance between asset returns and an investor's labor income. Our graphical approach easily handles the realistic case in which income shocks are partly, but not fully, hedgeable.[2]

We first show how covariance between income shocks and asset returns and persistence in the shocks affect portfolio choice over the life cycle. Next, we estimate the covariance and persistence parameters for occupation-level components of individual income using data from the Current Population Survey. After extracting the occupation-level components of individual income innovations, we investigate their covariance with aggregate equity and bond returns, selected industry-level equity returns, and the returns on portfolios formed on firm size and book-to-market equity values. We then apply the theoretical framework to the empirical results to calculate optimal portfolio allocations over the life cycle for selected occupations.

Our graphical approach captures several factors that influence portfolio choice over the life cycle: the drawdown of human capital as a worker ages, the impact of labor income innovations on the present value of lifetime resources, the increase in an investor's effective risk aversion as income smoothing ability declines with age, and systematic life cycle variation in the covariance between labor income shocks and asset returns. Each of these factors affects an investor's optimal level of risky asset holdings, as we show below.

According to the two-fund separation principle of traditional mean-variance portfolio analysis, every investor holds risky financial assets in the same proportions — only the level of holdings differs among investors. We show why and how that principle breaks down when an investor has a risky income stream (from work or business ownership) that is correlated with asset returns. We quantify this breakdown and several contributory factors. Our application of the theory shows that even moderate covariances between income shocks and asset returns can drive large differences between optimal portfolio shares and the shares implied by a more traditional approach that ignores labor income or other sources of income from nonmarketable assets.

The chief empirical inputs into our theoretical framework include the first two

[2]Bodie, Merton, and Samuelson (1992) derive analytical solutions for portfolio choice in a continuous time finite horizon setting with fully hedgeable labor income risks. Much other work adopts computationally intensive approaches to the portfolio implications of unhedgeable or partly hedgeable labor income risks. See, for example, Cocco, Gomes, and Maenhout (1999) for analysis in a finite horizon setting and Heaton and Lucas (1997), Viceira (1998), and Haliassos and Michaelides (1999) in infinite horizon settings.

moments of the asset return distribution and the covariance between income shocks and asset returns. While asset returns themselves receive enormous attention from researchers, only a handful of previous studies investigate their covariance with labor or proprietary business income. Campbell et al. (1999) consider the covariance between aggregate equity returns and the permanent component of household income for three education groups. Davis and Willen (2000) investigate the issue using a synthetic panel approach to demographic groups defined in terms of sex, educational attainment, and birth cohort. Although based on rather different empirical designs, both studies find that the correlation between labor income shocks and equity returns rises with education. Heaton and Lucas (2000) highlight the positive correlation between equity returns and the income of self-employed persons.[3]

Previous empirical research on the covariance between income shocks and asset returns relies on panel data sets or synthetic panels constructed from repeated cross sections. This paper pursues a somewhat different empirical approach. In particular, we rely on the repeated cross-section structure of the Current Population Survey to extract mean occupation-level income shocks, while controlling for a host of observable worker characteristics. We then focus the rest of the empirical investigation on the properties of the occupation-level shocks and their covariance with asset returns.

Our empirical approach has less demanding data requirements than panel-based approaches. It is also highly flexible in the sense that one can easily focus the empirical lens on any type of income shock that can be tied to observable characteristics of individuals, households, or businesses. We consider occupation-level income shocks in this paper, but the same method can be applied to income shocks related to industry, location, firm size, and worker characteristics like education, experience, and job tenure. Because its starting point is a standard human capital earnings regression fit to cross-sectional data, our approach offers a natural bridge between labor economics and finance.

The paper proceeds as follows. Section 1 develops the graphical approach in a two-period setting and explains how to handle multiple risky assets. Section 2 extends the graphical analysis to a many-period setting and analyzes several determinants of life cycle variation in optimal portfolio choice. Section 3 describes the data we use to identify occupation-level income innovations. Section 3 also characterizes the magnitude

[3]Other studies investigate the issue at a more aggregated level in an international setting. Botazzi, Pesenti, and van Wincoop (1996) consider the covariance of national labor income shocks with financial asset returns, and Baxter and Jermann (1997) consider their covariance with the returns on hypothetical claims to a country's capital stock. Davis, Nalewaik, and Willen (2000) consider the covariance between national output shocks and a variety of domestic and foreign asset returns for 18 industrialized countries.

and persistence of the occupation-level income innovations. Section 4 investigates the covariance between the occupation-level income innovations and a variety of asset return measures. Section 5 draws on the empirical results in Sections 3 and 4 to implement the theoretical framework developed in Sections 1 and 2. We calculate optimal portfolio allocations for several occupations under various assumptions about investor age and risk aversion, asset returns, and their covariance with labor income. We use the examples to illustrate life cycle variation in optimal portfolio allocations and the breakdown of two-fund separation.

2 Portfolio Choice with Risky Labor Income: A Graphical Approach

In this section, we develop a graphical approach to portfolio choice when investors face labor income shocks that are correlated with asset returns. Although our approach shares many features with textbook mean-variance analysis, it is fundamentally different. Rather than consider the mean and variance of a portfolio of risky assets, we consider the mean and variance of consumption. Wny do this? Because standard mean-variance analysis gives wrong answers when labor income is correlated with asset returns. Consider the following example: A standard mean-variance investor would never invest in a portfolio with negative expected excess returns and positive variance: a zero portfolio provides higher expected excess returns and lower variance. But if the portfolio is negatively correlated with labor income risk, an investor might well want to purchase such a portfolio: the portfolio reduces consumption but also reduces the variance of consumption. We show below that the failures of standard mean-variance analysis go beyond this simple example: for example, two-fund separation generally fails when asset returns and labor income shocks are correlated.

In Section 2.1, we construct a two-period model of portfolio choice and we show how to solve it graphically using indifference curves and budget sets. In Section 2.2, we explore two interesting aspects of the solution: the failure of two-fund separation and the gains from trade in risky assets. Section 3 extends our approach to a life cycle setting with many periods. Some new issues arise in the many-period life cycle setting, but all of the key points from the two-period setting carry over.

Some mathematical details are contained in the appendix. Willen (1999) and Davis and Willen (2000) provide a more thorough development of the mathematical analysis. Along with Davis, Nalewaik, and Willen (2000), they also consider asset pricing and risk sharing implications of the underlying theoretical model. This paper restricts attention to portfolio choice.

2.1 Portfolio Choice in a Two-Period Model

Consider an investor h who works in occupation i, lives for two periods (t=0,1), and initially has no financial assets. This period she receives labor income y_0^i, and next period she receives stochastic labor income $\tilde{y}_1^i$. Expected income next period is $E\big(\tilde{y}_1^i\big) = \bar{y}_1^i$, and the income innovation is $\eta^i = \tilde{y}_1^i - \bar{y}_1^i$. Our investor has access to $J+1$ financial assets: a riskless bond with certain gross return R_f; J risky securities, each with uncertain gross returns $\tilde{R}_j$. Let $\tilde{\mathbf{R}} = \begin{bmatrix}\tilde{R}_1 & \ldots & \tilde{R}_J\end{bmatrix}'$. We assume that labor income innovations η_1^i and risky asset returns $\tilde{\mathbf{R}}$ are jointly normally distributed. Investor h allocates B dollars to the riskless asset and S_j dollars to each risky asset. Let $\mathbf{S} = \begin{bmatrix}S_1 & \ldots & S_J\end{bmatrix}'$.

Let c_0 and $\tilde{c}_1$ denote consumption in periods zero and one, respectively. The intertemporal budget constraint (in expected value terms) follows from the definitions above:

$$\mathrm{c}_0 + \frac{1}{R_f}\mathrm{E}\big(\tilde{c}_1\big) = \mathrm{y}_0^i + \frac{1}{R_f}\mathrm{E}\big(\tilde{y}_1^i\big) + \frac{1}{R_f}\big(\mathrm{E}\big(\tilde{\mathbf{R}}\big) - R_f\big)'\mathbf{S} = Y + \frac{1}{R_f}\mathbf{ER}'\mathbf{S} = C, \tag{1}$$

where Y is the expected present value of lifetime labor income discounted at the risk-free rate ("human wealth"), $\mathbf{ER} = \mathrm{E}\big(\tilde{\mathbf{R}}\big) - R_f$. We call C, the value of consumption over the life cycle discounted at the riskless rate, "lifetime consumption."

Let the primitive utility function over c_0 and $\tilde{c}_1$ be time separable, and assume that the felicity functions defined over period consumption have the exponential form, $-\exp\big(-Ac\big)$, where $A^h > 0$ governs the degree of risk aversion. This functional form implies constant absolute risk aversion ("CARA") in the face of wealth shocks, although it is easy to handle variation in risk aversion across persons or over the life cycle. As a convenience, assume also that the subjective discount rate equals the riskless rate. Under these conditions, we can write the present discounted value of utility as a function of lifetime consumption and the variance of future consumption:

$$U^h\big(C, V\big) = -\frac{1}{aA^h}\exp\Big\{-aA^h\Big(C - \frac{A^h}{2R_f}V\Big)\Big\}, \tag{2}$$

where A^h measures absolute risk aversion, $a = 1/R_f$ is an annuitization factor, and $V = \mathrm{var}\big(c_1\big)$. And maximizing equation (2) is equivalent to maximizing:

$$C - \frac{A^h}{2R_f}V. \tag{3}$$

2.1.1 Indifference Curves

Figures 1 and 2 show indifference curves generated from equation (2). Each curve traces out combinations of lifetime consumption and consumption variance that leave utility unchanged. As one moves up and to the left, utility increases. Figure 1 shows, for a fixed degree of risk aversion, indifference curves that correspond to different levels of certain lifetime consumption. Figure 2 shows, for a fixed level of certain lifetime consumption, indifference curves that correspond to different degrees of risk aversion. Greater risk aversion steepens the slope of the indifference curve, because a more risk averse investor requires greater compensation for added consumption variance in order to maintain a given utility level.

Two aspects of these indifference curves merit attention. First, the indifference curves are straight lines — the tradeoff between lifetime consumption and consumption variance depends neither on the level of lifetime consumption nor on the variance of consumption.[4] Second, for a given level of risk aversion, all indifference curves in the top panel are parallel. See Figure 2. This means that an increase in lifetime consumption increases utility by the same amount regardless of the level of the variance.

2.1.2 Feasible Sets

In this section, we characterize the the *feasible set* — combinations of lifetime consumption and consumption variance that can be implemented by some feasible portfolio strategy — for any occupation i. To do this, we first consider an investor who chooses the portfolio that minimizes the variance of his or her consumption. Call the corresponding level of lifetime consumption C^i_{mv} and call the corresponding variance of period-one consumption V^i_{mv}. We call the point (C^i_{mv}, V^i_{mv}) the minimum variance point. Equation (4) shows how to use the minimum variance point to characterize the frontier of feasible set for any occupation:

$$C - C^i_{mv} = (1/R_f)\big(V - V^i_{mv}\big)^{1/2}\big(\mathbf{ER}'\mathbf{\Sigma}^{-1}\mathbf{ER}\big)^{1/2}. \qquad (4)$$

We now delve a little more deeply into equation (4). We first show how to derive it for the one-asset case (the more general case is in the appendix). We then discuss two aspects of the feasible set: (1) the shape of the feasible set is determined entirely by the distribution of asset returns and is independent of occupation; (2) the properties

[4] This feature is unique to CARA utility. With other preferences, the curvature of the indifference curves depends on both the level of lifetime consumption and the consumption variance. Specifically, for the common isoelastic specification (constant relative risk aversion), the slope of the indifference curves rises with the variance of consumption and decreases with lifetime consumption.

of occupational income — specifically its mean, variance and the covariance of labor income shocks with risky asset returns — determine the location of the feasible set.

To generate equation (4), we use some basic insights from regression analysis. Suppose there is only one asset and an investor in occupation i invests $\beta_y^i = \operatorname{cov}\big(\tilde{\eta}^i, \tilde{R}\big) / \operatorname{var}\big(\tilde{R}\big)$ in the that asset. Since β_y^i, is the coefficient on an OLS regression of $\tilde{\eta}^i$ on asset returns, the resulting consumption profile is the minimum variance combination of labor income and asset returns. So:

$$C_{mv}^i = Y - \tfrac{1}{R_f} ER\beta_y^i \qquad V_{mv}^i = \operatorname{var}\big(\tilde{\eta}^i - \beta_y^i \tilde{R}\big). \tag{5}$$

We can write any portfolio choice S as $S = \alpha_S - \beta_y^i$. For any portfolio choice S:

$$V = \operatorname{var}\big(\tilde{\eta}^i - \beta_y^i \tilde{R} + \alpha \tilde{R}\big) = V_{mv}^i + \alpha_S^2 \operatorname{var}(\tilde{R}), \tag{6}$$

where the second equality follows from the fact that $\tilde{\eta}^i - \beta_y^i \tilde{R}$ is a residual from a regression and thus orthogonal to $\tilde{R}$. And we can similarly write for any S,

$$C = Y - \tfrac{1}{R_f} ER\beta_y^i + \tfrac{1}{R_f} ER\alpha_S = C_{mv}^i + \tfrac{1}{R_f} ER\alpha_S. \tag{7}$$

Now, solve for α_S using equation (6) and substitute into equation (7) to get equation (4). Note that by setting $S = 0$, $\alpha_S = \beta_y^i$ and lifetime consumption equals lifetime income ($C = Y^i$) and variance of consumption equals variance of income ($V = \operatorname{var}(\tilde{\eta}^i)$). We call this point $(Y^i, \operatorname{var}(\tilde{\eta}^i))$ the *endowment point*.

What does equation (4) tell us? First, equation (4) tells us that the shape of the frontier of the feasible set is independent of occupation and entirely determined by the distribution of asset returns. A unit increase in the difference of the variance of consumption from the minimum variance always leads to the same increase in the level of lifetime consumption, which one can verify in Figures 3 and Figure 4. What does determine the shape? Equation (4) implies that the shape of the feasible set depends on $\big(\mathbf{ER}'\mathbf{\Sigma}^{-1}\mathbf{ER}\big)^{1/2}$, the Sharpe ratio of the tangency portfolio, which simplifies to ER/σ for the one-asset case. Figure 5 illustrates that an increase in the Sharpe ratio increases the size of the feasible set.

Second, equation (4) shows how three aspects of labor income affect the location of the feasible set: the level of income; the variance of income; and the covariance of income with asset returns. The analysis of the first two items is straightforward; the third, covariance, requires more insight and is one of the main contributions of the paper.

How do the mean and variance of occupational income affect the location of the feasible set? Figure 3 shows feasible sets for three different occupations: A, B, and C. None of the three occupations' income profiles covaries with asset returns, so $\beta_y^i = 0$

and $C^i_{MV} = Y^i$ and $V^i_{mv} = \text{var}(\tilde{\eta}^i)$. Occupation B has higher lifetime income and the same variance as occupation A. So the minimum variance of occupation B lies directly above that of occupation B and feasible set B lies directly above feasible set A. Occupation C has the same lifetime income but higher variance of income than occupation A, so by a similar argument, feasible set C lies directly to the left of feasible set A.

How does the covariance of occupational income shocks with asset returns affect the location of the feasible set? Figure 4 shows feasible sets for three occupations that differ only in terms of the covariance of labor income shocks with asset returns. To simplify matters, we again consider a model with a single risky asset. Income for occupation A is uncorrelated with stock returns. So the minimum variance point equals the endowment point. Income in occupation B is positively correlated with stock returns, implying that $\beta^B > 0$, which implies that $C^B_{mv} < Y^B = Y^A = C^A_{mv}$ and $V^B_{mv} < \text{var}(\tilde{\eta}^B) = \text{var}(\tilde{\eta}^A) = V^A_{mv}$. So feasible set B lies below and to the left of feasible set A. To understand the intuition here, consider the actual stock portfolios which correspond to the points in the feasible set. By equation (1) and the definition of Y, $S = \big(C - Y\big)\Big/\big(\text{E}\big(\tilde{R}_1\big) - R_0\big)$. For occupation A, a small long position and a small short position of equal absolute value lead to the same variance of consumption, but, assuming positive excess returns, the long position leads to higher consumption and the short position to lower consumption than the endowment point. For occupation B, the effects of such opposite choices are asymmetric. A small long position leads to an increase in consumption and in the variance of consumption. But a sufficiently small short position leads to a *reduction* in the variance of consumption, because portfolio returns are now negatively correlated with labor income shocks. Income in occupation C is negatively correlated with asset returns and a similar argument shows that the feasible set C is above and to the left of feasible set A.

2.1.3 Portfolio Choice

We solve for the optimal combination of lifetime consumption and variance, and thus the optimal portfolio, by combining the feasible set and the indifference curves in the usual way. Our discussion of portfolio choice proceeds in three steps. First, we introduce the notion of the exposure of a point in the feasible set to a particular risky asset. We then show that every investor has a "desired exposure" determined by his or her absolute risk aversion. And associated with every occupation is a level of "endowed exposure." Then, we show that an investor's demand for the risky asset equals the difference between his or her desired exposure and the endowed exposure associated with his or her chosen occupation.

What do we mean by the exposure of a point in the feasible set? Consider the one-asset case again. Exposure measures the sensitivity of consumption to the risky asset. Mathematically, we measure exposure as the coefficient on risky asset returns from a regression of consumption on risky asset returns and a constant. Consider an arbitrary location (C, V) in the feasible set. The exposure of (C, V) to risky asset returns $\tilde{R}$ equals

$$\beta_c = \frac{\operatorname{cov}\left(\tilde{c}_1, \tilde{R}\right)}{\operatorname{var}(\tilde{R})}. \tag{8}$$

By the regression analysis above:

$$V = V_{mv}^i + (\beta_c)^2 \operatorname{var}(\tilde{R}). \tag{9}$$

Using equation (4),

$$\beta_c = R_f \frac{C - C_{mv}^i}{ER}.$$

In Figure 6, the exposure of points in the feasible set is measured on the right axis. Note that the exposure is zero at the minimum variance point.

Desired exposure measures the level of exposure an investor wants. Mathematically, we can derive desired exposure from the standard consumption Euler equations, which imply, under the maintained assumptions, that at an optimum:

$$\beta_c^h = \frac{ER}{A^h \operatorname{var}(\tilde{R})}. \tag{10}$$

Equation (10) implies that desired exposure is invariant of occupation and depends only on risk aversion and the distribution of asset returns.[5] Graphically, we find desired exposure by finding the indifference curve in the feasible set that yields the highest utility. As usual, this is the indifference curve that is tangent to the feasible set. Desired exposure is the distance from 0 to the point labeled "D" on the right axis in Figure 7. The invariance of desired exposure to occupation follows from the fact that the indifference curves are parallel straight lines and the fact that the shape of the feasible set is invariant to occupation. Thus the position of the desired location in C-V space is always the same relative to the minimum variance point. Figure 8 shows how changes in risk aversion lead to changes in desired exposure. Higher risk aversion leads to steeper indifference curves and consequently lower desired exposure. Also note that as risk aversion increases to infinity, indifference curves converge to

[5]This is only true for CARA utility. In general, since the slope of the indifference curves depends on the variance of consumption, an increase in the variance of income leads to a reduction in desired exposure, a phenomenon called "crowding out." See Bodie, Merton, and Samuelson (1992).

vertical lines, and desired exposure converges to zero. Since risk aversion cannot be negative, desired exposure is always positive.

Endowed exposure measures the level of exposure an investor has if he or she invests nothing in the risky asset. Following the same logic as above, it is easy to see that:

$$\beta_y^i = R_f \frac{Y - C_{mv}^i}{ER}. \tag{11}$$

Graphically, endowed exposure is the distance on the right-hand axis from the minimum variance line to the endowment line. Figure 9 illustrates endowed exposure It is easy to verify that the four occupations listed in the figure all yield the same feasible set. Yet since the endowment points are different, they all yield different amounts of endowed exposure.

To solve for the optimal portfolio, use the budget constraint (equation (1)) and equations (8) and (11) as follows:

$$S = R_f \frac{C - Y}{ER} = R_f \frac{C - C_{mv}^i}{ER} - R_f \frac{Y - C_{mv}^i}{ER} = \beta_c^h - \beta_y^i. \tag{12}$$

In other words, demand for the risky asset equals the difference between desired and endowed exposure. Graphically, demand for the risky asset equals the distance on the right-hand axis between the endowed exposure point and the desired exposure point.

We can now see how risky labor income affects portfolio choice. Figure 10 combines desired exposure from Figure 8 with endowed exposure from Figure 9. Consider an investor with relative risk aversion of 3. His or her desired exposure, point D_1 in Figure 10, equals a little more than \$40 thousand. How much will she invest in the risky asset? That depends on his or her occupation. If he or she is in occupation 4, demand for the risky asset is high, approximately \$70 thousand. If he or she is in occupation 2, then endowed exposure exceeds desired exposure: our investor demands a short position in the risky asset.

For analytical simplicity, much of the preceding analysis focused on a version of the model in which there was only one risky asset. Now we explore portfolio choice in the more realistic case where investors can invest in a whole menu of risky assets. Most of our analysis carries over. The shape of the feasible set remains independent of occupation; the location of the feasible set is determined by the location of the minimum variance point. The main difference is that when there is more than one risky asset each point in the feasible set is associated with a vector of exposure measures, one for each risky asset. Demand for a particular risky asset equals the difference between desired and endowed exposure to that asset.

How do we measure exposure when there are many risky assets? In the single asset case, the exposure of a given point in $C - V$ space equals the coefficient on the risky asset in a regression of consumption on the risky asset and a constant. In the multi-asset case, the exposure of a given point in $C-V$ space to asset j equals the coefficient on risky asset j in a multiple regression of consumption on all the risky assets and a constant. In other words:

$$\boldsymbol{\beta}_c = \mathrm{var}(\tilde{\mathbf{R}})^{-1}\,\mathrm{cov}(\tilde{c}_1, \tilde{\mathbf{R}}).$$

Using the Euler equations again as in equation (10), desired exposure equals:

$$\boldsymbol{\beta}_c^h = \frac{1}{A^h}\,\mathrm{var}(\tilde{\mathbf{R}})^{-1}\mathbf{ER}.$$

And endowed exposure equals:

$$\boldsymbol{\beta}_y^i = \mathrm{var}(\tilde{\mathbf{R}})^{-1}\,\mathrm{cov}(\tilde{\eta}_1^i, \tilde{\mathbf{R}}). \tag{13}$$

And demand for the risky assets equals:

$$\mathbf{S}^h = \boldsymbol{\beta}_c^h - \boldsymbol{\beta}_y^i = \mathrm{var}(\tilde{\mathbf{R}})^{-1}\big[\frac{1}{A^h}\mathbf{ER} - \mathrm{cov}(\tilde{\eta}_1^i, \tilde{\mathbf{R}})\big]. \tag{14}$$

2.2 Features of the Solution

2.2.1 Two-Fund Separation

Two-fund separation holds with labor income if and only if there is no correlation between labor income shocks and asset returns. If income shocks are uncorrelated with asset returns, equation (14) implies that for any two investors, h and g:

$$\mathbf{S}^g = \frac{A^h}{A^g}\mathbf{S}^h.$$

In words, the proportion of total risky asset investment invested in any particular asset must be the same for all investors — two-fund separation holds. But if income shocks are correlated with asset returns, then two-fund separation always fails. It is easy to see why two-fund separation fails when investors are in different occupations. If one investor has relatively more endowed exposure to one asset than another, we would expect relatively less investment in that asset by the more exposed investor. Formally, consider two investors h and g in occupations i and k, respectively. Then equation (14) implies:

$$\mathbf{S}^g = \frac{A^h}{A^g}\mathbf{S}^h + \big(\frac{A^h}{A^g}\big)\boldsymbol{\beta}_y^i - \boldsymbol{\beta}_y^k.$$

What is more surprising is that two-fund separation fails even when two investors are in the *same* occupation. The intuition is that differences in risk aversion only affect desired exposure, not endowed exposure. Formally, the relationship of two investors h and g in the same occupation i is

$$\mathbf{S}^g = \frac{A^h}{A^g}\mathbf{S}^h + \big(\frac{A^h - A^g}{A^g}\big)\boldsymbol{\beta}_y^i.$$

2.2.2 Welfare Gains to Trade in Risky Assets

How valuable is the opportunity to trade risky assets, what we call the gains from trade, to an investor? Let C^* measure the certainty equivalent consumption that corresponds to a particular point in (C, V) space. Equation (2) implies that:

$$C^* = C - \frac{A^h}{2R_f} V.$$

Graphically, the intersection of the indifference curve that passes through (C, V) and the left-hand axis measures the certainty-equivalent consumption level. Equations (1), (9), and (12) imply that at an optimum:

$$C^* = Y^i + \frac{ER}{R_f}(\beta_c^h - \beta_y^i) - \frac{A^h}{2R_f}(V^i + (\beta_c^h - \beta_y^i)^2 \operatorname{var}(\tilde{R})).$$

Equation (10) allows us to eliminate ER and the we use some arithmetic to get:

$$C^* = Y^i - \frac{A^h}{2R_f} V^i + \frac{A^h \operatorname{var}(\tilde{R})}{2R_f}(\beta_c^h - \beta_y^i)^2 = Y^{i*} + \frac{A^h \operatorname{var}(\tilde{R})}{2R_f}(\beta_c^h - \beta_y^i)^2, \qquad (15)$$

where Y^{i*} is the certain equivalent consumption associated with the endowment point. Equation (15) implies that the gains from trade equal

$$C^* - Y^{i*} = \frac{A^h \operatorname{var}(\tilde{R})}{2R_f}(\beta_c^h - \beta_y^i)^2. \qquad (16)$$

We draw attention to three aspects of the gains from trade.

First, the gains from trade are always positive. Mathematically, this follows from the fact that the right-hand side of equation (16) is a square and always positive. Graphically and intuitively, the gains from trade are always positive because the endowment point is always in the feasible set. Since not trading risky assets is always an option, any alternative choice must be preferred by revealed preference. Second, only the difference matters to magnitude of the gain. The welfare gain is the same whether endowed exposure falls short of desired exposure by $25 thousand or exceeds desired exposure by $25 thousand.

Third, the gains to trade are nonlinear in the gap between desired and endowed exposure. For example, if we double the gap, we quadruple the gains from trade. What generates this nonlinearity? At the margin, a dollar investment in a risky asset (with positive excess returns) leads to an ER dollar increase in lifetime consumption, but the sign and magnitude of the change in the variance depend on the level of exposure. In Figure 11, going from exposure level E^4 to E^1 yields the same increase in lifetime consumption as a jump from E^1 to E^3, as does a jump from E^3 to E^2.

By contrast, an increase in exposure from E^4 to E^1 leads to a *fall* in the variance of consumption, a jump from E^1 to E^3 leads to a small increase in the variance, and a jump from E^3 to E^2 leads to a much bigger increase in the variance. Since the change in certain equivalent consumption is a weighted sum of the change in lifetime consumption and the change in variance, the marginal change in certain equivalent consumption (measured on the left axis) inherits the sensitivity of the marginal variance change to the level of exposure. Thus the marginal gains from trade depend on the level of exposure and the total gains from trade depend not only the difference between endowed and desired exposure but also on the level of an investor's endowed exposure.

Figure 11 illustrates the nonlinearity of the gains from trade. For an investor in occupation 1, the gains are roughly \$2 thousand. By contrast, for an investor in occupation 2, the gains are negligible. The difference in correlations between the two occupations is actually quite modest. Our parametric assumption for occupation 1 implies no correlation and for occupation 2 implies a correlation of 0.5. The above analysis suggests that, if there is a fixed cost to trading risky financial assets, even a moderate positive correlation might be enough to dissuade potential investors from participating in risky asset markets.

3 Many Periods and Other Extensions

In this section we show how to extend our model to a life cycle setting. We first show that we can redefine C and V so that all the analysis of section 2 carries over. We then examine how a life cycle setting affects desired and endowed exposure. And then we discuss some issues related to calculation of endowed exposure using time series data.

3.1 Multi-Period Model

We now consider an investor now lives for $T+1$ years ($t = 0, 1, ...T$). As above, the investor starts life with no financial assets. Each period an investor who works in occupation i receives labor income y_t^i, composed of a deterministic component and an income innovation $\eta_t^i = \tilde{y}_t^i - E\left(\tilde{y}_t^i\right)$. Each year except the last, an investor allocates $S_{j,t}$ dollars to each risky asset j and B_t dollars to a riskless bond. Risky asset j pays gross return $\tilde{R}_j$ and the riskless bond pays gross return R_f. Let $\tilde{c}_t$ denote consumption at time t.

At time t an investor chooses random sequences $\{B_s\}_{s=t}^{T-1}$ and $\{\mathbf{S}_s\}_{s=t}^{T-1}$ to maximize:

$$U((\{c_s\}_{s=t}^{T}) = \sum_{s=t}^{T} \delta^{s-t} u^h(c_t) \tag{17}$$

subject to the constraint that:

$$c_t = y_t + R_f B_{t-1} + \mathbf{R}_t' \mathbf{S}_{t-1} - B_t - \mathbf{1}' \mathbf{S}_t \tag{18}$$

for all t.

We make three additional assumptions, which allow for an analytical solution to the optimization problem defined by equations (17) and (18). First, as in Section 2, we assume that period utility is exponential. That is, $u(c) - \exp(-Ac)$. Second, we assume that the joint distribution of income innovations and risky asset returns evolves deterministically over the life cycle. And third, we assume that the joint distribution of income innovations and risky asset returns is normal.

For convenience, we continue to assume that the rate of time preference equals the risk-free rate of interest. We will find it convenient to use an operator that gives the expected present value of a random sequence discounted at the riskless rate:

$$\mathrm{PDV}_t(\{\tilde{z}_s\}_{s=t}^{T}) = \sum_{s=t}^{T} \frac{1}{R_f^{s-t}} E(\tilde{z}_s).$$

In particular, we will let $Y_t = \mathrm{PDV}_t(\{y_s\}_{s=t}^{T})$ and $C_t = \mathrm{PDV}_t(\{c_s\}_{s=t}^{T})$. The period-by-period by constraint (equation (18) implies that:

$$C_t = Y_t + R_f B_{t-1} + \mathbf{R}_t' \mathbf{S}_{t-1} + \mathrm{PDV}_t(\{\mathbf{ER}' \mathbf{S}_s\}_{s=t}^{T-1}).$$

In addition, we will often use the annuity factor $a_t = 1/(\Sigma_{s=t}^{T} 1/R_f^{s-t})$.

3.2 Portfolio Choice

Our maintained assumptions dramatically simplify the dynamic optimization problem defined by equations (17) and (18). Specifically, we can solve for the sequence $\{\mathbf{S}_t\}_{t=0}^{T-1}$ by solving T separate and independent optimization problems, one for each period, each one of which is identical in form to the two-period problem solved in Section 2. Specifically, at time t, choose any arbitrary values for $\mathbf{S}_s, s \neq t$ and for B_{t-1}. An investor chooses $\mathbf{S}_t$ to maximize:

$$C_t - \frac{1}{2} A_{t+1}^h V_{t+1}, \tag{19}$$

where $V_{t+1} = \mathrm{var}(\tilde{C}_{t+1})$, and $A_t^h = a_t A^h$.

It is not surprising that the optimization problem defined by equations (17) and (18), can be viewed as a series of two-period problems. Most well-posed dynamic portfolio choice problems can. What is surprising is that the two-period problems can be solved irrespective of whether the investor's future risky asset allocations are optimal. What generates this independence? Intuitively, we point to two things. First, equation (19) shows that portfolio choice at time t is independent of $\mathrm{PDV}_t(\{y_s\}_{s=t}^T) + R_f B_{t-1} + \mathbf{R}_t' \mathbf{S}_{t-1}$, since adding a constant to an optimization problem has no effect on the solution. Second, by the same logic, future risky asset choice doesn't depend on current risky asset choice, meaning that the investor can ignore the effects of his or her choices this period on his or her future choices. Formally, our independence result depends on two maintained assumptions. First, we assume that utility is exponential, so the level of wealth does not affect its risk tolerance. And second, we assume that the conditional joint distribution of asset returns and labor income shocks is state independent. At any time t, the covariance of asset returns and labor income, for example, is independent of the level of liquid wealth, the level of human wealth, or any previous investment decisions. Surprisingly, this independence result does not depend on the normality of income and asset return shocks. For details of the solution, see the appendix.

Since equation (19) has the same form as equation (3), all the analysis of Section 2 carries over. We can construct a feasible set of combinations of C_t and V_{t+1}. Associated with any point in the feasible set is a new measure of exposure:

$$\boldsymbol{\beta}_{C,t} = \mathrm{var}(\tilde{\mathbf{R}})^{-1} \,\mathrm{cov}(C_t, \tilde{\mathbf{R}}).$$

Again, we can use the Euler equations with respect to the risky assets to calculate desired exposure:

$$\boldsymbol{\beta}_{C,t}^h = \frac{1}{A_t^h} \,\mathrm{var}(\tilde{\mathbf{R}})^{-1} \mathbf{ER}.$$

And endowed exposure equals the exposure of an investor who invests nothing in risky assets:

$$\boldsymbol{\beta}_{Y,t}^i = \mathrm{var}(\tilde{\mathbf{R}})^{-1} \,\mathrm{cov}(Y_t, \tilde{\mathbf{R}}). \tag{20}$$

As in the two-period model, portfolio choice equals the difference between desired and endowed exposure:

$$\mathbf{S}_t = \boldsymbol{\beta}_{C,t}^h - \boldsymbol{\beta}_{Y,t}^i.$$

How does the solution to the multi-period model differ from the solution to the two-period model? In some ways, it does not differ. For example, lower absolute risk aversion leads to higher desired exposure and, all else equal, higher demand for risky assets. And higher covariance of labor income shocks with asset returns leads to higher endowed exposure and, all else equal, lower demand for risky assets. However, there are key differences in the dynamic model. First, an investor's time horizon affects desired exposure — what we call the income smoothing effect. And the the persistence of occupational income shocks affects endowed exposure — what we call the magnification effect. We discuss each in turn.

3.3 The Income Smoothing Effect and Desired Exposure

The shorter the planning horizon, the greater the utility loss caused by a single bad year for an investor. With only one more year to live, a $50 thousand investment loss means a $50 thousand cut in consumption during the last year of life. With a long time left to live, the investment loss can be spread over many years. Since investors ultimately care about consumption and the marginal utility of consumption is declining, a given-size shock to wealth has larger utility consequences for an investor with a shorter planning horizon.

Recall from the two-period case that desired exposure depends only on absolute risk aversion (the slope of the indifference curves). The same property holds in the many-period case, but effective risk aversion becomes $A_t^h = a_t A^h$, where A^h is the individual-specific measure of absolute risk aversion in the primitive utility function and a_t is the marginal propensity to consume (MPC) out of wealth.[6] The MPC is positive and rises monotonically with age, eventually reaching unity in the last period of life. In this model, as in traditional permanent income models, a dollar shock to wealth is spread over the rest of life. The longer an investor has to live, the more years over which to spread a shock. We refer to A_t^h as dynamic absolute risk aversion, because it changes over time as the investor ages and her planning horizon shrinks.

Figure 2 shows indifference curves for different levels of risk aversion. A picture showing different ages would look exactly the same — as an investor grows older, dynamic absolute risk aversion rises and the indifference curves steepen. If the investor's feasible set remains unchanged, she should reduce her holdings of risky assets as she ages.[7] Note, however, that we are talking about levels not proportions.

[6] A^h can also be allowed to vary with age.

[7] This effect arises in any permanent-income type model. That is, the higher the marginal propensity to consume out of wealth, the larger the impact of a dollar shock to wealth on consumption. With CRRA preferences, absolute risk aversion falls at the same rate as wealth, so that the proportion of total wealth invested in the risky asset remains constant (conditional on the covariance, magnification, and other life cycle considerations identified above). This constant-share implication of CRRA preferences is well known. However, since wealth falls (in expectation) over the life cycle, CRRA preferences also imply declining *levels* of risky asset holdings as an investor ages.

As people age, total wealth C_t tends to decline, so that investment in risky financial assets need not decline as a *fraction* of total wealth as an investor ages.

Financial wealth ($B_t + S_t$ at time t) typically grows over an investor's working life, as she converts human capital into financial wealth for retirement. (Financial wealth also tends to grow with the conversion of expected future excess returns into realized excess returns.) Since financial wealth grows over her working life and the level of risky financial asset holdings shrinks, the optimal share of financial wealth in risky assets falls over the life cycle — just as financial planners recommend.[8]

To sum up, the two-period analysis applies to the many-period situation with respect to income smoothing effects, if one replaces A^h with A^h_t.

3.4 The Magnification Effect and Endowed Exposure

In a dynamic model, shocks to income and asset returns affect more than an investor's financial wealth. If an investor can use shocks to asset returns or income to forecast, then shocks convey information about expected future income and affect an investor's beliefs about his or her lifetime income. Consider a tenure-track finance professor at a leading business school. If she is denied tenure and takes a position on Wall Street as a result, her pay will immediately jump up, and her expected future pay will also increase (perhaps even more). With the bad(?) news about tenure, her lifetime income grows by more than her current income. As a result, a modest shock to current income may magnify into a much more dramatic shock to lifetime income. For this reason, we refer to the effect of forecastability on lifetime income as the magnification effect.

Formally, we can see the magnification effect in our characterization of endowed exposure in equation (20). In this paper, we make two assumptions about the labor income process. First, we assume that it is an ARMA process. And second, we assume that asset returns have some forecasting power for future labor income shocks. Under

[8] On the advice of financial planners, see Canner, Mankiw, and Weil (1998) and Ameriks and Zeldes (2000). Since Bodie, Merton, and Samuelson (1992), many researchers have argued that the explanation in the text (growing financial wealth implies shrinking proportion in risky assets) is consistent with financial planner's advice. This is not quite correct — financial planners typically advise a falling proportion of wealth in risky assets even in retirement — *after* the drawdown of human capital is complete. Consider the financial planner's advice related in Ameriks and Zeldes, "The longer you have to invest, the more time you have to weather the market's inevitable ups and downs." This statement is inconsistent with the human capital drawdown explanation, but it is the correct explanation for why the level of investment in risky assets should fall over the life cycle — suggesting that financial planners are mixing up levels and proportions.

these two assumption, we can write an innovation to lifetime income as:

$$Y_t^i - \mathrm{E}_{t-1}(Y_t^i) = \mathrm{PDV}_t\big(\big\{\Psi_s(\mathrm{E}_t(\tilde{\eta}_s^i) - \mathrm{E}_{t-1}(\tilde{\eta}_s^i))\big\}_{s=t}^{T}\big), \tag{21}$$

where $\tilde{\eta}_s^i$ is the time-s innovation in labor income, and Ψ_s measures the impact on lifetime income of unit innovation. When income obeys an ARMA process, Ψ_t summarizes the impact of a current income innovation on the present value of lifetime resources.[9] Using equation (21), we can solve for the covariance of lifetime income and current asset returns:

$$\boldsymbol{\beta}_{Y,t}^i = \mathrm{var}(\tilde{\mathbf{R}}_t)^{-1}\,\mathrm{cov}(Y_t, \tilde{\mathbf{R}}_t) = \mathrm{PDV}_t\big(\big\{\Psi_s\,\mathrm{var}(\tilde{\mathbf{R}}_t)^{-1}\,\mathrm{cov}(\tilde{\eta}_s^i, \tilde{\mathbf{R}}_t)\big\}_{s=t}^{T}\big). \tag{22}$$

To illustrate the magnification effect, we consider three special cases. First, suppose that income shocks are white noise — so we cannot use current income shocks to forecast future income. Then $\Psi_s = 1$ for all s and

$$\boldsymbol{\beta}_{Y,t}^i = \mathrm{PDV}_t\big(\big\{\mathrm{var}(\tilde{\mathbf{R}}_t)^{-1}\,\mathrm{cov}(\tilde{\eta}_s^i, \tilde{\mathbf{R}}_t)\big\}_{s=t}^{T}\big). \tag{23}$$

The expression inside the brackets equals endowed exposure in the two-period model (see equation (13)) to asset returns at different lags. So endowed exposure in the dynamic model equals the present discounted value of the two-period endowed exposure measures. Second, suppose that asset returns do not enable investors to forecast future income but current income shocks are valuable for forecasting. Then

$$\boldsymbol{\beta}_{Y,t}^i = \Psi_t\,\mathrm{var}(\tilde{\mathbf{R}}_t)^{-1}\,\mathrm{cov}(\tilde{\eta}_t^i, \tilde{\mathbf{R}}_t). \tag{24}$$

Here, dynamic endowed exposure equals static endowed exposure multiplied by the effect of a shock to labor income today on lifetime income. Third, suppose that asset returns forecast income only one period ahead. Then endowed exposure equals:

$$\boldsymbol{\beta}_{Y,t}^i = \Psi_t\,\mathrm{var}(\tilde{\mathbf{R}}_t)^{-1}\,\mathrm{cov}(\tilde{\eta}_t^i, \tilde{\mathbf{R}}_t) + (1/R_f)\Psi_{t+1}\,\mathrm{var}(\tilde{\mathbf{R}}_t)^{-1}\,\mathrm{cov}(\tilde{\eta}_{t+1}^i, \tilde{\mathbf{R}}_t). \tag{25}$$

Now, endowed exposure is the sum of endowed exposure to current asset returns and endowed exposure to lagged asset returns discounted at the riskless rate.

3.5 Other Constraints on the Portfolio Allocation Decision

Investors may face a variety of other constraints on portfolio allocation decisions because of ownership positions in privately held firms, employment relationships that

[9] Formally, any ARMA process can be represented by an MA (∞).The MA coefficients ψ_i tell us that $E_t(y_{t+i}) - E_{t-1}(y_{t+1}) = \psi_1\,\tilde{\tilde{\eta}}_t^i$. This means that $\Psi_i = PDV\big(\{\psi_i\}_{i=1}^{T-t+1}\big)$.

require certain equity positions, short-sale constraints on risky assets, and limitations on borrowing ability. These constraints are easily handled in the two-period setting and often in the many-period setting as well.

Consider investors who must hold long positions in particular risky assets. For example, a small business owner is effectively endowed with a long position in her own business. This long position creates an endowed exposure for the small business owner that is analogous to the endowed exposure implied by a worker's human capital. Thus, we can treat the portfolio allocation decision in the same manner as before by simply re-defining income to include profits from the business. Of course, the size, variability, and covariance properties of a small business owner's income stream may differ from that of a worker's, but these facts introduce no new conceptual issues. Likewise, a senior executive at a large firm who must hold restricted stock as a condition of employment is also endowed with a particular exposure. Similarly, a pension fund with required holdings in certain firms, sectors, or geographic regions is effectively endowed with certain exposures. All of these cases can be handled by simply re-defining the endowed risky income stream in the analysis above.

Short-sale constraints on risky assets are also easily handled in the two-period and many-period settings. Geometrically, and with one risky asset, a short-sale constraint chops off the portion of the feasible set that lies below $S=0$. When a short-sale constraint binds for a particular risky asset, it effectively shuts down the investor's ability to participate in that asset market. Hence, her portfolio allocation can be re-computed after restricting attention to the subset of risky assets for which short-sale constraints do not bind. Because the optimal portfolio has an analytical solution in our model, candidate solutions are easily evaluated to determine which set of markets is effectively open to an investor subject to short-sale constraints.

In practice, short-sale constraints are less likely to bind than they might appear for a couple of reasons. First, higher expected returns on risky assets give every investor a motive to adopt a long position. Only when the correlation between income shocks and asset returns is positive and the hedging motive is strong enough will an investor want to adopt a net short position. Second, at the level of a pension fund, for example, short positions taken on behalf of some pension fund beneficiaries can be netted against long positions taken on behalf of other beneficiaries. Thus, a pension fund with a sufficiently diversified pool of beneficiaries can achieve the short positions desired by individual beneficiaries without adopting short positions at the fund level.

Borrowing constraints on the riskless asset are easily handled in the two-period setting. Geometrically, a no-borrowing requirement chops off the portion of the feasible set that lies above the investor's current level of financial assets. If the investor has access to limited borrowing, the constraint on her feasible set is further relaxed. In the many-period setting, borrowing constraints on the riskless asset are not as easy to handle. The added complexity arises because the possibility that borrowing constraints bind in the future alters the investor's attitude toward risky assets and the current consumption-savings choice.

4 Occupation-Level Income Innovations

4.1 Income Data and Selection Criteria

The Current Population Survey (CPS) randomly samples about 60,000 U.S. households every month. Among other items, the survey inquires about labor income, employment status, hours worked, educational attainment, occupation, and demographic characteristics of each household member. The Annual Demographic Files in the March CPS contain individual data on these items for the previous calendar year. Using the CPS March files, we estimate occupation-level components of individual annual earnings from 1967 to 1994.

To compute annual earnings, we use CPS data on wage and salary workers in the private and public sectors who were 23 to 59 years old in the earnings year. We exclude unincorporated self-employed persons from the earnings calculations, but we include self-employment and farm income for persons who were mainly wage and salary workers. We restrict the sample to persons who worked at least 500 hours during the year, and we exclude persons who were students or in the military at least part of the year.[10] In addition to these individual-level selection criteria, we also impose the occupation-level criteria described below.

The detailed occupational classification schemes in the CPS underwent major changes in 1970 and 1982. Where possible, we constructed a uniform classification scheme from 1967 or 1970 to 1994 based on the occupational descriptions in the CPS documentation and an examination of changes over time in occupational cell counts and mean occupational earnings. We dropped individual-level observations that met any of the following occupation-level selection criteria:

- The occupational group could not be extended back to 1970 or earlier in a consistent manner.
- Self-employed persons account for a large fraction of occupational employment (examples include physicians, dentists, lawyers, and farmers).

[10] We also exclude persons who report an hourly wage less than 75 percent of the federal minimum. We handle top-coded earnings observations in the same manner as Katz and Murphy (1992).

- The occupational category is vague (examples include "General Office Supervisors" and "Financial Managers").
- The number of individual-level observations in the occupation had a mean annual cell count less than 100 or a minimum annual cell count less than 50.

These criteria yield 57 detailed occupational classifications that extend from 1967 or 1970 to 1994. The occupational selection criteria reduced the number of individual-level observations by about one-half.

From these 57 occupations, we selected for further analysis 10 occupations with large cell counts and a consistent definition back to 1967. Table 1 lists these occupations and reports summary statistics on cell counts and average annual earnings in 1982 dollars.[11] As suggested by the table, the 10 occupations range widely in terms of educational requirements and annual labor income.

4.2 The Occupation-Level Component of Income Innovations

To extract the occupation-level component of individual earnings shocks, we first fit standard earnings regressions to the individual-level data. We fit separate earnings regressions for each occupation after pooling the data over all available years. For each occupation, we regress real earnings on sex, four educational attainment dummies, a quartic polynomial in age interacted with sex, and a full set of occupation-specific year effects. We estimate one set of regressions using annual earnings as the dependent variable and another using log earnings. The log earnings specification is more commonly used by empirical researchers, but the specification in natural units fits more closely with our theoretical model.

Our specification allows the age-earnings profile to vary freely across occupations (and sex) but not to shift over time. Effectively, we treat the occupation's average age-earnings profile over the 1967–1994 period, adjusted for sex and education, as predictable variation in a worker's expected earnings. As implied by the occupation-level earnings specifications described below, we also treat the average occupational earnings growth from 1967 to 1994 (conditional on worker characteristics) as part of expected earnings growth.

Let ε_t, $t=$ 1967, 1968, ..., 1994, denote the occupation-year effects estimated in the first-stage earnings regressions. To characterize the stochastic properties of the occupation-level component of individual earnings shocks, we fit simple ARMA models to the first-differenced values of the occupation-year effects. Following earlier work

[11] We express earnings in 1982 dollars using the GDP deflator for personal consumption expenditures.

by MaCurdy (1982) using panel data on individuals and by Davis and Willen (2000) using synthetical panel data for demographic groups, we fit second-order moving average processes of the form,

$$\Delta\varepsilon_t = \alpha + \eta_t + \psi_1 \quad \eta_{t-1} + \psi_2 \quad \eta_{t-2}, \tag{26}$$

where η_t denotes the time-t innovation to the occupation-level component of individual earnings shocks. These innovations and their covariance with asset returns are the main focus of the empirical investigation and the applied portfolio analysis in this paper.

It is apparent that our empirical approach ignores selection issues associated with worker mobility across occupational groups and between employment and not working. As a consequence, our estimates of the stochastic process for the occupation-level component of individual earnings may be incorrect even for infra-marginal workers who do not move. A proper treatment of these issues requires long panel data sets. In Davis and Willen (2000), we take the panel requirement seriously by constructing long time series for synthetic persons defined in terms of sex, birth cohort, and educational attainment. Alternatively, one can use true panel data sets such as the Panel Survey of Income Dynamics. In practice, the true panel approach has serious limitations imposed by the nature and size of available data sets.

In the absence of panel data sets that contain rich information about hundreds of thousands (better yet, millions) of persons over substantial portions of their life cycles, we think the empirical approach adopted here is a useful one. It can be readily adapted to investigate other components of individual-level earnings shocks that are correlated with observable worker characteristics — for example, age, job tenure, industry, and location. The main requirements for the approach are large cross-sectional individual-level datasets repeated over a number of years. Such datasets are staples of empirical studies in many countries.

4.3 The Magnitude and Persistence of the Innovations

The standard deviation of η_t in equation (26) quantifies the magnitude of innovations to the occupation-level component of individual earnings. The implied magnitude of the shock to the value of human capital depends on the persistence of η (a function of ψ_1 and ψ_2), the risk-free rate of interest, and the number of years remaining until retirement. By combining these elements, we can easily calculate the magnitude of a typical shock to the occupation-level component of human capital at a given age. The magnitude of this shock declines with age, because fewer years remain until retirement.[12]

Table 2 and Table 3 display the results of fitting (26) for wages measured in natural units and natural logs, respectively. The tables also report the implied present value multipliers on the occupation-level earnings shocks at ages 30 and 50, assuming a real discount rate of 2.5 percent per year and retirement after age 59.

To illustrate the calculation of the human capital shock implied by an occupation-level income innovation, consider the example of accountants and auditors at age 30. According to Table 2, the standard deviation of innovations to the occupation-level component of earnings is 1080 dollars, which equals 4.3 percent of annual earnings. At age 30, the present value multiplier on this innovation is 20.0, so that the implied impact on human capital amounts to 1080(20.0)= $21,600. This figure equals 87 percent of the average annual earnings for accountants and auditors reported in Table 1. As these calculations show, occupation-level earnings innovations are of modest size, but the implied effects on the present value of lifetime earnings are not.

Occupations differ quite a bit in terms of magnitude and persistence of occupation-level earnings innovations. The standard deviation of the occupation-level innovations in Table 3 ranges from 2.9 to 6.9 percent of annual earnings. Plumbers have the most volatile occupation-level earnings component in both dollar and percentage terms, while registered nurses and elementary school teachers have the least volatile.

In most cases, the occupation-level earnings process is less persistent than a random walk. For example, the long-run multiplier on an occupation-level earnings innovation for accountants and auditors equals 1 + (-.18) + (.11) = .93, according to the Table 2. The long run multiplier is much less persistent for electrical engineers (.28) and much more persistent for registered nurses (1.94). Likewise, the present value multiplier at age 30 is 6.8 for plumbers and 40.2 for registered nurses. These two occupations are outliers in terms of persistence. For the other occupations, the present value multipliers at age 30 range from 13 to 27 using the natural units wage measure and from 11 to 26 using the log measure.

The last two columns in Table 2 and Table 3 show how the present value multiplier declines between ages 30 and 50, given our assumptions about discounting and retirement. The age-50 multipliers are fairly sensitive to alternative assumptions about retirement age, but the basic point is not. As workers near retirement, earnings innovations have smaller and smaller effects on lifetime resources.

[12] As we mentioned in Section 2, this simple mechanical effect implies that a worker's endowed exposure to risky financial assets tends to decline with age. It must decline with age if the covariance between labor income innovations and asset returns is nonzero and independent of age. A covariance between labor income innovations and asset returns that rises with age works in the opposite direction of this horizon effect.

5 Covariance between Occupation-Level Income Innovations and Asset Returns

5.1 Covariance with Aggregate Equity Returns

To investigate the covariance between occupation-level earnings innovations and aggregate equity returns, we regress η_t from equation (26) on the realized market rate of return during period t. Recall that the slope coefficient in an ordinary least squares (OLS) regression of y on x can be written as COV(x,y)/VAR(x). Thus, we can use standard regression methods to quantify the covariance between income shocks and equity returns and to test whether the relationship is statistically significant. Other return measures can be introduced as additional regressors to investigate the covariance with multiple assets and to assess the scope for using financial assets to hedge occupation-level earnings risk. The goodness of fit (R^2 value) in this type of regression has an important economic intepretation: it is the estimated fraction of occupation-level earnings risk that can be hedged by a suitably structured asset portfolio.

In unreported regressions, we find little evidence that occupation-level income innovations and aggregate equity returns are linearly related in annual data from 1968 to 1994. At the 10 percent confidence level, none of the 10 occupations shows a statistically significant relationship between income innovations and returns on the value-weighted market portfolio.[13] As a check, we also considered the returns on several other broad-based equity indexes: the S&P 500, the New York Stock Exchange, the Wilshire 5000, and a value-weighted composite of the New York Stock Exchange, American Stock Exchange, and NASDAQ. For each measure, the results showed the same pattern of little or no evidence for a relationship between occupation-level income innovations and contemporaneous aggregate equity returns.

This result is quite puzzling from the vantage point of standard economic theories of growth, fluctuations, and asset pricing. Equilbrium models that obey standard asset-pricing relationships and that embed a conventional specification of the aggregate production technology imply a high positive correlation between aggregate equity returns and shocks to the aggregate value of human capital.[14] We take note of the

[13] As reported on Ken French's web site `http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html#HistBenchmarks`.

[14] By "conventional", we mean a production technology that is approximately Cobb-Douglas over capital and labor. Given a stable Cobb-Douglas technology and a competitive economy, factor income shares are constant over time. Hence, if the same discount rates apply to future capital and labor income, and asset prices reflect fundamentals, the unobserved value of aggregate human

puzzle here, but it is not necessary to resolve it to pursue this paper's agenda.

However, the difficulty of reconciling the empirical finding with standard equilibrium models might lead some readers to discount our evidence. Hence, it is worth remarking that other empirical studies find evidence with a similar flavor. For example, under the assumption that labor income growth follows a random walk, Fama and Schwert (1977) find a near-zero correlation between aggregate equity and human capital returns in the United States. Botazzi et al. (1996) report similar results for several countries. Davis, Nalewaik, and Willen (2000) find little correlation between aggregate output growth and domestic equity returns in regressions for 14 countries. Davis and Willen (2000) consider the correlation between asset returns and shocks to the value of human capital for synthetic persons defined in terms of sex, birth cohort, and educational attainment. The correlations with aggregate U.S. equity returns for these persons are centered near zero, and the goodness-of-fit never exceeds 5 percent of stochastic earnings variation for any group. While they find evidence of statistically significant correlations between equity returns and labor income innovations for some demographic groups, the correlations are rather modest, typically lying in the interval from -0.1 to 0.2. In sum, several studies that consider a variety of countries, time periods, and income components find zero or small correlations between aggregate equity returns and the value of human capital.

Empirical work based on larger samples, different components of labor income, different information sets, longer horizons, or more refined econometric techniques may yet uncover more powerful relationships between labor income innovations and aggregate equity returns. However, the evidence to date strongly suggests that the "market" portfolio is only weakly correlated with innovations in aggregate and group-level measures of labor income. It follows that the market portfolio has modest value as a hedge instrument for the average worker and probably for most occupational and demographic groups as well.

5.2 Other Asset Return Measures

We also investigated the covariance between occupation-level income innovations and the returns on long-term government bonds and other assets. Bond returns are significantly correlated with income innovations for a few occupations, as we report below. In most cases, bonds account for a greater fraction of occupation-level income innova-

capital fluctuates in a manner that is perfectly correlated with the observed value of claims to the aggregate capital stock. Models with these ingredients are standard, but they are hard to reconcile with the emerging body of work the finds low correlations between aggregate equity returns and labor income innovations.

tions when the returns are measured in nominal terms. Hence, we use nominal bond returns in the regressions below.[15]

We pursued two other ideas for hedging instruments. First, we sought to construct industry equity portfolios that respond sensitively to shocks to the value of human capital in particular occupations. For example, demand shocks in the construction sector induce a positive covariance between equity returns in construction industries (SICs 15, 16 and 17) and occupation-level income innovations for electrical engineers, electricians and plumbers. More generally, industry-level demand shocks and factor-neutral technology shocks impart a positive covariance between returns on industry equity and occupation-level income innovations.

However, prior reasoning alone cannot determine the sign, let alone the magnitude, of the covariance between industry equity returns and labor income innovations for industry workers. For example, labor-saving technological improvements in construction activity might be good for share holders but bad for the earnings of electricians and plumbers. As another example, the deregulation of the trucking industry during the 1970s and early 1980s was bad news for many truck drivers (Rose 1987) but good news for many trucking firms (Keeler 1989). The basic point is that factor-biased technology shifts (construction example) and rent shifting between owners and workers (trucking example) impart a negative covariance between industry-level equity returns and occupation-level income innovations.

The bottom line of this discussion is that the usefulness of industry-level equity portfolios as hedging instruments for workers is very much an empirical issue. Furthermore, if the mix of underlying shocks and economic response mechanisms changes over time, the covariance between industry-level equity returns and occupation-level income innovations is likely to change. The weight of this concern is also largely an empirical issue. No single study can definitively settle these empirical issues, so our results in this regard are best viewed as one installment in a broader empirical inquiry.[16]

We constructed the industry portfolios using firm-level equity returns and market values in the Center for Research in Security Prices (CRSP) database. For each occupation, except janitors and cleaners, we identified one or more industries that

[15] We can still specify the first moment of bond returns in real terms for the purposes of portfolio analysis. Data on bond returns are from "U.S. LT Gvt TR" in the "World Capital Market - Fixed" module of the Ibbotson Database.

[16] Davis and Willen (2000) take a different empirical approach to the same issue. They construct time-varying equity mutual funds for synthetic persons defined in terms of birth cohort, sex, and educational attainment. The weights for the equity mutual funds mirror the contemporaneous industry distribution of employment for the workers in the sex-education-cohort group.

account for a large fraction of the occupation's employment. In some cases, we had to omit natural SIC counterparts for particular occupations, because CRSP contains no firm-level observations during part of the sample period.[17] In the end, we identified the SIC industry groups listed in Table 4 for further analysis. We constructed value-weighted industry returns using firms in the CRSP data, and we updated the firm-level weights annually. The rightmost column in Table 4 shows the occupations to which we matched each industry-level return measure.

In another approach to hedging instruments, we considered the covariance between occupation-level income innovations and returns on equity portfolios formed on firm size (market equity value) and the ratio of book-to-market equity value. Fama and French (1993) construct these portfolios, and we use their data on returns.[18] The Fama-French SMB portfolio pays off the return on a portfolio of firms with small market values minus the return on a portfolio of firms with large market values. The Fama-French HML portfolio pays off the return on a portfolio of "value" stocks with a high ratio of book-to-market equity minus the return on a portfolio of "growth" stocks with a low ratio of book-to-market equity. The Fama-French portfolios are rebalanced quarterly and adjusted for transactions costs when firms are bought and sold.

Fama and French (1992, 1993, 1996) show that size and book-to-market factors account for much of the cross-sectional variation in returns on common stocks. Many other asset-pricing studies confirm an important role for these two factors.[19] The question naturally arises as to what types of risk are being priced by size and book-to-market value. In other words, why do small cap stocks earn a higher average return than large cap stocks? And, why do value stocks earn a higher average return than growth stocks? One possibility is that shocks to the value of human capital covary positively with the size and book-to-market factors. If so, then investors who are exposed to labor income risk will demand a return premium to hold small cap and value stocks. This asset-pricing logic suggests that labor income innovations might be correlated with the returns on the size or book-to-market portfolios. Following this logic, we investigate the covariance between occupation-level income innovations

[17]For example, SIC 872 (Accounting and Auditing) is a natural industry counterpart for the accounting and auditing occupation, but CRSP contains no firm-level observations for SIC 872 during much of the sample.

[18]We obtained the data from Ken French's web site `http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html#HistBenchmarks`.

[19]See the Fama and French studies for references to related work. Cochrane (2000) reviews the asset-pricing evidence related to size and book-to-market factors and provides references to more recent work.

and returns on the SMB and HML portfolios.

5.3 Covariance with Other Asset Returns

We examined bivariate and multivariate regressions of the occupation-level income innovations on returns for bonds, SMB and HML. Bond returns are significantly related to income innovations for a few occupations, and HML returns add modestly to the goodness of fit in regressions for truck drivers. However, only the SMB return exhibits a statistically significant relationship to the income innovations for most occupations.

Table 5 displays the bivariate regression results for SMB.[20] The table shows that the SMB portfolio accounts for 10 percent or more of income variation for about half the occupations. For several occupations, the regression results imply a fairly large positive correlation between income innovations and the SMB return. The correlation for accountants and auditors, for example, is $\sqrt{.14}$=.37.

Drawing on Table 2, Table 4, and Table 5, we can calculate the implied covariance between asset returns and innovations to the value of human capital for accountants and auditors as follows. The standard deviation of annual returns on SMB is 15.5 percent. So, a realized return on SMB that is one standard deviation above its mean is associated with an innovation in the value of human capital equal to (15.5)(-25.2)(20.0) = -7,812 dollars.

In unreported results, we reran the regressions in Table 5 including the return on the market portfolio. The market return is never significant at the 10 percent level in these regressions. The SMB coefficients and the corresponding t-statistics are typically somewhat larger when we include the market portfolio. We also examined regressions on the Fama-French SMB "factor," which differs from the SMB "portfolio" in that it includes no adjustment for the costs of portfolio rebalancing. These unreported results were similar to Table 5 but showed better fits for a few occupations. In the only dramatic difference, the R^2 value for electrical engineers is twice as large for the SMB factor as for the SMB portfolio.

The results in Table 5 suggest that the size portfolio offers some scope for hedging occupation-level income risk, as suggested by the asset-pricing logic outlined above. However, the pattern of results in Table 5 runs directly counter to our original motivation for investigating the SMB portfolio. Most of the slope coefficients in Table 5, and all of the statistically significant ones, imply that the relative return on small cap stocks covaries negatively with occupation-level income innovations. Thus, investors

[20] When we allow the small cap and big cap portfolios to enter the regressions separately, they do so with opposite signs and roughly equal magnitudes; likewise, for the growth and value portfolios.

who are exposed to labor income risk should be willing to hold small cap equities at a return discount relative to large cap equities. In fact, the average return on small cap stocks is higher.[21] So, while the findings in Table 5 are useful for portfolio allocation purposes, they heighten rather than resolve asset-pricing puzzles related to the return premium on small cap stocks.

Table 6 and Table 7 show regression results for the best-fitting set of asset return measures. We selected the best-fitting set based on the adjusted R^2 value in regressions on SMB, HML, bonds, and the industry portfolios listed in Table 4. Four of the industry meausures raised the adjusted R^2 value in at least one regression.[22] None of the assets we considered had explanatory power for auto mechanics.

Several results in Table 6 and Table 7 merit some attention. First, the results involving the SMB portfolio are typically strengthened by the inclusion of other assets. Second, the best-fitting set of asset returns accounts for 20 percent or more of occupation-level income risk for several occupations. Third, the covariance structure between income innovations and asset returns differs considerably across occupations. SMB is related to income innovations in most, but not all, occupations. Bonds are significantly related to income innovations in four occupations, but the sign of the relationship for registered nurses differs from the other occupations. Occupation-level income innovations for auto mechanics are unrelated to any of the asset returns we tried. Fourth, the industry equity portfolios are part of the best-fitting set of asset returns for about half of the occupations, although t-statistics for a test of the null hypothesis of no relationship to income innovations are usually below 2.

In summary, the regression results identify one or more assets for each occupation (except auto mechanics) that appear to provide some scope for hedging the occupation-level income innovations and shocks to the value of human capital for workers in those occupations. In the next section, we use these empirical results to construct optimal portfolios of risky assets according to the theory developed in Sections 2 and 3.

[21] Table 4 shows a very modest return premium on small cap stocks during our sample period. As others have observed, the realized premium on small cap stocks has declined in recent decades. The average annual value of the Fama-French SMB portfolio return was about 8 percentage points from 1964 to 1980 and -4 percentage points from 1981 to 1994.

[22] Aggregate equity returns are not statistically significant when added to the regression specifications shown in Table 6 and Table 7

6 Life Cycle Portfolio Choice with Risky Labor Income: Some Examples

We now implement the solution to the life cycle portfolio problem with risky labor income. We draw upon the empirical work in Sections 4 and 5 to characterize the magnitude, persistence, and covariance properties of labor income shocks.

6.1 Portfolio Allocations under Two-Fund Separation

Table 9 shows optimal portfolio allocations when asset returns and labor income are uncorrelated. The table considers three risky assets — the market, size, and value portfolios — and uses a real risk-free return of 3.5 percent per year. We do not impose short-sale contraints on risky asset holdings or restrictions on borrowing at the risk-free rate. Since two-fund separation holds under these conditions, every investor has the same risky asset portfolio shares, as shown in the top row. These shares depend on the joint return distribution for the three assets, which we fit to the first two sample moments in the data.

The table also displays optimal risky asset holdings at ages 40 and 60 for two occupations under various assumptions about relative risk aversion and expected returns. Given the coefficient of relative risk aversion (CRRA), we calculate the corresponding level of absolute risk aversion as

$$A^h = \frac{CRRA}{\sum_{a=23}^{59} y_a \big/ \big(75 - 22\big)} \ .$$

The denominator in this expression is a crude proxy for permanent income based on labor earnings from ages 23 to 59 and assuming that age 75 is the last year of life. The dynamic absolute risk aversion level that governs risky asset demand at each age equals the product of A^h and the marginal propensity to consume out of wealth, as discussed in Section 3.

This simple procedure neglects some issues that arise in a more careful calibration of the risk aversion coefficients (and their variation over the life cycle). First, for exponential utility, Davis and Willen (2000) show that consumption is proportional to a broad measure of wealth that includes the value of human capital, the discounted value of expected future excess returns on risky asset holdings, and a downward adjustment for consumption uncertainty that reflects precautionary behavior. The above procedure for calculating A^h treats human capital in a crude way and ignores the other components of the broad wealth measure. Second, changes in wealth and background risk over the life cycle influence the demand for risky assets when preferences do not have the exponential form. For example, preferences with constant relative risk aversion imply that absolute risk aversion falls with wealth and rises with background risk. The effects of *expected* life cycle variaton in wealth and background risk can be captured in an exponential framework by introducing life cycle variation in A^h. Third, mortality risk rises with age, so that an investor's effective time discount rate also rises with age. We set these issues aside here, because they are sufficiently involved as to merit an extended treatment in a separate paper.[23]

Table 8 shows that an electrical engineer with relative risk aversion of 3 should, according to the theory, hold a $1.03 million portfolio of risky assets. The portfolio consists of a $257 thousand short position in SMB and long positions in HML and the market portfolio. The optimal risky positions are smaller if we consider an otherwise identical investor who is 60 years old, or one who has relative risk aversion of 5. Optimal holdings are also about 40 percent smaller for a secondary school teacher, because her permanent income is about 40 percent smaller. In line with the two-fund separation principle, none of these changes alter the optimal portfolio shares.

In all of these cases, the optimal holdings are quite large relative to casual and systematic evidence regarding actual holdings — 40-year-old electrical engineers who hold million-dollar equity portfolios are not the norm. One important factor behind this gap between theory and evidence is the high returns on U.S. equities over the last century. Since many analysts believe that these high returns are unlikely to hold in the future, the last row in each panel of Table 8 shows the optimal allocations for expected returns on risky assets that are only half as large as the corresponding sample means. Investment positions drop by half as well, but the optimal allocations remain quite large compared to observed holdings for the typical person. This portfolio puzzle seems to have escaped attention in previous research because of the strong proclivity to focus on portfolio shares and to disregard theoretical implications for the level of risky asset holdings.[24]

We believe that the resolution of this puzzle rests at least partly on the opportunity cost of investor funds. In computing the portfolio allocations in Table 8, we allow investors to borrow unlimited amounts at the risk-free interest rate. If investors

[23]An interesting research question is how to best approximate the savings and portfolio choice behavior of a consumer-investor with constant relative risk aversion, by suitably specifying the life cycle path for A^h in a framework with exponential utility. A related question is how the best exponential approximation compares to approximate analytical solutions based on log linearization methods and to numerical approximation methods.

[24]Davis, Nalewaik and Willen (2000) discuss this portfolio puzzle in connection with the gains to international trade in risky financial assets.

must instead borrow at an interest rate that approximates the expected return on risky assets, then the optimal risky asset position is approximately *zero* when asset returns and labor income are uncorrelated. Since many (potential) investors face an opportunity cost of funds at least as great as the expected return on equities, it is unsurprising that half or more of all housholds have little or no holdings of risky financial assets.

6.2 Endowed Exposure and the Breakdown of Two-Fund Separation

Non-zero covariances between asset returns and labor income cause two-fund separation to break down in a particular way. To illustrate this point, Table 9 shows optimal allocations for seven occupations when we account for covariance with labor income shocks. Recall from Section 2 that optimal holdings in the zero-correlation case, "desired exposure," depend only on absolute risk aversion and asset returns. "Endowed exposure" gives the risky asset position implicit in the covariance between asset returns and the worker-investor's labor income.

The regression results in Section 4 show that most of our occupational groups have an endowed exposure to the SMB portfolio. As we explained in Section 2, the endowed exposure reflects the persistence of labor income innovations and their covariance with asset returns. So, while electrical engineers have much greater covariance of income innovations with SMB returns than secondary school teachers, income innovations are more pesistent for the latter and their endowed exposure is greater.

To calculate an investor's optimal portfolio, we simply subtract endowed exposure from desired exposure. Since endowed exposure is not proportional to desired exposure, two-fund separation fails. Other things being equal, the bigger the endowed exposure, the bigger the departure from the two-fund separation principle.

Table 10 illustrates this breakdown by showing optimal portfolio shares under different assumptions about risk aversion and excess returns for each occupation that has a non-zero covariance with one or more of the assets. The base case uses sample average excess returns and a relative risk aversion of 3. Given these assumptions, the departures from two-fund separation are modest. For example, the optimal shares for electrical engineers never differ from the zero-covariance optimum by more than three percentage points. For secondary school teachers, the traditional zero-covariance portfolio understates SMB holdings by 9 percentage points.

Because these effects are small, a portfolio manager might be forgiven for ignoring them. However, if one believes that high equity returns are an aberration, or that expected returns have declined in recent years, then the effects of covariance on optimal portfolio shares become more important. As an example, the second line for each occupation in Table 10 shows optimal portfolio shares when we set excess returns to one-half their sample averages. Recall that this change has no impact on the optimal shares when two-fund separation holds. In particular, the optimal SMB share is -25 percent under two-fund separation, regardless of whether we scale down excess returns. This invariance result fails when we take covariance into account.

As an example, the optimal SMB portfolio shares for secondary school teachers is +2 percent when excess returns are half their sample values and relative risk aversion is 5. To understand this result, recall that the level of excess returns has no effect on "endowed exposure." So, as we reduce excess returns and, hence, desired exposure, the relative size of endowed exposure goes up.

Higher risk aversion has the same effect, and for much the same reason. Greater risk aversion lowers desired exposure but does not affect endowed exposure. The last line in each panel of Table 10 shows optimal portfolio shares for the case of high risk aversion and low excess returns. In this case, the optimal portfolio shares sometimes deviate substantially from the two-fund separation principle. Based on traditional mean-variance analysis, a portfolio advisor would recommend a 25 percent short position in SMB. In contrast, the optimal position for secondary school teachers is a 17 percent long position in a plausible case that accounts for covariance betwen asset returns and labor income.

6.3 Life Cycle Variation in Endowed Exposure

Table 11 shows endowed exposure to the occupation-specific assets at different stages of the life cycle. Given an age-invariant covariance between labor income innovations and asset returns, the endowed exposure declines monotonically with age as the worker-investor draws down her human capital. This result follows immediately when the covariance is age invariant.[25] The rate of decline in endowed exposure is the same for the other risky assets.

As we discussed earlier, endowed exposure depends both on covariance and on the present value multiplier. Although the covariance with the health asset for registered nurses is much lower than the covariance with the build asset for electrical engineers, the present value multiplier on occupation-level income innovations is five times bigger

[25] Davis and Willen (2000) allow this covariance to vary smoothly with age in their empirical work but find only modest life cycle variation for demographic groups defined in terms of sex, education, and birth cohort. Given their findings, and since their empirical design is better suited for uncovering age effects of this sort, we imposed an age-invariant covariance structure in this paper.

for registered nurses. As a result, the endowed exposures to the industry-level assets for these two occupations are fairly similar early on in the life cycle.

A final issue involves life cycle variation in the extent of departures from two-fund separation. Other things being equal, a declining path of endowed exposure leads to ever smaller departures from two-fund separation as a consumer-worker ages. However, income smoothing capacity and "dynamic risk aversion" also decline with age, which creates a countervailing force. In particular, greater risk aversion intensifies the effect of covariance on optimal portfolio shares, as we showed above. So, for any given level of endowed exposure, the departure from two-fund separation is bigger for an older worker-investor.

7 Concluding Remarks

When labor income (or proprietary business income) and asset returns are correlated, investors are implicitly endowed with certain exposures to risky financial assets. These endowed exposures have important effects on optimal portfolio allocation.

We develop a simple graphical approach to portfolio choice over the life cycle that accounts for an investor's endowed exposure. Our graphical approach easily handles risky labor income, multiple risky assets, many periods, and several determinants of portfolio choice over the life cycle. As an added virtue, the chief empirical inputs into the framework are easily estimated using standard statistical procedures.

The two-fund separation principle that governs optimal portfolio choice in a traditional mean-variance setting breaks down when investors have endowed exposures to risky assets. In simple terms, an investor's optimal portfolio can be calculated as the difference between her desired exposure to risky assets and her endowed exposure. Because investors typically differ in their endowed exposures, they also differ in their optimal portfolio allocations (levels and shares), even when they have the same tolerance for risk and the same beliefs about asset returns.

The empirical approach to endowed exposure in this paper relies on repeated cross sections to extract occupation-level components of individual income innovations. Using annual data from 1968 to 1994, we find little evidence that occupation-level income innovations are correlated with aggregate equity returns. This finding and similar findings in other work present something of a puzzle for standard equilibrium models of fluctuations, growth, and asset pricing. Given rational asset pricing behavior, frictionless financial markets, and standard specifications of the aggregate production technology, dynamic equilibrium models imply a high correlation between aggregate equity returns and the value of human capital. That implication finds little support in our empirical results.

We do find evidence that several other asset return measures are correlated with occupation-level income innovations. The returns on portfolios formed on firm size (market capitalization) are correlated with occupation-level income innovations for about half the occupations we consider. In a few occupations, income innovations are correlated with returns on long-term bonds. In several instances, industry-level equity returns are correlated with the occupation-level income innovations of the workers in those industries. Both a priori reasoning and our empirical results suggest that industry-level equity returns can covary negatively or positively with labor income innovations for industry workers. It follows that the optimal hedge portfolio for occupation-specific and industry-specific components of risky labor income cannot be discerned without intensive empirical study.

When we apply the estimated covariances to our portfolio choice framework, we find sizable departures from the two-fund separation principle for plausible assumptions about expected asset returns and investor risk aversion. It is likely that future empirical research will more fully uncover the covariance structure between labor income and asset returns. If so, then the gap between optimal portfolio allocations and the uniform portfolio shares implied by the two-fund separation principle will also be larger.

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A Mathematical Appendix

For expositional convenience, the discussion in the text presumes that the risk-free interest rate equals the subjective discount rate. In the brief derivation of equations (1) and (2) that follow, we consider the more general case where the subjective discount factor δ is not necessarily equal to the reciprocal of the gross return on the riskless asset.

In the two-period model, the single-period budget constraints are $c_0 = \mathrm{y}_0 - B - S$and $\tilde{c}_1 = \tilde{y}_1 + R_f B + \tilde{R}_1 S$. Combining these two equations to eliminate B gives the intertemporal budget constraint:

$$c_0 + \frac{1}{R_f}\tilde{c}_1 = \mathrm{y}_0 + \frac{1}{R_f}\tilde{y}_1 + \frac{1}{R_f}\big(\tilde{R}_1 - R_f\big)S.$$

Taking expectations gives equation (1) in the main text. By definition,

$$U\big(c_0, \tilde{c}_1\big) = -\frac{1}{A^h}\left[\exp\big(-Ac_0\big) + \delta E \exp\big(-A^h \tilde{c}_1\big)\right] \quad . \tag{27}$$

The first-order condition of the optimization problem with respect to the riskless asset is

$$\exp\big(-Ac_0\big) = \delta R_f E\big(\exp\big(-A^h \tilde{c}_1\big)\big). \tag{28}$$

Substituting (28) into (27) characterizes utility entirely in terms of period-0 consumption:

$$U\big(c_0, \tilde{c}_1\big) = -\frac{1}{A^h}\big(1 + \frac{1}{R_f}\big)\exp\big(-Ac_0\big). \tag{29}$$

Since $\tilde{c}_1$ is the sum of normal random variables, it is also normal and we have

$$E\big(\exp\big(-A^h \tilde{c}_1\big)\big) = \exp\big(-AE\big(\tilde{c}_1\big) + \frac{1}{2}\operatorname{var}\big(A^h \tilde{c}_1\big)\big). \tag{30}$$

Taking logs of (28) and substituting in (30) yields

$$E\big(\tilde{c}_1\big) = c_0 + \frac{1}{2}Avar\big(\tilde{c}_1\big) + \ln \delta R_f. \tag{31}$$

Substituting (31) into (1) gives:

$$\big(1 + \frac{1}{R_f}\big)c_0 = C - \frac{1}{2}Avar\big(\tilde{c}_1\big) - \ln \delta R_f. \tag{32}$$

Substituting (32) into (29) and imposing $\ln \delta R_f = 0$ gives equation (2) in the text.

The many-period version follows by backward induction. The key insight is that since first-period consumption is affine in C, the distribution of consumption conditional on information in earlier periods is still normal and the above argument can be used with small adjustment. For details, see Davis and Willen (2000).

First, suppose a household satisfies its Euler equation with respect to the riskless asset for all $s \geq t$. Then:

$$\exp -A^h c_s = \mathrm{E}(\exp -A^h \tilde{c}_{s+1}) \text{ for all } s \geq t.$$

And it implies first that we can measure utility at time t along the optimal path entirely in terms of current consumption:

$$U(\{c_s\}_{s=t}^{T}) = \frac{-1}{a_t A^h} \exp -A^h c_t.$$

And the latter optimization problem is essentially the same as the one solved in section 2. In other words, we can use the graphical apparatus developed in Section 2 to solve each component of the multi-period problem. We illustrate this fact in three steps: first, we show that if a household satisfies its Euler equation with respect to the riskless asset, then expected utility at time t is proportional to period utility of consumption at time t. Second, we construct a consumption function that calculates current consumption conditional on current liquid wealth, current labor income, and current and future investment in the risky asset. And third, we use backward induction to prove our separability result.

We now use equation (x) to construct a consumption function. Taking logs of equation (x) implies that consumption follows a random walk with drift:

$$c_t = \mathrm{E}_{t-1}(\tilde{c}_{t+1}) + A^h \operatorname{var}(\tilde{c}_{t+1})/2.$$

Repeated forward substitution of equation (x) implies that consumption today equals:

$$c_t = a_t\big[\mathrm{PDV}_t(\{c_s\}_{s=t}^{T}) - \tfrac{A^h}{2}\,\mathrm{PDV}_t(\big\{\tfrac{\operatorname{var}(\tilde{c}_s)}{a_s}\big\}_{s=t+1}^{T})\big].$$

Substituting in the lifetime budget constraint yields our consumption function:

$$c_t = a_t\big[Y_t + R_f B_{t-1} + \tilde{\mathbf{R}}_t'\mathbf{S}_{t-1} + \mathrm{PDV}_t\big(\big\{\mathbf{ER}'\mathbf{S}_s\big\}_{s=t}^{T-1}\big) - \tfrac{A^h}{2}\,\mathrm{PDV}_t(\big\{\tfrac{\operatorname{var}(\tilde{c}_s)}{a_s}\big\}_{s=t+1}^{T})\big]. \quad (33)$$

We now use backward induction and our consumption function (equation (z)) to solve for optimal portfolios. Consider portfolio choice at time $T-1$. Since the household lives for only one more period, the problem is exactly like the problem in Section x, except that we need to include accumulated liquid wealth in our definition of current income. But since we showed that current income had no effect on desired or endowed exposure, it has no effect on portfolio choice and that's why our definition of C_t above ignores current income. So we have shown that Formally, $\mathbf{S}_{T-1}$

is independent of B_{T-2} and S_{T-2}. Consequently, when our household chooses how much of the risky asset to buy at time $T-2$, it can take risky asset choice at time $T-1$ as given. And that means that we can ignore all terms involving S_t, $t > T-2$ in maximizing equation (x). So at time $T-2$ the houshold maximizes:

$$Y_{T-2} + R_f B_{T-3} + \tilde{\mathbf{R}}_t' \mathbf{S}_{T-3} + (1/R_f)\mathbf{E}\mathbf{R}'\mathbf{S}_{T-2} - \frac{A^h}{2R_f} \frac{\mathrm{var}(\tilde{c}_{T-1})}{a_{T-1}}. \tag{34}$$

We can simplify further. Let $C_t = Y_t + R_f B_{t-1} + \tilde{\mathbf{R}}_t' \mathbf{S}_{t-1} + (1/R_f)\mathbf{E}\mathbf{R}'\mathbf{S}_t$. By equation (x) and the fact that S is non-stochastic: $\mathrm{var}(\tilde{c}_{T-1}) = a_{T-1}^2 \, \mathrm{var}(\tilde{C}_{T-1})$. So the household maximizes:

$$C_{T-2} - \frac{a_{T-1} A^h}{2R_f} \mathrm{var}(\tilde{C}_{T-1}). \tag{35}$$

Extending this argument by backward induction, one can show that at any period t, our household chooses S_t to maximize:

$$C_t - \frac{A_{t+1}^h}{2R_f} V_{t+1}. \tag{36}$$

Table 1: Occupational Classifications and Summary Statistics

Occupation	1980 Standard Occupational Classification	Sample Period	Mean Cell Count	Minimum Cell Count	1967-1994 Average Earnings
Accountants and Auditors	23	1967-94	542	327	24,881
Electrical Engineers	55	1967-94	246	150	33,923
Registered Nurses	95	1967-94	704	392	17,823
Teachers, Elementary School	156	1967-94	842	679	18,325
Teachers, Secondary School	157	1967-94	733	487	20,886
Janitors and Cleaners	453	1967-94	805	336	11,846
Auto Mechanics	505	1967-94	389	306	17,675
Electricians	575	1967-94	325	267	23,646
Plumbers	585	1967-94	220	168	22,437
Truck Drivers	804,805	1967-94	1079	744	18,665

Source: Authors' tabulations from the Annual Demographic Files of the March Current Population Survey using the selection criteria described in the text.

Note: Average earnings are computed as the 1967-1994 simple mean of unweighted mean annual earnings among persons who satisfy the selection criteria. Earnings are expressed in 1982 dollars using the GDP deflator for personal consumption expenditures.

Table 2: Stochastic Process for Occupational Component of Individual Earnings, Second-Order Moving Average Fit to First Differences, 1968-1994

Occupation	Intercept	MA(1) Coefficient	MA(2) Coefficient	Root Mean Squared Error	R-Squared Value	*Present Value Multiplier* Age 30	*Present Value Multiplier* Age 50
Accountants and Auditors	65	-0.18	0.11	1080	0.04	20.0	8.3
Electrical Engineers	67	-0.58	-0.14	1283	0.22	6.8	3.4
Registered Nurses	246	0.30	0.64	446	0.25	40.2	15.9
Elementary School Teachers	85	-0.08	0.38	525	0.10	27.2	11.0
Secondary School Teachers	15	0.08	-0.02	637	0.00	22.5	9.4
Janitors and Cleaners	-36	-0.35	-0.06	583	0.09	13.3	5.8
Auto Mechanics	-79	-0.02	-0.12	714	0.01	18.9	8.0
Electricians	-119	0.17	-0.60	951	0.16	13.2	6.1
Plumbers	-150	-0.22	-0.22	1453	0.06	12.8	5.7
Truck Drivers	-35	0.14	-0.30	790	0.06	18.5	8.0

Notes:

1. For each occupation, a second-order moving average process is fit to the occupational component of individual annual earnings in 1982 dollars. The moving average process is estimated by (conditional) nonlinear least squares. See the text for an explanation of how the occupational component of individual earnings is identified.

2. The standard errors on the moving average coefficients range from .17 to .23.

3. The present value multipliers are computed using a real discount rate of 2.5 percent per year and assuming retirement after age 59.

Table 3: Stochastic Process for Occupational Component of Individual Log Earnings, Second-Order Moving Average Fit to First Differences, 1968-1994

						Present Value Multiplier	
Occupation	Intercept $\times 100$	MA(1) Coefficient	MA(2) Coefficient	Root Mean Squared Error $\times 100$	R-Squared Value	Age 30	Age 50
Accountants and Auditors	0.0	-0.26	-0.04	4.3	0.06	15.3	6.6
Electrical Engineers	0.2	-0.67	-0.12	3.9	0.26	5.5	2.8
Registered Nurses	1.6	0.26	0.45	3.3	0.15	35.5	14.2
Elementary School Teachers	0.2	-0.09	0.32	2.9	0.05	26.0	10.6
Secondary School Teachers	0.0	-0.02	0.01	3.4	0.00	21.4	8.9
Janitors and Cleaners	-0.6	-0.38	-0.07	4.4	0.12	12.2	5.4
Auto Mechanics	-0.7	-0.02	0.00	4.3	0.00	21.0	8.8
Electricians	-0.8	0.17	-0.63	3.8	0.25	12.7	5.9
Plumbers	-1.1	-0.32	-0.18	6.9	0.09	11.4	5.2
Truck Drivers	-0.4	0.00	-0.15	4.3	0.01	18.7	8.0

Notes:

1. For each occupation, a second-order moving average process is fit to the occupational component of individual log annual earnings in 1982 dollars. The moving average process is estimated by (conditional) nonlinear least squares. See the text for an explanation of how the occupational component of individual log earnings is identified.

2. The standard errors on the moving average coefficients range from .17 to .24

3. The present value multipliers are computed using a real discount rate of 2.5 percent per year and assuming retirement after age 59.

Table 4: Asset Return Measures, Definitions, and Summary Statistics

Variable Name	Short Description	Mean Annual Return in Percent 1968-1994	Standard Deviation of Annual Returns	Occupation Match
Size	Fama-French Size Portfolio, Big - Small	-0.2	15.5	All
HML	Fama-French Book-to-Market Portfolio, Value - Growth Stocks	5.9	12.9	All
Bonds	Nominal Return on 10-Year Constant Maturity U.S. Government Bonds	8.5	10.1	All
Autos	Real Return on SIC 371 (Auto Mfg.)	6.4	25.0	Auto Mechanics
Elmach	Real Return on SIC 36 (Electrical Machinery Manufacturing)	5.8	21.4	Electrical Engineers
Build	Real Return on SICs 15, 16, 17 (Construction)	3.2	27.8	Electrical Engineers, Electricians, Plumbers
Freight	Real Return on SIC 42 and 472, ex. 4725 (Freight Transport by Road)	6.4	27.8	Truck Drivers
Technical	Real Return on SICs 871 and 7336 (Engineering, Architectural and Technical Services)	8.1	31.9	Electrical Engineers
Education	Real Return on SICs 82, ex. 823, and 833 (Education Services)	6.4	37.1	Elementary and Secondary Teachers
Health	Real Return on SIC 80 (Medical, Dental and Health Services)	12.8	37.1	Registered Nurses
Utility	Real Return on SICs 46 and 49, ex. 495 (Electricity, Gas, Steam, Water Works)	5.4	15.8	Electrical Engineers, Electricians, Plumbers
Finance	Real Return on SICs 62 and 67 (Investment Banking, Securities, Exchanges)	7.9	19.8	Accountants and Auditors

Notes:

1. Returns data for the Size and HML portfolios were obtained from Ken French's web site at `http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html#HistBenchmarks.` Fama and French (1993) describe the construction of these portfolios.

2. Returns data on bonds are from the Center for Research in Security Prices.

3. All industry-level return series are constructed from value-weighted portfolios of firm-level equity returns in the Center for Research in Security Prices database. See Davis and Willen (2000), especially Appendix A, for futher explanation. Nominal returns for the industry-level measures were converted to real returns using the GDP deflator for personal consumption expenditures.

4. There were insufficient firm-level equity securities to construct the returns for health in 1968 or for technical in 1987 and 1988. These data points are missing.

5. The last column lists the occupations for which we tried the returns measure as a regressor.

Table 5: Occupation-Level Earnings Innovations Regressed on Size Portfolio Returns, 1968-1994

	Natural Units Wage Measure			*Natural Log Wage Measure*		
	Slope	Standard	R-squared	Slope	Standard	R-Squared
Occupation	Coeff.	Error	Value	Coeff.	Error	Value
				$\times 1000$	$\times 1000$	
Accountants and Auditors	-25.2	12.4	.14	-1.0	0.5	.11
Electrical Engineers	-30.6	14.6	.11	-0.9	0.4	.09
Registered Nurses	-3.8	5.5	.02	-0.2	0.4	.01
Teachers, Elementary	-13.1	5.9	.13	-0.8	0.3	.16
Teachers, Secondary	-16.9	7.1	.15	-0.9	0.4	.14
Janitors and Cleaners	-13.5	6.7	.10	-0.6	0.5	.02
Auto Mechanics	-3.9	8.8	.01	-0.4	0.5	.02
Electricians	13.9	11.4	.05	0.4	0.4	.03
Plumbers	-25.4	17.3	.08	-1.5	0.8	.12
Truck Drivers	2.2	9.8	.00	-0.1	0.5	.00

Notes:

1. All regressions are estimated by ordinary least squares.
2. The dependent variables are the innovations from the fitted time-series processes in Table 2 (Natural Units) and Table 3 (Natural Logs). The regressor is the return on the Fama-French SMB portfolio.

Table 6: Occupation-Level Earnings Innovations Regressed on Best-Fitting Set of Asset Returns, Natural Units Wage Measure, 1968-1994

Occupation	Size	HML	Bonds	Industry	Measure	R-squared
Accountants and Auditors	-25.2 (12.4)					.14
Electrical Engineers	-47.0 (19.2)			13.9 (10.7)	Build	.20
Registered Nurses			16.1 (8.2)	-1.9 (2.3)	Health	.15
Teachers, Elementary	-22.3 (9.2)			5.0 (3.9)	Educ	.22
Teachers, Secondary	-32.3 (10.7)			8.4 (4.7)	Educ	.29
Janitors and Cleaners	-13.5 (6.7)					.14
Auto Mechanics						–
Electricians			-34.4 (16.3)	11.7 (5.9)	Build	.23
Plumbers	-47.5 (22.9)		-35.7 (26.7)	17.9 (12.9)	Build	.19
Truck Drivers		11.6 (11.3)	-27.3 (14.4)			.14

Notes:

1. All regressions are estimated by ordinary least squares.

2. The dependent variables are the innovations from the fitted time-series processes in Table 2.

3. No asset return measure is statistically significant in the regression for Auto Mechanics.

Table 7: Occupation-Level Earnings Innovations Regressed on Best-Fitting Set of Asset Returns, Natural Log Wage Measure, 1968-1994

Occupation	SMB	HML	Bonds	Industry	Measure	R-squared
Accountants and Auditors	-1.4 (0.6)			0.5 (0.5)	Finance	.19
Electrical Engineers	-1.3 (0.6)		0.4 (0.3)	Build	.18	
Registered Nurses			1.0 (0.6)			.10
Teachers, Elementary	-0.8 (0.3)					.20
Teachers, Secondary	-1.7 (0.6)			0.4 (0.2)	Educ	.28
Janitors and Cleaners	-0.6 (0.5)					.06
Auto Mechanics						–
Electricians			-1.9 (0.6)	0.5 (0.2)	Build	.35
Plumbers	-2.4 (1.0)		-2.5 (1.2)	0.8 (0.6)	Build	.28
Truck Drivers		0.8 (0.6)	-1.6 (0.8)			.18

Notes:

1. All regressions are estimated by ordinary least squares. The table entries report slope coefficients (standard errors) on the indicated asset returns.

2. The dependent variables are the innovations from the fitted time-series processes in Table 3.

3. No asset return measure is statistically significant in the regression for Auto Mechanics.

Table 8: Investment in Risky Assets with Zero Covariance between Earnings and Returns: Two-Fund Separation.

		RRA	Age	% reduction in returns	SMB	HML	Market	Total
Portfolio shares					-25	88	37	100
Level of investment	Electrical Engineers	3	40	0	-257	903	381	1027
		5	40	0	-154	542	229	616
		3	60	0	-148	520	220	592
		3	40	50	-129	451	191	514
	Secondary School Teachers	3	40	0	-158	556	235	632
		5	40	0	-95	334	141	379
		3	60	0	-91	320	135	364
		3	40	50	-79	278	117	316

Notes:

1. Portfolio shares are percentage of total investment in risky assets.
2. Level of investment is in thousands of 1982 dollars.

Table 9: Endowed Exposure, Desired Exposure, and Portfolio Choice.

		SMB	HML	Market	Total
Accountants and Auditors	Endowed exposure	-36	0	0	-36
	Desired exposure	-189	662	280	753
	Position	-153	662	280	789
Electrical Engineers	Endowed exposure	-28	0	0	-28
	Desired exposure	-257	903	381	1027
	Position	-229	903	381	1055
Elementary School Teachers	Endowed exposure	-42	0	0	-42
	Desired exposure	-139	488	206	555
	Position	-97	488	206	597
Secondary School Teachers	Endowed exposure	-52	0	0	-52
	Desired exposure	-158	556	235	632
	Position	-106	556	235	684
Janitors and Cleaners	Endowed exposure	-13	0	0	-13
	Desired exposure	-90	315	133	359
	Position	-76	315	133	372
Plumbers	Endowed exposure	-46	0	0	-46
	Desired exposure	-170	597	252	679
	Position	-124	597	252	725
Truck Drivers	Endowed exposure	0	16	0	16
	Desired exposure	-141	497	210	565
	Position	-141	481	210	550

Notes:

1. Entries show level of investment in risky assets in thousands of 1982 dollars.
2. Investor is 40 years old and has a relative risk aversion coefficient of 3.
3. See text for full discussion.

Table 10: Effects of Risk Aversion and the Level of Excess Returns on Portfolio Shares: The Breakdown of Two-Fund Separation.

	Percentage reduction in excess returns	RRA	SMB	HML	Market
Zero covariance	0	3	-25	88	37
Accountants and Auditors	0	3	-19	84	35
	50	5	-8	76	32
	75	5	5	67	28
Electrical Engineers	0	3	-22	86	36
	50	5	-15	81	34
	75	5	-6	74	31
Elementary School Teachers	0	3	-16	82	35
	50	5	0	70	30
	75	5	17	58	25
Secondary School Teachers	0	3	-16	81	34
	50	5	2	69	29
	75	5	19	57	24
Janitors and Cleaners	0	3	-21	85	36
	50	5	-11	78	33
	75	5	-0	70	30
Plumbers	0	3	-17	82	35
	50	5	-2	72	30
	75	5	14	61	26
Truck Drivers	0	3	-26	88	38
	50	5	-28	87	41
	75	5	-31	85	46

Notes:

1. Entries show portfolio shares; percentage of total investment in risky assets.
2. Investor is 40 years old and has a relative risk aversion coefficient of 3.
3. See text for full discussion.

Table 11: Endowed Exposure to Occupation Specific Assets.

	Age					
	30	35	40	45	50	55
Electrical Engineers	9.5	8.9	8.2	7.4	6.4	5.2
Registered Nurses	-6.7	-6.0	-5.1	-4.1	-2.9	-1.4
Elementary School Teachers	12.2	10.9	9.4	7.7	5.5	3.0
Secondary School Teachers	17.2	15.5	13.5	11.1	8.3	4.9
Electricians	14.5	13.3	11.9	10.2	8.2	5.8
Plumbers	21.2	19.5	17.3	14.8	11.8	8.2

Notes:

1. Entries show level of investment in risky assets in thousands 1982 dollars. got
2. See text for full discussion.

Figure 1: Indifference Curves for an Investor with $40,000 a Year in Income. This figure shows indifference curves equivalent to the same level of lifetime consumption with certainty for investors with different levels of relative risk aversion.

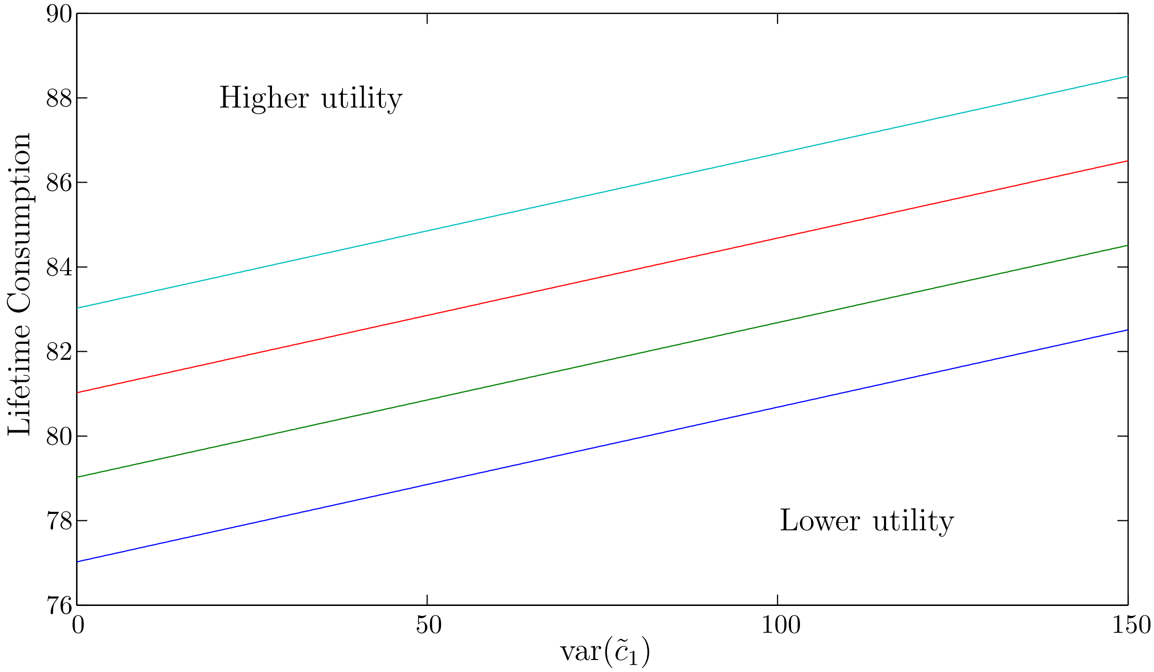

Figure 2: Indifference Curves for an Investor with $40,000 a Year in Income. This figure shows indifference curves equivalent to the same level of lifetime consumption with certainty for investors with different levels of relative risk aversion.

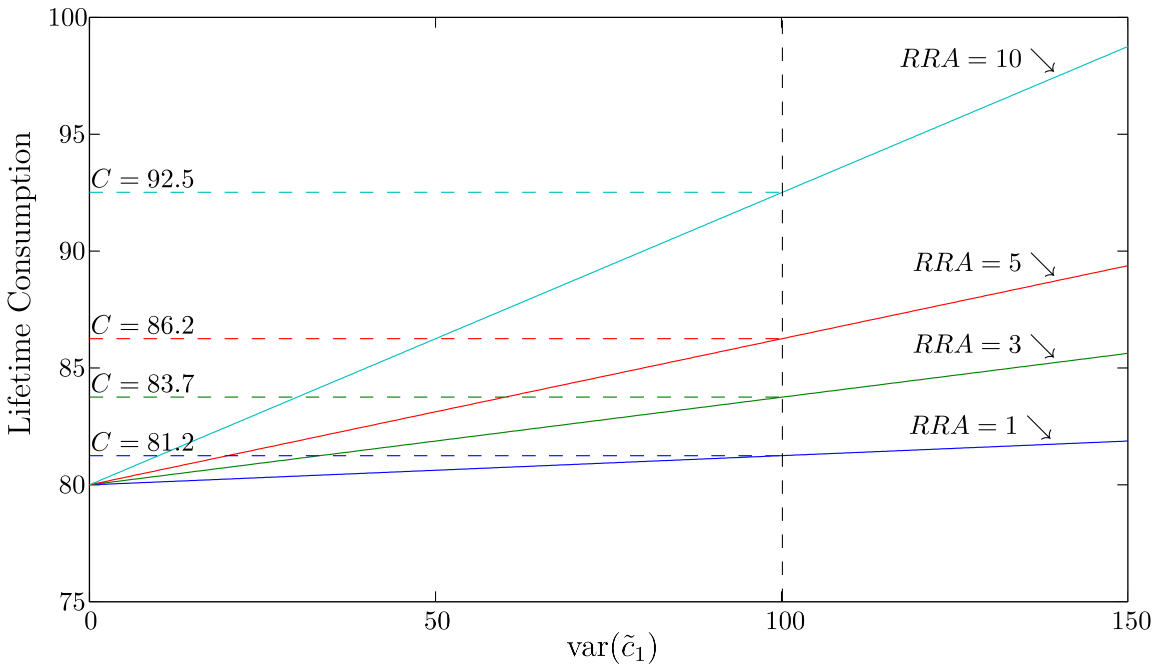

Figure 3: Feasible Sets for Different Occupations. $ER = 0.08$, $\beta^i = 0$, $\sigma(\tilde{R}) = 0.16$.

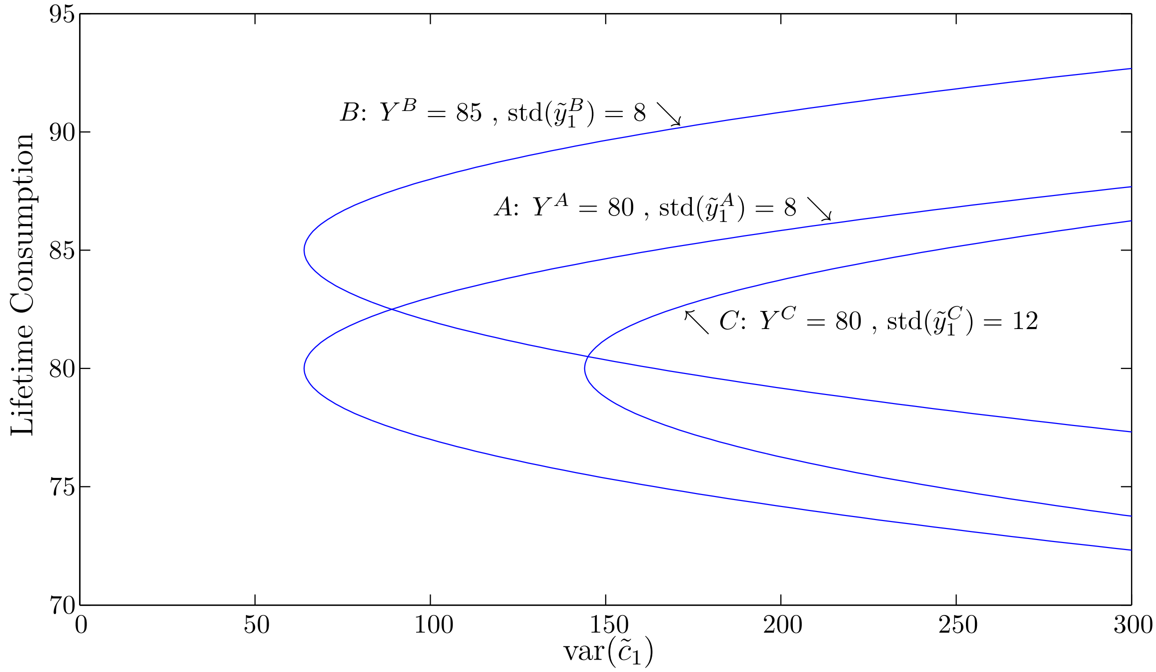

Figure 4: Feasible Sets for Different Occupations. Assume that $Y^i = 80$, $\mathrm{std}(\tilde{\eta}^i) = 8$, $ER = 0.08$, $\sigma(\tilde{R}) = 0.16$.

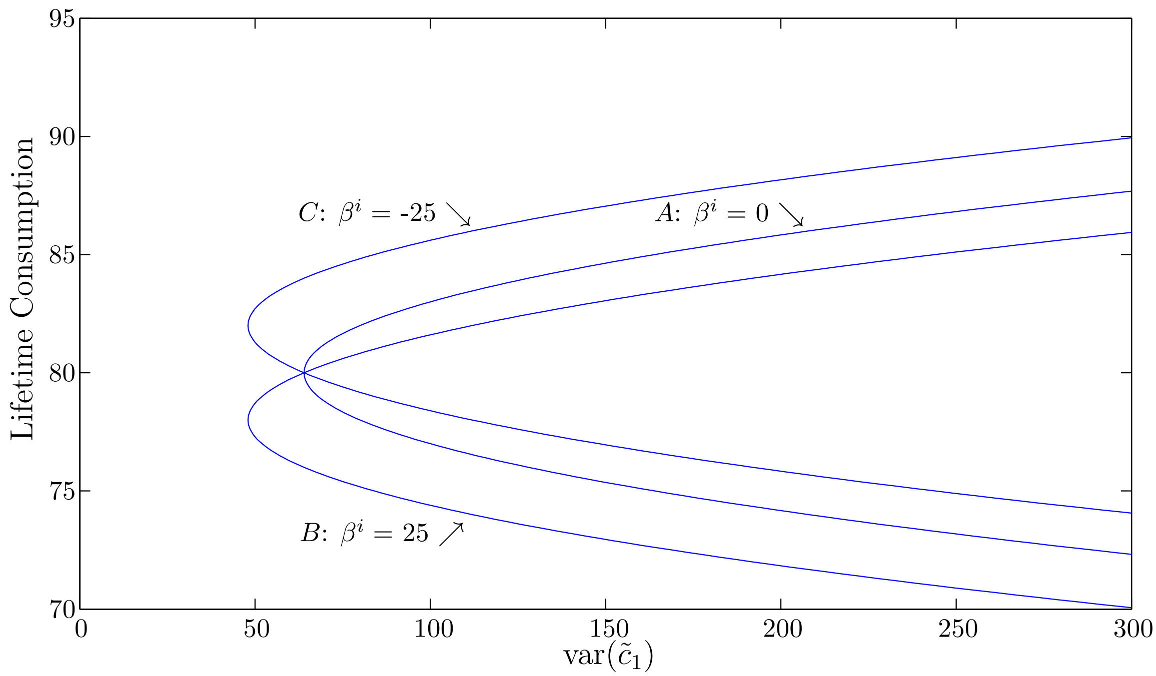

Figure 5: Feasible Set for a Single Occupation but Different Sharpe Ratios. Occupation characteristics: $Y^i = 80$, $\beta^i = 0$, $\text{std}(\tilde{\eta}^i) = 8$.

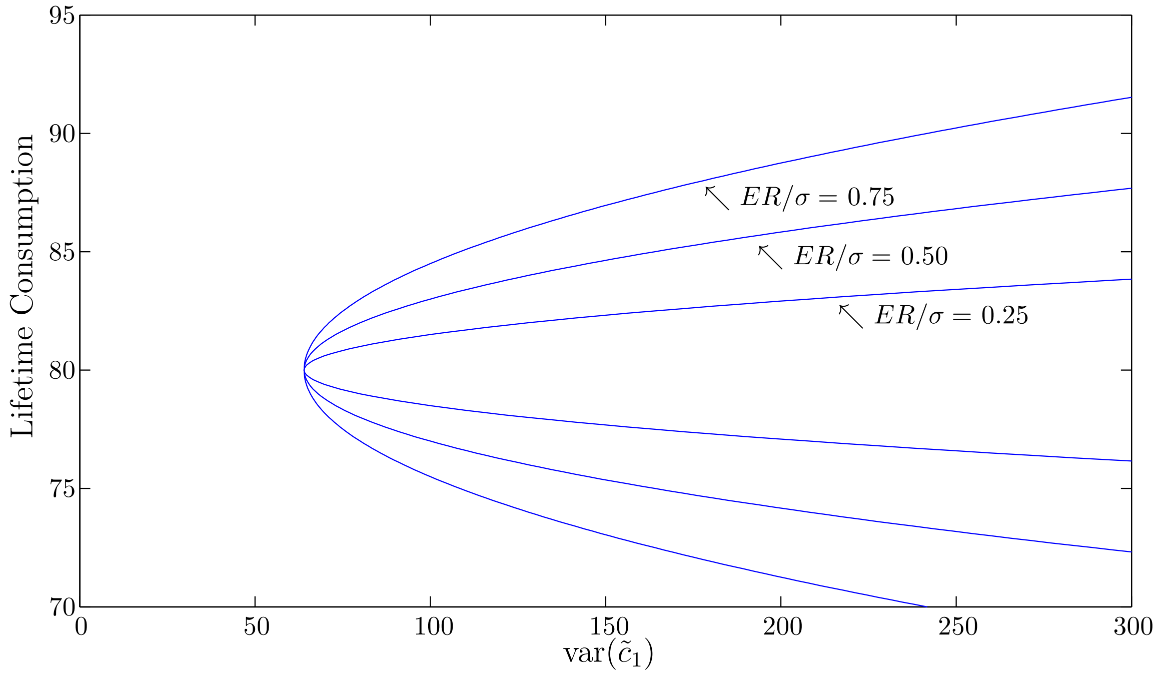

Figure 6: The "Exposure" of Points in the Feasible Set. $ER = 0.08$, $\sigma(\tilde{R}) = 0.16$.

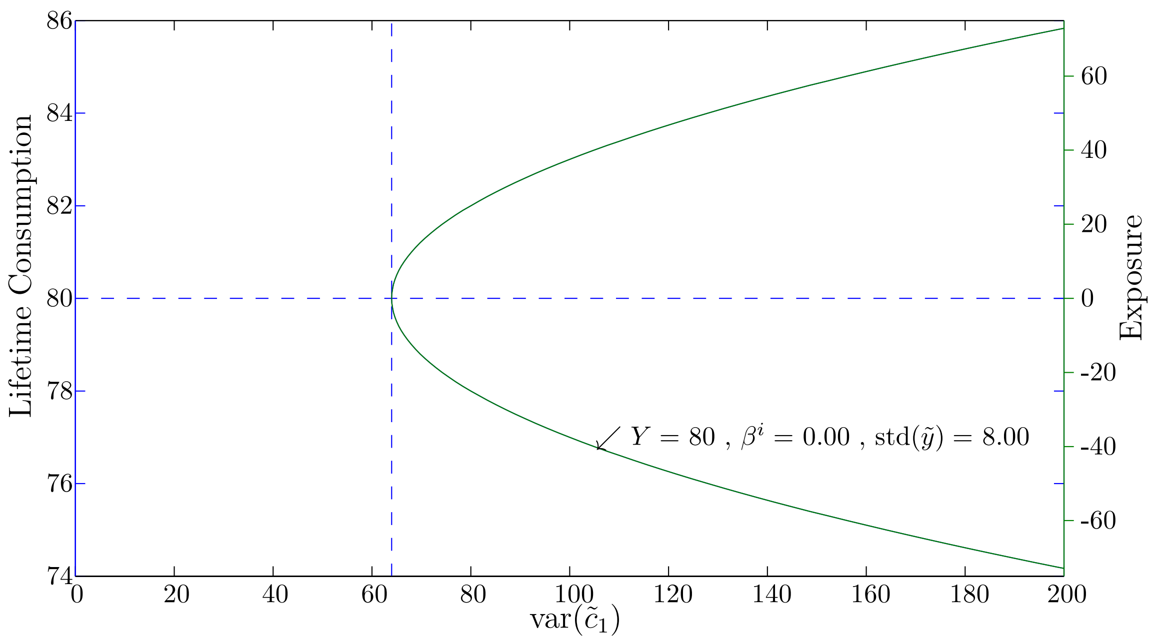

Figure 7: Desired Exposure. $A^h = 3/40$ which implies $RRA \approx 3$. $ER = 0.08$, $\sigma(\tilde{R}) = 0.16$.

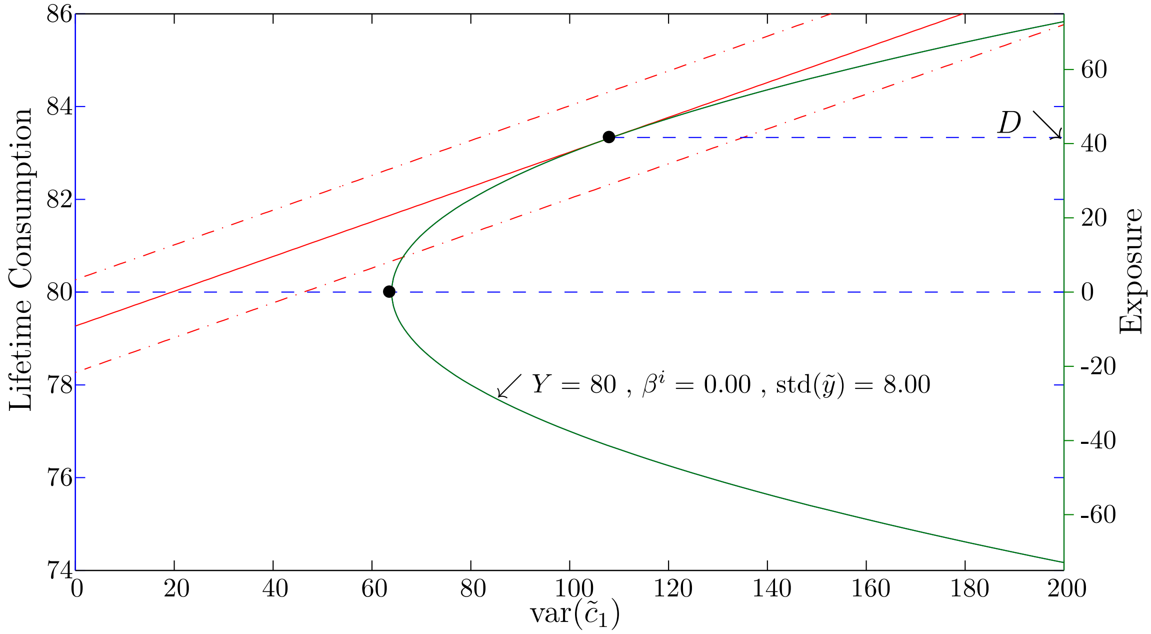

Figure 8: Desired Exposure for Households with Different Relative Risk Aversion. $A^1 = 3/40$ which implies $RRA^1 \approx 3$. $A^2 = 4/40$ which implies $RRA^2 \approx 4$. $A^3 = 2/40$ which implies $RRA^3 \approx 2$. $ER = 0.08$, $\sigma(\tilde{R}) = 0.16$.

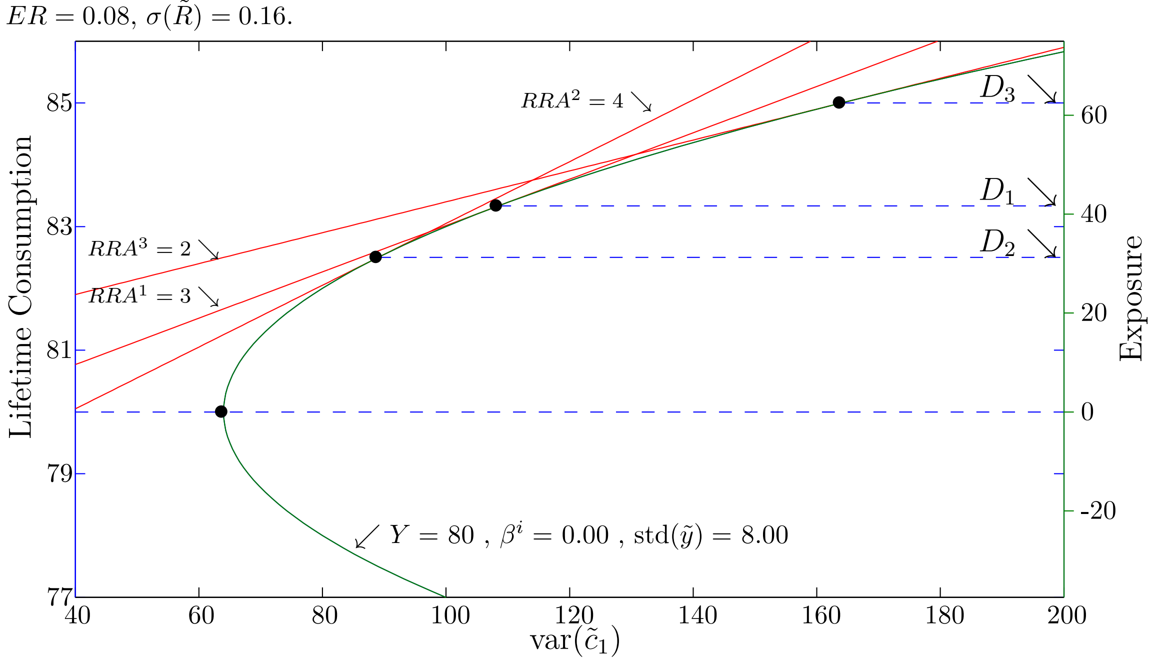

Figure 9: Endowed Exposure. Plot shows four profiles all of which give the same feasible set, but different levels of endowed exposure. $\mathrm{std}(\tilde{\eta}^i) = 8$, $ER = 0.08$, $\sigma(\tilde{R}) = 0.16$.

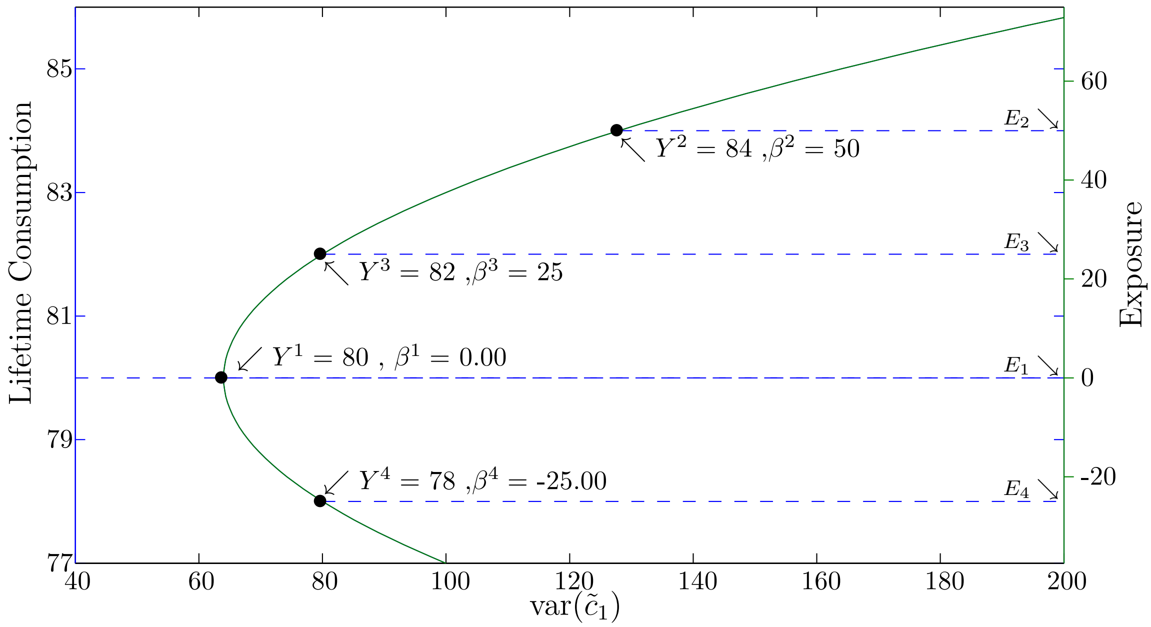

Figure 10: Portfolio Choice. Demand for risky asset equals distance between points marked D and E. For example, investor with $RRA = 3$ in occupation 4 demands $D_1 - E_4$ dollars of the risky asset. $\mathrm{std}(\tilde{\eta}^i) = 8$, $ER = 0.08$, $\sigma(\tilde{R}) = 0.16$.

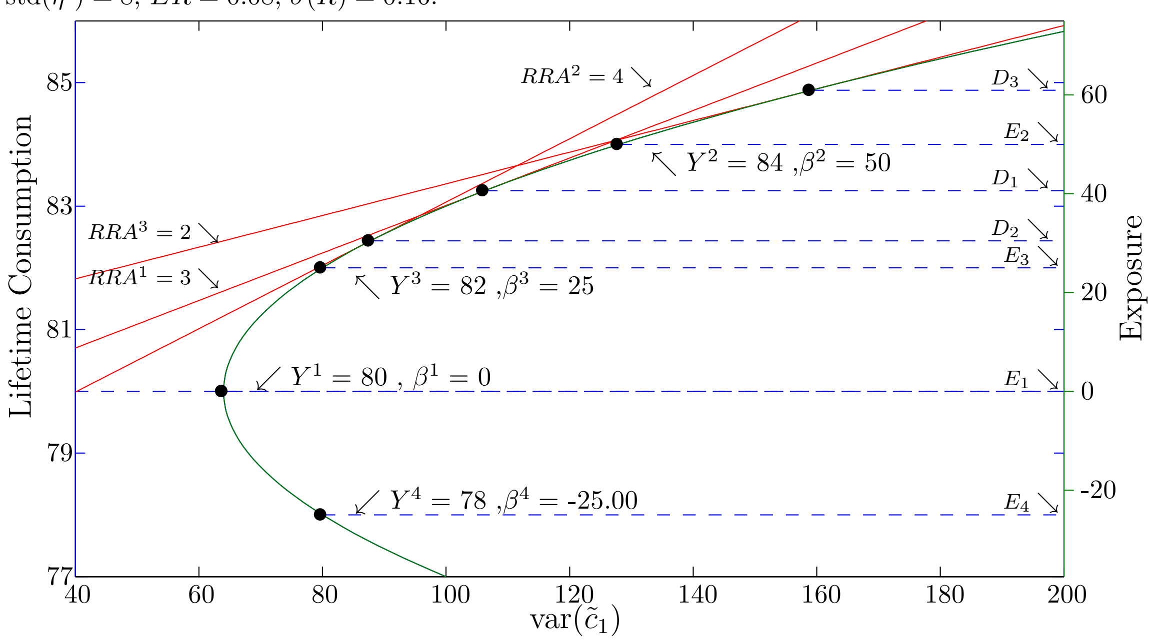

Figure 11: Gains from Trade. Intersection of indifference curves and left-axis measures certain equivalent consumption. std$(\tilde{\eta}^i) = 8$, $ER = 0.08$, $\sigma(\tilde{R}) = 0.16$.