An Evaluation of the Federal Reserve Estimates of the Natural Rate of Unemployment in Real Time

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Abstract:

The authors derive an estimate of the Federal Reserve’s assessment of the natural rate of unemployment in real time from the Greenbook forecast of inflation. The estimated natural rate starts to rise noticeably in the second half of the mid-1970s. It stays relatively high in the 1980s, and then declines noticeably in the second half of the 1990s. They compare the Greenbook estimates with the estimates obtained in real time from simple relationships that extract information about the natural rate of unemployment from the dynamics of inflation, aggregate demand, and the functioning of the labor market. When differences between these measures and the Greenbook arise, the improvement to the Greenbook inflation forecast that would have been achieved by using a different estimate of the natural rate of unemployment is typically small.

JEL Classifications: E37, E47, E5

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This paper presents preliminary analysis and results intended to stimulate discussion and critical comment. The views expressed herein are those of the authors and do not indicate concurrence by the Federal Reserve Bank of Boston, or by the principals of the Board of Governors, or the Federal Reserve System.

This paper, which may be revised, is available on the web site of the Federal Reserve Bank of Boston at http://www.bostonfed.org/economic/wp/index.htm.

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1. Introduction

The assessment of an economy’s degree of resource utilization is an important input to the conduct of monetary policy. This is the case not just when the monetary policy authority has, in addition to price stability, a full-employment mandate, but also when the mandate is specified only in terms of an inflation goal. As long as there is a link between the degree of resource utilization and inflation, inference about the amount of slack in the economy is in fact a relevant component of the inflation outlook, and as such it informs the conduct of monetary policy.

It is not always a straightforward proposition to evaluate how far the economy is from full employment, or from the natural rate of unemployment. This is especially true when the economy moves markedly away, and for an extended period of time, from a pre-existing notion of equilibrium. The high and persistent rates of unemployment in the most recent recession and recovery episodes have generated much debate on whether the natural rate of unemployment has changed as well. Inference about the degree of resource utilization has also been problematic in the past, and missteps in the conduct of monetary policy have often been attributed to an incorrect assessment of the economy’s distance from full employment.

The difficulties in assessing the degree of slack in an economy in real time have been documented extensively when the slack is measured by the output gap. Analysis of the Federal Reserve’s staff real-time assessment of the output gap in the 1980s and 1990s also points to the unreliability of the staff estimates.1 Less work, however, has been devoted to estimating economic slack in real time from a labor market perspective. Okun’s law relates the output gap to the unemployment rate gap, and thus the uncertainty about the real-time output gap has to translate, to some extent, into uncertainty about the real-time unemployment rate gap. Moreover, there is a large literature illustrating that estimates of the unemployment rate gap contain a considerable degree of uncertainty, even when these estimates benefit from data not available in real time.2 Nevertheless, focusing on the unemployment rate gap can have advantages. Contrary to GDP figures, the unemployment rate does not get revised. There is also

1 See, among others, Orphanides and Van Norden (2002) and Orphanides (1998). For an analysis of the more recent period with findings that contrast Orphanides and Van Norden’s, see Edge and Ruud (2012).

evidence indicating that the unemployment rate has predictive power for future revisions to GDP relative to the real-time GDP reading. Even more importantly, real-time inference about the unemployment rate gap can be drawn not just from the typical aggregate macro relationships such as the Phillips curve, but also from the functioning of the labor market.

A notion of the degree of resource utilization based on the labor market is also central to the FOMC’s current conduct of monetary policy. In addition to inflation developments, the FOMC’s recent policy statements have squarely focused on progress in the labor market as a guide for policy. Indeed, current guidance for the timing of the lift off of the federal funds rate from the zero lower bound is based on the economy reaching a specific value for the unemployment rate in a context of stable inflation. This kind of policy guidance relies on some notion of how the targeted value of the unemployment rate for the lift off date relates to the natural rate of unemployment. The recent debate about the amount of slack in the economy also has been heavily influenced by observations pertaining to the functioning of the labor market.

In this paper we revisit the issue of estimating activity slack in real time by focusing on real-time measures of the natural rate of unemployment. In particular, the paper provides an estimate of the Federal Reserve’s staff real-time assessment of the natural rate of unemployment. This assessment has been readily available since the 1990s, when the Greenbook explicitly stated the staff’s assumptions about the natural rate of unemployment. It is not, however, available from earlier periods, so it is necessary to infer the staff’s view about the natural rate of unemployment. We do so by backing out a pseudo-estimate of the Greenbook’s natural rate of unemployment from the Greenbook inflation forecast. We then compare the pseudo-Greenbook estimates with estimates obtained in real time from simple benchmark relationships. These relationships derive a measure of the natural rate of unemployment by estimating a Phillips curve and an aggregate demand equation together with a Phillips curve and a Beveridge curve relationship. We find that our Greenbook pseudo-estimates of the natural rate of unemployment are broadly consistent with these real-time estimates. There is little evidence suggesting that the Greenbook’s assessment of the natural rate of unemployment has been systematically lagging behind the benchmark estimates. When

3 See Aruoba (2008).
differences between the simple benchmark real-time estimates and the Greenbook estimates arise, the improvement to the Greenbook inflation forecast that would have been achieved by using a different estimate is typically small. However, this result depends on the sample period under consideration.

The rest of the paper proceeds as follows. In section 2 we illustrate our method for backing out estimates of the natural rate of unemployment from the Greenbook inflation forecast and discuss these estimates. In section 3 we consider a real-time exercise for estimating the natural rate of unemployment from simple benchmark relationships. Section 4 compares the estimates obtained in section 3 with the Greenbook pseudo-estimates. The performance of the different estimates is assessed in terms of the potential improvement to the Greenbook inflation forecast. Section 5 offers some concluding remarks, pertaining in particular to how our evaluation of the Greenbook’s assessment of the natural rate of unemployment would change if, instead of comparing the Greenbook estimates to real-time estimates, the comparison were made with ex-post estimates of the natural rate of unemployment.

2. Greenbook Estimates of the Natural Rate of Unemployment

In this section we extract real-time estimates of the natural rate of unemployment from the Federal Reserve’s economic projections reported in the Greenbook. The Greenbook forecast of the U.S. economy is produced by the research staff of the Federal Reserve Board before each FOMC meeting to support FOMC members in their policy deliberations. While an assessment of the size of the activity gap, be it in the form of an unemployment rate or in the form of an output gap, is a crucial element in the conduct of monetary policy, the Greenbook has been reporting a real-time assessment of the natural rate of unemployment in a consistent manner only since the 1990s. For earlier periods, when such an assessment was not readily available, it is necessary to draw inferences about the Board staff’s views of the real-time activity gap. The inference exercise retains some value even for the more recent period when the Board staff started to provide real-time estimates of the natural rate of unemployment. Because the Greenbook forecast is partly a judgment call, the inference exercise provides a measure of the
extent to which the Greenbook forecast conforms to the Board’s reported assessment of the natural rate of unemployment.

We infer the staff’s real-time assessment of the natural rate of unemployment from the Greenbook forecast of inflation. The basic premise of this exercise is that the activity gap plays an economically relevant role in driving inflation, and that this relationship is reflected in how the Board staff approaches the inflation forecast. In essence, we posit that the Greenbook inflation forecast can be described by a Phillips curve relationship, where the aggregate demand measure is defined in terms of an unemployment rate gap. Such a relationship is estimated on the Greenbook forecast of inflation, using information available in real time as explanatory variables. The equilibrium rate of unemployment is then backed out from the estimated relationship in the same way as it is when the relationship is estimated on actual data.

This is not the first study that tries to infer the Greenbook’s views about the natural rate of unemployment from a relationship that links real activity to inflation. In particular, Romer and Romer (2002) perform the same type of exercise using a simple back-of-the-envelope calculation that links changes in inflation to the deviation of the unemployment rate from its natural rate. Such an exercise results in noisy estimates of the natural rate of unemployment. Our approach imposes more structure, and arguably produces a somewhat clearer picture of the Federal Reserve’s assessment of the natural rate of unemployment in real time.

We estimate the following Phillips curve relationship based on the Greenbook forecasts of inflation:

\[
\pi_{t+4,GB}^A = \left( \alpha \pi_{t}^{RT} + (1-\alpha)\pi_{t}^{A,RT} \right) - \beta \left( u_{t}^{RT} - \bar{u}_t \right) + x_{t}^{RT} \delta + \epsilon_t,
\]

where \(\pi_{t+4,t}^{GB}\) denotes the Greenbook’s time \(t\) forecast of inflation over the next four quarters (with four-quarter inflation at any date \(t\) defined as \(\pi_t^4 = 0.25 \sum_{i=0}^{3} \pi_{t+i} \)). The independent variables in equation (1) are denoted with a superscript “RT” to indicate that these are observed in real time and thus are included in the Greenbook’s information set when forecasting inflation. The first bracketed term on the right-side of equation (1) captures inflation

\[4\] See their chart 2 on page 47.
expectations in the Phillips curve. We assume that these expectations are given by a weighted average between a measure of long-run inflation expectations, $\pi_{t}^{RT}$, and the average rate of inflation prevailing in the most recent four quarters, $\pi_{t}^{A,RT}$. The inclusion of long-run inflation expectations is meant to capture the FOMC’s inflation goal. The dependence of inflation expectations in the Phillips curve relationship (1) on a long-run expected measure of inflation does not necessarily imply an exploitable tradeoff between inflation and unemployment. Long-run inflation expectations could in fact respond quickly to certain inflation developments and/or policy actions. In this setup, the accelerationist view of inflation is nested as a limiting case when $\alpha = 0$.

The second term in equation (1) is the unemployment rate gap, where $u_{t}^{RT}$ denotes the unemployment rate and $\bar{u}_{t}$ is the unobserved natural rate of unemployment that we are interested in estimating. Finally, $x_{t}^{RT}$ is a vector of supply shocks. The relationship in (1) can be interpreted as featuring a time-varying intercept, $\beta \bar{u}_{t}$, which captures fluctuations in the natural rate of unemployment. In order to back out an estimate of $\bar{u}_{t}$ from (1), we assume that $\bar{u}_{t}$ evolves over time as a random walk,

\begin{equation}
\bar{u}_{t} = \bar{u}_{t-1} + \nu_{u,t}
\end{equation}

where the shock $\nu_{u,t}$ is uncorrelated with the shock $\epsilon_{t}$.

In the context of the Phillips curve relationship depicted in equation (1), the natural rate of unemployment can be thought of as the rate of unemployment that in the medium term stabilizes inflation at the level consistent with the perceived inflation goal. While we are backing out $\bar{u}_{t}$ from the Greenbook by assuming that the Greenbook outlook for inflation is based on a Phillips curve relationship summarized by equation (1), we are not implying that the Greenbook’s assessment of the natural rate of unemployment is informed exclusively by the relationship between inflation and real activity. The Greenbook estimates of the natural rate of unemployment are likely based on many different sources of information, such as micro-level information pertaining to the functioning of the labor market. However, as long as the Greenbook’s estimate of the natural rate of unemployment is a factor affecting the inflation
forecast, it is sufficient to consider the inflation forecast to infer the Greenbook’s assessment of the natural rate.

Data

Up to the end of 1985, Greenbook inflation forecasts refer to the GNP implicit price deflator. Starting in 1986, the inflation forecast refers to the core CPI. In the analysis we use all of the Greenbooks which feature the four quarters of inflation forecasts necessary to construct $\pi_{t+4}^{AGB}$. The Greenbook forecasts become publicly available with a five-year lag after publication, and as a result our analysis covers the period from 1970 to 2007. Real-time information on the most recent four quarters of inflation, $\pi_{t}^{4,RT}$, is as reported in each Greenbook, and so is the value for the unemployment rate $u_{t}^{RT}$. Supply shocks are captured by the change in real oil prices and by the change in real food prices. For $\pi_{t}^{RT}$, ten-year measures of inflation expectations are available only from the end of 1979. We use the Blue-Chip 10-year CPI inflation expectations from 1979:Q4 to 1991:Q2. Because these expectations were surveyed only twice a year, we interpolate for the missing quarters. From 1991:Q4 onward, we use the Survey of Professional Forecasters 10-year CPI inflation expectations. For the earlier period, we simulate real-time long-run inflation expectations according to the following relationship:

\[
\pi_{t}^{RT} = 0.965 \pi_{t-1}^{RT} + 0.0355 \pi_{t}^{4,RT}.
\]

The starting point for the simulation is 1955:Q4, with $\pi_{1955:Q4}^{RT}$ set at 1.7 percent. Given this starting point, simulated values are generated up to 1979:Q4, when $\pi_{1979:Q4}^{RT}$ reaches a reading that is very close to the first available data point for 10-year inflation expectations. The simple relationship assumed in equation (3) mimics well the Federal Reserve Board’s FRB/US model estimate of 10-year inflation expectations pre-1980, and can be thought of as an adaptive expectations mechanism of expectations’ formation.

5 The historical data for the 10-year inflation expectations from the Blue Chip Economic Indicators and from the Survey of Professional Forecasters are maintained by the Federal Reserve Bank of Philadelphia, at http://www.philadelphiafed.org/research-and-data/real-time-center/.

6 The 10-year inflation expectations series in FRB/US has the mnemonic “PTR.” Estimates of pre-1980 inflation expectations are obtained in FRB/US by backcasting a model of survey expectations fitted over
Estimation

Equations (1) and (2) are in state-space form, where (1) is the measurement equation and (2) is the state equation. Estimates of the relevant parameters can be obtained via maximum likelihood by implementing the Kalman filter. While we allow explicitly for time variation in the estimated natural rate of unemployment, the other coefficients in equation (1) are fixed over time. To capture some low-frequency changes in these other coefficients, we estimate equations (1) and (2) over four different subsample periods. The first subsample covers the 1970s. Since 1985 is the last year for which the Greenbook inflation forecasts refer to the GNP deflator, the next subsample considers the years from 1980 to 1985. We then split the remaining years into the period from 1986 to 1996 and the period from 1997 to 2007. We break the sample at the end of 1996 to allow for the possibility that around that time the slope of the Phillips curve became flatter.

At the estimation stage of a model such as the one in equations (1) and (2), relevant issues arise when assessing the standard error of the innovation to the time-varying component in equation (2), \( \sigma(v_u) \). These issues, and ways of addressing potential biases in in the estimation of \( \sigma(v_u) \), have been widely discussed in the literature.\(^7\) Here, rather than estimating \( \sigma(v_u) \), we calibrate the value to equal 0.07 in the first three subsamples that we consider and to equal 0.045 in the last subsample. The lower standard deviation for the latest subsample reflects the fact that during that period fluctuations in the unemployment rate have been relatively modest. Studies that back out the natural rate of unemployment for the United States from actual inflation data using a similar setup as the one considered in equations (1) and (2) rely on larger values for the standard deviation of \( v_u \) than the ones we have calibrated.\(^8\) However, Greenbook forecasts are typically available eight times a year.\(^9\) This higher frequency of the Greenbook

the 1981–2006 period. The relationship in equation (3) has been estimated over the period from 1968:Q1 to 1979:Q4 by using the “PTR” series as the dependent variable. The \( R^2 \) from the estimated relationship is 0.97.

\(^7\) See, for example, Stock and Watson (1998).

\(^8\) See, for example, Gordon (1997), Laubach (2001), and Staiger, Stock, and Watson (1997).

\(^9\) In the 1970s, the Greenbook was published at a monthly frequency.
forecasts relative to the quarterly frequency used in these other studies of actual inflation justifies a smaller calibrated $\sigma(v_u)$. The disturbance term, $\varepsilon_i$, in equation (1) is modeled as a first-order moving average process to account for the serial correlation generated by the overlap in the one-year-ahead inflation forecasts.

**Results**

Figures 1 to 4 depict the one-sided estimates of $\bar{u}$ against the unemployment rate for the four different subsamples we consider.¹⁰ For the 1970s (shown in figure 1), the estimated natural rate of unemployment increases from about 4.5 percent at the beginning of the sample to 7 percent in 1979. The standard error of the final observation in the sample for $\bar{u}$, which provides some indication of the imprecision of the one-sided estimates, is 0.4. A nontrivial portion of the increase in the estimated natural rate occurs in the mid-1970s. The inclusion of dummies to take into account the Nixon price controls does not appear to alter these findings. The estimation does place a significant weight on a positive $\alpha$ in (1), and the hypothesis that $\alpha = 0$ is rejected. In other words, a pure accelerationist specification of the Phillips curve does not fit the Greenbook inflation forecasts for this period as well as a specification that gives some weight to a proxy of the long-run inflation goal. It is still worth noting, however, that if one were to fit a pure accelerationist specification to the Greenbook forecasts for this period, the estimated $\bar{u}$ would be qualitatively similar, the only difference being that the estimated natural rate at the beginning of the decade would be somewhat lower, at about 4 percent. Still, by the end of 1975, $\bar{u}$ is estimated at 6 percent. As an additional check on the qualitative findings for this period, we split the 1970s into the years 1970–1973 and 1974–1979. We then estimate equation (1) with no time-varying intercept, backing out an average value of $\bar{u}$ for the two samples. Over the years 1970–1973, $\bar{u}$ is estimated near 5 percent. In the subsequent years the estimate rises to roughly 7 percent.

¹⁰ In order to facilitate comparison with later estimates, the figures report estimates from one Greenbook per quarter. We use the earliest Greenbook available in each quarter.
In the early to mid-1980s, depicted in figure 2, $\bar{u}_t$ hovers around 7 percent. The standard deviation of the last observation for $\bar{u}_t$ in the sample is 0.7, indicating a nontrivial amount of imprecision in the estimate. There continues to be a significant role for long-run inflation expectations in equation (1). In the second half of the 1980s, shown in figure 3, the estimated natural rate of unemployment declines to about 6 percent. It then increases somewhat in 1993–1994 before starting to decline again. In the late 1990s, shown in figure 4, $\bar{u}_t$ declines noticeably and stabilizes around 4.5 percent in the 2000s. In this latest sample period, the estimated slope $\beta$ is considerably smaller in absolute value than in the earlier samples, in accordance with those studies documenting a flattening of the Phillips curve.\textsuperscript{11} The standard deviation of the last observation for $\bar{u}_t$ in the sample is around 0.30, which is similar to the value estimated at the end of 1996.

The estimates for $\bar{u}_t$ that we have shown have been computed by fixing the standard deviation for the innovation term, $\nu_u$. Given the random-walk assumption for the evolution of $\bar{u}_t$, changing $\sigma(\nu_u)$ mainly impacts the high-frequency volatility of the estimated $\bar{u}_t$, but its lower-frequency movements remain qualitatively the same as in figures 1–4.

Information about the Board’s staff assumptions regarding the natural rate of unemployment is not readily available for the 1970s and 1980s, but it is for the most recent period beginning in the early 1990s.\textsuperscript{12} Coverage from 1989 to 1997 is irregular, but the Board’s assessment of the evolution of the natural rate of unemployment over this period is consistent with our estimates. For the post-1997 period, our estimates are near 4.5 percent, which is consistently below the Board’s reported staff assessment of 5 percent. Over the post-2002 period, however, the Board’s staff estimates of the natural rate did not include temporary productivity effects, which were allowing a lower rate of unemployment to be consistent with stable inflation. These productivity effects likely contributed to lowering the effective natural rate of unemployment implied by the Greenbook’s inflation forecast.

\textsuperscript{11} See, for example, Tetlow and Ironside (2007).

\textsuperscript{12} This information can be found at the Federal Reserve Bank of Philadelphia Real-Time Data Research Center: \url{http://www.phil.frb.org/research-and-data/real-time-center/greenbook-data/nairu-data-set.cfm}. 

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3. Real-Time Estimates of the Natural Rate of Unemployment

We now turn to consider estimates of the natural rate of unemployment based on two very stylized frameworks. In order to infer $\bar{u}_t$, we consider information available only up to time $t$. In this respect, our approach mimics as much as possible a real-time inference exercise. We do so because in order to have a fair comparison between the estimates of $\bar{u}_t$ obtained in this section with the Greenbook’s pseudo-estimates shown in the previous section, one needs to account for the fact that at each Greenbook date, the Board staff had to assess the natural rate of unemployment with information available only up to that date.

The simple models considered in this section draw inferences about the natural rate of unemployment jointly from the dynamics of inflation and real economic variables. While signals about $\bar{u}_t$ are not based solely on inflation, we believe that it is still important that the estimates of $\bar{u}_t$ that we back out from this exercise maintain some reference to the evolution of inflation. After all, the notion of a natural rate of unemployment is intimately linked to the dynamics of inflation.

3.1 Joint Estimation of the Natural Rate of Unemployment from the Phillips Curve and the IS Curve

The first setup we consider to estimate the natural rate of unemployment is based on a Phillips curve and a simple reduced-form IS equation. The Phillips curve takes the form:

$$\pi_t = \left(\alpha\pi_{t-1} + (1-\alpha)\pi_{t-4} - \sum_{i=1}^{2}\beta_i(u_{t-i} - \bar{u}_{t-i})\right) + \gamma x_t + \delta + \epsilon_{t},$$

In essence, we maintain the same relationship used in the previous section to fit the Greenbook forecasts of inflation, though here we are fitting actual inflation. Variable definitions are the same as before, with $\pi_t$ denoting the quarterly inflation rate at an annual rate. We drop the superscript “$RT$” for real time because to construct the variables we use the current vintage of data. However, the specific series we choose for our analysis are usually subject to only minor revisions involving seasonal adjustments. As a result, using the latest vintage should still retain the most salient features of the real-time vintages. As before, inflation expectations enter the

13 We use one-quarter inflation as the dependent variable rather than four-quarter inflation to avoid serial correlation in the error term.
Phillis curve as a weighted-average between trend inflation and recent inflation developments. A relatively small value for $\alpha$ should better capture inflation dynamics in the earlier part of the sample. A larger weight on trend inflation is instead associated with more recent inflation dynamics. If one solely considers the Phillips curve relationship in equation (4), the estimated evolution of the unemployment rate gap, and thus of $\bar{u}_t$, depends only on the dynamics of inflation, trend inflation, and the supply shocks. It is possible to bring other data to bear on the estimation of $\bar{u}_t$ by specifying an additional equation that features a role for the unemployment rate gap.

We do so here by specifying a simple reduced-form IS equation expressed in terms of the unemployment rate gap, which takes the form:

$$u_t - \bar{u}_t = \sum_{i=1}^{2} \rho_i (u_{t-i} - \bar{u}_{t-i}) + \sum_{i=1}^{2} \gamma_i (r_{t-i} - \bar{r}_{t-i}) + \varsigma r_p + \epsilon_{u,t}.$$  

(5)

In the above equation, the unemployment rate gap depends on its own lags, lags of the real interest rate gap, $r_t - \bar{r}_t$, and a risk premium $r_p$. The equilibrium real interest rate is denoted by $\bar{r}_t$. We assume that both the natural rate of unemployment and the equilibrium real interest rate are time-varying and evolve as random walks:

$$\bar{u}_t = \bar{u}_{t-1} + \nu_{u,t},$$  

(6)

$$\bar{r}_t = \bar{r}_{t-1} + \nu_{r,t}.$$  

(7)

The innovations $\nu_{u,t}$ and $\nu_{r,t}$ are independent from the innovations in the measurement equations (4) and (5). The setup described by the relationships among equations (4) through (7) shares some similarities with Laubach and Williams (2003). Our setup is, however, considerably simpler than Laubach and Williams, who focus on the output gap rather than the unemployment rate gap. As a result, their model imposes more structure, with the equilibrium real rate of interest affecting potential GDP growth and the output gap. In our framework, changes to the equilibrium real rate of interest can affect the unemployment rate gap via the IS equation (5). However, we do not model a relationship between the natural rate of unemploy-

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14 The risk premium is expressed in terms of deviations from its sample mean.

15 See also Orphanides and Williams (2002).
ment and the equilibrium real rate of interest, and the shocks $v_{u,t}$ and $v_{r,t}$ are assumed to be uncorrelated.

Data

We use data at a quarterly frequency. Inflation is measured by the core CPI. For the earlier part of the sample for which core CPI data is not available, we measure inflation with the CPI index that excludes food. The unemployment rate is based on the total civilian noninstitutional population 16 years of age and older. The trend inflation measure $\bar{\pi}$ is the same variable used in the previous section, which we refer to for the discussion on how the variable is constructed for the pre-1980 period. Supply shocks in the Phillips curve equation (4) consist of two lags for the change in real oil prices and one lag for the Q4/Q4 change in real food prices, where we use the CPI food price index. In the IS relationship shown in equation (5), the real rate of interest is expressed as the difference between the nominal yield on corporate BBB bonds and $\bar{\pi}$. The risk premium is the difference between the BBB corporate yield and the yield on 10-year Treasuries.

Estimation

The Phillips and IS relationships in equations (4) and (5), together with the assumed evolution for $\bar{u}_t$ and $\bar{\bar{r}}_t$ in equations (6) and (7), are estimated jointly by maximum likelihood using the Kalman filter. In order to capture potential changes in parameters other than $\bar{u}_t$ and $\bar{\bar{r}}_t$, the estimation uses a rolling window of 72 quarters. The cost of using such a relatively short window is the magnification of small-sample bias at the estimation stage. While the size of the window which in this context would provide a good balance between benefits and costs at the estimation stage is not obvious, allowing for the possibility of structural change can be important when estimating the Phillips curve. As already mentioned, the weight of long-run inflation expectations and the slope of the Phillips curve have changed over time, and the same consideration applies to the degree of pass-through of supply shocks to core inflation.\(^\text{17}\)

The setup of the exercise is meant to approximate inference about the natural rate of unemployment in real time, where at each point in time $t$ the most recent 72 quarters of

\(^{16}\) Core CPI data are available from 1957:Q1 onward.

\(^{17}\) See, for example, Fuhrer, Olivei, and Tootell (2009).
available data are used to estimate equations (4)–(7) and to evaluate $\bar{u}_t$. The first rolling window of estimation starts in 1955:Q1 and ends in 1972:Q4. As a result, the first real-time estimate of $\bar{u}_t$ is available for 1972:Q4.

When estimating a setup such as the one in equations (4)–(7), estimates of the standard deviation of the shocks to the permanent components, $\sigma(v_u)$ and $\sigma(v_r)$, can be biased toward zero. This so-called pile-up problem, whereby the Kalman filter underestimates the variance of the permanent component, has been discussed extensively in the literature. Failure to correct for this bias could lead to the wrong inference of too small of a variation in the natural rate of unemployment over time. To address this issue, we use the median unbiased estimator from Stock and Watson (1998) to obtain estimates of the signal-to-noise ratio for the natural rate of unemployment, $\lambda_u^2 \equiv \sigma^2(v_u) / \sigma^2(\varepsilon_u)(\beta_1^2 + \beta_2^2)$, and for the equilibrium real interest rate, $\lambda_r^2 \equiv \sigma^2(v_r) / \sigma^2(\varepsilon_u)(\gamma_1^2 + \gamma_2^2)$.$^{18}$ Since our approach uses a rolling window for estimating equations (4)–(7), we use the same rolling window to estimate the signal-to-noise ratios over time.$^{19}$ These ratios are then imposed when estimating the other parameters of the model via maximum likelihood.

**Results**

The estimates of the evolution of the natural rate of unemployment from this exercise, together with the actual unemployment rate, are depicted in figure 5. At each point in time we consider, $\bar{u}_t$ is the last observation in the rolling estimation window. As a result, the filtered measure that we obtain is one-sided. According to this exercise, the natural rate of unemployment started to rise in the mid-1970s, reaching a peak in the late 1970s. It then hovered near 6 percent in the 1980s and early 1990s, and then dropped to about 5 percent in the most recent sample period. It

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$^{18}$ The procedure uses functions of regression-based parameter stability test statistics, computed under the null hypothesis of no structural break.

$^{19}$ To offset some of the noise in the estimated ratios over the rolling moving window, we use a moving average of these estimates. If $\lambda_{u(t-h,t)}^2$ is the signal-to-noise ratio estimated over the window $[t-h,t]$, when estimating equations (4)–(7) over the sample window $[t-h,t]$, we use as signal-to-noise ratios the centered moving-average of the nine point estimates of $\lambda_r^2$ over the period $t-h-4$ to $t+4$.
is interesting to note that the estimated $\bar{u}_t$ remains low in the most recent five years, despite the sharp rise in the unemployment rate. Estimation results (not reported) do conform to the notion that over time the estimated slope of the Phillips curve has decreased, while the weight placed on trend inflation in the Phillips curve’s inflation expectations formation mechanism has increased.

3.2 Joint Estimation of the Natural Rate of Unemployment from the Phillips Curve and the Beveridge Curve

We now consider drawing inferences about the natural rate of unemployment in the context of a Phillips curve and a Beveridge curve framework. Such a framework has been proposed by Dickens (2009) to exploit the Beveridge curve as an additional source of information about fluctuations in the natural rate of unemployment. The benefit of this approach is to bring information about the functioning of the labor market to bear on $\bar{u}_t$, while at the same time preserving the signals from inflation. Dickens shows that when complementing the Phillips curve with the Beveridge curve, the information coming from the Beveridge curve plays an important role in the determination of the dynamics of the natural rate of unemployment. Needless to say, the Beveridge curve is featured prominently in the current debate about the level of the natural rate of unemployment. This, to some extent, may reflect the fact that with a flatter Phillips curve, it is becoming more difficult to draw signals about the natural rate of unemployment from inflation dynamics. Still, in the past the Beveridge curve has been a complement to the Phillips curve when assessing potential movements in the natural rate of unemployment.\footnote{See, for example, Katz and Krueger (1999) for a discussion of movements in the natural rate of unemployment in the 1990s.}

In this setup, we keep the same Phillips curve relationship as in equation (4). Following Dickens, the Beveridge curve takes the form

$$\ln \left( \frac{1 - u_t}{u_t} \right) = c - a\bar{u}_t + b \ln(u_t / \bar{u}_t) + \epsilon_{\nu_t},$$

where $\nu_t$ is the vacancy rate (the ratio of vacancies to the labor force). As previously, the natural
rate of unemployment is assumed to evolve as a random walk, with innovations $\nu_{u,t}$ that are uncorrelated with the innovations $\xi_{x,t}$ and $\xi_{v,t}$ in equations (4) and (8), respectively. An important assumption underlying the unemployment-vacancies relationship in equation (8) is that all persistent shocks to the unemployment rate other than those acting through vacancies are shocks to the natural rate of unemployment. Such a simplification is controversial. The ongoing debate about the Beveridge curve, for example, highlights potential factors that, while shifting the curve, do not necessarily imply a change in the natural rate of unemployment.21

Data
We use the help-wanted index, with the scale adjustment suggested by Zargosky (1998), to characterize the vacancy series until 1997:Q4. From 2001:Q1 on, we use data from the Job Openings and Labor Turnover Survey (JOLTS). There is, therefore, a gap in the estimation as the help-wanted series became less informative with the growth of the Internet. Data sources for the other variables have already been mentioned.

Estimation
The Phillips curve, the Beveridge curve, and the law of motion for the natural rate of unemployment in equation (6), are estimated jointly via maximum likelihood using the Kalman filter. We assume that the shocks to the Phillips curve and to the Beveridge curve, $\xi_{x,t}$ and $\xi_{v,t}$, are uncorrelated. This implies that the only source of unpredictable co-movement between equations (4) and (8) stems from innovations to the natural rate of unemployment, which is captured by the intercepts in both equations. Because time variation in these intercepts is common to both measurement equations, the “pile-up” problem is attenuated (see Basishta and Startz 2008), and the correction for bias applied in the previous exercise is not necessary. As before, in order to mimic inference about $\bar{u}_t$ in real-time, the estimation is performed over a rolling window of 72 quarters. The first real-time estimate of $\bar{u}_t$ is available for 1972:Q4.

21 For an overview of the recent debate about shifts in the Beveridge curve, see Daly, Hobijn, Sahin, and Valletta (2012).
Results

Estimates of the evolution of the natural rate of unemployment obtained from this exercise, together with the actual unemployment rate, are depicted in figure 6. For the same reasons as in the previous exercise, the reported estimate of \( \bar{u} \) obtained from our rolling estimation is always one-sided. The estimated values of the natural rate of unemployment are somewhat noisy, especially in the 1980s. One notable feature of the estimated \( \bar{u} \) is that it starts the sample period (the early 1970s) relatively high, at 6 percent. Such a level is similar to the estimates reported in Dickens (2009). The estimated natural rate of unemployment then climbs to about 7 percent by the end of the 1970s and stays between 6.5 and 7 percent for most of the 1980s. From the late 1980s to 1997, the estimated \( \bar{u} \) averages about 6 percent. As already mentioned, we lack data for the years from 1997 to 2002. In the most recent period, contrary to the estimates obtained via the Phillips curve–IS curve framework, the natural rate of unemployment is estimated to have risen, with the 2012:Q4 reading at 6.5 percent.

4. A Comparison of the Greenbook and the Real-Time Estimates of the Natural Rate of Unemployment

Figure 7 compares the Greenbook estimates of the natural rate of unemployment to the estimates obtained in the previous section. All of the estimates show an increase in the natural rate of unemployment in the 1970s. The Greenbook estimates increase earlier than the estimates derived from the Phillips curve–IS model (PC-IS henceforth). They instead catch up to the estimates derived from the Phillips curve–Beveridge curve model (PC-BC henceforth). Still, convergence between these two estimates is achieved by mid-1974. In the 1980s, there is a broad correspondence between the Greenbook and the PC-BC estimates. The estimates obtained from the PC-IS relationship, instead, are noticeably lower. From the late 1980s to the late 1990s, the three estimates are quite similar. One possible reason for this similarity is that during this period the Federal Reserve staff’s approach to estimating the natural rate of unemployment relied heavily on the signals coming from a Phillips curve similar to the one in equation (4). In

See Dickens (2009), figure 4.4.
the most recent years for which Greenbook data is publicly available, the three estimates of the natural rate of unemployment are all low and near 5 percent, with relatively minor differences.

Overall, the Greenbook estimates of the natural rate of unemployment appear to share the same lower-frequency properties of the estimates obtained in real time from the two simple specifications considered in the previous section. At a higher frequency, there are some differences, and the question then becomes whether the information contained in the estimates of the natural rate of unemployment obtained from the two simple models has useful informational content. To address this question, we assess the extent to which the Greenbook forecasts of inflation would have been improved by using the PC-IS or the PC-BC estimates of the natural rate of unemployment. To this end, we estimate the following equation:

\[ e_t^4 \equiv \pi_{t+4}^4 - \pi_{t+4}^{GB} = \alpha_0 + \alpha_1 (u_t - \bar{u}_t^{GB}) + \alpha_2 (\bar{u}_t^{GB} - \bar{u}_t^J) + \eta_{t+4}. \]

The dependent variable in equation (9) is the Greenbook forecast error of four-quarter inflation. In addition to a constant, the explanatory variables are given by the Greenbook pseudo-estimate of the unemployment rate gap, and by the difference between the Greenbook pseudo-estimate of the natural rate of unemployment and the estimate obtained from either the PC-IS or the PC-BC models. The Greenbook pseudo-estimate of the natural rate of unemployment is denoted by $\bar{u}_t^{GB}$. The notation $\bar{u}_t^J$ stands for either of the two other models (PC-IS or PC-BC). The disturbance $\eta_{t+4}$ follows a moving-average process of order 3.

The scope of the analysis is two-fold. We are interested in knowing whether divergences in the estimates of the natural rate of unemployment, $\bar{u}_t^{GB} - \bar{u}_t^J$, help to explain Greenbook inflation forecast errors. Equation (9) also controls for the Greenbook’s pseudo-estimate of the unemployment rate gap, as it is important to ascertain whether this estimate was factored efficiently into the forecast. A significant coefficient in equation (9) for the Greenbook pseudo-estimate of the unemployment rate gap has several possible interpretations. One interpretation is that our pseudo-estimate does not provide an accurate estimate of the Greenbook’s assessment of the unemployment rate gap, but the pseudo-estimate contains relevant information for forecasting inflation which was not being considered by the Board’s staff. As such, controlling for the pseudo-estimate of the unemployment rate gap in equation (9)
provides an important test of the plausibility of our approach to measuring the Greenbook assessment of the gap, as described in section 2.

Another interpretation for a significant coefficient in equation (9) for the Greenbook’s pseudo-estimate of the unemployment rate gap is that while such an estimate was accurate, the Greenbook’s assessment of the tradeoff between inflation and unemployment was not. From the exercise in section 2 we already know that the Greenbook forecasts of inflation have consistently been driven, among other factors, by the unemployment rate gap. The estimated sensitivity of inflation to the unemployment rate gap, captured by the coefficient $\beta$ in equation (1), has changed over time but has remained significant. Controlling for the unemployment rate gap in equation (9) then can also be interpreted as a way of testing whether the tradeoff between inflation and the unemployment gap was factored efficiently into the Greenbook’s inflation forecast.

Table (1) provides the results from estimating equation (9). Panel A considers the full sample from 1970 to 2007, while panel B excludes the 1970s from the estimation. The first column, in addition to the estimate for the constant $\alpha_0$, includes the Greenbook estimate of the unemployment rate gap only. The second column includes as an additional explanatory variable $\bar{u}^{GB} - \bar{u}'$, where $\bar{u}'$ is derived from the PC-IS model. In the third column, $\bar{u}'$ is derived from the PC-BC model. As concerns the full sample results, there is only modest evidence that the Greenbook did not factor our pseudo-estimate of the unemployment rate gap efficiently into the inflation forecast. In terms of factoring the tradeoff between inflation and unemployment efficiently into the forecast, the estimated negative coefficient would imply that the slope of the Phillips curve $\beta$ in equation (1) was being understated.

When including $\bar{u}^{GB} - \bar{u}'$ in the regression, a negative sign for $\alpha_2$ implies that when the Greenbook estimate of the natural rate of unemployment was higher than $\bar{u}'$, the Greenbook forecast of inflation was higher than the actual rate, and vice versa. The full sample results indicate some significance of the correction to the Greenbook estimate of the natural rate of unemployment using the estimate obtained from the PC-BC model. The correction using the
estimate of the natural rate of unemployment from the PC-IS model, is instead small and insignificant.

All of the reported findings for the full sample, however, are driven by the observations in the 1970s and change considerably when we consider only the period from 1980 to 2007. As panel B in table (1) shows, there is now little evidence suggesting that the Greenbook estimate of the unemployment rate gap was factored inefficiently into its inflation forecast. The difference between the Greenbook and the PC-IS estimates of the natural rate of unemployment becomes significant, while the correction from the PC-BC estimate is now insignificant. It should be noted that the standard errors reported in the table understate the uncertainty surrounding the estimated slopes in equation (9), as the explanatory variables are generated regressors. Moreover, the forecast improvement from correcting the Greenbook estimate of the natural rate of unemployment with the estimate obtained from the PC-IS model is small. This outcome is illustrated in figure (8), which shows actual four-quarter –inflation, the Greenbook predicted value, and the Greenbook predicted value with the adjustment to the natural rate of unemployment obtained from the PC-IS model using the estimates in equation (9) for the 1980–2007 period. We interpret the results as indicating that there is little evidence that the real-time estimates of the natural rate of unemployment obtained from the two models in the previous section would have consistently and meaningfully improved upon the Greenbook estimates in terms of forecasting inflation.

As concerns the decade of the 1970s, the regressions in table 2 show a potentially larger economic impact from using the natural rate of unemployment estimated by means of the Phillips curve and the Beveridge curve. According to the estimated $\alpha_2$, a 1 percentage point difference between the Greenbook estimate of the natural rate of unemployment and the estimate derived from the PC-BC model, would have affected the inflation forecast by roughly 2 percent. Moreover, the estimated $\alpha_1$ is also highly significant, suggesting that our pseudo-estimates of the unemployment rate gap are not factored efficiently into the Greenbook inflation forecast. As already mentioned, such a finding could indicate, among other things, that our

23 The contribution to the forecast error from the estimates of the natural rate of unemployment from the PC-IS model is not reported in this table, as it is not meaningful.
measure of the Greenbook’s assessment of the unemployment rate gap is incorrect. The robustness of these findings, however, is questionable. The significance of the adjustment to the PC-BC natural rate estimate diminishes once the regression controls for realized food inflation shocks, as the second column shows. Moreover, exclusion of the period 1973:Q3 to 1974:Q4 (in column 3), when oil price shocks were taking place, further reduces the importance of the adjustment. Similar effects are at work for the pseudo-estimate of the unemployment rate gap. Overall, we interpret these findings as suggesting that the improvement to the Greenbook forecast from considering the PC-BC estimate of the natural rate of unemployment is concentrated within a few quarters, and its identification is complicated by the presence of supply shocks. The same qualitative results hold for our pseudo-estimates of the unemployment gap, as the potential evidence that these estimates for the 1970s are not factored efficiently into the Greenbook inflation forecast is subject to the same limitations.

5. Concluding Remarks

In this paper we have compared the Greenbook estimates of the natural rate of unemployment to real-time estimates obtained from simple benchmark specifications. While there are differences across estimates, the estimates also share many similarities. There is little evidence suggesting that the Greenbook estimates of the natural rate of unemployment lag systematically behind the real-time estimates. When differences arise, we find little compelling evidence that the real-time measures we have considered would have led to systematic improvements to the Greenbook inflation forecast.

While the paper is concerned with a real-time assessment of the natural rate of unemployment, it is worthwhile to briefly comment on estimates of the natural rate of unemployment obtained with information not available in real time. These estimates, too, are generally surrounded by a high degree of uncertainty. An interesting feature of some of these estimates, however, is that they exhibit less volatility than the Greenbook estimates. This applies, for example, to the CBO estimate of the NAIRU, which we depict in figure (9) compared against the estimate of the natural rate of unemployment that we back out from the Greenbook. The low-frequency movements in the series are approximately the same, but
fluctuations in the CBO NAIRU occur within a much smaller range. Is there evidence that this less volatile measure would have better served policymakers? Table 3 repeats the simple exercise of the previous section, but this time the variable of interest is the difference between the Greenbook estimate of the natural rate of unemployment and the CBO NAIRU. For the full sample estimates reported in column (1), there is some indication that the adjustment leads to an improvement in the forecast. For the period from 1970 to 1979, there is evidence of an economically meaningful adjustment obtained from using the retrospective measure, though the estimated effect is not significant. However, similar to the results reported in table 2, the economic and statistical significance of all of the estimated coefficients in the regression (including the constant) is highly sensitive to controlling for supply shocks.\(^{24}\) There is some indication of an improvement in the Greenbook inflation forecast during the post-1980s sample, but similar to the results obtained with the real-time estimates of the natural rate of unemployment from the PC-IS model, the improvement is small. Also, as the estimates in column 4 indicate, the improvement is concentrated in the first half of the 1980s.

This lack of a significant improvement to the Greenbook inflation forecast should caution against concluding that the FOMC should have strived for a lower rate of unemployment whenever the CBO estimate of the NAIRU happened to be below the Greenbook estimate of the natural rate, and vice versa. After all, even the CBO estimate of the NAIRU is uncertain, and the lack of evidence that this measure has a clear advantage on the inflation forecast most likely reflects such uncertainty. Still, a caveat is in order. If inflation and unemployment are strictly related only when the unemployment rate is either very high or very low, then thinking about the lowest achievable unemployment rate with stable inflation is relevant given the Federal Reserve’s dual mandate. While there is some evidence supporting nonlinearities in the inflation-unemployment tradeoff,\(^ {25}\) the analysis and the conclusions drawn in this paper are based on a linear relationship existing between inflation and unemployment.

\(^{24}\) In particular, the same qualitative findings reported in the second and third column of Table 2 apply. Results are available upon request.

\(^{25}\) See, among others, Stock and Watson (2010).
References


Table 1

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<td>Constant</td>
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<td>-.206** (.074)</td>
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<td>.127 (.152)</td>
<td>-.163** (.059)</td>
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<td></td>
<td>.097 (.141)</td>
<td>-.230** (.081)</td>
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<td>$u_t - \bar{u}_t^{GB}$</td>
<td>-.267* (.122)</td>
<td>-.067 (.064)</td>
</tr>
<tr>
<td></td>
<td>-.261* (.123)</td>
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<tr>
<td></td>
<td>-.339* (.142)</td>
<td>-.062 (.065)</td>
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<tr>
<td>$\bar{u}_t^{GB} - \bar{u}_t^{PC,IS}$</td>
<td>-.099 (.184)</td>
<td>-.306** (.093)</td>
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<td>$\bar{u}_t^{GB} - \bar{u}_t^{PC,BC}$</td>
<td>1.262* (.504)</td>
<td>-.169 (.154)</td>
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Standard errors are in parentheses. The symbols * and ** denote significance at the 5 percent and 1 percent level, respectively.

Table 2
Predictability of Greenbook Inflation Forecast Errors: 1970 to 1979

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<td>Constant</td>
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<td>.391 (.198)</td>
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<td>-.770** (.256)</td>
<td>-.538 (.276)</td>
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<td>$\bar{u}_t^{GB} - \bar{u}_t^{PC,BC}$</td>
<td>-2.110** (.683)</td>
<td>-.929 (.778)</td>
<td>-.0078 (.602)</td>
</tr>
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</table>

Standard errors are in parentheses. The symbols * and ** denote significance at the 5 percent and 1 percent level, respectively.
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<tbody>
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<td>Constant</td>
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<td>1.04**</td>
<td>-.094</td>
<td>-.085</td>
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<tr>
<td></td>
<td>(.196)</td>
<td>(.356)</td>
<td>(.055)</td>
<td>(.057)</td>
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<tr>
<td>$u_t - \bar{u}^GB_t$</td>
<td>-.302*</td>
<td>-.894**</td>
<td>-.082</td>
<td>.025</td>
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<td></td>
<td>(.136)</td>
<td>(.277)</td>
<td>(.062)</td>
<td>(.093)</td>
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<tr>
<td>$\bar{u}^{GB}_t - \bar{u}^{NAIRU}_t$</td>
<td>-.634*</td>
<td>-.821</td>
<td>-.320**</td>
<td>-.140</td>
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<tr>
<td></td>
<td>(.258)</td>
<td>(.508)</td>
<td>(.106)</td>
<td>(.095)</td>
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</table>

Standard errors are in parentheses. The symbols * and ** denote significance at the 5 percent and 1 percent level, respectively.
Figure 1
Greenbook Estimate of the Natural Rate of Unemployment:
1970 to 1979

Figure 2
Greenbook Estimate of the Natural Rate of Unemployment:
1980 to 1985
Figure 3
Greenbook Estimate of the Natural Rate of Unemployment:
1986 to 1996

Figure 4
Greenbook Estimate of the Natural Rate of Unemployment:
1997 to 2007
Figure 5
Natural Rate of Unemployment Estimated from Phillips Curve and IS Curve

Natural Rate of Unemployment Estimated from Phillips Curve and IS Curve
Unemployment Rate

Figure 6
Natural Rate of Unemployment Estimated from Phillips Curve and Beveridge Curve

Natural Rate of Unemployment Estimated from Phillips Curve and Beveridge Curve
Unemployment Rate
Figure 7
Real-Time Estimates of the Natural Rate of Unemployment

Greenbook Estimate of the Natural rate of Unemployment
Natural Rate of Unemployment Estimated from Phillips Curve and IS Curve
Natural Rate of Unemployment Estimated from Phillips Curve and Beveridge Curve
Figure 8
Actual and Predicted Core CPI Inflation

Figure 9
CBO NAIRU and Greenbook Estimate of the Natural Rate of Unemployment: 1970 to 2007