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This Is What's in Your Wallet…and Here's How You Use It

Tamás Briglevics and Scott Schuh

Abstract:

Models of money demand, in the Baumol (1952)-Tobin (1956) tradition, describe optimal cash management policy in terms of when and how much cash to withdraw, an (s, S) policy. However, today, a vast array of instruments can be used to make payments, opening additional ways to control cash holdings. This paper utilizes data from the 2012 Diary of Consumer Payment Choice to simultaneously analyze payment instrument choice and withdrawals. We use the insights in Rust (1987) to extend existing models of payment instrument choice into a dynamic setting to study cash management. Our estimates show that withdrawals are rather costly relative to the benefits of having cash. It takes 3–8 transactions to recoup the fixed withdrawal costs. The reason is that the shadow value of cash decreases substantially with the number of available payment instruments and, correspondingly, individuals are less likely to make withdrawals.

Keywords: money demand, inventory management, payment instrument choice, payment cards, Diary of Consumer Payment Choice

JEL Classifications: E41, E42

Tamás Briglevics is a research associate in the Center for Consumer Payments Research in the research department of the Federal Reserve Bank of Boston and a graduate student at Boston College. Scott Schuh is a senior economist and policy advisor and the director of the Center for Consumer Payments Research in the research department of the Federal Reserve Bank of Boston. Their e-mail addresses are tbriglevics@gmail.com and scott.schuh@bos.frb.org, respectively.

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CONSUMER PAYMENTS Research Center

1 Introduction

A popular commercial campaign by the U.S. bank Capital One asks listeners, "What's in your wallet?" This paper attempts to answer this question using a panel of micro data from the new 2012 Diary of Consumer Payment Choice (DCPC). The question and answer offers fresh insights into (i) the transformation of the U.S. money and payment system from paper to electronics, and (ii) the effect of this transformation on liquid asset management. These days, U.S. consumers choose to adopt, carry, and use any of nearly a dozen payment instruments to buy goods and services.

There have been a number of recent contributions on payment instrument choice in various countries using transaction-level data; see, for example, Fung, Huynh, and Sabetti (2012) for Canada; Bounie and Bouhdaoui (2012) for France; von Kalckreuth, Schmidt, and Stix (2009) for Germany; Klee (2008) and Cohen and Rysman (2013) for the United States. In this paper, we begin by replicating the analysis of Klee (2008) on the DCPC data. The result shows that over the last decade payment instrument choice has undergone a remarkable transformation: checks have virtually disappeared from point-of-sale transactions.

Second, our data allow us to analyze sequences of consumer decisions about payment instrument choice and cash withdrawal. Hence, we can extend the existing literature on the inventory-theoretic models of money demand, which focused on describing individuals' optimal cash withdrawal policy assuming an *exogenously* given process of cash expenditures (see, for example, Baumol 1952; Tobin 1956; Miller and Orr 1966; Bar-Ilan 1990; Alvarez and Lippi 2009). It seems unrealistic to assume that these models provide a good description of the current U.S. payments system, where debit and credit card acceptance is almost universal at check-out counters. Therefore, we propose a model where, in addition to controlling the timing of withdrawals, agents also control how quickly they decrease their cash holdings.

This paper builds on the model of Koulayev et al. (2012), who analyze the adoption and use of payment instruments. In our model, a dynamic extension of their paper, the bundle of available instruments changes over time as consumers run out and replenish their cash holdings. On other dimensions (for example, the adoption of payment instruments, the correlation of the random utility terms), however, our setup is much less ambitious. These restrictions enable us to obtain closed-form solutions for the inventory management problem, as in Rust (1987). (See Chapter 7.7 in Train 2009, for a concise description.)

This framework can capture that when consumers make payments, they consider not only the current benefits of using a particular instrument, but also the effect this choice has on future transactions. To illustrate, take, for example, somebody with $10 in her purse, along with a credit card, who is planning to make two low-value transactions worth $8 and $3, respectively. Clearly, a choice to use cash for the $8 transaction will force her to either use the credit card for the $3 transaction or to withdraw cash, which might be inconvenient.

Preliminary results show that households value cash differently depending on the bundle of payment instruments they hold and their revolving credit balance. In particular, all else being equal, those with outstanding balances on their credit card are 7.3 percent more likely to use cash and 3.9 percent more likely to use debit cards to pay for median-sized transactions than those without credit balances. We also find meaningful variation in the estimated withdrawal costs by various withdrawal methods: It is about 18 percent more costly to get cash from a bank teller than to use an ATM, indicating that technological improvements are an important factor in keeping the number of cash transactions relatively high (these represent over 40 percent of all point-of-sale transactions).

The paper is organized as follows: Section 2 briefly reviews recent models that analyze the interactions between cash inventory management and payment instrument choice. Section 3 draws a quick comparison between the DCPC data and Klee (2008), and estimates simple multinomial logit models for various types of transactions. Section 4 describes the dynamic extension of the payment instrument choice model and discusses how the model can be solved. Section 5 extends that model to allow for withdrawals, linking payment instrument choice and cash demand. Section 6 describes the results of the estimation, and Section 7 concludes.

2 Related literature

There are a number of papers that have tried to embed payment instrument choice into money demand models: Sastry (1970), Bar-Ilan (1990), and Alvarez and Lippi (2012) all allow consumers to make costly credit transactions after they run out of cash. However, these models all imply a lexicographical ordering between cash and credit —cash is used until available, then credit— that is impossible to reconcile with the data.

Another strand of literature that analyzes different means of payment and money demand is New-Monetarism. In an example of this strand, (Nosal and Rocheteau 2011, Chapter 8) present a model in which consumers endogenously choose between credit and cash and can reset their cash holdings at a fixed cost. The tractability of their model makes it an appealing expository device of the issues we are studying and it is possible to re-interpret our model as an extension of their model with some additional randomness (for example, by adding random costs of using payment instruments) introduced to enable the model to explain the transaction-level data. Chiu and Molico (2010) extend the Lagos and Wright (2005) model in a different direction: By introducing a random fixed cost to making withdrawals, they are able to study an economy with a nontrivial distribution of cash holdings. In ongoing work we aim to extend the current version of our model along similar lines.

In recent empirical work Klee (2008) attempted to establish a link between payment instrument choice and money demand, but because of data limitations, was only able to link transaction values (not cash holdings) to payment instrument choice. Eschelbach and Schmidt (2013) find in a reduced-form estimation using German data that "the probability of a transaction being settled in cash declines significantly as the amount of cash available at one's disposal decreases," but they also fall short of explaining the link from cash use to cash demand.

3 Payment instrument choice

3.1 Payments transformation 2001–2012

This subsection replicates the econometric analysis in Klee (2008) using the DCPC data. First, we restrict our data to ensure that the results are comparable. The transactions used in Klee's estimation all come from a grocery store chain that accepted cash, check, debit, and credit cards (signature debit was recorded as credit card payment); moreover, she restricted her sample to transaction values between \$5 and \$150 (2001 dollar prices).[1] The DCPC has a much broader scope: it tries to cover all consumer transactions, not just purchases at grocery stores. In fact, it also includes information on not-in-person payments (such as on-line purchases), bill payments, and automatic bill payments. For the results in this subsection, we used only transactions made at grocery, pharmacy, liquor stores, and convenience stores (without gas stations), where cash, check, debit, or credit card was used,[2] and we kept the range of transaction values unchanged (in 2001 dollars).

As in Klee (2008), we estimate a multinomial logit model of payment instrument choice. The choice of respondent n to use payment instrument i in transaction t depends on the indirect utilities u_{nti}:

$$\begin{aligned} i^* &= \text{argmax}_i \quad u_{nti} \\ u_{nti} &= \mathbf{x}_t \beta_{1i} + \mathbf{z}_n \beta_{2i} + \epsilon_{nti}, \end{aligned}$$

[1] Note that Klee's data are not meant to be representative of the U.S. payment system.

[2] The DCPC also includes data on prepaid card, bank account number payment, money order, travelers' checks, text message, and other payments. For grocery stores, however, the share transacted with these instruments is negligible.

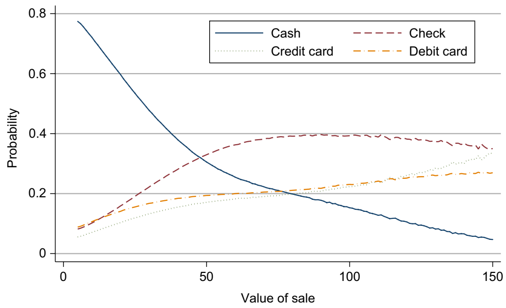
where vector $\mathbf{x}_t$ collects transaction-specific explanatory variables (for example, transaction value), while vector $\mathbf{z}_n$ denotes respondent-specific variables (for example, household income, age, education, gender, marital status), and ϵ_{nti} is assumed to be an independent and identically distributed (i.i.d.) Type I Extreme Value distributed error term. Note that since the variables in $\mathbf{x}_t$ and $\mathbf{z}_t$ do not vary across payment instruments, the coefficients β are assumed to be different for each payment instrument. The assumption about the error terms guarantees a closed-form solution for the choice probabilities:

$$\Pr(i|x_t, z_n) = \frac{\exp(u_{nti})}{\sum_i \exp(u_{nti})}.$$

The variables were chosen so as to match the specification in Klee (2008) as closely as possible.[3]

Figure 1 compares the estimated payment choice probabilities at different transaction values in 2001 and 2012. The left panel is taken from Klee (2008), while the right panel is obtained by carrying out the estimation on the DCPC data. The most striking difference between the two panels is that checks have virtually disappeared from grocery stores over the past decade. Second, the probability of choosing cash has roughly halved at all transaction values and cash is used overwhelmingly for low-value transactions. Credit and debit cards have stepped into the void left by the decline of cash for low-value transactions and checks for larger-value transactions. In particular, while the choice probability for PIN debit (orange dash-dotted line) levels off at large

[3]We have no information on the number of items bought and whether the respondent used a manufacturer's coupon to get a discount, nor do we have information on whether the respondent resides in an urban or rural area and whether or not she is a homeowner.



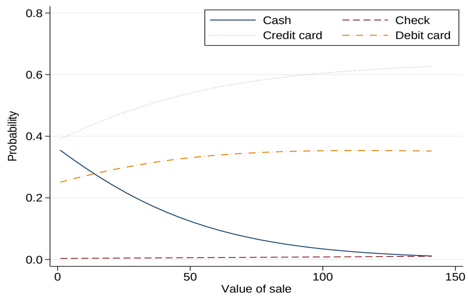


Figure 1: Payment instrument choice at grocery stores in 2001 (left, from Klee (2008)) and author's calculation based on DCPC data for 2012 (right)

transaction values, credit (including signature debit) increases monotonically over this range of purchases.

3.2 Payment instrument use in different contexts

In this subsection we drop the data restrictions in the previous subsection imposed by the need for comparability and re-do the same estimations on the broad range of payment contexts covered in the DCPC. The qualitative results from Figure 1 carry over to more general settings. For example, checks are largely absent from daily purchase transactions.[4] Cash transactions play a significant role in low-value in-person transactions, but, for obvious reasons, they are absent in not-in-person transactions.

Figure 2 shows payment instrument choice probabilities by transaction values. For in-person transactions (left panel) the graph tells a story similar to the one for grocery stores only (note that the scales of both axes have changed). Checks are rather unimportant; the change in the probability of cash use between low- and high-value transactions is by far the greatest among all payment

[4]They still play a role in bill payments, which we do not analyze in this paper.

instruments, although credit card use increases fairly quickly and does not level off even at transaction values as high as $1,000.

Unlike the studies that rely on scanner data, we are able to separate signature debit transactions from credit card transactions.[5] This is important, because according to Figure 2 signature debit transactions are very similar to PIN debit transactions, and quite different from credit transactions. There is not much difference between the two types of debit cards, although PIN debit use seems to level off at somewhat higher transaction values. This suggests, not surprisingly, that when making a payment, consumers are primarily concerned with the funds that debit and credit cards tap into; therefore, grouping payment methods by the network through which they are cleared may be misleading. The increase in the "Other" category as the transaction value increases is largely the result of a few fairly high-value purchases made using money order. Since there are few large-value transactions (the 99th percentile is at $341), these are a nontrivial portion of all large transactions.

Not-in-person purchases are dominated by credit and signature debit card payments, while bank account number payments (subsumed in the "Other" category) represent about 10 percent of all not-in-person transactions.

4 Dynamic model of consumer payment choice

The goal of our paper is to estimate a joint model of cash management and payment instrument choice. For expositional purposes, we think it is easier to

[5]Scanner datasets record the network through which transactions are routed by the merchant, not the actual payment instrument that consumers use.

Figure 2: Payment instrument choice at the point-of-sale (left) and not-in-person (right)

present the model in two steps: First, we describe the problem of a consumer who has a set amount of cash and has to make payment choices that respect the cash-in-advance constraint, but cannot make withdrawals. This will be done in the current section. Then, in Section 5, we extend this model, to give consumers a chance to change cash holdings, by making withdrawals.

In general, all types of payments, not just cash, are subject to constraints similar to the cash-in-advance constraint we focus on in this paper. Consumers may have minimum balance requirements on their checking account, or be up against their borrowing limit on their credit cards. Ideally, we would like to have a model that captures the availability of all payment instruments, but we do not have information about these other types of constraints. At the same time, we expect the cash-in-advance constraint to be the one that is most frequently binding.

4.1 The dynamic problem

Given that the availability of one of the payment instruments, cash, changes when it is used in a transaction, a link exists between current and future transactions: Choosing to use cash now, may limit the choice set in future transactions. Following Koulayev et al. (2012), we model this by stipulating that if cash balances are insufficient to settle a transaction, the consumer will no longer be able to take advantage of a (potentially) high realization of the random utility associated with cash transactions and therefore her expected utilities associated with future transactions will be lower. A forward-looking consumer will take into account this potential loss of utility when making the payment instrument choice in the current transaction. That is, she would maximize

$$\begin{aligned} V(m_t, t) &= \max_{i_t \in \{h,c,d\}} u^i_{ndt} + E\left[V(m_{t+1}, t+1)\right] \\ u^i_{ndt} &= \beta_i x_{ndt} + \gamma x_{ni} + \epsilon_{ndti} = \delta_{ndti} + \epsilon_{ndti}, \end{aligned}$$

where $V(m_t, t)$ denotes the value of having m_t amount of cash before making the tth transaction, and $E[.]$ is the mathematical expectation operator taken over the realizations of the shocks affecting future transactions. The instantaneous utility from using a payment instrument has three parts. Some variables x_{ndt} differ only across individuals (n) or days (d) or transactions (t), but not across payment instruments (i). Demographic variables and transaction values would be the obvious examples. For these variables, separate coefficients (β_i)

will have to be estimated for each payment instrument. Other explanatory variables are specific to a payment instrument (for example, whether a credit card gives rewards) and are included in the indirect utility function only for that instrument. For these variables, only a single parameter is estimated and these are collected in γ. The deterministic part of the indirect utility $\beta_i x_{ndt} + \gamma x_{ni}$ will be denoted by δ_{ndti}. Finally, there is a random component of the utility distributed independently and identically Type I generalized extreme value. The n and d subscripts will be dropped in what follows. The consumer chooses between **c**redit, **d**ebit, and cas**h** (if she has enough of it to pay for the tth transaction, $m_t \geq p_t$). The evolution of m is given by

$$m_{t+1} = m_t - p_t \cdot \mathcal{I}(i_t = h),$$

where $\mathcal{I}$ is an indicator function taking the value of 1 if cash is chosen ($i = h$) and 0 otherwise. The program has a finite number of "periods" (transactions) T, which is known to the consumer, and can be solved by evaluating the expectation on the right-hand side from the last period backwards.

Note that we assume throughout the model that the consumer knows with certainty at the beginning of the day, not only the number of transactions that she will make, but also the deterministic part δ_{ndti} of the indirect utility for each of these transactions. Although there are some variables in δ_{ndti}, such as the demographic characteristics, for which this information structure seems reasonable, assuming that the consumer knows exactly the dollar value of each transaction is clearly an extreme assumption.

To start the backward iteration, we need to fix the value of having an amount m of cash left *after* the last transaction (the terminal value of the value function). For simplicity, for now, assume that there is no value to carrying cash after the last transaction, resulting in

$$V(m_T, T) = \begin{cases} \max_{i \in \{h,c,d\}} u_T^i & \text{if} \quad m_T \geq p_T \\ \max_{i \in \{c,d\}} u_T^i & \text{if} \quad m_T < p_T \end{cases},$$

that is, the continuation value after transaction T is 0, regardless of the amount of cash on hand after the final transaction of the day. Note that given the simplifying assumption about the value of end-of-day cash holdings, the last period collapses to the multinomial logit choice problem, with expected utilities given by

$$E[V(m_T, T)] = \begin{cases} \ln\left(\sum_{i \in \{h,c,d\}} \exp(\delta_{Ti})\right) + \gamma & \text{if} \quad m_T \geq p_T \\ \ln\left(\sum_{i \in \{c,d\}} \exp(\delta_{Ti})\right) + \gamma & \text{if} \quad m_T < p_T \end{cases}, \tag{1}$$

just as in the static case of Section 3.

4.2 Transaction $T-1$

This means that, iterating backwards, the choice problem for $T-1$ is

$$V(m_{T-1}, T-1) = \\ \begin{cases} \max_{i \in \{h,c,d\}} u_{T-1}^i + E\left[V(m_{T-1} - p_{T-1} \cdot \mathcal{I}(i_{T-1} = h), T)\right] & \text{if} \quad m_{T-1} \geq p_{T-1} \\ \max_{i \in \{c,d\}} u_{T-1}^i + E\left[V(m_{T-1}, T)\right] & \text{if} \quad m_{T-1} < p_{T-1} \end{cases}. \tag{2}$$

While this function looks complicated, it is not hard to evaluate. Given m_{T-1} we know which of the two branches in equation (2) is relevant.

4.2.1 Insufficient cash for the current transaction, $m_{T-1} < p_{T-1}$

Starting with the simpler case, assume that $m_{T-1} < p_{T-1}$, meaning that: (i) in the current period only debit or credit can be chosen and therefore (ii) $m_T = m_{T-1}$. From (ii) we know which branch of $E\left[V(m_T, T)\right]$ in equation (1) is the relevant one, so all the terms in equation (2) are known and the choice probability of, for example, credit, will given by

$$\Pr\left(i_{T-1} = c | m_{T-1} < p_{T-1}\right) = \frac{\exp\left(\delta^c_{T-1} + E\left[V(m_{T-1}, T)\right]\right)}{\exp\left(\delta^c_{T-1} + E\left[V(m_{T-1}, T)\right]\right) + \exp\left(\delta^d_{T-1} + E\left[V(m_{T-1}, T)\right]\right)},$$

which collapses to the logit choice probability, since the expected utility terms for period T added to the δ_{T-1}s are the same and they all appear additively in the argument of the exp(.) operator, that is

$$\Pr(i_{T-1} = c | m_{T-1} < p_{T-1}) = \frac{\exp(\delta^c_{T-1}) \cdot \cancel{\exp\left(E\left[V(m_{T-1}, T)\right]\right)}}{\exp(\delta^c_{T-1}) \cdot \cancel{\exp\left(E\left[V(m_{T-1}, T)\right]\right)} + \exp(\delta^d_{T-1}) \cdot \cancel{\exp\left(E\left[V(m_{T-1}, T)\right]\right)}} = \frac{\exp(\delta^c_{T-1})}{\exp(\delta^c_{T-1}) + \exp(\delta^d_{T-1})}. \tag{3}$$

It is worth keeping this simple and intuitive principle in mind: *Dynamic considerations affect payment instrument choice only if the current choice reduces the* *expected utility when conducting the next transaction.* In this model, card use cannot do that.[6] The probability for debit card use will be analogous.

4.2.2 Cash is an option in $T-1$, $m_{T-1} \geq p_{T-1}$

Going back to equation (2), if $m_{T-1} \geq p_{T-1}$, then next period's expected utility, $E\left[V(m_T, T)\right]$, will be a bit more complicated to compute, since there are two possible values for m_T, depending on the payment instrument choice in the current transaction. With some probability cash will be chosen now, in which case $m_T = m_{T-1} - p_{T-1}$; otherwise $m_T = m_{T-1}$. Hence,

$$\begin{aligned} E\left[V(m_T, T)\right] = &\Pr(i_{T-1} = h) \cdot E\left[V(m_{T-1} - p_{T-1}, T)\right] + \\ &(1 - \Pr(i_{T-1} = h)) \cdot E\left[V(m_{T-1}, T)\right]. \end{aligned}$$

The expected value terms can be readily evaluated using equation (1), so all that needs to be calculated is the probability of using cash in the current transaction, which is given by a formula analogous to equation (3),

$$\begin{aligned} &\Pr(i_{T-1} = h | m_{T-1} \geq p_{T-1}) = \\ &\frac{\exp(\delta^h_{T-1} + E\left[V(m_{T-1} - p_{T-1}, T)\right])}{\exp(\delta^h_{T-1} + E\left[V(m_{T-1} - p_{T-1}, T)\right]) + \sum_{j=c,d} \exp(\delta^j_{T-1} + E\left[V(m_{T-1}, T)\right])}. \end{aligned} \tag{4}$$

Note the new first term in the denominator (the terms referring to credit and debit have been collapsed into a summation). Since cash can now be chosen in

[6] In reality, it could be the case that checking account balances drop to levels where they cannot be used, or that consumers max out their credit card(s). Unfortunately, we do not have data on that.

period $T-1$, debit and credit probabilities will decrease somewhat; hence the appearance of the new term.

Importantly, however, the formula reveals that the continuation utility after choosing cash may be different than the continuation utility after choosing cards. In particular, the first argument of $E\left[V(.,T)\right]$ is now $m_{T-1}-p_{T-1}$ if cash is chosen in $T-1$, whereas it is m_{T-1} if cards are used in period $T-1$. This is how consumers account for the fact that cash use now may limit their choices in the following transaction.

However, the principle stated above still applies: If (i) the consumer has enough cash to make both the $(T-1)$th and the Tth transaction with cash ($m_{T-1} \geq p_{T-1}+p_T$) or (ii) if she would not have enough cash to pay for the Tth transaction even if she did not use cash for transaction $T-1$ ($m_{T-1} < p_T$), then there is no effect of the payment instrument choice in $T-1$ on the value function in T. This argument extends to more transactions: If (i) $m_t \geq \sum_{s=t}^{T} p_s$ or (ii) $m_t < min_s\{p_s\}_{s=t+1}^{T}$, then the expected utilities in the formulas will be the same and the choice probabilities will collapse to the logit probabilities.[7]

Thus, we have demonstrated that the terms $E\left[V(m_{T-1}-p_{T-1}\cdot\mathcal{I}(i_{T-1}=h),T)\right]$ and $E\left[V(m_{T-1},T)\right]$ can be computed from functions that are readily known; hence, we are again left with the task of computing the choice probabilities in transaction $T-2$, given m_{T-2} using equation (4), and we can continue the recursion all the way up to the first transaction.

[7] Checking whether either of these special cases does in fact hold speeds up the evaluation of the expected utility tremendously for consumers who make many transactions a day.

5 Incorporating withdrawals

The dynamic model of Section 4 can be used to calculate the benefits of having a certain amount of cash on hand. The goal of this section is to use that information and data on withdrawals to estimate the costs associated with obtaining cash in order to characterize cash demand.

5.1 Simple model of withdrawals

Despite the availability of a closed-form solution for the dynamic model of Section 4, the evaluation of the value functions is computationally involved for individuals who report more than five transactions in a day and have an intermediate level of cash holdings. Therefore, we propose a simple model for withdrawals.

Consumers start the day with an exogenously given amount of cash. Before every purchase transaction they can decide whether they want to withdraw cash first. If they choose to do so, we assume that they withdraw enough cash to possibly settle all their remaining transactions with cash. That is, we assume, for now, that there is no variable cost of carrying cash within the day and that there is no limit on how much cash they can withdraw (clearly, a simplifying assumption for cashbacks). The fixed cost of making a withdrawal and the lack of a carrying/holding cost implies that consumers will make at most one withdrawal during the day; moreover, they have no reason to make a withdrawal after the last point-of-sale transaction.

Formally, if a consumer decides to make a withdrawal before transaction t, her new cash balances will be $m_t = \bar{m}_t \equiv \sum_{s=t}^{T} p_s$. The cost of making a withdrawal is modeled as

$$c_{ndjt} = \alpha z_{nd} + \alpha_j + \epsilon_{jt},$$

where z_{nd} is a vector of consumer- and day-specific explanatory variables, α_j is a withdrawal method-specific fixed-effect, and the ϵ_{jt} follow independent Type I extreme value distributions.

The consumer's choice before each transaction is given by,

$$E[\mathsf{W}(m_t, t, w_t = 0)] = \begin{cases} E\left[V(\bar{m}_t, t, w_{t+1} = 1)\right] - c_{jt} & \text{if} \quad \mathcal{I}^w_{jt} = 1 \\ E\left[V(m_t, t, w_{t+1} = 0)\right] & \text{if} \quad \sum_j \mathcal{I}^w_{jt} = 0 \end{cases}, \tag{5}$$

where $\mathcal{I}^w_{jt}$ is an indicator function for withdrawals (1 if a withdrawal is made using method j, 0 otherwise), where at most one of the $\mathcal{I}^w_{jt}$s might be bigger than 0. Note that due to the one-withdrawal-per-day limit, $w_{t+1} = w_t + \sum_j \mathcal{I}^w_{jt}$ is a new state variable: If a withdrawal was made earlier in the day, consumers do not have the option (nor the need) to make additional withdrawals, since they will be able to make all payments using cash. On the other hand, if they have not used up their withdrawal opportunity, then, in the current or in any one of the future transactions they may do so.

Formally,

$$E\left[V(\bar{m}_t, t, w_{t+1} = 1)\right] = \max_{i \in \{h,c,d\}} u^i_t + E\left[V(m_t - p_t \cdot \mathcal{I}(i_t = h), t+1, w_{t+2} = 1)\right],$$

with $m_t = \sum_{s=t}^{T} p_s$, meaning that the choice probabilities will not be affected by the cash-in-advance constraint, since it will not bind in the remaining transactions.

Also, since the withdrawal opportunity has already been used, the continuation value is given by $E[V(.)]$, not $E[W(.)]$.

The more computationally involved part will be the evaluation of

$$E\left[V(m_t, t, w_{t+1} = 0)\right] = \max_{i \in \{h,c,d\}} u_t^i + E\left[\mathsf{W}(m_t - p_t \cdot \mathcal{I}(i_t = h), t+1, w_{t+2} = 0)\right],$$

where the possibility of a future withdrawal will have to be included at each future transaction. However, the backward iteration described in Section 4 will still work in principle, with appropriate modifications. In particular, the random components of the withdrawal costs have been chosen to still yield closed-form solutions, similar to the payment instrument choice problem.

6 Results

The model is estimated by choosing parameters (α, β, γ) to maximize the likelihood of observing the sequence of payment instrument and withdrawal choices.

$$\log L(\tilde{i}, \tilde{j}; \alpha, \beta, \gamma) = \sum_n \sum_d \sum_t \left(\log(\Pr(\tilde{j}_{ndt})) + \log(\Pr(\tilde{i}_{ndt}))\right),$$

where $\tilde{i}, \tilde{j}$ denote the observed payment instrument and withdrawal method choices in the data. The estimated coefficients are reported in Table 1.

6.1 Marginal effects

The marginal effects are reported in Table 2. As noted earlier, there is a close connection between the multinominal choice model and our dynamic

	Indirect utilities			
	Debit card		Credit card	
Variable	Coeff.	S.E.	Coeff.	S.E.
Constant	-0.3199***	0.1286	-1.4100***	0.1738
TransVal	0.0057***	0.0013	0.0080***	0.0013
SmallVal	-0.7305***	0.0709	-1.0894***	0.0834
HHIncome	-0.0000***	0.0000	0.0000***	0.0000
Age	-0.0095***	0.0019	-0.0033	0.0023
Weekend	0.0809	0.0559	0.0660	0.0688
Female	0.0309	0.0568	-0.2688***	0.0679
PayDay	0.1933*	0.1003	0.0598	0.1237
RewardDC	0.0863	0.0613		
Revolver			-1.0980***	0.0671
RewardCC			1.2236***	0.0965
	Withdrawal costs			
Variable	Coeff.	S.E.		
IntRate	-0.0045	0.0171		
HHIncome	0.0000	0.0000		
Employed	0.2409*	0.1342		
PayDay	-0.7755***	0.1866		
Withdrawal methods				
ATM	4.4392***	0.1547		
Cashback	6.0196***	0.2160		
Bankteller	5.2709***	0.1777		
Family & friends	4.8565***	0.1642		
Other	5.0976***	0.1715		

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Table 1: Estimated coefficients

	Marginal effects		
	Cash	Debit	Credit
TransVal	-0.0016***	0.0007***	0.0009***
Under $10r	0.1977***	-0.1003***	-0.0974***
HHIncome	-0.0000***	-0.0000	0.0000***
Age	0.0017***	-0.0017***	0.0000
Weekend	-0.0180	0.0136	0.0044
Female	0.0209*	0.0162	-0.0371***
Payday	-0.0290	0.0323	-0.0033
RewardDC	-0.0145	0.0193	-0.0048
Revolver	0.0731***	0.0385***	-0.1117***
RewardCC	-0.1626***	-0.0857***	0.2483***

For dummy variables, marginal effect is a change from 0 to 1. TransVal=$12.53, income, age at sample average.
* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Table 2: Marginal effects for the final transaction on a day

specification: When continuation values across the three payment instruments are equal, the dynamic model collapses into the multinomial model. By assumption, the continuation values after the final transaction of a day are equal across payment instruments (they are all set to zero). Therefore, to facilitate comparison with previous results in the literature, we computed the marginal effects implied by our estimates for a hypothetical consumer (of average age, earning average income, and with all other variables set to zero) for her final transaction of a day. The main difference from Klee (2008) is that the effect of transaction values on cash use drops to about a quarter of what Klee found. Part of the explanation is obviously the inclusion of a dummy variable for small-value transactions, a choice that was motivated by the fact that some merchants accept only cash for small transactions. The other reason is that our dynamic framework controls explicitly for one of the main reasons transaction values might matter: the cash-in-advance constraint.

Moreover, the individual-level data show that other factors are just as important: Revolvers are much less likely to use credit cards than convenience users. On the other hand, credit card reward programs appear to be highly effective in steering consumers toward credit card use. Interestingly, debit card reward programs do not have the same effect.

6.2 Are consumers forward-looking?

Our model and the rest of the literature on payment choice can be thought of as addressing two extremes: We endow consumers with a great deal of information about their future transactions while the rest of the literature thinks of them as completely myopic. How important is this difference empirically? The simplest way to answer this question is to compare the choice probabilities of the two models. As noted before, the choice probabilities for the final transaction coincide with that of a multinomial logit model, but they may differ if the consumer plans to conduct more transactions.

Table 3 compares the payment instrument choice probabilities for the first transaction of the day for different total numbers of daily transactions. The same hypothetical consumer as in the previous subsection is assumed to start the day with $20, and all daily transactions are assumed to be for $12.53 (the median transaction value). Table 3 shows that the model predicts rather different choice probabilities in the five scenarios. In particular, the probability of using cash drops from 40 percent in the case of a single transaction, to just below 30 percent if she makes only one additional transaction. The drop in

Daily	Choice probabilities*		
transactions	Cash	Debit	Credit
1	0.4070	0.2397	0.3533
2	0.2947	0.2851	0.4202
3	0.2289	0.3117	0.4595
4	0.1827	0.3303	0.4870
5	0.1484	0.3442	0.5074

*Dummy variables set to 1, except for "Under$10;" transaction value at sample median (=$12.53); age, income at sample average.

Table 3: Choice probabilities for the first daily transaction for different total numbers of transactions

the probability of using cash is monotonic; in the case of a third transaction it is only roughly half what it would otherwise be. Since our choice model (like other multinomial logit models) possesses the independence-of-irrelevant-alternatives property, the *relative* probabilities of debit and credit do not change.

6.3 Withdrawal costs

Given the estimates of $\alpha, \alpha_j, \beta, \gamma$, the model can be used to conduct a cost-benefit analysis of cash withdrawals. In particular, given $\hat{\alpha}$ and $\hat{\alpha}_j$, we compute the average withdrawal cost by withdrawal methods in our sample:

$$\bar{c}_j = \frac{\sum_n \sum_d \hat{\alpha} z_{nd} + \alpha_j}{N_j},$$

where the denominator is the number of observed withdrawals using method j in the sample. This gives us a measure in units of consumer utility, which has no natural unit of measurement. To get a sense of the size of withdrawal

	Relative to			
Method	ΔEV^{DC}	ΔEV^{D}	ΔEV^{C}	$\bar{c}_{ATM}$
ATM	5.57	4.34	2.84	1.00
Cashback	7.51	5.85	3.82	1.35
Bank teller	6.59	5.13	3.35	1.18
Family & friend	6.08	4.74	3.10	1.09
Other	6.38	4.97	3.25	1.15

Table 4: Withdrawal costs

costs, we compare them with the expected benefit of having cash, defined as:

$$\Delta EV = E\left[V(p_T^{md}, 0, T)\right] - E\left[V(0,0,T)\right],$$

that is, the change in the expected utilities from making a payment of \$12.53 for the hypothetical consumer of the previous subsections. In fact, we compute this difference for debit and credit card holders (ΔEV^{DC}), debit card holders who do not have a credit card (ΔEV^{D}), and credit card holders who do not own a debit card (ΔEV^{C}).

Table 4 shows that, depending on the withdrawal method, it takes anywhere from 6 to 8 (median-sized) transactions to recoup the average withdrawal cost for consumers who have a debit card and a credit card (with no debt). For those who can only use a debit card instead of cash, withdrawals are less costly (cash is more useful): it takes them 4 to 6 (median-sized) transactions to make up for the withdrawal cost. Those with only a credit card recoup these same costs in 3–4 transactions.

The table also shows that ATM withdrawals are the least expensive, in utility terms, followed by getting cash from family and friends, other sources (including employers, check-cashing stores, cash refunds from returning goods, and unspecified locations), bank tellers, and retail store cash back. The difference between the least expensive and the most expensive source is about 35 percent.

6.4 Withdrawals

The solution to inventory theoretic models of cash demand (Baumol (1952), Tobin (1956), Alvarez and Lippi (2009)) is an (s, S) policy function, which specifies a level of cash balances s at which cash holdings are reset to S. As discussed above, consumers in our model do not optimize the size of their withdrawals, they withdraw just enough cash to carry them through the day. Therefore, a straightforward comparison between our model and the inventory theoretic studies does not exist. We can, however, compute the probability that someone makes a withdrawal before a particular transaction.

Figure 3 depicts these probabilities for consumers with different payment instruments in their portfolio. The hypothetical scenario behind the graph is that a consumer (average income, average age, employed, male) knows that he will have to make two, $12.50 transactions during the day. The horizontal axis denotes different amounts of cash in his wallet before the withdrawal opportunity preceding the first transaction, and the vertical axis denotes the probability that he will make a $25 withdrawal before the first transaction. The different lines correspond to different bundles of available payment instruments.

The solid line denotes the extreme case, where no credit or debit card is available to the consumer; therefore, he will have to make a withdrawal before the first transaction if he has less than $12.50 in his wallet. If he has more than

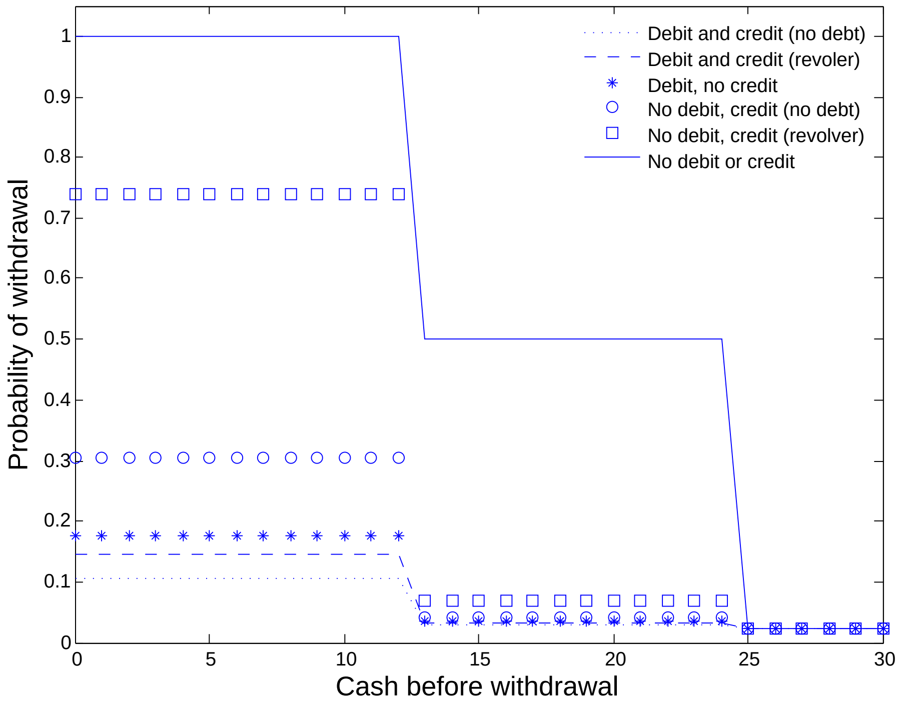


Figure 3: Withdrawal probabilities with different payment instrument bundles

that, he can afford to wait with the withdrawal until after the first transaction. Note that withdrawal costs also have a random component, so there is an option value of waiting. Figure 3 shows that consumers will use this option half the time. Finally, if he already has $25 or more in his pocket, there is no reason to get more cash.

The withdrawal decisions follow similar step-functions for every other payment instrument bundle. What stands out from the graph is that a person who revolves credit card debt and has no debit card (shown by squares) also values cash highly and is very likely to make a withdrawal if he is low on cash ($<$ $12.50). The option to delay a withdrawal, if he has enough cash ($\geq$$12.50) appears more valuable than for somebody with no alternative payment instrument, as indicated by the precipitous drop in the withdrawal probability. Since withdrawals are quite expensive, having just one additional option to complete a transaction already reduces the need for a withdrawal, especially if the benefits of the withdrawal (an expanded bundle of available payment instruments) can only be enjoyed in one additional transaction.

A similar line of reasoning explains why a convenience user of credit cards (shown by circles) will be not very likely to incur the cost of a withdrawal, even if his or her cash balances are low before the first transaction. Debit card users without a credit card (shown by stars), are even less likely to make a withdrawal, suggesting that debit cards are a closer substitutes for cash payments (at least at lower values) than credit cards are.

6.5 Shadow value of cash

Another way to measure the usefulness of cash, suggested by the monetary economics literature, is to compute the shadow value of cash, denoted by λ. This measures the change in utility from relaxing the cash-in-advance constraint by an infinitesimal amount. We measure it by adding $\Delta_\$ = \$1, \$5, \$12.53$ to the beginning-of-day cash holdings of each individual on each day and we compute the average of the resulting changes in expected utilities

$$\lambda_{\Delta_\$} = E\left[W(m_{nd} + \Delta_\$, t = 1, w_1 = 0)\right] - E\left[W(m_{nd}, t = 1, w_1 = 0)\right],$$

where m_{nd} is the actual amount of cash respondents had at the beginning of the day. Again, the same concept of ΔEV is used to normalize λ. That is, we normalize the average estimated benefits of adding $\Delta_\$$ dollars of cash to all consumers' beginning-of-day payment instrument bundles, by the expected utility that expanding the payment instrument bundle from {debit, credit} to $\{\Delta_\$,$ debit, credit} would give for a single $\Delta_\$$ transaction.

$$\begin{aligned}\frac{\lambda_{\$1}}{\Delta EV^{DC}} &\sim 0.0164\\ \frac{\lambda_{\$5}}{\Delta EV^{DC}} &\sim 0.1117\\ \frac{\lambda_{\$12.53}}{\Delta EV^{DC}} &\sim 0.2892.\end{aligned}$$

The costless relaxation of every consumer's budget constraint by the median transaction amount ($12.53) yields on average about a quarter of the expected utility of increasing the payment instrument choice set from debit and credit to cash, debit, and credit of the hypothetical consumer of subsection 6.3. The fact that this number is much lower than 1 suggests that a number of people in our sample are either already able to use cash for all of their transactions or only make transactions larger than $12.53; so for them the shadow value is zero. (Of course, doing away with the restriction of zero continuation value at the end of the day would change this.)

6.6 Simulations

Table 5 displays various moments in our data and shows how well the model replicates them. We considered several scenarios, the results for all of them based on 1,000 independent simulations.

First, to get an idea of how well the model explains the data, we ran a simulation of the model with the estimated parameters. Shocks were drawn

Payment instrument choice	Cash	Debit card	Credit card
Data	0.4990	0.2906	0.2104
Simulation			
DCPC starting cash	0.4657	0.3131	0.2212
$0 starting cash	0.2232	0.4567	0.3201
Simulation—No ATM			
DCPC starting cash	0.4589	0.3180	0.2231
$0 starting cash	0.1978	0.4729	0.3293
Simulation—Very Costly W			
DCPC starting cash	0.4175	0.3492	0.2333
$0 starting cash	0.0066	0.6004	0.3930

Withdrawals	Number	Share of methods				
		ATM	Cashback	Bank teller	Fam. & fr.	Other
Data	479	0.3549	0.0731	0.1545	0.2338	0.1837
Simulation						
DCPC starting cash	304	0.4863	0.0155	0.1052	0.2418	0.1513
$0 starting cash	824	0.4679	0.0182	0.1121	0.2442	0.1576
Simulation—No ATM						
DCPC starting cash	246	0	0.0427	0.2172	0.4433	0.2967
$0 starting cash	707	0	0.0485	0.2228	0.4314	0.2972
Simulation—Very Costly W						
DCPC starting cash	2	0.2115	0.1895	0.1905	0.2050	0.2035
$0 starting cash	35	0.1989	0.2019	0.1991	0.2010	0.1991

Table 5: Simulation results

according to the specified distributions, and the exogenous beginning-of-the-day cash balances were set to the values observed in the data (“DCPC starting cash"). Comparing the first two lines of the upper panel of the table shows that the model does fairly well in capturing the payment instrument choices. While the share of cash payments is somewhat underpredicted (46.57 percent vs. 49.9 percent in the data) and, correspondingly, debit and are credit overpredicted, the differences are fairly small.

On the other hand, the model performs much worse with withdrawals, a result that is not entirely surprising given our simplistic framework. Comparing the first two lines of the bottom panel of Table 5 shows that instead of the 479 withdrawals in the data, the model is able to predict only 304. We suspect three reasons for this. First, since the continuation value at the end of the day is set to zero and most individuals do not make more than two daily transactions, the high cost of withdrawals becomes prohibitive unless a very favorable shock is drawn. Second, we assume that agents start the day with an exogenous stock of cash, for which they do not have to pay. Therefore, many of them are able to make cash payments without making a withdrawal. Finally, of the 1,722 individuals in our sample, only 19 report not having a debit or credit card, meaning that the majority of the households are able to transact even without cash.

To understand better the role of the beginning-of-the-day cash balances, we re-ran the simulations with each consumer's beginning-of-day cash balances set to zero ("$0 starting cash"). This leads to a considerable drop in cash payments: 22.32 percent versus 46.57 percent in the previous simulation. These simulations yield many more cash withdrawals than in the data (824 versus 479), but the distribution across withdrawal methods is rather similar to the "DCPC starting cash" simulation. Both simulations overpredict ATM withdrawals at the expense of, for the most part, bank teller and cashback. Since cashbacks work differently in real life than the other methods, they require a preceeding debit payment and since we have not explicitly modeled this, it is no surprise that the prediction is off. The bank teller result is more discouraging.

A potential use of a structural model is to run policy experiments. The particular experiment we had in mind was to remove the possibility of ATM withdrawals (technically, we made ATM withdrawals very costly). That is, we asked what cash use would look like today had ATMs not been invented. The answer can be found in the "Simulation—No ATM" sections of each panel in Table 5. Surprisingly, cash use does not change much in either model compared with the respective baseline simulations: Cash use drops by less than a percentage point in the model with the observed starting cash balances and by about 2.5 percentage points in the model with $0 starting cash balances. The number of withdrawals drops by about a sixth in both simulations, and “Family and friends” become the primary source of cash. This highlights the partial equilibrium nature of our model: Where would family members and friends acquire this much additional cash?

Finally, to verify some of the above conjectures about what could be wrong with the model, we ran another experiment, where all withdrawal methods were made very expensive. In this case, the share of cash transactions dropped to 41.75 percent and 0.66 percent in the two simulations. This result confirms that the exogenous starting cash balances drive the results to a very large extent. Interestingly, even with the extremely high withdrawal cost in these scenarios, withdrawals do not disappear from the economy. The two withdrawals reported on the penultimate line of the bottom panel of Table 5 show that of the 19 respondents who had no debit or credit card there were two days when the exogenous starting cash balances were not able to cover the spending during those days. In these cases respondents are forced to make a withdrawal, regardless of the costs. Moreover, these 19 respondents recorded payments on 35 days, so when their beginning-of-day cash balances are set to \$0, they all have to make withdrawals on these days regardless of the withdrawal costs. The roughly uniform distribution across withdrawal methods shows that the random component of the cost drives this choice; the known components are equal(ly high).

All in all, the results of these simulations are mixed. On one hand, the model yields reasonable predictions for payment instrument choice, which is encouraging, but the simplistic framework for withdrawals clearly hinders it from providing a clear link between cash withdrawals and payment instrument choice. Future work will be directed toward extending the model so that it is able to explain observed withdrawal amounts, not just frequencies. This will help relax the assumption of only one withdrawal a day. Perhaps even more restrictive in the current formulation is that of the zero end-of-day continuation value. This was originally motivated by computational considerations: evaluating long sequences of transactions is still quite slow. A solution to this can be introducing a change in the information structure of the model: Not giving consumers full information about future transaction-specific variables enables us to recast the finite period model into an infinite horizon model. Solving for the value function in that model is more involved, however.

7 Conclusion

Using a new, transaction-level dataset of consumer payment choice, we are able to further our understanding of how consumers prefer to settle transactions. First, payment instrument bundles matter: Whether consumers earn rewards on their credit cards or pay interest on credit affects their choices markedly. Second, technology matters: Even in the simple model of this paper, we see substantial differences in the cost of obtaining cash. Third, payment instrument choice is ultimately a dynamic decision: Using an instrument for a transaction may limit its availability for future transactions. While much of monetary economics has focused on analyzing the optimal withdrawal policy that helps agents transact at minimal cost, an alternative margin that consumers can exploit in liquid asset management is payment instrument choice. As financial innovation blurs the boundary between transactions and savings accounts, this margin is likely to become even more important.

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