A Model of Real Estate Auctions

versus

Negotiated Sales

by Christopher J. Mayer
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Abstract

Real estate auctions have grown substantially in recent years, emerging as an alternative sales method for many institutions interested in selling large amounts of property quickly. This paper develops a framework for comparing auctions to the more traditional negotiated sales. The model shows that auctions will sell property at a discount because a quick sale results in a poorer "match" between house and buyer, on average, than could be obtained by waiting longer for a buyer. Furthermore, the model predicts that auction discounts should be larger in a down market with high vacancies, and in a smaller market with fewer buyers and sellers, when there is a larger difference between houses. Finally, the auction discount is smaller when property is more homogeneous, because the match between buyer and house matters less in the final price. Many of these results are verified empirically in other research.

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A Model of Real Estate Auctions versus Negotiated Sales

Real estate auctions have grown substantially in the United States in the last 15 years, mostly in regions that have suffered downturns in their local real estate markets. Many observers have suggested that auctions will disappear as the U.S. economy improves and financial institutions sell their glut of real estate. Others believe that U.S. auctions will continue to grow, following the pattern of countries such as Australia and New Zealand, where auctions are used more frequently in good times to sell property. Given that the U.S. government and private banks and developers have tens of billions of dollars of real estate in their portfolios, the performance of auctions has important public policy implications. These implications may extend well into the future, as the government looks for lessons on how to handle future crises involving financial institutions and real estate markets.

This paper develops a model to look at the ways auctions differ from traditional, negotiated sales techniques. By contrast, much of the previous literature analyzes these sales methods separately. The model helps to explain why auctions often sell real estate at a significant discount. It also permits analysis of conditions that affect the size of the auction discount. For example, properties auctioned in Dallas during the oil-price bust of the mid to late 1980s sold at a much larger discount (15 to 21 percent) than auctioned properties in Los Angeles during the boom of the mid 1980s (0 to 9 percent). (See Mayer 1993.)

The model uses a search framework in which buyers look for houses that "match" their preferences well. Sellers also look for buyers to arrive with a good "match" to their property, and they set an asking price that would appeal only to "well-matched" buyers. Auctions short-circuit this process; they sell
a property quickly, but only to buyers who are in the market at a given time. This smaller pool of auction buyers often results in poorer matches between buyers and houses. For sellers, the potential trade-off is clear: auctions provide a quicker sale, but at a lower price. Many sellers, such as the Resolution Trust Corporation, the Federal Deposit Insurance Corporation, and private banks, face large holding costs and may find that auctions bring them a better return than negotiated sales that would typically take longer. Buyers are attracted by the possibility of purchasing a house at a discount below the "market" price, but also realize that the house may match their preferences less well.

A further goal of this model is to explain why auction discounts seem larger in a downturn, such as that experienced by Dallas in the mid to late 1980s. In the context of this model, a "down" market is one in which the number of buyers decreases and the number of sellers increases. Fewer buyers and more sellers means that a given buyer will have a greater number of houses to choose from. With more alternatives available in the negotiated sale market, that buyer will pay less for an auction property that, on average, offers a poorer match. Using this framework, this model indicates that the decrease in auction prices during a downturn is not simply absolute, but also relative to prices in the negotiated sale market.

This paper compares auctions to negotiated sales, developing results that help explain the auction discounts. Section I surveys the previous literature and its implications for selling real estate. The basic model of negotiated sales is presented and solved in Section II, while Section III adds auctions to the model. Section IV gives the results of simulations assuming
that mismatch costs are distributed uniformly. Finally, the results are summarized and additions to the model are considered.

I. Previous Research

The theory of optimal auctions is an area that economists have studied heavily in recent years, focusing most often on the relative merits of different types of auctions. The initial motivation of much of the literature was Vickrey's (1961) famous revenue-equivalence result, in which he found that under certain conditions, including risk-neutral bidders, unaffiliated bids, and symmetrical valuations of buyers, four major auction types (English, first-price, second-price, and Dutch) all provide the seller with the same expected revenue. Much of the subsequent literature has focused on relaxing the above assumptions in attempts to understand the circumstances under which some auction types dominate others (from the

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1This section highlights theory that will be tested in subsequent sections of the paper. For a more complete survey of the auction literature, see Milgrom (1989) and McAfee and McMillan (1987).

2English auctions are open outcry sales in which the highest bidder receives the good at the bid price. The Dutch auction is the opposite of the English auction, with an auctioneer starting at a high price and continuing down to lower amounts. The first buyer to speak up claims the good at the last price mentioned by the auctioneer. Both first-price and second-price auctions involve potential buyers submitting sealed bids in advance to the seller, with the auctioned good going to the highest bidder. At a first-price auction the winner pays a price equal to his or her bid, while at a second-price sale the winner pays an amount equal to the highest losing bid.

3Actually, all auctions that fit the above conditions and have bids that are an increasing function of a bidder's valuation can be shown to be equivalent, in terms of both total surplus and seller's revenue.
perspective of the seller, buyer, or society) in maximizing surplus or making more efficient exchanges. 4

This literature has conflicting applications to the sale of real estate by auction (Lusht 1990; Vandell and Riddiough 1992). On one hand, the likely presence of risk-averse bidders causes first-price, sealed-bid auctions to have higher expected prices (Milgrom 1989; Riley 1989). 5 On the other hand, the fact that buyers have affiliated valuations suggests that English auctions might raise seller revenue by encouraging buyers to bid more aggressively than they would in a first-price auction (Milgrom and Weber 1982; McAfee and McMillan 1987; Milgrom 1989). 6 In addition, first-price auctions are more difficult for bidders to prepare for, as buyers not only must determine their own private valuation, but also must model the bids of other potential participants. The predominance of English auctions for selling most real estate (with the exception of some large commercial properties) suggests that the latter two concerns override the effects of risk aversion and indicates that English-style auctions are the logical choice for modeling real estate auctions.

4 English auctions are used most frequently in selling real estate, art, wine, used cars, and many other goods. First-price sealed bids are often used for procurement, drilling/mining rights, and selling a variety of financial instruments, including U.S. Treasury notes.

5 Intuitively, a potential buyer will likely increase his or her bid in response to uncertainty over the winning bid. The higher bid creates less expected surplus, but a greater probability of being the winning bidder.

6 Affiliation exists here because all buyers have a common value component in their valuation of a property (that is, all are concerned to some extent with a property’s resale value). Under English or second-price auctions, buyers pay only slightly more than the second highest bid, providing greater assurances that their valuation is not out of line with that of others in the market. Thus buyers may bid more aggressively because they are less likely to suffer from the “winner’s curse.” See Milgrom (1989) for a fuller description of the “winner’s curse.”
Another substantial literature has analyzed search markets. Early papers focus on labor markets, attempting to explain why prices do not seem to clear the market at any given time.⁷ These models generally assume that information is symmetric, but the matching technology is imperfect. When turnover occurs, workers (firms) cannot immediately find a replacement job (worker) that is a good match with their particular skills (needs). The search time creates unemployment and unfilled jobs, but also allows better matches to occur between workers and jobs. Wages are a byproduct of negotiations between workers and firms about how to split the surplus obtained by a good match. Stocks of workers are considered fixed in the short run, so shocks to demand (for workers) are only partially offset by wage changes. A second set of search models uses imperfect information and spatial differentiation to derive a market with positive vacancies and price dispersion.⁸

Many of the characteristics of labor markets described above apply to housing as well. Several recent papers have used a search framework to describe the workings of housing markets. Wheaton (1991) derives a model with two types of houses and two kinds of people. Households become mismatched with some probability, creating turnover. The paper shows that an imperfect matching technology leads to equilibrium vacancy rates, with some transitional households owning two houses. In this framework, small shifts in supply or demand lead to substantial price changes, but little change in quantity. Prices for a given type of house are identical. Read (1991) shows that

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⁷See Mortensen (1978), Diamond and Maskin (1979), Diamond (1982), and Hosios (1990) as examples of this literature.

⁸See Rothschild (1974), Butters (1977), and Burdett and Judd (1983).
spatial differentiation and imperfect information can lead to a housing market with positive vacancies and price dispersion. Arnott (1989) takes a different approach to explaining vacancies, relying upon the heterogeneity in rental housing units to give landlords market power. Because tenants are willing to pay a premium for their most preferred unit and all units are different, landlords set rents above the long-run replacement cost of housing. Free entry leads to equilibrium vacancies.

Despite the number of papers that look at auctions and at search markets, little attention has been given to markets in which both of these techniques exist simultaneously. Yet these markets may provide valuable insights into the advantages and disadvantages inherent in the choice of sales technique. Adams, Kluger, and Wyatt (1992) attempt to compare these two techniques by modeling negotiated sales as a slow Dutch auction. They show that if buyers arrive at an exogenous rate with independently, identically distributed valuations, the optimal strategy for a seller is to set a constant sales price rather than to lower the asking price over time. They conclude that a fixed asking price obtains a higher price than an auction. The prediction that auctions sell at a lower price is based on the fact that the buyer with the highest valuation at any given time will have a lower valuation than could be obtained by waiting for a longer period of time and drawing from a greater number of buyers. This result can be reversed, however, in the presence of a non-stationarity such as a seller who faces a penalty for not selling in a fixed period of time. Salant (1991) shows that such a non-stationarity changes the optimal strategy to one in which price declines over time.

9This question is particularly relevant given that sellers of items like wine, art, and real estate have a choice of sales technique, and this choice may have a substantial effect on the sales price and time to sale.
Adams, Kluger, and Wyatt (1992) also assume that auctions do not bring any additional buyers to the market.

This paper will develop a partial equilibrium model that compares auctions with negotiated sales. A framework in which buyers have different valuations of the same property is used because it seems to provide the best framework within which to compare the sales techniques. In particular, this model allows for an interesting trade-off. Auctions can sell a property more quickly, but at a "cost" of a poorer match than might occur in the search/negotiated sale market. Simulations of short-run variations in the vacancy rate will also allow predictions about the relative merit of auctions in different types of markets, something missing from the literature to this point.

II. The Model of Negotiated Real Estate Sales

The model described below is similar to that in Arnott's (1989) analysis of the rental housing market. Terms are defined in Table 1. Assume that N households exist in a market. In a given period, each household enters the market to search for a house with an (exogenous) arrival rate, μ, and departs from its current house at an equal rate. Thus, at any given time, n (= μN) households will be searching for a house, and an equal number will have departed from their current house. Once in the market, buyers costlessly observe all available properties, choosing the house with the lowest total

10 Salant (1991) could be interpreted as providing a framework in which auctions obtain a higher price than a negotiated sale. In his model, realtors get higher prices than houses for sale by owners because they increase the arrival rate of interested buyers. Many auctioneers claim that a large advantage of auctions is that they greatly increase the number of potential buyers who visit a property. If this were true, auctions might obtain higher prices, even in the Adams, Kluger, and Wyatt (1992) model.
<table>
<thead>
<tr>
<th>Variable</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>N</td>
<td>Total Number of Households</td>
</tr>
<tr>
<td>V</td>
<td>Number of Vacant (Excess) Houses</td>
</tr>
<tr>
<td>$\mu$</td>
<td>Rate of Households Arriving into and Departing from the Market</td>
</tr>
<tr>
<td>n</td>
<td>Number of Households Searching for a House ($= \mu N$)</td>
</tr>
<tr>
<td>A</td>
<td>Number of Available Houses ($= n + V$)</td>
</tr>
<tr>
<td>$m_i$</td>
<td>Money Price of House $i$</td>
</tr>
<tr>
<td>$x_i$</td>
<td>Mismatch Cost of House $i$</td>
</tr>
<tr>
<td>$p_i$</td>
<td>Total Price of House $i$ ($= m_i + x_i$)</td>
</tr>
<tr>
<td>$f(x_i);F(x_i)$</td>
<td>p.d.f., c.d.f. of $x_i$</td>
</tr>
<tr>
<td>$x^1, x^2$</td>
<td>1st Order Statistic, 2nd Order Statistic (minimum)</td>
</tr>
<tr>
<td>$g_1(x^1)$</td>
<td>p.d.f. of the 1st Order Statistic</td>
</tr>
<tr>
<td>$g_2(x^2)$</td>
<td>p.d.f. of the 2nd Order Statistic</td>
</tr>
<tr>
<td>$c$</td>
<td>Holding Cost of a Vacant Property</td>
</tr>
<tr>
<td>$r$</td>
<td>Real Interest Rate</td>
</tr>
<tr>
<td>$T$</td>
<td>Expected Time to Sale Given $m, A$ ($= 1/a$)</td>
</tr>
<tr>
<td>$F$</td>
<td>Replacement Cost of a House</td>
</tr>
<tr>
<td>$\pi$</td>
<td>Expected Proceeds from Selling a House</td>
</tr>
<tr>
<td>$m$</td>
<td>Market (Money) Price of a House</td>
</tr>
<tr>
<td>$v$</td>
<td>Market Vacancy Rate ($= V/U$)</td>
</tr>
<tr>
<td>$a$</td>
<td>Market Arrival Rate* ($= \mu N/A$)</td>
</tr>
<tr>
<td>$v_0(m_0;m,A)$</td>
<td>Seller 0’s Vacancy Rate Given $m_0 \neq m$</td>
</tr>
<tr>
<td>$a_0(m_0;m,A)$</td>
<td>Seller 0’s Arrival Rate* Given $m_0 \neq m$</td>
</tr>
<tr>
<td>$\pi_0(m_0;m,A)$</td>
<td>Seller 0’s (expected) Proceeds Given $m_0 \neq m$</td>
</tr>
</tbody>
</table>

*The arrival rate represents the probability that a buyer purchases a property in a given period.
price (defined below).\textsuperscript{11} There is no homelessness; buyers instantly choose their most preferred house among \( A \) available houses, which include a number of "excess" houses \( (V) \) that are unoccupied because the previous owner has already moved. In other words, \( A \) is equal to the number of houses that come on the market in the current period, \( n \), plus the number of vacant houses, \( V \), that were vacated previously but have not been sold (that is, \( A = n + V \)).

For a given buyer, the price of house \( i \), \( p_i \), is composed of a money price, \( m_i \), paid to the seller, and a mismatch cost, \( x_i \), incurred by the buyer because the house is not a perfect match \( (p_i = m_i + x_i) \). Each buyer draws \( x_i \) from the probability density function, \( f(x) \). The mismatch costs stem from characteristics of the house that do not match a given buyer’s preferences. For example, the house might have small bedrooms, but a large kitchen and family room, when a buyer prefers the opposite. It might have an old-fashioned kitchen instead of a modern one, or hardwood floors instead of carpeted ones. Many buyers will spend tens of thousands of dollars and hundreds of hours of work to transform a house according to their individual preferences. Some people even tear down an existing property and replace it with a custom-built home. Presumably buyers trade off a higher selling price against the quality of match. These preferences are independent, in that one person’s dream home is another’s nightmare. Thus, each draw from \( f(x) \) is assumed to be independently and identically distributed (i.i.d.) across both buyers and houses.

Sellers are households that have departed the market for some exogenous reason; they attempt to maximize the (expected) present discounted value of

\textsuperscript{11}Notice that this differs from the Wheaton (1991) and Read (1991) frameworks, in which search is costly and the matching technology is imperfect.
returns from the sale of the house. The model ignores the fact that many selling households are also buyers in this or another market and that this may affect their sales strategy. 12 Both buyers and sellers are fully informed about all the market parameters, including f(x) and all asking prices, but sellers do not know any single buyer's x_i and thus cannot discriminate among individuals. In setting their asking price, sellers take the market as given, including the number of households (N), the number of available units (A) and the number of searching households. Sellers then wait for a buyer willing to pay their asking price. 13 As Adams, Kluger, and Wyatt (1992) show, this is the optimal strategy so long as the arrival and departure rates are stationary. 14

In total, n buyers are each looking for a house among A available houses on the market. Each buyer chooses his or her best possible match. Buyers with a given match may be drawn to another house with a higher mismatch cost if the seller is willing to cut the price a little bit. Thus sellers trade off a quicker sale with a lower price.

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12 See Stein (1992) for a model in which down payment constraints, combined with the fact that buyers are also sellers, accentuate the real estate cycle. This occurs because sellers need to obtain a high price to get enough equity to purchase a new home. In a down market, this leads to some sellers setting artificially high asking prices, reducing transactions even further.

13 This model has no bargaining because sellers do not know a buyer's x_i and sellers have no residual valuation for their property. Any bargaining would likely take the form of offers at a percentage discount from the asking price. Thus, the asking price used here is just the original asking price minus discounts.

14 This result also depends on sellers having the financial ability to bear losses until a property sells. An owner who has purchased another house, for example, may face increasing pressure to sell his or her old house, leading to price cuts over time.
An equilibrium exists in this search model if, and only if, the following three conditions are satisfied:

(i) \( m_0 = m_0(m, A) \).

This equation says that each individual seller (in this case, seller 0) has set his or her optimal asking price, \( m_0 \), given the observed market price for a unit, \( m \), and the number of available units, and has no incentive to deviate by setting a higher or lower price.

(ii) \( m_0 = m \)

Any equilibrium must also be symmetric. We rule out any mixed-strategy equilibria where otherwise identical sellers might choose different asking prices.

(iii) \( \pi(A) = F \)

This condition states that the expected proceeds from selling a house equal the replacement cost, implying free entry and exit. This will be true only in long-run equilibrium. In subsequent sections this requirement is relaxed to examine the effects of temporary shocks to vacancies on prices when supply is fixed in the short run.

These three conditions permit a solution for the three unknowns—\( m \), \( m_0 \), and \( A \). The seller’s optimum is solved for first, taking the market as given (Condition i). Then the zero profit and symmetry conditions are imposed, yielding an equation in terms of vacancies.

The seller will maximize the (expected) present discounted value of profits, setting an asking price such that the interest cost of holding the house equals the expected return from waiting another unit of time. This is
shown in equation (1), and is equivalent to the equilibrium condition in an asset market.

\[(1) \quad r \pi_0 = a(m_0; m, A) [m_0 - \pi_0] - c(1 - a(m_0; m, A)).\]

The house sells in the next period with probability \(a(.)\). If it sells, the return is the difference between the asking price, \(m_0\), and the expected profits from holding the house another period; if the house doesn't sell, the owner also pays a holding cost, \(c\). The arrival rate \((a)\) is the rate at which a buyer arrives willing to purchase a property at the given asking price, \(m_0\), and hence is equivalent to the probability that a house sells in a given period. Simplifying (1), the seller will solve:

\[(2) \quad \max_{m_0} \pi_0 = \left( \frac{a(m_0; m, A)}{a(m_0; m, A) + r} \right) (m_0 + c) - \frac{c}{a(m_0; m, A) + r}.\]

The arrival rate depends on the market price \((m)\) and the number of available houses in the market \((A)\). If all asking prices were the same \((m_0 = m)\), then the arrival rate would be \(n/A\), or the number of households searching divided by the total number of available houses. But if some sellers raise their prices above the level set by the rest of the market, derivation of the arrival rate is more complicated. Consider the first order statistic, \(x^1(A; f(.))\), which is a random variable defined as the minimum of \(A\) draws from \(f(.)\). \(x^1\) has the probability density function:

\[15\text{Note that profits in this model, } \pi_0, \text{ are defined as the present discounted value of the sale price minus holding costs and do not include the fixed cost of building the house.} \]
The (conditional) second order statistic, \( x^2(x^1, A; f(.)) \), is also a random variable and is defined as the second lowest of \( A \) draws from \( f(x) \), given that \( x^1 \) is the minimum. Conditional on \( x^1 \), \( x^2 \) has p.d.f.:

\[
(4) \quad g^2(x^2 | x^1; A, f(.)) = (A - 1) \left( \frac{f(x^2)}{1 - F(x^1)} \right) \left( \frac{(1 - F(x^2))}{1 - F(x^1)} \right)^{A-2}.
\]

Following Arnott (1989), we define \( Q(m_0; m, A) \) as the probability a buyer who would otherwise prefer unit 0 instead chooses his or her second most preferred unit, because \( m_0 > m \). Thus \( Q(m_0; m, A) \) equals \( \Pr(m_0 + x^1 > m + x^2) \), which is equivalent to \( \Pr(m_0 > m + x^2 - x^1) \). The latter term is simply the probability that \( m_0 \) is greater than \( m \) plus the expected difference between the first and second order statistics. Thus

\[
(5) \quad Q(m_0; m, A) = \int_0^{x^1 + m_0 - m} g^1(x^1; A, f(.)) \int_{x^1}^{x^1 + m_0 - m} g^2(x^2 | x^1; A, f(.)) dx^2 dx^1.
\]

Combining (3) and (4) into (5) yields

\[
(6) \quad Q(m_0; m, A) = A(A - 1) \int_0^{x^1 + m_0 - m} f(x^1) \int_{x^1}^{x^1 + m_0 - m} f(x^2) (1 - F(x^2))^{A-2} dx^2 dx^1.
\]

A problem with creating \( Q(m_0; m, A) \) as a function of the difference between the first and second order statistics is that this definition ignores the possibility that a house is the most preferred unit of two or more buyers in the same period. In practice, the difference between the first and second order statistics should serve as a good approximation for the expected difference between the two most preferred houses available to a given buyer.
This assumption will have no substantial effect on the comparative statics in the subsequent simulations.\textsuperscript{16}

Define the arrival rate at unit 0 as follows:

\begin{equation}
\begin{aligned}
a_0(m_0; m, A) &= \left(\frac{\mu N}{A}\right) (1 - Q(m_0; m, A)).
\end{aligned}
\end{equation}

Equation (7) says that the arrival rate for house 0 is equal to the market arrival rate multiplied by the probability that a buyer is not deterred by \( m_0 > m \). Because there is no homelessness and all buyers match with a house in a given period, the market arrival rate is the number of buyers divided by the number of available houses.

Given the arrival rate, solving the seller's problem in equation (2) yields the following first order condition:

\begin{equation}
m_0 = \frac{n + rA - (\frac{\partial Q}{\partial m_0})cA(1 + r)}{(\delta Q / \delta m_0)rA}.
\end{equation}

The fact that sellers have symmetric positions implies that \( m = m_0 \) and \( Q = 0 \). Given the symmetry, a simplified version of equation (6) can then be used to solve for the derivative of \( Q \) with respect to \( m_0 \) as required in equation (8):

\textsuperscript{16}For example, consider the conclusion in Section III that the percentage discount at auctions increases in a down market. In terms of the search market, that result depends only on the fact that prices in the search market decrease with increased vacancies.
Putting (9) into (8) gives an equation that governs the short-run price, when supply is fixed. Because of shocks to supply or demand, vacancies can vary around equilibrium rates. For example, the economy might grow faster than expected, increasing demand for housing and reducing vacancies until more houses are built. Similarly, a downturn can lead to increased vacancies and reduced prices.

To look at long-run equilibria, we impose the free entry condition,

\[ \pi = \frac{a}{a + r} (m + c) - \frac{c}{a + r} = F, \]

which says that the proceeds from building a house equal the replacement cost (F). It is equivalent to saying that net profits after entry costs equal zero. Combining free entry with the short-run condition (8), gives the following (long-run) expression to solve for A:

\[ A^2 \left( \frac{\delta Q}{\delta m_0} (cr + F r^2) \right) + A \left( \frac{\delta Q}{\delta m_0} (nc + nrF) - nr \right) - n^2 = 0. \]

Equation (11) can be solved for many distributions, and simulated for others, to get an equilibrium vacancy rate and price level given the interest rate, holding costs, market size, and replacement cost of housing.

For example, Figure 1 graphs equations (8) and (10) using parameters from later simulations. As expected, in the short run, prices decrease with

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\[ ^{17} \text{See the base case in Section IV for the derivations of the exact equations used in Figure 1.} \]
Figure 1
Simulation Results
Base Case

House Price (Dollars)

Short-Run Price

Long-Run Price

Vacancy Rate (Percent)

Base Case: 3,000 Households, $80,000 Range of Mismatch Costs
increasing vacancies and increase with the number of households searching for a home. Both of these results are due to changes in competition. If the number of buyers increases or fewer houses are for sale, the competition for each available house is greater, causing sellers to raise their prices. The opposite is also true; less competition for each house leads to lower prices in the short run.

Prices in this model are very sensitive to changes in vacancies because the adjustments are assumed to be permanent. In hot markets, a small decline in the vacancy rate leads to large price increases, a factor seen in cities such as Boston and New York in the mid 1980s. In the long run, prices increase as vacancies increase, although the effect is small given the parameter values used to create Figure 1. This result follows from the fact that long-run prices are based on the cost of building and selling a property. A high vacancy rate translates into a longer time to sale and increased holding costs, causing a builder to charge higher prices to break even. The intersection of the free-entry condition (10) and short-run price equation (8) is the solution to (11), the equilibrium vacancy rate.\textsuperscript{18}

III. Auctions

This model of negotiated sales can be augmented to include property auctions. Think of a single seller holding an auction and assume that the auction price has no effect on \( m \), the market price of real estate sold in the search market (calculated in Section II). The auction is attended by all \( n \) buyers in the market at that time. Although buyers may "attend" the auction

\textsuperscript{18}Note that (11) may contain multiple equilibria. In simulations with the uniform distribution, only one of the three roots was ever positive, and with some extreme parameter values, none were.
by observing a house's characteristics, in practice only buyers who have a high valuation will actually go to the auction site. Each buyer determines his or her valuation by looking at the best alternative in the search market and is willing to bid up to the total cost of that alternative (that is, the market price plus the mismatch cost). The price of a house at auction can be compared to the market price to determine both the absolute discount or premium associated with auctions, and how the auction price varies with changes in vacancies.

The model is based on an English-style (ascending bid, open outcry), absolute (no reserve) auction.\textsuperscript{19} In the English auction, buyers will bid their valuation.\textsuperscript{20} A buyer’s valuation is positive if and only if $p^a < p^1$. That is, the total price of the auction property ($p^a$) is less than the total price of the buyer’s most preferred non-auction property ($p^1$) among the available negotiated sale properties. This implies that:

\begin{equation}
\begin{align*}
    m^a + x^a &< m^1 + x^1 \\
    m^a &< m - (x^a - x^1),
\end{align*}
\end{equation}

where $m^a$ is the auction (money) price, $x^1$ is the lowest mismatch cost of the $A$ vacant houses, $x^a$ is the mismatch cost of the auction property, and $m$ is the market price of all non-auction houses. Thus each buyer has a valuation equal to \((m - x^a + x^1)\). Buyers with a poor match in the search market or a good match with the auction house are likely to be the high bidders.

\textsuperscript{19}Because auctions in this model do not have reserve prices, which theory predicts will raise the price of the property sold at auction, the simulation results might overestimate the discount actually associated with reserve-price auctions. The comparative statics, however, will not change.

\textsuperscript{20}The possibility that a buyer might shade his or her bid in response to the winner's curse is irrelevant in this model because a buyer's valuation completely depends on his or her match with a property.
The winning bid at this auction will be approximately equal to the second highest valuation, assuming the bidding increment is close to zero. In expectation, the highest bid will be equal to the market price, $m$, minus the second lowest draw from $(x^a - x^1)$ with $n$ draws, where $n$ equals the total number of buyers in the market. From Section II we know that $x^1$ and $x^a$ have density functions $g^1(x^1)$ and $f(x^a)$, respectively, that also depend on the number of available houses and the number of buyers. Now define $z = x^a - x^1$, which has the following p.d.f.:

\[
(13) \quad h_{x^a-x^1}(z) = \int_{-\infty}^{\infty} f(x^a) g^1(x^a - z) dx^a,
\]

and c.d.f., $H(z)$. We can also define $z^2$ as the second lowest of $n$ draws from $h$, with p.d.f.:

\[
(14) \quad h^2(z^2) = n(n-1) h(z) H(z)(1-H(z))^{n-2}.
\]

Thus, the expected price at auction is equal to the market price minus the expectation of $z^2$, or:

\[
(15) \quad E\{m^a\} = m - \int_{-\infty}^{\infty} h^2(z^2) z^2 dz^2.
\]

This model can be used to describe the short-term dynamics of a market where vacancies vary around the equilibrium levels, possibly as a result of local economic shocks. This is of particular interest in predicting how the auction premium/discount varies with the economic cycle.

---

21 Since it is a buyer's dominant strategy to bid up to his or her valuation, a house at auction will be sold to the highest valuation buyer at a price just above the second highest valuation.
The next step is to show that the auction discount percentage rises in a bust market, when short-term shocks cause the number of vacancies and available houses to rise or the number of searching households to fall. This result is striking because it says that even though negotiated sale prices fall in a down market, auction prices fall further. Note, however, that the auction price (15) is always defined relative to the negotiated sale price. At the same time that the market price falls because of increased vacancies, the (expected) absolute discount at auction $E(z^2)$ actually rises, leading to a bigger percentage discount.

To show this result formally, define the (percentage) auction discount as follows:

$$\text{(16)} \quad \text{pct. auction discount} = 1 - \frac{m^a}{m} = 1 - \left( \frac{m}{m} - \frac{E(z^2)}{m} \right) = \frac{E(z^2)}{m}.$$ 

Thus the auction discount is the (expected) absolute auction discount divided by the market price in the negotiated sale market. In the short run, both the market price and the absolute auction discount are based upon the number of households searching for a house and the number of houses available.

As discussed earlier, in the short run (taking the number of houses as fixed) the market price in the search market increases with the number of households searching and decreases with the number of available houses. The absolute auction discount behaves in an inverse fashion, rising with vacancies and falling with the number of households searching; that is, $\delta(z^2)/\delta V > 0$ and $\delta(z^2)/\delta n < 0$.

As fewer households search, fewer draws are taken from the distribution of $z$, where $z$ is the difference in mismatch costs between the auction house and the most preferred house in the search market. With fewer draws, $E(z^2)$,
the expected second lowest mismatch cost (n draws), will increase, causing the auction discount to rise. A boom with more bidders will have the opposite effect. More buyers will increase the number of draws from the distribution of mismatch costs, lowering the expected difference in mismatch costs of the winning bidder and reducing the (absolute) auction discount.

As vacancies rise, the number of available houses also increases, reducing the (expected) mismatch cost of the best alternative house in the search market ($x^1$). Since the (expected) mismatch of the auction house ($x^a$) does not change, the (expected) difference in mismatch costs between the auction house and the best alternative will rise. (Note that $z = x^a - x^1$.) In other words, with more houses available the best alternative to the auction house is better and has a lower (expected) mismatch cost. Thus, the auction discount rises as vacancies increase.

These results from the negotiated sale and auction markets imply that the auction discount percentage will increase when a market is hit with a negative shock that increases vacancies or decreases the number of households searching. As vacancies rise, the negotiated sale market price (the denominator) falls and the absolute auction discount (the numerator) rises, leading to a bigger percentage discount at auction. The opposite occurs with changes in the number of households searching for a house. The intuition here is that buyers always choose their auction bids relative to prices and properties in the negotiated sale market. A downturn has two effects. It lowers prices in the negotiated sale market, and it also reduces the number of bidders at auction, raising the average difference in mismatch costs of the winning bidder. The latter result guarantees that auction prices fall faster.
than negotiated sale prices, leading to an auction discount that rises as a market suffers a short-term negative shock to vacancies.

IV. Simulations

The above model, combined with assumptions about the basic parameters, can be used to solve for the auction discount. Table 2 provides data on vacancy rates, market size, and house prices for 19 metropolitan areas in 1990. Clearly, vacancy rates vary across cities. Later simulations suggest that equilibrium vacancy rates can vary significantly in individual markets because of differing market sizes, mismatch costs, and replacement costs. Even accounting for local cycles, vacancy rates seem to take a range of values.

Table 3 shows national statistics over time on sales volume, median prices, and number of months' supply on the market. As Case and Shiller (1989) and others have shown, median prices do not fully reflect downturns in the market, as the mix of houses sold changes over time. During the downturn of the mid 1980s, volume seems to have fallen much more rapidly than median prices, with the volume having the predominant effect on average sales time. Even aggregated at the national level, the supply of houses on the market varied from 8 to 12.5 months over the period from 1982 to 1991.

The model can be simulated using parameters for the variables in the first order condition (11), including the mismatch cost, x. Unfortunately, no data are easily available on the range of mismatch costs between houses, so x

---

22The number of months' supply is calculated by dividing the number of units for sale at the end of the year by the average number of sales in a month.
<table>
<thead>
<tr>
<th>City</th>
<th>Vacancy Rate</th>
<th>Number of Homes Looked At</th>
<th>CT&amp;T Median Price ($000)</th>
<th>NAR Median Price ($000)</th>
<th>NAR Price, Percent Change, 1989-90</th>
<th>NAR Price, Percent Change, 1985-90</th>
</tr>
</thead>
<tbody>
<tr>
<td>Atlanta</td>
<td>4.0</td>
<td>15.0</td>
<td>99.1</td>
<td>86.4</td>
<td>2.9</td>
<td>n.a.</td>
</tr>
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<td>Boston</td>
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<td>15.1</td>
<td>166.2</td>
<td>174.2</td>
<td>.4</td>
<td>22.4</td>
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<td>132.1</td>
<td>116.8</td>
<td>9.2</td>
<td>44.0</td>
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<td>79.9</td>
<td>80.6</td>
<td>7.2</td>
<td>25.2</td>
</tr>
<tr>
<td>Dallas</td>
<td>3.3</td>
<td>12.1</td>
<td>90.6</td>
<td>89.5</td>
<td>-4.2</td>
<td>-4.8</td>
</tr>
<tr>
<td>Denver</td>
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<td>15.1</td>
<td>91.4</td>
<td>86.4</td>
<td>1.1</td>
<td>-2.5</td>
</tr>
<tr>
<td>Detroit</td>
<td>1.0</td>
<td>13.9</td>
<td>94.1</td>
<td>76.7</td>
<td>4.1</td>
<td>48.4</td>
</tr>
<tr>
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<td>n.a.</td>
<td>70.7</td>
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</tr>
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<td>70</td>
</tr>
<tr>
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<td>n.a.</td>
<td>89.3</td>
<td>2.8</td>
<td>10.9</td>
</tr>
<tr>
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<td>96.9</td>
<td>88.7</td>
<td>1.7</td>
<td>18</td>
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<td>New Orleans</td>
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<td>n.a.</td>
<td>67.8</td>
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<td>n.a.</td>
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<td>242.4</td>
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<td>80</td>
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<tr>
<td>Orlando</td>
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<td>86.6</td>
<td>82.8</td>
<td>3.8</td>
<td>17.8</td>
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<td>121.2</td>
<td>108.7</td>
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<td>46.9</td>
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<tr>
<td>Phoenix</td>
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<td>14.1</td>
<td>88.7</td>
<td>84</td>
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<td>12.3</td>
</tr>
<tr>
<td>San Francisco</td>
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<td>259.3</td>
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<td>78.7</td>
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<td>14.5</td>
<td>119.9</td>
<td>131.5</td>
<td>14.3</td>
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</tr>
<tr>
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<td>15.4</td>
<td>145.4</td>
<td>150.2</td>
<td>4.0</td>
<td>54.7</td>
</tr>
</tbody>
</table>

*All prices are in nominal terms.

*Source: U.S. Bureau of the Census, 1990, Vacancy Rate for Owner-Occupied Housing.

*Source: Chicago Title & Trust Company's Survey of Recent Home Buyers; includes single-family homes and condominiums.

*Source: National Association of Realtors: includes only single-family homes.
<table>
<thead>
<tr>
<th>Year</th>
<th>Sales Volume (Millions)</th>
<th>Percent Change in Volume, 1 Year</th>
<th>Median Sales Price, 1991 Dollars ($000)</th>
<th>Percent Change in Price, 1 Year</th>
<th>Average Supply of Houses, (Months)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1977</td>
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<td>19.1</td>
<td>96.4</td>
<td>5.7</td>
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<td>n.a.</td>
</tr>
<tr>
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<td>-3.6</td>
<td>104.5</td>
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<td>n.a.</td>
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<tr>
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<tr>
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<td>95.7</td>
<td>-3.8</td>
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<td>1983</td>
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<td>.5</td>
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<td>1984</td>
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<td>94.9</td>
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<td>1985</td>
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<td>8.9</td>
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<td>8.4</td>
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<td>1988</td>
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<td>.2</td>
<td>8.6</td>
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<td>1989</td>
<td>3.440</td>
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<td>102.3</td>
<td>-.5</td>
<td>8.0</td>
</tr>
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<td>1990</td>
<td>3.296</td>
<td>-4.2</td>
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<td>-2.7</td>
<td>9.2</td>
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<tr>
<td>1991</td>
<td>3.220</td>
<td>-2.3</td>
<td>100.3</td>
<td>.8</td>
<td>9.1</td>
</tr>
</tbody>
</table>

All prices are in real terms, 1991 dollars.
Source: National Association of Realtors.
will be assumed to be distributed uniformly over $[0, L]$. The upper limit between the best possible house ($x_i = 0$) and the worst possible unit ($x_j = L$) is $L$, which is initially assumed to be $80,000$, the same level as the replacement cost of a house. Even with such a large range, the average difference in mismatch costs between houses in a market with 100 available houses is only $800$. Given that some buyers will tear down an existing house to build another one, or build a custom house in an overbuilt market like Texas, the assumed range of mismatch costs might be low. Higher values of $L$ are experimented with in the simulations, which indicate that this variable has a significant effect on prices.

Using the uniform distribution for $0 < x < L$ gives $f(x) = 1/L$ and $F(x) = x/L$. Substituting $f(x)$ and $F(x)$ into the arrival rate equation (9) gives $\delta Q/\delta m_0 = A/L$. Combining this equation with (8) gives the following solution for the short-run market price:

$$m = \frac{L(n + rA) + cA^2(1 + r)}{A^2r}$$

(17)

and combining with the free entry condition from (10):

$$m = \frac{F(a + r) + c}{a}$$

(18)

gives the following equation for vacancies:

\[\text{Future research might look at other distributions, such as the normal, which have longer tails, giving a lower probability of finding houses that are extremely good or poor matches with an individual buyer.}\]
\[(19) \quad A^3(rc + r^2F) + A^2(rnF + nc) - A(rn) - n^2L = 0.\]

Simulations of (19) will show the order of magnitude of the auction discount, given reasonable parameters. First, it is necessary to establish a time period, here three months, within which buyers select a house. Although the model assumes that buyers immediately observe all available houses, in practice that search takes some period of time. The search time of three months is consistent with data from the National Association of Realtors, but is a bit lower than the period of time suggested by the Chicago Title and Trust survey. The lower number is used because the model specifies no rental market and no homelessness, so buyers must be able to match within a given period. In the base case, real interest rates are set at 0.5 percent per period, or 2.02 percent per year. The cost of holding a vacant unit, above and beyond the forgone interest, is initially assumed to be zero. For short vacancies, this is probably realistic. In a market where houses are vacant for well over a year, vandalism and physical depreciation become more of a problem and values of \(c>0\) would be appropriate.

Finally, the size of the initial search market is set at 3,000 houses. This says that after a buyer chooses the approximate house and lot size and the preferred neighborhood(s), about 3,000 households are in the market that fits that description. That seems to be reasonable for many markets. In practice, the search is narrowed considerably because only a small percentage of those households will actually place their house on the market. With a steady-state turnover/arrival rate of 1.67 percent per period, 50 buyers will be searching for housing in a given period. (A turnover rate of 1.67 percent means the average homeowner moves every 15 years.)
These parameters are summarized on the first line of Table 4, which also gives the results of simulating the auction model for the base case. These simulations suggest that in a market of 3,000 households, 100 units would be available for purchase, with half of those being vacant. The other half are occupied, but available for purchase in the next period when the current owner departs. With these assumptions, the market vacancy rate is 1.6 percent and the average time on the market is six months. A look at Table 2 suggests that these vacancy rates are a bit low relative to many U.S. cities. One possibility is that in 1990 many of the cities surveyed were suffering from "down" real estate markets, causing above-normal vacancy rates. Time to sale (Table 3) is harder to measure because of the discouraged seller effect, but the base case seems to underestimate this variable relative to actual time to sale.

Long-run (free entry) and short-run prices are graphed in Figure 2, with the latter describing what happens to prices when the stock of units is fixed. Seller behavior is characterized in the short run by the first-order condition (17), with the (long-run) zero profit condition (18) no longer binding. The seller maximizes his or her profit and those who own homes may actually earn positive or negative profits while the market adjusts back to equilibrium. The downturn in Dallas is a good example. Demand for housing fell sharply, with some households selling properties without being replaced by willing buyers. This led to an increase in vacant units and a decline in prices and profits. (See Table 2.)

24 The vacancy rate of 1.6 percent equals the number of available units (100) divided by the sum of the occupied units (3,000) and vacant units (50).
<table>
<thead>
<tr>
<th>Int. Rate (%)</th>
<th>Cost ($)</th>
<th>Households in Search</th>
<th>Market Size (Households)</th>
<th>Repl. Cost ($)</th>
<th>Max. Mismatch ($)</th>
<th>Vacancies</th>
<th>Price</th>
<th>Vac. Rate (%)</th>
<th>Avg. Sale Time (mths)</th>
</tr>
</thead>
<tbody>
<tr>
<td>.005</td>
<td>0</td>
<td>50</td>
<td>3,000</td>
<td>80,000</td>
<td>80,000</td>
<td>50</td>
<td>80,800</td>
<td>1.6</td>
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<td>80,000</td>
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<td>80,000</td>
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* Base Case, Figures 1-3
b Case II, Figures 2-3
Figure 2
Simulation Results
Base Case vs Case II

House Price (Dollars)

Short-Run Price
Auction Price
Long-Run Price

Base Case: 3,000 Households, $80,000 Range of Mismatch Costs
Case II: 1,500 Households, $120,000 Range of Mismatch Costs
The left-most set of lines in Figure 2 shows how the base case is affected by (short-run) shocks to vacancies, measured using the vacancy rate. (A similar graph could be drawn using time to sale.) Market and auction prices are graphed together. Clearly, prices fall as vacancies rise and vice versa. Price increases are particularly steep as the vacancy rate moves further below the long-run equilibrium level, marked by the intersection of the long-run and short-run equations. Note that the auction discount, the difference between the two price lines, increases as vacancy rates rise and prices fall.

Several factors might raise the predicted auction discount in a downturn. Auction properties may be in smaller, less common, market segments. A smaller market means that fewer houses will be available at any time, increasing the difference in mismatch costs between houses and leading sellers to set higher prices. Auction discounts will also be higher because the (expected) difference between the auction property and the next best alternative increases with fewer properties for sale. In this model a smaller market is equivalent to a larger range of mismatch costs. When mismatch costs rise, the equilibrium vacancy rate and the percentage auction discount increase. The results of these combined effects (smaller market and larger mismatch costs) are shown in Figure 2, labeled case II.

Figure 3 shows how the auction discount rises as vacancies rise, and compares case II to the base case. Although the base case and case II lines look like segments of the same line, they represent very different situations. The base case has a lower equilibrium vacancy rate and a lower auction discount when the vacancy rate deviates from the long-run equilibrium. Case
Figure 3
Simulated Auction Discount
Base Case vs Case II

Percent Discount

Vacancy Rate (Percent)

Base Case: 3,000 Households, $80,000 Range of Mismatch Costs
Case II: 1,500 Households, $120,000 Range of Mismatch Costs
II, by contrast, has significantly higher auction discounts in the long run, as well as in the short run when the vacancy rate moves away from equilibrium. Together, these simulations suggest that auctions might get large discounts in areas that have suffered significant downturns, particularly if auction properties have a greater variance in mismatch costs and a smaller market than other properties in the sample.

This last result is especially important, given the types of properties often sold in U.S. auctions by banks and government sellers. Many times, banks choose to auction properties that are hard to sell conventionally, either because the units are of lower quality or because they are different from other properties in the area. This suggests a smaller potential market, which could lead these properties to sell at a bigger discount relative to the average property. This is consistent with other empirical work on Los Angeles and Dallas (Mayer 1993), which finds that auctions of single-site (new) condominiums sold units at a smaller discount than auctions of scattered-site condominiums. The former were built as homogeneous units likely to appeal to a broader market, while many of the condos in the scattered-site auctions were older, of lower quality, and in smaller complexes, thus attracting a smaller group of potential buyers.

Looking at Figure 2, notice that short-run prices rise very quickly as the vacancy rate declines below the equilibrium level. This movement of prices occurs because entry is restricted and changes to vacancy rates are assumed to be permanent. In a boom, new houses can be built in as little as three to six months and new condos in a year or two. Thus, it is unlikely that permanent shocks to vacancies would occur as simulated in the model. Price increases in booms are due not only to shortages of existing houses, but
also to the fixed amount of land within close proximity of many desired locations. Prices rise at least in part because of increases in land values, which are reflected in the replacement cost, F. In busts, on the other hand, this model may do a better job of predicting price declines. The housing stock is fixed, so decreases in demand with little population growth can result in positive and permanent shocks to vacancy rates. This result is consistent with some price indices in depressed areas. Real condominium prices in Dallas County fell over 60 percent between 1985 and 1989.

Table 4 shows the effect of varying parameters on the equilibrium vacancy rate, house price, and time to sale. These are equilibrium results, meaning that the zero profit condition applies, and hence they are not directly comparable to the figures shown in Tables 2 and 3, which result from markets that may have been hit by short-term shocks.

Increasing the holding cost or interest rate, which makes it more expensive to keep a vacant property, reduces the equilibrium time to sale. A larger replacement cost cuts the vacancy rate, because entry is more expensive. However, if the maximum mismatch cost is scaled up by the same proportion as the replacement cost, then vacancy rates do not change. It is plausible that mismatch costs are greater in cities with high house prices. For example, San Francisco probably has higher repair costs and higher implicit wages for homeowners than Dallas or Houston.

The positive correlation between maximum mismatch cost and the equilibrium vacancy rate is attributable to increased monopoly power. A bigger range of mismatches means that a seller will be able to raise the price further above cost, giving higher profits. This leads to greater entry, more vacancies, and a longer time to sale. A similar relationship exists between
market size (total number of households) and vacancies. More houses being sold increases competition and reduces profits, resulting in a smaller vacancy rate. One measure of market power is the ratio of the market size to the maximum mismatch cost. A bigger market or a smaller mismatch cost means that houses are closer substitutes, leading to less market power, lower profits, fewer vacancies, and a lower vacancy rate.

The above results confirm that auctions sell property at a discount that increases in a down market. Why, then, are auctions used more frequently in down markets in the United States? The U.S. pattern of auctioning in down markets might well be explained by the existence of a few sellers (for example, the RTC or FDIC, or a private developer holding short-term "balloon" financing) that face higher holding costs than others in the market. The high-cost sellers will accept a lower price than the rest of the market in order to sell the property more quickly. For these sellers, auctions may be more attractive because of their even faster time to sale. Removing the cost symmetry could create a separating equilibrium in which auctions are attractive only for high-cost sellers. That describes the U.S. experience, where most sellers at auctions are large institutions. This discussion also suggests that sales prices of government properties will be below prices of

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25 The cost of running an auction is approximately the same as negotiated sales. For large enough sales (hundreds of units), auctions can actually be cheaper than realtor sales, involving an up-front payment of about 1 to 2 percent plus commission of 4 percent on all property sold.

26 Using $x$ distributed uniformly on $[0,L]$, it is possible to solve for the probability that a buyer is attracted by $m_0 < m$:

$$Q(m_0; m, V) = L^{-4} \left(1 - \left(1 - \frac{m_0 - m}{L}\right)^V\right)$$

Plugging into the first order condition (8), it can be shown that $\delta m_0 / \delta c < 0$. 34
other units in a market where most sellers have lower holding costs. In that case, other empirical work would overestimate the discount associated with auctions by comparing government auction prices with negotiated sale prices of individual (low-cost) homeowners.

This model assumes that the seller is unable to raise his or her price if two or more buyers are willing to pay that asking price. In extremely good markets, when prices are changing quickly, auctions can help a seller avoid setting "too low" an asking price. Potential buyers competing for a property could raise bids above an asking price that was set using an "expected" valuation. In hot markets, the probability of multiple bidders may increase enough to make competition through an auction perform better than setting an asking price. Sellers may be able to get the best of both techniques by setting an asking price and holding an auction if two or more buyers offer to buy the house. This happens quite frequently in boom markets, such as New York City in the mid 1980s when realtors requested that multiple prospective buyers submit bids for a popular property.

That policy is similar to a published-reserve auction, where a seller accepts bids above a reserve (asking price). In Australia, reserve auctions are a much larger percentage of the market in a boom than in a bust, although the reserves are rarely published. Lusht (1990) reports that during some booms, reserve auctions accounted for as much as 80 percent of sales in sub-markets of Melbourne. Economies of scale might explain why most sellers in the United States do not choose to auction in booms, but this is not
convincing. In England and Australia, auctioneers hold large sales in which they sell properties that belong to individual owners.\footnote{The vast majority of auctions in Australia are reserve sales, and a third or more of all properties do not sell at the auction because the high bid was rejected by the seller.}

V. Extensions and Conclusions

This model makes several predictions about prices at auction versus negotiated sale:

1. Auction prices should be lower than prices for houses sold at negotiated sales, with the possible exception of auctions held in very "hot" markets. Buyers do not bid up the price because, on average, the auction property is a poorer match (has a higher mismatch cost) than their best alternative in the negotiated sale market.

2. As a housing market improves and vacancies decline in the short term, possibly because of positive economic shocks, the auction discount falls. In a boom market, increased competition between buyers for a few houses raises the probability that multiple buyers will arrive with a good match for a single house, increasing the auction price. In a bust, auctions sell at a much larger discount.

3. Houses that have a lower range of mismatch costs, $L$, will be auctioned at a smaller discount. Units that are more homogeneous have a smaller relative mismatch cost and thus a smaller discount. Overall, a lower $L$ leads to a more efficient market with lower prices and vacancies, and a smaller time to sale. Sales technique matters less when buyers have similar valuations of the same property. Large markets have the same effect. The
difference in mismatch costs between houses declines as a buyer has more units to choose from, reducing mark-ups.

4. For high-cost sellers, comparing auction prices to "market" prices will exaggerate the auction discount. These sellers would normally cut the selling price below the "market" in order to sell more quickly and avoid additional holding costs.

Several possible extensions to this model should be considered in future research. Adding search costs, for example, may reduce the predicted auction discount. If buyers pay some cost to visit each property, they would prefer to visit auction properties because those units would sell, on average, at a lower price. From the buyer's perspective, each house has an equal probability of being a good match, but auction houses may be less expensive. In the aggregate, more buyers attending an auction will reduce the auction discount (by increasing competition for houses) and also reduce any one buyer's chance of being the winning bidder. Buyers will equate the marginal cost of visiting another house with the reduction in total price gained from buying the auction house multiplied by the probability of being the high bidder. 28

The model might be extended to consider different types of auctions, absolute and reserve. Alternatively, sellers could be given the option of auctioning property if two or more buyers are willing to meet the asking price, similar to a reserve auction. In a model where buyers are informed about all properties, such a strategy would eliminate the possibility that absolute auctions outperform negotiated sales. A shock-adjustment rule could

28 Auctioneers often claim to attract as many as a year’s worth of buyers in the six weeks preceding an auction.
be added to the first-order condition that governs short-term price movements. Such a rule would dictate the speed at which the market returns to the long-term equilibrium with zero profits and free entry. Finally, other distributions might be used to describe the mismatch costs. The normal distribution might increase the discount associated with auctions in a downturn by having a greater change in mismatch cost associated with a diminished number of bidders.

The results of this paper seem to fit nicely with the evidence in Australia, where auctions represent about one-quarter of all sales. As documented by Maher (1989) and Lusht (1990), auctions in Australia are used more frequently in boom markets and for "hot" properties. This pattern is exactly the opposite of that found in the United States. Although other empirical work has found that U.S. auctions do better in up markets, most American observers continue to view auctions as a sales method of last resort, for use in down markets. One can only guess as to whether the perceptions of market participants will change enough to allow auctions to continue their growth as the economy improves.
References


