Modeling Long-Term Nominal Interest Rates

by Jeffrey C. Fuhrer

Federal Reserve Bank of Boston

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Federal Reserve Bank of Boston

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PRELIMINARY DRAFT
Comments Welcome

Abstract

The Pure Expectations Hypothesis has long served as the preeminent benchmark model of the relationship between the yields on bonds of different maturities. When coupled with rational expectations, however, most empirical renderings of the model fail miserably. This paper explores the possibility that failure to account for changes in market participants' expectations about the monetary policy regime, including changes in the target rate of inflation and the response to inflation and output, may explain much of the failure of the PEH. Estimating explicit expectations for changing monetary policy regimes in conjunction with the PEH model goes a long way toward rescuing the PEH model. The long rate implied by the PEH for a stationary short rate process tracks the observed long rate. The predicted spread between the long and the short rate is highly correlated with the actual spread. The standard deviation of the theoretical spread is nearly identical to that of the actual spread. Overall, these results cast suspicion on the use of spread regressions to test the PEH.

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[1] The opinions expressed here are not necessarily shared by the Federal Reserve Bank of Boston or its staff. I thank Alicia Sasser for excellent research assistance.

The most widely-used description of the term structure of interest rates, the default in both casual and formal conversations among economists, is the pure expectations hypothesis. The tendency to fall back on this paradigm is so strong because candidates to replace it are so weak. Relatively minor differences between observed rates and the predictions of the theory can be attributed to term or risk premia that vary over time, perhaps in concert with the state of the business cycle. But the more profound failures of the pure expectations hypothesis documented by Macaulay (1938), Mankiw (1986), Mankiw and Miron (1986), Campbell and Shiller (1991), and presented below are far more difficult to explain.

This paper takes a different approach from previous papers to evaluating the pure expectations hypothesis. Most previous work uses regressions of the change in a short-term or long-term rate on the spread between the long and short rates as evidence of the failure of the pure expectations hypothesis. These regressions tend to demonstrate that, for some maturity combinations, short rates rise when the long-short spread is high, in conformance with the pure expectations hypothesis, and long rates tend to fall, in contradiction to the pure expectations hypothesis (see Campbell and Shiller (1991), Rudebusch (1995), among others). These papers may be viewed as indirectly examining the implications of the pure expectations hypothesis.

I more directly explore the implications of the pure expectations hypothesis, by examining the long-term nominal interest rate implied by the pure expectations hypothesis in conjunction with an explicit description of the process generating short-term nominal rates. The short rate is determined by the monetary authority in an attempt to achieve the standard objectives of monetary policy. In what follows, I will thus summarize Fed behavior as a "reaction function" in which the federal funds rate responds to deviations of inflation from a target and to deviations of output around potential, muted by an interest rate smoothing motive. Market participants will use their estimates of this reaction function, in conjunction with reduced-form equations for inflation and the output gap, to forecast short-term nominal rates and thus to determine the appropriate yield on a long-term bond, in conformance with the pure expectations hypothesis.[2]

As documented below, if we use the forecasts from a fixed-coefficient vector autoregression to estimate the long rate implied by the PEH, we obtain implied long rates that do not match observed long rates well at all. While Campbell and Shiller (1991) do not display the long rate implied by the PEH, their tables of the correlations of the actual and theoretical spreads and the ratios of their standard deviations tell the same story that I will tell graphically below. While the correlation between the theoretical and the actual spread can be fairly high, depending on the maturities involved, the theoretical spread is generally much less volatile than the actual spread.

The long rates implied by the PEH using forecasts from a fixed-coefficient VAR exhibit the following properties:

1. If short-term nominal rates are mean-reverting (even if they revert quite slowly), and participants in long bond markets are forward-looking with long horizons, the pure expectations hypothesis (PEH) predicts a long-term nominal rate that varies far too little compared to the actual data. The reason is that during much of the forecast horizon, the short rate is forecasted to be at or near its mean. Actual long rates exhibit "excess volatility" relative to the PEH's predictions.[3]

2. If short-term nominal rates contain an "important" unit root–i.e., a

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[2] This approach follows the spirit of Hamilton (1988). Hamilton's results, which as reported supported the PEH, were overturned by Driffil (1992), and thus cannot be considered a "fix" for the PEH.

[3] Shiller (1979) also documents the excess volatility of the long rate relative to the prediction of the PEH.

> unit root that dominates the variance of the series–then, to a first approximation, the PEH predicts long rates that look like short rates. The length of market participants' horizons is nearly irrelevant. Yet if the Fed targets the rate of inflation (as it almost surely has, at the very least since 1980), inflation will be stationary. This leaves the uncomfortable interpretation that most of the variance in long rates must be attributed to large and permanent shocks to an underlying long-term equilibrium real rate of interest.[4]

While it is possible that a reconciliation of the PEH with the data might be attained by assuming nonstationarity of the short rate, I explore only the stationary case in this paper.

Considerable effort has been devoted to augmenting the PEH with term premia and time-varying risk premia. While risk premia certainly exist and may well vary over time, they are unlikely to explain all of the difference between the PEH predictions and observed long-term rates. This will become evident in the following section. I take the approach of Campbell and Shiller (1991), attempting to "thoroughly explore the validity of the simple expectations theory before undertaking a detailed study of the source of predictable time variation in excess returns."

This paper pursues a monetary policy-based reconciliation of the data and the PEH model, as in Mankiw and Miron (1986), McCallum (1994), Dotsey and Otrok (1995), and Rudebusch (1995). If one is to take the PEH seriously, it is essential to understand the process that generates the short-term nominal interest rate. Since 1966, understanding the behavior of the short rate has been equivalent to understanding the behavior of the Fed, which has since that time essentially set the federal funds rate at a target

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[4] Proponents of real business cycle models might find comfort in such a proposition.

level, in response to movements in inflation and real activity.[5]

Unlike McCallum and Rudebusch, I do not focus primarily on the implications of particular fixed monetary policy rules for interest rate/spread regressions across the span of maturities. Unlike McCallum (1994), I do not consider a monetary policy response to a term structure spread itself to explain apparent conundrums in the data. Rather, I assume that the Fed responds only to the standard target variables of inflation and real activity. My goal is to see if one can use the PEH to derive reasonable predictions of long rates given a monetary policy-determined process for short rates. The question that this paper addresses is, how bad an approximation is the PEH for the determination of long-term rates, and can recognition of changes in Fed behavior make it a better approximation?

The next section depicts graphically the resounding failure of the PEH, using a methodology that is similar to Campbell and Shiller (1987), but focusing on the match between observed long rates and the long rates implied by the PEH. The second section motivates a structural interpretation of the failure of the PEH: if market participants' characterization of monetary policy changes over time, then the PEH predictions of the long rate may not be as bad as documented elsewhere. The third section presents estimates of the market's assumptions about Fed behavior. The fourth section explores the implications of the estimated long-term rates for existing empirical work, and the final section concludes.

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[5] Considerable debate focuses on the role of the federal funds rate in the period between October 1979 and November 1982; it is not widely agreed upon how the Fed used the federal funds rate during this period. See Goodfriend (1993) for detailed exposition of the issues.

1 Another Way of Looking at the Failure of the PEH

A simple version of the pure expectations hypothesis makes the *ex ante* long-term nominal rate of interest, R_t, equal to a weighted average of expected future short-term nominal interest rates, r_t over the life of the long-term bond. For a bond of maturity m,

$$R_t = \sum_{i=0}^{m} \omega_i E_t r_{t+i} \tag{1}$$

where the weights ω_i sum to unity. If we assume a geometric weighting pattern as in Shiller *et al* (1983), then $\omega_i = (1-\beta)\beta^i$ where $0 < \beta \leq 1$. We can approximate any finite-maturity bond by an infinite-maturity (consol) bond with geometric weights that make the *duration* of the bond equal to the duration of the finite-maturity bond. With a constant duration, the consol approximation of the finite-maturity bond with duration D is

$$R_t = \frac{1}{1+D} \sum_{i=0}^{\infty} \frac{D}{1+D}^{i} E_t r_{t+i} \tag{2}$$

This equation says little about how expectations of future short rates are formed, other than that they are "rational" and based on information up to and including period t. For this section, I assume that short-term nominal rates may be well modeled by a four-variable vector autoregression including the short rate, inflation, the long rate, and a measure of the output gap. Defining the vector of variables just described as x_t, the VAR may be written as

$$x_t = A(L)x_{t-1} + \epsilon_t \tag{3}$$

where the variable definitions are provided in Table 1, and lag lengths are chosen according to conventional criteria.

Rewriting equation 2 in its first-order form

$$R_t = \frac{D}{1+D} E_t R_{t+1} + \frac{1}{1+D} r_t . \tag{4}$$

Combining this equation with the VAR of equation 3, we can compute the long-term nominal rates that are consistent with the PEH and with the stationary vector autoregression.[6] Figure 1 displays the *ex ante* long-term nominal rate computed in this fashion. As the figure shows, the *ex ante* rate exhibits far less volatility than the actual long-term rate. The reason is that the forecasts of the nominal rate from the stationary VAR decay toward their mean well within the "forecast horizon" for a bond of 7-year duration, so that for much of the horizon, the funds rate is forecasted to be at its unconditional mean.

The correlation between the "theoretical" spread and the actual is .68, indicating that the movements in the implied long rate are often in the right direction, although as figure 1 indicates, they are far too small. This observation is consistent with the results for *ex ante* long-term real rates presented in Fuhrer and Moore (1995). Thus under the assumption of stationary short-term rates, the PEH applied to these maturities is strongly violated by the data. It would be possible, but difficult, to imagine time-varying risk premia of sufficient magnitude to close the gap between the long rates implied by the PEH in figure 1 and the observed long rates.

The use of a constant-coefficient VAR implicitly assumes a fixed-coefficient "reaction function" for Fed policy over the entire sample. The contribution of this paper will be that it allows for changing Fed reaction function co-

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[6] The method of computing *ex ante* rates, which is computationally identical to that of Campbell and Shiller (1987), is described in detail in Fuhrer and Moore (1995).

efficients across time, and that these estimated changes will be important enough to reconcile the PEH with the data on longer-term and shorter-term bonds.

1.1 Making Monetary Policy Explicit in the Determination of Long Rates

Kozicki (1994) examines the PEH model of the term structure in the context of re-specifying the MIT-PENN-SSRC (MPS) quarterly model. She observes the same mismatch of long rate forecasts implied by the PEH and observed long rates under the assumption of stationary short rate processes. She ultimately finds a remedy to the problem by using the long end of the term structure to pin down the "terminal expectation" of market participants. The intuition is that we can use the implied forward rates contained in long-maturity bonds to back out the "settling point" for expected future short rates. The endpoints implied in the long end of the term structure differ significantly from the unconditional expectations implied by a stationary VAR, and the *ex ante* rates implied by this procedure appear more closely linked to observed long rates.

While this procedure works in a mechanical sense, it begs the question of where the "terminal expectations"—essentially market participants' unconditional expectations for nominal rates—come from. One would expect the Fisher equation to hold in the long run, so that

$$E_t r_T = E_t \pi_T + E_t \rho_T \tag{5}$$

where T subscripts denote expectations many years out, for example at the maturity of the longest-term Treasury bonds, and π_T and ρ_T denote the corresponding long-term expectations for inflation and real rates, respectively.

In many structural models, and in the models that I consider here, the long-run value for inflation is determined solely by monetary policy. In expectations-augmented Phillips curve and contracting models of price determination, the price specification does not determine the equilibrium level of inflation. In the simplest version of the expectations augmented Phillips curve, for example,

$$\pi_t = c + E_t \pi_{t+1} + \gamma U_t \tag{6}$$

the specification determines the "non-accelerating-inflation rate of unemployment" as $-c/\gamma$, and constrains inflation to equal expected inflation in equilibrium. An auxiliary equation, such as a monetary policy function in which a monetary instrument is set in response to deviations of inflation from a target, is required to determine the equilibrium level of inflation.

In conformance with these widely-used descriptions of the inflation process, I will assume that monetary policy determines the long-term settling point for inflation. From the perspective of the PEH and the determination of long-term nominal rates, changes in market participants' expectations of the monetary authority's inflation target will alter the expectations of the "settling point" for short rates, and thus shift the implied long rate.

As important as the "settling point" in determining expectations of future short rates are the assumed responses of monetary policy to deviations of inflation and output from their desired values. For a given deviation of inflation from its target, a larger policy response will imply a larger deviation of the funds rate from its equilibrium. The corresponding argument holds for a given deviation of output from its equilibrium value.[7] Thus the market's assumptions about policy responses to inflation and output will alter their expectations of future short rates.

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[7] I assume throughout that the Fed cannot influence the equilibrium level of real output.

In the sections that follow, I present a simple model that explicitly includes the Fed's target inflation rate and that implies an equilibrium real interest rate. I allow the target inflation rate, the equilibrium real rate, and the short-run policy responses to shift over different monetary regimes. I compute the long-term nominal rates implied by the PEH, under the assumption of stationarity, allowing for shifts in the Fed's inflation target and in the equilibrium real rate. I compare the implied rates with these "endpoint expectations" identified from the structural model with the estimates presented in Figure 1, and determine the extent to which allowing for changing inflation targets and equilibrium real rates can "fix" the gross misbehavior of the PEH model.

2 A Model of Monetary Policy and Long-Term Interest Rates

2.1 Monetary Policy

Monetary policy is modeled as a federal funds rate reaction function, in the spirit of Fuhrer and Moore (1995) and Taylor (1993). The federal funds rate responds to deviations of inflation from its target and to deviations of the output gap from zero

$$f_t = \sum_{i=1}^{l} a_{f,i} f_{t-i} + \sum_{j=0}^{m} a_{\pi,j}(\pi_{t-j} - \bar{\pi}) + \sum_{k=0}^{n} a_{y,k}\tilde{y}_{t-k} + (1 - \sum_{i=1}^{l} a_{f,i})(\bar{\pi} + \bar{\rho}) \quad (7)$$

where $\bar{\pi}$ and $\bar{\rho}$ are the target and equilibrium rates of inflation and real interest rates, and where all of the coefficients may vary across monetary policy regimes. The lagged funds rate terms allow for an interest rate smoothing motive on the part of the monetary authority. The final term determines the steady-state value of the funds rate as the sum of the equilibrium real rate and the target inflation rate. The specification for real activity, detailed below, identifies the equilibrium real rate.

Monetary policy is assumed to follow an equation like 7 throughout the sample considered here. Market participants are assumed to know the form of the Fed reaction function, although their estimates of the parameters of the function can change over time.

2.2 Long Rates

Long-term nominal interest rates are assumed to be determined by the PEH. The first-order representation of the PEH with rational expectations, equation 4, defines the implied long-term rate, R_t^*, as

$$R_t^* = \sum_{i=0}^{\infty} \beta_i E_t r_{t+i} \tag{8}$$

where the weights β_i are constrained so that the duration of the implied long-term rate equals the duration, D, of the rate on the 10-year Treasury constant maturity bond:

$$\beta_i = \frac{1}{1+D}\left(\frac{D}{1+D}\right)^i .$$

For all of the empirical work that follows, I use a constant-duration approximation, with duration set equal to the average duration for the yield on the 10-year government constant maturity bond, which is 7 years in my sample.

2.3 Inflation and Real Activity

Inflation is defined as the quarterly log change in the GDP deflator. The output gap is defined as the deviation of log real per capita GDP from a segmented linear trend with a single breakpoint in 1973 as in Perron (1989). Both are determined by simple reduced-form equations from a vector autoregression (VAR) with four lags each of inflation, the federal funds rate, and the output gap. Lag lengths are determined according to conventional criteria.

3 Market-Perceived Monetary Regime Shifts and the PEH

Is it conceivable that shifts in market participants' expectations about monetary policy behavior—the target inflation rate and the response of monetary policy to inflation and output deviations—could alter the long rate implied by the PEH sufficiently to qualitatively improve Figure 1? The short answer is yes, changes in these parameters can substantially eliminate the discrepancy between the long rates implied by the PEH and observations on long rates. The question is whether relatively smooth and plausible shifts in the bond market's expectations of the inflation target, the equilibrium real rate, the response of the funds rate to inflation and real activity, and the degree of interest rate smoothing can "resurrect" the PEH to respectability.

To answer this question, I perform two exercises. In the first, I take the VAR equations for real activity and inflation, and the PEH arbitrage equation (equation 4) as given. For each observation, I solve for the monetary policy parameters that make the implied long-term nominal rate from the PEH "close" to the observed long-term rate. The definition of "closeness" varies across exercises, and I will define two types of closeness below. In addition, I impose a "smoothness" constraint on the sequence of parameters so obtained, under the assumption that actual and expected monetary policy changes occur somewhat gradually.

Formally, the optimization problem may be stated as

$$\underset{\theta_t}{min} \; \{\lambda(R^*/\sum\omega_i R_{t+i})^2 + (1-\lambda)(\theta_t/\theta_{t-1})^2\} \tag{9}$$

where θ_t is the vector of monetary policy parameters used in period t to forecast short rates, λ is the relative weight on the "fit" versus the smoothness of the parameter changes, and ω_i are the weights in the moving average of realized rates.

For the first set of results presented below, I use the squared difference between a centered nine-quarter moving average of the actual long rate and the implied long rate from the PEH as the measure of "fit".[8] In preliminary estimates, the interest rate smoothing parameter nearly always took a value very near unity. Thus the policy reaction function is simplified to

$$f_t - f_{t-1} = a_\pi(\pi_t - \bar{\pi}) + a_y\tilde{y}_t$$

Note that if we set $a_{f,1}$ to 1, the reaction function determines the change in the funds rate, and knowledge of the equilibrium level of the funds rate is not required, so $\bar{\rho}$ need not be estimated. The three parameters $[a_\pi, a_y, \bar{\pi}]$ are chosen to minimize the quantity in equation 9.[9]

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[8] Using a trailing eight-quarter moving average does not qualitatively affect the conclusions.

[9] Setting $a_{f,1}$ to 1 does *not* imply that the funds rate has a unit root. It does, however, imply that the funds rate moves *smoothly*. With any response to the inflation rate ($\pi \neq 0$), and with an equilibrium real rate determined elsewhere, the nominal rate will settle to the sum of the real rate and inflation in the long run. See the discussion in Fuhrer and Moore

A second exercise attempts to reconcile the PEH with the bond data by allowing for a small number of discrete one-time changes in the policy coefficients. In this exercise, I estimate the VAR across the discrete regime breaks, allowing the policy coefficients to change at each break, but holding the rest of the VAR coefficients constant at their full-sample estimates. I then compare the long rate implied by the PEH and this model with the actual long rate data.

3.1 Results: The Implied Long Rate and Reaction Function Coefficients

Moving Average of Long Rate For these estimates, the optimization criterion—the measure of "closeness"—is the squared difference between the rate implied by the PEH at some parameter settings, and the centered moving average of the observed long nominal rate. This criterion captures the idea that the PEH implied rate should equal the observed rate "on average", or more simply that it should lie approximately on top of the observed rate, but with somewhat lower volatility.

The results are presented in Figures 2–6; each figure assumes a different value of λ, the smoothing coefficient. As the figures demonstrate, relatively smooth changes in the emphasis on inflation and output in the reaction function, coupled with a smoothly declining expected inflation target, yield a PEH-implied long-term nominal rate that tracks the observed long-term rate quite closely.[10] The pattern of reaction function coefficients is plausible: For example, during the great disinflation of the early 1980s, the estimated coefficient on inflation rises dramatically. The implied inflation target declines

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(1995), especially pages 226–7, for more on this point.

[10] Table 3 provides the correlations between the actual and the theoretical spreads.

substantially over the sample.

No Average For these estimates, the optimization criterion is the squared difference between the rate implied by the PEH at some parameter settings, and the observed long nominal rate. This criterion reflects the notion that the PEH implied rate should equal the observed rate at every point in time. This leaves essentially no room for time-varying term premia to explain the differences between yields along the term structure, and thus is equivalent to the strictest form of the PEH.

Figures 7–8 display the results for this exercise. As expected, the implied long rate lies even closer to the observed long-term rate. Interestingly, the reaction function coefficients are not terribly different from those for the exercise above. The general pattern of increased emphasis on inflation in the early 1980s and a falling inflation target is upheld in these estimates.

These results can be viewed as a successful implementation of the motivation behind Hamilton (1988), later corrected by Driffil (1992). Hamilton posited different "regimes" for the univariate short rate process that were distinguished by different time-series properties of the short rate (its mean and variance). Driffil showed that, with the proper dataset, these changes were *not* sufficient to resuscitate the PEH. The regime shifts here are more economically motivated, tied to changes in monetary policy behavior, and apparently are sufficient to reconcile the PEH with the data.

3.1.1 Implications for Shorter-Term Rates

While the results discussed above show that modest and plausible changes in expectations about monetary policy can reconcile the PEH with the 10-year nominal bond data, these results would be of less interest if they implied nonsensical behavior for interest rates of other maturities. To put it somewhat differently, if entirely different sets of reaction function coefficients were required to reconcile the PEH with the 1- or 2-year maturity nominal bond yields, the foregoing results would be of little interest to anyone.

I compute the implied 6-month, 1-year, and 2-year rates using the policy coefficients estimated for the 10-year bond and compare them with the realized yields. The rates are defined as

$$r_{m,t} = (1/m) \sum_{i=0}^{m-1} E_t r_{1,t+i} \tag{10}$$

where m is the maturity, expressed in quarters, and $r_{1,t}$ is the $t-$period (quarterly) observation on the federal funds rate.

The results are displayed in Figure 9. As the figure shows, the PEH coupled with the reaction function estimates implies reasonable short- to medium-maturity yields. This lends more credibility to the reaction function coefficients estimated using only data for the 10-year bond.

3.2 A formal estimation of changing monetary policy regimes

The procedure described above may be viewed as an informal approach to estimating a "structural VAR" allowing for shifts in the one structural equation, the policy reaction function. To examine the importance of small quarter-to-quarter shifts in policy coefficients for reconciling the PEH with the data, as compared to discrete one-time changes in coefficients, I estimate the VAR across four regime breaks, allowing policy coefficients to change at each break, but holding the rest of the VAR coefficients constant at their full-sample estimates. I add the equation

$$R_t = R^* + \epsilon_R \tag{11}$$

in order to include the fit of the implied long rate to the observed long rate data in the estimation. This seems the proper estimation analogy to the procedure followed in the previous section.

I choose the break points based on the estimated coefficient patterns displayed in figures 2–6, so the estimates must be viewed as conditional on this assumption. The estimated parameters are summarized in table 2 below.[11]

The magnitude of the response parameters a_π and a_y are about the same as those estimated in the optimization in section 3, and the estimated inflation targets ($\bar{\pi}$) follow a similar pattern. Note that many of the coefficients in the middle subsamples are not estimated very precisely, although one could probably still reject the hypothesis of a constant inflation target throughout.

As figure 10 indicates, these discrete breaks in expectations about the three monetary policy parameters are perhaps better than the continuous time-varying parameters at reconciling the PEH with the long bond data. Thus the result that accounting for expectations about monetary policy can "fix" the PEH does not depend on important period-by-period fluctuations in the implied reaction function coefficients, but rather on a small number of significant changes in the Fed's reaction function.

To test the importance of the breakpoints chosen, I also estimate the same model, with breakpoints in 1979:III and 1982:IV, corresponding to the beginning and end of the nonborrowed reserves operating procedure. The results from this exercise are qualitatively similar to the preceding estimation. The long-term rate implied by the PEH lies quite close to the actual rate. The one significant difference is that the implied long rate from these estimates

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[11] A test based on results not shown here indicates that the estimated inflation and output response coefficients in the first two subsamples are insignificantly different from one another.

"overpredicts" the actual rate in 1990; it gets back on track by 1992. The implied long-term rate is shown in figure 11.

4 Implications for Existing Empirical Work

4.1 Spread regressions

Following Campbell and Shiller, I regress the long-run change in the short rate and the short-run change in the long rate on the actual spread and the theoretical spread. For the time-averaged fitted data, the PEH holds on average over a 9-quarter window. For the non-averaged fitted data, the PEH is very close to holding period-by-period. Table 5 displays the results of these standard regressions.

The results for the actual data conform reasonably well to the results presented in Campbell and Shiller and elsewhere. Of most interest are the regressions on the "theoretical" data, which of course conform to the pure expectations hypothesis. The coefficients in the regressions of the long-run change in the short rate on the spread are all greater than one, although they are not generally significantly different from one. The coefficient estimates for the short-run change in the long rate are all negative, although none are significantly different from zero. Because the PEH must hold by construction for the theoretical spreads, this exercise casts some suspicion on the usefulness of the standard spread regressions.

In one sense, these results confirm those of Rudebusch (1995). Because the spread may be expressed as the expectation of the weighted sum of the first-differences of future short rates, the coefficient in the test regression depends critically on the predictability of the change in the short rates. Rudebusch finds that if the Fed behaves in such a way as to make first-differences of the funds rate not very predictable (at least over some horizons), then the estimated spread coefficient in the test regression will be close to zero. In a sense, this paper makes the more general point that discrete shifts in Fed behavior can make mush of the standard test regressions, even though the PEH holds almost exactly in every period.

4.2 Comparisons of Actual and Theoretical Spreads

The ratios of the standard deviation of the theoretical spread to the actual spread are reported in Table 4. In Campbell and Shiller (1991), these ratios for bonds of 10-year maturity cluster around .5. For the theoretical spreads computed in this study, the standard deviation ratios are all quite close to one. Thus the realized spreads do *not* exhibit excess volatility relative to the theoretical spreads, once changes in policy are accounted for. This result overturns the result in Campbell and Shiller (1991).

Finally, the correlations between the actual and the theoretical spreads are of interest. Recall that in Campbell and Shiller, the correlations for this maturity spread were quite high. Here, for the variety of PEH-consistent spreads generated above, the correlation between the actual and the theoretical spread ranges from 0.89 to 0.99. By this metric, the PEH with smoothly-evolving policy parameters fits the data markedly better than the fixed-coefficient alternative.

4.3 How wrong could market expectations be?

If market participants had used the smoothly-evolving reaction function parameters estimated above to predict the funds rate, how far wrong would they have gone? The structural VAR estimates above suggest that the PEH-consistent reaction function coefficients could not be too far off, since the structural VAR estimates fit both the data on the funds rate and the long rate, and they appear to be not very different from the PEH-consistent estimates. But could a well-informed research department have shown "market participants" that their estimates of the policy response were statistically untenable?

I will not attempt a formal statistical test of this hypothesis here, as it is not obvious what the regression model that nests the fitting exercise of section 3 looks like. Instead, I will use two heuristic measures to address the question.

4.3.1 Information in the Funds Rate Prediction Errors

The first heuristic looks at errors made in predicting the funds rate with the evolving parameter estimates. If the predicted funds rate deviates significantly and persistently from the actual funds rate, then market participants would have been obviously foolish to use these estimates, and could have adjusted their estimates based on the structure in the prediction errors.

Figure 12 shows the autocorrelation functions for the predicted funds rates using estimates of the policy parameters for different smoothing parameters. As the figure shows, there is no strong evidence of structure in the prediction errors. There are just-significant autocorrelations at lags two and seven, both with magnitude of about .2. Thus the implied funds rate estimates from the time-varying parameters fit the actual funds rate data reasonably well.

4.3.2 Rolling regression estimates of reaction function

The second heuristic compares the sequence of reaction function parameter estimates obtained in section 3 with rolling regression estimates of the same parameters. Statistically, these estimates are not perfectly compatible, because the rolling regression estimates assume fixed parameters, while the PEH-consistent estimates explicitly assume time-varying parameters. Still, one can get a sense of what the estimated parameters from a widely-used technique look like compared to the PEH-consistent parameters, and whether they point to entirely different reaction function parameter estimates or to greater parameter stability.

Figure 13 displays the rolling regression parameter estimates for a 10-year data window for each of the three policy parameters, along with the PEH-consistent estimates for $\lambda = .2$ presented above. As the figure indicates, the range of estimates that arise from the rolling-regression methodology easily spans the estimates that arise from the optimization in section 3. Only in the early 1980s do the response coefficients fall outside the 2-standard-error bands around the rolling-regression estimates. However, the rolling-regression estimates themselves are a bit suspect during this period, as they imply that the Fed responded so as to *reinforce* rather than to lower inflation during the time of the great disinflation. By this simple gauge, then, the PEH-consistent policy parameters cannot be ruled out as statistically untenable.

5 Conclusion

The PEH imposes a very tight linkage between the process assumed to generate short-term nominal rates and the realizations of long-term nominal rates. As a result, a careful examination of the determinants of the short-term nominal rate is warranted. This paper takes the position that the primary–indeed, the sole–determinant of short nominal rate behavior is the behavior of the Federal Reserve. Thus a careful examination of the Fed's reaction function, and how it has changed over time, is required to understand the implications of the PEH for long rates.

Using a simple framework that uses reduced-form equations for inflation and output, combined with a very simple Fed reaction function and the PEH, I find that changes in the Fed's behavior over time can reconcile the PEH with the interest rate data quite nicely. In particular, the interest rates of various maturities that are implied by the PEH are highly correlated with their observed counterparts and exhibit nearly the same volatility. Thus, in Campbell and Shiller's terms, both the correlation between the theoretical and actual spread and the ratio of the standard deviations of the theoretical and actual spreads are close to one.

This paper also demonstrates that simple spread regression tests of the PEH may be very misleading. When monetary policy alters the process generating the short-term interest rate, there is little reason to believe that the coefficient in the canonical test regression should be one. In data for which the PEH is shown to hold nearly exactly, the test regressions yield coefficients not significantly different from zero.

Thus the Pure Expectations Hypothesis of the term structure may not be as awful as many empirical investigations have suggested. In fact, this paper develops results that suggest that it is a very good approximation of long-term bond behavior. The key to understanding previous failures of the PEH is in understanding the process that generates the short-term nominal interest rate. If allowance is made for shifts in the process used to form expectations of future short-term rates, then the PEH fares quite well.

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Table 1:
Quarterly data, 1966Q1–1994Q1

	Definition
π_t	log change in the implicit GDP deflator
r_t	Federal funds rate
y_t	log of per capita real GDP
$\tilde{y}_t$	deviation of y_t from segmented trend, breakpoint 1973
R_t	Yield on 10-year constant maturity treasury note

Table 2:
Maximum Likelihood Estimates of Monetary Policy Coefficients

	Sample Period							
Coefficient	1966-73		1974-80		1981-85		1986-94:1	
a_π	0.012	(0.007)	0.012	(0.007)	0.099	(2.789)	0.094	(0.040)
a_y	0.123	(0.020)	0.123	(0.020)	0.169	(0.056)	0.077	(0.029)
$\bar{\pi}$	0.130	(0.035)	0.010	(0.029)	0.000	(1.000)	0.040	(0.003)

Standard Errors in parentheses.

Table 3:

Correlation of Theoretical and Actual Spread

Fit to Moving Average				
λ				
0.05	0.10	0.20	0.30	0.50
.96	.96	.95	.93	.87
Fit to Data				
λ				
0.05	0.10	0.20	0.30	0.50
.99	.98	.96	.94	.89
FIML estimates, 4 regimes				
.93				

Table 4:

Ratio of Standard Deviations of Theoretical to Actual Spread

Fit to Moving Average				
λ				
0.05	0.10	0.20	0.30	0.50
1.09	1.08	1.09	1.10	1.12
Fit to Data				
λ				
0.05	0.10	0.20	0.30	0.50
.99	1.02	1.06	1.08	1.12
FIML estimates, 4 regimes				
.91				

Table 5:
Campbell-Shiller Test Regressions

Dependent variable: 10-year change in short rate				
	Actual Spread		Theoretical Spread	
Weight on Smoothing (λ)	Coeff.	Stand. Error	Coeff.	Stand. Error
0.05	1.08	0.29	1.09	0.24
0.10			1.12	0.25
0.20			1.12	0.26
0.30			1.17	0.26
0.50			1.30	0.23
FIML			1.10	0.30
Dependent variable: Change in long rate				
0.05	-1.76	0.77	-0.56	0.73
0.10			-0.47	0.73
0.20			-0.53	0.73
0.30			-0.52	0.72
0.50			-0.29	0.71
FIML			-1.67	0.78

The regression for the top panel is:

$$\sum_{i=1}^{m-1} r_{t+i}/(m-1) - r_t = c + b((m-1)/m)S_t$$

The regression for the bottom panel is:

$$R_{t+1} - R_t = c + bS_t/(m-1)$$

where m is the maturity of the long bond in quarters and S_t is the $t-$period observation on the spread between the long yield and the short yield.

Figure 1

Ten Year Constant Maturity vs. Implied PEH Rate

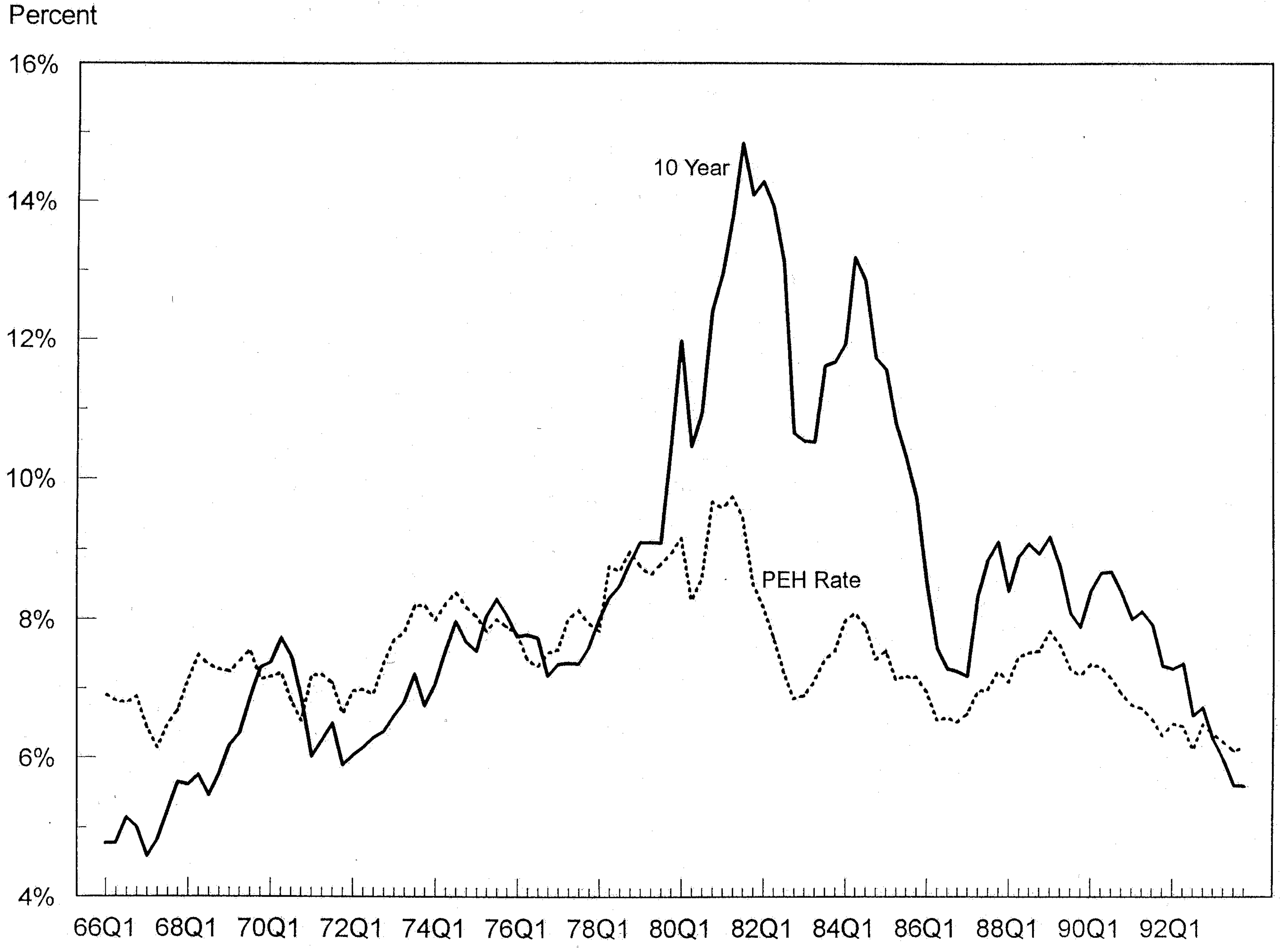
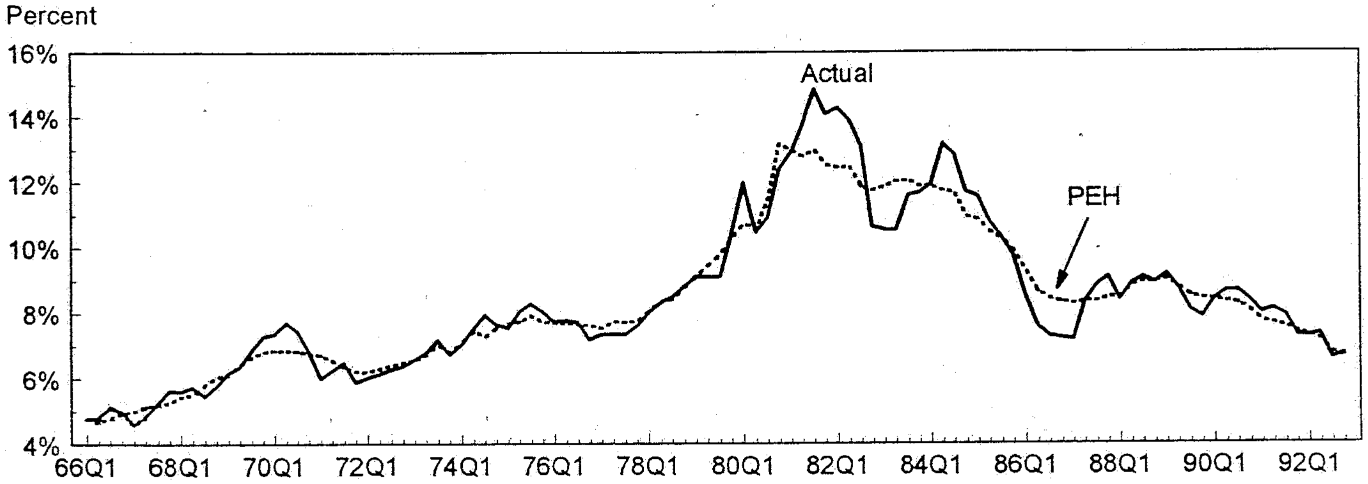


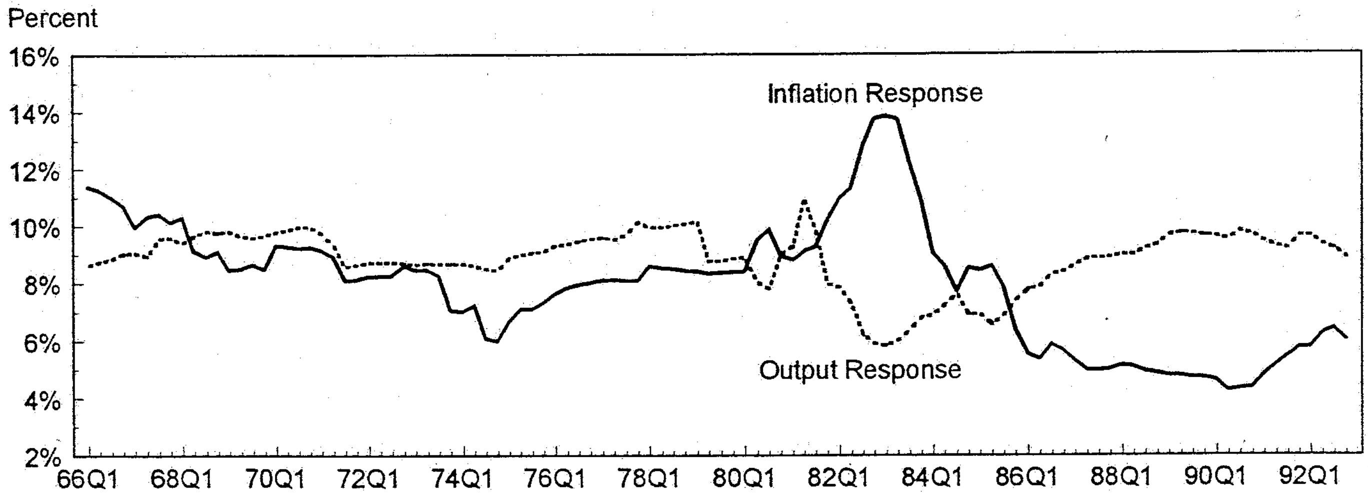
Figure 2

Long Rates and Reaction Function Coefficients Implied by PEH, Smoothing Coefficient = .05 Fit Moving Average of 10-Year Rate

Actual v. PEH Long Rates



Reaction Function Coefficients



Inflation Target

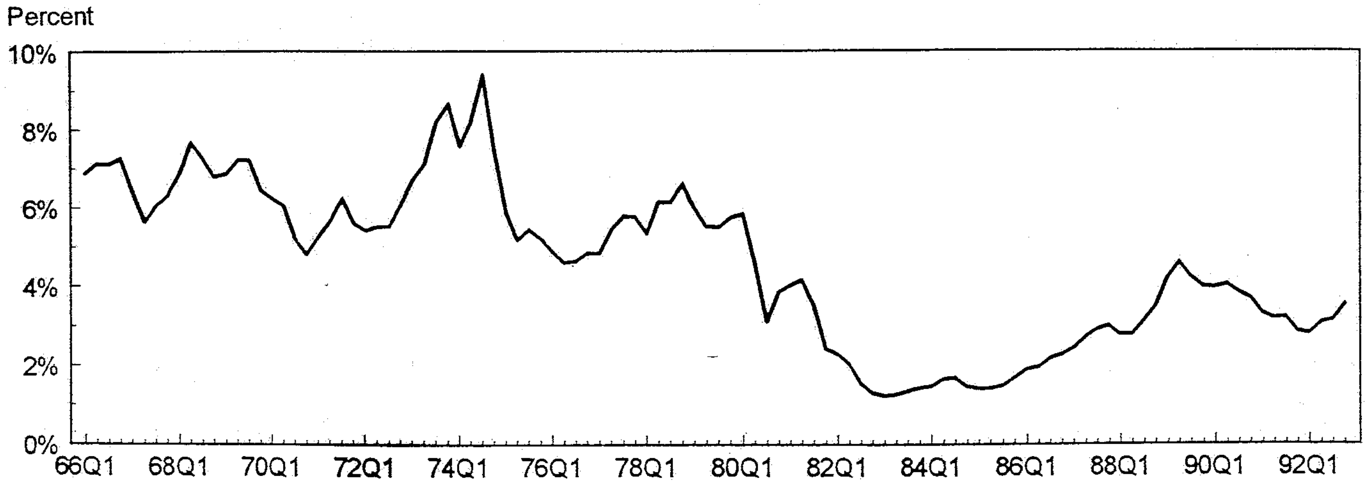
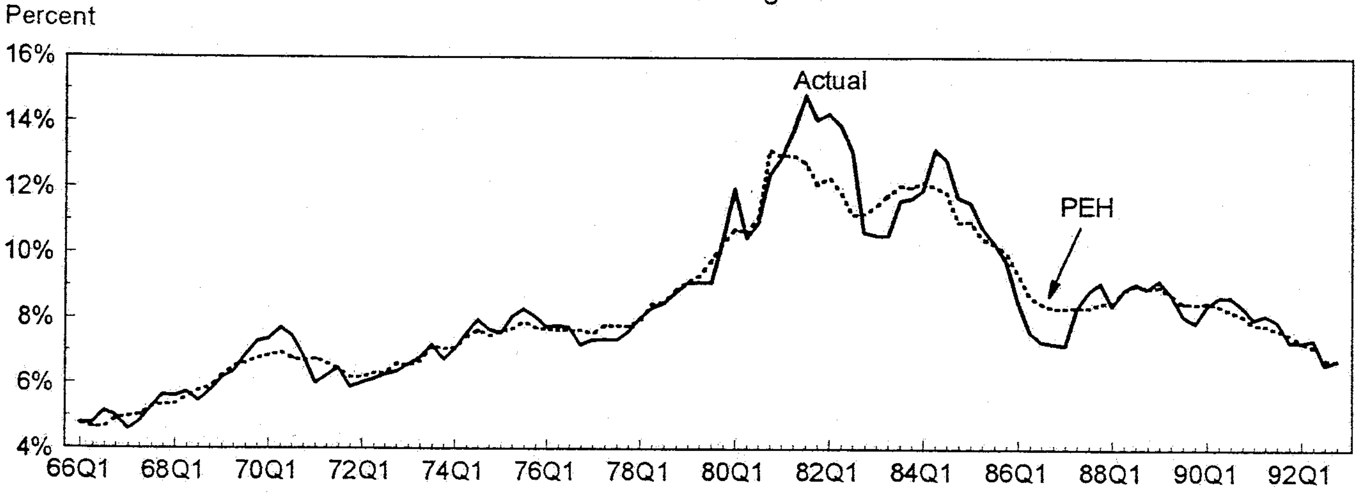


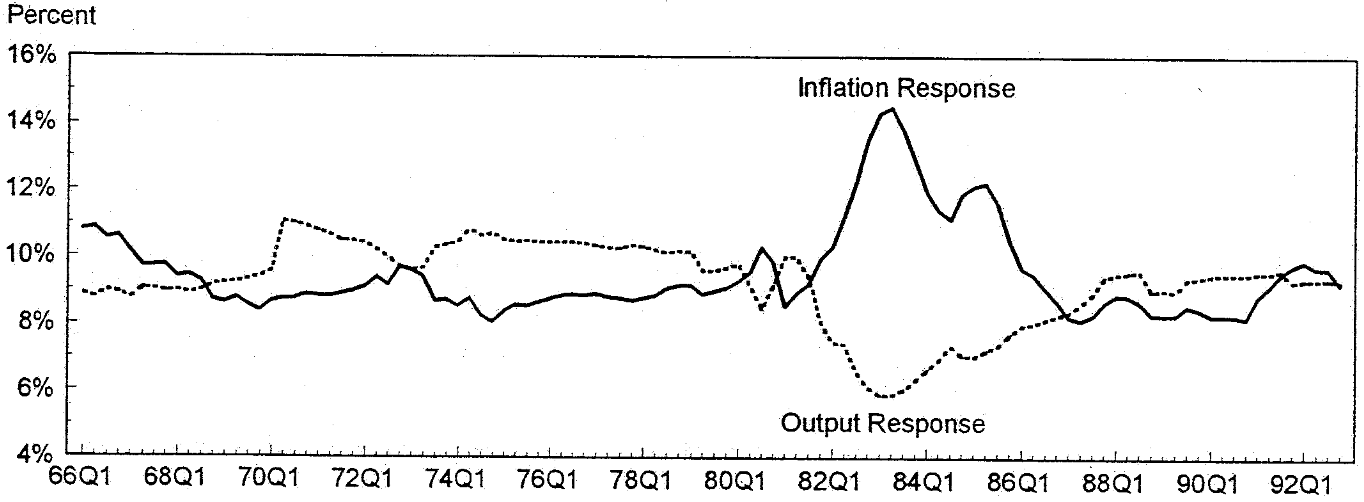
Figure 3

Long Rates and Reaction Function Coefficients Implied by PEH, Smoothing Coefficient = .1 Fit Moving Average of 10-Year Rate

Actual v. PEH Long Rates



Reaction Function Coefficients



Inflation Target

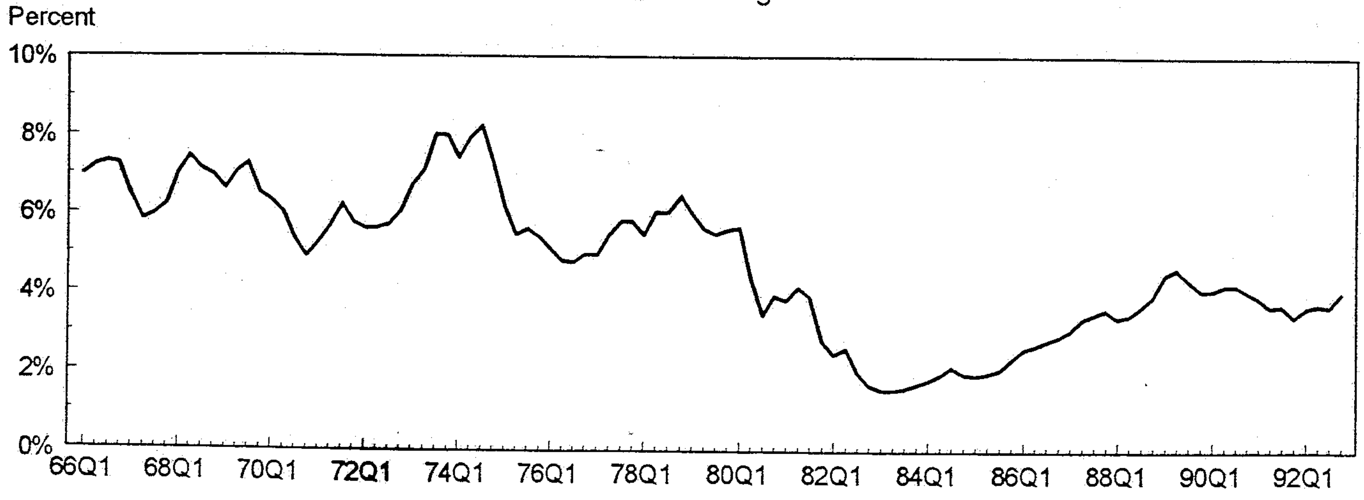
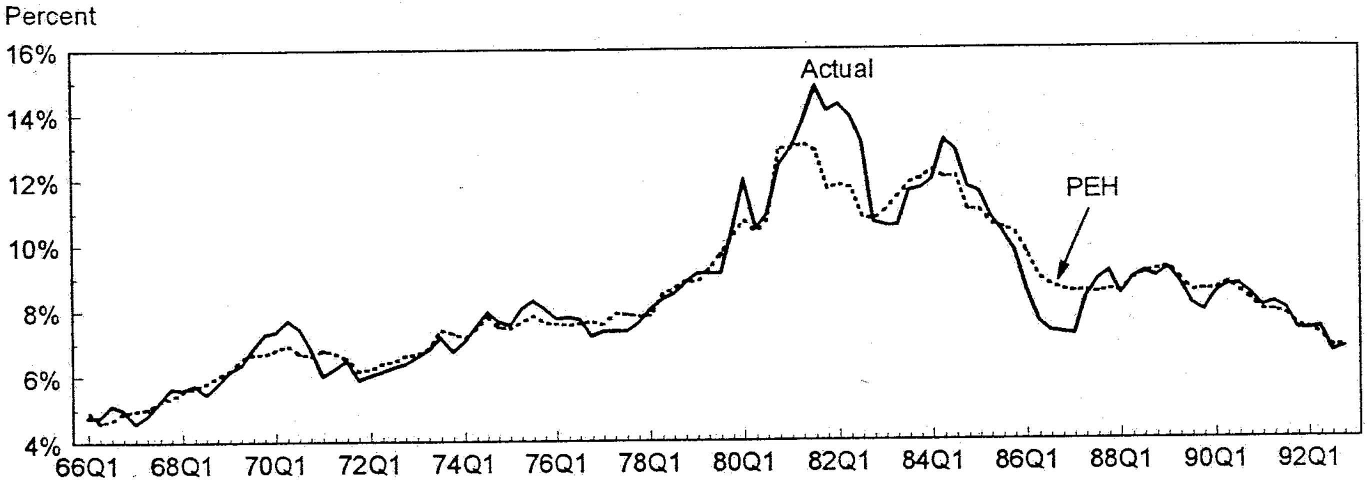


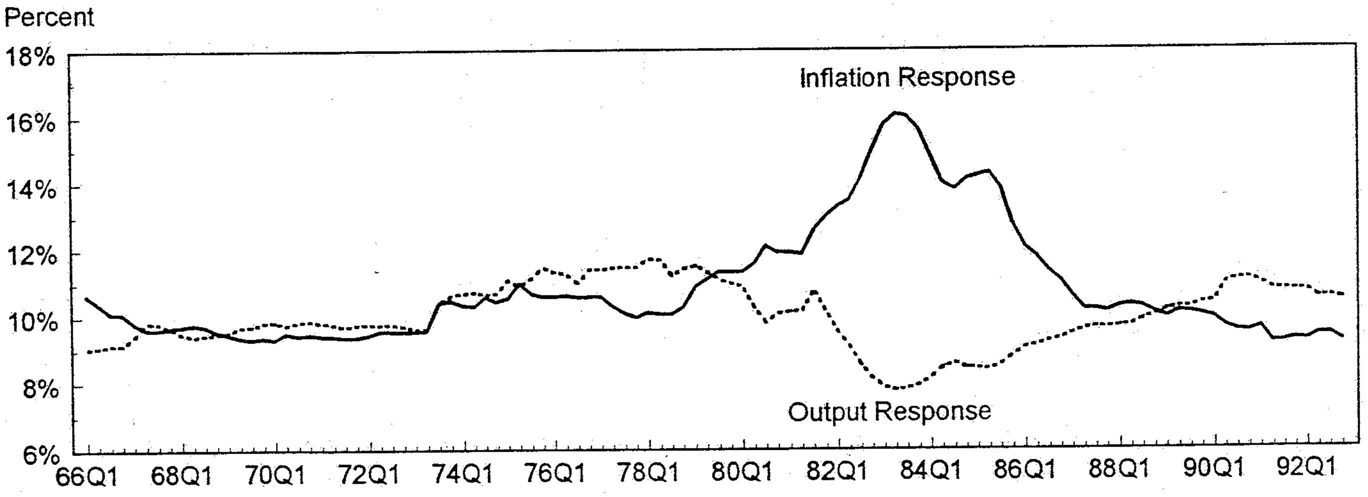
Figure 4

Long Rates and Reaction Function Coefficients Implied by PEH, Smoothing Coefficient = .2 Fit Moving Average of 10-Year Rate

Actual v. PEH Long Rates



Reaction Function Coefficients



Inflation Target

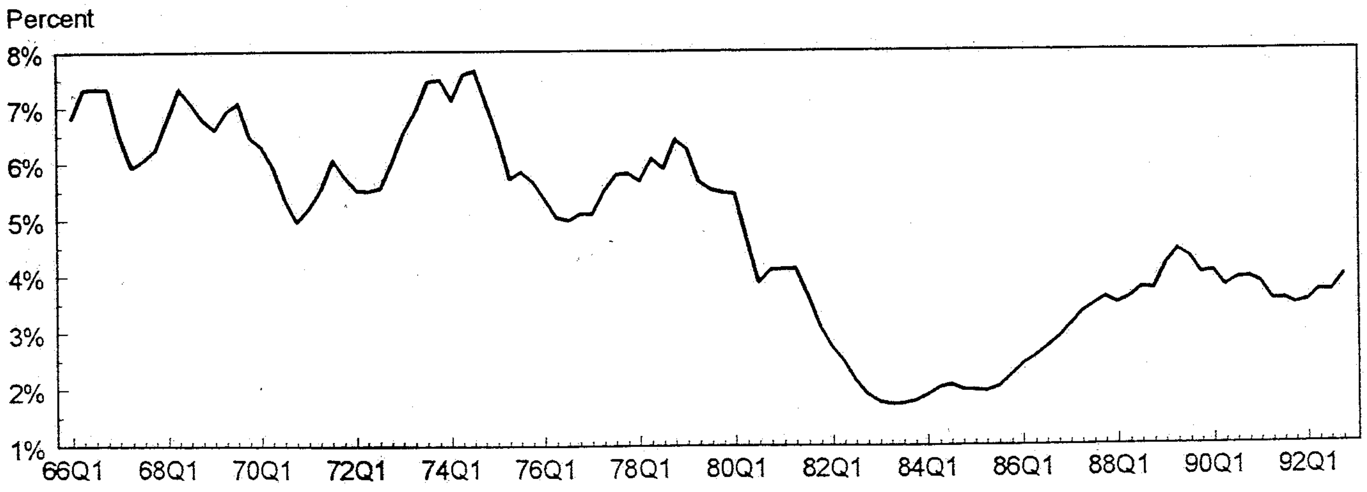
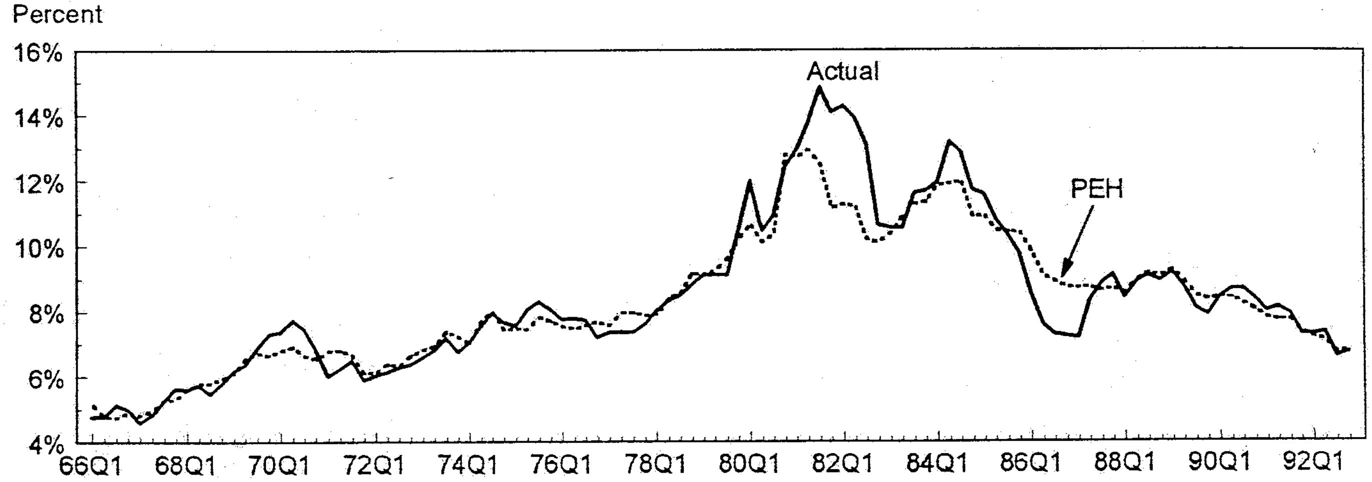


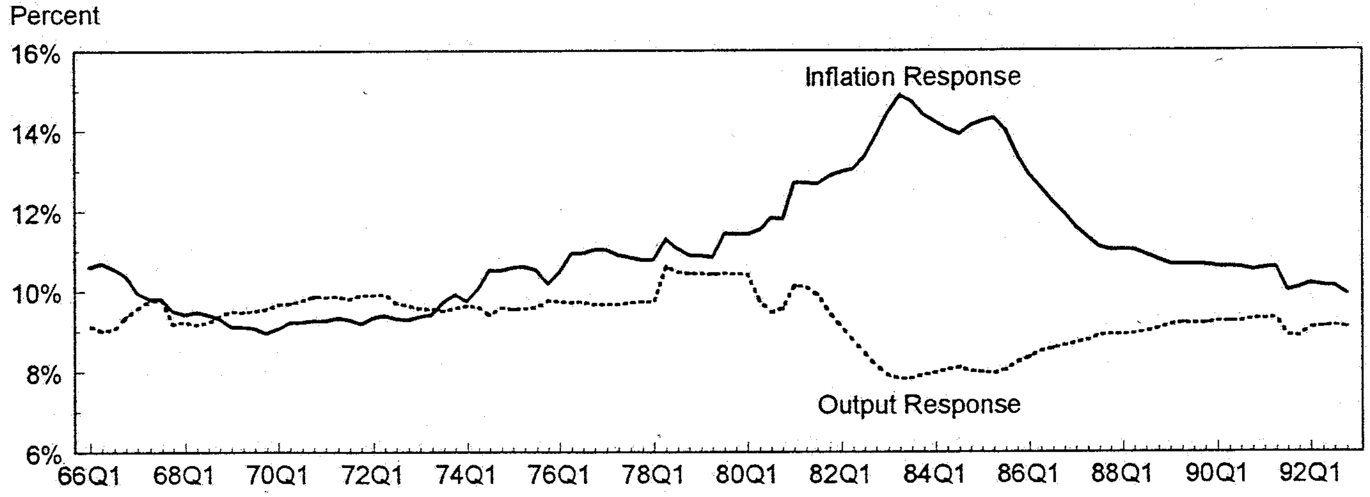
Figure 5

Long Rates and Reaction Function Coefficients Implied by PEH, Smoothing Coefficient = .3 Fit Moving Average of 10-Year Rate

Actual v. PEH Long Rates



Reaction Function Coefficients



Inflation Target

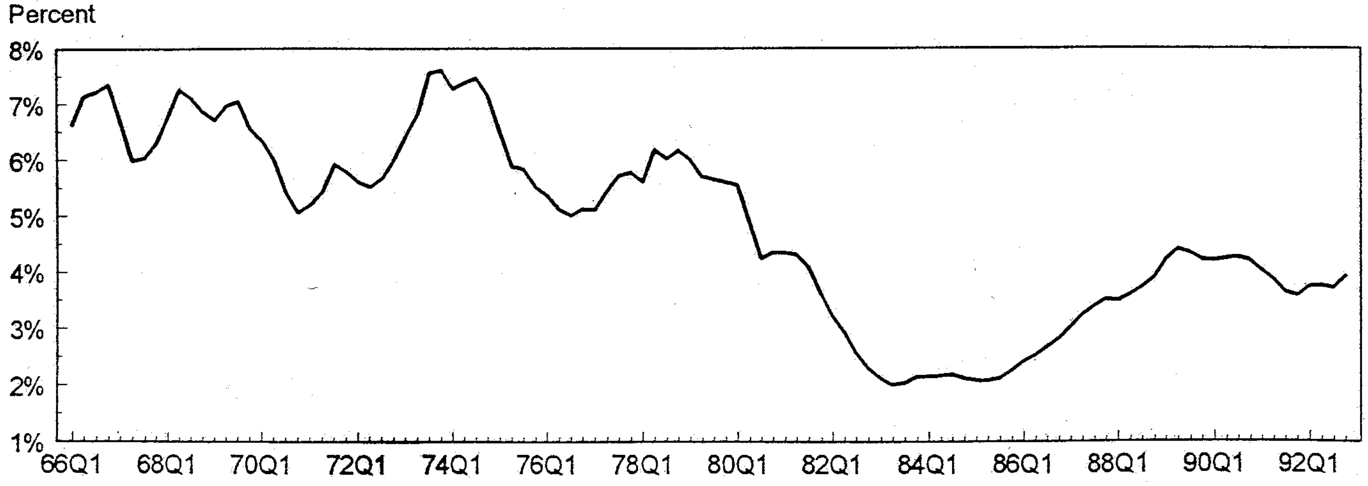
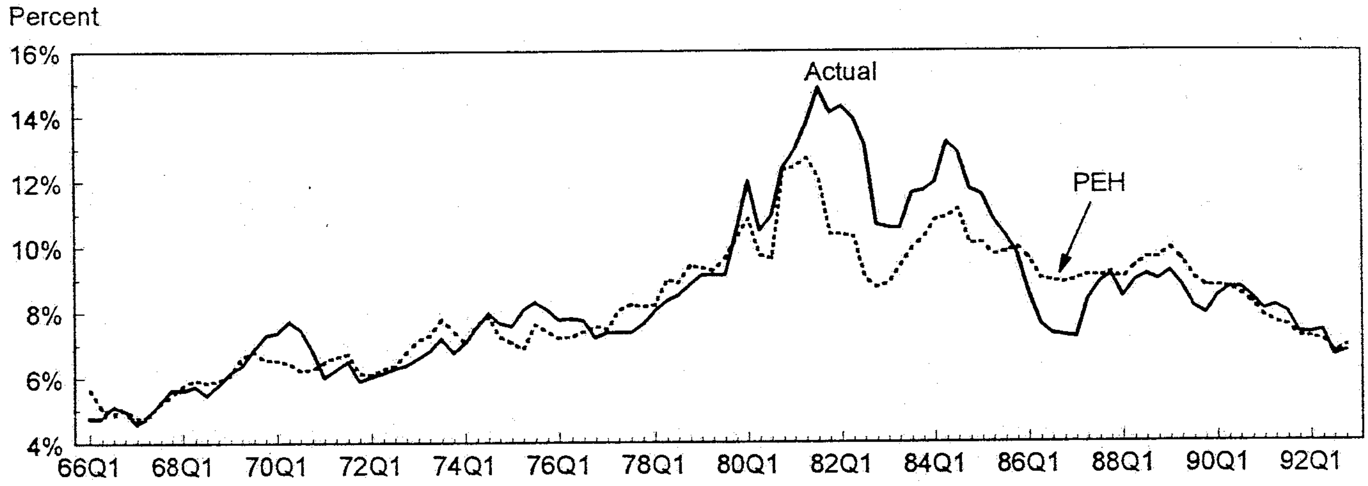


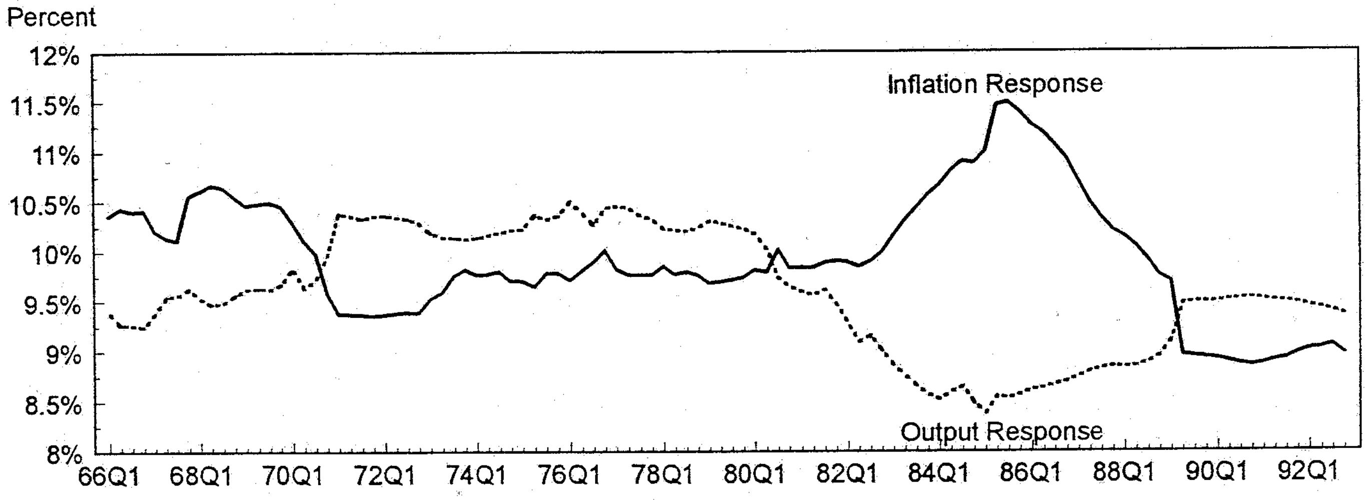
Figure 6

Long Rates and Reaction Function Coefficients Implied by PEH, Smoothing Coefficient = .5 Fit Moving Average of 10-Year Rate

Actual v. PEH Long Rates



Reaction Function Coefficients



Inflation Target

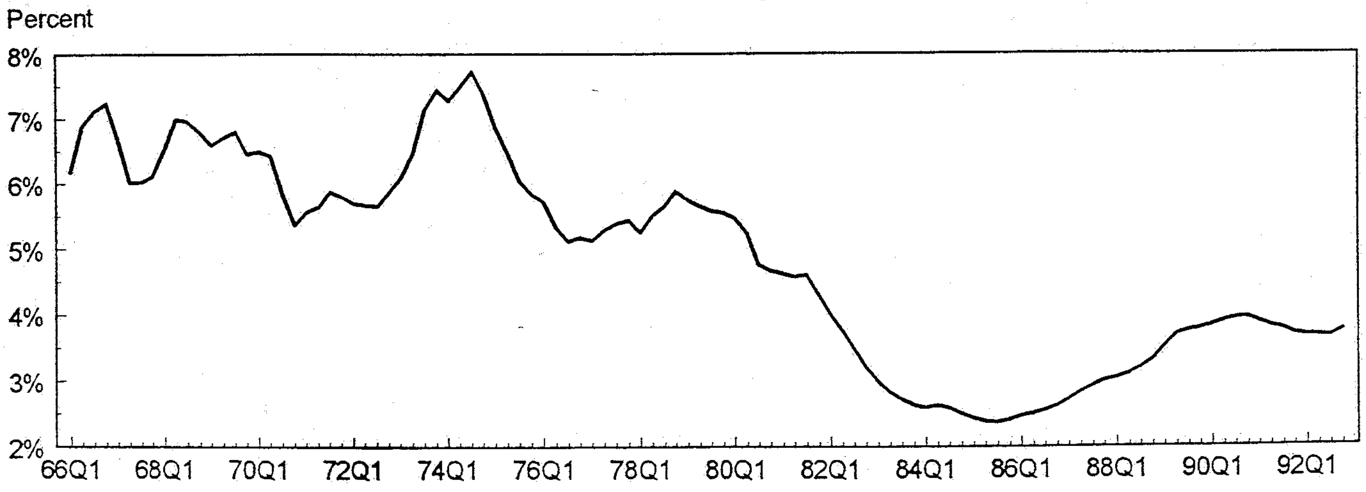
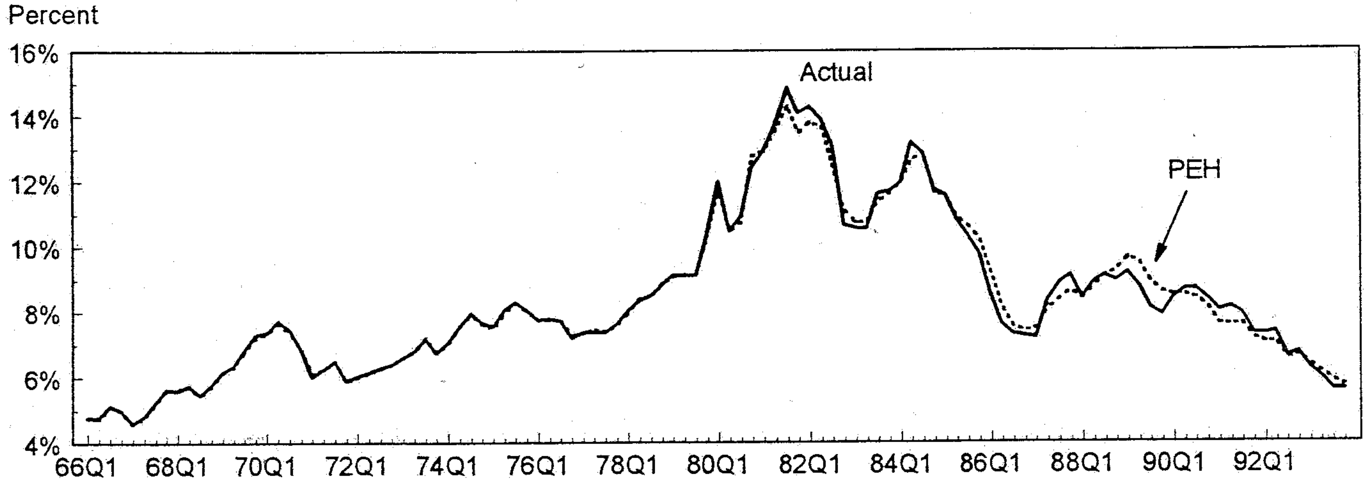


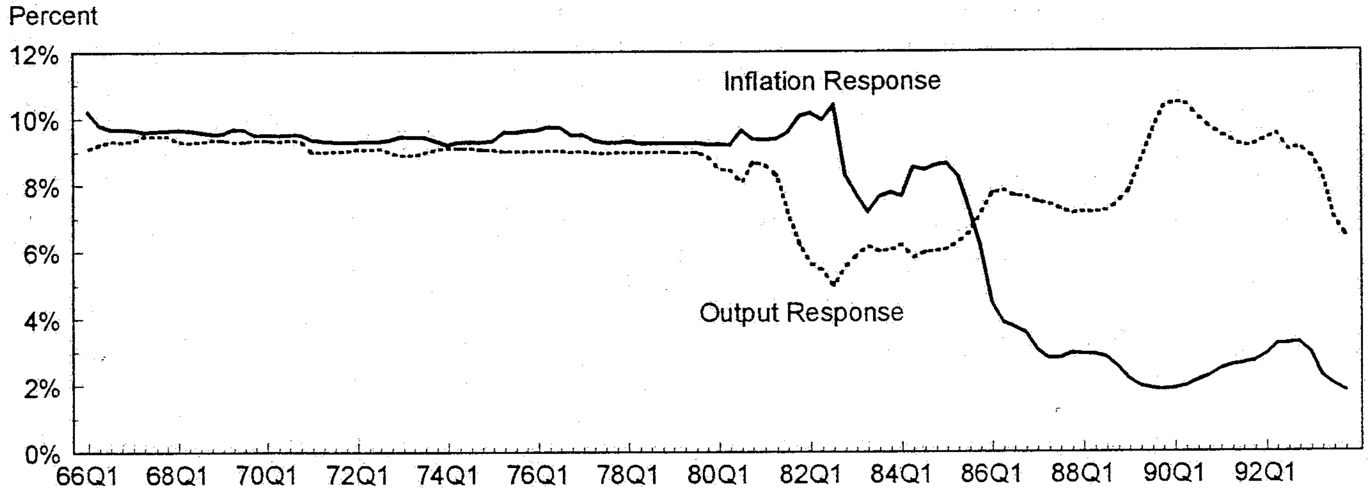
Figure 7

Long Rates and Reaction Function Coefficients Implied by PEH, Smoothing Coefficient = .05 Fit 10-Year Rate

Actual v. PEH Long Rates



Reaction Function Coefficients



Inflation Target

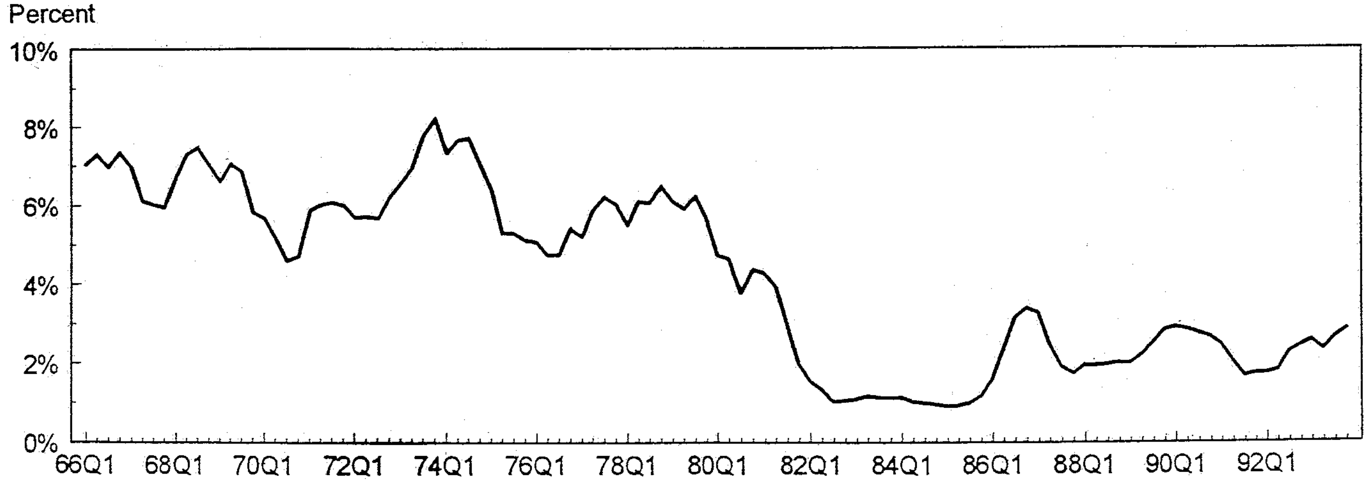
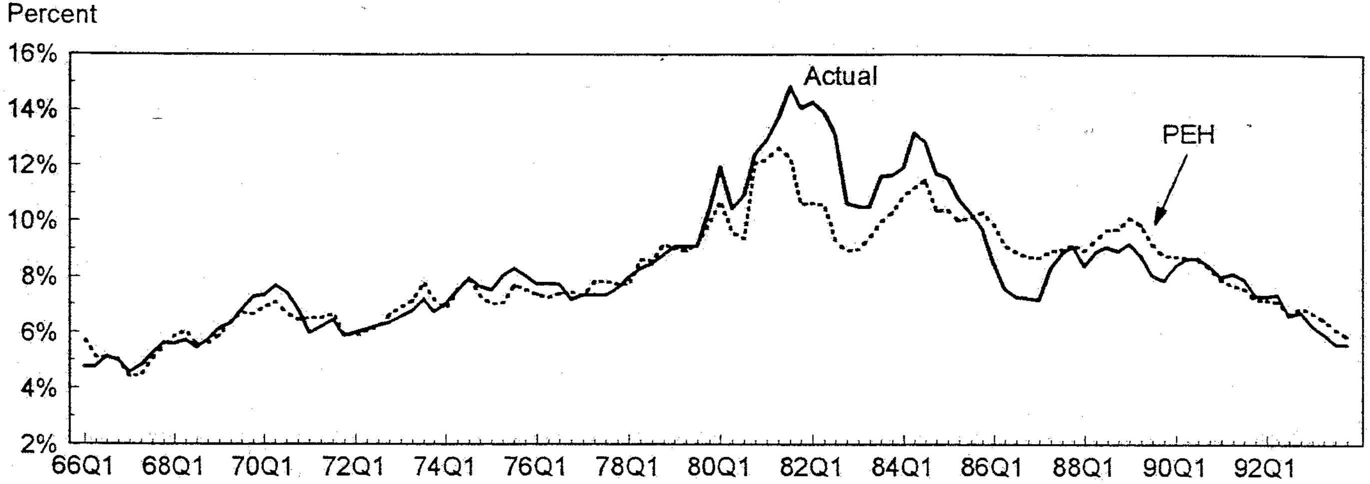


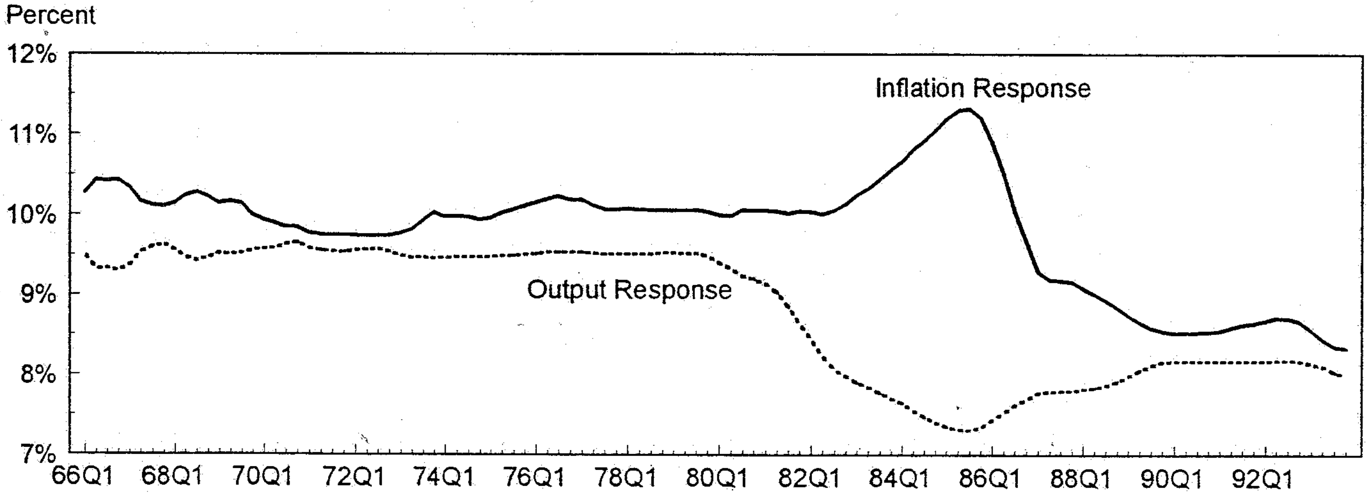
Figure 8

Long Rates and Reaction Function Coefficients Implied by PEH, Smoothing Coefficient = .5
Fit 10-Year Rate

Actual v. PEH Long Rates



Reaction Function Coefficients



Inflation Target

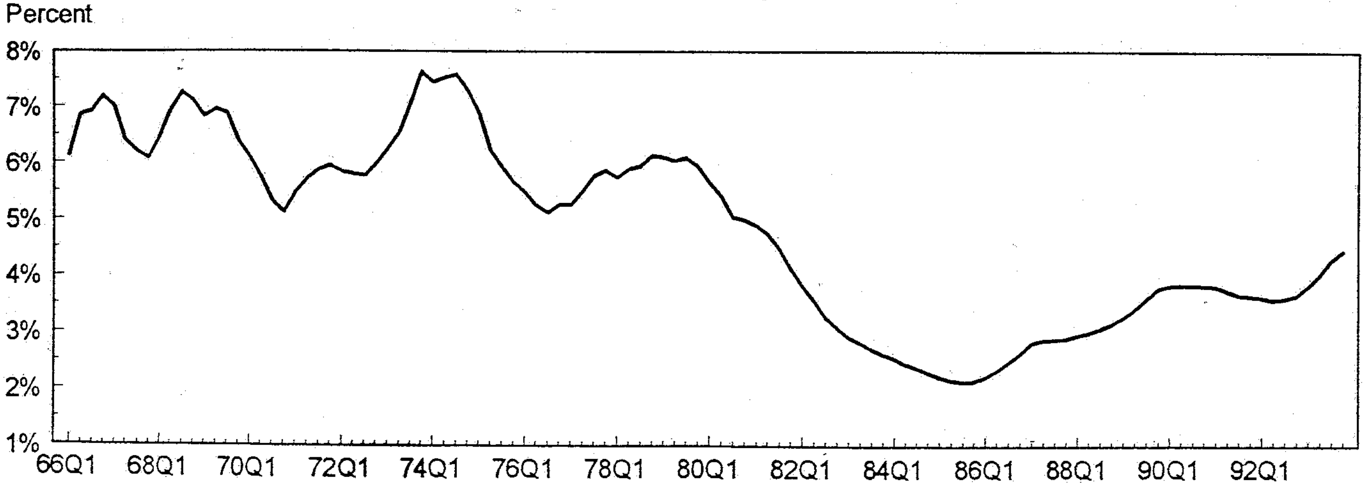
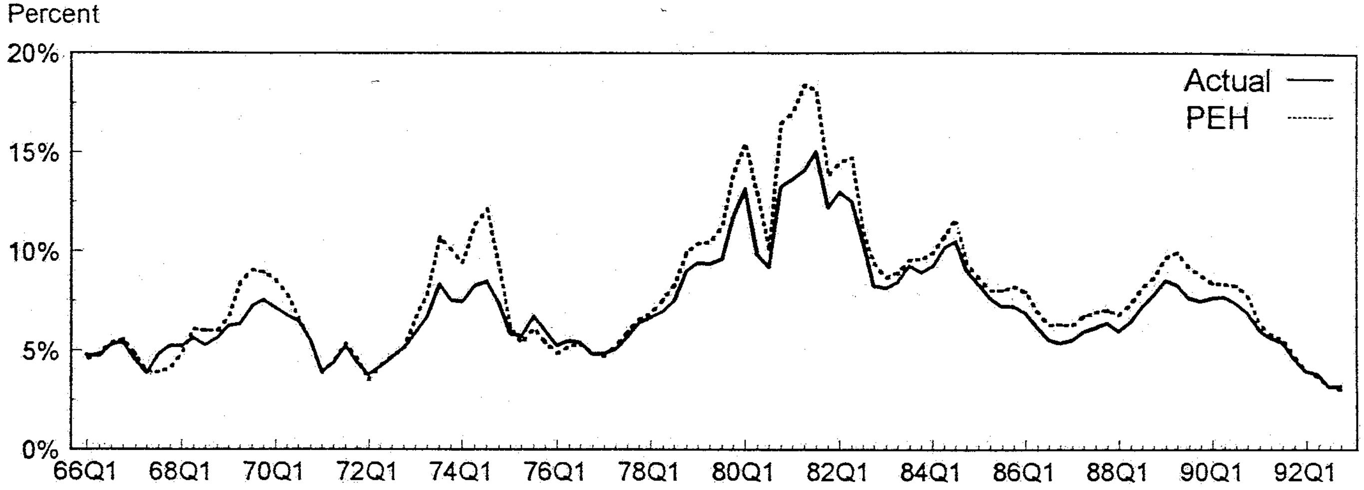


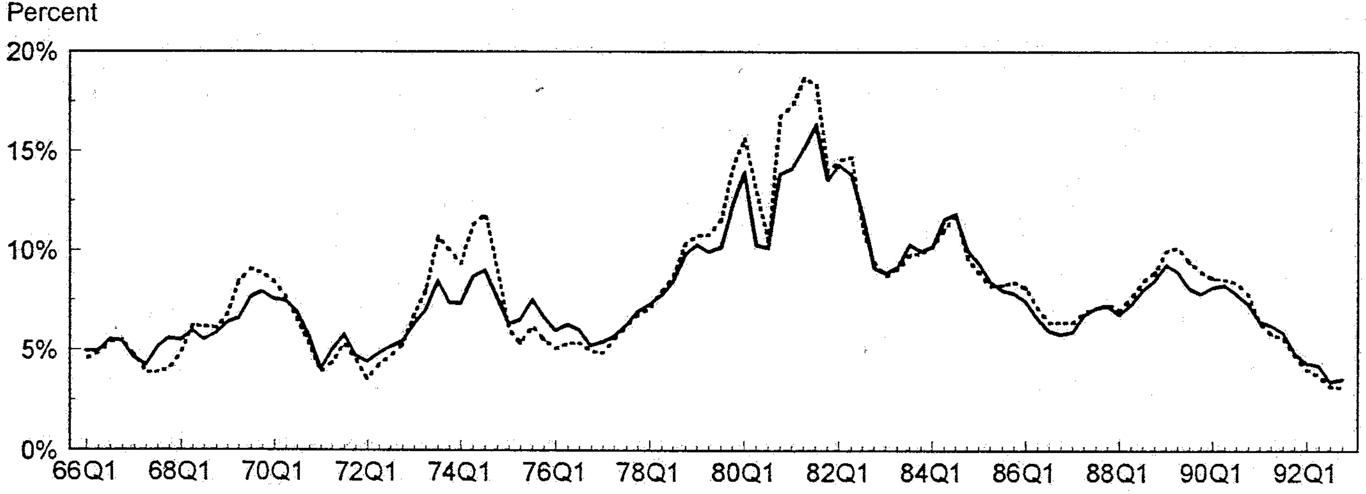
Figure 9

Shorter Maturity Treasury Yields Implied By PEH

Six Month, Secondary Market



One Year, Constant Maturity



Two Year, Constant Maturity

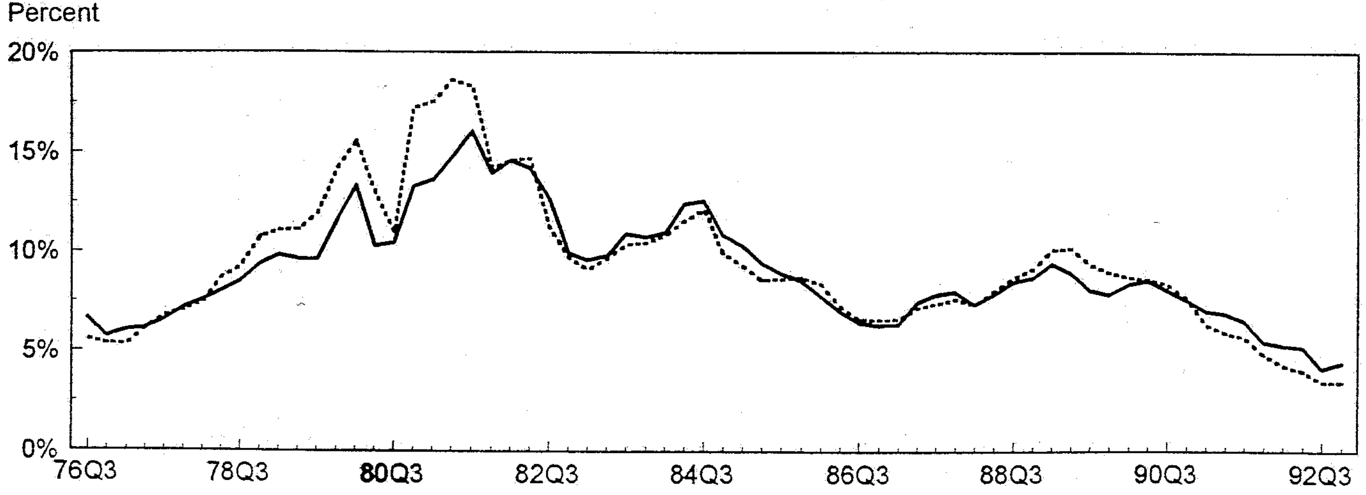


Figure 10

Actual and PEH-Implied Long Rates, from FIML Estimates

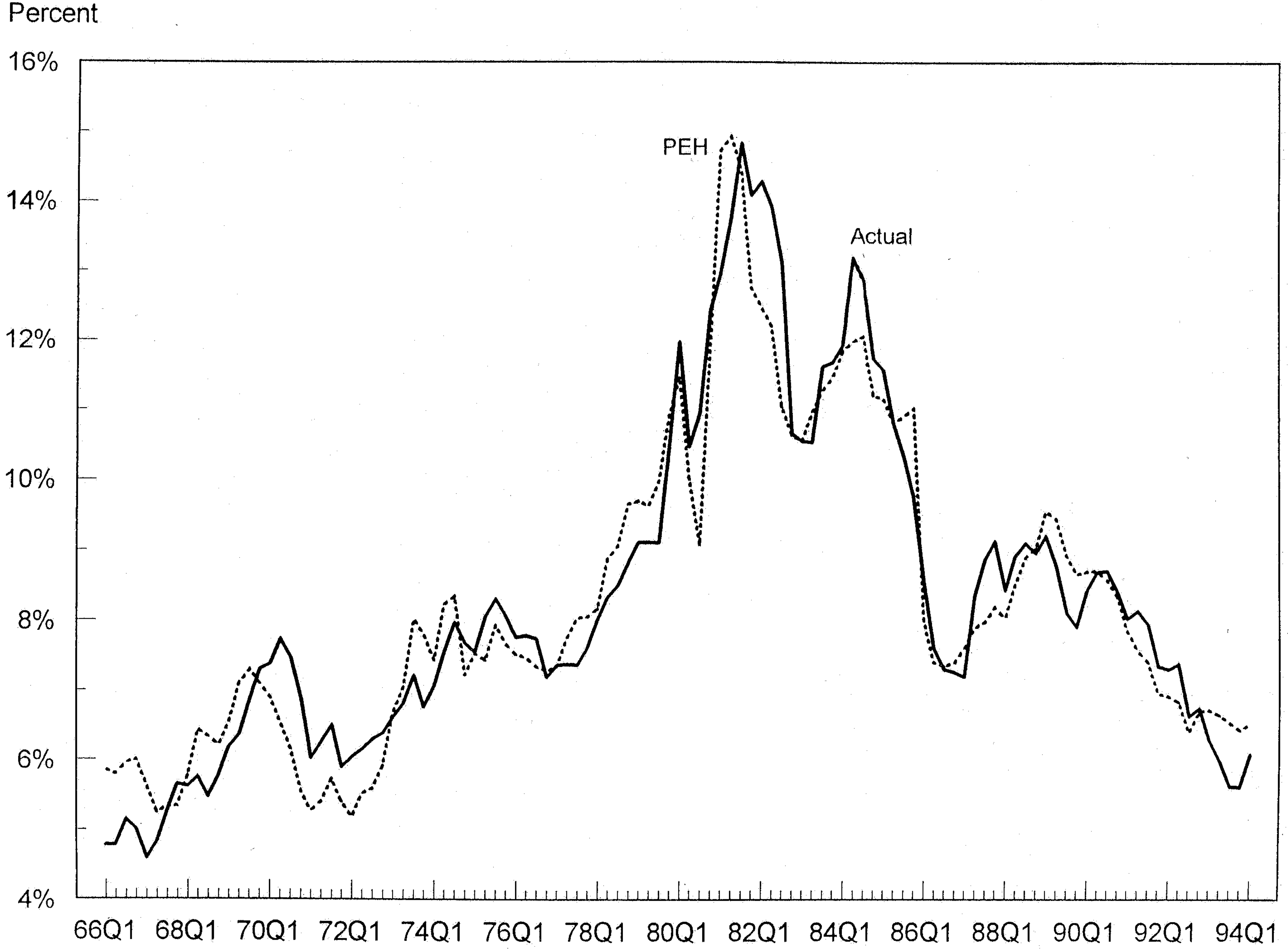


Figure 11

PEH-Implied Long Rates, from 3-sample FIML Estimates

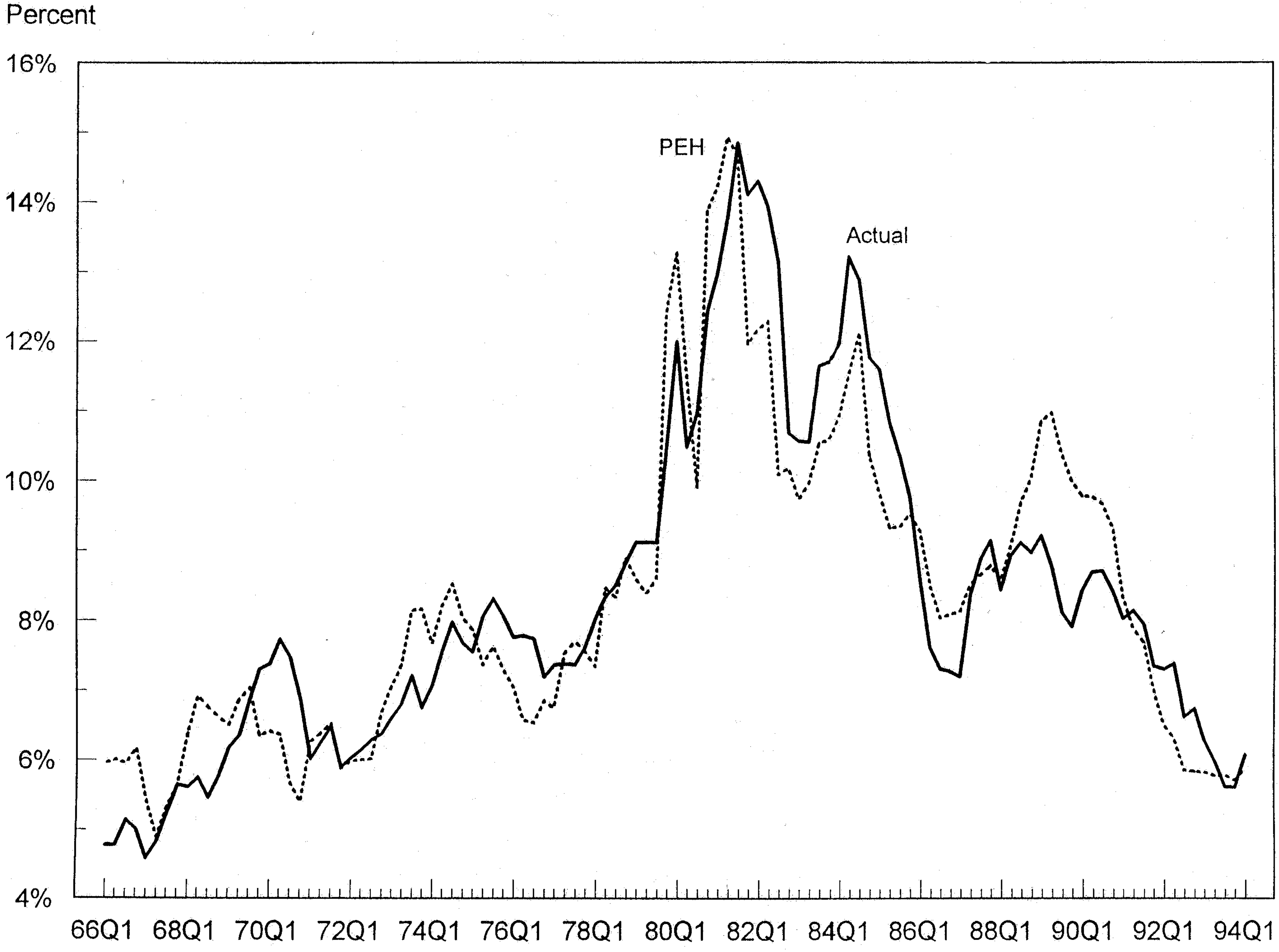


Figure 12

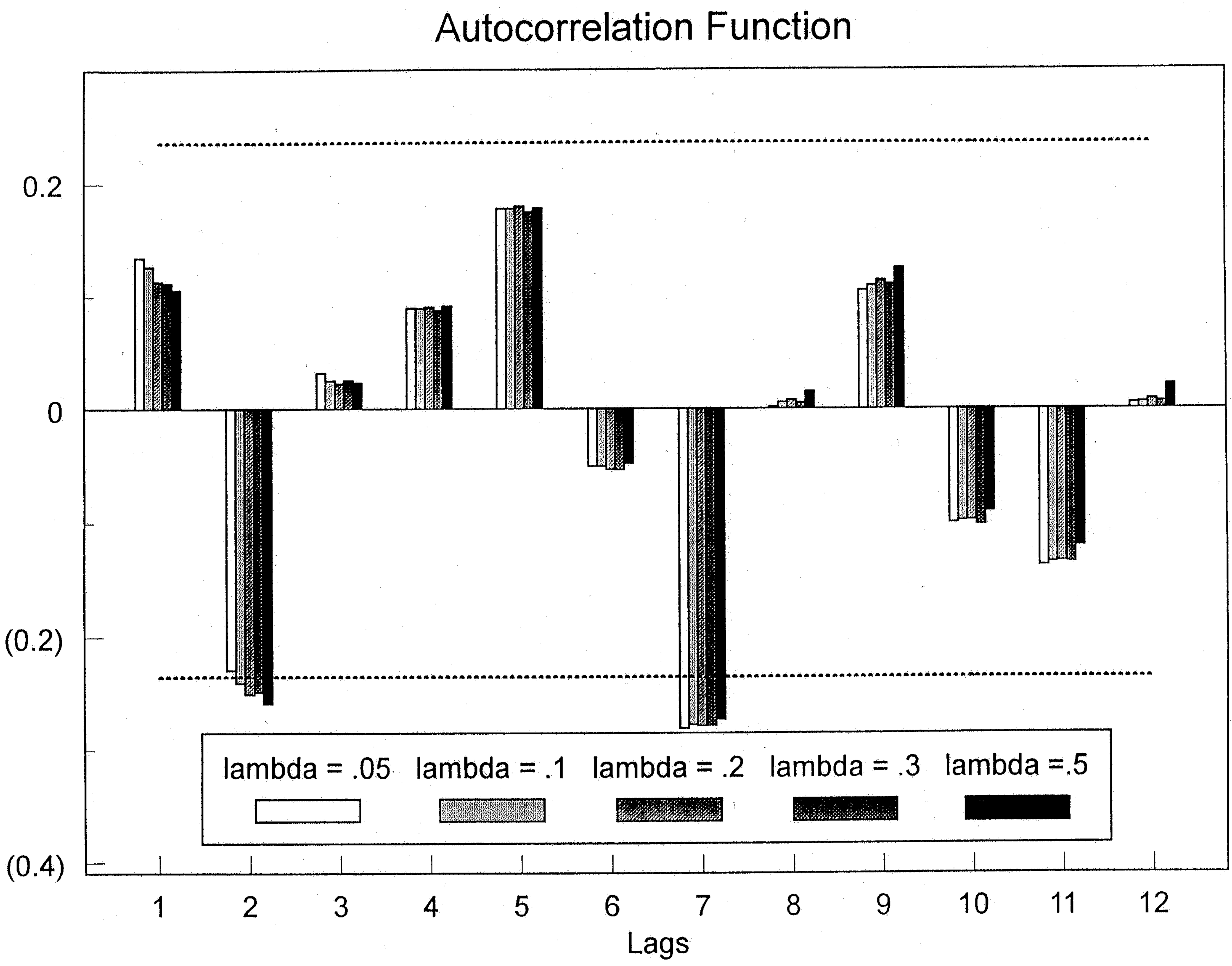
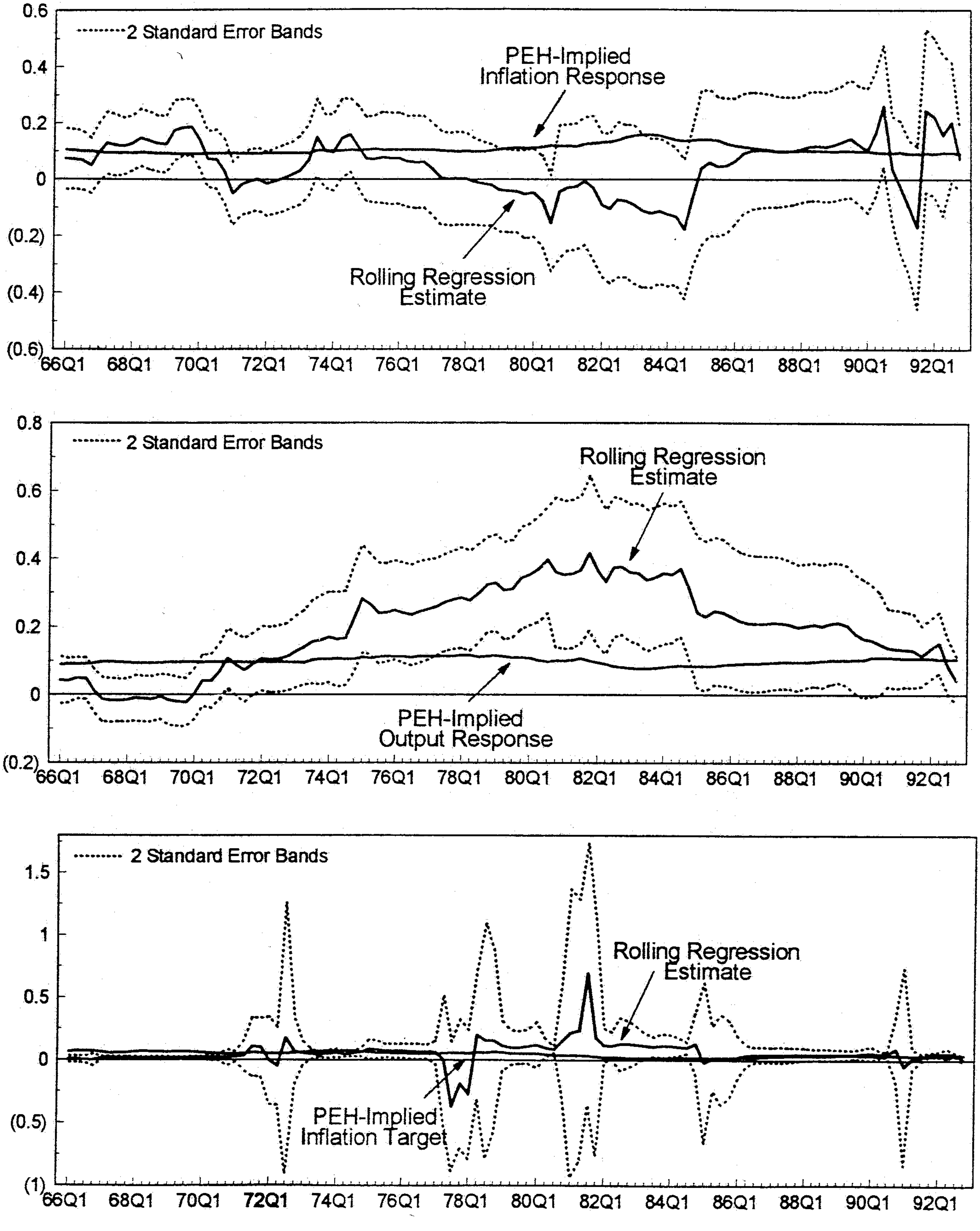

Autocorrelation Function
0.2
0
(0.2)
(0.4)
lambda = .05
lambda = .1
lambda = .2
lambda = .3
lambda =.5
1
2
3
4
5
6
7
8
9
10
11
12
Lags

Figure 13

Rolling Regression Reaction Function Parameter Estimates
With PEH Consistent Estimates



Note: Standard Errors were linearly interpolated for 71Q2-Q4, 72Q2-Q3, 77Q2-78Q2, and 81Q2-Q3.