Input and Output Inventories

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Abstract

This paper builds and estimates a new model of firm behavior that includes decisions to order, use, and stock input materials in a stage-of-fabrication environment with either gross production or value added technology. The model extends the traditional linear-quadratic model of output (finished goods) inventories by incorporating delivery and usage of input materials plus input inventory investment – features which largely have been ignored in the literature. Stylized facts indicate that input inventories are empirically more important than output inventories, especially in business cycle fluctuations. Firms simultaneously choose input and output inventories; thus, the model exhibits feedback between stocks induced by dynamic stage-of-fabrication linkages. Estimation of inventory decision rules shows the model is reasonably consistent with data in nondurable and durable goods industries. The results reveal inventory stock interaction, convex costs, viability of gross production and value added specifications, industrial differences, and input inventory-saving technology.

JEL Classification: E22, E23

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1 Introduction

Most firms produce goods in stages. A typical firm orders input materials from a supplier, takes delivery, and combines the materials with other factor inputs to produce finished output. Often during the production process the firm generates its own intermediate product as well. In addition, many firms sell their finished output to other firms, which view the output as input material. These stage-of-fabrication linkages – within and between firms – imply that rational, optimizing firms will be characterized by joint interaction among all aspects of production. Yet studies of firm behavior generally ignore such dynamic linkages, considering materials only to measure productivity. This paper begins to redress this oversight.

Nowhere is the neglect of stage-of-fabrication linkages more evident than in the inventory literature, where the vast bulk of work has focused almost exclusively on finished goods, or output, inventories. The literature, as summarized by Blinder and Maccini (1991), has been devoted primarily to understanding why the rational expectations version of the pure production smoothing model of output inventories seems to be inconsistent with the data.[1] Such intense scrutiny of output inventory investment largely has "crowded out" consideration of stage-of-fabrication linkages, such as ordering and usage of material inputs. As a consequence, input inventories – defined as raw materials and work-in-process – have been neglected almost entirely.

Neglect of input inventories is problematic for two main reasons. The first reason is conceptual: input inventories are the linchpin of the stage-of-fabrication production process. Input inventories arise whenever the delivery and usage of input materials differ; in other words, firms do not instantaneously obtain and use materials in production. Furthermore, since the usage of input materials is a factor of production, decisions about smoothing production and output inventory investment inherently are related to decisions about drawing down input inventories. In turn, the ability of the firm to draw down input inventories depends on supplier relationships (domestic and foreign), material prices, and factor substitutability – i.e., the spectrum of production decision making.

The second reason is empirical: simply put, input inventories are more important than output inventories. The Blinder-Maccini survey documents that inventory investment changes almost one-for-one with GDP during the average recession; Ramey (1989) shows that this recessionary decline in

[1]Various authors working with aggregate data have modified the basic model. Blanchard (1983), West (1986), Kahn (1987, 1992) and Fuhrer, Moore and Schuh (1995) add a stockout avoidance motive; Maccini and Rossana (1984), Blinder (1986), Miron and Zeldes (1988), and Durlauf and Maccini (1995) add cost shocks in the form of real input prices; Eichenbaum (1989) and Kollintzas (1992) add unobservable technology shocks; Ramey (1991) adds nonconvexities in production and West (1988) adds backlogs of unfilled orders. Other authors have turned to alternative data sources including Fair (1989), Ghali (1987), Haltiwanger and Maccini (1989), Kashyap and Wilcox (1993), Krane and Braun (1991), and Schuh (1996). A consensus explanation, however, is still lacking.

inventory investment occurs predominantly among input inventories. Romer (1986) demonstrates, in our terminology, that input inventory investment is strongly procyclical and thus the ordering and usage of input materials are *not* closely synchronized over the business cycle – a finding that argues for separate modeling of orders and usage. The Blinder-Maccini evidence, and our extended stylized facts, indicate that input inventories are twice as large as output inventories, and three times more variable. Moreover, the dominance of input inventories occurs primarily in *durable goods* industries, which typically have been excluded from applied inventory research. Finally, some econometric work with output inventories suggests interaction between inventory stocks. For example, Durlauf and Maccini (1995) find an important role for observable cost shocks (raw materials prices) in explaining output inventories, which likely arises through stage-of-fabrication linkages.

Despite the conceptual importance and empirical dominance of input inventories, the literature on input inventories is remarkably thin. Moreover, the limited attempts to develop optimizing models of stage-of-fabrication inventories invariably treat stocks as factors of production that yield service flows to the firm. Ramey (1989) includes materials, work-in-process, and finished goods inventory stocks in the production function and, using a restricted cost function, derives demand functions for each stock. Bils and Kahn (1996) propose a model in which procyclical work effort rationalizes procyclical marginal cost and inventory investment; the work-in-process inventory stock enters their production function (materials inventories are not considered). Furthermore, both studies involve estimation of Euler equations with generalized method of moments (GMM), which may give rise to small sample biases, as we discuss below.[2]

Three problems arise from treating inventory *stocks* as factors of production. First, finished goods inventories provide services to the *distribution*, not *production*, of goods. Second, the relevant factor of production is not the *stock* of materials and work-in-process inventories but the *flow* of these materials drawn out of inventory and used in production; the flow may not be well-described as a constant proportion of the stock, as assumed implicitly by previous work. Third, focussing on the stock of materials precludes examination of the separate decisions to order, take delivery of, and use input materials, which are integral aspects of dynamic firm behavior.

This paper extends the literature by developing and estimating a new simultaneous equations model of input and output inventories with separate decisions to order, use, and stock input ma-

[2]Related literature includes Husted and Kollintzas (1987), who offer a rational expectations model of the purchase and holding of imported raw materials inventories but ignore interaction with work-in-process or finished goods inventories, and West (1988), who introduces order backlogs and work-in-process inventories into the standard output inventory model. See also the unpublished work of Auerbach and Green (1980) and Mosser (1989). Other work explaining interaction among inventory types includes Lovell (1961), Maccini and Rossana (1984), Reagan and Sheehan (1985), Blinder (1986), Nguyen and Andrews (1988), Rossana (1990), and Bivin (1993), which rely on stock adjustment and reduced-form models.

terials. As a first step, we assume that materials and intermediate goods are inputs purchased from outside the firm, and that there are no input delivery lags or output order backlogs.[3] The model then makes several advances. First, and most prominently, only the *flow usage* of input materials enters the production function, as in the productivity literature.[4] Second, including the flow of materials admits alternative assumptions about the separability of materials in production: nonseparable (gross production) and separable (value added). Third, the firm simultaneously chooses output inventory investment and – as a consequence of choosing both deliveries and usage of raw materials – input inventory investment; thus linking output and inventory investment with cross-equation restrictions.

The model is fully structural with intertemporal cost minimization under rational expectations, but the total cost function employs several quadratic approximations similar to those employed in conventional output inventory models. The main reason for employing quadratic approximations is to obtain linear decision rules that can be estimated via maximum likelihood (ML). Recent Monte Carlo work by West and Wilcox (1994) and Fuhrer, Moore and Schuh (1995) demonstrates the existence of substantial small sample biases in generalized method of moments (GMM) parameter estimates of conventional output inventory Euler equations. In contrast, Fuhrer *et al.* demonstrate that ML estimation of conventional output inventory decision rules produces relatively unbiased and considerably more significant parameter estimates. Thus, we conduct the first joint ML estimation of input and output inventory decision rules.[5]

By and large, the data yield relatively strong econometric support for the model. Exploiting model identities, we overcome the lack of high-frequency data on deliveries and usage of materials and estimate versions of the model corresponding to: gross production and value added; nondurable and durable goods industry data; and joint and single-equation estimates of inventory decision rules. For all combinations of these features, the vast majority of parameter estimates are signed correctly and estimated significantly – in sharp contrast to most previous estimates of structural inventory models. Furthermore, joint estimation reveals substantial efficiency gains and provides evidence that the failure to impose stage-of-fabrication linkages may lead to misspecification biases.

[3]To some degree, of course, intermediate goods – and thus work-in-process inventories – are produced within the firm. Hence, an important extension of this paper is to model production of both intermediate and finished goods, which will require the firm to hold separate stocks of materials and work-in-process inventories. Extending the model to incorporate delivery lags and order backlogs may further improve the model's ability to fit the data.

[4]See Bruno (1984), Baily (1986), Basu (1996), Basu and Fernald (1995), and Basu and Kimball (1997), for discussions of the specification of materials in production functions and its role in explaining productivity movements.

[5]Eichenbaum (1984) also jointly estimated multiple decision rules for labor (hours) and output inventories, but did not consider input inventories. Although we do not explicitly estimate a labor equation, the model contains labor demand.

Although the data generally reject the overidentifying restrictions of the joint model, the degree of rejection is comparable to that experienced by similar previous estimation of output inventory models.

The econometric results generate numerous broad implications. First, the data clearly reveal evidence of stage-of-fabrication interactions between inventory stocks, and among inventory stocks and other facets of production. In particular, when the firm chooses output inventories it takes into consideration the gap between actual and target input inventories; at the same time, the firm's choice of output inventories affects the input inventory target stock and thus its choice of input inventories. Second, the data indicate that aggregate cost functions are convex, even for durable goods industries and even in the presence of input inventories. Third, gross production and value added specifications of the model both generally fit the data and provide evidence of inventory stock interaction. But the value added restrictions lead to a significantly simpler model that excludes some channels of propagation that are in the gross production model. Fourth, the model generally fits the data from both nondurable and durable goods industries; industrial differences may, however, stem from the model's simplifying assumptions. Finally, the data show evidence of input inventory-saving technology in both industries, especially in durable goods.

The remainder of the paper proceeds as follows. Section 2 updates and expands the stylized facts about inventory movements at different stages of fabrication. Section 3 presents the new stage-of-fabrication inventory model. Section 4 describes the econometric specification and estimation, and section 5 reports the econometric results. The paper concludes with a discussion of some implications for future research.

2 Motivation and Stylized Facts

This section presents key empirical facts about manufacturers' input and output inventories that motivate the stage-of-fabrication model developed in the next section. Table 1 lists the main variables in the paper. Data for these variables are in constant 1987 dollars, seasonally adjusted, and monthly for the period 1959:1 through 1994:5 (except for D_t and U_t, which are unobserved). All data are standard published government series except the materials price index, which we constructed from detailed price indexes. The data appendix contains complete details.

Tables 2 and 3 reveal several stylized facts about manufacturers' input and output inventories in nondurable goods and durable goods industries. Table 2 reports the change in inventory investment relative to the change in manufacturing sales during post-war recessions (changes measured from peak to trough). Table 3 reports the means and variances of inventory investment and inventory-to-sales ratios.

Table 1 — Variable Definitions

D_t	=	Deliveries of input materials
L_t	=	Labor input
M_t	=	Input inventories (materials and work-in-process)
N_t	=	Output inventories (finished goods)
T_t	=	Time trend (proxy for inventory technological change)
U_t	=	Usage of input materials
V_t	=	Real materials price
W_t	=	Real hourly wage rate
X_t	=	Sales (shipments)
Y_t	=	Production of finished goods ($X_t + \Delta N_t$)

Table 2

Change in Inventory Investment during Postwar Recessions

(Expressed as a Share (%) of the Change in Manufacturing Sales)

	Recessions						
	60:2-61:1	69:4-70:4	73:4-75:1	80:1-80:3	81:3-82:4	90:3-91:1	Average
Total Manufacturing	27	31	27	82	51	3	37
Input	7	30	32	54	26	10	27
Output	20	0	−5	28	25	−7	10
Nondurable Goods	−5	−3	6	40	8	−18	5
Input	5	−10	7	23	6	−6	4
Output	−10	7	−1	17	2	−12	0
Durable Goods	32	34	21	42	43	21	32
Input	1	40	26	31	20	17	22
Output	30	−6	−5	11	23	5	10

NOTES: The data are monthly, $1987 fixed-weighted. Recessions are defined by the National Bureau of Economic Research (NBER). Changes are calculated from business cycle peak (first date) to trough (second date).

Fact #1: *Input inventories are larger and more volatile than output inventories in manufacturing.*

Table 2 shows that changes in input inventory investment account for the bulk of the changes in total manufacturing inventory investment during recessions. During a typical recession, the decline in manufacturing inventory investment is more than one-third as large as the decline in manufacturing sales. Moreover, the decline in input inventory investment is responsible for nearly three-fourths of the overall inventory investment change. Table 3 indicates that input inventories are at least twice as large as output inventories in manufacturing, as measured by average inventory investment and inventory-to-sales ratios. Most importantly, the table shows that input inventory investment is more than three times more variable than output inventory investment in manufacturing. These facts suggest that an analysis of the cyclical behavior of manufacturing inventory investment should focus on input rather than output inventories, contrary to the overwhelming practice in the literature.

Fact #2: *Durable goods inventories are larger and more volatile than nondurable goods inventories.*

Table 2 shows that the decline in durable goods inventory investment during a typical recession accounts for nearly 90 percent of the decline in manufacturing inventory investment. Table 3 indicates that durable goods inventories are up to two times larger than nondurable goods inventories, as measured by average inventory investment and inventory-to-sales ratios. Moreover, the table shows that durable goods inventory investment is nearly five times more variable than nondurable goods inventory investment. These facts point to durable goods industries as the appropriate target of research on inventories, but empirical work has invariably emphasized nondurable goods industries.

Fact #3: *Input inventories are much larger and more volatile than output inventories in durable goods industries, but input and output inventories are similar in size and volatility in nondurable goods industries.*

This fact is a byproduct of the first two. Table 2 shows that the decline in input inventory investment during a typical recession accounts for the bulk of the decline in total inventory investment in both durable goods and nondurable goods industries. However, Table 3 indicates that input inventories are much larger than output inventories in durable goods industries, as measured by average inventory investment and inventory-to-sales ratios. Further, the table shows that input inventory investment is more than six times more variable than output inventory investment in durable goods industries. In nondurable goods industries, on the other hand, the magnitude and variability of input inventories are more even with those of output inventories. In nondurables, input inventory investment is a bit larger but a bit less variable than output inventory investment. The fact that output inventory investment is a bit more variable than input inventory investment provides some rationale for literature's focus on output inventory investment in nondurable goods industries. Nevertheless, it is difficult to rationalize the complete lack of attention to input inventories. More generally, if the ultimate goal of research is understanding the variability of total manufacturing investment, then the focus should be on input inventories, especially in durables.

Fact #4: *Interactions between input and output inventories are quantitatively significant, especially in durable goods industries.*

Table 3 reveals inventory stock interaction. Fifteen percent of the variance in manufacturing inventory investment is accounted for by the covariance between input and output inventory investment. When the inventory stocks are disaggregated into the three stages of processing (materials, work-in-process, and finished goods), the covariance term accounts for 25 percent of the variance. The table also shows that covariance among types of inventory investment is greater in durable goods industries than nondurable goods industries.

Together, these stylized facts suggest: (1) a complete analysis of inventory behavior requires the modeling of input inventories; (2) interaction between input and output inventories is empirically evident and potentially a significant feature of firm behavior; and (3) tests of the model should be conducted with durable goods, as well as nondurable goods, industries.

Before turning to the model, however, it is useful to point out a few additional features of the data from the time series plots in Figure 1. The upper panels of inventory ratios exhibit two notable features.[6] First, the output inventory ratio is more countercyclical than the input inventory ratio (correlations with detrended sales of $-.20$ versus $-.10$ in nondurables and $-.78$ versus $-.26$ in durables). Whereas the output inventory ratio rises sharply in virtually every recession, the input inventory ratio does not always rise in every recession. Furthermore, the increase in the input inventory ratio during recessions varies in magnitude considerably more than does the increase in the output inventory ratio. Second, the input inventory ratios in both industries appear somewhat to exhibit negative trends since the 1970s, trends which may reflect the introduction of inventory-saving technologies and production techniques.

The lower panels of real prices (the nominal price relative to the price of output) also exhibit some notable features. First, the real wage trends up in both industries, but it is much more cyclically sensitive in nondurables goods industries than in durable goods industries. Second, the real materials prices are essentially trendless in both industries, but they exhibit large, persistent increases during the 1970s and early 1980s. Most of these prices also are consistent with a split trend around 1973, as argued by Perron (1989). The trends in these prices turn out to have direct

[6]The input inventory ratio is calculated with production, rather than sales, in the denominator because production is assumed to be the primary target variable for input inventory stocks.

Table 3

Stylized Facts About Inventories

MEAN INVENTORY INVESTMENT

	Mfg.	%	Dur.	%	Non.	%
Total	0.51	100.0	0.33	100.0	0.18	100.0
Finished Goods	0.18	35.3	0.09	27.3	0.09	50.0
Input	0.33	64.7	0.24	72.7	0.09	50.0
Work-In-Process	0.17	33.3	0.14	42.4	0.03	16.7
Materials & Supplies	0.17	33.3	0.11	33.3	0.06	33.3

VARIANCE DECOMPOSITION OF INVENTORY INVESTMENT

	Mfg.	%	Dur.	%	Non.	%
Total	1.441	100.0	1.047	100.0	.210	100.0
Finished Goods	.264	18.3	.117	11.2	.114	54.3
Input	.958	66.5	.793	75.7	.079	37.6
Covariance	.218	15.1	.138	13.2	.017	8.1
Finished Goods	.264	18.3	.117	11.2	.114	54.3
Work-In-Process	.417	28.9	.384	36.7	.018	8.6
Materials and Supplies	.387	26.9	.276	26.4	.058	27.6
Covariance Terms	.373	25.9	.271	25.9	.019	9.0

MEAN INVENTORY/SALES RATIO (Std. Dev.)

	Mfg.		Dur.		Non.	
Total	1.66	(0.11)	1.98	(0.20)	1.29	(0.05)
Finished Goods	0.54	(0.03)	0.52	(0.04)	0.57	(0.03)
Input	1.11	(0.09)	1.46	(0.16)	0.73	(0.03)
Work-In-Process	0.53	(0.05)	0.83	(0.10)	0.19	(0.01)
Materials and Supplies	0.58	(0.04)	0.63	(0.07)	0.53	(0.03)

NOTES: Statistics are calculated with monthly $1987 data over the period 1959:01 to 1994:05.

implications for the input inventory target stock specification in the econometric estimation of the SOF model. Finally, a comparison of coefficients of variation indicates that the real materials price is relatively more variable than the real wage, especially in durables. This feature suggests that real materials price movements may play a more important role in explaining inventory movements.

3 The Stage-of-Fabrication Model

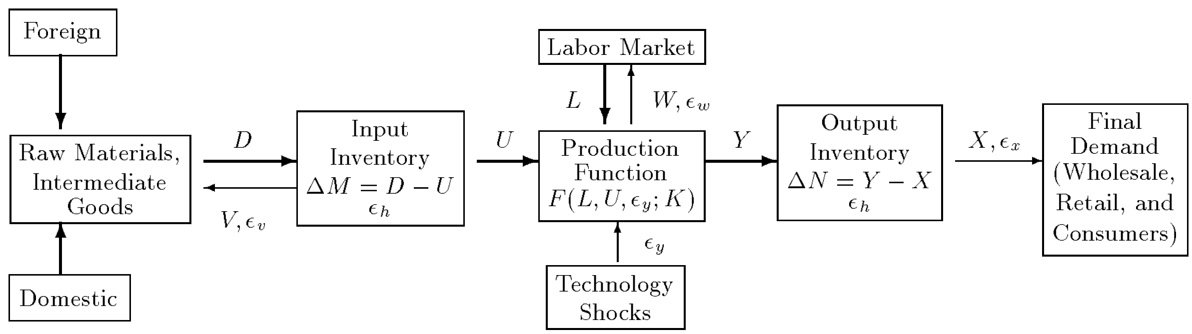
3.1 Overview

The model, illustrated schematically in Figure 2, focuses on flows through the stage-of-fabrication (SOF) production process employed by a firm to transform input inventories (raw materials and work-in-process) into output inventories (finished goods). Each period, the firm combines labor (L), materials used in production (U), and capital (K) to produce finished goods. Materials used in production are obtained from the on-hand stock of input inventories (M), which is continually replenished by deliveries (D) of materials from foreign and domestic suppliers. Production (Y) of final goods is added to the stock of output inventories (N), which are used to meet final demand (X). The firm takes final demand, the price of labor (W), and the price of deliveries (V) as exogenous (denoted by thin lines).

The firm optimizes in a dynamic stochastic environment. Five random shocks (ϵ) buffet the firm's production process. One shock is the traditional demand shock (ϵ_x). The other four shocks comprise a disaggregation of the traditional "supply" shock: (1) a technology shock (ϵ_y) affects the production function; (2) a holding cost shock (ϵ_h) affects the cost of carrying inventory stocks; (3) a real wage shock (ϵ_w) affects labor costs ; and (4) a real materials price shock (ϵ_v) affects the supply of materials. In the short run, with the capital stock fixed, the firm chooses U, M, and N to minimize the present value of total costs.[7] The first-order conditions are used to substitute out materials usage, which is empirically unobservable at high frequencies.

[7] Actually, the firm chooses L, U, and D, but because U and D are unobservable empirically we exploit the model flow identities to recast the problem with inventory stocks as choice variables.

Figure 2



The SOF model generalizes the traditional linear-quadratic framework that has served as the workhorse for models of output inventories and production. The central extension in the SOF model is the explicit introduction of input inventories, which must be chosen simultaneously with output inventories. Input inventory investment is controlled by varying the usage of materials in production and the deliveries of materials. Total costs—labor costs, inventory holding costs, and delivery costs—are approximated with a generalized quadratic form. The SOF model differs from the few inventory papers that include inventories other than finished goods by specifying the flow, rather than stock, of materials in the production function. For the purpose of testing the model with aggregate data, we adopt the convention of a representative firm, as is customary in the inventory literature.[8]

3.2 The Production Function

Following the literature on production functions and productivity – for example, Bruno (1984) and Baily (1986) – we assume that the production function contains as an input the flow of materials used in the production process. Specifically, the production function is

$$Y_t = F\left(K_t, L_t, U_t, \epsilon_{yt}\right) \ . \tag{1}$$

Note that U_t is the *flow* of materials used in the production process. Because Y_t is gross output, we refer to equation (1) as the gross production function.

An important specification issue for the production function is whether U_t is additively separable from the other factors of production. For example, Basu and Fernald (1995) show that evidence of externalities caused by productive spillovers exists in value added data but not gross production

[8]In future work, we intend to estimate the model with SOF inventory data from the firm-level M3LRD data base originally developed by Schuh (1992).

data. If U_t is separable, then the production function can be written as

$$Y_t - U_t = G\left(K_t, L_t, \epsilon_{yt}\right) \tag{2}$$

where $Y_t - U_t$ is value added. We refer to this form of the production function as the value added production function. Observe that equation (2) is a special case of equation (1) with the restrictions $F_U = 1$ and $F_{KU} = F_{LU} = F_{\epsilon_y U} = 0$.

To date, the limited number of inventory models that account for both input and output inventories have all, implicitly or explicitly, included the *stock* of input inventories as a factor of production. For example, the production functions in Ramey (1989) and Bils and Kahn (1996) are of the form

$$Y_t = F\left(K_t, L_t, M_t, \epsilon_{yt}\right) \ . \tag{3}$$

Ramey also includes the stocks of work-in-process and finished goods inventories in the production function. Bils and Kahn define input inventories to include only work-in-process, ignoring the larger and more volatile materials stock.

Three problems arise from including inventory stocks in the production function. First, as Ramey points out, finished goods inventories do not provide services to the production of goods because they are final goods. Finished goods inventories do provide services to the distribution of goods, and could be justified if "production" refers broadly to both production and distribution. Second, and more important, the production function is a flow concept; it expresses the flow of output produced during a period as a function of the flow of inputs used in the production process. Specification (3) implicitly presumes that the flow of materials used in the production process is proportional to the stock of materials held in inventories and that the factor of proportionality is constant over time. This presumption is accurate only under rather extreme conditions. Third, the specification precludes distinguishing among materials ordered, delivered, used, and held in storage. These individual actions comprise a richer and more realistic characterization of the firm's production environment. Dividing the input inventory decision into two parts – how much to order and how much to use – permits a more detailed analysis of the dynamics and a better opportunity to understand the sources of volatility in input inventories.

Two simplifying assumptions get the model off the ground. First, because inventory investment is a short-run decision, the capital stock is a fully fixed factor of production and hereafter suppressed in the functional form. Further, the remaining factors—materials usage and labor—possess positive and nonincreasing marginal products. Second, the firm purchases intermediate goods (work-in-process) from outside suppliers rather than producing them internally.[9] Thus, intermediate goods

[9] To allow for production of intermediate goods within the firm requires extending the production function to

are analogous to raw materials so work-in-process inventories can be lumped together with materials inventories.

3.3 The Cost Structure

The firm's total cost structure consists of three major components: labor costs, inventory holding costs, and materials costs. This section describes each component.

3.3.1 Labor Costs

Labor costs are

$$LC_t = W_t L_t + A(\Delta L_t) \tag{4}$$

with

$$\begin{aligned} A' &\lesseqgtr 0 \quad \text{as} \quad \Delta L_t \lesseqgtr 0 \\ A'' &> 0 \end{aligned}$$

where $\Delta L_t = L_t - L_{t-1}$. The first component, $W_t L_t$, is the standard wage bill. The second component, $A(\Delta L_t)$, is an adjustment cost function intended to capture the hiring and firing costs associated with changes in labor inputs. The adjustment cost function has the usual properties, including a rising marginal adjustment cost.

To reduce the number of decision variables and focus on the inventory decisions, we eliminate labor input. Inverting the production function, equation (1), yields

$$L_t = L(Y_t, U_t, \epsilon_{yt}) \tag{5}$$

with

$$\begin{aligned} L_Y &= 1/F_L > 0 \\ L_U &= -F_U/F_L < 0 \\ L_{\epsilon_y} &= -(F_{\epsilon_y}/F_L) < 0 \end{aligned} \tag{6}$$

where F_L, F_U, and F_{ϵ_y} are the marginal products of production with respect to the factor inputs. Substituting (5) into (4) yields

$$LC_t = W_t L(Y_t, U_t, \epsilon_{yt}) + A(L(Y_t, U_t, \epsilon_{yt}) - L(Y_{t-1}, U_{t-1}, \epsilon_{y,t-1})) \tag{7}$$

which is the central portion of the firm's cost function.

To construct an econometric model, it is necessary to parameterize the labor cost function, equation (7). In principle, we could use a specific model of production, such as Cobb-Douglas, to

incorporate joint production of final and intermediate goods. This extension is a substantial modification of the standard production process that we leave for future work.

parameterize the inverted production function, equation (5). However, such a specification would introduce nonlinearities and preclude estimation of the decision rules (unless the decision rules were linearized, which also entails approximation error). Instead, we follow the tradition of the inventory literature, which exploits generalized quadratic approximations of the form defined in Chambers (1988). Specifically, the quadratic approximation to labor costs is

$$\begin{aligned} LC_t = \left(\tfrac{\gamma_1}{2}\right) Y_t^2 + \left(\tfrac{\gamma_2}{2}\right) U_t^2 + \gamma_3 Y_t U_t + W_t[\gamma_4 Y_t + \gamma_5 U_t] \\ + \left(\tfrac{\varphi}{2}\right) [\gamma_6 \Delta Y_t + \gamma_7 \Delta U_t]^2 + \epsilon_{yt}(\gamma_8 Y_t + \gamma_9 U_t) \ . \end{aligned} \tag{8}$$

This equation omits products of inputs involving squared terms that would appear in a completely generalized quadratic approximation of equation (7). Such terms vitiate certainty equivalence and make it difficult—perhaps impossible—to solve for decision rules, which is necessary for our econometric strategy. Nevertheless, the approximation captures the essential elements of the production and adjustment cost functions, and is comparable to the most general approximations found in previous inventory work.

The signs of some, but not all, parameters in equation (8) are known without further assumptions. Parameters γ_1, γ_6, γ_8, and φ are all positive from the assumed convexity of the production and adjustment cost functions. Abstracting from dynamics, $\partial LC/\partial Y = \gamma_1 Y + \gamma_4 W$ should be positive from (6); given $\gamma_1 > 0$, $\gamma_4 > 0$ is a sufficient, though not necessary, condition. But $\partial LC/\partial W = \gamma_4 Y$ should also be positive from the wage bill, which indicates that γ_4 must be positive. In contrast, the signs of γ_2, γ_3, γ_5, γ_7, and γ_9 are unknown *a priori* because they depend on the specification of the production function.

3.3.2 Inventory Holding Costs

In line with much of the output inventory literature, holding costs for output inventories are a quadratic approximation to actual costs of the form

$$HC_t^N = (\delta_0 + \epsilon_{ht})N_t + \left(\frac{\delta}{2}\right)(N_t - N_t^*)^2 \tag{9}$$

where ϵ_{ht} is a white noise innovation to holding costs, N_t^* is the target level of output inventories that minimizes output inventory holding costs, and $\delta > 0$. We adopt an analogous formulation for input inventories; holding costs for these stocks are a quadratic approximation of the form

$$HC_t^M = (\tau_0 + \epsilon_{ht})M_t + \left(\frac{\tau}{2}\right)(M_t - M_t^*)^2 \tag{10}$$

where M_t^* is the target level of input inventories that minimizes input inventory holding costs, and $\tau > 0$.

The quadratic inventory holding cost structure balances two forces. Holding costs rise with the level of inventories, M_t and N_t, due to increased storage costs, insurance costs, etc. But holding costs fall with M_t and N_t because—given expected M_t^* and N_t^*—higher M_t and N_t reduce the likelihood that the firm will stock out of inventories. Note that equations (9) and (10) admit only a single homogeneous innovation to total inventory $(M_t + N_t)$ holding costs.

Finally, it remains to specify the inventory target stocks. Again following the literature, the output inventory target stock is

$$N_t^* = \alpha X_t \tag{11}$$

where $\alpha > 0$. The output inventory target depends on sales because the firm incurs costs due to lost sales when it stocks out of output inventories. For the input inventory target stock, the literature provides little guidance aside from Lovell's (1961) specification as a function of production, rather than sales. We adopt the more general input inventory target

$$M_t^* = \theta_Y Y_t + \theta_T T + \theta_V V_t + \theta_W W_t \tag{12}$$

where T is a linear time trend and $\theta_Y > 0$. The input inventory target depends mainly on production ($Y_t = X_t + \Delta N_t$) because stocking out of input inventories also entails costs associated with production disruptions—lost production, so to speak—that are distinct from the cost of lost sales. Lost production may be manifest in reduced productivity or even failure to realize production plans.

To summarize, the input and output inventory targets differ because the firm holds the two inventory stocks for different reasons. The firm stocks output inventories to guard against random demand fluctuations, but it stocks input inventories to guard against random fluctuations in productivity, materials prices and deliveries, and other aspects of production. Although sales and production are highly positively correlated, they differ enough at high frequencies for the difference in the target stock specification to be important.

The input inventory target also includes some variables that do not appear in the output inventory target. The time trend is included as a proxy for the introduction of technologies that affect the cost-minimizing level of stocks on hand. If firms adopt inventory-saving technologies or organizational processes, such as the so-called just-in-time production technique, then $\theta_T < 0$. Factor prices also influence the cost-minimizing level of materials inventories, given production and technology, because materials usage is a factor of production and there is likely to be substitutability among factors (especially in a representative agent model that encompasses heterogeneous industries). All else equal, higher material prices should reduce M_t^*, so $\theta_V < 0$. But the sign of θ_W is ambiguous because it depends on the relationship between labor and material usage. If labor and material usage are substitutes, a higher wage should increase M_t^*, so $\theta_W > 0$. But if labor and material usage are complements, then $\theta_W < 0$.[10]

3.3.3 Materials Costs

Input materials costs consist of purchase costs and adjustment costs. Specifically, materials costs are

$$MC_t = V_t D_t + \left(\frac{\kappa}{2}\right) (\Delta M_t)^2 . \tag{13}$$

Deliveries include materials used in the production process and materials inventory investment: $D_t = U_t + \Delta M_t$. The first term on the right side of equation (13) is the cost of ordering input materials at the "base" price each period. The second term is a quadratic approximation for adjustment costs on investment in materials inventories.

One rationale for the adjustment costs is that they are internal to the firm. They reflect the fact that, when the firm uses resources to bring about changes in the normal flow of materials rather than to produce output, it suffers the cost of forgone output. Examples include using labor to engage in the search cost of finding new suppliers of materials or the cost of finding buyers or otherwise getting rid of excess materials. Another rationale for the adjustment costs is that they are external to the firm. They reflect the idea that, due to monopoly power, the firm may face a rising supply price and thus an increasing marginal cost of acquiring materials more quickly.[11]

3.4 Cost Minimization

In standard output inventory theory, the firm maximizes profits given the *demand* curve it faces from consumers of its (final) product. Most applied work on output inventories avoids the difficulties of specifying the consumer's demand curve by assuming that demand is exogenous and recasting the problem in terms of cost minimization. Introducing input inventories adds another, equally complicated, dimension to the firm's profit maximization problem: the *supply* curve it faces from

[10] One final assumption has been made implicitly regarding both inventory types, namely that there is no depreciation of the inventory stock. This simplifying assumption may be unrealistic for certain industries and types of goods, such as food, but could easily be added to the model.

[11] The adjustment costs can also be motivated by aggregation. One rationale is that aggregation across heterogeneous firms induces persistence in aggregate inventories that requires an adjustment cost type of term in an aggregate model. This view is supported by the evidence in Schuh (1996) of aggregation bias in adjustment speeds of output inventory models. Another rationale pertains to aggregation across S,s policy rules. In this model, we ignore any fixed costs associated with the ordering of materials, which presumably would lead to S,s policies at the micro level. However, aggregation may smooth the micro S,s behavior into a relationship that looks like the partial adjustment mechanisms arising from quadratic costs of adjustment. Both of these issues raise difficult aggregation problems that we intend to explore in future work.

input materials producers. As a first step, we emulate the output inventory literature and assume that the firm not only takes final demand as exogenous but also the supply of materials (specifically, the materials price). Analogously, the wage is taken as exogenous, although the firm may influence the labor market as well.

Given demand (X_t) and factor prices (V_t and W_t) the firm's problem reduces to one of minimizing the discounted present value of total costs (TC),

$$E_0 \sum_{t=0}^{\infty} \beta^t (LC_t + HC_t^N + HC_t^M + MC_t) \ , \tag{14}$$

where $\beta = (1+r)^{-1}$ is the discount factor implied by the constant real rate of interest r. The two laws of motion governing inventory stocks,

$$\Delta N_t = Y_t - X_t \tag{15}$$

for output inventories and

$$\Delta M_t = D_t - U_t \tag{16}$$

for input inventories, can be used to substitute for production (Y_t) and deliveries (D_t). Thus, the firm chooses $\{U_t, M_t, N_t\}_{t=0}^{\infty}$ to minimize equation (14).

The model is derived as follows. Solve equation (15) for Y_t and equation (16) for D_t and then substitute these variables into equation (14). Take the derivatives of equation (14) with respect to U_t, M_t, and N_t to obtain the Euler equations. Define Δ^2 as the second-difference operator, i.e., $\Delta^2 X_t = \Delta X_t - \Delta X_{t-1}$.

Then the Euler equation for materials usage, U_t, is

$$\begin{aligned} &E_t \left\{ -\beta\varphi\gamma_7[\gamma_6(\Delta X_{t+1} + \Delta^2 N_{t+1}) + \gamma_7 \Delta U_{t+1}] \right. \\ &\left. +\gamma_2 U_t + \gamma_3(X_t + \Delta N_t) + \gamma_5 W_t + \varphi\gamma_7[\gamma_6(\Delta X_t + \Delta^2 N_t) + \gamma_7 \Delta U_t] + V_t + \gamma_9 \epsilon_{yt} \right\} = 0 \ . \end{aligned} \tag{17}$$

This optimality condition shows that the firm balances the marginal cost of ordering and using materials this period (second line) against the marginal cost of using materials next period (first line). Note that in period $t+1$ the only cost is the marginal cost of usage in production. All intertemporal costs associated with materials occur through the input inventory optimality condition.

The Euler equation for input inventories, M_t, is

$$\begin{aligned} &E_t \{ -\beta[V_{t+1} + \kappa\Delta M_{t+1}] \\ &+V_t + \kappa\Delta M_t + \tau(M_t - M_t^*) + \epsilon_{ht} + \tau_0 \} = 0 \ . \end{aligned} \tag{18}$$

This optimality condition shows that the firm balances the marginal cost of ordering and holding input inventories this period (second line) against the cost of ordering input inventories next period (first line).

Finally, the Euler equation for output inventories, N_t, is

$$\begin{aligned} & E_t\left\{\beta^2\varphi\gamma_6[\gamma_6(\Delta X_{t+2}+\Delta^2 N_{t+2})+\gamma_7\Delta U_{t+2}]\right. \\ & -\beta[\gamma_1(X_{t+1}+\Delta N_{t+1})+\gamma_3 U_{t+1}+\gamma_4 W_{t+1} \\ & +2\varphi\gamma_6(\gamma_6(\Delta X_{t+1}+\Delta^2 N_{t+1})+\gamma_7\Delta U_{t+1})+\gamma_8\epsilon_{y,t+1}-\tau\theta_Y(M_{t+1}-M^*_{t+1})] \\ & +\gamma_1(X_t+\Delta N_t)+\gamma_3 U_t+\gamma_4 W_t+\varphi\gamma_6(\gamma_6(\Delta X_t+\Delta^2 N_t)+\gamma_7\Delta U_t) \\ & \left.+\delta(N_t-N^*_t)-\tau\theta_Y(M_t-M^*_t)+\gamma_8\epsilon_{yt}+\epsilon_{ht}+\delta_0\right\}=0\;. \end{aligned} \tag{19}$$

This optimality condition shows that the firm balances the marginal cost of producing a good and storing it as output inventory this period (last two lines) against the cost of producing the good in the future (first three lines). The presence of adjustment costs on labor introduces an additional period over which costs are balanced.

Next, solve equation (17) for $E_t U_t$ and substitute it into equations (18) and (19) to eliminate the unobservable materials usage variable from the system. Define the lag operator as L, which works as a lead operator when inverted: e.g., $L^{-1}X_t = X_{t+1}$. Then, after substituting and collecting terms around common parameters, the Euler equation for input inventories, equation (18), becomes

$$E_t\left\{(1-\beta L^{-1})V_t+\kappa(1-\beta L^{-1})\Delta M_t+\tau(M_t-M^*_t)+\epsilon_{ht}+\tau_0\right\}=0 \tag{20}$$

and the Euler equation for output inventories, equation (19), becomes

$$\begin{aligned} & E_t\left\{(\gamma_1\gamma_2-\gamma_3^2)(1-\beta L^{-1})(X_t+\Delta N_t)\right. \\ & +\varphi(\gamma_2\gamma_6^2+\gamma_1\gamma_7^2-2\gamma_3\gamma_6\gamma_7)(1-\beta L^{-1})^2(\Delta X_t+\Delta^2 N_t) \\ & +(\gamma_4\gamma_2-\gamma_3\gamma_5)(1-\beta L^{-1})W_t+\varphi\gamma_7(\gamma_4\gamma_7-\gamma_5\gamma_6)(1-\beta L^{-1})^2\Delta W_t \\ & +\delta[\gamma_2(N_t-N^*_t)+\varphi\gamma_7^2(1-\beta L^{-1})(\Delta N_t-\Delta N^*_t)] \\ & -\tau\theta_Y[\gamma_2(1-\beta L^{-1})(M_t-M^*_t)+\varphi\gamma_7^2(1-\beta L^{-1})^2(\Delta M_t-\Delta M^*_t)] \\ & +(\gamma_2\gamma_8-\gamma_3\gamma_9)(1-\beta L^{-1})\epsilon_{yt}+\varphi\gamma_7(\gamma_8\gamma_7-\gamma_9\gamma_6)(1-\beta L^{-1})^2\Delta\epsilon_{yt} \\ & \left.+\gamma_2\epsilon_{ht}+\varphi\gamma_7^2(1-\beta L^{-1})\Delta\epsilon_{ht}\right\}=0\;. \end{aligned} \tag{21}$$

Terms involving $(1-\beta L^{-1})$ are called quasi-differences.

Observe that the interaction between input and output inventories operates somewhat differently in the two Euler equations. In equation (20), output inventories affect input inventories indirectly through the target stock, M_t^*. All else equal, an increase in output inventories raises production, Y_t, and M_t^*, thereby reducing the input inventory gap. In equation (21), input inventories affect output inventories directly, as an increase in input inventories raises the input inventory gap $M_t - M_t^*$ (and its first difference), all else equal. Note, however, that the same parameter product, $\tau\theta_Y$, governs the stock interaction in both equations, although other parameters also affect the output inventory equation.

3.5 Two Econometric Models

Euler equations (20) and (21) represent the most general form of the model that can be confronted with the available data. After substituting the target stocks M_t^* and N_t^* into the Euler equations, this general form contains eighteen parameters (excluding constants) to be estimated. Input inventory equation (20) is relatively parsimonious and contains: κ, the adjustment cost parameter on materials investment; τ, the holding cost parameter on input inventories; and the target stock parameters, θ_Y, θ_T, θ_W, and θ_V. Output inventory equation (21) includes the remaining parameters: γ_1 through γ_9, the labor cost parameters; φ, the labor adjustment cost parameter; δ, the holding cost parameter on output inventories; and α, the target stock parameter.

Although derivation of the general model is instructive, it is impractical for econometric work. Extensive parameterization induced by the generalized quadratic approximations and collinearity induced by the dynamics portend econometric difficulties. The number of parameters is large compared to variables in the system (M, N, V, W, and X), and estimation will rely heavily on multiple lags. Also, the output inventory equation is significantly more complex than the traditional model and it would be difficult to pinpoint whether differences between the general and traditional models were due to the generalizations or the SOF linkages. Consequently, we impose some parametric restrictions to achieve more parsimonious econometric models. Another advantage of these restrictions is the ability to distinguish between gross production and value added technologies.

3.5.1 Gross Production

One set of parameter restrictions pertains to a more parsimonious version of the gross production model. Specifically, let

$$\gamma_5 = \gamma_7 = \gamma_9 = 0 \qquad \gamma_6 = \gamma_8 = 1 \ .$$

Then the Euler equation for output inventories, equation (21), becomes

$$\begin{aligned} &E_t \left\{ \varphi(1-\beta L^{-1})^2(\Delta X_t + \Delta^2 N_t) + \overline{\gamma}(1-\beta L^{-1})(X_t + \Delta N_t) + \gamma_4(1-\beta L^{-1})W_t \right. \\ &\left. +\delta(N_t - N_t^*) - \tau\theta_Y(1-\beta L^{-1})(M_t - M_t^*) + (1-\beta L^{-1})\epsilon_{yt} + \epsilon_{ht} + \delta_0 \right\} = 0 \end{aligned} \tag{22}$$

where the entire equation has been divided through by γ_2 and $\overline{\gamma} = (\gamma_1 - \frac{\gamma_3^2}{\gamma_2})$. Regardless of the sign of γ_3, $\overline{\gamma}$ should be positive because $\gamma_1 > 0$ and $\gamma_2 < 0$ from the inverted production function. Here the parameters to be estimated are $\overline{\gamma}$, the parameter attached to the cost of producing output; φ, the adjustment cost parameter on labor and thus output; δ, the output inventory holding cost parameter; γ_4, the parameter capturing shifts in the real wage rate and thus marginal cost; and finally the product $\tau\theta_Y$, the parameter attached to the input inventory input gap. The input inventory gap allows an excess (or a deficiency) of input inventories to influence output inventory decisions.

Equation (22) is essentially the same as the standard output inventory equation except for the quasi-difference of the input inventory gap. The standard output inventory Euler equation is obtained by setting

$$\tau = 0 \qquad \text{or} \qquad \theta_Y = 0$$

which eliminates the input inventory gap term. The former assumption rules out input inventory stockout costs; the latter captures the interaction between input and output inventories. Such a restricted equation is equivalent to that fit by Eichenbaum (1984) and Durlauf and Maccini (1995). If one further restricts $\gamma_4 = 0$, then one obtains the Euler equations used by Blanchard (1983) and West (1986), among others.[12]

3.5.2 Value Added

The second set of parameter restrictions pertains to the value added production model. Specifically, let

$$\begin{aligned} &\gamma_1 = \gamma_2 = -\gamma_3 = \gamma \qquad &\gamma_4 = -\gamma_5 \\ &\gamma_6 = -\gamma_7 = \gamma_8 = 1 \qquad &\gamma_9 = 0 \end{aligned} .$$

This corresponds to the case where $Y_t - U_t$ becomes a factor in the inverted production function, rather than Y_t and U_t separately. Again, the Euler equation for input inventories remains the same,

[12] To elaborate, the standard output inventory model found in the inventory literature is a special case of the SOF model. The standard labor cost approximation can be obtained from equation (8) by letting $U_t = 0$ for all t. Then, equation (8) reduces to

$$LC_t = \left(\frac{\gamma_1}{2}\right) Y_t^2 + \gamma_4 W_t Y_t + \left(\frac{\varphi}{2}\right) \gamma_6 (\Delta Y_t)^2 + \gamma_8 \epsilon_{yt} Y_t$$

which is the cost function used in studies that allow input prices, such as the real wage, to impact production costs.

but the Euler equation for output inventories, equation (21), now becomes

$$\begin{aligned} & E_t \left\{ \delta[\gamma(N_t - N_t^*) + \varphi(1-\beta L^{-1})(\Delta N_t - \Delta N_t^*)] \right. \\ & -\tau\theta_Y[\gamma(1-\beta L^{-1})(M_t - M_t^*) + \varphi(1-\beta L^{-1})^2(\Delta M_t - \Delta M_t^*)] \\ & \left. +\gamma[(1-\beta L^{-1})\epsilon_{yt} + \epsilon_{ht}] + \varphi[(1-\beta L^{-1})^2 \Delta\epsilon_{yt} + (1-\beta L^{-1})\Delta\epsilon_{ht}] + \delta_0 \right\} = 0 \ . \end{aligned} \tag{23}$$

Here γ is the parameter attached to the cost of producing "value added", while φ, δ, and α are the adjustment cost, the output inventory holding cost, and target stock parameters. Further, $\tau\theta_Y$ again captures the interaction between input and output inventories, which in this model includes the input inventory gap and its first difference.

Equation (23) is markedly different from the standard output inventory models found in the literature, where there are no attempts to grapple with the distinction between gross production and value added. However, by introducing the ordering and usage of materials we are able to examine the impact of alternation production technologies. Unlike the gross production model, equation (23) includes only input and output inventory gaps and it includes first-differences of the technology and holding cost shocks. Note that (23) excludes γ_4, which does not appear as a result of the value added restrictions. By starting with the general model and imposing these restrictions, we can obtain an equation that permits testing of the value added assumption.

3.5.3 Complete SOF Models

Given our full-information approach to estimation, a complete SOF inventory model comprises five equations: an Euler equation for input inventories; an Euler equation for output inventories; and auxiliary equations for the exogenous variables V_t, W_t, and X_t. We consider two SOF models, gross production (GP) and value added (VA), which differ only in the output inventory equation (equation (22) for gross production and equation (23) for value added). The auxiliary equations for exogenous variables are specified as simple univariate autoregressive processes.

For comparison purposes, we also estimate single-equation SOF models for input and output inventories. Each single equation model is derived from the joint equation models by assuming all parameters and data associated with the other inventory stock are zero. This assumption implies that the single equation input inventory target, M_t^*, becomes a function of *sales* instead of production (i.e., $\Delta N_t = 0$). Consequently, there are two single equation models for output inventories (GP and VA) but only one single equation model for input inventories.

In principle, it would be informative to solve analytically for the decision rules of the complete SOF system. The decision rules would show the conceptual impact of the dynamic linkages imposed by the model on the behavior of production and inventory investment. Unfortunately, however, the SOF models are sixth-order difference equation systems in M and N. Such a system is very difficult—perhaps impossible—to solve analytically, though the numerical algorithm we use in estimation quickly and easily solves the model.

4 Econometric Specification and Estimation

4.1 Strategy

Following the tradition of Blanchard (1983) and Eichenbaum (1984), we estimate the SOF model using maximum likelihood on decision rules rather than generalized method of moments (GMM) on Euler equations. Three factors argue for maximum likelihood. First, instrumental variables estimators such as GMM tend to exhibit substantial biases and imprecision in small samples.[13] Second, Fuhrer, Moore, and Schuh (1995) demonstrates that maximum likelihood estimates of a benchmark linear-quadratic output inventory model are less biased and more significant than GMM estimates in small samples. Third, (unreported) attempts to estimate the SOF models with GMM produced the typical difficulties – parameter estimates are unstable, being highly sensitive to variations in normalization, instrument set, and other asymptotically irrelevant specifications [14].

Estimation of the new SOF model is the most comprehensive – and successful, as shown in the next section – to date. Key features are: (1) estimation of underlying structural parameters; (2) estimation of decision rules for both output and input inventories; and (3) joint estimation of input and output inventory equations, imposing all cross-equation restrictions. Furthermore, the joint estimation approach permits examination of the dynamic properties of the inventory system. Eichenbaum's (1984) joint estimation of labor and output inventories is the only other instance of joint estimation in the inventory literature. However, we extend Eichenbaum's work along two dimensions. First, we study input inventories in addition to output inventories and labor. Second, we attempt to link the parametric structure of the cost function approximations to the nature of the production function by considering gross production and value added.

Prior attempts to estimate SOF inventory models exhibit drawbacks relative to our approach. Ramey (1989), Mosser (1989), and Bils and Kahn (1996) estimate Euler equations with GMM. Many of their parameter estimates are imprecise or the wrong sign, and the overidentifying restrictions

[13] See West and Wilcox (1994) and Fuhrer, Moore, and Schuh (1995), and references therein. These studies suggest that the biases and imprecision can lead the econometrician to incorrectly reject the model or, perhaps worse, to draw incorrect conclusions about the signs of key structural parameters. Instrument irrelevance – a type of misspecification – appears to lead to poor small sample properties.

[14] See Humphreys (1995) for a discussion of problems associated with using GMM to estimate a similar inventory model.

usually are rejected. Husted and Kollintzas (1987) estimate a decision rule for input inventories with maximum likelihood, but they do not include output inventories. Maccini and Rossana (1984), Auerbach and Green (1980), Bivin (1989), (1993), Rossana (1990), Nguyen and Andrews (1988), and Reagan and Sheehan (1985) use various reduced-form estimation techniques. Although these studies find evidence of interaction between input and output inventories, they do not identify structural parameters.

4.2 Parametric Normalizations and Restrictions

In the recent applied inventory literature, which has been dominated by GMM estimation of Euler equations, parametric normalization is an important issue. The reason is that estimators such as GMM minimize an objective function formed from moments of the Euler equations derived from linear-quadratic approximations to the cost function. The Euler equations are homogeneous of degree zero in the structural parameters, and thus there is no "left-hand-side" variable determined by theory. Single equation GMM estimation of output inventory Euler equations requires at least one parametric restriction – for example, normalizing by one parameter. Thus only relative parameters are identified.

In contrast, maximum likelihood estimation of the decision rules identifies structural parameters from the restricted reduced-form parameters of the decision rules. These reduced-form parameters are functions of both the structural parameters of the SOF model *and* the parameters of the auxiliary models for the exogenous variables, as shown in section 7 of West (1993). For the SOF models, then, sufficiently long lags of the autoregressive models for the exogenous variables permit identification of all structural parameters. The extent to which the SOF models are overidentified depends on the difference between the number of reduced-form parameters and structural parameters. Using AR(1) models, we are able to estimate all equations with at least one overidentifying restriction, and as many as four, except for the single-equation input inventory model which is exactly identified.

For joint equation estimation of the gross production model, all structural parameters in equation (22) are estimated and identified. For all other estimation, it is necessary to further normalize by one parameter (we chose $\varphi = 1$; the choice is irrelevant for the maximum likelihood estimation). Although we estimated φ directly in the joint equation estimation of the gross production model, we report the parameters from the joint equation estimation relative to φ (where necessary) for comparability with parameters from the single equation estimation.

The discount factor, β, is preset at .995 rather than estimated. This assumption is common practice for structural estimation of this sort, and previous work indicates that estimates typically are not sensitive to sensible alternative values of β.

4.3 Model Specification

The SOF model can be written in matrix difference equation form as

$$E_t \left\{ \sum_{i=-lags}^{leads} H_i S_{t+i} - G\epsilon_t | \Omega_t \right\} = 0 \tag{24}$$

where

$$S_t = [M_t, N_t, V_t, W_t, X_t, 1, T]'$$

is the vector of system variables (1 refers to a constant); the H_i are conformable square matrices containing model parameters;

$$\epsilon_t = [\epsilon_{yt}, \epsilon_{ht}, \epsilon_{xt}, \epsilon_{vt}, \epsilon_{wt}, 0, 0]'$$

is the vector of structural disturbances; G is a conformable square matrix that may contain model parameters and/or the lag operator; and Ω_t is the information set available at time t.

The fundamental random shock ϵ_t is distributed iid N(0,Σ) and the rank of Σ equals the number of stochastic equations in the model (five). For the gross production (g) model, G is

$$G_g = \begin{bmatrix} \Psi_{g1} & \Psi_{g2} & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

where

$$\begin{aligned} \Psi_{g1} &= (1 - \beta L^{-1}) \\ \Psi_{g2} &= 1\ . \end{aligned}$$

For the value added (v) model, G is

$$G_v = \begin{bmatrix} \Psi_{v1} & \Psi_{v2} & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

where

$$\Psi_{v1} = \gamma(1-\beta L^{-1}) + \varphi(1-L)(1-\beta L^{-1})^2$$
$$\Psi_{v2} = \gamma + \varphi(1-L)(1-\beta L^{-1}) \ .$$

Although the G matrices are relatively simple for this model, they embody three novel aspects of the error structure: (1) the holding cost shock appears in multiple equations; (2) structural parameters appear in G_v; and (3) G_g and G_v are dynamic and forward-looking.

4.4 Information, Expectations, and Innovations

The firm forms expectations rationally using all available information, Ω_t, and the full model, equation (24). Further, the firm's information set includes all variables dated period t and earlier; in particular, $X_t \in \Omega_t$.[15] Under these assumptions, the model can be rewritten as:

$$\sum_{i=-lags}^{0} \overline{H}_i S_{t+i} + \sum_{i=1}^{leads} \overline{H}_i E_t \{S_{t+i}|\Omega_t\} = \epsilon_t \ . \tag{25}$$

where $\overline{H}_i = (\overline{G})^{-1} H_i$ and $\overline{G}$ is defined by: $E_t\{G\epsilon_t\} = \overline{G}\epsilon_t$.[16] Because ϵ_t is white noise and not serially correlated, $E_t\{\epsilon_t|\Omega_t\} = \epsilon_t$ and $E_t\{\epsilon_{t+i}|\Omega_t\} = 0$ for all $i > 0$. Therefore, $\overline{\Psi}_{g1} = \overline{\Psi}_{g2} = 1$ and $\overline{\Psi}_{v1} = \overline{\Psi}_{v2} = \gamma + \varphi(1-L)$. Unfortunately, the model solution and estimation procedures employed in this study (described next) cannot handle G matrices with nonzero off-diagonal elements or

[15] The information set assumption is standard for most maximum likelihood applications and consistent with the Blanchard and Fuhrer *et al.* studies. However, it is inconsistent with the traditional buffer stock assumption in the output inventory literature that current sales are unknown when the firm chooses production plans (i.e., $X_t \ni \Omega_t$). If current-period variables are unknown, equation (25) also would include an expectational error term. See Blanchard (1983), pp. 382-385, for details.

[16] Although not indicated explicitly, premultiplication of H_i by $(\overline{G})^{-1}$ is restricted to the model's stochastic equations so as not to introduce innovations into deterministic equations (the constant and time trend).

diagonal elements other than zero or one. Thus we are forced to assume that $\overline{\Psi}_{g1} = \overline{\Psi}_{v1} = 1$ and $\overline{\Psi}_{g2} = \overline{\Psi}_{v2} = 0$ in the estimation reported in section 5. These assumptions prohibit identification of the technology and inventory holding cost shocks, ϵ_{yt} and ϵ_{ht}. Instead, we define ϵ_{mt} and ϵ_{nt} as innovations for the Euler equations.

4.5 Solution and Estimation

The SOF model is a system of linear, rational expectations equations which can be solved using the numerical procedure developed by Anderson and Moore (1985).[17] In contrast to GMM, which substitutes actual data for (unobserved) expectations of future variables, the Anderson-Moore procedure relies on model-consistent expectations. It does so by converting equation (25) into first-order companion form then performing an eigensystem calculation to generate stable, model-consistent expectations. Using these expectations, the procedure solves the model and derives the observable structure, which is a set of decision rules containing only current and lagged variables:

$$\sum_{i=-lags}^{0} A_i S_{t+i} = \epsilon_t \tag{26}$$

where the A_i matrices embody all of the parametric restrictions implied by the structural model. Multiplying equation (26) by A_0^{-1} produces the restricted reduced-form system.

The observable structure is used to form the concentrated log likelihood function

$$\mathcal{L} = T(\log|\mathcal{J}| - 0.5\log|\hat{\Sigma}|) \tag{27}$$

where

$$\hat{\Sigma} = T^{-1}\sum_{t=1}^{T} \hat{\eta}_t \hat{\eta}_t' \tag{28}$$

is the estimated covariance matrix, $\eta_t = [\epsilon_{mt}, \epsilon_{nt}, \epsilon_{xt}, \epsilon_{vt}, \epsilon_{wt}]'$ is the subset of ϵ_t pertaining to the structural disturbances of the stochastic equations, and $\mathcal{J}$ is the Jacobian linking $\hat{\eta}_t$ to the portion of the S_t data pertaining to the stochastic equations. The likelihood function is maximized with a sequential quadratic programming algorithm using numerical derivatives. The Hessian of the log-likelihood function is computed with the optimization routine's BFGS update formula. The covariance matrix is obtained by numerically evaluating the Hessian at the final parameter estimates.[18] Standard errors are the roots of the diagonal elements of the inverted Hessian.

[17] The Anderson-Moore procedure is a generalization of Blanchard and Kahn (1980), and is similar to the methods of Sims (1996) and King and Watson (1995). Solution and estimation of the SOF model follows the technique described in Fuhrer *et al* (1995), to which the reader is referred for more complete details.

[18] The Hessian is evaluated with differencing intervals equal to 1 percent of the estimated parameter values. In

Following Blanchard (1983), we use a two-step approximation to full-information maximum likelihood estimation. In the first step, parameters of the AR(1) auxiliary models are estimated with OLS. In the second step, the structural parameters are estimated with maximum likelihood conditional on the OLS estimates of the AR(1) models. The two-step estimator is asymptotically equivalent to full-information estimation but less efficient; the standard errors are not corrected for the first stage uncertainty. More importantly, the two-step estimator is considerably faster—a major consideration given the increased complexity of the SOF model over the standard output inventory model. Relative to a standard single-equation output inventory equation, the joint SOF model takes much longer to estimate (a few minutes versus an hour—or much more—on a Sparc 10 workstation).

5 Econometric Results

This section reports the econometric results for the SOF inventory models. It contains results for the following variations of the SOF model: (1) gross production (GP) and value added (VA) assumptions; (2) single equation estimation and jointly estimated equations; and (3) nondurable goods and durable goods industry data.[19] All regressions cover the period 1959:1 through 1994:5, less appropriate lags. The appendix spells out the data sources and details.

5.1 Auxiliary Equation Estimates

Table 4 contains regression results for the first-step OLS estimates of the auxiliary equations for the exogenous variables. Following the bulk of the applied inventory literature, each variable is assumed to be a trend-stationary autoregressive process. The results shown for AR(1) models generally support this assumption. All lagged coefficients are significant and less than one. As expected, the trend is significant for sales and real wages, except for the nondurables real wage

some instances, notably smaller differencing intervals increase standard error estimates enough to cause a small number of parameter estimates to become insignificant. The reason is that although the likelihood surface of the SOF model has a globally well-defined maximum, it becomes flat very near the maximum due to the extensive parameterization induced by the quadratic approximations. Differencing intervals of 1 percent represent parameter changes of economically meaningful magnitude in the SOF model.

[19] For several reasons, we deviate from tradition in the output inventory literature of estimating the model with data for the six production-to-stock 2-digit SIC industries. First, joint decision-rule estimation is much more intricate and time consuming than previous estimation methods. Second, including the remainder of the manufacturing sector increases the number of industries to twenty, many of which have interesting idiosyncrasies worth studying in more detail. Third, this simple version of the model omits features that ultimately will be required for a more complete understanding of inventory fluctuations. Our strategy is to briefly investigate whether the simple model is roughly consistent with all manufacturing data and leave more detailed industry-based investigations for future work.

(t-statistic of 1.3). However, the materials price does not exhibit a significant trend.[20] The main shortcoming is serial correlation in the errors, which largely can be eliminated with additional lags. However, we found the parameter estimates of the SOF models generally to be insensitive to lag length, which is consistent with the findings of Fuhrer *et al.* (1995). [21] The estimates in Table 4 are imposed on the maximum likelihood estimation of the SOF models.

Table 4

Auxiliary Equation Estimates for Exogenous Variables

	Nondurables			Durables		
	X	V	W	X	V	W
X_{t-1}	.943**			.964**		
	(.016)			(.014)		
V_{t-1}		.971**			.977**	
		(.012)			(.010)	
W_{t-1}			.989**			.978**
			(.007)			(.011)
T (×1000)	8.79**	−.012	.071	6.46**	.003	.181**
	(2.53)	(.009)	(.056)	(2.51)	(.007)	(.091)
R^2	.998	.955	.994	.992	.957	.993
S.E.E.	.875	.020	.077	1.95	.018	.090
$Q(36)$	156.	50.7	207.	139.	93.8	137.
	(.00)	(.05)	(.00)	(.00)	(.00)	(.00)

NOTES: The models are estimated over the period 1959:1 through 1994:5, less appropriate lags. Asymptotic standard errors and p-values are in parentheses. Q(p) is the Box-Pierce statistic for serial correlation with p lags. A * indicates significance at the 10 percent level, and ** indicates significance at the 5 percent level. See the text for more details.

[20] Data for all exogenous variables exhibit evidence of a split trend with a break around 1973, as argued by Perron (1989). However, two problems arise from a split trend specification. First, the timing of the split is much less well-defined in these monthly data than in Perron's annual data. Second, the split trend introduces a nonlinearity that cannot be handled by our model solution methodology.

[21] We also experimented with more general auxiliary equations for exogenous variables, including vector autoregressive (VAR) and vector error correction (VEC) models. These vector auxiliary models do not change the SOF model results dramatically, but some of the SOF parameter estimates and standard errors vary a bit across auxiliary models. The data clearly exhibit evidence of interaction among the exogenous variables, as well as among exogenous variables and the inventory stocks. However, our sense is that unrestricted reduced-form specifications such as VAR and VEC models are not likely to yield satisfactory and interpretable results. We leave the task of imposing more structure and endogenizing V_t, W_t, and X_t for future work.

5.2 Parameter Estimates

Tables 5a and 5b report the results from the maximum likelihood estimation of the SOF models. The parameters are from equations (20), (22), and (23) and the parameter sign predicted by the theory is included in the tables. The tables also report the product, $\tau\theta_Y$, which captures the interaction between input and output inventories in equation (22). The slope of the marginal cost of production in the tables is obtained from the second derivative of the total cost function, which is

$$\partial^2 TC/\partial Y^2 = \overline{\gamma} + (1+\beta)\varphi$$

for the gross production model; the formula for value added is the same except that γ replaces $\overline{\gamma}$. This statistic is also reported relative to the parameter φ in the tables. The term $2(\mathcal{L} - \mathcal{L}^{\mathcal{R}})$ is the χ^2 statistic from the likelihood ratio test of the model's overidentifying restrictions (p-values in parentheses), where $\mathcal{R}$ indicates the likelihood obtained from restrictions imposed by the SOF model. The $Q(12)$ number is a χ^2 test statistic for general serial correlation at 12 lags (10 percent critical value of 18.6).

5.2.1 Single Equations

Column one shows the estimates from a conventional single-equation output inventory model with gross production. In general, the parameter estimates are quite consistent with the model, with all but one parameter positive and significant. Furthermore, the estimates indicate that the marginal cost of production slopes upward. These results are consistent with ML estimates of decision rules for finished goods inventories reported in the literature.

Two shortcomings are apparent in the estimates, however. First, the point estimates of the wage cost parameter, γ_4, are negative – significantly so in nondurables-which is inconsistent with the model. A second shortcoming is the overall rejection of the model apparent from the tests of overidentifying restrictions and serial correlation. Although discouraging, these test results are nevertheless comparable to similar results reported in the literature for estimation of structural models like the SOF model.

Column two shows the estimates from the single-equation output inventory model under the value added restriction. A first look at this novel form of the output inventory model shows that the literature seems to have overlooked a viable alternative to the gross production specification. The parameter estimates are all correctly signed and significantly estimated, and hence quite supportive of the model as well. Test statistics for overidentifying restrictions and serial correlation are better relative to the gross production model, but still weigh against the model.

Table 5a – SOF Model Estimates for Nondurables

		Single Equations			Joint Equations	
	Predicted	N		M	M & N	M & N
Parameter	Sign	GP	VA	GP & VA	GP	VA
α	+	.68**	.64**		.66**	.65**
		(.02)	(.04)		(.01)	(.03)
θ_Y	+			3.59**	3.75**	3.79**
				(1.03)	(.09)	(.07)
θ_T	−			−.35**	−.38**	−.39**
				(.16)	(.02)	(.02)
θ_V	−			81.0**	65.3**	66.0**
				(22.9)	(10.5)	(8.3)
θ_W	?			−1.34	−1.47	−1.29
				(2.03)	(.98)	(.85)
$\overline{\gamma}/\varphi$	+	.120**			.060	
		(.043)			(.037)	
γ/φ	+		.0014*			.0026*
			(.0008)			(.0014)
γ_4/φ	+	−2.12**			−1.97**	
		(.71)			(.69)	
δ/φ	+	.015**			.015**	.084
		(.004)			(.004)	(.060)
τ	+			.0024	.230**	.122**
				(.0020)	(.005)	(.002)
κ	+			1.33	133.3**	70.4**
				(1.23)	(30.6)	(15.6)
$\tau\theta_Y$	+			.0086	.864**	.462**
				(.0075)	(.058)	(.006)
$\partial^2 TC/\partial Y^2$	+	2.115**	1.996**		2.055**	1.998**
		(.043)	(.001)		(.037)	(.001)
$2(\mathcal{L}-\mathcal{L}^{\mathcal{R}})$		78.3	18.1	0	110.0	42.5
		(.00)	(.00)	(na)	(.00)	(.00)
$Q(12)\ \epsilon_M$				19.6	19.5	19.4
$Q(12)\ \epsilon_N$		53.9	47.5		53.6	31.7

NOTES: The models are estimated over the period 1959:1 through 1994:5, less appropriate lags. GP stands for gross production, and VA stands for value added. Asymptotic standard errors are in parentheses. Q(p) is the Box-Pierce statistic for serial correlation with p lags. A * indicates significance at the 10 percent level, and ** indicates significance at the 5 percent level. See the text for more details.

Table 5b – SOF Model Estimates for Durables

		Single Equations			Joint Equations	
	Predicted	N		M	M & N	M & N
Parameter	Sign	GP	VA	GP & VA	GP	VA
α	+	.74**	.66**		.74**	.66**
		(.01)	(.02)		(.01)	(.02)
θ_Y	+			24.1**	23.2**	24.1**
				(11.9)	(2.2)	(1.5)
θ_T	−			−2.59	−2.43**	−2.58**
				(1.60)	(.40)	(.10)
θ_V	−			436.3**	429.4**	429.7**
				(179.7)	(87.8)	(155.5)
θ_W	?			−165.6**	−165.7**	−167.6**
				(76.5)	(9.9)	(30.3)
$\overline{\gamma}/\varphi$	+	.434**			.421**	
		(.106)			(.115)	
γ/φ	+		.0024**			.0033**
			(.0005)			(.0005)
γ_4/φ	+	−.48			−.22	
		(.71)			(.66)	
δ/φ	+	.024**			.025**	2.69
		(.006)			(.007)	(17.6)
τ	+			.016	.0014**	.033
				(.042)	(.0004)	(.031)
κ	+			59.1	4.69**	114.0
				(156.9)	(.66)	(110.6)
$\tau\theta_Y$	+			.392	.033**	.786
				(1.03)	(.011)	(.744)
$\partial^2 TC/\partial Y^2$	+	2.429**	1.997**		2.416**	1.998**
		(.106)	(.001)		(.115)	(.001)
$2(\mathcal{L}-\mathcal{L}^{\mathcal{R}})$		202.0	135.2	0	97.5	20.0
		(.00)	(.00)	(na)	(.00)	(.00)
$Q(12)\ \epsilon_M$				152.0	149.9	149.6
$Q(12)\ \epsilon_N$		56.6	45.2		57.0	42.8

NOTES: The models are estimated over the period 1959:1 through 1994:5, less appropriate lags. GP stands for gross production, and VA stands for value added. Asymptotic standard errors are in parentheses. Q(p) is the Box-Pierce statistic for serial correlation with p lags. A * indicates significance at the 10 percent level, and ** indicates significance at the 5 percent level. See the text for more details.

Single-equation estimation of the new input inventory equation, shown in column three, yields decidedly mixed results. In support of the model, the target stock parameter estimates are mostly of the correct sign and significantly estimated.[22] The key parameter, θ_Y, is positive and significant, although here sales replaces production because output inventory investment is assumed to be zero. In addition, θ_T is negative in both industries (nearly significant in durables), suggesting the presence of inventory-saving technological change. Negative estimates of θ_W indicate complementarity between labor and materials, but the coefficient is only significant in durables.

Less encouraging results are that the non-target stock parameters are not significantly estimated and that estimates of θ_V have the wrong sign. The point estimates of the input inventory holding cost parameter, τ, and adjustment cost parameter, κ, are correctly signed but all are estimated insignificantly. Further, the estimate of θ_V is positive and significant, contrary to the predictions of the model. Finally, the single-equation input inventory equation exhibits serial correlation, though it is nearly insignificant in nondurables.

5.2.2 Joint Equations

Columns four and five report the joint estimation of the input and output inventory equations. Before turning to specific results, two general points are worth noting about the joint estimates relative to the single-equation estimates. First, joint estimates of some key parameters differ considerably in magnitude. This result suggests that the single-equation estimates may suffer from misspecification bias because they fail to account for stock interaction. Second, most joint estimates are estimated considerably more precisely. This result indicates significant efficiency gains from accounting for stock interaction. Together, these points provide evidence of important interaction between input and output inventories, and highlight the benefits of joint estimation.

Joint estimates of the target stock parameters are virtually identical to the single-equation estimates but even more significant. The output inventory target parameter, α, is essentially unchanged in magnitude or precision, but all of the input inventory target parameters (the θ's) are more precisely estimated. Note especially that the magnitude of θ_Y is about the same but is now much more precisely estimated. This finding is particularly important because in the joint model the input inventory target depends on production, not just sales, and the data indicate that production is the more appropriate variable. Thus, output inventories influence input inventory

[22] Estimation with $\theta_V = \theta_W = \theta_T = 0$ does not converge in single equation or joint equations estimation because the adjustment cost parameter κ diverges from zero. The reason is that the restricted input inventory gap $(M_t - \theta_Y Y_t)$ is extremely persistent, exhibits a negative trend, and rarely crosses zero. The only way for the SOF model to rationalize this behavior is for adjustment costs on M_t to become infinitely large. Thus, empirically, it appears that M_t^* must include variables other than Y_t.

decisions by raising output and the input inventory target stock. Similarly, the negative estimate of θ_T in durables becomes significant in the joint estimation as well. Estimates of θ_V decline a bit in the joint estimation for both industries, but nowhere near enough to become negative as predicted by the model. Finally, observe that there is little difference between target stock estimates for the gross production and value added models.

Joint estimates of the remaining parameters, however, are generally different from the single-equation estimates, especially the key parameters τ and κ. These parameters, which were insignificant in the single equation estimation, are significant in three out of the four cases in the joint estimation. Furthermore, the magnitudes of the joint estimates are notably different from those of the single equation estimates, unlike other parameters of the model.

A particularly important result is that the product, $\tau\theta_Y$, which captures the interaction between input and output inventories in the output inventory equation, is now highly significant in all but one case. This is strong evidence that input inventories influence output inventory decisions. Further, the labor cost parameters, $\overline{\gamma}$ in the gross production model and γ in the value added model, are generally significant (though $\overline{\gamma}$ is not quite so in nondurables), and the slope of marginal cost is clearly positive and very significant in all cases.

The impact of joint estimation on the models' overidentifying restrictions and residual serial correlation is, however, modest. The jointly estimated equations result in very small improvement, if any, in tests of overidentifying restrictions and serial correlation.

5.2.3 Comparisons and Implications

Looking beyond specific parameters, the econometric results generate a number of broad implications:

1. **Stage-of-fabrication interactions** – Input and output inventories are intertwined through SOF production linkages. The data provide strong evidence that output inventories influence input inventory holdings through the target stock for input inventories. At the same time, input inventory gaps influence output inventory decisions. In other words, firms do *not* choose each stock in isolation, but jointly consider both stocks – as well as other factors of production – when making production plans. Excluding the dynamic interactions associated with the delivery and usage of material inputs appears to be inconsistent with the data and limits our understanding of total inventory movements.

2. **Convexity** – Aggregate cost functions are convex. Our results reconfirm the important finding that the marginal cost of production slopes *upward*, not downward. This issue has

been debated in the inventory literature at least since Ramey's (1991) claim to the contrary, but is also of interest to macroeconomics in general. Our results extend the evidence against aggregate nonconvexity in two ways. First, we find aggregate convex costs even in durable goods industries, where nonconvexities are most often surmised to arise, at least at the micro level.[23] In fact, the key parameters ($\overline{\gamma}/\varphi$ and γ/φ) are actually more significantly positive in durables than nondurables. Second, the results indicate convex costs even in the presence of input inventories. If material costs are linear, or there are fixed ordering costs, input inventories would follow nonconvex (S,s) rules. Presumably, this nonconvexity could spill over into production behavior through SOF linkages.

3. **Production function** – The specification of materials usage does not affect the model's ability to fit the data but it does affect the nature of the model's dynamics. Gross production and value added specifications of the model are both reasonably consistent with the data, though statistical tests indicate the value added model fits modestly better. However, the dynamics implied by the two specifications are notably more distinct. The value added restrictions significantly simplify the model and eliminate some channels of propagation that are present in the gross production model. These differences are particularly notable in the models' ability to generate buffer stock behavior in response to demand shocks. This result suggests that a more detailed treatment of the production environment, as in Basu (1996) and Basu and Kimball (1997), may yield even more different implications from the materials specification. In general, though, broad inferences from the model about SOF interactions, convexity, etc., are insensitive to the materials specification.

4. **Industrial heterogeneity** – Nondurable and durable goods industries exhibit some interesting differences. First, the input inventory target stock parameter θ_Y is considerably larger than the output inventory target stock parameter α in both industries, but the disparity is much larger in durables. Second, input inventory stockout costs, measured by τ, are larger in nondurables, but output inventory stockout costs, measured by (unreported) δ from the joint gross production model, are larger in durables. Third, input inventory adjustment costs, measured by κ, are larger in nondurables, but labor adjustment costs, measured by (unreported) φ from the joint estimates of the gross production model, are larger in durables.

5. **Inventory-saving technology** – Manufacturing firms tend to conserve on input inventories over time. The data show that, conditional on other factors, the input inventory target

[23]See, for example, Bresnahan and Ramey (1994) who report evidence of nonconvexities in auto production plants resulting from fixed costs.

stock has a negative trend in both nondurable and durable goods industries. The trend is more pronounced in durable goods industries, presumably because the technological advances designed to reduce inventory holdings are concentrated in these industries. There does not appear to be such a trend in output inventories.

6. **Prices** – The SOF models' predictions regarding real wages and materials prices generally are not confirmed. Regarding wages, perhaps the incompleteness of the quadratic labor cost approximation or simplifications of the model eliminated important wage terms and introduced a misspecification bias. Or perhaps the real wage (i.e., productivity) needs to be adjusted for variation in capacity utilization, as argued by Bils and Kahn (1996). Regarding materials prices, the results suggest that a more complex role for speculative forces may be at work.

5.3 Dynamic Properties

This section explores the dynamic properties of the SOF models and their unrestricted reduced form by examining impulse response functions, which appear in Figure 3.[24] The gross production and value added models are compared with each other and with the reduced form of the gross production model (the more general of the two SOF models). The analysis focuses on the observable endogenous variables, input and output inventories (M and N). Recall that the solution and estimation procedures prohibit identification of the fundamental technology and holding cost shocks, ϵ_{yt} and ϵ_{ht}, so we identify ϵ_{nt} and ϵ_{mt} instead. These latter shocks are less interpretable and thus not discussed in any detail.

A broad conclusion emerging from Figure 3 is that the behavioral restrictions imposed by the SOF models affect output inventory dynamics much more than input inventory dynamics. The input inventory responses (rows two and four) of the SOF and reduced-form models are virtually identical in eight out of ten cases. However, the output inventory responses vary widely across models and industries in terms of their shape, persistence and even sign.

Perhaps the most striking behavior is the dynamic response of inventories to a positive sales

[24] The responses are calculated under the assumption that the innovations (ϵ_t) are fundamental to the SOF model and therefore do not require orthogonalization. This assumption is sensible from the point of view of the SOF model, where two of the innovations come from Euler equations and the remainder rest on the assumption of exogeneity.For the unrestricted reduced form model, one might argue for orthogonalized innovations because the model is a "near" VAR. We elected not to do so for two main reasons. First, given the extensive number of zero restrictions, the unrestricted reduced form is more "near" the structural model than it is "near" a VAR in M, N, V, W, and X. Second, impulse responses are notoriously sensitive to aspects of orthogonalization (e.g., variable ordering) and such additional assumptions reduce our ability to isolate the implications of the SOF theory.

shock (ϵ_x), shown in the last column of the figure. In the gross production model, output inventories drop sharply before returning to equilibrium after one year – about the length of an average recession. This response is the kind of "buffer stock" behavior long hypothesized for output inventories but rarely, if ever, documented in the data. Neither the value added nor the unrestricted reduced form model exhibits this buffer stock behavior. In contrast, the response of input inventories to a sales shock is quite similar across models. Input inventories build up slowly in both industries, peaking after about two years.

Real materials price shocks (ϵ_v) evoke asymmetric responses across inventory types. Output inventories tend to decline following the shock, but the response is very weak, particularly in the SOF models. Given firms' ability to alter deliveries of materials and (potentially) the mix of materials and labor in production, it seems reasonable for output inventories to be largely unaffected by materials price shocks. However, input inventories *increase* notably in response to the price shock, rather than decreasing. This feature of the model is puzzling and remains to be explained.

Real wage shocks, ϵ_w, also tend to evoke asymmetric responses across inventory types. Output inventories are flat in the value added model, which has no substantive channel for wage dynamics, and increase only mildly in the gross production model. In contrast, input inventories decline in response to a wage shock, especially in durable goods industries where the decline is large and protracted. If the wage shock signaled a rise in underlying labor productivity, the firm would draw down input inventories by increasing production (materials usage) and build up output inventories. To get the observed mild response of output inventories, though, demand also would have to rise almost in tandem with production. On the other hand, if the wage shock signaled a rise in the cost of labor, then presumably the firm would draw down input inventories by substituting materials for labor in production – if such substitutions were technologically feasible. In this case, production and output inventories would not respond to the shock.

6 Summary

This paper takes a step toward redressing the inventory literature's general neglect of input (materials and work-in-process) inventories. The paper presents stylized facts about input inventories that show they play a more important role empirically than do output inventories. It also offers a viable new stage-of-fabrication (SOF) model that extends the venerable linear-quadratic inventory model for output inventories. Finally, the paper provides econometric evidence that the SOF model does reasonably well at matching the data, although there is obvious room for improvement, as with all econometric models. This evidence is particularly striking in light of the tight restrictions imposed by the joint estimation of input and output inventory decision rules.

Several broad conclusions emerge. First and foremost, it seems exceedingly clear that material inputs play an important role in understanding producer behavior both theoretically and empirically. Producers' decisions of how much materials to order and how much materials to use in production affect – and are affected by – all aspects of production through dynamic SOF linkages. Failure to impose these linkages appears to be inconsistent with the data. Less clear, at least empirically, is the extent to which the specification of materials usage in the production function matters. This conclusion is attributable to the limitations of the approximations underlying the model. The data confirm evidence of industrial differences, but the simplifications of the model do not permit a full and precise accounting of them yet.

By all means, the model in this paper should be viewed as a first step toward a more general SOF theory. Several simplifications need to be relaxed. First, order backlogging (unfilled orders) needs to be introduced. Second, input inventories need to be disaggregated into materials and work-in-process, and the production process must be generalized to yield production of intermediate goods. Third, it would be desirable to incorporate general equilibrium linkages by explicitly modeling both sides of the upstream (materials) and downstream (finished goods) markets. Finally, further complexity of the model will put increasing stress on the heavily parameterized linear-quadratic framework and may necessitate taking a more direct approach to specifying production and cost functions.

A Data Appendix

The real inventory and shipments (sales) data are from the Census Bureau's Manufacturers' Shipments, Inventories, and Orders (M3) survey. The M3 data are seasonally adjusted and deflated in constant \$1987 by the Bureau of Economic Analysis (BEA), as described by Hinrichs and Eckman (1981). Also, we marked up the inventory data from cost basis to market basis using the procedure outlined by West (1983). An implicit price index for shipments (final goods) is obtained from the ratio of real shipments to nominal shipments.

The nominal wage data are average hourly earnings of production or nonsupervisory workers from the Bureau of Labor Statistics' (BLS) establishment survey. The wage data are seasonally adjusted. Real wages are obtained by deflating with the shipments implicit price index.

For raw materials prices, we had to construct new indexes for disaggregated industries because the BLS's Producer Price Program contains only aggregate manufacturing materials price indexes. Our materials price indexes are constructed from highly detailed commodity Producer Price Indexes (PPI) aggregated to the 2-digit SIC industry level using the information on the manufacturing industrial input-output structure from the *1982 Benchmark Input-Output Tables of the United States*. These disaggregated materials price indexes are available upon request.

To elaborate, the indexes are constructed as follows. Let i index 2-digit industries and j index raw material inputs (typically defined as groupings of 3- and 4-digit SIC industries) for the 2-digit industries.[25] Using the input-output tables, construct the dollar-value shares of material inputs into industry i, denoted M_{ij}, as

$$s_{ij} = \frac{P_j M_{ij}}{\sum_{j=1}^{J_i} P_j M_{ij}}$$

where P_j denotes the detailed input materials price. Then the industry materials price (P_i) is the weighted sum of materials prices:

$$P_i = \sum_{j=1}^{J_i} s_{ij} P_j \ .$$

In practice, a difficulty arises in calculating P_i because the input-output analysis (s_{ij}) is based on SIC *industries* but the PPI program (P_j) is based on *commodity* groupings. However, these classification systems correspond at low levels of aggregation, as described in the BLS "PPI-SIC Translator."[26] We imposed the following criteria for matching PPI and SIC commodities:

1. The SIC code commodity must match a PPI code commodity at the 4- or 5-digit SIC level or above.

2. The corresponding PPI code commodity must appear as one of the "Crude Materials for Further Processing" or "Intermediate Materials, Supplies and Components" price indexes.

3. The PPI data series must be available from 1959 through 1995.

Table A.1 lists the detailed PPIs, and their weights, which were used to construct the materials price indexes.

Finally, the product durability level materials price indexes are weighted aggregates of the 2-digit industry level indexes. Let k index product durability, i.e., nondurable (n) and durable (d) goods industries. Using the materials purchased data from the Census Bureau's *1994 Annual Survey of Manufactures*, construct the dollar-value share of materials purchased as

$$s_i = \frac{P_i M_i}{\sum_{i \in k} P_i M_i} \ .$$

[25] Materials inputs excludes: (1) output from industry i that is an input into industry i, to remove intra-industry prices; and (2) commodities comprising less than 5% of total inputs to a given industry.

[26] SIC-based prices became part of the Producer Price Program in the mid 1980s, but these data series do not go back far enough in time to use with the inventory and shipments data (available from 1959).

Table A.1 – Materials Price Indexes and Weights

Nondurable Goods			Durable Goods		
SIC Industry	PPI Component	Weight	SIC Industry	PPI Component	Weight
20 Food	Livestock	0.50	25 Furniture	Mill Work	0.71
	Fluid Milk	0.19		Dressed Wood, other	0.13
	Grains	0.17		Dressed Douglas Fir	0.09
	Live Poultry	0.07		Dressed Southern Pine	0.07
	Fruits & Vegetables	0.03	32 Stone/Clay/Glass	Concrete Ingredients	0.58
	Raw Cane Sugar	0.01		Gypsum Products	0.23
	Eggs	0.01		Ceramic Products	0.19
	Unprocessed Fin Fish	0.01	33 Primary Metals	Iron and Steel Scrap	0.51
21 Tobacco	Leaf Tobacco	1.00		Nonferrous Scrap	0.41
22 Textiles	Raw Cotton	0.97		Iron Ore	0.08
	Wool	0.03	34 Fabricated Metals	Iron and Steel	0.83
23 Apparel	Raw Cotton	0.53		Primary Nonferrous Metals	0.17
	Cattle Hides	0.44	35 Nonelect. Machinery	Misc. Metal Products	0.86
	Wool	0.03		Secondary Nonferrous Metals	0.11
26 Paper	Wastepaper	0.68		Blast/Electric Furnace Prod.	0.03
	Woodpulp	0.32	36 Electrical Machinery	Iron and Steel	0.52
27 Printing	Pulp and Paper Prod.	0.85		Nonferrous Metals	0.48
	Industrial Chemicals	0.15	37 Transportation	Automotive Components	0.29
28 Chemicals	Fertilizer Materials	0.81		Iron and Steel Foundry Prod.	0.27
	Paint Materials	0.19		Blast Furnace Products	0.14
29 Petroleum	Crude Petroleum	0.71		Engines	0.13
	Bituminous Coal	0.29		Primary Nonferrous Metals	0.09
30 Rubber	Crude Rubber	1.00		Tires	0.08
31 Leather	Beef and Veal	0.78	38 Instruments	Misc. Electronic Prod.	0.35
	Hides and Skin	0.22		Primary Copper	0.35
				Instr. to Measure Electricity	0.15
				Other Electrical Components	0.15

NOTES: Industries 24 (Lumber) and 39 (Miscellaneous) are excluded because no PPI commodities met the required criteria.

Then the durability level materials price index is

$$P_k = \sum_{i \in k} s_i P_i$$

Real materials prices also are obtained by deflating with the shipments implicit price index.

Figure 1

Basic Data For the Stage of Fabrication Model

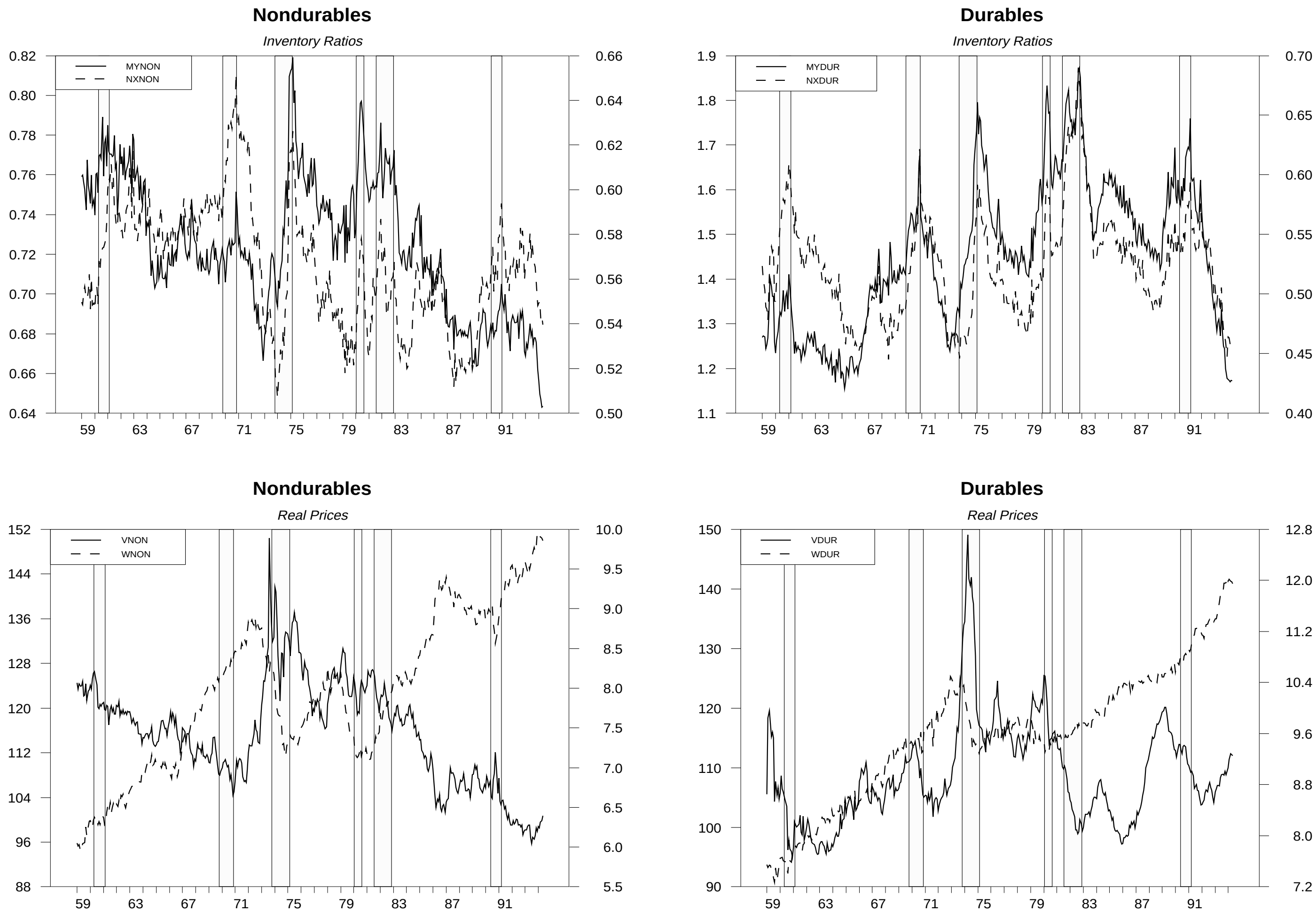
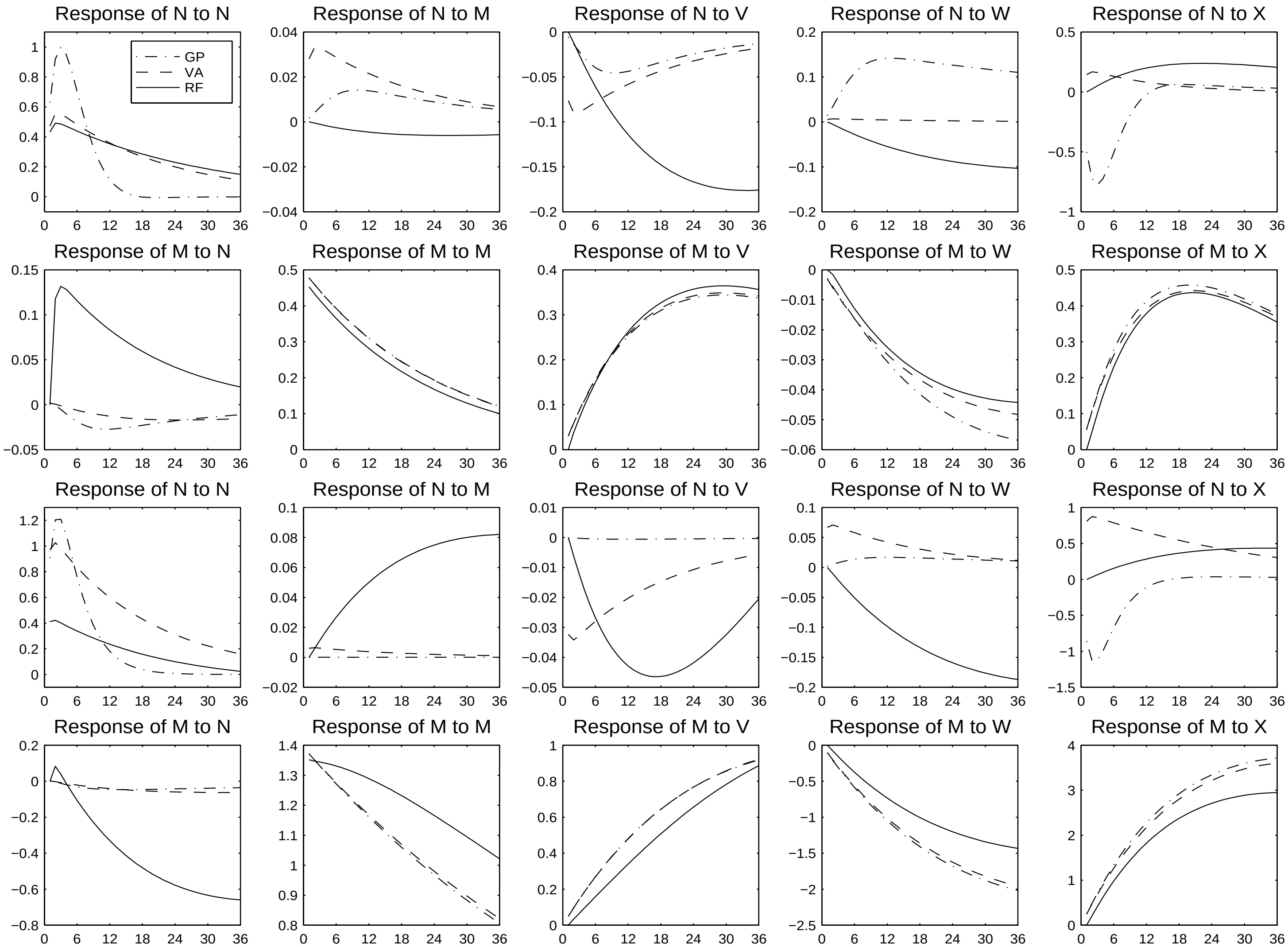


Figure 3 − Impulse Response Functions for the SOF Models

Nondurables (rows 1−2) and Durables (rows 3−4)



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