Wealth, Liquidity and Consumption

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I. Monetary Influences on Consumption and Saving

In discussion of the effects on aggregate demand of monetary policies and events, investment spending has been the main focus of attention. Economists have devoted a great deal of theoretical and empirical effort to tracing monetary influences on plant and equipment expenditure and on residential construction. They have paid relatively less attention to monetary effects on consumption and saving. One reason has been the wide currency of a simple Keynesian consumption function, a mechanical relation of consumption to disposable income. It has not been easy empirically to improve on the approximation that consumption is a constant fraction of disposable income, although the short-run volatility of this fraction is a major source of uncertainty and error both in forecasting and -- as the unhappy memory of the 1968 surcharge reminds us -- in policy.

In this paper we consider various monetary influences on consumption and attempt to estimate their importance. We do not have a new aggregate consumption function to propose, and we cannot at this point hope to explain the instability of the propensity to consume that has been so troublesome to forecasters and policy makers. Our approach is semi-realistic simulation. Instead of postulating a macro-economic consumption function, we derive aggregate consumption explicitly from a model of the decisions of individual households. We simulate a population of households with semi-realistic demographic and economic characteristics. We assume that

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these households make consumption decisions and plans in accordance with certain rules of behavior and market constraints. More specifically, the households conform to a life-cycle model of consumption and saving.

Each of our simulations generates a hypothetical path of consumption and saving for the population as a whole. The simulations differ from each other in the economic environment to which the households are adapting. Some of the environmental differences can be associated with monetary policies. Any change in monetary policy alters the households' constraints and expectations, and its global impact is gauged by the difference in the resulting simulated aggregate path of consumption and saving.

"Semi-realistic" means that the overall characteristics of the hypothetical population resemble those of the population of the United States, and that parameters have been chosen so that the magnitudes of aggregate variables have a familar ring. But we cannot of course begin to mimic the actual population in detail, and we have necessarily made many untested *a priori* assumptions. Compared with usual studies of consumption, our work contains a much greater and bolder theoretical component and a much weaker component of conventional statistical estimation and testing. We do not defend this methodology here, nor do we regard it as a substitute for customary econometric methods. But the conventional methods have not been dramatically successful, and we do believe that microeconomic simulations can provide some interesting macroeconomic insights.

There are two major recognized channels of monetary influence on consumption: (A) changes in wealth and in interest rates, (B) changes in liquidity constraints. We shall also address ourselves to (C) changes in taxes, temporary and permanent. The third would traditionally be regarded as an aspect of fiscal rather than monetary policy. But the impact of a tax change depends, in our model, on the

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monetary environment in which it occurs, and for comparative purposes it is instructive to examine it within the same general framework.

A. Wealth and Interest Rates

Wealth, of course, has frequently been proposed as an argument in theoretical and statistical consumption functions.[1] Early in the Keynesian controversy the wealth effect on the propensity to consume became prominent as the vehicle for the "Pigou effect." Currently popular econometric consumption functions for the United States are essentially, suppressing lags, of the form

$$C = aY_d + bW \quad (I.1)$$

where C is real consumption, Y_d real disposable income, and W real net nonhuman wealth of households. With coefficients a and b of the order of .5-.7 and .03-.05 respectively, and with W normally five times Y_d, an equation of this kind is consistent with the observation that consumption is normally of the order of 90 percent of disposable income. At the same time, the equation implies a much lower marginal propensity to consume from changes in disposable income unaccompanied by changes in wealth.[2] In this respect it appears to be consistent with the abundant evidence that the marginal propensity to consume from income is lower in the short run than in the long run.[3]

[1]See Ackley, 1961, pp. 554-561 for a good summary.

[2]Ando-Modigliani (1963) and Arena (1964) have estimated consumption functions of this form. The consumption function of the MIT-Penn-SSRC econometric model is also essentially of this type.

One difficulty with the equation is that, although it requires a W/Y_d ratio the order of 5 or more in order to obtain a realistic C/Y_d ratio, it does not generate enough saving to maintain so high a wealth/income ratio. If the normal saving ratio is .10 and the growth rate of the economy is .035-.04, the equilibrium wealth/income ratio is only 2-1/2 or 3. The answer may be that household wealth grows by capital gains, some of which reflect corporate saving, as well as by personal saving as measured in the national income accounts. In principle these gains should be included in the disposable income used in the equation, but Arena's attempts to do so were not successful.

[3]This is not always true. In some cyclical fluctuations, the market value of household wealth has moved as much as, or more than, disposable income. Stickiness of consumption must then be attributed to inelasticity of income expectations rather than to stability in nonhuman wealth.

Monetary policy can affect household wealth by changing interest rates and the market value of securities and of other assets. Evidently this mechanism was important in the 1969-70 decline in stock and bond prices, and in the 1971 recovery of these markets. In the MIT-Penn-SSRC model, the consumption consequence of such asset revaluations in a very important component of the power of monetary policy over aggregate demand.

There is, however, some danger in applying a consumption function like (I.1) in this context. The historical variations of W which yield an empirical estimate of the propensity to consume from wealth have not been solely or even principally the kind of variations generated by monetary policy. The historical path of household wealth results from: (a) planned accumulation, the consequence of the very saving behavior that wealth is supposed to help explain, (b) unexpected gains or losses due to changes, actual or expected, in the capacity of the economy's capital stock to earn income for its owners, and (c) unexpected gains or losses due to changes in the discount rates at which the market capitalizes prospective earnings. These sources of change in wealth should not be expected to have identical effects on consumption. In particular, the changes engineered by monetary policy are of type (c) and necessarily involve changes in interest rates, while the other types do not.

Interest rates determine the terms on which households can make substitutions between present and future consumption. In theory a change in wealth connected with a change in interest rates will have not only "income effects" on consumption but also intertemporal "substitution effects." These are not included in equation (I.1), and indeed econometric studies of consumption and saving have been notably unsucessful in detecting them.[4] But in view of the formidable identification problems involved, we are not entitled to assume they do not exist. The model used in our simulations allows for a modest amount of intertemporal substitution. Therefore it is necessary and possible to specify various packages of changes in interest rates and asset valuations and to distinguish among their consumption effects.

The effects on current consumption of changes in wealth and in interest rates may depend on the importance of liquidity constraints, about to be discussed in section I.B. Capital gains which are realizable in cash or in enlarged credit lines may permit households

[4]As, for example, assumed by Ando and Modigliani (1963).

to escape from constraints on their current consumption. In these circumstances the apparent marginal propensity to consume from wealth will be higher than in a perfect capital market.

B. Liquidity Constraints

In macroeconomics there has always been tension between "wealth" and "liquidity" theories of consumption and saving. Should the income variables in consumption functions be liquidity measures -- disposable income, disposable income less contractual saving, etc. -- or should they be human wealth measures -- permanent or lifetime income? Should the stock variables be liquidity measures -- liquid assets -- or wealth measures -- net worth?

In a theoretically perfect capital market, the consumption plans of households are constrained only by their wealth, human as well as nonhuman. Households can turn future income from the assets they own and from their own labor into current consumption on the same terms on which they can convert current income into future consumption. Within the bounds of solvency, they can dissave and borrow at the same interest rates at which they can save and lend. In such a world, the wealth of households, including the "permanent income" from their labor, is the only relevant measure of their consumable resources.

Additional constraints arise when households cannot substitute one kind of wealth for another, or can do so only with a penalty. Human wealth may be illiquid because households are not allowed to have a negative nonhuman net worth position even when it is offset by the value of their future labor income. Alternatively, they may be allowed to borrow against prospective wages and salaries, but only at a penalty rate. The threshold at which liquidity constraints apply may indeed be a positive level of nonhuman wealth. Borrowing is often possible, or possible without penalty, only on a fraction of the value of real estate, securities, and other assets. Mortgage contracts and retirement plans typically require the household to build up its nonhuman wealth at a prescribed rate. The market inposes penalties not just for dissaving but also for saving at less than the contracted rates.

Monetary policy is one determinant of the tightness of such liquidity constraints. Easy money conditions induce lenders to liberalize their down payment and margin requirements, to reduce penalty rates, to make consumer credit available on easier terms, to take more chances on unsecured personal IOU's. In tight money periods lenders move in the opposite direction.

C. Permanent and Temporary Changes of Taxes

The effects of tax changes on consumption depend on the importance of liquidity constraints. In the hypothetical world of perfect capital markets, increases of tax rates reduce human and nonhuman wealth by lowering expected incomes from labor and property. They may also, by lowering after-tax interest rates, have substitution effects in favor of present consumption against future consumption. Temporary tax increases diminish wealth calculations very little and will have weak income effects.

The situation is quite different for taxpayers whose current consumption is constrained by liquidity. An increase in taxes withheld or required to be paid in cash will have a powerful effect; in principle the marginal propensity to consume will be 1.0. This will be true whether the tax increase is permanent or temporary, a distinction that is much less important in a "liquidity" theory of consumption than in a "wealth" theory.

One of the difficulties of aggregation that confronts macroeconomic specifications of the consumption function is that there are undoubtedly both liquidity-constrained and liquidity-unconstrained households in the economy, in proportions that vary from time to time. The younger and poorer households are more likely to be liquidity-constrained. One advantage of the microeconomic simulation method of this paper is that differential incidence of liquidity constraints can be systematically introduced and its consumption effects calculated.

II. The Life Cycle Model As a Framework of Analysis

Our framework for analysis of the questions raised in section I is the life cycle model of household consumption.[5] We begin with a simplified exposition of this model, in two stages. Many of the essential points can be illustrated by the familar textbook example of a consumer with a two-period lifetime. This is done in section II.A; section II.B sketches the extension of the model to multi-period consumption and saving decisions; section II.C points out some of its aggregative implications.

[5]The basic idea goes back to Fisher (1907, 1930). Its modern elaboration begins with Modigliani-Brumberg (1954). Our approach in this paper is a sequel to Tobin (1967).

A. Two-period Consumption Decisions

Consider a consumer with a two-period lifetime. In Figure 1 the horizontal axis measures first period consumption c_0 and the vertical axis second period consumption c_1. Labor incomes in the two periods are (y_0,y_1), marked as point y. Coordinate axes are also shown with origin at y. On these axes, W_0 is the value in first period consumption of the consumer's nonhuman wealth, and W_1 is its value in second period consumption. W_0 and W_1 are related by the one-period interest rate: $W_1 = W_0\ (1+r)$. The point $(y_0,y_1 + W_1)$, labelled W_1, represents one feasible consumption combination, one involving zero current saving. In the assumed perfect capital market, the household can move in either direction from this point, on terms of 1+r units of deferred consumption for one unit of initial consumption. The point Y_0 measures the present value of total consumable resources, equal to $y_0 + \frac{y_1 + W_1}{1 + r} = y_0 + \frac{y_1}{1+r} + W_0$. The point Y_1 is the value of total resources in terms of second-period consumption. The consumer can choose any point on the opportunity locus Y_0Y_1. In the illustration he chooses point c.

A liquidity constraint would be illustrated by a kink in the opportunity locus. For example, if the consumer could not consume in period 0 more than $y_0 + W_0$, the locus would be vertical from point W_0 to the horizontal axis. If he could exceed $y_0 + W_0$ only by borrowing at a rate $r_b > r$, the locus Y_1W_0L would have a steeper slope, $-(1+r_b)$ instead of $-(1+r)$, from W_0 to the horizontal axis at L. The kink could occur further to the left if the consumer were required to carry a positive amount of wealth into period two, or penalized to the extent he did not.

The consumer is assumed to have a preference ordering of consumption points (c_0,c_1) with the usual properties, and to choose a point on the highest attainable indifference curve. In the later sections of the paper we have represented these preferences by a

particular utility function, and we will introduce that representation here. We assume that the consumer's prospective utility U is a discounted sum of utilities of amounts consumed in each period:

$$U = \sum_{i=0}^{a^*} u(c_i) \left(\frac{1}{1+\delta}\right)^i \qquad \text{(II. 1).}$$

The same one-period utility function u applies to every period; the marginal utility $u'(c_i)$ is positive and declines with c_i. Future utility is discounted at a subjective rate δ, the pure rate of time preference. In Figure 1, for example, the slope of an indifference curve is:

$$-\frac{u'(c_0)}{u'(c_1)}(1+\delta),$$

and in particular it is $-(1+\delta)$ for $c_1 = c_0$, i.e., along the 45° ray. The curvature of the indifference curves is related to the substitutability between consumption in different periods. We take for marginal utility

$$u'(c_i) = Bc_i^{-\rho} \qquad \rho > 0 \qquad \text{(II. 2)}$$

so that $-\rho$ is the elasticity of (undiscounted) marginal utility with respect to c_i. The slope of a (c_0, c_1) indifference curve is then $\left(\frac{c_0}{c_1}\right)^{-\rho}(1+\delta)$. The larger the value of ρ, the faster the slope of the indifference curve changes as the ratio c_1/c_0 moves to the left or right of the 45° ray. A high value of ρ means high curvature and low intertemporal substitutability. Following Fellner (1967) and others, we take $\rho = 1.5$ in our calculations below.[6]

[6]Tobin (1967) assumed $\rho = 1$, as would follow from a logarithmic utility function. Ando and Modigliani, (1963, p. 59), on the other hand, assumed perfect complementarity, i.e., L-shaped indifference curves with the corner of the 45° line.

In a perfect capital market, a consumer maximizes U subject only to the budget constraint

$$\sum_{i=0}^{a^*} c_i d_i - \sum_{i=0}^{a^*} y_i d_i - W_0 = 0$$

$$\sum_{i=0}^{a^*} (c_i - y_i) d_i - W_0 = 0 \qquad \text{(II. 3)}$$

where the d_i are the market discount factors that convert consumption and income in period i to present values. In the two-period illustration $d_0 = 1$ and $d_1 = \frac{1}{1+r}$. The first order conditions of the constrained maximum are:

$$u'(c_i) \left(\frac{1}{(1+\delta)}\right)^i - \lambda d_i = 0 \qquad i = 0, 1, 2, \ldots a^* \qquad \text{(II. 4)}$$

where λ, the Lagrange multiplier, is the marginal utility of consumable resources. If market interest rates are constant, so that $d_i = \left(\frac{1}{1+r}\right)^i$, we have

$$\frac{u'(c_{i+j})}{u'(c_i)} = \left(\frac{1+\delta}{1+r}\right)^j \qquad \text{(II. 5)}.$$

From (6) we know that undiscounted marginal utility must rise, fall, or remain constant with age according as δ is greater than, smaller than, or equal to r. If, for example, the market interest rate r exceeds the subjective discount rate δ, second-period consumption must exceed first-period consumption. The chosen combination will be to the left of the 45° line, as in Figure 1.

For our specific utility function, condition (II.4) becomes:

$$c_i = \left(\frac{B}{\lambda} d_i \frac{1}{(1+\delta)^i}\right)^{\frac{1}{\varphi}} \qquad i = 0, 1, 2, \ldots a^* \qquad \text{(II. 6)}$$

For example, in the two-period case $c_1 = c_0\left(\frac{1+r}{1+\delta}\right)^{\frac{1}{\rho}}$. The elasticity of c_1/c_0 with respect to 1+r is $\frac{1}{\rho}$, or .67 for our numerical assumption. This means roughly that a 100-basis-point rise in the interest rate will increase c_1 relative to c_0 by two-thirds of 1 percent.

An increase in consumable resources with no change of interest rates would be represented in Figure 1 by a parallel outward shift of the budget constraint. On our assumptions it would lead to a proportionate increase in c_1 and c_0, because the slope of an indifference curve derived from (II.2) depends only on the ratio of the two consumptions, not their absolute amounts. The same implication -- proportionate shift in all c's -- holds for the multi-period case.

A fall in the interest rate will tilt the opportunity locus counterclockwise and lead to intertemporal substitution. In general an interest rate decline will also have an income effect, enlarging the opportunity set for dissavers and restricting it for savers.

Both income and substitution effects are different if liquidity constraints are operative. So long as the consumer is at a kink in his opportunity locus, he will consume immediately 100 percent of any increment in currently available resources. The substitution effect, however, will be zero for small changes in interest rates.

As our discussion in section I.A indicated, changes in wealth induced by monetary policy are associated with interest rate changes, while other changes in wealth need not be. In Figure 1 the shift of locus from Y_0Y_1 to $Y_0'Y_1'$ reflects pure capital gain, with no change of interest rates. W_0 and W_1 increase in the same proportion, to W_0' and W_1'. However, the shift of locus from Y_0Y_1 to $Y_0''Y_1''$, involves the same capital gain from W_0 to W_0' but provides no increase in W_1.

In the first case, the income effect is positive, and proportionately of the same magnitude whether the initial consumption choice was c or any other point on the budget constraint Y_0Y_1. In the second case, whether the income effect is positive, zero, or negative depends on whether the initial consumption choice was to the right of W_1, at W_1, or to the left of W_1. Only if the initial choice was to the right of W_1, involving dissaving in the first period, does the income effect work in favor of current consumption. In the illustration of Figure 1,

c was to the left of W_1 and the income effect is negative. But while there is no substitution effect in the case of pure capital gain, the reduction of the interest rate in the second case always favors current consumption.

Obviously there are other possibilities. In the second case, $(Y_0''Y_1'')$, wealth consists entirely of claims that mature in the second period, claims that do not outlive the household. To the extent that claims are longer-lived, a smaller reduction of the interest rate will suffice to accomplish the given gain in initial wealth $W_0' - W_0$, and there will be a positive increment in W_1.[7] The two-period example does not permit us to exhibit the opposite case, where wealth consists of claims which mature short of the household's horizon. There will be some periods for which W_1 is reduced -- as if the budget constraints cut below W_1 in Figure 1. Saving for consumption in late periods is less fruitful because of the low yield at which maturing claims must be reinvested.

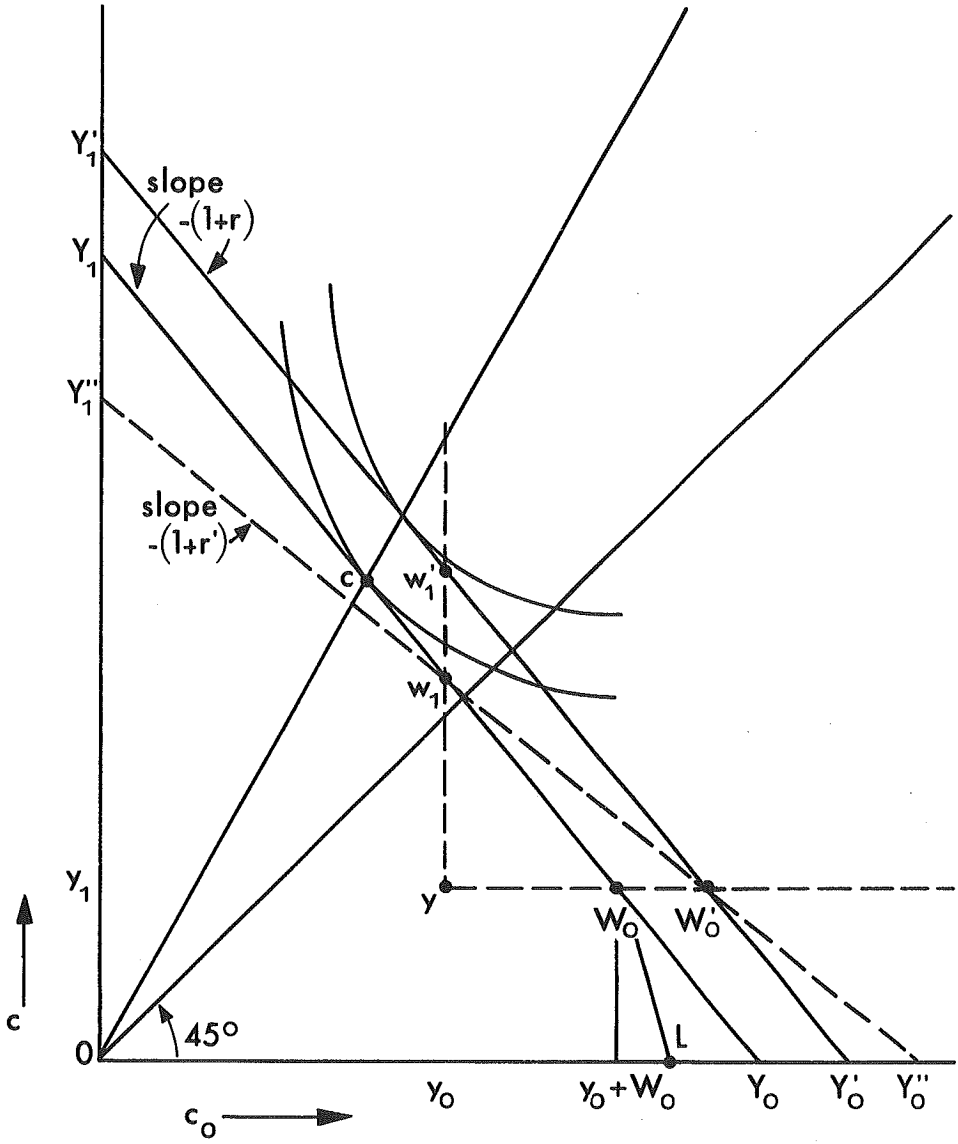
A case similar to the shift of opportunity locus to $Y_0''Y_1''$ arises when asset revaluations in security markets are regarded as temporary. This means that they are associated with temporary rather than permanent changes in discount rates. Consider, for example, consol-like claims that rise in value because of a decline in the interest rate connecting period zero and period one, while subsequent rates remain unchanged. These claims will revert to their old value after period one. The value of the household's wealth in current consumption is increased, but its value in future consumption is not.

[7]If wealth takes the form of consol-like claims, the new discount rate is:

$$r' = \frac{rW_0}{W_0'}, \text{ and } \frac{W_1'}{W_1} = \frac{(1+r')W_0'}{(1+r)W_0} = \frac{W_0'/W_0 + r}{1+r}.$$

In this case W_1 increases almost in proportion to W_0.

Figure 1.


Y_1'
slope
-(1+r)
Y_1
Y_1''
slope
-(1+r')
c
w_1'
w_1
y_1
y
W_o
W_o'
c
0
45°
L
y_o
y_o+W_o
Y_o
Y_o'
Y_o''
c_o

It is possible that capital gains may accompany *increases* in interest rates, so that substitution effects oppose, while income effects favor, current consumption. This combination would be the result not of monetary policy but of optimistic revisions of expected future profits.

Finally, the modeling of tax changes in the two-period illustration is obvious. A permanent tax on labor increase reduces both y_0 and y_1, while a temporary tax lowers only y_0. The income effect on current consumption is obviously greater for the permanent tax except when the household is liquidity-constrained. Taxes on property income are like interest rate reductions.

B. Multi-period Lifetime Consumption Decisions

Consider a household at the beginning of its career, anticipating a sequence of labor incomes and deciding on a sequence of consumption rates within the limits set by its income prospects. In Figure 2 an expected income sequence is illustrated, and along with it a chosen consumption plan. Both the income sequence and the consumption plan are pictured in two ways, in current real dollars (dashed curves) and in dollars discounted to the decision date (solid curves).

The consumption plan is shown as smoother than the income sequence. The spirit of the life-cycle hypothesis is that consumers prefer steady consumption to fluctuating consumption. The one-period marginal utility of consumption, like (II.2) above, is declining. Households save and dissave in order to smooth out their income paths. Saving for retirement is the clearest example of such behavior, but certainly not the only one. Another example is debt financing by young people to obtain a standard of life beyond their current means but consistent with their occupational status and income prospects. Of course the household is not free to choose any paths for c that it desires. It is limited by its income sequence. Specifically, the sum of the differences between discounted y_i and discounted c_i -- the present value of its savings and dissavings from labor income -- must add up to zero over the lifetime, as in equation (II.3) above.

Figure 3 provides the same information as Figure 2 in different form. The curves are the integrals of the "discounted y" and "discounted c" curves. The Y curve shows for each age the cumulative total of labor income earned until that age, discounted to household age zero. Similarly the C curve shows the present value, as of age zero, of consumption through age a. At the terminal age a^* Y

Figure 2.

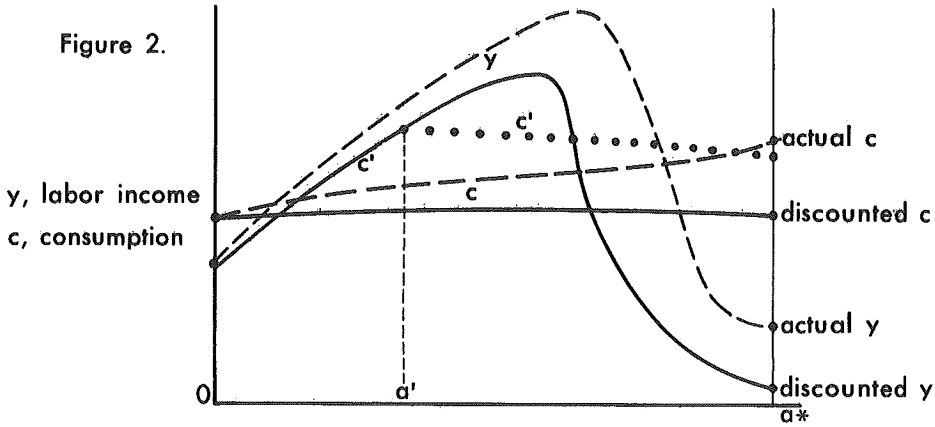


Figure 3.

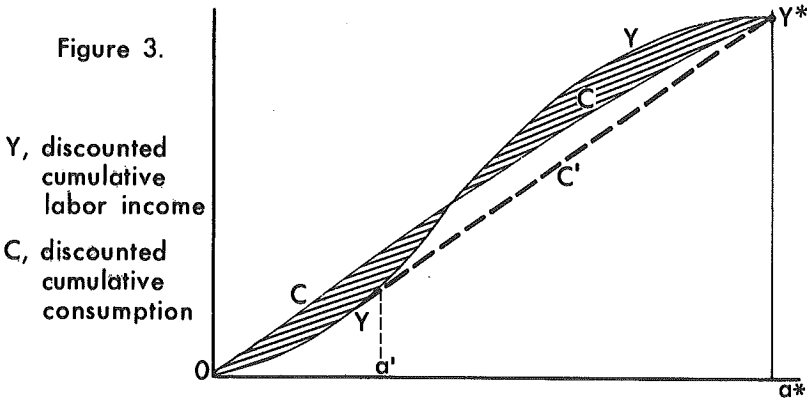
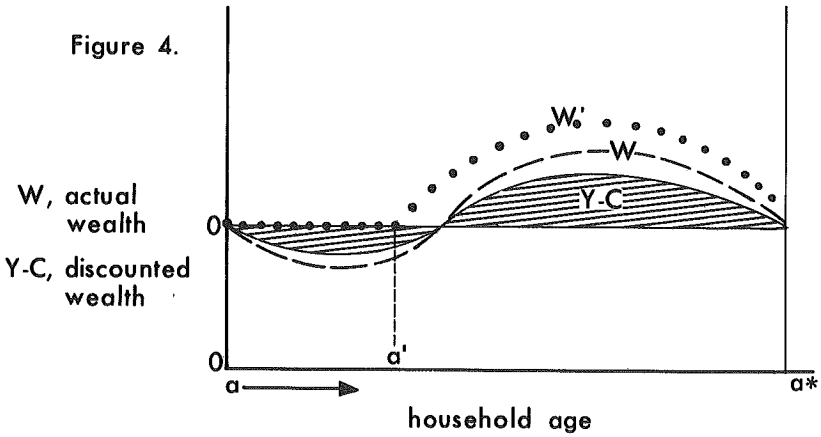


Figure 4.



and C meet. This is the budget constraint: The present value of lifetime consumption must be the same as the present value of lifetime income. Actual consumption, cumulated at current dollars, will generally exceed actual labor income summed over the whole life. The household will earn and consume some interest.

From the income and consumption paths the wealth profile of the household can be easily derived. In present value terms, nonhuman wealth W is just the vertical difference, positive or negative, between Y and C. These differences are shaded in Figure 3 and plotted in Figure 4 as "discounted wealth." By putting and discounting process in reverse, this present value wealth profile can be converted into a current dollar wealth profile -- the dashed curve "actual W" in Figure 4. If the household's expectations are realized, this is the course its wealth will follow as its plans are carried out.

This account has assumed that the household can save and dissave in a perfect capital market -- in particular, that the household can borrow against future labor income at the same interest rates at which it can save. The only constraint has been the lifetime budget constraint. Terminal wealth must not be negative, a restriction that limits total lifetime consumption but not its allocation among ages. In Figure 3 curve C must start at 0 and end at Y^*, but in between it may have any shape the household desires.

Consider, on the other hand, a simple liquidity constraint, that nonhuman wealth W can never be negative. The best the household can do, so constrained, is to consume its cash income in early years until a′and then follow the dotted curves c′and C′in Figures 2 and 3. Correspondingly, in Figure 4, discounted W will be O until age a′and then follow the dotted path. The less drastic constraint of a penalty borrowing rate, finite instead of infinite, would move the household in the same direction. In general, as the example illustrates, liquidity constraints *raise* the household's wealth profile.

In the illustration, the household begins and ends with zero wealth. The model can easily accommodate other assumptions. For a household beginning with inherited wealth, the Y and W curves of Figures 3 and 4 will start with positive intercepts. Inheritances anticipated at later ages would be shown as jumps in the Y curve. Similarly any planned or required bequest at a^* would be indicated by a positive difference between Y^* and C at a^*.

The plan made at age zero can be reconsidered and remade in the same manner at every subsequent age a. If external constraints and market interest rates conform to original expectations, and if the household's preferences are unchanged, the new decisions simply

confirm the old, and the original plan will be executed. But if conditions and expectations change, the household will make a new plan for the remainder of its life.

In this introductory exposition of the model for a single household we have ignored some complications which we have to face in the applications of the model described later in the paper. These include allowance for life cycle variation of the size and composition of the household -- as children are born, grow up, and leave -- and actuarial allowance for mortality.

C. Macroeconomic Implications

The life cyle model has interesting implications for the economy as a whole. The income, consumption, saving, and wealth of a household depend on what profiles it is following and on its age. Aggregates of these variables can be obtained by summing over all households. Households differ both in profile and in age, but of course their age differences are much easier to observe. Specific results can be obtained by calculating the aggregate income, consumption, saving, and wealth of a population of households of different ages, all following essentially the same life-cycle profiles. The aggregate value of any variable is the sum of the profile variables for different ages, weighted by the number of households of each age. The aggregate will change from year to year as the population grows and its age distribution changes.

Allowance can also be made for steady growth of labor productivity. The expected income profiles of Figures 2 and 3 take general gains in labor income into account, as well as increases which are simply related to experience and seniority. A similar household starting a year later would face a higher income profile, shifted upward, as a first approximation, by the proportion γ at every age. With everything else equal, the model of consumption choice implies a similar proportionate shift in every other profile of Figures 2 to 4. The income, consumption, saving, and wealth of 10-year-old households in 1975 will all be $(1+\gamma)$ times as large as those of the 10-year-old households of 1974. The aggregate consequences is that all the macro variables will grow at the rate γ per year, plus any changes that may occur because of changes in the population of households of various ages.

In a demographic "golden age," the population is growing at a steady rate n per year and its relative age distribution is constant. Consequently the number of households of each age is growing at

rate n. If it is also an economic "golden age," interest rates are constant and likewise so is the growth of labor productivity γ. The model then implies that all the aggregates are growing at the rate $n + \gamma$. Since this is the natural growth rate of the economy, the life-cycle model provides an explanation of saving behavior consistent with a neoclassical growth equilibrium.[8]

III. Description of the Simulations

In this section we describe more specifically the modeling of the consumption decision and the variables which influence it. The appendix contains a more complete mathematical treatment and indicates our data sources.

A. Demographic Assumptions

We distinguish among individuals by only three characteristics. The first of these is age, the central variable of the life cycle model. The second distinguishing characteristic is sex, since realistic calculations require some recognition of family structure and of the work habits and consumption requirements of different family members.

Finally we have divided the population into two income classes. The relative proportions of the population in the groups are those that existed in 1963 between the population above and below the poverty line. If different income groups face different opportunity sets (e.g., differential ease of access to capital markets), then aggregate consumption may depend on the income distribution. We have assumed that the two income classes differ only in the relative levels of their income profiles, not in the time shapes of the profiles or other demographic and economic circumstances.

The basic behavioral unit is the cohort, which consists of all adult females of a given age plus associated adult males and children of various ages. All cohorts are actuarially average. There are no unattached individuals or families of larger or smaller size.

An individual lives with his parental family until age 21 (in the case of males) or 18 (in the case of females). Any income earned as a teenager is contributed to the household, which in turn makes provision for the child's consumption needs until he leaves the household. At 18 the females form the nucleus of a new cohort, to which a

[8]See Tobin (1967).

complement of males, including newly matured 21-year-olds, are assigned. As the cohort ages it will gain some adult males from each new group of 21-year-olds. Some of the current crop of 21-year-old males are, in turn, assigned to older cohorts.

With a minor exception discussed in the appendix, a cohort loses its adult members only by death. Each cohort is disbanded when the female becomes 85. A specific, unchanging, perfectly anticipated mortality table is assumed. All people expect to die before age 85. The cohorts will include some adult males who are younger than the female and who will thus outlive the cohort. These men are assigned to new cohorts. No children are reassigned in this manner since the last age at which females bear children -- 49 according to the birth table assumed -- is such that all children have matured and left the cohort before it disbands. It is assumed for convenience that women do not bear children before age 18. The birth vector has been adjusted accordingly.

Although a number of demographically unrealistic simplifications have been made, none of them is quantitatively significant. The simplifications are necessary to make the computational burden manageable.

B. Income Expectations and the Consumption Allocation

In making its lifetime consumption plan the cohort is constrained not to allocate more than the present value of its lifetime resources. These total resources consist of human and nonhuman wealth. The former is the accumulated savings -- including capital gains -- of the cohort;[9] the latter is the present value of future labor income.

The evaluation of both sources of wealth involves expectations about their future income streams. For a number of reasons these income streams may be expected to vary with time.

Because of age-related differences in participation rates and in productivity, labor earnings vary with age, generally rising until about age 40 or 50 and then declining. For women, on the average, there is a slight decline related to reduced participation in the primary child-rearing years. We assume that the labor earnings of an individual of a given age and sex in any year will be a constant proportion of the labor income of a 40-year-old male in that year. Thus the relative income profile by age, for both men and women, will be assumed constant over time.

[9]Inheritances and bequests are ignored.

The absolute level of the profile, however, will change. We assume labor-augmenting technological change at a constant annual rate $\gamma = .0225$. Although factor rewards might be expected to be influenced by variations in the capital-labor ratio, we have not assumed an explicit production technology and such effects will not be considered.

A final source of variation in income streams will be changes in tax rates, both on property and on labor income. It is disposable labor income which is to be allocated to consumption or to saving, and it is after-tax property yields which are relevant to this allocation.

Having estimated the present value -- at current and expected rates of discount -- of its lifetime resources, the cohort then allocates these resources among all its members for all the years that they are expected to live.

The optimal allocation will be one for which the prospective marginal utility of a unit of consumption is the same in every year, so that total utility cannot be increased by shifting a unit from one year to another. We assume, of course, that the marginal utility of consumption in a given year declines with amount of that consumption. That is why the household seeks to avoid large differences in consumption between years. The marginal utility of a unit of consumption will also vary with the year in which it is to occur: we assume a pure rate of time preference of δ. Thus the value of a unit of utility from consumption t years hence has only $1/(1+\delta)^t$ times the value of a unit of utility today. In our simulations we have used $\delta = .02$.

The utility of consumption will also vary from year to year with household size and composition. This variation reflects economies of scale in household life and differences in the needs and priorities of various household members. To allow for these phenomena we weight the utility of consumption for children and teenagers differently from adults. In this calculation of household size, adults receive a weight of $w_a = 1.0$, while the weights for teenagers and children, w_t and w_c, are .5 and .2 respectively. Thus a consumption-year for a child is equal to $w_c = .2$ "equivalent adult years."[10]

Barring the complications discussed in section (III.C), the cohort maximizes its utility if it allocates its consumption -- discounted by a transformation of the difference between the expected interest rate and the rate of time preference -- so as to equalize consumption per

[10]For our purposes teenagers are defined as those children who earn incomes, aged 15-17 (female) or 15-20 (male).

equivalent adult year, where the equivalent adult years, too, are discounted by transformations of the interest rate, the rate of time preference, and birth and death rates.

C. Capital Gains and Interest Rate Changes

In the two previous sections, III.A and III.B, we have explained our model of the household sector of the economy. The households make the consumption decisions, and our purpose is to see how those decisions are affected by monetary policies and other events exogenous to the household sector. In Part II we discussed in general terms the policy and environmental changes of interest, and now we explain how we have modeled these "shocks" in our simulations. In this section we discuss capital gains and interest rate changes. In the two sections following we discuss how we have modeled liquidity constraints and their relaxation or tightening, and how we have modeled tax changes.

As we pointed out in Part II, capital gains and interest rate changes are intimately bound together. It is not possible to trace the effects of shocks of this kind without being explicit about the nature of the assets whose yields are assumed to change, and about the expected asset prices and interest rates.

We are assuming that the wealth of the household sector consists of various direct and indirect claims on the economy's capital stock. Monetary policies and events can change the valuation of the stock, and so can changes in the real earnings of capital due to technological or macroeconomic developments. But in the long run adjustments in the size of the capital stock or in monetary interest rates, or in both, keep market valuations of capital in line with reproduction costs. We do not provide a model of those adjustments, but we assume that our households know they will occur and we provide them accordingly with a plausible mechanism of expectations.

The present discounted value of the earning stream of capital per dollar of reproduction cost is

$$q = \frac{R_1^e}{(1+r_1^e)} + \frac{R_2^e}{(1+r_1^e)(1+r_2^e)} + \ldots + \frac{R_n^e}{(1+r_1^e)(1+r_2^e)\ldots(1+r_n^e)} + \ldots$$

where R_i^e is the expected net earnings i years hence and r_i^e is the expected one-year rate of interest in the ith year. The R_i^e are net of depreciation and operating costs.

For a finite-lived piece of capital directly owned the R_i^e become zero at some point. If the R_i^e represent earnings on equity shares in a firm, however, they may not be expected to be zero. Rather it may be expected that the firm's shares will yield earnings in perpetuity. In the special case in which both R_i^e and r_i^e are expected to be constant forever at R and r, respectively, we know that $q = \frac{R}{r}$.

In long-run equilibrium q must equal one, i.e., the market value of a unit of capital stock must equal its reproduction cost.

Both R_i^e and r_i^e represent expectations about the future. For generating expectations we have assumed a mechanism which distinguishes between long run and temporary phenomena. Essentially, expectations are assumed to be regressive in the short run and adaptive in the long run. Suppose that rates of return have been constant for some time at a level $\bar{r}$. This $\bar{r}$ will come to be regarded as a normal level. Suppose, however, in some period, r rises above $\bar{r}$. It might then seem reasonable to believe that r will stay above $\bar{r}$ for a while but will eventually decline to $\bar{r}$: expectations in the short run are regressive. If r continues to exceed $\bar{r}$ for some time, however, it will be less reasonable to expect a return to $\bar{r}$. In fact, $\bar{r}$ will no longer be regarded as the normal level, and estimates of the normal level will be revised upward.

If earnings on equities (the capital stock) R diverge from what has been a normal level, an entirely analogous mechanism operates. The two processes are in fact linked since, as we have noted above, long-run equilibrium requires $R = r = \bar{R} = \bar{r}$ (i.e., $q = 1$). Thus the normal level of earnings on capital and the normal level of interest rates must be identical.

We will assume that R and r, if they differ from $\bar{R}$, will be expected to converge geometrically to $\bar{R}$ with 85 percent of the remaining difference expected to be eradicated in each year. We assume an adaptive mechanism for $\bar{r}$, where 80 percent of the weight is on $\bar{R}_{-1}$ and 10 percent each on the current levels of R and r:

$$R_i^e = \bar{R} + \theta_R^i (R - \bar{R}) \qquad \text{(III. 1)} \qquad r_i^e = \bar{R} + \theta_r^i (r - \bar{R}) \qquad \text{(III. 2)}$$

$$\bar{R} = (1 - \eta_R - \eta_r)\, \bar{R}_{-1} + \eta_R R + \eta_r r \qquad \text{(III. 3)},$$

where we assume $\theta_R = \theta_r = .85$ and $\eta_R = \eta_r = .10$.

Actual values of R and r are assumed to be known and to be exogenously determined. Monetary policy will influence r in the first instance, while changes in R will be due to capital-augmenting technical change and other factors affecting the earnings of firms. Such effects are dynamically interrelated, as both affect $\bar{r}$ and hence each other. In part because they are interrelated, differences are viewed as temporary, since there exist natural forces in the economy causing R and r to reconverge to each other. As we have indicated above, however, there may be times when a permanent change in the earnings on capital is expected. This corresponds to a shift in $\bar{R}$ overriding the adaptive expectations of equation (III.3). In our simulations we will investigate the effects of changes in $\bar{R}$ as well as the effects of changes in R and r.

D. Liquidity Constraints

Monetary policy will affect consumption through its effects on borrowing conditions and liquidity constraints as well as through its influence on wealth. The monetary authority's ability to affect such credit conditions will be parameterized in two variables in our simulations. One of these will be a borrowing rate r_b charged on funds borrowed. In general r_b will exceed r, the market rate of interest (lending rate for individuals). The second instrument will involve quantitative restrictions as discussed below.

Foreseen dissaving, for example in the retirement years, presumably does not pose a liquidity problem, there having been sufficient time to reallocate the portfolio to provide necessary liquidity. It is in the younger years that liquidity constraints may be of consequence, forcing the household to save more, or dissave less, than it desires.

For the purposes of our simulations it will be assumed that a cohort undertakes at age $u_a = 25$ an illiquid investment of amount A, financed by debt on which the cohort commits itself to make annual payments of principal of A/T in each of T consecutive years.[11] Cohorts are not permitted to make advanced payments on their contracts. Both the illiquid investment and the debt bear the market rate of interest.

[11] A will be assumed to be $30,000 per adult female for new group 1 cohorts and $7500 for new group 2 cohorts in the first year of the simulations. It will be assumed to grow at the constant rate γ, the rate of growth of per capita income. The simulations assume T=20.

We introduce the concept of contractual saving, $\hat{s}$, saving required of the cohort in a given year. The contractual payments of principal, A/T, are one source of obligatory saving, but not the only one. If in some year the cohort wishes to save less than $\hat{s}$, it will have one borrowing option available to it. It will borrow at a penalty rate r_b, the principal to be reduced in $\hat{T}$ equal payments of $1/\hat{T}$ times the amount borrowed. We have used $\hat{T} = 5$.

$\hat{s}$ may differ from A/T, the amount due on the initial agreement, for two reasons. First, if any secondary borrowing has occurred in the past $\hat{T}$ years, the current obligation is the sum of the amounts due on the primary and such secondary obligations. Note that any borrowing in the last $\hat{T}$ of the T years of the initial contract extends the period in which the cohort is susceptible to saving constraints, since the secondary obligations are subject to the same stipulations as the primary contract.

The second reason $\hat{s}$ may differ from A/T is related to a second credit rationing instrument. Suppose E, possibly zero, is the amount currently due at secondary loan repayments, so that the total due is A/T + E. Lenders may require that only a fraction φ of the amount due actually be paid. Equivalently, lending institutions make available loans at the market rate of interest r in the amount:

$$(1-\varphi)\ (A/T + E).$$

φ cannot exceed 1 if advanced repayment cannot be required. Or the other hand, in order not to be a constraint under any circum stances, φ must equal negative infinity, or else r_b must equal the market interest rate r. For $\varphi > -\infty$, any borrowing in excess of:

$$(1-\varphi)\ (A/T + E)$$

occurs at the penalty rate r_b.

Monetary policies operate on consumption through these two parameters, the penalty rate for borrowing r_b and the range of it applicability φ. Presumably by altering policy mix and institutiona structure the two parameters can be varied relative to one another. In our simulations, such variations create a wide range of credit marke

opportunity loci facing cohorts. Borrowing can be prevented altogether with $\varphi = 1$ and r_b set prohibitively high. Algebraically smaller φ with r_b still prohibitively high corresponds to direct quantitative limits on borrowing. A lower r_b will permit price allocation beyond $(1-\varphi)(A/T + E)$.

The discussion has been in terms of a liquidity constraint faced only in the current period, and this is the basis on which our calculations have been made. But very likely a household expects also to be bound by similar constraints in the future. Calculation of the truly optimal consumption plan would then require explicit recognition of all possible future constraints and their costs. Indeed the timing of the undertaking of large illiquid investments should also be endogenous. The solution of such a nonlinear dynamic programming problem, however, is not computationally feasible for the present investigation.

E. Tax Rate Changes

A final element of the economic environment which affects consumption decisions is tax policy. All of the income streams above, both property and labor, are after-tax disposable incomes. We will consider uniform percentage reductions in incomes from each source separately and from the two together. We also examine the effects of temporary and permanent taxes. In both cases it will be assumed that the timing of the tax changes are perfectly anticipated. Interest payments are assumed tax deductible. Capital gains are taxed on an accrual basis.[12]

IV. Results of Specific Simulations

In this section we discuss the simulated effects of changes in policy instruments and of changes in expectations about the earnings stream of capital. Simulation 1, termed the "neutral" case for shorthand reference, represents the standard against which the other cases will be compared. The various simulations are defined in Table 1 and their differences relative to simulation 1 are noted. The actual time paths of r, R, q, and $\bar{R}$ for those cases in which they vary are presented in Table 2. The resulting time paths of aggregate consumption (C), aggregate wealth (W), and the personal saving ratio (S) are presented in Table 3.

[12]For computational convenience, taxes on future labor income of teenagers are not anticipated, though such taxes are imposed at the time the income is actually earned.

TABLE 1

DESCRIPTION OF EXOGENOUS CHANGES DEFINING THE VARIOUS SIMULATIONS

Simulation	Description
1	"Neutral." $r = R = \bar{R} = .0525$, $q = 1$ throughout. $r_b = .07$, $\varphi = 1.0$. No tax surcharges.
2	Interest rate changes. Starting in fourth period r declines, then rises back to initial level by ninth period.
3	Profit rate changes, short run. Starting in fourth period R rises, then declines to initial level by ninth period.
4	Profit rate changes, long run. Same short-run movement of R as in case 3. In addition, in sixth period long-run expectations change, $\bar{R}$ rises.
5	Eased liquidity constraint. $\varphi = .5$.
6	Differential liquidity constraints, $\varphi = .5$, $r_b = .07$ for higher income group. $\varphi = 1.0$, $r_b = .10$ for lower income group.
7	Tax surcharge plus capital gains. Five period increase in taxes on all income, reducing disposable income by 2 percent, coupled with an increase in R in the second period, later followed by a return to its initial level.
8	Temporary labor income tax surcharge. Labor income reduced by 2 percent for five periods.
9	Temporary property income tax surcharge. Property income reduced by 2 percent for five periods.
10	Temporary income tax surcharge. Combination of cases 8 and 9.
11	Permanent labor income tax surcharge, of same size as in case 8.
12	Permanent property income tax surcharge, of same size as in case 9.
13	Permanent income tax surcharge, of same size as in case 10.
14	Eased liquidity constraint. $\varphi = 0$.
15	Temporary labor income tax surcharge, of same size as in case 8, plus eased liquidity constraint. $\varphi = 0$.
16	Interest rate changes, same as case 2, plus eased liquidity constraint. $\varphi = 0$.
17	Profit rate changes, same as case 3, plus eased liquidity constraint. $\varphi = 0$.

TABLE 2

TIME PATHS FOR r, R, q, $\bar{R}$ IN THE SIMULATIONS WHERE THEY ARE NOT CONSTANT*

(YEARS)

Simulation	Variable	1969	1970	1971	1972	1973	1974	1975	1976	1977	1978	1979
2,16	r	.0525	.0525	.0525	.0425	.0325	.0325	.0325	.0425	.0525	.0525	.0525
	q	1.0	1.0	1.0	1.043	1.089	1.089	1.090	1.044	1.0	1.0	1.0
	$\bar{R}$	.0525	.0525	.0525	.0515	.0497	.0483	.0471	.0472	.0483	.0491	.0498
3,17	R	.0525	.0525	.0525	.0625	.0725	.0725	.0725	.0625	.0525	.0525	.0525
	q	1.0	1.0	1.0	1.042	1.083	1.083	1.083	1.041	1.0	1.0	1.0
	$\bar{R}$	.0525	.0525	.0525	.0535	.0553	.0567	.0579	.0578	.0568	.0559	.0552
4	R	.0525	.0525	.0525	.0625	.0725	.0725	.0725	.0625	.0525	.0525	.0525
	q	1.0	1.0	1.0	1.042	1.082	1.082	1.082	1.041	1.0	1.0	1.0
	$\bar{R}$	.0525	.0525	.0525	.0535	.0625**	.0625	.0625	.0615	.0597	.0583	.0571
7	R	.0525	.0625	.0725	.0725	.0725	.0625	.0525	.0525	.0525	.0525	.0525
	q	1.0	1.042	1.083	1.083	1.083	1.041	1.0	1.0	1.0	1.0	1.0
	$\bar{R}$	.0525	.0535	.0553	.0567	.0579	.0578	.0568	.0559	.0552	.0547	.0542

*In other simulation these variables have the constant values they have in simulation No. 1, namely $r = R = \bar{R} = .0525$, $q = 1$, except that in simulations 9, 10, 12, 13, after-tax yields are 98% of .0525.

** Represents a change in long-run expectations other than as represented by the adaptive expectations mechanism of equation (III.3).

TABLE 3

TIME PATHS FOR AGGREGATE CONSUMPTION (C), MARKET VALUE OF WEALTH (W), AND THE SAVING RATIO (S), FOR VARIOUS SIMULATIONS

Simulation Number and Type	1 Neutral			2 Easy Money			3 Capital Gains-- Short Run			4 Capital Gains-- Long Run			5 Eased Liquidity Constraint			6 Differential Liquidity Constraints		
YEARS	C	W	S	C	W	S	C	W	S	C	W	S	C	W	S	C	W	S
1969	592.5	1894.8	.061	592.5	1894.8	.061	592.5	1894.8	.061	592.5	1894.8	.061	596.2	1894.8	.056	592.0	1894.8	.062
1970	607.9	1933.3	.066	607.9	1933.3	.066	607.9	1933.3	.066	607.9	1933.3	.066	614.3	1930.1	.057	607.2	1934.0	.068
1971	624.2	1976.3	.071	624.2	1976.3	.071	624.2	1976.3	.071	624.2	1976.3	.071	633.0	1967.4	.059	623.5	1978.2	.073
1972	641.3	2023.8	.076	655.4	2111.2	.054	650.6	2108.6	.089	650.6	2108.6	.089	652.1	2007.0	.061	640.7	2027.5	.079
1973	660.1	2076.3	.079	687.2	2243.5	.038	681.3	2258.9	.103	676.2	2256.1	.109	671.8	2049.2	.063	659.7	2082.1	.083
1974	683.4	2133.2	.078	706.2	2271.4	.042	705.6	2336.4	.101	701.7	2339.3	.107	693.4	2094.4	.064	683.0	2141.5	.081
1975	707.6	2190.8	.076	729.5	2302.9	.040	730.7	2415.8	.101	727.7	2423.6	.105	715.9	2141.6	.064	707.2	2202.1	.080
1976	732.1	2249.2	.075	738.5	2234.8	.057	744.9	2402.0	.088	742.7	2414.4	.092	739.3	2190.7	.065	731.8	2263.5	.079
1977	756.8	2308.6	.073	748.3	2184.5	.076	758.0	2376.0	.077	756.4	2391.8	.080	763.2	2241.8	.065	756.6	2326.3	.079
1978	781.5	2369.8	.075	772.5	2246.0	.077	782.7	2439.1	.077	781.6	2457.6	.080	787.8	2294.9	.065	781.5	2391.1	.079
1979	806.8	2433.1	.075	796.6	2310.4	.079	807.9	2504.4	.077	807.2	2525.3	.080	812.9	2350.1	.066	807.0	2458.2	.079

TABLE 3 (cont'd)

TIME PATHS FOR AGGREGATE CONSUMPTION (C), MARKET VALUE OF WEALTH (W), AND THE SAVING RATIO (S), FOR VARIOUS SIMULATIONS

Simulation Number and Type	Neutral			7 Temporary Surcharge Plus Capital Gains			8 Temporary Labor Income Tax			9 Temporary Property Income Tax			10 Temporary General Income Tax			11 Permanent Labor Income Tax		
YEARS	C	W	S	C	W	S	C	W	S	C	W	S	C	W	S	C	W	S
1969	592.5	1894.8	.061	590.0	1894.8	.046	589.5	1894.8	.050	593.0	1894.8	.057	590.0	1894.8	.046	584.6	1894.8	.058
1970	607.9	1933.3	.066	612.9	2002.9	.066	604.4	1925.6	.054	608.1	1930.8	.063	604.5	1923.1	.051	599.5	1930.6	.063
1971	624.2	1976.3	.071	637.4	2127.2	.084	620.3	1960.4	.059	624.0	1971.4	.068	620.1	1955.5	.056	615.3	1970.7	.068
1972	641.3	2023.8	.076	654.9	2185.7	.089	637.0	1999.1	.063	640.7	2016.8	.073	636.4	1992.1	.060	631.9	2015.4	.073
1973	660.1	2076.3	.079	674.5	2250.1	.093	654.4	2042.1	.067	658.7	2067.4	.078	653.3	2033.2	.065	649.6	2065.0	.077
1974	683.4	2133.2	.078	692.6	2230.9	.092	678.7	2089.8	.081	682.2	2122.9	.079	677.5	2079.1	.082	672.1	2119.4	.076
1975	707.6	2190.8	.076	706.4	2209.7	.078	702.9	2149.7	.079	706.5	2181.3	.077	701.7	2139.6	.080	695.7	2175.0	.075
1976	732.1	2249.2	.075	731.0	2269.8	.077	728.1	2210.5	.078	731.2	2240.2	.076	727.0	2201.0	.078	719.7	2231.3	.074
1977	756.8	2308.6	.075	755.4	2331.0	.077	753.5	2271.8	.076	756.0	2300.1	.075	752.6	2263.0	.077	744.0	2288.6	.073
1978	781.5	2369.8	.075	780.0	2394.1	.077	778.9	2334.2	.076	780.8	2361.7	.075	778.1	2325.9	.076	768.2	2347.5	.073
1979	806.8	2433.1	.075	805.0	2459.6	.078	804.3	2398.2	.076	806.1	2425.3	.075	803.6	2390.2	.076	792.8	2408.4	.074

TABLE 3 (cont'd)

Simulation Number and Type	Neutral			12 Permanent Property Income Tax			13 Permanent General Income Tax			14 No Liquidity Constraint			15 No Liquidity Constraint, Labor Tax			16 No Liquidity Constraint Easy Money			17 No Liquidity Constraint Capital Gains		
YEARS	C	W	S	C	W	S	C	W	S	C	W	S	C	W	S	C	W	S	C	W	S
1969	592.5	1894.8	.061	594.3	1894.8	.055	586.3	1894.8	.052	597.7	1894.8	.054	595.2	1894.8	.041	597.7	1894.8	.054	597.7	1894.8	.054
1970	607.9	1933.3	.066	609.4	1929.5	.060	600.9	1926.8	.057	617.0	1928.7	.053	614.4	1920.6	.040	617.0	1928.7	.053	617.0	1928.7	.053
1971	624.2	1976.3	.071	625.5	1968.6	.065	616.5	1963.2	.062	637.0	1963.4	.053	634.3	1946.4	.040	637.0	1963.4	.053	637.0	1963.4	.053
1972	641.3	2023.8	.076	642.3	2012.3	.070	632.8	2004.0	.067	657.7	1998.9	.053	654.9	1972.4	.038	675.7	2085.2	.027	665.5	2082.6	.068
1973	660.1	2076.3	.079	660.0	2060.8	.075	649.9	2049.6	.072	679.0	2035.4	.053	676.1	1998.5	.038	717.3	2196.3	.002	694.8	2216.3	.084
1974	683.4	2133.2	.078	682.6	2114.4	.075	671.5	2100.3	.074	700.9	2073.1	.053	697.9	2025.0	.054	740.2	2195.4	.005	717.0	2279.0	.085
1975	707.6	2190.8	.076	706.2	2169.6	.074	694.4	2153.0	.072	723.5	2112.1	.053	720.5	2064.4	.054	763.4	2192.6	.006	740.2	2345.1	.086
1976	732.1	2249.2	.075	730.9	2225.8	.072	718.2	2206.9	.071	746.8	2152.5	.053	743.8	2105.4	.054	766.7	2095.5	.019	755.6	2322.5	.073
1977	756.8	2308.6	.075	755.7	2282.8	.071	742.7	2262.1	.070	770.7	2194.5	.054	767.7	2147.9	.055	768.2	2021.8	.046	771.6	2287.3	.058
1978	781.5	2369.8	.075	780.6	2341.2	.071	767.0	2318.3	.070	795.2	2238.2	.054	792.1	2192.3	.055	790.9	2059.0	.049	797.3	2335.0	.057
1979	806.8	2433.1	.075	805.7	2401.4	.072	791.7	2376.3	.070	820.2	2283.6	.054	817.1	2238.3	.055	814.5	2099.3	.050	823.3	2383.4	.057

The simulations are hypothetical even though they are labelled with real calendar years. The first year corresponds to 1969. In particular, actual disposable income of historical 1969, $631.6 billion, will be disposable income for our 1969 as well, except in those simulations where tax surcharges are imposed. The 11 periods of each simulation are labelled 1969-1979.

Time paths for C and S are graphed for selected simulations in Figures 5 and 6. In examining these, it should be recalled that generally we have simulated both halves of a cycle in whatever exogenous variable is being changed. Thus in case 2, r first declines, then rises. In deriving estimates of various marginal propensities and elasticities, however, only the first period in which a change occurs is of interest to us, since it is only that period that *ceteria paribus* really obtains. By the next period people have begun to react to the changed environment.

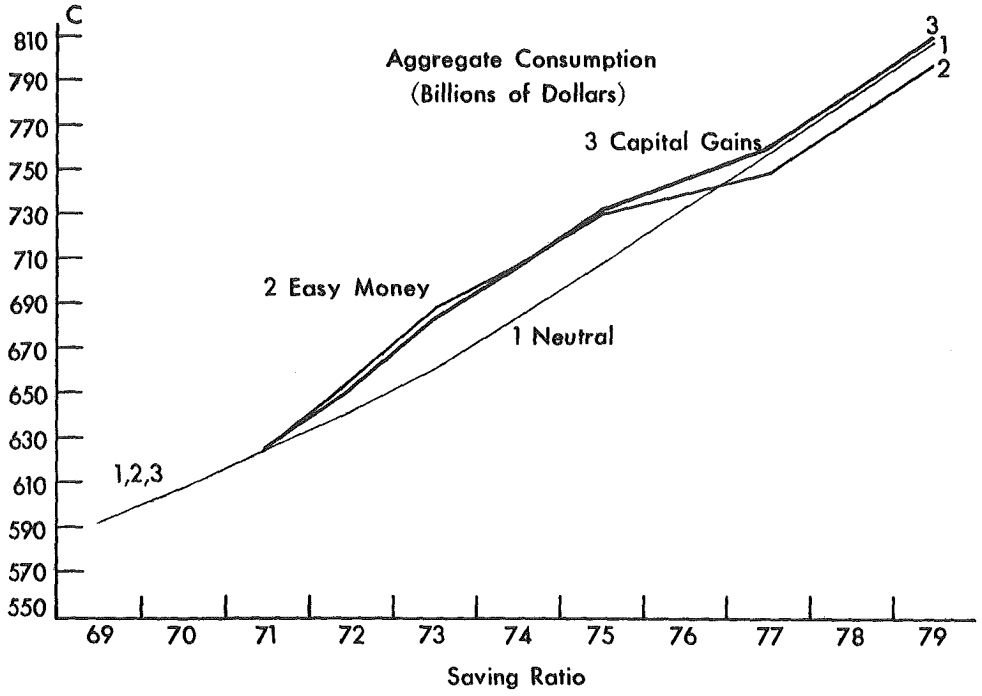
It is assumed that in the years prior to the start of the simulations r and R have been constant at .0525 long enough for .0525 to be regarded as the normal level for both. Hence $\bar{R} = .0525$ and $q = 1$ initially. With regard to the obligatory saving required of younger cohorts, it will be assumed that all previous payments have been made on schedule and that no secondary borrowing has occurred.

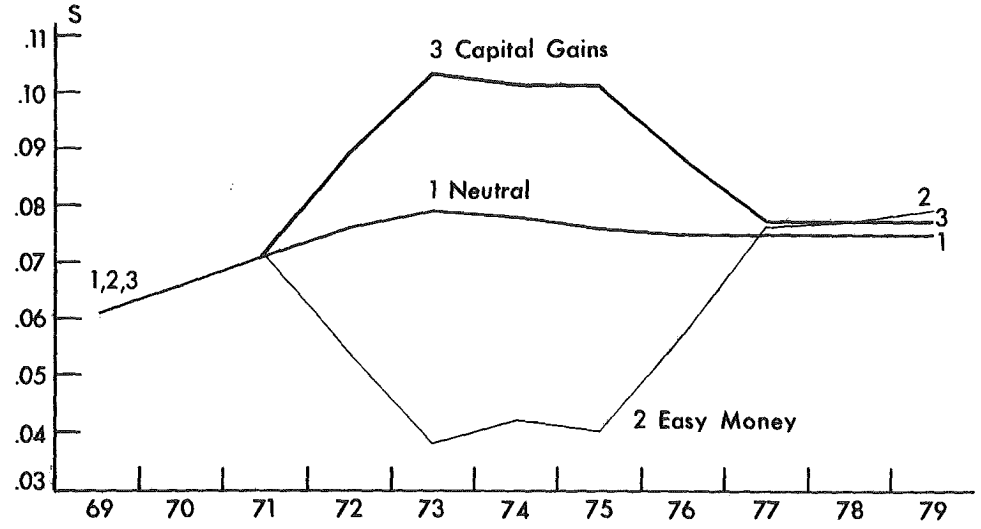
Comparisons of certain of the simulations below will permit us to obtain approximate estimates of the marginal propensity to consume from total resources and its components. For reference we have calculated the average propensity for 1969 in our simulations. This average, the ratio of aggregate consumption to the present value of aggregate total lifetime resources, is .055. In a world from which liquidity constraints are absent, the marginal and average propensities are equal for a life cycle model.

An examination of simulations 1, 2, and 3 indicates that both lower interest rates and higher capital incomes stimulate consumption. In the former case (2) the actual disposable income of individuals has not changed (relative to case 1). Income streams from capital and from labor have not changed, though they are discounted at a new interest rate. In the latter case (3) disposable income has increased, since R, the earning stream from capital, has risen.

In both cases there has been an unanticipated increase in W, having a positive income or wealth effect on present consumption. The substitution effect works in opposite directions in cases 2 and 3, favoring current consumption in the former where r declines and

CHART V
SIMULATION PATHS 1-3

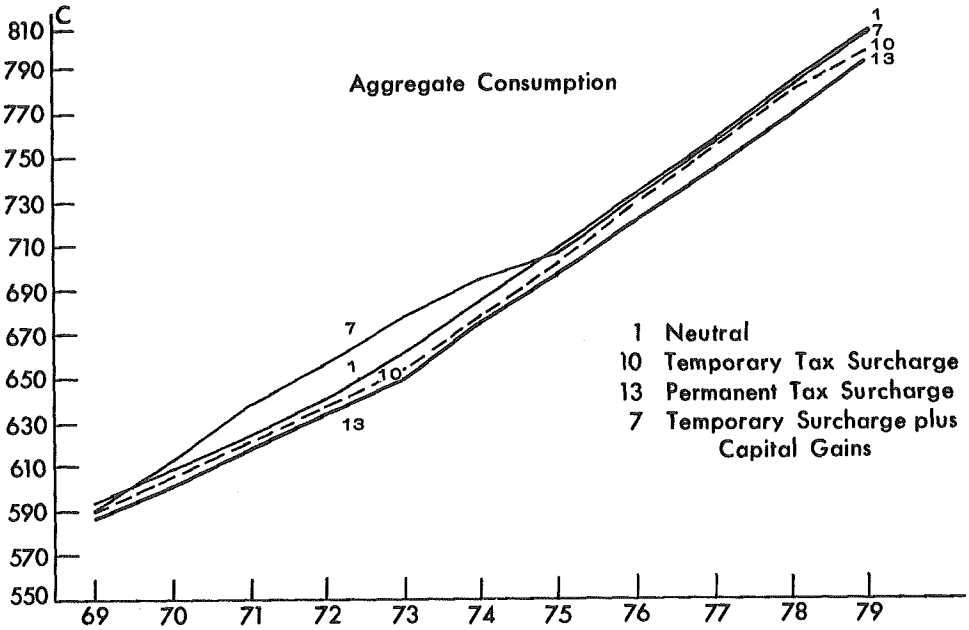


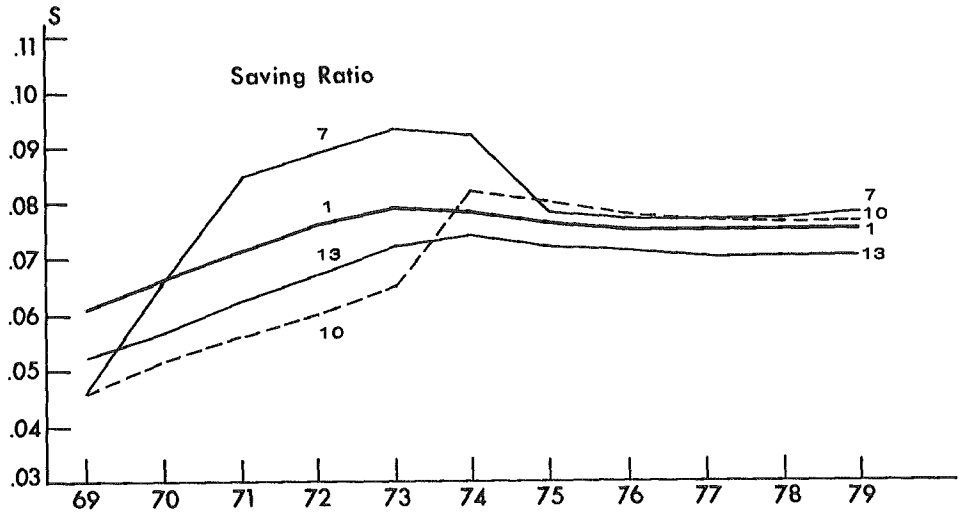


SOURCE: Table III; Paths 1, 2 and 3

Chart VI

SIMULATION PATHS 1, 7, 10, 13





SOURCE: Table III, Paths 1, 10, 13, 7 respectively

working against current consumption in the latter case where r rises.[13]

Because we have observations on the two different cases we can derive approximate magnitudes for an aggregate marginal propensity to consume from wealth and for an interest-elasticity of consumption. Our technique as demonstrated in the appendix indicates that the marginal propensity to consume from wealth is of the order of .09 to .12.

This is our second estimate of the marginal propensity to consume from total resources. It is considerably higher than our finding above, affirming the theoretical reasoning about the effects of liquidity constraints. Some caution is required in attaching significance to the magnitude of the difference, however, since the current estimate, for reasons indicated in the appendix, is perhaps the least precise we have attempted. The indicated interest elasticity of consumption is between -.02 and -.43.

In an attempt to evaluate the influence of liquidity constraints on the marginal propensity to consume from wealth, we have repeated simulations 1, 2, and 3 in simulations 14, 16, 17 with the liquidity constraint relaxed sufficiently to insure that no cohorts were constrained in the years 1969-1972. Repeating the calculations described in the appendix, we find the probable values of the marginal propensity to consume out of wealth to be bracketed by .08 to .14. The interest elasticity is -.04 to -.72.

The changes in r and R in cases 2 and 3 occur in two equal steps in 1972 and in 1973 in the simulations. The percentage capital gains, measured by $\Delta q/q$, are roughly the same in both cases and roughly equal in the two years. Consumption also is increased in two roughly equal steps, but more in case 2 than in case 3. The consumption increments over the neutral case are $14.1 billion for 1972 and $27.1 billion for 1973 for case 2; $9.3 billion for 1972 and $21.2 billion for 1973 for case 3. As q declines in two steps in 1976 and 1977, the excesses of C over the neutral case also decline. Why does simulation 3 exhibit a smaller impact on consumption? The difference is in the direction of the substitution effects. It is shown even more dramatically by the saving ratios (.103 in 1973 for case 3 as against .038 for case 2.) The correspondingly greater capital formation in case 3 eventually leads to greater consumption there despite the substitution effect favoring saving.

[13]Recall that because expectations are such that equilibrium will be re-established with $R = r = \bar{R}$, and because $\bar{R}$ is influenced by R, r is expected to rise even in the case where it is R that has changed initially.

Simulation 4 differs from simulation 3 only in that in 1973 long-run profit expectations change. The substitution effect favoring current saving is greater, but certainly more moderate than the difference between the long-run profits rates (72 basis points in 1973) might superficially indicate. The explanation lies in the fact that the current levels -- and hence the expected levels for the immediately following years -- of R and r are the same in the two cases (R = .0725, r = .0525 in 1973). Since the expected rates in the near future have more influence than those further distant, the effects on consumption are not too dissimilar.

Simulations 5, 6, and 14 examine the influence of changes in the instruments affecting liquidity. Case 5 differs from case 1 only in that φ is .5 in the former rather than 1.0. In case 14, φ is zero. That is, it is possible in case 5 (14) for individuals to borrow up to half (the entirety) of their contractually required saving at the market rate of interest r rather than at a penalty rate r_b. In case 1 all of the borrowing incurs the penalty rate. The result in case 5 is to increase consumption by $4 billion to $11 billion in various years, with wealth accumulation suffering a concomitant decrease ($83 billion over the 10-year period). In case 14 consumption exceeds that of case 1 by $5.2 billion to $18.9 billion. Accumulated wealth is less by $149.5 billion.

In simulation 6 the two income groups face different liquidity constraints. As in case 5, the higher income group is assumed to be able to forego half of its required saving costlessly (φ = .5) and to be able to borrow beyond that at a rate of 7 percent (r_b = .07). The lower income group may not borrow costlessly (φ = 1.0), and they must pay more for the funds they do borrow (r_b = .10). Relative to those in case 1, the credit market conditions are eased for the higher income group and are more stringent for the lower income group. The negative incentive on the consumption of the poorer group has a stronger influence, as aggregate consumption declines slightly relative to case 1.

The savings ratios are better indicators of the effects in the later years. By then greater disposable income due to more capital accumulation permits more absolute consumption. By 1979 wealth in case 6 exceeds wealth in case 1 by $15.1 billion.[14]

[14]Most of this, however, reflects an artificiality in the simulations. In our calculations, not only reduced consumption but also reduced penalty interest payments permit greater accumulation. The institutions engaging in lending are considered exogenous to the household sector. Hence disposable income equals not just consumption plus saving, but rather consumption plus saving plus penalty interest premiums on loans. We intend in further calculations to redistribute these payments as incomes to wealth-owners.

A comparison of cases 8 to 13 with case 1 indicates that a labor income tax reduces consumption, a property income tax increases it, and a general income tax -- a combination of labor and property income taxes -- decreases consumption, but less than the labor income tax alone initially. In these simulations the time paths of other variables, including the before-tax rates of return, r, R, and $\bar{R}$, are the same as in the reference simulation, case 1. The variable q remains at par, in the face of the tax on property income, because it is assumed that R and r are lowered in the same proportion.

The taxes on labor income have only wealth or income effects on consumption. They do not affect rates of return, do not have a substitution effect. The property income tax has both, with the substitution effect in favor of current consumption (since after-tax rates have declined) being stronger than the effect of the income lost in tax payments.

In principle a tax surcharge that is expected to be temporary should have little effect on current consumption, the effect being spread over the remaining years of life. Comparing cases 1 and 8 for 1969, we find consumption reduced by $3.0 billion. The aggregate expected reduction of lifetime resources is $47.8 billion (not shown). Thus our third estimate for the marginal propensity to consume from total resources is .063.

To test our theoretical proposition that operative liquidity constraints may increase this marginal propensity, we have duplicated the comparison of cases 1 and 8 with a relaxed liquidity constraint ($\varphi = 0$) in cases 14 and 15. We find a reduction in first period consumption of $2.5 billion, indicating that the tighter credit market conditions of simulation 8 enhance the effectiveness of the tax increase by about 20 percent. The corresponding marginal propensity to consume is .052, close to the average propensity (and theoretical unconstrained marginal propensity) of .055 and somewhat lower than the liquidity-constrained .063 found in simulations 1 and 8.

In simulation 11, in which labor income streams are reduced uniformly for all years, a $162.4 billion decrease in total resources leads to a $7.9 billion decrease in first period consumption. The corresponding marginal propensity, which is roughly the marginal propensity to consume out of total resources, is .049.

As we noted above, a general income tax increase does not initially lower consumption as much as a labor income tax alone because of the disincentive effect on saving of lower expected rates of return. The decreased capital accumulation eventually leads to a reversal,

however, with more consumption occurring in the case of the labor income tax alone. It must be recalled, however, that we do not attempt to take into account the system-wide response of before-tax rates of return to variations in the size of the capital stock.

One of the explanations offered for the apparent ineffectiveness of the tax surcharge of the 1960's is that capital gains enjoyed by individuals had a more than offsetting effect on consumption. We have found results consistent with this explanation in simulation 7. There we imposed a temporary (five-year) reduction of 2 percent on all income as in case 10. The corresponding tax revenue is $12.6 billion for 1969. In addition, we assumed increases in capital earnings starting in the second year as indicated in Table 2. In the first year, before the first increments to wealth, consumption is less than in the standard case by $2.5 billion. With the first capital gains, however, consumption increases by $5.0 billion, and ultimately by $14.4 billion, relative to case 1.

In the discussion of the use of temporary changes in taxes as stabilization policies, a consumption tax has been suggested as a more powerful alternative to an income tax.[15] A temporary consumption tax contains, as an income tax does not, an incentive to postpone spending. It has a substitution effect as well as an income effect. For illustration, we have simulated (but not tabulated) the results of a flat rate consumption tax, unexpectedly imposed and known to last only one year. As expected, this tax is much more effective than an equivalent income tax in discouraging current consumption. Comparing equal yield ($14.5 billion) one year consumption and income taxes, we found the former reduced consumption by $13.2 billion (relative to case 1) while the latter was only one-eighth as effective, cutting consumption by only $1.7 billion.

V. Conclusions

1. The method is promising. The model generates aggregates which are realistic and plausible in magnitude and in their simulated time paths. We are certainly not entitled to conclude that American households are actually conforming to the life-cycle model, much less to our specialization of it. But assuming that they are doing so gives reasonable results. In further work more attention should and can be paid to sources of differences among households other than

[15]See Tobin, 1969, pp. 211-2.

age; to the effects of uncertainties on consumption and accumulation plans; to the diversity of assets available for saving; and to other features of the "real world" that the model of the present paper omits or oversimplifies.

2. Revaluations of nonhuman wealth do, according to the model, have important effects on consumption and saving. But these effects depend significantly on the nature of the revaluation, in particular on the concomitant changes in current and expected interest rates. In our "easy money" simulation (2), a reduction of interest rates brought about by monetary policy increased consumption by 16.1 percent of the increase in wealth it accomplished. In simulation 4, wealth and consumption both rise because of a nonmonetary shock: profits and expected profits rise. The increase of consumption is 8.8 percent of the increment of wealth.

3. Liquidity constraints make a difference. In our simulations they are binding on younger and poorer segments of the population. In their absence, the marginal propensity to consume currently from an increase in consumable resources -- current wealth plus the present value of labor income -- would be the same as the average, about .055. Our simulations indicate the marginal propensity to consume from current wealth to be .09 to .12. The excess is attributable to the role of realizable capital gains in relieving liquidity constraints on current consumption.

For the same reason, the marginal propensity to consume from current disposable income is higher than it would be in a perfect capital market. Our simulations of tax changes give permanent changes 2.6 times as much effect on current consumption as temporary (five-year) changes. This difference is in the expected direction, but in a model without liquidity constraints it would be larger, 3.4 times instead of 2.6 times. These comparisons would be more striking if our "temporary" tax rise lasted a shorter time.

4. Monetary policies tighten or relax liquidity constraints. Changes in those constraints, including the differential between borrowing and lending interest rates, have important effects in themselves, as comparison of simulations 1, 5, 6, and 14 indicates. Moreover, the tightness of liquidity constraints helps to determine the effectiveness of other policy instruments. A temporary tax increase, for example, is 1.2 times as powerful in the tight credit simulation (8 relative to 1) as in the easy credit simulations (15 relative to 14).

To construct a complete story of the linkages of monetary policy to the propensity to consume, it would be necessary to specify how given Federal Reserve operations simultaneously change interest

rates, capital values, and liquidity constraints. We have not attempted to provide those links in the chains of causation.

APPENDIX

DATA SOURCES AND INITIAL CONDITIONS

The initial population was that of the United States on July 1, 1969 as estimated in *Current Population Reports* (Series P-25, 1970, Table 1, p. 12) for ages 0-84. The estimated 1.29 million people aged 85 and above were ignored. The birth rates by single age of mother were interpolated from grouped data for 1967 reported in the *Statistical Abstract of the United States* (1969, p. 48). As noted in the text the birth rates for women younger than 18 were set at zero. To compensate, birth rates for ages 18-21 were increased slightly.

Mortality rates for 1967 for ages 0-69 also came from the *Statistical Abstract* (1969, p. 54). For ages 70-84, mortality rates were interpolated from crude death rates calculated from grouped data in *Demographic Yearbook of the United Nations* (1969, pp. 169, 603). The interpolations from the two sources were constrained to be continuous at age 69.

The simulations required the assigning of all males and of females younger than 18 to cohorts. No direct observations were available on the initial values of the

$$N_{pm}(x, a),\ N_{pmc}(x, a),\ N_{pmt}(x, a),\ N_{pfc}(x, a),\ \text{or}\ N_{pft}(x, a)$$

(number of adult males, male children, male teenagers, female children, female teenagers, respectively, aged a in cohorts with adult females age x). The $N_{pm}(x, a)$ were approximated by frequency distributions $\pi_m(x, a)$ of husbands aged a by age of wife (x).

Unmarried males were assigned with the same distribution used for husbands. Thus for $N_m(a)$ the total number of males aged a, $N_{pm}(x, a)$ is given by

$$N_{pm}(x,a) = \pi_m(x,a) \cdot N_m(a)$$

Note

$$\sum_x N_{pm}(x,a) = \sum_x \pi_m(x,a) \cdot N_m(a)$$

$$= N_m(a) \sum_x \pi_m(x,a) = N_m(a)$$

since for the frequency distribution $\pi_m(x, a)$ the sum $\sum_x \pi_m(x, a) = 1$. The $\pi_m(x, a)$ were interpolated from tables grouped both by x and by a in *Current Population Reports* (Series P-20, 1969, Table 17, p. 83).

The distributions for children and teenagers were interpolated from tables in *Current Population Reports* (Series P-20, 1969, Table 6, p. 51) which strictly applied only to age distributions of youngest children. This distortion was somewhat offset, however, by the fact that the distributions as used were assumed to apply to the age of the mother (cohort age) while the reported distributions were by age of head of household. Further, the distributions were restricted to be consistent with the assumptions that females do not bear children before age 18 nor after age 49.

The preceding discussion applies to the derivation of the initial distributions among cohorts. Distributions for later years of the simulations are generated through the use of the appropriate birth and mortality rates as the simulations progress. The only additional demographic assumptions required concern the assignment of new 21-year-old males and 18-year-old females, and of the males surviving the disbanding of the 84-year-old cohort. It is assumed that the initial distribution of 21-year-old males, $\pi_m(x, 21)$ applies in all future years as well. Similarly, the new cohort forming with 18-year-old females is assigned $\pi_f(a) \cdot N_f(18) = N_{pm}(18, a)$ males where the π_f are constant over time and are interpolated from *Current Population Reports* (Series P-20, Table 17, p. 83).

The implication of assigning to the new cohort a full complement of males of various ages is that some males originally assigned to one cohort are reassigned to a younger cohort. A more realistic model of household formation would of course resolve the problem, but such

completeness is not feasible. The current simplification has only minor effects and only on the younger cohorts.

The two income groups correspond to groups above and below the Level 1 Poverty Line as defined in Projector and Weiss (1966, p. 37) (roughly $3000 for a family of four in 1963). The indication there, and the assumption we have used, is that 70 percent of the population is in the higher income group and that their income is four times as great as that of individuals of the same age and sex in the lower income group. After approximate adjustment for omitted items (life insurance cash balances, pension rights, annuities) the average net worth of group I was also about four times that of group II.

The actual wealth profiles by age used as initial conditions were interpolated from Projector (1968, Table S17, p. 316), then scaled up to give a wealth-disposable income ratio of 3.0 for the first year simulated. The four-to-one ratio between net worth of individuals in the two groups was maintained. In interpolating, net worth of zero for cohorts aged 18 and 85 was assumed.

In a similar manner, interpolated labor earnings by age for 1967 from Projector and Weiss (1966, pp. 162-6) were scaled up to $532.1 billion. This is the labor share of disposable income in 1969 consistent with the wealth-disposable income ratio and rate of return on capital assumed. Again the four-to-one ratio between the earnings of the two groups was maintained. Thus a male aged i in group j (j=1,2) earns $\beta^m_{ji} \cdot y^m_{1,40}$ and a female earns $\beta^f_{ji} \cdot y^m_{1,40}$, where $y^m_{1,40}$ represents the labor income of a forty-year-old male in the first group. We have $\beta^m_{2,i} = 1/4 \cdot \beta^m_{1,i}$ and $\beta^f_{2,i} = 1/4\beta^m_{1,i}$. The β^m and β^f are assumed constant over time, while $y^m_{1,40}$ grows exponentially at the rate γ:

$$y^m_{1,40}(t+1) = (1+\gamma)y^m_{1,40}(t) \ .$$

In deriving the β^m and β^f from Current Population Reports (Series P-60, 1969, Table 3, p. 26), the median incomes reported there were

multiplied by the percentage of the age-sex group receiving income to account for participation rates. Since the estimates do not exclude property income, we have the set β^{m}_{ji}, β^{f}_{ji} for $i \geq 65$ equal to zero.

DERIVATION OF AN ALLOCATION RULE: NO LIQUIDITY CONSTRAINTS

For ease of explication we shall present the analysis of this section in terms of a behavioral unit consisting of a single individual. The grand utility function of a cohort will be a sum of individual utility functions, weighted by appropriate equivalent adult weights.

We assume the utility function $u(c_0, c_1 \ldots, c_{a^*-x})$ for an individual aged x has the specific form

$$u(c_0, c_1, \ldots) = \sum_{i=0}^{a^*-x} u(c_i)(1+\delta)^{-i} \frac{s(x+i)}{s(x)} \qquad \text{(A. 1)}$$

where

$$u(c) = A - Bc_i^{-\rho+1} \qquad \text{(A. 2)}$$

and a* is the last age to which individuals survive given the mortality table assumed, δ is the pure rate of time preference, s(x) is the probability of surviving from birth to age x, A and B are arbitrary constants of no consequence (except that B must be positive), and $-\rho$ is the (constant) elasticity of marginal utility. We assume $-\rho = 1.5$. (Assuming a form for u of $u(c) = \log c$ as in Tobin [1967] is equivalent to choosing $\rho = 1$.

Assuming first a world of perfect capital markets with no constraints on dissaving and no divergence between the borrowing and lending rate, the optimal consumption plan results from maximizing the Lagrangean

$$\mathcal{L} = \Sigma u(c_i)\,(1+\delta)^{-i} \frac{s(x+i)}{s(x)} + \lambda \left(W(1+r_0) + W_h - \Sigma c_i (1+\tilde{r}_i)^{-i} \frac{s(x+i)}{s(x)} \right) \qquad \text{(A. 3)}^*$$

*W is multiplied by $(1+r_0)$ since in this discrete model there is a distinction between beginning of period and end of period stocks. W is interpreted as the beginning of period stock and thus earns r_0W in the current period. The model is recursive rather than simultaneous: first production occurs, using the beginning of period capital stock, then the savings decision allocates output between consumption and investment.

W is the market value of non-human wealth

$$W = (\sum_{i=1}^{\infty} R_i^e (1+\tilde{r}_i)^{-i})K = qK \,, \qquad \text{(A. 4)}$$

and W_h is human wealth

$$W_h = \sum_{i=0}^{a^*-x} y_i (1+\tilde{r}_i)^{-i} \frac{s(x+i)}{s(x)} \,. \qquad \text{(A. 5)}$$

The expected labor income i years hence is y_i . $\tilde{r}_i$ is the expected i period rate of interest

$$(1+\tilde{r}_i)^i = (1+r_1^e)(1+r_2^e) \ldots (1+r_i^e) \qquad \text{(A. 6)}$$

where r_j^e is the expected one period rate of interest j periods hence. Differentiating (A.3) and eliminating λ from the resulting first-order conditions yields

$$c_i = \left(\frac{1+\tilde{r}_i}{1+\delta}\right)^{\frac{i}{\rho}} \cdot \frac{W(1+r_0) + W_h}{\sum \left[(1+\tilde{r}_i)^{\frac{\rho-1}{\rho}} (1+\delta)^{\frac{1}{\rho}} \right]^{-i} \frac{s(x+i)}{s(x)}} \qquad \text{(A. 7)}$$

The second factor on the right of (A.7) is a constant independent of i. Consumption per person-year in the ith year exceeds (is the same as, is less than) c_0 if $\tilde{r}_i > \delta$ ($\tilde{r}_i = \delta$, $\tilde{r}_i < \delta$). Since recalculation occurs every year, only the first year of the consumption plan need be actually realized.

DERIVATION OF AN ALLOCATION RULE: WITH LIQUIDITY CONSTRAINTS

Let $\hat{s}$ represent the amount of saving the individual is obligated to do in the current period. As indicated in the text, $\hat{s}$ will equal a fraction φ of A/T plus any payments due on secondary loans undertaken. If an individual wishes to save less than $\hat{s}$, he has only one option. He may borrow at a rate r_b, paying back by making equal

payments against the principal of $1/\hat{T}$ in each of the next $\hat{T}$ years.

Let $\hat{c}$ be the amount of consumption which would result if exactly $\hat{s}$ were saved out of current income. Then the Lagrangean is

$$\mathcal{L}' = \Sigma u(c_i)(1+\delta)^{-i}\frac{s(x+i)}{s(x)} + \lambda_1\left(W + W_h - (c_0 - \hat{c})\Sigma(r_b - r_i^e)\frac{\hat{T}-i}{\hat{T}}(1+\tilde{r}_i)^{-i-1}\right.$$

$$\left. - \Sigma c_i(1+\tilde{r}_i)^{-i}\frac{s(x+i)}{s(x)}\right) \qquad \text{(A. 8)}$$

$$+ \lambda_2(c_0 - \hat{c})$$

The term $(c_0 - c)\Sigma(r_b - r_i^e)\frac{T-i}{T}(1+\tilde{r}_i)^{-i-1}$ represents the net interest loss on new borrowing, $c_0 - \hat{c}$. The constraint associated with the multiplier λ_2 insures that individuals cannot lend at the higher borrowing rate r_b, i.e. $c_0 - \hat{c}$ must be non-negative. Differentiating and solving for the c_0 yields

(A. 9)

$$c_0 = \frac{1}{1+Q}^{\frac{1}{\rho}} \frac{W + W_h + \hat{c}Q}{\Sigma\frac{s(x+i)}{s(x)}\left[(1+\tilde{r}_i)^{\frac{\rho-1}{\rho}}(1+\delta)^{\frac{1}{\rho}}\right]^{-i} + [1+Q]^{\frac{\rho-1}{\rho}} - 1}, \quad \lambda_2 = 0$$

or

$$c_0 = \hat{c}, \quad \lambda_2 \geq 0,$$

where

$$Q = \Sigma(r_b - r_i^e)\frac{\hat{T}-i}{\hat{T}}(1+\tilde{r}_i)^{-i-1}$$

AN APPROXIMATE ESTIMATE OF THE MARGINAL PROPENSITY TO CONSUME FROM WEALTH AND THE INTEREST ELASTICITY OF CONSUMPTION

Let us summarize the influence of r_1^e, r_2^e, ..., r_n^e, ... on aggregate consumption by a single variable r^e. Similarly let us indicate the impact of the R_i^e by R^e. Let W denote aggregate wealth. Then we can write current aggregate consumption as

$$C = C(W(R^e, r^e), r^e, \pi, \ldots) \qquad \text{(A. 10)}$$

π, the current aggregate earnings on capital enter separately because, as noted above, W as a valuation of the earning stream from non-human sources does not include earnings in the current period. We need not specify the other variables affecting consumption since they will be held constant in obtaining our estimates.

Differentiating (A.10) with respect to R^e, r^e, and π yields

$$dC = C_W(W_R dR^E + W_r dr^e) + C_\pi d\pi + C_r dr^e \qquad \text{(A. 11)}$$

where C_W, W_R, W_r, C_π, C_r represent partial derivatives of the functions C and W with respect to the subscripted variables. Now $C_\pi = C_W$ since an extra dollar of market value of wealth and an extra dollar of income -- income in the present period only -- both command the same consumption value today and hence both augment the present value of cohort lifetime resources by the same amount.

Assume that the various partial derivatives C_W etc. are approximately constant in the neighborhood of variation of the values of C, W, π , r^e, and R^e involved in our simulations. Then we may apply equation (A.11) to the non-infinitesimal changes in variables between two of our simulations for which all other variables are unchanged. The 1972 values of the variables for simulations 1, 2, and 3 meet this criterion. Thus dC is the difference in the 1972 values of aggregate consumption between cases 2 and 1: dC = 655.4 - 641.3 = 14.1. For case 3, dC = 9.3. The expression $(W_R dR^E + W_r dr^e)$ is dW, and is 87.4

and 84.8 for the two cases respectively. $d\pi$ is zero for case 2 and 20.3 (not shown) for case 3. Since $C_\pi = C_W$ we may write

$$14.1 = 87.4\, C_W + C_r dr_2^e \qquad \text{(A. 12)}$$

$$9.3 = 105.1\, C_W + C_r dr_3^e \qquad \text{(A. 13)}$$

where dr_2^e and dr_3^e are the changes in r^e from case 1 for cases 2 and 3 respectively.

The dr_i^e are unobservable, but we can bracket their values. From table 2 we see that in case 2, the current interest rate in 1972 is .0425, which is less than that in case 1 by .01. $\bar{R}$, the normal rate to which future rates are expected to return, is less in case 2 than in case 1 by .001 = .0525 - .0515. The differential between the two cases in expected one period rates of return in 1972 is nearly -.01 for early periods and is closer to -.001 for later periods. dr_2^e is some weighted average of these differentials, and is thus bounded by -.01 and -.001. Similarly dr_3^e is a weighted average of 0.0 (differential between cases 1 and 3 in r) and 0.001 (differential in $\bar{R}$). Solving the two equations (A.12, A.13) by using the four sets of boundary values dr_2^e = (-.01, -.001), dr_3^e = (0, .001) yields the solutions for C_W and $C_r \cdot \frac{r}{C}$ presented in the text. In converting to the interest elasticity values of r and C for 1972 in simulation 2 were used.

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DISCUSSION

GARDNER ACKLEY

Before getting down to my specific assignment, I want to offer my congratulations to Frank Morris and his associates for centering the attention of this Conference on consumer spending.

Consumer spending is one of those subjects about which my own intellectual history can best be described as "the more I know the less I know." To some extent I think this description summarizes the experience of the entire economics profession -- or at least of that generation whose careers began, as mine did, just in time to be caught up in the Keynesian revolution. It didn't take most of us very long -- after digesting Keynes, and after looking at early national income and product data -- to be convinced that we knew all we needed to know about aggregate consumer spending, and that our real puzzles lay in investment spending, and in the macroeconomics of prices, wages, and income distribution.

But confidence -- my own, at least -- that consumer spending is easy to understand, has progressively deteriorated. The deterioration of my confidence has, if anything, accelerated in the last few years as I have been asked to pose as a GNP forecaster, and as I have realized more concretely than ever before how much difference the shift of a few tenths of a percentage point in the saving rate can make for a forecast, and thereby for a policy prescription. It does seem to me that economists have been continually peeling away one after another layer of mystery or misunderstanding about consumer spending, only to find another, denser one below. I hope -- but I

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must say I am not very confident -- that our discussion here today will get us at least a little closer to the kernel of truth. In any case, I regard the effort as extremely high on the priorities list for macroeconomics. That is why I am glad that Frank and his associates planned this Conference on the theme of consumer spending.

Part of our subject matter traces back to the resolution of one of our earliest puzzles about consumption. Macroeconomic observation appeared to show that, in the long run, aggregate consumption was roughly proportional to income, not a declining fraction of a rising income as Keynes' introspection, the evidence of budget studies, and national income and product data for the 1930s had all suggested. The belief that the marginal propensity to consume was less than the average -- and perhaps even declined as income rose -- was of course a powerful strand in the Stagnationist position. It supported apocalyptic visions of an inevitable crisis of capitalist development -- as well as a cheap case for income redistribution.

Causes of Differences in Savings Rates

But Jim Duesenberry, Jim Tobin, Franco Modigliani (who are here), Milton Friedman (who is at least well represented here), and others helped us to see how we could reconcile data that related to differences in individual consumption behavior at different levels of individual income with data relating to changes in aggregate consumption as aggregate income changed. One part of this reconciliation involved the recognition that much of the *difference* in savings rate at different incomes had to do with *stages in the life cycle:* many of the dissavers or low savers with low incomes were young families, whose consumption reflected not only current income but also expected future income; others were families with retired earners engaged in planned dissaving. Many of the high-income, high savers were people at the peak of their earning capacity, repaying their early debts and preparing nest eggs for retirement. Further cases of high saving at high incomes and low or negative savings at low incomes perhaps represented persons with substantial temporary or windfall components of income, positive or negative. Still another part of the reconciliation consisted of correcting the misclassification, as "consumption," of consumers' investment in durable goods.

Now it is one thing to recognize that these factors may well provide all or an important part of the reconciliation between

cross-section and time-series evidence on consumption; or to hypothesize on the basis of them that consumption depends not only on current but also on past and expected future incomes; or even perhaps to conclude that changes in the age distribution and the family structure of the population might powerfully affect aggregate personal saving. But it is quite a further step to hypothesize, as Modigliani explicitly does in his paper here that "the consumption of a representative household over some arbitrary short period of time, such as a year or a quarter, reflects a more or less conscious attempt at achieving the preferred distribution of consumption over the life cycle, subject to the constraint imposed by the size of the resources accruing to the household over its lifetime"; and then to use this hypothesis as the entire basis for a theory of consumption behavior -- as does Modigliani, and as do Tobin and Dolde.

I happen to be singularly unconvinced as to either the correctness or the usefulness of this hypothesis. But my problems with it are made rather more acute in the case of Dolde and Tobin than they are in the case of Modigliani and collaborators. For the empirical hypothesis that aggregate consumption spending depends on wealth as well as on income does not require full acceptance of the individual behavioral hypothesis which I have just quoted. It could be based on far less restrictive -- possibly even on quite unrelated -- behavioral hypotheses. But the Dolde and Tobin simulations, and their implications for prediction and policy, are based on the full and complete acceptance of this individual behavioral principle (more royalist than the King, as Jim Duesenberry suggested) -- but with one major modification, namely, "subject to liquidity constraints."

Three Different Impacts of Wealth on Consumption

Now if one is going to simulate the life cycle hypothesis of consumer behavior, I doubt that one could do a better job, at least as a starter, than Dolde and Tobin have done. They clearly show that the theory requires at least three different kinds of impacts of wealth on consumption, depending on whether a change in wealth consists of (a) wealth accumulated from saving, with no change in the parameters that are assumed to effect the desired level and pattern of accumulation; or (b) changes in wealth associated with changes in the expected earning power of real assets; or (c) changes in asset values associated with changes in interest rates. This should make us cautious about empirical macro-functions which use a single undifferentiated wealth variable, and that is all to the good.

Nevertheless, let me make a couple of complaints regarding their methodology which seem largely independent of my basic difficulties with the underlying hypothesis. Then I will try to explain briefly some of my more fundamental reservations about the entire exercise.

As I understand it, Dolde and Tobin assume a population which consists of 134 different kinds of families. These families are cross-classified according to 67 different ages of the wife (ranging from 18-85), and according to two income brackets (poor and non-poor), the average income of the former being taken as 1/4 that of the latter. The initial number of families in each class and the initial differences among their characteristics more or less correspond -- in many but not all respects, and with various adjustments and simplifications -- to the actual population, income, wealth, and debt structures of 1969. But each family within each of these 134 types is assumed to be "actuarily average." That is, it has the same size, age and sex composition, income level, lifetime income profile, debt-repayment schedule, access to borrowing, and wealth as every other family in its class. Therefore, it also has the same consumption and saving.

Realism of the "Average" Liquidity Constraint Assumption

One hundred thirty-four family types must be a fairly large number to manipulate, and I can well understand the reasons for not having twice, or 10 times, as many. Yet I cannot agree that the crudeness of the division is of slight consequence for the results, which is what the authors sometimes suggest. The model might be adequate if it were not for the very great importance of the one qualification that the authors impose on the strict life cycle hypothesis, namely, "subject to liquidity constraints." The reason is that liquidity constraints are not symmetrical in their effects. They constrain consumer spending when liquidity is inadequate, but an excess of liquidity does not, I think, symmetrically expand such spending.

Take a specific example. Assume that the average non-poor family, wife aged 48, is marked by a structure of family size and composition, income and income expectations, wealth, debt and required debt repayment -- and hence of consumption and saving -- such that the total impact on consumption of a given temporary tax increase could be spread in unconstrained fashion over the remaining years of the family's life in a way that would maximize discounted lifetime utility. For the *average* family in this class, in other words,

there is no liquidity constraint. But when we recognize that non-poor families, wife aged 48, comprehend a wide variety of different family sizes and compositions, incomes and income expectations, wealth, required debt repayment -- and hence of individual consumption and saving -- then liquidity constraints might require or induce a great many of these families to meet the added tax payment all or largely out of current consumption. The remaining families in the class would not, however, spend correspondingly more. Simulating the behavior of the *average* family in each of 134 classes thus seems to me to fail to do justice to the wide variety of individual situations that must impose liquidity constraints on a great number of them. If liquidity constraints are important -- and I am sure that I believe they are more important than do our authors -- then the "actuarily-average" assumption must greatly underplay their impact.

While we are on the liquidity constraint assumption, let me also complain that in the simulations used, these constraints are not assumed to be substantially, or at all realistically, constraining. As I understand it, all families are assumed to have quantitatively unlimited access to credit; and have no hesitation in using it, so long as the substitution of current for future consumption will increase discounted utility. The constraint lies in the fact that, if families have contractual debt repayments, then all or some part of their borrowing -- at the margin -- may be at a penalty interest rate. In most of the simulations, the penalty interest rate is 7 percent. In one case the penalty rate for the poor is 10 percent. I wonder if that realistically or significantly reflects the kind of liquidity constraints which exist.

Now the more severe the constraint imposed by a family's illiquidity, and the more numerous are the families exposed to such constraint, then, as the authors show, the less significant will be the effects on consumption of changes in wealth and/or interest rates, and the greater the effect of temporary tax changes. If the authors have greatly underrepresented the effects and the incidence of these constraints, their simulations must give quite misleading quantitative results. I may add that I am puzzled by the assumption that monetary policy can significantly and differentially affect -- or ever afford to be aimed to affect -- the two particular dimensions in which the authors assume liquidity constraints to vary. Structural policies, yes; direct controls, yes; but general monetary policy? I very much doubt it.

Consideration of Changes in Earned Income

My second general problem with the simulation model relates to its neglect of what I had assumed was the most powerful implication of the life-cycle hypothesis, namely: that consumption should vary with changes in the expected level and time pattern of future labor income. In the present treatment, consumption is seen to vary with changes in the expected earning power of non-human wealth but not in that of human wealth. Yet the latter quantity is far larger and -- I should guess -- every bit as variable. The model contains no parameter (other than the rate of taxation of labor income) which is readily variable to explore the implications of changed expectations of labor income. My own guess is that small changes in expected labor earnings should swamp the effects of substantial changes in all the other variables considered in the model, and that changes in these expectations often -- even typically -- accompany changes in monetary and fiscal policy, and changes in asset yields.

To be sure, the present study is aimed at the effects of policy, and of monetary policy in particular. But as Tobin and Dolde continue their simulation studies, I hope they will find some way to expand their model to permit systematic analysis of variations in expected labor incomes.

Now let me turn to my problems with the hypothesis itself in its broadest form. What does it assume?

First: It assumes that each family has at all times a *definite, conscious* vision

- of the family's future size and composition, including the life expectancy of each member;

- of the entire lifetime profile of the income from work of each member -- after the then applicable taxes;

- of the present and future extent and terms of any credit available to it;

- of the future emergencies, opportunities, and social pressures which will impinge on its consumption spending;

- of present and future interest rates and rates of return on any equities it owns (after taxes); and (as one part of the solution of a system of simultaneous equations)

> of the amount of its saving or dissaving in every future year, and of the wealth it will own and the debts it will have at all future points of time.

Does each family have at all times such a definite, conscious vision of its economic future? I wonder. Indeed I wonder if it even always knows a number of the *current* facts assumed to be most important: the current available rates of return on savings; the current cost and availability of credit; even its current income, consumption, saving, and asset values. Yet for the assumed process of intertemporal allocation to be meaningful, and for it to have most of the consequences deduced, the family needs to have a reasonably clear vision on these and other matters extending well into the future.

I have been trying to use that greatest of all research techniques of the economist -- introspection -- to discover whether I now or ever possessed any reasonably clear vision on most of these matters. I may be a pecularily unimaginative guy, but I think that my answer has to be negative. To be sure, I never assumed that someday I would be a millionaire and therefore never thought very much about borrowing a half million dollars to buy a yacht. But I believe that the theory requires a somewhat more sensitive vision than that.

Second: The theory assumes that the family's vision, whether correct or incorrect, is held with sufficient *certainty* for more than a short period ahead to be meaningful. Otherwise, the discount for uncertainty -- and the changing discount for changing degrees of uncertainty -- would appear to prevent any rational or stable lifetime planning of consumption. It may be that there are (or have been) more stable societies than ours, in which people live much as their fathers did, where one is reasonably certain what he can expect and what will be expected of him. But in a world numb with "future shock" does this behavioral hypothesis really have meaning for our society? My own uncertainty discounts for the year after next are exceedingly high!

Third: The theory assumes that each family makes rational, conscious, and rather complex calculations based on its vision, which result in a lifetime plan for spending and saving; and that it repeats these calculations, and alters its lifetime spending plan, on the basis of every significant change in the information which it receives or the expectations which it holds. These must be conscious and rather sophisticated calculations, not at all intuitive, and ones that use fairly precise -- i.e., quantitative -- information and fairly precisely formulated expectations. Do people actually go through these

assumed calculations, and repeat them each time there is a change in monetary or fiscal policy, as well as each time there are independently occuring changes in the economic environment? Some do, I am sure. But how many? The past five years have seen some of the most pronounced changes we are ever likely to witness in monetary and fiscal policy, as well as in levels of equity prices, values of owner-occupied houses and other consumer assets, and, of course, in incomes and (I assume) income expectations. I suppose that, as an economist, I should be ashamed to confess that I cannot think of a single consumption or saving decision which I have made in this period which was consciously or (so far as I can reconstruct) unconsciously influenced by these changes -- other than in income and income expectations. If *you* have made such changes in your consumption spending over the past five years, was it a continuous revaluation based on each 10 basis point change in interest rates and each 10 point change in the Dow Jones? This would seem to be what is required if monetary policy is going to accomplish any even reasonably fine tuning through consumption effects.

Fourth: The hypothesis assumes that each family starts with nothing and ends with nothing. Its lifetime consumption equals its lifetime income. Dolde and Tobin casually remark that if the family plans to make bequests or if it receives or expects to receive them, this circumstance can easily enough be accommodated into the theory. But does this not dismiss without analysis what may well be a large part of the phenemenon of saving? If each family actually had zero lifetime saving, and income and population were constant, aggregate personal saving would have to be zero. Positive saving, of course, arises from population and income growth. But does it not also arise in significant amounts from net accumulations passed from one generation to the next? If even one generation planned to and did consume over its lifetime its entire resources, could not much of our wealth be wiped out? Bequests and capital preservation seem to me to be major facts of life. Certainly not all bequests and all failures to consume capital are accidental or irrational. How then do they fit into the pattern of rational allocation assumed by the theory? What aspects of economic evolution or economic policy might influence the size of such planned bequests? How might they influence the strength of the desire to save, or the pressure to preserve or to spend capital? Surely the theory leaves what seem to me to be exceedingly crucial questions dangling.

Let me conclude by suggesting a fifth assumption that life-cycle theorists must be making. They must assume that alterations of

current consumption behavior arising from altered lifetime spending plans based on lifetime utility maximization are not swamped by those changes in consumption that respond to other influences, and therefore that allocating research effort to this approach is an efficient investment in seeking to understand and thus better to forecast consumer spending. My previous questions must imply that I have some doubts about this assumption. However, I think that we all must follow with great interest the continuing work of those who assume that lifetime utility maximization provides a fruitful framework for analyzing consumption and saving behavior.

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REBUTTAL

JAMES TOBIN

I will begin with Gardner's last remark. One response to it is that there is no difficulty in accommodating initial wealth and bequests into the life cycle model. One can, for example, extend the life cycle to include the famous three generations from shirt sleeves to shirt sleeves, instead of just one generation. That could be done, and I think that we will try to do that in further work if we are not altogether discouraged by what Gardner said.

But the more important response is this: What is wrong about what Gardner said is the assertion that it is not possible to account for the existing volume of wealth in a country like the United States without assigning a substantial part of that wealth to the bequest motive. The fact is I have made some calculations of this in a paper published a few years ago called "Life Cycle Saving and Balanced Growth."[1] With the growth rates of population and productivity in the United States, and with the demographic structure and income profiles characteristic of the U.S. population, one can account for -- without any bequest motive whatsoever -- the holding of wealth in relation to income of the order of magnitude which exists in the United States. Even though no generation saves, the economy as a whole saves because of the growth of population and per capita income. There is a revolving fund which is saved up by young people and middle-aged people and then turned over by old people to younger people who are coming along in the next generation. The fund would be constant in an economy stationary in both population and productivity. But it is not constant -- it is forever growing in a

[1]See: *Ten Economic Studies in the Tradition of Irving Fisher,* Wm. Fellner, ed., New York: J. Wiley & Sons, 1967, pp. 231-256.

growing economy, and it can easily be of the magnitude of the wealth/income ratios we have observed. So I think that Gardner has misunderstood the situation on that point.

Many of Gardner's complaints about the modeling of the things we were trying to take into account are well taken. I think that in subsequent work we would like to include more income classes and more differences among consumers within income classes. As Gardner correctly pointed out, there are some important nonlinearities in the model, particularly those introduced by liquidity constraints. These mean that averages of the results are not the results of the average determinants. He is correct in that, and we can do more to avoid biases of aggregation.

Nevertheless, I think we did a lot of disaggregation. And in doing a lot of disaggregation we called attention to a number of phenomena which are usually concealed in completely aggregative approaches to the same problem: for example, age differences and liquidity constraints. It is a little annoying, I must say, when you engage in a certain amount of disaggregation, to be told that you should have done more: and that since you didn't do more you might as well rely on the old aggregative relationships which didn't do any. I don't know what the optimal amount of disaggregation is. There are limits of space in the storage of computers and even limits to the time of people like Walt Dolde, but we probably can do more and will push along that line.

The logic of the consumption functions used by practicing economists has shown for a good many years a tension between a liquidity theory and a wealth theory. The Keynesian consumption function, the basis of the Keynesian multiplier, relates consumption to disposable income. It really is largely a liquidity theory. Otherwise there is no explanation why you should expect consumption to be related to the current flow of income in any constant systematic manner. Rather you would expect it to be related to these longer run calculations of resources that we have tried to deal with.

Now if it had been true that the income-flow theory of consumption was a resounding success, and that its indications were being borne out all the time, then we wouldn't need to go into the wealth theory or the life cycle theory and all that. We wouldn't need to seek a more fundamental theory about why saving ratios are what they are and how they relate to various parameters. But we all know that the cash income theory is not a resounding success. The assumption that in our economy there is a tight income-consumption linkage enforced by the fact that people don't have any other source of

money to spend is false. The linkage is not that tight. We saw an example in the miscalculation made in regard to effects of tax surcharge.

So we face the necessity of formulating a more fundamental model. There is ample evidence that the permanent income or life cycle theory is moving in the right direction, away from the strictly mechanical disposable income consumption relationship.

The general issues of methodology of economics would take us into a long argument. I think that much of what Gardner said about that would be a criticism of all economics. Economists do make rather pure abstract assumptions about the way decisions are made by business firms and by households, for all kinds of problems. If you are going to say, "Well, *I* don't make them exactly that way and I don't know people that make them that way," then we are really left looking for some other method of constructing hypotheses and figuring out their implications. In my opinion, these things do work out at the margin and in the aggregate in a good enough way to justify our procedures. Pretending, even if somewhat counterfactually and unrealistically, that decisions are made on a rational basis -- or with some purpose to them, as Jim Duesenberry said -- is a worthwhile way of proceeding. If it isn't a worthwhile way of proceeding, then I would like to know what other way we should proceed.