The Effectiveness of Alternative Monetary Policy Tools in a Zero Lower Bound Environment

James D. Hamilton    Jing (Cynthia) Wu

Department of Economics
UC San Diego
What more can monetary policy do when:

- the fed funds rate is 0.18%
- reserves are over a trillion dollars?
Policy options

1. Communicate expansionary intentions after escape from the zero lower bound
Policy options

1. Communicate expansionary intentions after escape from the zero lower bound

2. Purchase assets other than T-bills
   a. foreign assets
   b. risky assets
   c. long-term assets
Preferred habitat model of Vayanos and Vila

- preference of some borrowers or lenders for certain maturities
- arbitrageurs ensure that each risk factor is priced the same across assets
Preferred habitat model of Vayanos and Vila

- preference of some borrowers or lenders for certain maturities
- arbitrageurs ensure that each risk factor is priced the same across assets
- decreased preference of Treasury to borrow long-term
  ⇒ reduced exposure of arbitrageurs to long-term risk factors
  ⇒ reduced price of this risk (flatter yield curve)
## Outline

1. Model
2. Data
3. Empirical results prior to crisis
4. Model and empirical results at the ZLB

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Hamilton and Wu (UCSD)
Discrete-time version of Vayanos and Vila (2009)

Arbitrageurs’ objective:

$$\max E_t(r_{t,t+1}) - (\gamma/2) \Var_t(r_{t,t+1})$$

- first-order condition:

$$y_{1t} = E_t(r_{n,t,t+1}) - \gamma \vartheta_{nt}$$

where $y_{1t} =$ return on riskless asset

$\vartheta_{nt} = (1/2)$ change in variance from one more unit of asset $n$
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Rate of return

$$r_{n,t,t+1} = \frac{P_{n-1,t+1}}{P_{nt}} - 1$$

$$r_{t,t+1} = \sum_{n=1}^{N} z_{nt} r_{n,t,t+1}$$
Suppose that log of bond price is affine function of macro factors $f_t$,

$$\log P_{nt} = \bar{a}_n + \bar{b}'_n f_t$$

and factors follow Gaussian VAR(1):

$$f_{t+1} = c + \rho f_t + \Sigma u_{t+1}$$
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and factors follow Gaussian VAR(1):

$$f_{t+1} = c + \rho f_t + \Sigma u_{t+1}$$

Then variance of return on portfolio is approximately

$$d_t' \Sigma \Sigma' d_t$$

$$d_t = \sum_{n=2}^{N} z_{nt} \bar{b}_{n-1}$$

and $(1/2)$ derivative of variance with respect to asset $n$ is

$$\vartheta_{nt} = \bar{b}_{n-1}' \Sigma \Sigma' d_t$$
Discrete-time version of Vayanos and Vila (2009)

If preferred-habitat borrowing is also an affine function of $f_t$, then in equilibrium, prices of risk are an affine function of factors as well, and framework implies a standard affine-term-structure model.
Data

- Treasury yields (weekly and end-of-month, Jan 1990 - Aug 2010)
- Separate estimates of Fed holdings
Maturity structure: December 31, 2006
Average maturity

Hamilton and Wu (UCSD)
Prior to Crisis
3-factor model, estimated weekly Jan 1990 - July 2007, assuming only that prices of risk are affine in the factors

Factors $f_t$: level, slope and curvature

$$\text{level} = \frac{y_{6m} + y_{2y} + y_{10y}}{3}$$
$$\text{slope} = y_{10y} - y_{6m}$$
$$\text{curvature} = y_{6m} + y_{10y} - 2y_{2y}$$

Yields measured with error: 3m, 1y, 5y and 30y

generates estimates of factor dynamic parameters $(c, \rho, \Sigma)$, risk-pricing parameters $(\lambda, \Lambda)$, and how each maturity loads on factors $\bar{b}_n$. 
### Results

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Estimate 1</th>
<th>Estimate 2</th>
<th>Estimate 3</th>
</tr>
</thead>
<tbody>
<tr>
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<td>-0.0034</td>
<td>-0.0003</td>
<td>0.0006</td>
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<tr>
<td></td>
<td>(0.0089)</td>
<td>(0.0074)</td>
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<tr>
<td>$\rho$</td>
<td>0.9895</td>
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<td>$a_1 \times 5200$</td>
<td>4.1158</td>
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<td>(0.0058)</td>
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<td>$\Sigma$</td>
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<td></td>
<td>(0.0236)</td>
<td>(0.0045)</td>
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<tr>
<td>$\Sigma_e \times 5200$</td>
<td>0.0978</td>
<td>0.0025</td>
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<tr>
<td></td>
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<td>$\Lambda$</td>
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<tr>
<td>$\Lambda$</td>
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<td>-0.0480</td>
<td>-0.0948</td>
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<tr>
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<td>(0.0594)</td>
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<td>0.0847</td>
<td>-0.0266</td>
<td>0.1773</td>
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<tr>
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<td>(0.0455)</td>
<td>(0.0825)</td>
<td>(0.1200)</td>
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<td></td>
<td>-0.0567</td>
<td>0.0531</td>
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<tr>
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<td>(0.0436)</td>
<td>(0.0596)</td>
<td>(0.1594)</td>
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</table>
Preferred habitat

Preferred-habitat-implied price of risk

$$\sum \lambda_t = \gamma \sum \Sigma' \sum_{n=2}^{N} z_{nt} b_{n-1}$$
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Preferred-habitat-implied price of risk

\[ \sum \lambda_t = \gamma \sum \sum' \sum_{n=2}^{N} z_{nt} \bar{b}_{n-1} \]

Suppose that:

- arbitrageurs correspond to entire private sector
- U.S. Treasury debt is sole asset held by arbitrageurs
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Suppose that:

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Then:

\[ z_{nt} = \text{share of publicly-held debt represented by maturity } n \]
Preferred habitat

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\[ \sum \lambda_t = \gamma \sum \sum' \sum_{n=2}^{N} z_{nt} \bar{b}_{n-1} \]

Suppose that:

- arbitrageurs correspond to entire private sector
- U.S. Treasury debt is sole asset held by arbitrageurs

Then:

\[ z_{nt} = \text{share of publicly-held debt represented by maturity } n \]

\[ q_t = 100 \sum \sum' \sum_{n=2}^{N} z_{nt} \bar{b}_{n-1} \]
Excess holding returns

- Excess holding return
e.g. hold 5 year bond over 1 year

\[ h_{5,1,t} = \log \frac{P_{4,t+1}}{P_{5,t}} - y_{1,t} \]
Excess holding returns

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$$h_{5,1,t} = \log \frac{P_{4,t+1}}{P_{5,t}} - y_{1,t}$$

- Regression

$$h_{nkt} = c_{nk} + \beta'_{nk} f_t + \gamma'_{nk} x_t + u_{nkt}.$$
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- Expectation hypothesis: excess holding returns are unpredictable
- ATSM: \( f_t \) contains all the information at \( t \)
<table>
<thead>
<tr>
<th>Regressors</th>
<th>6m over 3m</th>
<th>1yr over 6m</th>
<th>2y over 1y</th>
<th>5y over 1y</th>
<th>10y over 1y</th>
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<td>$f_t^*$</td>
<td>0.357</td>
<td>0.356</td>
<td>0.331</td>
<td>0.295</td>
<td>0.331</td>
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<td></td>
<td>(0.000)</td>
<td>(0.000)</td>
<td>(0.000)</td>
<td>(0.000)</td>
<td>(0.000)</td>
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<tr>
<td>$f_t, z_t^{A*}$</td>
<td>0.410</td>
<td>0.420</td>
<td>0.373</td>
<td>0.300</td>
<td>0.336</td>
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<tr>
<td></td>
<td>(0.020)</td>
<td>(0.119)</td>
<td>(0.311)</td>
<td>(0.728)</td>
<td>(0.665)</td>
</tr>
<tr>
<td>$f_t, z_t^{L*}$</td>
<td>0.428</td>
<td>0.501</td>
<td>0.524</td>
<td>0.398</td>
<td>0.357</td>
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<td>(0.003)</td>
<td>(0.008)</td>
<td>(0.006)</td>
<td>(0.035)</td>
<td>(0.196)</td>
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<tr>
<td>$f_t, q_t^*$</td>
<td>0.444</td>
<td>0.568</td>
<td>0.714</td>
<td>0.617</td>
<td>0.549</td>
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<tr>
<td></td>
<td>(0.002)</td>
<td>(0.000)</td>
<td>(0.000)</td>
<td>(0.000)</td>
<td>(0.001)</td>
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<tr>
<td>$f_t, z_t^{A}, z_t^{L}, q_t^*$</td>
<td>0.476</td>
<td>0.597</td>
<td>0.741</td>
<td>0.670</td>
<td>0.634</td>
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<tr>
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<td>(0.000)</td>
<td>(0.001)</td>
<td>(0.000)</td>
<td>(0.002)</td>
<td>(0.054)</td>
</tr>
</tbody>
</table>

$f_t$: term structure factors
$z_t^{A}$: average maturity
$z_t^{L}$: fraction of outstanding debt over 10 years
$q_t$: Treasury factors
Endogeneity

Goal: if maturities of outstanding debt change, how would yields change?
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Conventional regression

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Concerns:

- Is \( f_t \) responding to \( q_t \), or is \( q_t \) responding to \( f_t \)?
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- Is \( f_t \) responding to \( q_t \), or is \( q_t \) responding to \( f_t \)?
- Spurious regression
Yield factor forecasting regressions

Our approach:

\[ f_{t+1} = c + \rho f_t + \phi q_t + \epsilon_{t+1} \]
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Advantages:

- answers forecasting question of independent interest
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Advantages:

- answers forecasting question of independent interest
- avoids spurious regression problem
- nonzero \( \phi \) does not reflect response of \( q_t \) to \( f_t \)
- estimate incremental forecasting contribution of \( q_t \) beyond that in \( f_t \)
Significance of Treasury factors

\[ f_{t+1} = c + \rho f_t + \phi q_t + \varepsilon_{t+1} \]

\[ F \text{ test that } \phi = 0 \]

<table>
<thead>
<tr>
<th></th>
<th>( F \text{ test} )</th>
</tr>
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<tbody>
<tr>
<td>level</td>
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</tr>
<tr>
<td></td>
<td>(0.023)</td>
</tr>
<tr>
<td>slope</td>
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</tr>
<tr>
<td></td>
<td>(0.005)</td>
</tr>
<tr>
<td>curvature</td>
<td>2.672</td>
</tr>
<tr>
<td></td>
<td>(0.049)</td>
</tr>
</tbody>
</table>
Fed sells all Treasury securities < 1 year, and uses proceeds to buy up long-term debt

E.g. in Dec. 2006, the effect would be to sell $400B short-term securities and buy all bonds > 10 year
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<table>
<thead>
<tr>
<th></th>
<th>$\phi_i \Delta$</th>
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</thead>
<tbody>
<tr>
<td>level</td>
<td>0.005</td>
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<tr>
<td></td>
<td>(0.112)</td>
</tr>
<tr>
<td>slope</td>
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</tr>
<tr>
<td></td>
<td>(0.116)</td>
</tr>
<tr>
<td>curvature</td>
<td>-0.073</td>
</tr>
<tr>
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<td>(0.116)</td>
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</table>

$\Delta$: average change in $q_t$
Impact on yield curve 1-month ahead
Financial Crisis and Zero Lower Bound
Zero Lower Bond

- Short term yields near zero
- Longer term yields considerable fluctuation.
- Explanation: when escape from ZLB (with a probability), interest rates will respond to $f_t$ as before
Parsimonious Model of ZLB

- Same underlying factors $f_t$

\[ f_{t+1} = c + \rho f_t + \Sigma u_{t+1} \]

same $(c, \rho, \Sigma)$
Parsimonious Model of ZLB

- Same underlying factors $f_t$
  \[ f_{t+1} = c + \rho f_t + \Sigma u_{t+1} \]
  same $(c, \rho, \Sigma)$

- Once escape from ZLB
  \[ \tilde{y}_{1t} = a_1 + b'_1 f_t \]
  \[ \tilde{p}_{nt} = \bar{a}_n + \bar{b}'_n f_t \]

  $\bar{a}_n$ and $\bar{b}_n$ calculated from the same difference equations
Parsimonious Model of ZLB

- At ZLB

\[ y_{1t}^* = a_1^* \]
\[ p_{nt}^* = \bar{a}_n^* + \bar{b}_n^* f_t. \]

\( \pi^Q \): probability still at ZLB next period

No-arbitrage:
Can calculate \( \bar{b}_n^* \) (how bond prices load on factors at ZLB) as functions of \( \bar{b}_n^* \) (how they’d load away from the ZLB) along with \( \pi^Q \) (probability of remaining at ZLB), \( \rho \) (factor dynamics), and \( \Lambda \) (risk parameters).
Parsimonious Model of ZLB

Assume: \((c^Q, \rho^Q, a_1, b_1, \Sigma)\) as estimated pre-crisis

\[ \Rightarrow (\bar{a}_n, \bar{b}_n) \text{ same as before} \]

Estimate two new parameters \((a_1^*, \pi^Q)\) to describe 2009:M3-2010:M7 data from

\[ Y_{2t} = A_2^\dagger + B_2^\dagger Y_{1t} + \varepsilon_t \]

- \(Y_{1t} = 6\)-month, 2-year, 10-year
- \(Y_{2t} = 3\)-month, 1-year, 5-year, 30-year
- \(A_2^\dagger, B_2^\dagger\) functions of \((c^Q, \rho^Q, a_1, b_1, \Sigma)\) and \((a_1^*, \pi^Q)\)
- Estimation method: minimum chi square (Hamilton and Wu, 2010)
Parameter estimates for ZLB

Slightly better fit if allow new value for $a_1$ after escape from ZLB

$5200a_1^* = 0.068$ (ZLB = 0.07% interest rate)

$\pi^Q = 0.9907$ (ZLB may last 108 weeks)

$5200a_1 = 2.19$ (compares with $5200a_1 = 4.12$ pre-crisis—market expects lower post-ZLB rates than seen pre-crisis)
Actual and fitted values
# Model Fit

<table>
<thead>
<tr>
<th></th>
<th>Contemporaneous $R^2$</th>
<th>Forecast $R^2$</th>
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<td>restricted</td>
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<tr>
<td>3m</td>
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<tr>
<td>1y</td>
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<tr>
<td>5y</td>
<td>0.961</td>
<td>0.975</td>
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<td>30y</td>
<td>0.965</td>
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Factor Loadings
One-month-ahead predicted effect of Fed swapping short-for long-term
One-month-ahead predicted effect of Fed swapping short-for long-term
<table>
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<tr>
<th>Study</th>
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<th>ZLB</th>
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<td>Gagnon, et. al.</td>
<td>10 yr yield</td>
<td>20</td>
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<td>Greenwood-Vayanos</td>
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<td>D'Amico-King</td>
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<td>Deutsche Bank</td>
<td>10yr yield</td>
<td>20</td>
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<td>14</td>
<td>13</td>
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</tbody>
</table>

Table 5: Comparison of different estimates of the effect of replacing $400 billion in long-term debt with short-term debt.
Caveats

- The effects come in the model from investors’ assumption that the changes are permanent.
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- The Treasury is better suited to implement than Fed.
Caveats

- The effects come in the model from investors’ assumption that the changes are permanent.
- The Treasury is better suited to implement than Fed.
- Operation works by transferring risk from government's creditors to the Treasury-Fed.