Have We Underestimated the Probability of Hitting the Zero Lower Bound?

PELIMINARY DRAFT
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I. Introduction

The zero lower bound (ZLB) on nominal interest rates limits the ability of central banks to add monetary stimulus to offset adverse shocks to the real economy and to check unwelcome disinflation. The experience of Japan in the 1990s motivated a great deal of research on the macroeconomic consequences of the ZLB and monetary policy strategies to overcome these effects. Economic theory has provided important insights about both the dynamics of the economy in the vicinity of the ZLB and possible policy strategies for mitigating its effects. But theory alone cannot provide a quantitative assessment of the practical importance of the ZLB threat, which depends critically on the frequency and degree to which the lower bound constrains the actions of the central bank as it seeks to stabilize real activity and inflation, thereby impinging on the unconstrained variability and overall distribution of the nominal funds rate that would otherwise arise. These factors in turn depend on the expected magnitude and persistence of adverse shocks to the economy; the dynamic behavior of real activity, inflation, and expectations; and the monetary policy strategy followed by the central bank, including its inflation target. (The latter factor plays a key role in ZLB dynamics, because the mean of the unconstrained distribution of the nominal funds rate equals the inflation target plus the economy's equilibrium real short-term rate of interest.) The quantitative evaluation of these factors requires one to use a model of the economy with sound empirical foundations.

Previous research was generally sanguine about the practical risks posed by the ZLB, as long as the central bank did not target too low an inflation rate. Reifschneider and Williams (2000) used stochastic simulations of the Federal Reserve's large-scale rational-expectations macroeconometric model, FRB/US, to evaluate the frequency and duration of episodes when policy was constrained by the ZLB. They found that if monetary policy followed the prescriptions of the standard Taylor (1993) rule with an inflation target of 2 percent, the federal funds rate would be near zero about 5 percent of the time and the "typical" ZLB episode would last four quarters. Their results also suggested that the ZLB would have relatively minor effects on macroeconomic performance under these policy assumptions. In addition, Reifschneider and Williams found that monetary policy rules with larger responses to output and inflation than the standard Taylor rule encountered the ZLB more frequently, with relatively minor macroeconomic consequences as long as the inflation target did not fall too far below 2 percent. Other studies reported similar findings although they, if anything, tended to find even smaller effects of the ZLB (see, for example, Coenen 2003 and Schmitt-Grohe and Uribe 2007, and other papers in the reference list). Finally, research in this area suggested that monetary policies could be crafted that greatly mitigated any effect of the ZLB. Proposed strategies to accomplish this goal included responding more aggressively to economic weakness and falling inflation, or promising to run an easier monetary policy for a time once the ZLB is no longer binding (see Reifschneider and Williams (2002) and Eggertsson and Woodford (2003) and references therein).

The events of the past few years call into question the reliability of those analyses. The federal funds rate has been at its effective lower bound for almost two years, and futures data suggest that market participants currently expect it to remain there until early 2012. The current episode thus is much longer than those typically generated in the simulation analysis of Reifschneider and Williams (2000). The same study suggested that recessions as deep as that just experienced would be exceedingly rare—on the order of once a century or less frequent. Of course, recent events could be interpreted as just bad luck—after all, five hundred year floods do eventually happen. Alternatively, recent events could be flashing a warning sign that previous estimates of ZLB effects significantly understated the inherent volatility of the economy that arises from the interaction of macroeconomic disturbances and the economy's dynamics.

The goal of this paper is to examine and attempt to answer four key questions regarding the frequency and duration of ZLB episodes using a range of econometric models, including structural and time series models. First, how surprising have recent events been? Second, has the estimated probability of hitting the ZLB changed much over time? Third, how severely did the ZLB bind during the crisis? And, finally, what lessons do we take for the future in terms of the expected frequency, duration, and magnitude of ZLB episodes?

One contribution of this paper is to apply a variety of structural and statistical models to analyze these questions, rather than using a single structural model as was done in past research. Research on the ZLB has generally focused on results from structural models because many of the issues in this field have monetary policy strategy and expectational dynamics at their core. For example, studies have typically employed structural models run under rational expectations to assess expected macro performance under, say, different inflation targets or under price-level targeting in order to ensure consistency between the central bank's actions and private agents' beliefs. In this paper, we use two empirical macroeconomic models developed at the Board of

Governors—one a more traditional large-scale model and the other an optimization-based dynamic stochastic general equilibrium model—to analyze the extent that the ZLB is likely to constrain policies. Because both models have strong empirical foundations, they should provide informative quantitative estimates of the risks posed by the ZLB.

However, a potential drawback to using structural models to quantify the likelihood of the risks confronting policymakers is that such models impose stringent constraints and priors on the data that may inadvertently provide flawed empirical characterizations of the economy. In particular, they are constructed to yield "well-behaved" long-run dynamics, as long as the monetary policy rule satisfies certain conditions such as the Taylor principle, and the fiscal authorities (explicitly or implicitly) pursue stable policies that, say, target a fixed debt-to-GDP ratio. In addition, they tend to abstract from structural change and generally assume that the parameters and the shock processes are constant over time and known by policymakers. As a result of these features, structural models may significantly understate the persistence of episodes of low real interest rates, because they implicitly assume that the medium- to long-run equilibrium real interest rate—a key factor underlying the threat posed by the ZLB—is constant.[1] This is because the asymmetric nature of the ZLB implies that low frequency variation in the equilibrium real interest rate raises the overall probability of hitting the ZLB, all else equal.

Because of these potential limitations of structural models, in this paper we include in our analysis three statistical models that impose fewer theoretical constraints on the data and allow for a wider set of sources of uncertainty. One is a vector autoregression model with time-varying parameters (TVP-VAR); the second is a model that allows for unit-root behavior in both potential output growth and the equilibrium real interest rate (Laubach-Williams 2003); and the third is a univariate model that allows for GARCH error processes. In selecting these statistical models, one of our aims is to use models that arguably provide more scope than structural models do for taking into account uncertainty about the range and persistence of movements in the equilibrium real interest rate.

[1] Whether or not real interest rates are stationary is, admittedly, not obvious. Ex post measures for the United States display no clear trend over the past sixty years, and the fact that U.S. real short-term rates were on average low during the 1970s, and high during the 1980s, is in part an artifact of excessively loose monetary policy in the former period and corrective action during the latter period. But phenomena such as the persistent step-down in Japanese output growth since the early 1990s, the global savings glut of the past decade, and secular trends in government indebtedness illustrate that there are many reasons to view the equilibrium real interest rate as a series that can shift over time.

In summary, our findings are as follows. First, the events of the past few years have been generally well outside the forecast confidence bands of empirical macroeconomic models. Second, model-based analyses that ignore uncertainty regarding parameters and variances of shocks are noticeably more surprised by recent events than ones that take account of such uncertainty. Third, all of the models are "fooled" by the Great Moderation period. Indeed, a striking and disconcerting finding is that some of the lowest values for the estimated probability of hitting the ZLB over the subsequent five years occur in 2006 and 2007, right before the onset of the crisis and recession. This is true for all the models that we study. Fourth, our estimates suggest that the ZLB had a first-order impact on macroeconomic outcomes in the United States, although the magnitude of the estimates depends on the preferences of the central bank and the assumed persistence of the shocks affecting the economy (see also Williams 2009).

In assessing the probability of hitting the ZLB going forward, one must confront a number of issues that we identify in this paper. First, the reliance on model stochastic simulations that assume constant parameters and variances and abstract from data and parameter uncertainty contributes to an underestimate of the probability of encountering the ZLB. Our results indicate that time-varying parameters, measurement error, and parameter uncertainty can significantly raise the estimated probability of hitting the zero lower bound, indicating that future research should incorporate these factors in the analysis. Second, researchers need to find ways to ensure that model-generated probability distributions adequately account for relatively rare tail events, even if the data in the model's estimation sample does not include any such events. This adjustment can be accomplished by using long samples in estimating the shock variances, as we do in some of our models, or by using methods that incorporate a prior on tail events and making the distribution of these events less sensitive to recent data. Finally, our analysis shows that one can obtain quite different answers depending on the model used in the analysis. For example, we find that an estimated DSGE model predicts that it is extremely unlikely that the Fed could get stuck at the ZLB for a year or longer, while other models that feature stronger intrinsic persistence view such outcomes as much more likely. This range of results indicates that research on the ZLB should explicitly integrate a range of models, including models that allow for structural change.

Our analysis suggests that the expected future probability of hitting the ZLB and the expected duration of ZLB episodes are both somewhat higher than reported in Reifschneider and

Williams (2000, 2002), but not dramatically so. This increase is primarily due to two factors. First, including the shocks from the past two years in our sample raises our estimate of the underlying variability of the economy, implying that monetary policy will be constrained by the ZLB slightly more frequently in the future. Second, as noted above, our analysis highlights the important role that time-varying model parameters, measurement error (especially for latent variables), and parameter uncertainty can have on one's estimates of macroeconomic variability. Past analysis of the ZLB has mostly abstracted from these issues, leading to a downward bias in estimates of the frequency of hitting the zero bound. In some cases, this bias appears to be large. We take some comfort in the fact that our estimates based on models that incorporate these factors are generally close to those estimates from FRB/US (which does not incorporate them), suggesting that the bias from this source may be relatively small in the case of FRB/US.

II. Models and methodological issues

As noted, we use five different models to evaluate the likely incidence of encountering the ZLB. Each of these models is "off the shelf", in that we have taken models already in use at the Federal Reserve or that are well-established in the academic literature. In this section, we provide brief descriptions of the models and references for more detailed information. Table 1 provides a summary of the key features of the models.

FRB/US

The FRB/US model is a large-scale estimated model of the U.S. economy with a detailed treatment of the monetary transmission mechanism. We include the FRB/US model because it has good empirical foundations and has long been used at the Fed for forecasting and policy analysis. In addition, FRB/US has the advantage of having been used in previous analyses of the ZLB. Although it is not a DSGE model, the main behavioral equations are based on the optimizing behavior of forward-looking households and firms subject to costs of adjustment. The model displays sluggish adjustment of real activity and inflation in response to shocks (see Brayton et al 1997 for details).

We assume rational expectations for those parts of our analysis where we explore the macroeconomic effects of systematic changes in monetary policy, such as changes in the inflation target. We also assume rational expectations in situations involving pronounced changes in monetary policy, such as would occur under policies that aim to minimize an expected loss function. In forecasting exercises, however, we simulate the model using the expectational assumption commonly used at the Fed for this type of work. Under this assumption, agents base their expectations on the forecasts of a small VAR model rather than the full FRB/US model. This approach has the virtue of computational simplicity; it also has a proven track record in forecasting.

Another noteworthy aspect of the FRB/US projections presented in this paper concerns the extrapolation of shocks and exogenous variables. Although shocks to behavioral equations are assumed to be serially uncorrelated with mean zero in the estimation of the model, we do not follow the standard approach used with the other models and set the baseline projected values of the stochastic innovations to zero. Instead, we extrapolate these shocks at their weighted average value over the preceding sixty quarters, using weights that decline geometrically at a rate of 1 percent per quarter. Analysis at the Federal Reserve indicates that this type of intercept-adjustment procedure—which has been the standard approach to forecasting with FRB/US since the inception of the model in the mid-1990s—increases real-time predictive accuracy. As for exogenous variables, again we follow standard practice in FRB/US forecasting and extrapolate these series using simple univariate time-series equations.

EDO (Estimated Dynamic Optimization-based model)

The EDO model is a DSGE model of the US economy developed and used at the Board of Governors for forecasting and policy analysis; see Chung, Kiley and Laforte (2010) for documentation on the current version of the model, and Edge et al (2008) for additional information. Like FRB/US, EDO—which represents the current standard approach to macro modeling—has strong empirical foundations and is used by the Federal Reserve for forecasting and policy analysis. Although the model has not been in service long enough to compile a reliable track record, pseudo real-time forecasting exercises suggest that it has good forecasting properties.

EDO builds on the Smets and Wouters (2007) model. Households have preferences over nondurable consumption services, durable consumption services, housing services, and leisure and feature internal habit in each service flow. Production in the model takes place in two distinct sectors that experience different (stochastic) rates of technological progress—an assumption that allows the model to match the much faster rate of growth in constant dollar-terms observed for some expenditure components, such as nonresidential investment. As a result, growth across sectors is balanced in nominal, rather than real, terms. Expenditures on nondurable consumption, durable consumption, residential investment, nonresidential investment are modeled separately while the remainder of aggregate demand is represented by an exogenous stochastic process.

Wages and prices are sticky in the sense of Rotemberg (1982), with indexation to a weighted average of long-run inflation and lagged inflation. A simple estimated monetary policy reaction function governs monetary policy choices. The exogenous shock processes in the model include the monetary policy shock; the growth rates of economy-wide and investment-specific technologies; financial shocks, such as a stochastic economy-wide risk premium and stochastic risk premia that affect the intermediaries for consumer durables, residential investment, and nonresidential investment; shocks to autonomous aggregate demand; and price and wage markup shocks.

The model is estimated using Bayesian methods over the sample period 1984Q4 to 2009Q4. Accordingly, the model's estimates are guided entirely by the Great Moderation period. The data used in estimation include the following: real GDP; real consumption of nondurables and services excluding housing; real consumption of durables; real residential investment; real business investment; aggregate hours worked in the nonfarm business sector (per capita); PCE price inflation; core PCE price inflation; PCE durables inflation; compensation per hour divided by GDP price index; and the federal funds rate. Each expenditure series is measured in per capita terms, using the (smoothed) civilian non-institutional population over the age of 16. We remove a very smooth trend from hours per capita prior to estimation.

TVP-VAR

The specification of the TVP-VAR (time-varying parameter vector autoregression) model closely follows Primiceri (2005). The VAR model contains a constant and two lags of the four-quarter change in the GDP price index, the unemployment rate, and the 3-month Treasury bill rate. Let X_t denote the column vector consisting of these variables, ordered as listed. The system obeys

(1.1) $$A_t^0 X_t = \bar{A}_t + \sum_{s=1}^{2} A_t^1 X_{t-s} + B_t \varepsilon_t$$

where A_t^0 is lower triangular and each non-zero element of the A matrices follows an independent Gaussian unit-root process. Consequently, both the equilibrium real interest rate and the variances of the shocks are time-varying. The matrix B_t is diagonal and the logarithm of an entry on the diagonal follows an independent Gaussian unit-root process, i.e., the volatility of structural shocks is stochastic. Estimation is Bayesian, with the prior constructed as in Primiceri (2005), using a 40 quarter training window starting in 1953Q3.[2]

Laubach-Williams

The Laubach-Williams (LW) model includes estimated equations for the output gap, core PCE price inflation, the funds rate, and relative non-oil import and oil prices. (See Laubach and Williams, 2003). Potential GDP, its growth rate, and the equilibrium real interest rate are all nonstationary unobservable latent variables. The other parameters of the model, including those describing the variances of the shock processes, are assumed to constant.[3] We estimate the LW model by maximum likelihood using the Kalman filter using data starting in 1961.[4] Unlike FRB/US and EDO, the LW model implicitly assumes adaptive expectations, features very gradual dynamic responses to shocks, and includes permanent shocks to the equilibrium real interest rate.

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[2] The prior setting is identical to Primiceri (2005), with one exception: we have set the prior mean of the covariance matrix for innovations to the log-variances substantially higher than in that paper. Specifically, the prior mean is [0.05, 0.05, 0.001], versus [0.0004, 0.0004, 0.0004] with the original prior. Relative to the original, this prior favors drift in volatilities more so than in VAR coefficients. The estimation algorithm also follows Primiceri (2005) exactly, except that we use the approach of Jacquier, Polson and Rossi (1994) to draw the log-variance states. The MCMC sample was 20000 draws, following a burn-in run of 10000 iterations.

[3] In order to conduct stochastic simulations of the model, we appended AR(1) equations (without constants) for relative oil and nonoil import prices to the model and estimated the additional parameters jointly with the other model parameters.

[4] The Kalman gain parameters for the growth rate of potential output and the latent variable that influences the equilibrium real interest rate are estimated using Stock and Watson's (1998) median unbiased estimator as described in Laubach and Williams (2003). We do not incorporate uncertainty about these gain parameters in our analysis in this paper. Doing so would imply even greater uncertainty about interest rates and raise the probability of hitting the ZLB.

GARCH model

We estimate univariate GARCH models of the 3-month Treasury bill rate, the inflation rate of the GDP price index, and the unemployment rate. Specifically, each series is assumed to follow an auto-regressive process of order two

(1.2) $$x_t = c + a_1 x_{t-1} + a_2 x_{t-2} + e_t ,$$

where the conditional variance of the innovation, e_t, is given by

(1.3) $$\sigma_t^2 = \kappa + \sum_{i=1}^{p} G_i \sigma_{t-i}^2 + \sum_{j=1} A_j e_{t-j}^2$$

and each equation is estimated subject to the constraints

(1.4) $$\sum_{i=1}^{p} G_i + \sum_{j=1} A_j < 1,\ \kappa > 0,\ G_i \geq 0,\ A_j \geq 0 .$$

The lag structure of the GARCH model was selected on the basis of the Bayesian information criterion over the sample 1968q1-2007q4.[5] See Engle (2001) for further details on the estimation of GARCH models.

Simulation Methodology

We use stochastic simulations to construct estimated probability distributions. The ultimate goal is to derive the best characterization of future uncertainty using historical data. In the EDO and LW simulations, we incorporate both parameter uncertainty and measurement error. In the case of LW, uncertainty about the equilibrium real interest rate and the output gap, two variables that enter in the monetary policy reaction function, is substantial, as discussed in Laubach and Williams (2003). The stochastic simulations of the TVP-VAR also take account of parameter uncertainty. The sheer size of FRB/US makes it computationally infeasible to incorporate parameter uncertainty and measurement error into the uncertainty estimates.[6]

Imposing the non-linear ZLB constraint on EDO and FRB/US imposes no major problems, although special code is needed to ensure that expectations are consistent with the

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[5] The optimal values p and q for the bill rate and inflation innovations are both one. For the unemployment rate, the optimal value of p remains one while the BIC assigns a value of four to q.

[6] Some of the statistics reported in this preliminary version of the paper are based on relatively small samples of stochastic simulations, on the order of thousands or tens of thousands of iterations; hence the reported estimates are to subject to revision. Future versions of the paper will report results using much larger sample sizes to minimize numerical imprecision.

possibility of positive future shocks to the policy reaction function. Because LW is a backward-looking model, there is no difficulty in enforcing the ZLB. Imposing the ZLB constraint on the TVP-VAR and the GARCH models can be quite problematic, and so we allow nominal short-term interest rates to fall below zero in our analysis.[7] Failure to impose the constraint in these two models will bias downward the estimates of the adverse effects of the ZLB on output and inflation that we derive from them. However, such understatement is less of an issue with the GARCH model because its equations are univariate.

We use the monetary policy reaction functions embedded in each structural model or we append an estimated rule to the model as needed. In EDO, FRB/US, and LW, the estimated policy reaction functions assume that the federal funds rate depends on core PCE inflation, the assumed inflation target (2 percent under baseline assumptions), and the model-specific estimate of the output gap; in addition, the EDO and FRB/US models assume that monetary policy is inertial. The specific reaction functions for these three models are:

(1.5) FRB/US: $$R_t = .82R_{t-1} + .18\left[R^* + \pi_t + .65\left(\pi_t - \pi_t^*\right) + 1.04Y_t\right]$$

(1.6) EDO: $$R_t = .66R_{t-1} + .34\left[R^* + \pi_t + .46\left(\pi_t - \pi_t^*\right) + .21Y_t + .33\Delta Y_t\right]$$

(1.7) LW: $$R_t = R_t^* + \pi_t + .50\left(\pi_t - \pi_t^*\right) + .72Y_t$$

For these rules, the concept of potential output underlying Y is not the flex-price level of output but a measure that evolves more smoothly over time—specifically, a production-function measure in the case of FRB/US, a Beveridge-Nelson measure in the case of EDO, and a Kalman filter estimate in LW. In LW simulations, we assume the policymaker does not know the true value of the equilibrium real interest rate and the output gap, but instead uses the Kalman filter estimates of these objects in the setting of policy.[8] We do not include shocks to the policy rules, except for those owing to the ZLB, in stochastic simulations of FRB/US, EDO, and LW but do in the case of the TVP-VAR and GARCH models.

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[7] Formally, we may regard the ZLB as a shock to the monetary policy rule. Imposing it on a reduced form model therefore requires being able to identify a monetary policy shock—indeed, in principle, to identify a vector of anticipated shocks out to the horizon at which the ZLB is expected to bind. In the case of a univariate GARCH model, no widely accepted benchmark identification exists. The TVP-VAR does assume a triangular structural form at every time, but the resulting "monetary policy shock" does not appear to have reasonable properties over the entire distribution at the dates of interest.

[8] In this way we allow for policymaker misperceptions of potential output and the equilibrium real interest rate. See Orphanides et al (2000) and Orphanides and Williams (2002) for analyses of this issue. We abstract from policymaker misperceptions of this type in the other models analyzed in this paper.

Past research has generally used large sets of stochastic simulations to estimate in an unconditional sense how often the ZLB is likely constrain monetary policy. Such an approach requires that the model yield a stationary steady state with well-behaved long-run dynamics. The particular specification choices made in order to impose these restrictions may inadvertently bias the estimate of the incidence of hitting the ZLB. For example, in the FRB/US and EDO models, the long-run equilibrium real interest rate is constant. In contrast, the LW and TVP-VAR models allow for low-frequency variation in the equilibrium real interest rate. Indeed, the TVP-VAR allows for nonstationary time-variation in all parameters and variances, which implies the absence of any meaningful steady state and unconditional moments.[9]

Given that some of the models we consider do not have well-defined unconditional moments, in this paper we focus primarily on conditional probabilities of policy being constrained by the ZLB. Specifically, we compute five-year-ahead model forecasts conditional on the state of the economy at a given point in time. We then use these simulations to describe the model's prediction regarding the incidence of hitting the ZLB and the resulting macroeconomic outcomes. Later, we compare these conditional statistics to unconditional statistics that are comparable to those in previous research.

III. How surprising have recent events been?

We start our analysis by comparing the actual course of events over the past few years with what each of the models would have predicted prior to the crisis, hopping off from conditions in late 2007. With the exception of the FRB/US model, the projections are based on model parameters estimated with historical data only through 2007. In addition, we also compute confidence intervals for the projections, based on the sort of shocks encountered prior to 2008; these shocks extend back to the 1960s for all the models except EDO. In the case of the EDO, TVP-VAR, LW and GARCH models, the confidence intervals also take account of parameter uncertainty (as well as measurement uncertainty in the case of EDO). By comparing the actual evolution of the economy with these confidence intervals, we can judge whether the models view recent events as especially unlikely.

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[9] We could modify the TVP-VAR and LW models so that they generate stationary steady states. Such an undertaking lies outside the scope of the present paper and we leave this to future research.

Figure 1 reports results from the FRB/US model for the output gap, the unemployment rate, core PCE price inflation, and the federal funds rate. As can be seen, the model prior to the crisis would have viewed the subsequent evolution of real activity and short-term interest rates as extremely improbable, in that actual conditions by 2010 fall far outside the 95 percent confidence band about the late 2007 projection. In contrast, the model is not surprised by the behavior of inflation during the downturn, given the modest degree of disinflation that has occurred to date. This probability assessment is based on the sort of shocks seen from 1968 to 2007; if the analysis were re-run using shocks from only the early 1980s through 2007, the model would be even more surprised.[10]

EDO is also quite surprised by recent events, as seen in figure 2A. Specifically, if we assume that macroeconomic volatility is characterized by the sort of shocks seen from late 1984 through 2007, and if we allow for parameter uncertainty and measurement error, then the results look quite similar to the picture painted by FRB/US. Presumably, EDO would be somewhat less surprised if we conditioned the confidence intervals on the sorts of shocks seen since the late 1960s, as we do with FRB/US. Unfortunately, extending the model's sample period back in time raises difficult estimation issues because of changes in the monetary policy regime and other factors. For this reason, we have not attempted to generate results using a longer sample period, and so cannot say how the model's assessment of uncertainty would increase if it were based on a period less dominated by the Great Moderation experience.[11]

Additional analysis carried out with the EDO model reveals the importance of parameter uncertainty and measurement error to assessments of the risk posed by the ZLB. Figure 2B reports confidence intervals for the post-2007 EDO projections when the effects of these two sources of uncertainty are not taken into account. As can be seen, ignoring these two factors makes it appear as if the model views recent events as even more improbable. Additional information on this point is provided by table 2, which reports various model estimates of the

[10] With hindsight, FRB/US sees the economy as having been hit primarily by huge shocks to the demand for new houses and to the value of residential real estate. By themselves, these shocks account for about half of the widening of the output gap seen since late 2007. In addition, shocks to risk premiums for corporate bonds, equity and the dollar account for another third of the fall in aggregate output.

[11] With hindsight, EDO sees the economy as primarily having been hit with a big, persistent risk-premium shock in late 2008 and during the first half of 2009. In 2008Q4, the estimated economy-wide risk premium was two standard deviations away from its mean under the stationary distribution; by the first half of 2009, the premium was three standard deviations away from its mean. Although other shocks also contributed to the economic downturn, their quantitative importance to the decline in aggregate output is estimated to be much less important.

likelihood of a ZLB event and related statistics, with and without allowance for the effects of parameter uncertainty. In the case of EDO, expanding the sources of uncertainty triples the estimated probability of hitting the ZLB sometime between early 2008 and mid-2010, and widens the confidence intervals for the 2010Q2 projections between 18 and 25 percent.

Figures 3A and 3B present comparable results from the TVP-VAR model, respectively with and without allowance for the effects of parameter uncertainty. Like EDO and FRB/US, this statistical model see the marked rise in the unemployment rate since late 2007 as quite surprising, based on the shocks that have hit the economy since the mid-1960s. But unlike the two structural models, the TVP-VAR model is not completely surprised that short-term interest rates fell to zero, in that the actual path of the T-bill rate falls inside the 95 percent confidence interval when allowance is made for the effects of parameter uncertainty (figure 3A). But when this source of uncertainty is inappropriately ignored, the confidence bands shrink appreciably and misleadingly suggest that hitting the ZLB was a near impossibility from the perspective of late 2007. As indicated in table 2, projection confidence intervals that take account of parameter uncertainty are between 21 and 34 percent wider than those that do not.

Figure 4A and 4B present results from the LW model, again with and without controlling for the effects of parameter uncertainty. The LW model, which is estimated over a sample starting from the early 1960s and incorporating greater intrinsic inertia than the two structural models, yields relatively high probabilities of hitting the ZLB over the next five years. In addition, it places high probabilities of ZLB episodes lasting four or more quarters. In the LW model, failing to account for the effects of parameter and latent variable uncertainty sharply biases down the estimated width of the confidence intervals about the LW model's projections. For example, table 2 shows that the 95 percent confidence interval for the 2010Q2 projection of inflation is 22 percent wider when these factors are taken into account, while the confidence interval for the output gap more than doubles. These relatively large effects stem from the presence of both parameter and filter uncertainty that affect uncertainty about the equilibrium real interest rate and the output gap in this model.[12]

Like the two structural models, the LW model is not surprised by the movements in inflation but is surprised by the sharp decline in short-term interest rates, given the sort of disturbances that hit the economy from the early 1960s on. However, the LW model departs

[12] See Hamilton (1986) for a discussion of these two sources of uncertainty.

from the two structural models in not being especially surprised by the fall in the estimated output gap, even when no allowance is made for parameter uncertainty. In part, the lack of surprise arises because the LW stochastic simulations, unlike the FRB/US ones, treat the output gap as an unobserved variable that is always uncertain. In addition, the LW model estimates that the output gap is measured with less precision than EDO estimates, implying that this source of uncertainty has relatively less of an effect on the width of the EDO confidence intervals. Another contributing factor is the manner in which the LW model interprets the recent co-movement of output and inflation and their implications for potential output. With actual output contracting precipitously but inflation declining only modestly, the LW model infers that potential output growth was quite weak during the recession and the early stages of the subsequent recovery; as a result, the LW output gap declines only modestly over the last few years. (See Weidner and Williams 2009 for a further discussion of this point.) In contrast, the statistical filtering procedures used by both EDO and FRB/US put less weight on movements in inflation in inferring movements in potential output. Finally, we should emphasize that even though the LW model was not surprised by the estimated evolution of the output gap, it was quite surprised by the evolution of actual output.

Figure 5 presents results for our last statistical model, the GARCH univariate equations. In this case, the confidence intervals around the projections are quite wide because they allow for the possibility of time-varying variances of the shocks. As a result, the model is not all that surprised that short-term interest rates fell almost to zero after 2007, although the degree to which the unemployment rate rose is still seen as remarkable.

The bottom line of this analysis is that recent events would have been judged very unlikely prior to the crisis, based on analyses from a variety of models and statistical approaches using U.S. data on conditions over the past several decades. A second clear finding is the importance of parameter uncertainty—and in the case of the LW model, uncertainty about persistent latent variables—to any statistical assessment of the likelihood of recent events or hitting the ZLB in general. Finally, the various models give quite different estimates of the probability of hitting the ZLB—especially the probability of being stuck there for a year or longer, as will be demonstrated in the next section. These findings strongly suggest that researchers need to take account of uncertainty with respect to parameters, models and the persistence of shocks if they wish to provide policymakers with reasonable estimates of the threat posed by the ZLB.

IV. Has the estimated probability of hitting the ZLB changed much over time?

We address this question by using the various models to estimate how the likelihood of hitting the ZLB within the next five years would have looked at different points in the past, given assessments at the time of actual and expected economic conditions and the types of shocks that could hit the economy. Ideally, we would use real-time data and real-time versions of the models to carry out such an analysis, because after-the-fact projections based on revised data sometimes provide a very misleading picture of the actual outlook at the time. Such a real-time analysis is beyond the scope of this paper, however, and so we restrict ourselves to probability assessments based on model projections and error-variance assessments constructed using the vintage of data available at the time of the writing of this paper.

Specifically, we generate the model projections and error-variance estimates using historical data through the prior quarter, for each quarter from 2000Q1 on. In this exercise, each model is used to generate a sequence of rolling 20-quarter projections and accompanying probability distributions centered on those projections. With the exception of FRB/US, rolling estimates of model parameters are generated using historical data through the prior quarter; FRB/US' coefficients are instead held fixed at the estimates derived from data through 2008, because the size of the model makes repeated re-estimation infeasible. In the stochastic simulations of all the models, the rolling estimates of the shock distributions are based on an expanding sample of historical model errors. From these rolling estimates of the probability distributions for real activity, inflation, and short-term interest rates, we compute the probability at each point in time of two different ZLB events. The first probability is the likelihood at each point in time that the nominal federal funds rate or T-bill rate will fall below 26 basis points at least once within the next 20 quarters. The second probability is the likelihood that the funds rate or the T-bill rate will be below 26 basis points for at least four consecutive quarters sometime within the next 20 quarters. Figure 6 shows the evolution over the past decade of the odds of either hitting the ZLB or being persistently at the ZLB within a few years, as gauged by the various models.

The results show considerable variation across time in the risk of hitting the ZLB over the medium term but roughly the same pattern across models (upper panel). All the models except one show the odds of a ZLB event as falling to 10 percent or less in 2006 after having run at an elevated level during the sluggish recovery that followed the 2001 recession. From a low in 2006, the probability estimates then began to rise sharply, coming near or reaching 100 percent by late 2008. The two structural models thereafter show the odds remaining extraordinarily high through to the present, while the three statistical models show the estimated probabilities gradually declining but still remaining quite elevated.

To varying degrees, these estimated probabilities of hitting the ZLB are influenced by the Great Moderation period. In particular, there is a tendency for the models to mark down the likelihood of encountering extremely low interest rates as their estimates of the variance of macroeconomic shocks is based on samples that include more data from the Great Moderation period. This sensitivity to the Great Moderation period is perhaps greatest in the TVP-VAR model, in which the innovation variances are allowed to vary over time—an additional flexibility in model specification that makes the model more sensitive to small-sample variation. (In the case of EDO, of course, estimated probabilities prior to the crisis are entirely based on conditions during the Great Moderation period.)

Differences across models are more pronounced regarding the probability of being persistently stuck at the ZLB (bottom panel). EDO shows these odds consistently remaining close to zero until mid-2008, whereupon they rise modestly to about 15 percent as the model's assessments of macroeconomic volatility begin to incorporate the events of the crisis. In contrast, FRB/US shows the odds climbing to almost 50 percent in early 2003, then declining to 5 percent in 2007, and then skyrocketing to 100 percent by late 2008. This profile differs markedly from that exhibited by EDO primarily because output is much more inertial in FRB/US; this result may help to explain why some researchers working with DSGE models in the past have not viewed the ZLB as a serious concern, assuming that such models tend to be as non-inertial as EDO. Results from the TVP-VAR, LW, and GARCH models are broadly in line with those generated with FRB/US through late 2008, although these statistical models judge that the odds of a persistent ZLB event within the next five years have since moved noticeably lower.[13]

The estimated probabilities plotted in figure 6 reflect the effects of time-variation in both the economic outlook and estimated macroeconomic volatility. To illustrate the importance of the latter factor alone, we re-run the FRB/US and EDO stochastic simulations around a steady-state baseline in which the economy has been and is expected to remain in equilibrium, and allow only the estimated shock process to change over time. In the steady-state baseline, the output gap is constant at zero, inflation is 2 percent, and the nominal funds rate is 4¼ percent. As before, the assessment of macroeconomic volatility evolves over time using an expanding sample of historical shocks. We do not rerun the stochastic simulations of the statistical models, however, because forcing a steady-state baseline on them is conceptually problematic given their data-filtering procedures, and hence generates odd results.

Results from this exercise are summarized in figure 7. Starting from an initial state of equilibrium, the estimated probability of hitting the ZLB within the next 20 quarters is roughly flat from 2000 through 2007 at about 3 percent according to FRB/US (upper panel). The same model judges the likelihood of being persistently stuck at the ZLB as even lower over the same period, at about 2 percent (lower panel). Starting in the second half of 2008, however, both FRB/US probabilities jump markedly, respectively to 9 percent and 8 percent, as the implications of recent shocks for macroeconomic volatility are incorporated into the model's assessment of macroeconomic volatility. Qualitatively, the probabilities generated by the EDO model display a similar time profile as the FRB/US estimates. However, EDO consistently judges the risk of hitting the ZLB bound as lower than FRB/US does when starting from a position of equilibrium. In fact, EDO judges the odds of a persistent ZLB event as essentially zero, even when the model's assessment of macroeconomic volatility takes accounts of recent events.

With the exception of the EDO probabilities, the estimates reported in figures 6 and 7 implicitly assume that the modeler takes a relatively long view about the types of shocks that could hit the economy in the medium term, in that the models' assessments of macroeconomic volatility are based on conditions back to the 1960s. That this is not always the case is demonstrated by the willingness of many observers to see the Great Moderation period as a sign

[13] Of the various model estimates of the probability of a persistent ZLB event, the ones generated by FRB/US appear to be closest to the current views of financial market participants, given that options on Eurodollar futures and interest rate caps currently indicate very low odds of a hike in the federal funds rate before 2012.

that the economy had become permanently less volatile. Figure 8 illustrates how the FRB/US' estimates of the probability of hitting the ZLB within the next 20 quarters would have evolved over time if assessments of future volatility were based on the sort of shocks experienced within a relatively short historical window—specifically, the last twenty-five years. As in figure 7, the estimated probabilities are conditional on the economy being in an initial state of equilibrium.

As the estimated shock variance becomes increasingly dominated by events during the Great Moderation, the probability of hitting the ZLB at least once over the medium term falls steadily, from 5 percent in 1992 to 1 percent in early 2008. With the onset of the Great Recession and the resultant increase in the estimated volatility of the economy, this probability leaps to 11 percent. The probability of experiencing a persistent ZLB event shows a similar time profile.

V. How severely did the ZLB bind during the crisis?

The evidence presented so far suggests that monetary policy may have been importantly constrained by the ZLB during the crisis, given that FRB/US and the statistical models show the probability of experiencing a persistent ZLB episode rising to a very high level during the crisis. By themselves, however, these statistics do not directly measure the degree to which monetary policy was constrained by the ZLB during the crisis, nor the resultant deterioration in economic performance. To address this issue, we now consider results from counterfactual simulations of FRB/US in which we explore how conditions over the last few years might have evolved had it been possible to push nominal interest rates below zero.

What monetary policy would have done in the absence of the zero lower bound constraint depends, of course, on policymakers' judgments about how best to respond to changes in current and projected economic conditions in order to promote price stability and maximum sustainable employment. Such judgments would have depended on many factors, including assessments of overall resource utilization, the outlook for employment growth and inflation, the risks to that outlook, and the perceived responsiveness of real activity and prices to additional monetary stimulus. Although we cannot hope to account for all the factors that influence the FOMC's decision process, we can give a flavor of what might have occurred in the absence of the ZLB constraint by employing optimal-control techniques of the sort advocated by Lars Svensson (2003, 2005).

Specifically, we ask what path of the *unconstrained* nominal funds rate from 2009Q1 on would have been expected to minimize deviations of resource utilization from zero and inflation from 2 percent, subject to two factors:[14]

- the baseline economic outlook as it stood in early 2009, where the forecast is conditioned on the prescriptions of a policy rule or some other presumably non-optimal path for monetary policy; and
- an economic model that links deviations of the funds rate from its baseline path to the expected response of real activity and inflation.

As discussed below, we use two different baseline forecasts for this exercise—one generated after-the-fact using the FRB/US model and the current vintage of historical data, and one derived from real-time private forecasts. In addition, we solve the optimal control problem subject to the dynamics of the FRB/US model under the assumption that agents have model-consistent (i.e., rational) expectations, in the sense that any differences in expectations from their baseline values are fully consistent with any deviations from baseline in the simulated paths of interest rates, inflation, and other factors. Using FRB/US for this analysis seems appropriate to us given that Federal Reserve staff have regularly used the model for exercises of this sort at the Federal Reserve, as discussed in Svensson and Tetlow (2005).

Our base-case specification of the loss function used in the optimal-control exercise takes the form

$$L = E_t \sum_{j=0}^{m} .99^j \left[\left(U_{t+j} - U^*_{t+j} \right)^2 + \left(\pi_{t+j} - 2 \right)^2 + \Delta R^2_{t+j} \right], \tag{1.8}$$

where U denotes the unemployment rate; U^* is the estimated long-run sustainable rate of unemployment (i.e., the NAIRU); π denotes inflation, measured by the four-quarter percentage change in the chain-weighted core PCE price index; and R denotes the nominal funds rate. As can be seen, this specification penalizes large quarter-to-quarter changes in the nominal funds rate in addition to deviations of U from U^* and π from the assumed 2 percent target; this third term serves to damp excessively sharp movements in the funds rate that might otherwise occur. In solving the optimal-control problem, we set the length of the evaluation window, m, at 100 quarters but only minimize the function with respect to the values of R over the first 60 quarters;

[14] In the economic projections published quarterly by the FOMC since early 2009, most Committee participants reported that their long-run inflation projection (as measured by the PCE price index) equaled 2 percent.

beyond that point, the funds rate is assumed to follow the path prescribed by the estimated policy rule discussed earlier.[15]

Because the real-activity leg of the Federal Reserve's dual mandate is expressed in terms of maximum sustainable employment, we think it appropriate to define resource utilization in the base-case specification of the loss function in terms of the unemployment rate. However, a case could be made for focusing on resource utilization in the overall economy and not just the labor market. Accordingly, we also generate optimal policy paths using a version of the loss function expressed in terms of the output gap, Y:

(1.9) $$L = E_t \sum_{j=0}^{m} .99^j \left[.25Y_{t+j}^2 + \left(\pi_{t+j} - 2\right)^2 + \Delta R_{t+j}^2 \right]$$

In this alternative specification, the relative weight placed on stabilizing the output gap is only one-fourth the weight placed on stabilizing inflation and damping quarter-to-quarter movements in the funds rate. This adjustment in relative weights is needed to ensure comparability of results across the two specifications of *L*, given that the variability of the output gap is roughly twice as great as the variability of the unemployment rate. Alternatively put, the Okun's Law coefficient that links movements in the unemployment gap to movements in the output gap is roughly equal to 0.5, and square of this coefficient equals 0.25.

As noted above, our analysis considers two different characterizations of the economic outlook in early 2009—one derived from the FRB/US model and one based on private forecasts released at the time. We generate the FRB/US baseline forecast by hopping off from conditions in 2009Q1 and simulating the model subject to automated projections of various equation errors and exogenous variables, using the same extrapolation procedure discussed earlier.[16] Other important conditioning assumptions for the baseline model projection include:

- The federal funds rate equals the prescriptions of the estimated policy rule described earlier, subject to an effective lower bound of 12.5 basis points.
- No explicit adjustment is made for large-scale asset purchases. As a result, the baseline model projection does not incorporate any significant implicit effects, given that the FOMC did not announce the full dimensions of the 2009 LSAP program until the

[15] These settings are sufficient to ensure that the simulated path of the funds rate follows an essentially flat trajectory at the end of the 60-quarter optimization window and beyond.

[16] Although this forecasting procedure has in some respects the spirit of a real-time forecasting exercise, we emphasize again that the data used in the analysis are the currently published estimates of GDP and inflation, not the data as they were published in early 2009.

meeting in mid-March 2009. Prior to that time, the FOMC had announced its intention to purchase a more modest volume of agency MBS, and so the 2009Q1 economic and financial conditions from which the model projections hops off were only slightly influenced by asset purchases.

- Fiscal policy assumptions incorporate the tax cuts, grants to S&L governments, extended unemployment benefits, and other increases in federal spending that were enacted in 2009.

Because the extrapolation procedure assumes a low persistence of the shocks experienced in 2008 and 2009, the FRB/US model predicts that a strong recovery in consumer durables, housing, and business investment should have begun in mid-2009. At the time, the model predicts that a marked deceleration in prices should have occurred in 2009 and 2010, in part reflecting FRB/US' view that economic slack was very high and would remain persistently so for several years.

Our alternative baseline forecast is based on the Blue Chip consensus projections released in early March 2009. In that release, survey participants provided forecasts of quarterly real GDP growth, the unemployment rate, overall CPI inflation, the rate on 3-month Treasury bills, and the yield on 10-year Treasury bonds through the end of 2010; in addition, participants provided annual projections for these series for the period 2011 through 2020. Because our optimal-control procedure uses the federal funds rate instead of the T-bill rate, we translate the projections of the latter into forecasts of the former by assuming they were equal. In the case of inflation, we translate Blue Chip forecasts for the CPI into projections for core PCE prices using the projections of the spread between these two series that were reported in the spring 2009 Survey of Professional Forecasters.

The Blue Chip projections also contain information related to the supply side of the economy. In particular, the long-run consensus forecasts showed unemployment stabilizing late in the decade at 5½ percent, accompanied by stable output growth and inflation. This combination suggests that the NAIRU implicit in the March 2009 outlook was about 5½ percent. In contrast, the Blue Chip long-run projections made in early 2008 and 2009 suggest that private forecasters estimated the NAIRU to be about 4¾ percent—about the same as its value in the FRB/US baseline. Thus, private forecasters by early 2009 apparently thought that the financial crisis would have permanent adverse supply-side effects sufficient to raise U^* 3/4 percentage point. Based on the extended Blue Chip projections published in March 2010, private forecasters subsequently upped their estimate of the fallout from the crisis further, as evidenced by a revised long-run projection for the unemployment rate equal to 6 percent.

The Blue Chip projections for real GDP growth, coupled with the projected path of the unemployment rate, can also be used to infer private forecasters' views about potential output via the prism of Okun's Law and the assumption that private forecasters expect both the output gap and the unemployment gap to be closed during the last few years of the decade. With the caveat that such inferences are more speculative than the one made about U^* because other factors could be at play, the Blue Chip projections seem to imply that private forecasters saw a sharp, temporary slowdown in trend multi-factor productivity growth from 2008 to 2010, in contrast to the relatively stable growth assumed in the FRB/US baseline projection. If this inference is correct, private forecasters in early 2009 saw the output gap as having widened to only minus 4 percent, in contrast to the minus 8 percent gap estimated by FRB/US.

The final step in generating a Blue Chip baseline projection suitable for use in optimal-control model simulations is to force the FRB/US model to replicate the various aspects of the private-sector consensus forecast just discussed, including the implicit supply-side assumptions. This matching step is done through adjustments to equation intercepts. In addition, we extend the Blue Chip projection beyond 2020 by assuming that the economy expands along a steady-state; this extension step is needed because the optimal-control procedure requires a long baseline. (We make a similar assumption with respect to the FRB/US baseline projection.)

Figure 9A compares the actual paths of the funds rate, real activity, and inflation to what would have been expected to occur in 2009Q1 and beyond given the baseline FRB/US forecast, conditional on three different assumptions for monetary policy. Specifically, the figure reports results for the baseline projection run under the constrained estimated policy rule (red lines), for expected conditions under an unconstrained optimal policy that targets the unemployment gap (blue lines), and for expected conditions under an unconstrained optimal policy that targets the output gap (green lines).

Hopping off from early 2009 conditions, the baseline FRB/US forecast shows the output gap closing only gradually and the unemployment rate slowly falling back to the model's estimate of the NAIRU. As a result, the model would have expected inflation to turn negative for a time before gradually moving back towards the 2 percent inflation target. Under these conditions, the model would have predicted monetary policy, following the prescriptions of the estimated policy rule with the ZLB imposed, to hold the funds rate close to zero into early 2012. However, if the Federal Reserve could have instead implemented an unconstrained optimal monetary policy that targeted inflation and the unemployment gap, it would have responded to the persistently weak outlook by driving the nominal funds rate far below zero from 2009 through 2011. Based on the dynamics of the FRB/US model under rational expectations, such a policy would have been expected to induce a sharp drop in long-term interest rates in early 2009 that would have directly stimulated real activity. In addition, the optimal policy would have pushed inflation above 2 percent in the 2017 to 2020 period before subsequently returning it to the inflation target (not shown). The resultant boost to inflation expectations in the early years of the optimization period would have had the beneficial effect of raising actual inflation and providing further stimulus to real activity in the near-to-medium term.

A comparison of the red and blue lines suggests that the severity of the ZLB constraint was indeed great. Given the FRB/US outlook, short-term interest rates would ideally have been about 600 basis points lower in 2009 than was actually possible. Under such a policy, FRB/US predicts that the improvement in expected macroeconomic performance relative to the baseline outlook would have been considerable. In particular, the unemployment rate would have been expected to fall back to 4¾ percent by late 2011 instead of early 2014, and core inflation would have bottomed out at ¾ percent in early 2010 instead of turning negative.

As indicated by the green lines of figure 9A, an optimal monetary policy that targeted the output gap would resulted in almost the same path of the funds rate, and hence the same expected improvement in macroeconomic conditions relative to the baseline projection. This result follows because FRB/US (from the standpoint of early 2009) neither expected Okun’s Law to be appreciably violated during the recession nor anticipated that the financial crisis would generate a significant slowdown in the level of trend multi-factor productivity. As a result, the model’s assessment was that the estimated/projected co-movement of the output gap and the unemployment rate over the current business cycle were and would continue to be similar, in contrast to the assessment implicit in the Blue Chip consensus forecast.

Figure 9B reports the corresponding results when the optimal policy path is conditioned on the Blue Chip consensus projection. Like FRB/US, private forecasters in early 2009 anticipated only a slow decline in the unemployment rate back to the implicit NAIRU under the baseline monetary policy (red lines). Unlike FRB/US, however, these forecasters anticipated both less deflation through 2010 and a sharper subsequent snapback in inflation to its target level, perhaps because they thought that the output gap had not widened as much as the model did. These conditions likely help to explain why the Blue Chip consensus forecast showed the FOMC starting to raise short-term interest rates as early as the latter half of 2010.

Nevertheless, the Blue Chip outlook at the time still would have justified pushing the federal funds rate well below zero, had that been possible. For example, an optimal monetary policy that targeted inflation and the unemployment gap would have called for pushing nominal short-term interest rates down to almost -4 percent (blue lines). As a result, the unemployment rate would have been expected to fall much more rapidly than in the Blue Chip baseline even as inflation fluctuated around 2 percent. Targeting the output gap in place of the unemployment gap would have implied less need to push the funds rate into negative territory (green lines). Nevertheless, the counterfactual simulation predicts that pushing the funds rate modestly below zero, and then keeping the funds rate well below the Blue Chip baseline path through 2012, would have yielded a noticeable improvement in projected macroeconomic conditions relative to baseline.

These optimal-control results indicate that one's sense of the severity of the ZLB constraint during the crisis depends importantly on the amount of slack that had opened up, expectations for the length of the recession and the subsequent pace of the recovery in real activity, and views about the potential for future disinflation.

Regarding the first of these considerations, many observers have argued that the financial crisis likely had a significant and adverse effect on the economy's productive potential. For example, the IMF estimates that potential output growth in the United States slowed from close to 2½ percent in 2006 to a bit above 1½ percent on average in 2009 and 2010 (IMF, 2010). Other forecasters, including the OECD (2010) and the Congressional Budget Office (2010), also judge that potential output growth slowed appreciably following the financial crisis, reflecting the effects of less capital investment, smaller gains in trend multi-factor productivity, and impairments to labor market functioning that have boosted U^*. Despite the estimated hit to potential output, however, these same organizations estimate that the severity of the recession was enough to push the output gap in 2009 into the range of minus 5.1 percent to minus 7.2 percent. By comparison, the FRB/US estimate used in the optimal-control simulations shows somewhat more product-market slack than the upper end of this range of estimates, while the real-time estimate derived from private forecasts show somewhat less slack than indicated by the lower end of the range (at least based on our interpretation of the Blue Chip data). Note, however, that some of the output gap estimates reported by Weidner and Williams (2009) show even less slack in 2009 than the Blue Chip-consistent estimate.

Evidence has also accumulated that the financial crisis and deep recession may be having persistent adverse effects on the functioning of the labor market and on workers' skills, thereby increasing the NAIRU—a development that would be consistent with the Blue Chip baseline but not the FRB/US baseline. For example, Barnichon and Figura (2010) estimate that reduced matching efficiency may have added 1½ percentage points to the unemployment rate during the recent recession, based on recent shifts in the Beveridge curve. In addition, the duration of unemployment has increased much more markedly in the current slump than in previous downturns. Although the implications of these developments for the long-run natural rate of unemployment are still unclear—for example, past cyclical reductions in matching efficiency were later reversed—they are generally consistent with the upward revisions to extended Blue Chip projections of the unemployment rate recorded since 2007.

Given that the real activity leg of the Federal Reserve's dual mandate is expressed in terms of employment, we put more weight on optimal-control results based on targeting the unemployment gap instead of the output gap. As a result, we tend towards the view that the ZLB has significantly constrained monetary policy, thereby resulting in a deeper and more prolonged slump and more disinflation than would otherwise have been the case. Of course, the results plotted in both figures 5A and 5B are predicated on a lower estimate of the NAIRU than private forecasters now appear to hold, suggesting that both figures may overstate the severity of the ZLB constraint. With hindsight, however, the effects of a higher NAIRU on the optimal path of the funds rate are offset by the fact that the unemployment rate over the course of 2009 and 2010 rose appreciably higher than was projected in early 2009. In addition, the projected recovery in real activity is now somewhat slower and more delayed than before (although the outlook for inflation is little changed). As a result, re-running the Blue Chip optimal-control experiment with the benefit of hindsight about economic conditions through the spring of 2010, but again starting the optimization in 2009Q1, generates a peak reduction in the nominal funds rate of -400 basis points, despite the fact that the NAIRU is assumed to equal 6 percent.[17]

Before leaving this section, we would be remiss if we did not note some important caveats and limitations to our optimal-policy analysis:

- *Model uncertainty*. Other models might give materially different assessments of the severity of the ZLB constraint, either because their projections are more optimistic or pessimistic than the FRB/US outlook, or because they have a different characterization of the monetary transmission mechanism. EDO, for example, would have been more optimistic than FRB/US about the outlook in early 2009, and so would have seen less need for additional monetary stimulus. Moreover, even conditional on the Blue Chip outlook at the time, EDO's dynamics are such that it still would have called for less monetary accommodation than FRB/US.
- *Monetary transmission mechanism*. In the current environment, FRB/US' dynamics may not provide an accurate characterization of the likely responses of real activity and inflation to changes in the federal funds rate. For example, the interest-sensitivity of aggregate demand may now be unusually low because of reduced access to credit or heightened uncertainty about the economic outlook. If so, an unconstrained optimal policy response might call for a larger peak decline in the nominal funds rate.
- *Policymaker preferences*. The loss function may mischaracterize policymakers' actual preferences. For example, policymakers could implicitly score the loss from a percentage point increase in the unemployment gap as different from that associated with a percentage point increase in the inflation gap, instead of weighing them as equal as the loss function assumes. Moreover, the quadratic loss function abstracts from any special concerns that policymakers might have about extreme tail risks; such risks are of particular concern in current circumstances, given the probability asymmetries created by the zero lower bound and the resultant increase in the likelihood of deflation.

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[17] To generate these figures, we use the Blue Chip long-run economic forecasts published in March 2010. Since that time the economic outlook has worsened somewhat, suggesting that the estimated severity of the ZLB constraint during the recession may now in hindsight appear even greater. This Blue Chip forecast, in contrast to the one made in early 2009, would have fully incorporated the past and projected effects of the Federal Reserve's large-scale purchases of mortgage-backed securities, agency debt, and longer-term Treasury securities. In the absence of these purchases, a hindsight evaluation of the severity of the ZLB constraint would have presumably shown an even larger optimal unconstrained decline in the federal funds rate.

- *Unconventional monetary policy*. Our optimal-control analysis does not address the FOMC's ability to ease the ZLB constraint through large-scale asset purchases and other unconventional monetary policy actions. Although this topic is beyond the scope of this paper, we would note in passing that the LSAP program appears to have significantly lowered long-term interest rates through some combination of portfolio balance effects, improved confidence, and altered expectations for the future path of short-term interest rates.[18]

VI. What are the lessons from this analysis for the future?

In retrospect, we and other researchers should have been leery of basing our analyses solely on the U.S. postwar experience. If we had put more weight on the cross-country evidence, which shows a fairly frequent pattern of financial crises followed by deep recessions, we might have been less sanguine about ZLB risks in the United States.[19] Instead, our models and those of other researchers of zero bound issues are estimated to match the behavior of the U.S. economy over portions of the postwar period. As a result, the models may severely underestimate the frequency of very bad recessions, such as the Great Depression in the United States and financial crises in other countries.

To investigate this possibility, we use simulations of EDO and a linearized version of FRB/US to generate the unconditional moments for real activity and other variables under each model's estimated policy rule with the inflation target set to 2 percent. In generating these simulated moments, we assume that the economy is buffeted with the sorts of shocks experienced from 1968 through mid-2010 in the case of FRB/US, and from the mid-1980s through mid-2010 in the case of EDO; the EDO estimates also take account of parameter

[18] For example, Gagnon et al (2010) estimate that the program to purchase up to $1.75 trillion in longer-term Treasury securities, GSE debt and agency mortgage-backed securities lowered long-term interest rates somewhere between 30 and 80 basis points. Based on model simulations and some simple regressions, achieving such a reduction in long-term rates through conventional monetary policy might have required a cut in the federal funds rate of about 120 to 320 basis points, assuming that such a reduction would have been expected to persist for several years.

[19] For example, a recent BIS review of the cross-country evidence suggests that the frequency of financial crises is roughly 4 to 5 percent per year—that is, about four or five times a century. The BIS review also found that, following a financial crisis, the cumulative shortfall in output relative to pre-crisis trends was about 20 percent of GDP on average, assuming no permanent reduction in potential output following the crisis. By contrast, no such event occurred in the United States during the postwar period.

uncertainty and measurement error.[20] We then compare the results from this exercise to similar estimates produced by Reifschneider and Williams (2000). The earlier results, some of which were not reported in the published 2000 paper, were generated using an earlier version of the FRB/US model run with two different simple funds rate rules—the Taylor (1993) rule and the Henderson-McKibbon rule. In addition, the 2000 simulation results were based on shocks of the sort experienced from 1966 through 1995.

Table 3 summarizes our results, some of which are quite striking. First, the latest estimates do not show that the frequency of ZLB events or their expected duration has risen appreciably from those reported in 2000. Admittedly, comparisons with the earlier results are difficult because the estimated policy rules used in EDO and the current version of FRB/US differ from those used in the earlier analysis. Still, one might have expected recent events to have raised the estimated frequency of ZLB events noticeably relative to the FRB/US figure reported in the 2000 study using the Taylor (1993) rule, given that the latter responds less aggressively to movements in output and inflation than either of the current estimated rules. And in fact, unconditional estimates of the likelihood of hitting the ZLB (not shown) do rise when the recent events are taken into account, holding the structure of FRB/US and EDO constant. But in the case of FRB/US, this effect is apparently offset by other changes to the model over the past decade; among other things, these changes result in a noticeably smaller estimate of the unconditional standard deviation of core inflation.

Another striking result concerns the simulated frequency of deep recessions, defined here as an output gap wider than minus 6 percent. All the estimates, old and new, suggest that deep recessions are highly infrequent, perhaps occurring only once a century; in addition, the estimates of the joint frequency of deep recessions and a near-zero funds rate indicate that monetary policy is typically constrained by the zero lower bound during severe slumps. Such low simulated probabilities may suggest that the model simulations are not making sufficient allowance for the possibility of extreme events in which the economy experiences a sequence of highly adverse shocks. Alternatively, both FRB/US and EDO may not adequately capture the nonlinear changes in dynamics that occur under extreme conditions, with the result that severe

[20] Because the simulated moments reflect the 2009-2010 shocks as well as earlier disturbances, they implicitly incorporate the effects of the recent Federal Reserve's unconventional policy actions as well as the historically-unusual component of fiscal policy actions during the recession.

shocks are followed by prolonged and painful economic adjustments as, for example, households and firms slowly repair their balance sheets.[21]

Looking forward, both modelers and policymakers will face the difficult problem of deciding how much to revise their estimates of macroeconomic volatility in light of recent events. On the one hand, we all should now have a greater appreciation of the persistent distress that can arise in the wake of a financial crisis, as well as the unexpected ways such a crisis can emerge and metastasize in developed economies with complicated financial systems. As we have noted, this experience suggests that macroeconomists need to take a more expansive view than they probably did before about the sorts of shocks that might potentially hit the economy. On the other hand, there may be a risk of going too far in this direction in light of the regulatory reforms now being implemented here and abroad. To the extent these reforms are successful in checking the emergence of excesses of the sort seen in recent years and in increasing the resilience of the financial system and the broader economy to future shocks, they may help to reduce the odds of seeing future downturns of the magnitude that we have just witnessed.

VII. Conclusion

The zero lower bound has been an important constraint on monetary policy in many countries over the past several years. The fact that many central banks have encountered the ZLB should not have come as a surprise—previous research using empirical models that were not based primarily on the Great Moderation period did predict that the ZLB would be a relatively frequent constraint on monetary policy in a low inflation environment. What has been a surprise is the magnitude and duration of the constraint imposed by the ZLB in the United States and in some other countries.

Our analysis has identified a number of factors that may have led to an underestimation of the extent to which the ZLB may affect macroeconomic outcomes. First, past research on the ZLB has generally focused on "well-behaved" models that are characterized by relatively rapid return to a stationary steady state following shocks. These models probably did not make

[21] To address this concern, we are currently examining various methods that could allow us to capture conditional variation in the persistence of the exogenous drivers, such as block bootstrapping; we are also investigating richer model structures which allow for this kind of variation during estimation. Such procedures may allow the higher degree of persistence of low equilibrium real rate states arguably seen in the data to show through to the simulated moments.

sufficient allowance for highly persistent episodes where the equilibrium real interest rate is low or periods when the volatility of shocks is high. Second, previous analyses abstracted from important sources of uncertainty about the economy, including uncertainty about parameters and latent variables such as natural rates. We find that these forms of uncertainty can significantly increase the expected variability of interest rates and therefore the incidence of hitting the ZLB. Third, the Great Moderation period can have a sizable effect on estimates of underlying macroeconomic variability used in model simulations. Indeed, analysis that relies entirely on data from this tranquil period cannot help but reach the conclusion that the ZLB is of little consequence. Recent events provide compelling evidence that 25 years is too small a sample for evaluating tail risks, and in recognition of this fact, research on the ZLB must take into account the evidence from history and other countries that the economy can be hit by persistent large negative shocks.

Finally, we should note that we have ignored a number of factors in our analysis. First, the effects of the ZLB can depend on the assumption regarding expectations formation. Roberts and Reifschneider (2006) and Williams (2006) find that deviations from rational expectations magnify the effects of the ZLB. Second, in most cases we abstracted from the problems of real-time measurement of data and natural rates that add additional uncertainty to the setting of policy and macroeconomic outcomes. Third, we did not take into account fiscal and unconditional monetary policy actions that may step in when the economy is constrained by the ZLB. And fourth, we have not explored the implications of recent events for the potential effectiveness of various strategies that have been proposed to mitigate the effects of the ZLB, such as price level targeting. Future research—including papers at this conference—will help improve our understanding of these and related issues.

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Table 1
Summary of Models

	EDO	FRB/US	TVP-VAR	LW	GARCH
Estimation sample start	1984	1968	1964	1961	1968
Estimation method	Bayes	OLS	Bayes	ML	ML
Estimated equations	28	56	3	8	3
Time-varying r*	No	No	Yes	Yes	No
Time-varying parameters	No	No	Yes	No	No
Time-varying variances	No	No	Yes	No	Yes

Table 2
Effects of Parameter Uncertainty on Estimates of the Probability of Hitting the ZLB
Based on Model Projections that Start from 2007Q4 Conditions

	Estimates Include Parameter Uncertainty			Estimates Exclude Parameter Uncertainty		
	EDO	TVP-VAR	LW	EDO	TVP-VAR	LW
Probability of reaching the ZLB on or before 2010Q2	1.5	3.5	23	0.5	1.6	16
Probability of having been at the ZLB for four consecutive quarters on or before 2010Q2	.4	2.5	12	.2	.7	6
Width of 95 percent confidence intervals for projections of conditions in 2010Q2:						
Short-term interest rate[1]	7.3	7.7	11.2	6.2	6.0	9.9
Inflation rate	3.4	5.7	5.9	2.7	4.7	4.8
Output gap	8.1	—	15.9	6.7	—	7.6
Unemployment rate	—	3.5	—	—	2.6	—

1. Federal funds rate for the EDO and LW models, 3-month Treasury bill rate for the TVP-VAR model.

Table 3
Alternative Estimates of Macroeconomic Performance with an Inflation Target of 2 Percent

	FRB/US[1]			EDO[1]
	Current version	2000 version, 1993 Taylor rule	2000 version, Henderson-Mckibbon rule	Current version
Frequency of ZLB episodes[2]	6	5	17	4
Mean duration of ZLB episodes[3]	5	4	4	2
Frequency of deep recessions[4]	1	2	1	1
Frequency of deep recessions and binding ZLB[5]	1	2	1	1
Standard deviations of:				
Output gap	2.5	3.0	1.9	2.2
Core PCE inflation	1.0	1.9	1.9	1.0
Federal funds rate	2.7	2.5	3.9	2.0

1. FRB/US stochastic simulations run with the current version and the 2000 version of the model based on shocks observed from 1968Q1 to 2010Q1 and 1966Q1 to 1995Q4, respectively. EDO stochastic simulations based on shocks observed from 1984Q4 to 2010Q2 and take account of parameter uncertainty and measurement error. The current versions of FRB/US and EDO use the estimated policy rules described in the main text.
2. Percent of time that the federal funds rate is at or below 25 basis points.
3. Mean number of consecutive quarters that the federal funds rate is at or below 25 basis points.
4. Percent of time the output gap is at or below -6 percent.
5. Percent of time the output gap is at or below -6 percent and the federal funds rate is at or below 25 basis points.

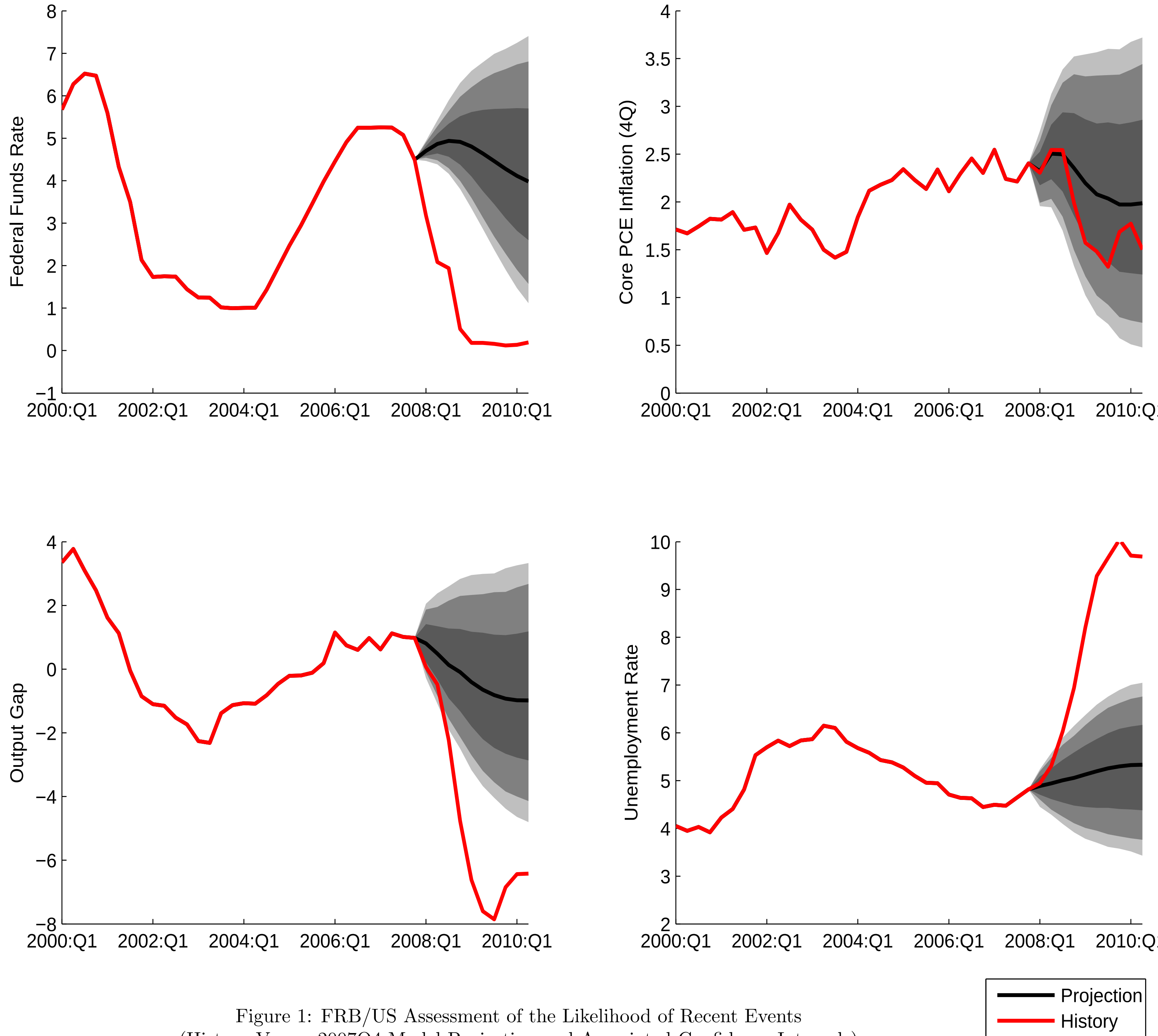

Figure 1: FRB/US Assessment of the Likelihood of Recent Events
(History Versus 2007Q4 Model Projection and Associated Confidence Intervals)
Shaded areas: 68, 90 and 95 percent confidence intervals

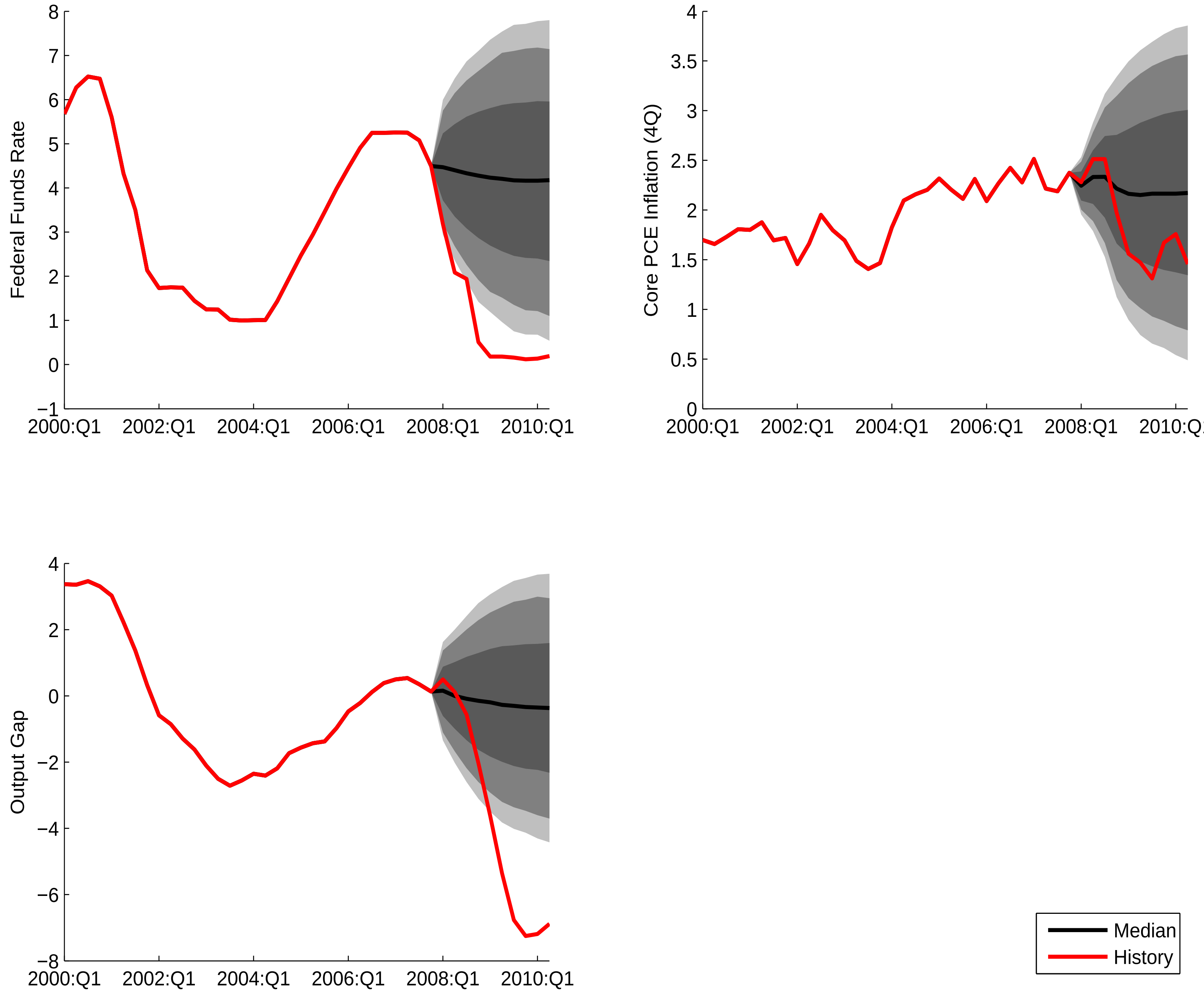

Figure 2a: EDO Assessment of the Likelihood of Recent Events
(History Versus 2007Q4 Model Projection and Associated Confidence Intervals)
Shaded areas: 68, 90 and 95 percent posterior credible sets

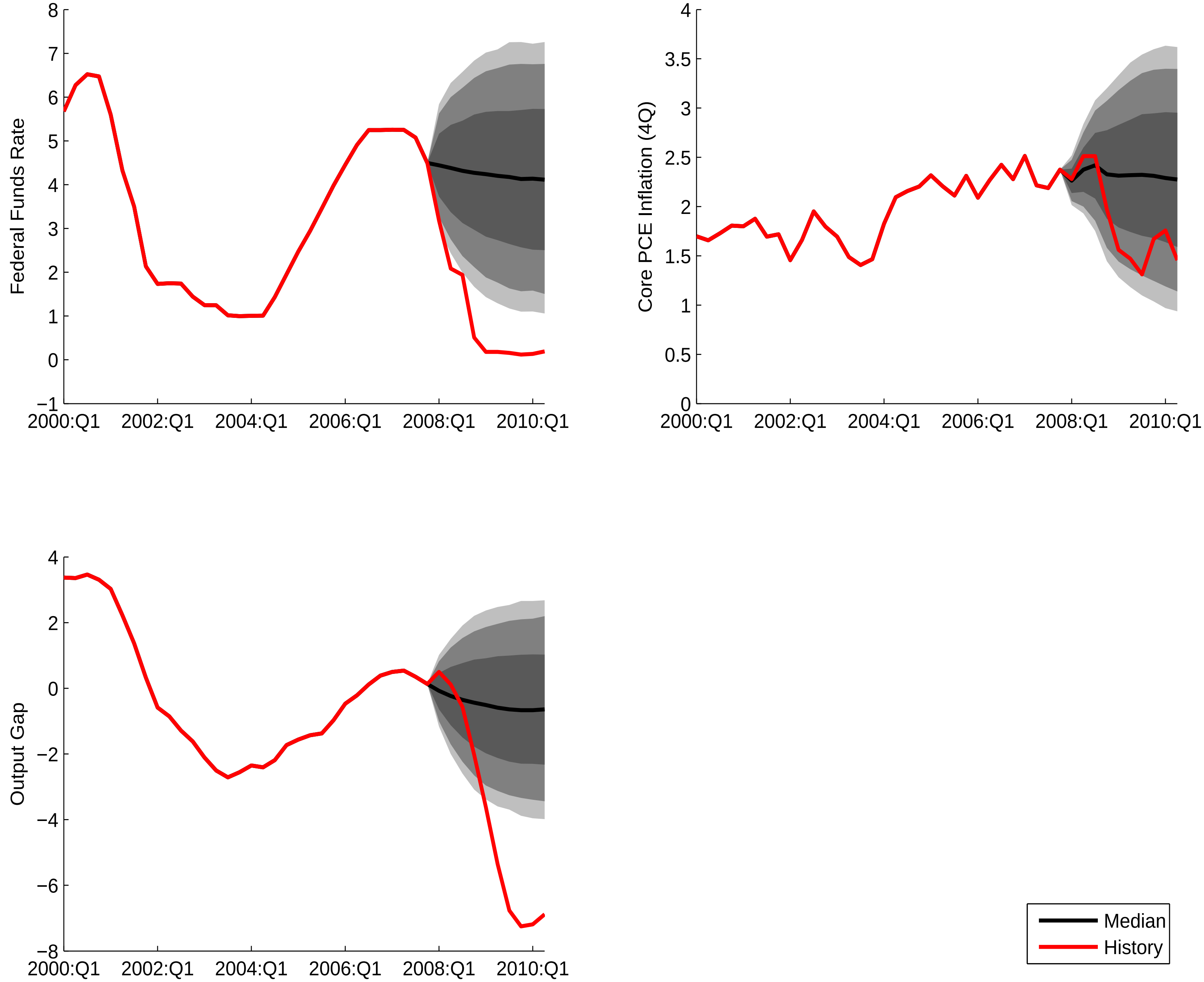

Figure 2b: EDO Assessment of the Likelihood of Recent Events
(History Versus 2007Q4 Model Projection and Associated Confidence Intervals)
Initial Condition (Posterior Mean) Known with Certainty
Shaded areas: 68, 90 and 95 percent posterior credible sets

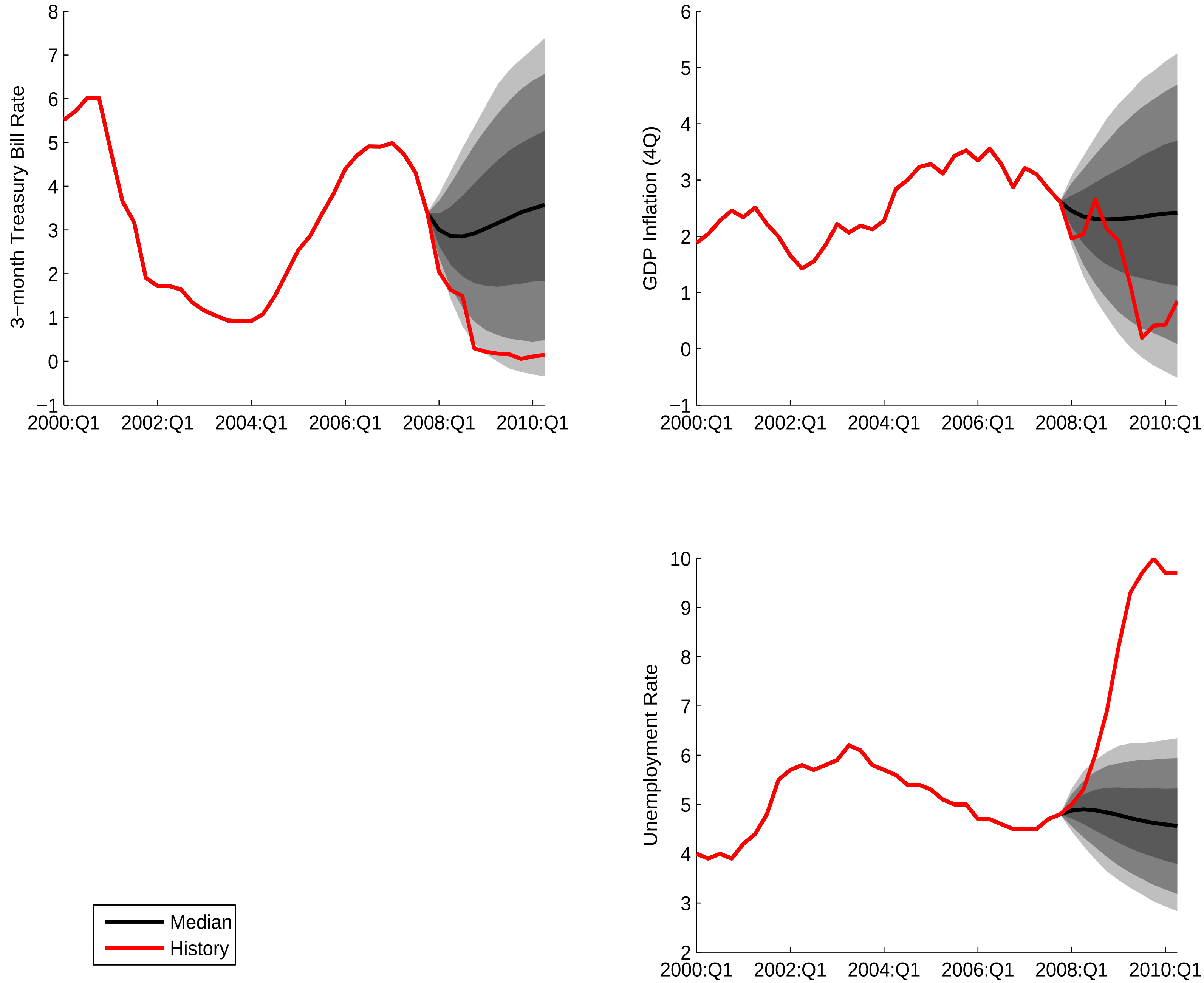

Figure 3a: TVP-VAR Assessment of the Likelihood of Recent Events
(History Versus 2007Q4 Model Projection and Associated Confidence Intervals)
Shaded areas: 68, 90 and 95 percent posterior credible sets

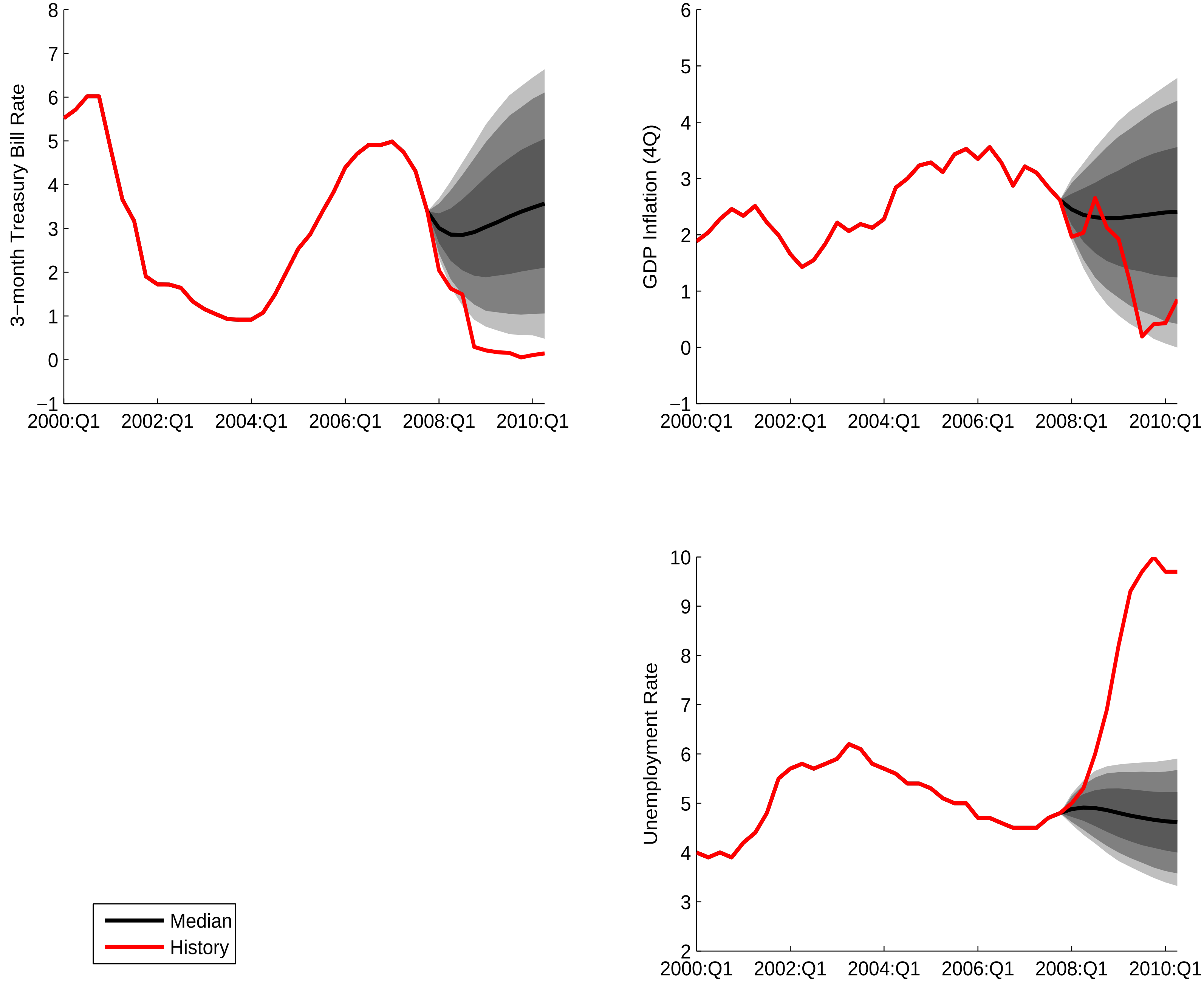

Figure 3b: TVP-VAR Assessment of the Likelihood of Recent Events
(History Versus 2007Q4 Model Projection and Associated Confidence Intervals)
Initial Condition (Posterior Mean) Known with Certainty
Shaded areas: 68, 90 and 95 percent posterior credible sets

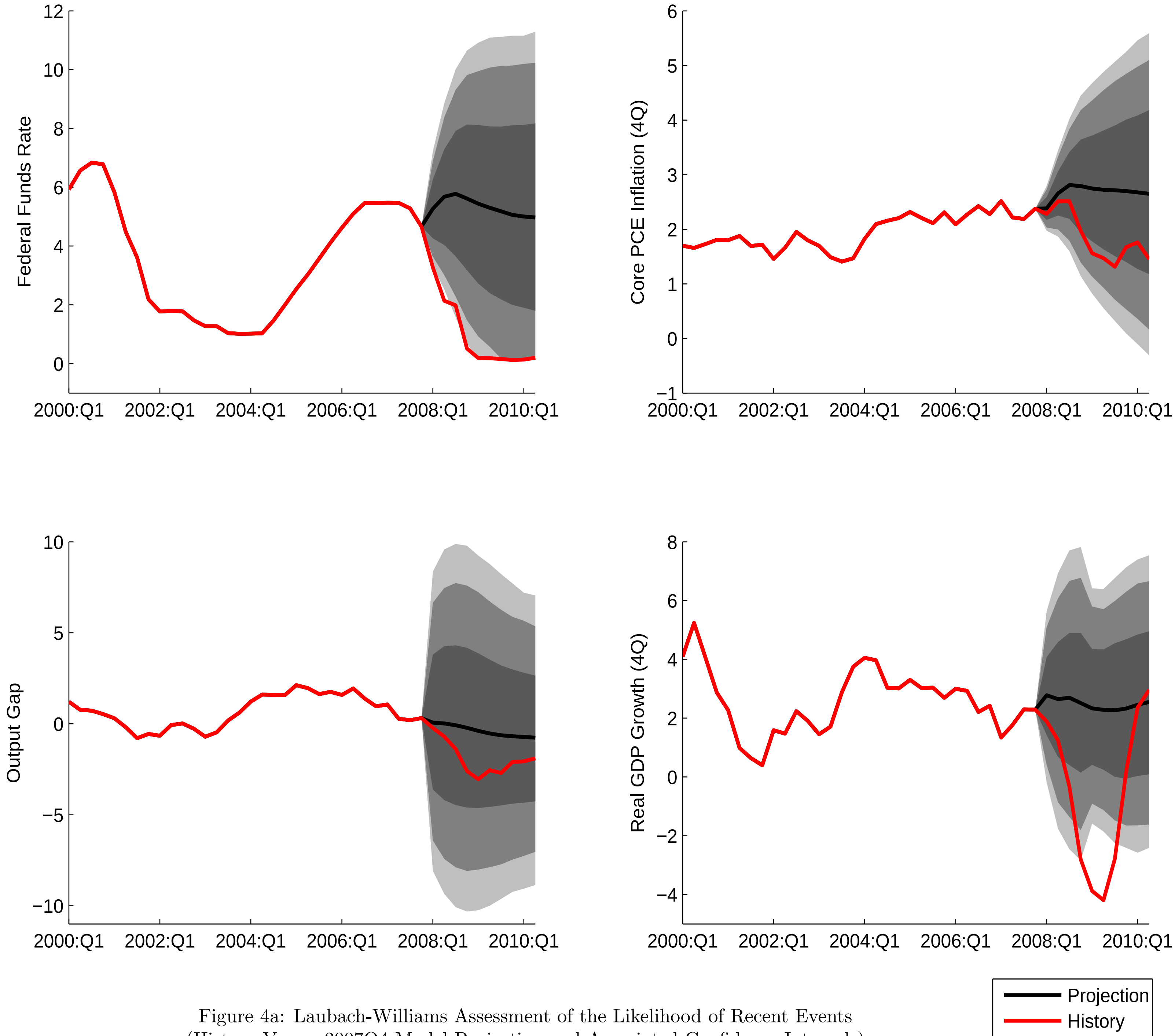

Figure 4a: Laubach-Williams Assessment of the Likelihood of Recent Events
(History Versus 2007Q4 Model Projection and Associated Confidence Intervals)
Shaded areas: 68, 90 and 95 percent confidence intervals

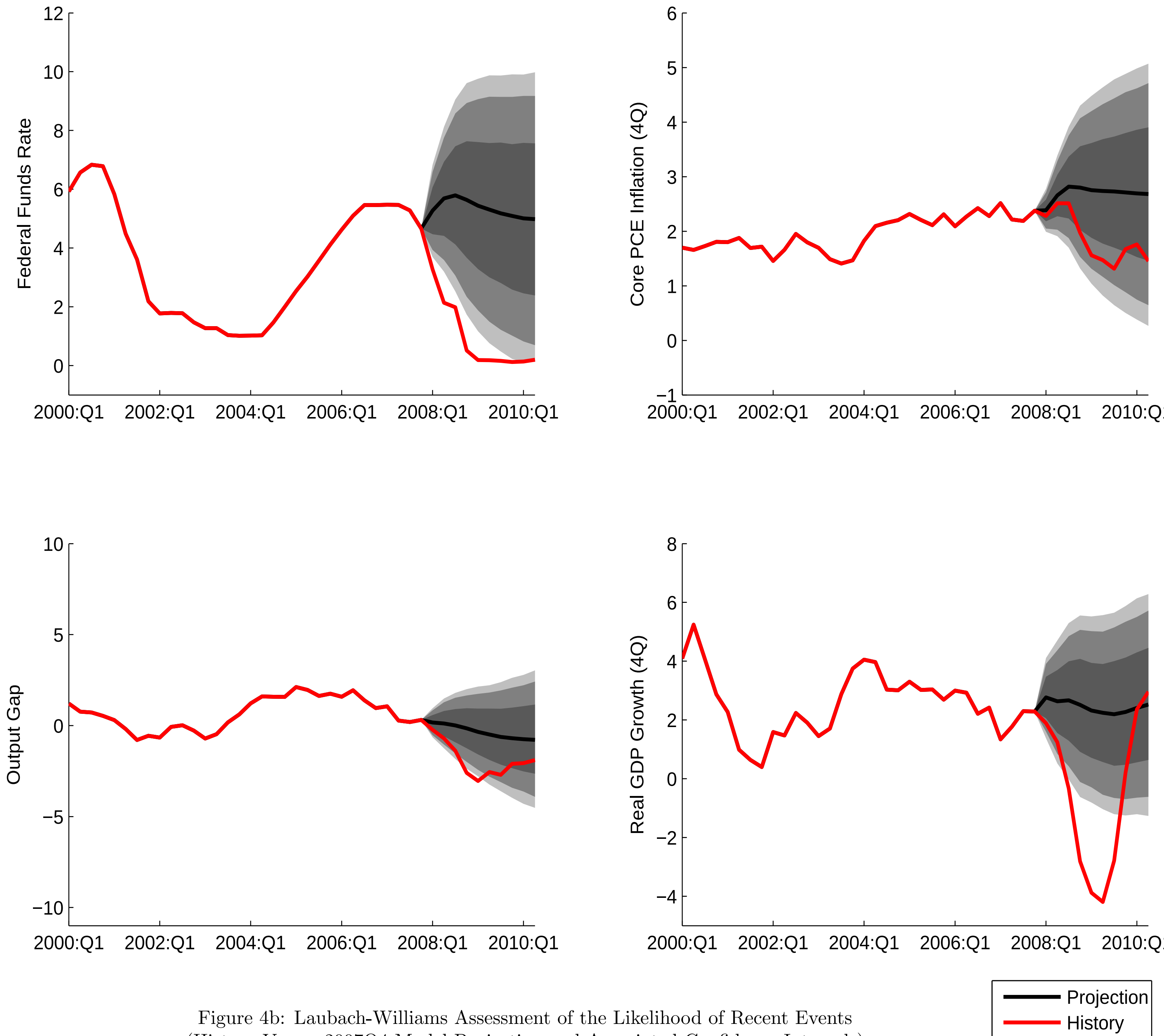

Figure 4b: Laubach-Williams Assessment of the Likelihood of Recent Events
(History Versus 2007Q4 Model Projection and Associated Confidence Intervals)
Initial Condition (Median) Known with Certainty
Shaded areas: 68, 90 and 95 percent confidence intervals

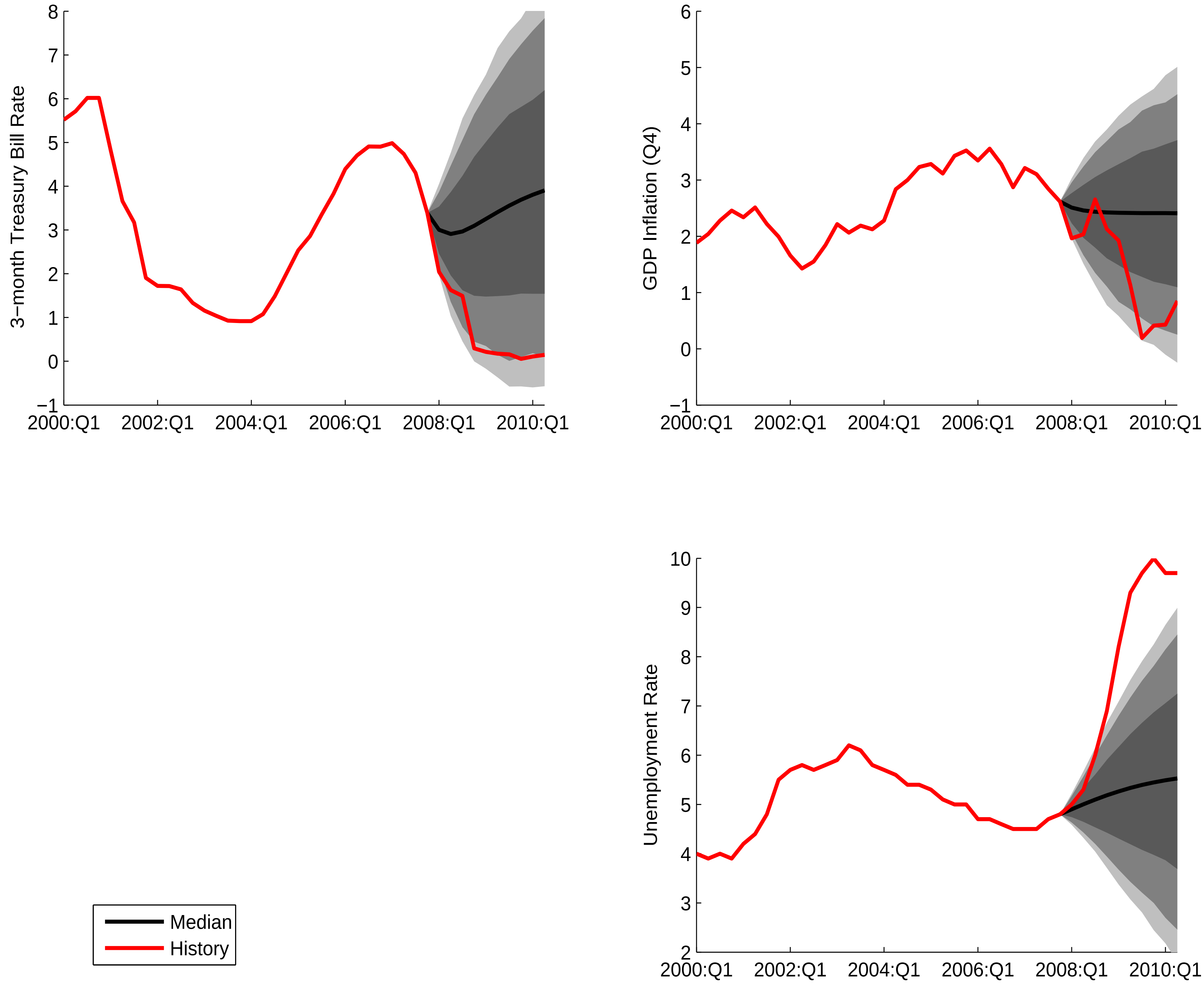

Figure 5: GARCH Assessment of the Likelihood of Recent Events
(History Versus 2007Q4 Model Projection and Associated Confidence Intervals)
Shaded areas: 68, 90 and 95 percent confidence intervals

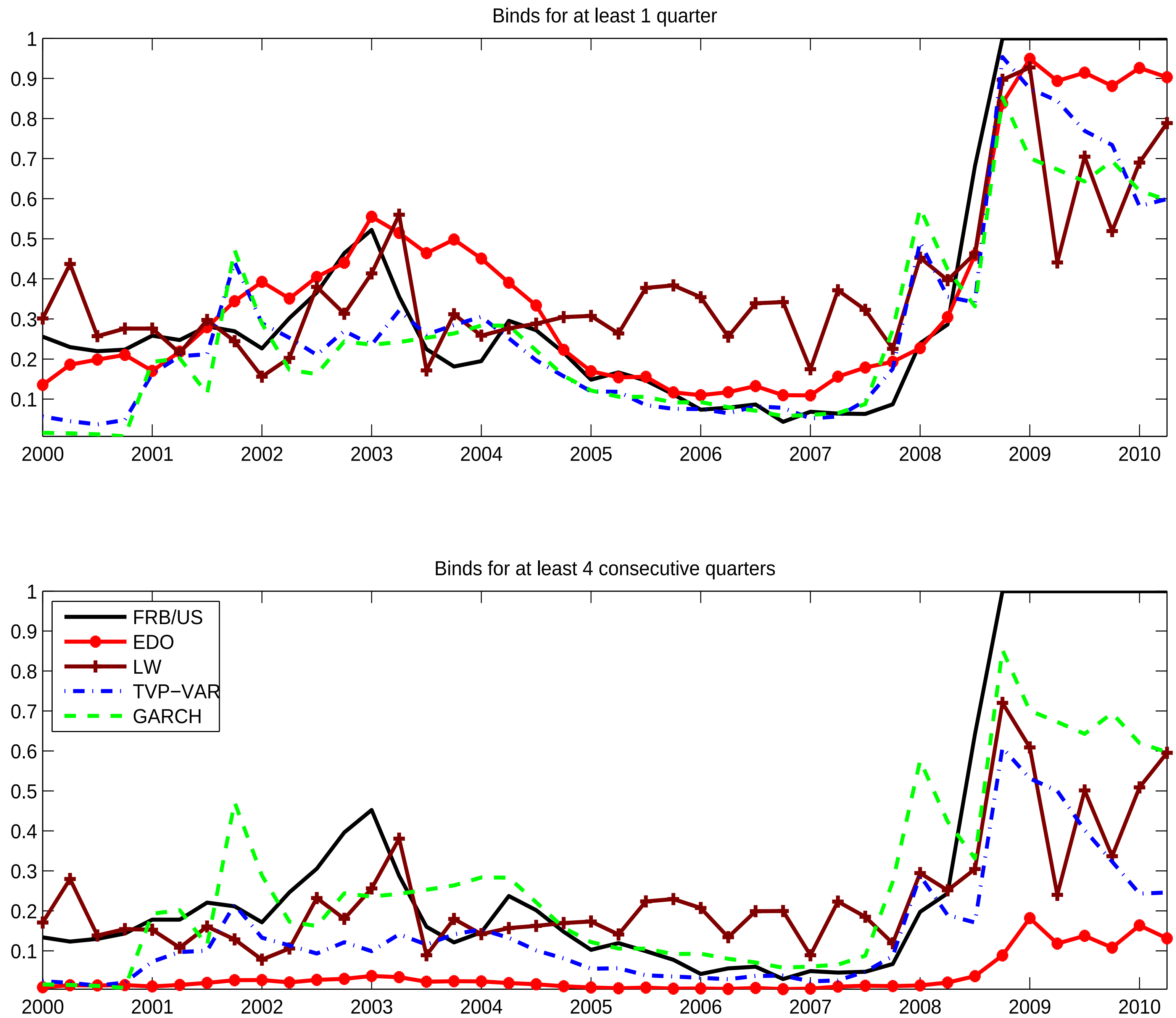

Figure 6: Probability of ZLB event within next 20 quarters
(Estimates Conditional on Rolling Baseline Projections and Assessments of Macroeconomic Volatility)

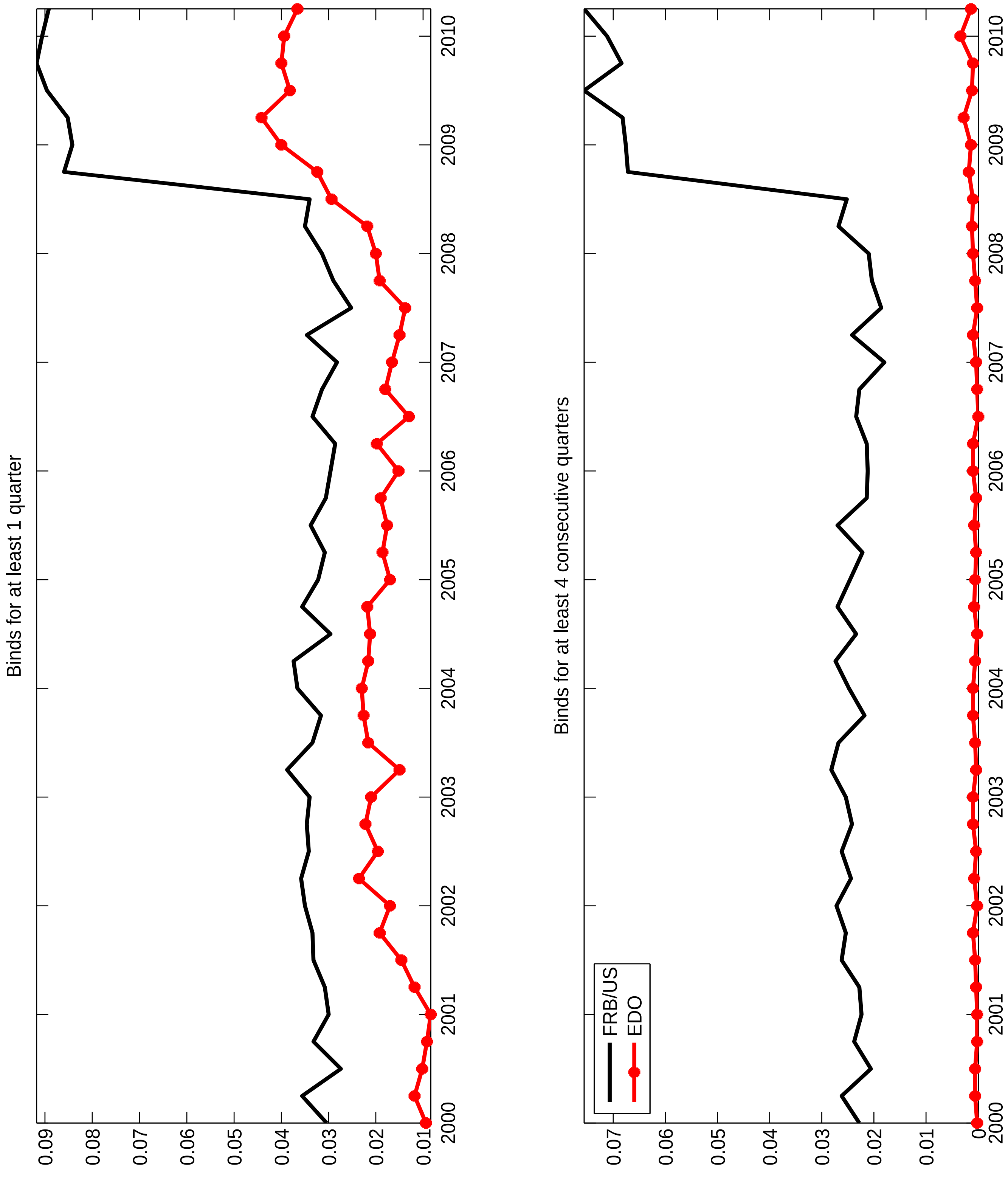

Figure 7: Probability of ZLB event within next 20 quarters
(Estimates Conditional on Rolling Assessments of Macroeconomic Volatility with Initial Conditions Fixed at Steady-State Values)

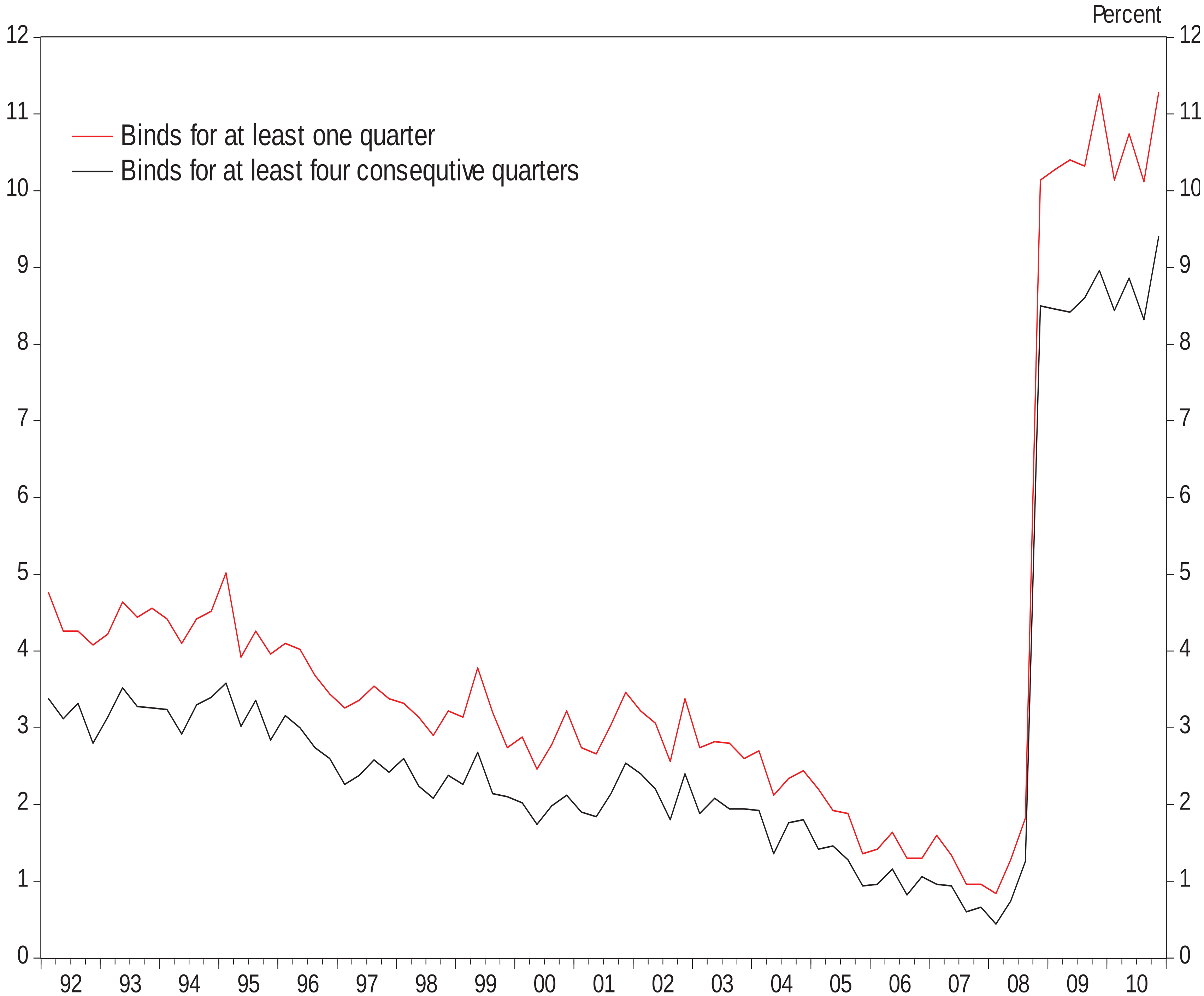

Figure 8: Probability of ZLB event within next 20 quarters in FRB/US Using a Rolling 25-Year Window to Assess Volatility
(Initial Conditions Fixed at Steady-State Values)

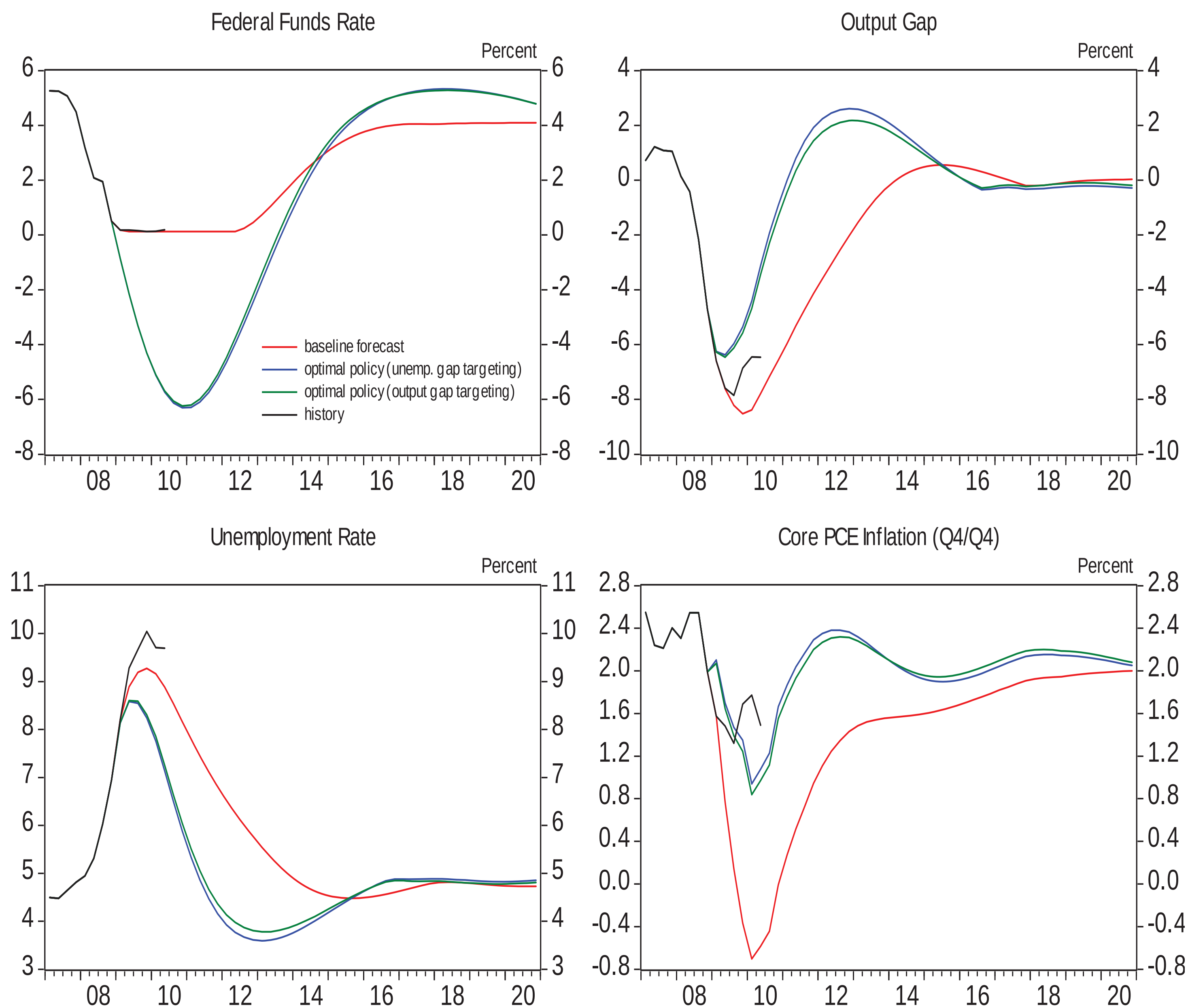

Figure 9a: Unconstrained Optimal Monetary Paths
Conditional on the Early 2009 FRB/US Baseline Projection and FRB/US Dyanmics

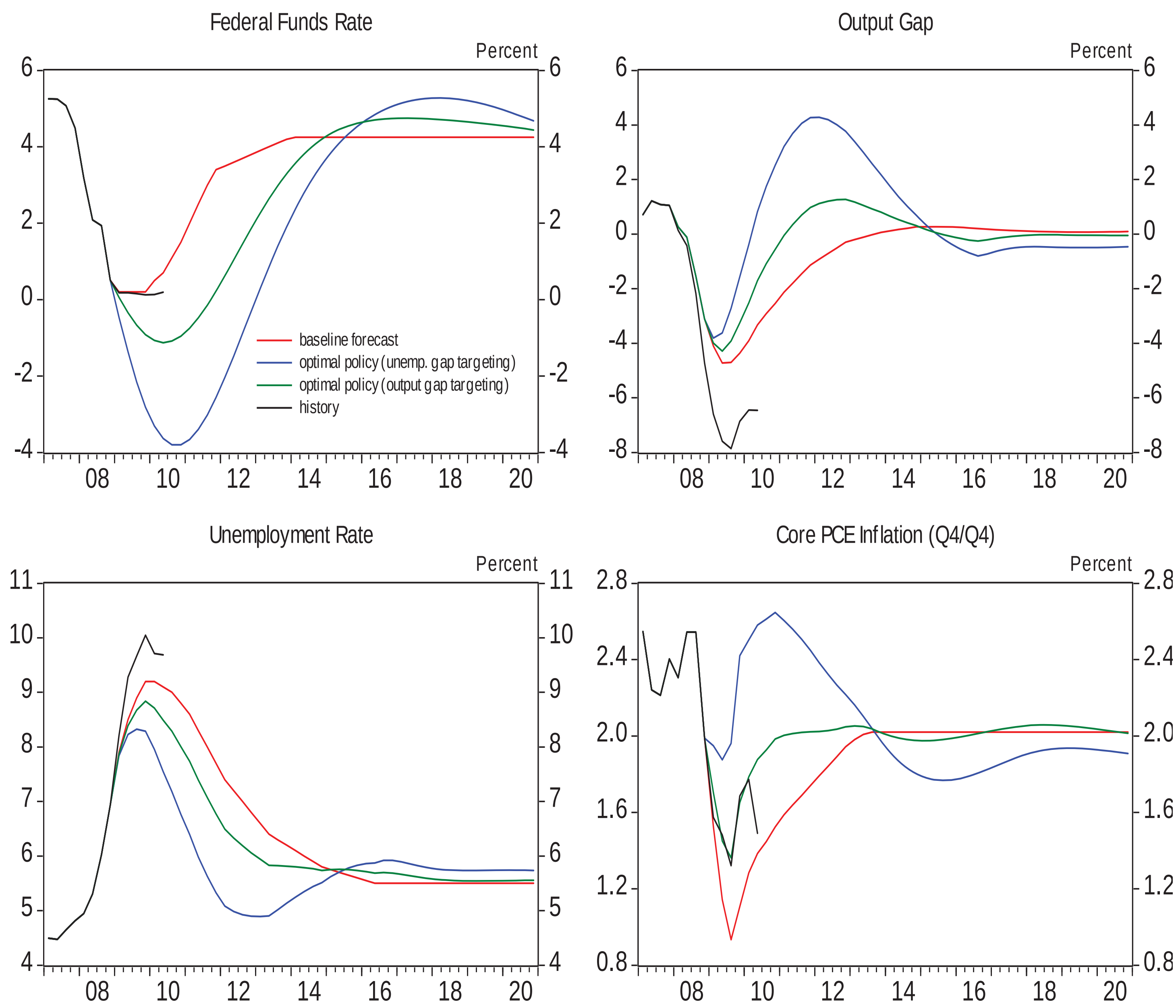

Figure 9b: Unconstrained Optimal Monetary Paths
Conditional on the Early 2009 Blue Chip Consensus Projection and FRB/US Dyanmics