

The Appropriate Time Frame for Controlling Monetary Aggregates: The St. Louis Evidence

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Since the mid-1960s increasing attention has been paid to the effect of monetary actions on the pattern of economic activity. An important issue has been the role of monetary aggregates in the conduct of stabilization policy. Monetary actions, measured by changes in monetary aggregates, are now generally recognized as powerful in their effect on economic activity and are considered at least as an equal partner with fiscal actions in economic stabilization programs. For purposes of monetary policy, however, questions remain regarding the nature of the temporal response of important economic variables to monetary shocks. Knowledge of the length of the lags and whether or not the lags are variable is required in order to ascertain the appropriate time period for controlling monetary aggregates. This study investigates the effect on economic activity of one monetary aggregate, the money stock defined as currency plus private demand deposits.

The Fisherian interpretation of monetary actions affecting primarily the price level is widely accepted as descriptive of the long-run effect of changes in the stock of money. However, monetary actions are conducted primarily in the pursuit of achieving much shorter-term goals. If the long-run neutrality of money is accepted, it is important for policy purposes to investigate the nature of the intermediate adjustments to changes in the money stock. Of prime concern is the relative effect of changes in money on prices and output. Specifically, do monetary actions affect output, and thus employment in the short run and, if so, what is the time frame of this effect? These questions relate to the policy problem of the extent to which monetary authorities can secure short-run increases in output without incurring the cost of later inflation.

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This paper investigates, first, the time patterns of effect of monetary actions on several aspects of economic activity, concentrating on the response of output and the average price of output. The second section presents the implications of these results for the proper time frame for controlling money.

RESPONSES OF OUTPUT AND PRICES TO CHANGES IN THE MONEY STOCK

In order to investigate the question of the appropriate period for controlling money, it is necessary to consider the dynamics of the economic system. Among the important issues that must be considered are the magnitude of the response of various economic variables to monetary shocks, the length and variability of the lags, and the relative effect on different variables. This section, first, develops a hypothesis regarding the nature of short- and long-run responses of prices and output to a change in money. Regression analysis is then used to test the hypothesis.

Development of the Hypothesis

In the commonly accepted general equilibrium system, market trading is typically assumed to be conducted with the benefit of perfect information and behavior is assumed to adjust instantly and costlessly to changes in economic conditions as they occur. Where money is included in the system, it is often assumed to function only as a medium of exchange. Excess demand functions for non-money assets are assumed to be homogenous of degree zero in money prices, and thus autonomous changes in the stock of money result only in equiproportionate changes in the general level of prices. "Money" in this context closely resembles a simple accounting device. Variations in the amount of money outstanding have no effect on production, employment, or relative prices in the system.

Economic decisions in the real world are not made with complete knowledge of market opportunities, however, and adjustments to new or better information can generally not be made without cost. The *tatonnement* process is not representative of day-to-day trading procedures where transactions are made on the basis of expectations

and trades at non-equilibrium prices are the rule.¹ Thus the implications of equilibrium analysis are indicative of long-run effects of monetary shocks. The nature and speed of adjustment in economic activity to autonomous monetary actions are important issues to be considered. What are the intermediate effects of changes in the stock of money and how long do they endure? For this study, neo-classical monetary theory is expanded to include imperfect information and adjustment costs, providing a hypothesis regarding the long- and short-run responses of prices and output to a monetary shock.

Long-Run Response

As with any exogenous force, monetary actions which change the money stock set off adjustment processes by shifting supply and demand relationships in various markets. The aggregate long-run effect of a permanent change in the stock of money can be summarized in two variables, total output and the average price level. The commonly accepted interpretation of the neo-classical monetary theory holds that money is neutral in its long-run effect on output; the long-run expansion path of output is determined by the supply of productive resources, technology and the relative efficiency of labor and capital. There is no long-run effect of a monetary shock on the factors which influence the trend growth of output. It is generally accepted that the trend growth of money, productivity, resource endowment, and money demand influence the trend rate of price increase.

Commonly drawn implications from this theory are that:

A permanent change in the rate of growth of money has a permanent, equiproportionate effect on the equilibrium *rate of change* of prices and no effect on the equilibrium *rate of change* of output.²

Short-Run Responses

The nature of the short-run effect of a change in the stock of money depends on the manner in which economic units adjust their

¹For an interesting attempt to incorporate these considerations in a general equilibrium framework see H. Grossman, "Aggregate Demand and Employment," presented at the meetings of the Western Economic Association, University of Santa Clara, August 1972.

²According to this statement, the implications are stated in terms of the slopes of the output expansion path and the price level time path and is not concerned with the important question of whether money shocks result in permanent displacements of the paths.

behavior to disequilibrium between actual and desired money holdings. With less than perfect information on current and future market opportunities, economic decisions are based on expectations of conditions in the various markets. These estimates are related not only to prices and quantities in markets for current consumables and producers' goods, but also for financial assets which are means to future consumption. The speed of adjustment to exogenous shocks reflects the time required to change expectations and the costs of altering behavior in response to new expectations. Legal, institutional and technological constraints often preclude adjustment without penalty, and the advantages of rapid adjustment must be traded off against these costs. Thus the effect of an autonomous shock will tend to be distributed over time and markets, first as information about the change in market conditions is disseminated and expectations are generated, and secondly as economic units adjust their behavior to new information and expectations.

Changes in the stock of money relative to its demand is an important exogenous shock. Demand for "money" is a consequence of the fundamental social service which money provides, i.e., reduced cost of trading. With imperfect information and positive costs of adjustment, society will adopt as money that asset or set of assets which is thought to minimize the amount of resources which must be devoted to exchange.³ Thus the factors which affect the demand for money balances in a period of time include those factors which affect the demand for current and future consumables, such as current human and nonhuman wealth, estimates of market opportunities, and the pattern of time preference relative to the real rental rate on funds and the expected rate of change of prices on consumables.

Following the introduction of an excess stock of money balances into the system, economic units will attempt to divest themselves of excessive money balances, given current estimates of prices and rates of return. Demand for real and non-money financial assets will increase. As a result, businesses experience some increase in demand for goods and services, and consequently inventories will be drawn down and backlogs of orders will tend to increase.

The problem faced by business firms is one of determining whether the increased demand is permanent or temporary. There are several possible adjustments — continue to run down inventories, build up an inventory of orders, increase output, or increase prices.

³K. Brunner and A. Meltzer, "The Uses of Money: Money in the Theory of an Exchange Economy," *American Economic Review*, LVI, December 1971, pp. 784-805.

Each adjustment bears a cost, and miscalculations can be expensive. It is postulated that, on average, firms find it less costly to gather information about demand conditions by changing output first rather than prices, and then, in the case of a permanent change in demand, as more information becomes available, prices are changed.

The implications from the postulated short-run behavior which incorporates imperfect information and adjustment costs are that:

The adjustment of prices to a monetary shock is not instantaneous and that temporary output effects can be expected as the economy moves to a long-run equilibrium.

Testing the Hypothesis

Testing a hypothesis involves confronting logical implications of the hypothesis with empirical evidence. If the evidence is in good agreement with all of the implications considered, the hypothesis is judged to be confirmed; if it is not, the hypothesis is rejected.

The operational form of the implications which were presented above are as follows:

1. The long-run elasticity of the price level with respect to money, $\bar{\epsilon}(P,M)$, is unity.
2. The long-run elasticity of output with respect to money, $\bar{\epsilon}(Q,M)$, is zero.
3. The short-run elasticity of output with respect to money, $\epsilon(Q,M)$, is greater than zero.
4. The short-run elasticity of the price level with respect to money, $\epsilon(P,M)$, is zero.

General Considerations

These implications are tested by means of regression analysis of data for the U.S. economy over the period from 1955/I to 1971/II. Observations on the rate of change of various price indices and measures of output are regressed against current and lagged values of the rate of change of money and other exogenous variables.

The basic specification tested is of the form:

$$(1) \dot{X} = F(\dot{M}, \dot{Z})$$

where (X) is a matrix of observations on the rates of change of prices and output, (\dot{M}) is a matrix of contemporaneous and lagged rates of

change of the money stock, (Z) is a matrix of contemporaneous and lagged rates of change of other exogenous factors which might affect (X). This relationship should be recognized as a *final* form, relating endogenous variables to exogenous variables only.⁴

In testing the hypothesis it is necessary to take into consideration initial conditions. There are two aspects to the set of initial conditions which are important in the analysis. The first aspect is the effect of current non-monetary exogenous forces. These are factors originating outside of the economic system and generally beyond the control of the monetary authorities, including fiscal actions, world trade conditions, technology, consumer time preferences, the legal framework, and random events such as strikes and weather conditions. The second aspect is the stage of adjustment of endogenous behavior to prior economic shocks. In this area are the influences of market expectations, demand for information, the costs of adjustment, and capacity utilization.

In evaluating the effect of monetary actions on economic activity, it is important to separate these two aspects. The second aspect mentioned above represents the lagged effect of prior shocks and thus should not be considered as a separate influence. The first aspect remains as an independent consideration.

Choice of the final form as the vehicle for testing the hypothesis implies that the set of initial conditions is summarized in current and lagged values of the exogenous variables. The length of the lag specified for the exogenous variables in a final form regression is a postulate describing the period over which the dependent variable adjusts to changes in the independent variable. For example, to specify

$$\dot{X} = a_0 \dot{M} + a_1 \dot{M}_{-1} + a_2 \dot{M}_{-2}$$

is to postulate that the variable X adjusts completely in three periods to a change in M . This specification can then be interpreted as a statement about the time span over which a new equilibrium is reached.

There is no *a priori* knowledge, however, regarding the exact time period required for adjustment to a monetary shock. Thus, in testing the hypothesis, determination of the appropriate length lag becomes an important consideration. For the purpose of this study, the

⁴The form differs from the *reduced* form in that the reduced form allows the inclusion of non-contemporaneous endogenous variables.

regressions with the lag structure which produces the highest \bar{R}^2 (minimum standard error) are used to test the hypothesis. The length of the lag, at that point, is taken as the time span over which adjustment of prices and output to a monetary shock takes place. Over this period of adjustment, the signs and estimated magnitudes of the sums of the coefficients on money are used to test the long-run implications of the hypothesis. The distribution of estimated coefficients on current and lagged values of money and their level of significance within the first part of this period are used to test the short-run implications.

If the difference between the estimated coefficient and the postulated coefficient for each of the four implications is found to be not statistically significant from zero (at the 5 percent level), the hypothesis is considered confirmed. On the other hand, if the difference for any one implication is found to be statistically significant from zero, the hypothesis is rejected.

Definition of Variables

Monetary policy attempts to secure national economic goals by actions which influence behavior in the private sector of the economy, i.e., households and businesses. While total economic activity is composed of more than the actions of those sectors, the other sectors are generally beyond the control of the monetary authorities. The Federal Government, for example, engages directly in all aspects of economic activity, from production to consumption, but there is little that monetary actions can do to influence this portion of the economy. Government actions are a separate autonomous force influencing the private sector, and monetary actions should be considered in conjunction with this and other exogenous factors.

Three measures of spending are used in this study. The first one is the standard expenditure definition of gross national product (Y).

$$(1) Y = C + I + G + Ex - Im$$

where C = private expenditure on consumer goods and services, I = private expenditure on investment goods, G = government expenditures on goods and services, Ex = exports, or foreign expenditures on U.S. goods, and Im = imports, or domestic spending on foreign produced goods and services. As measured in the National Income Accounts, GNP is a measure of all expenditures which generate income or employment in the U.S. economy.

This measure is too broad for measuring the response of domestic spending to a monetary shock, since it includes variables essentially beyond the influence of the monetary authorities. As mentioned earlier, autonomous changes in the stock of money affect economic activity by altering private demand for real and financial assets. A money shock would produce, in part, increased demand for consumables and investment goods, both domestically and foreign produced. That is, some portion of the effect of an increase in the money stock would be on import demand and thus is not directly on domestic output and employment. Imports should be included to measure private spending, while government expenditure and exports should be eliminated.

To reflect these considerations, total private expenditures in the economy (Y_p) are defined as:

$$(2) Y_p = C + I = C^p + C^f + IP + I^f$$

where superscripts refer to the sector where production originates, p = private domestic sector and f = foreign sector. The demand for domestic versus foreign output would depend on relative prices of domestic and foreign output, and the quality of the respective output.

Since this study is concerned with the response of domestic output and the corresponding price level, private expenditures on domestic output are defined as:

$$(3) Y_p^p = Y_p - Im = C^p + IP.$$

Although this study is primarily interested in private spending on domestic output, tests of the hypothesis are made using all three of these spending concepts and the measures of output and prices related to each. These are related to total expenditure in the economy (Y), total private expenditure (Y_p), and private expenditure on domestic production (Y_p^p). Thus the matrix of prices includes the GNP deflator (P), the deflator for private consumption and investment expenditure (P_p), and the deflator for private expenditures less imports (P_p^p).⁵ The associated output measures are (Q), ($Q_p = Y_p/P_p$), and ($Q_p^p = Y_p^p/P_p^p$).

⁵The price variable (P_p) is constructed as the weighted sum of the national income account deflators for private consumption expenditures and gross private domestic investment. The variable (P_p^p) is equal to (P_p) minus the weighted deflator for expenditures on imported goods and services.

Testing Long-Run Implications

The first long-run implication that the long-run elasticity of prices, $\bar{\epsilon}(P,M)$, is unity was tested by means of the following regression:⁶

$$\Delta \ln P = a_0 + a_1 D_1 + a_2 D_2 + \sum_{i=0}^n m_i \Delta \ln M_{-i} + \sum_{i=0}^k e_i \Delta \ln E_{-i}$$

The variables are expressed in changes in the logarithms, which for small movements approximate the percentage rate of change. The estimated coefficients can thus be interpreted as elasticities. The sum of the coefficients on the money terms, here defined as currency plus private demand deposits, is taken as evidence on $\bar{\epsilon}(P,M)$, the long-run elasticity of prices with respect to money.

The variable D_1 and D_2 are dummy variables which are nonzero in the quarter of a major labor strike and the following quarter, respectively. The variable (E) is high-employment government expenditure which is included to take account of a potentially important exogenous policy variable, but the estimated coefficients are not emphasized.⁷ The constant term is the average influence of all other systematic forces influencing the rate of growth of the price level.

The estimated sum of the money coefficients for each of the price measures and for various lengths of lag on money are presented in Exhibit I.⁸ The results were little affected by changes in the length of the lag on government expenditures and thus the table includes only those results for a lag of four quarters on that variable. In each case, the lag on the money terms which gives the maximum adjusted \bar{R}^2 also yields a sum which is not significantly different from unity, at the 5 percent level of significance. The evidence is thus in good agreement with the long-run implication that $\bar{\epsilon}(P,M) = 1.0$. The sum of the money coefficients on all three price measures are found to be

⁶The tests were run using all combinations of the Almon constraints. The criteria were not significantly affected by the degree of polynomial or the end-point constraints. Only a small portion of the results are thus presented here.

⁷The estimates for high-employment expenditures were similar to those found by Andersen-Jordan. See L.C. Andersen and J.L. Jordan, "Monetary and Fiscal Actions: A Test of Their Relative Importance in Economic Stabilization," *Review*, Federal Reserve Bank of St. Louis, November 1968, pp. 11-24.

⁸See footnote 9 for an explanation of the very small constant term in the price regressions reported in Exhibit I.

EXHIBIT I
1955 I – 1971 II
ALMON CONSTRAINTS: d=3, t+1≠0

$$(1) \Delta \ln P = a_0 + a_1 D_1 + a_2 D_2 + \sum_{i=0}^n m_i \Delta \ln M_{-i} + \sum_{i=0}^4 e_i \Delta \ln E_{-i}$$

n	Σm_i	Σe_i	a_0	\bar{R}^2	D.W.	$H_0: \Sigma m_i = 1.0$
4	.324*	.048	.361*	.141	.600	
8	.559*	.043	.214*	.451	.817	
12	.686*	.012	.206*	.584	1.023	
16	.800*	.004	.163*	.652	1.232	
20	.912*	.015	.078	.704	1.429	Accept
24	1.030*	.027	-.022	.750	1.676	Accept
28	1.150*	.025	-.091	.776	1.881	Accept

$$(2) \Delta \ln P_p = a_0 + a_1 D_1 + a_2 D_2 + \sum_{i=0}^n m_i \Delta \ln M_{-i} + \sum_{i=0}^4 e_i \Delta \ln E_{-i}$$

4	.281*	.063	.277*	.168	.666	
8	.505*	.056	.137	.456	.944	
12	.631*	.027	.127	.597	1.209	
16	.717*	.021	.094	.637	1.362	
20	.829*	.026	.021	.683	1.550	
24	.936*	.036	-.067	.720	1.732	Accept
28	1.058*	.035	-.141	.745	1.892	Accept

$$(3) \Delta \ln P_p^p = a_0 + a_1 D_1 + a_2 D_2 + \sum_{i=0}^n m_i \Delta \ln M_{-i} + \sum_{i=0}^4 e_i \Delta \ln E_{-i}$$

4	.166	.059	.309*	.017	1.280	
8	.418*	.054	.149	.336	1.765	
12	.526*	.027	.147	.404	2.058	
16	.586*	.028	.116	.412	2.125	
20	.689*	.029	.056	.439	2.217	
24	.809*	.040	-.041	.473	2.309	Accept
28	.941*	.039	-.122	.500	2.408	Accept

*Significant at 5% level.

P = GNP deflator

P_p = Deflator for private consumption expenditure and gross domestic investment

P_p^p = Deflator for private expenditure less imports

not significantly different from unity only after 20 quarters, implying the adjustment process of prices to a monetary shock takes over five years to run its course.

The second long-run implication, that $\bar{\epsilon}(Q,M) = 0$, was tested by means of the regression:

$$\Delta \ln Q = a_0 + a_1 D_1 + a_2 D_2 + \sum_{i=0}^n m_i \Delta \ln M_{-i} + \sum_{i=0}^k e_i \Delta \ln E_{-i}$$

The constant term is the average influence of all other systematic forces influencing the rate of growth of output.

The results of this test are presented in Exhibit II. The sums of the money coefficients are not significantly different from zero for lags longer than four quarters; the evidence is consistent with the long-run implication that $\bar{\epsilon}(Q,M) = 0$. The explanatory power of the relationship is maximized between lags of 12 to 20 quarters, implying a somewhat faster adjustment in output than in prices.

What are the implications of the lag in the output regression being shorter than in the price regression? This question may be examined with reference to changes in velocity. Consider the equation of exchange

$$MV = PQ$$

where V = expenditure velocity of money. Expressing the relationship in elasticity form gives:

$$\epsilon(P,M) + \epsilon(Q,M) - \epsilon(V,M) = 1.0$$

This shows that there need not be a close correspondence at all times between the elasticities of prices and output, as money shocks may also affect velocity. This latter effect was tested as a check on the consistency of the price and output regressions by means of the following regression:

$$\Delta \ln Y = a_0 + a_1 D_1 + a_2 D_2 + \sum_{i=0}^n m_i \Delta \ln M_{-i} + \sum_{i=0}^k e_i \Delta \ln E_{-i}$$

where Y is a measure of spending, $Y = PQ$.

EXHIBIT II

1955 I - 1971 II

ALMON CONSTRAINTS: $d=3, t+1 \neq 0$

$$(1) \Delta \ln Q = a_0 + a_1 D_1 + a_2 D_2 + \sum_{i=0}^n m_i \Delta \ln M_{-i} + \sum_{i=0}^4 e_i \Delta \ln E_{-i}$$

n	Σm_i	Σe_i	a_0	\bar{R}^2	D.W.	$H_0: \Sigma m_i = 0$
4	.664*	-.085	.446	.329	1.105	
8	.149	-.085	.793*	.515	1.397	Accept
12	-.126	-.015	.798*	.586	1.649	Accept
16	-.274	-.013	.867*	.596	1.720	Accept
20	-.359	-.004	.895*	.595	1.718	Accept
24	-.462	-.007	.956*	.571	1.614	Accept
28	-.538	-.032	1.063*	.549	1.505	Accept

$$(2) \Delta \ln Q_p = a_0 + a_1 D_1 + a_2 D_2 + \sum_{i=0}^n m_i \Delta \ln M_{-i} + \sum_{i=0}^4 e_i \Delta \ln E_{-i}$$

4	1.054*	-.280*	.550	.411	1.240	
8	.431	-.282*	.960	.574	1.595	Accept
12	.153	-.212*	.964*	.613	1.746	Accept
16	.075	-.211*	.998*	.614	1.763	Accept
20	.036	-.170	.934*	.597	1.717	Accept
24	-.050	-.164	.958*	.554	1.560	Accept
28	.006	-.203	1.018*	.527	1.451	Accept

$$(3) \Delta \ln Q_p^P = a_0 + a_1 D_1 + a_2 D_2 + \sum_{i=0}^n m_i \Delta \ln M_{-i} + \sum_{i=0}^4 e_i \Delta \ln E_{-i}$$

4	1.053*	-.280*	.550	.411	1.241	
8	.430	-.282*	.960*	.574	1.596	Accept
12	.152	-.212	.964*	.614	1.747	Accept
16	.074	-.211*	.998*	.614	1.765	Accept
20	.035	-.170	.934*	.597	1.718	Accept
24	-.052	-.164	.958*	.554	1.562	Accept
28	.005	-.203	1.018	.527	1.453	Accept

*Significant at 5% level

Q = GNP at 1958 prices

Q_p = Real private consumption expenditure plus real gross private domestic investment,
 $Q_C + Q_I$.

Q_p^P = Real private expenditure less real imports, $Q_C^P + Q_I^P$

EXHIBIT III

1955 I - 1971 II

ALMON CONSTRAINTS: $d=3, t+1 \neq 0$

$$(1) \Delta \ln Y = a_0 + a_1 D_1 + a_2 D_2 + \sum_{i=0}^n m_i \Delta \ln M_{-i} + \sum_{i=0}^4 e_i \Delta \ln E_{-i}$$

n	Σm_i	Σe_i	a_0	\bar{R}^2	D.W.	$H_0: \Sigma m_i = 1.0$
4	.989*	-.037	.816*	.500	1.576	Accept
8	.707*	-.042	1.007*	.547	1.692	Accept
12	.560*	-.003	1.004*	.567	1.756	
16	.526*	-.009	1.031*	.564	1.785	
20	.552*	.011	.973*	.562	1.799	Accept
24	.568*	.019	.934*	.536	1.714	Accept
28	.612	-.007	.972*	.509	1.584	

$$(2) \Delta \ln Y_P = a_0 + a_1 D_1 + a_2 D_2 + \sum_{i=0}^n m_i \Delta \ln M_{-i} + \sum_{i=0}^4 e_i \Delta \ln E_{-i}$$

4	1.335*	-.217*	.827*	.553	1.522	Accept
8	.936*	-.225	1.097*	.583	1.696	Accept
12	.783*	-.185	1.091*	.588	1.724	Accept
16	.792*	-.190	1.093*	.586	1.726	Accept
20	.865*	-.144	.955*	.578	1.718	Accept
24	.885*	-.128	.891*	.535	1.582	Accept
28	1.064*	-.168	.877*	.512	1.480	Accept

$$(3) \Delta \ln Y_P^P = a_0 + a_1 D_1 + a_2 D_2 + \sum_{i=0}^n m_i \Delta \ln M_{-i} + \sum_{i=0}^4 e_i \Delta \ln E_{-i}$$

4	1.219*	-.221*	.860*	.501	1.793	Accept
8	.848*	-.228*	1.109*	.555	1.969	Accept
12	.678*	-.185	1.111*	.572	2.017	Accept
16	.659*	-.182	1.115*	.571	2.019	Accept
20	.724*	-.141	.990*	.564	2.004	Accept
24	.757*	-.124	.917*	.529	1.872	Accept
28	.946*	-.164	.896*	.508	1.779	Accept

*Significant at 5% level

Y = Nominal GNP

Y_P = Private consumption expenditure plus gross private domestic investment,
 $P_P(Q_C + Q_I)$

Y_P^P = Private expenditure less imports, $P_P^P(Q_C^P + Q_I^P)$

Since $\epsilon(P,M) + \epsilon(Q,M) = \epsilon(Y,M)$, this regression tests (indirectly) the effect of money shocks on velocity. The results are presented in Exhibit III.⁹ The sum of the estimated coefficients is taken as evidence of $\bar{\epsilon}(Y,M)$, which in turn can be considered as $\bar{\epsilon}(P,M) + \bar{\epsilon}(Q,M)$ for all three measures of spending. In the case of the rate of change of spending, the adjustment apparently is completed after about four quarters. In terms of the sum coefficients, these results are generally consistent with those implied by the price and output regressions, i.e., $\bar{\epsilon}(Y,M) = \bar{\epsilon}(P,M) = 1.0$ and $\bar{\epsilon}(Q,M) = 0$.

Testing Short-Run Implications

The third and fourth implications are that in the short-run $\epsilon(Q,M) > 0$ and $\epsilon(P,M) = 0$. These are tested by examining the patterns and the levels of significance of individual coefficients of the regressions for the first few quarters after a change in the rate of money growth. Chart I presents the distribution of coefficients from the regressions for $\Delta \ln Q_p^p$ and $\Delta \ln P_p^p$, with 24 lagged money terms. Those variables, which relate to spending by the private sector on domestically produced output, represent the portion of aggregate demand which is probably most directly influenced by monetary actions. The responses of the other definitions of the variables are the same.

The chart shows that there is a sharp and substantial positive response of output growth for five quarters following a permanent change in the rate of increase of money, then the effect becomes negative and remains less than zero out to 19 lagged terms. The price response, however, is essentially zero for over five quarters and builds slowly from there. For the first several quarters after a change in the

⁹The exceedingly small constant in the $\Delta \ln P$ equation should not be taken as an indication that only money growth influences the price level. Changes in the trend growth of money demand, productivity, and factor endowment also influence the price level.

The small constant in the $\Delta \ln P$ equations reported in Exhibit I can be explained in the following manner. The constant in the $\Delta \ln Y$ equation can be interpreted as measuring the trend rate of change in velocity, which reflects trend movements in the demand for money. The constant in the $\Delta \ln Q$ equation can be considered as the trend rate of growth of output, which reflects trend increases in productivity and factor endowment. Since $\Delta \ln P = \Delta \ln Y - \Delta \ln Q$, the constant in the $\Delta \ln P$ equation equals the constant in the $\Delta \ln Y$ equation less the constant in the $\Delta \ln Q$ equation. In the sample period, these two constant terms are approximately equal; therefore, the constant in the $\Delta \ln P$ equation is very small.

rate of money growth the output coefficients are positive and statistically significant from zero and the price coefficients are not. The evidence is in good agreement with the short-run implications that $\epsilon(Q,M) > 0$ and $\epsilon(P,M) = 0$.

Conclusion From Test Results

Since the evidence in each test is consistent with the implications under consideration, the neo-classical hypothesis expanded to include imperfect information and costs of adjustment is judged to be confirmed. The conclusion to be drawn from this exercise is that a change in the trend of money growth has no permanent effect on the rate of growth of output and results instead in an equiproportionate change in the rate of increase of prices. There is, however, a substantial short-run effect of money shocks on output growth. The adjustment of output, while zero in the long-run, is extremely volatile compared to the adjustment pattern in prices. Prices show a relatively slow adjustment, which does not begin to appear, on average, until almost a year after a monetary shock. The length of the adjustment period for both prices and output to a monetary shock was found to be about 24 quarters. It should be remembered that this evidence relates only to the U.S. economy during the sample period and thus to the magnitude of changes in money growth experienced during that period.¹⁰

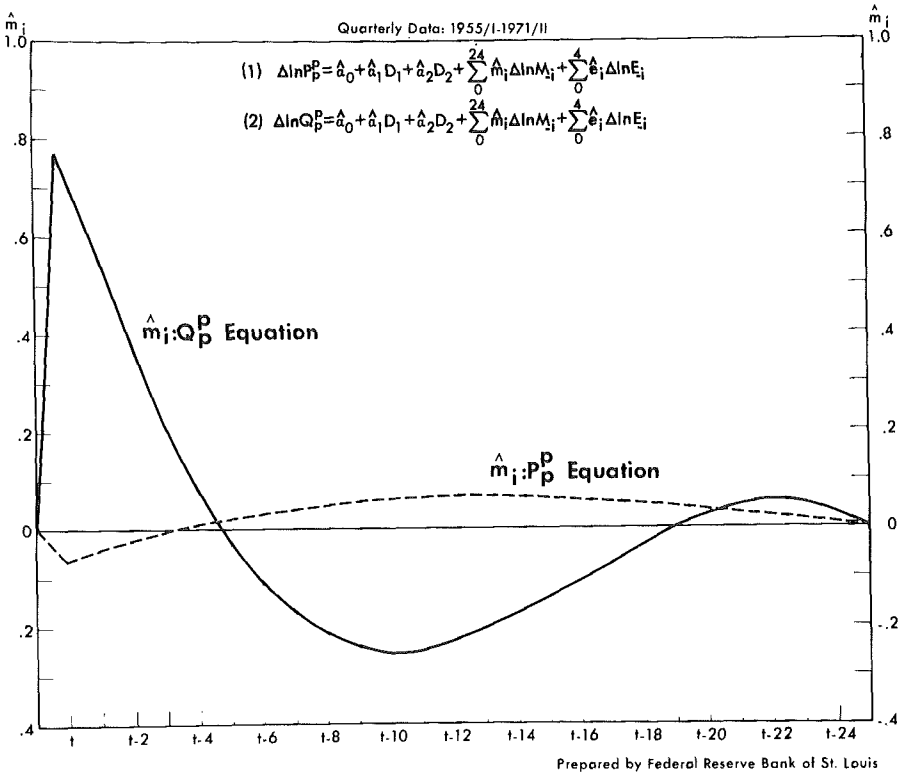
IMPLICATIONS FOR CONTROLLING MONEY

Having accepted the hypothesis regarding the responses of output and the price level to a monetary shock, the regressions of the previous section are used to develop some implications for monetary analysis and control of the money stock. In particular, problems of ascertaining the magnitude of response to a change in money growth and the length of the appropriate time period for analyzing the response are investigated.

This section, first, develops regression equations for the responses of output and the price level to a change in the rate of money growth. Next, these equations are used to simulate the time path of

¹⁰As with any hypothesis, that considered here is subject to scrutiny under a wide range of experience. For further confirming evidence see John L. Scadding, "The Relationship between Changes in the Stock of Money and Changes in Output and Prices: Canada 1954-1969," unpublished manuscript, Stanford University, June 1972.

Chart I
Estimated Coefficients

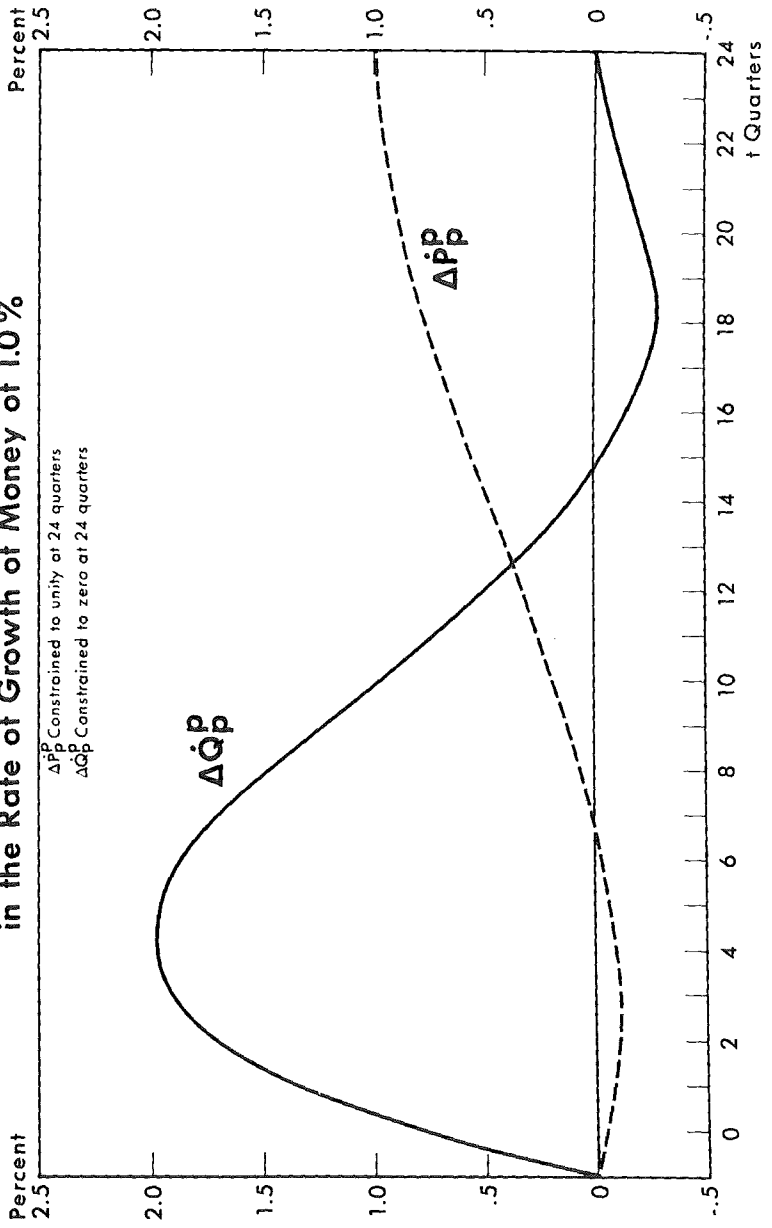


output and the price level in response to various types of monetary shocks. The simulations provide important insights into the factors which must be taken into consideration in judging the most likely responses and in determining the appropriate time period for monetary control.

General Nature of Responses

While the estimated sums of the money coefficients are not exactly equal to the implied long-run elasticities of the hypothesis, the tests reported in the preceding section showed the differences not to be statistically significant. These long-run elasticities and the patterns of response over the adjustment period are accepted as the best representation of the real world. In developing empirical measures of the responses of output and the price level to a change in the rate of money growth, the coefficients reported in Chart I are,

Chart II
**Cumulative Effect of a Permanent Increase
 in the Rate of Growth of Money of 1.0%**



Prepared by Federal Reserve Bank of St. Louis

therefore, re-estimated subject to the constraint that the sum of the money coefficients in the price equation equals unity and in the output equation equals zero.¹¹ The responses over time are measured by cumulating the sums of the coefficients for each equation. On basis of the evidence presented in the previous section, a 24-quarter lag period is taken as an approximation of the time required for full adjustment.

The cumulative effect of a permanent increase of 1 percent in the trend of rate of money growth on the rates of output and price increase is presented in Chart II. The response of output is in the form of deviations from its trend rate of growth as measured by the constant term in that equation. The price level in the sample period was found to have no trend independent of the rate of money growth, because the constant term in the price equations is very small and not statistically significant from zero. Thus the response of the price level is measured in deviations from its rate of growth consistent with the prior trend rate of money growth.

Chart II indicates that the immediate response to accelerated money growth is a rapid and substantial increase in the rate of output for five quarters, with essentially no price response. Growth of output then ceases to accelerate and falls rapidly while the rate of price increase rises moderately. The rate of output expansion falls below its trend rate about 15 quarters after the monetary shock. Price increases continue to accelerate and reach a permanently higher trend level after 24 quarters, just as output growth returns permanently to its initial trend rate.

Simulation Experiments

Two general types of simulation experiments are performed. They are accelerations in the rate of money growth which change its trend growth rate and deviations of money growth around a constant trend. In the second set of simulations the length of time over which money growth deviates from its trend is considered. These simulations provide evidence regarding the influence of changes in one important initial condition — the stage of adjustment to prior monetary shocks — on the response of output and the price level. The

¹¹Introduction of these additional constraints into the Almon procedure did not alter the timing of the estimated distribution. The magnitude of individual coefficients was somewhat affected, however.

influence of changes in other initial conditions, such as trends of productivity, resource endowment, and money demand, is not examined.

Accelerations in Money Growth Producing Changes in its Trend

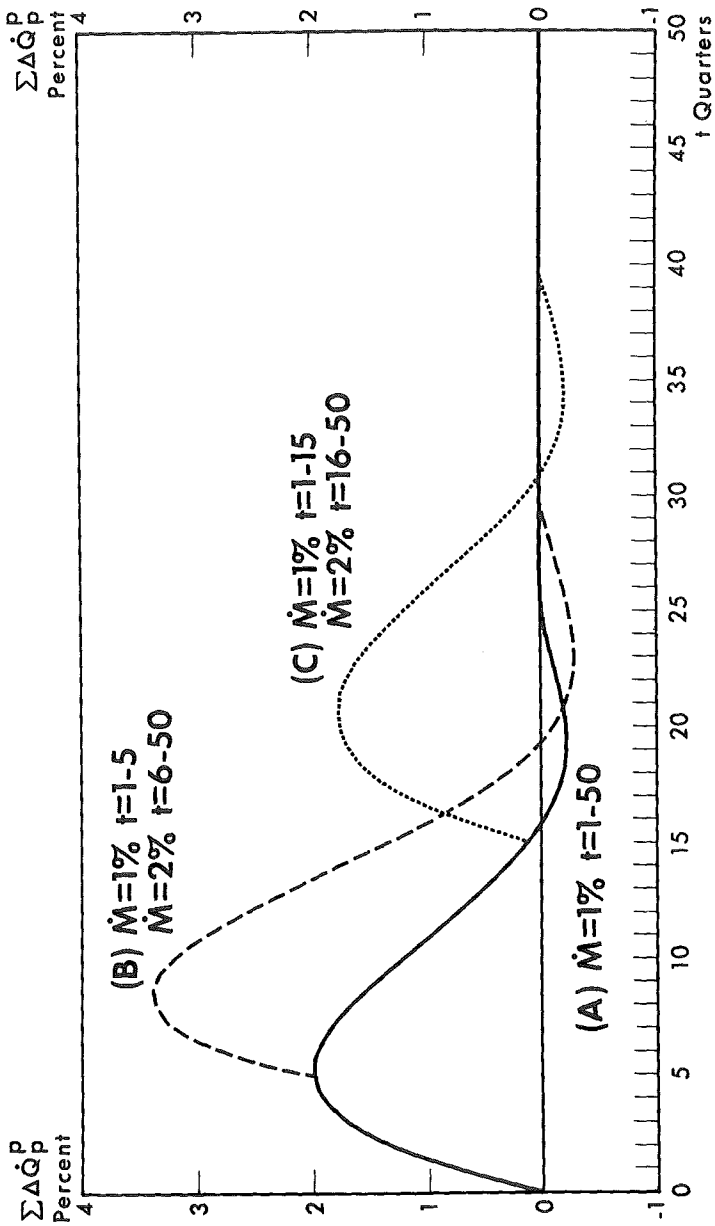
Three simulations are presented in Charts III and IV which indicate the short-run and the long-run responses of output and price to an increase in the trend growth of money. All three start from an equilibrium situation in which there has been a constant rate of increase in the money stock for an extended period. Output is growing at its trend rate, which is determined by growth of factor endowment and technological advance, and the price level is rising at the rate determined by the prior trend of money growth. The first simulation is a permanent 1 percentage point increase in the quarterly rate of money growth, exactly as reported in Chart II.¹² The second one involves a 1 percentage point increase in the quarterly money growth rate maintained for five quarters and then an additional 1 percentage point increase which is maintained thereafter. The third simulation is the same as the second except that the additional acceleration in the quarterly money growth rate occurs after 15 quarters.

These three simulations indicate the responses of output and price to a permanent acceleration of money growth with three different sets of initial conditions, stated in terms of the stage of adjustment to previous monetary shocks. In Simulation 1, a 1 percentage point increase in the rate of money growth begins at a point of full adjustment. In Simulation 2, the rate of money growth is permanently increased following five quarters of adjustment to a prior acceleration. This is the point of maximum, positive short-run adjustment of *output* to the prior shock. In Simulation 3, the second acceleration is introduced at the mid-point of the period of *price* adjustment to a prior shock.

The short-run response of output (arbitrarily set at 5 quarters) to the increase in the rate of monetary expansion varies depending on the initial conditions (Chart III). Beginning at a point of full adjustment to the prior trend rate of money growth, there is a very strong

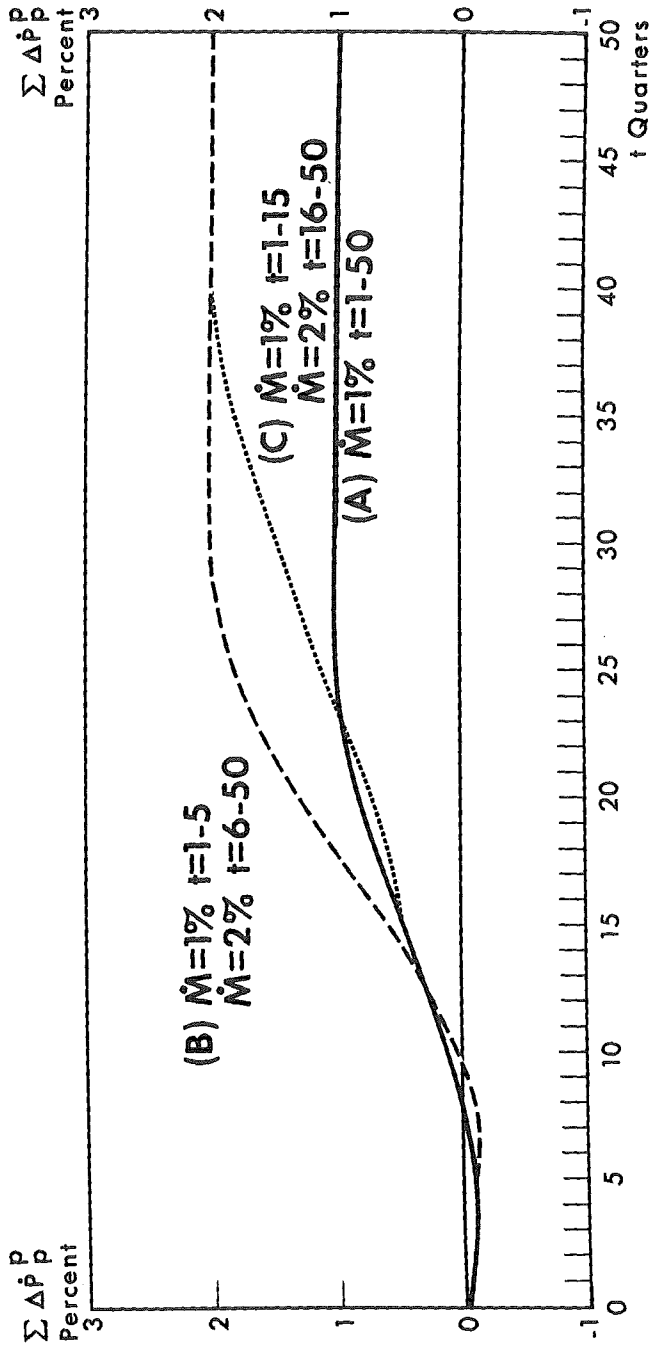
¹²Throughout this presentation it should be remembered that the variables are defined in quarterly rates of change which can imply substantial *annual* rates of increase. A change of 1 percent in the quarterly rate of increase of a variable, for example, translates into a change of over 4 percentage points in the annual rate, i.e., from a 4 percent rate of increase to an 8 percent rate.

Chart III
 Cumulative Effect on Output Growth of
 Permanent Increases in Rate of Growth of Money



Prepared by Federal Reserve Bank of St. Louis

Chart IV
**Cumulative Effect on Rate of Price Increase of
 Permanent Increase in Rate of Growth of Money**



Prepared by Federal Reserve Bank of St. Louis

short-run response of output to the 1 percentage point increase in the rate of money growth, line (A). The rate of growth of output is almost 2 percentage points above trend after five quarters. The incremental response to the additional increase in money growth which occurs after five quarters of adjustment to the previous 1 percentage point shock is somewhat less, line (B). The rate of output expansion rises another 1.5 percentage points after five quarters, reaching 3.5 percentage points above trend 10 quarters after the initial money shock. In the case of the additional change in money growth which does not come until after 15 quarters of adjustment to the previous 1 percentage point shock, the incremental output response, line (C), lies between those of the other two cases.

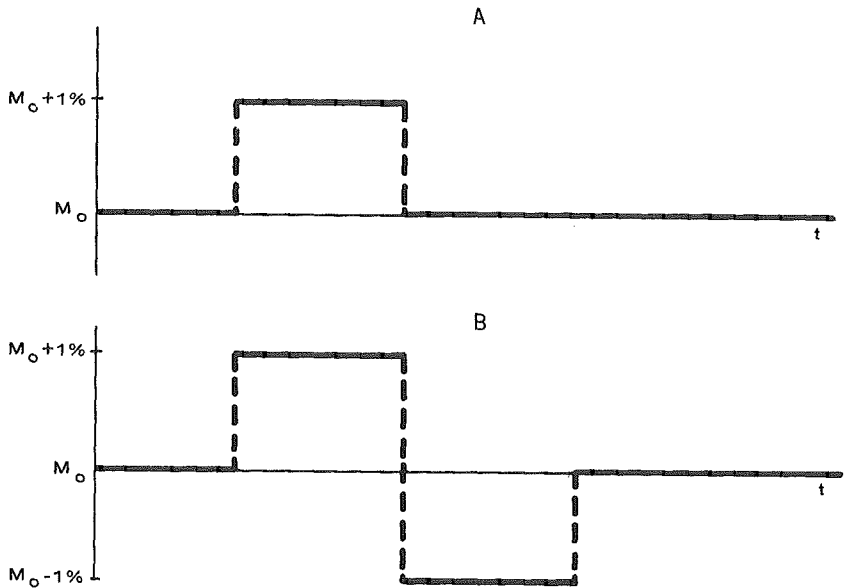
Since the price level responds more slowly to the pattern of money growth over 24 quarters, its short-run response will be measured by the difference shown in Chart IV between the price change for the fifth quarter following the final acceleration in money growth and the change which prevailed at the last quarter before the final acceleration. In the first simulation (starting at full adjustment), there is virtually no short-run response in the price level. The rate of price increase after five quarters is unchanged. The second simulation (starting after five quarters of adjustment) shows a short-run response of an increase of about .5 of a percentage point from the zero beginning rate. The third simulation (starting after 15 quarters of adjustment) shows a smaller short-run response than the second one, an increase from 0.4 percent to 0.6 percent.

In all three simulations, new long-run trend rates of money growth are produced. In the first case, the trend of money growth is increased 1 percent a quarter, and in the last two cases it is increased by 2 percent a quarter. As a consequence, new trend rates of price increase are also produced. There are substantial short-run gains in output growth stemming from monetary acceleration but there is no change in the trend of output.

Short-Run Deviations Around Constant Trend

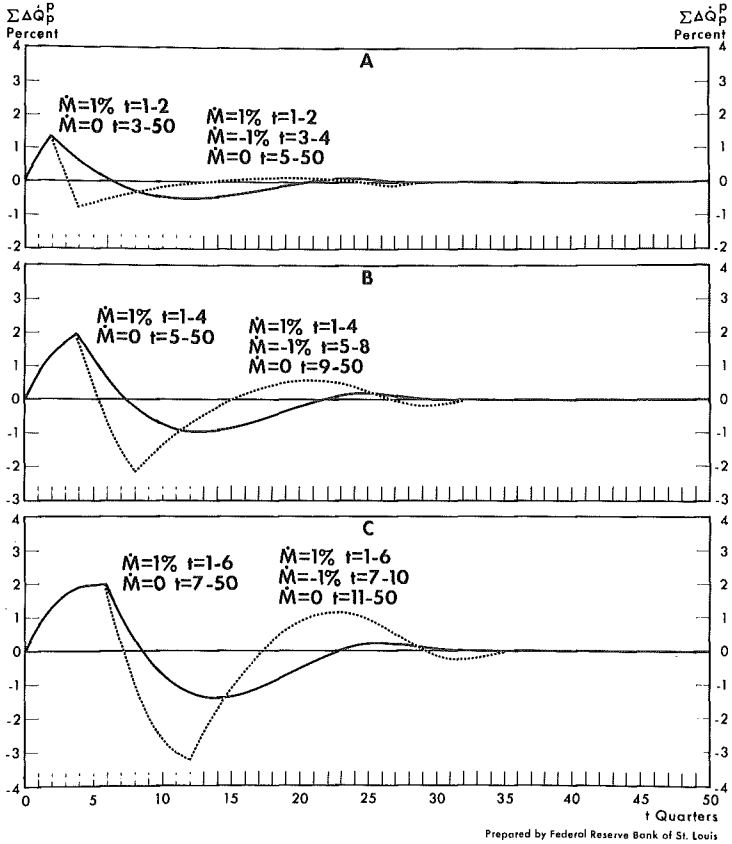
Two types of simulations of short-run deviations of money growth around a constant trend rate are examined in this section. The first type, as illustrated in panel A below, is a 1 percentage point increase in the growth rate of money for various length periods followed by a deceleration to the initial trend growth rate. The second type, panel B, is a positive 1 percentage point change from trend for six or less quarters, followed by a negative 1 percentage point change from

trend for corresponding periods of time, and then a return to trend. In this last case the short-run rate of money growth averages the same as the trend. Chart V presents the simulation results for periods with temporary accelerations of two, four, and six quarters.



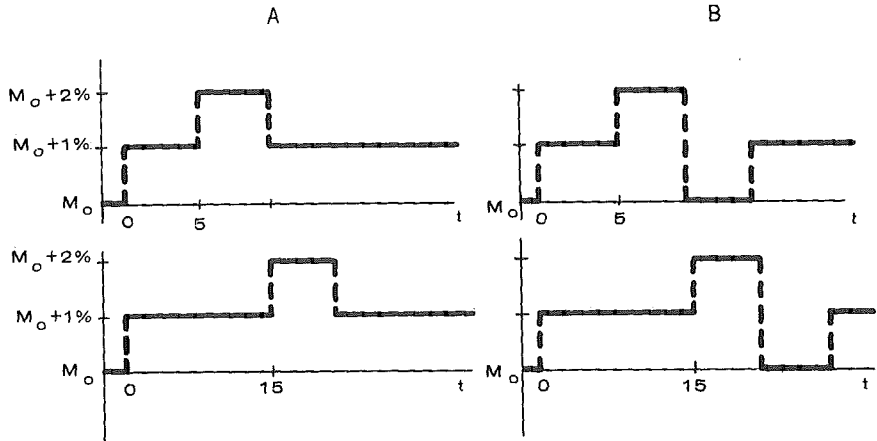
Results from both types of simulations indicate that, *starting from equilibrium with output growing at its trend rate*, a temporary increase of 1 percentage point in the quarterly change in money for a period even as short as two quarters has a substantial, positive impact on output. The positive response of output to a two quarter acceleration of money growth above trend is a rate of 1.3 percent per quarter after those two quarters, over a four quarter period of acceleration output growth increases by 1.9 percentage points, and for six quarters the rate of growth of output increases by 2.0 percent. The following downswing in output, as the rate of growth of money returns permanently to its trend value, and subsequent variations until equilibrium is achieved again are greater when the acceleration in money growth is offset by an equal decrease and then returned to trend (the dashed line) than when money growth is returned immediately to its trend rate after the acceleration.

Chart V
Cumulative Effect on Output Growth of
Temporary Increases in Rate of Growth of Money



It should be remembered that these simulations were performed starting from a position of equilibrium growth of output. As shown in the previous set of simulations, the short-run output response also depends on initial conditions in terms of the stage of adjustment to prior shocks. To investigate the effect of temporary accelerations in money growth with different initial conditions, the simulations reported previously in the present section were run starting with a 1 percentage point increase in the money growth rate and then the two types of simulations under consideration were performed. In one case the rate of money growth is increased by an additional 1 percent after five quarters, and in the other case the second acceleration comes 15 quarters after the initial increase. Panel A below illustrates

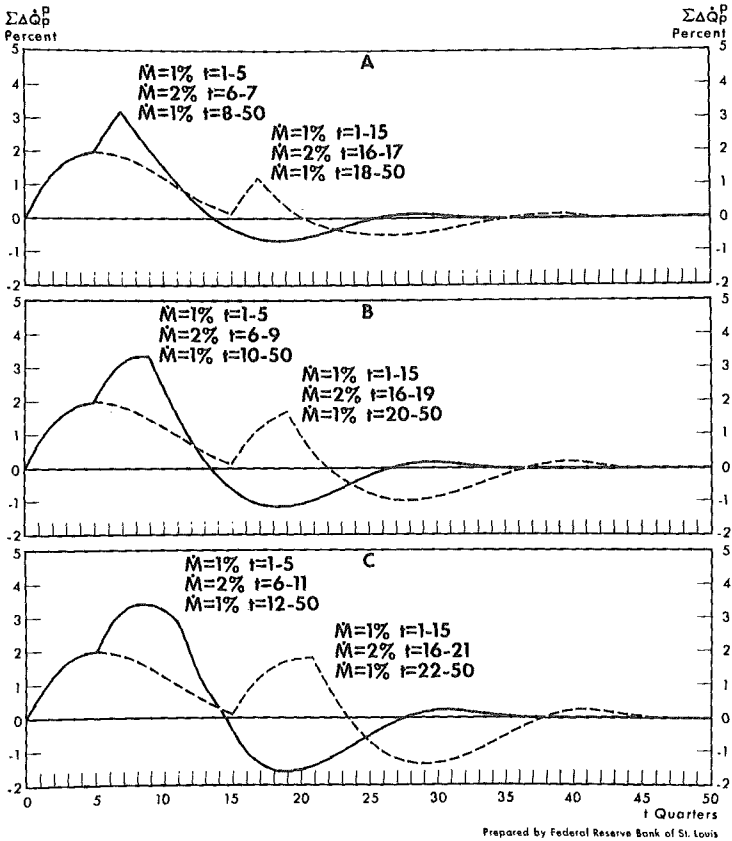
the patterns of money growth used to generate the results presented in Chart VI and panel B shows those associated with Chart VII. The final trend rate in these simulations is 1 percentage point greater than in the simulations from equilibrium, but there is no change in trend over the time interval of the two simulations.



In the first tier of Charts VI and VII are the results of a temporary deviation ($\dot{M}=2\%$) of two quarters in length. The second tier shows the results of deviations of four quarters in length and the third shows the results of deviations of six quarters. The solid lines plot the effect of these deviations after five quarters of accelerated money growth, and the dashed lines show the effect when the deviations occur after 15 quarters of accelerated money growth.

In Chart VI, where the rate of money growth is returned immediately to its new trend after the temporary increases, one obvious effect of the different initial conditions is the much greater variability of \dot{Q}_p^p in the case where the deviation comes five quarters after the initial acceleration. The effect of counteracting the temporary deviation of money growth above the new trend by a similar deviation below trend is presented in Chart VII. Compared to Chart VI the effect of this policy is a much longer decline in the rate of output growth. The end result is the same, however, as in each case output growth returns to its initial rate of increase.

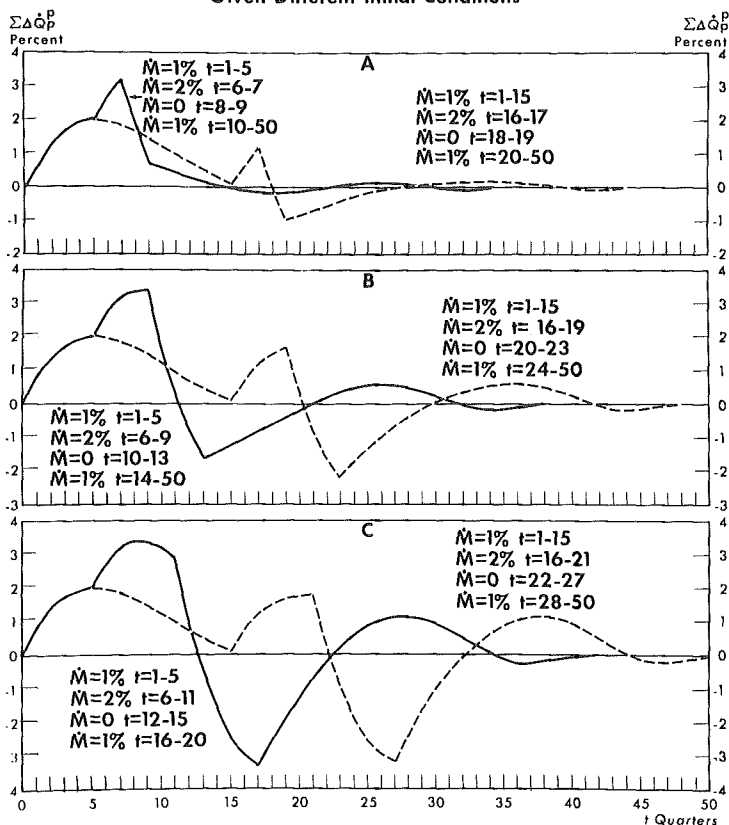
Chart VI
 Cumulative Effect on Output Growth of
 Temporary Increases in Rate of Growth of Money
 Given Different Initial Conditions



Longer-Run Deviations Around a Constant Trend

The responses of output and prices to longer-run deviations in money growth around a constant trend are investigated in this section. The deviations in money growth are in the form of a sine wave, oscillating with an amplitude of 1 percentage point about the trend rate of money growth. The responses of output and prices were simulated for different frequencies of the sine function. These simulations demonstrate two interesting properties of the responses of output and prices to longer-run variations in money growth around a constant trend.

Chart VII
 Cumulative Effect on Output Growth of
 Temporary Deviations in Rate of Growth of Money
 Given Different Initial Conditions



Prepared by Federal Reserve Bank of St. Louis

One property is that lengthening the wave length of M changes the lead-lag relationship of the responses to changes in money. These are presented in Exhibit IV. For wave lengths from 8 to 20 quarters, the peak growth of output follows shortly after that of money; at 24 quarters, the peak of money lags that of output. Peaks of price change lead those of money growth for wave lengths up to 24 quarters; the lead and the lag is the same for 28 quarters, and the peaks in prices lag the peaks in money growth for longer wave lengths.

The second property is that changes in the wave length have a discernible influence on the amplitude of the response of output and to a lesser extent on the amplitude of price response (Chart VIII). As the wave length is increased from 12 to 20 quarters, the amplitude of the output response rises sharply, but there is little change in the amplitude of the price response. Then as the wave length is increased further, the output amplitude slowly decreases, while the price amplitude slowly rises.

EXHIBIT IV

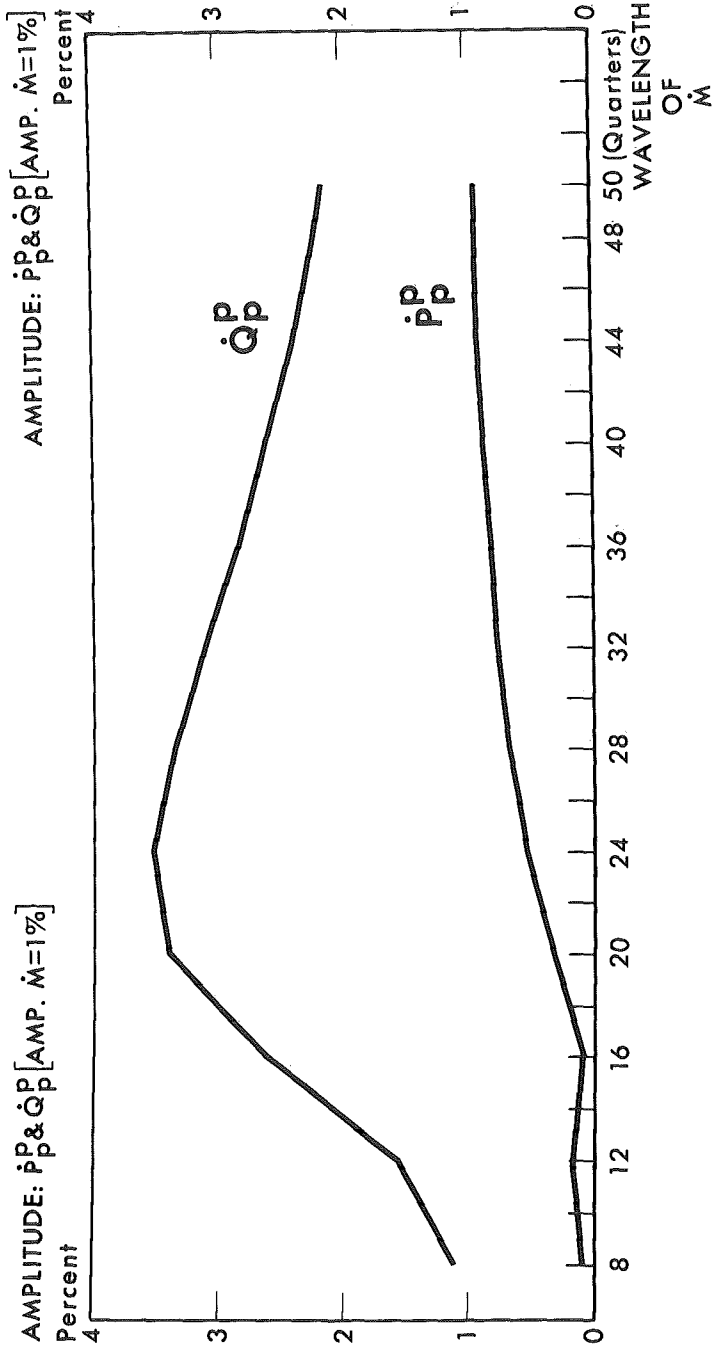
Wave Length of \dot{M} Series	Number of Periods		Number of Periods	
	Peak of \dot{P}_p^p		Peak of \dot{Q}_p^p	
	Precedes	Follows	Precedes	Follows
	Peak of \dot{M}	Peak of \dot{M}	Peak of \dot{M}	Peak of \dot{M}
8	3	5	7	1
12	5	7	11	1
16	6	10	14	2
20	7	13	19	1
24	10	14	Coincident	
28	14	14		27
32	18	14	2	30
36	21	15	3	33
40	25	15	4	36
44	29	15	5	39
48	33	15	7	41
50	35	15	7½	42½

Conclusions

This study has presented evidence consistent with the view that in the long-run changes in money growth predominately affect prices, but the response of output and price to a change in money growth is distributed over a fairly long period of time. The regression equations indicate that a period of about 24 quarters may be a good approximation of the period of adjustment. It also presented evidence consistent with the view that these two responses are distributed in a different manner within the period of adjustment to a maintained change in the money growth rate. Simulation experiments were then performed as a means of developing the implications of these response patterns for ascertaining the expected responses to a change in money growth and for selecting the appropriate period for monetary analysis.

Chart VIII

The Relation Between the Amplitude of \dot{P}_p and \dot{Q}_p ,
and the Wavelength of \dot{M}



The simulations demonstrate that the variable in which an analyst is interested — output, the price level, or nominal GNP — has an important bearing on the manner in which one analyzes the effects of movements in money. In the case of output, short-run variations in money growth are of foremost importance, while for the price level, the trend rate of money growth should be emphasized. Since a change in GNP is the sum of output and price changes, if one is interested in this variable both short-run variations and the trend growth of money must be taken into consideration.

It was shown that the stage of adjustment to past monetary shocks, an important type of initial condition, has a very important bearing on the observed response of output and price to a change in money growth. Therefore, one must take into consideration the pattern of money growth rates over the previous 24 quarters. This is particularly crucial for assessing the most likely short-run responses of output and nominal GNP.

The type of shocks expected in the future along with initial conditions are very important for monetary analysis. For a given stage of adjustment to prior monetary shocks, it was shown that there are different short-run responses of output depending on whether a short-run change in the money growth rate is permanent or if it is a deviation around a constant trend. The type of deviations over long periods, with a constant trend, change the observed lead-lag relationships between money growth and changes in output and price. They also change the amplitude of variations in output and price.

All of these considerations lead to the conclusion that empirical knowledge of the responses, which allows one to take into consideration the stage of adjustment to previous monetary shocks and other initial conditions, is essential in assessing the impact of a change in money growth on output, price, and nominal GNP. In the absence of such knowledge, it would be difficult to develop a general rule of thumb with regard to either the expected short-run responses or to a fixed, short-run period of analysis.

Some analysts contend that the often observed variable lag to a change in money makes it difficult to use monetary actions in economic stabilization. Although the equations estimated in this study indicate a fixed and predictable response of output and price level to changes in the money stock, the simulations using various types of money shocks and stages of adjustment to prior monetary shocks demonstrate that it is possible to observe a so-called "variable lag" response. This study indicates that such a response is to be expected and can be measured. Therefore, the frequently observed

variable lag does not mean that controlling money is a "will-o'-the-wisp" tool of economic stabilization.

DISCUSSION

BENJAMIN M. FRIEDMAN*

The purpose of this conference is to talk about the control of monetary aggregates. The principal question which we have addressed thus far has been how the Federal Reserve System can so control any given monetary aggregate as to keep the values over time of that aggregate as close as possible to an appropriately-selected target path. A related, and in some sense prior, question is how the Federal Reserve should go about choosing the appropriate target path itself. An intelligent selection of the monetary aggregate target path depends upon knowledge of the relationship between the monetary aggregate and income, prices, employment or whatever aspects of the economy may represent the ultimate goals of monetary policy.

The basic thrust of the Andersen-Karnosky paper is to search for evidence on the relationships between one monetary aggregate — the money stock — and three familiar policy goals — income, real output, and prices. Hence the subject matter of this paper is a necessary precursor to the determination of the appropriate time frame for monetary aggregate control, and the paper's attempt to refine our knowledge of several key relationships is clearly a step in a useful direction. The paper itself, however, does not go on to use its empirical estimates of various linkages to address the time-frame question directly, and so I want to spend a few moments considering how to go about solving a problem of this type.

Two fundamental inputs (in addition to others of less interest at the moment) influence the choice of the time frame for monetary aggregate control: The first input, as we are already aware, is our estimate of the relationship between the monetary aggregate which we seek to control and the policy goal variables for the sake of which we undertake to do so. The second input, the unfortunate or uncomfortable aspect of the situation, is the degree of either confidence or uncertainty which we have in our estimates of these key relationships. In other words, the proper time frame for

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monetary-aggregate control depends not only on the pattern of simulation results which emerges from our best available economic model but also on the extent to which we are sure or unsure of the validity of that pattern. To what extent do we believe that this pattern is an accurate and reliable representation of the economic system, or, alternatively, to what extent do we allow for the presence of uncertainties and possible disturbances of the economic system?

Are Alternatives Incompatible?

The Morris paper, presented at the outset of this conference, emphasized the distinction between a fixed monetary policy regime as advocated by Milton Friedman, characterized by a constant rate of money growth, and a flexible monetary policy regime, characterized by discretionarily variable rates of growth of money or reserves or whatever the relevant aggregate may be. I suggest that, in fact, these two positions are not so incompatible as the Morris paper implies. In particular, allowing for the presence of uncertainty — that is, for our ignorance about the functioning of the economy — in the design of a monetary-aggregate control scheme leads inevitably to a policy system which represents a compromise between the rigid and the fixed monetary policy positions.

A simple model may serve to illustrate this point. Following the notation of the Andersen-Karnosky paper, let M and Q represent the rates of growth of the money stock and real output, respectively. Suppose, as in the material summarized by Table II of the Andersen-Karnosky paper, that the value of M , together with several other factors, determines the value of Q :



The rigid monetary policy regime advocated by Milton Friedman is always to set M equal to some fixed value, say, MF :

$$M = MF \tag{1}$$

Doing so results in a particular value of real output, say, Q_F :



The motivation for the flexible monetary policy regime is that the other factors which influence real output may, independently of the value of M , cause the value Q_F of Q to differ from its preferred value, say, Q_P . Under these circumstances, advocates of the flexible regime recommend replacing expression (1) by

$$M = MF + MP \quad (2)$$

where MP is a discretionary policy component of (M). Choosing a positive MP renders M greater than MF (e.g., $M = 4\% + 2\% = 6\%$), and choosing a negative MP renders M less than MF (e.g., $M = 4\% - 1\% = 3\%$). In the context of this simplified flexible monetary policy, relationships such as those in the Andersen-Karnosky paper indicate the impact on Q of any given choice of MP . If some fixed coefficient V represents this impact, the economic system which monetary policy makers confront is

$$Q = Q_F + V \cdot MP \quad (3)$$

Assuming that we know the key coefficient V , choosing MP so as to render Q equal to the preferred value Q_P is straightforward:

$$Q_P = Q \quad (4)$$

$$Q_P = Q_F + V \cdot MP \quad (5)$$

$$MP = \frac{1}{V} (Q_P - Q_F) \quad (6)$$

Expression (6) indicates the appropriate value of the discretionary policy component of M , and using this value in expression (2) yields the appropriate target value for M itself.

If we are not perfectly sure of the value of coefficient V , however, the situation is somewhat different. We may think that some value V is the most likely value, but we also usually recognize that the true value of the coefficient describing the impact of MP on Q may be either somewhat greater or somewhat smaller than our best estimate.

In the context of econometric equations such as those in the Andersen-Karnosky paper, we typically think that the true value is likely (with two-thirds probability) to be no greater than $V + SE$ and no less than $V - SE$, where SE is the relevant coefficient standard error. Hence the presence of uncertainty changes expression (3) to

$$Q = QF + (V \pm SE) \cdot MPU \quad (3')$$

where MPU indicates the discretionary policy component of M chosen under the explicit recognition of uncertainty.

If monetary policy makers confront the economic system described by expression (3'), they cannot be sure of rendering Q equal to the preferred value QP , regardless of the value of MPU which they choose. They can, however, choose MPU so as to make the expected value of the discrepancy $QP - Q$ as small as possible. If large discrepancies $QP - Q$ are even more than proportionally undesirable than small discrepancies, it may be appropriate to choose MPU so as to render the expected value of the squared discrepancy $(QP - Q)^2$ as small as possible. Doing so changes expression (6) to

$$MPU = \frac{V}{V^2 + SE^2} (QP - QF) \quad (6')$$

At this point we may ask what influence the presence of uncertainty has on the choice of the discretionary policy component of M . In other words, how does MPU in expression (6') differ from MP in expression (6). The ratio of the two values is

$$\frac{MPU}{MP} = \frac{\frac{V}{V^2 + SE^2}}{\frac{1}{V}} \quad (7)$$

and straightforward algebraic manipulation yields

$$\frac{MPU}{MP} = \frac{\left(\frac{V}{SE}\right)^2}{\left(\frac{V}{SE}\right)^2 + 1} \quad (8)$$

Since the value of this ratio is clearly less than unity, expression (8) indicates that the influence of uncertainty is to lead monetary policy makers to choose a value of M which differs from the fixed MF by

less than does that corresponding value of M which they would choose if they were perfectly sure of the impact of their actions. This result is intuitively both plausible and appealing; ignorance dictates caution.

In the context of econometric models, the ratio of a coefficient to its standard error — that is, the $\frac{V}{SE}$ ratio in expression (8) — is simply the t -statistic associated with the coefficient. Hence expression (8), which indicates the extent of caution dictated by a particular degree of ignorance, is in fact simply

$$\frac{MPU}{MP} = \frac{t^2}{t^2 + 1} \quad (9)$$

From expression (9) it is easy to consider the influence of a variety of degrees of uncertainty. A t -statistic of $t = 2$ for example, a familiar minimum standard in econometric work, warrants choosing a value of MPU equal to four-fifths of the corresponding value of MP which we would choose in the absence of uncertainty. A t -statistic of $t = 1$ warrants choosing a value of MPU equal to only one-half of the corresponding value MP.

Although the Andersen-Karnosky paper does not indicate the t -statistics associated with the coefficients of the regression equation it reports, Denis Karnosky kindly gave me this information for the third equation in the paper's Table II; this equation is the source of the output simulations described in the latter half of the paper. The t -statistic for the coefficient in this equation which is equivalent to coefficient V in expression (3) is approximately $t = 8$. Using this t -statistic in expression (9) implies that the appropriate value of MPU in the presence of this degree of uncertainty is $\frac{64}{65}$, or more than 98%, of the corresponding value of MP which would be appropriate if we were perfectly sure of the impact of discretionary monetary policy on Q . If this t -statistic is an accurate description of our ignorance, therefore, it is appropriate to proceed almost as if we were not ignorant at all; and the compromise between a rigid and a flexible monetary policy becomes almost indistinguishable from the flexible policy itself.

Additional Questions

At this point I want to ask, without attempting to answer, several questions about the equations presented in the Andersen-Karnosky paper.

First, the equations reported in this paper differ in several respects from previous monetarist equations. The Andersen-Karnosky equations show, for example, that it is necessary to take account of the lagged impact of monetary growth on income, real output and prices for twenty-four quarters — that is, six years — which is quite a long time. This result differs substantially from the implications of the Andersen-Jordan equation or the Andersen-Carlson model. Which is correct? The t -statistic of $t = 8$ in the Andersen-Karnosky paper is predicated on the assumption that we know the length of lag involved in these relationships, but a comparison of different monetarist results suggests that in fact we do not know.

Secondly, the three equations reported in Table I of the Andersen-Karnosky paper examine the relationship between money and prices. Even the versions of these three equations with a twenty-eight quarter lag on the money variable probably do not maximize the equation's adjusted coefficient of determination (\bar{R}^2). The reported value of \bar{R}^2 for each of the three equations rises as each additional four quarters increment the length of the lag. In this case, why stop at twenty-eight quarters?

Thirdly, a key object of these equations is to test the proposition that, in the long run, the elasticity of prices with respect to changes in money is unitary. Confirmation of this hypothesis depends upon the closeness to unity of the coefficient sums Σm_i reported in the first column of the table. For each of the three equations, however, the value of this coefficient sum is not only rising but actually accelerating as each additional four quarters increment the length of the lag. If the lag were sufficiently long so as to maximize any or all of the three equations' \bar{R}^2 values, would the resulting coefficient sums Σm_i be so much in excess of unity as to warrant rejecting the hypothesis of unitary elasticity?

Fourthly, the three equations reported in Exhibit III examine the relationship between money and income. A key object of these equations is to test the proposition that, in the long run, the elasticity of income with respect to changes in money is unitary. Once again, confirmation of this hypothesis depends upon the closeness to unity of the coefficient sums Σm_i reported in the first column of the table. For the first equation reported in Exhibit III,

the two lag lengths which yield the greatest \bar{R}^2 values (twelve quarters, for which $\bar{R}^2 = .567$, and sixteen quarters, for which $\bar{R}^2 = .564$) are precisely those lag lengths for which it is necessary to reject the hypothesis of unitary elasticity. Does this result mean that we must in fact reject the hypothesis? Alternatively, what does this result imply about our knowledge of the proper lag length for equations of this type?

The point of these questions is not simply to pick holes in the Andersen-Karnosky paper's equations. These equations are useful, and they offer some interesting evidence on several relationships which are central to the formulation of monetary policy. Instead I am asking whether the extent of our uncertainty about a number of aspects of these key relationships is not greater than that implied by the t-statistic $t = 8$ which warrants setting a policy almost (98%) equivalent to the policy that would be appropriate in the absence of uncertainty.

At the conclusion of a conference such as this one, it is appropriate to ask where we should go from here. These questions which I have asked all relate to the nature of the research which would be useful for the Federal Reserve System and independent researchers to emphasize, in the interest of furthering the art of making monetary policy. We are already aware that we need to learn more about the relationships between monetary aggregates or other monetary policy variables and the variables which represent the ultimate goals of policy. Indeed, as Karl Brunner's discussion has suggested, we must think carefully about whether or not the overall theoretical structure which underlies our empirical work is correct. The somewhat paradoxical point which I have tried to emphasize in addition, however, is that we also need to learn more about our ignorance so that we may allow for it in formulating policy. Once we do so, the seemingly inconsistent positions which advocate either a rigid or a flexible monetary policy become not so inconsistent after all.