Intermediary Leverage, Macroeconomic Dynamics, and Macroprudential Policy[1]
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Abstract

We develop a macroeconomic model in which financial intermediaries choose their privately optimal lending and leverage. The privately-optimal leverage choices of intermediaries imply significant and socially-inefficient macroeconomic effects of changes in the risk facing intermediaries and the cost of their external funds. We show how various fiscal and monetary policy tools can be used to remedy the situation. In particular, we demonstrate a near-equivalence of reserve requirements and a Pigovian tax on intermediary leverage, and then show how alternative strategies for adjusting reserve requirements improve equilibrium allocations in a fully-specified dynamic-general-equilibrium model with financial intermediation. In doing so, we evaluate several macroprudential policy suggestions, demonstrating that policies which lean against broad measures of asset values may hinder the economy's response to productivity. Policy strategies focused on mitigating shifts in the spread between borrowing rates and a risk-free interest rate appear to have robust stabilization properties, yielding better performance than strategies which lean against fluctuations in the ratio of credit to output.

JEL Classification Code: E32, E44, E58, E61, G18 and G21

1 Introduction

In order to understand how shocks to financial conditions influence macroeconomic outcomes and the possible role of policies in mitigating such effects, we develop a macroeconomic model in which financial intermediaries choose the mix of debt and equity used to finance their lending that is privately optimal. These privately-optimal choices imply socially-inefficient fluctuations in response to shocks, especially financial shocks.

The financial intermediaries in our model find debt financing attractive due to preferential tax treatment. Intermediary leverage is limited by the fact that increased leverage increases the probability of a costly default, making investors unwilling to accept excessive debt levels. In addition, we acknowledge the distinction between internal cash flow and outside equity by incorporating a premium on raising external equity. Finally, our model incorporates an inherent asymmetry between intermediaries assets (lending) and liabilities: Lending commitments cannot be adjusted quickly in response to changes in intermediaries' balance sheet condition. This friction, in conjunction with the premium on external funds, makes aggregate lending decisions sensitive to idiosyncratic risk within the intermediary sector or shocks to the external finance premium.

We first demonstrate the socially-inefficient fluctuations in response to shocks, especially financial shocks implied by intermediaries' decisions. We then show how a social planner would adjust intermediary leverage in response to shocks in order to maximize household welfare. The *Ramsey* policy implies leverage choices substantially different from privately-optimal decisions, reflecting the pecuniary externality associated with asset prices in our model. Moreover, we show that a reserve requirement acts in a manner similar to a Pigovian tax on leverage, and can nudge allocations toward efficient outcomes if designed properly. To illustrate such design issues, we consider cyclical macroprudential policies as envisaged in the Basel 3 process – which has developed a framework in which capital requirements may be adjusted in response to cyclical conditions such as excessive aggregate credit growth (as considered in research such as BCBS

(2010), Drehmann, Borio, Gambacorta, Jiminez, and Trucharte (2010), Repullo and Saurina (2011), and Edge and Meisenzahl (2011), which offer differing perspectives on this issue). Our analysis considers policies that lean against fluctuations in credit, asset values, and borrowing spreads. We show that policies that lean against credit limit the adverse consequences of shocks to financial conditions, but may induce undesirable investment cycles in response to movements in total factor productivity because credit cycles do not move in tandem with the business cycle. Policies that lean against fluctuations in broad measures of asset values/prices can mitigate the financial accelerator associated with financial shocks, but may substantially inhibit investment in response to technological innovation, which is counterproductive.[1] A policy that leans against credit spreads mitigates the financial cycle without leading to undesirable implications for investment that takes advantage of improvements in productivity.

Our analysis of cyclical macroprudential policies expands upon previous work along several dimensions. As in Cúrdia and Woodford (2009), our model illustrates the potential benefits of policies that lean against movements in credit spreads, while significantly expanding their analysis through a model with debt and equity frictions and, hence, a much richer description of intermediary leverage. As in Christensen, Meh, and Moran (2011), our framework provides an illustration of how policies that simply lean against credit movements may distort the economy's desirable adjustment to other shocks. Our consideration of a range of indicators (credit, asset market values, and credit spreads) expands considerably the range of policies, and associated pitfalls, that have been studied in macroeconomic models relative to the analyses in, e.g., Christensen, Meh, and Moran (2011) and Angelini, Neri, and Panetta (2011). Finally, our inclusion of reserve requirements as a macroprudential instrument echoes a suggestion of Stein (2011) and Hanson, Kashyap, and Stein (2011) in a large, dynamic-general-equilibrium model.

[1] The monetary policy literature has focused on the merits and pitfalls associated with responding to broad measures of asset values–for example, Bernanke and Gertler (1999).

2 Model

The model economy consists of (i) a representative household, (ii) a representative firm producing intermediate goods, (iii) a continuum of monopolistically competitive retailers, (iv) a representative firm producing inv in order to gauge if such simple policiesestment goods, and (v) a continuum of financial intermediaries.

The representative household lacks the skill necessary to directly manage financial investment projects. As a result, the household saves through financial intermediaries. In addition to the assumed role of intermediation, we will adopt a framework in which raising equity from external funds is costly – a key financial friction in our model. As we discuss further below, a distinction between internal and external funds lies at the heart of much research in corporate finance (e.g., Myers and Majluf (1984)).

Finally, we assume a timing convention in intermediaries' financing decisions that is designed to highlight risks associated with intermediation. A key aspect of intermediation is that financial intermediaries make long-term commitments despite short-run funding risks. For example, a substantial portion of commercial and industrial lending by commercial banks are in the form of loan commitments; more generally, banks have substantial mismatches between the maturities of their assets and liabilities. Rather than introducing long-term assets, we adopt a simple framework which splits a time period into two. Lending and borrowing (e.g., asset and liability) decisions of intermediaries have to be made in the first half of the period t; idiosyncratic shocks to the returns of the projects made at time $t-1$ are realized in the second half of the period t, at which point lending and borrowing decisions cannot be reversed (until period $t+1$).[2]

This set of assumptions has two advantages: First, the intra-period irreversibility in

[2] Another related approach would be the following. One can assume that a random fraction of households require early redemption of their debts/deposits at intermediaries in the second half of the period. In this case, the idiosyncratic redemption rate replaces the idiosyncratic shocks to the return on lending. Owing to the illiquidity of the investment project, the intermediary has to raise additional funds in interbank market or stock market to meet the "run". This will create a similar effect on the lending decision of the intermediary.

lending and borrowing decisions, in conjunction with costs of external equity financing, creates balance sheet risk and generates *precaution* in lending decisions; second, the timing convention helps us derive an analytical expression for the equity issuance and default triggers of intermediaries, allowing a sharp characterization of the equilibrium.

We now walk through the financing decisions (for debt, equity, and payouts) of intermediaries. The model discussion of non-financial activities (of households and non-financial firms) is relatively brief, as those aspects of our model follow standard practice.

2.1 Financial Intermediary Sector

Financial intermediaries are necessary to finance investment projects. These projects are financed with debt and equity raised from investors – in our model, the representative household. Intermediaries wish to use debt – that is, to be leveraged – because a corporate income tax makes debt financing attractive and because raising equity from outside investors is costly.

In presenting the model, we first walk through the costs facing each intermediary in raising funds (via debt and equity) to finance the investment projects. Given these constraints on funding, we then turn to the intermediaries optimal choices of lending to investment projects, financing of these projects via borrowing and equity, and dividend payout policies to maximize shareholder value.

2.1.1 Intermediary Debt Contract

The cost and terms of borrowing are determined by the terms of the debt contract between a financial intermediary $i \in [0,1]$ and an investor. Denote the fraction of a lending/investment project financed via intermediary borrowing by $1 - m_t$. The debt is collateralized by the total investment project. If the intermediary does not default in the next period, it repays this debt (in amount $(1 + r^B_{t+1})(1 - m_t)$, where r^B_{t+1} is the interest rate on borrowing). In the event of default, the investor receives the collateral asset, whose per unit market value is Q_{t+1}. Because investors are assumed to lack the skill to manage investment projects, immediate liquidation of the investment project is required at a distressed sales cost, a fraction $\eta \in (0,1)$ of the asset value.

The intermediary's investment project delivers a random gross return, $1 + r^F_{t+1}$ after tax. The return on lending/investment projects consists of an idiosyncratic component ϵ_{t+1} and an aggregate component $1 + r^A_{t+1}$ such that $1 + r^F_{t+1} = \epsilon_{t+1}(1 + r^A_{t+1})$ where the idiosyncratic component has a time-varying distributions $F_{t+1}(\cdot)$. In particular, we assume that the second moment of the distribution follows a Markov process (detailed further below), while the first moment is time-invariant (and normalized to equal one, $\mathbb{E}_t[\epsilon_{t+1}] = 1$). The time-variation in the second moment of the idiosyncratic return will have aggregate implications under the financial market frictions considered herein.

After the tax deduction on interest expenses, the debt burden of the intermediary is equal to $[1 + (1 - \tau_c)r^B_{t+1}](1 - m_t)$, where τ_c denotes the corporate income tax rate. A default occurs when the realized asset return $\epsilon_{t+1}(1 + r^A_{t+1})$ falls short of the value of the debt obligation $(1 + (1 - \tau_c)r^B_{t+1})(1 - m_t)$. This implies a default trigger ϵ^D_{t+1} – realizations of the idiosyncratic return below this value imply default

$$\epsilon_{t+1} \leq \epsilon^D_{t+1} \equiv (1 - m_t)\frac{1 + (1 - \tau_c)r^B_{t+1}}{1 + r^A_{t+1}}. \tag{1}$$

Intermediary Debt Pricing Households discount future cash flows with their stochastic discount factor, denoted by $M_{t,t+1}$. Given the default trigger (1) and the assumption regarding the bankruptcy costs, the no-arbitrage condition for the household should satisfy

$$1 - m_t = \mathbb{E}_t \left\{ M_{t,t+1} \left[(1 - \eta) \int_0^{\epsilon^D_{t+1}} \epsilon_{t+1}(1 + r^A_{t+1}) dF_{t+1} + \int_{\epsilon^D_{t+1}}^{\infty} (1 - m_t)[1 + r^B_{t+1}] dF_{t+1} \right] \right\}. \tag{2}$$

The first term inside the parentheses on the right-hand side is the default recovery, where the recovery rate $1 - \eta$ owes to the costs of bankruptcy/liquidation. The second term is the non-default income. The discounted value of this total return must equal the value of funds lent to intermediaries by the household, $1 - m_t$.

This equation works as the households' participation constraint in the intermediary's optimization problem for capital structure – that is, intermediaries must take into account the required return to households on debt in deciding leverage. For later use, it is useful to replace r^B_{t+1} in the participation constraint with an expression including ϵ^D_{t+1}. Using the definition of the default trigger, we can express the borrowing rate as $r^B_{t+1} = \frac{1}{1-\tau_c}\left[\frac{\epsilon^D_{t+1}(1+r^A_{t+1})}{1-m_t} - 1\right]$. Substituting this in the participation constraint yields

$$1 - m_t = \mathbb{E}_t\left\{M_{t,t+1}\left[\int_0^{\epsilon^D_{t+1}}(1-\eta)\epsilon_{t+1}dF_{t+1} + \int_{\epsilon^D_{t+1}}^{\infty}\frac{\epsilon^D_{t+1} - \tau_c(1-m_t)}{1-\tau_c}dF_{t+1}\right](1+r^A_{t+1})\right\} \tag{3}$$

2.1.2 Intermediary Equity Finance

We now turn to the problem of intermediary equity financing. The equity stake in the intermediary entitles the representative household to the profits from the lending capacity of the intermediary and, as is standard in the corporate finance literature, we assume that the managers of the intermediaries maximize the value of incumbent shareholders. However, intermediaries may find themselves short of cash flow (in circumstances described below) in which case they must either enter bankruptcy or raise additional equity. If the intermediary raises equity capital, we assume that they must sell new shares *at a discount*, which generates a dilution effect: issuing new equity with a notional value of a dollar reduces the value of *existing* shares more than a dollar. Our approach, based on Bolton and Freixas (2000), is to assume a parametric form for the dilution cost: issuing new equity involves a constant per-unit issuance cost, $\varphi \in (0, 1)$.[3] We denote equity related cash flow by D_t. D_t is dividends paid when positive, and equity issuance when negative. With

[3]Our approach, based on Bolton and Freixas (2000), can be considered standard in corporate finance literature: See Gomes (2001) and Cooley and Quadrini (2001), for example. Pursuing a structural motivation for the existence of the dilution costs is beyond the scope of this analysis. See Myers and Majluf (1984) and Myers (2000) for a more formal presentation.

our assumption of costly equity issuance, actual cash inflow from the issuance ($-D_t$) is $-(1-\varphi)D_t$. Total equity related cash flow for the intermediary is $-D_t + \varphi \min\{0, D_t\}$.

Suppose that the intermediary invests in S_t units of asset whose market price is given by Q_t. The intermediary borrows $1 - m_t$ for each dollar of its lending/investment project. The cash inflow associated with this debt financing from households is given by $(1 - m_t)Q_tS_t < Q_tS_t$. To close the funding gap, the intermediary has three other sources: internal funds, N_t, equity issuance $-D_t + \varphi \min\{0, D_t\}$, and a (potential) lump-sum government transfer T_t such that

$$Q_tS_t = (1 - m_t)Q_tS_t + N_t - D_t + \varphi \min\{0, D_t\}, \tag{4}$$

which is simply the flow of funds constraint facing the intermediary.

Without default, the internal funds of the intermediary are given by the difference between the total return from the asset minus the debt payment, i.e., $N_t = \epsilon_t(1 + r_t^A)Q_{t-1}S_{t-1} - (1 + (1 - \tau_c)r_t^B)(1 - m_{t-1})Q_{t-1}S_{t-1}$. However, owing to the limited liability of the intermediary, internal funds are truncated by zero. Hence internal funds are given by

$$\begin{aligned} N_t &= [\max\{\epsilon_t, \epsilon_t^D\}(1 + r_t^A) - (1 - m_{t-1})(1 + (1 - \tau_c)r_t^B)]Q_{t-1}S_{t-1} \\ &= [\max\{\epsilon_t, \epsilon_t^D\} - \epsilon_t^D](1 + r_t^A)Q_{t-1}S_{t-1}. \end{aligned} \tag{5}$$

Combining the flow-of-funds constraint 4 with the definition of internal funds 5 yields the cash-flow equation, inclusive of the impact of limited liability, that governs the link between past lending, equity and debt issuance, and dividend payouts that will constrain intermediary decisions.

$$[\max\{\epsilon_t, \epsilon_t^D\} - \epsilon_t^D](1 + r_t^A)Q_{t-1}S_{t-1} - D_t + \varphi \min\{0, D_t\} - m_tQ_tS_t = 0 \tag{6}$$

2.1.3 Value Maximization

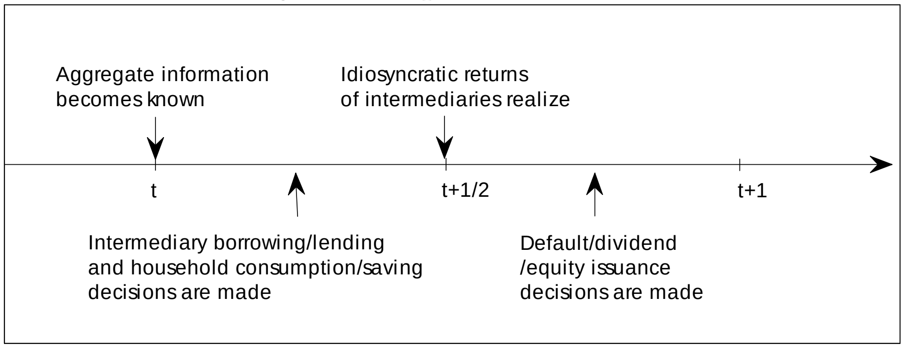
Symmetric Equilibrium In order to present a sharp characterization of the equilibrium, the timing convention mentioned earlier is important. Formally, we assume: (i) all aggregate information is known at the beginning of each period; (ii) based on aggregate information, intermediaries make lending/borrowing decisions, which are irreversible within a given period; (iii) idiosyncratic shocks are realized after the lending/borrowing decision; (iv) some intermediaries undergo the default/renegotiation process; (v) finally, equity issuance/dividend payout decisions are made. Figure 1 shows the timing of information and decisions.

This timing convention, the risk neutrality of intermediaries, and the absence of persistence in the first moment of idiosyncratic shock imply a symmetric equilibrium in which all intermediaries choose the same lending/investment level and capital structure. The symmetric equilibrium also implies that all intermediaries face the same borrowing cost and default trigger at the borrowing/lending stage (e.g., the first half of period t). The shadow value of the participation constraint (the no-arbitrage condition for a bond investor, (3)), denoted by θ_t, also has a degenerate distribution since the borrowing decision is made before the realization of the idiosyncratic shock.

However, the distribution of dividends and equity financing do depend on the realization of idiosyncratic shocks, and thus has a non-degenerate distribution. Since the flow of funds constraint depends on the realization of the idiosyncratic shock, the shadow value of the constraint, denoted by λ_t, also has a non-degenerate distribution.

To simplify the dynamic problem, we decompose the intermediary problem into two stages in a way that is consistent with the timing convention: in the first stage, the intermediary solves for the value maximizing strategies for lending and borrowing without knowing its realization of net-worth. In the second stage, the intermediary solves for the value maximizing dividend/issuance strategy based upon all information, including the realization of its net-worth.

Figure 1: Sequence of Events



Formally, we define two value functions, J_t and $V_t(N_t)$. J_t is the ex-ante value of the intermediary before the realization of idiosyncratic shock while $V_t(N_t)$ is the ex-post value of the intermediary after the realization of idiosyncratic shock. In our symmetric equilibrium, the ex-ante value function does not depend on the intermediary specific state variables and J_t is a function of aggregate state variables only. The ex-post value function $V_t(\cdot)$, however, depends on the realized internal funds, N_t, which is a function of the realized idiosyncratic shock as shown by (5). Since the first stage problem is based upon the conditional expectation of net-worth, not the realization, it is useful to define an expectation operator $\mathbb{E}_t^{\epsilon}(\cdot) \equiv \int \cdot dF_t(\epsilon)$, the conditioning set of which includes all information up to time t, except the realization of the idiosyncratic shock. Because of the assumed conditioning set, all aggregate state variables at time t can be taken out of the expectation operator. From the perspective of households, there is no new information in the second half of the period because of the law of large numbers: At the beginning of each period, the household exactly knows how much additional equity funding is required for the intermediary sector as a whole (as indicated by the timing of household decisions in figure 1). This ensures that the lending and borrowing decisions of intermediaries are consistent with the savings decisions of households.

All financial intermediaries are owned by the representative household, and hence discount future cash flows by the stochastic pricing kernel of the representative household, $M_{t,t+1}$. Before the realization of the idiosyncratic shock, the intermediary maximizes shareholder value by solving for the size of its lending, debt, and equity from retained earnings (through choices for the aggregate project size S_t, leverage m_t, and the default trigger ϵ_t^D, subject to the households' participation constraint 3 and the intermediary's cash-flow constraint 6),

$$J_t = \max_{S_t, m_t, \epsilon_{t+1}^D} \{\mathbb{E}_t^{\epsilon}[D_t] + \mathbb{E}_t[M_{t,t+1} \cdot V_{t+1}(N_{t+1})]\}. \tag{7}$$

After the realization of the idiosyncratic shock, the intermediary solves

$$V_t(N_t) = \max_{D_t} \{D_t + \ \mathbb{E}_t[M_{t,t+1} \cdot J_{t+1}]\} \tag{8}$$

subject to its cash-flow constraint 6. Problem (7) solves for the optimal lending/borrowing /default choices based upon $\mathbb{E}_t^{\epsilon}[N_t]$ and $\mathbb{E}_t^{\epsilon}[D_t]$, which are aggregate information. At this stage, the intermediary does not know whether default or issuance or distribution of dividends will occur under its optimal strategy. In contrast, problem (8) solves for the optimal level of distribution/issuance based on the realization of net-worth. In the second stage problem, the truncated net worth N_t becomes a state variable through the intermediary's cash-flow constraint 6.

Denote the multiplier on the households participation constraint equation 3 by λ_t and the multiplier on the intermediary's cash-flow constraint 6 by $\theta_t Q_t S_t$. The first-order conditions associated with problems 8 and 7 are given by the following (The appendix provides details of the derivation).

- FOC for D_t :

$$\lambda_t = \begin{cases} 1 & \text{if } D_t \geq 0 \\ 1/(1-\varphi) & \text{if } D_t < 0 \end{cases} \tag{9}$$

- FOC for S_t :

$$m_t \mathbb{E}_t^{\epsilon}[\lambda_t] = \mathbb{E}_t\{M_{t,t+1}\mathbb{E}_{t+1}^{\epsilon}[\lambda_{t+1}(\max\{\epsilon_{t+1}, \epsilon_{t+1}^D\} - \epsilon_{t+1}^D)](1 + r_{t+1}^A)\} \tag{10}$$

- FOC for m_t :

$$\mathbb{E}_t^{\epsilon}[\lambda_t] = \theta_t \left[1 + \frac{\tau_c}{1-\tau_c}\mathbb{E}_t\left(M_{t,t+1}[1 - F_{t+1}(\epsilon^D_{t+1})]\right)\right] \tag{11}$$

- FOC for ϵ^D_{t+1} :

$$\begin{aligned} 0 = {} & \mathbb{E}_t\left\{M_{t,t+1}\mathbb{E}^{\epsilon}_{t+1}\left[\lambda_t\left(\frac{\partial \max\{\epsilon_{t+1}, \epsilon^D_{t+1}\}}{\partial \epsilon^D_{t+1}} - 1\right)\right]\right\} \\ & + \theta_t \mathbb{E}_t\left\{M_{t,t+1}\left[\left((1-\eta) - \frac{1}{1-\tau_c}\right)\epsilon^D_{t+1} f_{t+1}(\epsilon^D_{t+1})\right](1 + r^A_{t+1})\right\} \\ & + \theta_t \mathbb{E}_t\left\{M_{t,t+1}\left[\frac{1}{1-\tau_c}[1 - F_{t+1}(\epsilon^D_{t+1})](1 + r^A_{t+1}) + (1 - m_t)\frac{\tau_c}{1-\tau_c} f_{t+1}(\epsilon^D_{t+1})\right]\right\} \end{aligned} \tag{12}$$

2.1.4 Discussion

Equation (9) states that the shadow value of the internal funds depends on the intermediary's realized equity regime: the marginal valuation of an additional dollar is equal to one as long as it does not face any difficulty in closing the funding gap, and as a result, distributes a strictly positive amount of dividends; the shadow value can be strictly greater than 1 if the intermediary faces a short-term funding problem and has to raise equity funds.

To see the economic effects of the time-variation in the value of intermediaries internal funds associated with balance sheet risk, we solve for the value of the idiosyncratic portion of the lending return that requires raising external equity by determining the level of the idiosyncratic return that implies zero dividend in the flow of funds constraint (4). Idiosyncratic returns below this "issuance trigger"require raising external funds

$$\epsilon^E_t \equiv (1 - m_{t-1})\frac{1 + (1-\tau_c)r^B_t}{1 + r^A_t} + m_t \frac{Q_t S_t + T_t}{(1 + r^A_t)Q_{t-1}S_{t-1}} = \epsilon^D_t + m_t \frac{Q_t S_t + T_t}{(1 + r^A_t)Q_{t-1}S_{t-1}}. \tag{13}$$

(13) shows that the support of the idiosyncratic shock is divided into three parts: (i) $(0, \epsilon^D_t]$, (ii) $(\epsilon^D_t, \epsilon^E_t]$, and (iii) (ϵ^E_t, ∞). In the first interval, the intermediary defaults. In the second interval, the intermediary avoids default, but needs to raise new funds externally. In the third interval, the intermediary pays dividends to the shareholders.

Since the shadow value takes one with probability $1 - F_t(\epsilon_t^E)$ and $1/(1-\varphi)$ with probability $F_t(\epsilon_t^E)$, the expected shadow value is given by

$$\mathbb{E}_t^{\epsilon}[\lambda_t] = 1 - F_t(\epsilon_t^E) + \frac{F_t(\epsilon_t^E)}{1-\varphi} = 1 + \mu F_t(\epsilon_t^E) \geq 1,\ \mu \equiv \frac{\varphi}{1-\varphi}. \tag{14}$$

The inequality is strict as long as $\varphi > 0$. The fact that the expected shadow value of internal funds is always greater than 1 shows that the intra-period irreversibility of the lending decision creates *caution* on the part of the risk-neutral intermediaries. Though intermediaries know that they may be swamped with excess cash flow *ex-post*, they follow a conservative lending strategy due to potential balance sheet risks. Moreover, the degree of conservatism is endogenously time-varying as a function of macroeconomic developments (as captured in the aggregate state variables).

Effects of Costly Equity Finance and the Default Option To see how the financial market frictions affect the prices of assets and the lending decisions of the intermediaries, it is useful to focus on the FOC for S_t, which, after dividing through by $m_t \mathbb{E}_t^{\epsilon}[\lambda_t]$, is

$$1 = \mathbb{E}_t \left\{ M_{t,t+1} \frac{\mathbb{E}_{t+1}^{\epsilon}[\lambda_{t+1}]}{\mathbb{E}_t^{\epsilon}[\lambda_t]} \cdot \frac{1}{m_t} \left[\frac{\mathbb{E}_{t+1}^{\epsilon}[\lambda_{t+1} \max\{\epsilon_{t+1}, \epsilon_{t+1}^D\}]}{\mathbb{E}_{t+1}^{\epsilon}\lambda_{t+1}} - \epsilon_{t+1}^D \right] (1 + r_{t+1}^A) \right\} \tag{15}$$

(15) can be considered an asset pricing formula. It is different from the text book version for three reasons. First, it is a levered asset pricing formula. Second, the pricing kernel of the financial intermediaries is a filtered version of the representative household's stochastic discount factor (that is, there is a role for λ_t). And third, the default option affects the lending decisions.

Let's consider the filtering of the stochastic discount factor in more detail. The wedge between the pricing kernels of the intermediaries, which we will denote by $M_{t,t+1}^B$, and that of the representative household ($M_{t,t+1}$) is determined by the liquidity condition measured by the ratio of expected shadow values of internal funds, today vs tomorrow,

$$M^B_{t,t+1} \equiv \frac{\mathbb{E}^{\epsilon}_{t+1}[\lambda_{t+1}]}{\mathbb{E}^{\epsilon}_{t}[\lambda_{t}]} M_{t,t+1}, \tag{16}$$

a ratio summarizing the intermediary's expectation about their dynamic balance sheet condition. Specifically, a large value for λ_t relative to λ_{t+1}, all else equal, is equivalent (within 15) to an decrease in the household's stochastic discount factor, as might occur, for example, if the household were to become more impatient. As a result, anything that increases the shadow value of cash flow for intermediaries today versus tomorrow will boost required asset returns, crimp lending, and lead to weaker economic activity. A prime example of such a factor would be an increase in idiosyncratic uncertainty today versus tomorrow – a factor that is irrelevant in the absence of costly outside equity ($\varphi = 0$, hence $\lambda_t = 1$) and illiquidity/short-run commitments in lending. In this sense, (15) and (16) can be thought of as an application of liquidity-based asset pricing model (LAPM, Holmström and Tirole (2001)) in a dynamic general equilibrium economy. [4].

Now consider the possibility the intermediary is fully equity financed and hence the default options is irrelevant. In this case ($m_t = 1$ and $\epsilon^D_t = 0$), 15 becomes

$$1 = \mathbb{E}_t \left\{ M_{t,t+1} \frac{\mathbb{E}^{\epsilon}_{t+1}[\lambda_{t+1}]}{\mathbb{E}^{\epsilon}_{t}[\lambda_{t}]} \left[\frac{\mathbb{E}^{\epsilon}_{t+1}[\lambda_{t+1}\epsilon_t]}{\mathbb{E}^{\epsilon}_{t+1}[\lambda_{t+1}]} (1 + r^A_{t+1}) \right] \right\}.$$

Assuming costly equity financing ($\varphi > 0$), $\mathbb{E}^{\epsilon}_{t+1}[\lambda_{t+1}\epsilon_{t+1}]$ is always less than $\mathbb{E}^{\epsilon}_{t+1}[\lambda_{t+1}]$. This occurs because high realizations of the idiosyncratic return ϵ_{t+1} will be associated with higher internal funds and hence lower shadow values for cash (λ_{t+1}), and this negative covariance implies, via Jensen's inequality, that the former expression is less than the latter expression.[5] This means that the asset return $1 + r^A_{t+1}$ must be higher than it would

[4]See He and Krishnamurthy (2008), who derive an intermediary specific pricing kernel by assuming risk aversion for the intermediary. Also see Jermann and Quadrini (2009), who derives a similar pricing kernel by assuming a quadratic dividend smoothing function.

[5]In fact, we considered a related case in our earlier work (Kiley and Sim (2011a) and Kiley and Sim (2011b)), in which we consider the financial intermediary sector under a regulatory capital requirement.

be in a frictionless market, in which the covariance between cash-flow realizations and the value of cash is irrelevant, and that, under a diminishing marginal rate of return from capital, capital is under-accumulated because of capital market frictions.

Returning to the effect of default directly (with debt financing $m_t < 1$ and $\epsilon_t^D > 0$), it is clear that default occurs under low realizations of idiosyncratic returns (ϵ_{t+1} low) and high values for the shadow value of intermediary cash flow (λ_{t+1} high). More specifically, the default creates an option value as the limited liability makes the return of the intermediary a convex function of idiosyncratic return. The option value is given by the difference between $\mathbb{E}_{t+1}^{\epsilon}[\lambda_{t+1} \max\{\epsilon_{t+1}, \epsilon_{t+1}^D\}]$ and $\mathbb{E}_{t+1}^{\epsilon}[\lambda_{t+1}\epsilon_{t+1}]$; this difference is always greater than zero and the value of the option is strictly positive as long as uncertainty is present. In contrast to the equity market friction ($\varphi > 0$), the default option associated with debt financing encourages risk-taking, pushing down the required return to capital ($1 + r_{t+1}^A$) and inducing over-investment in capital assets, *ceteris paribus*.

This default option is more valuable when uncertainty regarding the asset return increases. This, however, does not imply that the financial intermediaries will increase their lending to risky assets at a time of heightened uncertainty: while a greater uncertainty boosts the risk appetite of the intermediaries through the default option, the same increase in uncertainty boosts the expected shadow values of cash flow (the effect of the costly equity financing friction ($\varphi > 0$)), thereby elevating the required return to lending for the intermediaries, which then reduces lending to risky assets.

2.2 The Rest of the Model Economy

To close the model, we now turn the production, capital accumulation, and the consumption/labor supply decisions of non-financial firms and households. Regarding the structure of production and capital accumulation, we assume that the production of consumption and investment goods are devoid of financial frictions. This assumption, while strong, helps us focus on the friction facing the financial intermediaries in their funding markets rather than the friction in their lending (investment) market. [6]

2.2.1 Production and Investment

There is a competitive industry that produces intermediate goods using a constant returns to scale technology; without loss of generality, we assume the existence of a representative firm. The firm combines capital (K) and labor (H) to produce the intermediate goods using a Cobb-Douglas production function, $Y_t^M = a_t H_t^{\alpha} K_t^{1-\alpha}$, where the technology shock follows a Markov process, $\log a_t = \rho_a \log a_{t-1} + \sigma_a v_t$, $v_t \sim N(0,1)$.

The intermediate-goods producer issues state-contingent claims S_t to a financial intermediary, and use the proceeds to finance capital purchases, $Q_t K_{t+1}$. A no-arbitrage condition implies that the price of the state-contingent claim must be equal to Q_t such that $Q_t S_t = Q_t K_{t+1}$. After the production and sale of products, the firm sells its undepreciated capital at the market value, returns the profits and the proceeds of the capital sale to the intermediary. The competitive industry structure implies that the firm's static profit per capital is determined by the capital share of revenue, i.e., $r_t^K = (1-\alpha) P_t^M Y_t^M / K_t$, where P_t^M is the price level of the intermediate goods. Hence the after-tax return for the intermediary is given by

$$1 + r_t^A = \frac{(1-\tau_c)(1-\alpha) P_t^M Y_t^M / K_t + [1 - (1-\tau_c)\delta] Q_t}{Q_{t-1}}. \tag{17}$$

We assume costs of adjusting investment at the aggregate level to allow for time-variation in the price of installed capital (K_t) relative to investment. More specifically, we assume that there is a competitive industry producing new capital goods combining the existing capital stock and consumption goods using a quadratic adjustment cost of investment, $\chi/2(I_t/I_{t-1} - 1)^2 I_{t-1}$.

[6] Other recent studies of intermediaries, notably Gertler and Karadi (2011) and Gertler and Kiyotaki (2010), adopt a similar assumption.

2.2.2 Households

We specify a constant-relative-risk-aversion utility function for the representative household. with habit persistence. Preferences are given by

$$\sum_{j=0}^{\infty} \beta^j \left[\frac{1}{1-\gamma} [(C_{t+j} - hC_{t+j-1})^{1-\gamma} - 1] - \frac{\zeta}{1+\nu} H_{t+j}^{1+\nu} \right], \tag{18}$$

where C_t is consumption, H_t is hours worked, β is the time discount factor, γ governs curvature in the utility function, h is the habit parameter, ν is the inverse Frisch elasticity of labor supply, and ζ determines the relative weighting on hours worked in overall utility. The representative household earns a wage by providing labor hours. The efficiency condition for labor hours is given by $\frac{W_t}{P_t}\Lambda_t = \zeta H_t^{\nu}$, where P_t is the composite price level of the retail goods and Λ_t is the marginal utility of consumption. Households must save through financial intermediaries, investing in either their debt or equity shares.

We denote the total outstanding debt of financial intermediaries by B_t. In equilibrium, $B_t = \int [1 - m_{t-1}(i)] Q_{t-1} S_t(i) di = (1 - m_{t-1}) Q_{t-1} K_{t+1}$, where i is an index for an intermediary and the last equality is due to the symmetric equilibrium and the equality between aggregate lending and aggregate capital. We presented the no-arbitrage condition determining households' willingness to own intermediary debt previously (3). Investment in the equity shares of the financial intermediary satisfies the equilibrium condition

$$1 = \mathbb{E}_t \left[M_{t,t+1} \frac{\mathbb{E}_{t+1}^{\epsilon}[\max\{D_{t+1}, 0\} + (1-\varphi)\min\{D_{t+1}, 0\}] + P_{t+1}^S}{P_t^S} \right] \tag{19}$$

where P_t^S is the ex-dividend price of an intermediary share. This is a standard dividend-price formula for the consumption CAPM, taking into account the effect of the equity issuance cost on dividend related cash flows to investors (as shown in the appendix). Note that in our symmetric equilibrium, $P_t^S(i) = P_t^S$ for all $i \in [0, 1]$ because $P_t^S(i) = \mathbb{E}_t[M_{t,t+1} \cdot J_{t+1}]$ does not depend on intermediary specific variables. Finally, note that in general equilibrium, the existing shareholders and the investors in the new shares are the same entity, the representative household. Hence, costly equity financing does not create a wealth effect for the household, but affects the aggregate allocation through the marginal efficiency conditions of the intermediaries.

Finally, we assume that the representative household has no access to central bank funding. Instead, the representative household has access to a nominal bond whose one-period return equals the policy interest rate set by the central bank, R_t, adjusted for an exogenous aggregate "risk" premium Ξ_t (reflecting unmodeled distortions between the central bank and households). Under these assumptions, the condition linking households stochastic discount factor and the policy interest rate is given by

$$1 = \mathbb{E}_t \left[M_{t,t+1} R_t \Xi_t \frac{P_t}{P_{t+1}} \right] \tag{20}$$

We assume that the "risk premium" follows a Markov process, $\log \Xi_t = \rho_\Xi \log \Xi_{t-1} + \sigma_\Xi w_t$, $w_t \sim N(0,1)$. Other models, most notably Smets and Wouters (2007) and Chung, Kiley, and Laforte (2010), have also used this aggregate risk premium shock to explain economic fluctuations. As we will demonstrate below, it is a convenient proxy for a range of potential shocks to the debt and equity frictions facing intermediaries. [7]

2.2.3 Nominal Rigidity and Monetary Policy

We assume that a continuum of monopolistically competitive firms take the intermediate outputs as inputs and transform them into differentiated retail goods $Y_t(j)$, $j \in [0,1]$. To generate nominal rigidity, we assume that the retailers face a quadratic cost in adjusting their prices $P_t(j)$ given by $\frac{\chi^p}{2} \left(\frac{P_t(j)}{P_{t-1}(j)} - \bar{\Pi} \right)^2 P_t Y_t$, where Y_t is the CES aggregate of the

[7] Moreover, as we highlight below further, we find it preferable to focus on this aggregate risk premium shock to illustrate some basic properties of our model. This preference arises because there are also important differences between shocks to the debt and equity frictions in our model, which we highlight, and the aggregate risk premium shock allows us to discuss predictions that are robust to the consideration of debt/equity frictions.

differentiated products with an elasticity of substitution ε, $\bar{\Pi}$ is the steady state inflation rate, and ς is the indexation weight. (Nominal wages are perfectly flexible). In order to make the equilibrium of our model in the absence of nominal price rigidity and financial frictions "first best", we further assume that a system of distortionary subsidies offsets the markup associated with monopolistic competition.

For monetary policy, we specify a Taylor-type interest rule given by

$$1 + r_t = (1 + r_{t-1})^{\rho^r} \left[\left(\frac{Y_t}{Y_t^*} \right)^{\kappa^y} \left(\frac{\Pi_t}{\bar{\Pi}} \right)^{\kappa \Delta p} \right]^{1-\rho^r} \epsilon_t^R \tag{21}$$

where Y_t^* is specified as the capacity level of output, given by $Y_t^* = K_t^{1-\alpha}(z_t H^*)^{1-\alpha}$, where z_t is the current level of technology and H^* denotes steady-state work hours.[8]

2.2.4 Fiscal Policy

In our baseline model, the fiscal policy is simply dictated by the period-by-period balanced budget constraint, which is given by

$$T_t = \tau^c r_t^K K_t - \tau^c [\delta + (1 - m_{t-1})] Q_{t-1} K_t. \tag{22}$$

The first item is the proceeds of the corporate income tax and the second item is the sum of depreciation allowances and the tax refund on debt holdings.

3 Model Properties: The Financial Accelerator

We first explore the role of the financial accelerator within our model, both as a mechanism to amplify the real effects of financial shocks and as a source of volatility through disturbances to the debt and equity frictions facing intermediaries.

[8] Boivin, Kiley, and Mishkin (2010) discuss how a policy reaction function involving this type of output gap, which adjusts for shifts in technology, facilitates good economic performance and may account for the successful stabilization of inflation since the 1980s.

Table 1: Baseline Calibration

Description	Calibration
Preferences and production	
Time discounting factor	$\beta = 0.985$
Constant relative risk aversion	$\gamma = 2$
Habit persistence	$h = 0.75$
Elasticity of labor supply	$1/\nu = 0.25$
Value added share of labor	$\alpha = 0.6$
Depreciation rate	$\delta = 0.025$
Real/nominal rigidity and monetary policy	
Investment adjustment cost	$\chi = 5$
Price adjustment cost	$\chi^p = 125$
Monetary policy inertia	$\rho^r = 0.75$
Taylor rule coefficient for output gap	$\kappa^y = 0.125$
Taylor rule coefficient for inflation gap	$\kappa^{\Delta p} = 1.5$
Financial Frictions	
Liquidation cost	$\eta = 0.03$
Dilution cost	$\varphi = 0.15$
Corporate tax	$\tau_c = 0.20$
Long run level of uncertainty	$\bar{\sigma} = 0.05$
Exogenous Stochastic Process	
Persistence of aggregate risk premium	$\rho_\Xi = 0.90$
Volatility of shock to risk premium	$\sigma_\Xi = 0.0005$
Persistence of technology shock	$\rho_a = 0.90$
Volatility of shock to technology process	$\sigma_a = 0.0025$

3.1 Calibration

Many of our parameters are set at standard values (see table 1). The discount factor β is set to 0.985, implying a steady state real return to capital near 6 percent per year (given that we calibrate to a quarterly frequency). The households' risk aversion parameter γ is set to 2, a modest value, while habit persistence (h) is set to 0.75, within the typical range. The Frisch labor-supply elasticity η is set to 0.25. We set the labor share in production α to 0.60 and the depreciation rate δ to 0.025. With regard to adjustment costs for investment and prices, we adopt a moderate value for investment adjustment costs (χ equal to 5) and a large value for prices (χ^p equal to 125); these values deliver reasonable responses of investment and price inflation to shocks, are broadly consistent with empirical work suggesting very flat "Phillips curves", and, for reasonable variations, have little influence on our main results.

The parameters for the interest rate rule assume moderate persistence/inertia (ρ_r equal to 0.75), a response to inflation equal to 0.5 ($\kappa_{\Delta p}$ equal to 1.5), and a response to the deviation of output from the level consistent with technology, the level of productive capital, and steady-state labor input, κ_y, equal to 0.125; the long-run output and inflation responses equal those of Taylor (1993) (at annual rates).

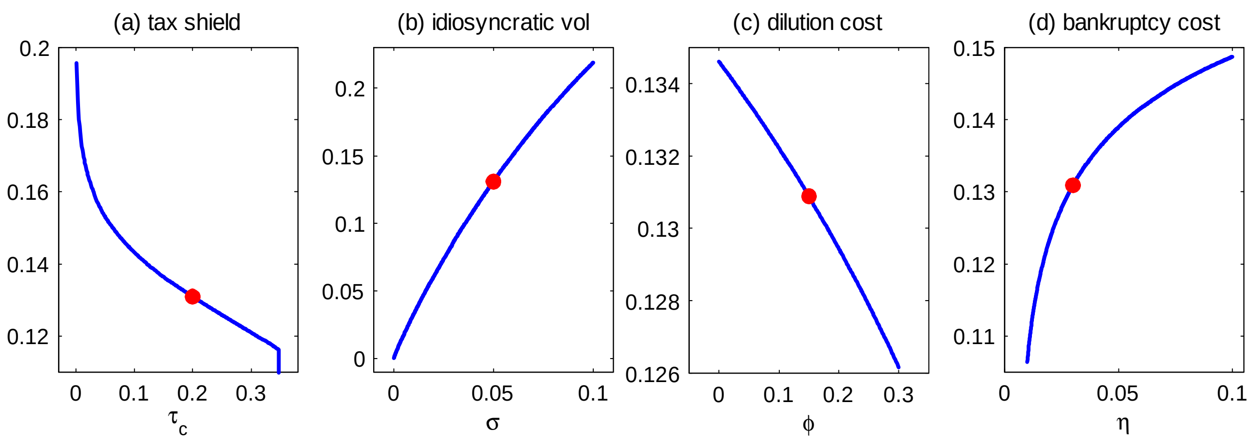
3.2 Equilibrium Leverage

There are several aspects of our calibration that govern predictions for equilibrium leverage within the financial sector and the macroeconomic effects of credit policies. Figure 2 presents the steady-state relationships between leverage and the key structural parameters of the model–namely, the tax shield for debt τ_c (which is a central factor influencing leverage in our model), idiosyncratic risk (σ), the cost of outside equity (φ), and the bankruptcy cost (η). The red circles highlight the calibration for the parameters used throughout our quantitative analysis.

The tax advantage of debt financing associated with the deductibility of interest in a corporate tax framework provides one of the rationales for leverage by financial intermediaries and is the focus of the first panel of the figure. A larger tax preference for debt encourages intermediaries to increase leverage. As indicated by the curve in the panel, the relationship is highly nonlinear. In particular, as the tax shield approaches zero from above, the marginal benefit of debt falls rather quickly. We set the income tax shield to 0.2, which, in our model, leads to the high leverage/low margin ratios around 12 percent observed for financial institutions in the United States. [9]

[9] In practice, the tax shield reflects the deductibility of interest from corporate tax liability and is given by the differential between the corporate income tax rate and the interest income tax rate imposed on the bond investors. For simplicity, our model only includes a corporate income tax τ_c. The tax shield calibration, at

Figure 2: Determination of Steady State Capital Structure (Leverage)



In the description of the model, we pointed out that an increase in uncertainty is detrimental to the financial intermediaries as borrowing costs are boosted by the increased default probability. We assume that ϵ_t follows a log-normal distribution. As shown in the second panel, even a small increase in idiosyncratic risk can result, in our model, in a substantial deleveraging of the financial industry. To calibrate the long run level of idiosyncratic risk, we observe that the standard deviation of the return on assets within a quarter across the top 100 commercial banks in United States since 1986 is about 0.035 percent.[10] We choose a slightly higher level of risk, 0.05, since the dispersion measure from the Call Reports likely understates the downside risk facing financial institutions given the highly fat-tailed aspect of the distribution in the data and the possibility that institutions are able to smooth income reports relative to the underlying performance of loans, at least somewhat.

The dilution cost associated with raising outside equity is another important determinant of leverage. In the model, the cost of raising equity is given by the expected shadow value of internal funds, $\mathbb{E}_t(\lambda_{it}) = 1 + \varphi/(1-\varphi)F_t(\epsilon_t^E)$. Holding the probability of equity

0.2, is low for the corporate tax rate, but high for the overall tax shield.

[10]This calculation is based on data from the Federal Financial Institutions Examination Council's Report of Condition and Income (the Call Reports).

issuance $F_t(\epsilon_t^E)$ constant, a higher level of dilution cost directly increases the marginal cost of raising equity. The higher cost of equity capital then induces the intermediaries to increase leverage, as shown by the decline in the equilibrium margin in the third panel. There exists a wide range of estimates/calibrations regarding the size of dilution effect. In particular, Gomes (2001) provides a particularly low estimate of 0.06. We choose a relatively high value, $\varphi = 0.15$; such a value leads to a substantial financial accelerator in our model and would seem especially relevant to capture the the harsh funding climate that may accompany financial distress, such as that observed in the Great Recession. However, our choice is still on the moderate side of estimates/calibrations found in literature. For instance, Cooley and Quadrini (2001) adopts $\varphi = 0.15$.

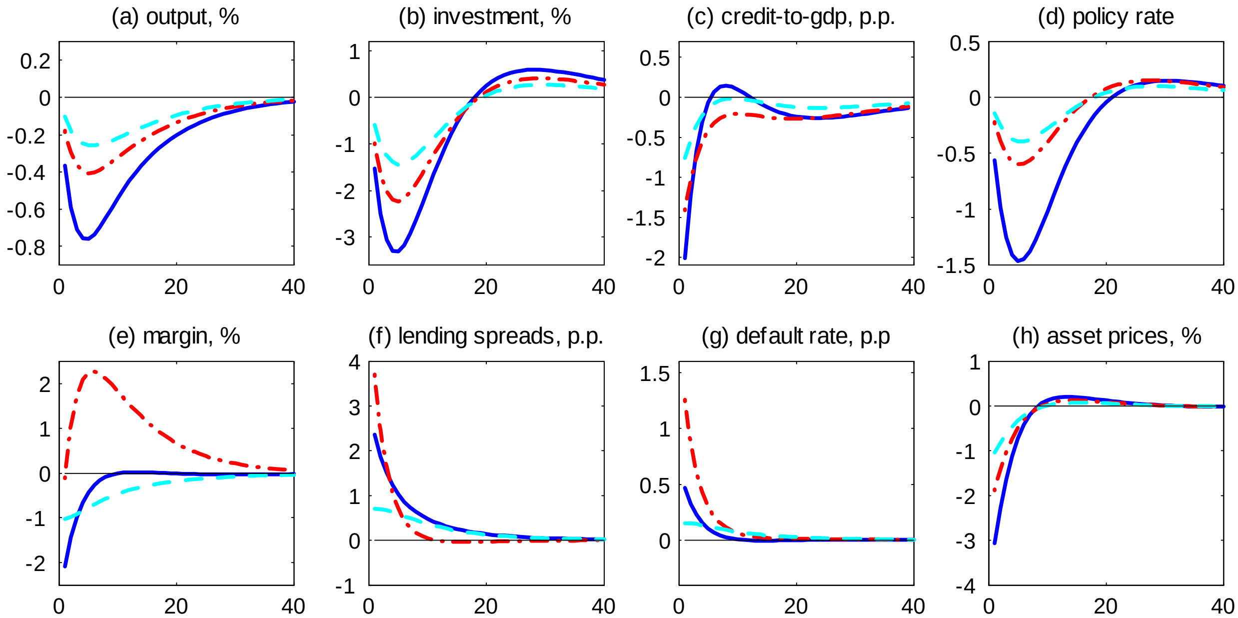
The fourth panel shows the relationship between the bankruptcy cost and leverage. In general, holding leverage constant, a greater bankruptcy cost increases borrowing costs. However, an increase in borrowing costs induces the intermediaries to reduce borrowing, raising the equilibrium margin as shown in the last panel. We choose a moderate level of bankruptcy cost, 0.03, which is a substantially lower level than considered for manufacturing firms in the financial accelerator literature (for instance, 0.12 in Bernanke, Gertler, and Gilchrist (1999)). This choice reflects the fact that the liquidation costs of financial institutions may be low owing to a much lower degree of asset specificity.[11]

3.3 Dynamic Responses to Financial Disturbances

We first present the response of the economy to a shift in the aggregate risk premium – Ξ_t. The solid blue line in figure 3 shows the response of output, investment, the monetary policy rate, asset prices, the equity margin (i.e., m_t, the inverse of leverage), the credit-to-output (GDP) ratio, the lending spread defined as $R^L - R$, and intermediary default

[11]Note that here we refer to the resource cost of liquidation. The "fire sale cost" of liquidation can be quite substantial depending on market condition. This latter aspect of the cost, however, is captured in the endogenous fluctuation of asset values, rather than in the bankruptcy cost.

Figure 3: Impact of Financial Shocks



Note: Blue solid, red solid-dot and cyan dashed lines correspond to risk premium, uncertainty and dilution cost shocks.

rate.[12] Importantly, note that there are *no real effects* from a shock to the aggregate risk premium in an economy with flexible prices and no distortions related to raising outside equity ($\varphi = 0$), as this shock is a disturbance to the "equilibrium nominal interest rate" as defined in Woodford (2003), and hence only has real effects because of nominal rigidities.

The shock to the risk premium Ξ is calibrated such that the exogenous component increases by 100 basis points (not shown). This exogenous disturbance is amplified significantly by the financial frictions, with the lending spread rising more than 200 basis points (solid blue line, panel (h)). As a result of this amplification, output, investment, and credit all decline notably; monetary policy eases in response. This illustrates the substantial financial accelerator in response to financial disturbances.

The intermediary frictions, in addition to amplifying financial disturbances, give rise

[12] The lending rate is defined as the return which satisfies the asset pricing formula for lending 15 given the intermediary discount factor,

$$1 = \mathbb{E}_t\{M^B_{t,t+1} R^L_{t+1}\}.$$

to important economic effects from disturbances to the debt and equity costs facing intermediaries. To see these effects, we consider two shocks to intermediaries that highlight how financial frictions create additional sources of business cycle risk. The first such disturbance we consider is time-variation in the idiosyncratic risk facing intermediaries. Specifically, we assume that the standard deviation of the distribution of idiosyncratic risk σ changes over time (with an adjustment to the central tendency of the distribution to ensure the change in risk is a mean-preserving spread – that is, that the mean of ϵ always equals one). To model time-varying uncertainty, we assume the following process:

$$\log \sigma_t = (1 - \rho_\sigma) \log \bar{\sigma} + \rho_\sigma \log \sigma_{t-1} + u_t. \tag{23}$$

We set ρ_σ equal to 0.90.

The second disturbance to intermediaries' financing conditions we consider is time-variation in the cost of outside equity, φ. We assume the following process,

$$\log \varphi_t = (1 - \rho_\varphi) \log \bar{\varphi} + \rho_\varphi \log \varphi_{t-1} + u_t^*. \tag{24}$$

We set ρ_σ equal to 0.90.

Each of these shocks has important economic implications. An increase in risk raises the probability of default and the probability that an intermediary will need to raise outside equity. These effects boost the value of cash and lead a desire to increase the equity margin; in addition, intermediaries reduce lending. Such dynamics are shown by the red-dashed line in 3: In this figure, the shock to risk is chosen to lead to an increase in the default probability of a bit more than 1/2 percentage point, implying a 200 basis point increase in the lending spread (panels (f) and (g)); the jump in the spread reflects the increase in the current value of cash, λ_t, relative to future values associated with high current uncertainty. The decline in credit is immediate (panels (c)); in contrast, the equity margin (panel (e)) increases only after a few periods – as the initial decline in asset prices (panel (h)) offsets the incentive to cut leverage in the short run. As a consequence. output and investment decline.

Qualitatively, an increase in the cost of outside equity, φ_t has similar effects on intermediaries cost of funds and hence lending decisions, as illustrated by the cyan-dashed line in figure 3. (Note that the increase in φ_t is chosen to lead to an increase in the lending spread somewhat less than 200 basis points, which illustrates the similarity in responses to those of the aggregate risk premium and the risk shocks while also ensuring a bit of distance between the reported lines, to aid the reader in visually assessing the effects). One difference is that an increase in the cost of equity finance leads to a persistent decline in intermediaries' equity margin (i.e., increase in financial leverage), as reported in panel (e). This occurs because equity has become more expensive, all else equal, relative to debt. The higher cost of funds facing intermediaries leads to a contraction in credit, lending, output, and investment.

Overall, these "microeconomic" shocks – to idiosyncratic risk and to the cost of outside equity – are qualitatively similar on most dimensions to those of the aggregate risk premium – highlighting the overall sense of how financial shocks influence lending and hence real activity. Because of these similarities, our policy experiments will focus on dynamics following an aggregate risk premium shock.

4 Macroprudential Policies

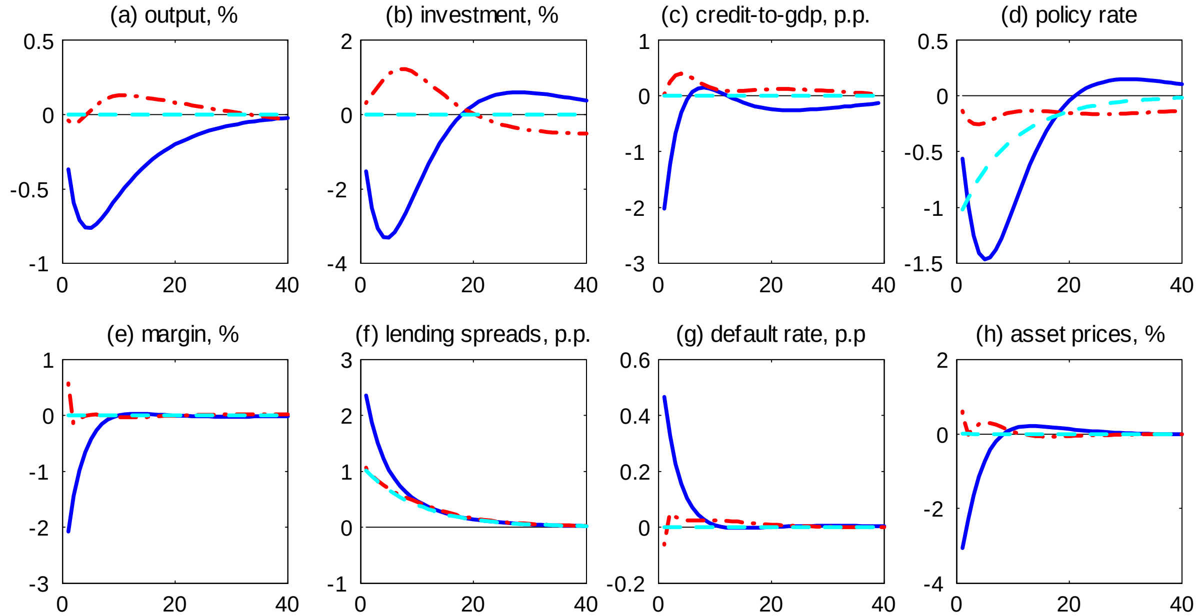
The integration of intermediary balance sheet management, lending decisions, and macroeconomic fluctuations makes our framework ideal for analysis of macroprudential policies. We focus on stabilization policies. In particular, we consider the possibility of macroprudential rules to lean against financial imbalances in a manner that mitigates the influence of shocks to the financial sector, consistent with the framework being developed as part of the Basel 3 process and recent research focusing on policies to raise bank capital in response to an increase in aggregate credit (lending) relative to output (GDP) (e.g., BCBS (2010), Drehmann, Borio, Gambacorta, Jiminez, and Trucharte (2010), Christensen, Meh, and Moran (2011), and Edge and Meisenzahl (2011)). In order to highlight the central economic mechanisms, we first consider the nature of financial frictions in our model and their implications for an optimal Ramsey policy designed to maximize welfare. We then consider simple rules for the macroprudential instrument that capture some features of the Ramsey policy, along with the types of strategies suggested by previous research. Importantly, we emphasize that a reserve requirement acts in a manner similar to a Pigovian tax on leverage.

4.1 A Ramsey Approach

Consider a social planner who maximizes household welfare through direct choice of the equity margin. While infeasible, this approach highlights the socially desirable movements in leverage in response to technology and financial shocks, thereby highlighting the inefficiencies that simpler (in concept and in implementation) policy strategies should attempt to mimic.

To understand the rational for such policy strategies, remember that the intermediary's pricing kernel ($M^B_{t,t+1}$) deviates from the representative household's stochastic discount factor, $M^B_{t,t+1} = M_{t,t+1} \cdot \mathbb{E}^{\epsilon}_{t+1}[\lambda_{t+1}]/\mathbb{E}^{\epsilon}_{t}[\lambda_t]$. When today's funding condition becomes unusually adverse, the required return of the intermediaries can be elevated to a level that leads to a sharp cut back in intermediary investment. We expect the Ramsey policy to "lean against" fluctuations in the inefficiency wedge, $\mathbb{E}^{\epsilon}_{t+1}[\lambda_{t+1}]/\mathbb{E}^{\epsilon}_{t}[\lambda_t]$, as household welfare would be maximized (absent other frictions) by discounting investment projects using the household stochastic discount factor. Moreover, an unintended consequence of private intermediaries cutting lending in response to an elevated shadow value of internal funds is downward pressure on the market prices of balance sheet assets, which further deteriorates the funding conditions of the intermediaries. In a competitive

Figure 4: Impact of Financial Shock: Private, Frictionless, and Ramsey allocations



Note: Blue solid, red solid-dot and cyan dashed lines correspond to Private, Ramsey and RBC benchmark allocation.

equilibrium, individual intermediaries do not internalize the impact of their actions on the market prices of existing assets, a *pecuniary externality*. A social planner can internalize the effects of its actions on market prices

Figure 4 compares the impact of the financial shock on the competitive equilibrium (blue solid lines) and on the Ramsey allocation (red dashed lines). The Ramsey allocation essentially eliminates the fluctuations in output and investment initiated by the financial shock. The most striking difference in the two allocations can be found in the response of asset prices, shown in panel (h) of the figure: while the competitive equilibrium is associated with a large drop in the asset prices of about 3.5 percent on the impact, the Ramsey allocation actually increases the asset prices about 1 percent.

In the competitive equilibrium, the shadow cost of capital $\mathbb{E}_t^{\epsilon}[\lambda_t]$ increases significantly as a result of the financial shock. In response, the financial intermediaries choose an undesirable mix: shrinking the size of the balance sheet and increasing the leverage/decreasing their equity margin (panel (e)) at the same time to avoid the higher cost of capital. This is undesirable for two reasons: it increases the default probability of the financial institutions, which are associated with the dead-weight loss of resources in the form of bankruptcy costs; more importantly, the financial intermediaries pass through the increase in the value of internal funds to the final user of the credit chain, as shown by the higher lending spread in panel (f). One can see that the Ramsey planner achieves the superior allocation by not shrinking the intermediary's balance sheet and instead, slightly expanding the balance sheet and the equity base of the balance sheet in a period of *financial distress*. These differences highlight the direction in which simpler, implementable policies should lean – namely, toward offsetting the effects of adverse changes in financial conditions on the funding and lending incentives facing intermediaries to limit balance sheet contraction in response to financial shocks.

4.2 Simple Macroprudential Policy Rules

The Ramsey policy, in which the central planner optimally chooses leverage, is complicated and model specific. To implement a *simple* cyclical tool within our framework, we first assume that the government imposes a *Pigovian* tax on leverage/equity margin τ_t^m (that is, a tax on $(1 - m_t)$, so that higher leverage/lower margin increases tax owed for a positive tax rate). (Tax revenue is transferred back to the intermediaries by a lump sum transfer T_t). This tax rate, which we call the *leverage tax*, is equal to zero in the steady state, but can become positive (which encourages intermediaries to raise equity and reduce leverage) or negative (which encourages intermediaries to lower equity and increase leverage) in response to a rule linked to an indicator of potential financial imbalances.

To understand the rational for such policy strategies, remember that the potentially inefficient propagation of financial shocks stems from the discrepancy between the economic cost ($\mathbb{E}_t^{\epsilon}[\lambda_t]m_t$) and the accounting cost (m_t) of funding. With the proposed leverage tax, the flow of funds constraint (4) are modified into

$$Q_t S_t = (1 - m_t)(1 - \tau_t^m) Q_t S_t + T_t + N_t - D_t + \varphi \min\{0, D_t\} \tag{25}$$

To offset the financial accelerator associated with fluctuations in the shadow value of intermediary internal funds, the leverage tax transforms the economic cost into

$$\mathbb{E}_t^\epsilon[\lambda_t][m_t + \tau_t^m(1 - m_t)] \gtreqless \mathbb{E}_t^\epsilon[\lambda_t] m_t \quad \text{if} \quad \tau_t^m \gtreqless 0. \tag{26}$$

The amount of funding required for each dollar of investment increases if the tax rate is positive, and decreases if the tax rate it negative (a subsidy). Such a strategy, if designed well, could mitigate any undesirable effects of "non-fundamentals".

Such a tax policy is similar to a reserve requirement: This similarity is notable, as reserve requirements have long been part of the policy toolkit; Earlier work, such as Stein (2011), also highlighted this similarity, but we explore such policies in a quantitative, general equilibrium model. To see the similarity, suppose that the intermediaries are required to set aside an exogenously given amount X_{t+1} as reserves. Under this policy, the flow of funds constraint now takes the form of $Q_t S_t + X_{t+1} = (1 - m_t) Q_t S_t + X_t + N_t - D_t + \varphi \min\{0, D_t\}$. Note that T_t in (25) is replaced by X_t, the reserve requirement from time $t - 1$. By requiring $X_{t+1} = r_t^m (1 - m_t) Q_t S_t$, one can transform the above flow of funds constraint into

$$Q_t S_t = (1 - m_t)(1 - r_t^m) Q_t S_t + X_t + N_t - D_t + \varphi \min\{0, D_t\} \tag{27}$$

where r_t^m can be interpreted as a time-varying reserve requirement ratio. Comparing (25) and (27), one can see immediately the resemblance of the two policies. While the remainder of our snalysis uses a leverage tax, quantitative results are similar for a reserve requirement.[13]

The most important question is then how the macroprudential instrument (in our case, the tax rate or the reserve requirement) should respond to macroeconomic conditions such that it can effectively limit undesirable fluctuations in the value of intermediary cash-flow. We focus on macroprudential rules tied to three indicators. The first indicator is the lending-to-output ratio, which is the approach suggested in BCBS (2010) and Drehmann, Borio, Gambacorta, Jiminez, and Trucharte (2010) and considered in some other research, such Christensen, Meh, and Moran (2011). Our analysis also includes two other approaches: A macroprudential rule linked to broad measures of asset values (the market value of firms) and a rule linked to the lending rate spread over the risk-free rate.

We first analyze the response of the economy under a macroprudential rule in which the leverage tax leans against the credit-to-output gap as in

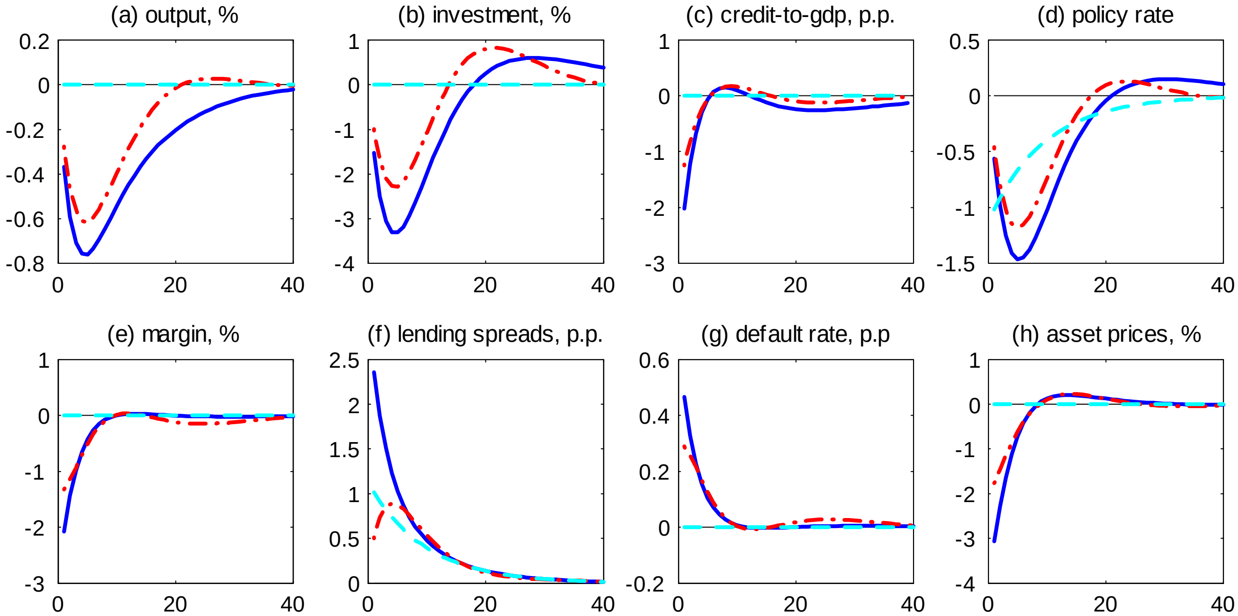
$$\tau_t^M = 0.25[ln(Q_t S_t/\bar{QS}) - ln(Y_t/\bar{Y})] \tag{28}$$

The sensitivity of the leverage tax to increases in credit relative to output is chosen to illustrate the core properties of the rules; we revisit the choice of this sensitivity, using formal criteria, below.

Figure 5 presents impulse responses to the risk premium shock. The solid-blue line illustrate the response in the absence of a macroprudential rule, as in figure 3, while the dashed cyan line shows the response in a frictionless economy (that is, the first-best response); as emphasized previously, the first-best response involves no movement in output. The red-dashed line provides the response under the macroprudential rule. It is clear

[13] Despite the similarities, there are two subtle differences. First, we assume that the proceeds from the leverage tax are transferred back to the intermediaries *within the period* for a balanced budget, i.e., $T_t = t_t^m(1 - m_t)Q_t S_t$ (assuming zero corporate income tax for simplicity), whereas the reserve holdings from the last period are given by $X_t = r_{t-1}^m(1 - m_{t-1})Q_{t-1}S_{t-1}$. More importantly, being a lump sum transfer, T_t is taken as given by the intermediaries, while the intermediaries understand $X_t = r_{t-1}^m(1 - m_{t-1})Q_{t-1}S_{t-1}$. As a consequence, the intermediaries realize that today's reserve requirement increases the shadow cost of investment to $\mathbb{E}_t^\epsilon[\lambda_t][m_t + t_t^m(1 - m_t)]$, but also recognize that the same policy reduces the effective borrowing cost per investment to $[1 + (1 - \tau_c)r_{t+1}^B - r_t^m](1 - m_t)$.

Figure 5: Effect of Credit-to-Output Policy Rule on Response to Financial Shock

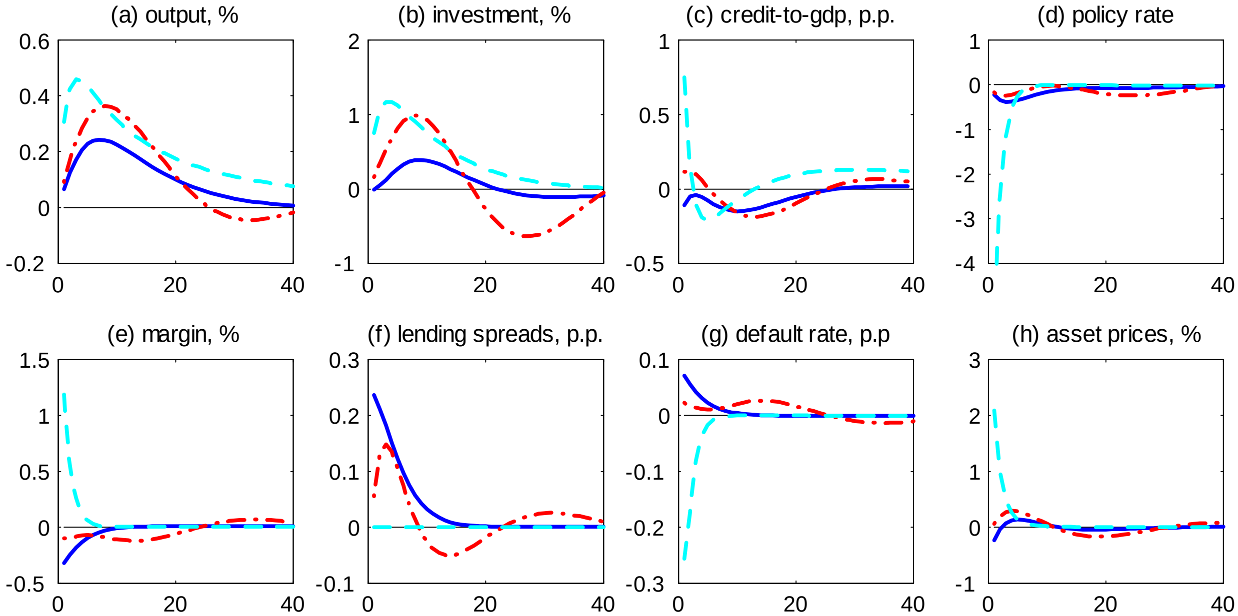


Note: Blue solid, red solid-dot and cyan dashed lines correspond to Private, Credit-to-GDP policy and RBC.

that leaning against credit is beneficial in response to this financial disturbance, as the increase in the risk premium decreases credit (panel (c)), and leaning against this decline by subsidizing leverage encourages lending somewhat, mitigating the decline in investment and output (panels (b) and (a)).

However, leaning against credit may be less desirable in response to non-financial disturbances. For example, improvements in productivity bring about an increase in the desired capital stock, which requires an expansion in credit; moreover, the dynamics of the capital stock are substantially different from those of output. Leaning against the desirable shifts in productive capital may induce unwanted effects. Figure 6 illustrates this possibility, reporting the response to a 1 percent increase in productivity in the absence of a macroprudential rule (the solid-blue line), in a frictionless economy (the first best, in the cyan-dashed line), and under the macroprudential rule (the red-dashed line). Because lending finances capital accumulation and the capital accumulation cycle is different from the business cycle in output (as one is a stock, and the other a flow), lending and output

Figure 6: Effect of Credit-to-Output Policy Rule on Response to Productivity Shock



Note: Blue solid, red solid-dot and cyan dashed lines correspond to Private, Credit-to-GDP policy and RBC.

are out of phase (as illustrated by the movement in credit relative to output, panel (c)), and responding to fluctuations in credit relative to output induces undesirable movements in investment (panel (b)). This suggests such an approach may not have robust stabilization properties.

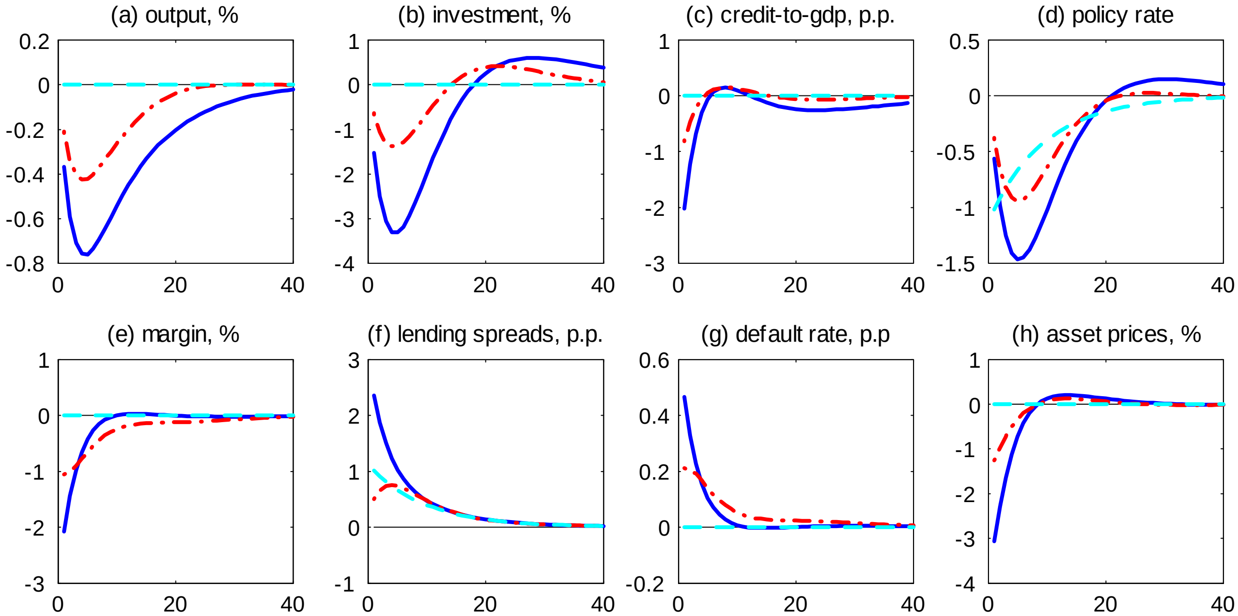
Another possible policy strategy involves leaning against broad movements in asset market values – that is, shifts in the market value of firms, as suggested in the monetary policy debate over responding to the stock market or house prices. We explore this possibility by considering the following macroprudential rule,

$$\tau_t^M = 0.25ln(Q_t K_t / \bar{Q}K). \tag{29}$$

This strategy raises the cost of funding a loan – that is, increases τ_t^m, when the market value of firms increases.[14] As earlier, the response coefficient allows for illustrative exam-

[14]Note that the response is to the market value of firms, an observable, rather than simply an unobservable asset price.

Figure 7: Effect of Asset-Values Policy Rule on Response to Financial Shock



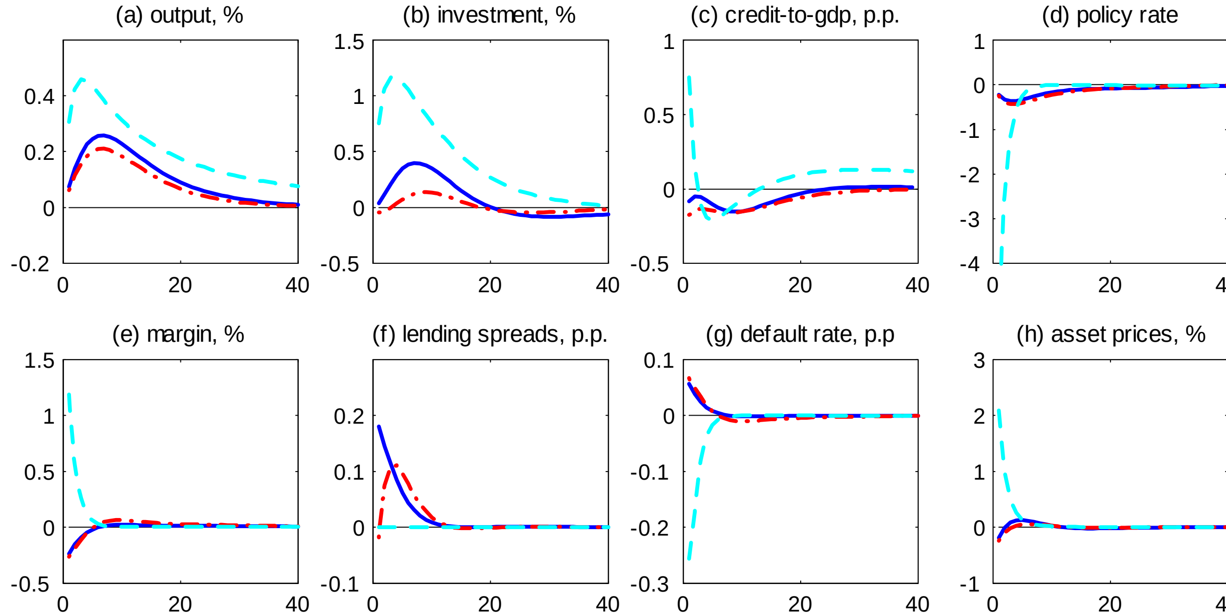
Note: Blue solid, red solid-dot and cyan dashed lines correspond to Private, Asset Value policy and RBC.

ples of the economic effect.

Figure 7 presents impulse responses to the risk premium shock. The solid-blue line illustrate the response in the absence of a macroprudential rule, while the dashed cyan line shows the response in a frictionless economy and the red-dashed line provides the response under the macroprudential rule. It is clear that leaning against asset market values is beneficial in response to this financial disturbance, as the increase in the risk premium depresses asset prices (panel (h)), and leaning against this decline by subsidizing leverage encourages lending somewhat, mitigating the decline in investment and output (panels (b) and (a)).

As with the credit-based policy, leaning against asset market values may be less desirable in response to non-financial disturbances. Improvements in productivity brings about an increase in the desired capital stock and asset prices (panel (h)) in figure 8, where the solid-blue line reports the response in the absence of the macroprudential rule and the red-dashed line reports the response under the macroprudential rule). In this case, lean-

Figure 8: Effect of Asset-Values Policy Rule on Response to Productivity Shock



Note: Blue solid, red solid-dot and cyan dashed lines correspond to Private, Asset Value policy and RBC.

ing against the rise in asset values substantially decreases the movements in investment (panel (b)), which is highly undesirable, as the first-best response (the cyan-dashed line) calls for a considerable increase in investment to take advantage of higher productivity.

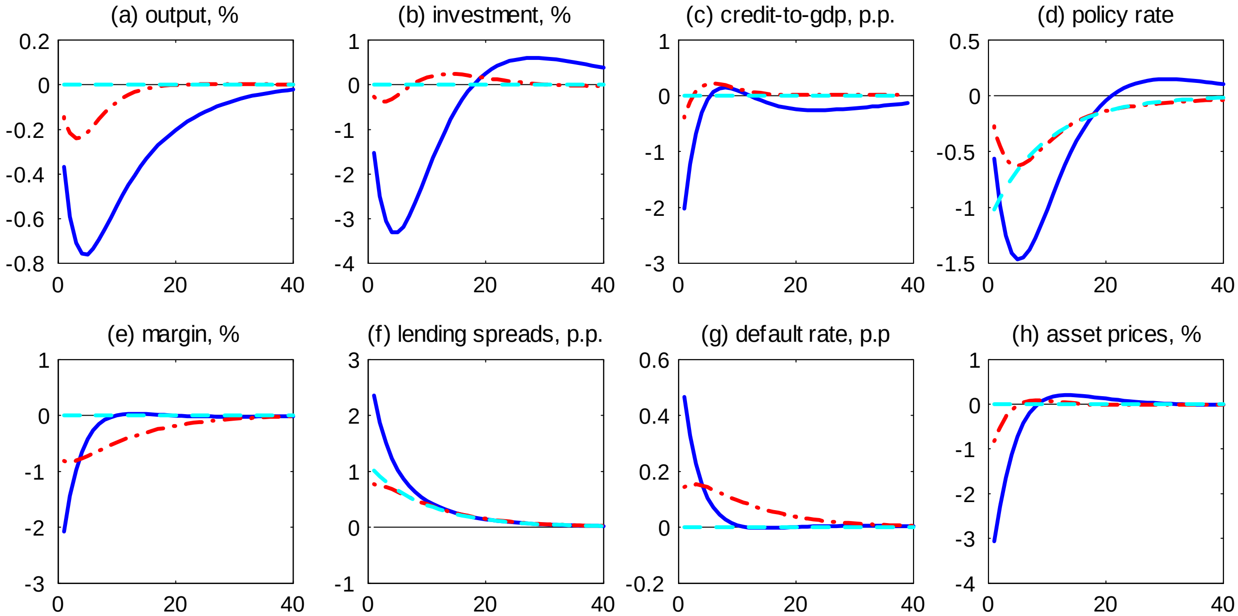
Finally, we consider the response of the economy under a macroprudential rule in which the leverage tax leans against the (annualized) lending spread gap as in

$$\tau_t^M = -0.25[4(R_t^L - R_t)] \tag{30}$$

The sensitivity of the leverage tax to increases in the lending spread is chosen to illustrate the core properties of the rules; we revisit the choice of this sensitivity below. Importantly, the basic structure of the approach is simple – when lending spreads widen, indicating strains in the intermediation sector, lower the tax to encourage lending.

Figure 9 presents impulse responses to the risk premium shock (with each line color/style illustrating the same cases as before). It is clear that leaning against the lending spread shifts the economy's response in the desirable direction in response to this financial dis-

Figure 9: Effect of Lending-Spread Policy Rule on Response to Financial Shock



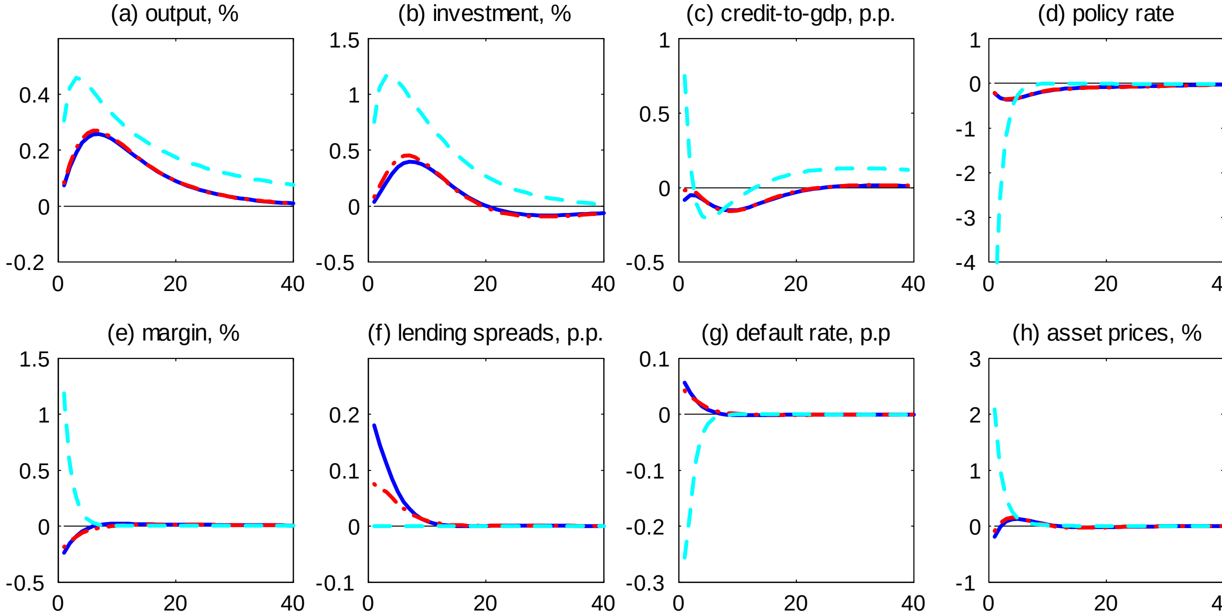
Note: Blue solid, red solid-dot and cyan dashed lines correspond to Private, Lending-Spread policy and RBC.

turbance. By leaning against the increase in the spread, the widening in lending spreads is lessened (panels (f)) and the decline in lending mitigated (panel (c)); as a result, output and investment decline substantially less. Moreover, the spread-based macroprudential rule does not perform badly in response to a productivity disturbance (figure 10). Specifically, such a shock has only modest effects on the lending spread, and hence only calls for a modest adjustment to macroprudential policy; consequently, output and investment are not adversely affected in any significant way (although the policy similarly does not lead these responses to lie closer to the first-best responses).

4.3 "Optimal" Simple Policy Strategies

As we emphasized earlier, the calibrations of the macroprudential rules were made to illustrate the core principles underlying the strategies. In part, the calibrations were informed by a more rigorous approach, albeit one whose results are highly model specific, in which we found the parameters that minimized the deviation of output from its effi-

Figure 10: Effect of Lending-Spread Policy Rule on Response to Productivity Shock



Note: Blue solid, red solid-dot and cyan dashed lines correspond to Private, Lending-Spread policy and RBC.

cient level. Specifically, we computed the parameter in each simple rule that minimized the variance of output around its flexible-price, financially unconstrained value (that is, the "real business cycle" benchmark, which is first-best in our model because we assume that distortions associated with monopolistic competition are addressed by a system of taxes/subsidies that render the flexible-price equilibrium efficient).

We considered three cases – one in which fluctuations solely reflected technology shocks, one in which fluctuations solely reflected risk premium shocks, and one in which the variance of output was due, in equal proportions, to productivity and risk premium shocks. In each case, the variance of the exogenous shock was set to imply that the standard deviation of output was 2.5 percent; for the case with both technology and risk premium shocks, this parameterization also implied that the standard deviation of inflation (at an annual rate) was 2.5 percent.[15] (Note that only the shares of the variance of

[15] Such values are consistent, roughly, with the standard deviation of the output gap used in the Federal Reserve Board's FRB/US model and of price inflation in the United States (as measured by the GDP price index) from 1970-2010.

	Rule	Standard Deviations of		
	Coefficient	Output	Output Gap	Inflation (a.r.)
		Technology Shock Only		
No Policy	0.00	2.50	2.30	1.85
Optimal CY policy	0.31	3.29	2.03	1.65
Calibrated CY policy	0.25	3.17	2.03	1.64
Optimal QK policy	-0.02	2.89	1.96	1.60
Calibrated QK policy	0.25	2.01	2.70	2.20
Optimal SPR policy	287.4	4.17	1.18	1.06
Calibrated SPR policy	1.00	2.61	2.20	1.78
		Risk Premium Shock Only		
No Policy	0.00	2.50	2.50	3.00
Optimal CY policy	1.27	1.68	1.68	2.24
Calibrated CY policy	0.25	1.83	1.83	2.36
Optimal QK policy	5775	0.45	0.45	1.42
Calibrated QK policy	0.25	1.27	1.27	1.96
Optimal SPR policy	1.26	0.39	0.39	1.07
Calibrated SPR policy	1.00	0.60	0.60	1.39
		Technology and Risk Premium Shocks (50/50)		
No Policy	0.00	2.50	2.40	2.49
Optimal CY policy	0.39	2.74	1.91	2.01
Calibrated CY policy	0.25	2.59	1.94	2.03
Optimal QK policy	152	1.34	2.01	1.95
Calibrated QK policy	0.25	1.68	2.11	2.08
Optimal SPR policy	1.31	1.89	1.57	1.44
Calibrated SPR policy	1.00	1.89	1.61	1.60

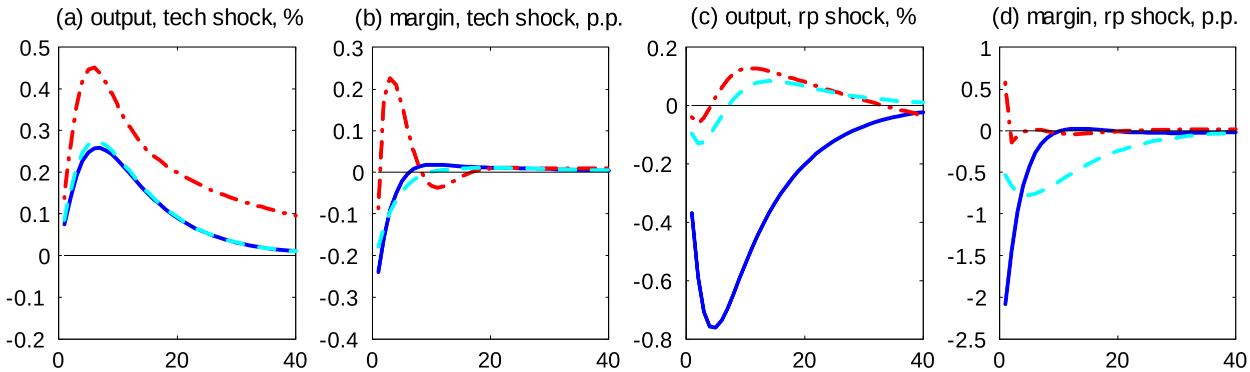
Table 2: Stabilization Effects of Alternative Macroprudential Policy Rules

output attributable to these shocks are relevant for the "optimal policy" the minimizes the variance of the "output gap"). The results are presented in table 2.

Focusing first on the credit-to-output based policy, the optimized coefficient in the simple rule is 0.3-0.4 in either case containing risk-premium shocks – a bit below the illustrative example considered above. This value decreases the distance between output and first-best (as can be seen from the lower standard deviation of the "output gap").

This occurs regardless of the source of the shock. In contrast, the "optimal" coeffi-

Figure 11: Optimized Spread Policy vs Ramsey Policy



Note: Blue solid, red solid-dot and cyan dashed lines correspond to Private, Ramsey Policyand Optimal Spread policy.

cient for the asset-values (QK) policy is not robust to the source of disturbance – the sign switches between technology shocks and risk premium shocks. This occurs for the reason discussed above and shown in the impulse responses: In response to an increase in productivity, leaning against asset prices is counterproductive–indeed, the best policy is to amplify the asset price response.

The spread-based policy has a consistent sign, although the "optimal" response to a technology shock has an implausible magnitude. In response to a risk premium shock or when the source of shocks is balanced, on average, across productivity and risk premium shocks, a simple rule leaning fairly aggressively against spreads does well at lowering the volatility of output, the output gap, and inflation. The finding that a spread-based policy may have good properties reflects the intuition we emphasized earlier – spreads indicate distortions between households' marginal valuation of consumption across periods and that of intermediaries which are inefficient. (This finding is also similar to that in, for example, Cúrdia and Woodford (2009)).

It is also notable that the spread-based policy does a better job at stabilizing inflation then the other policies on a fairly-consistent basis. As our model shares core distortions with those in the New-Keynesian literature on monetary policy, where fluctuations in inflation are a key source of inefficiency, this result further suggests at the good performance of spread-based policies. This result suggests that a more careful examination of the coordination of macroprudential and monetary policy strategies is warranted, which we leave for future research.

Finally, we can comparing these optimal simple rules to the Ramsey optimum discussed earlier, in which policymakers maximize household welfare through direct choice of the equity margin. As shown in figure 11, such an approach (the red, dashed line) is very similar to the optimal simple rule using lending spreads (the black, dashed line) in response to a risk premium shock (panels (c) and (d)); the strategy based on lending spreads is less effective at matching the Ramsey outcome following a technology shock (panels (a) and (b)), although the simple strategy nudges responses in the right direction.

5 Conclusions

We have developed a macroeconomic model in which financial intermediaries optimally choose their leverage – that is, the mix of debt and equity that finance their balance sheet. The leverage choices of intermediaries are privately-optima, but lead to socially inefficient and significant macroeconomic effects of changes in the risk facing intermediaries and the cost of their external funds.

Our analysis demonstrated that policy strategies that lean against asset market values may limit adverse cyclical effects from shifts in financial conditions, but could inhibit investment responses to shifts in the desired capital stock associated with, for example, improvements in productivity. Policies linked to the credit-to-output ratio should only respond very weakly to changes in credit relative to output if it hopes to lower output gap volatility, with only moderate associated gains in welfare in our model framework. Finally, our analysis suggests improved economic stability from a macroprudential policy that leans against credit spreads in lending rates, because such spreads capture shifts in financial frictions and fluctuate little in response to non-financial disturbances such as improvements in productivity.

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Appendices

A Intermediary Value Maximization and First-Order Conditions

The net-worth of an intermediary is given by $N_t = \max\{0, \epsilon_t(1+r_t^A)Q_{t-1}S_{t-1} - (1-m_{t-1})[1+(1-\tau_c)r_t^B]Q_{t-1}S_{t-1}\}$, where the max operator is due to the limited liability condition. Using (1), we simplify the net-worth equation as $N_t = [\max\{\epsilon_t, \epsilon_t^D\} - \epsilon_t^D](1+r_t^A)Q_{t-1}S_{t-1}$. With that in mind, we express the intermediary optimization problem as

$$\begin{aligned}
J_t = \min_{\theta_t} \max_{S_t, m_t, \epsilon_{t+1}^D} \Bigg\{ & \mathbb{E}_t^\epsilon[D_t] + \mathbb{E}_t\left[M_{t,t+1} \cdot \mathbb{E}_{t+1}^\epsilon[V_{t+1}(N_{t+1})]\right] \\
& + \mathbb{E}_t^\epsilon\left[\lambda_t\left(N_t + T_t + \bar{\varphi}(D_t) - [m_t + \tau_t^m(1-m_t)]Q_tS_t\right)\right] \\
& + \theta_t Q_t S_t \mathbb{E}_t\Bigg[M_{t,t+1}\left((1-\eta)\Phi(s_{t+1}^D - \sigma_{t+1}) + \frac{1}{1-\tau_c}\epsilon_{t+1}^D(1-\Phi(s_{t+1}^D))\right)(1+r_{t+1}^A) \\
& - (1-m_t)\left(1 + \frac{\tau_c}{1-\tau_c}\mathbb{E}_t\left(M_{t,t+1}[1-\Phi(s_{t+1}^D)]\right)\right)\Bigg]\Bigg\}
\end{aligned}$$

and

$$V_t(N_t) = \min_{\lambda_t} \max_{D_t}\left\{D_t + \mathbb{E}_t\left[M_{t,t+1} \cdot J_{t+1}\right] + \lambda_t\left(N_t + T_t + \bar{\varphi}(D_t) - [m_t + \tau_t^m(1-m_t)]Q_tS_t\right)\right\}$$

where we modify the flow of funds constraint to include the macroprudential policy τ_t^m. Differentiating these expressions yields the following four optimization conditions:

- FOC for D_t : $\lambda_t = \begin{cases} 1 & \text{if } D_t \geq 0 \\ 1/(1-\varphi) & \text{if } D_t < 0 \end{cases}$
- FOC for S_t : $m_t \mathbb{E}_t^\epsilon[\lambda_t] = \mathbb{E}_t\left\{M_{t,t+1}\mathbb{E}_{t+1}^\epsilon[\lambda_{t+1}(\max\{\epsilon_{t+1}, \epsilon_{t+1}^D\} - \epsilon_{t+1}^D)](1+r_{t+1}^A)\right\}$
- FOC for m_t : $\mathbb{E}_t^\epsilon[\lambda_t] = \theta_t\left[1 + \frac{\tau_c}{1-\tau_c}\mathbb{E}_t\left(M_{t,t+1}[1-F_{t+1}(\epsilon_{t+1}^D)]\right)\right]$
- FOC for ϵ_{t+1}^D : $$\begin{aligned} 0 = \; & \mathbb{E}_t\left\{M_{t,t+1}\mathbb{E}_{t+1}^\epsilon\left[\lambda_t\left(\frac{\partial \max\{\epsilon_{t+1}, \epsilon_{t+1}^D\}}{\partial \epsilon_{t+1}^D} - 1\right)\right]\right\} \\ & + \theta_t \mathbb{E}_t\left\{M_{t,t+1}\left[\left((1-\eta) - \tfrac{1}{1-\tau_c}\right)\epsilon_{t+1}^D f_{t+1}(\epsilon_{t+1}^D)\right](1+r_{t+1}^A)\right\} \\ & + \theta_t \mathbb{E}_t\left\{M_{t,t+1}\left[\frac{1}{1-\tau_c}[1-F_{t+1}(\epsilon_{t+1}^D)](1+r_{t+1}^A) + (1-m_t)\frac{\tau_c}{1-\tau_c}f_{t+1}(\epsilon_{t+1}^D)\right]\right\} \end{aligned}$$

A computational appendix (not for publication, Appendix B) is available; it provides further derivation of analytical expressions used for model simulations.[16]

B Analytical Expressions for Model Simulations (Not for Publication)

This appendix presents some expressions used for model simulations.

[16]Computations performed in Dynare 4.2.1 (Adjemian, Bastani, Juillard, Mihoubi, Perendia, Ratto, and Villemot (2011)). Programs available from the authors.

B.1 Introducing Lognormal Distribution to Intermediary FOCs

As discussed in the text, we assume a lognormal distributions for ϵ_t. Denoting this cumulative distribution function by $\Phi$and probability density function by ϕ, the first-order conditions for $m_t$and ϵ^D_{t+1}become

- FOC for m_t : $(1-\tau^m_t)\mathbb{E}^\epsilon_t[\lambda_t] = \theta_t \left[1 + \frac{\tau_c}{1-\tau_c}\mathbb{E}_t\left(M_{t,t+1}[1-\Phi(s^D_{t+1})]\right)\right]$
- FOC for ϵ^D_{t+1} : $$0 = \theta_t Q_t S_t \mathbb{E}_t\left\{M_{t,t+1}\left[(1-\eta)\frac{\phi(s^D_{t+1}-\sigma_{t+1})}{\sigma_{t+1}\epsilon^D_{t+1}}(1+r^A_{t+1})\right.\right.$$ $$+\frac{1}{1-\tau_c}\left((1-\Phi(s^D_{t+1})) - \frac{\phi(s^D_{t+1})}{\sigma_{t+1}}\right)(1+r^A_{t+1})$$ $$\left.\left.-\ (1-m_t)\left(1+\frac{\tau_c}{1-\tau_c}[1-\Phi(s^D_{t+1})]\right)\right]\right\} + \mathbb{E}_t\left[M_{t,t+1}\cdot\frac{\partial N_{t+1}}{\epsilon^D_{t+1}}V'_{t+1}(N_{t+1})\right]$$

B.2 Bond Pricing Equation

The default condition that defines the trigger level of idiosyncratic shock $\epsilon^D_{t+1}$is obtained by equating the (gross) investment return and the after-tax debt burden

$$\epsilon^D_{t+1}(1+r^A_{t+1}) \equiv (1-m_t)[1+(1-\tau_c)r^B_{t+1}]. \tag{A1}$$

Solving for the borrowing rate yields $r^B_{t+1} = \frac{1}{1-\tau_c}\left[\frac{\epsilon^D_{t+1}(1+r^A_{t+1})}{1-m_t} - 1\right]$. For simplicity, we assume that the bond investor is not subject to interest rate income tax. Using the expression above, the debt payment to the investor can be expressed as

$$\begin{aligned}(1-m_t)(1+r^B_{t+1}) &= (1-m_t)\left\{1+\frac{1}{1-\tau_c}\left[\frac{\epsilon^D_{t+1}(1+r^A_{t+1})}{1-m_t}-1\right]\right\}\\ &= (1-m_t)\left(1-\frac{1}{1-\tau_c}\right)+\frac{1}{1-\tau_c}\epsilon^D_{t+1}(1+r^A_{t+1}).\end{aligned}$$

Substituting the above in the bond pricing equation and rearranging the terms yields

$$\begin{aligned}&(1-m_t)\left[1-\left(1-\frac{1}{1-\tau_c}\right)\mathbb{E}_t\left(M_{t,t+1}\int_{\epsilon^D_{t+1}}^{\infty} dF_{t+1}\right)\right]\\ &= \mathbb{E}_t\left\{M_{t,t+1}\left[\int_0^{\epsilon^D_{t+1}}(1-\eta)\epsilon_{t+1}dF_{t+1} + \int_{\epsilon^D_{t+1}}^{\infty}\frac{1}{1-\tau_c}\epsilon^D_{t+1}dF_{t+1}\right](1+r^A_{t+1})\right\}.\end{aligned}$$

We define the standardized default trigger as

$$s^D_{t+1} \equiv \sigma^{-1}_{t+1}[\log\epsilon^D_{t+1} + 0.5\sigma^2_{t+1}]. \tag{A2}$$

Using this and the property of truncated lognormal distribution, $\int_0^{\epsilon_{t+1}^D} \epsilon_{t+1} dF_{t+1} = \Phi(s_{t+1}^D - \sigma_{t+1})$, one can rewrite the bond pricing equation as

$$(1-m_t)\left[1+\frac{\tau_c}{1-\tau_c}\mathbb{E}_t\left(M_{t,t+1}[1-\Phi(s_{t+1}^D)]\right)\right] \tag{A3}$$
$$= \mathbb{E}_t\left\{M_{t,t+1}\left[(1-\eta)\Phi(s_{t+1}^D-\sigma_{t+1})+\frac{1}{1-\tau_c}\epsilon_{t+1}^D(1-\Phi(s_{t+1}^D))\right](1+r_{t+1}^A)\right\}$$

B.2.1 Expected Shadow Value of Internal Funds

We define the equity issuance trigger ϵ_t^Eas the value of idiosyncratic shock that exactly satisfies the flow of funds constraint when $D_t = 0$, i.e., $0 = \max\{0, \epsilon_t^E - \epsilon_t^D\}(1+r_t^A)Q_{t-1}S_{t-1} + T_t - [m_t + \tau_t^m(1-m_t)]Q_t S_t$, or equivalently

$$\epsilon_t^E \equiv (1-m_{t-1})\frac{1+r_t^B}{1+r_t^A} + [m_t+\tau_t^m(1-m_t)]\frac{Q_t S_t + T_t}{(1+r_t^A)Q_{t-1}S_{t-1}}. \tag{A4}$$

When $\epsilon_t \geq \epsilon_t^E$, $D_t \geq 0$and $\lambda_t = 1$while $\epsilon_t < \epsilon_t^E$, $D_t < 0$and $\lambda_t = 1/(1-\varphi)$. We denote the standardized issuance trigger by

$$s_t^E \equiv \sigma_t^{-1}[\log \epsilon_t^E + 0.5\sigma_t^2]. \tag{A5}$$

We can then compute the expected shadow value of internal funds as a weighted average,

$$\mathbb{E}_t^\epsilon[\lambda_t] = 1 - F_t(\epsilon_t^E) + \frac{1}{1-\varphi}F_t(\epsilon_t^E) = 1-\Phi(s_t^E)+\frac{1}{1-\varphi}\Phi(s_t^E) = 1+\mu\Phi(s_t^E), \quad \mu \equiv \frac{\varphi}{1-\varphi}. \tag{A6}$$

B.2.2 FOC for Investment

Directly differentiating the net-worth equation yields

$$\frac{\partial N_{t+1}}{\partial S_t} = [\max\{\epsilon_{t+1}, \epsilon_{t+1}^D\} - \epsilon_{t+1}^D](1+r_{t+1}^A)Q_t.$$

Applying Benveniste-Scheinkman's formula, $V_t'(N_t) = \lambda_t$, updating one period and combining it with the marginal effect of investment on the net-worth shows that the FOC for $S_t$is equivalent to

$$\begin{aligned}[m_t+\tau_t^m(1-m_t)]\mathbb{E}_t^\epsilon[\lambda_t] &= \mathbb{E}_t\{M_{t,t+1}\lambda_{t+1}[\max\{\epsilon_{t+1},\epsilon_{t+1}^D\}-\epsilon_{t+1}^D](1+r_{t+1}^A)\} \\ &= \mathbb{E}_t\{M_{t,t+1}\mathbb{E}_{t+1}^\epsilon[\lambda_{t+1}(\max\{\epsilon_{t+1},\epsilon_{t+1}^D\}-\epsilon_{t+1}^D)](1+r_{t+1}^A)\}\end{aligned}$$

where the law of iterated expectation is used in the second line. Dividing through by $E_t^\epsilon[\lambda_t]$yields

$$[m_t+\tau_t^m(1-m_t)]\mathbb{E}_t^\epsilon[\lambda_t] = \mathbb{E}_t\left[M_{t,t+1}\frac{\mathbb{E}_{t+1}^\epsilon[\lambda_{t+1}]}{\mathbb{E}_t^\epsilon[\lambda_t]}\left(\frac{\mathbb{E}_{t+1}^\epsilon[\lambda_{t+1}\max\{\epsilon_{t+1},\epsilon_{t+1}^D\}]}{\mathbb{E}_{t+1}^\epsilon[\lambda_{t+1}]}-\epsilon_{t+1}^D\right)(1+r_{t+1}^A)\right]$$

Dividing through by $\tilde{m}_t \equiv m_t + \tau_t^m(1-m_t)$and , substituting (A6) for $E_t^\epsilon[\lambda_t]$and replacing $\epsilon_{t+1}^D(1+r_{t+1}^A)$with $(1-m_t)(1+(1-\tau_c)r_{t+1}^B)$yields

$$1 = \mathbb{E}_t\left\{M_{t,t+1}^B\frac{1}{\tilde{m}_t}\left[1+\tilde{r}_{t+1}^A-(1-m_t)[1+(1-\tau_c)r_{t+1}^B]\right]\right\} \tag{A7}$$

where

$$M^B_{t,t+1} \equiv M_{t,t+1} \frac{\mathbb{E}^{\epsilon}_{t+1}[\lambda_{t+1}]}{\mathbb{E}^{\epsilon}_{t}[\lambda_{t}]} = M_{t,t+1} \frac{1+\mu\Phi(s^E_{t+1})}{1+\mu\Phi(s^E_t)} \tag{A8}$$

and

$$1+\tilde{r}^A_{t+1} \equiv \frac{\mathbb{E}^{\epsilon}_{t+1}[\lambda_{t+1}\max\{\epsilon_{t+1}, \epsilon^D_{t+1}\}]}{\mathbb{E}^{\epsilon}_{t+1}[\lambda_{t+1}]}(1+r^A_{t+1}).$$

To derive an analytical expression for the modified return $1+\tilde{r}^A_{t+1}$, we first rewrite it as

$$1+\tilde{r}^A_{t+1} = \left\{ \frac{\mathbb{E}^{\epsilon}_{t+1}[\lambda_{t+1}\epsilon_{t+1}]}{\mathbb{E}^{\epsilon}_{t+1}[\lambda_{t+1}]} + \frac{\mathbb{E}^{\epsilon}_{t+1}[\lambda_{t+1}\max\{0, \epsilon^D_{t+1} - \epsilon_{t+1}\}]}{\mathbb{E}^{\epsilon}_{t+1}[\lambda_{t+1}]} \right\} (1+r^A_{t+1}).$$

The first term inside the curly bracket can be evaluated as

$$\begin{aligned}
\mathbb{E}^{\epsilon}_{t+1}[\lambda_{t+1}\epsilon_{t+1}] &= \int_0^{\epsilon^E_{t+1}} \frac{\epsilon_{t+1}}{1-\varphi} dF_{t+1} + \int_{\epsilon^E_{t+1}}^{\infty} \epsilon_{t+1} dF_{t+1} \\
&= \frac{1}{1-\varphi}\Phi(s^E_{t+1}-\sigma_{t+1}) + 1 - \Phi(s^E_{t+1}-\sigma_{t+1}) = 1+\mu\Phi(s^E_{t+1}-\sigma_{t+1}).
\end{aligned}$$

Similarly, we can derive the analytical expression for the second term as

$$\mathbb{E}^{\epsilon}_{t+1}[\lambda_{t+1}\max\{0, \epsilon^D_{t+1}-\epsilon_{t+1}\}] = \int_0^{\epsilon^D_{t+1}} \frac{\epsilon^D_{t+1}-\epsilon_{t+1}}{1-\varphi} dF_{t+1} = \frac{1}{1-\varphi}[\epsilon^D_{t+1}\Phi(s^D_{t+1}) - \Phi(s^D_{t+1}-\sigma_t)]$$

where we use the fact that $\lambda_{t+1} = 1/(1-\varphi)$when $\epsilon_{t+1} \leq \epsilon^D_{t+1} < \epsilon^E_{t+1}$. Combining the two expressions above with (A6) yields

$$1+\tilde{r}^A_{t+1} \equiv \left\{ \frac{1+\mu\Phi(s^E_{t+1}-\sigma_{t+1})}{1+\mu\Phi(s^E_{t+1})} + \frac{\epsilon^D_{t+1}\Phi(s^D_{t+1}) - \Phi(s^D_{t+1}-\sigma_t)}{(1-\varphi)[1+\mu\Phi(s^E_{t+1})]} \right\} (1+r^A_{t+1}) \tag{A9}$$

B.2.3 FOC for margin

Simply substituting in (A6) yields

$$(1-\tau^m_t)[1+\mu\Phi(s^E_t)] = \theta_t \left[1 + \frac{\tau_c}{1-\tau_c}\mathbb{E}_t\left(M_{t,t+1}[1-\Phi(s^D_{t+1})] \right) \right] \tag{A10}$$

B.2.4 FOC for default trigger

To transform the FOC for ϵ^D_{t+1}into a form that is more convenient for computation, we need to evaluate the following differentiation

$$\begin{aligned}
\mathbb{E}_t\left[M_{t,t+1} \cdot \frac{\partial N_{t+1}}{\epsilon^D_{t+1}} V'_{t+1}(N_{t+1}) \right] &= \mathbb{E}_t\left[M_{t,t+1} \frac{\partial \max\{\epsilon_{t+1}, \epsilon^D_{t+1}\} - \epsilon^D_{t+1}}{\partial \epsilon^D_{t+1}} (1+r^A_{t+1}) Q_t S_t V'_{t+1}(N_{t+1}) \right] \\
&= \mathbb{E}_t\left\{ M_{t,t+1}\mathbb{E}^{\epsilon}_{t+1}\left[\lambda_{t+1}\left(\frac{\partial \max\{\epsilon_{t+1}, \epsilon^D_{t+1}\}}{\partial \epsilon^D_{t+1}} - 1 \right) \right] (1+r^A_{t+1}) Q_t S_t \right\}
\end{aligned}$$

where we used the envelope condition $V'_{t+1}(N_{t+1}) = \lambda_{t+1}$and the law of iterated expectation in the third line. To that end, first, we think of $\max\{\epsilon_{t+1}, \epsilon^D_{t+1}\}$as a function of a 'variable' $\epsilon^D_{t+1}$for a given 'parameter' $\epsilon_{t+1}$and take a differentiation of $\max\{\epsilon_{t+1}, \epsilon^D_{t+1}\}$with respect to ϵ^D_{t+1}as follows

$$\frac{\partial \max\{\epsilon_{t+1}, \epsilon^D_{t+1}\}}{\partial \epsilon^D_{t+1}} = \begin{cases} 0 & \text{if } \epsilon^D_{t+1} \leq \epsilon_{t+1} \\ 1 & \text{if } \epsilon^D_{t+1} > \epsilon_{t+1} \end{cases}.$$

Second, we now think of the above as a function a 'variable' $\epsilon_{t+1}$for a given 'parameter' ϵ^D_{t+1}since we now need to integrate this expression over the support of ϵ_{t+1}. Reminding that the shadow value is equal to $1/(1-\varphi)$when $\epsilon_{t+1} \leq \epsilon^D_{t+1} < \epsilon^E_{t+1}$, one can see immediately that

$$\mathbb{E}^{\epsilon}_{t+1}\left[\lambda_{t+1}\frac{\partial \max\{\epsilon_{t+1}, \epsilon^D_{t+1}\}}{\partial \epsilon^D_{t+1}}\right] = \int_0^{\epsilon^D_{t+1}} 1 \cdot \frac{dF_{t+1}}{1-\varphi} = \frac{\Phi(s^D_{t+1})}{1-\varphi}.$$

Using this expression, we can rewrite the FOC for ϵ^D_{t+1}as

$$\begin{aligned} 0 \;=\; & \theta_t \mathbb{E}_t\left\{M_{t,t+1}\left[(1-\eta)\frac{\phi(s^D_{t+1}-\sigma_{t+1})}{\sigma_{t+1}\epsilon^D_{t+1}} + \frac{1}{1-\tau_c}\left([1-\Phi(s^D_{t+1})] - \frac{\phi(s^D_{t+1})}{\sigma_{t+1}}\right)\right](1+r^A_{t+1})\right. \\ & \left. -(1-m_t)\left(1+\frac{\tau_c}{1-\tau_c}[1-\Phi(s^D_{t+1})]\right)\right]\right\} + \mathbb{E}_t\left\{M_{t,t+1}\left[\frac{\Phi(s^D_{t+1})}{1-\varphi} - [1+\mu\Phi(s^E_{t+1})]\right](1+r^A_{t+1})\right\} \end{aligned} \tag{A11}$$

B.3 Household Optimization Conditions

We denote the total outstanding of intermediary debts by B_t. In equilibrium, $B_t = \int[1-m_{t-1}(i)]$ $Q_{t-1}S_t(i)di = (1-m_{t-1})Q_{t-1}K_t$, where $i \in [0,1]$is an index for intermediary. The last equality is due to the symmetric equilibrium and the no-arbitrage condition mentioned in the main text. The realized aggregate return on intermediary debts, denoted by $1+\tilde{r}^B_t$, is given by

$$1+\tilde{r}^B_t \equiv \left[\int_0^{\epsilon^D_t}(1-\eta)\epsilon_t dF_t + \int_{\epsilon^D_t}^{\infty}(1-m_t)(1+r^B_t)dF_t\right]\frac{1+r^A_t}{1-m_{t-1}}.$$

Using $1+\tilde{r}^B_t$, we can express the household's budget constraint as

$$0 = W_tH_t + (1+\tilde{r}^B_t)B_t - B_{t+1} - P_tC_t - \int_0^1 P^S_t(i)S^F_{t+1}(i)di + \int_0^1[\max\{D_t(i),0\} + P^S_{t-1,t}(i)]S^F_t(i)di$$

where $W_t$is a nominal wage rate, $H_t$is labor hours, and $S^F_t(i)$is the number of shares outstanding at time t. $P^S_{t-1,t}(i)$is the time $t$value of shares outstanding at time $t-1$.[17] $P^S_t(i)$is the ex-dividend value of equity at time t. The two values are related by the following accounting identity, $P^S_t(i) = P^S_{t-1,t}(i) + X_t(i)$where $X_t(i)$is the value of new shares issued at time t. The costly equity finance

[17]In our actual computation, we assume that the bankruptcy cost $\eta\Phi(s^D_t - \sigma_t)(1+r^A_t)Q_{t-1}K_t$ is transferred back to the household. This is to focus on the implications of the debt market friction through the FOCs of the intermediaries. Our main conclusions are not affected by this assumption.

assumption adopted for the financial intermediary implies that $X_t(i) = -(1-\varphi)\min\{D_t(i), 0\}$. Using the last two expressions, one can see that the budget constraint is equivalent to

$$0 = W_t H_t + (1+\tilde{r}_t^B)B_t - B_{t+1} - P_t C_t - \int_0^1 P_t^S(i) S_{t+1}^F(i) di$$
$$+ \int_0^1 [\max\{D_t(i), 0\} + (1-\varphi)\min\{D_t(i), 0\} + P_t^S(i)] S_t^F(i) di.$$

The household's FOCs for asset holdings are summarized by two conditions,

- FOC for $B_{t+1}: 1 = \mathbb{E}_t\left[M_{t,t+1}(1+\tilde{r}_{t+1}^B)\right]$
- FOC for $S_{t+1}^F(i): 1 = \mathbb{E}_t\left[M_{t,t+1}\frac{\mathbb{E}_{t+1}^{\epsilon}[\max\{D_{t+1},0\}] + (1-\varphi)\mathbb{E}_{t+1}^{\epsilon}[\min\{D_{t+1},0\}] + P_{t+1}^S}{P_t^S}\right]$.

where $E_{t+1}^{\epsilon}[\max\{D_{t+1}, 0\}] = \int_0^1 \max\{D_t(i), 0\} di$and $E_{t+1}^{\epsilon}[\min\{D_{t+1}, 0\}] = \int_0^1 \min\{D_t(i), 0\} di$.

It is straightforward to verify that the FOC for intermediary debts is equivalent to the participation constraint of the household in the intermediary debt contract. In our actual computation, we use the following analytical expressions to compute the return on equity.

$$1 = \mathbb{E}_t\left[M_{t,t+1}\frac{D_{t+1}^+ - (1-\varphi)D_{t+1}^- + P_{t+1}^S}{P_t^S}\right] \tag{A12}$$

where using the flow of funds constraint for intermediaries, one can show that

$$D_{t+1}^+ \equiv \mathbb{E}_t^{\epsilon}[\max\{D_t, 0\}] = \{1 - \Phi(s_t^E - \sigma_t) - \epsilon_t^D[1 - \Phi(s_t^E)\}R_t^A Q_{t-1}S_{t-1}$$
$$- [1 - \Phi(s_t^E)]\{[m_t + \tau_t^m(1-m_t)]Q_t S_t - T_t\} \tag{A13}$$

$$D_{t+1}^- \equiv -\mathbb{E}_t^{\epsilon}[\max\{D_t, 0\}] = -1/(1-\varphi)[\Phi(s_t^E - \sigma_t) - \Phi(s_t^D - \sigma_t) - \epsilon_t^D \Phi(s_t^E)]R_t^A Q_{t-1}S_{t-1}$$
$$+ \Phi(s_t^E)/(1-\varphi)\{[m_t + \tau_t^m(1-m_t)]Q_t S_t - T_t\} \tag{A14}$$

B.4 Derivation of Steady State

B.4.1 Equilibrium Rates of Return

Using a numerical root finder, one can jointly solve for ϵ^D, r^B, r^K, ϵ^E, s^E, s^D, and $m$that satisfy the followings:

$$1 - m = \left\{(1-\eta)\Phi(s^D - \sigma) + \frac{\tau_c}{1-\tau_c}\epsilon^D[1 - \Phi(s^D)]\right\}\frac{(1-\tau_c)r^K + 1 - (1-\tau_c)\delta}{1 - \beta(1 - \tau_c/(1-\tau_c))[1 - \Phi(s^D)]}$$

$$1 = \frac{\beta}{m + \tau^m(1-m)}\left\{\left[\frac{1 + \mu\Phi(s^E - \sigma)}{1 + \mu\Phi(s^E)} + \frac{\epsilon^D\Phi(s^D) - \Phi(s^D - \sigma)}{1 - \varphi + \varphi\Phi(s^E)}\right]\right.$$
$$\left.\times[(1-\tau_c)r^K + 1 - (1-\tau_c)\delta] - (1-m)[1 + (1-\tau_c)r^B]\right\}$$

$$0 = \theta\beta\left[(1-\eta)\frac{\phi(s^D - \sigma)}{\sigma\epsilon^D} + \frac{1}{1-\tau_c}\left([1 - \Phi(s^D)] - \frac{\phi(s^D)}{\sigma}\right)\right]$$
$$\times[(1-\tau_c)r^K + 1 - (1-\tau_c)\delta] - \beta(1-m)\left(1 + \frac{\tau_c}{1-\tau_c}[1 - \Phi(s^D)]\right)$$
$$+ \beta\left[\frac{\Phi(s^D)}{1-\varphi} - [1 + \mu\Phi(s^E)]\right][(1-\tau_c)r^K + 1 - (1-\tau_c)\delta]$$

$$
\begin{aligned}
\epsilon^D &= (1-m)\frac{1+(1-\tau_c)r^B}{(1-\tau_c)r^K+1-(1-\tau_c)\delta}, \\
\epsilon^E &= \epsilon^D + \frac{[m+\tau^m(1-m)](1+t)}{(1-\tau_c)r^K+1-(1-\tau_c)\delta}, \\
s^E &= \frac{1}{\sigma}(\log \epsilon^E + \sigma^2), \\
s^D &= \frac{1}{\sigma}(\log \epsilon^D + \sigma^2)
\end{aligned}
$$

where the last three equations are the steady state versions of (A3), (A7) and (A11).

B.4.2 Levels

Once the equilibrium returns are obtained, we can analytically solve for endogenous quantities. From the assumption that subsidies eliminate the distortions associated with monopolistic competition, $p^M = 1$. From the FOC for capital rental decision, we have $y/k = r^K/[(1-\alpha)p^M] = \varepsilon r^K/[(1-\alpha)(\varepsilon-1)] \equiv \rho_{y/k}$. Substituting this in the resource constraint of the steady state, we can compute the consumption/capital ratio as $c/k = y/k - i/k = \rho_{y/k} - \delta$. By dividing the production function by k, we have $y/k = (h/k)^\alpha$or equivalently, $h/k = (y/k)^{1/\alpha} = \rho_{y/k}^{1/\alpha}$. Hence, $h/c = \rho_{y/k}^{1/\alpha}/[\rho_{y/k} - \delta]$, or equivalently, $h = \{\rho_{y/k}^{1/\alpha}/[\rho_{y/k} - \delta]\}c$. The wage curve in the steady state is given by $\zeta h^\nu = w(1-a\beta)(1-a)^{-\gamma}c^{-\gamma}$. From the FOC for the labor hours, we have $w = \alpha p^M y/h = \alpha p^M (y/k)/(h/k) = \alpha p^M \rho_{y/k}^{(\alpha-1)/\alpha}$. Substituting this in the wage curve yields $\zeta h^\nu = \alpha p^M \rho_{y/k}^{(\alpha-1)/\alpha}(1-a\beta)(1-a)^{-\gamma}c^{-\gamma}$Substituting $h = \{\rho_{y/k}^{1/\alpha}/[\rho_{y/k} - \delta]\}c$in the above yields $c = [\zeta^{-1}\alpha p^M(1-a\beta)(1-a)^{-\gamma}\rho_{y/k}^{(\alpha-1-\nu)/\alpha}\left(\rho_{y/k} - \delta\right)^\nu]^{1/(\gamma+\nu)}$The levels of the other variables can be computed straightforwardly from this.