Uncertainty Shocks
In A Model Of Effective Demand

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Preliminary
Can Higher Uncertainty Reduce Overall Economic Activity?

“I believe that overall uncertainty is a large drag on the economic recovery.” – Narayana Kocherlakota, November 22, 2010

Can increased uncertainty generate simultaneous drops in output, consumption, investment, and hours worked?

Study uncertainty shocks in representative-agent DSGE model

Uncertainty shock is exogenous increase in volatility of aggregate shocks
Transmission of Uncertainty to Macroeconomy

**Increased uncertainty in typical partial equilibrium models**

- Reduces consumption through precautionary saving
- Decreases investment via real options effects

**Intuitive economy-wide effects of increased uncertainty**

- Reduction in consumption and investment
- Fall in output via $Y = C + I$
- Decrease in hours through $Y = F(K, ZN)$
From Intuition to Model

Does the partial-equilibrium intuition hold in general equilibrium?

Use one-sector closed economy representative-agent framework

Standard flexible price models struggle to generate business-cycle comovements in response to changes in uncertainty

Countercyclical markups via sticky prices can restore comovement

Data-driven quantitative exercise in reasonably calibrated DSGE model
Model Summary

Representative-agent New-Keynesian sticky price model with capital


Household holds equity shares and one-period risk-free bonds

Epstein-Zin preferences over streams of consumption and leisure

1st & 2nd moment shocks to household discount factors (demand shocks)

\[
\ln(a_t) = \rho_a \ln(a_{t-1}) + \sigma_a^a \varepsilon_t^a \quad \varepsilon_t^a \sim N(0, 1)
\]

\[
\ln(\sigma_t^a) = (1 - \rho_{\sigma^a})\ln(\sigma^a) + \rho_{\sigma^a} \ln(\sigma_{t-1}^a) + \sigma_{\sigma^a}^a \varepsilon_t^\sigma_a \quad \varepsilon_t^\sigma_a \sim N(0, 1)
\]
Model Summary

Firms own capital stock, issue debt, & pay dividends

Quadratic cost of adjusting nominal price

Adjustment costs to changing rate of investment

Monetary authority follows standard Taylor rule

1st & 2nd moment shocks to technology

Stochastic process for technology

\[
\ln(Z_t) = \rho_z \ln(Z_{t-1}) + \sigma_t \varepsilon_t \sim N(0, 1)
\]

\[
\ln(\sigma_t) = (1 - \rho_{\sigma z}) \ln(\sigma^{\varepsilon}) + \rho_{\sigma z} \ln(\sigma_{t-1}^{\varepsilon}) + \sigma^{\varepsilon} \varepsilon_t \sim N(0, 1)
\]
Model Calibration and Solution

Calibrate model parameters to estimates of Ireland (2003, 2010)

Examine impulse responses of uncertainty shocks under two cases:

1. Flexible Prices
2. Sticky Prices

Solve model using 3rd-order approximation to policy functions
Flexible Price Model Intuition

Under flexible prices, can increased uncertainty generate simultaneous drops in output and all of its components?

Increased uncertainty $\Rightarrow$ Precautionary saving & lowers $C_t$

Precautionary saving $\Rightarrow$ Precautionary working

$Y_t = F(K_t, Z_t N_t)$ $\Rightarrow$ Increase in total output

$Y_t = C_t + I_t$ $\Rightarrow$ Investment must rise

Increased uncertainty lowers $C_t$, but raises $Y_t$, $I_t$, and $N_t$
Flexible Price Model Intuition

\[ W_t \]
\[ N_t \]
\[ LD (K_t, Z_t) \]
\[ LS (\lambda_t) \]
Flexible Price Model Intuition

\[ W_t, N_t, LD(K_t, Z_t), LS(\lambda_t) \]
Second Moment Technology Shock with Flexible Prices

Output

Consumption

Investment

Markup

Hours Worked

Volatility of Technology Shock

Flexible Prices
Second Moment Preference Shock with Flexible Prices

Output

Consumption

Investment

Markup

Hours Worked

Volatility of Preference Shock

Flexible Prices
Effects of Uncertainty with Demand-Determined Output

How to restore primacy of reasoning from $Y_t = C_t + I_t$?

Examine uncertainty shocks in model where output is demand-determined in the short run (the *Effective Demand* of the title)

Introduce endogenously-varying markups via nominal price rigidity
Sticky Price Model Intuition

Increased uncertainty $\Rightarrow$ Precautionary saving & working

Precautionary working $\Rightarrow$ Higher markup which lowers labor demand

Labor demand may fall enough to reduce $N_t$ and $Y_t$

Can lead to further reduction in $C_t$ and decline in $I_t$
Sticky Price Model Intuition

\[ L^D(K_t, Z_t, \mu_t) \]

\[ L^S(\lambda_t) \]

\[ L^S(\lambda_t \uparrow) \]
Sticky Price Model Intuition

\[ W_t \]

\[ N_t \]

\[ LD(K_t, Z_t, \mu_t) \]

\[ LS(\lambda_t) \]

\[ LS(\lambda_t \uparrow) \]
Second Moment Technology Shock with Sticky Prices

Output

Consumption

Investment

Markup

Hours Worked

Volatility of Technology Shock

Flexible Prices

Sticky Prices
Second Moment Preference Shock with Sticky Prices

Output, Consumption, Investment, Markup, Hours Worked, Volatility of Preference Shock

Flexible Prices vs. Sticky Prices

Percentage Points

Volatility of Preference Shock

Hours Worked

Markup

Volatility of Preference Shock

Flexible Prices
Sticky Prices
Calibrating Magnitude of Uncertainty Shocks

Increased uncertainty can reduce $Y$, $C$, $I$, & $N$ under sticky prices

What is a reasonable-sized uncertainty shock in the data?

What does model predict for a reasonable-sized uncertainty shock?

Use VIX as measure of ex ante aggregate uncertainty

VIX is forward-looking measure of S&P 500 return volatility
VIX & VIX-Implied Uncertainty Shocks

Estimate reduced-form AR(1) model for quarterly VIX $V_t^D$

$$\ln(V_t^D) = (1 - \rho_V)\ln(V_t^D) + \rho_V \ln(V_{t-1}^D) + \sigma^D \varepsilon_t^V$$

$\varepsilon_t^V \triangleq$ VIX-implied uncertainty shock
Model-Implied VIX

Use 3rd-order perturbation method to generate model-implied VIX

Household Euler equation for equity holdings

\[
\frac{P_t^E}{P_t} = E_t \left\{ M_{t+1} \left( \frac{D_{t+1}^E}{P_{t+1}} + \frac{P_{t+1}^E}{P_{t+1}} \right) \right\}
\]

Return on equity

\[
R_{t+1}^E \triangleq \frac{D_{t+1}^E/P_{t+1} + P_{t+1}^E/P_{t+1}}{P_t^E/P_t}
\]

Model-implied VIX

\[
V_t^M \triangleq 100 \times \sqrt{4 \times \text{Var}_t (R_{t+1}^E)}
\]
Uncertainty Shock Calibration

1 standard deviation VIX-implied uncertainty shock in data
⇒ Raises level of the VIX to 24.4% from sample average of 20.5%

Calibrate size of uncertainty shocks in model to match VIX-implied results

Match average VIX & equity premium using risk aversion & leverage ratio

Model-implied VIX is approximately AR(1) in volatility shocks

Calibrate each volatility shock process to match observed VIX fluctuations
Uncertainty Shock Calibration

**Output**: Percentage Points

**Consumption**: Percentage Points

**Investment**: Percentage Points

**Hours Worked**: Percentage Points

**Volatility of Shock**: Percentage Points

**VIX**: Annualized Equity Return Volatility

Models:
- Technology
- Preference

Data: Red dashed line
Quantitative Implications of Uncertainty Shocks

Did uncertainty play a role in the Great Recession?

3+ standard deviation VIX-implied uncertainty shock in Fall of 2008

Little evidence of change in the ex post volatility of technology shocks

3 standard deviation uncertainty shock to demand in model
⇒ Peak drop in output of 1.8 percentage points

Results suggest uncertainty contributed to severity of Great Recession
Uncertainty or Financial Market Disruptions?

A false choice

A financial market disruption is an event, which can have multiple effects

Most analysis has focused on first-moment effects
  (higher cost of capital, tighter borrowing constraints, etc.)

We analyze likely effects of the concurrent rise in uncertainty

Increased uncertainty might also be due to financial disruptions

Modeling 2nd-moment shocks complements other work on crisis
Conclusions

Uncertainty can decrease $Y$, $C$, $I$, & $N$ under reasonable assumptions.

Decline in output and its components is quantitatively significant.

Modeling 2nd-moment shocks complements other work on crisis.
Additional Details
Representative Household (I)

Household maximizes lifetime utility from consumption and leisure

\[ V_t = \max \left[ a_t \left( C_t (1 - N_t)^\eta \right)^{\frac{1-\sigma}{\theta_V}} + \beta \left( E_t V_{t+1}^{1-\sigma} \right)^{\frac{1}{\theta_V}} \right]^{\frac{\theta_V}{1-\sigma}} \]

\[ \psi \triangleq \text{IES} \quad \theta_V \triangleq \frac{1 - \sigma}{1 - \frac{1}{\psi}} \]

Household stochastic discount factor

\[ M_{t+1} = \beta \frac{a_{t+1}}{a_t} \left( \frac{C_{t+1} (1 - L_{t+1})^\eta}{C_t (1 - L_t)^\eta} \right)^{\frac{1-\sigma}{\theta_V}} \frac{C_t}{C_{t+1}} \left( \frac{V_{t+1}}{E_t \left[ V_{t+1}^{1-\sigma} \right]} \right)^{1-\frac{1}{\theta_V}} \]
Representative Household (II)

Household budget constraint

\[ C_t + \frac{P_t^E}{P_t} S_{t+1} + \frac{1}{R_t^R} B_{t+1} = W_t N_t + \left( \frac{D_t^E}{P_t} + \frac{P_t^E}{P_t} \right) S_t + B_t \]

Stochastic process for preference (demand) shocks

\[ \ln(a_t) = \rho_a \ln(a_{t-1}) + \sigma_a^a \varepsilon_t^a \quad \varepsilon_t^a \sim N(0, 1) \]

\[ \ln(\sigma_t^a) = (1 - \rho_{\sigma^a}) \ln(\sigma^a) + \rho_{\sigma^a} \ln(\sigma_{t-1}^a) + \sigma_{\sigma^a}^a \varepsilon_t^\sigma^a \quad \varepsilon_t^\sigma^a \sim N(0, 1) \]
Representative Goods-Producing Firm (I)

Firm owns capital stock $K_t(i)$ & employs labor $N_t(i)$

Quadratic cost of changing nominal price $P_t(i)$

$$\frac{\phi_P}{2} \left[ \frac{P_t(i)}{\Pi P_{t-1}(i)} - 1 \right]^2 Y_t$$

Cobb-Douglas production function subject to fixed costs

$$Y_t(i) = K_t(i)^\alpha [Z_t N_t(i)]^{1-\alpha} - \Phi$$

Adjustment costs to changing rate of investment

$$K_{t+1}(i) = (1 - \delta)K_t(i) + I_t(i) \left( 1 - \frac{\phi I}{2} \left( \frac{I_t(i)}{I_{t-1}(i)} - 1 \right)^2 \right)$$
Representative Goods-Producing Firm (II)

Firm $i$ chooses $N_t(i)$, $K_{t+1}(i)$, $I_t(i)$, and $P_t(i)$ to maximize cash flows

$$\max E_t \left\{ \sum_{s=0}^{\infty} M_{t+s} \left( \frac{D_{t+s}(i)}{P_{t+s}} \right) \right\}$$

Definition of firm cash flows

$$\frac{D_t(i)}{P_t} = \left[ \frac{P_t(i)}{P_t} \right]^{1-\theta} Y_t - \frac{W_t}{P_t} N_t(i) - I_t(i) - \frac{\phi P}{2} \left[ \frac{P_t(i)}{\Pi P_{t-1}(i)} - 1 \right]^2 Y_t$$

Firm issues 1-period bonds to finance fraction of capital stock each period

$$B_{t+1}(i) = \nu K_{t+1}(i)$$

Bonds earn 1-period real risk-free rate $R^R_t$
Representative Goods-Producing Firm (III)

Total cash flows divided between payments to debt or equity

Payments to equity

\[ \frac{D_t^E(i)}{P_t} = \frac{D_t(i)}{P_t} - v \left( K_t(i) - \frac{1}{R_t^R} K_{t+1} \right) \]

Leverage does not affect firm value or optimal firm decisions

(Modigliani & Miller (1963) theorem holds)

Equity becomes more volatile with leverage
Aggregation

All users of final output assemble the final good $Y_t$ using the range of varieties $Y_t(i)$ in a CES aggregator

$$Y_t = \left[ \int_0^1 Y_t(i) \frac{\theta_{\mu} - 1}{\theta_{\mu}} \, di \right]^{\frac{\theta_{\mu}}{\theta_{\mu} - 1}}$$

Aggregate production function

$$Y_t = K_t^{\alpha} (Z_t N_t)^{1-\alpha} - \Phi$$

Stochastic process for technology

$$\ln(Z_t) = \rho_z \ln(Z_{t-1}) + \sigma_z \varepsilon_t^{z} \quad \varepsilon_t^{z} \sim N(0, 1)$$

$$\ln(\sigma_t^{z}) = (1 - \rho_{\sigma z}) \ln(\sigma^{z}) + \rho_{\sigma z} \ln(\sigma_{t-1}^{z}) + \sigma_{\sigma}^{z} \varepsilon_t^{\sigma_z} \quad \varepsilon_t^{\sigma_z} \sim N(0, 1)$$
Monetary Policy & National Income Accounting

Nominal interest rate rule

\[ \ln(R_t) = \rho_R \ln(R_{t-1}) + (1 - \rho_R) (\ln(R) + \rho_\pi \ln(\Pi_t/\Pi) + \rho_y \ln(Y_t/Y_{t-1})) \]

National income accounting

\[ Y_t = C_t + I_t + \frac{\phi_P}{2} \left( \frac{\Pi_t}{\Pi} - 1 \right)^2 Y_t \]
Second Moment Technology Shock with Sticky Prices
Second Moment Preference Shock with Sticky Prices

- **Inflation**
  - Percentage Points
  - Flexible Prices: Blue
  - Sticky Prices: Red

- **Nominal Interest Rate**
  - Percentage Points
  - Flexible Prices: Blue
  - Sticky Prices: Red

- **Real Interest Rate**
  - Percentage Points
  - Flexible Prices: Blue
  - Sticky Prices: Red

- **Real Wage**
  - Percentage Points
  - Flexible Prices: Blue
  - Sticky Prices: Red

- **Rental Rate of Capital**
  - Percentage Points
  - Flexible Prices: Blue
  - Sticky Prices: Red

- **Volatility of Preference Shock**
  - Percentage Points
  - Flexible Prices: Blue
  - Sticky Prices: Red
VIX & VIX-Implied Uncertainty Shocks

Estimate reduced-form AR(1) model for quarterly VIX $V^D_t$

$$\ln(V^D_t) = (1 - \rho_V)\ln(V^D) + \rho_V\ln(V^D_{t-1}) + \sigma^D \varepsilon^V_t, \quad \varepsilon^V_t \sim N(0, 1)$$

Results: $V^D = 20.4\%$  $\rho_V = 0.83$  $\sigma^V = 0.19$

$\varepsilon^V_t$: VIX-implied uncertainty shock
VIX & VIX-Implied Uncertainty Shocks

VIX

VIX–Implied Uncertainty Shocks

Annualized S&P 500 Return Volatility

VIX


Standard Deviations

Uncertainty Shocks, Monetary Policy, & ZLB

Monetary authority follows conventional active interest rate rule

Helps stabilize economy by offsetting 2nd moment preference shock

What if monetary authority is constrained by zero lower bound on nominal interest rates?

Preliminary results
Second Moment Preference Shock at ZLB

Output

Consumption

Investment

Markup

Hours Worked

Volatility of Preference Shock

Unconstrained

Constrained