Ambiguous Business Cycles

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Motivation

- Role of uncertainty shocks in business cycles?
- usually: uncertainty = risk
  ⇒ agents confident in probability assessments
  ⇒ shocks to uncertainty = shocks to volatility
- this paper: uncertainty = risk + ambiguity (Knightian uncertainty)
  ⇒ allows for lack of confidence in prob assessments
  ⇒ shocks to uncertainty can be shocks to confidence
Overview

- Standard business cycle model with ambiguity aversion
  - recursive multiple priors preferences
  - ambiguity about mean aggregate productivity
  ⇒ 1st order effects of uncertainty

- Methodology
  - study uncertainty shocks with 1st order approximation
  - simple estimation strategy based on linearization
  - motivate & bound set of priors by concern w/ nonstationarity

- Properties
  - ambiguity shocks work like “unrealized” news shocks with bias.
  - in medium scale DSGE model estimated on US data, ambiguity shocks
    ★ generate comovement and account for $> \frac{1}{2}$ of fluctuations in $Y, C, I, H$.
    ★ imply countercyclical asset premia.
Literature


Ambiguity aversion & preferences

- $S =$ state space
  - one element $s \in S$ realized every period
  - histories $s^t \in S^t$

- Consumption streams $C = (C_t (s^t))$

- Recursive multiple-priors utility (Epstein and Schneider (2003))

$$U_t (C; s^t) = u (C_t (s^t)) + \beta \min_{p \in P_t (s^t)} E^p [U_{t+1} (C; s^{t+1})],$$

- Primitives:
  - felicity $u$ (possibly over multiple goods) & discount factor $\beta$
  - one-step-ahead belief sets $P_t (s^t)$ – size captures (lack of) confidence

- Why this functional form?
  - preference for known odds over unknown odds (Ellsberg Paradox)
  - formally, weaken Independence Axiom
A stylized business cycle model with ambiguity

- Representative agent with recursive multiple priors utility.
- Felicity from consumption, hours
  \[ u(C_t, N_t) = \frac{C_t^{1-\gamma}}{1-\gamma} - \beta N_t \]
- Output \( Y_t \) produced by
  \[ Y_t = Z_t N_{t-1} \]
- Labor chosen one period in advance
- Belief sets that enter utility
  - specify ambiguity about exogenous productivity
  - beliefs about endogenous variables derived from “structural knowledge” of economy
  - true TFP process: iid lognormal with \( E[Z_t] = 1 \), \( var(\log Z_t) = \sigma_z^2 \)
Belief set: time variation in ambiguity

- Agents experience changes in confidence, described by process $a_t$
  \[ \Rightarrow \text{Representation of one-step-ahead belief set } \mathcal{P}_t \]
  \[ \log Z_{t+1} = \mu_t - \frac{1}{2} \sigma_z^2 + \sigma_z \varepsilon_{z,t+1} \]
  \[ \mu_t \in [-a_t, a_t] \]

- Examples for evolution of (lack of) confidence $a_t$
  1. Linear, homoskedastic law of motion
     \[ a_t = (1 - \rho_a) \bar{a} + \rho_a a_{t-1} + \varepsilon_{a,t} \]
     Interpretation: intangible information affects confidence
  2. Feedback from realized volatility
     \[ a_t = \sqrt{2 \eta \sigma_z,t} \]
     Interpretation: observed turbulence lowers confidence

Follows if $\mathcal{P}_t$ is "constant entropy" ball around true DGP:

\[ \mu_t \in [-a_t, a_t] \Leftrightarrow R_t = \frac{\mu_t^2}{2 \sigma_{z,t}^2} \leq \eta \]
Social planner problem

- Bellman equation

\[ V(N, Z, a) = \max_{N'} \left[ u(ZN, N') + \beta \min_{\mu \in [-a, a]} E_{\mu} V(N', Z', a') \right] \]

- Worst-case belief: future technology is low

\[ \mu^* = -a \]

⇒ planner acts as if bad times ahead

- Interpretation: precautionary behavior

- First order effects of ambiguity.
Characterizing equilibrium

Two Steps

1. Solve planner problem under worst case belief \( \mu^* = -a \)
   Optimal hours from FOC
   \[
   1 = E^{-a} \left[ \beta (Z'N')^{-\gamma} Z' \right]
   \]

2. Characterize variables under true shock process (in logs)
   \[
   n_t = -(1/\gamma - 1) \left( a_t + \frac{1}{2} \gamma \sigma_z^2 \right)
   \]
   \[
   y_{t+1} = z_{t+1} + n_t
   \]

\( \Rightarrow \) Worst case belief reflected in action \( n_t \), but not in shock realization \( z_{t+1} \)
Properties of equilibrium

- Dynamics

\[ y_{t+1} = z_{t+1} + n_t \]
\[ n_t = -(1/\gamma - 1)(a_t + \frac{1}{2}\gamma\sigma_z^2) \]
\[ a_t = (1 - \rho_a)\bar{a} + \rho_a a_{t-1} + \epsilon_{a,t} \]

- first order effects of uncertainty on output, even as \( \sigma_z^2 \to 0 \)
- if substitution effect is strong enough \( (1/\gamma > 1) \):

1. loss of confidence generates a recession
2. increase in confidence leads to an expansion

- TFP is not unusual in either cases
- hours do not forecast TFP if \( \text{cov}(a_t, z_{t+1}) = 0 \)

- Asset prices reflect time varying ambiguity premia

\[ \text{price of 1-step-ahead consumption claim} = \frac{E_t C_{t+1}}{R_t} \exp (-a_t - \gamma\sigma_z^2) \]
Comparison of shocks

- Ambiguity shocks
  \[ n_t = -(1/\gamma - 1) \left( a_t + \frac{1}{2} \gamma \sigma_z^2 \right) \]
  - loss of confidence generates recession if \( 1/\gamma > 1 \)
  - hours do not forecast TFP: regress \( z_{t+1} \) on \( n_t \) to get slope 0
  - time varying ambiguity premium on consumption claim = \( a_t \)

- News & noise shocks
  \[ n_t = (1/\gamma - 1) \left( \frac{1}{2} \gamma (1 - \pi) \sigma_z^2 \right) \]
  - signal about productivity \( s_t = z_{t+1} + \sigma_s \varepsilon_{s,t} \) with \( \pi := \sigma_z^2 / (\sigma_z^2 + \sigma_s^2) \)
  - bad signal (news or noise) generates recession if \( 1/\gamma > 1 \)
  - hours forecast TFP: regress \( z_{t+1} \) on \( n_t \) to get slope \( \gamma / (1 - \gamma) \)
  - constant risk premium on consumption claim

- Volatility shocks
  \[ n_t = -(1/\gamma - 1) \frac{1}{2} \gamma \sigma_z^2, t \]
  - volatility process \( \sigma_z^2, t = \text{var}_t(z_{t+1}) \), mean adjusts so \( E_t Z_{t+1} = 1 \)
  - volatility increase generates recession if \( 1/\gamma > 1 \)
  - hours do not forecast TFP (but forecast turbulence)
  - time varying risk premium on consumption claim = \( \gamma \sigma_z^2, t \)
General framework: Rep. agent & Markov uncertainty

- Notation (as before $s = \text{exogenous state}$)
  - $X = \text{endogenous states (e.g. capital)}$
  - $A = \text{agent actions (e.g. consumption, investment)}$
  - $Y = \text{other endogenous variables (e.g. prices)}$

- Recursive equilibrium
  - Functions for actions $A$, other endog vars $Y$, value $V$ s.t., for all $(X, s)$:

$$V(X, s) = \max_{A \in B(Y, X, s)} \left\{ u(c(A)) + \beta \min_{p \in \mathcal{P}(s)} E^p \left[ V(X', s') \right] \right\}$$

$$s.t. \ X' = T(X, A, Y, s, s')$$

- endog var determination: $G(A, Y, X, s) = 0$
- true exogenous Markov state process $p^*(s) \in \mathcal{P}(s)$

- Analysis again in 2 steps
  - find recursive equilibrium
  - characterize variables under “true” state process
Characterizing equilibrium: a guess-and-verify approach

- basic idea
  1. guess the worst case belief $p^0$
  2. find recursive equilibrium under expected utility & belief $p^0$
  3. compute value function under worst case belief, say $V^0$
  4. verify that the guess $p^0$ indeed achieves the minimum

- “essentially linear” economies & productivity shocks
  - environment $T, B, G$ s.t. 1st order approx. ok under expected utility
  - ambiguity is about mean of innovations to $s$

- simplification in essentially linear case
  - in step 1, guess that worst case mean is linear in state variables
  - step 2 uses loglinear approximation around “zero risk” steady state (sets risk to zero, but retains worst case mean)
  - step 4 then checks monotonicity of value function
An estimated DSGE model with ambiguity

Similar to CEE (2005), SW (2007)

1 Intermediate goods producers
   ▶ Price setting monopolist; competitive in the factor markets
     ★ mark-up shocks.

2 Final goods producers.
   ▶ Combines intermediate goods to produce a homogenous good.

3 Households: ambiguity-averse
   ▶ Own capital stock, consume, monopolistically supply specialized labor
   ▶ investment adjustment costs, internal habit in consumption.
     ★ efficiency of investment and price of investment shocks.

4 “Employment agencies”
   ▶ aggregate specialized labor into homogenous labor.

5 Government
   ▶ Taylor-type interest rule: reacts to inflation, output gap and growth.
     ★ government spending and monetary policy shocks.
Technology

- The intermediate good $j$ is produced using the function:

$$Y_{j,t} = Z_t K_{j,t}^\alpha (\epsilon_t L_{j,t})^{1-\alpha} - F_t$$

- Final goods:

$$Y_t = \left[ \int_0^1 Y_{j,t} \frac{1}{\lambda_{f,t}} dj \right]^{\lambda_{f,t}}$$

- Capital accumulation:

$$\bar{K}_{t+1} = (1 - \delta) \bar{K}_t + \left[ 1 - S \left( \frac{l_t}{l_{t-1}} \right) \right] l_t.$$ 

- Beliefs about technology:

$$\log Z_{t+1} = \rho_z \log Z_t + \mu_t + \sigma_u \epsilon_{Z, t+1}$$

$$\mu_t \in [-a_t, a_t]$$

$$a_t - \bar{a} = \rho_a (a_{t-1} - \bar{a}) + \sigma_a \epsilon_{a, t}$$
A bound on ambiguity

- Beliefs about technology

\[
\log Z_{t+1} = \rho_z \log Z_t + \mu_t + \sigma_u \varepsilon_{z,t+1}
\]

\[
\mu_t \in [-a_t, a_t]
\]

\[
a_t - \bar{a} = \rho_a (a_{t-1} - \bar{a}) + \sigma_a \varepsilon_{a,t}
\]

- agents view innovations as risky \((\sigma_u \varepsilon_{z,t+1})\) and ambiguous \((\mu_t)\)
- they know empirical moments of true \(\{\mu^*_t\}\), but not the exact sequence
- empirical moments say \(\log Z_{t+1} - \rho_z \log Z_t \sim i.i.N(0, \sigma_z^2)\); \(\sigma_z^2 > \sigma_u^2\)
- agents respond to uncertainty about \(\mu^*_t\) as if minimizing over \([-a_t, a_t]\)

- Constrain \(a_t\) to lie in a maximal interval \([-a_{\text{max}}, a_{\text{max}}]\)

- require that “boundary” beliefs \(\pm a_{\text{max}}\) imply “good enough” forecasts:

- there exists \(\sigma_u^2\) s.t. for every potential true DGP \(\{\mu^*_t\}\),

\(a_{\text{max}}\) or \(-a_{\text{max}}\) is best forecasting rule at least \(\alpha\) of the time

- \(a_{\text{max}}\) is best forecasting rule at date \(t\) if true mean \(\mu^*_t > a_{\text{max}}\)

- for example, \(\alpha = 5\%\) implies \(a_{\text{max}} = 2\sigma_z = \text{bound used in estimation}\)
Estimation

- Linearization $\rightarrow$ estimation using standard Kalman filter methods.
- Data: US 1984Q1-2010Q1: Output, consumption, investment, price of investment growth, hours, FFR, inflation.
- Law of motion for $a_t$ is estimated
- Estimates:
  - productivity dynamics
    \[ \rho_z = 0.95, \sigma_z = 0.0045 \]
  - confidence dynamics
    \[ \bar{a} = 0.0043, \rho_a = 0.96, \sigma_a = 0.00041 \]
  - other parameters broadly consistent with previous studies.
Role of ambiguity in fluctuations

- Variance decompositions: business cycle frequency

<table>
<thead>
<tr>
<th>Shock/Var.</th>
<th>Output</th>
<th>Consumption</th>
<th>Investment</th>
<th>Hours</th>
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<tr>
<td>Ambiguity</td>
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<td>51</td>
<td>14</td>
<td>30</td>
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<tr>
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<td>13</td>
<td>9</td>
<td>2</td>
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<tr>
<td>Efficiency of invest.</td>
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<td>32</td>
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<tr>
<td>Stochastic Growth</td>
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<td>7</td>
<td>6</td>
<td>13</td>
</tr>
<tr>
<td>Price mark-up</td>
<td>12</td>
<td>12</td>
<td>12</td>
<td>13</td>
</tr>
</tbody>
</table>

- any other shocks < 5% for above variables.
- long-run theoretical decomposition: $\varepsilon_{a,t}$ about 50% of fluctuations.
Dynamics: loss of confidence

- Technology
- Ambiguity
- GDP
- Consumption
- Investment
- Hours worked
- Ex-post excess return
- Price of capital
- Net real interest rate

C. Ilut, M. Schneider
(Duke, Stanford)
Ambiguous Business Cycles
BU/Boston Fed, 2011
Estimated ambiguity path
Historical shock decomposition

Output Growth

Data (year over year)
Model implied: only ambiguity shock

Consumption Growth

Investment Growth

Log Hours

C. Ilut, M. Schneider (Duke, Stanford)
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Welfare cost of fluctuations through ambiguity

Setting $\sigma_z = 0$ vs estimate $\Rightarrow \bar{a} = 0$ vs estimate

- Welfare: $\bar{V}$ ≡ Value function under ”zero risk steady state” (with estimated $\bar{a}$)
  - Welfare cost of fluctuations, as % of $C_{SS}(\bar{a} = 0)$, due to:
    1. ambiguity:
       $$\lambda^{ambig} = [\bar{V} - V^{SS}(\bar{a} = 0)] (1 - \beta)\beta^{-1} = 13\%$$
    2. risk (known probability distributions):
       $$\lambda^{risk} = V_{\sigma\sigma}(1 - \beta)\beta^{-1} = 0.01\%$$

- $V_{\sigma\sigma}$ : effect of fluctuations in $\varepsilon_{z, t+1}$ in a second order approx. of $V(\cdot)$.

- Other vars: Output, Capital, Consumption, Hours lower by 15\%
Conclusion

- Standard business cycle model with ambiguity aversion:
  - recursive multiple priors preferences.
  - ambiguity about mean productivity.
  - discipline from modeling concern with nonstationarity

- With ambiguity, uncertainty shocks have 1st order effects:
  - can apply standard linearization techniques for solution and estimation
  - work like “unrealized” news shocks with bias
  - potentially large role in business cycle

- Next
  - characterize further essentially linear settings
Parametrization

- Recall
  \[ a_t - \bar{a} = \rho_a (a_{t-1} - \bar{a}) + \sigma_a \varepsilon_{a,t} \]

- Restrictions: process \( a_t \) s.t.
  1. \( a_t \) is positive:
     \[ \bar{a} - m \frac{\sigma_a}{\sqrt{1 - \rho_a^2}} \geq 0 \]
  2. \( a_t \) is bounded by the discipline of the non-stationary argument
     \[ \bar{a} + m \frac{\sigma_a}{\sqrt{1 - \rho_a^2}} \leq 2\sigma_z \]

- Scale
  \[ \bar{a} = n\sigma_z, \ n \in [0, 1] \]

- Directly estimate \( n, \rho_a \).
Solution method: Steps

- Find deterministic “distorted” steady state \( x_o \) based on
  \[ z_o = \exp \left( \frac{-\bar{a}}{1 - \rho_z} \right) \]

- Linearize around distorted SS: Find \( A, B : \)
  \[ x_t - x_o = A(x_{t-1} - x_o) + BP(s_{t-1} - s_o) + B(\Xi_t - \Xi_o) \]

- Equilibrium:
  - True DGP dynamics:
    \[ \hat{Z}_t = \tilde{E}_{t-1} \hat{Z}_t + a_{t-1} \]
  - Endogenous variables:
    \[ x_t - x_o = A(x_{t-1} - x_o) + BP(s_{t-1} - s_o) + B(\hat{\Xi}_t - \Xi_o) \]

\[ \hat{\Xi}_t = \begin{bmatrix} a_{t-1}/\sigma_z \\ 0_{(n-1)\times1} \end{bmatrix} \]
Ellsberg Paradox

- bet on black from risky urn $\succ$ bet on black from ambiguous urn
- bet on white from risky urn $\succ$ bet on white from ambiguous urn
- expected utility cannot capture choices, but $\min_{p \in \mathcal{P}} E^p[u(c)]$ can!