
Andrew Atkenson, Andrea Eisfeldt, Pierre-Olivier Weill

Discussion by Joao Gomes
Wharton School

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Organization

1. Objectives and Methodology

2. Findings

3. Comments and Suggestions
Background and Objectives

Many models suggest that financial market conditions play an important role in determining the magnitude of business cycles.
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Leverage and the capital structure become important determinants of firm decisions and aggregate outcomes.
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- These might be much more important for small and unlisted/unrated firms

In a few papers the entire cross-sectional distribution of firms is a key state variable

- But unfortunately still not in most of them
- Early generation models rely only on the mean
Estimating the Financial Health of U.S. Firms

The focus here is on the “Distance to Insolvency”
Estimating the Financial Health of U.S. Firms

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- Builds on Leland (1994)
- Relies only on readily available firm-level data on equity return volatility
The Classic Model of the Levered Firm

The market value of **equity**

\[ e(z, k, b) = \max_{k', b'} \left[ d(z, k, k', b, b') + \beta \mathbb{E}_z e(z', k', b') \right] , \]
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The enterprise or **asset** value

\[
v(z, k, b) = \max_{k', b'} \left[ \pi(z, k) - i(k, k') + \beta \mathbb{E}_z v(z', k', b') \right],
\]

The market value of the **liabilities**

\[
v(z, k, b) - e(z, k, b)
\]
The Classic Model with Optimal Default

The value of equity

\[ e(z, k, b) = \max_{k', b'} \left[ d(z, k, k', b, b') + \beta \max\{\mathbb{E}_z e(z', k', b'), 0\} \right], \]
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Optimal Insolvency or Default

\[ z^d = \max \{z : e(z, k, b) = 0\} \]

Probability of Default

\[ F(z^d(k, b)) \]
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\[ F(z^d(k, b)) \approx \alpha_1 \ln(k) + \alpha_2 \ln(b/k) + \ldots \]

The Classic Model with Optimal Default - Simplified

Key simplifications: Leland (1994)

- No investment, $k' = k = 1$
- Infinite horizon debt with no issuance, $b' = b$

The value of equity $e(z, b) = \left[ \pi(z) - cb + \beta \max\{E_z e(z', b), 0\} \right]$.

Optimal Insolvency or Default

$z_{ds} = \max\{z: e(z, b) = 0\}$

But now the value of assets and liabilities are trivial to compute

$v(z) = E_z \sum \beta t \pi(z_t) - \frac{cb}{1 - \beta} - V_B$
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$$v(z) = E^z \sum \beta^t \pi(z_t) - - V^A$$

$$cb/(1 - \beta) - - V^B$$
Equity Value and Default

Equity value

\[ e(z, b) = v(z) - \frac{cb}{1 - \beta} \]

Even if \( v(z) < \frac{cb}{1 - \beta} \) there may be value in continuing to operate - gambling for resurrection.

When equity is free to pick (optimal default) then \( x(z, b) > 0 \). However it is also possible that \( x(z, b) \) is negative - equity is a short position on the default option.

If default is triggered by some covenant violation, or some liquidity shortage.
Equity Value and Default

Equity value

\[ e(z, b) = v(z) - \frac{cb}{1 - \beta} + x(z, b) \]

where \( x(z, b) \) an be seen as the option value associated to operating the levered firm.
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Default and Asset Values

Default threshold is obtained in terms of asset values

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Default and Asset Values

Default threshold is obtained in terms of asset values

\[ v(z^{ds}) = v^d = cb/(1 - \beta) - x(z^{ds}, b) \]

Probability of Default

\[ G(v(z) - v^d) \]
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Empirical Issues

- What is the empirical distribution of asset values, \( G \)?
Distance to Insolvency

Assume $G(\cdot)$ is lognormal with variance $\sigma_v$.

- Probability of Default is

$$N \left[ \frac{v(z) - v^d}{v(z)} \frac{1}{\sigma_v} \right]$$
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In continuous time

- The number of steps to reach default
Implementation

However we would still need to compute

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- The market value of assets, $v(z)$
- Its variance, $\sigma_v$
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- The market value of assets, $v(z)$
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However we would still need to compute

- The market value of assets, $v(z)$
- Its variance, $\sigma_v$
- And the default threshold $v^d$

The KMV-style estimates

- Just assume $v^d = cb/(1 - \beta) = V^B$
- The so-called *Distance to Default*
The Big Idea

Use the fact that:

\[
\frac{v(z) - v^d}{v(z)} \frac{1}{\sigma_v} \approx \frac{1}{\sigma_e}
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\[
\sigma_e = \frac{v(z)}{v(z) - v^d} \sigma_v
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\[ \sigma_e = \frac{v(z)}{v(z) - v^d} \sigma_v \]

Unfortunately this is the case where its also uninteresting
Bounding Distance to Insolvency

In general

\[ v^d = cb/(1 - \beta) + x(z, b) \]
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Depending on the option value this default cut-off could be larger or smaller than the value of outstanding liabilities, \( cb/(1 - \beta) \).
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- Formally the equity value can be above or below

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Practically this means \( 1/\sigma_e \) could actually be either smaller or larger than the Distance to Insolvency

\[ \frac{v(z) - v^d}{v(z)} \frac{1}{\sigma_v} \]
Empirical Findings

The estimated distance to insolvency measure (inverse equity volatility) correlates fairly well with

- Credit ratings and
- Credit default swaps
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But a few empirical issues also arise

- Equity volatility is volatile: e.g. the crash of October 1987
- Equity volatility does not mean much for unlevered firms
Some Final Thoughts on Solvency Measures

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1. Effects of credit markets show up even if firms never default
   - Models with collateral constraints

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1. Effects of credit markets show up even if firms never default
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2. Default risk is often associated with lack of liquidity and not solvency

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- Between 2000 and 2011, 33% of US firms who failed to make payments on their debt (a default event) never filed for bankruptcy.