Designing a Simple Loss Function for the Fed: Does the Dual Mandate Make Sense?*

Davide Debortoli  Jinill Kim  Jesper Lindé
University of California, San Diego  Korea University  Federal Reserve Board and CEPR
Ricardo Nunes
Federal Reserve Board

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Abstract

Yes. Using the workhorse Smets and Wouters (2007) of the U.S. economy, we find that the role of the output gap is as important or even more important than inflation. Moreover, we document that a loss function with wage inflation and the hours gap provides an even better approximation of the household true welfare function than a standard objective based on inflation and the output gap. Our results hold up when we introduce interest rate smoothing in the objective to capture the observed gradualism in policy behavior and ensure that the probability of the federal funds rate hitting the zero lower bound is negligible.

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Keywords: Central banks’ objectives, simple loss function, monetary policy design, Smets-Wouters model

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Contact: Debortoli: <ddebor@ucsd.edu>; Kim: <jinillkim@korea.ac.kr>; Lindé: <jesper.l.linde@frb.gov>; Nunes: <ricardo.p.nunes@frb.gov>.
1. Introduction

Some decades back—before DSGE modeling was used in the analysis of monetary policy—a popular setting to do normative exercise in the monetary policy literature was to adopt a linear-quadratic framework: the objective function is (purely) quadratic, and the constraints are linear. Another trend of optimal policy literature was to analyze Ramsey policy in a micro-founded model, which typically did not involve many variables.

During the last couple of decades, much progress has been made in linking these two trends by transforming a general (i.e. non-linear) policy problem into an LQ problem. Rotemberg and Woodford (1998) showed in detail that—under the assumption that the steady state satisfies certain efficiency conditions—the objective function of households can be transformed into a quadratic function. With this quadratic objective function, optimization subject to linearized constraints would be sufficient in obtaining legitimate results from a normative perspective. Some assumptions about efficiency are “unpalatable”, as exemplified by the presence of a positive subsidy that would make the steady state of the market equilibrium equivalent to that of the social planner. This shortcoming was subsequently addressed by e.g. Benigno and Woodford (2012), who extended the LQ transformation to a general setting without the presence of such subsidies.

This paper uses their approach to analyse optimal monetary policy within the workhorse Smets and Wouters (2007) model – SW07 henceforth – of the US economy. Specifically, we examine how a simple objective for the central bank, i.e. the Federal Reserve, should be designed in order to approximate the true welfare of households in the model economy as closely as possible.

Even though it is “optimal” and ideal to implement the Ramsey policy directly, it is clear that most countries do not ask their central bank to implement such a policy for society. Instead, central banks are mandated to follow a simple objective that involves only a small number of economic variables; in the case of the United States, for example, the Federal Reserve’s objective is a dual mandate with maximum employment and stable prices.\(^1\) One important theoretical reason to stay away from the Ramsey policy in favor of a simple mandate is related to the problem of time inconsistency associated with the Ramsey policy. In practice, policymakers have also emphasized that it is hard to formulate and communicate a complicated state-contingent policy in a world full of uncertainties.\(^2\)

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\(^1\) The dual mandate was codified only in the Federal Reserve Reform Act of 1977. However, the transition of emphasis from financial stability—at the founding of the Federal Reserve in 1913—to macroeconomic stability was mostly influenced by the Great Depression. Of course, the tide has shifted again since the Global Financial Crisis erupted in 2007. See Bernanke (2013) for a summary of Federal Reserve’s one hundred years.

\(^2\) Taylor and Williams (2009) argues for simple and robust policy rules.
In light of the simple ad hoc mandates that are apparently prevalent in the world of central banking, we analyze how to improve performance of central banking when constrained by such simple mandates. In our analysis, the central bank has to announce and follow a strategy consistent with optimizing the given simple objective subject to the private behavior of the economy. The only degree of freedom the central bank has is to assign particular weights of each term in the (exogenously) assigned simple objective function, and it does so by maximizing the level of welfare for the households populating the economy. Thus, a central question in our analysis is what weights central banks should put on variables in a simple objective function if they want monetary policy to mimic the Ramsey policy as closely as possible. For instance, does the Federal Reserve’s dual mandate improve households’ welfare relative to strict inflation targeting?

As noted earlier, we adopt the SW07 model in our analysis. This model represents a prominent example of how the U.S. economy can be described by a system of dynamic equations consistent with optimizing behavior. A conventional procedure for estimating such a model, following the seminal work of Smets and Wouters (2003), is to form the likelihood function for a first-order approximation of the dynamic equations and use Bayesian priors for the deep parameters. Doing so yields a posterior distribution for the parameters.

In a normative analysis that involves an evaluation of a specific criterion function, it may be important to allow for both parameter and model uncertainty. However, before doing such a full-fledged analysis, it is instructive start out by performing a normative exercise in the context of a specific model and particular parameter values. This is the also the approach adopted in this paper: we use the SW07 model and calibrate benchmark parameter values at their posterior mode. Conditional on the specific model and the particular parameter values, the Ramsey policy can be formed by maximizing the level of household welfare. The optimal policy takes all the model equations except the equation describing monetary policy as given and implements the policy that is best for the households.

Our main findings are as follows. First, we find that adding a term involving a measure of real activity appears to be much more important than previously thought. A positive weight on any of the typical variables like the output gap, the level of output, and the growth rate of output improves welfare significantly; but among them, the model-consistent output gap performs the best. Specifically, we find that in a simple loss function—with the weight on annualized core inflation normalized to unity—the optimized weight on the output gap is about 1. This is considerably

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3 See Walsh (2005) as an example.
higher than the reference value of 0.048 derived in the graduate textbook of Woodford (2003) and the value of 0.25 assumed in the speech by Yellen (2012).\textsuperscript{4} The key mechanism why this result obtains is the presence of price and wage markup shocks which introduces a trade-off between stabilizing inflation and the output gap. Second, we find that a loss function with wage inflation and the hours gap provides an even better approximation of the household true welfare function relative to a simple standard inflation-output gap based objective. As is the case with the inflation-output gap based simple objective, the hours gap, defined as the difference between actual and potential hours worked per capita, should be assigned a large weight in such as loss function. The reason why targeting labor market variables provides a better approximation of the Ramsey policy is that the labor market in the SW07 model features nominal wage frictions and mark-up shocks. Third and finally, we show that our basic result is robust to a number of important perturbations of the simple loss function; notably when imposing realistic limitations of the extent to which monetary policy makers change interest rates.

This paper proceeds as follows. The ensuing subsection starts with a discussion of the previous literature to place our contribution in a proper context. After that, we present the SW07 model and the parameters we use in our analysis. Section 2.1 shows how to compute the Ramsey policy and to evaluate the alternative monetary policies. Section 3 reports our benchmark results. The robustness of our results along some key dimensions are subsequently discussed in Section 4, while the comparison with simple rules is discussed in Section 5. Finally, Section 6 provides some concluding remarks and suggestions for further research.

1.1. Relationship with Previous Research

Our paper is related to previous research on optimal monetary policy from the following three perspectives. First, we try to synthesize the developments in both positive and normative economics. From the positive side, most of the empirical literature has been based on models that are not derived from microfoundations. Therefore, though some models have been successful in explaining the data, they are prone to be subject to the Lucas (1976) critique. Especially normative analysis or policy recommendations based on such ad hoc models have been subject to heavy criticism.\textsuperscript{5} Smets and Wouters (2007) is viewed as one of the most successful attempts in explaining the data.

\footnotesize
\textsuperscript{4} Yellen (2012) assumed a value of unity for the unemployment gap, which by Okun’s law translates into a value of 0.25 for the output gap.

\textsuperscript{5} Leeper and Zha (2003) defined a modest policy intervention as a change in policy that does not significantly shift agents’ beliefs about policy regime and does not generate quantitatively important expectations-formation effects a la Lucas (1976).
while being consistent with microfoundations. From the perspective of normative economics, most of the existing papers are based on simple linear-quadratic models. Even though policy recommendations are model consistent, the simplicity of the models being used has been criticized as unrealistic. In doing normative analysis with an empirically realistic model, this paper achieves the objective of providing theoretically coherent yet empirically relevant policy recommendations.

The second perspective of locating this paper in the literature—related with the preceding point—comes from a choice of the criterion function for a central bank. In optimal policy analysis with simple models, usually in analyzing fiscal policy rather than monetary policy, the tradition of Ramsey policy has been adopted and the utility function of households has been used as the criterion for the social planner. In most normative analysis on monetary policy, an ad hoc criterion—usually quadratic and involving only a few variables—has been employed in the context of linear(ized) models. This paper tries to move the state-of-the-art one step forward by using a medium-scale DSGE model to achieve both theoretical consistency and empirical relevance; furthermore, we assume that central banks follow a simple criterion function as in the real world and that the welfare of the economy is evaluated based on the unconditional expectation of the utility function of households.

The third angle to view this paper is from the perspective of methodology for normative analysis. The spirit of our exercise is closest to that of Jensen (2002) in that we search over the space of weight parameters that are attached to different terms of an ad hoc loss function. In choosing a model which the minimization exercise is subject to, our choice is similar to Ilbas (2008) in that both papers follow the works of Smets and Wouters (2003, 2007) and eliminate the policy rule equation from their model. The second part of our exercise—that involve simple interest rules rather than objective functions—is similar in spirit to Kim and Henderson (2005) in that optimal search is over the space of parameters in simple rules with observables only.

2. The Model and Our Exercise

In this section, we lay out the model we use and describe our exercise to find optimized simple objectives.

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6 See e.g. the classical paper by Clarida, Gali and Gertler (1999).
7 One difference is that we assume commitment, while Jensen (2002) analyzed in the discretion setting.
8 While we analyze a parameter space based on the welfare metric, Ilbas (2008) estimates the parameters of a loss function—that is, based on the likelihood metric.
9 While this paper adopts numerical techniques to optimize over a parameter space, Kim and Henderson (2005) derived welfare implications analytically based on a closed-form solution.
2.1. The Smets and Wouters (2007) Model

Below, we describe the firms’ and households’ problem in the model, and state the market clearing conditions.\textsuperscript{10} The discussion of the conduct of monetary policy is deferred to Section 2.2.

2.1.1. Firms and Price Setting

Final Goods Production The single final output good $Y_t$ is produced using a continuum of differentiated intermediate goods $Y_t(f)$. Following Kimball (1995), the technology for transforming these intermediate goods into the final output good is

$$\int_0^1 G_Y \left( \frac{Y_t(f)}{Y_t} \right) df = 1. \quad (1)$$

Following Dotsey and King (2005) and Levin, Lopez-Salido and Yun (2007), Smets and Wouters (2007) assume that $G_Y(.)$ is given by the following strictly concave and increasing function:

$$G_Y \left( \frac{Y_t(f)}{Y_t} \right) = \frac{\phi_p}{1-(\phi_p-1)\epsilon_p} \left[ \frac{\phi_p(1-\phi_p)\epsilon_p}{\phi_p} \frac{Y_t(f)}{Y_t} + \frac{(\phi_p-1)\epsilon_p}{\phi_p} \frac{Y_t(f) - (\phi_p-1)\epsilon_p}{\phi_p} + \left[ 1 - \frac{\phi_p}{1-(\phi_p-1)\epsilon_p} \right] \right], \quad (2)$$

where $\phi_p \geq 1$ denotes the gross markup of the intermediate firms. The parameter $\epsilon_p$ governs the degree of curvature of the intermediate firm’s demand curve. When $\epsilon_p = 0$, the demand curve exhibits constant elasticity as with the standard Dixit-Stiglitz aggregator. When $\epsilon_p$ is positive—as in SW07—the firm’s instead face a quasi-kinked demand curve, implying that a drop in its relative price only stimulates a small increase in demand. On the other hand, a rise in its relative price generates a large fall in demand. Relative to the standard Dixit-Stiglitz aggregator, this introduces more strategic complementarity in price setting which causes intermediate firms to adjust prices less to a given change in marginal cost. Finally, we notice that $G_Y(1) = 1$, implying constant returns to scale when all intermediate firms produce the same amount.

Firms that produce the final output good are perfectly competitive in both product and factor markets. Thus, final goods producers minimize the cost of producing a given quantity of the output index $Y_t$, taking as given the price $P_t(f)$ of each intermediate good $Y_t(f)$. Moreover, final goods producers sell units of the final output good at a price $P_t$, and hence solve the following problem:

$$\max_{\left\{ Y_t, Y_t(f) \right\}} P_t Y_t - \int_0^1 P_t(f) Y_t(f) df \quad (3)$$

\textsuperscript{10} For a more detailed description of the model, we refer the reader to the on-line appendix to the Smets and Wouters paper, which is available online at http://www.aeaweb.org/aer/data/june07/20041254_app.pdf.
subject to the constraint (1). Note that for $\epsilon_p = 0$, this problem leads to the usual expressions

$$\frac{Y_t(f)}{Y_t} = \left[ \frac{P_t(f)}{P_1} \right]^{-\frac{\phi_p}{\epsilon_p}}, P_t = \left[ \int P_t(f) \frac{1}{1-\phi_p} df \right]^{1-\phi_p}.$$  

**Intermediate Goods Production** A continuum of intermediate goods $Y_t(f)$ for $f \in [0,1]$ is produced by monopolistically competitive firms, each of which produces a single differentiated good. Each intermediate goods producer faces a demand schedule from the final goods firms through the solution to the problem in (3) that varies inversely with its output price $P_t(f)$ and directly with aggregate demand $Y_t$.

Each intermediate goods producer utilizes capital services $K_t(f)$ and a labor index $L_t(f)$ (defined below) to produce its respective output good. The form of the production function is Cobb-Douglas:

$$Y_t(f) = \varepsilon_t^a K_t(f)^{\alpha} \left[ \gamma^t L_t(f) \right]^{1-\alpha} - \gamma^t \Phi,$$  

where $\gamma^t$ represents the labour-augmenting deterministic growth rate in the economy, $\Phi$ denotes the fixed cost (which is related to the gross markup $\phi_p$ so that profits are zero in the steady state), and $\varepsilon_t^a$ is total factor productivity which follows the process

$$\ln \varepsilon_t^a = (1 - \rho_z) \ln \varepsilon_0^a + \rho_z \ln \varepsilon_{t-1}^a + \eta_t^a, \eta_t^a \sim N(0, \sigma_a).$$  

Firms face perfectly competitive factor markets for hiring capital and the labor index. Thus, each firm chooses $K_t(f)$ and $L_t(f)$, taking as given both the rental price of capital $R_{K_t}$ and the aggregate wage index $W_t$ (defined below). Firms can costlessly adjust either factor of production. Thus, the standard static first-order conditions for cost minimization imply that all firms have identical marginal cost per unit of output.

The prices of the intermediate goods are determined by Calvo-Yun (1996) style staggered nominal contracts. In each period, each firm $f$ faces a constant probability, $1 - \xi_p$, of being able to reoptimize its price $P_t(f)$. The probability that any firm receives a signal to reset its price is assumed to be independent of the time that it last reset its price. If a firm is not allowed to optimize its price in a given period, it adjusts its price by a weighted combination of the lagged and steady state rate of inflation, i.e.,

$$P_t(f) = (1 + \pi_{t-1})^{1-p} (1 + \pi)^{1-\epsilon_p} P_{t-1}(f)$$  

where $0 \leq \epsilon_p \leq 1$ and $\pi_{t-1}$ denotes net inflation in period $t-1$, and $\pi$ the steady state inflation rate. A positive value of $\epsilon_p$ introduces structural inertia into the inflation process. All told, this leads to the following
optimization problem for the intermediate firms

$$\max_{P_t(f)} \mathbb{E}_t \sum_{j=0}^{\infty} (\beta \xi_p)^j \frac{\mathbb{E}_t P_t}{\mathbb{E}_t P_{t+j}} \left[ \hat{P}_t(f) \left( \prod_{s=1}^{j} \left( 1 + \pi_{t+j-1} \right)^{\nu_p} \left( 1 + \pi \right)^{1-\nu_p} \right) - MC_{t+j} \right] Y_{t+j}(f), \quad (6)$$

where $\hat{P}_t(f)$ is the newly set price; notice that with our assumptions all firms that re-optimize their prices actually set the same price.

Because a non-linear version of the model cannot be written on recursive form when time-varying price markup shocks are included like in the SW07 model, we instead introduce directly a shock $\varepsilon^P_t$ in the first-order condition to the problem in (6). And following SW07, we assume the shock is given by an exogenous ARMA(1,1) process:

$$\ln \varepsilon^P_t = (1 - \rho_p) \ln \varepsilon^P + \rho_p \ln \varepsilon^P_{t-1} + \eta^p_t - \mu_p \eta^p_{t-1}, \eta^p_t \sim N(0, \sigma_p). \quad (7)$$

When this shock is introduced in the non-linear model, we put a scaling factor on it so that it enters exactly the same way in a log-linearized representation of the model as the price markup shock does in the SW07 model.\footnote{11 Alternatively, we should have followed Adjemian et al. (2008) and introduced the shock as a tax on the intermediate firm’s revenues in the problem (6) directly. An advantage with such an approach would be that the shock is “microfounded” in the sense that we know what it represents. The drawback would be that it is not obvious that the log-linearized representation of the model is the same as in SW07.}

### 2.1.2. Households and Wage Setting

We assume a continuum of monopolistically competitive households (indexed on the unit interval), each of which supplies a differentiated labor service to the production sector; that is, goods-producing firms regard each household’s labor services $L_t(h), h \in [0, 1]$, as an imperfect substitute for the labor services of other households. It is convenient to assume that a representative labor aggregator combines households’ labor hours in the same proportions as firms would choose. Thus, the aggregator’s demand for each household’s labor is equal to the sum of firms’ demands. The aggregated labor index $L_t$ has the Kimball (1995) form:

$$L_t = \int_0^1 G_L \left( \frac{L_t(h)}{L_t} \right) dh = 1 \quad (8)$$

where the function $G_L(.)$ has the same functional form as (2), but is characterized by the corresponding parameters $\epsilon_w$ (governing convexity of labor demand by the aggregator and $\phi_w$ (gross wage markup). The aggregator minimizes the cost of producing a given amount of the aggregate

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labor index $L_t$, taking each household’s wage rate $W_t(h)$ as given, and then sells units of the labor index to the intermediate goods sector at unit cost $W_t$, which it is natural to interpret as the aggregate wage rate. From the FOCs, the aggregator’s demand for the labor hours of household $h$ – or equivalently, the total demand for this household’s labor by all goods-producing firms – is given by

$$\frac{L_t(h)}{L_t} = G_{L}^{-1} \left[ \frac{W_t(h)}{W_t} \int_{0}^{1} G'_{L} \left( \frac{L_t(h)}{L_t} \right) \frac{L_t(h)}{L_t} \, dh \right]$$  \hspace{1cm} (9)$$

The utility functional of a typical member of household $h$ is

$$E_t \sum_{j=0}^{\infty} \beta^j \left[ \frac{1}{1-\sigma_c} (C_{t+j}(h) - \kappa C_{t+j-1}) \right]^{1-\sigma_c} \exp \left( \frac{\sigma_c - 1}{1+\sigma_c} L_{t+j}(h)^{1+\sigma_c} \right)$$  \hspace{1cm} (10)$$

where the discount factor $\beta$ satisfies $0 < \beta < 1$. The period utility function depends on household $h$’s current consumption $C_t(h)$, as well as lagged aggregate per capita consumption to allow for external habit persistence. The period utility function also depends inversely on hours worked $L_t(h)$.

Household $h$’s budget constraint in period $t$ states that its expenditure on goods and net purchases of financial assets must equal its disposable income:

$$P_tC_t(h) + P_tI_t(h) + \frac{B_{t+1}(h)}{\varepsilon_t R_t} + \sum_{s} \xi_{t,t+1}B_{D,t+1}(h) - B_{D,t}(h)$$  \hspace{1cm} (11)$$

$$= B_t(h) + W_t(h) L_t(h) + R_t^F Z_t(h) \bar{K}_t(h) - a(Z_t(h)) \bar{K}_t(h) + \Gamma_t(h) - T_t(h)$$

Thus, the household purchases part of the final output good (at a price of $P_t$), which it chooses either to consume $C_t(h)$ or invest $I_t(h)$ in physical capital. Following Christiano, Eichenbaum, and Evans (2005), investment augments the household’s (end-of-period) physical capital stock $\bar{K}_{t+1}(h)$ according to

$$\bar{K}_{t+1}(h) = (1-\delta)\bar{K}_t(h) + \varepsilon_t \left[ 1 - S \left( \frac{I_t(h)}{I_{t-1}(h)} \right) \right] I_t(h)$$  \hspace{1cm} (12)$$

The extent to which investment by each household $h$ turns into physical capital is assumed to depend on and exogenous shock $\varepsilon_t$ and how rapidly the household changes its rate of investment according to the function $S \left( \frac{I_t(h)}{I_{t-1}(h)} \right)$ which we specify as

$$S(x_t) = \frac{\gamma}{2} (x_t - \gamma)^2$$

\hspace{1cm} (12)$$

Note that we deviate slightly from the notation in SW07 by using $h$ to index households and using $\kappa$ to denote the degree of habit formation.
Notice that this function satisfies $S(\gamma) = 0$, $S'(\gamma) = 0$ and $S''(\gamma) = \tilde{\varphi} \gamma^2 > 0$. Since SW07 define $S''(1) \equiv \varphi$, our adjustment cost function implies we need to set $\tilde{\varphi} \equiv \varphi / \gamma^2$. The stationary investment-specific shock $\varepsilon^i_t$ follows

$$
\ln \varepsilon^i_t = \rho_i \ln \varepsilon^i_{t-1} + \eta^i_t \sim N(0, \sigma_i).
$$

In addition to accumulating physical capital, households may augment their financial assets through increasing their government nominal bond holdings $(B_{t+1})$, from which they earn an interest rate of $R_t$. The return on bonds is also subject to a risk-shock $\varepsilon^b_t$, which follows

$$
\ln \varepsilon^b_t = \rho_b \ln \varepsilon^b_{t-1} + \eta^b_t \sim N(0, \sigma_b).
$$

In addition, we assume that agents can engage in frictionless trading of a complete set of contingent claims to diversify away idiosyncratic risk. The term $\int_{s_t} x_{t+1} B_{D,t+1}(h) - B_{D,t}(h)$ represents net purchases of state-contingent domestic bonds, with $x_{t+1}$ denoting the state price, and $B_{D,t+1}(h)$ the quantity of such claims purchased at time $t$. Each member of household $h$ earns after-tax labor income $W_t(h) L_t(h)$, after-tax capital rental income of $R_t^k Z_t(h) K_t(h)$, and pays a utilization cost of the physical capital equal to $a(Z_t(h)) \bar{K}_t(h)$ where $Z_t(h)$ is the capital utilization rate, so that capital services provided by household $h$, $K_t(h)$, equals $Z_t(h) \bar{K}_t(h)$. The capital utilization adjustment function $a(Z_t(h))$ is assumed to be given by

$$
a(Z_t(h)) = r^k \frac{1}{z_1} [\exp(\tilde{z}_1 (Z_t(h) - 1)) - 1],
$$

where $r^k$ is the steady state net real interest rate ($\bar{R}_t^K / \bar{P}_t$). Notice that the adjustment function satisfies $a(1) = 0$, $a'(1) = r^k$, and $a''(1) \equiv r^k z_1$. Following SW07, we want to write $a''(1) = z_1 = \psi / (1 - \psi) > 0$, where $\psi \in [0, 1]$ and a higher value of $\psi$ implies a higher cost of changing the utilization rate; our parameterization of the adjustment cost function then implies that we need to set $\tilde{z}_1 \equiv z_1 / r^k$. Finally, each member also receives an aliquot share $\Gamma_t(h)$ of the profits of all firms, and pays a lump-sum tax of $T_t(h)$ (regarded as taxes net of any transfers).

In every period $t$, each member of household $h$ maximizes the utility functional (10) with respect to its consumption, investment, (end-of-period) physical capital stock, capital utilization rate, bond holdings, and holdings of contingent claims, subject to its labor demand function (9), budget constraint (11), and transition equation for capital (12).

Households also set nominal wages in Calvo-style staggered contracts that are generally similar to the price contracts described previously. Thus, the probability that a household receives a signal
to reoptimize its wage contract in a given period is denoted by $1 - \xi_w$. In addition, SW specify the following dynamic indexation scheme for the adjustment of the wages of those households that do not get a signal to reoptimize: $W_t(h) = \gamma (1 + \pi_{t-1})^{\xi_w} (1 + \pi)^{1-\xi_w} W_{t-1}(h)$. All told, this leads to the following optimization problem for the households

$$\max_{\tilde{W}_t(h)} \mathbb{E}_t \sum_{j=0}^{\infty} (\beta \xi_w)^j \hat{\Pi}_{t+j} \hat{P}_t \left[ \tilde{W}_t(h) \left( \Pi_{s=1}^j (1 + \pi_{t+j-s})^{\xi_w} (1 + \pi)^{1-\xi_w} \right) - W_{t+j} \right] L_{t+j}(h),$$

where $\tilde{W}_t(h)$ is the newly set wage; notice that with our assumptions all households that re-optimize their wages will actually set the same wage.

Following the same approach as with the intermediate goods firms, we introduce a shock $\varepsilon_t^w$ in the resulting first-order condition. This shock, following SW07, is assumed to be given by an exogenous ARMA(1,1) process

$$\ln \varepsilon_t^w = (1 - \rho_w) \ln \varepsilon_{t-1}^w + \rho_w \ln \varepsilon_{t-1}^w + \eta_t^w - \mu_w \eta_{t-1}^w, \eta_t^w \sim N(0, \sigma_w). \quad (15)$$

As discussed previously, we use a scaling factor for this shock so that it enters in exactly the same way as the wage markup shock in SW07 in the log-linearized version of our model.

### 2.1.3. Market Clearing Conditions

Government purchases $G_t$ are exogenous, and government spending relative to trend output, i.e. $g_t = G_t / (\gamma Y)$, are assumed to follow the following exogenous AR(1) process:

$$\ln g_t = (1 - \rho_g) \ln g + \rho_g (\ln g_{t-1} - \rho_g \ln \varepsilon_{t-1}^a) + \varepsilon_t^g, \varepsilon_t^g \sim N(0, \sigma_g) \quad (16)$$

Government purchases have no effect on the marginal utility of private consumption, nor do they serve as an input into goods production. The consolidated government sector budget constraint is

$$\frac{B_{t+1}}{R_t} = G_t - T_t + B_t \quad (17)$$

By comparing the debt terms in the household budget constraint in eq. (11) with the equation above, we understand that receipts from the risk shock is subject to iceberg costs, and hence does not add income to the government. But even if they did, it would not matter as we follow SW and assume that the government balances its expenditures each period through lump-sum taxes, $T_t = G_t + B_t - B_{t+1}/R_t$, so that government debt $B_t = 0$ in equilibrium.$^{13}$

$^{13}$ As Ricardian equivalence holds in the model, it does not matter for equilibrium allocations whether the government balances it debt or not each period.
Total output of the service sector is subject to the resource constraint:

\[ Y_t = C_t + I_t + G_t + a(Z_t) \bar{K}_t \]  

(18)

where \( a(Z_t) \bar{K}_t \) is the capital utilization adjustment cost.

Finally, we need to specify the aggregate production constraint. To do that, we note that the unweighted sum of the intermediate firms output equals

\[ Y_{t}^{sum} = \int_{0}^{1} Y_t(f) df, \]

which from eq. (4) can be rewritten as

\[ Y_{t}^{sum} = \int_{0}^{1} \left[ \varepsilon_t^a K_t(f)^{\alpha} \left[ \gamma^t L_t(f) \right]^{1-\alpha} - \gamma^t \Phi \right] df \]

\[ = \varepsilon_t^a \left( \frac{K_t}{\gamma^t L_t} \right)^{\alpha} \int_{0}^{1} \gamma^t L_t(f) df - \gamma^t \Phi \]

where the second equality follows from the fact that every firm’s capital-labor ratio will be the same in equilibrium.

Also, from the FOCs to the final goods aggregator, it follows that

\[ Y_{t}^{sum} = Y_t \int_{0}^{1} \frac{\phi_p}{\phi_p - (\phi_p - 1) \epsilon_p} \left( \left[ \frac{P_t(f)}{P_t} \right]^{1 - \frac{\phi_p - (\phi_p - 1) \epsilon_p}{\phi_p - 1} \epsilon_p} + \frac{(1 - \phi_p)\epsilon_p}{\phi_p} \right) df, \]

so that

\[ \varepsilon_t^a \left( \frac{K_t}{\gamma^t L_t} \right)^{\alpha} \int_{0}^{1} \gamma^t L_t(h) dh - \gamma^t \Phi = Y_t \int_{0}^{1} \frac{\phi_p}{\phi_p - (\phi_p - 1) \epsilon_p} \left( \left[ \frac{P_t(f)}{P_t} \right]^{1 - \frac{\phi_p - (\phi_p - 1) \epsilon_p}{\phi_p - 1} \epsilon_p} + \frac{(1 - \phi_p)\epsilon_p}{\phi_p} \right) df \]

In the expression above, \( \Lambda_t^p \) is the lagrangean multiplier in the final goods aggregator problem. It can be shown that when \( \epsilon_p = 0 \), \( \Lambda_t^p = 0 \) for all \( t \). By inserting the expression for the unweighted sum of labor, \( \int_{0}^{1} \gamma^t L_t(h) dh \), into this last expression, we may finally derive the aggregate production constraint which depends of aggregate technology, capital, labor, fixed costs, as well as the price and wage dispersion terms.\(^{14}\)

2.1.4. Model Parameterization

When solving the model, we adopt the parameter estimates (posterior mode) in Tables 1.A and 1.B in SW07. We also use the same values for the calibrated parameters. Table 1 provides the relevant values.

\(^{14}\) We refer the interested reader to Adjemian, Paries and Moyen (2008) for further details.
Table 1: Parameter Values in Smets and Wouters (2007).

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Calibrated Description</th>
<th>Value</th>
<th>Parameter</th>
<th>Estimated Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \delta )</td>
<td>Depreciation rate</td>
<td>0.025</td>
<td>( \varphi )</td>
<td>Investment adj. cost</td>
<td>5.48</td>
</tr>
<tr>
<td>( \phi_w )</td>
<td>Gross wage markup</td>
<td>1.50</td>
<td>( \sigma_c )</td>
<td>Inv subs. elas. of cons.</td>
<td>1.39</td>
</tr>
<tr>
<td>( g_y )</td>
<td>Gov’t ( G/Y ) ss-ratio</td>
<td>0.18</td>
<td>( \kappa )</td>
<td>Degree of ext. habit</td>
<td>0.71</td>
</tr>
<tr>
<td>( \varepsilon_p )</td>
<td>Kimball Elas. GM</td>
<td>10</td>
<td>( \xi_w )</td>
<td>Calvo prob. wages</td>
<td>0.73</td>
</tr>
<tr>
<td>( \varepsilon_w )</td>
<td>Kimball Elast. LM</td>
<td>10</td>
<td>( \sigma_l )</td>
<td>Labor supply elas.</td>
<td>1.92</td>
</tr>
<tr>
<td>( \xi_p )</td>
<td>Calvo prob. prices</td>
<td>0.65</td>
<td>( t_w )</td>
<td>Ind. for non-opt. wages</td>
<td>0.59</td>
</tr>
<tr>
<td>( \pi )</td>
<td>Steady state net infl. rate</td>
<td>0.0081</td>
<td>( t_p )</td>
<td>Ind. for non-opt. prices</td>
<td>0.22</td>
</tr>
<tr>
<td>( \psi )</td>
<td>Capital utilization cost</td>
<td>0.54</td>
<td>( \psi )</td>
<td>Ind. for non-opt. prices</td>
<td>0.22</td>
</tr>
<tr>
<td>( \phi_p )</td>
<td>Gross price markup</td>
<td>1.61</td>
<td>( \phi_p )</td>
<td>Gross price markup</td>
<td>1.61</td>
</tr>
<tr>
<td>( \alpha )</td>
<td>Capital production share</td>
<td>0.19</td>
<td>( \gamma )</td>
<td>Steady state gross growth</td>
<td>1.0043</td>
</tr>
</tbody>
</table>

Shock Processes

<table>
<thead>
<tr>
<th>Shock</th>
<th>Persistence</th>
<th>MA(1)</th>
<th>Std. of Innovation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Neutral Technology</td>
<td>( \rho_a )</td>
<td>0.95</td>
<td>-</td>
</tr>
<tr>
<td>Risk premium</td>
<td>( \rho_b )</td>
<td>0.18</td>
<td>-</td>
</tr>
<tr>
<td>Gov’t spending</td>
<td>( \rho_g )</td>
<td>0.97</td>
<td>( \rho_{ga} )</td>
</tr>
<tr>
<td>Inv. Specific Tech.</td>
<td>( \rho_i )</td>
<td>0.71</td>
<td>-</td>
</tr>
<tr>
<td>Price markup</td>
<td>( \rho_p )</td>
<td>0.90</td>
<td>( \mu_p )</td>
</tr>
<tr>
<td>Wage markup</td>
<td>( \rho_w )</td>
<td>0.97</td>
<td>( \mu_w )</td>
</tr>
<tr>
<td>Monetary policy</td>
<td>( \rho_r )</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

Note: SW07 estimates \( \rho_c = 0.12 \) and \( \sigma_c = 0.24 \), but in our optimal policy exercises we set these parameters to nil.

There are two things notice with regards to the parameters in Table 1. First, we scale the standard deviations of the innovations to the price and wage markup shocks so that they have the same variances as they have in the linearized representation of the model in its original form. Second, we set the monetary policy shock parameters to nil, as we restrict our analysis to optimal policy rules (as further discussed below).

2.2. Our Exercise: Finding Optimized Simple Objectives

Rotemberg and Woodford (1998) showed—under the assumption that the steady state satisfies certain efficiency conditions— that the objective function of households can be transformed into a quadratic function. With this quadratic objective function, optimization subject to linearized constraints would be sufficient in obtaining legitimate results from the normative perspective. Some assumptions about efficiency were “unpalatable” as exemplified by the presence of positive subsidy that would make the steady state of the market equilibrium equivalent to that of the social plan-
Therefore, many researchers—including Benigno and Woodford (2012)—extended the LQ transformation to a general setting without the presence of such subsidies. Benigno and Woodford (2012) demonstrates that the objective function of the households can be transformed as follows:

\[
\sum_{t=0}^{\infty} E_0 \left[ \beta^t U(X_t) \right] \simeq \text{constant} - \sum_{t=0}^{\infty} E_0 \left[ \beta^t X_t' W^{society} X_t \right],
\]

where \( X_t \) is a \( N \times 1 \) vector with model variables measured as their deviation from the steady state; therefore, \( X_t' W^{society} X_t \) on the RHS is referred to as the linear-quadratic approximation of the household utility function \( U(X_t) \) on the LHS.

We define Ramsey policy as a policy which maximizes (19) subject to the N-1 constraints of the economy. While \( N \) is the number of variables, there are only N-1 constraints provided by the SW07 model because the monetary policy rule is omitted. Unlike the efficient steady-state case of Rotemberg and Woodford (1998), second-order terms of the constraints do influence the construction of the \( W^{society} \) matrix in (19), and as explained in Section 2.1 we made assumptions on the functional forms for the various adjustment functions (for example, the capital utilization rate \( a(Z_t) \), the investment adjustment cost function \( S(I_t/I_{t-1}) \), and the Kimball aggregators) that are consistent with the linearized behavioral equations in SW07.

The Ramsey policy—which is complicated because the \( W^{society} \) matrix is complicated—is defined by

\[
\tilde{X}_{t}^{\text{optimal}} \left( W^{society}; \tilde{X}_{t-1} \right) = \arg \min_{X_t} \sum_{t=0}^{\infty} \beta^t X_t' W^{society} X_t,
\]

where \( \tilde{X}_t \) now includes Lagrange multipliers as well.

We adopt the unconditional expectations operator as a basis for welfare evaluation, which implies that the loss under the Ramsey optimal policy is given by

\[
\text{Loss}^{\text{Ramsey}} = E \left[ \left( X_t^{\text{optimal}} (W^{society}) \right)' W^{society} \left( X_t^{\text{optimal}} (W^{society}) \right) \right],
\]

where \( X_t^{\text{optimal}} \) is the subset of \( \tilde{X}_t^{\text{optimal}} \) by excluding the Lagrange multipliers. Our choice of an unconditional expectation as the welfare measure—despite its inconsistency with the weighted sum of the conditional expectations—is for the sake of simplicity in the exposition of results.\(^{16}\) Using

\(^{15}\) Even when the theoretically motivated research imposed this undesirable assumption, most empirically oriented papers including Christiano, Eichenbaum and Evans (2005) and Smets and Wouters (2003, 2007) did not assume the existence of such positive subsidies.

\(^{16}\) The criterion is closely related to maximization of the conditional welfare when the society’s discount factor, \( \beta \) in the expression \( \left( 1 - \beta \right)^{-1} E_0 \left[ \sum \beta^t X_t^{C_{B}} (W^{C_{B}}; \tilde{X}_{t-1}) \right]' W^{society} \left[ X_t^{C_{B}} (W^{C_{B}}; \tilde{X}_{t-1}) \right] \), is approaching unity. In our case, we have that \( \tilde{\beta} = \beta \gamma^{-\sigma_{c}} = 0.993 \) using the parameter values in Table 1.
as the metric would require a stance on the initial conditions in the welfare evaluations, and we opt for simplicity since ordering among policy options under unconditional welfare is independent of initial conditions.\textsuperscript{17}

Most central banks, however, are subject to a mandate that is much simpler than the Ramsey policy. To be concrete, we assume that a central bank maximizes

\[ E_0 \left[ \sum_{t=0}^{\infty} \beta^t X_t' W^{CB} X_t \right], \]

where \( W^{CB} \) is a sparse matrix with many zeros. Given a simple mandate, the optimal behavior of the central bank is

\[ X_{t}^{optimal} (W^{CB}; X_{t-1}) = \arg \min_{X_t} E_0 \left[ \sum_{t=0}^{\infty} \beta^t X_t' W^{CB} X_t \right]. \tag{21} \]

We assume that congress, or another government body with executive powers, institutes a law that governs the objective and behavior of the central bank. Two alternative assumptions about the specificity of the law will be entertained. In the first case, we assume that the law only specifies both the variables in the quadratic objective and the weights the central bank should assign to them. For example, it can specify (strict) inflation targeting or a dual mandate involving a significant weight on real activity in addition to inflation. As an alternative, we assume the law specifies the form of the objective function but not the weights that should be assigned to each variable in the objective. In this latter case, we assume the central bank determines the weights in \( W^{CB} \) by maximizing the welfare of the households. Formally, the central bank in this case determines the best simple mandate among the class of possible mandates as the solution to the following problem:

\[ W^* (\Omega) = \arg \min_{W^{CB} \in \Omega} E \left[ X_{t}^{optimal} (W^{CB})' W^{society} (X_{t}^{optimal} (W^{CB})) \right], \]

where the weighting matrix \( W^{society} \) is defined by (19) and \( \Omega \) denotes the set of possible simple mandates consistent with the law. Under the assigned or optimized \( W^* \), the realized loss for the society is given by

\[ Loss^{objective} = E \left[ X_{t}^{optimal} (W^*)' W^{society} (X_{t}^{optimal} (W^*)) \right]. \tag{22} \]

To measure the welfare performance the simple mandate, we calculate the level of its loss relative to the loss under the Ramsey policy. In our presentation of results, we express this welfare difference

\textsuperscript{17} See Jensen and McCallum (2010) for a detailed discussion about this criterion—with a comparison to the timeless perspective. They motivate the optimal unconditional continuation policy based on the presence of time inconsistency, since the policy would reap the credibility gains successfully. We note, however, that our approach does not exactly follow theirs in that their optimal steady state could be different from the steady state under the Ramsey policy in a model with steady-state distortions.
in consumption units as follows:

$$CEV = 100 \left( \frac{Loss^{objective} - Loss^{Ramsey}}{\bar{C} \left( \frac{\partial L}{\partial C} | s.s. \right)} \right),$$  \hspace{1cm} (23)$$

where $\bar{C} \left( \frac{\partial L}{\partial C} | s.s. \right)$ can be interpreted as how much welfare increases when consumption is increased by one percent. That is, CEV represents the percentage point increase in households’ consumption, at every state of the world, that makes them—in expectation—equally well-off under the simple mandate as they would be under the Ramsey policy.  \(^{18}\)

### 3. Benchmark Results: Importance of Real Activities

In Table 2 below, we report our benchmark results. The benchmark simple mandate we consider reflects the standard practice of monetary policy, and is what Svensson (2010) refers to as “flexible inflation targeting”. Specifically, we use the textbook treatment in Woodford (2003) and assume the simple mandate can be captured by the following period loss function

$$L_t^q = (\pi_t - \pi)^2 + \lambda^q x_t^2$$ \hspace{1cm} (24)$$

where $\pi_t$ denotes quarterly inflation and $x_t$ is a measure of economic activity. Based on the deep parameters in his benchmark model, Woodford derives a value of 0.003 for $\lambda^q$. Most central banks, however, has a target for the annualized inflation rate. Thus, in practice the relevant weight for the measure of resource utilization is given by

$$16L_t^a = (4 (\pi_t - \pi))^2 + 16\lambda^a x_t^2$$  

$$\equiv (\pi_t^a - \pi^a)^2 + \lambda^a x_t^2$$ \hspace{1cm} (25)$$

where $\pi_t^a$ denotes the annualized rate of inflation, and $\lambda^a \equiv 16\lambda^q$ the rescaled weight on economic activity taking into account that the target variable is the annualized inflation rate. For this case, Woodfords quarterly weight of 0.003 translates into $\lambda^a = 0.048$. In all the tables below (Table 2 included), we will report values for $\lambda^a$.

In the first row in Table 2, we apply Woodford’s weight on three different measures of economy activity. Our first measure is the output gap. The output gap is defined as output minus the level of output that would prevail when prices and wages are fully flexible and the inefficient markup

\(^{18}\) Given the presence of habits, there are two ways to compute CEV. Our measure is calibrated to the case that only the current consumption is increased by a certain percentage.
The second measure we consider is simply the level of output (as deviation from the deterministic labor augmenting trend). Finally, we also consider output growth in the spirit of the work on “speed-limit” policies by Walsh (2003).

Turning to the results in the first row we see, as expected, that adopting a target for the output gap yields the lowest loss; the CEV compensation needed for households is in this case well below that required to be indifferent when having output growth in the loss function. Another important lesson from the first row of the table is that the absolute magnitude of the CEV numbers are quite small; only for output growth the simple mandate generates a somewhat sizeable loss equivalent to a compensation with 0.5 percent of steady state consumption in every period. Given the previous literature on the welfare costs of business cycles (e.g. the seminal work by Lucas, 1987, and subsequent work of Otrok, 2001), this result is not surprising.

Table 2: Benchmark Results for “Flexible Inflation Targeting” Mandate (25).

<table>
<thead>
<tr>
<th>Simple Mandate</th>
<th>Weight (λ*)</th>
<th>CEV (%)</th>
<th>Weight (λ*)</th>
<th>CEV (%)</th>
<th>Weight (λ*)</th>
<th>CEV (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Woodford (2003)</td>
<td>0.048</td>
<td>0.1381</td>
<td>0.048</td>
<td>0.1624</td>
<td>0.048</td>
<td>0.4886</td>
</tr>
<tr>
<td>Dual Mandate</td>
<td>0.250</td>
<td>0.0411</td>
<td>0.250</td>
<td>0.0808</td>
<td>0.250</td>
<td>0.2552</td>
</tr>
<tr>
<td>Optimized Weight</td>
<td>1.042</td>
<td>0.0128</td>
<td>0.542</td>
<td>0.0715</td>
<td>47.088</td>
<td>0.0885</td>
</tr>
</tbody>
</table>

Note: CEV denotes the consumption equivalent variation (in percentage points) needed to make households indifferent between the Ramsey policy and the simple mandate under consideration; see eq. (23). The “Dual Mandate” refers to a weight of unity for the unemployment gap in the loss function (25), which translates into λ* = 0.25 when applying a variant of Okun’s law. Finally, “Optimized Weight” refers to minimization of eq. (22) w.r.t. λ* in eq. (25).

However, the conventional loss criteria that are used in policy circles are rather different from those in academic circles. For examples, the simple mandate for the U.S. Federal Reserve the so-called dual mandate, which stipulates that Fed jointly should pursue stable prices and maximum employment. Prominent academics like Svensson (2011) have interpreted this mandate is a simple loss function in inflation and the unemployment gap (i.e. actual unemployment minus the NAIRU). And in a recent speech, Federal Reserve Board Vice Chair Jane Yellen (2012 - provide citation) applied specific variant in the FRB/US model which assigned equal weights to annualized inflation and the unemployment gap. In the following, we refer to this specification of the loss function as the “Dual Mandate”. However, since a simple empirical specification of Okun’s law converting the unemployment gap to the output gap is given by

\[ u_t - u_t^* = \frac{y_t - y_t^*}{2}, \]

19 Justiniano, Primiceri and Tambalotti (2011) refers to out output gap as actual output minus potential output because it excludes the inefficient shocks. However, notice that our measure of potential output is still below the efficient level because we assume that no steady state subsidy removes the output distortion induced by the steady state price and wage markups. The efficient level of output refers to an outcome when the presence of subsidies restore the steady-state efficiency; it is different from potential output up to a constant (in logs). The deviation of actual output from the efficient or potential output is sometimes referred to as the welfare-relevant output gap.
we have that the unit weight on the unemployment gap converts into a weight of $\lambda^a = 0.25$ on the output gap when inflation is annualized.\footnote{Moreover, Gali, Smets and Wouters (2011) argue within a variant of the SW07 model with unemployment that fluctuations in their estimated output gap are near mirror image of those experienced by the unemployment rate. Therefore, the Okun’s law we apply can also find support in structural modeling framework.} This is roughly five times bigger than the value derived by Woodford. Interestingly, we can see from the second row in Table 2 that the welfare losses are two times smaller in the "Dual Mandate" case for output and output growth, and more that three times smaller for the output gap. Clearly, our analysis suggest that a larger weight than previously recognized should be assigned to stabilize economic activity in addition to inflation, especially when targeting the output gap.

Finally, let’s consider the case when the law governing the central bank implies that it is “free” to optimize over the coefficient in the loss function (but not which variables it targets). As previously noted, we assume the central bank optimize $\lambda^a$ in the simple loss function to maximize household welfare, i.e. minimize $\text{Loss}^{\text{objective}}$ in eq. (22) w.r.t. $\lambda^a$ in eq. (25). The results of this final exercise, which by contraction will improve on the Woodford and Dual Mandate, are reported in the third column in Table 2 under the label “Optimized Weights”. As we can see from the last row, the optimized coefficient for the output gap is 1.05—much higher than in the two preceding ad hoc criterias. In the case when the level of output is used instead of the output gap, the optimized coefficient is about 0.5; in the case of output growth, the optimized coefficient is much higher at 47.1, which is essentially a speed limit regime (see Walsh 2003, AER). Still, responding to the model consistent output gap is the preferred measure from a welfare perspective, and results in a very low loss of 0.01 percentage points relative to Ramsey policy.

3.1. Reason for the Importance of Real Activity

The key message from Table 2 is that the rationale for targeting some measure of real activity is much more important than previously thought in policy circles and recognized in previous influential academic work by e.g. Woodford (2003), Walsh (2005) and others. By perturbing the parameter values (i.e. turning off some bells and whistles) in the model, we seek to nail down why the model suggests to place such a high weight. Without going into to much details, our (tentative) conclusion is that the feature of indexation to lagged inflation for non-optimizing price and wage setters is the single most important reason why the our measures of real activity receive more attention.

We begin the analysis by using the parameters provided in Table 1 to recompute $\lambda^a$ according
to the analytic formula provided in Woodford (2003), which is given by

$$\lambda^a \equiv 16 \frac{\kappa_x}{\phi_p - 1},$$

where $\kappa_x$ is the coefficient for the output gap in the linearized pricing schedule (i.e. in the New Keynesian Phillips curve), and $\frac{\phi_p}{\phi_p - 1}$ is the elasticity of demand of intermediate goods. In the SW07 model, the NKPC is given by

$$\pi_t - \pi_{t-1} = \beta \gamma^{1-\sigma_e} (E_t \pi_{t+1} - \pi_t) + \left(1 - \beta \gamma^{1-\sigma_e} \xi_p \right) \frac{1 - \xi_p}{\xi_p ((\phi_p - 1) \epsilon_p + 1)} \lambda_{ct} + \varepsilon_{p,t}. \tag{27}$$

However, because the SW07 model features endogenous capital and sticky wages, there is no simple mapping between the output gap and real marginal costs within the fully-fledged model. But by dropping capital and the assumption of nominal wage stickiness, we can derive a value of $\kappa_x = 0.143$ in the (simplified) SW07 model.\textsuperscript{21} From the estimated average mark-up $\phi_p$, we then compute $\lambda^a = 0.87$. This value is considerably higher than Woodford (2003) value (0.048) used in Table 2 for mainly two reasons. First, Woodford’s $\kappa_x$ becomes substantially lower due to the assumption of firm-specific labor (the “Yeoman-Farmer” model of Rotemberg and Woodford, 1997). Second, the estimated mark-up in SW07 implies a substantially lower substitution elasticity ($\frac{\phi_p}{\phi_p - 1} = 2.64$) compared to Woodford’s assumed value (7.88).

We caution that this analysis is only tentative, as it by necessity only considers a simplified model without some of the key features in the fully-fledged model. As a consequence, the $\lambda^a$ we computed will only partially reflect the true structure of the entire SW07 model. Yet, the analysis is indicative that a large part of the gap between Woodford’s (2003) value and our benchmark finding of $\lambda^a = 1.042$ stems from differences in household preferences and the substitution elasticity between the intermediate goods.

After this initial exercise, we turn to exploring the mechanisms within the context of the fully-fledged model. Our approach is to turn off or reduce some of the frictions and shocks featured in the model one at a time to isolate the drivers of our results. The findings are provided in Table 3. In the table, the first row restates our benchmark value of $\lambda^a$, i.e. the “Optimized” row of Table 2. That is, this row corresponds to the results when the weights are optimized based on the benchmark calibration of the model. The second row, denoted as "No Indexation," presents the optimal weight on the real-activity term in the case when dynamic indexation is shut down, i.e. calibrated to zero.

\textsuperscript{21} More specifically, we derive $\pi_t - \pi_{t-1} = \beta \gamma^{1-\sigma_e} (E_t \pi_{t+1} - \pi_t) + \kappa_x \left[ x_t - \frac{\xi_p}{1 + \sigma_t (1 - x_t) x_{t-1}} \right] + \varepsilon_{p,t}$ where $x_t$ is the output gap and the slope coefficient $\kappa_x$ equals $\frac{(1 - \beta \gamma^{1-\sigma_e} \xi_p) (1 - \xi_p)}{\xi_p ((\phi_p - 1) \epsilon_p + 1)} \left( \frac{1 + \sigma_t (1 - x_t)}{1 - x_t} \right)$.  

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All the other parameters of the model are kept unchanged. The optimized weight on the output gap in the benchmark case is about 1.05, but the “No Indexation” calibration lowers the optimized weight to roughly 0.3 – less than a third of the benchmark value. In the other columns where real activities are captured by the level and growth rate of detrended output, the optimized weights are also found to be only about a third of the benchmark values.

<table>
<thead>
<tr>
<th>Simple Mandate</th>
<th>$x_t \rightarrow$ Output gap</th>
<th>$x_t \rightarrow$ Output (dev from trend)</th>
<th>$x_t \rightarrow$ Output growth</th>
</tr>
</thead>
<tbody>
<tr>
<td>Benchmark</td>
<td>Weight ($A^2$)</td>
<td>CEV (%)</td>
<td>Weight ($A^2$)</td>
</tr>
<tr>
<td>No Indexation</td>
<td>0.318</td>
<td>0.0122</td>
<td>0.179</td>
</tr>
<tr>
<td>No $\varepsilon_t^p$ Shocks</td>
<td>0.914</td>
<td>0.0115</td>
<td>0.343</td>
</tr>
<tr>
<td>No $\varepsilon_t^w$ Shocks</td>
<td>2.094</td>
<td>0.0058</td>
<td>0.355</td>
</tr>
<tr>
<td>No $\varepsilon_t^p$ and $\varepsilon_t^w$ Shocks</td>
<td>N.I.</td>
<td>0.0047</td>
<td>0.161</td>
</tr>
</tbody>
</table>

Note: “No Indexation” refers to setting $\varepsilon_p = \varepsilon_w = 0$. No $\varepsilon_t^p$ ($\varepsilon_t^w$) refers to setting the variance of the “price markup shock” (“wage markup shock”) to 1/100 of its variance in the benchmark calibration, whereas no $\varepsilon_t^p$ and $\varepsilon_t^w$ refers to setting the variance of both shocks to 1/100 of their baseline values. N.I. stands for not identified.

To understand why indexation makes the real-activity term much more important than in a model with no indexation, it is instructive to consider the following simple New Keynesian model with indexation and sticky prices only. If we compute a micro-founded welfare-based approximation to the household utility function following Woodford (2003), such a model would feature the following terms in the approximated loss function

$$\left(\pi_t - \varepsilon_p\pi_{t-1}\right)^2 + \lambda \left(y_{t \text{ gap}}\right)^2,$$

where $\varepsilon_p$ is the indexation parameter in the pricing equation. Suppose further, for simplicity, that inflation dynamics in equilibrium can be represented by an AR(1) process:

$$\pi_t = \rho \pi_{t-1} + \varepsilon_t.$$

In this simple setup, the welfare metric would include the terms

$$E_0 \left[\left(\rho - \varepsilon_p\right)^2 \left(\pi_{t-1}\right)^2 + \lambda \left(y_{t \text{ gap}}\right)^2\right].$$

In more empirically relevant models like SW07, intrinsic inflation persistence ($\rho$) is in large part explained by the indexation parameter ($\varepsilon_p$). Therefore, these two parameter values are pretty similar, and the coefficient in front of the inflation term tends to become smaller. Hence, in a loss function like ours (eq. 25) where the inflation coefficient is set to unity, the coefficient in front of real activities tend to become relatively larger—as evidenced in Table 2.
Finally, rows 3-5 in Table 3 examine the role of the inefficient “markup” shocks in the model. By comparing the results in the third and fourth rows, we see that the wage markup shock contributes the most to the welfare costs of the simple mandate. But even when one of these shocks are taken out of the model, the CB should still respond vigorously to economic activity in order to maximize welfare of the households inhabiting the economy. Only if both shocks are taken out jointly, \( \lambda^a \) falls sharply for output and output growth. However, the weight on the output gap is in this case essentially not identified when considering the model consistent output gap, since there is no-longer any trade-off between responding to inflation and the output gap. As a consequence, any \( \lambda^a > 0 \) produces roughly the same CEV (0.0047). [Jesper: We should have a graph with this case and the benchmark case].

4. Robustness Analysis

In this section, we explore the robustness of our results along some key dimensions. We first examine to what extent adding labor market variables like e.g. hours worked and wage inflation to the loss make improves welfare. Second, we consider the merits of so-called speed limit policies analyzed by Walsh (2003). Third and finally, we consider the extent to which the implied interest rate movements for the considered simple mandates are reasonable.

4.1. The Quest for a Different Objective: Should Labor Market Variables be considered?

One of the reasons for the popularity of inflation targeting comes from the results in the New Keynesian literature – importantly Clarida, Gali and Gertler (1999) and Woodford (2003) – that inflation in the general price level is costly to the economy. However, the old Keynesian literature emphasized the importance of wage inflation in addition to price inflation as an important element on both positive and normative grounds, see e.g. Erceg, Henderson and Levin (2000) and Gali (2011).\(^{22}\) In the SW07 model where both wages and prices are sticky, it is possible that wage inflation may be equally or even more important to stabilize than price inflation. In addition to consider wage inflation, we think it is of interest to examine to what extent other labor market variables like hours worked can substitute for overall economic activity within the model. Hence,

\(^{22}\)Kim and Henderson (2005) compared different types of rigidities and included older citations.
we propose to study the following augmented loss function:

\[ L^a_t = \lambda_n^a (\pi^a_t - \pi^a)^2 + \lambda^a x^2_{1,t} + \lambda^a_{\Delta w} (\Delta w^a_t - \Delta w^a)^2 + \lambda^a_{\Delta l} x^2_{2,t}, \tag{28} \]

where \( \Delta w^a_t \) denotes annualized wage inflation, \( x_t \) is – as before – a measure of overall economic activity, and \( l_t \) is a measure of activity in the labor market.

In Table 4, we report results for this augmented loss function (28). The first row restates the “Benchmark” results, i.e. the optimized coefficients in Table 2. The second row adds wage inflation to the loss function by allowing \( \lambda^a_{\Delta w} > 0 \). Relative to the unitary weight on the inflation term, the optimized objective function would ask for a weight of roughly 3.2 for the output gap term, and an optimal weight of about 1.5 for the wage inflation term—higher than the price inflation term. The level of welfare is also substantially higher (32.8 percent) than under the benchmark case. Unlike wage inflation, inclusion of the number of hours from its potential level does not change the welfare results as dramatically. However, responding to price inflation is still important, a feature we learn from the third row in the table where \( \Delta w_t \) replaces \( \pi_t \); in this case the welfare are slightly more modest.

Next, we introduce the labor market gap, defined as \( l_t - l^pot_t \), as an additional target variable in the fourth column of Table 4. Unlike wage inflation, inclusion of the labor market gap does not, by itself, increase welfare much. Moreover, given that price inflation is the nominal anchor, replacing the output gap with the hours gap results in a welfare loss as can be seen from the fifth row. However, when price inflation is replaced by wage inflation as a target variable, the labor gap performs much better than the output gap (third row), and this labor market oriented simple mandate generates substantial gains relative to our benchmark specification.

<table>
<thead>
<tr>
<th>Table 4: Variations of the Loss Function: Gap Variables in (28).</th>
</tr>
</thead>
<tbody>
<tr>
<td>Loss Function</td>
</tr>
<tr>
<td>-------------------------------</td>
</tr>
<tr>
<td>Benchmark</td>
</tr>
<tr>
<td>Adding ( \Delta w_t )</td>
</tr>
<tr>
<td>Replacing ( \pi_t ) with ( \Delta w_t )</td>
</tr>
<tr>
<td>Adding ( l_{t}^gap )</td>
</tr>
<tr>
<td>Replacing ( y^gap_{t} ) with ( l_{t}^gap )</td>
</tr>
<tr>
<td>Replacing ( [\pi_t, y^gap_{t}] ) with ( [\Delta w_t, l_{t}^gap] )</td>
</tr>
</tbody>
</table>

Note: The table reports variations of the simple objective (28). Only “gap” variables are used as measures for \( x_{1,t} \) and \( x_{2,t} \). The numbers in the “Gains” column are computed as \( 100 \left(1 - \frac{CEV_{L_Falt}}{0.0128}\right) \) where \( CEV_{L_Falt} \) is the CEV for the alternative loss function and 0.0128 is the “Benchmark” objective CEV (row 1).

The role played by wage inflation is not as prominent in the case when \( x_{1,t} \) in (28) is represented by output (as deviation from trend) instead of the output gap. As shown in Table 5, the welfare
gain from the benchmark case is only 5.3 percent higher when wage inflation is included in (28). Accordingly, welfare is reduced by one percent – the third row – when inflation is omitted. On the other hand, adding hours worked per capita enhances the welfare of households by nearly 30 percent - a substantial gain. Finally, we see from the last row that a mandate with only wage inflation and hours worked performs the best, and reduces the welfare costs of the simple mandate by over 60 percent relative to the benchmark objective.

Jesper: We should have a graph here with the CEV on the y-axis and the value of \( \lambda^a \) for the benchmark specification and the wage inflation and hours gap specification, to examine if the good performance of the latter is more sensitive to a specific value of hours.

Table 5: Variations of the Loss Function: Level Variables in (28).

<table>
<thead>
<tr>
<th>Loss Function</th>
<th>( \lambda^a ) - ( \pi^a_t )</th>
<th>( \lambda^a - y_t - \bar{y}_t )</th>
<th>( \lambda^a_w - \Delta w_t )</th>
<th>( \lambda^a_l - l_t - \bar{l} )</th>
<th>CEV (%)</th>
<th>Gains</th>
</tr>
</thead>
<tbody>
<tr>
<td>Benchmark</td>
<td>1.000</td>
<td>0.544</td>
<td>-</td>
<td>-</td>
<td>0.0715</td>
<td>-</td>
</tr>
<tr>
<td>Adding ( \Delta w_t )</td>
<td>1.000</td>
<td>0.954</td>
<td>0.463</td>
<td>-</td>
<td>0.0674</td>
<td>5.3%</td>
</tr>
<tr>
<td>Replacing ( \pi_t ) with ( \Delta w_t )</td>
<td>-</td>
<td>1.054</td>
<td>1.000</td>
<td>-</td>
<td>0.0722</td>
<td>-1.0%</td>
</tr>
<tr>
<td>Adding ( l_t - \bar{l} )</td>
<td>1.000</td>
<td>0.392</td>
<td>-</td>
<td>1.344</td>
<td>0.0502</td>
<td>29.8%</td>
</tr>
<tr>
<td>Replacing ( y_t - \bar{y}_t ) with ( l_t - \bar{l} )</td>
<td>1.000</td>
<td>-</td>
<td>-</td>
<td>2.947</td>
<td>0.0616</td>
<td>13.8%</td>
</tr>
<tr>
<td>Replacing ( [\pi_t, y_t - \bar{y}_t] ) with ( [\Delta w_t, l_t - \bar{l}] )</td>
<td>-</td>
<td>-</td>
<td>1.000</td>
<td>3.475</td>
<td>0.0473</td>
<td>33.8%</td>
</tr>
</tbody>
</table>

Note: The table reports variations of the simple objective (28). Only ‘level’ variables are used as measures for \( x_{1,t} \) and \( x_{2,t} \). The numbers in the “Gains” column are computed as 100 \( \frac{CEV_{LFalt} - CEV_{LFalt}}{CEV_{LFalt}} \) where CEV_{LFalt} is the CEV for the alternative loss function and 0.0715 is the “Benchmark” objective CEV (row 1).

Our conclusion is that while a standard objective with price and inflation and the output gap generates small welfare losses relative to Ramsey policy (around 0.01% of steady state consumption), it makes sense within the SW07 model – which features substantial frictions in the labor market – to target wage inflation and a labor market gap instead. Doing so will reduce the welfare costs of the simple mandate even further. Moreover, we have shown that this conclusion is robust even if one considers the level of output and hours worked instead of their deviations around potential. Furthermore, regardless if the objective focus on price or wage inflation, we always find a robust role for responding vigorously to economic activity (may it be output or hours worked), in line with our benchmark results in Table 2.

4.2. The Quest for a Different Objective: Speed Limit Policies

In this subsection, we examine the performance of so-called speed limit policies (SLP henceforth) advocated by Walsh (2003). Although Walsh formulation of SLP was growth relative to potential, we report results for the growth relative to steady state to understand how contingent the results are on measuring the change in potential accurately. Moreover, since the results in the previous
subsection suggested that labor market based objective performed very well within our framework, we study the performance of SLP for a labor market based simple mandate as well.

The results are reported in Table 6. As can be seen from the table, we report results for two parameterizations of the objective: the benchmark weight derived in Woodford (2003) – row 1 – and a weight that is optimized to maximize household welfare (row 2). Obviously, the results for $\Delta y_t$ are identical to those reported in Table 2. Interestingly, we see that when replacing the level of output growth with the growth rate of the output gap ($\Delta y_t^{gap}$), welfare is increased substantially conditional on placing a sufficiently large coefficient on the growth rate of the output gap. However, by comparing these results with those for $y_t^{gap}$ in Table 2, we see it is still better to target the output gap in levels.

Turning to the objectives based on wage inflation and hours, we see that they perform less well than the standard inflation-output objectives unless the weight on the labor concept is sufficiently large. As is the case for output, the growth rate of the labor gap is preferable to the growth rate of labor itself, but by comparing with the results in Table 4 we see that targeting the level of the labor gap is still highly preferable in terms of maximizing welfare of the households.

<table>
<thead>
<tr>
<th>Parameterization</th>
<th>Price Inflation Objective</th>
<th>Wage Inflation Objective</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$x_t - \Delta y_t$</td>
<td>$x_t - \Delta y_t^{gap}$</td>
</tr>
<tr>
<td></td>
<td>$x_t - \Delta \lambda^a$</td>
<td>$x_t - \Delta \lambda^a$</td>
</tr>
<tr>
<td>Woodford</td>
<td>0.048 0.4886</td>
<td>0.048 0.4764</td>
</tr>
<tr>
<td>Optimized</td>
<td>47.088 0.0885</td>
<td>11.090 0.0222</td>
</tr>
</tbody>
</table>

Note: The loss function under price inflation is specified as in (25), while the loss function with the annualized nominal wage inflation rate wage is specified as $(\Delta w_t^\pi - \Delta w^\pi)^2 + \lambda^a x_t^2$. $\Delta y_t$ denotes output growth as deviation from the s.s. growth rate ($\gamma$). $\Delta y_t^{gap}$ is the first difference of output as deviation from potential, i.e. $\Delta y_t - \Delta y_t^{pot}$. Same definitions apply to hours worked. See notes to Table 2 for further explanations.

4.3. The Quest for a Different Objective: Price- and Wage-Level Targeting

Several important papers in the previous literature have stress the merits of price level targeting as opposed to the standard inflation targeting loss function. Price level targeting, i.e. a commitment to eventually bring back the price level to a baseline path in the face of shocks that creates a trade-off between stabilizing inflation and economic activity. Our benchmark flexible inflation targeting objective in eq. (25) can be reformulated into a price level targeting objective as follows:

$$L_t^a = (p_t - \bar{p}_t)^2 + \lambda^a x_t^2,$$

where $p_t$ is the log-price level in the economy and $\bar{p}_t$ is the target price level path which grows with the steady state net inflation rate $\pi$ according to $\bar{p}_t = (1 + \pi) \bar{p}_{t-1}$. We adopt a specification
isomorphic to that in (29) when we consider wage level targeting, but replace the first term with \( w_t - \bar{w}_t \) where \( w_t \) is the nominal wage and \( \bar{w}_t \) is the nominal target wage which grows according to \( \bar{w}_t = \gamma (1 + \pi) \bar{w}_{t-1} \) where \( \gamma \) is the gross technology growth rate of the economy (see Table 1).

In Table 6.b, we report results for both price- and wage level targeting objectives. As can be seen from the table, there are no welfare gains from pursuing price-level targeting relative to our benchmark objective in Table 2 (which yielded av CEV of 0.0128 for the price-inflation output gap specification), regardless if targeting the output or the hours gap. For wage-level targeting, we obtain the same finding (in this case, the relevant comparison is the wage-inflation hours gap specification in Table 4 which yields a CEV of 0.0047). This finding are not perhaps surprising, given that the welfare costs in our model is more associated to changes in prices and wages (because of indexation) as opposed to accumulated price- and wage-inflation rates.

<table>
<thead>
<tr>
<th>Table 6.b: Sensitivity Analysis: Merits of Price and Wage Level Targeting.</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Parameterization</strong></td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>Woodford</td>
</tr>
<tr>
<td>Optimized</td>
</tr>
</tbody>
</table>

Note: The loss function under price-level targeting is given by (29), while the loss function with the annualized nominal wage inflation rate wage is specified as \((w_t - \bar{w}_t)^2 + \lambda^a x_t^2\). See notes to Table 2 for further explanations.

4.4. Volatility of Interest Rates

As previously discussed, the main message of our benchmark results in Table 2 is that real activities are more important than previously recognized by academics (e.g. Woodford, 2003) and policymakers (e.g. Yellen, 2012). In addition to inflation and some measure of resource utilization, simple objectives often includes a term involving the volatility of interest rates; see e.g. Rudebusch and Svensson (1999). In practice, this term is often motivated by reference to “aversion to interest-rate variability”. From a theoretical perspective, Woodford (2003) derives an extended version of (25) augmented with an interest rate gap term

\[ \lambda_r (r_t^a - r^a)^2 \]

when allowing for monetary transactions frictions (\( r_t^a - r^a \) is the deviation of the annualized nominal policy rate \( r_t^a \) around the steady state annualized policy rate \( r^a \)). As an alternative, many researchers (e.g. Rudebusch and Svensson, 1999) and policymakers (e.g. Yellen, 2012) instead consider augmenting the objective function with the variance of the change in the short-run interest
rate, i.e.

$$\lambda_r (\Delta r_t^a)^2$$

where $\Delta$ denotes the first-difference operator. By allowing for a lag of the interest rate in the loss function, the specification introduces interest rate smoothing, as the reduced form solution will feature the lagged interest rate in the central bank’s reaction function. Both specifications, however, will reduce volatility of policy rates because the central bank will, ceteris paribus, tend to be less aggressive in the conduct of monetary policy when $\lambda_r > 0$.

The first row in Table 7 consider the standard Woodford (2003) specification with only $x_t$ as an additional variable to inflation in (25). The second row in the table includes the $(r_t^a - r^a)^2$ term in the loss function and uses Woodfords’ (2003) weights for economic activity and the interest rate (0.048 and 0.077, respectively). The third row reports results for Yellen’s (2012) specification of the loss function which includes the $(\Delta r_t^a)^2$ term in the loss function instead of $(r_t^a - r^a)^2$ and uses different weights on economic activity and the interest rate (0.25 and 1.00, respectively). Finally, the last two rows present results when the coefficient on $x_t$ and the interest rate gap – row 4 – and the change in the interest rate gap – row 5 – are optimized to maximize welfare of the households.

Turning to the results, we see by comparing the first and second rows in the table that the CEV is not much affected by the introduction of the interest term for the output gap and output, but for output growth the inclusion of the interest rate volatility reduces the welfare costs by more than a factor of 2. Comparing the third row – the Yellen parameterization – with the Woodford specification in the second row, we see that while welfare improves considerably for all three different $x_t$ variables, it is only for output growth that this improvement stems from the interest rate term. For the output gap and output, the improvement is all due to the higher $\lambda^a$, which can be confirmed by comparing the “Dual Mandate” row in Table 2 with the Yellen row in Table 7.

When allowing the central bank to optimize the weights, we find that the objective function that includes three terms – the last two rows in Table 7 – we find that the optimized weight on the interest rate term in both cases are driven towards a very low numbers, implying that the welfare improvements are marginal. Only for output, we find a modest improvement from 0.0715 to about 0.063 from including any of the two interest rate terms. Hence, our main message that the real activities should carry a large weight still holds up.
Table 7: Sensitivity Analysis: Minimization of $\pi_t - \pi^a$ with an interest rate term.

<table>
<thead>
<tr>
<th>Loss Function</th>
<th>Output Gap $x_t$</th>
<th>Output (dev from trend) $x_t$</th>
<th>Output Growth $x_t$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Woodford: only $x_t$</td>
<td>$\lambda^a$</td>
<td>$\lambda_p$</td>
<td>CEV (%)</td>
</tr>
<tr>
<td>Woodford: $r_t^a - r^a$</td>
<td>0.048</td>
<td>0.1381</td>
<td>0.048</td>
</tr>
<tr>
<td>Yellen: $\Delta r_t^a$</td>
<td>0.250</td>
<td>1.0000</td>
<td>0.0546</td>
</tr>
<tr>
<td>Optimized: $\Delta r_t^a$</td>
<td>1.042</td>
<td>0.0001</td>
<td>0.0127</td>
</tr>
<tr>
<td>Optimized: $\Delta r_t^a$</td>
<td>1.042</td>
<td>0.0001</td>
<td>0.0127</td>
</tr>
</tbody>
</table>

Note: The loss function with the level of the interest rate is specified as $(\pi_t^a - \pi^a)^2 + \lambda^a x_t^2 + \lambda_p (r_t^a - r^a)^2$, while the loss function with the change in the interest rate is specified as $(\pi_t^a - \pi^a)^2 + \lambda^a x_t^2 + \lambda_p (\Delta r_t^a)^2$. See notes to Table 2 for further explanations.

One of the concerns on financial stability is related to the fact that the nominal interest rate is commonly used as an instrument of monetary policy. In this vein, high volatility of optimal interest rates will be problematic if such an optimal policy is to be implemented; especially if the probability distribution of nominal rates for the mandates under consideration covers the negative range in a nontrivial way. A standard approach to check whether the zero lower bound raises a problem is to compute the standard deviation of the nominal interest rate: Rotemberg and Woodford (1998) adopts the rule-of-thumb that the steady state nominal rate minus two standard deviation (std) of the rate should be non-negative. Others, like Adolfson et al. (2011) have adopted a three std non-negativity constraint. Since our adopted parameterization of the SW07 model implies an annualized nominal interest rate of 6.25 percent, the allowable std is 3.125 under the Rotemberg and Woodford rule-of-thumb and slightly below 2.1 under the stricter three std criterion adopted by Adolfson et al. (2011).

Table 8 report the result of our exercise. For brevity of exposition we focus on the output gap only, but the results can be shown to be very similar for output and output growth as well. As seen from the first three rows in the table, the objective functions in Table 2 that involve only inflation and the output gap produce high interest rate volatilities. The std’s are all around 9 percentage points – a few times bigger than our thresholds. Hence, these loss functions are contingent on unrealistically large movements in the policy rate. Turning to the fourth and fifth rows, which report results for the Woodford and Yellen loss functions augmented with an interest rate terms, we see that the std’s shrink by almost a factor of ten; these specifications are hence clearly consistent with the non-negativity constraint on nominal rates.
Table 8: Interest Volatility for Output Gap in Loss Function.

<table>
<thead>
<tr>
<th>Loss Function</th>
<th>$\lambda^a$ − Output Gap</th>
<th>$\lambda_r$</th>
<th>CEV (%)</th>
<th>std($r_t^a$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Woodford</td>
<td>0.048</td>
<td>−</td>
<td>0.1381</td>
<td>8.92</td>
</tr>
<tr>
<td>Dual Mandate</td>
<td>0.250</td>
<td>−</td>
<td>0.0411</td>
<td>8.76</td>
</tr>
<tr>
<td>Optimized</td>
<td>1.042</td>
<td>−</td>
<td>0.0128</td>
<td>9.00</td>
</tr>
<tr>
<td>Woodford: $r_t^a − r^a$</td>
<td>0.048</td>
<td>0.0770</td>
<td>0.1353</td>
<td>0.98</td>
</tr>
<tr>
<td>Yellen: $\Delta r_t^a$</td>
<td>0.250</td>
<td>1.0000</td>
<td>0.0546</td>
<td>1.24</td>
</tr>
<tr>
<td>Optimized*: $r_t^a − r^a$</td>
<td>1.161</td>
<td>0.0770*</td>
<td>0.0222</td>
<td>2.24</td>
</tr>
<tr>
<td>Optimized*: $\Delta r_t^a$</td>
<td>1.110</td>
<td>1.0000*</td>
<td>0.0246</td>
<td>2.04</td>
</tr>
</tbody>
</table>

Note: std($r_t^a$) denotes the standard deviation for the annualized nominal interest rate. The * in the last two rows denote that these values have been fixed, and are hence not optimized.

The last two rows in the table report results when we re-optimize the weight on the output gap ($\lambda^a$) given certain weights (0.077 for ($r_t^a − r^a$)² in the next-to-last row and 1 for ($\Delta r_t^a$)² in the last row) on the interest rate terms in the loss function. As seen from the last column, these policies generate considerably lower interest volatility relative to the optimized loss function which excludes any interest rate terms, and the obtained std’s are in line with even the three std threshold applied by Adolfson et al. (2011). To compensate for the presence of the interest rate term, the optimization results in a slightly higher $\lambda^a$ compared to the simple loss function with the output gap only. Despite this, the lower flexibility to adjust policy rates is associated with a lower welfare; the CEV roughly doubles in both cases. We conclude that our benchmark result on the importance of the real activity term survives the test regarding the concern on interest rate volatility.

5. Simple Interest Rate Rules

Up to this point, we have considered how simple mandates would fare in terms of welfare metric and tried to find the best simple mandate among a certain of class. In this section, we instead consider a simple class of policy rules. We do so for two reasons. First, we are interested to what extent simple rules can approximate the Ramsey policy equally well as a simple loss function based approach. Second, we are interested to know how a simple policy rule should be parameterized to implement our benchmark simple mandate.

To be concrete, the central bank is posited to implement monetary policy by following a certain simple rule. Once a rule is adopted, the model will become complete, and the solution would describe the behavior of the economy. The central bank, however, is assumed to be able to choose the response coefficients in the simple rule to maximize either household welfare or minimize the loss of the simple mandate.

If we denote the solution as $X_t (R; X_{t-1})$, where $R$ represent a specific policy rule. Then, the
optimized simple rule that maximizes household welfare is defined by

\[ R^{\text{Ramsey}}(\Lambda) = \arg \min_{R \in \Lambda} \mathbb{E} \left[ (X_t (R; X_{t-1})') W^{\text{society}} (X_t (R; X_{t-1})) \right], \]

where \( \Lambda \) is the set of possible parameterizations of the rule. The resulting loss for the society is

\[ \text{Loss}^{R-\text{rule}}(\Lambda) = \mathbb{E} \left[ (X_t (R^* (\Lambda)))' W^{\text{society}} (X_t (R^* (\Lambda))) \right]. \]  \( \text{(30)} \)

Given our previous results that an objective with inflation and the output gap provides a good approximation of household welfare, we consider variants of the following simple rule:

\[ r_t^a - r^a = (1 - \rho_r) [\varrho_\pi (\pi_t^a - \pi^a) + \varrho_y (y_t^{\text{gap}}) + \varrho_{\Delta y} \Delta y_t^{\text{gap}}] + \rho_r (r_{t-1}^a - r^a). \]  \( \text{(31)} \)

But since the results in Table 4 documented that an objective with wage inflation and the hours gap even better approximated the true household welfare function, we also entertain variants of the following simple interest rate rule:

\[ r_t^a - r^a = (1 - \rho_r) [\varrho_{\Delta w} (\Delta w_t^a - \Delta w^a) + \varrho_l (l_t^{\text{gap}}) + \varrho_{\Delta l} \Delta l_t^{\text{gap}}] + \rho_r (r_{t-1}^a - r^a). \]  \( \text{(32)} \)

The results of this exercise are reported in Table 9. In the table, Panel A report results when the objective is to minimize (30) and the rule is given by (31). Panel B, on the other hand, report results when the rule is given by (32).

Turning to the results the first panel in Table 9, we see that neither a Taylor (1993) nor a Taylor (1999) version of (31), which sets \( \varrho_\pi = 1.5, \varrho_y = 0.5 \) and \( \varrho_{\Delta y} = \rho_r = 0 \), can approximate the Ramsey policy well using the 0.05 CEV cut-off rule by Schmitt-Grohe and Uribe. Both policy rules increase CEV substantially relative to the simple optimized objective in Table 2, as shown by the loss values in the last column. When we allow for interest rate smoothing - the third row in the table - we see that welfare is improved notably. When we include the change in the output gap in the simple rule (fourth row), we find that welfare is again improved markedly, and the optimal coefficient for \( \varrho_{\Delta y} \) is very high. [Jesper: We should recompute the optimized coefficients to ensure that \( \text{std}(r_t^a) \) is not unreasonable (i.e. are than the interest rate thresholds assumed to Adolfson et al. (2011) and Schmitt-Grohé and Uribe).

Finally, we see from Panel B that the performance of (32) is similar to the standard inflation-output gap based rule in Panel A.
Table 9: Performance of Simple Rules (31).

<table>
<thead>
<tr>
<th>Panel A: Inflation and Output Gap Rule (31)</th>
</tr>
</thead>
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<tr>
<td>Parameterization</td>
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<td>Optimized: $\varphi_{\Delta y} = 0$</td>
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<td>Optimized</td>
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<tr>
<th>Panel B: Labor Market Rule (32)</th>
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<tbody>
<tr>
<td>Parameterization</td>
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<tr>
<td>Taylor (1993)</td>
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<td>Optimized: $\varphi_{\Delta l} = 0$</td>
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<tr>
<td>Optimized</td>
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Note: The “Optimized” coefficients in the panels are found by maximizing household welfare, i.e. minimizing CEV. The value in “Loss” column is computed as $100 \left(1 - \frac{0.0128}{\text{CEV}} \right)$, where 0.0128 is CEV for the benchmark objective in Table 2 (“Optimized”). The loss for the policy rule under consideration relative to the CEV for the benchmark optimized simple objective in Table 2. We constrain the smoothing coefficient $\rho_r$ to be between 0-0.99, and the response coefficients $\varphi_{\Delta y}$ and $\varphi_{\Delta l}$ to be between 0-10.

6. Conclusions

- Introduce financial frictions and see if this introduces a rationale for leaning against the wind when monetary policy is approximated with simple objectives and/or simple rules.

- Hardwire transmission lags of monetary policy into the model would be an interesting extension for future work.

[Remains to be written.]
References


Rudebusch, Glenn and Lars E.O. Svensson (1999), Inflation Targeting


