Central bank credibility and the persistence of inflation and inflation expectations*

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Abstract

The standard new Keynesian model with rational expectations has trouble accounting for the observed persistence in inflation and volatility of inflation expectations. Modifications like price indexation help the model match the observed dynamics of inflation and short-run inflation expectations, but the model still cannot account for the observed variability of long-run inflation expectations. This paper introduces a model where agents are unsure about the central bank's inflation target. They believe that the central bank's inflation target could lie between two extremes, and their beliefs vary depending on the central bank's stock of credibility. They form the expectations used in price and wage setting using this perceived inflation target, and they use past observations of inflation to update their beliefs about the credibility of the central bank. Thus a series of high inflation observations can lead them to believe (incorrectly) that the central bank has adopted a high target. The model can match the observed volatility and persistence of both inflation and long-term inflation expectations. The model is then calibrated to match the observed levels of Federal Reserve credibility in the 1980's and the 2000's. By simply changing the level of credibility, holding all else fixed, the model can explain nearly all of the observed changes in the volatility and persistence of inflation and inflation expectations in the U.S. since the 1980's.

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1 Introduction

Milton Friedman said that "Inflation is always and everywhere a monetary phenomenon" (1968). Friedman was careful to qualify that inflation is a "steady and sustained rise in prices", for while a number of factors can lead to a transitory movement in prices in the short run, only monetary policy can cause a sustained rise in the price level over the medium to long run. Movements in current inflation or even agents' expectations of inflation over the next year could be driven by a number of factors unrelated to monetary policy, their expectation of inflation over the long run should be entirely driven by their perception of monetary policy.

The benefit of setting a credible inflation target is that it anchors long-run inflation expectations (Bernanke, Laubach, and Mishkin, 2001). If the central bank announces that it will keep inflation at $x\%$ over the medium to long run, and agents believe them, then long-run inflation expectations should be $x\%$. Even without a formal inflation target the central bank can still communicate to the public its desired inflation rate over the long run and if the central bank is credible, then long-run inflation expectations should be constant at the announced rate. Most developed country central banks express their desire for low and stable inflation, but comparing the evidence both across time and across countries shows that their record in anchoring long-run inflation expectations is mixed.

Williams (2006) and Stock and Watson (2007) find that U.S. inflation is less responsive to its own lags now than in the 1970s. They argue this is because inflation expectations are better anchored now than they were in the 1970's, and thus transitory fluctuations in inflation do not affect inflation expectations. Similarly, Blanchard and Gali (2007), Blanchard and Riggi (2009), and Evans and Fisher (2011) argue that the reason that oil price shocks in the 1970's had a large effect on inflation but that shocks of similar magnitude in the 2000's did not is because improved central bank credibility which has served to better anchor inflation expectations. Leduc, Sill, and Stark (2007), Mehra and Herrington (2008), and Clark and Davig (2011) find that U.S. inflation expectations are much less volatile and much less responsive to macroeconomic news and commodity prices now than they were in the 1970's. Goodfriend and King (2005) examine public statements by Federal Reserve policy makers and the transcripts of FOMC meetings during the Volcker disinflation in the early 1980's and show that the Fed saw regaining credibility as the key step towards anchoring inflation expectations.

Gürkaynak, Sack, and Swanson (2005) find that in the U.S., long-run inflation expectations, proxied by far forward Treasury yields, respond to macroeconomic news. Long-forward rates, which they argue are mainly composed of inflation expectations, should not respond to macroeconomic news if long-run inflation expectations are truly anchored. Gürkaynak, Levin, and Swanson (2006) do a similar exercise but compare the response of far forward rates in the U.S., the UK, and Sweden to macroeconomic news. They find that far forward rates respond very little to news in inflation targeting Sweden and respond the most in the U.S. Their sample contains data from the UK from both before and after the independence of the Bank of England. They find that far forward rates from pre-independence UK behave more like those from the U.S., but far forward rates from post-independence UK behave more like Sweden. Similarly Beechey, Johannsen, and Levin (2011) use far forward inflation expectations derived from inflation swaps and find that far forward inflation expectations in the U.S. are more sensitive to current macroeconomic news than far forward expectations in a number of European countries.

Since inflation expectations are incorporated into wage and price setting, which then affect the price level in the future, the unanchoring of inflation expectations is closely related to the persistence of inflation. Benati (2008) estimates inflation persistence in many different countries across many different monetary regimes. He finds that inflation persistence was near zero in many of the countries on the gold standard, while he cannot reject the hypothesis that in many developed countries inflation followed a random walk throughout much of the post-WW2 period. He finds that in the post-Volcker United States, inflation does not follow a random walk but the persistence parameter is still positive and significant, while persistence is near zero in many inflation targeting countries.

Despite all of the attention paid to the persistence and variability of inflation and inflation expectations in the empirical literature, the standard New Keynesian model with rational expectations cannot reproduce many of the dynamics of inflation and inflation expectations that we observe in the data. Authors usually include rule-of-thumb pricing behavior, as in Gali and Gertler (1999), or price indexation, as in Christiano, Eichenbaum, and Evans (2005), to introduce what Fuhrer (2006; 2011) refers to as "intrinsic" inflation persistence.[1] These features usually help the model explain the persistence of inflation or the dynamics of short-run inflation expectations, but as this paper will show, the New Keynesian model with rational expectations cannot explain the variability in long-run inflation expectations that we observe in many countries.

To explain the persistence of inflation, but most importantly the persistence and variability of long-run inflation expectations, this paper will construct a model where agents are unsure about the central bank's inflation target. Agents believe the central bank's inflation

[1] Mankiw and Reis (2002) introduce a model of sticky information, as opposed to sticky prices, where agents slowly accumulate the information necessary to set prices. They find that this sticky information model can reproduce the inflation persistence that we observe in the data as well as the output cost of disinflation.

target could vary between a high target and a low target, and expectations about the future are based on a weighted average of outcomes under the high and low target scenarios. The weight they place on the low target is referred to as the central bank's stock of credibility. Agents use past observations of inflation to update this stock of credibility, and thus their belief about the central bank's target.[2] A period of high inflation can shift agent's beliefs about the inflation target, and they will place more weight on the high target. This high expected inflation will be incorporated into price and wage setting decisions, and thus a string of high inflation observations can lead to high inflation expectations, which become self-fulfilling.[3]

Throughout this paper we will refer to the model where agents form expectations about the future based on a weighted average of two scenarios, and the weight is endogenous, as the *endogenous credibility* model. This paper will show that a New Keynesian model with endogenous credibility preforms much better than the benchmark model in its ability to explain the volatility and persistence of inflation and inflation expectations that we observe in the data. We then compare the results from model with endogenous credibility to the benchmark New Keynesian model with either price and wage indexation or near permanent shocks, which are two features that researchers use to add inflation persistence to the benchmark New Keynesian model. The models with indexation or with permanent shocks do just as well as the model with endogenous credibility in matching the persistence in current inflation and the dynamics of short-run inflation expectations, but these two models preform rather poorly in explaining the behavior of long-run inflation expectations. Only the model with endogenous credibility can match the volatility and co-movement of long-run measures of inflation expectations. We then calibrate the model to match the observed levels of Federal Reserve credibility in the 1980's and the 2000's. We show that by simply changing the level of central bank credibility, holding all else fixed, the model can explain nearly all of the observed changes in the volatility and persistence of inflation and inflation expectations in the U.S. from the 1980's to today.

A number of authors have proposed modifications to the standard New Keynesian model to account for observed shifts in trend inflation and long-term inflation expectations. Cogley and Sbordone (2008) estimate a model with a role for both variable trend inflation and

[2] In the way that agents update their beliefs about the central bank's target, this model is very similar to Barro (1986), where agents are unsure whether or not the central banker can commit, and thus use past observations of inflation to update their beliefs about the central banker's type, and thus their expectations for the long-run level of inflation.

[3] The mechanism is similar to, but not identical to the expectations trap in Albanesi, Chari, and Christiano (2003). The difference is that the formal expectations traps literature is based on discretionary policy. Here the central bank can commit (it follows a Taylor rule policy function), but agents are unsure about the cental bank's target.

price indexation. They find that variable trend inflation is responsible for the persistence of inflation in the data, and after accounting for variable trend inflation, price indexation is unimportant.[4] Similarly, Ireland (2007) estimates a model that allows for variable trend inflation and finds that the Fed's inflation target was low during the 1950's, rose throughout the 60's and 70's, and since then has fallen back to pre-1970's levels.

Recently, some authors have modified the standard New Keynesian model to say that agents don't have complete information about the central bank's inflation target, and must learn this from observations of past inflation. Milani (2007) incorporates "learning" into the standard New Keynesian model, estimates the model, and finds that when learning is included, you do not need to incorporate features like price indexation or habit formation in consumption to get the persistence of macroeconomic variables. Similarly, Lansing (2009) constructs a model where agents use a Kalman filter approach to deduce whether a shock to the policy function is permanent or transitory, and he shows that this model can reproduce the observed time-varying persistence and volatility of U.S. inflation. Andolfatto and Gomme (2003) and Erceg and Levin (2003) construct models where agents are unsure about either the money growth rule or the central bank's inflation target, and must infer the target from past observations of inflation. They show how this learning is necessary to explain the large output loss that accompanies a transition from a high inflation regime (high money growth rate or high inflation target) to a low inflation regime (low money growth rate or low inflation target). Similarly Schorfheide (2005) and Del Negro and Eusepi (2012) estimate a DSGE model with either complete information or a role for learning and finds that the model with complete information does well in explaining most of the historical experience in the U.S., but the model with learning is necessary to explain the Volcker disinflation of the early 1980's.

This paper will proceed as follows. Some statistics describing the behavior of inflation and inflation expectations in both the U.S. and the UK are presented in section 2. Here we pay particular attention to how the volatility and persistence of inflation and inflation expectations have changed over time, we compare the statistics from the U.S. in the 1980's to the statistics from the U.S. today and the statistics from the UK pre-1997 to those from the UK post-1997. The theoretical model is described in section 3. Basically the model is the benchmark New Keynesian model described in Christiano, Eichenbaum, and Evans (2005), but expectations are formed using this concept of endogenous credibility. The calibration of the model is discussed in section 4. Here special attention is paid to exactly how to calibrate

[4] In a related empirical study, Levin and Piger (2004) find that once you allow for a structural break in the level of inflation, which occurs in most countries in the late 1980's - early 1990's, in most countries, fluctuations in inflation are simply transitory fluctuations around the variable mean, and the inflation process has very little persistence.

the model to reflect historical observations of central bank credibility and the anchoring of inflation expectations. The results from the model are presented in section 5. Here we will examine both impulse responses and simulated moments from the model to see how the model with endogenous credibility preforms much better than the model with rational expectations in matching the volatility and persistence of inflation and inflation expectations, especially the behavior of long-run inflation expectations. Finally section 6 concludes with some directions for further research.

2 Volatility and Persistence of Inflation Expectations

In this section, we'll present some descriptive statistics related to the volatility and persistence of inflation and inflation expectations. In order to appreciate how these statistics can vary, we'll look at these statistics in both the U.S. and the UK across multiple time periods.

We will consider measures of both short-run inflation expectations and long-run inflation expectations. The three measures we consider are: the expected change in the price level over the next year (one year ahead inflation expectations, $E_t(\pi_{t+1})$), the expected annual inflation rate over the next ten years (10 year ahead inflation expectations, $E_t\left(\frac{1}{10}\sum_{i=1}^{10}\pi_{t+i}\right)$), and the expected inflation rate over a period beginning five years from now and ending ten years from now (5 year - 5 year forward inflation expectations, $E_t\left(\frac{1}{5}\sum_{i=6}^{10}\pi_{t+i}\right)$).

Table 1 presents some evidence about the cross-time evolution of the volatility and persistence of inflation and inflation expectations in the U.S. and the UK. In the table, U.S. inflation is defined as the year-over-year percentage change consumer price index (CPI), and inflation expectations are taken from the dataset compiled by the Federal Reserve Bank of Cleveland and described in Haubrich, Pennacchi, and Ritchken (2011). This dataset contains measures of n year ahead inflation expectations for the U.S. for $n = 1...30$. Expectations are observed monthly from January 1982 to the present. To produce the descriptive statistics in table 1 we use the Cleveland Fed's measures of 1 year ahead expectations, 5 year ahead expectations, and 10 year ahead expectations.[5] UK inflation is defined as the year-over-year percentage change in the UK retail price index, and expectations are taken from the difference between 5 and 10 year real and nominal UK government bonds and are published at monthly frequency starting in 1985 by the Bank of England.

In table 1 the sample for the U.S. is split into an early sample, from 1982 to 1989, and a later sample, from 2000 to 2007. The data from the UK is split into the 1985-1996 sample

[5] We use the measures of 5 and 10 year ahead expectations to back out the 5 year-5year forward inflation rate.

and the 1997-2007 sample. The first thing to notice is that in both the U.S. and the UK, the volatility of inflation fell dramatically between the early sample and the later sample. In the U.S., the volatility of 1 year ahead inflation expectations proportionally fell in line with the fall in inflation volatility, in both the 80's and in the 00's, 1 year ahead inflation expectations is about half as volatile as inflation. In the U.S., the volatility of long-run inflation expectations actually fell by more than the fall in inflation volatility. Both 10 year ahead inflation expectations and 5 year-5 year forward expectations went from being about half as volatile as inflation to about a third as volatile. In the UK, the volatility of long-run inflation expectations fell in line with the drop in inflation volatility, and the 10 year ahead expectations and the 5 year-5 year forward expectations are around 40% as volatile as inflation in both the earlier and later periods.

In both the U.S. and the UK, there is a sharp reduction in the correlations between current inflation and future inflation expectations between the earlier and the later time periods. In the U.S. in the 1980's, the correlation between current inflation and year ahead inflation expectations was over 0.7, while the correlations between current inflation and long-run measures of expectations were greater than 0.5. In the 2000's, the correlation between current inflation and 1 year ahead expectations drops to about 0.4, and the correlations between current inflation and longer term measures of expectations drop even more. The correlation between current inflation and 10 year ahead expectations falls to about 0.25, and the correlation with 5 year-5 year forward expectations drops below 0.2. Similarly the statistics for the UK show that the long-run inflation expectations were highly correlated with current inflation in the period before the Bank of England's independence with correlation coefficients around 0.6 to 0.7, but in the period after independence, long-run inflation expectations are largely uncorrelated with current inflation, with correlation coefficients only about 0.2 to 0.3.

Table 2 also presents some evidence about the cross-time evolution of the volatility and persistence of inflation and inflation expectations in the U.S. and the UK, but this time the expected inflation data is taken from 1, 5 and 10 year inflation swaps. Since the data is taken from inflation swaps, the sample begins in July 2004. The first column for each country corresponds to the data from July 2004 to December 2007 and the second column corresponds to the data from January 2008 to November 2011.

Thus the data is split into pre-crisis and crisis/post-crisis samples. The first and most obvious difference between the two samples is that inflation volatility tripled in both the U.S. and the UK in the crisis sample, and the volatility of 1 year-ahead inflation expectations nearly quadrupled in the U.S. and increased by a factor of six in the UK during the crisis.

However, the recent crisis had much less of an effect on the volatility of measures of long-run inflation expectations, especially those in the UK. In the U.S. the volatility of 10 year-ahead inflation expectation nearly tripled between the pre-crisis and crisis periods, but they only increased by about 75% in the UK. Similarly, the volatility of the 5 year - 5 year forward expectation increased by about 75% in the U.S. but only around 40% in the UK.

In both the U.S. and the UK, the correlation between current inflation and year ahead inflation expectations is nearly the same in both the pre-crisis and the crisis samples. In the U.S., the correlation between current inflation and 10 year ahead expectations is largely unchanged in the two samples, but in the UK, the correlation between current inflation and 10 year ahead expectations is 0.75 prior to 2008, but the correlation nearly drops to zero in the post-2008 data. Similarly, the correlation between current inflation and 5 year-5 year forward expectations changes from positive to negative between the two sample periods in both the U.S. and the UK. In the UK, the correlation between the two is greater than 0.7 prior to 2008 but less than -0.6 after 2008.

3 The Model

The model with endogenous central bank credibility is based on the standard New Keynesian model in Christiano, Eichenbaum, and Evans (2005). There are monopolistically competitive intermediate goods firms that produce a differentiated product that is then aggregated into a final good used for consumption, investment and government purchases. There are also households that supply a differentiated type of labor. Calvo (1983) pricing in both the intermediate goods sector and the household sector gives rise to nominal wage and price rigidities.

Due to these wage and price rigidities, a firm or a household knows that if given the opportunity to change their price today, their new nominal price will most likely be in place for at least a few periods into the future. Thus when setting an optimal price or wage, price setters have to take into account not only current conditions, but the expectation of future conditions. In the standard New Keynesian model, the expectation of future variables is determined using rational expectations. We abstract from that here. Instead we assume that agents are unsure about the central bank's inflation target. They believe the target could vary between two extremes, and agents' belief about the target is determined by the central bank's stock of credibility. Every period agents update their belief about the central bank's credibility using past observations of inflation. Thus agents will lower their beliefs about the central bank's credibility following a series of high inflation observations, and they will revise upward their beliefs about the central bank's inflation target. If agents form expectations expecting high inflation, then these high expectations get incorporated into the price and wage setting decisions, leading to higher inflation.

3.1 Production

Final goods, used for private consumption, government consumption, and investment are formed through a Dixit and Stiglitz (1977) aggregation of intermediate goods from firms $i \in [0\ 1]$:

$$C_t + I_t + G_t = \left(\int_0^1 y_t(i)^{\frac{\sigma-1}{\sigma}} di \right)^{\frac{\sigma}{\sigma-1}} \tag{1}$$

where $y_t(i)$ is the quantity produced by firm i, and σ is the elasticity of substitution between intermediate goods from different firms. When considering the results from simulations of the model, in one set of simulations we will simulate the model under stochastic government spending shocks. There will be more about the calibration of the exogenous process for G_t in section 4, but the steady state value of G_t is set such that in the steady state, government spending is 20% of GDP.

From the aggregator function in (1), the demand for the intermediate good from firm i is:

$$y_t(i) = \left(\frac{P_t(i)}{P_t} \right)^{-\sigma} (C_t + I_t + G_t) \tag{2}$$

where $P_t(i)$ is the price set by firm i, and $P_t = \left(\int_0^1 (P_t(i))^{1-\sigma} di \right)^{\frac{1}{1-\sigma}}$.

The firm produces finished goods by combining capital and labor in the following Cobb-Douglas production technology:

$$y_t(i) = A_t h_t(i)^{1-\alpha} k_t(i)^{\alpha} - \phi \tag{3}$$

where $h_t(i)$ and $k_t(i)$ are the labor and capital employed by the firm in period t, ϕ is a small fixed cost term that is calibrated to ensure that firms earn zero profit in the steady state, and A_t is a stochastic productivity parameter common to all firms.

From the firm's cost minimization problem, the demand from firm i for labor and capital is given by:

$$\begin{aligned} h_t(i) &= (1-\alpha) \frac{MC_t}{W_t} (y_t(i) + \phi) \\ k_t(i) &= \alpha \frac{MC_t}{R_t} (y_t(i) + \phi) \end{aligned} \tag{4}$$

where W_t is the wage rate, R_t is the capital rental rate and $MC_t = \frac{1}{A_t}\left(\frac{W_t}{1-\alpha}\right)^{1-\alpha}\left(\frac{R_t}{\alpha}\right)^{\alpha}$.

Price setting by intermediate goods firms In period t, the firm will be able to change its price with probability $1-\xi_p$. If the firm cannot change prices then they are reset automatically according to $P_t(i) = \pi^I_{t-1}P_{t-1}(i)$, where $\pi^I_{t-1} = \pi_{ss}$, the steady state gross inflation rate. In an alternative version of the model we will consider the case where prices are indexed to the previous period's inflation rate, $\pi^I_{t-1} = \frac{P_{t-1}}{P_{t-2}}$.

Thus if allowed to change their price in period t, the firm will set a price to maximize:

$$\max_{P_t(i)} \tilde{E}_t\left(\sum_{\tau=0}^{\infty}\beta^{\tau}\left(\xi_p\right)^{\tau}\lambda_{t+\tau}\left\{\Pi^I_{t,t+\tau}P_t(i)\,y_{t+\tau}(i) - MC_{t+\tau}y_{t+\tau}(i)\right\}\right)$$

where λ_t is the marginal utility of income in period t and

$$\Pi^I_{t,t+\tau} = \begin{cases} 1 & \text{if } \tau = 0 \\ E_t\left(\pi^I_{t+\tau-1}\right)\Pi^I_{t,t+\tau-1} & \text{if } \tau > 0 \end{cases}$$

As discussed in this paper's technical appendix, the firm that is able to change its price in period t will set its price to:

$$P_t(i) = \frac{\sigma}{\sigma-1}\frac{\tilde{E}_t\left(\sum_{\tau=0}^{\infty}\beta^{\tau}\left(\xi_p\right)^{\tau}\lambda_{t+\tau}MC_{t+\tau}\left(\Pi^I_{t,t+\tau}\right)^{-\sigma}\left(P_{t+\tau}\right)^{\sigma}y_{t+\tau}\right)}{\tilde{E}_t\left(\sum_{\tau=0}^{\infty}\beta^{\tau}\left(\xi_p\right)^{\tau}\lambda_{t+\tau}\left(\Pi^I_{t,t+\tau}\right)^{1-\sigma}\left(P_{t+\tau}\right)^{\sigma}y_{t+\tau}\right)} \tag{5}$$

If prices are flexible, and thus $\xi_p = 0$, then this expression reduces to:

$$P_t(i) = \frac{\sigma}{\sigma-1}MC_t$$

which says that the firm will set a price equal to a constant mark-up over marginal cost.

Notice that the optimal price $P_t(i)$ does not involve the usual rational expectations operator, $E_t(\cdot)$, but a modified operator $\tilde{E}_t(\cdot)$.

Instead of assuming, as in most rational expectations models, that private agents know the central bank's inflation target with certainty, assume that agents are unsure about the inflation target. Specifically, they believe the target could vary anywhere between a low target, $\bar{\pi}^L$, or a high target, $\bar{\pi}^H$.

Agents know the distribution of actual inflation around the two targets. Figure 1 shows the distribution of actual inflation around the target value $\bar{\pi}^L = 0\%$ or around the target value $\bar{\pi}^H = 10\%$. Thus they believe that the central bank has an inflation target $\tilde{\pi}$, where:

$$\tilde{\pi}_t = c_t\bar{\pi}^L + (1-c_t)\,\bar{\pi}^H$$

Let Ω_t be the set of information about the structure of the economy, all parameters (other than the inflation target), and the sequence of shocks to affect the economy up to and including shocks in period t, then for any variable x_{t+i} for all $i = 1...\infty$ in the model:

$$\tilde{E}\left(x_{t+i}|\Omega_t, c_t\right) = E\left(x_{t+i}|\Omega_t, \tilde{\pi}_t\right)$$

and for notational simplicity define $\tilde{E}_t\left(x_{t+i}\right) \equiv \tilde{E}\left(x_{t+i}|\Omega_t, c_t\right)$. The central bank's stock of credibility, c_t, is a function of previous inflation rates, specifically suppose that the observed value of inflation in period $t-1$ is $\mathring{\pi}$, then agents will update their perception of the central bank's credibility according to:

$$c_t = \rho\left(\frac{c_{t-1}P\left(\pi_{t-1} = \mathring{\pi}|\bar{\pi}^L\right)}{c_{t-1}P\left(\pi_{t-1} = \mathring{\pi}|\bar{\pi}^L\right) + \left(1 - c_{t-1}\right)P\left(\pi_{t-1} = \mathring{\pi}|\bar{\pi}^H\right)}\right) + \left(1-\rho\right)\bar{c} \tag{6}$$

where $P\left(\pi_{t-1} = \mathring{\pi}|\bar{\pi}^L\right)$ is the probability that inflation in period $t-1$ would be $\mathring{\pi}$ given that the central bank is targeting the low inflation rate, $P\left(\pi_{t-1} = \mathring{\pi}|\bar{\pi}^H\right)$ is the probability of the same event given that the central bank is targeting a high inflation rate, $\bar{c}$ is the steady state level of the central bank's credibility, and ρ is a parameter that measures how responsive is the central bank's credibility to past realizations of inflation. Thus ρ determines the anchoring of inflation expectations.

In this updating function, this model is very similar to Barro (1986) where agents are unsure whether or not the central banker can commit and use past observations of inflation to update their beliefs about the central banker's type, and thus their expectations for the long-run level of inflation.

How the observed value of inflation in $t-1$ affects the central bank's credibility is illustrated by the example in figure 1. The figure shows two distributions of inflation around the target values $\bar{\pi}^L = 0\%$ and $\bar{\pi}^H = 10\%$. Suppose that the rate of inflation in period $t-1$ was 6%, as shown by the vertical line. Agents will see that a 6% inflation rate is more likely under $\bar{\pi}^H = 10\%$ than $\bar{\pi}^L = 0\%$, and thus the central bank's credibility would fall, $\frac{c_{t-1}P\left(\pi_{t-1}=\mathring{\pi}|\bar{\pi}^L\right)}{c_{t-1}P\left(\pi_{t-1}=\mathring{\pi}|\bar{\pi}^L\right)+\left(1-c_{t-1}\right)P\left(\pi_{t-1}=\mathring{\pi}|\bar{\pi}^H\right)} < c_{t-1}$.[6]

Write the price set by the firm that can reset prices in period t as $P_t^*(i)$ to denote it as

[6]Since inflation follows a continuous distribution around $\bar{\pi}^L$ or $\bar{\pi}^H$ then $P\left(\pi_{t-1} = \mathring{\pi}|\bar{\pi}^L\right) = P\left(\pi_{t-1} = \mathring{\pi}|\bar{\pi}^H\right) = 0$, which leads to a $\frac{0}{0}$ in equation (6). If however we assume that inflation approximately follows a normal distribution around $\bar{\pi}^L$ or $\bar{\pi}^H$, then $\frac{c_{t-1}P\left(\pi_{t-1}=\mathring{\pi}|\bar{\pi}^L\right)}{c_{t-1}P\left(\pi_{t-1}=\mathring{\pi}|\bar{\pi}^L\right)+\left(1-c_{t-1}\right)P\left(\pi_{t-1}=\mathring{\pi}|\bar{\pi}^H\right)} = \lim_{\varepsilon\to 0}\frac{c_{t-1}\left(\Phi\left(\pi_{t-1}\leq\mathring{\pi}|\bar{\pi}^L\right)-\Phi\left(\pi_{t-1}\leq\mathring{\pi}-\varepsilon|\bar{\pi}^L\right)\right)}{c_{t-1}\left(\Phi\left(\pi_{t-1}\leq\mathring{\pi}|\bar{\pi}^L\right)-\Phi\left(\pi_{t-1}\leq\mathring{\pi}-\varepsilon|\bar{\pi}^L\right)\right)+\left(1-c_{t-1}\right)\left(\Phi\left(\pi_{t-1}\leq\mathring{\pi}|\bar{\pi}^H\right)-\Phi\left(\pi_{t-1}\leq\mathring{\pi}-\varepsilon|\bar{\pi}^H\right)\right)}$, where Φ is the c.d.f. of the normal distribution, and by l'Hospital's rule this equals $\frac{c_{t-1}\phi\left(\pi_{t-1}=\mathring{\pi}|\bar{\pi}^L\right)}{c_{t-1}\phi\left(\pi_{t-1}=\mathring{\pi}|\bar{\pi}^L\right)+\left(1-c_{t-1}\right)\phi\left(\pi_{t-1}=\mathring{\pi}|\bar{\pi}^H\right)}$, where ϕ is the p.d.f. of the normal distribution.

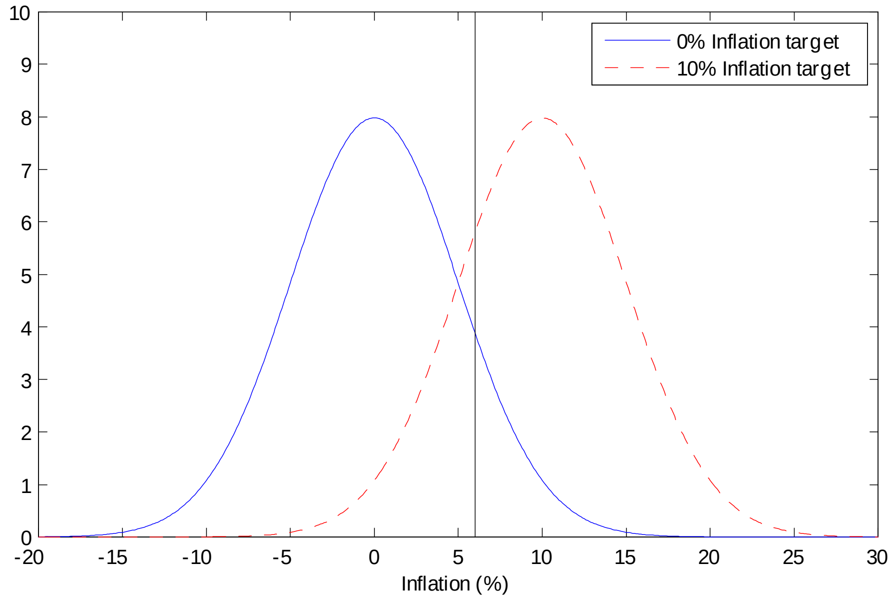

Figure 1: The distribution of actual inflation around the two target values, $\bar{\pi}^L = 0\%$ and $\bar{\pi}^H = 10\%$.

an optimal price. Firms that can reset prices in period t will all reset to the same level, so $P_t^*(i) = P_t^*$. Substitute this optimal price into the price index $P_t = \left(\int_0^1 (P_t(i))^{1-\sigma} di\right)^{\frac{1}{1-\sigma}}$. Since a firm has a probability of $1-\xi_p$ of being able to change their price, then by the law of large numbers in any period $1-\xi_p$ percent of firms will reoptimize prices. Thus the price index, P_t, can be written as:

$$P_t = \left(\xi_p \left(\pi_{t-1}^I P_{t-1}\right)^{1-\sigma} + \left(1-\xi_p\right)\left(P_t^*\right)^{1-\sigma}\right)^{\frac{1}{1-\sigma}} \tag{7}$$

After combining the expression for the optimal price in (5) and the equation describing the evolution of the price index in (7), one can derive the usual New Keynesian Phillips Curve (NKPC) that relates inflation this period to current marginal costs and the expected value of inflation next period:

$$\hat{\pi}_t = \beta \tilde{E}_t\left(\hat{\pi}_{t+1}\right) + \frac{\left(1-\xi_p\right)\left(1-\beta\xi_p\right)}{\xi_p}\left(m\hat{c}_t\right) \tag{8}$$

Notice in this Phillips curve the expectation of next period's inflation is arrived at when agents are unsure about the central bank's target inflation rate and central bank credibility is endogenous, $\tilde{E}_t\left(\hat{\pi}_{t+1}\right)$. If instead agents had full information about the central bank's inflation target then this NKPC simply condenses to its usual form where $E_t\left(\hat{\pi}_{t+1}\right)$ replaces $\tilde{E}_t\left(\hat{\pi}_{t+1}\right)$.

In a later section we will compare the results of the model with endogenous central bank credibility to the model with full information and price indexation. As discussed earlier, full price indexation implies that firms that cannot reset their price in period t simply scale up their existing price by the previous period's inflation rate π_{t-1}. In this case the NKPC becomes:

$$\hat{\pi}_t = \frac{1}{1+\beta}\hat{\pi}_{t-1} + \frac{\beta}{1+\beta}E_t\left(\hat{\pi}_{t+1}\right) + \frac{\left(1-\xi_p\right)\left(1-\beta\xi_p\right)}{\xi_p\left(1+\beta\right)}\left(m\hat{c}_t\right) \tag{9}$$

From equation (9) it is easy to see how the price indexation introduces the lagged inflation term $\hat{\pi}_{t-1}$ into the Phillips curve and thus introduces persistence into the inflation process. It is not as obvious, but the fact that the future inflation term is denoted $\tilde{E}_t\left(\hat{\pi}_{t+1}\right)$ instead of $E_t\left(\hat{\pi}_{t+1}\right)$ also introduces the lagged inflation rate and thus persistence into the Phillips curve under endogenous credibility in equation (8). Recall that the expectations operator in the model with endogenous credibility, $\tilde{E}_t\left(\cdot\right)$, depends on the central bank's stock of credibility c_t. Recall from equation (6) that the formula to update the central bank's stock of credibility depends on the lagged inflation rate, π_{t-1}.

If c_t depends on the lagged inflation rate, then $\tilde{E}_t\left(\hat{\pi}_{t+1}\right)$ depends on the lagged inflation rate, and thus the lagged inflation rate is a part of the Phillips curve under endogenous central bank credibility. The effect of the lagged inflation rate on c_t, and thus $\tilde{E}_t\left(\hat{\pi}_{t+1}\right)$, is increasing in ρ. If ρ is positive but small then the lagged inflation rate is part of the Phillips curve under endogenous credibility, but the effect is small. If ρ is close to one then the lagged inflation rate has a much greater presence in the Phillips curve, and thus inflation and inflation expectations are more volatile and persistent.

3.2 Households

Households, indexed $l \in [0\ 1]$, supply labor, own capital, and consume from their labor income, rental income, and interest on savings. Furthermore they pay lump sum taxes to the government to finance government expenditures.

The household maximizes their utility function:

$$\max \sum_{t=0}^{\infty} \beta^t \left[\ln\left(C_t\left(l\right)\right) - \psi\left(H_t\left(l\right)\right)^{\frac{1+\sigma_H}{\sigma_H}}\right] \tag{10}$$

subject to their budget constraint:

$$
\begin{aligned}
& P_t C_t(l) + P_t I_t(l) + T_t(l) + B_{t+1}(l) \\
= \; & W_t(l) H_t(l) + R_t K_t(l) + (1 + i_t) B_t(l)
\end{aligned} \tag{11}
$$

where $C_t(l)$ is consumption by household l in period t, $H_t(l)$ is the household's labor effort in the period, $T_t(l) = P_t G_t(l)$ are the lump sum taxes paid by the household to finance government consumption, $B_t(l)$ is the household's stock of bonds at the beginning of the period[7], $W_t(l)$ is the wage paid for the household's heterogenous labor supply, and $K_t(l)$ is the stock of capital owned by the household at the beginning of the period.

The household's capital stock, $K_t(l)$, evolves according to the usual capital accumulation equation:

$$
K_{t+1}(l) = (1 - \delta) K_t(l) + I_t(l)
$$

where market clearing in the market for physical capital requires that the sum of the physical capital stock across households is equal to the sum of physical capital demand across firms, $\int_0^1 K_t(l)\, dl = \int_0^1 k_t(i)\, di$.

Each household supplies a differentiated type of labor. The function to aggregate the labor supplied by each household into the aggregate stock of labor employed by firms is:

$$
H_t = \left(\int_0^1 H_t(l)^{\frac{\theta-1}{\theta}} dl \right)^{\frac{\theta}{\theta-1}} \tag{12}
$$

where market clearing in the labor market requires that $H_t = \int_0^1 h_t(i)\, di$. Since the household supplies a differentiated type of labor, it faces a downward sloping labor demand function:

$$
H_t(l) = \left(\frac{W_t(l)}{W_t} \right)^{-\theta} H_t
$$

3.2.1 Wage setting by households

In any given period, household l faces a probability of $1 - \xi_w$ of being able to reset their wage. If the household cannot change its wage then it is reset automatically according to $W_t(l) = \pi_{t-1}^I W_{t-1}(l)$, where $\pi_{t-1}^I = \pi_{ss}$, the steady state gross inflation rate. In an alternative version of the model we will consider the case where wages are indexed to the previous period's inflation rate, $\pi_{t-1}^I = \frac{P_{t-1}}{P_{t-2}}$.

[7] Market clearing in the bond market requires that the sum of bond holdings across all households equals zero, $\int_0^1 B_t(l)\, dl = 0$.

Assume that complete asset markets exist that allow households to pool risk. The wage rate and the labor effort will be different across households due to nominal wage rigidity, but all other variables that appear in the household budget constraint are equal across households. Thus all households have the same level of consumption, $C_t(l) = C_t$ and the same marginal utility of consumption.

If household l is allowed to reset their wages in period t they will set a wage to maximize the expected present value of utility from consumption minus the disutility of labor.

$$\tilde{E}_t \left(\sum_{\tau=0}^{\infty} \beta^{\tau} (\xi_w)^{\tau} \left\{ \lambda_{t+\tau} \Pi^I_{t,t+\tau} W_t(l) H_{t+\tau}(l) - \psi \left(H_{t+\tau}(l) \right)^{\frac{1+\sigma_H}{\sigma_H}} \right\} \right)$$

Thus after technical details which are located in the appendix, the household that can reset wages in period t will choose a wage:

$$W_t(l)^{\frac{\theta}{\sigma_H}+1} = \frac{\theta}{\theta-1} \frac{1+\sigma_H}{\sigma_H} \psi \frac{\tilde{E}_t \left(\sum_{\tau=0}^{\infty} \beta^{\tau} (\xi_w)^{\tau} \left(\frac{W_{t+\tau}}{\Pi^I_{t,t+\tau}} \right)^{\frac{\theta}{\sigma_H}+\theta} (H_{t+\tau})^{\frac{1+\sigma_H}{\sigma_H}} \right)}{\tilde{E}_t \left(\sum_{\tau=0}^{\infty} \beta^{\tau} (\xi_w)^{\tau} \lambda_{t+\tau} \Pi^I_{t,t+\tau} \left(\frac{W_{t+\tau}}{\Pi^I_{t,t+\tau}} \right)^{\theta} H_{t+\tau} \right)} \tag{13}$$

If wages are flexible, and thus $\xi_w = 0$, this expression reduces to:

$$W_t(l) = \frac{\theta}{\theta-1} \frac{\frac{1+\sigma_H}{\sigma_H} \psi (H_t)^{\frac{1}{\sigma_H}}}{\lambda_t}$$

When wages are flexible the wage rate is equal to a mark-up, $\frac{\theta}{\theta-1}$, multiplied by the marginal disutility of labor, $\frac{1+\sigma_H}{\sigma_H} \psi (H_t)^{\frac{1}{\sigma_H}}$, divided by the marginal utility of consumption, λ_t.

Notice again that when expectations of future variables are used to calculate the current optimal wage, agents use the modified expectations operator, $\tilde{E}_t(\cdot)$, instead of the rational expectations operator, $E_t(\cdot)$.

Write the wage rate for the household that can reset wages in period t, $W_t(l)$, as $W_t^*(l)$ to denote it as an optimal wage. Also note that all households that can reset wages in period t will reset to the same wage rate, so $W_t^*(l) = W_t^*$.

All households face a probability of $(1-\xi_w)$ of being able to reset their wages in a given period, so by the law of large numbers $(1-\xi_w)$ of households can reset their wages in a given period. Substitute W_t^* into the expression for the aggregate wage rate $W_t = \left(\int_0^1 W_t(l)^{1-\theta} dl \right)^{\frac{1}{1-\theta}}$, to derive an expression for the evolution of the aggregate wage:

$$W_t = \left(\xi_w \left(\pi^I_{t-1} W_{t-1} \right)^{1-\theta} + (1-\xi_w) (W_t^*)^{1-\theta} \right)^{\frac{1}{1-\theta}}$$

In the model with endogenous credibility, the New Keynesian Phillips Curve relating wage inflation this period to expected future wage inflation and the marginal disutility of labor this period is given by:

$$\hat{\pi}_t^w = \beta \tilde{E}_t\left(\hat{\pi}_{t+1}^w\right) + \frac{(1-\xi_w)(1-\beta\xi_w)}{\xi_w}\left(\frac{\sigma_H}{\theta+\sigma_H}\right)\left(\frac{1}{\sigma_H}\hat{H}_t - \hat{\Lambda}_t - \hat{w}_t\right)$$

where $\pi_t^w = \frac{W_{t+1}}{W_t} - 1$.

If wages that could not be changed in a given period were reset using the previous period's inflation rate, but central bank credibility was fixed, then the New Keynesian Phillips curve would be:

$$\hat{\pi}_t^w = \hat{\pi}_{t-1} - \beta\hat{\pi}_t + \beta E_t\left(\hat{\pi}_{t+1}^w\right) + \frac{(1-\xi_w)(1-\beta\xi_w)}{\xi_w}\left(\frac{\sigma_H}{\theta+\sigma_H}\right)\left(\left(\frac{1}{\sigma_H}\right)\hat{H}_t - \hat{\Lambda}_t - \hat{w}_t\right)$$

Just as before in the Phillips curve with price inflation, persistence is added to the model with indexation by the presence of the lagged inflation rate in the Phillips curve equation. In the model with endogenous credibility, the lagged inflation rate has an effect on the stock of central bank credibility and thus on $\tilde{E}_t\left(\pi_{t+1}^w\right)$. The full derivation of both Phillips curves is presented in the appendix.

3.3 Monetary Policy

The monetary policy instrument is the short-run risk free rate, i_t, which is determined by the central bank's Taylor rule function:

$$i_{t+1} = i_{ss} + \theta_p\left(\pi_t - \bar{\pi}\right) + \theta_y\hat{y}_t + m_t \tag{14}$$

where $\hat{y}_t = \frac{GDP_t}{G\tilde{D}P_t} - 1$, where $G\tilde{D}P_t$ is the level of GDP at time t in an economy with the same structure as the one just described and subject to the same shocks, only there are no price or wage frictions, $\xi_p = \xi_w = 0$, and m_t is an exogenous monetary policy shock. $\bar{\pi}$ is the central bank's inflation target, which is not known by the private agents in the economy, in order to ensure that private agents don't make systematic mistakes in predicting the long-run level of inflation, $\bar{\pi} = \bar{c}\bar{\pi}^L + (1-\bar{c})\,\bar{\pi}^H$.

4 Calibration

4.1 Parameter Values

The various parameters used in the model and their values are listed in table 3. The first five parameters, the discount factor, capital's share of income, the capital depreciation rate, the elasticity of substitution across varieties from different firms, the elasticity of substitution between labor from different households, and are all set to values that are commonly found in the literature.

The next two parameters are the Calvo wage and price stickiness parameters. The wage and price stickiness parameters are set to 0.75, implying that a household expects to change their wage and firms expect to change their prices once a year. We use the standard Taylor rule parameters for the parameters in the monetary policy function. The central bank places a weight of 0.5 on the output gap and 1.5 on the inflation rate.

The next three parameters in the table are the central bank's inflation target and the public's perception of the bounds for the central bank's inflation target. We assume that the central bank targets an annual inflation rate of 5%. The public doesn't know this, and believes that the annual inflation target drifts between 0% and 10%. Note that this combination of real and perceived inflation targets determines that the steady state level of central bank credibility, $\bar{c} = 0.5$.

The last two parameters in the table $\phi\left(\pi|\bar{\pi}^L\right)'$ and $\phi\left(\pi|\bar{\pi}^H\right)'$ are the first derivatives of the p.d.f.'s of inflation distributed around the two targets, $\bar{\pi}^L$ and $\bar{\pi}^H$. These parameters are the slope of the two distribution functions in figure 1, evaluated at the steady state level of inflation. As can be seen from updating equation for c_t in equation (6) and the accompanying footnote in the text, the value of these two first derivatives, evaluated at the steady state inflation rate $\bar{\pi}$, are all that we need for a linearization of the updating equation in (6). The value of these two first derivatives are found by calculating the p.d.f.'s of the distribution of inflation around the two target values and assuming that the standard deviation of inflation around these two targets is 0.7%, which approximately the standard deviation of inflation in the benchmark version of the model without endogenous credibility.

As shown by the updating function for c_t, the central bank's stock of credibility will eventually return to $\bar{c}$. The central bank's actual inflation target, $\bar{\pi} = \bar{c}\bar{\pi}^L + (1-\bar{c})\,\bar{\pi}^H$, and thus while agents may believe the target is $\tilde{\pi}_t = c_t\bar{\pi}^L + (1-c_t)\,\bar{\pi}^H$, with enough time $\tilde{\pi}_t \to \bar{\pi}$ and agents form expectations around the true target, but in the short to medium term $\tilde{\pi}_t \neq \bar{\pi}$. The effect that a series of inflation surprises can have on agent's perceptions of c_t and thus $\tilde{\pi}_t$, or alternatively the rate at which c_t returns to $\bar{c}$, and thus $\tilde{\pi}_t$ returns to $\bar{\pi}$ depends on the parameter ρ, which measures the "anchoring" of inflation expectations.

In the version of the model where central bank credibility is fixed and long-run inflation expectations are perfectly anchored, $\rho = 0$. In the version of the model with endogenous credibility, we can calibrate the model such that the model can match the observed responsiveness of long-run inflation expectations to a surprise in current observed inflation.

As discussed earlier, the Cleveland Fed calculates n year ahead inflation expectations for $n = 1...30$ for the U.S. at a monthly frequency. From the 5 and 10 year ahead inflation expectations at time t we can calculate 5 year-5 year forward inflation expectations at time t, which is the average of the monthly inflation rates that we expect to observe between 5 years from now and 10 years from now, $E_t\left(\frac{1}{5}\sum_{i=6}^{10}\pi_{t+i}\right)$. Similarly we can use the 6 and 11 year ahead expectations at time $t-1$ to calculate 6 year-5 year forward inflation expectation at time $t-1$, $E_{t-1}\left(\frac{1}{5}\sum_{i=7}^{11}\pi_{t-1+i}\right)$, which by the law of iterative expectations is the expectation taken at time $t-1$ of the 5 year-5 year forward inflation expectation at time t, $E_{t-1}\left(\frac{1}{5}\sum_{i=7}^{11}\pi_{t-1+i}\right) = E_{t-1}\left(E_t\left(\frac{1}{5}\sum_{i=6}^{10}\pi_{t+i}\right)\right)$.

Similarly, the Cleveland Fed calculates the 1 year ahead expected inflation rate. $E_{t-1}(\pi_t)$ is the one year ahead inflation expectation taken last year. Subtract that from the current observed inflation rate to find unexpected inflation over the previous year, $\pi_t - E_{t-1}(\pi_t)$.

A simple OLS regression is used to calculate by how much to agents update their long-run inflation expectations in response to unexpected inflation:

$$E_t\left(\frac{1}{5}\sum_{i=6}^{10}\pi_{t+i}\right) - E_{t-1}\left(E_t\left(\frac{1}{5}\sum_{i=6}^{10}\pi_{t+i}\right)\right) = \alpha + \gamma\left(\pi_t - E_{t-1}(\pi_t)\right) + \varepsilon_t$$

From this regression, if actual inflation in period t is 1 percentage point higher than expected the previous year, agents increase their expectations of inflation between 5 and 10 years from now by γ percentage points. Using monthly data, we run this regression over three time periods, from 1982-1989, from 1990-1999, and from 2000-2007. The estimated γ's are presented in table 4. The table shows that in the 1980's, when inflation was 100 basis points above what was expected the previous year, people would raise their 5 year-5 year inflation expectations by 27 basis points. In the 2000's, the same 100 basis point unexpected inflation would only lead to an 8 basis point increase in long-run inflation expectations. Between the 1980's and the 2000's, long-run inflation expectations had become better anchored and thus current inflation had less of an influence on long-run inflation expectations.

The parameter ρ that measures the responsiveness of central bank credibility to innovations to current inflation is set such that when the parameter γ is calculated from simulations of the model, in the model with endogenous central bank credibility, $\gamma = 0.279$.

4.2 Shock Processes

In the next section, we will examine the responses of inflation and inflation expectations to both productivity and government spending shocks. For simplicity, we only consider the effect of one shock at a time, and we assume that each shock follows an AR(1) process with an autoregressive coefficient of 0.9. In one alternative version in the next section we will consider the case where the shock is nearly permanent with an autoregressive coefficient of 0.9999.

Since the model is solved with a first-order approximation around the steady state, and only one shock is active at any time, the variance of the shock doesn't matter for most of the dynamics in the model. To ease the comparison between the model and the data, the variance of each shock is calibrated so that the standard deviation of inflation in the model with endogenous credibility is 1.357%, which the same as that in the U.S. during the 1980's as seen in table 1.

5 Results

To assess the effect of endogenous central bank credibility, we will present the results from the model in two steps. First, with impulse responses, we will chart the path of inflation and inflation expectations following a productivity or government spending shock. Here we will not only compare the model with endogenous credibility to the model with fixed credibility (and thus perfectly anchored long-run inflation expectations), but we will consider additional features of the New Keynesian model that researchers have used to increase the volatility and persistence of inflation and inflation expectations; namely price and wage indexation or a highly persistent shock process. Then with simulations of the model we will calculate the same statistics that are presented in tables 1 and 2 and see how only the model with endogenous credibility can replicate the features observed in the data like the volatility of long-run inflation expectations or the high correlation between current inflation and long-run inflation expectations.

5.1 Impulse responses

The responses of current inflation, 1 year ahead inflation expectations, 10 year ahead expectations, and 5 year-5 year forward expectations to a negative TFP shock are presented in figure 2.

First, let us compare the impulse responses in the model with endogenous credibility to those from the benchmark model where agents know the central bank's inflation target with certainty. Following the negative TFP shock, current inflation jumps about 20 basis points in both models. However, in the benchmark model, inflation quickly returns to its steady state level, but in the model with endogenous credibility, inflation is much more persistent.

The persistence of inflation in the endogenous credibility model can also be seen in the responses of inflation expectations. One year ahead inflation expectations initially jump by about 10 basis points in both models, but in the benchmark model they quickly return to the steady state. When agents know the inflation target with certainty, long-run measures of inflation expectations barely move following the shock, but when credibility is endogenous these long-run measures react positively following a positive shock to current inflation and are quite persistent.

Thus endogenous central bank credibility leads to greater volatility and persistence in both inflation and inflation expectations. In the standard New Keynesian model, researchers have used either price and wage indexation or a very persistence forcing process to help the model match observed levels of inflation persistence.[8] Figure 2 also plots impulse responses for the version of the model with rational expectations but full price and wage indexation, and the version of the model with fixed credibility but where the persistence of shock to TFP is set to 0.9999.

Both additional features of the model lead to greater inflation volatility and persistence. Current inflation is slightly more volatile and persistent in the model with endogenous credibility than it is in the model with the near permanent forcing process, but inflation is less volatile in the model with endogenous credibility than it is full price and wage indexation. Following the TFP shock, current inflation jumps by about 20 basis points in the version of the model with endogenous credibility, but it jumps nearly 35 basis points in the model with full price and wage indexation.

However, with rational expectations, the shock to inflation quickly dissipates and despite the fact that initially inflation was so much greater in the model with full indexation, after about 30 quarters, inflation in the model with endogenous credibility is higher. Since the initial response of inflation is greater under full indexation than under endogenous credibility, the initial response of 1 year ahead expectations is also greater under full indexation. This result is even true for 10 year ahead expectations. However, since inflation is more persistent in the model with endogenous credibility than in the model with full indexation, the response of the 5 year-5 year forward expectation is much greater in the model with endogenous credibility.

The responses of the same four variables to a government spending shock are presented

[8] See Fuhrer (2006)

in figure 3. The same pattern continues to hold under demand shocks as under supply shocks. Inflation and inflation expectations are far more volatile and persistent in the model with endogenous credibility than they are in the benchmark version of the model with fixed credibility. In addition, versions of the model with fixed credibility but either full indexation or near permanent shocks can produce more volatile inflation responses in the short run, but inflation is far more persistent under endogenous central bank credibility, and thus only under endogenous credibility is there much response in the long-run measures of inflation expectation like the 5 year-5 year forward expectation.

5.2 Moments from model simulations

The volatility and persistence of current and expected inflation taken from simulations of the model under productivity shocks is presented in table 5. The table presents simulated moments from four versions of the model. The version of the model with endogenous credibility, the version of the model with full price and wage indexation, the version of the model where the exogenous productivity shock follows close to a unit root process, and the benchmark version of the model with rational expectations, no price and wage indexation, and non-permanent productivity shocks.

The table is meant to compare the model with endogenous credibility with the other modifications of the New Keynesian model authors have proposed to raise persistence of inflation. First, from the table it is clear that all three modifications, endogenous credibility, indexation, and permanent shocks, increase the persistence of inflation over the benchmark New Keynesian model. These three modifications also raise the relative volatility of one year ahead inflation expectations, and they improve the model's ability to match the positive co-movement between current inflation and inflation expectations (particularly long-run inflation expectations).

However, the model with price and wage indexation or permanent shocks fail to match the relative volatility of long-run inflation expectations. As is shown in table 1, in the United States in the 1980's long-run inflation expectations, either the 10 year ahead expected inflation rate or the 5 year-5 year forward expected inflation rate are around half as volatile as current inflation. In the benchmark New Keynesian model they are around a tenth as volatile as current inflation. Adding intrinsic or inherited inflation persistence does go some way towards explaining the volatility of 10 year ahead expectations, but these two modifications fail to raise the relative volatility of the 5 year-5 year forward expected inflation rate. Introducing price and wage indexation actually leads to a fall in the relative volatility of the 5 year-5 year forward rate. Only the model with endogenous credibility, parameterized to match the anchoring of inflation expectations observed in the United States in the 1980's, can produce the observed relative volatility of long-run expected inflation.

Table 6 presents the same model simulation results, only now the model is driven by government spending shocks instead of productivity shocks. The results are broadly similar, just as in the case where the model is driven by productivity shocks, only the version of the model with endogenous credibility can replicate the volatility of long-run inflation expectations. Just as before, the versions of the model with indexation or near permanent shocks bring a slight improvement in the ability of the New Keynesian model to match the relative volatility of 10 year ahead inflation expectations, but do not begin to explain the volatility of the 5 year-5 year forward rate.

Comparing changes in credibility Table 7 presents the moments that describe the behavior of inflation and inflation expectations in the United States during the 1980's and the 2000's, and it also presents the results from simulations of the model under both productivity shocks and government spending shocks where the model is calibrated to match the anchoring of inflation expectations, described by the γ parameter from table 4, during the 1980's and the 2000's. The first three columns in the table present the U.S. data and the percent change in the data from the 1980's to the 2000's. The second set of three columns present the results from simulations of the model under productivity shocks. The first two columns in the middle section present the results from the model where ρ parameter in equation (6) is calibrated to match the anchoring of inflation expectations described by the γ parameter, as seen in the last row of the table. The third set of three columns present the same results but in the model where business cycles are driven by government spending shocks.

Thus table 7 is meant to show whether or not the observed changes in the dynamics of U.S. inflation and inflation expectations from the 1980's to today can be explained by changes in central bank credibility, holding all else constant.

The first thing to notice is that the model, particularly the model with productivity shocks, can explain nearly all of the fall in U.S. inflation volatility from the 1980's to today. In the data, U.S. inflation volatility fell by 38% over this period. In the model with productivity shocks, changes in the anchoring of inflation expectations, holding all else fixed, led to a 30% fall in inflation volatility. Under government spending shocks the change in volatility is not quite as large, but the observed change in anchoring will still lead to a 13% fall in inflation volatility.

The change in anchoring can also explain the fall in the relative volatility of various measures of expected inflation. In the data, the relative volatility of one year ahead inflation expectations fell by about 10% and that for long-run expectations fell by 30%. The model with productivity shocks actually over-predicts this fall in relative volatility and predicts that the relative volatility of one year ahead expectations should fall by about 20% and the volatility of long-run expectations should fall by 60-80%. The model with government spending shocks is nearly perfect in predicting that the observed changes in the anchoring parameter should lead to a 30% fall in the relative volatility of long-run measures of inflation expectations.

The models do a relatively poor job in explaining the fall in the contemporaneous correlation between current inflation and the various measures of inflation expectations. However, improvements in anchoring can explain the observed fall in the correlation between inflation expectations and the observed inflation rate in the previous period. In the data, this correlation fell by nearly a third when considering one year ahead expectation and by over half when considering long-run measures of expectations. In the model with productivity shocks, changes in the anchoring parameter, holding all else constant, also predict about a 50% fall in the correlation between measures of expected inflation and the lagged inflation rate.

Finally, a change in the level of central bank credibility in the model can explain nearly all of the change in the persistence of inflation and various measures of expected inflation from the 1980's to today. The one period autocorrelation of current inflation has fallen by about 11% over this time. Improvements in anchoring alone should cause this autocorrelation to fall by about 8%. Similarly, the model nearly matches the observed changes in the persistence of long-run measures of inflation expectations.

6 Summary and conclusion

This paper provides a mechanism through which past observations of inflation can influence the public's perception of the central bank's inflation target and thus can influence inflation expectations into the future. This paper shows how this mechanism can lead to an increase in the volatility and persistence of both inflation and inflation expectations in the benchmark New Keynesian model. Other features added to the standard New Keynesian model, like price and wage indexation, can improve on the model's ability to explain the volatility and persistence of current inflation and short-run inflation expectations, but only the model with endogenous credibility can match the volatility of long-run inflation expectations that we see in the data.

This concept of endogenous central bank credibility gives rise to two interesting directions for further research. The first is in an open economy. As described in the first paragraph of the introduction, when Milton Friedman said that "inflation is always and everywhere a monetary phenomenon", he was careful to qualify that inflation is a sustained increase in the general price level. Exogenous shocks, like an increase in commodity prices, could lead to a transitory increase in the price level, but a sustained increase over the long run must be driven by monetary policy, or at least the public's perception of monetary policy.

Thus an interesting extension of this endogenous credibility model to an open economy would be to consider how foreign shocks that cause a transitory increase in domestic inflation might affect the public's belief about the credibility of the central bank, and then that change in credibility would affect expectations and price setting by domestic agents into the future. Thus the transitory increase in prices due to the foreign shock could have a long-lasting effect on domestic inflation.

The second, and closely related direction for further research, relates to the optimal conduct of monetary policy when the central bank's stock of credibility is endogenous. Orphanides and Williams (2004; 2007) and Gaspar, Smets, and Vestin (2006; 2011) present models where agents' have imperfect information about the parameters in the central bank's policy rule function, or where they are unsure if a shock to inflation is transitory or permanent. These models all show that in this environment, the central bank should be more aggressive when responding to changes in inflation. The mechanism in this model with endogenous credibility is very similar to the mechanism in a learning model, only the interpretation is different.

Posen (2011) argues that the central bank's reaction to a transitory increase in prices should depend on the anchoring of inflation expectations. If the central bank's stock of credibility is very sensitive to the observed inflation rate (in terms of the model, a high ρ parameter) then then central bank will want to be very aggressive in responding to transitory increases in inflation, but as expectations become better anchored and the stock of central bank credibility is less responsive to the observed inflation rate (a lower ρ parameter) then the central bank may not want to be as aggressive in responding to transitory movements in prices. Thus an interesting direction for further research would be to quantify how the central bank's optimal monetary policy depends on this "anchoring" of inflation expectations.

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A Technical Appendix - Not for publication

This appendix will present some of the more technical derivations in the paper related to the nominal rigidities present in the model. The first section in the appendix will solve for the household's optimal wage and present the derivation of the New Keynesian Phillips Curve (NKPC) for wage inflation. The second section will solve for the firm's optimal price and present the derivation of the NKPC for core inflation.

A.1 Sticky Wages

In any given period, household j faces a probability of $1 - \xi_w$ of being able to reset their wage.

If household j is allowed to reset their wages in period t they will set a wage to maximize the expected present value of utility from consumption minus the disutility of labor.

$$\tilde{E}_t \left(\sum_{\tau=0}^{\infty} \beta^\tau (\xi_w)^\tau \left\{ \lambda_{t+\tau} W_t(l) H_{t+\tau}(l) - \psi (H_{t+\tau}(l))^{\frac{1+\sigma_H}{\sigma_H}} \right\} \right) \tag{15}$$

where $\lambda_{t+\tau}$ is the marginal utility of consumption in period $t+\tau$.[9]

The imperfect combination of labor from different households is described in (12). Use this function to derive the demand function for labor from a specific household:

$$H_t(l) = \left(\frac{W_t(l)}{W_t} \right)^{-\theta} H_t \tag{16}$$

where $W_t = \left(\int_0^1 W_t(l)^{1-\theta} dl \right)^{\frac{1}{1-\theta}}$ is the average wage across households, and H_t is aggregate labor supplied by all households.

Substitute the labor demand function into the maximization problem to express the maximization problem as a function of one choice variable, the wage rate, $W_t(l)$:

[9] We assume complete contingent claims markets among households within a country. This implies that the marginal utility of consumption is the same across all households within a country, regardless of their income. Therefore the total utility from the consumption of labor income in any period is simply the country specific marginal utility of comsumption, λ_t, multiplied by the household's labor income, $W_t(l) N_t(l)$.

$$\tilde{E}_t\left(\sum_{\tau=0}^{\infty}\beta^{\tau}\left(\xi_w\right)^{\tau}\left\{\lambda_{t+\tau}\Pi^I_{t,t+\tau}W_t\left(l\right)\left(\frac{W_t\left(l\right)}{W_{t+\tau}}\right)^{-\theta}H_{t+\tau}-\psi\left(\left(\frac{W_t\left(l\right)}{W_{t+\tau}}\right)^{-\theta}H_{t+\tau}\right)^{\frac{1+\sigma_H}{\sigma_H}}\right\}\right)$$

After some rearranging, the first order condition of this problem is:

$$W_t\left(l\right)^{\frac{\theta}{\sigma_H}+1}=\frac{\theta}{\theta-1}\frac{1+\sigma_H}{\sigma_H}\psi\frac{\tilde{E}_t\left(\sum_{\tau=0}^{\infty}\beta^{\tau}\left(\xi_w\right)^{\tau}\left(\frac{W_{t+\tau}}{\Pi^I_{t,t+\tau}}\right)^{\frac{\theta}{\sigma_H}+\theta}\left(H_{t+\tau}\right)^{\frac{1+\sigma_H}{\sigma_H}}\right)}{\tilde{E}_t\left(\sum_{\tau=0}^{\infty}\beta^{\tau}\left(\xi_w\right)^{\tau}\lambda_{t+\tau}\Pi^I_{t,t+\tau}\left(\frac{W_{t+\tau}}{\Pi^I_{t,t+\tau}}\right)^{\theta}H_{t+\tau}\right)}$$

If wages are flexible, and thus $\xi_w = 0$, this expression reduces to:

$$W_t\left(l\right)=\frac{\theta}{\theta-1}\frac{\frac{1+\sigma_H}{\sigma_H}\psi\left(H_t\right)^{\frac{1}{\sigma_H}}}{\lambda_t}$$

Thus when wages are flexible the wage rate is equal to a mark-up, $\frac{\theta}{(\theta-1)}$, multiplied by the marginal disutility of labor, $\frac{1+\sigma_H}{\sigma_H}\psi\left(H_t\right)^{\frac{1}{\sigma_H}}$, divided by the marginal utility of consumption, λ_t.

Write the wage rate for the household that can reset wages in period t, $W_t\left(l\right)$, as $W_t^*\left(l\right)$ to denote it as an optimal wage. Also note that all households that can reset wages in period t will reset to the same wage rate, so $W_t^*\left(l\right) = W_t^*$.

All households face a probability of $\left(1-\xi_w\right)$ of being able to reset their wages in a given period, so by the law of large numbers $\left(1-\xi_w\right)$ of households can reset their wages in a given period. The wages of the other ξ_w will automatically reset by the previous periods inflation rate.

So substitute W_t^* into the expression for the average wage rate $W_t = \left(\int_0^n W_t\left(l\right)^{1-\theta}dl\right)^{\frac{1}{1-\theta}}$, to derive an expression for the evolution of the average wage:

$$W_t=\left(\xi_w\left(\pi^I_{t-1}W_{t-1}\right)^{1-\theta}+\left(1-\xi_w\right)\left(W_t^*\right)^{1-\theta}\right)^{\frac{1}{1-\theta}}$$

A.1.1 Derivation of the NKPC for wage inflation

Recall that $w_t(l) = w_t^*$ for all households that can change their wage in period t. As presented in the text, the optimal wage in real terms is given by:

$$(w_t^*)^{\frac{\theta}{\sigma_H}+1} = \frac{\theta}{\theta-1}\frac{1+\sigma_H}{\sigma_H}\psi\frac{\tilde{E}_t\left(\sum_{\tau=0}^{\infty}\beta^{\tau}(\xi_w)^{\tau}\left(w_{t+\tau}\frac{\Pi_{t,t+\tau}}{\Pi_{t,t+\tau}^I}\right)^{\frac{\theta}{\sigma_H}+\theta}(H_{t+\tau})^{\frac{1+\sigma_H}{\sigma_H}}\right)}{\tilde{E}_t\left(\sum_{\tau=0}^{\infty}\beta^{\tau}(\xi_w)^{\tau}\Lambda_{t+\tau}(w_{t+\tau})^{\theta}\left(\frac{\Pi_{t,t+\tau}}{\Pi_{t,t+\tau}^I}\right)^{\theta-1}H_{t+\tau}\right)}$$

Furthermore, the expression for the evolution of the average wage in real terms is:

$$w_t = \left(\xi_w\left(\frac{\pi_{t-1}^I w_{t-1}}{\pi_t}\right)^{1-\theta} + (1-\xi_w)(w_t^*)^{1-\theta}\right)^{\frac{1}{1-\theta}}$$

The linearized forms of these two expressions are given by:

$$\begin{aligned}
\hat{w}_t^* &= (1-\beta\xi_w)\frac{\sigma_H}{\theta+\sigma_H}\left(\left(\frac{\theta}{\sigma_H}\right)\hat{w}_t + \left(\frac{1}{\sigma_H}\right)\hat{H}_t - \hat{\Lambda}_t\right) + \beta\xi_w\left(\left(\tilde{E}_t(\pi_{t+1}) - \pi_t^I\right) + \tilde{E}_t\left(\hat{w}_{t+1}^*\right)\right) \\
\hat{w}_t &= (1-\xi_w)\hat{w}_t^* + \xi_w\left(\hat{w}_{t-1} + \pi_{t-1}^I - \pi_t\right)
\end{aligned}$$

After a few substitutions, the New Keynesian Phillips Curve equation describing wage inflation is given by:

$$\pi_t^w = \pi_{t-1}^I + \frac{(1-\xi_w)(1-\beta\xi_w)}{\xi_w}\left(\frac{\sigma_H}{\theta+\sigma_H}\right)\left(\left(\frac{1}{\sigma_H}\right)\hat{H}_t - \hat{\Lambda}_t - \hat{w}_t\right) + \beta\tilde{E}_t\left(\pi_{t+1}^w\right) - \beta\pi_t^I$$

In the model with fixed central bank credibility, $\tilde{E}_t\left(\pi_{t+1}^w\right) = E_t\left(\pi_{t+1}^w\right)$, and in the model without wage indexation, $\pi_t^I = 0$.

A.2 Sticky Output Prices

In period t, the firm will be able to change it's price with probability $1-\xi_p$.

The firm that can reset prices in period t will choose $P_t(i)$ to maximize discounted future profits:

$$\max_{P_t(i)} \tilde{E}_t \left(\sum_{\tau=0}^{\infty} \beta^{\tau} \left(\xi_p\right)^{\tau} \lambda_{t+\tau} \left\{ \Pi^I_{t,t+\tau} P_t(i)\, y_{t+\tau}(i) - MC_{t+\tau} y_{t+\tau}(i) \right\} \right)$$

where $MC_{t+\tau}$ is marginal cost of production in period $t+\tau$.

The demand function faced by the firm is given in (**??**). Substitute this demand function into the maximization problem to express this problem as a function of one choice variable, $P_t(i)$:

$$\max_{P_t(i)} \tilde{E}_t \left(\sum_{\tau=0}^{\infty} \beta^{\tau} \left(\xi_p\right)^{\tau} \lambda_{t+\tau} \left\{ \Pi^I_{t,t+\tau} P_t(i) \left(\frac{P_t(i)}{P_{t+\tau}} \right)^{-\sigma} y_{t+\tau} - MC_{t+\tau} \left(\frac{P_t(i)}{P_{t+\tau}} \right)^{-\sigma} y_{t+\tau} \right\} \right)$$

After some rearranging, the first order condition with respect to $P_t(i)$ is:

$$P_t(i) = \frac{\sigma}{\sigma - 1} \frac{\tilde{E}_t \left(\sum_{\tau=0}^{\infty} \beta^{\tau} \left(\xi_p\right)^{\tau} \lambda_{t+\tau} MC_{t+\tau} \left(\Pi^I_{t,t+\tau}\right)^{-\sigma} \left(P_{t+\tau}\right)^{\sigma} y_{t+\tau} \right)}{\tilde{E}_t \left(\sum_{\tau=0}^{\infty} \beta^{\tau} \left(\xi_p\right)^{\tau} \lambda_{t+\tau} \left(\Pi^I_{t,t+\tau}\right)^{1-\sigma} \left(P_{t+\tau}\right)^{\sigma} y_{t+\tau} \right)}$$

If prices are flexible, and thus $\xi_p = 0$, then this expression reduces to:

$$P_t(i) = \frac{\sigma}{\sigma - 1} MC_t$$

which says that the firm will set a price equal to a constant mark-up over marginal cost.

Write the price set by the firm that can reset prices in period t as $P^*_t(i)$ to denote that it is an optimal price. Firms that can reset prices in period t will all reset to the same level, so $P^*_t(i) = P^*_t$. Substitute this optimal price into the price index $P_t = \left(\int_0^1 \left(P_t(i)\right)^{1-\sigma} di \right)^{\frac{1}{1-\sigma}}$ and use the fact that in any period $1 - \xi_p$ percent of firms will reoptimize prices to derive an expression for the price index, P_t:

$$P_t = \left(\xi_p \left(\pi^I_{t-1} P_{t-1}\right)^{1-\sigma} + \left(1-\xi_p\right)\left(P^*_t\right)^{1-\sigma}\right)^{\frac{1}{1-\sigma}}$$

A.2.1 Derivation of the NKPC for core inflation

As presented in the text, the optimal wage in real terms is given by:

$$p^*_t = \frac{\sigma}{\sigma-1} \frac{\tilde{E}_t\left(\sum\limits_{\tau=0}^{\infty} \beta^\tau \left(\xi_p\right)^\tau \Lambda_{t+\tau} mc_{t+\tau} \left(\frac{\Pi^I_{t,t+\tau}}{\Pi_{t,t+\tau}}\right)^{-\sigma} \left(p_{t+\tau}\right)^\sigma y_{t+\tau}\right)}{\tilde{E}_t\left(\sum\limits_{\tau=0}^{\infty} \beta^\tau \left(\xi_p\right)^\tau \Lambda_{t+\tau} \left(\frac{\Pi^I_{t,t+\tau}}{\Pi_{t,t+\tau}}\right)^{1-\sigma} \left(p_{t+\tau}\right)^\sigma y_{t+\tau}\right)}$$

Furthermore, the expression for the evolution of the price index in real terms is:

$$1 = \left(\xi_p \left(\frac{\pi^I_{t-1} p_{t-1}}{\pi_t}\right)^{1-\sigma} + \left(1-\xi_p\right)\left(p^*_t\right)^{1-\sigma}\right)^{\frac{1}{1-\sigma}}$$

Recall that $p_t(i) = \tilde{p}_t$ for all firms that can change their price in period t. The linearized form of these two expressions is given by:

$$\begin{aligned} \hat{p}^*_t &= \left(1-\beta\xi_p\right)\left(m\hat{c}_t\right) + \beta\xi_p\left(\tilde{E}_t\left(\pi_{t+1}\right) - \pi^I_t\right) + \beta\xi_p\tilde{E}_t\left(\hat{p}^*_{t+1}\right) \\ 0 &= \xi_p\left(\hat{p}_{t-1} + \pi^I_{t-1} - \pi_t\right) + \left(1-\xi_p\right)\hat{p}^*_t \end{aligned}$$

Furthermore note that the linearization of the price index can be rewritten as $\hat{p}^*_t = \frac{-\xi_p}{1-\xi_p}\left(\hat{p}_{t-1} + \pi^I_{t-1} - \pi_t\right)$ and $\tilde{E}_t\left(\hat{p}^*_{t+1}\right) = \frac{-\xi_p}{1-\xi_p}\left(\hat{p}_t + \pi^I_t - \tilde{E}\left(\pi_t\right)\right)$. After a few substitutions, the New Keynesian Phillips Curve equation is given by:

$$\pi_t = \pi^I_{t-1} + \frac{\left(1-\beta\xi_p\right)\left(1-\xi_p\right)}{\xi_p}\left(m\hat{c}_t\right) + \beta\tilde{E}_t\left(\pi_{t+1}\right) - \beta\pi^I_t$$

In the model with fixed central bank credibility, $\tilde{E}_t\left(\pi^w_{t+1}\right) = E_t\left(\pi^w_{t+1}\right)$, and in the model without price indexation, $\pi^I_t = 0$. Furthermore note that in the model with price indexation, $\pi^I_t = \pi_t$.

Table 1: Volatility and Persistence of inflation and inflation expectations

		US		UK	
		$80's$	$00's$	$85-96$	$97-07$
Standard deviation	π_t	1.357	0.838	2.396	0.899
Standard deviation	$E_t(\pi_{t+1})$	0.598	0.535		
relative to π_t	$\frac{1}{10}\sum_{i=1}^{10} E_t(\pi_{t+i})$	0.536	0.370	0.427	0.405
	$E_t\left(\frac{1}{5}\sum_{i=6}^{10} \pi_{t+i}\right)$	0.490	0.330	0.361	0.418
Correlation with π_t	$E_t(\pi_{t+1})$	0.707	0.438		
	$\frac{1}{10}\sum_{i=1}^{10} E_t(\pi_{t+i})$	0.588	0.249	0.767	0.236
	$E_t\left(\frac{1}{5}\sum_{i=6}^{10} \pi_{t+i}\right)$	0.560	0.176	0.567	0.327
Correlation with π_{t-1}	$E_t(\pi_{t+1})$	0.704	0.479		
	$\frac{1}{10}\sum_{i=1}^{10} E_t(\pi_{t+i})$	0.582	0.276	0.754	0.201
	$E_t\left(\frac{1}{5}\sum_{i=6}^{10} \pi_{t+i}\right)$	0.552	0.195	0.568	0.317
Autocorrelation	π_t	0.965	0.855	0.985	0.962
	$E_t(\pi_{t+1})$	0.922	0.727		
	$\frac{1}{10}\sum_{i=1}^{10} E_t(\pi_{t+i})$	0.969	0.917	0.968	0.941
	$E_t\left(\frac{1}{5}\sum_{i=6}^{10} \pi_{t+i}\right)$	0.965	0.907	0.942	0.941

Table 2: Volatility and Persistence of inflation and inflation expectations

		US		UK	
		Pre-08	Post-08	Pre-08	Post-08
Standard deviation	π_t	0.765	1.984	0.710	2.438
Standard deviation	$E_t\left(\pi_{t+1}\right)$	0.541	0.826	0.424	0.719
relative to π_t	$\frac{1}{10}\sum_{i=1}^{10} E_t\left(\pi_{t+i}\right)$	0.167	0.182	0.268	0.138
	$E_t\left(\frac{1}{5}\sum_{i=6}^{10} \pi_{t+i}\right)$	0.174	0.112	0.315	0.130
Correlation with π_t	$E_t\left(\pi_{t+1}\right)$	0.657	0.566	0.665	0.607
	$\frac{1}{10}\sum_{i=1}^{10} E_t\left(\pi_{t+i}\right)$	0.585	0.433	0.763	0.092
	$E_t\left(\frac{1}{5}\sum_{i=6}^{10} \pi_{t+i}\right)$	0.116	-0.363	0.717	-0.617
Correlation with π_{t-1}	$E_t\left(\pi_{t+1}\right)$	0.395	0.345	0.643	0.467
	$\frac{1}{10}\sum_{i=1}^{10} E_t\left(\pi_{t+i}\right)$	0.537	0.201	0.722	-0.059
	$E_t\left(\frac{1}{5}\sum_{i=6}^{10} \pi_{t+i}\right)$	0.167	-0.460	0.685	-0.609
Autocorrelation	π_t	0.725	0.940	0.953	0.965
	$E_t\left(\pi_{t+1}\right)$	0.634	0.869	0.693	0.920
	$\frac{1}{10}\sum_{i=1}^{10} E_t\left(\pi_{t+i}\right)$	0.630	0.793	0.952	0.647
	$E_t\left(\frac{1}{5}\sum_{i=6}^{10} \pi_{t+i}\right)$	0.593	0.491	0.950	0.837

Table 3: Parameter Values

Symbol	Value	Description
β	0.99	discount factor
α	.36	capital share in production of value added
δ	0.025	capital depreciation rate
σ	10	elasticity of substitution (eos) across varieties from different firms
θ	21	eos between labor from different households
ξ_p	0.75	probability that a firm cannot reset prices
ξ_w	0.75	probability that a household cannot reset wages
θ_p	1.5	coefficient on inflation in the Taylor rule
θ_y	.5	coefficient on the output gap in the Taylor rule
$\bar{\pi}$	$(1.05)^{.25}$	Central bank's true inflation target
$\bar{\pi}^H$	$(1.10)^{.25}$	Public's perception of the high inflation target
$\bar{\pi}^L$	$(1.00)^{.25}$	Public's perception of the low inflation target
$\phi\left(\pi\|\bar{\pi}^L\right)'$	-45.065	Derivative, taken at $\bar{\pi}$ of p.d.f. of inflation under $\bar{\pi}^L$
$\phi\left(\pi\|\bar{\pi}^H\right)'$	45.065	Derivative, taken at $\bar{\pi}$ of p.d.f. of inflation under $\bar{\pi}^H$

Table 4: The effect of unexpected inflation on long term infaltion expectations

Period	$\hat{\gamma}$
1982-2007	0.182
1982-1989	0.279
1990-1999	0.178
2000-2007	0.087

Table 5: Simulated moments of inflation and inflation expectations from different versions of the model with TFP shocks.

		Endogenous Credibility	Indexation	Persistent Shock	Benchmark
Standard deviation	π_t	1.357	2.035	0.853	0.631
Standard deviation	$E_t\left(\pi_{t+1}\right)$	0.917	0.820	0.660	0.479
relative to π_t	$\frac{1}{10}\sum_{i=1}^{10} E_t\left(\pi_{t+i}\right)$	0.752	0.268	0.238	0.110
	$E_t\left(\frac{1}{5}\sum_{i=6}^{10}\pi_{t+i}\right)$	0.697	0.074	0.147	0.084
Correlation with π_t	$E_t\left(\pi_{t+1}\right)$	0.970	1.000	0.993	0.972
	$\frac{1}{10}\sum_{i=1}^{10} E_t\left(\pi_{t+i}\right)$	0.910	0.993	0.882	0.125
	$E_t\left(\frac{1}{5}\sum_{i=6}^{10}\pi_{t+i}\right)$	0.873	0.870	0.661	-0.500
Correlation with π_{t-1}	$E_t\left(\pi_{t+1}\right)$	0.750	0.564	0.354	0.081
	$\frac{1}{10}\sum_{i=1}^{10} E_t\left(\pi_{t+i}\right)$	0.753	0.563	0.358	0.181
	$E_t\left(\frac{1}{5}\sum_{i=6}^{10}\pi_{t+i}\right)$	0.744	0.502	0.308	0.134
Autocorrelation	π_t	0.929	0.922	0.830	0.729
	$E_t\left(\pi_{t+1}\right)$	0.975	0.922	0.856	0.723
	$\frac{1}{10}\sum_{i=1}^{10} E_t\left(\pi_{t+i}\right)$	0.991	0.915	0.932	0.975
	$E_t\left(\frac{1}{5}\sum_{i=6}^{10}\pi_{t+i}\right)$	0.995	0.905	0.980	0.955
Gamma	γ	0.280	0.098	0.082	-0.064

Table 6: Simulated moments of inflation and inflation expectations from different versions of the model with government spending shocks.

		Endogenous Credibility	Indexation	Persistent Shock	Benchmark
Standard deviation	π_t	1.357	2.802	1.159	0.696
Standard deviation	$E_t\left(\pi_{t+1}\right)$	0.901	0.912	0.552	0.672
relative to π_t	$\frac{1}{10}\sum_{i=1}^{10} E_t\left(\pi_{t+i}\right)$	0.436	0.394	0.277	0.144
	$E_t\left(\frac{1}{5}\sum_{i=6}^{10} \pi_{t+i}\right)$	0.225	0.166	0.271	0.132
Correlation with π_t	$E_t\left(\pi_{t+1}\right)$	0.991	0.999	0.957	0.996
	$\frac{1}{10}\sum_{i=1}^{10} E_t\left(\pi_{t+i}\right)$	0.990	0.977	0.304	0.560
	$E_t\left(\frac{1}{5}\sum_{i=6}^{10} \pi_{t+i}\right)$	0.901	0.878	0.070	−0.518
Correlation with π_{t-1}	$E_t\left(\pi_{t+1}\right)$	0.676	0.744	0.201	0.304
	$\frac{1}{10}\sum_{i=1}^{10} E_t\left(\pi_{t+i}\right)$	0.686	0.742	0.286	0.268
	$E_t\left(\frac{1}{5}\sum_{i=6}^{10} \pi_{t+i}\right)$	0.640	0.692	0.259	−0.046
Autocorrelation	π_t	0.948	0.966	0.761	0.847
	$E_t\left(\pi_{t+1}\right)$	0.967	0.960	0.796	0.847
	$\frac{1}{10}\sum_{i=1}^{10} E_t\left(\pi_{t+i}\right)$	0.952	0.943	0.994	0.919
	$E_t\left(\frac{1}{5}\sum_{i=6}^{10} \pi_{t+i}\right)$	0.939	0.921	0.996	0.983
Gamma	γ	0.278	0.286	−0.065	−0.077

Table 7: Comparing the moments of U.S. inflation and inflation from the 1980's and the 2000's to simulations of the model calibrated to match levels of credibility in the 1980's and the 2000's

		US Data			Model - A_t			Model - G_t		
		80's	00's	% Change			% Change			% Change
Standard deviation	π_t	1.357	0.838	−38.25%	1.357	0.938	−30.84%	1.357	1.180	−13.01%
Standard deviation	$E_t(\pi_{t+1})$	0.598	0.535	−10.51%	0.917	0.723	−21.12%	0.901	0.861	−4.44%
relative to π_t	$\frac{1}{10}\sum_{i=1}^{10} E_t(\pi_{t+i})$	0.536	0.370	−30.94%	0.752	0.258	−65.71%	0.436	0.314	−28.11%
	$E_t\left(\frac{1}{5}\sum_{i=6}^{10}\pi_{t+i}\right)$	0.490	0.330	−32.62%	0.697	0.110	−84.30%	0.225	0.158	−29.62%
Correlation with π_t	$E_t(\pi_{t+1})$	0.707	0.438	−37.98%	0.970	0.971	0.01%	0.991	0.993	0.20%
	$\frac{1}{10}\sum_{i=1}^{10} E_t(\pi_{t+i})$	0.588	0.249	−57.62%	0.910	0.950	4.43%	0.990	0.932	−5.90%
	$E_t\left(\frac{1}{5}\sum_{i=6}^{10}\pi_{t+i}\right)$	0.560	0.176	−68.67%	0.873	0.730	−16.30%	0.901	0.230	−74.51%
Correlation with π_{t-1}	$E_t(\pi_{t+1})$	0.704	0.479	−31.96%	0.750	0.393	−47.57%	0.676	0.581	−14.08%
	$\frac{1}{10}\sum_{i=1}^{10} E_t(\pi_{t+i})$	0.582	0.276	−52.59%	0.753	0.394	−47.73%	0.686	0.587	−14.47%
	$E_t\left(\frac{1}{5}\sum_{i=6}^{10}\pi_{t+i}\right)$	0.552	0.195	−64.72%	0.744	0.317	−57.47%	0.640	0.234	−63.46%
Autocorrelation	π_t	0.965	0.855	−11.42%	0.929	0.853	−8.22%	0.948	0.932	−1.65%
	$E_t(\pi_{t+1})$	0.922	0.727	−21.20%	0.975	0.917	−5.97%	0.967	0.953	−1.47%
	$\frac{1}{10}\sum_{i=1}^{10} E_t(\pi_{t+i})$	0.969	0.917	−5.42%	0.991	0.925	−6.70%	0.952	0.933	−1.96%
	$E_t\left(\frac{1}{5}\sum_{i=6}^{10}\pi_{t+i}\right)$	0.965	0.907	−5.97%	0.995	0.950	−4.59%	0.939	0.974	3.79%
Gamma	γ	0.279	0.087	−68.82%	0.280	0.087	−68.88%	0.278	0.089	−67.89%

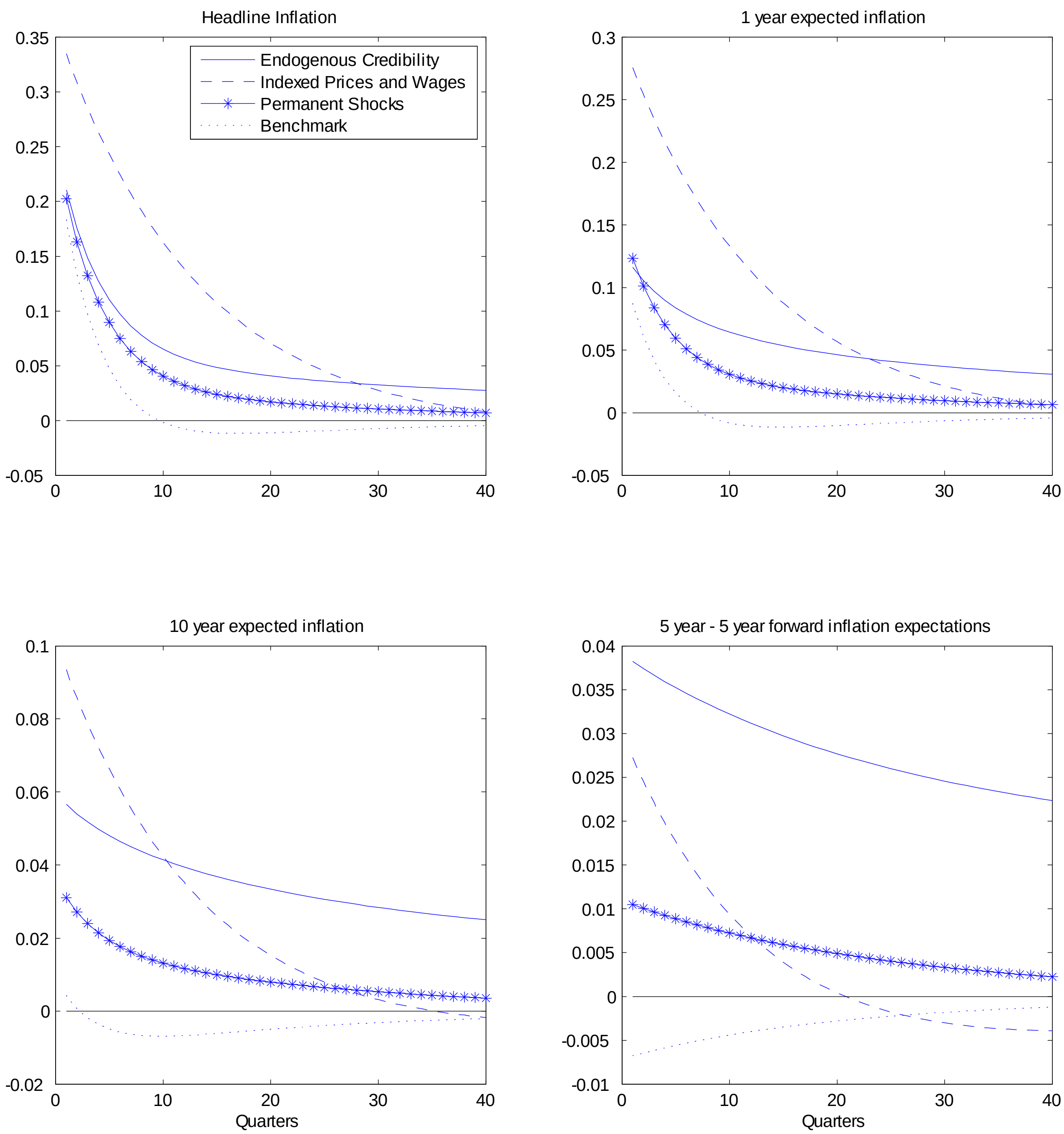

Figure 2: Response of inflation and inflation expectations to a negative TFP shock.

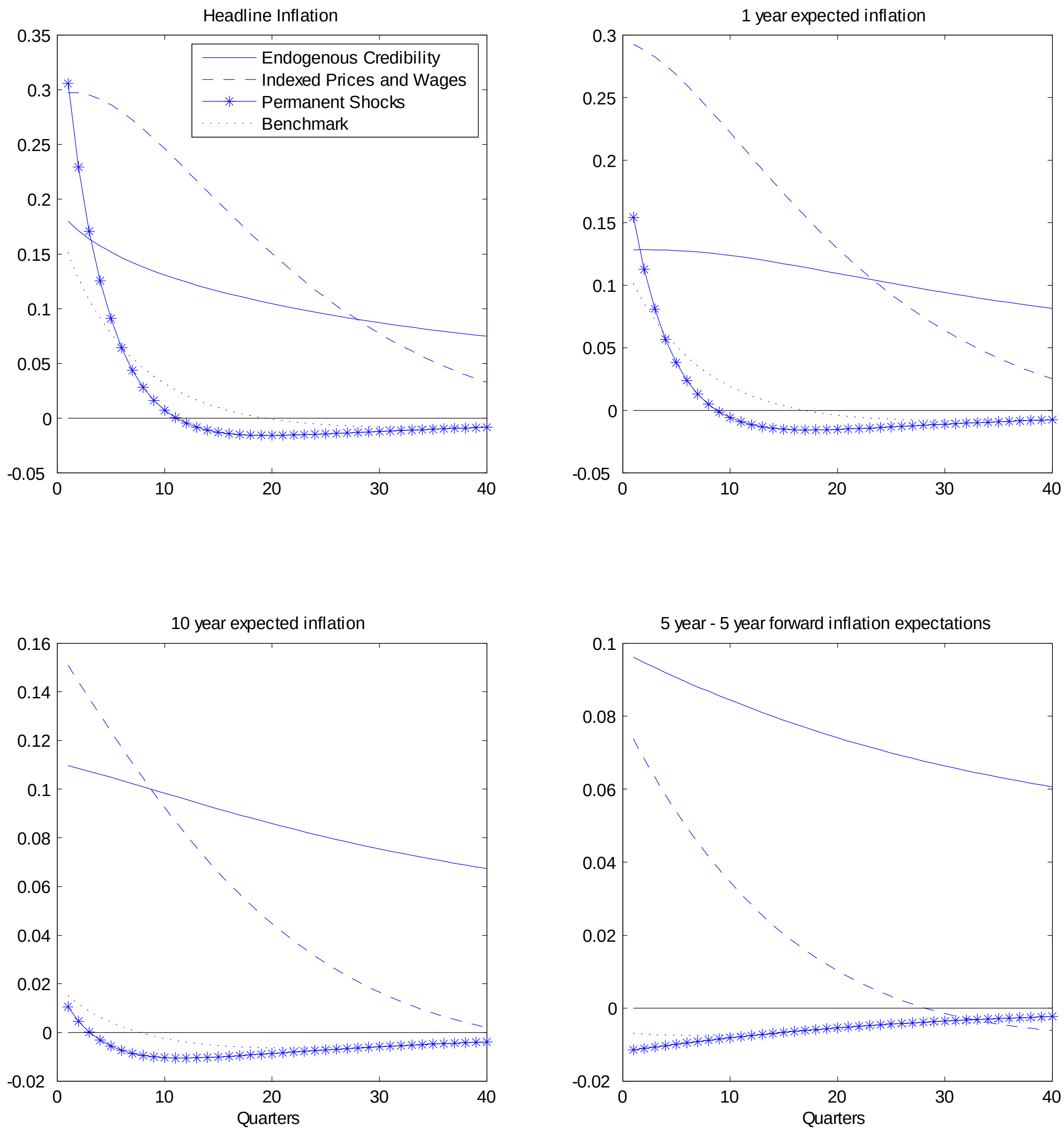

Figure 3: Response of inflation and inflation expectations to a positive government spending shock.