

Targeting Long Rates in a Model with Segmented Markets

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Abstract: This paper develops a model of segmented financial markets in which the net worth of financial institutions limits the degree of arbitrage across the term structure. The model is embedded into the canonical Dynamic New Keynesian (DNK) framework. Our principle results include the following. First, there are welfare gains to having the central bank respond to the term premium, eg., including the term premium in the Taylor Rule. But the sign of the preferred response depends upon the type of shocks driving the business cycle. Second, a policy that directly targets the term premium sterilizes the real economy from shocks originating in the financial sector.

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1. Introduction.

In the aftermath of the 2008 financial crisis many central banks have adopted unconventional policies, including outright purchases of long term government debt. These bond purchases raise a number of research questions for macro theory. Under what conditions can such purchases have aggregate effects? If they have aggregate consequences, how do term premia movements affect inflation and economic activity? What are appropriate policy rules for such interventions? To answer such questions, this paper develops a model of the term premium in which central bank purchases can affect the yield structure independently of the anticipated path of short term interest rates. The model is embedded into an otherwise canonical medium-scale DNK model where long-term bonds are necessary to finance investment purchases. This implies that both new and old policy questions can be examined in a unified framework.

The key features of the model include the following. First, the short term bond market is segmented from the long term bond market in that only financial intermediaries can purchase long term debt. Households can access the long-term debt instruments indirectly by providing deposits to intermediaries. Second, the ability of intermediaries to arbitrage the yield gap between the short term deposit rate and long term lending rate is limited by net worth. That is, a simple hold-up problem constrains the amount of deposits that can be supported by a given level of intermediary net worth. Third, the intermediary faces adjustment costs in rapidly varying the size of its portfolio in the wake of shocks. These assumptions imply that central bank purchases of long-term bonds will have a significant effect on long yields. Finally these long-term yields affect real economic activity because of our final assumption: capital investment is financed by the issuance of long term bonds which sell in the same market that absorbs long term Treasuries. Taken together, these assumptions imply that central bank purchases of long-term bonds will have a significant and persistent effect on long yields and real activity.

We use the model to consider the efficacy of alternative policies linked to the term premium. This is a natural policy in the context of the model as the distortion arising from market segmentation is, to a linear approximation, equal to the term premium. Hence, we show that there are significant welfare gains to including the term premium in a traditional Taylor rule operating on the short term rate. We also consider policies that

utilize a Taylor-type rule over the long rate. Such a long rate policy sterilizes the rest of the model economy from shocks originating in the financial system. This sterilization directly analogous to the classic Poole (1970) result that a FFR targets sterilize the economy from money demand shocks.

The two papers closest in spirit to the current work are Gertler and Karadi (2011, 2013) and Chen, Curdia, and Ferrero (2013). There are two crucial similarities between these papers and the present work. First, there is some friction that limits the ability to arbitrage across the short-term and long-term bond markets. This implies that the long rate is not the expected average of short rates, i.e., there is a term premium. Second, the market segmentation has real effects because some portion of real activity is financed in the segmented market. Gertler and Karadi (2013) assume that the entire capital stock is re-financed each period by the purchase of equity claims in this market by intermediaries. Chen et al. (2013) assume that a small subset of consumers finance their consumption in the segmented market. In contrast, the current paper assumes that new investment is financed in the segmented market with the issuance of long term debt. Both of these assumptions will magnify the effects of segmentation because investment is the most interest-sensitive component of aggregate expenditure, and the long term debt assumption implies that persistent interest rate movements have larger effects. Hence, a central bank bond purchase policy will have a much larger effect in the present paper than in the models of Gertler and Karadi (2013) and Chen et al. (2011).

The paper proceeds as follows. The next section develops the theoretical model, culminating in a discussion of calibration. Section 3 presents our quantitative results including how the segmentation affects the IRFs to shocks, and the efficacy of central bank policies that directly or indirectly target the term premium. Section 4 concludes.

2. The Model.

The economy consists of households, financial intermediaries (FI's), and firms. We discuss each in turn.

Households.

Households are infinitely lived with preferences over consumption (C_t) and labor (L_t) given by:

$$E_0 \sum_{j=0}^{\infty} \beta^j \left\{ \ln(C_{t+j} - hC_{t+j-1}) - B \frac{L_{t+j}^{1+\eta}}{1+\eta} \right\} \quad (1)$$

The household earns income by selling its labor services and renting capital to the intermediate goods firm. The household has two means of intertemporal smoothing: short term deposits (D_t) in the financial intermediaries (FI), and accumulation of physical capital (K_t). Households also have access to the market in short term government bonds (“T-bills”). But since T-bills are perfect substitutes with deposits, and the supply of T-bills moves endogenously to hit the central bank’s short-term interest rate target, we treat D_t as the household’s net resource flow into the FI’s. To introduce a need for intermediation, we assume that all investment purchases must be financed by issuing new “investment bonds” that are ultimately purchased by the FI. We find it convenient to use the perpetual bonds suggested by Woodford (2001). In particular, these bonds are perpetuities with cash flows of 1, κ_I , κ_I^2 , etc. Let Q_t^I denote the time-t price of a new issue. Given the time pattern of the perpetuity payment, the new issue price Q_t^I summarizes the prices at all maturities, eg., $\kappa_I Q_t^I$ is the time-t price of the perpetuity issued in period t-1. The duration and (gross) yield to maturity on these bonds are defined as: duration = $(1 - \kappa_I)^{-1}$, gross yield to maturity = $Q_t^{-1} + \kappa_I$. Let CI_t denote the number of new perpetuities issued in time-t to finance investment. In time-t, the household’s nominal liability on past issues is given by:

$$F_{t-1} = CI_{t-1} + \kappa_I CI_{t-2} + \kappa_I^2 CI_{t-3} + \dots \quad (2)$$

We can use this recursion to write the new issue as

$$CI_t = (F_t - \kappa_I F_{t-1}) \quad (3)$$

The household constraints are thus given by:

$$C_t + \frac{D_t}{P_t} + P_t^k I_t + \frac{F_{t-1}}{P_t} \leq W_t L_t + R_t^k K_t - T_t + \frac{D_{t-1}}{P_t} R_{t-1} + \frac{Q_t^I (F_t - \kappa_I F_{t-1})}{P_t} + div_t \quad (4)$$

$$K_{t+1} \leq (1 - \delta) K_t + I_t \quad (5)$$

$$P_t^k I_t \leq \frac{Q_t^I (F_t - \kappa_I F_{t-1})}{P_t} = \frac{Q_t^I CI_t}{P_t} \quad (6)$$

where P_t is the price level, P_t^k is the real price of capital, R_{t-1} is the gross nominal interest rate on deposits, W_t is the real wage, R_t^k is the real rental rate, T_t are lump-sum taxes, and div_t denotes the dividend flow from the FI’s.

The household also receives a profit flow from the intermediate goods producers and the new capital producers,

but this is entirely standard so we dispense from this added notation for simplicity. The “loan-in-advance” constraint (6) will increase the private cost of purchasing investment goods. Although for simplicity we place capital accumulation within the household problem, this model formulation is isomorphic to an environment in which household-owned firms accumulate capital subject to the loan constraint. In any event, the first order conditions to the household problem include:

$$\Lambda_t = E_t \beta \Lambda_{t+1} \frac{R_t}{\Pi_{t+1}} \quad (7)$$

$$BL_t^\eta = W_t \Lambda_t \quad (8)$$

$$\Lambda_t P_t^k M_t = E_t \beta \Lambda_{t+1} [R_t^k + (1 - \delta) P_{t+1}^k M_{t+1}] \quad (9)$$

$$\Lambda_t Q_t^I M_t = E_t \beta \Lambda_{t+1} \frac{[1 + \kappa_I Q_{t+1}^I M_{t+1}]}{\Pi_{t+1}} \quad (10)$$

where Λ_t denotes the marginal-utility of time-t consumption, and $\Pi_t \equiv \frac{P_t}{P_{t-1}}$ is gross inflation. Expressions (7) and (8) are the familiar Fisher equation and labor supply curve. The capital accumulation expression (9) is distorted relative to the familiar by the time-varying distortion M_t , where $M_t \equiv 1 + \frac{\vartheta_t}{\Lambda_t}$, and ϑ_t is the multiplier on the loan-in-advance constraint (6). The endogenous behavior of this distortion is fundamental to the real effects arising from market segmentation.

Financial Intermediaries.

The FI’s in the model are a stand-in for the entire financial nexus that uses accumulated net worth (N_t) and short term liabilities (D_t) to finance investment bonds (F_t) and the long-term government bonds (B_t). The FIs are the sole buyers of the investment bonds and long term government bonds. We again assume that government debt takes the form of Woodford-type perpetuities that provide payments of 1, κ , κ^2 , etc. Let Q_t denote the price of a new-debt issue at time-t. The time-t asset value of the current and past issues of investment bonds is:

$$Q_t^I C I_t + \kappa_I Q_t^I [C I_{t-1} + \kappa_I C I_{t-2} + \kappa_I^2 C I_{t-3} + \dots] = Q_t^I F_t \quad (11)$$

The FIs balance sheet is thus given by:

$$\frac{B_t}{P_t} Q_t + \frac{F_t}{P_t} Q_t^I = \frac{D_t}{P_t} + N_t \quad (12)$$

Note that on the asset side, investment lending and long term bond purchases are perfect substitutes to the FI. Let

$R_t^L \equiv E_t \left(\frac{1+\kappa Q_{t+1}}{Q_t} \right) = E_t \left(\frac{1+\kappa_I Q_{t+1}^I}{Q_t^I} \right)$, denote the common one-period gross return on these assets. The financial

friction arises on the other side of the balance sheet: FI's ability to attract deposits will be limited by their net worth. We will use a simple hold-up problem to generate this constraint, but a wide variety of informational restrictions will generate the same constraint. Let X_t denote the bank's real asset portfolio:

$$X_t \equiv \frac{B_t}{P_t} Q_t + \frac{F_t}{P_t} Q_t^I. \quad (13)$$

This portfolio has expected return R_t^L during time t . At the beginning of period $t+1$, but before aggregate shocks are realized, the FI can choose to default on its planned repayment to depositors. In this event, depositors can seize at most fraction Φ_t of the FI's assets. If the FI chooses to repay depositors, the FI is left with $(R_t^L X_t - R_t \frac{D_t}{P_t})$. If the FI defaults, the FI is left with $(1 - \Phi_t) R_t^L X_t$, but is otherwise free to continue functioning in subsequent periods.¹ To ensure that the FI will always re-pay the depositor, the time- t hold-up constraint is thus given by:

$$R_t \frac{D_t}{P_t} \leq \Phi_t R_t^L X_t \quad (14)$$

Using the balance sheet identity and re-arranging we have:

$$X_t \leq N_t L \left(\frac{R_t^L}{R_t} \right) \quad (15)$$

where the leverage function is given by:

$$L \left(\frac{R_t^L}{R_t} \right) \equiv \left[1 - \Phi_t \frac{R_t^L}{R_t} \right]^{-1} \quad (16)$$

(ψ_n Log-linearizing this expression we have:

$$(r_t^L - r_t) = \nu l_t - \phi_t \quad (17)$$

¹ This is in contrast to Gertler and Karadi (2011, 2013) who assume that the bank exits the industry.

where $\nu \equiv (L_{ss} - 1)^{-1}$, is the elasticity of the interest rate spread to leverage, and $\phi_t \equiv \ln(\Phi_t)$, follows an AR(1) process:

$$\phi_t = (1 - \rho_\phi)\phi_{ss} + \rho_\phi\phi_{t-1} + \varepsilon_{\phi,t}, \quad (18)$$

Decreases in ϕ_t will exacerbate the hold-up problem, and thus are “credit shocks” which will increase the spread and lower real activity. Qualitatively this log-linearized expression (14) for leverage is identical to the corresponding relationship in the more complex costly-state- verification (CSV) environment of, for example, Bernanke, Gertler, and Gilchrist (1999). One important quantitative difference is that interest rate spreads are much more responsive to leverage in our framework than in the CSV model calibrated to the same steady state leverage. In CSV models with leverage calibrated to 2 the elasticity ν is typically less than 0.05, whereas it would be 0.5 in our model for the same steady state leverage. This is part of the reason why financial frictions have a comparably large real effect in our model.

The hold-up problem would not remain a constraint if the FI could accumulate sufficient net worth. At the beginning of period t , the FI has profits on its portfolio equal to

$$prof_t \equiv \left[\frac{Q_{t-1}B_{t-1}}{P_t} \left(\frac{1+\kappa Q_t}{Q_{t-1}} \right) + \frac{Q_{t-1}^I F_{t-1}}{P_t} \left(\frac{1+\kappa_I Q_t^I}{Q_{t-1}^I} \right) - R_{t-1} \frac{D_{t-1}}{P_t} \right] \quad (19)$$

The FI will pay out some of these as dividends to the household, and retain the rest as net worth for subsequent activity. In making this choice, the FI discounts dividend flows using the household’s pricing kernel, augmented with additional impatience. The FI’s decision problem is given by:

$$\max E_0 \sum_{j=0}^{\infty} (\beta\zeta)^j \Lambda_{t+j} div_{t+j} \quad (20)$$

subject to

$$div_t + N_t[1 + N_t f(N_t)] = prof_t \quad (21)$$

where $f(N_t) \equiv \frac{\psi_n}{2} \left(\frac{N_t - N_{ss}}{N_{ss}} \right)^2$. The FI’s net worth decision is given by:

$$\Lambda_t [1 + N_t f'(N_t) + f(N_t)] = E_t \beta \zeta \Lambda_{t+1} \frac{R_t^L}{\Pi_{t+1}} \quad (22)$$

Equations (15) and (22) are fundamental to the model as they summarize the limits to arbitrage between the return on long term bonds and the rate paid on short term deposits. The net worth constraint (15) limits the FI's ability to attract deposits and eliminate the arbitrage opportunity between the deposit and lending rate. Hence, the expectations theory of the term structure holds within the long bond market, but not between the short and long debt market. In essence, the segmentation decouples the short rate from the rest of the term structure. Increases in net worth allow for greater arbitrage and thus can eliminate this market segmentation. Equation (22) limits this arbitrage in the steady-state ($\zeta < 1$) and dynamically ($\psi_n > 0$).² Since the FI is the sole means of investment finance, this market segmentation means that central bank purchases that alter the supply of long term debt will have repercussions for investment loans because net worth and deposits cannot quickly sterilize the purchases.

Final good producers.

Perfectly competitive firms produce the final consumption good Y_t combining a continuum of intermediate goods according to the CES technology:

$$Y_t = \left[\int_0^1 Y_t(i)^{1/(1+\epsilon_p)} di \right]^{1+\epsilon_p} \quad (23)$$

Profit maximization and the zero profit condition imply that the price of the final good, P_t , is the familiar CES aggregate of the prices of the intermediate goods.

Intermediate goods producers.

² In particular, comparing equations (7) and (22) we see that without adjustment costs the spread between the loan rate and the return on short-term T-bills would be constant over time.

A monopolist produces the intermediate good i according to the production function

$$Y_t(i) = A_t K_t(i)^\alpha L_t(i)^{1-\alpha} \quad (24)$$

where $K_t(i)$ and $L_t(i)$ denote the amounts of capital and labor employed by firm i . The variable $\ln A_t$ is the exogenous level of TFP and evolves according to:

$$\ln A_t = \rho_A \ln A_{t-1} + \varepsilon_{a,t}, \quad (25)$$

Every period a fraction θ_p of intermediate firms cannot choose its price optimally, but resets it according to the indexation rule

$$P_t(i) = P_{t-1}(i) \Pi_{t-1}^p, \quad (26)$$

where $\Pi_t = \frac{P_t}{P_{t-1}}$ is gross inflation. The remaining fraction of firms chooses its price $P_t(i)$ optimally, by maximizing the present discounted value of future profits

$$E_t \left\{ \sum_{s=0}^{\infty} \theta_p^s \frac{\beta^s \Lambda_{t+s} / P_{t+s}}{\Lambda_t / P_t} \left[P_t(i) \left(\prod_{k=1}^s \Pi_{t+k-1}^p \right) Y_{t+s}(i) - W_{t+s} L_{t+s}(i) - P_{t+s} \rho_{t+s} K_{t+s}(i) \right] \right\} \quad (27)$$

where the demand function comes from the final goods producers.

New Capital Producers.

New capital is produced according to the production technology that takes I_t investment goods and transforms them into $\mu_t \left[1 - S \left(\frac{I_t}{I_{t-1}} \right) \right] I_t$ new capital goods. The time- t profit flow is thus given by

$$P_t^k \mu_t \left[1 - S \left(\frac{I_t}{I_{t-1}} \right) \right] I_t - I_t \quad (28)$$

where the function S captures the presence of adjustment costs in investment, as in Christiano et al. (2005), and is given by:

$$S\left(\frac{I_t}{I_{t-1}}\right) \equiv \frac{\psi_i}{2} \left(\frac{I_t}{I_{t-1}} - 1\right)^2$$

These firms are owned by households and discount future cash flows with Λ_t . The investment shock follows the stochastic process

$$\log \mu_t = \rho_\mu \log \mu_{t-1} + \varepsilon_{\mu,t}, \quad (29)$$

where $\varepsilon_{\mu,t}$ is *i.i.d.* $N(0, \sigma_\mu^2)$. Following Justiniano, Primiceri, and Tambalotti (2010,2011), we call these MEI shocks, for “marginal efficiency of investment.” The chief source of the business cycle in the model will be TFP and MEI shocks.

Central Bank Policy.

We assume that the central bank follows a familiar Taylor rule over the short rate (T-bills and deposits):

$$\ln(R_t) = (1 - \rho) \ln(R_{ss}) + \rho \ln(R_{t-1}) + (1 - \rho)(\tau_\pi \pi_t + \tau_y y_t^{gap}) \quad (30)$$

where $y_t^{gap} \equiv (Y_t - Y_t^f)/Y_t^f$, denotes the deviation of output from its flexible price counterpart. We will think of this as the Federal Funds Rate (FFR). Below we will also investigate the efficacy of putting the term-premium into the Taylor rule. The supply of T-bills is endogenous, varying as needed to support the FFR target. As for the long term policy, the central bank will choose between: (i) an exogenous path for the quantity of long term debt available to FIs, or (ii) a policy rule for the long term bond yield. We will return to this below.

Fiscal policy is entirely passive. Government expenditures are set to zero. Lump sum taxes move endogenously to support the interest payments on the short and long debt.

Loglinearized Model.

To gain further intuition and to derive the term premium, we first log-linearize the model. Let $b_t \equiv [\ln(Q_t \frac{B_t}{P_t}) - \ln(Q_{ss} \frac{B_{ss}}{P_{ss}})]$, and $f_t \equiv [\ln(Q_t^I \frac{F_t}{P_t}) - \ln(Q_{ss}^I \frac{F_{ss}}{P_{ss}})]$, denote the real market value of the bonds

available to FIs. We will focus on bonds of 10-year maturities, so R_t^{10} will denote their gross yield. Using lower case letters to denote log deviations, the log-linearized model is given by the following:

$$\lambda_t = \frac{1}{(1-\beta h)(1-h)} E_t[\beta h c_{t+1} - (1 + \beta h^2)c_t + h c_{t-1}] \quad (32)$$

$$\eta L_t = w_t + \lambda_t \quad (33)$$

$$\lambda_t = \lambda_{t+1} + r_t - \pi_{t+1} \quad (34)$$

$$\lambda_t + p_t^k + m_t = E_t\{\lambda_{t+1} + [1 - \beta(1 - \delta)]r_{t+1}^k + \beta(1 - \delta)(p_{t+1}^k + m_{t+1})\} \quad (35)$$

$$m_t + r_t + q_t^i = \beta \kappa_I E_t(q_{t+1}^i + m_{t+1}) \quad (36)$$

$$(1 - \kappa_I)(p_t^k + i_t) = f_t - \kappa_I(f_{t-1} + q_t^i - q_{t-1}^i - \pi_t) \quad (37)$$

$$r_t^L = E_t \frac{\kappa_I q_{t+1}^i}{R_{SS}^L} - q_t^i \quad (38)$$

$$r_t^L = E_t \frac{\kappa q_{t+1}}{R_{SS}^L} - q_t \quad (39)$$

$$r_t^{10} = -\left(\frac{R_{SS}^L - \kappa}{R_{SS}^L}\right) q_t \quad (40)$$

$$(r_t^L - r_t) = \left(\frac{1}{L_{SS} - 1}\right) \left[\frac{b_{SS}}{L_{SS} n_{SS}} b_t + \left(1 - \frac{b_{SS}}{L_{SS} n_{SS}}\right) f_t - n_t\right] - \phi_t \quad (41)$$

$$\psi_n n_t = (r_t^L - r_t) \quad (42)$$

$$\frac{b_{SS}}{n_{SS}} b_t + \left(L_{SS} - \frac{b_{SS}}{n_{SS}}\right) f_t = n_t + (L_{SS} - 1) d_t \quad (43)$$

$$w_t = m c_t + m p l_t \quad (44)$$

$$r_t^k = m c_t + m p k_t \quad (45)$$

$$\pi_t = \frac{\kappa \pi}{1 + \beta l_p} m c_t + \frac{\beta}{1 + \beta l_p} E_t \pi_{t+1} + \frac{l}{1 + \beta l_p} \pi_{t-1} \quad (46)$$

$$p_t^k = \psi_i [(i_t - i_{t-1}) - \beta E_t (i_{t+1} - i_t)] - \mu_t \quad (47)$$

$$\left(1 - \frac{I_{SS}}{Y_{SS}}\right) c_t + \frac{I_{SS}}{Y_{SS}} i_t = a_t + \alpha k_t + (1 - \alpha) L_t \quad (48)$$

$$k_{t+1} = (1 - \delta) k_t + \delta (\mu_t + i_t) \quad (49)$$

$$r_t = \rho r_{t-1} + (1 - \rho) (\tau_\pi \pi_t + \tau_y y_t^{gap}) \quad (50)$$

To close the model, we need one more equation outlining the policy rule for the long term debt market.

Before a discussion of these policy options, several comments are in order.

First, equation (35) highlights the economic distortion, m_t , arising from the segmented markets. Solving this forward we have:

$$p_t^k + m_t = \sum_{j=0}^{\infty} [\beta(1-\delta)]^j \{ [1 - \beta(1-\delta)] r_{t+j}^k - (r_{t+j} - \pi_{t+j+1}) \} \quad (51)$$

As is clear from (51), the segmentation distortion, m_t , acts like a mark-up or excise tax on the price of new capital goods. What is this distortion? Using (36) and (38) we have

$$m_t = \sum_{j=0}^{\infty} (\beta\kappa_I)^j \Xi_{t+j}, \quad (51)$$

where

$$\Xi_{t+j} \equiv \beta\kappa_I q_{t+j+1}^i - q_{t+j}^i - r_{t+j} \approx r_{t+j}^L - r_{t+j} \quad (52)$$

The distortion is thus the discounted sum of the future one-period loan to deposit spreads. As discussed above, this spread exists because of the assumed market segmentation.

Second, the *term premium* can be defined as the difference between the observed yield on a 10-year bond (see (40)) and the corresponding yield implied by applying the expectation hypothesis (EH) of the term structure to the series of short rates. The price of this hypothetical EH bond satisfies

$$r_t = E_t \frac{\kappa q_{t+1}^{EH}}{R^L} - q_t^{EH} \quad (53)$$

while its yield is given by

$$r_t^{EH,10} = \left(\frac{R_{SS} - \kappa}{R_{SS}} \right) q_t^{EH}. \quad (54)$$

Using these definitions, the term premium can be expressed as

$$\text{term premium} \equiv tp_t \equiv (r_t^{10} - r_t^{EH,10}) = - \left(\frac{R_{SS}^L - \kappa}{R_{SS}^L} \right) q_t + \left(\frac{R_{SS} - \kappa}{R_{SS}} \right) q_t^{EH} \quad (55)$$

Solving the bond prices in terms of the future short rates, we have

$$\begin{aligned} tp_t &= \left(\frac{R_{SS}^L - \kappa}{R_{SS}^L} \right) \sum_{j=0}^{\infty} \left(\frac{\kappa}{R_{SS}^L} \right)^j r_{t+j}^L - \left(\frac{R_{SS} - \kappa}{R_{SS}} \right) \sum_{j=0}^{\infty} \left(\frac{\kappa}{R_{SS}} \right)^j r_{t+j} \\ &\approx (1 - \beta\kappa) \sum_{j=0}^{\infty} (\beta\kappa)^j (r_{t+j}^L - r_{t+j}) \end{aligned} \quad (56)$$

Comparing (51) and (56), the distortion m_t is closely proxied by the term premium. One minor difference is that the weights in the term premium are linked to the duration of the government bond via κ , while the segmentation distortion (51) is linked the duration of the investment bond (κ_I). In any event, a policy that eliminates fluctuations in the term premium will largely eliminate fluctuations in the market segmentation distortion.

Third, the loan-deposit spread arises because of the segmentation effects summarized in (41)-(42). Equation (41) expresses the endogenous response of leverage to higher expected returns on intermediation, while equation (42) summarizes the FIs desire to accumulate more net worth in response to the profit opportunity of the spread. The model's dynamics collapse to the familiar DNK model if we set $\psi_n = 0$, so that net worth can move instantaneously to eliminate all arbitrage opportunities.

As noted above, a policy that stabilizes the term premium will be welfare improving (at least without any other distortion, e.g. sticky prices). This suggests the efficacy of a central bank directly targeting this premium. But even if the central bank is unable to target the term premium, under a long rate policy the supply of long debt held by FIs will be endogenous. In particular, (41) and (43) separate out from the rest of the model, and define the behavior of long bonds and FI deposits that move endogenously to support the long rate target. This implies that “credit shocks”, those proxied by ϕ_t in (41), will have no effect on real activity or inflation. That is, a long rate policy sterilizes the real economy from financial shocks. This is analogous to the classic result of Poole (1970) in which an interest rate target sterilizes the real economy from shocks to money demand.

Debt market policies.

To close the model, we need one more restriction that will pin down the behavior in the long debt market. We will consider two different policy regimes for this market: (i) exogenous debt, and (ii) endogenous debt. We will discuss each in turn.

Exogenous debt. The variable b_t denotes the real value of long term government debt on the balance sheet of FI's. There are two distinct reasons why this variable could fluctuate. First, the central bank could engage in

long bond purchases (“quantitative easing,” or QE). Second, the fiscal authority could alter the mix of short debt to long debt in its maturity structure. We will model both of these scenarios as exogenous movements in long debt. Under either scenario, the long yield r_t^{10} will be endogenous. To model a persistent and hump-shaped QE policy shock we will use an AR(2):

$$b_t = \rho_1^b b_{t-1} + \rho_2^b b_{t-2} + \epsilon_t^b \quad (57)$$

Within such an exogenous debt regime, we will also consider policies in which the Taylor rule for the short rate responds to some measure of the term premium:

$$r_t = \rho r_{t-1} + (1 - \rho)(\tau_\pi \pi_t + \tau_y y_t^{gap} + \tau_{tp} tp_t) \quad (58)$$

where the term premium (tp_t) is defined as in (56). As noted earlier, there are reasons to think that such a policy may be welfare-improving.

Endogenous debt. The polar opposite scenario is a policy under which the central bank targets the term-premium tp_t , in a fashion similar to the Taylor rule for the short rate:

$$tp_t = \rho_{10} tp_{t-1} + (1 - \rho_{10})(\tau_\pi^{10} \pi_t + \tau_y^{10} y_t). \quad (55)$$

Under this policy regime the level of long debt b_t will be endogenous. We will focus on policies that peg the term premium at steady state, ie., $tp_t = 0$.

When the government pegs the term premium at steady state it effectively becomes the marginal lender to the private sector for corporate investment. To see this, note from (56) that one way to achieve a term premium peg is to hold constant the spread of the period return on corporate bonds over the deposit rate at all times. But then from the balance sheet of the intermediary and the leverage constraint, intermediary net worth and household deposits are constant. Hence, any increase of intermediary holdings of corporate debt is achieved via a selling government bonds back to the treasury. The proceeds from this sale effectively finances loans to corporates.

Calibration.

Much of the calibration is standard and is similar to Gertler and Karadi (2013): $\beta = 0.99$, $h = 0.8$, $\eta = 0.25$, $\iota_p = 1$, $\epsilon_p = 5$, $\theta_p = 0.85$, $\psi_i = 2$. Monetary policy over the funds rate is given by $\rho = 0.8$, $\tau_\pi = 1.5$, and $\tau_y^{gap} = 0.5$. The atypical parameters for calibration are those surrounding the FI. We will use evidence on interest rate spreads and leverage to pin down two primitive parameters.³ The steady-state loan-deposit spread and leverage ratio are given by:

$$\zeta = \left(\frac{R_{SS}^L}{R_{SS}} \right)^{-1}$$

$$L_{SS} = \left[1 - \Phi_{SS} \left(\frac{R_{SS}^L}{R_{SS}} \right) \right]^{-1}$$

We will choose the parameters ζ and ψ , to match an interest rate spread of 100 annual bp, and a leverage level of 6. This is the same calibration as in Gertler and Karadi (2013). The government and investment bonds will both be calibrated to a duration of 40 quarters, $(1 - \kappa_f)^{-1} = (1 - \kappa)^{-1} = 40$. We also need to calibrate the balance sheet proportion $\frac{b_{SS}}{n_{SS}}$. This is proportional to the fraction of FI assets held as long term debt:

$$\frac{b_{SS}}{n_{SS}} = \frac{b_{SS}}{n_{SS} + d_{SS}} * \frac{n_{SS} + d_{SS}}{n_{SS}} = \frac{b_{SS}}{f_{SS} + b_{SS}} * L_{SS}$$

Consistent with studies of bank balance sheets, we set the ratio of government securities to total bank assets to

$$\frac{b_{SS}}{f_{SS} + b_{SS}} = 40\%.$$

Finally, the adjustment cost parameter ψ_n drives the link between net worth accumulation and the loan-deposit spread. We will choose this parameter to be consistent with the empirical evidence on the effect of the Fed's QE policies on the 10 year bond rate. We will later provide sensitivity analysis showing that only a modest

³ The hold-up constraint is (linearly) isomorphic to a more elaborate CSV problem between depositors and FI's. In a CSV model, the primitives include: (i) idiosyncratic risk, (ii) death rate, and (iii) monitoring cost. One typically chooses these to match values for (i) leverage, (ii) interest rate spread, and (iii) default rate. The hold-up model has only two primitives: (i) the impatience rate ζ , and (ii) the fraction of assets that can be seized Φ . In comparison to the hold-up model, the extra primitive in the CSV framework allows it to match one more moment of the financial data (default rates).

degree of adjustment costs are necessary to produce a significant change in the term premium and real economic activity.

Figure 1 graphs the change in the Fed's bond portfolio relative to the government debt in the hands of the domestic public. The QE policies are quite apparent. We will consider a QE shock that increases b_t by 6.5%, comparable to the magnitude in Figure 1 (roughly \$300 billion). To match the persistent nature of this expansion we set $\rho_1^b = 1.8$, and $\rho_2^b = -0.81$. Empirical estimates of the response of the 10 year yield to these QE shocks vary from no effect to over 45 bp (eg., the evidence discussed in Chen et al. (2013)). We set $\psi_n = 1$, implying that the long term yield moves by 23 bp in response to our QE shock, a response in the middle of the estimates in the literature.

3. Quantitative Results.

a. QE shocks.

The impulse response to the QE shock is exhibited in Figure 2. The policy shock has a persistent effect on the 10 year yield, with all of this initial movement being driven by changes in the term premium. This term premium effect dissipates as net worth responds and segmentation returns to steady state levels, so that the long rate is eventually driven by the path of the short rate. The policy has a persistent and significant effect on investment and output, while consumption is little changed for the first 10 quarters. The demand component of the shock naturally leads to an increase in inflation, and thus a policy-induced increase in the funds rate. The funds rate eventually overshoots its long run level, thus leading to a persistent decline in the long rate. The real-effects of the QE shock is larger and more persistent than in other models of QE for two reasons. First, the long term nature of the investment debt (10 year maturity) amplifies the effect of a given movement in the yield. And second, the FIs finance investment purchases, the most elastic portion of aggregate output. In contrast, Gertler and Karadi (2011, 2013) have the entire capital stock financed by the FIs, while Chen et al. (2011) have consumption of one class of agents linked to the segmented market.

Sensitivity analysis on the QE experiment is reported in Table 1. The first observation is that the quantitative results are relatively unaffected by the size of the adjustment costs on net worth, ψ_n . The peak investment response only varies from 8.1% to 10.4% as we vary the adjustment cost parameter from 0.25 to 2. Similarly, the degree of price stickiness has only modest effects on the response of investment and the long yield. Fundamentally, the “loan-in-advance” constraint is a real constraint, so that QE has an effect even in a flexible price world. The remaining parameters in Table 1 have a bigger quantitative impact. If FIs hold little government debt, then purchases of debt have a smaller effect on their balance sheet and thus a smaller effect on real activity. This suggests a form of diminishing returns to QE activities in that the stock of government debt held by FIs is negatively related to past QE expansions. The effect of higher leverage is in the opposite direction. From (41), higher levels of steady state leverage dampen the effect of movements in leverage on the loan-deposit spread, so that QE shocks have a smaller effect.

Finally, Table 1 demonstrates the quantitative importance of the duration of the investment bonds. These different durations imply significantly different steady state distortions as M_{ss} is increasing in the duration of the investment bond. Figure 3 plots this relationship, with M_{ss} varying from 1.04 at 20 quarters, 1.07 at 40 quarters, and 1.11 at 80 quarters. This steady state effect is also manifested in the dynamic effect of a QE shock as longer maturities for the investment bonds lead to much larger effects on both real activity and the long bond term premium. For example, as we move from 5 year bonds to 20 year bonds, the peak investment response increases from 7.2% to 12.5%, and the long rate response increases from 20bp to 29bp. Recall that without a borrowing constraint for investment, changes in the term premium would have no real effects.⁴

b. Other shocks under an exogenous debt policy.

Figures 6-9 look at the effect of an investment shock, a TFP shock, a credit shock, and a monetary shock, respectively. The real shocks are both set to 1%, and the monetary shock is 100 annual bp. The TFP shock has

⁴ If we vary the duration of government debt, κ , with an exogenous debt policy there will be no real impact. Government bond duration separates out from the rest of the system (see equations 39 and 40) so κ only affects bond pricing and the term premium.

an autocorrelation of 0.95, the investment and credit shocks have autocorrelation equal to 0.8, and the monetary shock has autocorrelation of 0.6. The term premium moves only modestly in response to the investment shock so that the basic dynamics resemble those of the DNK model. But the TFP shock leads to a significant increase in the term premium, so that investment declines and thus dampens the response of output to the shock. This increase in the term premium is a response to the 59 bp decline in the short rate induced by the deflationary pressure of the TFP increase. The inertial movement in the FI's balance sheet implies that the long rate cannot move as sharply as the short rate leading to the increase in the term premium. Similarly, the credit shock leads to a significant increase in the long rate (32 bp) and term premium (36 bp), with corresponding real effects that would not arise in a simple DNK model. As with a QE shock, the term premium returns to steady state before the long rate because the overshooting of the short rate eventually dominates the determination of the long rate. Finally, the monetary shock induces a significant increase in the term premium which amplifies the effect of the monetary shock (compared to its DNK counterpart).

c. **Shocks under a term premium peg.**

Figures 10-13 look at the same experiment as Figures 6-9, but with a debt policy in which the central bank pegs the term premium at steady state. By construction, a term premium peg implies that debt levels move endogenously to anchor the term premium at steady state. As noted earlier, a term premium peg will largely eliminate all fluctuations in the segmentation distortion m_t , so that the response to shocks will closely mirror the DNK counterpart to the present model.

Under the exogenous debt policy, the investment shock had only a small effect on the term premium, so the IRF to an investment shock under a term premium peg is essentially the same as under the exogenous debt policy. But things are much different for the other shocks. In particular, the TFP shock induces the central bank to purchase long term debt so that FI holdings of government debt fall by more than 2%. The behavior of the real quantities are now more typical: consumption, investment and output now co-move, with investment responding

the most sharply. As for the credit shock, the central bank prevents a movement in the term premium by engaging in a sizeable asset purchase, exceeding 12%. This endogenous response sterilizes the rest of the model economy from the credit shock: there are no real effects of the credit shock. Finally, since movements in the term premium are eliminated, the real effects of the shock are modest compared to Figure 9 in which the term premium rises by 15 bp.

d. **Asset purchases at the zero lower bound**

Large scale asset purchases have been conducted in an environment where short term rates have reached their effective lower bound. We study the macroeconomic effects of such bond purchases in the context of our model by considering a shock to households discount factor that increases their desire to save. Such a shock is often invoked in linear models as a way to mimic precautionary savings coming from increased uncertainty in fully nonlinear models. The discount factor shock also directly affects the labor supply decision of households whenever there are habits. Hence, the shock causes a fall in labor supply as well as an increases in desired savings.

We consider a negative discount factor shock that is sufficiently large to drive the short term rate to the lower bound for several quarters. We consider the following truncated Taylor rule responding to inflation and output.⁵

$$r_t = \max(0, \rho r_{t-1} + (1 - \rho)(\tau_\pi \pi_t + \tau_y y_t + \tau_{tp} tp_t))$$

We solve for the equilibrium in this quasi-linear model by adding fully anticipated exogenous disturbances to the interest rate rule over future periods such that the resulting interest rate path obeys the lower bond. This idea was first used in Reifschneider and Williams (1999); see also Hebden, Linde and Svensson (2009). For the numerical implementation, we follow Holden and Paetz (2012). We set price indexation to zero in order to avoid the problems outlined in Carlstrom, Fuerst and Paustian (2012).

Figure XXX compares the behavior of the economy under two different rules for long term debt; a quantity peg and term premium peg. Under the quantity peg, the term premium rises persistently. The premium rises, because

⁵ We let policy respond to output instead of the output gap for the following reason. Monetary policy in the flexible price economy has real effects due to nominal debt contracts, hence the discount factor shock could also drive the flexible price economy to the lower bound. We wanted to avoid having to solve two lower bound problems sequentially.

the increased desire to save can only translate into more deposits if intermediaries become more leveraged. This in turn requires them to earn a higher spreads of loan rates over deposit rates which drive term premia up. That spread acts like a tax on investment reducing investment below steady state. The fall in investment increases marginal cost and hence puts some upward pressure on inflation relatively to the policy of a term premium peg. Under a term premium peg, there is a large rise in investment brought about by purchases of bonds that free up room on the intermediaries balance sheet to finance investment. The term premium peg results in a larger fall in inflation because of the increase in aggregate supply brought about by investment. Hence, a policy of bond purchases supporting the peg does not necessarily shorten the spell at the zero lower bound. On the contrary with endogenous bond purchases, the economy stays at the lower bound for an additional quarter and the funds rate stays lower after the exit compared to a quantity peg.

e. **Welfare consequences of a Taylor rule including the term premium.**

In this section we consider the effect of a central bank including the term premium in its FFR Taylor rule. In particular, suppose that the Taylor Rule is given by:

$$r_t = \rho r_{t-1} + (1 - \rho)(\tau_\pi \pi_t + \tau_y y_t^{gap} + \tau_{tp} tp_t)$$

where the term premium (tp_t) is defined as in (56). As an initial experiment, we set the remainder of the Taylor rule at the benchmark parameter values ($\rho = 0.8$, $\tau_\pi = 1.5$, $\tau_y = 0.5$) and consider the welfare consequences of alternative values for τ_{tp} .

The first step in the analysis is to ensure equilibrium determinacy. Figure 14 looks at equilibrium determinacy for the Taylor rule that includes the term premium. For determinacy under a term premium rule, you cannot have too large of a response to the term premium. At the baseline calibration, this restriction is $\tau_{tp} < 0.76$. The reason is that responding positively to the term premium implies a negative response to the future path of the funds rate. But a negative response to the term premium is typically consistent with determinacy. Further,

Figure 14 implies that as long as this response is not too negative, then the response to inflation can be significantly below unity.

Figures 15-17 looks at the welfare consequence of alternative τ_{tp} for the baseline parameter calibration. We consider two real shocks: TFP shocks, and MEI shocks. We choose the standard deviation of the shocks so that the standard deviation of output equals 0.02 under the baseline parameter calibration. Figure 15 focuses only on MEI shocks and sets the MEI SD = 0.033. Figure 16 does the complementary exercise for the TFP shocks and sets the TFP SD = 0.015. Figure 17 looks at both shocks. In this case we set the relative SDs so that the 80% of the variability of investment comes from the MEI shock. This is consistent with the evidence in Justiniano, Primiceri, and Tambalotti (2010,2011). This calibration implies SD of MEI = 0.027, and SD of TFP = 0.009. The units are in consumption perpetuities, i.e., 0.2 means a perpetual increase in consumption equal to 0.2% of steady-state consumption, or a one-time increase of 20%.

Figure 15 considers the case of MEI shocks. Although the preferred response is positive, there is only a modest gain to having the central bank respond to the term premium. This is anticipated by Figure 6 where an MEI shock led to a trivial movement in the term premium (and hence the distortion, m). Further, this small but positive welfare gain is potentially dangerous. As we increase the term premium response beyond 0.5, we start approaching indeterminacy and the existence of welfare-reducing sunspot equilibria.

With TFP shocks in Figure 16, the welfare gain of a term premium response is significant (roughly 20 times the magnitude of MEI shock). The optimal response is negative, so that indeterminacy of equilibrium is not an issue. The explanation comes from Figure 7. The TFP shock leads to a significant increase in the term premium, driven by the decline in inflation and the short rate. This distorts the economy by an endogenous movement in the implicit tax on investment, m_t . But figure 9 suggests that the central bank can ameliorate these effects by lowering the policy rate in response to the increased term premium. By decreasing the funds rate monetary policy offsets this distortion, and the decrease in marginal cost also encourages labor. The welfare gains can be significant, in excess of 0.3% consumption perpetuity.

Figure 17 considers both shocks, calibrated to match a SD of 2%, with 80% of the variance of investment coming from MEI shocks. The results are as anticipated: there is a modest gain to responding negatively to increases in the term premium. The welfare gain is on the order of a 0.1% consumption perpetuity.

We have conducted a similar analysis for the case of flexible prices, $\theta_p = 0$. The results are qualitatively and quantitatively similar. This suggests that the welfare gain of responding to the term premium is independent of the degree of price stickiness in the model.

Finally, Table 2 considers two stark policies. In both cases, the central bank uses the baseline Taylor rule (without a response to the term premium). In terms of long debt policy, we consider two extremes: (i) the level of long debt in circulation is held fixed (so that the term premium is endogenous), vs. (ii) the term premium is pegged (so that the level of long debt is endogenous). Note that the term premium peg does not completely stabilize the market segmentation distortion m_t , but its variability is lowered by an order of magnitude. As the previous results suggest, there are welfare gains to stabilization of the term premium, although these gains become more modest if MEI shocks are the principle source of the business cycle. Table 2 also confirms the hunch that the welfare gains are largely independent of the degree of price stickiness.

4. Conclusion.

This paper has built a model to analyze the Quantitative Easing policy used by the Fed during the recent ZLB environment. At the core of any such model is an assumption about market segmentation. In the present model, we assume the short term money market is segmented from the long term bond market. Households buy long-term debt instruments indirectly by providing deposits to intermediaries. But intermediaries are limited in their ability to arbitrage the return differentials because the amount of deposits is constrained by an intermediary's net worth. Risk neutral intermediaries would immediately increase net worth to eliminate these movements but the intermediary faces adjustment costs in varying the size of its portfolio. Finally these long-term yields affect real economic activity because of a loan in advance constraint for capital investment.

We show that the real impact of this segmentation is meaningful. These real effects arise because the assumed segmentation introduces a time-varying wedge or distortion on the cost of investment goods. But any wedge needs a remedy. We emphasize two results. First, a monetary policy that targets the term premium in a Taylor rule can largely eliminate movements in the distortion from market segmentation. In particular, welfare is improved modestly when the short-term rate responds negatively to the term premium. Second, a policy that makes the balance sheet endogenous by directly targeting the term premium will sterilize credit shocks. The advantage of this sterilization depends quite naturally on the importance of credit shocks in the business cycle.

A few comments are in order when using the model to guide policy analysis. We have assumed that government and private sector bonds are perfect substitutes. Hence, when government bonds are purchased from intermediaries, they respond by replacing public with private debt one for one. In practice, this link is less strong because of imperfect substitutability. Hence, our model is likely to give an upper bound on the impact of asset purchases. Our model suggests that a natural criterion to judge the success of asset purchase programs is not the term premium per se, but rather the spread of corporate loan rates over funding costs of intermediaries. It is the latter distortion that drives economic activity in our model, with the term spread merely being a mirror of its expected future path. In practice, there are situations where the term spread on sovereign debt is small or even negative, but significant spreads of risk-adjusted loan rates over funding costs still hold back real activity. In those circumstances, one would have to model the imperfect spillover from the compression of term spreads to the compression of loan - deposit spreads and analyze further policy interventions.

APPENDIX.

A. Nonlinear equilibrium conditions:

$$\Lambda_t = \frac{1}{C_t - hC_{t-1}} - E_t \frac{\beta h}{C_{t+1} - hC_t} \quad (\text{A1})$$

$$\Lambda_t = E_t \beta \Lambda_{t+1} \frac{R_t}{\Pi_{t+1}} \quad (\text{A2})$$

$$BL_t^n = W_t \Lambda_t \quad (\text{A3})$$

$$\Lambda_t P_t^k M_t = E_t \beta \Lambda_{t+1} [R_{t+1}^k + (1 - \delta) P_{t+1}^k M_{t+1}] \quad (\text{A4})$$

$$\Lambda_t Q_t^l M_t = E_t \beta \Lambda_{t+1} \frac{[1 + \kappa_l Q_{t+1}^l M_{t+1}]}{\Pi_{t+1}} \quad (\text{A5})$$

$$R_t^k = MC_t MPK_t \quad (\text{A6})$$

$$W_t = MC_t MPL_t \quad (\text{A7})$$

$$\Pi_t^* = \frac{\epsilon_p}{\epsilon_p - 1} \frac{X_{1t}}{X_{2t}} \Pi_t \quad (\text{A8})$$

$$X_{1t} = MC_t \Lambda_t Y_t + \beta \theta_p \Pi_t^{-l_p \epsilon_p} \Pi_{t+1}^{\epsilon_p} X_{1t+1} \quad (\text{A9})$$

$$X_{2t} = \Lambda_t Y_t + \beta \theta_p \Pi_t^{l_p(1-\epsilon_p)} \Pi_{t+1}^{\epsilon_p - 1} X_{2t+1} \quad (\text{A10})$$

$$\Pi_t^{1-\epsilon_p} = (1 - \theta_p) (\Pi_t^*)^{1-\epsilon_p} + \theta_p \Pi_{t-1}^{l_p(1-\epsilon_p)} \quad (\text{A11})$$

$$d_t = \Pi_t^{\epsilon_p} [(1 - \theta_p) (\Pi_t^*)^{-\epsilon_p} + \theta_p \Pi_{t-1}^{-l_p \epsilon_p} d_{t-1}] \quad (\text{A12})$$

$$C_t + I_t = Y_t \quad (\text{A13})$$

$$Y_t = \frac{A_t K_t^\alpha L_t^{1-\alpha}}{d_t} \quad (\text{A14})$$

$$K_t = (1 - \delta) K_{t-1} + \mu_t \left[1 - S \left(\frac{I_t}{I_{t-1}} \right) \right] I_t \quad (\text{A15})$$

$$P_t^k \mu_t \left[1 - S \left(\frac{I_t}{I_{t-1}} \right) - \frac{I_t}{I_{t-1}} S' \left(\frac{I_t}{I_{t-1}} \right) \right] = 1 - \frac{\beta \Lambda_{t+1}}{\Lambda_t} P_{t+1}^k \mu_{t+1} \left(\frac{I_{t+1}}{I_t} \right)^2 S' \left(\frac{I_{t+1}}{I_t} \right) \quad (\text{A16})$$

$$\bar{B}_t + \bar{F}_t \leq N_t \left[1 - \Phi_t \left(\frac{R_t^k}{R_t} \right) \right]^{-1} \quad (\text{A17})$$

$$P_t^k I_t \leq \bar{F}_t - \kappa_I \frac{\bar{F}_{t-1}}{\Pi_t} \frac{Q_t^I}{Q_{t-1}^I} \quad (\text{A18})$$

$$\Lambda_t [1 + N_t f'(N_t) + f(N_t)] = E_t \beta \zeta \Lambda_{t+1} \frac{R_t^L}{\Pi_{t+1}} \quad (\text{A19})$$

$$R_t^L = E_t \left(\frac{1 + \kappa_I Q_{t+1}^I}{Q_t^I} \right) \quad (\text{A20})$$

$$R_t^L = E_t \left(\frac{1 + \kappa Q_{t+1}}{Q_t} \right) \quad (\text{A21})$$

$$R_t^{10} = Q_t^{-1} + \kappa \quad (\text{A22})$$

$$\ln(\bar{B}_t) = (1 - \rho_1^b - \rho_2^b) \ln(\bar{B}_{ss}) + \rho_1^b \ln(\bar{B}_{t-1}) + \rho_2^b \ln(\bar{B}_{t-2}) + \varepsilon_t^b \quad (\text{A23})$$

$$\ln(R_t) = (1 - \rho) \ln(R_{ss}) + \rho \ln(R_{t-1}) + (1 - \rho)(\tau_\pi \pi_t + \tau_y y_t) + \varepsilon_t^r \quad (\text{A24})$$

$$\ln(A_t) = \rho_a \ln(A_{t-1}) + \varepsilon_t^a \quad (\text{A25})$$

$$\ln(\Phi_t) = (1 - \rho_\phi) \ln(\Phi_{ss}) + \rho_\phi \ln(\Phi_{t-1}) + \varepsilon_t^\phi \quad (\text{A26})$$

$$\ln(\mu_t) = \rho_\mu \ln(\mu_{t-1}) + \varepsilon_t^\mu \quad (\text{A26})$$

where

$$f(N_t) \equiv \frac{\psi_n}{2} \left(\frac{N_t - N_{ss}}{N_{ss}} \right)^2$$

$$S\left(\frac{I_t}{I_{t-1}}\right) \equiv \frac{\psi_i}{2} \left(\frac{I_t}{I_{t-1}} - 1 \right)^2$$

$$\bar{B}_t \equiv Q_t \frac{B_t}{P_t}$$

$$\bar{F}_t \equiv Q_t^I \frac{F_t}{P_t}$$

B. Steady State:

We choose B so that $L_{SS} = 1$. We also normalize $\mu_{SS} = A_{SS} = 1$.

$$\Lambda_{SS} = \frac{(1 - \beta h)}{(1 - h)C_{SS}}$$

$$1 = \beta R_{SS}$$

$$B = W_{SS}\Lambda_{SS}$$

$$R_{SS}^k = \frac{M_{SS}[1 - \beta(1 - \delta)]}{\beta}$$

$$M_{SS} = \frac{\beta}{(1 - \beta\kappa_I)Q_{SS}^I}$$

$$R_{SS}^k = MC_{SS}MPK_{SS}$$

$$W_{SS} = MC_{SS}MPL_{SS}$$

$$1 = \frac{\epsilon_p X_{1SS}}{\epsilon_p - 1 X_{2SS}}$$

$$X_{1SS} = \frac{MC_{SS}\Lambda_{SS}Y_{SS}}{1 - \beta\theta_p}$$

$$X_{2SS} = \frac{\Lambda_{SS}Y_{SS}}{1 - \beta\theta_p}$$

$$d_{SS} = 1$$

$$C_{SS} + I_{SS} = Y_{SS}$$

$$Y_{SS} = K_{SS}^\alpha$$

$$\delta K_{SS} = I_{SS}$$

$$P_{SS}^k = 1$$

$$\bar{B}_{SS} + \bar{F}_{SS} = N_{SS} \left[1 - \Phi_{SS} \left(\frac{R_{SS}^L}{R_{SS}} \right) \right]^{-1}$$

$$I_{SS} = \bar{F}_{SS}(1 - \kappa_I)$$

$$1 = \beta\zeta R_{SS}^L$$

$$Q = (R_{SS}^L - \kappa)^{-1}$$

$$Q_I = (R_{SS}^L - \kappa_I)^{-1}$$

$$R_{SS}^{10} = R_{SS}^L$$

Some simplifications:

$$M_{SS} = \frac{(\beta R_{SS}^L - \beta \kappa_I)}{(1 - \beta \kappa_I)} > 1$$

$$MC_{SS} = \frac{\epsilon_p - 1}{\epsilon_p} < 1$$

$$\frac{K_{SS}}{Y_{SS}} = \frac{\beta \alpha MC_{SS}}{M_{SS} [1 - \beta(1 - \delta)]}$$

$$K_{SS} = \left(\frac{K_{SS}}{Y_{SS}} \right)^{1/(1-\alpha)}$$

$$C_{SS} = Y_{SS} - \delta K_{SS}$$

C. Calibration:

$$\beta = 0.99, h = 0.8, \eta = 0.25, \iota_p = 1, \epsilon_p = 5, \theta_p = 0.85, \psi_i = 2, \psi_n = 1.$$

$$\beta R_{SS}^L = 1.0025$$

$$\Phi_{SS}(1.0025) = \frac{L_{SS} - 1}{L_{SS}}$$

$$L_{SS} = 6$$

$$\text{duration} = 40 = (1 - \kappa_I)^{-1}$$

$$\text{duration} = 40 = (1 - \kappa)^{-1}$$

$$\frac{\bar{B}_{SS}}{\bar{B}_{SS} + \bar{F}_{SS}} = 40\%$$

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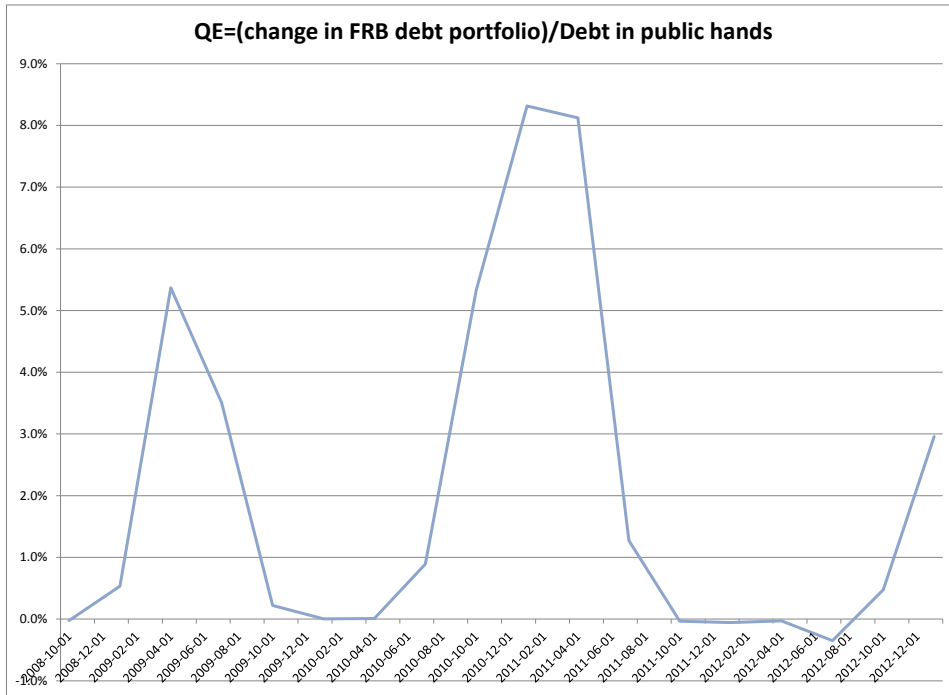
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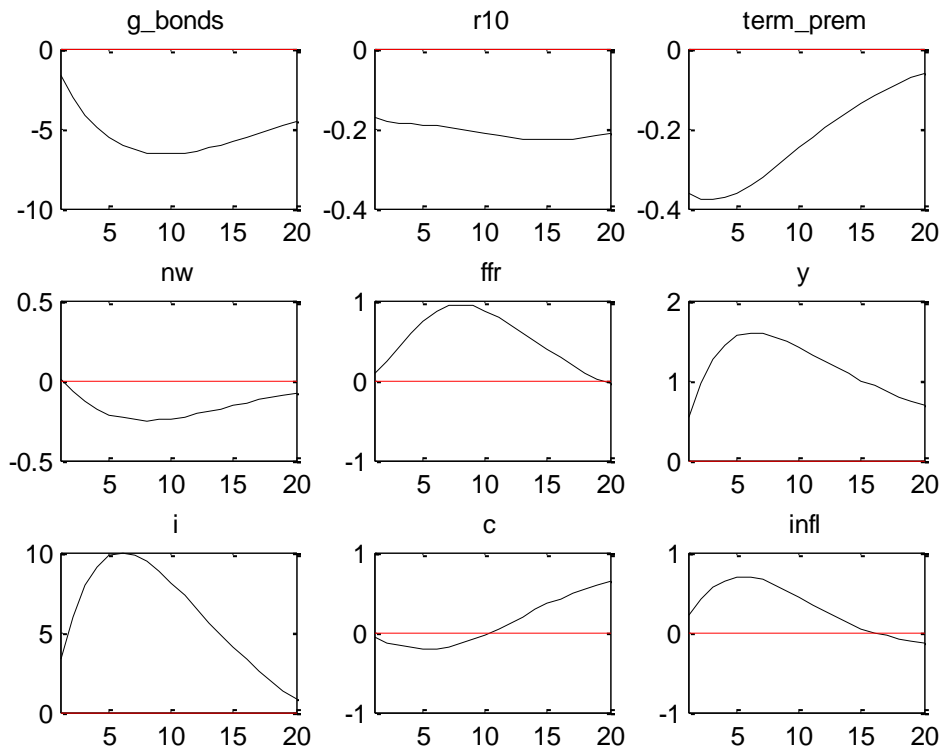
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Figure 1:



Source: FRB St. Louis.

Figure 2: OE experiment, baseline parameter values.



Legend: All variables are in percentage points and all rates are annualized. g_bonds denotes the amount of government bonds on the balance sheet of FI's.

Figure 3: Effect of investment bond duration on M_{ss} .

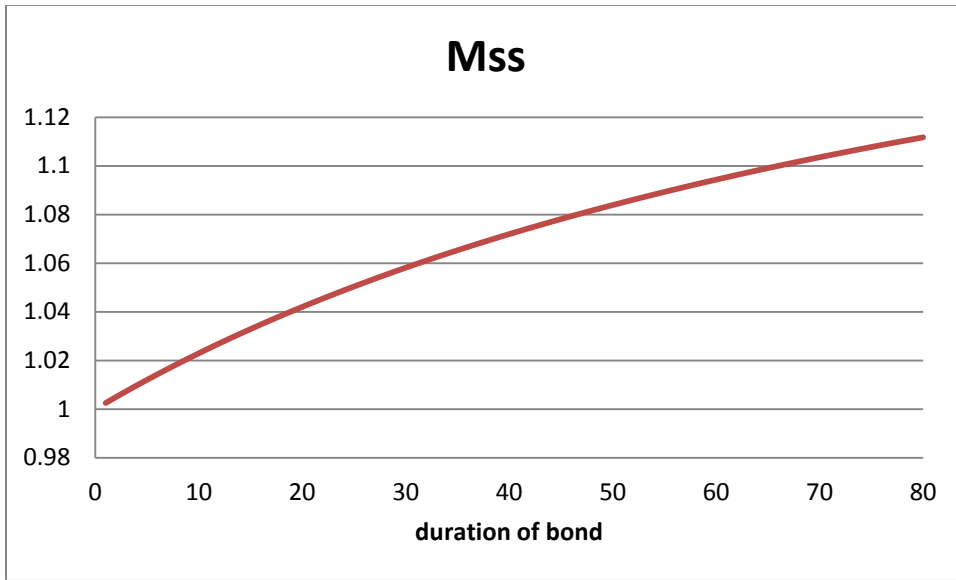


Figure 5: Discount factor shock and zero lower bound.

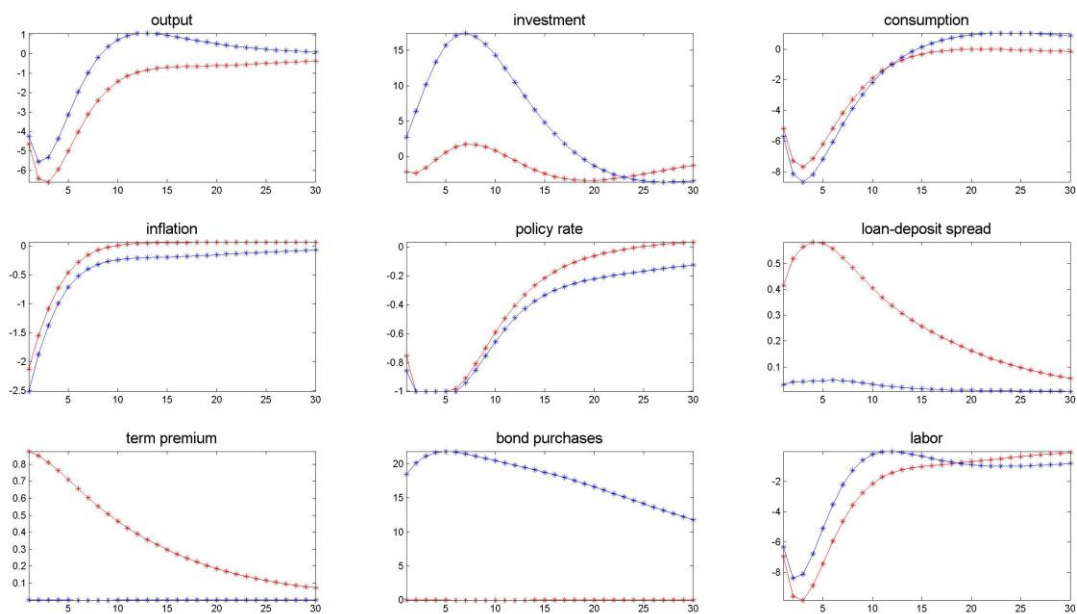
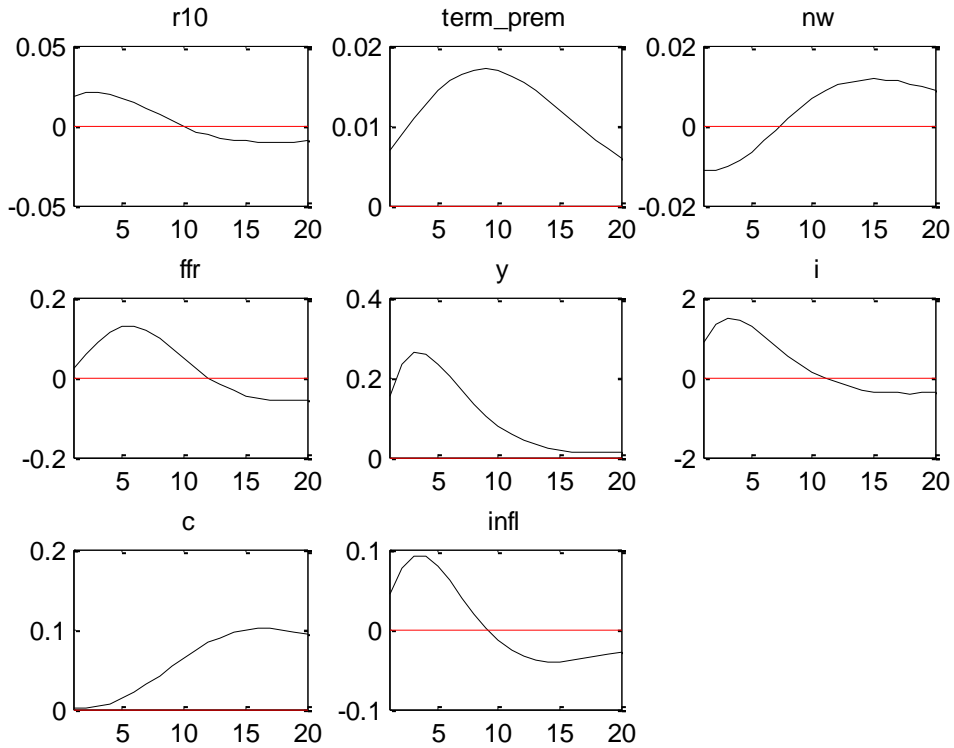
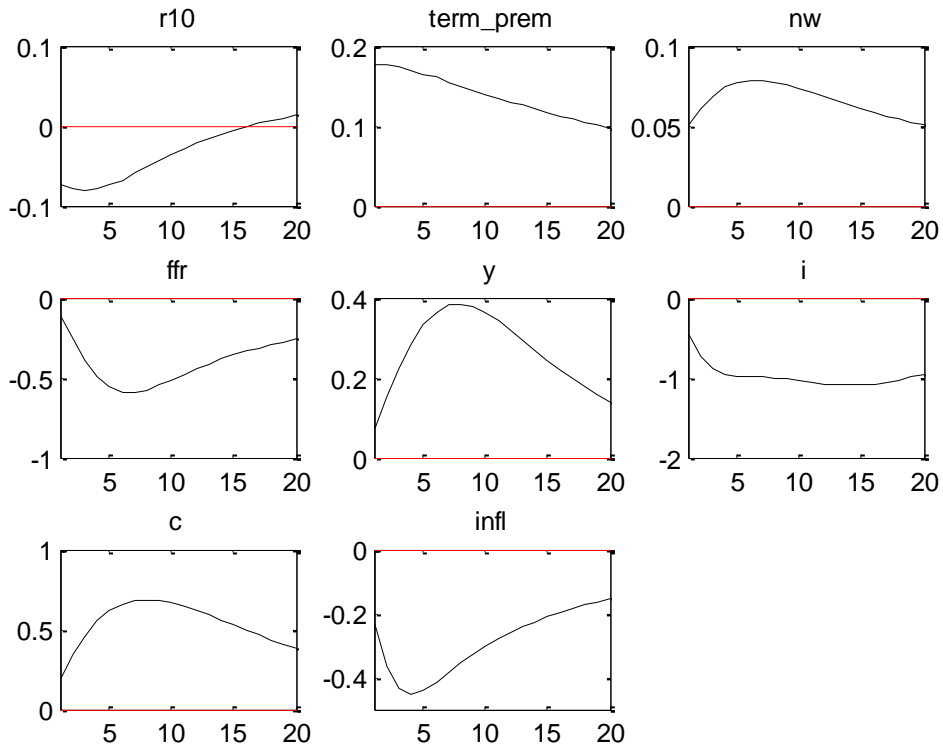


Figure 6: A 1% investment shock under an exogenous debt policy.



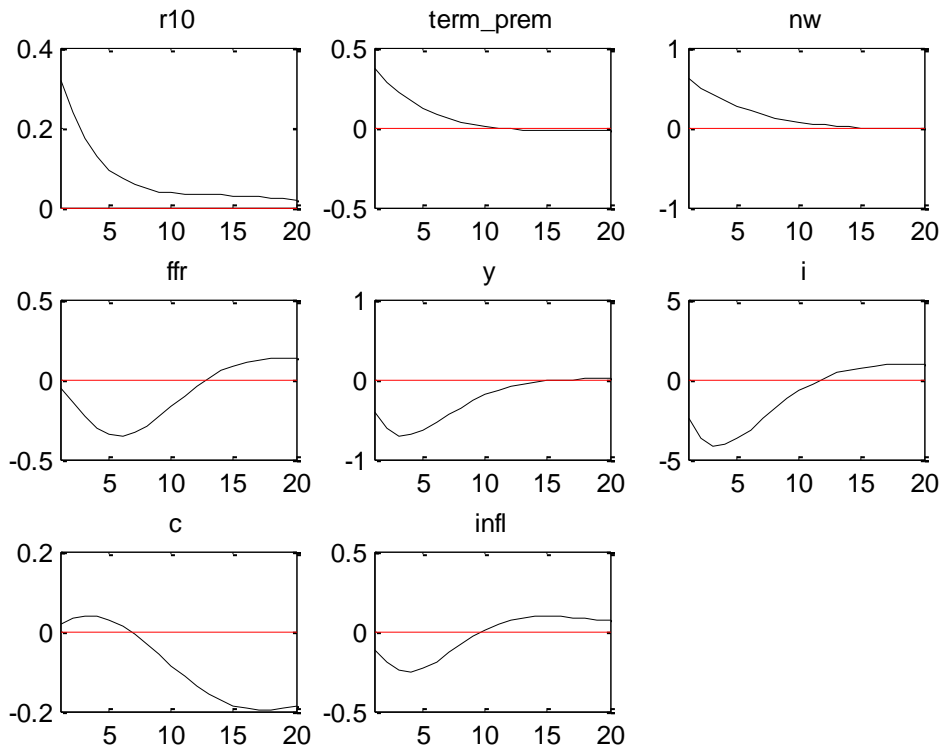
Legend: All variables are in percentage points and all rates are annualized. g_bonds denotes the amount of government bonds on the balance sheet of FI's.

Figure 7: A 1% TFP shock under an exogenous debt policy.



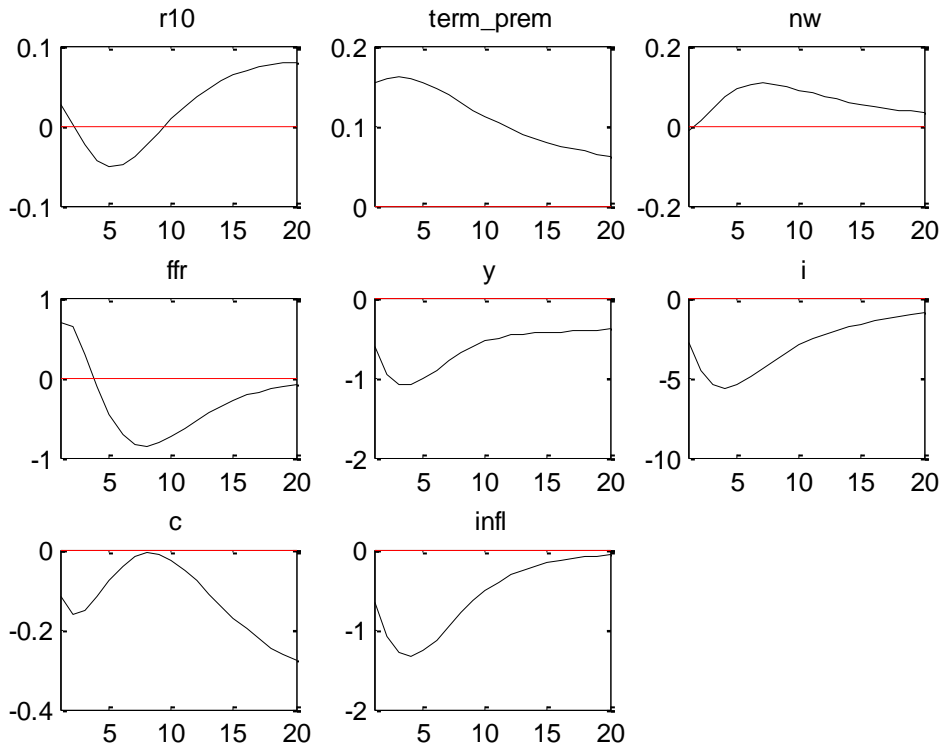
Legend: All variables are in percentage points and all rates are annualized. `g_bonds` denotes the amount of government bonds on the balance sheet of FI's.

Figure 8: A 1% credit shock under an exogenous debt policy.



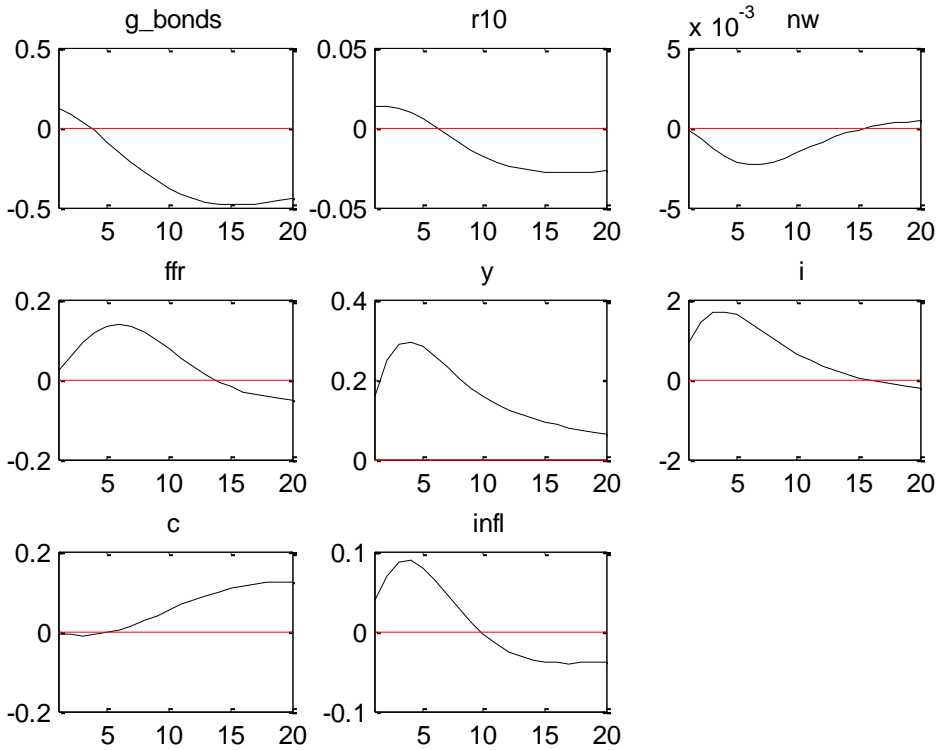
Legend: All variables are in percentage points and all rates are annualized. `g_bonds` denotes the amount of government bonds on the balance sheet of FI's.

Figure 9: A 100 bp monetary shock under an exogenous debt policy.



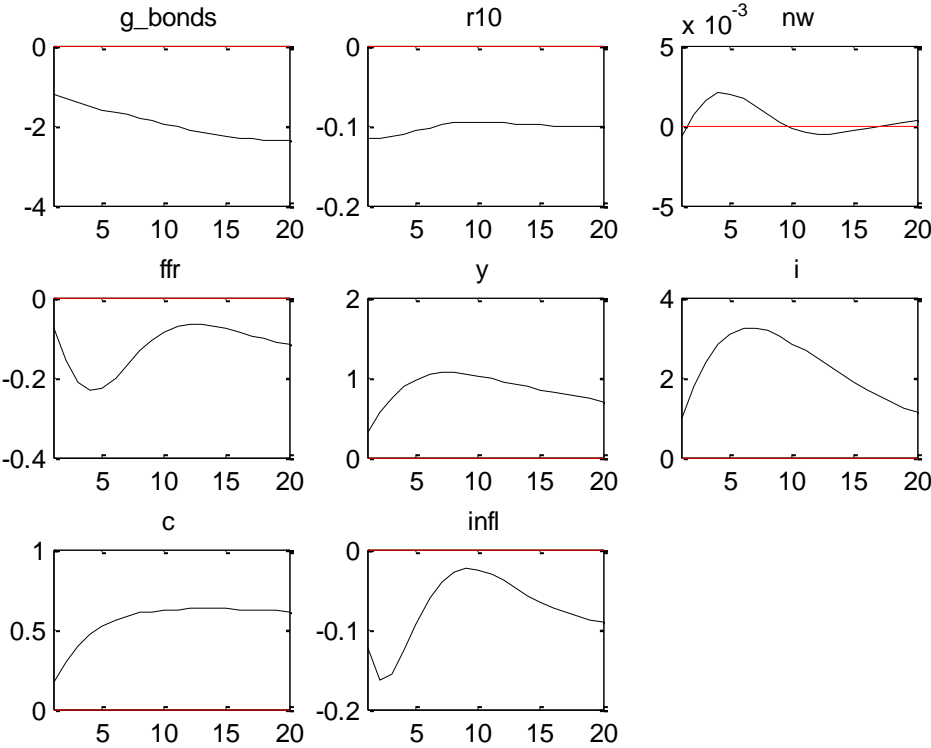
Legend: All variables are in percentage points and all rates are annualized. g_bonds denotes the amount of government bonds on the balance sheet of FI's.

Figure 10: A 1% investment shock under a term premium peg.



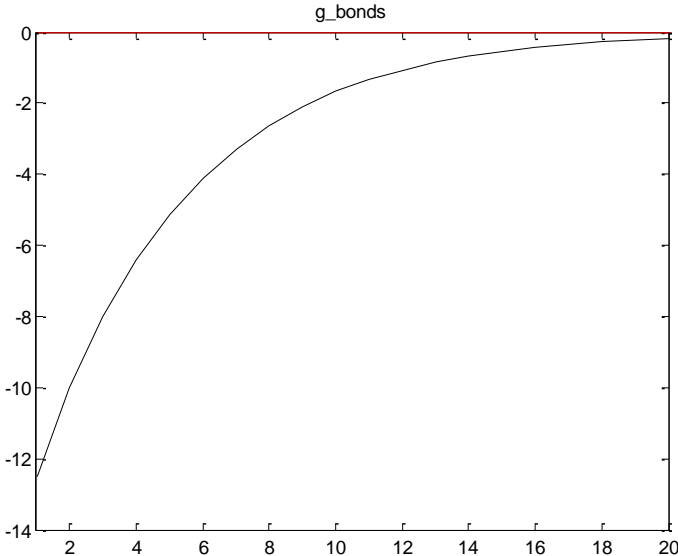
Legend: All variables are in percentage points and all rates are annualized. g_bonds denotes the amount of government bonds on the balance sheet of FI's.

Figure 11: A 1% TFP shock under a term premium peg.



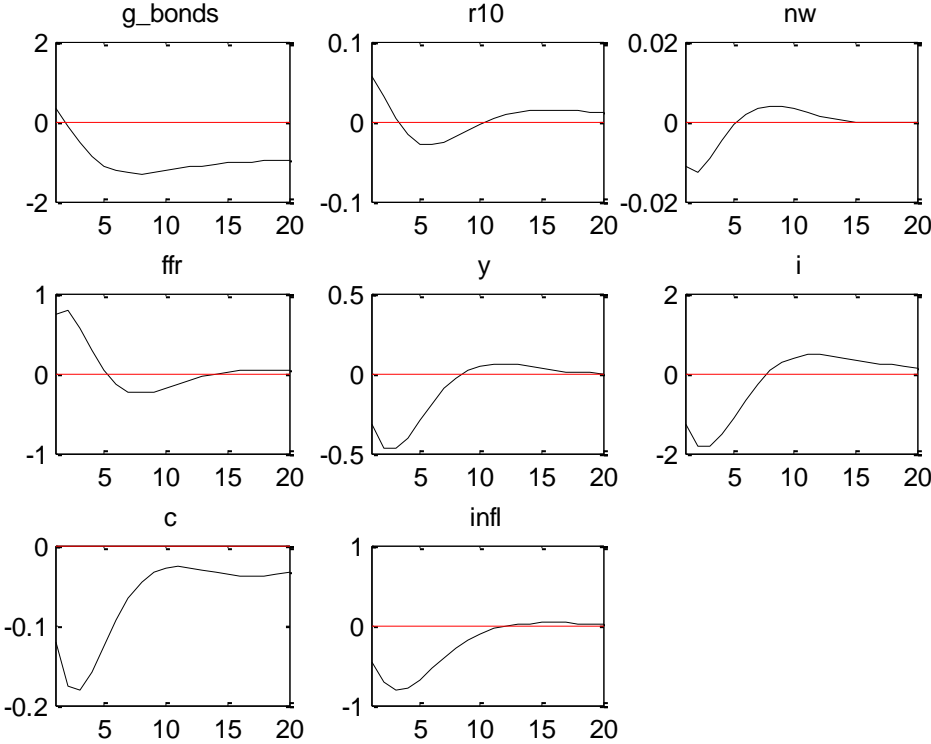
Legend: All variables are in percentage points and all rates are annualized. g_bonds denotes the amount of government bonds on the balance sheet of FI's.

Figure 12: A 1% credit shock under a term premium peg.



Legend: All variables are in percentage points and all rates are annualized. g_bonds denotes the amount of government bonds on the balance sheet of FI's.

Figure 13: A 100 bp monetary shock under a term premium peg.



Legend: All variables are in percentage points and all rates are annualized. g_bonds denotes the amount of government bonds on the balance sheet of FI's.

Figure 14: Determinacy under a term premium Taylor Rule.

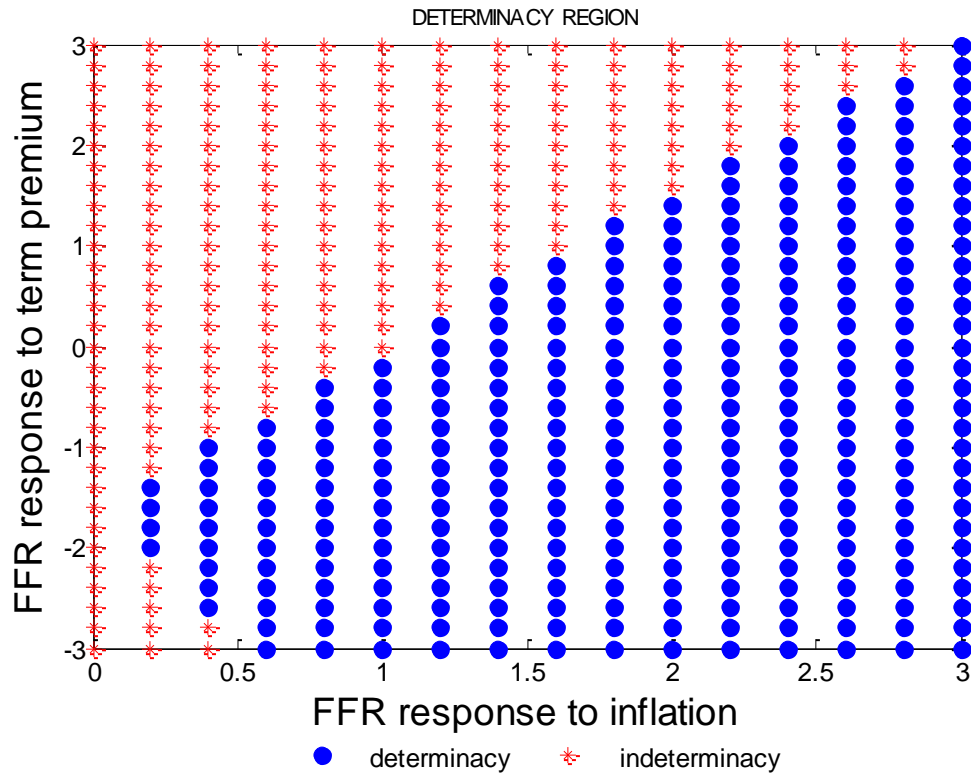
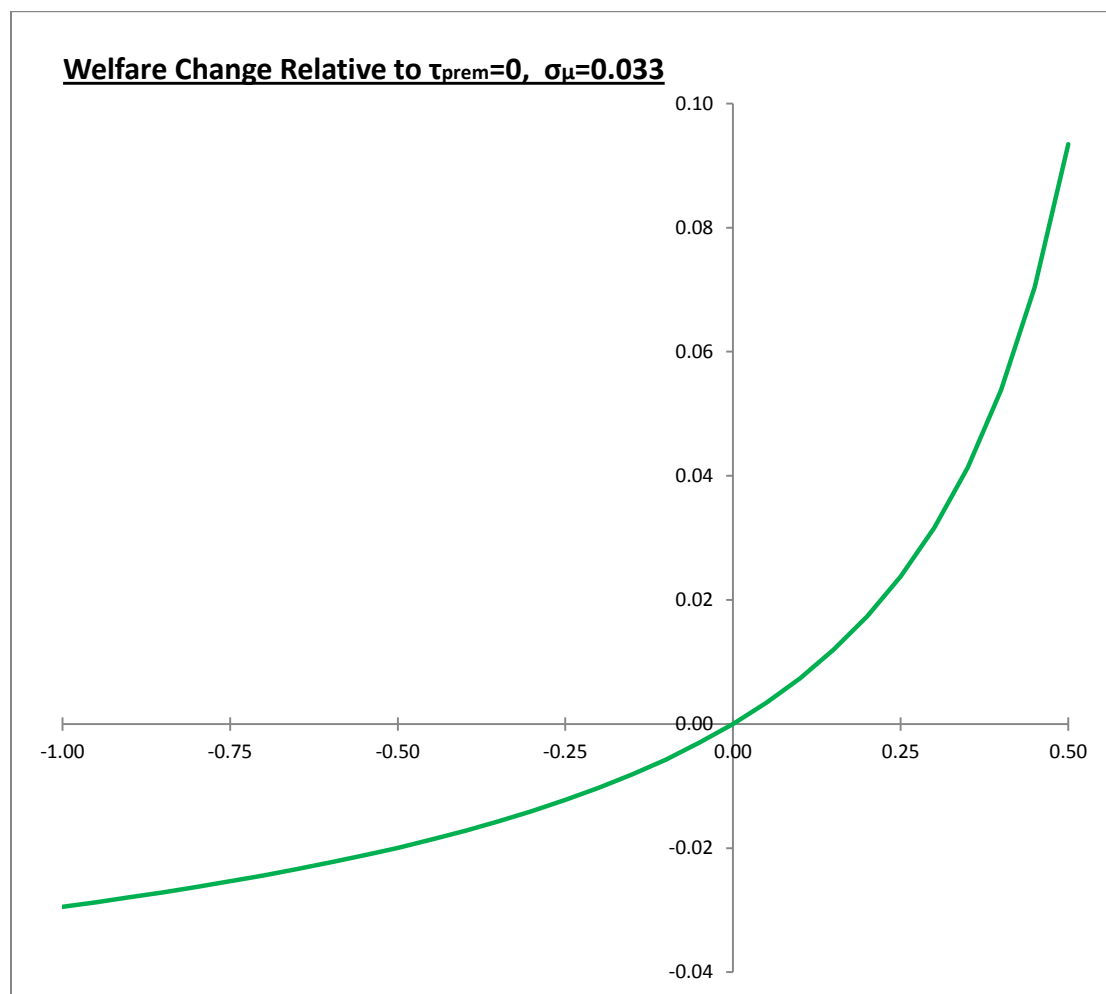


Figure 15: Welfare Consequences of Taylor Rule with term premium response.
(Only MEI shocks)



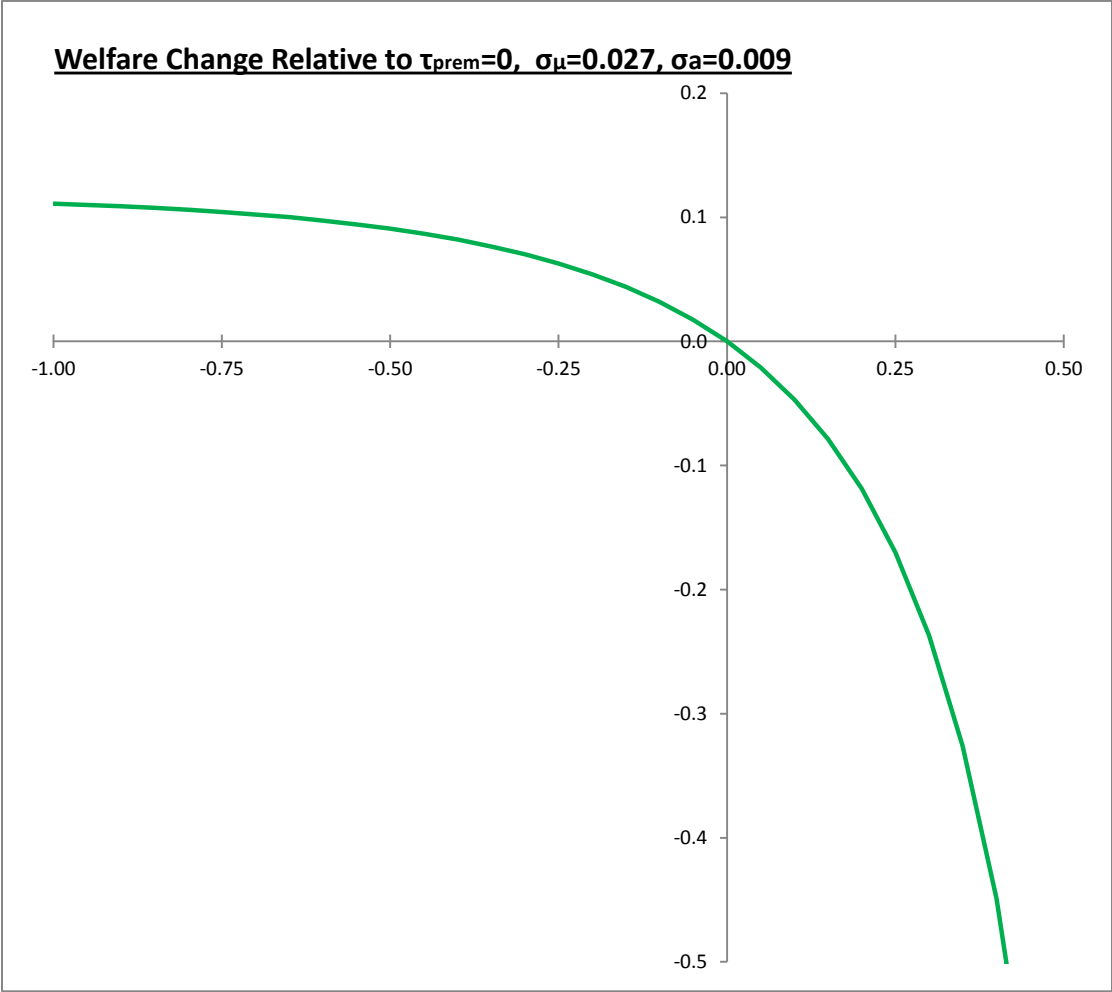
The units are in consumption perpetuities, ie., 0.2 means a perpetual increase in consumption equal to 0.2% of steady-state consumption, or a one-time increase of 20%.

Figure 16: Welfare Consequences of Taylor Rule with term premium response.
(Only TFP shocks)



The units are in consumption perpetuities, ie., 0.2 means a perpetual increase in consumption equal to 0.2% of steady-state consumption, or a one-time increase of 20%.

Figure 17: Welfare Consequences of Taylor Rule with term premium response.
(TFP and MEI shocks)



The units are in consumption perpetuities, ie., 0.2 means a perpetual increase in consumption equal to 0.2% of steady-state consumption, or a one-time increase of 20%.

Table 1: Sensitivity Analysis

This table contains sensitivity analysis of the effect of model parameters on the peak impact of a QE shock.

Parameter Value	Peak Investment Response	Peak Ten-Year Yield Response	Peak Inflation Response
Baseline*	9.98	-0.23	0.69
$\psi_n = 0$	0	0	0
$\psi_n = 0.25$	8.05	-0.19	0.54
$\psi_n = 0.5$	9.23	-0.21	0.63
$\psi_n = 2$	10.40	-0.24	0.7278
Dur_Inv = 20	7.17	-0.20	0.5049
Dur_Inv = 80	12.54	-0.29	0.8389
$\theta_p = 0$	9.70	-0.28	1.34
$\theta_p = 0.75$	9.80	-0.25	0.95
$\theta_p = 0.95$	9.93	-0.19	0.24
$\frac{Govt\ Debt}{Bank\ Assets} = 0$	0	0	0
$\frac{Govt\ Debt}{Bank\ Assets} = 0.2$	4.27	-0.10	0.30
$\frac{Govt\ Debt}{Bank\ Assets} = 0.8$	31.18	-0.78	2.01
Leverage = 3	13.37	-0.30	0.98
Leverage = 9	8.05	-0.19	0.54

*The baseline parameter values are $\psi_n = 1$, Dur_Inv = 40, $\theta_p = 0.85$, $\frac{Debt}{Assets} = 0.4$, leverage = 6. All variables are expressed in percent, and rates are annualized.

Table 2: Comparing two stark policies.

Here we consider two stark policy choices: holding the balance sheet fixed ($b_t = 0$), vs. a term premium peg, ($tp_t = 0$). The calibration of the SD of the shocks is as in Figures 15-17.

<u>Sticky prices</u> ($\theta_p = 0.85$)			
	Both shocks	TFP alone	MEI alone
Welfare gain of term premium peg*	0.144	0.479	-0.042
SD of m with exogenous debt	4.8%	7.8%	1.4%
SD of m with term premium peg	0.3%	0.5%	0.2%

<u>Flexible prices</u> ($\theta_p = 0$)			
	Both shocks	TFP alone	MEI alone
Welfare gain of term premium peg*	0.164	0.494	-0.020
SD of m with exogenous debt	5.1%	8.4%	1.3%
SD of m with term premium peg	0.3%	0.5%	0.2%

*The welfare units are in consumption perpetuities, i.e., 0.2 means a perpetual increase in consumption equal to 0.2% of steady-state consumption, or a one-time increase of 20%.