# Fiscal Multipliers in a Nonlinear World*
# (Preliminary and Incomplete)

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**Abstract**

Previous work has shown that, in a liquidity trap, aggressive government spending cuts can be self-defeating in the short-run due to a higher-than-normal multiplier. A potentially serious drawback of the existing literature is the use of linearized models. Recently, Braun, Koerber and Waki (2012) and others claim that in a liquidity trap, a model can behave qualitatively different depending on whether it has been linearized or not. We examine their claim with a focus on whether fiscal austerity can be self-defeating - i.e. austerity causes government debt to rise due to adverse effects on aggregate demand. Specifically, we compare the government debt and output effects due to changes in fiscal spending in linearized and nonlinear general equilibrium models. We start with a variant of the simple benchmark model in Woodford (2003), which allows us to carefully parse out the differences between the linear and nonlinear solutions. Finally, we examine the robustness of our results in the workhorse model of Christiano, Eichenbaum and Evans (2005) augmented with the Bernanke, Gertler and Gilchrist (1999) financial accelerator.

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*This version: October 25, 2013. The views expressed in this paper are solely the responsibility of the authors and should not be interpreted as reflecting the views of the Board of Governors of the Federal Reserve System or of any other person associated with the Federal Reserve System. E-mail addresses: jesper.l.linde@frb.gov and mathias.trabandt@gmail.com

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## 1. Introduction

We assess the implications of taking model non-linearities explicitly into account when calculating fiscal multipliers in an environment when the zero lower bound of nominal interest rates is binding.

Recent work on the effects of fiscal stimulus suggests that the fiscal spending multiplier can be much higher when monetary policy is constrained by the ZLB, see e.g. Eggertsson (2010), Davig and Leeper (2011), Christiano, Eichenbaum and Rebelo (2011), Woodford (2011), Coenen et al. (2012) and Erceg and Linde (2012). Erceg and Linde (2012) show that spending hikes can be associated with a "fiscal free lunch" in a sufficiently long-lived liquidity trap. The flip side of this finding is that it is hard to reduce government debt in the short-run through aggressive spending cuts.

The bulk of the existing literature has analyzed fiscal multipliers in models that are linearized around the steady state (apart from the ZLB constraint on the monetary policy rule). The implicit assumption with this procedure is that the linearized solution is accurate even far away from the steady state. But recent work (see e.g. Braun, Koerber and Waki, 2012) suggests that analyses based on linearized supply and demand schedules might produce misleading results at the zero lower bound. Essentially, Braun et al. are arguing that extrapolations of decision rules far away from the steady state are invalid.

In this paper we address the following question: can fiscal austerity be self-defeating in a liquidity trap in a fully nonlinear environment? We undertake a *positive* analysis of the effects of spending-based fiscal consolidations on *output* and *government debt*. The modeling starting point is a variant of the workhorse New Keynesian DSGE model of Woodford (2003). This model features monopolistic competition and Calvo sticky prices and the central bank follows a Taylor rule subject to the ZLB constraint on nominal rates. We rule out the well-known problems associated with steady state multiplicity emphasized by Benhabib, Schmitt-Grohe and Uribe (2001) by restricting our attention to the steady state with a positive inflation rate. We document and analyze the key differences between the linearized and fully nonlinear solutions of this model.

Next, we examine the differences in multiplier schedules in an empirically plausible model developed by Christiano, Eichenbaum and Evans (2005) which we augmented with the Bernanke, Gertler and Gilchrist (1999) financial accelerator mechanism. Our analysis allows us to study potential fiscal free lunches in a liquidity trap in a model which has a spending multiplier in line with the VAR evidence in times when monetary policy is unconstrained. [More literature to be discussed: Christiano and Eichenbaum (2012), Fernandez-Villaverde et al. (2012).]

In our analysis, we compare fiscal multipliers on output and debt in nonlinear and linearized representations of the model. We focus on features which account for the discrepancies between the nonlinear and linearized solution. Relative to the existing literature, we focus on the implications for government debt in a model with real rigidities. In particular, we introduce real rigidities through the Kimball (1995) state-dependent demand elasticity for the intermediate goods firms which allows our model to simultaneously account for the *macroeconomic evidence* of a low linearized Phillips curve slope (0.01) and the *microeconomic evidence* of frequent price re-optimization (3-4 quarters) at the same time.

Our analysis points toward important quantitative differences between output and debt multipliers in linearized and nonlinear DSGE models when the model is calibrated to reflect microeconomic evidence on the frequency of price changes only. More importantly, when the model is calibrated to account for macroeconomic evidence of the slope of the Phillips curve and microeconomic evidence on the frequency of price changes jointly, the quantitative differences between the linear and nonlinear model appear to be much smaller. Another key finding is that linearization of pricing equations accounts for the bulk of the differences between nonlinear and linearized solutions. [Remains to be written.]

The remainder of this paper is organized as follows. Section 2 presents the stylized New Keynesian model and discusses how the model is parameterized. Section 3 presents our benchmark results. Section 4 examines the implications of a more empirically-realistic model, and Section 4 concludes.

## 2. The Stylized New Keynesian Model

The simple model we study is very similar to the one developed in Erceg and Linde (2012), which in turn is closely related to the model studied by Eggertsson and Woodford (2003). We deviate from Erceg and Linde (2012) by allowing for a Kimball (1995) aggregator (with the standard Dixit-Stiglitz specification as a special case) as well as a discount factor shock. Below, we outline the model. In the appendix we describe the linear and non-linear versions in greater detail.

### 2.1. The Model

#### 2.1.1. Households

The utility functional for the representative household is

$$\mathbb{E}_t \sum_{j=0}^{\infty} \beta^j \varsigma_t \left\{ \frac{1}{1-\frac{1}{\sigma}} \left(C_{t+j} - C\nu_{t+j}\right)^{1-\frac{1}{\sigma}} - \frac{N_{t+j}^{1+\chi}}{1+\chi} + \mu_0 F\left(\frac{MB_{t+j+1}(h)}{P_{t+j}}\right) \right\} \tag{1}$$

where the discount factor $\beta$ satisfies $0 < \beta < 1$ and is subject to an exogenous component $\varsigma_t$. The period utility function depends on the household's current consumption $C_t$ as deviation from a "reference level" $C\nu_{t+j}$, where the exogenous positive taste shock $\nu_t$ raises this reference level and thus the marginal utility of consumption associated with any given consumption level. The period utility function also depends inversely on hours worked $N_t$. Following Eggertsson and Woodford (2003), the subutility function over real balances, $F\left(\frac{MB_{t+j+1}(h)}{P_{t+j}}\right)$, is assumed to have a satiation point for $\overline{MB}/P$. Hence, inclusion of money − which is a zero nominal interest asset − provides a rationale for the zero lower bound on nominal interest rates. However, we maintain the assumptions that money is additive and that $\mu_0$ is arbitrarily small so that changes in real money balances have negligible implications for seigniorage. Together, these assumptions imply that we can disregard the implications of money for government debt and output.

The household's budget constraint in period $t$ states that its expenditure on goods and net purchases of (zero-coupon) government bonds $B_{G,t}$ must equal its disposable income:

$$P_t\left(1+\tau_{C,t}\right)C_t + B_{G,t} + MB_{t+1} = \left(1-\tau_{N,t}\right)W_t N_t + \left(1+i_{t-1}\right)B_{G,t-1} + MB_t - T_t + \Gamma_t \tag{2}$$

Thus, the household purchases the final consumption good (at a price of $P_t$) and subject to a sales tax $\tau_{C,t}$. Each household earns after-tax labor income $\left(1-\tau_{N,t}\right)W_t N_t$ ($\tau_{N,t}$ denotes the tax rate), pays a lump-sum tax $T_t$ (this may be regarded as net of any transfers), and receives a proportional share of the profits $\Gamma_t$ of all intermediate firms.

In every period $t$, the household maximizes the utility functional (**??**) with respect to its consumption, labor supply and bond holdings. Forming the Lagrangian and computing the first-order conditions w.r.t. $\left[\begin{array}{ccc} C_t & N_t & B_{G,t} \end{array}\right]$, we obtain the standard consumption Euler equation

$$\frac{\left(C_t - C\nu_t\right)^{-\frac{1}{\sigma}}}{\left(1+\tau_{C,t}\right)} = \beta\delta_{t+1} E_t \frac{\left(1+i_t\right)}{1+\pi_{t+1}} \frac{\left(C_{t+1} - C\nu_{t+1}\right)^{-\frac{1}{\sigma}}}{\left(1+\tau_{C,t+1}\right)}, \tag{3}$$

where we have defined

$$\delta_{t+1} = \frac{\varsigma_{t+1}}{\varsigma_t} \tag{4}$$

and introduced the notation $1 + \pi_{t+1} = P_{t+1}/P_t$. We also have the following labor supply schedule

$$\frac{N_t^{\chi}}{\left(C_t - C\nu_t\right)^{-\frac{1}{\sigma}}} = \frac{(1 - \tau_{N,t})}{(1 + \tau_{C,t})} \frac{W_t}{P_t}. \tag{5}$$

Equations (3) and (5) are the key equations for the household side of the model.

#### 2.1.2. Firms and Price Setting

*Final Goods Production* The single final output good $Y_t$ is produced using a continuum of differentiated intermediate goods $Y_t(f)$. Following Kimball (1995), the technology for transforming these intermediate goods into the final output good is

$$\int_0^1 G_Y\left(\frac{Y_t(f)}{Y_t}\right) df = 1. \tag{6}$$

Following Dotsey and King (2005) and Levin, Lopez-Salido and Yun (2007), we assume that $G_Y(.)$ is given by the following strictly concave and increasing function:

$$G_Y\left(\frac{Y_t(f)}{Y_t}\right) = \left(\frac{\phi_p}{1-(\phi_p-1)\epsilon_p}\left[\left(\frac{\phi_p+(1-\phi_p)\epsilon_p}{\phi_p}\right)\frac{Y_t(f)}{Y_t} + \frac{(\phi_p-1)\epsilon_p}{\phi_p}\right]^{\frac{1-(\phi_p-1)\epsilon_p}{\phi_p-(\phi_p-1)\epsilon_p}} + \left[1 - \frac{\phi_p}{1-(\phi_p-1)\epsilon_p}\right]\right), \tag{7}$$

where $\phi_p \geq 1$ denotes the gross markup of the intermediate firms. The parameter $\epsilon_p$ governs the degree of curvature of the intermediate firm's demand curve. When $\epsilon_p = 0$, the demand curve exhibits constant elasticity as with the standard Dixit-Stiglitz aggregator. When $\epsilon_p$ is positive—as in SW07—the firm's instead face a quasi-kinked demand curve, implying that a drop in its relative price only stimulates a small increase in demand. On the other hand, a rise in its relative price generates a large fall in demand. Relative to the standard Dixit-Stiglitz aggregator, this introduces more strategic complementarity in price setting which causes intermediate firms to adjust prices less to a given change in marginal cost. Finally, we notice that $G_Y(1) = 1$, implying constant returns to scale when all intermediate firms produce the same amount.

Firms that produce the final output good are perfectly competitive in both product and factor markets. Thus, final goods producers minimize the cost of producing a given quantity of the output index $Y_t$, taking as given the price $P_t(f)$ of each intermediate good $Y_t(f)$. Moreover, final goods producers sell units of the final output good at a price $P_t$, and hence solve the following problem:

$$\max_{\{Y_t, Y_t(f)\}} P_t Y_t - \int_0^1 P_t(f)\, Y_t(f)\, df \tag{8}$$

subject to the constraint (6). Note that for $\epsilon_p = 0$, this problem leads to the usual expressions

$$\frac{Y_t(f)}{Y_t} = \left[\frac{P_t(f)}{P_t}\right]^{-\frac{\phi_p}{\phi_p - 1}}, \; P_t = \left[\int P_t(f)^{\frac{1}{1-\phi_p}} df\right]^{1-\phi_p}$$

*Intermediate Goods Production* A continuum of intermediate goods $Y_t(f)$ for $f \in [0,1]$ is produced by monopolistically competitive firms, each of which produces a single differentiated good. Each intermediate goods producer faces a demand schedule from the final goods firms through the solution to the problem in (8) that varies inversely with its output price $P_t(f)$ and directly with aggregate demand $Y_t$.

Aggregate capital ($K$) is assumed to be fixed, so that aggregate production of the intermediate good firm is given by

$$Y_t(f) = K(f)^{\alpha} N_t(f)^{1-\alpha}. \tag{9}$$

Despite the fixed aggregate stock $K \equiv \int K(f)\, df$, shares of it can be freely allocated across the $f$ firms, implying that real marginal cost, $MC_t(f)/P_t$ is identical across firms and equal to

$$\frac{MC_t}{P_t} \equiv \frac{W_t/P_t}{MPL_t} = \frac{W_t/P_t}{(1-\alpha) K^{\alpha} N_t{}^{-\alpha}}, \tag{10}$$

where $N_t = \int N_t(f)\, df$.

The prices of the intermediate goods are determined by Calvo-Yun (1996) style staggered nominal contracts. In each period, each firm $f$ faces a constant probability, $1 - \xi_p$, of being able to reoptimize its price $P_t(f)$. The probability that any firm receives a signal to reset its price is assumed to be independent of the time that it last reset its price. If a firm is not allowed to optimize its price in a given period, it adjusts its price according to the following formula

$$\tilde{P}_t = (1+\pi) P_{t-1}, \tag{11}$$

where $\pi$ is the steady-state (net) inflation rate and $\tilde{P}_t$ is the updated price.

Given Calvo-style pricing frictions, firm $f$ that is allowed to reoptimize its price ($P_t^{opt}(f)$) solves the following problem

$$\max_{P_t^{opt}(f)} \mathrm{E}_t \sum_{j=0}^{\infty} (\beta\xi_p)^j \varsigma_{t+j} \Lambda_{t,t+j} \left[(1+\pi)^j P_t^{opt}(f) - MC_{t+j}\right] Y_{t+j}(f)$$

where $\Lambda_{t,t+j}$ is the stochastic discount factor (the conditional value of future profits in utility units, recalling that the household is the owner of the firms), and demand $Y_{t+j}(f)$ from the final goods firms is given by:

$$Y_{t+j}\left(f\right)=\frac{\phi_p}{\phi_p-\left(\phi_p-1\right)\epsilon_p}\left(\left[\frac{\left(1+\pi\right)^j P_t^{opt}\left(f\right)}{P_{t+j}}\frac{1}{\vartheta_{t+j}}\right]^{-\frac{\phi_p-\left(\phi_p-1\right)\epsilon_p}{\phi_p-1}}+\frac{\left(1-\phi_p\right)\epsilon_p}{\phi_p}\right)Y_{t+j}, \quad (12)$$

where $\vartheta_{t+j}$ is the Lagrangian multiplier from the final good firms problem (8).

### 2.1.3. Monetary and Fiscal Policies

The evolution of nominal government debt is determined by the following equation

$$B_{G,t}=\left(1+i_{t-1}\right)B_{G,t-1}+P_tG_t-\tau_{C,t}P_tC_t-\tau_{N,t}W_tN_t-T_t \quad (13)$$

where $G_t$ denotes real government expenditures on the final good $Y_t$. Scaling with $1/\left(P_tY\right)$, we obtain

$$\frac{B_{G,t}}{P_tY}=\frac{\left(1+i_{t-1}\right)}{\left(1+\pi_t\right)}\frac{B_{G,t-1}}{P_{t-1}Y}+\frac{G_t}{Y}-\tau_{C,t}\frac{C_t}{Y}-\tau_{N,t}\frac{W_tN_t}{P_tY}-\frac{T_t}{P_tY}. \quad (14)$$

Following the convention in the literature on fiscal multipliers, we start out by assuming that lump-sum taxes stabilize the evolution of government debt (as share of nominal trend GDP, $b_{G,t}\equiv\frac{B_{G,t}}{P_tY}$). Specifically, we follow Erceg and Linde (2012) and assume that lump-sum taxes as share of nominal trend GDP, $\tau_t\equiv\frac{T_t}{P_tY}$, follow the simple rule

$$\tau_t-\tau=\varphi_b\left(b_{G,t-1}-b_G\right) \quad (15)$$

Government spending, $g_{y,t}\equiv\frac{G_t}{Y}$, $\tau_{C,t}$ and $\tau_{N,t}$ are kept exogenous.

Turning to the central bank, it is assumed to adhere to a Taylor-type policy rule that is subject to the zero lower bound:

$$1+i_t=\max\left(1,\left(1+i\right)\left[\frac{1+\pi_t}{1+\pi}\right]^{\gamma_\pi}\left[\frac{Y_t}{Y_t^{pot}}\right]^{\gamma_x}\right) \quad (16)$$

where $Y_t^{pot}$ denotes the level of output that would prevail if prices were flexible, and $i$ the steady-state (net) nominal interest rate, which is given by $r+\pi$ where $r\equiv 1/\beta-1$. In the linearized model, (16) is written

$$i_t=\max\left(0,i+\gamma_\pi\left(\pi_t-\pi\right)+\gamma_x x_t\right)$$

where $x_t\equiv\ln\left(Y_t/Y_t^{pot}\right)$.

### 2.1.4. The Aggregate Resource Constraint

We now turn to discuss the derivation of the aggregate resource constraint. Let $Y_t^{sum}$ denote the unweighted average (sum) of output for each firm $f$, i.e.

$$Y_t^{sum} = \int_0^1 Y_t(f)df.$$

which from (9) and the fact that all firms have the same capital-labor ratio can be rewritten as

$$\begin{aligned} Y_t^{sum} &= \int \left(\frac{K(f)}{N_t(f)}\right)^{\alpha} N_t(f)df \\ &= \left(\frac{K}{N_t}\right)^{\alpha} \int N_t(f)df \qquad (17) \\ &= K^{\alpha} N_t^{1-\alpha} \end{aligned}$$

Recalling that $Y_{t+j}(f)$ is given from (12), it follows that

$$Y_t^{sum} = Y_t \int_0^1 \frac{\phi_p}{\phi_p - (\phi_p - 1)\epsilon_p} \left( \left[\frac{P_t(f)}{P_t}\frac{1}{\vartheta_t}\right]^{-\frac{\phi_p - (\phi_p - 1)\epsilon_p}{\phi_p - 1}} + \frac{(1-\phi_p)\epsilon_p}{\phi_p} \right) df,$$

or equivalently, using (17):

$$Y_t = (p_t^*)^{-1} K^{\alpha} N_t^{1-\alpha}, \qquad (18)$$

where

$$p_t^* \equiv \int_0^1 \frac{\phi_p}{\phi_p - (\phi_p - 1)\epsilon_p} \left( \left[\frac{P_t(f)}{P_t}\frac{1}{\vartheta_t}\right]^{-\frac{\phi_p - (\phi_p - 1)\epsilon_p}{\phi_p - 1}} + \frac{(1-\phi_p)\epsilon_p}{\phi_p} \right) df.$$

In the technical appendix, we show how to develop a recursive formulation of the sticky price distortion term $p_t^*$.

Now, because actual output $Y_t$ is what is available for private consumption and government spending purposes, it follows that:

$$\underbrace{C_t + G_t}_{\equiv Y_t} \leq (p_t^*)^{-1} \underbrace{K^{\alpha} N_t{}^{1-\alpha}}_{\equiv Y_t^{sum}}. \qquad (19)$$

The sticky price distortion clearly introduces a wedge between input use and the output available for consumption (including by the government).[1] Even so, this term vanishes in the log-linearized version of the model.

[1] As the economy is assumed to be endowed with the fixed aggregate capital stock $K$ which does not depreciate, no resources is devoted to investment. An alternative formulation would have embodied a constant capital depreciation rate in which case output would have been used for $C_t$, $I$ and $G_t$.

### 2.2. Parameterization

Our benchmark calibration – essentially adopted from Erceg and Linde (2012) – is fairly standard at a quarterly frequency. We set the discount factor $\beta = 0.995$, and the steady state net inflation rate $\pi = .005$; this implies a steady state interest rate of $i = .01$ (i.e., four percent at an annualized rate). We set the intertemporal substitution elasticity $\sigma = 1$ (log utility), the capital share parameter $\alpha = 0.3$, the Frisch elasticity of labor supply $\frac{1}{\chi} = 0.4$, and the steady state value for the consumption taste shock $\nu = 0.01$.[2] As a comprise between the low estimate of $\phi_p$ in Altig et al. (2011) and the higher estimated value by Smets and Wouters (2007), we set $\phi_p = 1.1$. This leaves us with two additional deep parameters to pin down; the price contract duration parameter $\xi_p$, and the Kimball elasticity demand parameter $\epsilon_p$. To pin down these parameters, our starting point is the New Keynesian Phillips Curve

$$\pi_t - \pi = \beta \left( \mathrm{E}_t \pi_{t+1} - \pi \right) + \kappa_{mc} \widehat{mc_t}, \tag{20}$$

which obtains in our model where $\widehat{mc_t}$ denotes marginal cost as log-deviation from its steady state value. The parameter $\kappa_{mc}$, i.e. the slope of the Phillips curve, is given by

$$\kappa_{mc} \equiv \frac{\left(1-\xi_p\right)\left(1-\beta\xi_p\right)}{\xi_p} \frac{1}{1+\left(\phi_p - 1\right)\epsilon_p}.$$

A large body of *microeconomic* evidence, see e.g. Klenow and Malin (2010) and Nakamura and Steinsson (2012) and the references therein, suggest that firms change their prices rather frequently, on average somewhat more often than once a year. Based on this micro evidence, we set $\xi_p = 0.667$, implying an average price contract duration of 3 quarters $(\frac{1}{1-0.667})$. On the other hand, the *macroeconomic* evidence suggest that the sensitivity of aggregate inflation to variations in marginal cost is very low, see e.g. Altig et al. (2011). To capture this, we adopt a value for $\epsilon_p$ so that the slope of the Phillips curve ($\kappa_{mc}$) – given our adopted values for $\beta$, $\xi_p$ and $\phi_p$– equals 0.012.[3] This calibration allows us to match both the micro- and macroevidence on price setting behavior and are aimed at capturing the resilience of core inflation, and measures of expected inflation, during the recent global recession.

We assume a government debt to annualized output ratio of 0.6 (consistent with U.S. pre-crisis federal debt level), implying a quarterly value for $b_G = 2.4$. From (14), the steady labor income

[2] By setting the steady value ofo the consumption taste shock to a small value, we ensure that the dynamics for alternative shocks are roughly invariant to the presence of $-C\nu_t$ in the period consumption utility function.

[3] The median estimates of the Phillips Curve slope in recent empirical studies by e.g. Adolfson et al (2005), Altig et al. (2011), Galí and Gertler (1999), Galí, Gertler, and López-Salido (2001), Lindé (2005), and Smets and Wouters (2003, 2007) are in the range of $0.009 - .014$.

tax rate $\tau_N$ equals

$$\tau_N = \left(\frac{\phi_p}{1-\alpha}\right)(r \times b_G + g_y - \tau_C (1-g) - \tau).$$

Under the additional assumptions that $\tau_C = 0$, the government consumption share of steady state output $g_y = 0.2$, and that net lump-sum taxes $\tau = 0$, the above steady state relationship implies $\tau_N = 0.33$, i.e. an average labor income tax of 33 percent. The parameter $\varphi_b$ in the tax rule (15) is set equal to 0.01, which implies that the contribution of lump-sum taxes to the response of government debt is extremely small in the first couple of years following a shock (so that almost all variation in tax revenue comes from fluctuations in labor tax revenues). For monetary policy, we use the standard Taylor (1993) rule parameters $\gamma_\pi = 1.5$ and $\gamma_x = .125$.

In order to facilitate comparison between the non-linear and linear model, we specify processes for the exogenous shocks such that there is no loss in precision due to an approximation. In particular, the preference, discount and government spending shocks are assumed to follow AR(1) processes:

$$\begin{aligned}\left(\frac{G_t - G}{G}\right) &= \rho_G \left(\frac{G_{t-1} - G}{G}\right) + \sigma_G \varepsilon_{G,t},\\ (\nu_t - \nu) &= \rho_\nu (\nu_{t-1} - \nu) + \sigma_{\nu,t},\\ \left(\frac{\delta_t - \delta}{\delta}\right) &= \rho_\delta \left(\frac{\delta_{t-1} - \delta}{\delta}\right) + \sigma_\delta \varepsilon_{\delta,t}.\end{aligned} \tag{21}$$

Our baseline parameterization of these processes adopts a persistence coefficient of 0.95, so that $\rho_\nu = \rho_G = \rho_\delta = 0.95$ in (21). But following some prominent papers in the literature on fiscal multipliers, we also investigate the sensitivity of our results when the processes are assumed to be general Markov processes.

### 2.3. Solving the Model

We confine ourself to study perfect foresight simulations, i.e. solutions where uncertainty about future shock realizations is irrelevant for the dynamics of the economy. While other papers − see for instance Adam and Billi (2006, 2007) within a linearized framework and Fernández-Villaverde et al. (2012) and Gust, Lopez-Salido and Smith (2013) within a nonlinear framework − have shown that allowing for uncertainty can potentially have important implications for equilibrium dynamics, we nevertheless choose not to do so for the following two main reasons. First, because the bulk of the existing literature have used a perfect foresight approach, retaining this approach allows us to parse out the effects of going from a linearized to a nonlinear framework. Second, the perfect foresight assumption allows us to readily study the robustness in a larger scale model with many state variables. So far, the solution algorithms used to solve models with shock uncertainty have typically not been applied to models with more than 4-5 state variables.[4]

To solve the model, we feed the relevant equations in the nonlinear and log-linearized versions of the model to Dynare. Dynare is a pre-processor and a collection of MATLAB routines which can solve non-linear models with forward looking variables. For perfect foresight simulations like ours, Dynare uses a Newton-type algorithm, and the details of the algorithm used can be found in Juillard (1996). For the linearized model, we used the algorithm outlined in Hebden, Linde and Svensson (2012) to check for uniqueness. However, for the nonlinear version of the model, we cannot rule out the possibility that there exists other solutions in addition to the one found by Dynare. We note, however, that this problem pertains to all papers in the literature which study nonlinear models.

## 3. Results for the Stylized Model

In this section, we report our main results in the linearized and non-linear solution of the model outlined in the Section above. We start out by reporting how we construct the baseline scenarios and then report the marginal fiscal multipliers.

### 3.1. Baseline Scenario

As mentioned earlier, our aim is to compare fiscal spending multipliers in linearized and nonlinear versions of the model economy. Specifically, we seek to characterize how the difference between the multiplier in the linear and nonlinear frameworks varies with the expected duration of the liquidity trap.

To construct a baseline where the interest rate is bounded at zero for $\text{ZLB}_{DUR} = 1, 2, 3, ..., T$ periods, we follow the previous fiscal multiplier literature (e.g. Christiano, Eichenbaum and Rebelo, 2011) and assume that the economy is hit by a large adverse shock that triggers a deep recession and drive interest rates to zero. The larger value of $\text{ZLB}_{DUR}$ we want to have, the larger the adverse shock has to be. The particular shock we consider is a negative consumption taste shock $\nu_t$ in (21) following Erceg and Linde (2012), but we present results in Appendix A when the recession is instead assumed to be triggered by the discount factor shock $\delta_t$ that was used in the seminal

[4] A recent paper by Judd, Maliar and Maliar (2011) provides a promising avenue to compute the stochastic solution of larger scale models efficiently.

papers by Eggertsson and Woodford (2003) and Christiano, Eichenbaum and Rebelo (2011).

To provide clarity on how we pick the shock sizes, Figure 1 reports how the linear and nonlinear specifications react to a negative taste shock which generates a liquidity trap of length $\text{ZLB}_{DUR} = 8$ quarters in the linearized model variant. The economy is in the deterministic steady state in period 0, and then the shock hits the economy in period 1. As is evident from Figure 1, the same-sized shock has a rather different impact on the economy depending on whether the model is linearized or solved in its original nonlinear form. For instance, we see from panel 3 that while the nominal interest rate is bounded by zero from periods 1 to 8 in the linearized model, the equally-sized consumption demand shock (panel 9) only generates a two quarter trap in the non-linear model. Hence, we need to subject the nonlinear model to a more negative consumption demand shock − as shown in panel 9 in Figure **??** − to generate $\text{ZLB}_{DUR} = 8$ for the interest rate (panel 3).[5]

A lot of intuition about the differences between the linearized and nonlinear variants can be gained from Figures 1 and **??**. Starting with Figure 1, we see from the fifth panel that the potential real rate falls roughly about the same in both models. Still, the linearized model generates a much longer liquidity trap because inflation and expected inflation falls much more (panel 2), which in turn causes the real interest rate (panel 4) to rise much more initially. The larger initial rise in the real interest rate triggers a larger fall in the output gap (panel 1) and consequently real GDP falls more in the linearized model as well (because the impact on potential GDP is about the same, as implied by the similarity of the potential real interest rate response).

Turning to Figure **??**, we first note from the third panel that the paths for the policy rate are bounded at zero for 8 quarters and display a very similar path upon exit from the liquidity trap. Moreover, panel 9 shows that it takes a much larger adverse consumption demand shock in the nonlinear model to trigger a liquidity trap of the same expected duration as in the linearized model. This implies that the drop in the potential real rate and real GDP (panels 5 and 7) is much more severe in the nonlinear model. Even so, and perhaps most important, we see that inflation − panel 2 − falls substantially less in the nonlinear model. This implies that the differences between the linearized and nonlinear version of the model too a large extent is driven by the linearization of the pricing block of the model.

[5] Figure 2 also depicts a third line ("Nonlinear model with linearized price block"), which we will discuss further in Section 3.2.

### 3.2. Marginal Fiscal Multipliers

As previously noted, we are seeking to compare fiscal multipliers in liquidity traps of same expected duration in the linearized and nonlinear frameworks. Accordingly, we allow for differently sized shocks in the linearized and nonlinear models so that each model variant variants generate a liquidity trap with the same expected duration $\text{ZLB}_{DUR} = 1, 2, 3, ..., T$. Let $B_t^{linear}\left(\sigma_{\nu,i}^{linear}\right)$ and $B_t^{nonlin}\left(\sigma_{\nu,i}^{nonlin}\right)$ denote vectors with simulated variables in the linear and nonlinear models, respectively. The baseline paths are functions of the size of the consumption demand shock $\nu_t$, $\sigma_\nu$, which as explained in the previous section are set so that $\sigma_{\nu,i}^{j}$ generates a

$$\sigma_{\nu,i}^{linear} \Rightarrow \text{ZLB}_{DUR} = i,$$

and

$$\sigma_{\nu,i}^{nonlin} \Rightarrow \text{ZLB}_{DUR} = i,$$

where we consider $i = 1, 2, ..., T$. In the specific case of $i = 8$, panel 9 in Figure **??** shows that $\sigma_{linear,8}^{\nu} = -.18$ and $\sigma_{nonlin,8}^{\nu} = -.42$.

To these different baseline paths, we add the fiscal response in the first period ($t = 1$); that is, the same period as the adverse shock hits. By letting $S_t^{linear}\left(\sigma_{\nu,i}^{linear}, \sigma_G\right)$ and $S_t^{nonlin}\left(\sigma_{\nu,i}^{nonlin}, \sigma_G\right)$ denote vectors with simulated variables in the linear and nonlinear models when both the negative baseline shock $\sigma_\nu$ and the positive government spending shock $\sigma_G$ hits the economy, we can compute the partial impact of the fiscal spending shock as

$$I_t^j\left(\text{ZLB}_{DUR}\right) = S_t^j\left(\sigma_{\nu,i}^j, \sigma_G\right) - B_t^j\left(\sigma_{\nu,i}^j\right)$$

for $j = \{linear, nonlin\}$ and where we write $I_t^j\left(\text{ZLB}_{DUR}\right)$ to highlight its dependence on the liquidity trap duration. Notice that the fiscal spending shock is the same for all $i$ and is scaled so that $\text{ZLB}_{DUR}$ remains unaffected. By setting the fiscal impulse so that the liquidity trap duration remains unaffected, we retrieve "marginal" spending multiplier in the sense that they show the impact of a "tiny" change in the fiscal instrument.[6]

In Figure 3 we report the results of our exercise. The upper left panel report the impact spending multiplier, i.e. simply

$$m_i = \frac{1}{g_y}\frac{\Delta Y_{t,i}}{\Delta G_{t,i}}$$

[6] Had we considered a larger fiscal intervention that altered the duration of the liquidity trap, there would have been an important distinction between the *average* (i.e. the total response) and *marginal* (i.e. the impact of a small change in $g_t$ which leaves $\text{ZLB}_{DUR}$ unchanged) multiplier as discussed in further detail in Erceg and Linde (2012).

where the $\Delta$-operator represents the difference between the scenario with the spending change and the baseline without the spending change. We compute $m_i$ for $\text{ZLB}_{DUR} = 1, ..., 12$, but also include results for the case when the economy is at the steady state, so that $\text{ZLB}_{DUR} = 0$.

As the linear approximation should be more accurate the closer the economy is to the steady state, it is not surprising that the difference between the "linear" and "nonlinear" multiplier increases with the duration of the liquidity trap. For a three year liquidity trap, the recorded multiplier is more than twice as large in the linearized model relative to the nonlinear model. For shorter-lived liquidity traps, the differences are notably more modest, and in the special case when the economy is in the steady state ($\text{ZLB}_{DUR} = 0$ in the figure) we note that the multipliers are identical (as they should) in both economies. The difference in the government debt (as share of actual annualized GDP) response after 1 year, shown in the upper right panel, largely follows the pattern for $m_i$ and increases with $\text{ZLB}_{DUR}$.[7]

The substantial differences in the output and debt responses begs the question of which factors account for them. The middle upper panel, which shows the response of the one-period ahead expected annualized inflation rate (i.e., $4\text{E}_t\pi_{t+1}$) sheds some light on this. As can be seen from the panel, expected inflation responds much more in a long-lived trap in the linearized model than in the nonlinear model. The sharp increase in expected inflation triggers a larger reduction in real rates in the linearized model, and thereby induces a more favorable response of private consumption which helps to boost output relative to the nonlinear model.

But an important question still remains, why does expected inflation respond more in the linearized economy? To shed light on this, we simulated a variant of the nonlinear model in which we linearized the price block of the model, e.g. replaced all pricing equations in the nonlinear model with the standard linearized Phillips curve and removed the price distortion term from the aggregate resource constraint (19). Following the approach with the linear and nonlinear models, we construct baseline scenarios for this variant of the model as described in section 3.1 for $\text{ZLB}_{DUR} = 1, ..., 12$. The dash-dotted lines in Figure 2 depicts the eight quarter liquidity trap baseline in this variant of the model. Clearly, the simulated paths in this model are very similar to those in the linearized model. Hence, it is perhaps not that surprising that the results in Figure 3 for this model (referred to as "Pseudo-linear Model") also displays a striking similarity with the linearized model. Hence, we draw the conclusion that linearization of the price block, and not the aggregate demand part of the model, accounts for the bulk of the differences between the effects of fiscal spending in a

[7] For ease of interpretability, we have normalized the response of debt and inflation so that they correspond to a initial change in government spending (as share of steady state output) by one percent.

long-lived liquidity trap in the linear and nonlinear models.

All results so far have been developed in the benchmark model which employs the Kimball (1995) aggregator. In the lower panel of Figure 3 we present results for the standard Dixit and Stiglitz aggregator, keeping all other parameters unchanged. This implies that we are considering a higher substantially higher slope (i.e. $\kappa_{mc}$ of the New Keynesian Phillips curve in eq. (20). As can be seen from the results, the differences between the linear and nonlinear models are even more pronounced in this case, with the multiplier in an 8-quarter trap being over 5 times larger than in the nonlinear model. Taken together, the results in Figure 3 suggest that the findings of the papers in the previous literature which relied on linearized models were more distorted to the extent that they relied on a calibration with a higher slope of the Phillips curve and thus a larger sensitivity of expected inflation.

As a final experiment, Figure 4 compares Kimball vs. Dixit-Stiglitz variants of the model when the sticky price parameter $\xi_p$ is adjusted in the Dixit-Stiglitz version so that the slope of the linearized Phillips curve (20) is the same as in our benchmark calibration although the Kimball elasticity $\epsilon_p = 0$. Both the Kimball and Dixit-Stiglitz versions hence feature a linearized Phillips curve with an identical slope coefficient ($\kappa_{mc} = 0.012$), but the Dixit-Stiglitz version of the model achieves this with a substantially higher value of $\xi_p$ (0.90). Since only the value of $\kappa_{mc}$ matters in linearized versions of the model, the results with this variant are invariant w.r.t. the mix of $\xi_p$ and $\epsilon_p$ that achieves a given $\kappa_{mc}$. Hence, the multiplier schedules in the upper panels in Figure 4 are identical. But in the nonlinear versions of the model, shown in the lower panels, the results differ. In particular, we see that when the Dixit-Stiglitz aggregator implies that expected inflation and output multiplier responds more when the duration of the liquidity trap increases relative to the benchmark Kimball variant of the model. Thus, when the Kimball parameter $\epsilon_p$ is reduced, the more will expected inflation and output multiplier respond when $\text{ZLB}_{DUR}$ increases; conversely, increasing $\epsilon_p$ and lowering $\xi_p$ flattens the output multiplier schedule even more. Our intuition behind these results is that a higher value of $\epsilon_p$ induces the elasticity of demand to vary more with the relative price differential among the intermediate good firms. Thus, intermediate firms which only infrequently are able to re-optimize their price will optimally choose to respond less to a given fiscal impetus as they may experience a much larger impact on their demand for a given change in their relative price. As a result, aggregate current and expected inflation is less affected.

## 4. Robustness in a Workhorse New Keynesian Model

Here we will report results in the Christiano, Eichenbaum and Evans (2005) model augmented with the Bernanke, Gertler and Gilchrist (1999) financial accelerator mechanism.[**Remains to be written.**]

## 5. Conclusions

[**Remains to be written.**]

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Figure 1: Baselines in Linear and Nonlinear Models for an Equally−Sized Consumption Demand Shock

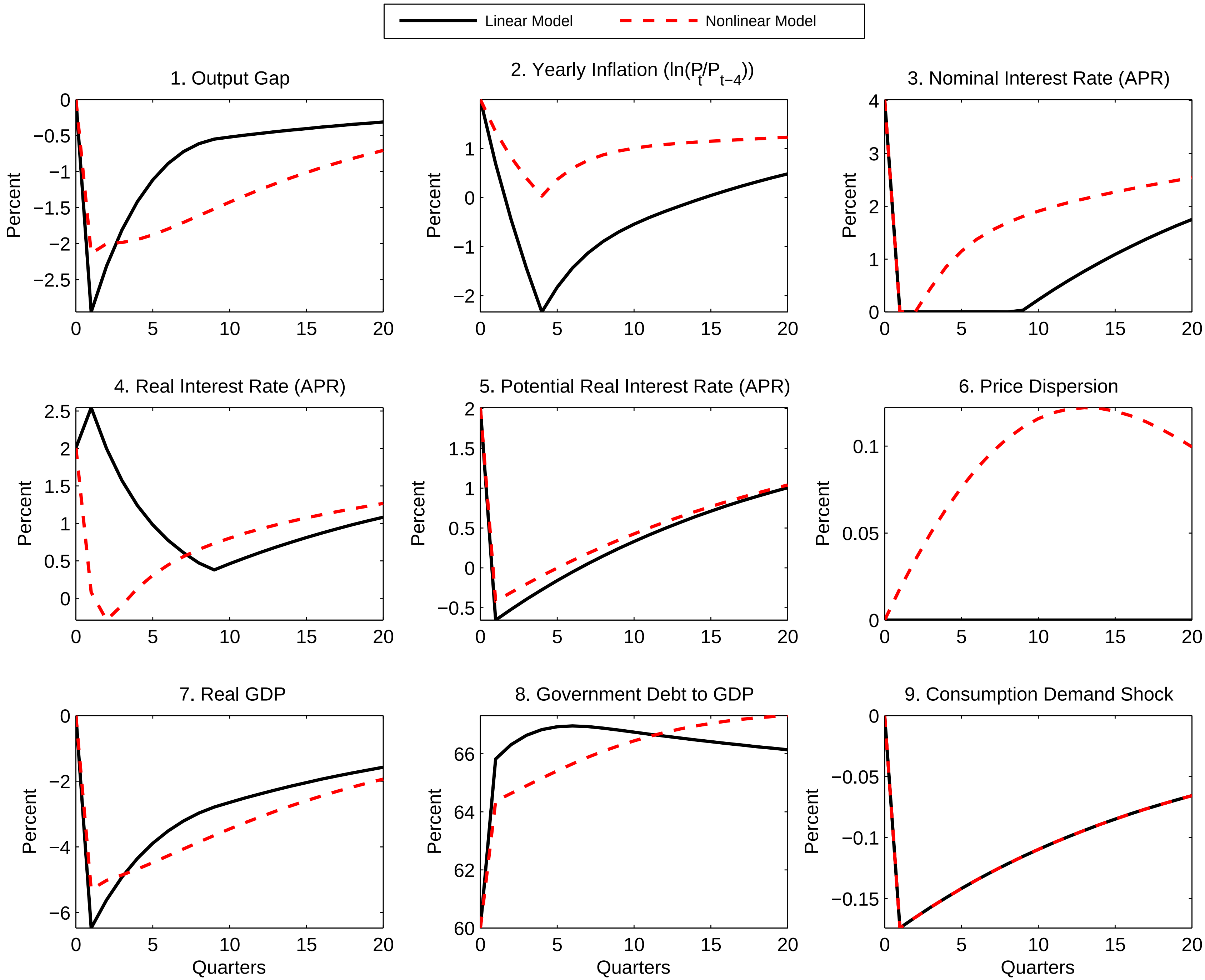

Figure 2: Baselines for 8−Quarter Liquidity Trap

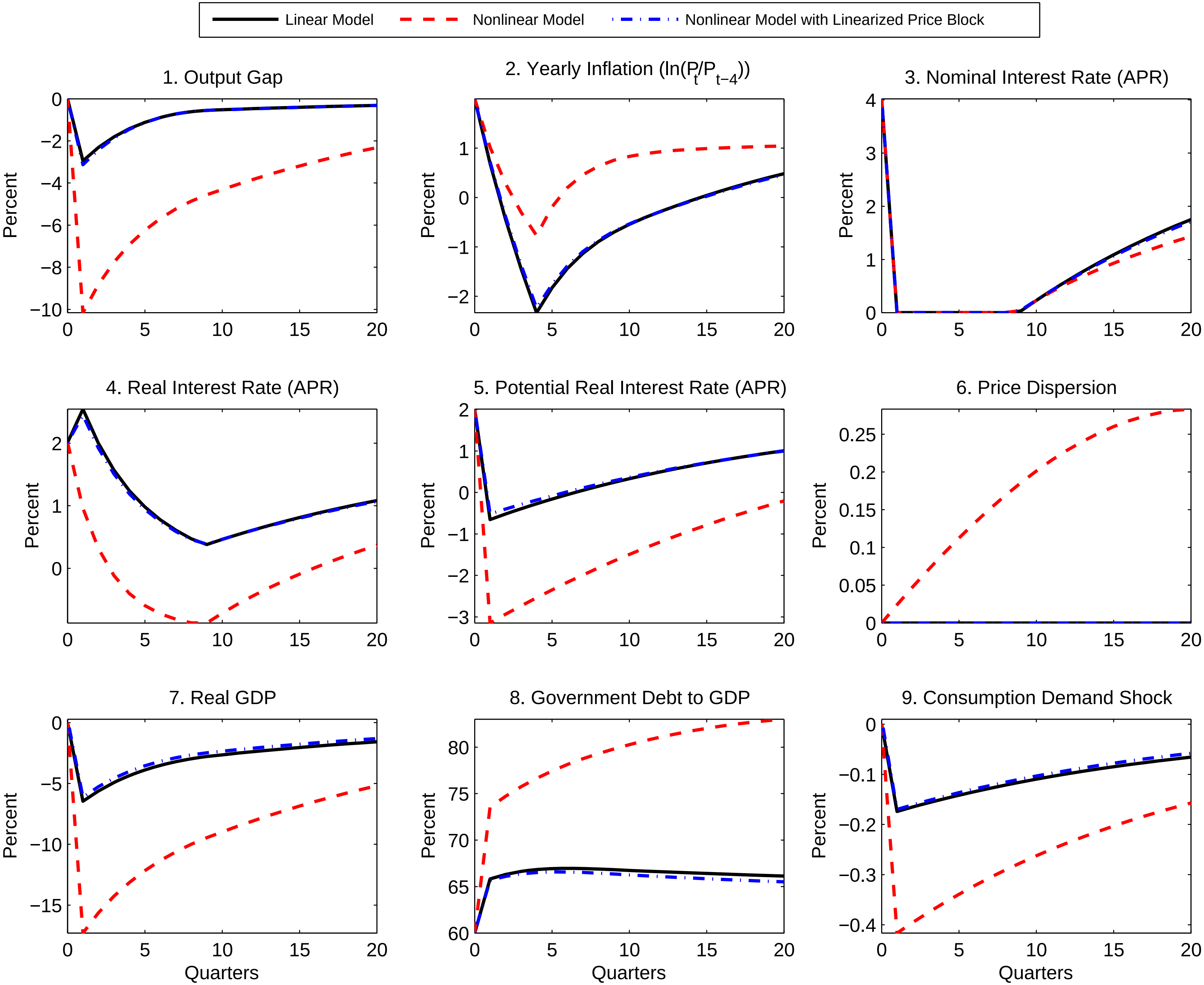

Figure 3: Marginal Multipliers

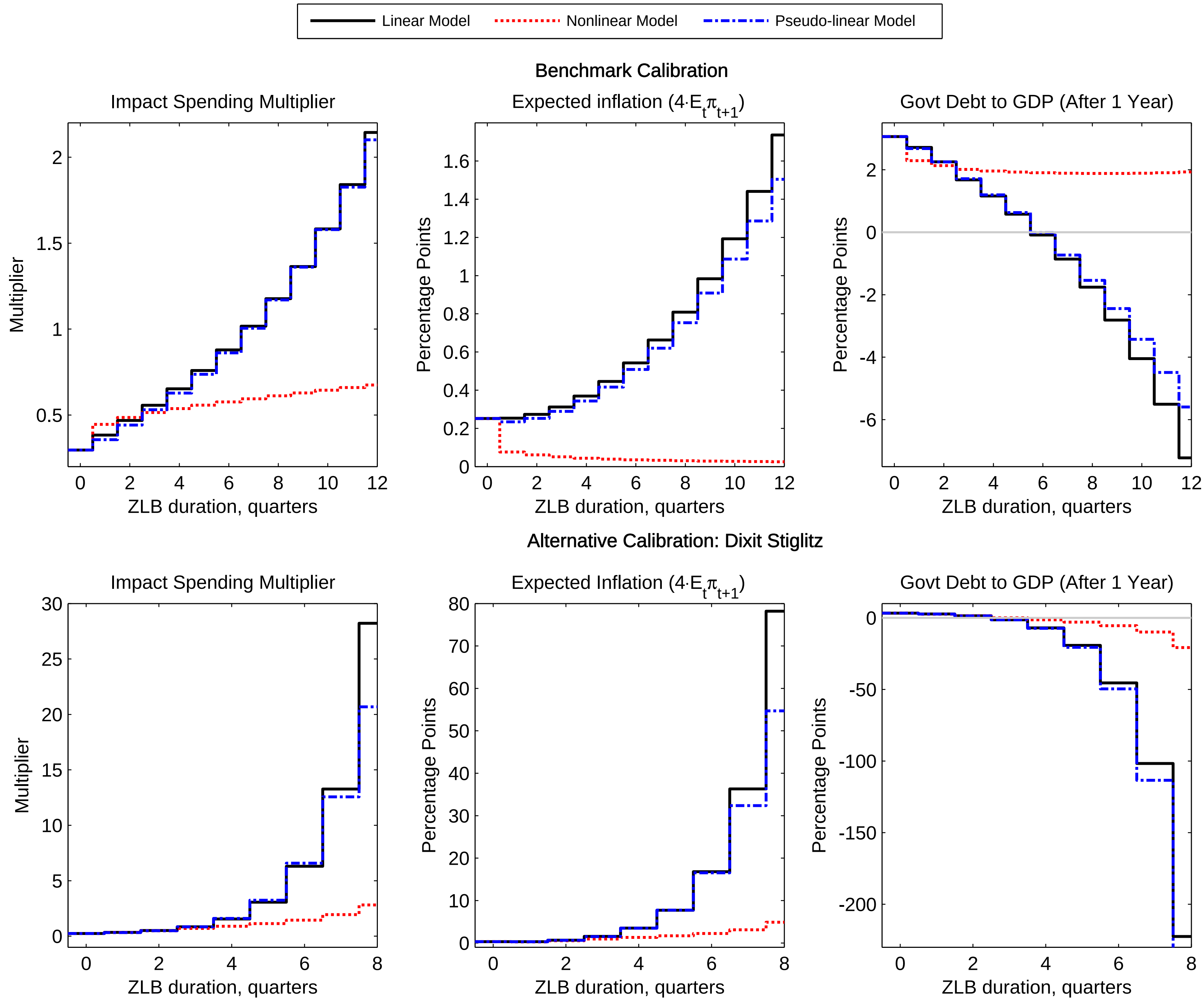

Figure 4: Marginal Impact of Changes in Spending: Kimball Vs. Dixit-Stiglitz