The Expansionary Lower Bound: a Theory of Contractionary Monetary Easing

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Abstract

We provide a theory of the interaction between monetary policy and borrowing constraints. In particular, we characterize sufficient conditions for the existence of an “Expansionary Lower Bound” (ELB), defined as the interest rate below which monetary easing becomes contractionary. The ELB can be positive, thus acting as a more stringent constraint than the Zero Lower Bound (ZLB), and can be affected by international monetary conditions, therefore giving rise to important departures from Mundell’s trilemma. We provide three applications under which the ELB may arise: first, in the presence of currency mismatches; second, due to carry-trade capital flows; third, because of the effects of monetary policy on bank profitability.

JEL Codes: E5, F3, F42

Keywords: Monetary policy, collateral constraints, currency mismatches, carry trade, spillovers

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1 Introduction

The last few years have witnessed a growing skepticism about the ability of monetary policy to stabilize the economy cycle. A first contentious issue is whether emerging markets (EMs) can effectively use domestic monetary policy to insulate themselves from changes in US monetary conditions. On the one hand, Bernanke (2015) argues that if EMs do not attach intrinsic value to exchange rate stability, they can effectively respond to shocks arising from the international monetary system, in line with Mundell’s trilemma. On the other hand, Rey (2015, 2016) and Rajan (2015) claim that this is not necessarily the case broadly due to the presence of financial frictions. A somewhat related concern is that monetary policy may also be undermined by carry trade flows. For example, policy makers in EMs are often reluctant to lower interest rates during an economic downturn because they fear spurring capital outflows, a symmetric argument to the one presented in Blanchard et al. (2016). Finally, the effectiveness of monetary policy has come under scrutiny even in advanced economies. There is indeed an intense debate on whether the prolonged period of ultra low interest rates since the global financial crisis may have adverse effects on the financial sector and the economy at large (Borio and Zabai, 2016; Brunnermeier and Koby, 2016).

In this paper we provide a theory of the interaction between monetary policy and borrowing constraints, and show how it can help to rationalize these controversial issues. Our key insight is that, when monetary policy affects borrowing limits, a central bank might be constrained in its ability to achieve the efficient level of output by an “Expansionary Lower Bound” (ELB). The ELB is the interest rate below which further monetary easing becomes contractionary. Importantly, the ELB can occur at positive interest rates and is therefore a potentially tighter constraint for monetary policy than the zero lower bound (ZLB). Furthermore, the ELB can be a function of foreign interest rates and therefore provide a novel channel through which monetary policy spills over across countries.

We begin by establishing the conditions for the existence of the ELB in the context of a general framework that encompasses both closed and open economy models. The key aspect of our theory is that monetary easing determines a tightening of borrowing constraints. We show that for the ELB to arise two conditions have to be satisfied. First, monetary policy needs to be able to affect whether borrowing constraints are binding or not. In particular, a large enough monetary easing should make constraints become binding. For this to be the case, monetary easing should tighten borrowing constraints by more than it possibly reduces the unconstrained demand for borrowing. Second, once constraints become binding, monetary easing should tighten borrowing constraints enough to curb the aggregate demand of constrained agents by more than monetary easing stimulates the demand of the unconstrained ones. If these two conditions are satisfied, the IS curve that links the policy rate to aggregate demand becomes backward bending. The turning point occurs at the ELB interest rate.\footnote{See also Obstfeld (2015) for an intermediate view.}
rate and this places an upper bound on the level of output achievable through monetary stimulus.

We then present three applications of our general model where the borrowing constraints take the form of collateral constraints on financial intermediaries. We use these applications to shed light on the three controversial aspects of monetary policy discussed above. The first application highlights the role of currency mismatches. This is a proverbial concern in EMs that in recent years have again accumulated large amounts of US dollar debt attracted by low US rates (Acharya et al., 2015; McCauley, McGuire and Sushko, 2015). In the model, unhedged currency mismatches are held by financial intermediaries that borrow internationally in US dollars and lend domestically in local currency. Importantly, these banks are subject to collateral constraints that limit domestic lending to a certain proportion of their networth. When collateral constraints are not binding, monetary accommodation is expansionary. Lower rates boost domestic demand and, by depreciating the exchange rate, they also strengthen foreign demand. In this case, EMs with flexible exchange rates can effectively insulate themselves from international monetary shocks, as under Mundell’s trilemma.

However, a sufficiently large monetary easing in EMs can make collateral constraints become binding due to the erosion of banks’ networth arising from the exchange rate depreciation, thus satisfying the first condition for the existence of the ELB. Furthermore, if foreign-currency debt is sufficiently large, further monetary easing has contractionary effects on output in line with the second condition. This is because the tightening of collateral constraints generates an increase in lending rates and a reduction in domestic spending that outweighs the boost in foreign demand from the exchange rate depreciation. Importantly, the ELB depends on US monetary conditions, as it increases with US policy rates. A monetary tightening in the US can thus determine an economic downturn in EMs by pushing them against the ELB. Note that this is the case even if EMs have flexible exchange rates, thus providing a key departure from the trilemma.

We also explore possible policy tools that EMs can use to escape the ELB. We show that forward guidance, despite being effective against the ZLB, is powerless with respect to the ELB. This is because the ELB is an endogenous constraint that responds to both the current and future monetary stance. EMs can instead relax the ELB by recapitalizing banks or by using capital controls and foreign exchange rate intervention to delink the exchange rate from domestic monetary conditions. Finally, we analyze the implications of the ELB for the ex-ante conduct of monetary policy. We show that the possibility of the ELB becoming binding in the future weakens the effectiveness of monetary policy in the prior periods. Furthermore, it requires monetary authorities to keep the economy below potential to better support output in the future should the ELB become binding. The ex-ante analysis provides also interesting insights about spillovers from US monetary policy. From an ex-post perspective, the US can relax the ELB in EMs by lowering policy rates. However, this becomes much less effective if it is anticipated, since EMs would increase foreign currency borrowing. EMs can instead effectively relax the risk of a future ELB becoming binding by using
capital controls to reduce foreign-currency debt.

The second application considers the implications of carry-trade flows for the ability of monetary policy to stimulate the economy. When foreign investors follow carry-trade strategies based on interest rate differentials, a reduction in the policy rate can trigger a sell-off of domestic government bonds. As domestic banks absorb the excess supply of bonds, their collateral constraints tighten and eventually become binding. From this point onwards, further monetary easing becomes contractionary since bond purchases by domestic banks crowd out domestic lending. In this model application, fiscal policy can play an important role in overcoming the ELB. In particular, by reducing the stock of government debt, fiscal consolidation can relax collateral constraints and re-establish conventional monetary transmission to lending rates and domestic demand. However, these effects are contingent on how fiscal consolidation is implemented since it should not place excessive burden on domestic borrowers.

Finally, we consider a third application where monetary policy affects collateral constraints because of the impact on bank profitability. Several advanced economies have recently lowered policy rates below zero. However, banks have been reluctant to pass negative rates to depositors. As long as collateral constraints are not binding, reducing policy rates below zero can still lead to a decline in lending rates and thus provide monetary stimulus. However, this comes at the cost of lower profitability which can eventually make collateral constraints binding. Once constraints bind, further monetary easing is unable to provide additional stimulus thus giving rise to an ELB.

The paper is structured as follows. After reviewing the relevant literature, we present our general model in section 2. We then analyze the applications based on currency mismatches, carry trade capital flows, and bank profitability in sections 3, 4, and 5, respectively. We summarize the key findings and avenues for further research in the concluding section.

Literature review. We develop the analysis in the context of models with borrowing constraints and heterogeneity between constrained and unconstrained agents. The paper is thus related to a growing literature that analyzes monetary policy in models with incomplete financial markets and heterogeneous agents (Auclert, 2016; Gornemann, Kuester and Nakajima, 2016; Kaplan, Moll and Violante, 2016; McKay, Nakamura and Steinsson, 2016; Guerrieri and Lorenzoni, 2016; Werning, 2015). These models reveal important departures from the monetary transmission mechanism in representative agent models. For example, they tend to find a stronger responsiveness of consumption to income effects and uncover novel channels of monetary transmission through redistributive effects. Nonetheless, in all these papers, monetary easing remains expansionary. On the contrary, our analysis emphasizes the possibility that monetary policy may actually be constrained in its ability to stimulate output because of adverse effects on collateral constraints.

The notion that borrowing constraints can place limits on the ability of monetary policy to stabilize output is reminiscent of the literature spurred by the 1997 East Asian financial crisis. Despite
solid fiscal positions, East Asian countries suffered a severe crisis because sharp exchange rate depreciations impaired the balance sheets of banks and firms with unhedged dollar liabilities. This motivated the development of a third generation of currency crisis models to explain how the interplay between borrowing constraints and currency mismatches can give rise to self-fulfilling currency runs (Krugman, 1999; Aghion, Bacchetta and Banerjee, 2000, 2001). Particularly related to our paper was the intense debate about the appropriate monetary response, with some arguing in favor of monetary stimulus to support domestic demand, while others calling for a monetary tightening to limit balance-sheet disruptions. These trade-offs are analyzed in Céspedes, Chang and Velasco (2004) and Christiano, Gust and Roldos (2004) whose focus is on the effects of monetary policy once borrowing constraints are binding and in the context of models where monetary policy cannot influence whether agents are constrained or not. These models can generate situations in which monetary easing becomes contractionary, but they do not feature an ELB: even if monetary easing is contractionary, monetary policy can still achieve any desired level of output by raising rather than lowering policy rates. We depart from this earlier literature by considering models in which monetary policy itself affects whether constraints are binding or not which is essential to generate an ELB or equivalently an upper bound on the output level that monetary policy can achieve.

Our paper is also related to the work by Ottonello (2015) and Farhi and Werning (2016) that show how currency mismatches and borrowing constraints can complicate the conduct of monetary policy. In their models monetary easing remains expansionary, but by depreciating the exchange rate it tightens borrowing constraints and forces a reduction in domestic consumption. Therefore, monetary policy faces a trade-off between supporting output and stabilizing domestic consumption. We instead emphasize that under certain conditions monetary easing can lead to a sufficiently strong contraction in domestic consumption that it also causes a reduction in output.

Our third application shows that monetary policy can affect borrowing constraints through its impact on bank profitability. A similar idea is developed in Brunnermeier and Koby (2016) that point out that, when banks have market power, monetary easing can erode their intermediation margins. This can in turn make collateral constraints become binding at which point further monetary easing can lead to an increase in lending rates. Our model reveals an alternative channel through which monetary accommodation can harm bank profitability and that can be at play also in a competitive banking system. The recent experience with negative interest rate policies in several advanced economies has shown that banks are reluctant to pass negative interest rates to depositors. This is likely due to the fear that depositors may react very strongly against negative rates, by transferring money to other banks or holding cash. Such a floor on deposit rates can adversely impact bank profitability and possibly lead to an ELB whereby monetary accommodation is no longer able to

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2 Ottonello (2015) considers also an extension of his model in which borrowing constraints limit the country’s ability to import intermediate goods. In this case, monetary easing can in principle have contractionary effects on output by depreciating the exchange rate and tightening borrowing constraints. Nonetheless, in his calibration monetary accommodation remains expansionary.
stimulate the economy. Note that, differently from Brunnermeier and Koby (2016), we characterize how profitability losses affect not only lending rates, but aggregate demand thus taking into account the impact on savers too.

2 A general model of the Expansionary Lower Bound

In this section, we characterize the conditions for the existence of the ELB in the context of a generic macro model in which monetary policy affects borrowing constraints. In particular, we aim to place broad restrictions on how monetary policy should move borrowing constraints for the ELB to arise. We show that this requires two key conditions: first, a sufficiently deep monetary easing should make borrowing constraints become binding; second, further monetary easing should tighten borrowing constraints enough to determine an overall contraction in aggregate demand. The model is cast in very general terms and thus encompasses various possible settings in both closed and open economy. Note also that in this section we do not take a stand on the particular mechanism through which monetary policy affects borrowing constraints, focusing only on how strong the effect should be for the ELB to arise. We will provide specific examples of borrowing constraints that can satisfy the conditions derived in this section by considering three different model applications in the remaining of the paper.

2.1 Model setup

Consider an economy populated by two classes of agents $j \in \{B, S\}$ both with unitary mass that we refer to as borrowers and savers, respectively. Each agent produces a variety of the consumption good $C_{jt}$, where subscripts denote the variety and time of consumption. Agents maximize the present discounted utility from consuming the bundle $C_j^t = C_{B}^j, C_S^j$ where the superscript indicates the agent that is consuming, according to

$$\sum_{t \geq 0} \beta^t U \left( C_j^t \right)$$

The maximization problem is subject to a conventional intertemporal budget constraint

$$\sum_{t \geq 0} Q_t \left( p_j^t C_j^t - \Pi_j^t \right) \leq 0$$

where $p_j^t$ is the price of the consumption bundle, $\Pi_j^t$ is total nominal income, including both labor and capital returns, and $Q_t$ is the time-0 price of a bond paying one unit of the numeraire at time $t$. The nominal interest rate is thus equal to $I_t = Q_t / Q_{t+1}$. For the sake of simplicity, we assume that agents enter time-0 with no financial assets or liabilities. At time-0, agents face the following borrowing constraint
They key feature of the model that can give rise to the ELB is that we allow the borrowing limit $\Phi^j_0$ to depend on the monetary policy rate

$$d\Phi^j_0 = \varepsilon^j_0 d\phi^j_0$$

where lower-case letters denote percentage changes of their upper-case counterparts, for example $d\Phi^j_0 = d\Phi^j_0/\Phi^j_0$. Note that we do not take a stand on the specific mechanism through which monetary policy affects borrowing constraints. The purpose of this section is merely to characterize how monetary policy should affect borrowing constraints for the ELB to arise, by placing restrictions on the elasticity of the borrowing limit to the policy rate $\varepsilon^j_0$. The model applications following this section will provide examples of particular forms of borrowing constraints that satisfy these restrictions.

We are interested in analyzing the model responses to a transitory monetary policy shock at time 0. Similarly to Auclert (2016), we assume that the monetary shock affects output and prices only at time 0 by imposing that $d\pi^j_t = dp^j_t = d_i = 0$ for all $t \geq 1$. Since we focus the analysis on the model responses at time-0, from point onward we drop time subscripts to ease the notation. The first-order consumption response at time 0 of an unconstrained agent can be expressed as

$$d\tilde{c}^j = \delta \left[ (1 - \lambda^j) (d\pi^j - dp^j) - \lambda^j (d_i + dp^j) \right] - (1 - \delta) \sigma (d_i + dp^j)$$

where $\lambda^j = 1 - \Pi^j/\Pi^c$ is the fraction of consumption expenditures financed by borrowing, $\delta$ represents the marginal propensity to consume, and $\sigma$ is the intertemporal elasticity of substitution. Note that we use a tilde to denote the unconstrained consumption level. The term $d\pi^j - dp^j$ captures variations in real income as well as in the terms of trade if we are considering an open economy. Taking into account that the monetary shock is assumed to have no effects of the long-run price level, the term $d_i + dp^j$ is the percentage change in the real interest rate. The first part in equation (1) captures a conventional wealth effect: consumption increases with income and declines with an increase in the real rate. These two effects are weighed in proportion to how much the agent is borrowing. The second second term in equation (1) reflects instead the intertemporal substitution response to a change in the real interest rate.

If the borrowing constraint is binding, the consumption response is given by

$$d\bar{c}^j = (1 - \lambda^j) d\pi^j + \lambda^j d\phi^j - dp^j$$

where the upper bar denotes the consumption level in the constrained equilibrium. Consumption increases with nominal income and declines with the price level. Furthermore, monetary policy affects the consumption level of constrained agents by possibly moving the borrowing limit $\phi^j$.

Regarding changes in the consumption of a specific variety, these are characterized by the fol-
Following conditions

\[
dc^j_i = dc^j + \eta^j \left( dp^j_i - dp^j_j \right) \\
\]

\[
dp^j_i = \left( 1 - \alpha^j_i \right) dp^j_j + \alpha^j_k dp^j_k \\
\]

where \( \eta^j \) is the elasticity of substitution across the two goods and \( \alpha^j_k = \frac{p^j_k C^j_k}{p^j_j C^j_j} \) is the share of nominal spending on good \( k \) by agent \( j \). We close the model by imposing conventional market clearing conditions. These allow us to express the responses of aggregate nominal income and real output as follows

\[
d\pi^j_i = \frac{1 - \alpha^j_k}{1 - \lambda^j} \left( dp^j_j + dc^j_j \right) + \left( 1 - \frac{1 - \alpha^j_k}{1 - \lambda^j} \right) \left( dp^j_k + dc^j_k \right) \\
\]

\[
dy_k = \gamma^j_k dc^j_k + \left( 1 - \gamma^j_k \right) dc^j_k \\
\]

where \( \gamma^j_k = C^j_k / Y_k \) is the share of aggregate output \( Y_k \) consumed by the agent \( j \). Finally, we allow prices to depend on both aggregate output and on the nominal policy rate according to

\[
dp^j_k = \varepsilon^{p^j_i} y_k + \varepsilon^{p^j_i} d_i \\
\]

The elasticity of the price level with respect to output is assumed to be positive, \( \varepsilon^{p^j_i} y_k \geq 0 \), to capture a conventional upward sloping aggregate supply. The dependence of the price level on the policy rate captures instead possible exchange rate effects. For example, if the price is set in a currency different from the numeraire unit, a monetary accommodation that leads to an exchange rate depreciation would increase the price expressed in numeraire units.

### 2.2 Conditions for the existence of the ELB

We define the Expansionary Lower Bound as a nominal interest rate below which further monetary accommodation becomes contractionary. Therefore, the ELB places a limit on the ability of monetary policy to stimulate output. To formally derive the conditions for its existence, we consider the problem of a social planner that uses monetary policy to maximize output and ask under which conditions the problem features an internal solution. The planner sets the nominal interest rate, taking as given the agents’ first order conditions, the borrowing constraints, and the economy’s resource...
We solve the planner’s problem in appendix [] showing that two conditions must be satisfied for the ELB to arise. These are summarized in the following proposition:

**Proposition 1 (Existence conditions for the ELB.).** Monetary policy is constrained in its ability to stimulate \(Y_j\) by the ELB if two conditions are satisfied. First, monetary accommodation should push unconstrained borrowers against their borrowing constraint which requires

\[
\lambda^B d\phi^B > d\bar{c}^B + dp^B - (1 - \lambda^B) d\pi^B \tag{2}
\]

Second, once borrowing constraints are binding, further monetary easing should determine a contraction in aggregate demand for the good \(j\) so that

\[
\gamma_S^j d\bar{c}_S^j + (1 - \gamma_S^j) d\bar{c}^B_j > 0 \tag{3}
\]

**Proof.** See Appendix [] for the formal description and solution of the planner’s problem. \(\square\)

The left and right sides of condition (2) capture how monetary tightening affects respectively the maximum and desired level of borrowing. For monetary easing to push borrowers into the constrained region, a policy rate cut must narrow the gap between the maximum and desired borrowing level. Depending on the responsiveness of prices and output, in some model applications monetary easing leads to an increase in the desired borrowing level. In this cases, condition (2) can be satisfied even if monetary policy does not affect borrowing constraints. If instead monetary easing reduces the desired borrowing level, it must also lead to a sufficiently strong tightening of borrowing constraints for the ELB to arise.

Condition (3) places additional restrictions on how monetary policy should affect borrowing constraints. Specifically, monetary easing should tighten borrowing constraints enough so that it reduces spending by constrained borrowers more than it stimulates spending by unconstrained savers. This is essential to determine an overall reduction in aggregate demand, thus leading to contractionary effects on output. Note that condition (3) is not sufficient for the ELB arise. In some classes of models, it is indeed possible that borrowers are never constrained so that it is irrelevant that aggregate demand declines if constraints are binding. Similarly, there are model setups in which borrowers are always constrained in which case monetary policy can still stimulate output unboundedly by simply increasing (rather than lowering) policy rates if condition (3) is satisfied. Therefore, for the ELB to emerge it is essential that both conditions (2) and (3) are satisfied.\(^4\)

\(^3\)Note that we do not allow the planner to circumvent borrowing constraints or manipulate agents’ first order conditions. We will allow for this possibility in the context of the model applications to analyze how alternative policies, for example capital controls or targeted fiscal transfers, can help to overcome the ELB.

\(^4\)Note also that these conditions are technically local restrictions that characterize the existence of the ELB in a particular point of the state space. In principle, there could models in which these conditions are violated over part of the state space. For example, this is the case if a moderate monetary easing makes constraints become binding, but additional
We now consider how the ELB conditions simplify depending on whether we are considering closed or open economies. This is helpful to build further intuition and to place more explicit restrictions on how monetary policy should affect borrowing constraints, i.e. on the elasticity $\varepsilon^\theta_i$. For the sake of simplicity, we assume unitary elasticity of substitution across goods and intertemporally, so that $\sigma = \eta_j = 1$. Consider first a closed economy where both agents produce the same homogeneous good and face the same price $P^j = P$ and $\Pi^j = \omega^j \Pi$. Then,

**Corollary 1 (Existence conditions for the ELB in a closed economy.).** In a closed economy, the conditions for the existence of the ELB reduce to

$$
\varepsilon^\theta_i > \frac{\bar{\lambda}^B}{\bar{\lambda}^B} \\
\varepsilon^\theta_i > \left( \frac{1}{\bar{\lambda}^B} - 1 \right) \left( \frac{1}{\omega^B} - 1 \right) (1 - \delta^S) - 1
$$

where $\bar{\lambda}^B$ is the maximum fraction of consumption that can be financed by borrowing and $\omega^j = \Pi^j / \Pi$ is the share of revenues earned by agent $j$.

*Proof.* See Appendix [ ].

**Corollary 2 (Existence conditions for the ELB in a small open economy.).** TBC

*Proof.* See Appendix [ ].

### 3 The ELB and currency mismatches

The first application of our general model shows that monetary easing can have contractionary effects on output due to the presence of currency mismatches. We consider a small open economy in which households consume domestic and foreign goods, and borrow abroad through domestic financial intermediaries. Financial firms are exposed to currency mismatches since they issue foreign-currency debt $D^*_t$, while providing local-currency loans $L_t$.\(^5\) We assume that, due to a standard agency problem, creditors impose a collateral constraint on financial intermediaries which

\(^5\) Alternatively, we could assume that unhedged exposures are actually borne by domestic non-financial firms. Emerging markets firms have indeed considerably increased the issuance of dollar bonds since the global financial crisis, as for example documented in Acharya et al. (2015) and McCauley, McGuire and Sushko (2015). We prefer our interpretation based on financial intermediaries for two reasons. First, even if currency mismatches are concentrated in the non-financial corporate sector, an exchange rate depreciation tends to ultimately generate losses in the financial sector too, as firms default on their loans. Second, there is compelling empirical evidence (Caballero, Panizza and Powell, 2015; Bruno and Shin, 2015) that non-financial firms in emerging markets have behaved recently like financial intermediaries, by issuing dollar debt at low rates while holding large positions in domestically denominated financial assets.
might limit their lending ability. To streamline the presentation and focus on the key insights of the model, we will focus on the first two periods of the model, \( t = \{0, 1\} \), and assume that collateral constraints may become binding only at time 1. From time 2 onwards, we assume that the economy is at its efficient deterministic steady state. Variables without a time subscript denote steady state values, while the superscript \( * \) denotes foreign variables and prices in foreign currency.

### 3.1 Model setup

Home households choose consumption \( C_t \) and labor \( H_t \) to maximize the intertemporal utility function

\[
E_0 \left[ \ln C_0 - \frac{H_0^{1+\varphi}}{1+\varphi} + \beta_0 \left( \ln C_1 - \frac{H_1^{1+\varphi}}{1+\varphi} \right) + \frac{\beta_1}{1-\beta} \left( \ln C - \frac{H^{1+\varphi}}{1+\varphi} \right) \right]
\]

where \( \varphi > 0 \) is the inverse of the wage elasticity of labor supply and \( \beta_t \) is the intertemporal discount factor which can be time-varying. The consumption index \( C_t \) is defined as \( C_t = C_{H,t}^{-\alpha} C_{F,t}^{\alpha} \), where \( \alpha \in (0, 1) \) and \( C_{H,t} \) and \( C_{F,t} \) are consumption aggregators of domestic and foreign goods.\(^6\)

Households supply labor at the competitive wage rate \( W_t \) and own domestic firms. Therefore, by receiving labor payments and profit distributions, households appropriate all revenues from production that we denote with \( \Pi_{H,t} \). Households provide financial intermediaries at time-0 with an initial level of networth \( N_0 \) and they smooth consumption by raising local-currency loans \( L_t \) on which they pay the lending rate \( I_t^L \). The household budget constraints at \( t = \{0, 1\} \) are thus given by

\[
P_0 C_0 - \Pi_{H,0} = L_0 - N_0
\]

\[
P_1 C_1 - \Pi_{H,1} = L_1 - L_0 I_0^L
\]

where \( P_t C_t = P_{H,t} C_{H,t} + P_{F,t} C_{F,t} \) is total nominal spending and \( P_{F,t} \) and \( P_{H,t} \) are the prices of domestic and foreign goods in domestic currency.

From time 2 onwards, the economy is assumed to be in a deterministic steady state, so that total nominal spending is simply equal to \( PC = \Pi_H + (1-\beta) \left( -L_1 I_1^L + N_2 \right) \). Domestic households demand for the domestic good is given by the standard intratemporal condition \( P_{H,t} C_{H,t} = (1-\alpha) P_t C_t \), where total spending is determined by the Euler equation \( 1/P_t C_t = \beta_t I_t^L \mathbb{E}_t \left[ 1/P_{t+1} C_{t+1} \right] \). Foreign households behave symmetrically, so that their demand for Home the home good is \( P_{H,t}^* C_{H,t}^* = \alpha^* P_t^* C_t^* \), where total spending satisfies \( 1/P_t^* C_t^* = \beta_t I_t^L \mathbb{E}_t \left[ 1/P_{t+1}^* C_{t+1}^* \right] \). Note that we are allowing foreign households to borrow or save at the foreign policy rate \( I_t^* \). This is because we assume that

\(^6\)Formally, we assume that firms produce differentiated varieties of the domestic good indexed by \( j \in [0, 1] \). Therefore, the consumption aggregators for domestic and foreign goods are equal respectively to \( C_{H,t} = \left( \int_0^1 C_{H,t} (j) d j \right)^{\frac{\varepsilon}{\varepsilon-1}} \) and \( C_{H,t} = \left( \int_0^1 C_{H,t} (j) d j \right)^{\frac{\varepsilon}{\varepsilon-1}} \) where \( \varepsilon > 1 \) is the elasticity of substitution among varieties.
foreign banks are competitive and do not face binding collateral constraints, so that deposit and lending rates are simply equal to the prevailing policy rate.

The Home economy includes a continuum of monopolistically competitive firms, each producing a different variety of the domestic good, using a linear technology with total factor productivity $A_t$. As in conventional New Keynesian models, each firm faces downward sloping demand curve for its own variety and chooses prices to maximize its profits.\(^7\) For the sake of tractability, we introduce nominal rigidities only at $t = \{0, 1\}$ by assuming that all firms keep prices at a common pre-determined level $\bar{P}_H$. From time 2 onwards all prices are flexible and output is at its efficient level.\(^8\) In this first application we assume that firms price their goods in domestic currency and the law of one price holds, so that the foreign currency price of the domestic good is $P^*_H = P_H/e_t$, where $e_t$ is the nominal exchange rate, defined as the domestic price of one unit of the foreign currency. This implies that $\Pi_H = \bar{P}_HY_H$, for $t = \{0, 1\}$. The production sector in the foreign country follows a perfectly symmetric structure.

The domestic financial sector includes a measure one of competitive banks that issue foreign currency debt $D^*_t$ on which they pay the foreign monetary policy rate $I^*_t$. On the assets side, banks have domestic currency loans $L_t$ that pay the lending rate $I^*_L$, and central bank reserves $R_t$ that are remunerated at the domestic policy rate $I_t$. Banks’ balance-sheet is thus given by $N_t + e_tD^*_t = L_t + R_t$ and networth evolves according to

$$N_{t+1} = L_tI^*_L + R_tI_t - e_{t+1}D^*_tI^*_t$$

Note that an increase in the exchange rate $e_{t+1}$ denotes a depreciation of the domestic currency. This leads to an increase in the local-currency value of foreign liabilities which reduces banks’ networth. We assume that at time 1 banks face the following collateral constraint that limits lending to a certain multiple of networth

$$L_1 \leq \phi N_1$$

where $\phi \geq 1$. Banks take interest rates as given and choose assets and liabilities to maximize the present discounted value of their networth to the household. No arbitrage between central bank

\(^7\)Formally, each firm $j$ produces a different variety $Y_H(j)$ of the domestic good using the technology $Y_H = A_tH_t(j)$. The domestic and foreign demand for variety $j$ are given by the downward sloping demand curves

$$C_{H,t}(j) = \left( \frac{P_{H,t}}{P_{H,t}(j)} \right) \epsilon C_{H,t}$$

where $P_{H,t} = \left( \int_0^1 P_{H,t}(j)^{1-\epsilon} \, dj \right)^{\frac{1}{1-\epsilon}}$ and $P_{H,t}(j) = \left( \int_0^1 P_{H,t}(j)^{1-\epsilon} \, dj \right)^{\frac{1}{1-\epsilon}}$.

\(^8\)Formally, $P_H = \frac{1}{1-\tau_H}MC$, where $MC \equiv W/A$ is the marginal cost of production and $\tau_H$ is a labor subsidy set by the domestic planner to make the steady state level of output efficient.
reserves and foreign currency debt implies the conventional Uncovered Interest Parity (UIP) condition, $E_t \left[ (e_I - e_{I+1}^*) (I_{t+1} + \phi I_{t+1}) \right] = 0$, while the first order condition with respect to domestic lending implies $I_{t+1}^L \geq I_t$. If the constraint is not binding, the domestic lending rate is equal to the policy rate. If instead the constraint binds, the lending rate increases above the policy rate to ensure market clearing in the loan market. In order to simplify the algebra and abstract from the central bank balance sheet, we analyze this first model application by considering the limit for $R_t \downarrow 0$.

The model is closed by imposing market clearing for domestic goods: $Y_{H,t} = C_{H,t} + C_{H,t}^*$. In the next sections we characterize the equilibrium of the model at time 0 and 1 and study the effects of both conventional and unconventional policies. Before doing so, however, we need to model how nominal prices are determined in steady state. In order to pin down the nominal part of the economy under flexible prices, we assume the in steady state both the Home and Foreign central banks commit to constant money supply rules with anchor nominal spending, such that $PC = M$ and $P^* C^* = M^*$. Using the households steady state budget constraint and the Home goods market clearing condition, we can compute the steady state value of the nominal exchange rate: $e = \frac{\alpha M}{\alpha^* M^* - (1 - \beta) \beta^* I_1}$.

### 3.2 Model equilibrium at time 1

In this section, we solve for the model equilibrium at time 1, taking as given the amount of loans and deposits with which banks enter the period. This allows us to characterize the conditions under which monetary policy is constrained by the ELB. We first focus on the implications of conventional changes in policy rates at time 1. We then consider to what extent unconventional policy tools may relax the ELB, including using forward guidance to vary the level of steady state spending. To streamline the presentation and focus on the key insights of the model, we assume that the model is deterministic from time 1 onwards and make a few parametric restrictions. In particular, we solve the model taking the limit of $\beta \uparrow 1$. This eliminates the role of wealth effects in the determination of consumption, since households roll over loans at a zero interest rate from time 2 onwards. In Appendix [ ] we show that the results derived in this section, in particular regarding the condition for the existence of the ELB, carry forward even without these parametric restrictions.

#### 3.2.1 Monetary Policy and the ELB

Market clearing for domestic goods implies that the level of output at time 1 is equal to

$$Y_{H,1} = \frac{1}{P_H} \left( \frac{(1 - \alpha) M}{\beta^* I_1^*} + e_1 \frac{\alpha^* M^*}{\beta^* I_1^*} \right)$$

The first term in the right side parenthesis captures nominal spending by domestic households which is decreasing in the lending rate. The second term represents foreign households spending on Home goods which is increasing in the weakness of the domestic currency. Consider first how monetary easing affects output if banks are unconstrained, so that lending rates are equal to the policy rate,
In this case, a reduction in the policy rate $I_1$ stimulates output through two channels. First, it boosts spending by domestic households through a conventional intertemporal substitution effect. Second, it leads to a depreciation of the exchange rate, $e_1 = I_1 \frac{\alpha M}{\alpha M^*}$, that boosts foreign demand. Therefore, as long as banks are unconstrained monetary easing is necessarily expansionary.

Due to currency mismatches, however, the exchange rate depreciation caused by monetary easing leads to an erosion of time-1 bank networth which is given by

$$N_1 = L_0 - e_1 D_0^*$$

where $L_0 \equiv L_0 I_0$ and $D_0^* \equiv D_0 I_0^*$ are the value of loans and debt repayments, respectively, and represents the state variables of the model. This leads to a tightening of the collateral constraint (4) that becomes binding for a sufficiently low domestic policy rate. Once banks are constrained, the lending rate increases above the policy rate since it has to ensure equilibrium between loan demand and the constrained loan supply. In particular, the lending rate is given by

$$I_1^{\text{con}} = \frac{\alpha M / \beta_1}{(\phi - 1) L_0 - e_1 \left( \phi D_0^* - \frac{\alpha' M^*}{\beta_1 I_1^*} \right)}$$

where the superscript con denotes that this definition is conditional on banks being constrained. The expression above shows that, when banks are constrained, the exchange rate depreciation triggered by monetary easing may actually lead to an increase rather than a decline in lending rates. This is the case in so far as banks have large foreign currency liabilities relative to foreign demand, i.e. $\phi D_0^* > \frac{\alpha' M^*}{\beta_1 I_1^*}$. The intuition is simple: by depreciating the exchange rate, monetary accommodation reduces banks’ networth thus curbing the constrained supply of loans and requiring a commensurate increase in the lending rate to preserve market clearing.

Therefore, once banks are constrained, if currency mismatches are severe enough, monetary easing leads to a contraction in domestic spending because of the rising lending rates. This negative effect on aggregate demand has to be compared with the positive effect on foreign demand due to the exchange rate depreciation. Can the contractionary effects on domestic spending outweigh the expansionary effects from foreign spending so that monetary easing has contractionary effects on output? This is indeed possible if foreign currency debt is sufficiently high as formalized in the following proposition.

**Proposition 2 (Currency mismatches and the ELB.).** If foreign currency liabilities are sufficiently large to satisfy

$$\phi (1 - \alpha) D_0^* > \frac{\alpha' M^*}{\beta_1 I_1^*}$$

monetary policy faces an “expansionary lower bound” (ELB) on the domestic policy rate below
which monetary easing becomes contractionary. The ELB is given by

\[ I_1^{ELB} = I_1^* \frac{\alpha M}{\alpha^* M^*} \frac{\phi \underline{D}_0}{(\phi - 1) \underline{L}_0} \]  

(8)

and the maximum attainable output level is

\[ Y_{H,1}^{ELB} = \alpha^* M^* \frac{(\phi - 1) \underline{L}_0}{I_1^* \beta_1^* \phi \underline{D}_0 \alpha \bar{P}_H} \]  

(9)

Proof. To obtain condition (7), first solve for the output level when banks are constrained by replacing the exchange rate and the constrained lending rate (6) into equation (5). Then, solve for the conditions under which the derivative of output relative to the policy rate is positive. To derive the definition of the ELB (8), solve for the policy rate at which banks become constrained, by setting \( I_{1,con}^L = I_1 \). Finally, the output level at the ELB (9) is obtained by solving for equation (5) when the policy rate is equal to the ELB.

The insights of proposition 2 are illustrated in Figure 1 that shows how domestic policy rates affect output. If the domestic interest rate is sufficiently high, above the ELB \( I_1^{ELB} \), collateral constraints are not binding since banks’ networth is supported by a strong exchange rate. In this case, monetary accommodation through a reduction in the policy rate \( I_1 \) has conventional expansionary effects on output. However, if \( I_1 \) declines below \( I_1^{ELB} \), the associated depreciation of the exchange rate erodes banks’ networth so that collateral constraints become binding. If foreign currency debt is sufficiently high to satisfy condition (7), further monetary accommodation becomes contractionary since the reduction in domestic demand caused by rising lending rates outweighs the increase in foreign demand arising from the exchange rate depreciation. This also implies that monetary policy is unable to raise output above the level associated with the ELB, \( Y_{H,1}^{ELB} \).

![Figure 1: Domestic monetary policy and the ELB under currency mismatches.](image)

How should monetary policy be set in the presence of the ELB? In the current setting, where prices are rigid and there is no inflation, optimal monetary policy involves bringing output as close as possible to the efficient level. In particular, let \( Y_{H,1}^{FB} = A_1 (1 - \alpha) \frac{1}{1+\phi} \) denote the first-best
level of output and $I_{FB}^H = \frac{M}{\bar{P}_H Y_{FB}^H} \left( \frac{1 - \alpha}{\bar{P}_1^H} + \alpha \frac{\bar{P}_1^H}{\bar{P}_1} \right)$ be the interest rate that implements $Y_{FB}^H$ absent the collateral constraint. Then

**Corollary 3 (Optimal monetary policy under the ELB.).** Assume that condition (7) is satisfied, so that the model features an ELB. If $I_{ELB}^1 \leq I_{FB}^1$, the central bank ensures that output is at first best, $Y_{H,1} = Y_{FB}^H$, by setting $I_1 = I_{FB}^1$. If instead $I_{ELB}^1 > I_{FB}^1$, the central bank brings output to the maximum attainable level, $Y_{H,1} = Y_{ELB}^H$, by setting $I_1 = I_{ELB}^1$.

**Proof.** The monetary authority chooses $I_1$ to maximize household welfare subject to the households and banks’ first order conditions, budget and balance-sheet constraints, and market clearing conditions. The maximization problem is presented in Appendix [A].

It is also important to note that the ELB can act as a tighter constraint to monetary policy than the zero lower bound (ZLB). Equation (8) shows indeed that the ELB increases with the level of foreign liabilities $D^*_0$, so that

**Corollary 4 (ELB versus ZLB.).** If foreign currency liabilities are sufficiently high to satisfy

\begin{equation}
D^*_0 > \frac{L_0 (\phi - 1) \alpha^* M^*}{I_1^* \phi} \frac{\alpha M}{\alpha M}
\end{equation}

the ELB occurs at positive interest rates, i.e. $I_{ELB}^1 > 1$, and becomes a stricter constraint for monetary policy than the ZLB.

**Proof.** The condition is derived by imposing $I_{ELB}^1 > 1$ in equation (8). 

Finally, the ELB provides valuable insights for the ongoing debate on the relevance of Mundell’s trilemma for emerging markets. In the presence of currency mismatches, foreign monetary policy can indeed affect the ELB and thus influence the ability of domestic monetary policy to stabilize output in the Home country. This is formalized in the following corollary

**Corollary 5 (Foreign monetary policy and the ELB.).** An increase in the foreign policy rate rises the ELB and reduces the maximum achievable output level in the home country

\[ \frac{\partial I_{ELB}^1}{\partial I^*_1} > 0, \quad \frac{\partial Y_{ELB}^H}{\partial I^*_1} < 0 \]

**Proof.** The result is obtained by taking partial derivatives of equations (8) and (9) with respect to $I^*_1$. 

The model implications that underpin corollary 5 are illustrated in Figure 2. The left chart shows how the ability of domestic monetary policy to stimulate the economy depends on the level of the
foreign interest rate. If collateral constraints are not binding, changes in foreign monetary policy do not affect domestic output since they are offset by exchange rate movements.\footnote{As shown in Appendix [], changes in foreign monetary policy can have effects on the domestic economy even when constraints are not binding if we allow for wealth effects by not taking the limit of $\beta \uparrow 1$. However, as long as constraints do not bind, the effects on domestic output can be offset with appropriate changes of the domestic policy rate.} This is an implication of Mundell’s trilemma whereby exchange rate flexibility insulates the country from foreign monetary conditions. Note that this is true even in the presence of currency mismatches, but only as long as constraints do not bind. However, by depreciating the domestic currency, an increase in foreign policy rates leads to an erosion in banks’ networth that tightens collateral constraints and raises the ELB. Therefore, as illustrated in the right chart, if foreign policy rates increase sufficiently, the ELB becomes binding and further foreign monetary tightening pushes the domestic economy into a recession. The interplay between currency mismatches and collateral constraints can therefore generate a significant departure from Mundell’s trilemma, providing support to the idea that emerging markets may be unable to isolate themselves from US monetary conditions (Rey, 2015, 2016 and Rajan, 2015).

### 3.2.2 Policies to escape the ELB

In this section, we consider several policy tools that can potentially be used to escape the ELB. We begin by considering the scope for forward guidance that can be interpreted in the model as a commitment to change the level of steady state nominal spending $M$. Forward guidance can play an important role in overcoming the ZLB (Krugman, Dominquez and Rogoff, 1998; Svensson, 2003; Eggertsson and Woodford, 2003). To see this, note that when banks are unconstrained, domestic output is simply given by $Y_{\text{unc}}^{H,1} = M/I_1 \beta_1 \bar{P}_H$. Therefore, if the central bank cannot lower $I_1$ because of the ZLB, it can stimulate the economy by committing to higher future price level. Is forward guidance effective also against the ELB? The answer is no. Equation (8) shows that the ELB moves proportionally with $M$, so that the central bank can in principle lower the ELB by committing to a tighter future monetary stance that reduces $M$. This generates an appreciation of the domestic exchange rate that relaxes banks’ collateral constraint and allows the central bank to reduce time-
policy rates. However, the overall effect on output is null, as can be seen by the fact that the output level at the ELB in equation (9) is not a function of $M$. Intuitively, this is because the central bank can lower the ELB and time-1 interest rates only by committing to a tighter future monetary stance. Since agents behave in a forward looking manner, this has no effect on aggregate demand. Then, why is forward guidance effective in dealing with the ZLB, but not with the ELB? The reason is that the ELB is an endogenous bound that depends on both the current and future stance of monetary policy.

A policy tool that is instead quite effective in overcoming the ELB is the recapitalization of the banking sector, as also analyzed in Kollmann, Roeger and in’t Veld (2012) and Sandri and Valencia (2013). Assume that the recapitalization involves lump sum transfers from households to banks, so that it can be interpreted as an increase in the amount of loan repayments, i.e. an increase in $L_0$. Then, equations (8) and (9) show that an increase in $L_0$ lowers the ELB and increases the maximum attainable level of nominal spending. The intuition is straightforward: the recapitalization of the banking sector relaxes collateral constraints, thus reducing lending rates and stimulating domestic demand. However, using bank recapitalizations to overcome the ELB can entail various costs that are absent from the model. First, rather than using lump sum transfers, policy makers have to finance recapitalizations through distortionary taxation. Second, recapitalizations can involve substantive moral hazard costs. Third, the presence of currency mismatches and collateral constraints is not limited to banks. Households and firms can themselves hold unhedged currency positions in which case recapitalizations become much harder to implement.

Finally, we consider the role of capital controls that can be used to de-link the exchange rate from domestic monetary conditions. In particular, the government can stimulate capital inflows and support the domestic exchange rate by providing banks with a subsidy $\tau^{cc}_1$ on foreign currency debt. This places a wedge in the UIP condition, $e_1 = e(1 - \tau^{cc}_1)I^*_1/I_1$, that supports the exchange rate, relaxes the ELB, and allows for greater monetary stimulus. Policy makers may also try to support the exchange rate while pursuing domestic monetary accommodation by using foreign exchange intervention. This involves selling international reserves to stem the depreciation pressures arising from lower domestic rates. To the extent that foreign exchange intervention is effective because of market frictions, it operates very similarly to capital controls by essentially placing a similar wedge in the UIP condition as for example discussed in Gabaix and Maggiore (2015) and Cavallino (2016).

### 3.3 Model equilibrium at time 0

We now analyze the model equilibrium from the perspective of time 0. This allows us to show how the possibility of the ELB becoming binding in the future has important consequences for monetary

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10 We could also consider credit easing policies, whereby the government provides lending subsidies or try to operate itself as a financial intermediary as in Gertler and Karadi (2011), Gertler, Kiyotaki and Queralto (2012) and Negro et al. (2011). These measures would also help to relax lending constraints and stimulate aggregate demand.
The equilibrium levels of foreign currency debt and domestic loans carried into period 1 are equal to

\[ D_0^* = \delta \frac{\alpha^* M^*}{E_0[I_1]} \]  
\[ L_0 = N_0 I_0 + \delta \frac{\alpha M}{E_0[I_1]} \]  

where \( \delta \equiv \frac{1}{\beta_0 \beta_1} - \frac{1}{\beta_0^* \beta_1^*} \). We assume that \( \delta \) is large enough such that \( D_0^* \) satisfies condition (7) for any \( I_1^* \). This implies the existence of an ELB at time 1 and that domestic monetary policy behaves according to corollary 3. Note that we allow the domestic and foreign interest rates at time 1 to be stochastic, as central banks attempts to keep output at the efficient level in response to TFP shocks.

Taking into account the endogenous levels of \( D_0^* \) and \( L_0 \), the ELB can be expressed as

\[ I_{ELB}^1 = \phi \frac{\delta \alpha M I_0 I_1 / E_0[I_1]}{\phi - 1 N_0 I_0 + \delta \alpha M / E_0[I_1]} \]  

This equation shows a first important interaction between monetary policy at time 0 and the ELB. Lower policy rates \( I_0 \) reduce the returns on bank networth so that banks enter period 1 with less capital. This tightens collateral constraints and leads to an increase in the ELB.

Regarding time-0 output, this is given by

\[ Y_{H,0} = \frac{(1 - \alpha) M / \beta_0}{\beta_0^* \beta_1 I_0 E_0[I_1]} + e_0 \frac{\alpha^* M^* / \beta_0^*}{\beta_0^* \beta_1^* I_0 E_0[I_1]} \]

where the exchange rate is \( e_0 = e_{I_0^* E_0[I_1]} / (I_0 E_0[I_1]) \). This equation reveals that if the ELB at time 1 is binding in at least some states of the world, monetary accommodation becomes less effective in stimulating time-0 output. To see this, note that a reduction in \( I_0 \) determines an increase in the time-1 ELB as shown in equation 13. If the ELB is binding with positive probability \( \rho > 0 \), this determines an increase in \( E_0[I_1] \) which in turn reduces the stimulative impact on time-0 output. In fact, if the ELB is binding for sure at time 1, \( \rho = 1 \), monetary policy becomes completely ineffective in stimulating time-0 output, since any reduction in \( I_0 \) is offset by a proportional increase in \( I_{ELB}^1 \).

These rich interactions between policy rates at time 0 and the ELB have important implications for the optimal conduct of monetary policy at time 0. In choosing \( I_0 \), the central banks has indeed to trade off the effects on time-0 output with the implications for the ELB and time-1 output. Therefore, optimal policy involves setting rates above the level consistent with the efficient level of output \( Y_{H,0}^{FB} = A_0 (1 - \alpha) \frac{1}{\beta_0^*} \) in order to reduce the ELB and raise output at time 1. In other words, the central bank tolerates a negative output gap at time 0 to raise output at time 1 in case the ELB becomes binding.

The model equilibrium from time 0 is also helpful to revisit the implications of foreign monetary
conditions for the ELB taking into account anticipatory effects. From the perspective of time 1, Corollary 5 showed that a reduction in foreign policy rates could lower the ELB and allow the domestic economy to achieve higher output. However, if this reduction is expected, it can become much less effective in relaxing the ELB. The reason is that the expectation of lower foreign policy rates leads to a higher accumulation of foreign currency debt, as shown in equation (11). This provides an interesting perspective on the ongoing debate regarding the impact of US monetary policy on emerging markets. The model supports the recent concerns that emerging markets may be unable to insulate themselves from US monetary conditions, even if they have flexible exchange rates. However, it also shows that any commitment by the US to refrain from policy rate changes that can destabilize emerging markers would be partially undone by endogenous changes in foreign currency borrowing.

We conclude the analysis by considering the scope for macro-prudential capital controls that can be put in place in anticipation of the ELB becoming binding. As shown in appendix [], by taxing capital inflows at time 0, policy makers can effectively reduce the amount of foreign currency debt carried into period 1. This lowers the time-1 ELB, $I_{ELB}^1$, and allows for a higher level of output. Therefore, the model provides additional support for the use of macro-prudential capital controls, that have been so far justified in the literature because of the ZLB and exchange rate rigidities (Farhi and Werning, 2016; Korinek and Simsek, 2016) or because of pecuniary externalities in the context of real models (Jeanne and Korinek, 2010; Bianchi, 2011; Korinek and Sandri, 2016).

4 The ELB and carry traders

In this section, we consider a second application of our general model in which the ELB can arise because of carry trade flows. The key idea is that monetary easing can trigger capital outflows which can in turn lead to a domestic credit crunch. We develop the analysis using a small open economy model similar to the one presented in the previous section. However, this application features heterogeneity also between domestic savers and borrowers. Furthermore, domestic banks are no longer exposed to currency mismatches: they collect domestic deposits, provide loans, and invest in government bonds, all denominated in domestic currency. The key feature of the model is that government bonds are also purchased by foreign investors according to an unhedged carry-trade strategy: their demand for bonds is increasing in the spread between the domestic and foreign interest rate. As in the previous section, we assume that banks face collateral constraints at time 1 and that from time 2 onwards the economy is in steady state.

4.1 Model setup

The household sector mimics the one presented in section 3, but now there are two types of domestic agents: borrowers and savers. Borrowers receive a fraction $\omega_b$ of the total value of domestic
production $\Pi_{H,t}$ and pay a fraction $\eta_t$ of the aggregate tax bill $T_t$. Their time-1 budget constraint and Euler equation are equal to

$$P_1C_1^B = \Pi_{H,1}^B + L_1 - L_0 - T_1^B$$
$$P_1C_1^B = \frac{PC^B}{\beta_1 I_1^D}$$

where $\Pi_{H,1}^B = \omega_1 \Pi_{H,1}$, $T_1^B = \eta_1 T_1$, $L_0$ are loan repayments at the beginning of the period, and $L_1$ are new loans whose interest rate is $I_1^D$. The budget constraint and Euler equation of savers are instead

$$P_1C_1^S + D_1 = \Pi_{H,1}^S + D_0 - T_1^S$$
$$P_1C_1^S = \frac{PC^S}{\beta_1 I_1^D}$$

where $D_0$ and $D_1$ denote deposits respectively at the beginning and end of the period, $I_1^D$ is the deposit rate, $\Pi_{H,1}^S = (1 - \omega_1) \Pi_{H,1}$, and $T_1^S = (1 - \eta_1) T_1$. As in the previous application, we neglect wealth effect by considering the model equilibrium under the limit for $\beta \uparrow 1$. Therefore, in steady state nominal spending by each agent is simply equal to their nominal income, so that $PC^B = \omega M$ and $PC^S = (1 - \omega) M$.

The production sector is identical to the one presented in section 3 with one exception. In the previous application, we assumed producer-currency pricing so that the export price of the Home good was denominated in the domestic currency. Under that assumption, monetary easing boosts the foreign demand for the Home good since, by depreciating the exchange rate, it lowers the foreign currency price. In this application we are instead interested in the effects of monetary easing on domestic demand, taking into account the impact on both domestic borrowers and savers. We thus assume that producers set prices in the local currency where the good is sold, so that the export price of the Home good is rigid in foreign currency at the level $\bar{P}_{H}^*$. The value of domestic production can then be expressed as $\Pi_{H,j} = \bar{P}_{H} \left( C_{H,j}^B + C_{H,j}^S \right) + \epsilon_t \bar{P}_{H}^* C_{H,j}^S$.

The domestic financial sector includes a continuum of competitive banks that collect domestic deposits $D_t$ and provide domestic-currency loans $L_t$. Furthermore, banks buy domestic government bonds $B_t$ whose gross return is $I_t^B$, and hold central bank reserves $R_t$ that are remunerated at the domestic policy rate $I_t$. Therefore, the banks’ balance-sheet constraints and networth evolution are given by

$$N_t = L_t + B_t + R_t - D_t$$
$$N_{t+1} = L_t^D + B_t^D + R_t I_t - D_t I_t^D$$
At time 1, banks face a collateral constraint that limits lending and bond holdings to a multiple of net worth

\[ L_1 + \xi B_1 \leq \phi N_1 \]  

(14)

where \( N_1 = L_0 + B_0 + R_0 - D_0 \) and \( \xi \in (0, 1) \) captures the regulatory or market-based capital requirements against the holdings of government bonds. No arbitrage between central bank reserves and deposits requires \( I_D^t = I_1^t \), while the first order conditions with respect to loans and domestic government bonds imply

\[ I_L^t \geq I_t \]

\[ I_B^t = \xi I_L^t + (1 - \xi) I_t \]

When the borrowing constraint is not binding, the domestic lending rate and the bond return are equal to the policy rate. If instead the constraint binds, the lending rate increases above the policy rate to ensure market clearing in the loan market. As a result, the bond rate must increase as well in proportion to the capital requirement \( \xi \).

Government bonds are also purchased by foreign investors that follow an unhedged carry-trade strategy. Their demand for domestic government bonds is increasing in the differential between the yields on domestic bonds and the foreign policy rate

\[ B_F^t = \propto (I_B^t - I^*_t) \]

where \( \frac{\partial B_f^t}{\partial I_B^t} > 0 \) and \( \frac{\partial B_f^t}{\partial I^*_t} < 0 \). As explained in the next section, this demand is consistent with the notion that foreign investors increase their holdings of domestic government bonds when their excess return relative to foreign assets rises.

Finally, the model features a public sector that includes the government and the central bank. The central bank collects reserve deposits from private banks and uses them to buy government bonds, so that \( R_t = B_{CB}^t \), where \( B_{CB}^t \) denotes the holdings of government bonds by the central bank. We assume that the central bank has no capital and that all profits are rebated lump-sum to the government. The consolidated budget constraint of the public sector is thus given by

\[ B_{i-1}^G I_{i-1}^B = B_i^G + B_{CB}^t (I_{i-1}^B - I_{i-1}) + T_i \]

We close the model by imposing conventional market clearing conditions for domestic goods and government bonds

\[ Y_{H,t} = C_{H,t}^B + C_{H,t}^S + C_{H,t}^* \]

\[ B_i^G = B_i + B_i^F + B_i^{CB} \]
4.2 Model equilibrium

To solve the model, we need to specify a functional form for the demand of government bonds by foreigners. In order to obtain analytical solutions, we set

\[ B^F_t = B^F_t + \gamma \left( \frac{1}{I^*_t} - \frac{1}{\tilde{I}^B_t} \right) \]

where \( B^F_t \) captures a component of foreign demand that may depend on variables other than the interest rate differential, and \( \tilde{I}^B_t \) is the harmonic mean of the lending and policy rate, \( 1/\tilde{I}^B_t = (\xi / I^L_t + (1 - \xi) / I_t) \).\(^{11}\) Note that to solve the model analytically, \( \tilde{I}^B_t \) differs slightly from the exact definition of the government bond yield, \( I^B_t = \xi I^L_t + (1 - \xi) I_t \). However, this does not alter the key implications of the model and in particular the potential for the ELB to arise.

We characterize the model equilibrium at time 1 starting with the behavior of the exchange rate. This warrants some discussion since the assumption that foreigner investors follow a carry-trade strategy departs from conventional models based on the UIP condition. Under UIP, the exchange rate is pinned down by a no-arbitrage condition between domestic and foreign bonds and foreign investors are willing to absorb any net supply of domestic bonds. In our model foreigners are instead willing to hold a finite amount of government bonds which is pinned down by the gap between the domestic and foreign interest rate. In turn, the exchange rate has to ensure that the net supply of domestic bonds is consistent with the amount that foreigners are willing to purchase. The exchange rate performs this role by affecting the value of production in domestic currency. Specifically, a depreciation of the exchange rate increases the domestic currency value of national income and thus allows the country to absorb a higher amount of government debt.

The exchange rate responds to monetary policy in a conventional manner. A monetary easing that reduces the yield on domestic government bonds \( \tilde{I}^B_t \) leads to a depreciation of the exchange rate. The exchange rate depreciates also in response to an increase in the foreign interest rate \( I^*_t \). This is because a reduction in \( \tilde{I}^B_t \) or increase in \( I^*_t \) trigger capital outflows, as foreigners reduce their holdings of domestic bonds. To restore equilibrium in the bond market, the exchange rate has to depreciate, thus raising the local currency value of domestic income and reducing the net supply of bonds abroad. The fact that the exchange rate depreciates when \( \tilde{I}^B_t \) declines may raise the concern that in the model carry traders reduce their holdings of domestic bonds when the exchange-rate-adjusted excess return over foreign bonds increase. This would occur if the exchange rate depreciation is stronger than the reduction in \( I^*_t \), so that the foreign-currency return on domestic bonds \( \tilde{I}^B_t e_t / e \) increases. However, under a broad range of parameters, the model predicts that excess returns on domestic bonds co-move with the yield \( \tilde{I}^B_t \), thus validating a carry-trade strategy based on the interest rate differential.

\(^{11}\)For the sake of simplicity, we treat \( B^F_t \) as constant in this section. However, the ELB can arise even if \( B^F_t \) is a function of the exchange-rate-adjusted excess return of domestic relative to foreign bonds, as in Gabaix and Maggiori (2015).
Regarding the equilibrium level of output, this can be expressed as

\[ Y_{H,1} = \left( \frac{\omega}{\bar{I}_1} + \frac{1 - \omega}{\bar{I}_1} \right) \frac{1 - \alpha}{\bar{PH}_1} + \frac{\alpha^*}{\bar{PH}_1 \bar{I}_1^*} \]  

(15)

where we simply set \( M = M^* = 1 \). The first and second additive terms on the right side of the equation capture the consumption of Home goods by domestic and foreign households, respectively. Because of the assumption of local-currency pricing, monetary policy has no effect on foreign demand. Therefore, the impact on output depends only on the effect on domestic demand. Monetary easing unequivocally boosts the demand of domestic savers since deposit rates decline in line with the policy rate. The effect on borrowers is instead not clear cut since it depends on the effect that monetary easing has on the lending rate. For monetary easing to become contractionary, it should then raise the lending rate enough to generate a contraction in borrowers’ demand that outweighs the increase in savers’ demand.

If borrowing constraints are not binding, the lending rate is equal to the policy rate and monetary easing is necessarily expansionary. However, a reduction in the domestic policy rate lowers foreign demand for government bonds and implies that domestic banks have to absorb more government debt. This brings banks closer to their borrowing constraint (14), provided that monetary easing does not cause at the same time a strong contraction in borrowing demand. The latter can occur if borrowers’ income raises sharply. To rule out this possibility and simplify the analytical solutions, we assume that borrowers do not earn any income at time 1 by setting \( \omega_1 = 0 \). Therefore, a sufficiently large monetary accommodation pushes banks again their borrowing limit.

Once banks are constrained, further monetary easing forces them to curtail domestic lending in order to absorb the government bonds offloaded by carry traders. Note that this is the case as long as \( \xi < 1 \): banks tilt their portfolio towards less capital-expensive government bonds, as bond yields keep declining with monetary accommodation while lending rates increase.\(^{12}\) When banks are constrained the lending rate can be expressed as

\[
\frac{1}{\bar{I}^{L,con}_1} = \Theta_1 + \xi \left( \frac{\bar{R}_1 + \bar{B}^F}{\bar{I}_1} + \gamma \left( \frac{1 - \xi}{\bar{I}_1} \right) \right) - (\xi - \eta_1) \bar{B}^G_1
\]

where \( \Theta = \phi \bar{N}_1 - \bar{I}_0 - \eta \bar{B}^G_0 \) is a function of state variables. This expression shows that a reduction in the policy rate leads to an increase in the lending rate. The strength of this effect depends on the sensitivity \( \gamma \) of carry-trade flows to interest rate changes. If lending rates increase sufficiently with monetary accommodation, the contractionary impact on borrowers’ demand outweighs the increase in demand by savers. This leads to an overall reduction in output, thus giving rise the ELB as

\(^{12}\)In instead \( \xi = 1 \), yields on government bonds and lending rates are equalized. The equilibrium implies that once banks become constrained, further monetary easing has no effect on lending and bond rates, leaving foreign holdings of government bonds and domestic lending unchanged.
formalized in the following proposition

**Proposition 3 (Carry-trade capital flows and the ELB).** If the sensitivity of foreign investors’ demand for domestic government bond is high enough to satisfy

\[
\gamma > \frac{\omega (1 - \omega)}{\xi (\omega - \xi)} \beta_1
\]

(16)

with \( \omega > \xi \), then monetary policy faces an “expansionary lower bound” (ELB). The ELB is given by

\[
\frac{1}{I_{ELB}^1} = \frac{\Theta_1 + \xi \left( R_1 + B_{1c}^F + \frac{\gamma}{P_t} \right) - (\xi - \eta_1) B_{1c}^F}{\omega + P_t + \xi \gamma} \frac{\omega}{\eta_1} + \xi \gamma \beta_1
\]

(17)

and the maximum attainable output level is

\[
Y_{H,1}^{ELB} = \frac{1 - \alpha}{P_t \beta_1} \Theta_1 + \xi \left( R_1 + B_{1c}^F + \frac{\gamma}{P_t} \right) - (\xi - \eta_1) B_{1c}^F + \frac{1}{I_{ELB}^1} \left( \frac{1 - \alpha}{\omega + P_t + \xi \gamma} + \frac{\alpha^*}{\bar{P} \beta_1} \right)
\]

(18)

**Proof.** TBC

The intuition is simple: the reduction in the policy rate triggers a reversal in the carry trade positions of foreign investors that cut their holdings of government bonds. Domestic banks must employ their limited capital to absorb the excess supply of government debt. In order to do so they are forced to reduce lending to households, therefore causing a reduction in the consumption of borrowers. If the capital outflow is large enough, i.e. the sensitivity of foreign investors demand to interest rate changes is high enough, the contractionary effect of an interest rate cut on borrowers demand outweighs its expansionary effect on savers. Note that the presence of the ELB hinges upon condition (16) alone. This restriction ensures that monetary easing becomes contractionary once banks are constrained, thus satisfying the second condition for the existence of the ELB as in equation (3). As in the case of the first application, this also implies that monetary easing brings unconstrained banks closer to their borrowing constraints, as required by the first existence condition (2).

As in the previous application, optimal monetary policy in the presence of an ELB takes a conveniently simple form. If the sensitivity of foreign investors demand is high enough to satisfy condition 16, the central bank sets \( I_1 = I_1^{FB} \) if \( I_1^{FB} \geq I_1^{ELB} \) and \( I_1 = I_1^{ELB} \) otherwise. The interest rate that implements the first-best level of output is given by

\[
I_1^{FB} = \frac{(1 - \alpha)/(\bar{P} \beta_1)}{P_t \beta_1^*}.
\]

Similarly, we can derive the condition under which the ELB is tighter than the ZLB. This happens if the sensitivity of
foreign investors demand exceeds the following threshold

\[ \gamma > \frac{I^{\ast}}{I^{\ast} - 1} \left( \frac{\Theta_1 + \xi (R_1 + \bar{R}_1^2) - (\xi - \eta_1) B_1^{\xi}}{\xi} - \frac{\omega}{\xi \beta_1} \right) \]

provided that \( I^{\ast} > 1 \).

We conclude this section by analyzing the effects of foreign monetary policy. Equation 15 reveals that an increase in the foreign interest rate contracts foreign demand. This channel, absent in the first application, is due to our assumption of local-currency pricing. In principle, the central bank could compensate the loss in foreign demand by lowering the policy rate and stimulating domestic spending. However, if condition (16) is satisfied, the central bank may end up being constrained by the ELB. This concern is particularly severe become a foreign monetary tightening not only depresses foreign demand for the Home good, but it also raises the ELB as carry traders pull out of the country.

### 4.2.1 Policies to escape the ELB and time-0 equilibrium

The model provides interesting insights about the impact of fiscal policy on the ELB. As we have seen, once constraints become binding, private lending is crowded out by the need to absorb the government bonds sold by foreign investors. This seems to suggest a clear role for fiscal consolidation that by reducing the stock of government debt can free up financial resources for lending. However, the model shows that the effects of fiscal consolidation crucially depend on the implementation details. A reduction in the level of government bonds is able to lower the ELB and allow for a higher level of output only if it does not impose an excessive tax burden on domestic borrowers.

This can be seen by observing in equation 17 that the ELB declines with a reduction of government debt only if \( \xi > \eta_1 \). This requires that the extra lending obtained by removing government debt from the banks’ balance-sheets should exceed the increase in the tax burden imposed on borrowers.

The model incorporates also an important role for quantitative easing in relaxing the ELB. Equations 17 and 15 show that monetary authorities can lower the ELB and raise output by purchasing government bonds from banks in exchange for central bank reserves. This operation relaxes collateral constraints and allows banks to increase domestic lending. Interestingly, part of the gains from quantitative easing are eroded by the actions of carry traders. By lowering yields on government bonds, quantitative easing leads indeed to stronger capital outflows as foreign investors further reduce their holdings of domestic debt.

As in the first application, we now characterize the time-0 equilibrium, focusing on the interplay between ex-ante monetary policy and the potential of a future ELB. Agents enter period 0 with no loans and deposits and the government has no outstanding debt. We assume that borrowers have no income, \( \omega_0 = 0 \), and capitalize banks with an equity injections equal to \( N_0 \). Therefore borrowers
enter period 1 with loan repayment obligations given by

$$L_0 = \frac{\omega M}{\beta_0 \beta_1 \mathbb{E}_0 [I_1]} + I_0 (N_0 - \eta_0 B_G^0)$$

Using the definition above and considering that $N_1 = I_0 N_0$, we can re-write the time-1 ELB as follows

$$\frac{1}{T^{ELB}_1} = \frac{1}{T_{1}^{ELB}} \left( I_0 (N_0 (\phi - 1) + B_G^0 (\eta_0 - \eta_1)) \right) - \frac{\omega}{\beta_0 \beta_1 \mathbb{E}_0 [I_1]} + \xi \left( R_1 + \tilde{B}_1^C + \gamma \right) - (\xi - \eta_1) B_G^1$$

This expression is helpful to verify whether, similarly to the first application, a time-0 monetary tightening can lower a future ELB. We still observe that an increase in $I_0$ is able to lower the ELB since it increases the return on bank network and thus $N_1$. However, $I_0$ now impacts the ELB also as a function of the stock of government debt $B_G^0$ and the distributional effects of fiscal policy. Assume that the government issues debt at time 0, $B_G^0 > 0$, to provide domestic transfers and raises taxes at time 1 to meet debt repayments. A time-0 monetary tightening contributes to lower the ELB through the effects on government debt only if borrowers receive proportionally more government transfers the time-0 than they are taxed at time-1, i.e. if $\eta_0 > \eta_1$. If instead fiscal policy is tilted in favor of savers, an increase in $I_0$ raises the ELB by increasing the repayment burden imposed on borrowers.

Equation (19) shows also an additional effect that dampens the effect of $I_0$ on a future ELB. Assume that $\eta_0 \geq \eta_1$ so that an increase in $I_0$ lowers the ELB and thus the expected future lending rate $\mathbb{E}_0 [I_1]$. The latter effect provides an incentive for borrowers to raise $L_0$ which tends to increase the ELB, thus weakening the beneficial effects from monetary tightening. To the extent that the ELB declines with $I_0$, the model implies that monetary policy at time 0 becomes less effective in affecting time-0 output $Y_{H,0}$. The reason is analogous to the first application. Monetary easing at time-0 raises the ELB and thus the expected future lending rate. In turn, this weakens the overall stimulative effect borrowers’ demand.

5 The ELB and bank profitability

In this section, we develop a third version of the model in which monetary easing can become contractionary because of its effect on bank profitability. The key feature of the model that can give rise to the ELB is that deposit rates cannot decline below a certain level. This is consistent with the recent experience with negative interest rate policies in several advanced economies where banks were reluctant to pass negative rates to depositors. We develop the analysis using a closed economy
model that features domestic heterogeneity between borrowers and savers, similar to the one presented in section 4. As in previous applications, we assume that banks face collateral constraints at time 1 and that from time 2 onwards the economy is in steady state.

5.1 Model setup

The production and household sector are identical to the ones presented in section 4, with the exception that the economy is closed. Therefore, households consume only domestic goods, \( C_i^t = C_{H,t}^i \), and all production is sold domestically, \( Y_{H,t} = C_{H,t}^H + C_{H,t}^S \). The financial sector is made of a continuum of banks whose networth and balance sheets are defined as in section 4. Since government bonds play no particular role in this application, we consider the model solution taking the limit of \( B^G_t \) going to zero. Banks face a collateral constraint that depends not only on current networth, but also on profits \( \Upsilon_1 \) net of dividend payments \( \Delta_1 \)

\[
L_1 \leq \phi^N N_1 + \phi^T (\Upsilon_1 - \Delta_1) \tag{20}
\]

where profits are defined as \( \Upsilon_1 = L_1 I_1^L - D_1 I_1^D - N_1 \). Regarding dividends, banks remunerate shareholders by paying an equity premium \( \nu \) on top of the deposit rate, so that \( \Delta_1 = N_1 (I_1^D + \nu - 1) \). As we shall see, the higher is the equity premium, the stricter the ELB becomes.

The model allows for the possibility that banks have market power in period 1. When collateral constraints are not binding, the lending rate includes indeed a spread \( \varepsilon^L \geq 0 \) on top of the policy rate

\[
I_1^L \geq I_1 + \varepsilon^L \tag{21}
\]

where the inequality sign applies in the case in which constraints are binding. Similarly, banks try to set deposit rates below the policy rate by the margin \( \varepsilon^D \geq 0 \). However, deposit rates cannot decline below a certain floor \( I_1^D \) so that

\[
I_1^D = \text{Max} [I_1 - \varepsilon^D, I_1^D] \tag{22}
\]

Note that the assumption that spreads are additive with respect to the policy rate eliminates the channel through which monetary policy affects bank profitability in Brunnermeier and Koby (2016). In their paper, spreads are multiplicative in which case monetary easing leads to an immediate reduction in intermediation margins. Using additive spreads, we turn off this channel and focus instead on how profitability is impacted by the floor on deposit rates.

5.2 Model equilibrium at time 1

Time-1 output is given by
\[ Y_{H,1} = \left( \frac{\omega}{\beta_1} + \frac{1 - \omega}{\beta_1} P_{H} \right) M \]  

(23)

where the first and second terms in parenthesis capture spending by borrowers and savers, respectively. If collateral constraints are not binding, monetary easing is necessarily expansionary since it involves a reduction in lending rates that stimulating spending by borrowers. Furthermore, if deposit rates are above their floor \( I_{D1} \), monetary easing leads also to a reduction in deposit rates that boosts spending by savers too.

Regarding the impact on bank profitability, note that we can write profits net of dividend payments as

\[ Y_1 - \Delta_1 = L_1 (I_{L1} - I_{D1}) - \nu N_1 \]

If banks are unconstrained and deposit rates are above their floor, net profits tend to increase with monetary easing. Specifically, this is the case as long as banks have some degree of market power and monetary accommodation increases aggregate lending. However, once the deposit rate reaches its lower bound \( I_{D1} \), further monetary accommodation harms bank profitability since, by lowering lending rates, it compresses intermediation margins. The erosion in profitability can in turn push banks against their collateral constraint at which point conventional monetary easing becomes ineffective in stimulating aggregate demand. This is because collateral constraints prevent any pass-through to lending rates, thus giving rise to an ELB as summarized in the following proposition

**Proposition 4 (The ELB and bank profitability).** Assume that the deposit rate cannot decline below a certain lower bound \( I_{D1} \) so that

\[ I_{D1} \geq I_{ELB1} \]

then monetary policy faces an ELB given by

\[ (\phi^N - \nu \phi^\Pi) N_1 = \left( L_0 + \left( \frac{\omega (1 - \omega)}{I_{ELB1} + \epsilon^I} - (1 - \omega) \omega \right) M \right) (1 - \phi^\Pi (I_{ELB1} + \epsilon^I - 1)) \]  

(25)

provided that the lower bound on deposit rates is binding, i.e. that \( I_{ELB1}^D \leq I_{D1} + \epsilon^D \)

**Proof.** See Appendix

Note that if banks have no market power in the deposit market, \( \epsilon^D = 0 \), and the lower bound on deposit rates is zero, so that \( I_{D1} = 1 \), then the ELB can arise only at negative policy rates. The ELB thus provides the true limit to monetary stimulus in the case of negative interest rate policies. However, the ELB can arise also at positive policy rates. This is the case if banks have market power in the deposit market so that the lower bound on deposit rates can become binding even at positive policy rates.
Regarding policies to escape the ELB, this model application provides important insights about the role of competition policy. Measure to foster competition in deposit markets can indeed be helpful to reduce the ELB, as they reduce deposit spreads and thus the policy rate at which the lower bound on deposit becomes binding. On the contrary, stronger competition in lending markets can rise the ELB since it reduces bank profitability. The model provides also an important role for policy intervention aimed at restricting dividend payments, captured by a reduction in $v$. Retained earnings can indeed increase future bank capital and relax borrowing constraints.

6 Conclusion

TBC
References


