Income Inequality, Financial Crises and Monetary Policy*

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Abstract

We construct a general equilibrium model in which income inequality results in insufficient aggregate demand and deflation pressure by allocating a greater share of national income to a group with the least marginal propensity to consume, and if excessive, can lead to an endogenous financial crisis. The effectiveness of monetary policy during financial crises is severely distorted by the zero lower bound (ZLB) constraint. Such an economy generates left-skewed distributions for equilibrium prices and quantities, creating disproportionately large downside risks. Consequently, symmetric monetary policy rules that are designed to minimize the fluctuations in equilibrium quantities and prices around fixed means become inefficient. We find that a mix of lenient policy against inflation during normal times and forward guidance of promising a prolonged period of low interest rate during financial crises can be effective in correcting the deflation bias caused by the skewed distributions. While we assume no direct preferences of central bankers over income inequality, monetary policy rules correcting the skewed distributions also lessen the degree of income inequality substantially as low income households suffer the most from the asymmetric macroeconomic risks.

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1 Introduction

Income inequality plays an important role in macroeconomic dynamics. As shown by the compelling evidence collected by Jappelli and Pistaferri (2014), there exists vast heterogeneity of Marginal Propensity to Consume, MPC henceforth. According to their findings, the average MPC of the most affluent income group is substantially lower than the MPCs of lower income groups. In fact, a half of the most affluent income group has the MPC out of transitory income equal to zero.[1] If the economy allocates a growing share of national income to a group with the lowest MPC, this can produce insufficient aggregate demand. In fact, Summers (2015), in his attempt to revive the underconsumption theory of Hansen (1939), points out income inequality as one of the most important factors driving secular stagnation. In fact, the concern for the link between income inequality and underconsumption dates back to almost 100 years ago:

> "...society was so framed as to throw a great part of the increased income into the control of the class least likely to consume it. The new rich...preferred the power which investment gave them to the pleasures of immediate consumption...And so the cake increased; but to what end was not clearly contemplated... – the virtue of the cake was that it was never to be consumed, either by you nor by your children after you." – Keynes (1919)

Another important macroeconomic consequence of income inequality is the deflation pressure. As is well known, labor income share of the U.S. economy has steadily declined over the last four decades as real wage growth has fallen behind labor productivity growth for long time.[2] Labor income share is correlated with the income share of the bottom 95 percent income groups. As such, the decline of labor income share is another indicator of growing income inequality. Labor income share is also a good indicator of real marginal cost, and as such it predicts inflation dynamics very well as shown by Sbordone (2002) and Woodford (2001). Following the New Keynesian literature, current inflation is simply the present value of future marginal costs. Thus, if the current trend decline of labor income share continues, it might be much more challenging to achieve the inflation target, which is a grave concern for central bankers.

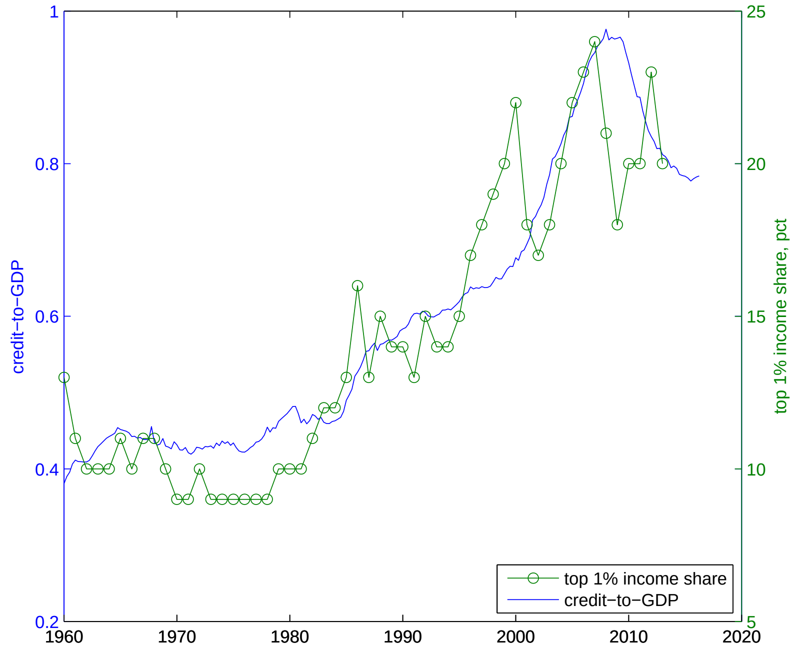
Perhaps most importantly, income inequality plays a more important role for the overall stability of financial system. When a growing share of national income is allocated to the groups with the least MPCs, unused income should be stored in financial claims. The financial claims may represent investment in real assets such as physical capital. If this were the case, the economy would grow faster through capital deepening and more job creation. However, there is no evidence supporting such a hypothesis. This means that the bulk of this saving flows into the borrowing of lower income groups.

Figure 1 suggests a close link between income inequality and credit-to-GDP ratio, which is considered a good indicator of excessive credit. The blue solid line shows the evolution of the household

[1] We are assuming that there is a high correlation between top 1 percent income earners in Atkinson, Piketty, and Saez (2011) and the last percentile of cash-on-hand distribution in Jappelli and Pistaferri (2014).

[2] In this regard, Solow (2015) points out the declining bargaining power of U.S. workers since 1980s as the origin of the decline in the labor income share.

Figure 1: Credit and Income Inequality



Sources: (a) Financial Accounts of the United States, Federal Reserve System; (b) Economic Policy Institute.

sector credit-to-GDP ratio since 1960. The green-circled line shows the evolution of the income share of the top 1 percent earners based upon the data collected by Atkinson, Piketty, and Saez (2011). During 1960s and 1970s, both time series moved more or less sideways. Since the early 1980s, however, both series continued to rise over more than three decades. In 2007, on the eve of the Global Financial Crisis, the credit-to-GDP ratio reached an unprecedented level and the top 1 percent income share reached 24 percent, a level unseen since 1928, which was also on the eve of the Great Depression.

Such credit expansion maintains aggregate demand for a while by allowing lower income households to finance their spending. It is in this sense in which Summers (2015) argues that the economy can maintain the capacity level of output only with some sorts of financial froths. Many researchers and policy institutions, however, view the credit growth in late 1990s and early 2000s as "excessive" or "unsustainable" in the sense that it indicates a substantial degree of vulnerability of the financial system. For instance, Drehmann, Borio, Gambacorta, Jimenez, and Trucharte (2010), Drehmann, Borio, and Tsatsaronis (2011) and Adrian, Covitz, and Liang (2015) point out that the excessive credit growth as measured by the private sector credit-to-GDP gap is a high quality indicator for the likelihood of financial instability.[3] Jorda, Schularick, and Taylor (2011) and Schularick and Taylor (2012) established a statistical link between the credit growth and the probability of financial crisis. If excessive credit growth is a good predictor of the vulnerability to financial crisis, and if income inequality is the main driver of excessive credit growth, then one could conclude that: a monetary

[3]The credit-to-GDP gap is defined as the deviation of the credit-to-GDP ratio from its secular trend, which is constructed using the Hodrick-Prescott filter. See Adrian and Liang (2014) and Edge and Meisenzahl (2011) for more references.

authority with a financial stability mandate should be concerned about income inequality and its implications for financial stability.

In this paper, we develop a theoretical model that allow us to study the link among income inequality, secular stagnation, excessive credit growth, financial crisis and deflation pressure. To that end, we build upon the recent breakthrough made by Kumhof, Ranciere, and Winant (2015), KRW henceforth. Using a fully optimization-based model, KRW show that income inequality can lead to excessive leverage, which then endogenously increases the probability of a total meltdown of the financial system. We adapt their model in an environment in which monetary policy is well defined. This requires introducing nominal rigidities and endogeneizing the production and income distribution of the KRW model. The introduction of nominal rigidities is crucial because without it, one cannot generate insufficient aggregate demand even in the presence of income inequality. Also in contrast to the KRW, the size of a financial crisis is endogenous and is affected by monetary policy. Using the resulting model, we investigate the performances of alternative monetary policy rules in controlling both the probability and the size of financial crises.

In investigating the role of monetary policy, we pay close attention to the role played by the Zero Lower Bound, ZLB henceforth, constraint. This constraint restricts the ability of the monetary authority to stabilize the economy during a financial crisis. Knowing that its stabilization function during financial crises may be paralyzed significantly by the binding ZLB, the monetary authority may have a greater incentive to adopt a policy that reduces the probability of crises, even when doing so may sacrifice its traditional mandate of price stability. Our main findings are summarized as follows.

First, in the baseline model economy, in which the monetary authority follows a strict inflation targeting and the model parameters are calibrated to match the debt-to-income ratio of the bottom 95 percent income earners in the data, income inequality, defined as the income share of the top 5 percent income earners relative to the bottom 95 percent income earners, is negatively correlated with aggregate output, consumption, investment and inflation rate. Our results show that the secular stagnation owing to underconsumption is a real possibility. The underconsumption in our model does not exist as low consumption level of a representative agent. The underconsumption is a consequence of composition of aggregate MPC. The decline of consumption share of the agents with greater MPCs is responsible for insufficient aggregate demand and deflation pressure.

Second, in the baseline model economy, we find a strong, positive correlation between the debt-to-income ratio of the bottom 95 percent income earners and the probability of financial crisis. This confirms the findings of empirical researchers: the excess credit growth is a high quality indicator of the likelihood of financial crises. We also find that the income inequality is positively correlated with the probability of financial crises.[4] This suggests that stabilization policies that are designed to lean against the excessive credit growth to prevent financial crises may not be successful if such policies do not address the root cause of the excessive credit growth: the income inequality. This is because the economy with sufficient amount of savings will find ways to channel the excess savings

[4]The results in this paper justifies the approach of Gourio, Kashyap, and Sim (2016), which models the probability of Barro (2006)-type disaster as a reduced form function of credit variables.

into borrowings of low income households through regulatory arbitrage. If the economy fails to do so, then the economy will avoid financial crises, but face another problem: insufficient aggregate demand and deflation pressure. Thus, our results show that most of the predictions made by Summers (2015) can be verified by a model that is constructed with the first-order principles.

Third, we find that the distributions of endogenous prices and quantities are highly skewed to the left, i.e., to the downside. The skewed distribution in our environment is not due to the skewed distributions of underlying shocks as in Adrian, Boyarchenko, and Giannone (2016) or Adrian and Duarte (2016). In our model, all underlying shocks follow Gaussian distributions. However, the model economy has two important sources of nonlinearity: occasional eruptions of financial crises in which all borrowers default on their liabilities; occasionally binding ZLB constraint.[5] Since these two sources of nonlinearity work only to the downside of the economy, each friction generates a significant degree of skewness for the distributions of equilibrium prices and quantities. While the exact mechanism behind the skewed distribution is different from Adrian and Duarte (2016), we share their conclusion: the skewed distribution, i.e., the presence of asymmetrically large downside risks raises the issue of suboptimality of Taylor rule. This prompts us to search for monetary policy rules that can undo the skewed distributions induced by financial crises and the ZLB constraint. Such policies may bring first-order difference in welfare because correcting the skewed distribution may bring a sizable change in the mean.

Fourth, regarding strict inflation targeting, we find that while Rogoff's conservative central banker that "places a large, but finite, weight on inflation" (Rogoff, 1985) minimizes the volatility of both inflation and output, the conservative central banker tend to increase the probability of financial crises. This is because more aggressive responses to inflation requires larger increases in the policy rate, which makes the debt servicing costs of low income households go up, thus elevating the low income households' incentive to default. However, the same conservative central banker minimizes the damage upon a financial crisis as the crisis brings a substantial deflation pressure to which the conservative central bank provides decisive accommodation by holding the nominal interest rate at zero for an extended period of time.

Our results suggests that the optimal inflation targeting requires a particular mix: more lenient response to inflation during normal times to minimizes the probability of default and decisive accommodation during financial crisis to ameliorate the damage brought on by the crisis. The latter may involve an extended period of binding ZLB constraint. Thus such a conservative central banker is opportunistic in the sense that he is conservative only to the downside risks. It is the skewed distributions of employment and inflation that calls for such an opportunistic central banker. The economy is distributed asymmetrically, and therefore monetary policy should be asymmetric.

For this reason, we propose an alternative loss function that penalizes not only the variance of inflation around the target, but also the skewness of inflation and analyze alternative policies that work asymmetrically across crisis and non-crisis periods. Our results indicate that a policy rule that

[5]See Dupraz, Nakamura, and Steinsson (2016) for a plucking model of business cycle fluctuations owing to the downward nominal wage rigidity and Basu and Bundick (2015) for the role of binding ZLB constraint in generating endogenous volatility.

augments the strict inflation targeting rule by forward guidance of promising a prolonged period of extremely low interest rate during financial crises can effectively eliminate the skewness in inflation and deflation bias owing to financial crises and the binding ZLB constraint.

Furthermore, such a rule can make a first order difference for the means of equilibrium quantities and prices. The rule, which we call time-varying inflation targeting rule, commits not to raise the intercept to a normal level even after a long period since the eruption of a financial crisis and even after the inflation rate reaches the target in order to offset the deflationary periods with inflationary periods. In principle, the rule is in the same spirit of Reifschneider and Williams (2000)' rule of providing extra stimulus according to the past duration and the strength of binding ZLB constraint. Such a monetary policy rule is a fundamentally asymmetric policy rule because the asymmetric adjustment is activated only by an eruption of a financial crisis, which is only one-direction risk.

In our analysis, we assume that the central bankers do not have any specific preferences over the degree of income inequality. However, we find that the time-varying inflation targeting rule greatly lessens the degree of income inequality although that is not an intended consequence. This is not a coincidence because it is the low income households in the model that suffer the most from the presence of disproportionately large downside risks, and a monetary policy rule that is designed to fight the skewed distributions brings the most benefits to low income households without necessarily worsening the outcome for high income households. On the contrary, a misspecified inflation targeting rule in the sense of failing to provide sufficient amount of monetary accommodation during crises may be responsible for deterioration in income inequality. This means, as suggested by Coibion, Gorodnichenko, Kueng, and Silvia (2012), that the monetary policy may not be an "innocent bystander" for growing income inequality in the U.S.

The rest of the paper is organized as follows: Section 2 develops the model; Section 3 explains our calibration strategy; Section 4 presents the model dynamics; Section 5 discusses our main findings regarding the linkage among income inequality, aggregate demand and financial stability in detail; Section 6 analyzes the relationship between monetary policy and financial stability using alternative monetary policy rules; Section 7 proposes an alternative loss function for central banks and how optimized policy rules under this alternative loss function specification improves macroeconomic outcomes. Finally, Section 8 concludes. A complete description of the non-stochastic steady state, the solution method and the complete system of equations are described in Appendix B, C and D, respectively.

2 The Model

There are two groups of agents: the top 5 percent earners and the bottom 95 percent earners. Broadly speaking the top 5 percent earners play the role of the shareholders of corporations and the bottom 95 percent earners play the role of workers in production. In the financial market, the top 5 percent plays the role of lender and the bottom 95 percent plays the role of borrowers. The population shares of the top 5 percent and bottom 95 percent earners are denoted by $\chi = 0.05$ and $1 - \chi = 0.95$, respectively. Each group contains a continuum of agents and forms a large family that shares the budget and consumption. Employed workers work for wage income and unemployed workers search for jobs.

We assume a segmented asset market structure such that only shareholders, i.e. the top 5 percent earners, own production firms and accumulate physical capital. Shareholders also accumulate private bonds and government bonds. Workers do not participate in capital market. The only instrument available to workers to smooth their consumption profile is borrowing from the private bond market. In equilibrium, shareholders lend money to workers.

Monetary policy determines the interest rate on government bonds. While only shareholders accumulate government bonds, monetary policy also affects workers' consumption profiles due to general equilibrium effects. We closely follow KRW in modeling preferences of the two groups except that the shareholders also accumulate physical capital and government bonds as well as private bonds. Below, superscripts T and B are used to indicate the top 5 percent and bottom 95 percent earners.

2.1 Top 5 Percent of Income Distribution

We model the top 5 percent of the income distribution as agents who own the shares of the firms and accumulate financial assets and physical capital. Per capita consumption level of the top 5 percent is denoted by c_t^T. Per capita private bond issuance by the bottom 95 percent and per capital government bond holdings by the top 5 percent are denoted by b_t and b_t^G, respectively. Hence in this notation, per capita holdings of private bond by the top 5 percent is constructed as $b_t(1-\chi)/\chi$. The preferences of top 5 percent agent are specified as

$$U_t^T = \mathbb{E}_t \sum_{t=0}^{\infty} (\beta^T)^t \left\{ \frac{(c_t^T)^{1-1/\sigma_c} - 1}{1 - 1/\sigma_c} + \psi^B \frac{\left[1 + b_t \left(\frac{1-\chi}{\chi}\right)\right]^{1-1/\sigma_b} - 1}{1 - 1/\sigma_b} + \psi^G \frac{(1 + b_t^G)^{1-1/\sigma_g} - 1}{1 - 1/\sigma_g} \right\}, \quad (1)$$

where the composite consumption good is given by

$$c_t^T = \left[\int_0^1 c_t^T(i)^{1-1/\gamma} di \right]^{1/(1-1/\gamma)}, \quad (2)$$

with γ being the elasticity of substitution between different varieties and σ_c the elasticity of intertemporal substitution. ψ^B and ψ^G are strictly positive weights given to financial asset holdings. These preferences, adopted from KRW, exhibit "love of financial wealth" or "the spirit of capitalism" (Max Weber). We simply generalize them by allowing government bond holdings to have a similar effect on the utility as in the case of private bond holdings. This is necessary to create a transmission channel for monetary policy, which does not exist in KRW framework. σ_b and σ_g determine how fast the marginal utility of financial asset holdings decline. Finally β^T is the discount factor of shareholders.

At time t shareholders offer q_t^B units of consumption to the bottom 95 percent earners in return for $\mathbb{E}_t[\pi_{t+1}^{-1}]$ unit of consumption tomorrow. When the borrowers default, the shareholders get paid back only $(1-h)\mathbb{E}_t\pi_{t+1}^{-1}$, where h is the hair-cut associated with the default. We denote the actual payment by

$$l_t = (1 - h\delta_t^B)\frac{b_{t-1}}{\pi_t}, \tag{3}$$

where $\delta_t^B \in \{0, 1\}$ is the default indicator. When the default is optimal for one borrower, it is so for the other borrowers. Therefore in this economy, a default event can be interpreted as the total meltdown of the financial system where all borrowers default at the same time.

Per capita budget constraint for the top 5 percent earners can then be expressed as

$$c_t^T = \frac{b_{t-1}^G}{\pi_t} - \frac{b_t^G}{1+i_t} + (l_t - q_t^B b_t)\frac{1-\chi}{\chi} - q_t^K\left[\frac{k_t}{\chi} - (1-\delta)\frac{k_{t-1}}{\chi}\right] + r_t\frac{k_{t-1}}{\chi} + \frac{\Pi_t^Y}{\chi} + \frac{\Pi_t^K}{\chi} - \frac{T_t}{\chi} \tag{4}$$

where i_t is the nominal interest rate controlled by the monetary authority, r_t is the rental rate of capital, k_t/χ is per capita capital stock, δ is the depreciation rate of capital, q_t^K is the relative price of capital, Π_t^Y/χ is the per capita profit of intermediate-goods firms, Π_t^K/χ is the per capita profits of investment-goods firms and T_t/χ is the per capita lump sum tax. We denote the shadow value of the budget constraint by Λ_t^T. The FOCs of the shareholders are given by

$$\Lambda_t^T = (c_t^T)^{-1/\sigma_c}, \tag{5}$$

$$q_t^B = \beta^T \mathbb{E}_t\left[\frac{\Lambda_{t+1}^T}{\Lambda_t^T}(1 - hp_{t+1}^\delta)\frac{1}{\pi_{t+1}}\right] + \frac{\psi^B}{\Lambda_t^T}\left[1 + b_t\left(\frac{1-\chi}{\chi}\right)\right]^{-1/\sigma_b}, \tag{6}$$

$$1 = \beta^T \mathbb{E}_t\left[\frac{\Lambda_{t+1}^T}{\Lambda_t^T}\left(\frac{r_{t+1} + (1-\delta)q_{t+1}^K}{q_t^K \lambda_t}\right)\right], \tag{7}$$

and

$$\frac{1}{1+i_t} = \beta^T \mathbb{E}_t\left[\frac{\Lambda_{t+1}^T}{\Lambda_t^T}\frac{\lambda_t}{\pi_{t+1}}\right] + \frac{\psi^G}{\Lambda_t^T}(1 + b_t^G)^{-1/\sigma_g}, \tag{8}$$

where

$$p_{t+1}^\delta \equiv \mathbb{E}_t[\delta_{t+1}^B],$$

i.e., the probability of default tomorrow. We assume that the economy is subject to Smets and Wouters (2007)'s risk premium shock to create aggregate demand disturbances, which is denoted by λ_t and follows an AR(1) process:[6]

$$\log \lambda_t = \rho_\lambda \log \lambda_{t-1} + \epsilon_{\lambda,t}, \quad \epsilon_{\lambda,t} \sim N(0, \sigma_\lambda^2). \tag{9}$$

[6] Note that the risk premium shock appears in the denominator of equation (7) and the numerator of equation (8). The risk premium shock appears in the denominator of Lucas tree equation (7) such that the expected real return falls. However, the same risk premium shock appears in the numerator of the consumption Euler equation (8) such that the shock is equivalent with monetary policy shock implemented by "nature"and not by the monetary authority. Fisher (2015) provides a structural interpretation of such a shock as "money demand shock".

For later use, we define the stochastic discounting factor of the shareholders as

$$m_{t,s}^T \equiv (\beta^T)^{s-t} \frac{\Lambda_s^T}{\Lambda_t^T}. \tag{10}$$

2.2 Bottom 95 percent of Income Distribution

We denote per capita consumption level of the bottom 95 percent earners by c_t^B. Preferences of the bottom 95 percent earners are specified as

$$U_t^B = \mathbb{E}_t \sum_{t=0}^{\infty} (\beta^B)^t \left\{ \frac{(c_t^B)^{1-1/\sigma_c}}{1-1/\sigma_c} \right\}, \tag{11}$$

where per capita composite consumption is given by

$$c_t^B = \left[\int_0^1 c_t^B(i)^{1-1/\gamma} di \right]^{1/(1-1/\gamma)}. \tag{12}$$

We assume that the bottom 95 percent earners do not develop preferences over financial wealth. We assume that there is a pecuniary default cost for the bottom 95 percent earners. As in KRW, we assume that the default cost is a fraction ν_t of aggregate output y_t and and the fraction follows the following process:

$$\nu_t = \rho_\nu \nu_{t-1} + \gamma_\nu \delta_t^B. \tag{13}$$

Since the bottom 95 percent earners are assumed not to participate in capital markets, per capita budget constraint for the bottom 95 percent earners is given by

$$c_t^B = q_t^B b_t - l_t + \frac{1}{1-\chi} \left[\int_0^1 w_t(i) n_t(i) di + (1-\chi-n_t) b^U - \nu_t y_t \right], \tag{14}$$

where $w_t(i)n_t(i)$ are wage income employed by firm i, $n_t = \int_0^1 n_t(i)di$ is the total employment and b^U are unemployment benefits. The FOCs for the bottom 95 percent earners are

$$\Lambda_t^B = (c_t^B)^{-1/\sigma_c}, \tag{15}$$

and

$$q_t^B = \beta^B \mathbb{E}_t \left[\frac{\Lambda_{t+1}^B}{\Lambda_t^B} (1 - hp_{t+1}^\delta) \frac{1}{\pi_{t+1}} \right]. \tag{16}$$

For later use, we define the stochastic discounting factor of the bottom 95 percent earners as

$$m_{t,s}^B \equiv (\beta^B)^{s-t} \frac{\Lambda_s^B}{\Lambda_t^B}. \tag{17}$$

Note that neither the lender's efficiency condition (6) nor the borrower's efficiency condition (16) incorporates the effect of increasing debt on the probability of default or the price of bond. In other words, both agents behave as if $\partial p^{\delta}_{t+1}/\partial b_t = \partial q^B_t/\partial b_t = 0$. This is because there exists a continuum of agents in each type, and the agents view their individual actions inconsequential for aggregate macroeconomic outcomes. Thus this assumption creates an important source of pecuniary externality.

2.3 Financial Crisis

Default involves pecuniary and non-pecuniary costs. The latter is the direct cost to workers' utility. Default occurs when the utility gain from defaulting is greater than the direct utility cost of default χ_t, which follows an iid process. We denote the workers' values of default and non-default by U^D_t and U^N_t. We assume the following timing convention. At the beginning of each period, aggregate shocks except χ_t are realized. New borrowing decisions are made based upon this information set that does not include χ_t, thus the decision on b_t is not conditioned upon the realization of χ_t. Then χ_t realizes and default decision is made.

Formally, default occurs when

$$\chi_t < U^D_t - U^N_t. \tag{18}$$

We define the value of default and the value of non-default as the sum of one period utility and the continuation value in each case:

$$U^D_t = \frac{(c^D_t)^{1-1/\sigma_c} - 1}{1-1/\sigma_c} + V^D_t, \tag{19}$$

and

$$U^N_t = \frac{(c^N_t)^{1-1/\sigma_c} - 1}{1-1/\sigma_c} + V^N_t. \tag{20}$$

The consumption values under default and non-default strategies are given by

$$c^D_t = q^B_t b_t - (1-h)\frac{b_{t-1}}{\pi_t} + \frac{1}{1-\chi}\left[w_t n_t + (1-\chi-n_t)b^U - (\rho_\nu \nu_{t-1} + \gamma_\nu) y_t\right], \tag{21}$$

and

$$c^N_t = q^B_t b_t - \frac{b_{t-1}}{\pi_t} + \frac{1}{1-\chi}\left[w_t n_t + (1-\chi-n_t)b^U - \rho_\nu \nu_{t-1} y_t\right]. \tag{22}$$

The continuation values under default and non-default strategies are

$$\begin{aligned} V^D_t = {} & \beta^B \mathbb{E}_t\left[p^{\delta}_{t+1}\left(\frac{(c^{DD}_{t+1})^{1-1/\sigma_c} - 1}{1-1/\sigma_c} + V^D_{t+1}\right)\right] \\ & + \beta^B \mathbb{E}_t\left[(1-p^{\delta}_{t+1})\left(\frac{(c^{DN}_{t+1})^{1-1/\sigma_c} - 1}{1-1/\sigma_c} + V^N_{t+1}\right)\right]; \end{aligned} \tag{23}$$

$$\begin{aligned} V^N_t = {} & \beta^B \mathbb{E}_t\left[p^{\delta}_{t+1}\left(\frac{(c^{ND}_{t+1})^{1-1/\sigma_c} - 1}{1-1/\sigma_c} + V^D_{t+1}\right)\right] \\ & + \beta^B \mathbb{E}_t\left[(1-p^{\delta}_{t+1})\left(\frac{(c^{NN}_{t+1})^{1-1/\sigma_c} - 1}{1-1/\sigma_c} + V^N_{t+1}\right)\right], \end{aligned} \tag{24}$$

where c^{DD}_{t+1}, c^{DN}_{t+1}, c^{ND}_{t+1} and c^{NN}_{t+1} are defined as the consumption value tomorrow in the following four cases: (i) default again tomorrow after today's default; (ii) non-default tomorrow after today's default; (iii) default tomorrow after non-default today; (iv) non-default to morrow after non-default today. Each case can be constructed as follows:

$$c_{t+1}^{DD} = q_{t+1}^{B} b_{t+1} - (1-h)\frac{b_t}{\pi_{t+1}} + \frac{1}{1-\chi}\left\{w_{t+1}n_{t+1} + (1-\chi-n_{t+1})b^U - [\rho_\nu(\rho_\nu \nu_{t-1}+\gamma_\nu)+\gamma_\nu]y_{t+1}\right\}; \tag{25}$$

$$c_{t+1}^{DN} = q_{t+1}^{B} b_{t+1} - \frac{b_t}{\pi_{t+1}} + \frac{1}{1-\chi}\left\{w_{t+1}n_{t+1} + (1-\chi-n_{t+1})b^U - \rho_\nu(\rho_\nu \nu_{t-1}+\gamma_\nu)y_{t+1}\right\}; \tag{26}$$

$$c_{t+1}^{ND} = q_{t+1}^{B} b_{t+1} - (1-h)\frac{b_t}{\pi_{t+1}} + \frac{1}{1-\chi}\left\{w_{t+1}n_{t+1} + (1-\chi-n_{t+1})b^U - (\rho_\nu\rho_\nu \nu_{t-1}+\gamma_\nu)y_{t+1}\right\}; \tag{27}$$

$$c_{t+1}^{NN} = q_{t+1}^{B} b_{t+1} - \frac{b_t}{\pi_{t+1}} + \frac{1}{1-\chi}\left\{w_{t+1}n_{t+1} + (1-\chi-n_{t+1})b^U - \rho_\nu\rho_\nu \nu_{t-1}y_{t+1}\right\}. \tag{28}$$

Regarding the distribution of utility cost of default, we assume that χ_t follows a modified logistic distribution as in KRW:

$$\Xi(\chi_t) = \begin{cases} \dfrac{\varrho}{1+\exp(-\varsigma\chi_t)} & \text{if } \chi_t < \infty \\ 1 & \text{if } \chi_t = \infty \end{cases} \tag{29}$$

The probability of default is then simply given by

$$p_{t+1}^{\delta} \equiv \text{prob}(\delta_{t+1}^{B} = 1) = \mathbb{E}_t[\Xi(U_{t+1}^{D} - U_{t+1}^{N})]. \tag{30}$$

For later purpose, we define $\Delta U_t^B \equiv U_t^D - U_t^N$.

2.4 Labor Market

The matching process between vacancies and unemployed workers is assumed to be governed by a constant returns to scale matching function:

$$m(v_t, u_t) = \zeta v_t^{\epsilon} u_t^{1-\epsilon}, \tag{31}$$

where $v_t \equiv \int v_{it} di$ denotes the measure of vacancies and u_t denotes the measure of unemployed workers searching for a job at the beginning of period t. The parameter ζ stands for matching efficiency and $(1-\epsilon)$ is the matching function elasticity with respect to unemployment. The matching function is assumed to be concave and increasing in both of its arguments. Labor market tightness is defined as

$\theta_t \equiv v_t/u_t$. The endogenous probability for an unemployed worker to meet a vacancy is given by:

$$p(\theta_t) = \frac{m(v_t, u_t)}{u_t} = \zeta\theta_t^{\epsilon}, \tag{32}$$

and the endogenous probability for a vacancy to meet with an unemployed worker is:

$$q(\theta_t) = \frac{m(v_t, u_t)}{v_t} = \zeta\theta_t^{\epsilon-1}. \tag{33}$$

Note that firms consider these flow probabilities as given when deciding their optimal level of employment.

The unemployment rate is determined as

$$u_t = 1 - \chi - (1-\rho)n_{t-1}. \tag{34}$$

2.5 Firm problem

There exists a continuum of measure one of firms indexed by i. Each firm produces a differentiated good, using an identical Cobb-Douglas production function with capital and labor, denoted by $k_t(i)$ and $n_t(i)$. The production technology is represented by:

$$y_t(i) = z_t k_{t-1}(i)^{\alpha} n_t(i)^{1-\alpha}, \tag{35}$$

where z_t is an aggregate productivity shock following an AR(1) process,

$$\log z_t = \rho_z \log z_{t-1} + \epsilon_{z,t}, \quad \epsilon_{z,t} \sim N(0, \sigma_z^2). \tag{36}$$

Firms make three decisions: pricing decision subject to infrequent price adjustments, which will be detailed below; hiring decision in a frictional labor market; and capital rental decision.[7] The timing of events for the firm's problem is summarized as follows. At the end of time $t-1$, a fraction ρ of workforce is exogenously separated from the firms. Then, aggregate shocks realize and the firms post vacancies $v_t(i)$ at a flow vacancy posting cost ξ per period, which will be filled with probability $q(\theta_t)$. Firms then make the capital rental decision, which is assumed to be frictionless. As will be shown, this assumption helps establish the equalization of marginal costs across heterogeneous firms.

It is assumed that vacancies posted at the beginning of the period can be filled in the same period before production takes place, and that recently separated workers can search for a job. Thus, the law of motion for the firm's workforce is given by:

$$n_t(i) = (1-\rho)n_{t-1}(i) + q(\theta_t)v_t(i). \tag{37}$$

After the matching process is complete, the wage is determined through Nash wage bargaining. Finally,

[7] Similar to Thomas and Zanetti (2009) and Thomas (2011), we assume that firms are subject both to staggered price adjustment and search frictions.

production takes place, and wages, capital rents and dividends are paid.

Note that firms' cost minimization problem can be separated from the optimal pricing problem as we assume that the production technology satisfies constant return-to-scale, capital allocation is frictionless and the real wage can be renegotiated each period even though subject to quadratic adjustment costs.

2.5.1 Cost minimization

The firm's cost minimization problem can be separated from the optimal pricing problem. We assume that the firm faces real wage rigidities by assuming a quadratic adjustment cost of changing the real wage. A firm i minimizes its production costs subject to equations (35) and (37). The cost minimization problem can be formalized by means of the Lagrangian:

$$\begin{aligned}\mathcal{L} = {} & \mathbb{E}_t \sum_{s=t}^{\infty} m_{t,s}^{T} \left[w_s(i) n_s(i) + \frac{\upsilon}{2} \left(\frac{w_s(i)}{w_{s-1}(i)} - 1 \right)^2 w_{s-1}(i) n_s(i) + r_s k_{s-1}(i) + \xi v_s(i) \right] \\ & + \mathbb{E}_t \sum_{s=t}^{\infty} m_{t,s}^{T} J_s(i) \{ n_s(i) - [(1-\rho) n_{s-1}(i) + q(\theta_s) v_s(i)] \} \\ & + \mathbb{E}_t \sum_{s=t}^{\infty} m_{t,s}^{T} \mu_s(i) \{ y_s(i) - z_t k_{t-1}(i)^{\alpha} n_t(i)^{1-\alpha} \},\end{aligned} \tag{38}$$

where $\mu_t(i)$ and $J_t(i)$ are the shadow values of the constraints (35) and (37), respectively. Thus, $\mu_t(i)$ represents the real marginal cost of production and $J_t(i)$ is the real marginal value of employment. As indicated by the quadratic adjustment cost in the first line of the Lagrangian, we assume that the economy faces real wage rigidity.[8]

The associated first-order conditions with respect to vacancies, $v_t(i)$, employment, $n_t(i)$, and capital $k_t(i)$ are, respectively

$$\frac{\xi}{q(\theta_t)} = J_t(i), \tag{39}$$

$$J_t(i) = \mu_t(i)(1-\alpha)\frac{y_t(i)}{n_t(i)} - w_t(i) - \frac{\upsilon}{2}\left(\frac{w_t(i)}{w_{t-1}(i)} - 1\right)^2 w_{t-1}(i) + (1-\rho)\mathbb{E}_t[m_{t,t+1}^{T} J_{t+1}(i)], \tag{40}$$

and

$$r_t = \mu_t(i)\alpha \frac{y_t(i)}{k_{t-1}(i)}. \tag{41}$$

Equation (39) equalizes the costs of posting a vacancy (recall that ξ is per period vacancy posting cost and $1/q(\theta_t)$ is the expected vacancy duration) with the benefit of filling the vacancy. The benefit of filling a vacancy is given by equation (40) and equals the gap between the cost reduction of an

[8] Whether to put this cost in firms' problem or in workers' problem is inconsequential as the two maximize the joint share as will be shown later.

additional worker and the wage, plus the continuation value of the employment relationship when surviving exogenous job separation. Note that $J_t(i)$ measures the value of marginal worker, which equals the average value of workers owing to the constant returns to scale technology. Finally, given that the capital allocation decision is frictionless, equation (41) equals the real rental rate to the cost reduction of using an additional unit of capital.

Note that by combining equations (35) and (41), one can show that the labor productivity can be expressed as

$$\frac{y_t(i)}{n_t(i)} = z_t \left[\frac{r_t}{\mu_t(i)\alpha z_t}\right]^{-(1-\alpha)/\alpha} \equiv \phi(z_t, r_t, \mu_t(i)).$$

Hence the marginal value of worker satisfies

$$\begin{aligned} J_t(i) = {} & \mu_t(i)(1-\alpha)\phi(z_t, r_t, \mu_t(i)) - w_t(i) \\ & -\frac{\upsilon}{2}\left(\frac{w_t(i)}{w_{t-1}(i)} - 1\right)^2 w_{t-1}(i) + (1-\rho)\mathbb{E}_t[m^T_{t,t+1} J_{t+1}(i)] \end{aligned} \tag{42}$$

Equation (39) shows that $J_t(i)$ does not depend on firm-specific variables. Then, equation (42) implies that not only the marginal cost and current wage, but also the entire path of wage over any given history should be equalized across all firms, hence we have a symmetric equilibrium.

2.5.2 Wage bargaining

The worker's surplus at a firm i is given by the sum of the surplus of the contract wage w_{it} over the worker's outside option value $\underline{w}_t$ and the continuation value at the firm i,

$$W_t(i) = w_t(i) - \underline{w}_t + (1-\rho)\mathbb{E}_t[m^B_{t,t+1} W_{t+1}(i)]. \tag{43}$$

The worker's outside option $\underline{w}_t$ is given by the sum of unemployment benefit and the expected value of new job next period,

$$\underline{w}_t = b^U + (1-\rho)\mathbb{E}_t\left[m^B_{t,t+1} p(\theta_{t+1}) \int_0^1 \frac{v_{t+1}(j)}{v_{t+1}} W_{t+1}(j) dj\right], \tag{44}$$

where $v_{t+1}(j)/v_{t+1}$ is the probability of the separated worker finding a job at firm j.

We assume that the firm and the marginal worker bargain over the wage to maximize the Nash product:

$$w_t(i) = \arg\max_{w_t(i)} W_t(i)^{\eta_t} J_t(i)^{1-\eta_t}, \tag{45}$$

where η_t is the bargaining power of the worker, which we assume to follow an AR(1) process:

$$\log \eta_t = (1-\rho_\eta)\log\eta + \rho_\eta \log\eta_{t-1} + \epsilon_{\eta,t}, \quad \epsilon_{\eta,t} \sim N(0, \sigma^2_\eta). \tag{46}$$

The first order condition to the Nash bargaining problem is given by the surplus sharing rule,

$$0 = \eta_t \frac{\partial W_t(i)}{\partial w_t(i)} \frac{J_t(i)}{W_t(i)} + (1-\eta_t)\frac{\partial J_t(i)}{\partial w_t(i)}, \tag{47}$$

where

$$\frac{\partial W_t(i)}{\partial w_t(i)} = 1,$$

and

$$\frac{\partial J_t(i)}{\partial w_t(i)} = -1 - \upsilon\left(\frac{w_t(i)}{w_{t-1}(i)} - 1\right) + (1-\rho)\mathbb{E}_t\left\{\beta^S_{t,t+1}\frac{\upsilon}{2}\left[\left(\frac{w_{t+1}(i)}{w_t(i)}\right)^2 - 1\right]\right\}.$$

We now define the generalized workers' bargaining power Ω_t as

$$\Omega_t \equiv \frac{\eta_t}{\eta_t + (1-\eta_t)\Gamma^J_t/\Gamma^W_t},$$

where $\Gamma^W_t \equiv -\partial W_{it}/\partial w_{it}$ and $\Gamma^J_t \equiv \partial J_{it}/\partial w_{it}$. Note that no real wage rigidity implies $\Gamma^J_t/\Gamma^W_t = 1$ and $\Omega_t = \eta_t$. Using the generalized bargaining power, the surplus sharing rule (47) can be rewritten as a generalization of the standard Nash-wage bargaining condition:

$$\Omega_t J_t(i) = (1-\Omega_t)W_t(i). \tag{48}$$

To find the equilibrium wage, we plug in the surplus functions of the firm and worker into the surplus sharing condition (48)

$$\begin{aligned} 0 = \Omega_t &\left[\mu_t(i)(1-\alpha)\frac{y_t(i)}{n_t(i)} - w_t(i) - \frac{\upsilon}{2}\left(\frac{w_t(i)}{w_{t-1}(i)} - 1\right)^2 w_{t-1}(i) + (1-\rho)\mathbb{E}_t[m^T_{t,t+1}J_{t+1}(i)]\right] \\ &-(1-\Omega_t)\left[w_t(i) - \underline{w}_t + (1-\rho)\mathbb{E}_t m^B_{t,t+1}W_{t+1}(i)\right]. \end{aligned}$$

Since the surplus sharing condition holds in the future as well, we can rewrite the worker's outside option as

$$\underline{w}_t = b^U + (1-\rho)\mathbb{E}_t\left[m^B_{t,t+1}p(\theta_{t+1})\int_0^1 \frac{v_{t+1}(j)}{v_{t+1}}\frac{\Omega_{t+1}}{1-\Omega_{t+1}}J_{t+1}(j)dj\right].$$

The first order condition for vacancies given by equation (39) implies that the marginal value of worker is equalized across all firms, and hence we can further simplify the above expression to

$$\underline{w}_t = b^U + (1-\rho)\mathbb{E}_t\left[m^B_{t,t+1}p(\theta_{t+1})\frac{\Omega_{t+1}}{1-\Omega_{t+1}}J_{t+1}\right]. \tag{49}$$

This and (42) also imply that the marginal cost of production is equalized across all firms. We can then combine equation (39) and (47)∼(49) to derive the equilibrium wage as

$$
\begin{aligned}
w_t = {} & \Omega_t \mu_t (1-\alpha)\frac{y_t}{n_t} + (1-\Omega_t) b^U - \Omega_t \frac{\upsilon}{2}\left(\frac{w_t}{w_{t-1}} - 1\right)^2 w_{t-1} \\
& + (1-\rho)\mathbb{E}_t\left[\left(\Omega_t m^T_{t,t+1} - (1-\Omega_t) m^B_{t,t+1}[1-p(\theta_{t+1})]\frac{\Omega_{t+1}}{1-\Omega_{t+1}}\right)\frac{\xi}{q(\theta_{t+1})}\right]
\end{aligned} \tag{50}
$$

Note that if we assume a single representative agent such that $m^S_{t,t+1} = m^W_{t,t+1} = m_{t,t+1}$ and further assume a constant bargaining power and no real wage rigidity, the equilibrium wage will be given by a more conventional form,

$$
w_t = \eta_t \mu_t (1-\alpha)\frac{y_t}{n_t} + (1-\eta_t) b^U + \eta(1-\rho)\mathbb{E}_t\left\{m_{t,t+1}\xi\theta_{t+1}\right\}.
$$

2.5.3 Optimal pricing

Each firm produces a differentiated good and faces an identical isoelastic demand curve given by

$$
y_t(i) = \left(\frac{P_t(i)}{P_t}\right)^{-\gamma} y_t, \tag{51}
$$

where $P_t(i)$ is the firm's price, P_t is the aggregate price level, y_t is the aggregate demand, and γ is the elasticity of substitution between differentiated goods.

We assume that firms set prices according to a variant of the formalism proposed by Calvo (1983). In particular, each firm has a constant probability $1-\varphi$ to reset its price in any given period, which is independent across firms and time. Firms that cannot reset their price in a given period partially index their price to lagged inflation. Thus, the firm's price in period t is

$$
P_t(i) = \begin{cases} P_t^* & \text{with probability } 1-\varphi \\ P_{t-1}(i)\pi_{t-1}^{v} & \text{with probability } \varphi \end{cases}
$$

where P_t^* is the optimal reset price, $\pi_t = P_t/P_{t-1}$ is the inflation rate, and v is the inflation indexation parameter. Note that the assumption of partial indexation for firms that cannot reset their price can be thought of as a fraction v of firms following a full indexation by setting $P_t(i) = P_{t-1}(i)\pi_{t-1}$ (as in Christiano, Eichenbaum, and Evans (2005)) and a fraction $1-v$ of firms following a traditional Calvo (1983) framework by setting $P_t(i) = P_{t-1}(i)$.

Given this environment, a firm that reoptimizes its price in period t chooses the reset price P_t^* that maximizes the expected present value of the profits generated while the price remains effective. Formally, P_t^* satisfies:

$$
P_t^* = \arg\max_{P_t^*} \mathbb{E}_t \sum_{s=t}^{\infty} \varphi^{s-t} m^T_{t,s}\left(\frac{P_t^*}{P_s}\prod_{j=t}^{s-1}\pi_j^{v} - \mu_s\right)\left(\frac{P_t^*}{P_s}\prod_{j=t}^{s-1}\pi_j^{v}\right)^{-\gamma} y_s, \tag{52}
$$

with $0 < v < 1$ and $\prod_{j=t}^{s-1}\pi_j^{v} = 1$ for $s \leq t$. Note that in equation (52), P_t^* does not depend on firm index i because we have already established the equalization of the marginal cost across different firms and hence all firms that reset prices at time t choose an identical price.

Let $p_{0,t} \equiv P_t^*/P_{t-1}$ denote the optimal reset price inflation rate. The first order condition to (52) is then expressed as the optimal reset price inflation rate

$$p_{0,t} = \frac{\mathcal{P}_t^N}{\mathcal{P}_t^D}, \tag{53}$$

where the numerator and the denominator of the reset price inflation rate satisfy the following recursions:

$$\mathcal{P}_t^N = \pi_t^{(1-v)\gamma}\Big\{\pi_t^{v\gamma}\gamma\mu_t y_t + \varphi\mathbb{E}_t\left[m_{t,t+1}^T\mathcal{P}_{t+1}^N\right]\Big\}, \tag{54}$$

and

$$\mathcal{P}_t^D = \pi_t^{(1-v)(\gamma-1)}\Big\{\pi_t^{v(\gamma-1)}(\gamma-1)y_t + \varphi\mathbb{E}_t\left[m_{t,t+1}^T\mathcal{P}_{t+1}^D\right]\Big\}. \tag{55}$$

Combining the aggregate price index implied by the Dixit-Stiglitz aggregator and the partial indexation assumption, one can summarize the aggregate inflation dynamics as

$$\pi_t = \left[(1-\varphi)p_{0,t}^{1-\gamma} + \varphi\pi_{t-1}^{v(1-\gamma)}\right]^{1/(1-\gamma)}. \tag{56}$$

It is straightforward to show that the inflation dynamics represented by equations (54)∼(56) are consistent with the following log-linearized Phillips curve:

$$\hat{\pi}_t = \frac{v}{1+\beta^T v}\hat{\pi}_{t-1} + \frac{\beta^T}{1+\beta^T v}\mathbb{E}_t[\hat{\pi}_{t+1}] + \frac{(1-\varphi)(1-\beta^T\varphi)}{\varphi(1+\beta^T v)}\hat{\mu}_t, \tag{57}$$

where the hat represents log deviations from the steady state.

2.6 Closing the model

2.6.1 Investment-good industry

We assume that a competitive investment-good industry is presided by a continuum of firms with CRS technology, which transforms consumption goods to capital goods. The combination of perfect competition and CRS technology makes the scale of firms indeterminate. For this reason, we assume the presence of a representative firm, which solves

$$\max_{x_s}\mathbb{E}_t\sum_{s=t}^{\infty} m_{t,s}^T\left\{q_s^K x_s - \left[x_s + \frac{\kappa}{2}\left(\frac{x_s}{x_{s-1}}-1\right)^2 x_{s-1}\right]\right\}.$$

The first order condition to the above problem is given by an investment Euler equation:[9]

$$q_t^K = 1 + \kappa \left(\frac{x_t}{x_{t-1}} - 1 \right) - \mathbb{E}_t \left\{ m_{t,t+1}^T \frac{\kappa}{2} \left[\left(\frac{x_{t+1}}{x_t} \right)^2 - 1 \right] \right\}. \tag{58}$$

2.6.2 Government Budget Constraint

We assume that the government balances its budget in each period. Government spending is composed of two elements: the unemployment benefits and the interest rate expenses on government debt. We assume that the government spending is financed by lump sum tax on shareholders. The balanced budget constraint then implies

$$T_t = (1 - \chi - n_t) b^U + \chi \left(\frac{b_{t-1}^G}{\pi_t} - \frac{b_t^G}{1 + i_t} \right). \tag{59}$$

In all computations considered in this paper, we assume that the amount of government bond issuance remains constant. This implies that when the monetary authority changes the interest rate, it is the demand for government bond that should jump up and down to clear the bond market.

2.6.3 Market clearing

We define the aggregate consumption c_t as

$$c_t = (1 - \chi) c_t^B + \chi c_t^T. \tag{60}$$

Substituting equations (4) and (14) in equation (60) yields

$$\begin{aligned} c_t = {} & (1 - \chi)(q_t^B b_t - l_t) + w_t n_t + (1 - \chi - n_t) b^U - \nu_t y_t \\ & + \chi \left(\frac{b_{t-1}^G}{\pi_t} - \frac{b_t^G}{1 + i_t} \right) + \left(l_t - q_t^B b_t \right)(1 - \chi) - q_t^K x_t + r_t k_{t-1} + \Pi_t^Y + \Pi_t^K - T_t. \end{aligned} \tag{61}$$

Imposing the balanced budget condition (59) and substituting the profits of intermediate-goods firms and investment-goods firms:

$$\Pi_t^Y = y_t - w_t n_t - \frac{\upsilon}{2} \left(\frac{w_t}{w_{t-1}} - 1 \right)^2 w_{t-1} n_t - r_t k_{t-1} - \xi v_t, \tag{62}$$

and

$$\Pi_t^K = q_t^K x_t - \left[x_t + \frac{\kappa}{2} \left(\frac{x_t}{x_{t-1}} - 1 \right)^2 x_{t-1} \right], \tag{63}$$

[9] We assume that the investment-good firms are owned by shareholders, and hence the stochastic discounting factor $m_{t,s}^T$. However, since this industry is a zero profit industry, the profits of these firms do not appear in the shareholders problem.

in equation (61), one can verify that equation (60) implies the following resource constraint:

$$(1-\nu_t)y_t = c_t + x_t + \xi v_t + \frac{\upsilon}{2}\left(\frac{w_t}{w_{t-1}} - 1\right)^2 w_{t-1}n_t + \frac{\kappa}{2}\left(\frac{x_t}{x_{t-1}} - 1\right)^2 x_{t-1}. \tag{64}$$

Hence we do not impose equation (64) in our system of equations. We simply impose equation (60) together with equations (4) and (14). Note that the resource constraint is comparable with the counterpart of KRW. If capital accumulation and labor market friction do not exist, the right hand side collapses into c_t, which is the case in KRW. The essential difference from KRW is that the production is endogenous. Equating (35) and (51) and integrating both sides of the equality yields

$$y_t = \Delta_t^{-1} z_t n_t^{\alpha} k_{t-1}^{1-\alpha}, \tag{65}$$

where

$$\Delta_t = (1-\varphi)\left(\frac{P_t^*}{P_t}\right)^{-\gamma} + \varphi\left(\frac{P_{t-1}\pi_{t-1}^{\upsilon}}{P_t}\right)^{-\gamma} = \pi_t^{\gamma}[(1-\varphi)p_{0,t}^{-\gamma} + \varphi\pi_{t-1}^{-\upsilon\gamma}]. \tag{66}$$

Δ_t is due to the price dispersion under the staggered pricing.

Note that the default penalty $\nu_t y_t$ of the bottom 95 percent income earners does not accrue to anybody else's account in the model economy. In this sense, the default penalty is a resource loss. In KRW's original formulation, in which aggregate income is determined exogenously without nominal rigidity, (64) is an equilibrium condition, i.e., a market clearing condition, equating aggregate supply, the left hand side of (64) and aggregate demand, the right hand side of (64). As a consequence, when the default occurs, the financial crisis works like a supply shock.

However, when the aggregate output is endogenously determined in an environment with nominal rigidity, this assumption produces a surprising outcome. In this environment, (64) is no longer an equilibrium condition. It simply plays the role of aggregate demand, which should be equalized with aggregate supply given by (65). Consequently, the default penalty boosts aggregate demand by a factor of $(1-\nu_t)^{-1}$ and the default increases production and employment, and results in positive inflation pressure.[10] This is because, from the perspective of production firms, it does not matter whether their products are going to be used for consumption and fixed capital formation or just to be wasted as long as they generate revenues.

For this reason, we deviate from KRW and assume that the default penalty is transferred to the top 5 percent income earners in a lump-sum fashion. Under this assumption, $(1-\nu_t)$ term drops out of the left hand side of (64), and the crisis is associated with large drops in output and employment and deflation pressure. The transfer of the default penalty to the lender can be viewed as debt restructuring: a default relieves the budget constraint of a borrower by the amount of haircut, i.e., hb_{t-1}/π_t, but postpones the payments over a long period of time with a decay rate ρ_ν. Note that the nature of the borrower's problem is not altered at all by this assumption. While the budget constraint of the lender has one additional item $\nu_t y_t/\chi$, the efficiency conditions characterizing the lender's problem remain the same.

[10] The impulse response functions in this case are available upon request.

2.6.4 Monetary Policy

In the baseline analysis, we assume that the monetary authority adopts an inertial Taylor rule that only reacts to the inflation gap:

$$i_t = \max\left\{0, \rho_i i_{t-1} + (1-\rho_i)\left[i^* + \rho_\pi \left(\frac{\pi_t^Y - \pi^*}{4}\right)\right]\right\} \tag{67}$$

where $\pi_t^Y \equiv \pi_t \pi_{t-1} \pi_{t-2} \pi_{t-3}$ is the annual inflation rate and π^* is the usual annual inflation target.

Two remarks are necessary regarding the choice of the monetary policy reaction function (67). First, in the absence of shocks that affect the output gap and inflation gap in opposite directions, such as price mark-up shocks, the model economy exhibits the divine coincidence. As a result, assigning a strictly positive coefficient to the output gap or the unemployment gap is equivalent to raising the inflation coefficient if the natural level of output or the natural rate of unemployment is measured without observation errors. We simplify the analysis by assuming that the monetary authority reacts only to the inflation gap.

Second, we strictly impose the ZLB constraint on the policy interest rate. This allows us to analyze how the ZLB affects the ability of the central bank to control the probability of financial crisis as well as to stabilize inflation rate and unemployment rate during a financial crisis. In turn, we also analyze how the reaction coefficient to inflation rate affects the frequency of financial crises and binding ZLB constraint. Since we use linear perturbation solution method, the ZLB constraint cannot be implemented by equation (67). We use the combination of current monetary policy shock and future news shocks. For this purpose, we express the monetary policy rule as

$$i_t = \rho_i i_{t-1} + (1-\rho_i)\left[i^* + \rho_\pi \left(\frac{\pi_t^Y - \pi^*}{4}\right)\right] + \sigma_m \sum_{j=1}^{n} \epsilon_{j,t-j} + \sigma_m \epsilon_{0,t}, \tag{68}$$

where $\epsilon_{0,t}$ is the current monetary policy shock and $\epsilon_{j,t-j}$ is $j-$periods ahead news shock to the monetary policy rate. While both $\epsilon_{0,t}$ and $\epsilon_{j,t-j}$ are called "shocks", we only use them only to satisfy the ZLB constraint, which can be expressed as a combination of

$$i_t \geq 0, \mathbb{E}_t[i_{t+1}] \geq 0, \ldots, \mathbb{E}_t[i_{t+n}] \geq 0. \tag{69}$$

Note that we ensure that the current interest rate satisfies the ZLB, but also that the future interest rates in expectations are to be consistent with the ZLB constraint. For a sufficiently large value for n, $\mathbb{E}_t[i_{t+n+1}] \geq 0$ can be safely assumed to be non-binding. In each period, $n+1$ shocks should be *endogenously* determined to satisfy the $n+1$ constraints given by (69). This means

$$\epsilon_{k,t} = \sigma_m^{-1} \max\left\{0, -\rho_i \mathbb{E}_t[i_{t+k-1}] - (1-\rho_i)\left[i^* + \rho_\pi \left(\frac{\mathbb{E}_t[\pi_{t+k}^Y] - \pi^*}{4}\right)\right] - \sigma_m \sum\nolimits_{j=k+1}^{n} \epsilon_{j,t-j}\right\} \tag{70}$$

for $k = 0, ..., n$. In words, we use the current monetary policy shock ($k = 0$) to satisfy the ZLB today and the news shocks ($k > 0$) to satisfy the ZLB in expectations. Note that $\mathbb{E}_t[i_{t+j-1}]$ and $\mathbb{E}_t[\pi_{t+j}]$

Table 1: Structural Parameters for Preferences and Default

	Target/Source	Value
Population share of top 5 percent	5 percent	$\chi = 0.05$
Discount factor of bottom 95 percent	Literature	$\beta^B = 0.99$
Discount factor of top 5 percent	Income share of top 5 percent earners = 0.46	$\beta^T = 0.89$
Elasticity of subs. between goods	Literature	$\gamma = 3$
Utility weight on private bond	1× KRW	$\psi^B = 0.05$
Utility weight on public bond	1× KRW	$\psi^G = 0.05$
Elasticity of intertemporal subs	Literature	$\sigma_c = 1$
Wealth elasticity of top 5 percent	Debt to income ratio of bottom 95 percent = 1.5	$\sigma_b = 1.95$
Wealth elasticity of top 5 percent	0.92× KRW	$\sigma_g = 1$
Persistence of default cost	Quarterly version of KRW	$\rho_\nu = 0.65^{0.25}$
Impact default cost	1.625× KRW	$\gamma_\nu = 0.065$
Haircut	0.5× KRW	$h = 0.05$
Steady state interest rate	Literature	$i_{ss} = 0.005$
Default probability	1× KRW	$\varrho = 0.15$
Default probability	1× KRW	$\varsigma = 18$

for $j = 1, ..., n$ should be consistent with $\epsilon_{0,t}$ and $\epsilon_{j,t}$ in the sense that when $\epsilon_{0,t}$ and $\epsilon_{j,t}$ are given to the economy, the agents' expectations about $\mathbb{E}_t[i_{t+j-1}]$ and $\mathbb{E}_t[\pi_{t+j}]$ for $j = 1, ..., n$ should be identical with those showing up on the right hand sides of (70).[11]

3 Calibration

Table 1 summarizes our choices of the structural parameters regarding structure of preferences. While we closely follow Kumhof, Ranciere, and Winant (2015), we adjust the values of some parameters because of three reasons. First, our model is calibrated with quarterly frequency, and for this reason, we set the persistence of default cost ρ_ν equal to $0.65^{0.25} \approx 0.9$. For the same reason, we calibrate the steady state risk-free interest rate equal to 0.5 percent at quarterly level. In contrast to KRW, in which the risky private bond is the only object of financial investment, the risk-free bond is an important financial investment vehicle through which monetary policy is transmitted to the model economy. Once we calibrate the steady state value of risk-free rate, (8) then pins down the level of per capital government bond holdings by the top 5 percent earners. Our calibration implies that the total government debt-to-income ratio ($\chi b^G/(1-\nu)y$) of 16 percent. This value is much smaller than in the data. However, our goal is to set the risk-free rate at a realistic level and let (8) determine whatever value for b^G that is consistent with the first order condition.

Second, national income is endogenously determined in our economy, where as it is exogenously determined in KRW. As will be shown, on the impact of the financial crisis, there will be endogenous reaction to production. This endogenous reaction in production arises because of endogenous monetary stimulus in our economy, which does not exist in their case. Such endogenous adjustment can be nontrivial if there is a significant degree of nominal rigidity. For this reason, we assume that the

[11]This requires some iterations. See Appendix C for details.

output loss due to default events is somewhat greater, and we set $\gamma_\nu = 0.065$, which is greater than their calibration 0.04 by a factor of 1.25.

Third, we calibrate some parameters such that the steady state debt-to-income rate of the bottom 95 percent earners is much higher than in their analysis. While they target 62.3 percent for the debt-to-income ratio of 1983, we target 150 percent, which is close to the debt-to-income ratio on the eve of the Great Recession. This choice leads us to set σ_b the wealth elasticity of top 5 percent earners much higher than in their choice, which is slightly greater than 1. As shown by KRW, the higher the value for σ_b, the higher the marginal propensity to save for the top 5 percent earners. This is because a higher value for σ_b implies that the marginal utility from holding private bonds declines more slowly. We also choose a slightly lower value of discount factor for the top 5 percent earners such that there is a greater incentive to invest in financial assets. However, we make the same choice for the utility weights given to the financial wealth holdings, and set $\psi^B = \psi^G = 0.05$ as in their analysis. For the counterpart of σ_b for government bond holding, we choose $\sigma_b = 1$, a similar value to their choice for private bond holdings. With these choices, the consumption and income share of the bottom 95 percent earners is determined as 0.53 and 0.52 in the steady state.

We use the same values ϱ and ς for the modified logistic distribution of utility cost of default, which are calibrated by Kumhof, Ranciere, and Winant (2015) to match the default probability reported in Schularick and Taylor (2012). This choice makes the default probability much higher than in their analysis given our higher target value for the debt-to-income ratio. Indeed, we want the probability of default to be higher than their 2 percent in annual frequency. However, we need to lower the incentive to default to hit our quarterly target of 1.3 percent such that the probability in annual frequency remain close to the value reported by Schularick and Taylor (2012) on the eve of the Global Recession. To that end, we halve the haircut due to the default by setting $h = 0.05$. Finally, the population share of the top 5 percent earners is naturally set equal to 5 percent.

Table 2 summarizes the standard parameters pertaining to the structure of production, labor market friction, nominal rigidity, monetary policy and shock processes. We set α the capital share of production equal to 0.2. This calibration is somewhat lower than the convention of $0.3 \sim 0.4$. The conventional calibration results in too a low labor income share in our environment, in which there exist rents due to monopolistic competition and the rents are divided into different agents according to their bargaining power.

We set κ the coefficient of investment adjustment cost equal to 5 to make investment three times more volatile than output. δ the depreciation rate of capital stock is set equal to 0.025. ρ the exogenous separation rate of worker and firm is calibrated as 0.05. ζ the matching function efficiency is set equal to 2.3 to hit the job finding rate of 0.85 from the data. Regarding the matching function elasticity of vacancy ϵ, we follow the conventional choice of 0.5. Given this choice, we set the steady state bargaining power of the worker η equal to 0.5 to satisfy the Hosios condition. We calibrate b^U the unemployment benefit equal to 0.5, which is about 90 percent of equilibrium wage. We set ξ the vacancy posting cost equal to 0.11. This choice is inconsequential for the dynamics of the model.

We choose a substantially low value 0.05 for the probability of resetting price $1-\varphi$ to be consistent

Table 2: Structural Parameters for Technology, Labor Market and Nominal Rigidity

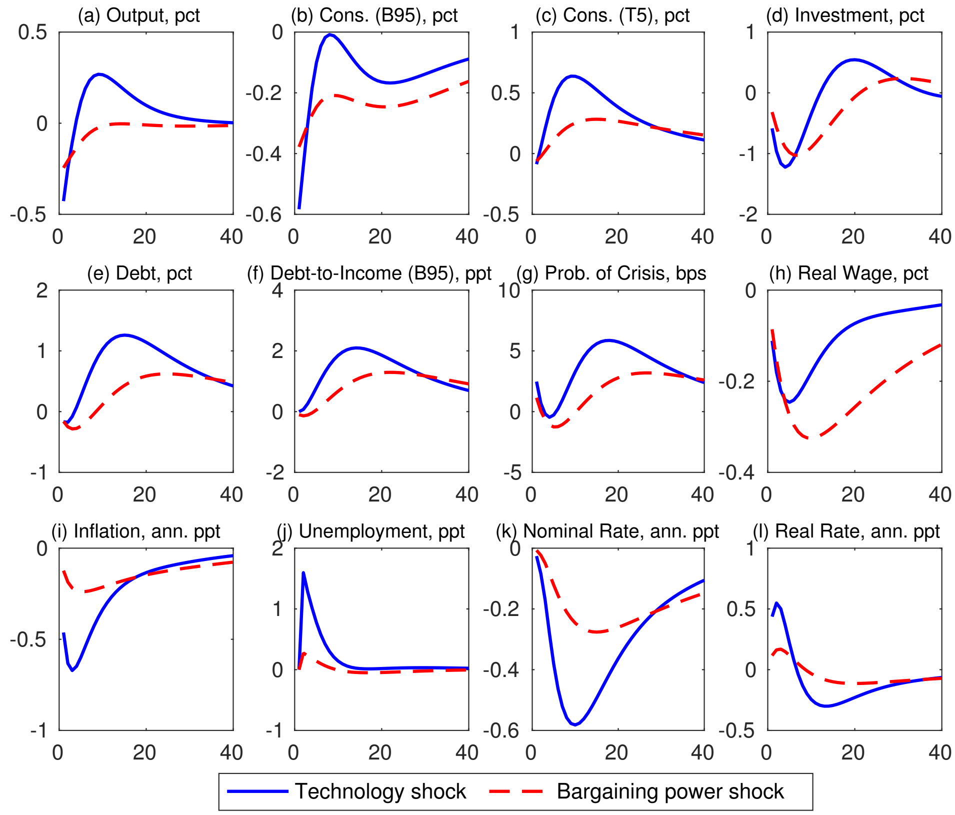
	Target/Source	Value
Capital share of production	Labor income share = 0.54	$\alpha = 0.20$
Investment adjustment cost	std(investment)/std(ouput)=3	$\kappa = 5$
Depreciation rate	Literature	$\delta = 0.025$
Separation rate	Literature	$\rho = 0.05$
Matching efficiency	Job finding probability=0.85	$\zeta = 2.3$
Matching function elasticity	Literature	$\epsilon = 0.50$
Worker's bargaining power	Hosios condition	$\eta = 0.50$
Unemployment benefit	$b^U\ /w = 0.9$	$b^U = 0.5$
Vacancy posting cost	Literature	$\xi = 0.11$
Real wage stickiness	S.D. of real comp. 4qtr. growth of non-farm bus. sec. = 1.6 percent	$\vartheta = 1000$
Inflation indexation	Literature	$\nu = 0.5$
Price stickiness	S.D. of inflation rate = 1 percent	$\varphi = 0.93$
Taylor rule: Inertia	Literature	$\rho_i = 0.85$
Taylor rule: Inflation gap	Literature	$\rho_\pi = 1.5$
Persistence of technology shock	Literature	$\rho_z = 0.90$
Persistence of bargaining power shock	Near Random Walk	$\rho_\eta = 0.95$
Persistence of risk premium shock	Literature	$\rho_\lambda = 0.90$
S.D. of technology shock	Literature	$\sigma_z = 0.010$
S.D. of bargaining power shock	1/3 of output variance share	$\sigma_\eta = 0.110$
S.D. of risk premium shock	1/3 of output variance share	$\sigma_\lambda = 0.001$

with so called "flat" Phillips curve. For the same reason, we choose a substantial degree of price indexation, setting $\upsilon = 0.5$. Even with these calibrations, inflation rate is substantially more volatile than in the data with its quarterly standard deviation being slightly greater than 1 percent, which is 2 times greater than in the data, and 4 times greater than in the Great Moderation period (1985-2006). However, we decide not to go all the way to fit the aggregate data given that our calibration is deviating from the micro-level data regarding price adjustment. We set ϑ the real wage adjustment cost equal to 1,000. While this seems an extreme calibration, this helps matching the standard deviation of 4 quarter growth rate of real compensation in the non-farm business sector. It also helps smoothing inflation dynamics.

As mentioned earlier, we let the monetary policy rule react to inflation gap only given the nature of the shocks being consistent with the so called divine coincidence. We set the inertial term $\rho_i = 0.85$ and calibrate the inflation coefficient ρ_π as 1.5 following Taylor (1999) in our baseline. Note that in an economy with the divine coincidence, setting $\rho_\pi = 1.5$ without the reaction term for the output gap implies a more lenient monetary policy rule than implied by Taylor (1999). However, we analyze a range of values for ρ_π in our analysis.

Finally, we calibrate the persistence of technology shock equal to 0.9 as is conventional. We then set the standard deviation of the technology shock equal to 1 percent. We set the persistence of risk premium shock equal to that of technology shock. Regarding the persistence of bargaining power shock, we set $\rho_\lambda = 0.95$. This is due to our prior that changes in bargaining power happen through

Figure 2: Impacts of Technology and Bargaining Power Shocks



changes in social norms and institutions and therefore the process should be a slow moving one. With these choices, we set the standard deviations of risk premium shock and bargaining power shock such that three shocks have equal variance decomposition shares for output in the long run. We use monetary policy shock only to satisfy the ZLB constraint.

4 Model Dynamics

4.1 Without Crisis and ZLB

In this subsection, we characterize the model dynamics using impulse response functions. In doing so, we assume that neither default nor binding ZLB constraint occur. We then show how these two sources of nonlinearities modify the model dynamics.

Figure 2 shows the effects of a positive technology shock (blue solid line), and a negative bargaining power shock (black dash-dotted line). Panel (a) shows that, despite the shock being positive, the response of aggregate output is initially negative before it turns positive. This is because our baseline monetary policy fails to provide enough monetary stimulus (see Gali (1999)). The positive technology shock lowers marginal costs and generates deflationary pressure as shown in panel (i). In response to the deflationary pressure, the monetary policy rate is lowered in panel (k). However, monetary accommodation is not strong enough and, as a result, the real interest rate initially raises as shown in panel (l), which then leads to a decline of real investment (panel (d)) and aggregate output.

Overall, the decline of aggregate demand and higher technology then leads to a decline of job creation, rising the unemployment rate in panel (j), and lowering the real wage in panel (h). Both the increase in unemployment and decline of real wage lead to a persistent decline of consumption for the bottom 95 percent earners. To smooth out the decline in consumption, the bottom 95 earners increase borrowing and the debt-to-income ratio of the bottom 95 percent is elevated, which then increases the probability of crisis in panel (g). In contrast, the consumption level of the top 5 percent earners increases persistently owing to higher profits. The decline of real investment also contributes to the increase in the consumption of the top 5 percent earners because the reduced investment level creates a slack in the budget constraint of this group.

In the literature, a shock to bargaining power affects the aggregate output through labor market channel. For instance, a lower bargaining power leads to an improvement in the job creation condition, expanding both employment and investment (see Gertler, Sala, and Trigari (2008)). Such a labor market channel exists in our model economy. However, the negative bargaining power shock also redistributes income from the bottom 95 percent earners to the top 5 percent earners. This redistribution may lead to a decline of aggregate demand if the marginal propensity to save of the top 5 percent earners is strong enough such that the reduced consumption of the bottom 95 percent earners is not completely offset by the increased consumption of the top 5 percent earners. Panel (b) and (c) show that this is indeed the case in our baseline calibration. Such an aggregate demand channel does not exist in a representative agent model.

Despite the fundamental improvement in job creation, employment actually decreases because of the fall in aggregate demand. For the same reason, investment decreases in panel (d). Since the lower bargaining power reduces the real wage level persistently, marginal costs decline and the economy faces a mild deflationary pressure. Monetary policy reacts by lowering the interest rate in (k), but the real interest rate is slightly increased in panel (l). The bottom 95 percent earners try to smooth out the decline in consumption by issuing more bond. Since the top 5 percent earners have to increase lending, they do not increase their own consumption sufficiently. As the debt-to-income ratio goes up persistently, the probability of default increases.

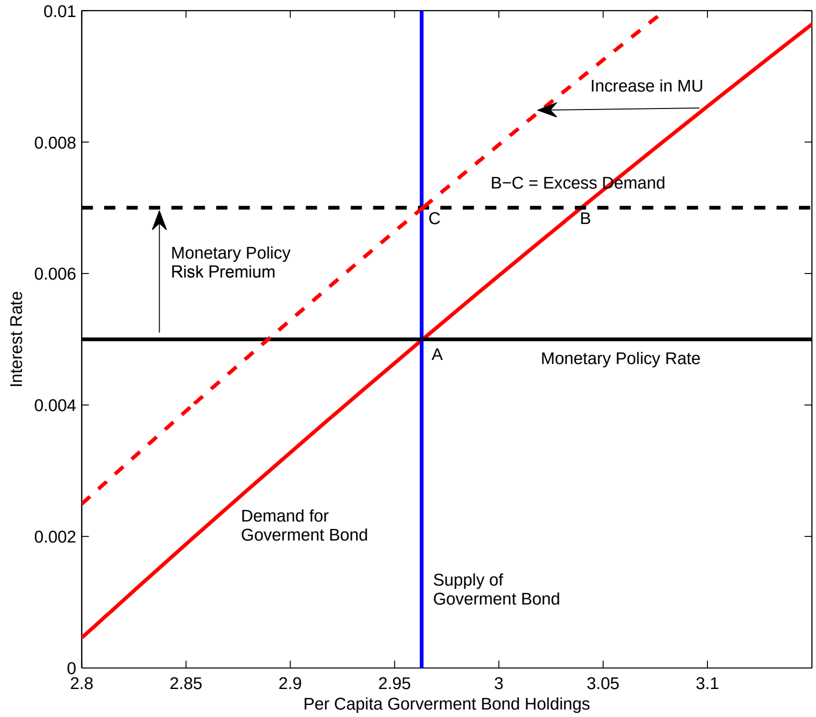
Before we move on to the effects of risk premium and monetary policy shocks, we find it useful to illustrate how these shocks are transmitted into the model economy. This is because the conventional transmission channel of monetary policy and risk premium shocks works somewhat differently owing to the additional term in the consumption Euler equation given by equation (8). We illustrate this using a monetary policy shock. The case of risk premium shock can be understood similarly given that the two shocks are isomorphic to each other.

In the nonstochastic steady state, the consumption Euler equation becomes

$$b^G = \left\{ \frac{\Lambda^T}{\psi^G} [(1+i^*)^{-1} - \beta^T] \right\}^{-\sigma_g} - 1. \tag{71}$$

This shows that the demand for government bond is an increasing function of risk-free rate. Since the

Figure 3: Monetary Policy and Government Bond Market Clearing

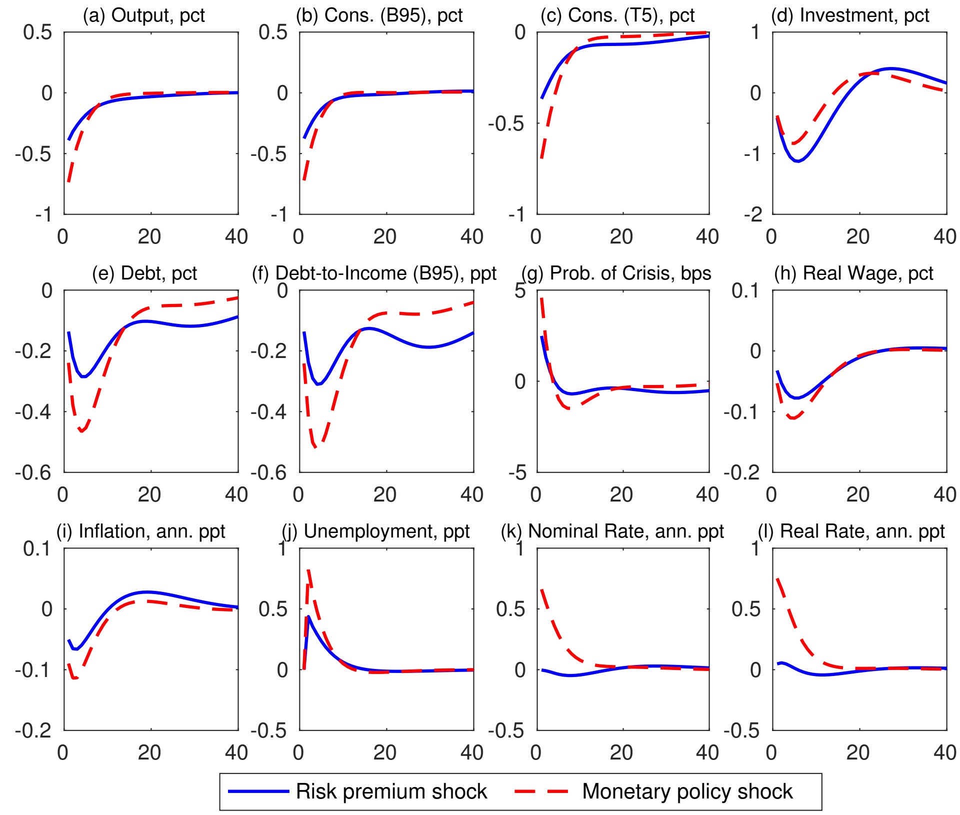


supply of government bond is fixed, the equilibrium interest rate is determined as the intersection of the upward sloping demand curve and the vertical supply as marked by point A in Figure 3. When the monetary policy rate is raised, the horizontal line shifts from black solid line to black dashed line. Given that the supply of government bond is fixed, this generates excess demand for government bond, marked by B-C. To clear the market, the marginal utility of the top 5 percent earners Λ^T should go up, i.e., the consumption level of the top 5 percent earners should go down such that the demand curve move to the left as indicated by red dashed line. Thus the new equilibrium is found at point C.

Figure 4 shows the impacts of risk premium shock (blue solid line) and monetary policy shock (black dash-dotted line). The two shocks are very similar in nature in that both affect the nominal return on risk-free bonds. There are two subtle differences in our context, however. First, the two shocks have different persistence: the monetary policy shock has a persistence of 0.85 due to the inertial term while the risk premium shock process has a persistence of 0.95 by assumption. More importantly, we assume that risk premium shock also affects the Euler equation for capital accumulation for the top 5 percent earners. Hence, we conjecture that if the two shocks are calibrated with similar standard deviation, the risk premium shock has stronger impact and propagation than the monetary policy shock.

As we conjectured, panel (a) shows that risk premium shock exhibit slightly more persistent dynamics than the monetary policy shock. Panel (d) also shows that the risk premium shock affects investment more substantially than the monetary policy shock because it disturbs capital accumulation decision more directly. The reason why the top 5 percent earners' consumption is negatively affected by this shock in panel (c) is no different from the New Keynesian literature. The shock alters

Figure 4: Impacts of Risk Premium and Monetary Policy Shocks



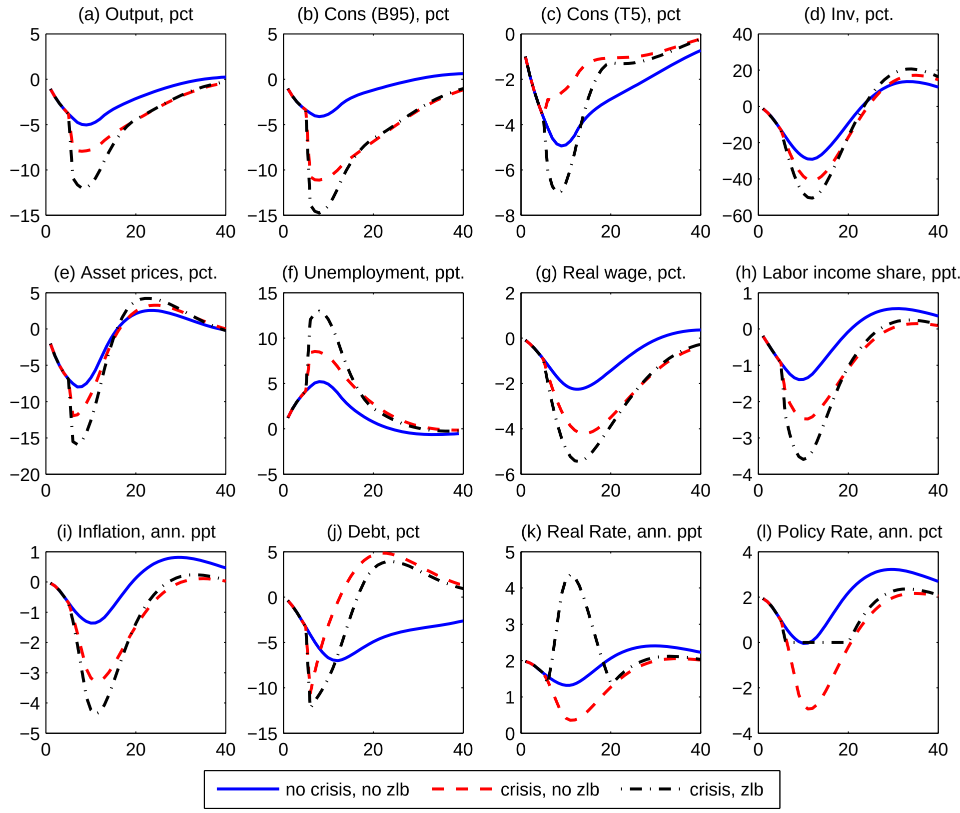
the dynamic profile of consumption of the agents whose consumption Euler equation is affected by this shock.

What is different from the literature is that in panel (b), the consumption level of the bottom 95 percent earners declines even more than the top 5 percent earners even though this shock does not directly affect the consumption Euler equation of the bottom 95 percent. This is because the decline of aggregate demand leads to fall in real wage and employment in panel (h) and (j). It is also because the bottom 95 percent earners find it difficult to smooth out their consumption decline in response to these two aggregate demand shocks in contrast to the case of technology shock and bargaining power shock. In both cases, the debt issuance of the bottom 95 percent earners declines in panel (e) as the borrowing cost is elevated by these shocks. The decline of consumption in the context of higher borrowing cost increases the incentive to default and the probability of default goes up in panel (g).

4.2 With Crisis and ZLB

Figure 5 illustrates how occasional eruptions of financial crises make a difference in the response of endogenous variables to a given series of random shocks. It also makes clear how the ZLB constraint works in conjunction with the endogenous crisis mechanism. To create an environment in which a financial crisis and a binding ZLB constraint may occur, we assume a sequence of risk premium shocks

Figure 5: Nonlinear Effects of ZLB and Financial Crisis



that are large enough to turn the economy into a deep recession.[12] We also assume that the inertial coefficient ρ_i is equal to zero to highlight the role of the binding ZLB constraint.[13]

The blue solid line shows the response of the economy under the assumption that a financial crisis does not occur during this episode. Whether or not a financial crisis occurs depends on the specific random draw that the economy is given for the utility cost of default and the difference between the values of default and non-default for the bottom 95 percent income earners. Note that the sequence of risk-premium shocks is hostile enough to send the nominal interest rate near zero as shown in panel (l). Also note that without a financial crisis, which the economy avoids by pure luck, the ZLB constraint does not bind in this example.

Panels (a)$\sim$(c) show that aggregate output, the consumption of the bottom 95 percent and the consumption of the top 5 percent all decline about 5 percent from their steady states. In panel (d), investment in physical assets declines nearly 30 percent. Panel (f) shows that the unemployment rate

[12]In particular, we assume $\epsilon_{\lambda,1:12} = \sigma_\lambda \times [\ 3\quad 3\quad 3\quad 3\quad 3\quad 3\quad 3\quad 2.5\quad 2\quad 1.5\quad 1\quad 0.5\ ]'$.

[13]Note that the inertial coefficient makes the ZLB constraint binding less frequently. This is because once the ZLB constraint binds, the interest rate in the next period assign $\rho_i \times 100$ percent weight to zero, which then makes the interest rate much higher than the level without the inertial coefficient. In our stochastic simulation, removing the inertial coefficient makes the ZLB constraint bind 5 times more than in the baseline with the inertial coefficient. While the role of the inertial coefficient is not the main focus of our analysis, we note that the presence of the inertial coefficient also makes the probability of financial crisis lower than in the baseline. Hence, we find a rationale for policy inertia that does not rely on the preference of the monetary authority regarding variability of policy instrument as in Woodford (1999) or on the determinacy and learnability as in Bullard and Mitra (2007).

rises up to 5 percentage points above its steady state level. Such a high unemployment rate creates a considerable downward pressure on the real wage in panel (g). The combination of real wage and employment responses is greater than the output response, as a result, the labor income share falls persistently as shown in panel (h). In our model, like in many New Keynesian models, the labor income share closely follows real marginal costs. Hence such a persistent decline in the labor income share generates substantial deflation pressure in panel (i). Finally panel (k) shows that the bottom 95 percent income earners face a long and painful process of deleveraging.

The red dashed line shows the case where the economy is given a particularly low realization of the random draw for the utility cost of default in the same situation, and consequently, a financial crisis occurs. This case assumes that the ZLB constraint does not exists as indicated by the fact that the level of nominal interest rate falls to -3 percent. Overall, the responses of all endogenous variables increase substantially except the consumption level of the top 5 percent income earners. In response to the financial crisis, the top 5 percent earners reduce investment in physical assets and financial assets to an extent that there exists a surplus cash flow that can be used to smooth out consumption expenditure.

The black dash-dotted line shows the case when the ZLB constraint exists in addition to the financial crisis, and thus the constraint becomes binding endogenously in response to the crisis. Our numerical procedure reverse-engineers a sequence of contemporaneous and news shocks to monetary policy that make the path of nominal interest rate consistent with the ZLB.[14]

The economy in this environment undergoes a much deeper recession. Aggregate output, for instance, can deviate as much as 7 percentage points from the baseline without the crisis and the binding ZLB constraint. Note that the size of financial crisis should be measured by the difference between the black dash-dotted line and the blue solid line rather than by the difference between the red dashed line and the black solid line. This example shows that the size of financial crises depends on overall economic environment at the time of crisis eruption. A financial crisis can happen even during a boom period. Depending on other shocks hitting the economy, the binding ZLB constraint may or may not be a part of the propagation channels of financial crises.

5 Income Inequality, Aggregate Demand and Financial Crises

In this section, we simulate the economy for a long period of time and compute the unconditional moments of the economy. This exercise can be thought of as Monte-Carlo simulations that obtain unconditional moments by simulation in an environment where analytical moments are not available due to the nonlinearities of the model. This exercise has the potential to uncover important links between income inequality, secular stagnation, deflation pressure and the likelihood of financial crises. The links have been conjectured but not yet quantified in the existing literature.

[14]Due to the endogenous feedback loop between the monetary policy and the economy, this requires an iteration that looks for a solution to a fixed point problem. Crudely speaking, the fixed point problem can be described as $i(\boldsymbol{\epsilon}_t) = f(i(\boldsymbol{\epsilon}_t))$, where $\boldsymbol{\epsilon}_t = [\epsilon_{0,t}, \ldots, \epsilon_{n,t}]$ is contemporaneous and news shocks to the policy rate, and $f(\cdot)$ represents the model economy, including the monetary policy reaction function. We look for a particular fixed point, in which $i(\boldsymbol{\epsilon}_t) = f(i(\boldsymbol{\epsilon}_t)) = 0$.

Table 3: Moments of Key Variables Under Baseline Monetary Policy Rule

	w/o ZLB	w/ ZLB
1. corr(income inequality, y)	-0.496	-0.787
2. corr(income inequality, c)	-0.465	-0.777
3. corr(income inequality, x)	-0.595	-0.695
4. corr(income inequality, π)	-0.952	-0.912
5. corr(income inequality, b/y)	0.719	0.384
6. corr(income inequality, p^{δ})	0.621	0.277
7. corr(consumption inequality, y)	-0.686	-0.807
8. corr(consumption inequality, c)	-0.678	-0.815
9. corr(consumption inequality, x)	-0.468	-0.502
10. corr(consumption inequality, π)	-0.809	-0.762
11. corr(consumption inequality, b/y)	0.903	0.695
12. corr(consumption inequality, p^{δ})	0.830	0.619
13. corr(b , y)	-0.277	0.667
14. corr(b , π)	-0.567	0.638
15. corr(b/y , p^{δ})	0.982	0.938
16. $E(p^{\delta})$	3.232	2.803
17. $E(i)$, quarterly, percent	-0.015	0.628
18. $E(\pi)$, quarterly, percent	-0.343	-0.698
19. $E(y)$	0.895	0.856
20. E(debt-to-income, bottom 95 percent)	1.701	1.547
21. E(labor income share)	0.521	0.508
22. s.d.(y) , percent	2.626	7.069
23. s.d.(π) , quarterly, ppts	0.635	1.162
24. corr($\hat{y}, \pi$)	0.437	0.894
25. frequency of ZLB, percent	-	2.123
26. skewness(i)	-0.199	0.841
27. skewness(π)	-0.202	-1.140
28. skewness(y)	-1.033	-1.713

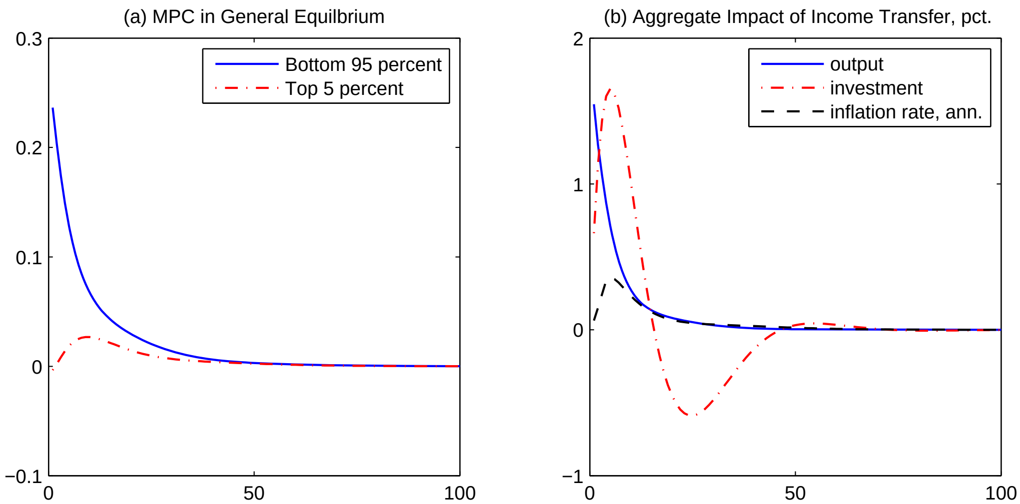
Note: We compute the moments using 500,000 simulations with identical set of random draws for shocks in each column. Each case assumes an inertial term coefficient for the interest rate of 0.85. For the method of generating financial crises, see the appendix.

Table 3 summarizes the moments of the model simulated for 500,000 periods under the baseline monetary policy rule, which takes the form of strict inflation targeting in that it only responds to the inflation gap.[15] To show the effects of the ZLB constraint, the table reports the results without and with the ZLB constraint separately on the first and second column, respectively.

Table 3 reports that income inequality, defined as the ratio of the income of the top 5 percent earners relative to the income of the bottom 95 percent earners, is negatively correlated with aggregate output and the degree of correlation is strong (on the order of -0.5 without the ZLB constraint and -0.8 with the ZLB constraint in line 1). When a growing share of national income is distributed to the group of agents whose MPC is much lower than the majority of agents, aggregate demand falls. This is why not only aggregate output, but also aggregate consumption and investment are negatively correlated with the income inequality in lines 2 and 3. Lines 7∼9 show that consumption inequality is also negatively correlated with aggregate quantities with similar degrees of strength. Lines 4 and 10

[15] In all comparisons of the moments below, we use an identical set of shocks for all alternative simulations.

Figure 6: Impact of Transitory Income Transfer Shock



point to the strong deflationary bias of the income/consumption inequality. We want to emphasize the role played by nominal rigidity. The negative link between income inequality and aggregate demand cannot be shown in the original framework of KRW because KRW's environment is a real business cycle type, and thus Say's Law applies. Exogenously given aggregate supply always determines the level of aggregate demand.

The composition effects of heterogeneous MPCs and the link between income inequality and aggregate demand can be shown in a more definitive way, in Figure 6, where we show the impact of redistributing income from the top 5 percent earners to the bottom 95 percent earners. We denote the per capita income transfer for the bottom 95 percent earners by ω. We calibrate this number as 10 percent of wage income of the bottom 95 percent earners in the steady state. Per capita transfer from the top 5 percent is then given by $-\omega(1-\chi)/\chi$. Panel (a) shows the marginal impacts of income redistribution on consumptions of the two agents, i.e., dc^W/ω (blue solid line) and $dc^S/[-\omega(1-\chi)/\chi]$ (red dash-dotted line).[16] Panel (a) shows that the marginal impact of transitory income shock is tremendously larger for the bottom 95 percent earners. Panel (b), displaying the impacts on aggregate output (blue solid line), investment (red dash-dotted line) and inflation rate (black dashed line), shows that improvement in income distribution, even when the shock is completely transitory, may bring a persistent boom by changing the composition of aggregate MPC.

Our results indicate that the secular stagnation due to income inequality and underconsumption, originally raised by Hansen (1939) and recently re-taken by Summers (2015), is a real possibility. Summers (2014), for instance, suggests that the decline of effective demand owing to the increase in income inequality is one factor behind secular stagnation:

> "Third, changes in the distribution of income, both between labor income and capital income and between those with more wealth and those with less, have operated to raise the

[16]The marginal impact meausures dc^W/ω and $dc^S/[-\omega(1-\chi)/\chi]$ are not the MPC discussed in micro-level. This is because the income distribution in our experiment brings a large general equilibrium effects as shown by the aggregate output response.

propensity to save, as have increases in corporate-retained earnings. (...) An increase in inequality and the capital income share operate to increase the level of savings."– Summers (2014)

Lines 5 and 11 of Table 3 show that the strong correlation between income/consumption inequality and excess credit, defined as the deviations of credit-to-GDP ratio from its steady state, such as seen in the data, can be replicated by our model. More importantly, lines 6 and 12 indicate a strong correlation between inequality and probability of financial crisis. Most importantly, in line 15, we find a near perfect linear relationship between the excess credit and the probability of financial crises regardless of the presence of the ZLB constraint. On the one hand, such a linear relationship replicates the statistical findings by Jorda, Schularick, and Taylor (2011) and Schularick and Taylor (2012). On the other hand, the near perfect linear relationship justifies the reduced form approach that assumes a (log) linear relationship between the excess credit and the probability of disaster without structural modeling. An approach, for instance, recently taken by Gourio, Kashyap, and Sim (2016).

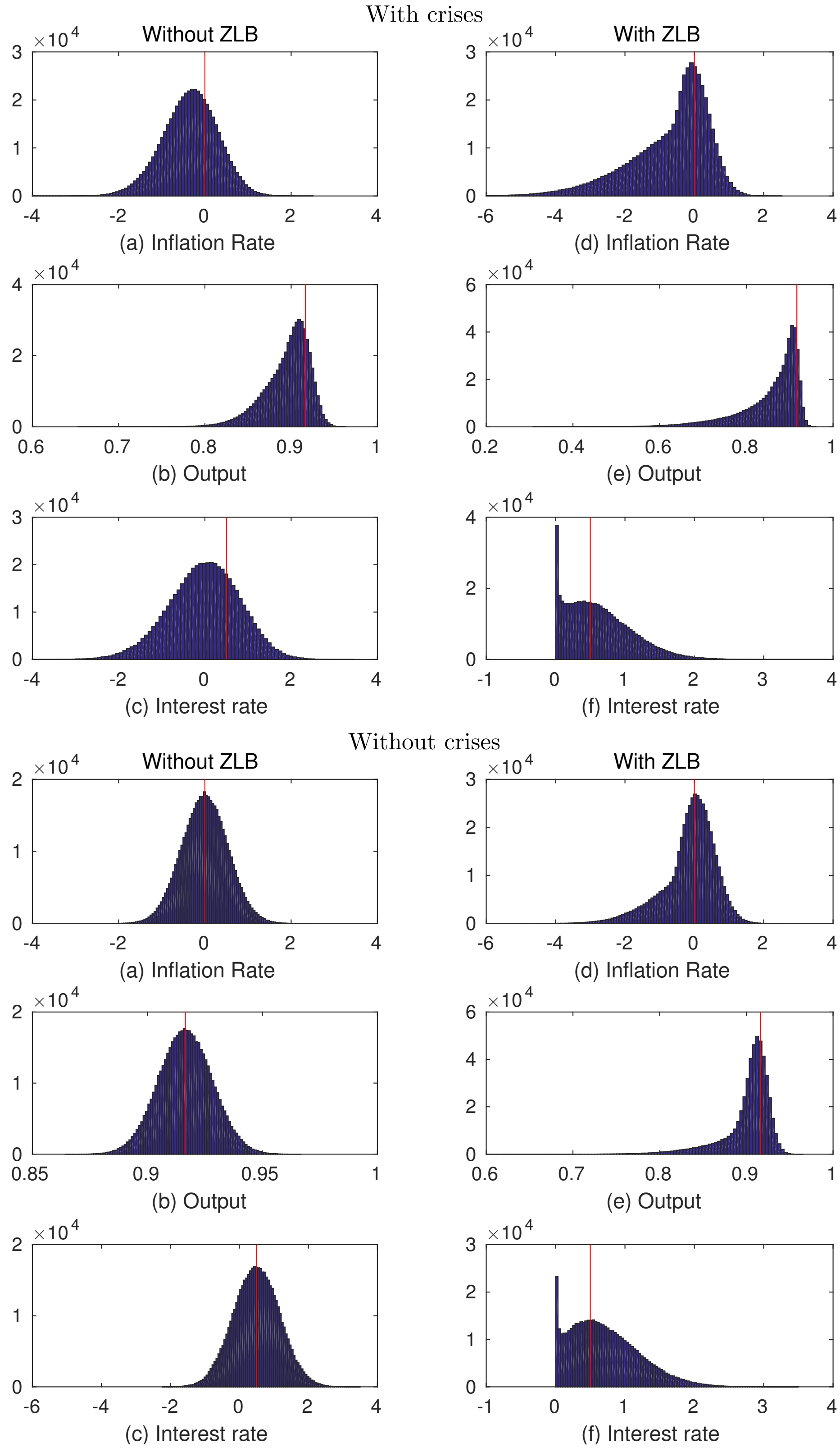
We finish this section by noting several consequences of the ZLB constraint. First, the presence of the ZLB constraint substantially strengthens the nexus between income/consumption inequality and aggregate demand as shown in lines 1∼3 and 7∼9. Despite this stronger tie between the inequality and aggregate demand, the ZLB constraint lowers the mean probability of financial crises from 3.23 to 2.80. This is because the ZLB constraint raises the mean level of interest rate, which increases borrowing cost and reduces the incentive of the bottom 95 percent earners to borrow. This can be seen in the decline of the debt-to-income ratio of these agents with the presence of the ZLB in line 20. This lowers the probability of financial crises.

Second, the ZLB constraint modifies the distributions of aggregate quantities. It increases the volatilities of aggregate output and inflation rate dramatically, from 2.63 percent to 7.07 percent and from 0.64 percentage points to 1.16 percentage points, respectively as shown by lines 22 and 23. More importantly, the ZLB constraint makes the distributions of endogenous quantities more skewed to the downside while it makes the distribution of the monetary policy rate more skewed to the upside, thus creating deflation bias.

Figure 7 shows how the exact shapes of the distributions change depending on the presence of financial crises and the presence of the ZLB constraint using histograms of simulated data. First, consider the lower 6 panels titled as "Without crises". The probability of financial crises, by construction of the logistic distribution (29), never approaches unity. This means that we can consider a special situation in which the random draws of utility cost are sufficiently high that crises never happen during the simulation. The left column shows the distributions of inflation rate, output and interest rate without the ZLB constraint. In this case, the skewness is almost exactly equal to zero for all three variables, which should not be surprising given the assumed Gaussian shocks.

Now consider the right column of the lower panels. In this case, we still assume that financial crises do not materialize, but the economy is subject to the ZLB constraint. Now the distributions of inflation rate and output are negatively skewed and the distribution of nominal interest rate is positively skewed. Compare this case with the left column of the upper 6 panels titled as "With

Figure 7: Nonlinear Effects of ZLB and Financial Crisis



crises", which is the case where we let financial crises happen according to the actual random draws of utility cost of default, but the economy is not subject to the ZLB constraint. The distributions of inflation and output are more negatively skewed in the latter case and the distribution of nominal interest rate is now negatively skewed. This shows that the crises themselves create deflation bias even without the ZLB constraint. This is not surprising because financial crises generate discrete jumps only to the down side of the economy.

Finally, consider the right column of the upper panels. The combination of crises and the ZLB constraint creates extraordinary degrees of skewness for the endogenous variables. Since negative skewness assigns more probability mass to the downside of the economy, this means that the mean levels of aggregate output and inflation rate are substantially lower either due to crises or the ZLB constraint. Indeed, line 19 in Table 3 shows that the mean level of output is 4.5 percent lower with the ZLB constraint. Line 18 shows that the mean inflation rate is -0.343 without the ZLB constraint and is -0.698 with the ZLB constraint. Hence, roughly speaking, the deflation bias created by the crises without the ZLB constraint and the additional deflation bias with the ZLB constraint appear to be similar.

The skewness of the distributions of endogenous variables and the resulting implication for the means of the distributions raise an important issue: if a particular policy has a benefit of reducing the probability or the size of financial crises, but if the policy also increases the frequency of the ZLB constraint binding, such a benefit should be weighed against the cost of increasing the frequency of binding ZLB constraint because the two aspects of the policy have opposite implications for the skewness, and hence for the mean of a variable of interest. For instance, if a particular policy rule is much more efficient in reducing either the probability or the size of financial crises, it may reduce the skewness of aggregate output despite the increased frequency of the ZLB constraint. This may result in first-order difference in welfare.

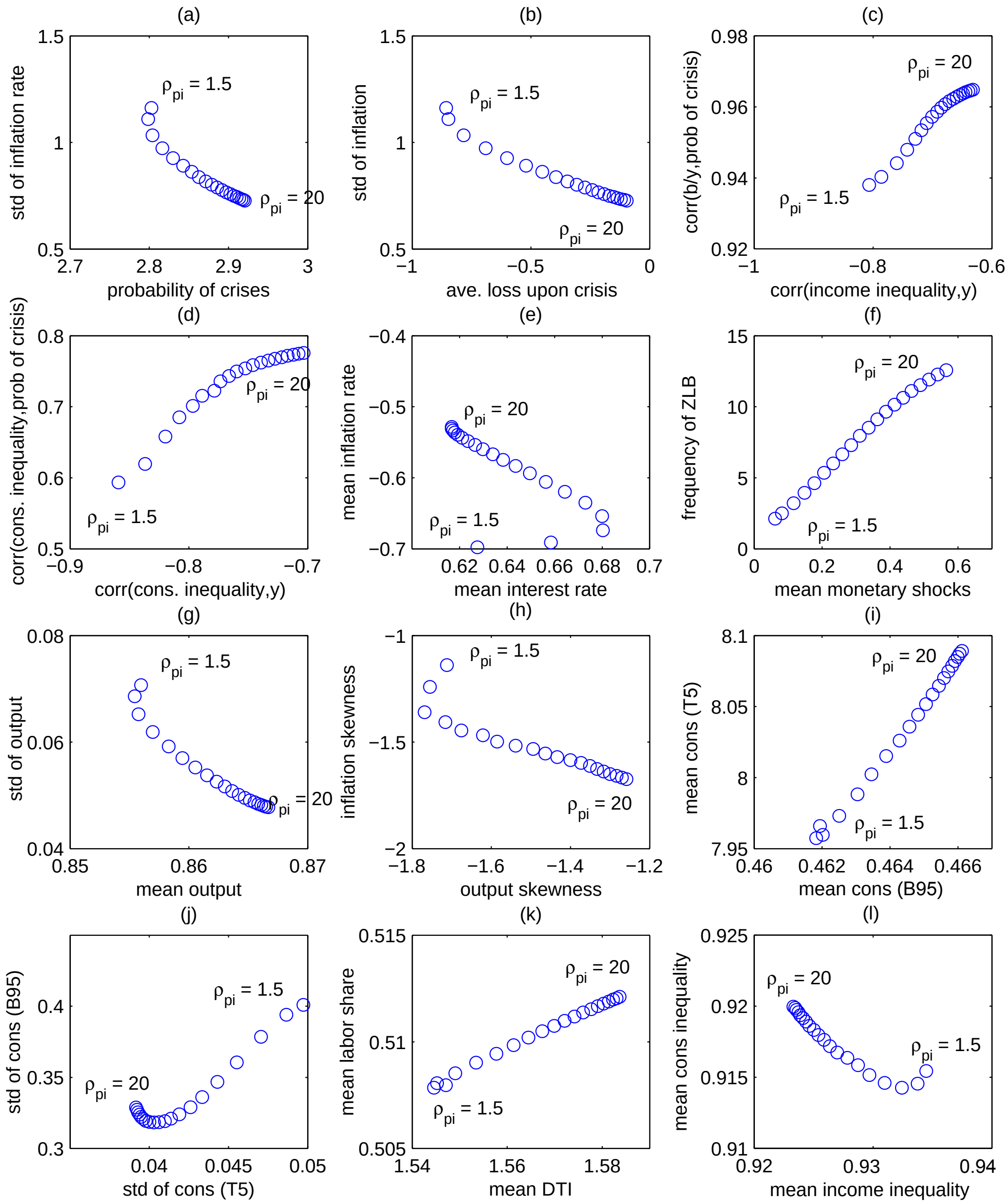
6 Monetary Policy and Financial Stability

In this section, we analyze how alternative monetary policy rules affect the probability and the size of financial crises, the link between income inequality and aggregate demand, and the link between excess credit and the probability of financial crises. We also investigate how changes in the monetary policy rule affect the skewed distributions of endogenous quantities, the means of aggregate output and consumption, and the frequency of ZLB constraints. We start with traditional inflation targeting regime.

6.1 Two Sides of Rogoff's Conservative Central Banker

In this section, holding the inertial coefficient of the baseline monetary policy rule (68) constant, we vary the strength of inflation reaction coefficient ρ_π from the baseline value of 1.5 to 20. Figure 8 summarizes the changes in the key moments of the model economy. In each panel, each dot shows the moments of 500,000 simulations. Panel (a) shows that increasing the inflation coefficient results in

Figure 8: Inflation Targeting and Financial Stability

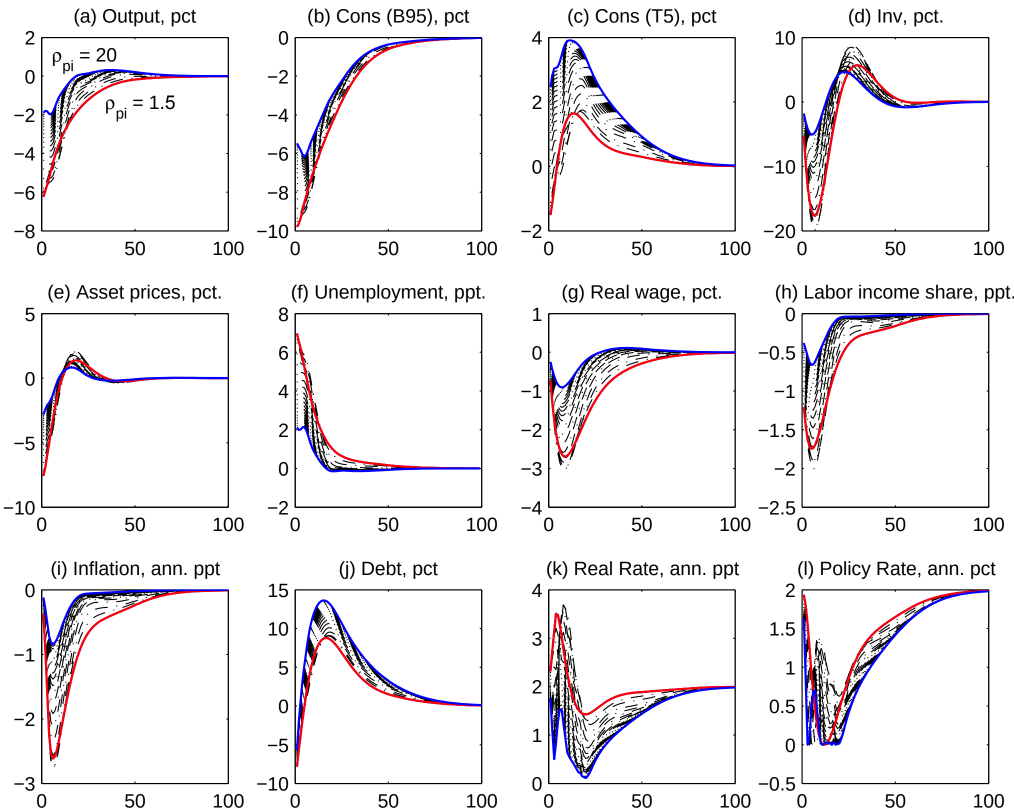


Note: We compute the moments using 500,000 simulations with the same set of random draws for shocks used in Table 3. These simulations assume an inertial term for the interest rate with a coefficient of 0.85. For the method of generating financial crises, see the appendix.

smaller standard deviation for inflation volatility on the vertical axis, but increases mean probability of crises on the horizontal axis. The latter aspect is due to the fact that decisive tightening in response to inflation pressure tends to raise the borrowing costs of lower income households and strengthens the incentive to default.

The result in panel (a) seemingly suggests that there exists a tradeoff between financial stability and price stability: to reduce the probability of financial crises, the monetary authority has to accept a greater volatility in inflation. However, the results shown in panel (b), showing the standard deviation

Figure 9: Impact of Crisis under Alternative Inflation Targeting Rules



of inflation on the vertical axis and the average output loss during a financial crisis on the horizontal axis,[17] suggests a different interpretation: while strong response to inflation increases the probability of crises, such a policy is also very effective in minimizing the output and employment loss during a crisis. In other words, so called Rogoff's conservative central banker trades high damage/low probability regime for low damage/high probability regime.

Figure 9 illustrates the impact of financial crisis shock under alternative inflation targeting rules. Panel (a) shows that by increasing the inflation coefficient from 1.5 to 20, the monetary authority can reduce the maximum damage to aggregate output from 6 percent to 2 percent during a typical crisis episode. Note that such an improvement requires the monetary authority to hold the nominal interest rate at the zero lower bound more than 20 quarters. In contrast, the baseline monetary policy rule barely touches the ZLB and lifts off from the ZLB too soon as shown in panel (l). Also note that the rule that minimizes the output and employment loss substantially boosts credit growth during the recovery ensuing the crisis as shown in panel (j). This helps reduce the drop in the consumption level of the bottom 95 percent income earners from 10 percent to 6 percent.

Panel (c) and (d) of Figure 8 show that strengthening the inflation response substantially weakens the negative link between income/consumption inequality and aggregate demand on the horizontal axes. However, this desirable feature comes at a cost of stronger positive link among excess credit,

[17]The average output loss is measured by the mean of the impulse response of output to a crisis shock for 100 periods. It is important to measure this output loss in an environment where the ZLB constraint is imposed.

consumption inequality and probability of crises shown on the vertical axes.

In panel (e), we find that Rogoff's conservative central banker can reduce substantially the deflation bias measured by the deviation of the mean inflation from zero. Note that in our environment, the deflation bias exists even without the ZLB constraint. This is because financial crises work only to the downside. Panel (f) shows the mean monetary policy shocks that are needed to implement the ZLB constraint on the horizontal axis and the frequency of binding ZLB constraint.[18] One can see that Rogoff's conservative central banker increases the frequency of the binding ZLB constraint from 2 percent to 12 percent. This shows that the conservative central banker reduces the skewness of output and inflation distributions due to financial crises, but increases the skewness owing to the ZLB constraint. Overall impact on the skewed distributions is mixed as shown in panel (h): the conservative central banker reduces overall skewness of aggregate output, but increases the skewness for inflation.

In panel (g), moving inflation coefficient from 1.5 to 20 increases the mean level of aggregate output 1.4 percent. The same move also reduces the volatility of aggregate output almost 50 percent. Both of these outcome may have a first-order impact on welfare. Furthermore, in panels (i) and (j), the same policy move increases the mean consumption levels and decreases their volatilities for both types of agents.[19] Finally, in panels (k) and (l), the conservative inflation targeting has the benefits of elevating the mean level of labor income share and reducing income inequality. However the same panels show the mixed results of increasing debt-to-income ratio of the bottom 95 percent earners and increasing consumption inequality. These last two aspects certainly contribute to the elevated probability of crises seen in panel (a).

So far, we have shown various aspects of Rogoff's conservative central banker. Despite many desirable aspects of the conservative central banker, we find the undesirable tendency of increasing the probability of crises, while the same central banker wants to accommodate the damage upon crisis decisively. These two different aspects of the conservative central banker suggests that the optimal use of inflation targeting may involve an opportunistic mix: adopt lenient policy rules against inflation during normal times to maintain the probability of financial crises at low levels, but adopt Rogoff's conservative central banker once a financial crisis erupts. Such an asymmetric and opportunistic policy mix may correct the consequences of asymmetrically large downside risks.

6.2 Promising to Lower the Natural Rate of Interest

We have shown that financial crises may create disproportionately large downside risks by themselves and by increasing the likelihood of binding ZLB constraint. In response to such skewed distributions of endogenous prices and quantities, symmetric monetary policy rules such as Taylor rule may not be optimal.

For this reason, in this section, we consider an asymmetric monetary policy rule. Since the skewed distributions are due to the crises that only work to the downside of the economy, we consider a

[18]The mean monetary policy shock includes both contemporaneous and news shocks.

[19]Panel (i) reports per capita consumption levels for each agent type whereas income/consumption inequality is constructed as the ratio of aggregates.

monetary policy rule that deviates from its normal form only in response to financial crises. In particular we consider the following variation of (68):

$$i_t = \rho_i i_{t-1} + (1-\rho_i)\left[i^* + i_t^* + \rho_\pi \left(\frac{\pi_t^Y - (\pi^* + \pi_t^*)}{4}\right)\right] + \sigma_m \sum_{j=1}^{n} \epsilon_{j,t-j} + \sigma_m \epsilon_{0,t}, \tag{72}$$

where we now assume that the monetary authority deviates from its long-run intercept of Taylor rule and the deviation is determined by the following rule:

$$i_t^* = \rho_{i^*} i_{t-1}^* - \sigma_{i^*} \delta_t^B, \quad \sigma_{i^*} > 0,\ 0 < \rho_{i^*} < 1. \tag{73}$$

In response to the eruption of a crisis, the monetary authority announces that its short-run intercept of monetary policy rule will be lowered from its long run value substantially and persistently and go back to the long run level only gradually. How low and how long depend on σ_{i^*} and ρ_{i^*}. This is an extraordinary form of a forward guidance in that lowering the interest rate through the adjustment of the intercept in response to a crisis implies that the central bank promises to let the nominal rate deviate from the long-run natural rate even when the inflation rate reaches its long-run target if ρ_{i^*} is sufficiently close to 1. Exactly the same effect can be achieved by announcing that the short-run inflation target will be raised for an extended period of time in response to the eruption of a financial crisis. For this reason, we call the policy rule given by (72) and (73) 'time-varying inflation targeting rule'.[20] Note that δ_t^B takes only 0 or 1. This means that the time-varying inflation target only deviates to the upside and thus works as an antidote the deflation bias caused by financial crises and/or the ZLB constraint.

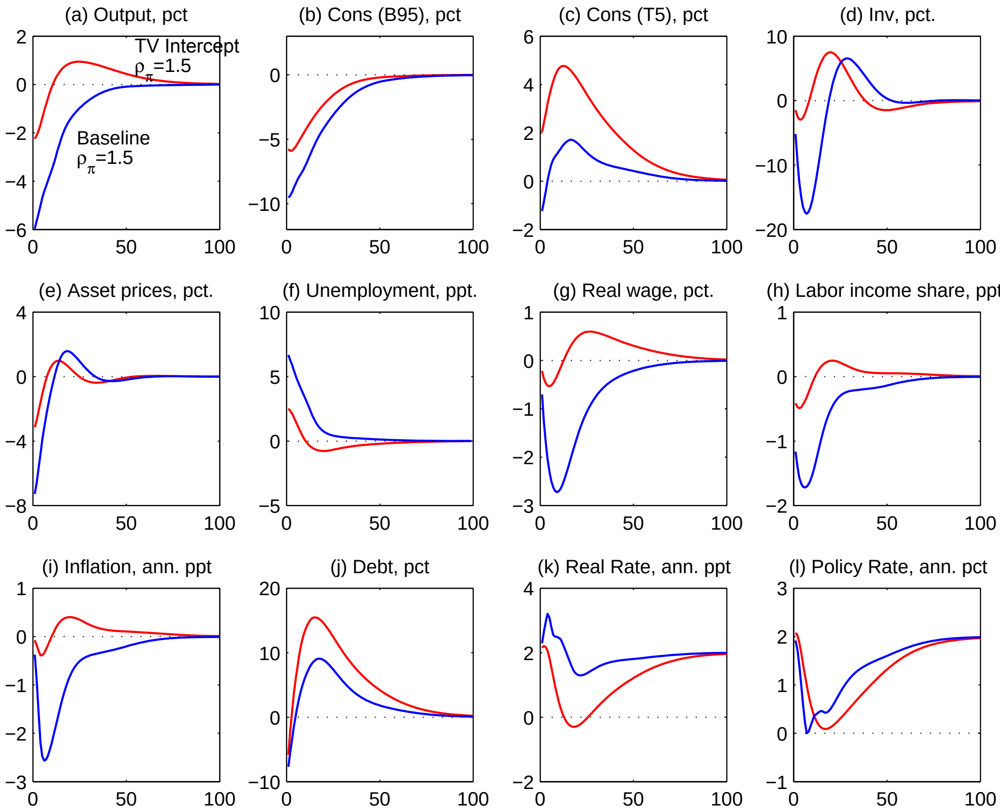
Figure 10 shows the results of implementing the time-varying inflation targeting rule in response to a financial crisis. For this experiment, we assume $\sigma_{\pi^*} = 0.005$ and $\rho_{\pi^*} = 0.99$. In words, the monetary authority lowers the intercept to zero immediately in response to the crisis, and raises it back in an extremely gradual fashion with a decay rate of 0.01. We assume that the monetary authority otherwise follows a strict inflation targeting with its other policy coefficients fixed as in the baseline, i.e., $\rho_i = 0.85$ and $\rho_\pi = 1.5$. For a comparison, blue line shows the case of the baseline with its intercept fixed at 0.005.

Figure 10 shows that such an unconventional and asymmetric monetary policy can be extremely effective in stabilizing the aftermath of financial crises. The policy limits the drop of output at 2 percent level and restrains inflation rate from falling more than 40 basis points, which is in stark contrast to 250 basis points drop in the baseline case. One may be puzzled by the results because the path of nominal interest rate shown in panel (l) is not significantly different from the baseline. However the path of nominal interest rate is somewhat misleading. The path of real interest rate shown in panel (k) indicates considerably more accommodative policy stance.

The time-varying inflation targeting rule may bring a sizable welfare gain by correcting the skewed distribution. For instance, the skewness of output under the time-varying inflation targeting rule is

[20] For an obvious reason, the rule can be called "inflation targeting rule augmented with time-varying intercept".

Figure 10: Augmented Inflation Targeting Rule with Time-Varying Intercept During a Crisis



reduced to -1.384 from -1.713 of the baseline. Consequently, the mean level of output under the time-varying inflation targeting rule is increased more than 7 percent. Similarly, the skewness of inflation rate is reduced to -0.777 from -1.140 and the mean inflation rate is now small positive of 0.4 percent. The time-varying inflation targeting rule incarnates the spirit of the opportunistic conservative central banker: implementing lenient monetary policy against inflation during normal times without financial crises, but providing decisive accommodation to the economy by lowering the intercept persistently. Our results indicate that the benefits of such an asymmetric monetary policy can be dramatic even at this case when the policy coefficients are arbitrarily calibrated. In the next section, we show the case of fully optimized simple rules.

7 Fighting the Skewed Distribution

7.1 A New Loss Function

The conventional notion of stabilization policy is to minimize the fluctuations of target variables around their provided. This notion implicitly assumes that the means of equilibrium prices and quantities are determined outside the control of stabilization policies. This conventional view is summarized by a symmetric loss function such as

$$L = \lambda \mathbb{E}_0[(\pi_t - \pi^*)^2] + (1-\lambda)\mathbb{E}_0[(u_t - u^*)^2]. \tag{74}$$

However, our results suggest that the conventional loss functions composed of variances of target variables such as inflation and output gap (or unemployment gap) may need to be reconsidered because the underlying data generating process may be severely distorted to the downside and the degree of negative skewness, and consequently, the mean of target variable may depend on a specific monetary policy rule.

In this paper we propose an alternative loss function to take into account the presence of disproportionately large downside risks. For simplification, we first omit the term for unemployment deviation. This is because, in the absence of markup shock, the divine coincidence holds in our model economy. Second, we propose an additional term to capture the degree of skewness of inflation rate:

$$L = \lambda \mathbb{E}_0[(\pi_t - \pi^*)^2] + (1-\lambda)\mathbb{E}_0 \left\{ \left| \left(\frac{\pi_t - \bar{\pi}}{\sigma_\pi} \right)^3 \right| \right\} \tag{75}$$

where π^* is the target of the central bank and $\bar{\pi}$ and σ_π are the mean and the standard deviation of the inflation rate under a given monetary policy rule. This specification of the loss function makes explicit the costs of failing to control the excessive skewness that is generated by the combination of financial crises and limited effectiveness of monetary policy in responding to the downside risks under the ZLB constraint. Panels (b) and (h) of Figure 8 suggest that there exists a tradeoff between the standard deviation and the skewness of inflation.

Note that the second moment of the loss function (75) is not a centered moment as the mean of the distribution may deviate from the target substantially and the central bank wants to penalize the deviation from the target, not from the mean of the distribution. However the third moment is a centered moment as we assume that the central bank wants to penalize the pure statistical asymmetry of the distribution.

7.2 Optimal Simple Rules under the New Loss Function

In this section, we optimize the two monetary policy rules considered in this paper, namely, strict inflation targeting policy (68), the augmented inflation targeting rule that we call natural rate adjustment policy (72) to minimize the loss function (75).

The optimization results crucially depend on the weights given to the two moments of the loss function. For this reason we optimize the policy rules under three specifications: $\lambda = 0.0$, 0.5 and 1.0. With $\lambda = 1.0$, the loss function becomes identical with the conventional loss function. With $\lambda = 0.0$, the loss function penalizes only the skewness. With $\lambda = 0.5$, the loss function assigns an equal weight to the two moments. We take this last case as our benchmark specification.

Table 4 reports the optimization results. Consider the middle panel, which shows the case $\lambda = 0.5$. The two monetary policy rules find it optimal to choose a relatively low inflation coefficient. This is

Table 4: Optimal Simple Rules

	$\lambda = 0.0$		$\lambda = 0.5$		$\lambda = 1.0$	
	R1	R2	R1	R2	R1	R2
ρ_π (inflation coeff.)	1.000	1.496	1.630	1.252	20.00	11.74
ρ_i (inertial term)	0	0.493	0.000	0.000	0.000	0.063
ρ_i^* (persistence of r^*)	–	0.995	–	0.984	–	0.995
σ_i^* (vol. of shocks to r^*)	–	0.005	–	0.005	–	0.005
L (loss function value)	0.676	0.000	1.111	0.769	0.715	0.539

Note: R1 refers to strict inflation targeting policy, R2 to time-varying inflation targeting rule. Minimization program simulates the model economy 500,000 simulations with identical set of random draws for shocks for each set of candidate coefficients. We then compute the loss function values and search for minimizer of the loss function value for each parameter of a given monetary policy rule. The ranges of parameter values given to the minimization programs are the followings: for inflation coefficient, [1.0,20]; for the inertial term, [0.0, 0.9]; for persistence of r-star, [0.0, 0.995]; for volatility of shocks to r-star, [0.0,0.005].

because the loss function assigns an equal weight to the skewness and it is beneficial not to raise the mean probability of default by keeping the inflation coefficient low.

Regarding the inertial coefficient, both rules choose zero level of policy inertia. This is because the minimum level of policy inertia helps the economy reach the ZLB as soon as possible during financial crises to give maximum degree of crisis accommodation especially given that the inflation coefficients are low. This outcome suggests that the reduction in negative skewness due to financial crises is greater than the increase in negative skewness owing to hitting the ZLB more frequently.

The time-varying inflation targeting rule chooses to drop the natural rate to zero immediately in response to an eruption of financial crisis as can be seen in the fact that $\sigma_i^* = 0.005$. Furthermore the rule chooses an extreme persistence level of $\rho_i^* = 0.984$ in bringing back the natural rate to a normal level. Such a policy commits itself not to raise the interest rate too fast even when the inflation rate reaches or overshoots the target. Since this policy is called for only to cope with financial crises, such policy can provide an effective correction to the skewed distributions.

The optimized loss shows that in the benchmark case of equal weights on the second and third order moments of inflation rate, the optimized time-varying inflation targeting rule performs substantially better than the strict inflation targeting rule. This casts doubts on the consensus upon the desirability of symmetric inflation targeting rule once we introduce asymmetric business cycle trough financial crises and the ZLB constraint.

The first panel of Table 4 shows what happens if the central bank ignores the volatility of inflation and simply focuses on reducing the large downside risks. Not surprisingly, the strict inflation targeting rule hits the lower bound on inflation coefficient of 1.0 in order to reduce the probability of financial crises. (see the note of Table 4 for the full ranges of parameter values given to the minimization programs). The time-varying inflation targeting rule also chooses a relatively low coefficient for inflation rate of about 1.5.

Regarding the policy inertia, again, the strict inflation targeting chooses the lower bound for policy inertia as the rule does not have any other shock absorption other than hitting the ZLB as soon as possible in repose to financial crises. In contrast, the optimized time-varying inflation targeting rule now chooses a substantial degree of policy inertia as the optimized inertial term is close to 0.5. This is in stark contrast to the cases where strictly positive weights are given to inflation volatility in which optimized inertial coefficient is essentially zero. This suggests that the more the policymakers concern about the presence of asymmetric macroeconomic risks, the greater the preferences for policy gradualism.

The time-varying inflation targeting rule chooses the maximum degree of crisis accommodation (i.e., $\sigma_i^* = 0.005$) and an extreme degree of persistence as well to provide a strong monetary accommodation during financial crises. In terms of the loss function values, the time-varying inflation targeting rule outperforms the strict inflation targeting rule by a decisive margin as the optimized time-varying inflation targeting rule essentially removes the skewness completely.

The last panel of Table 4 shows the case where the central bank completely ignores the presence of larger downside risks. Not surprisingly, inflation coefficients of all cases increase substantially. It is an irony that the strict inflation targeting rule underperforms the time-varying intercept adjustment rule in terms of minimized loss function value even when the loss function consists entirely of the volatility of the inflation rate even though the strict inflation targeting rule is designed to minimize the volatility of inflation. This shows the limit of symmetric monetary policy even for the stabilization of volatility of inflation rate when the distribution of inflation rate is severely skewed to the downside.

Table 5 summarizes the changes of the distributions of equilibrium quantities and prices. The first row shows how widely the mean of aggregate output can change depending on the weights given to the loss function terms and depending on a specific policy. For instance, the mean level of output is maximized under the combination of zero weight on the volatility of the inflation rate and the adoption of the time-varying inflation targeting rule. The mean level of output in this case is 1.068, which is 15 percent above the mean level of output under the baseline inertial Taylor rule. Such a large gain in the mean level of output is possible because the policy rule under these preferences of the central bank eliminates the skewness of output and inflation almost perfectly. However this gain comes at a cost of elevating both the mean and the standard deviation (vs the target) of inflation rate to more than 3 percent.[21]

Such a volatile and high inflation rate may face substantial resistance from policymakers who are leaning toward the traditional mandate of price stability. However, consider adopting the time-varying inflation targeting rule, and moving from zero weight ($\lambda = 1.0$) to one half weight ($\lambda = 0.5$) for the skewness measure. The transition does not require such a dramatic deterioration in the mean and the standard deviation of inflation rate. Table 5 shows that under the optimized time-varying inflation targeting rule, the transition increases inflation volatility only marginally. In contrast, the transition can still bring large benefits in terms of the means of aggregate output and inflation rate: the mean of aggregate output is more than 3 percent higher and the deflation bias almost completely disappears as the mean inflation rate is close to zero. Overall, our results indicate that there may be a large room

[21] Figure 13 in Appendix A shows the detailed shapes of the distributions of inflation rate, output and nominal interest rate under the optimized strict inflation targeting and the r-star adjustment rule both in the case of $\lambda = 0.0$ (upper two rows) and in the case of $\lambda = 1.0$ (bottom two panels).

Table 5: The Moments of Equilibrium Prices and Quantities under Optimized Policy Rules

	$\lambda = 0.0$		$\lambda = 0.5$		$\lambda = 1.0$		Baseline
	R1	R2	R1	R2	R1	R2	
Output							
1. Mean	0.866	1.068	0.867	0.907	0.868	0.883	0.856
2. Std. dev.	6.018	6.880	5.352	4.743	4.726	4.578	7.069
3. Skewness	-1.441	-0.124	-1.598	-1.335	-1.065	-1.088	-1.713
Inflation							
4. Mean	-0.571	3.059	-0.567	0.110	-0.493	-0.241	-0.698
5. Std. dev.	1.126	1.317	0.984	0.978	0.687	0.694	1.162
6. Std. dev. (vs target)	1.276	3.331	1.136	0.984	0.846	0.734	1.355
7. Skewness	-0.676	0.000	-0.933	-0.569	-1.686	-1.625	-1.140
Crisis/ZLB							
8. Mean prob. of crises	2.903	5.413	2.918	3.403	2.947	3.123	2.803
9. Frequency of binding ZLB	9.810	9.727	11.39	13.24	19.72	18.20	2.123
10. Ave. output loss (crisis)	-4.375	-0.483	-3.297	-1.075	-0.802	-0.588	-3.457
11. Skewness of prob. of crisis	0.573	0.477	0.258	0.602	-0.297	-0.186	0.696
Income Inequality							
12. Mean income inequality	0.914	0.819	0.914	0.894	0.911	0.904	0.919
13. Mean con. (T5)	8.090	10.50	8.094	8.573	8.109	8.283	7.966
14. Mean con. (B95)	0.466	0.540	0.466	0.480	0.467	0.472	0.462
15. Coeff. of var. of con. (T5)	0.032	0.072	0.029	0.033	0.043	0.041	0.050
16. Coeff. of var. of con. (B95)	0.099	0.084	0.093	0.084	0.083	0.082	0.108
17. Skewness of con. (T5)	-1.917	0.428	-1.317	-0.047	-1.081	-1.059	-2.000
18. Skewness of con. (B95)	-1.166	-0.843	-1.284	-1.153	-1.274	-1.304	-1.407
19. Real Interest Rate, ann	4.425	-10.07	4.494	1.486	4.664	3.522	5.300

Note: R1 refers to strict inflation targeting policy, R2 to time-varying inflation targeting rule. The moments reported in the table are based on 500,000 simulations of the model economy under each optimized policy rule with identical set of random draws for shocks used in the minimization of the loss function. Ave. output loss (crisis) is measured by the 20 period average of the impulse response of output to crisis shock. The moments of consumption (con. (T5) and con. (B95)) refer to the moments of per capita consumption values of the two agents.

for improvement in terms of welfare and costs of business cycle by adopting an asymmetric monetary policy designed to fight the skewed distributions of endogenous quantities and prices.

In contrast, even when the central bank adopts the equal weights on the volatility and the skewness in its loss function, sticking to the strict inflation targeting may not reap the same benefit. The transition from $\lambda = 1.0$ to $\lambda = 0.5$ does not increase the mean of the output distribution and the deflation bias as measured by the mean deviation of inflation rate from the target actually deteriorates. Even simply adopting the time-varying inflation targeting rule with no explicit weight to the skewness measure results in non-trivial gains in the means of aggregate output and inflation with respect to the baseline.

Note that the transition to $\lambda = 0.5$ or $\lambda = 0.0$ results in a substantial increase in the probability of financial crises under the time-varying inflation targeting rule as shown by row 8. This is because the transition under the time-varying inflation targeting rule increases the mean inflation rate substantially even while decreasing the mean nominal interest rate slightly, lowering the real interest rate significantly. The much more lower real interest facilitates the borrowing of the bottom 95 percent income earners, which then creates a greater incentive to default.

Does that mean that the transition under the time-varying inflation targeting rule strengthens the vulnerability of financial system? The answer is quite the opposite. To see this consider row 10, which reports the average output loss during a financial crisis.[22] The expected output loss due to financial crises is given by the probability of crisis times the average output loss conditional upon a crisis. While such a transition under the time-varying inflation targeting rule increases the probability of crises, the same transition lowers the average output loss conditional upon a crisis even more dramatically such that the actual expected loss of output due to financial crises goes down substantially with the transition under the time-varying inflation targeting rule.

We note that the transition to a regime that assigns a strictly positive weight to the skewness measure of inflation rate under the natural rate adjustment can greatly lessen the degree of income inequality as measured by the ratio of the income of top 5 percent earners relative to the income of bottom 95 percent earners (shown in row 12). This outcome is not because the central banker in the model economy is assigned direct preferences for low degree of income inequality. Rather the improvement in income inequality is simply the result of transitioning to a regime that takes into account the asymmetrically large downside risk and implementing a policy rule that can fight against the asymmetry by responding to crises in an asymmetric way.

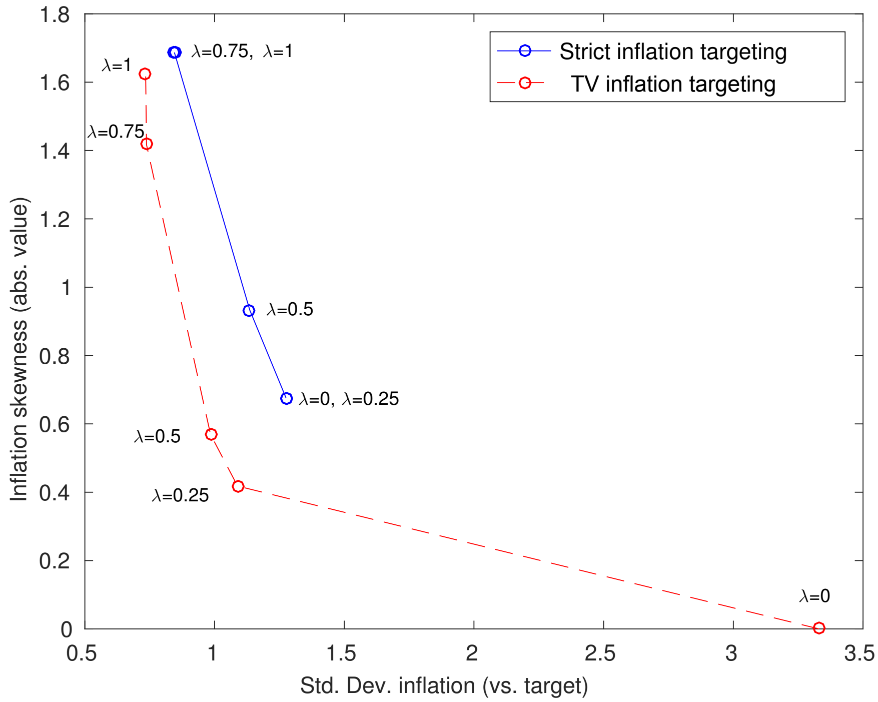
However, there is a sense in which this outcome is not an intended consequence, but a predictable consequence nonetheless. Using the baseline economy, we have shown that income inequality is behind many of the undesirable consequences such as insufficient aggregate demand, deflation bias, overborrowing and financial instability. Therefore our analysis suggests that the transition to a new loss function regime under the time-varying inflation targeting rule could not ameliorate such undesirable consequences of income inequality without addressing the root cause of these consequences.

We also note that the improvement in income inequality is achieved in a way that does not deteriorate the outcome for the top 5 percent income earners. In fact, moving from $\lambda = 1.0$ to $\lambda = 0.5$ under the time-varying inflation targeting rule increases the mean level of per capita consumption of top 5 percent income earners more than 4 percent while reducing its volatility (in the sense of the coefficient of variation) more than 25 percent. This shows a large room for Pareto improvement once the monetary policy gives up symmetric inflation targeting rules. It is notable in this sense that the same transition from $\lambda = 1.0$ to $\lambda = 0.5$ under the strict inflation targeting does not reduce the income inequality. It neither improves the mean consumption level of top 5 percent earners nor the mean consumption level of bottom 95 percent earners. It reduces the volatility of consumption for top 5 percent earner, but increases it for bottom 95 percent earners.

One notable phenomenon is that while moving from $\lambda = 1.0$ to $\lambda = 0.0$ under the time-varying inflation targeting rule reduces the income inequality about 10 percent, the same transition also results in much greater consumption level of the top 5 percent income earners. This suggests that

[22]This is measured by average decline of output for 20 quarters in response to a crisis shock.

Figure 11: Efficient Policy Frontiers



the transition increases the MPC for the top 5 percent earners by holding the real interest rate at a substantially lower level on average. For instance, row 19 shows that the level of annual real interest rate is reduced as much as 13 percentage points by moving from $\lambda = 1.0$ to $\lambda = 0.0$. This shows that if the root cause of insufficient aggregate demand lies at the low MPC of group with large income shares, the asymmetric monetary policy rule proposed in this paper can provide an effective tool to fight insufficient aggregate demand.

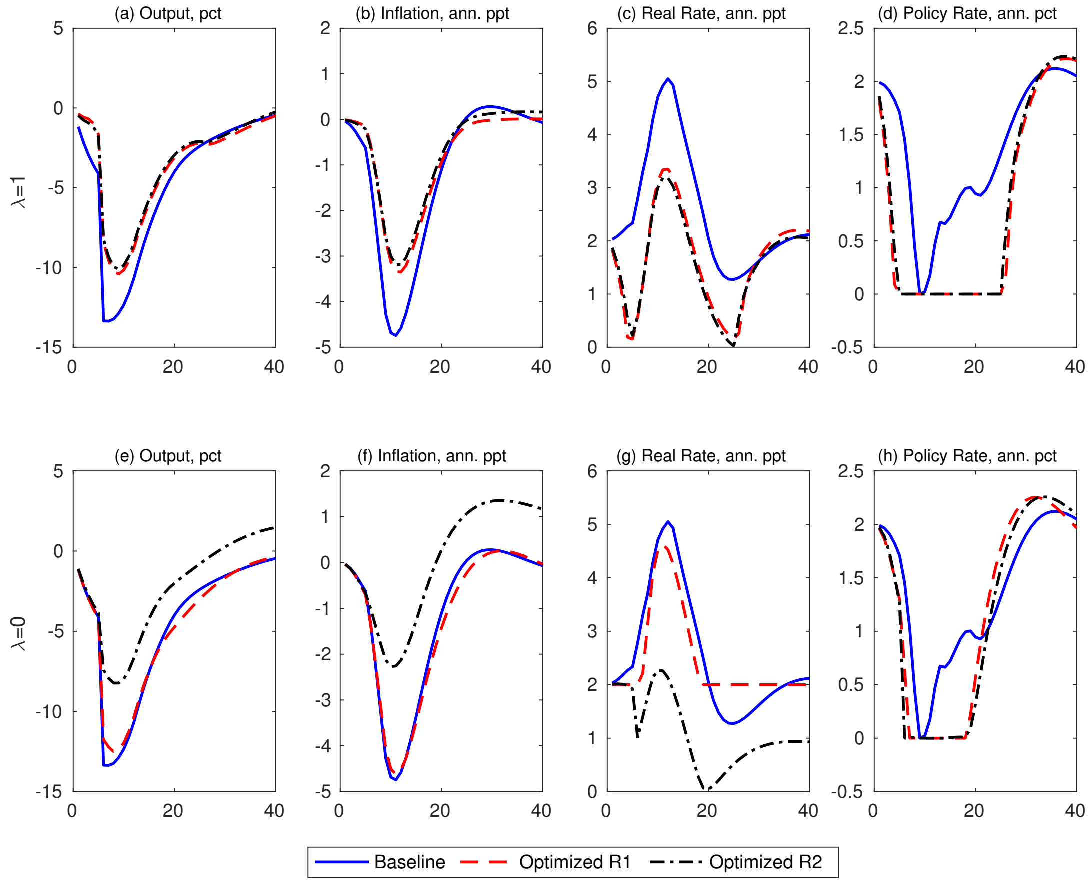
7.3 Tradeoff between Stabilizing Inflation and Limiting the Downside Risks

We finish our discussion by showing the properties of tradeoff in stabilizing the volatility of inflation rate and reducing the degree of skewed distribution.

Figure 11 illustrates the tradeoffs between inflation volatility and its skewness. The red line shows the combinations of optimized standard deviation (from the target) of inflation rate on the horizontal axis and the optimized skewness of inflation rate on the vertical axis in the cases of $\lambda = 0.0$, 0.25, 0.5, 0.75 and 1.0 for the time-varying inflation targeting rule. The blue line shows the counterpart for the strict inflation targeting rule.[23] The figure shows three important properties of the tradeoff. First, both frontiers are sloped downward, indicating that a central bank can reduce the size of downside risks only by accepting greater inflation volatility. Second, the figure also illustrates how the time-varying inflation targeting rule improves the nature of the tradeoff. For instance, under $\lambda = 0.5$

[23] The blue line appears to show only three dots, but it actually shows five dots. This happens because under the strict inflation targeting rule, $\lambda = 0.0$ and $\lambda = 0.25$ results in exactly the same optimized coefficients and thus the same moments. The same thing happens when $\lambda = 0.75$ and $\lambda = 1.0$. See Table 6 of Appendix A.

Figure 12: Stabilization Properties of Optimized Simple Rules



regime, switching from the strict inflation targeting rule to the time-varying inflation targeting rule reduces both the volatility and the skewness of inflation rate. Third, the figure shows the limit of the strict inflation targeting rule in the sense that under the regime, there is a clear limit to reducing the skewness of inflation rate: no degree of willingness to accept greater inflation volatility results in reduction in skewness further below 0.68 (in absolute value).

Figure 12 replicates the experiment we performed in Figure 5 using the same sequence of shocks under the assumption that the crisis occurs and the ZLB constraint exists and compares the macroeconomic performances under the baseline monetary policy rule (blue solid line), optimized inflation targeting rule (red dashed line) and optimized time-varying inflation targeting rule (black dash-dotted line). We first consider the upper panels, which show the cases of $\lambda = 1.0$, that is, when the central bank assigns the maximum weight on inflation volatility as is standard. In this case, despite the augmented term in the time-varying inflation targeting rule, both rules behave similarly and this is precisely because the optimization chooses the policy coefficients such that the inflation volatility is minimized even for the time-varying inflation targeting rule. Both rules have the minimum level of policy inertia and a large coefficient for inflation rate and consequently the augmented term of the time-varying inflation targeting rule hardly makes any difference.

We now consider the bottom panels, which are the cases under $\lambda = 0.0$, that is, when the central bank assigns the maximum weight on limiting the size of downside risks. The bottom panels clearly show that the time-varying inflation targeting rule outperforms the strict inflation targeting rule by a large margin both in terms of output stabilization and in terms of inflation stabilization. This is somewhat surprising given that the paths of policy rate under the two optimized rules shown in panel (h) are almost indistinguishable from each other.

The key to the success of the time-varying inflation targeting rule can be found in panel (g), showing the paths of real interest rate. Under the time-varying inflation targeting rule, the central bank's forward guidance of the path of the natural rate of interest helps economic agents expect that the downside of the economy will be limited and consequently, the real marginal cost will be much higher than in the baseline, which then increases inflation expectations. Such a higher path of expected inflation rate lowers the real interest rate substantially as shown in panel (g). This limits the downside risk of the economy. Comparing the upper panels and the bottom panels reveals that the maximum stabilization function of an asymmetric policy rule designed to limit the size of the downside risks during financial crises can be reaped only when the central bank assigns a sizable weight on the skewness of inflation rate.

8 Conclusion

In this paper, using a quantitative general equilibrium model, we show a possibility that income inequality may be behind many of the problems facing the economy currently: excessive credit growth, secular stagnation, financial instability and deflation pressure. We also show that monetary policy's stabilization function during financial crises may be severely distorted due to the ZLB constraint. Consequently, the distributions of aggregate output, employment and inflation are considerably skewed to the left, representing the presence of disproportionately large downside risks. Such an asymmetric risk distribution calls for an asymmetric approach to monetary policy. An optimal use of inflation targeting may require an opportunistic mix of adopting lenient policy against inflation during normal times to minimize the probability of financial crises and adopting Rogoff's conservative central banker during crises to fight the deleveraging cycle and deflation pressure. One way of implementing such an asymmetric inflation targeting rule is to augment the strict inflation targeting rule with a time-varying natural rate of interest that is strongly and persistently lowered in response to an eruption of financial crisis. Our analysis indicates that such an asymmetric monetary policy rule can bring large benefits of eliminating deflation bias by correcting the skewed distribution.

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Appendices

A Additional Figures and Tables

Figure 13: Distributions of Output, Inflation and Nominal Interest Rate Under Optimized Rules

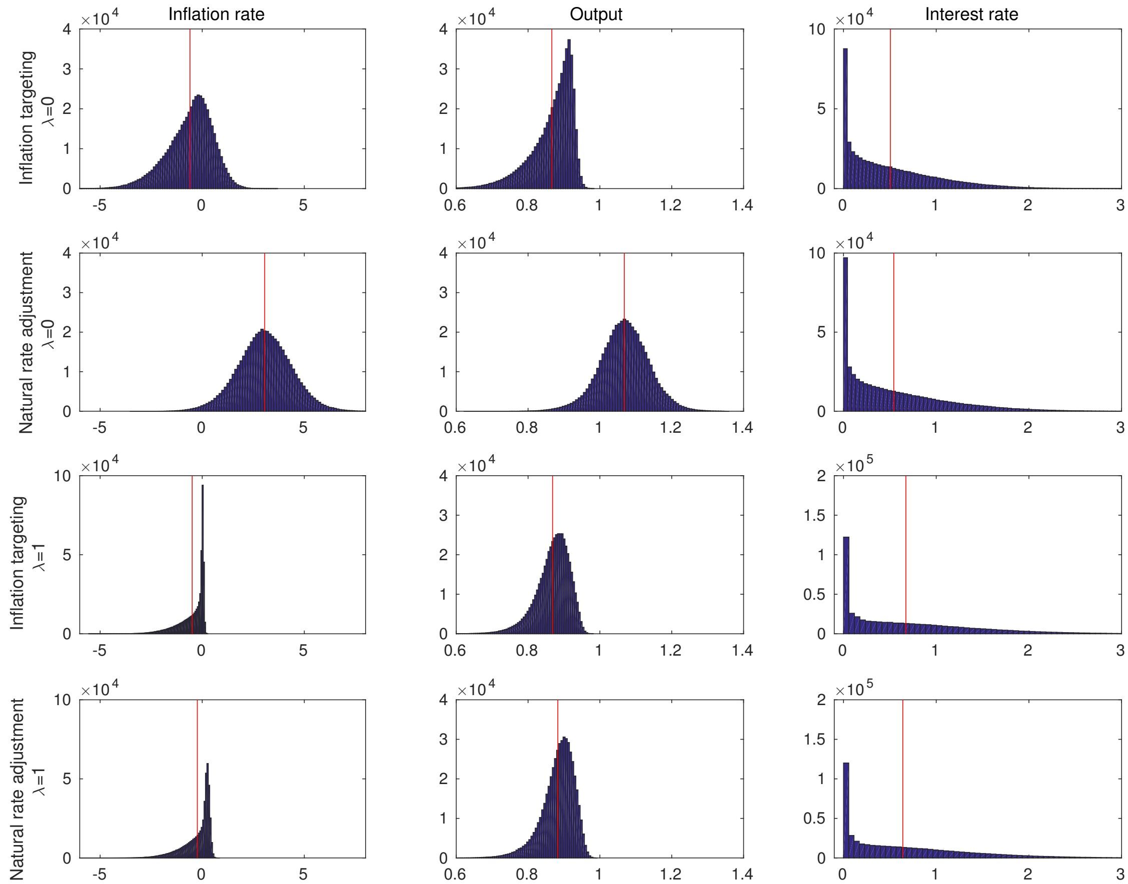

Table 6: Optimal Simple Rules

	$\lambda = 0.0$		$\lambda = 0.25$		$\lambda = 0.5$		$\lambda = 0.75$		$\lambda = 1.0$	
	R1	R2	R1	R2	R1	R2	R1	R2	R1	R2
ρ_π	1.000	1.496	1.000	1.000	1.630	1.252	20.00	5.860	20.00	11.74
ρ_i	0.000	0.493	0.000	0.063	0.000	0.000	0.000	0.000	0.000	0.063
ρ_i^*	–	0.995	–	0.984	–	0.984	–	0.995	–	0.995
σ_i^*	–	0.005	–	0.005	–	0.005	–	0.005	–	0.005
L	0.676	0.000	0.914	0.610	1.111	0.769	0.958	0.762	0.715	0.539

Note: R1 refers to strict inflation targeting policy, R2 to time-varying inflation targeting rule. Minimization program simulates the model economy 500,000 simulations with identical set of random draws for shocks for each set of candidate coefficients. We then compute the loss function values and search for minimizor of the loss function value for each parameter of a given monetary policy rule. The ranges of parameter values given to the minimization programs are the followings: for inflation coefficient, [1.0,20]; for the inertial term, [0.0, 0.9]; for persistence of r-star, [0.0, 0.995]; for volatility of shocks to r-star, [0.0,0.005].

B Non-Stochastic Steady State

B.1 Pinning Down the Endogenous Variables

To determine the steady state of the model, we guess the values of borrowing level b and default probability p^δ. We assume the zero inflation steady state, i.e., $\pi = 1$. Equation (56) then satisfies

$$1 = \left[(1-\varphi)p_0^{1-\gamma} + \varphi\right]^{1/(1-\gamma)}, \tag{B.1}$$

resulting in $p_0 = 1$. Equations (53)∼(55) imply

$$p_0 = \frac{\gamma\mu y\pi^\gamma/(1-\beta\varphi)}{(\gamma-1)y\pi^{(\gamma-1)}/(1-\beta\varphi)} = 1, \tag{B.2}$$

which results in

$$\mu = \frac{\gamma-1}{\gamma}. \tag{B.3}$$

In the steady state $q^K = 1$ from equation (58). Then, equation (7) determines

$$r = 1/\beta^T - (1-\delta). \tag{B.4}$$

Using this in equation (41), we define the steady state output-capital ratio as

$$\rho_{y/k} \equiv \frac{y}{k} = \frac{1/\beta^T - (1-\delta)}{\mu\alpha}. \tag{B.5}$$

From the production function, we have

$$\frac{y}{k} = z\left(\frac{n}{k}\right)^{1-\alpha}$$

or equivalently

$$\rho_{n/k} \equiv \frac{n}{k} = \left(\frac{\rho_{y/k}}{z}\right)^{1/(1-\alpha)}. \tag{B.6}$$

Again from the production function, we derive output-labor ratio as

$$\rho_{y/n} \equiv \frac{y}{n} = z\left(\frac{n}{k}\right)^{-\alpha} = z\rho_{n/k}^{-\alpha}. \tag{B.7}$$

These ratios can be used to determine the levels of output and capital once the level of employment n is determined.

We determine the steady state job market tightness by calibrating the job finding rate from the data, i.e., $p(\theta) = p$. Using equation (B.7), we simplify the expression for the wage as

$$w = \eta\mu(1-\alpha)\rho_{y/n} + (1-\eta)b^U + (1-\rho)\eta[\beta^T - \beta^B(1-p)]\frac{\xi}{q}. \tag{B.8}$$

Equations (39) and (40) imply in steady state

$$\frac{\xi}{q} = \mu(1-\alpha)\rho_{y/n} - w + (1-\rho)\beta^T\frac{\xi}{q}. \tag{B.9}$$

Equations (B.8) and (B.9) provide us with two equations for two unknowns w and ξ/q. Substituting (B.8) in (B.9) and solving it for ξ/q yields

$$J = \frac{\xi}{q} = \frac{(1-\eta)\left[\mu(1-\alpha)\rho_{y/n} - b^U\right]}{1-(1-\rho)\beta^T\{1-\eta[1-(1-p)\beta^B/\beta^T]\}}. \tag{B.10}$$

Using this in (50) yields the steady state wage.

Since $\xi/q = \xi\theta/p(\theta) = \xi\theta/p$, we have

$$\theta = \frac{p}{\xi}\left[\frac{(1-\eta)\left[\mu(1-\alpha)\rho_{y/n} - b^U\right]}{1-(1-\rho)\beta^T\{1-\eta[1-(1-p)\beta^B/\beta^T]\}}\right]. \tag{B.11}$$

This determines the matching function efficiency as

$$\zeta = \frac{p}{\theta^\epsilon}. \tag{B.12}$$

Note that we pin down ζ endogenously by matching the job finding rate of the model with the data. This requires us to treat ζ as an endogenous variable, which always takes the same value as in the steady state.

From equation (37), we have the steady state employment stock as

$$n = \frac{q}{\rho}v = \frac{q}{\rho}\theta u. \tag{B.13}$$

Substituting this expression in equation (34) yields

$$u = 1 - \chi - (1-\rho)\frac{q}{\rho}\theta u.$$

Solving this for u yields

$$u = \frac{1-\chi}{1+(1-\rho)q\theta/\rho}.$$

The vacancy posting is determined as

$$v = \theta u$$

and using this in equation (B.13) gives us the steady state employment stock. The levels of capital and output are then given by

$$y = \rho_{y/n} n \tag{B.14}$$

and

$$k = \rho_{n/k}^{-1} n. \tag{B.15}$$

In a steady state where a default event does not occur,

$$c^B = c^N = (q^B - 1)\, b + \frac{1}{1-\chi}\left[wn + (1-\chi-n)b^U\right] \tag{B.16}$$

and

$$c^T = b\,(1-q^B)\,\frac{1-\chi}{\chi} - q^K\frac{\delta k}{\chi} + \frac{rk}{\chi} + \frac{\Pi^Y}{\chi} + \frac{\Pi^K}{\chi} + \frac{T}{\chi} \tag{B.17}$$

where

$$\Pi^Y = y - wn - rk - \xi v \tag{B.18}$$

and

$$\Pi^K = q^K x - x = 0. \tag{B.19}$$

Substituting (B.18), (B.19), the steady state version of the FOC for borrowing,

$$q^B = \beta_W(1-p^\delta h) \tag{B.20}$$

and the balanced budget constraint for government,

$$T = -(1-\chi-n)b^U + \chi b^G\left(1 - \frac{1}{1+i}\right)$$

in equation (B.17), we express the steady state consumption level of shareholders as

$$c^T(b) = \frac{1}{\chi}\{b[1-\beta_W(1-p^\delta h)](1-\chi) - q^K\delta k + y - wn - \xi v - (1-\chi-n)b^U\}. \tag{B.21}$$

Note that the right hand side of the above has only one unknown, b. In order to pin down b, we equate the FOCs of the two agents regarding borrowing and lending, which results in

$$(\beta^B - \beta^T)(1 - p^\delta h) = \frac{\psi}{[(1-s)c^T(b)]^{-1/\sigma_c}} \left[1 + b\left(\frac{1-\chi}{\chi}\right)\right]^{-1/\sigma_b}. \tag{B.22}$$

Our initial guess b should satisfy (B.22) given our guess p^δ. We now discuss how the guess solution p^δ can be validated.

B.2 Probability of Default

We envision a steady state in which there has been no default for a long time such that the lagged value of pecuniary default cost has converged to zero, i.e., $\nu_{-1} = 0$. In such an environment, if a default occurs today, the value of default cost becomes $\nu = \gamma_\nu$. If a default does not occur today, the value of default cost becomes $\nu = 0$. If a default occurs again tomorrow after an episode of default today, $\nu' = (1 + \rho_\nu)\gamma_\nu$. If a default does not occur tomorrow after an incidence of default, $\nu' = \rho_\nu \gamma_\nu$. If a default occurs tomorrow after no incidence of default today, $\nu' = \gamma_\nu$. If a default does not occur either today or tomorrow, $\nu' = 0$. Note that all of these 4 cases are hypothetical in that a default does not occur in our steady state.

We denote 4 cases of consumption level tomorrow under the 4 scenarios by c^{DD}, c^{DN}, c^{ND} and c^{NN}. These are given by

$$c^{DD} = [q^B - (1-h)]b + \frac{1}{1-\chi}\left[wn + (1 - \chi - n)b^U - (1 + \rho_\nu)\gamma_\nu y\right], \tag{B.23}$$

$$c^{DN} = (q^B - 1)b + \frac{1}{1-\chi}\left[wn + (1 - \chi - n)b^U - \rho_\nu \gamma_\nu y\right], \tag{B.24}$$

$$c^{ND} = [q^B - (1-h)]b + \frac{1}{1-\chi}\left[wn + (1 - \chi - n)b^U - \gamma_\nu y\right], \tag{B.25}$$

and

$$c^{NN} = (q^B - 1)b + \frac{1}{1-\chi}\left[wn + (1 - \chi - n)b^U\right]. \tag{B.26}$$

Note that all right hand side variables are already determined above under the initial guess solution of p^δ. However this guess solution needs to be verified.

In the steady state, the continuation values of default and non-default are given by

$$V^D = \beta^B p^\delta \left(\frac{(c^{DD})^{1-1/\sigma_c} - 1}{1 - 1/\sigma_c} + V^D\right) + \beta^B (1 - p^\delta)\left(\frac{(c^{DN})^{1-1/\sigma_c} - 1}{1 - 1/\sigma_c} + V^N\right) \tag{B.27}$$

and

$$V^N = \beta^B p^\delta \left(\frac{(c^{ND})^{1-1/\sigma_c} - 1}{1 - 1/\sigma_c} + V^D\right) + \beta^B (1 - p^\delta)\left(\frac{(c^{NN})^{1-1/\sigma_c} - 1}{1 - 1/\sigma_c} + V^N\right). \tag{B.28}$$

By subtracting (B.27) from (B.28), one can derive ΔV as

$$\begin{aligned}
\Delta V^B(p^\delta) &= V^D - V^N \\
&= \beta^B p^\delta \left(\frac{(c^{DD} - sc^D)^{1-1/\sigma_c}}{1 - 1/\sigma_c} - \frac{(c^{ND} - sc^N)^{1-1/\sigma_c}}{1 - 1/\sigma_c}\right) \\
&\quad + \beta^B (1 - p^\delta)\left(\frac{(c^{DN} - sc^D)^{1-1/\sigma_c}}{1 - 1/\sigma_c} - \frac{(c^{NN} - sc^N)^{1-1/\sigma_c}}{1 - 1/\sigma_c}\right).
\end{aligned}$$

Hence ΔU^B is given by

$$\Delta U^B(p^\delta) = \frac{(c^D - sc^B)^{1-1/\sigma_c}}{1 - 1/\sigma_c} - \frac{(c^N - sc^B)^{1-1/\sigma_c}}{1 - 1/\sigma_c} + \Delta V^B(p^\delta).$$

Note that we express ΔU and ΔV functions of our initial guess p^δ. Since default probability is given by

The default probability is then given by

$$p^{\delta} = \frac{\varrho}{1 + \exp(-\varsigma \Delta U^{B}(p^{\delta}))} \tag{B.29}$$

(B.29) can then be viewed as a fixed point problem. Hence, the steady state needs to be pinned down by the simultaneous equations of (B.22) and (B.29). Once this is done, the consumption level of the opt 5 percent earners can be determined by (B.21). Using this consumption level, we evaluate the marginal utility of the top 5 percent earners and then finally determine the level of government debt consistent with the calibrated of risk-free bond rate i :

$$b^{G} = \left\{ \frac{\Lambda^{T}}{\psi^{G}} [(1+i)^{-1} - \beta^{T}] \right\}^{-\sigma_g} - 1.$$

C Solution Method

The model is inherently nonlinear for two reasons, the ZLB constraint and the binary nature of the default. However, given the large number of state variables, we simulate the model piecewise linearly. This section illustrates how such simulations can be executed. Suppose that we solve the model linearly, ignoring the ZLB constraint and the default. In our model economy, $\delta_t^B \in \{0, 1\}$, has both endogenous and exogenous natures. δ_t^B is endogenous in the sense that the probability of $\delta_t^B = 1$ is endogenously determined. However given the probability of default, whether or not the default occurs is stochastic. In this latter sense, δ_t^B can be viewed as an exogenous variable.

Step 1. We linearize the model around the non-stochastic steady state in which we assume that the economy has not experienced a crisis for a long period of time and the default cost ν has converged to zero. We can express the state space representation for the system of equations as

$$\begin{bmatrix} \mathbf{s}_t \\ \mathbf{x}_t \end{bmatrix} = \begin{bmatrix} \mathbf{A}\mathbf{s}_{t-1} + \mathbf{B}\boldsymbol{\epsilon}_t \\ \mathbf{C}\mathbf{s}_t \end{bmatrix}, \ \boldsymbol{\epsilon}_t = \begin{bmatrix} \boldsymbol{\epsilon}_t^{EX} \\ \boldsymbol{\epsilon}_t^{EN} \end{bmatrix} \tag{C.1}$$

and

$$\begin{aligned} \boldsymbol{\epsilon}_t^{EX} &= [\ \epsilon_{z,t} \quad \epsilon_{\eta,t} \quad \epsilon_{\lambda,t}\ ]' \\ \boldsymbol{\epsilon}_t^{EN} &= [\ \delta_t^B \quad \epsilon_{0,t} \quad \epsilon_{1,t} \quad \cdots \quad \epsilon_{n,t}\ ]' \end{aligned} \tag{C.2}$$

where $\mathbf{s}_t$ and $\mathbf{x}_t$ are state and policy vectors with dimension Note that $\boldsymbol{\epsilon}_t^{EX}$ follows Gaussian distributions. In contrast, $\boldsymbol{\epsilon}_t^{EN}$, although can be viewed as 'shocks' from a mechanical point of view, we treat $\boldsymbol{\epsilon}_t^{EN}$ as endogenous variables since the sizes of the shocks should be determined endogenously.

Step 2. In each period, we simulate the economy with random draws for $\hat{\boldsymbol{\epsilon}}_t$ under the assumption that $\delta_t^B = 0$ and $\epsilon_{j,t} = 0$. The system of equations will return the default probability

$$p_t^{\delta(0)} = \Xi(\Delta U_t^B) = \frac{\vartheta}{1 + \exp(-\varsigma \Delta U_t)}$$

and the endogenous component of monetary policy

$$i_t^{(0)} = \rho_i i_{t-1} + (1-\rho_i) \left[i + \rho_\pi \left(\frac{\pi_t^Y - \pi^*}{4} \right) \right].$$

Step 3. We then need to check if a default occurs and/or the ZLB is violated. To see if the ZLB is violated or not, we simply need to check if i_t is negative or not. We also need to check whether or not $\mathbb{E}_t[i_{t+j}] \geq 0$ is satisfied. A default can be detected in the following way. Take a random draw from a uniform distribution over a support $[0, 1]$. If this draw is strictly greater than $p_t^{\delta(0)}$, we conclude that a default did not occur. If the draw is strictly less than $p_t^{\delta(0)}$, we conclude that a default did occur. There can be four cases: (i) no default, no ZLB violation; (ii) default, no ZLB violation; (iii) no default, ZLB violation; (iv) default, ZLB violation.

Case (i). Since there is no default and the ZLB is not violated, we move to the next period.

Case (ii). Since a default occurs, we simulate the economy again with $\delta_t^B = 1$ and check if no ZLB violation is still valid both in current period and in expectations. If the ZLB constraint binds or is expected to bind in future, then move to Case (iv). Otherwise we move to the next period.

Case (iii). Since the ZLB constraint is violated ($i_t^{(0)} < 0$), where the superscript (0) indicates the number of iteration in ZLB loop, we set $\epsilon_{0,t}^{(1)} = -i_t^{(0)}$ $\epsilon_{j,t}^{(1)} = \max\{0, -\mathbb{E}_t[i_{t+j}^{(0)}]\}$ to satisfy $\mathbb{E}_t[i_{t+j}^{(1)}] \geq 0$. We then simulate the economy again and check if the ZLB in current period or in expectations is violated or not. This process cannot be done in one step because once the monetary policy shock or news shock is given to the system, the economy reacts to the shocks, and the initial amounts of monetary policy shocks to satisfy the ZLB may not be enough. If the constraint is still violated, we set $\epsilon_{0,t}^{(2)} = -i_t^{(0)} - i_t^{(1)}$ and $\epsilon_{j,t}^{(1)} = -\mathbb{E}_t[i_{t+j}^{(0)}] - \mathbb{E}_t[i_{t+j}^{(1)}]$, and simulate the economy again and check if the ZLB is violated in current or in future expectations. If the ZLB is finally satisfied after S simulation, the final level of monetary policy shock is given by

$$\epsilon_{0,t}^{(S)} = -\sum_{s=0}^{S} i_t^{(s)} \text{ and } \epsilon_{j,t}^{(S)} = -\sum_{s=0}^{S} \mathbb{E}_t[i_{t+j}^{(s)}] \text{ for } j = 1, \ldots N. \tag{C.3}$$

An important thing in this iteration is to check whether or not a default occurs owing to the endogenous reaction of the economy to the monetary policy shock to satisfy the ZLB for a given simulation number j. If a default occurs for this reason, then move to Case (iv). If not, move to the next period.

Case (iv). In this case a crisis occurs and the ZLB binds. Hence we set $\delta_t^B = 1$ and determine the size of monetary policy shock according to (C.3). For a given simulation number j, we need to check if a crisis occurs or not. In principle, it is logically possible, although not likely, that the economy gets itself out of the default mode due to the monetary policy shock. If this happens, move to Case (iii).

D System of Equations

There are variables 50 endogenous variables within the system:

$$X_t \equiv \begin{bmatrix} c_t^T & b_t & l_t & c_t & \pi_t & q_t^B & i_t & q_t^K & k_t & r_t \\ T_t & \Lambda_t^T & \Lambda_t^B & w_t & n_t & \nu_t & y_t & \Lambda_{W,t} & m_{t,t+1}^T & \Delta U_t \\ p_t^\delta & v_t & u_t & \theta_t & p_t & q_t & z_t & m_{t,t+1}^B & J_t & \mu_t \\ \underline{w}_t & W_t & \eta_t & p_{0,t} & \mathcal{P}_t^N & \mathcal{P}_t^D & x_t & \lambda_t & \Gamma_t^W & \Delta_t \\ c_t^D & c_t^N & \Pi_t^Y & \Pi_t^K & c_t^{DD} & c_t^{DN} & c_t^{ND} & c_t^{NN} & \Omega_t & \Gamma_t^J \end{bmatrix} \tag{B-i}$$

The following provides the complete list of 50 equations for 50 endogenous variables:

$$l_t = (1 - h\delta_t^B)\frac{b_{t-1}}{\pi_t} \tag{D.1}$$

$$c_t = \chi c_t^T + (1-\chi)c_t^B \tag{D.2}$$

$$T_t = -b^U(1 - \chi - n_t) \tag{D.3}$$

$$\Lambda_t^T = (c_t^T - sc_{t-1}^T)^{-1/\sigma_c} \tag{D.4}$$

$$m_{t,t+1}^T \equiv \beta^T \frac{\Lambda_{t+1}^T}{\Lambda_t^T} \tag{D.5}$$

$$1 = \mathbb{E}_t\left[m_{t,t+1}^T\left(\frac{r_{t+1} + (1-\delta)q_{t+1}^K}{q_t^K \lambda_t}\right)\right] \tag{D.6}$$

$$\frac{1}{1+i_t} = \beta^T \mathbb{E}_t \left[\frac{\Lambda^T_{t+1}}{\Lambda^T_t} \frac{\lambda_t}{\pi_{t+1}} \right] + \frac{\psi^G}{\Lambda^T_t}(1+b^G_t)^{-1/\sigma_g} \tag{D.7}$$

$$q^B_t = \mathbb{E}_t \left[m^T_{t,t+1}(1-hp^\delta_{t+1})\frac{1}{\pi_{t+1}} \right] + \frac{\psi}{\Lambda_{T,t}} \left[1 + b_t \left(\frac{1-\chi}{\chi} \right) \right]^{-1/\sigma_b} \tag{D.8}$$

$$q^B_t = \mathbb{E}_t \left[m^B_{t,t+1}(1-hp^\delta_{t+1})\frac{1}{\pi_{t+1}} \right] \tag{D.9}$$

$$c^B_t = q^B_t b_t - l_t + \frac{1}{1-\chi} \left[w_t n_t + (1-\chi-n_t)b^U - \nu_t y_t \right] \tag{D.10}$$

$$\nu_t = \rho_\nu \nu_{t-1} + \gamma_\nu \delta^B_t \tag{D.11}$$

$$\Lambda^B_t = (c^B_t - sc^B_{t-1})^{-1/\sigma_c} \tag{D.12}$$

$$m^B_{t,t+1} \equiv \beta^B \frac{\Lambda^B_{t+1}}{\Lambda^B_t} \tag{D.13}$$

$$\begin{aligned} \Delta U^B_t = & \frac{(c^D_t - sc^B_{t-1})^{1-1/\sigma_c}}{1-1/\sigma_c} - \frac{(c^N_t - sc^B_{t-1})^{1-1/\sigma_c}}{1-1/\sigma_c} \\ & + \beta^B \mathbb{E}_t \left[p^\delta_{t+1} \left(\frac{(c^{DD}_{t+1} - sc^D_t)^{1-1/\sigma_c}}{1-1/\sigma_c} - \frac{(c^{ND}_{t+1} - sc^N_t)^{1-1/\sigma_c}}{1-1/\sigma_c} \right) \right] \\ & + \beta^B \mathbb{E}_t \left[(1-p^\delta_{t+1}) \left(\frac{(c^{DN}_{t+1} - sc^D_t)^{1-1/\sigma_c}}{1-1/\sigma_c} - \frac{(c^{NN}_{t+1} - sc^N_t)^{1-1/\sigma_c}}{1-1/\sigma_c} \right) \right]. \end{aligned} \tag{D.14}$$

$$c^D_t = q^B_t b_t - (1-h)\frac{b_{t-1}}{\pi_t} + \frac{1}{1-\chi} \left[w_t n_t + (1-\chi-n_t)b^U - (\rho_\nu \nu_{t-1} + \gamma_\nu) y_t \right] \tag{D.15}$$

$$c^N_t = q^B_t b_t - \frac{b_{t-1}}{\pi_t} + \frac{1}{1-\chi} \left[w_t n_t + (1-\chi-n_t)b^U - \rho_\nu \nu_{t-1} y_t \right] \tag{D.16}$$

$$c^{DD}_t = q^B_t b_t - (1-h)\frac{b_{t-1}}{\pi_t} + \frac{1}{1-\chi} \left[w_t n_t + (1-\chi-n_t)b^U - [\rho_\nu(\rho_\nu \nu_{t-2} + \gamma_\nu) + \gamma_\nu] y_t \right] \tag{D.17}$$

$$c^{DN}_t = q^B_t b_t - \frac{b_{t-1}}{\pi_t} + \frac{1}{1-\chi} \left[w_t n_t + (1-\chi-n_t)b^U - \rho_\nu(\rho_\nu \nu_{t-2} + \gamma_\nu) y_t \right] \tag{D.18}$$

$$c^{ND}_t = q^B_t b_t - (1-h)\frac{b_{t-1}}{\pi_t} + \frac{1}{1-\chi} \left[w_t n_t + (1-\chi-n_t)b^U - (\rho_\nu \rho_\nu \nu_{t-2} + \gamma_\nu) y_t \right] \tag{D.19}$$

$$c^{NN}_t = q^B_t b_t - \frac{b_{t-1}}{\pi_t} + \frac{1}{1-\chi} \left[w_t n_t + (1-\chi-n_t)b^U - \rho_\nu \rho_\nu \nu_{t-2} y_t \right] \tag{D.20}$$

$$p^\delta_t = \frac{\vartheta}{1+\exp(-\varsigma \Delta U_t)} \tag{D.21}$$

$$u_t = 1 - \chi - (1-\rho) n_{t-1} \tag{D.22}$$

$$p_t = \zeta \theta^\epsilon_t \tag{D.23}$$

$$q_t = \zeta \theta^{\epsilon-1}_t \tag{D.24}$$

$$\frac{\xi}{q_t} = J_t \tag{D.25}$$

$$J_t = \mu_t(1-\alpha)\frac{y_t}{n_t} - w_t - \frac{\upsilon}{2}\left(\frac{w_t}{w_{t-1}} - 1\right)^2 w_{t-1} + (1-\rho)\mathbb{E}_t[m^{T}_{t,t+1}J_{t+1}] \tag{D.26}$$

$$\Gamma^W_t = -1 \tag{D.27}$$

$$\Gamma^J_t = -1 - \upsilon\left(\frac{w_t}{w_{t-1}} - 1\right) + (1-\rho)\mathbb{E}_t\left\{\beta^S_{t,t+1}\frac{\upsilon}{2}\left[\left(\frac{w_{t+1}}{w_t}\right)^2 - 1\right]\right\} \tag{D.28}$$

$$\Omega_t = \frac{\eta_t}{\eta_t + (1-\eta_t)\Gamma^J_t/\Gamma^W_t} \tag{D.29}$$

$$W_t = \frac{\Omega_t}{1-\Omega_t}J_t \tag{D.30}$$

$$\underline{w}_t = b^{U} + (1-\rho)\mathbb{E}_t\left[m^{B}_{t,t+1}p_{t+1}\frac{\Omega_{t+1}}{1-\Omega_{t+1}}J_{t+1}\right] \tag{D.31}$$

$$\begin{aligned} w_t = {} & \Omega_t\mu_t(1-\alpha)\frac{y_t}{n_t} + (1-\Omega_t)b^{U} \\ & +(1-\rho)\mathbb{E}_t\left[\left(\Omega_t m^{T}_{t,t+1} - (1-\Omega_t)m^{B}_{t,t+1}(1-p_{t+1})\frac{\Omega_{t+1}}{1-\Omega_{t+1}}\right)\frac{\xi}{q(\theta_{t+1})}\right] \end{aligned} \tag{D.32}$$

$$\mathcal{P}^{N}_t = \pi_t^{(1-v)\gamma}\left\{\pi_t^{v\gamma}\gamma\mu_t y_t + \varphi\mathbb{E}_t\left[m^{T}_{t,t+1}\mathcal{P}^{N}_{t+1}\right]\right\}, \tag{D.33}$$

$$\mathcal{P}^{D}_t = \pi_t^{(1-v)(\gamma-1)}\left\{\pi_t^{v(\gamma-1)}(\gamma-1)y_t + \varphi\mathbb{E}_t\left[m^{T}_{t,t+1}\mathcal{P}^{D}_{t+1}\right]\right\}. \tag{D.34}$$

$$p_{0,t} = \frac{\mathcal{P}^{N}_t}{\mathcal{P}^{D}_t} \tag{D.35}$$

$$\pi_t = \left[(1-\varphi)p_{0,t}^{1-\gamma} + \varphi\pi_{t-1}^{v(1-\gamma)}\right]^{1/(1-\gamma)}. \tag{D.36}$$

$$q^{K}_t = 1 + \kappa\left(\frac{x_t}{x_{t-1}} - 1\right) - \mathbb{E}_t\left\{m^{T}_{t,t+1}\frac{\kappa}{2}\left[\left(\frac{x_{t+1}}{x_t}\right)^2 - 1\right]\right\} \tag{D.37}$$

$$i_t = \rho_i i_{t-1} + (1-\rho_i)\left[i + \rho_\pi\left(\frac{\pi^{Y}_t - \pi^*}{4}\right) + \rho_u\left(\frac{u_t - u^{N}_t}{4}\right)\right] + \epsilon_{m,t} \tag{D.38}$$

$$\log z_t = \rho_z \log z_{t-1} + \epsilon_{z,t} \tag{D.39}$$

$$\log\eta_t = (1-\rho_\eta)\log\eta + \rho_\eta\log\eta_{t-1} + \epsilon_{\eta,t} \tag{D.40}$$

$$\log\lambda_t = \rho_\lambda\log\lambda_{t-1} + \epsilon_{\lambda,t} \tag{D.41}$$

$$y_t = \Delta_t^{-1}z_t k_t^{\alpha}n_t^{1-\alpha} \tag{D.42}$$

$$\Delta_t = \pi_t^{\gamma}[(1-\varphi)p_{0,t}^{-\gamma} + \varphi\pi_{t-1}^{-v\gamma}] \tag{D.43}$$

$$n_t = (1-\rho)n_{t-1} + q_t v_t \tag{D.44}$$

$$k_t = (1-\delta)k_{t-1} + x_t \tag{D.45}$$

$$r_t = \mu_t \alpha \frac{y_t}{k_t} \tag{D.46}$$

$$\theta_t = \frac{v_t}{u_t} \tag{D.47}$$

$$\Pi_t^Y = y_t - w_t n_t - \frac{\upsilon}{2}\left(\frac{w_t}{w_{t-1}} - 1\right)^2 w_{t-1} n_t - r_t k_{t-1} - \xi v_t \tag{D.48}$$

$$\Pi_t^K = q_t^K x_t - \left[x_t + \frac{\kappa}{2}\left(\frac{x_t}{x_{t-1}} - 1\right)^2 x_{t-1}\right] \tag{D.49}$$

$$c_t^T = (l_t - q_t^B b_t)\frac{1-\chi}{\chi} - q_t^K\left[\frac{k_t}{\chi} - (1-\delta)\frac{k_{t-1}}{\chi}\right] + r_t\frac{k_{t-1}}{\chi} + \frac{\Pi_t^Y}{\chi} + \frac{\Pi_t^K}{\chi} + \frac{T_t}{\chi} + \frac{\nu_t y_t}{\chi} \tag{D.50}$$