Persuading Multiple Audiences: An Information Design Approach to Banking Regulation *

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Abstract

A policy maker concerned with the potential default of a bank conducts an *asset quality review* and a *stress test* under the scrutiny of multiple types of market participants (audiences). Surprisingly, the optimal *comprehensive assessment* (asset quality review and stress test) is opaque when the bank has high-quality assets, and transparent when the bank has poor-quality assets. The optimal policy also imposes contingent recapitalizations. Without the latter, disclosure of information may backfire and the bank may fare worse than under laissez faire. The paper also studies the interplay between information disclosure and the policy maker's role as a Lender of Last Resort (LOLR). The optimal mechanism consists of an *emergency lending* facility that (a) provides funds to banks in exchange for assets, and (b) discloses information about the bank's liquidity position. Comprehensive interventions display a non-monotone pecking order: the private sector funds banks with either high or poor-quality assets, while institutions with intermediate-quality assets participate in the government's emergency lending program. Interestingly, imposing capital requirements hurts the effectiveness of LOLR policies. My results shed light on the role information disclosure as a regulatory tool in environments with multiple audiences and multi-dimensional fundamentals.

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1 Introduction

How much information should policy-makers disclose about banks' balance sheets during financial crises? What is the optimal level of transparency to calm down the market? Should policy-makers provide more information about banks with stronger fundamentals or about those with weaker fundamentals? What is the interplay between disclosure of information and the policy makers' role as *Lenders of Last Resort* (LOLR)?

Information disclosure has become a prominent tool in banking supervision since the global financial crisis. In February 2009, the Federal Reserve introduced the Supervisory Capital Assessment Program (SCAP) (commonly known as the Fed's *stress test*). The objective of the program was to assess whether the capital buffers of the 19 largest bank holding companies were enough to sustain lending in the event of an unexpectedly severe recession, and to communicate these results to the public (Hirtle and Lehnert [2015]).[1] Many scholars and policy-makers believe that the disclosure of stress tests results was a critical inflection point in the financial crisis because it provided market participants with credible information about potential losses which helped restore market confidence (Bernanke [2013]).

Since their introduction, *stress tests* and *asset quality reviews* have been regularly conducted both in the US and in the Eurozone.[2] Despite the consensus that transparency may impose market discipline on the otherwise opaque banking system (Morgan [2002], Flannery et al. [2013]),[3] there exists fundamental disagreement concerning the amount of information that should be disclosed as well as the set of policies that should accompany such disclosures. While the *stress tests* conducted by the Fed, for example, have combined granular data with a pass/fail grade,[4] the European Central Bank decided in 2016 to not assign grades to banks in order to avoid stigmatization. Moreover, while both regulatory authorities complement their disclosures with capital requirements, American regulators have chosen to publicly announce their decisions while their European counterparts have opted for private recommendations.[5],[6]

A crucial difficulty associated with the design of such disclosures is the complexity of the interactions among the multiple types of market participants involved. In fact, when a policy-maker discloses information about a bank, it speaks to multiple *audiences* who care about different aspects of the bank's private information. Namely, potential investors interested in the long-term profitability of the bank's assets; short-term

[1] The supervisors' disclosure came at a time when informational asymmetries between inside and outside market participants regarding the soundness of the banking system had disrupted credit channels, leading to unprecedented interbank lending rates, abrupt haircuts in the repo market, and the freeze of capital markets for banks(Morgan et al. [2014]).

[2] See Morgan et al. [2014], Flannery et al. [2017] and Petrella and Resti [2013] for evidence on the effect of such disclosure policies. The first two papers show that the stress tests conducted in the US provided information not previously available to the rest of market participants. The last paper provides similar evidence for the tests conducted in the EU.

[3] See Babus and Farboodi [2018] for a theory where opacity endogenously emerges as part of banks' strategy to create information asymmetry with external investors.

[4] In 2018, the Fed introduced for the first time an intermediate third grade: *conditional non-objection*, assigned to Goldman Sachs and Morgan Stanley. Both bank holding companies had to cut by half the amount they intended to distribute among shareholders in order to avoid failing the test.

[5] The privacy policy does not apply to those companies publicly listed for which capital requirements count as inside information and must be disclosed.

[6] Goldstein and Sapra [2014] offer an excellent review of the costs associated with information disclosures.

creditors concerned by the bank's liquidity position; speculators interested in the fate of the bank; counterparties exposed to a potential default; taxpayers concerned with the use of public funds if a bailout takes place; the bank itself, which strategically chooses its funding strategy in response to the information publicly disclosed, among others. As a result, an optimally designed disclosure policy must necessarily account for the strategic reactions it induces on these multiple audiences.

Despite the recent attention that stress tests (and, more generally, disclosure policies as regulatory tools) have drawn from the theoretical literature, the natural question concerning the *optimal* degree of transparency of such exercises remains elusive. The reason behind this surprising observation is the standard assumption, usually encountered in the literature, of a single audience (i.e., a single receiver) for the policy-maker's disclosure which, to a large extent, simplifies the policy-maker's problem.[7] When this is the case, the optimal policy is *opaque* and consists of an action recommendation to the single audience.[8] In most cases this takes the form of a pass/fail test (i.e., a recommendation whether to keep pledging funds to the bank). In contrast, with multiple audiences, disclosures intended for a particular audience are *observed* simultaneously by the rest of market participants, generating an endogenous reaction. As a result, it is no longer clear what the optimal degree of transparency of such disclosures should be, or how to design them. Put differently, a crucial ingredient to discuss about the optimal degree of transparency of regulatory disclosures is accounting for the strategic interaction between the multiple audiences concerned about the banks' multi-dimensional private information. This paper aims to shed light on this question and to inform the debate on the optimal design of such disclosures.

To tackle this issue, I consider the minimal model that preserves the richness of the problem. The model consists of a bank, a policy-maker and two audiences: long-term investors and short-term creditors. The bank has private information about two dimensions, namely, (i) the long-term profitability of its assets and (ii) its liquidity position. Throughout the paper I refer to these two variables as the bank's fundamentals. Uncertainty about the bank's fundamentals is gradually resolved. While the quality of the bank's assets is determined early, the amount of liquid funds is determined at a later stage after a shock materializes. The timing is meant to reflect the idea that the quality of the bank's assets depends on investment decisions made in the past, while the liquidity position of the bank is subject to shocks and may vary precipitously. The policy-maker's technology allows her to learn the realization of these variables and to disclose information about them to the multiple audiences.[9]

The first audience, long-term investors, are primarily interested in learning about the long-term profitability of the bank's assets (e.g., the amount of non-performing loans). The second audience, short-term creditors, on the other hand, are concerned by the bank's liquidity position and its ability to repay short-term debt. Nevertheless, long-term investors also care about disclosures concerning the bank's liquidity position, as such

[7] An important exception is the paper by Inostroza and Pavan [2019], where the authors consider the optimal design of stress tests in environments with heterogeneously informed investors.

[8] This is a simple manifestation of the Revelation Principle (Myerson [1982], Myerson [1986]).

[9] As is standard in the *information design* literature, I assume that the policy-maker has commitment power and chooses the information disclosure policy before observing the true realization of the bank's fundamentals.

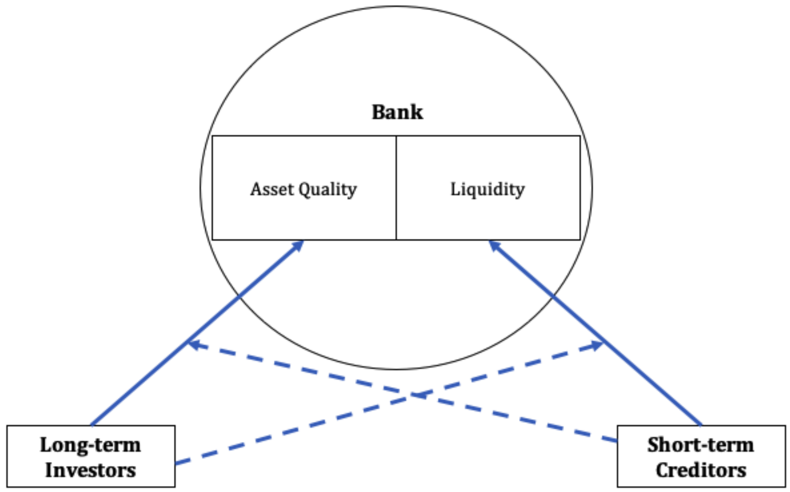

Figure 1: Persuading Multiple Audiences.

information affects short-term creditors' beliefs about the bank's liquidity buffers and, hence, their decisions of whether to keep rolling-over the bank's debt. Given that short-term creditors' claims are senior to those of long-term investors, the latter may be wiped out if short-term creditors decide to stop pledging to the bank. Therefore, long-term investors are *indirectly* affected by disclosures about the bank's liquidity position. In turn, short-term creditors *indirectly* care about the long-term profitability of the bank's assets. Disclosures about this dimension determine how much funds long-term investors are willing to provide for claims on the bank's assets, and hence the bank's ability to raise funds to cover liquidity shortages in the future.

Thus, optimally designed disclosures should have a fixed point structure in the sense that disclosures about one of the dimensions (e.g., the long-term profitability of the bank's assets) should account not only for the reaction of the audience who directly cares about it (e.g., long-term investors), but also for the endogenous reaction of the audiences who *indirectly* care about that dimension (e.g., short-term creditors) and, thus, should be a *best response* to the optimal disclosure about the other dimensions that the latter audiences are *directly* concerned about (e.g., the bank's liquidity position), and vice versa.

The optimal *comprehensive assessment* can be described as follows. The policy-maker first examines the long-term profitability of the bank's assets by conducting an *asset quality review*. When the profitability of the bank's assets is above a threshold, the asset quality review assigns a *unique passing grade*. Conditional on passing the asset quality review, no further disclosures about the bank's liquidity position are necessary. When the quality of the assets, instead, falls below such a threshold, the asset quality review assigns one of *multiple failing grades*. The optimal asset quality review has a monotone partitional structure in which adjacent quality levels are pooled together under the same grade. To improve the bank's chances of survival, and conditional on the bank having failed the asset quality review, the policy-maker conducts a *liquidity stress* *test*. When the liquidity position of the bank is sufficiently good, the bank is assigned a *pass* grade, which convinces short-term creditors to keep rolling over the bank's debt. In the opposite case, the bank is given a *failing* grade, which prompts short-term creditors to run.

The asymmetrical structure of the optimal comprehensive assessment (a unique passing grade and multiple failing grades) stems from the strategic interaction of the two audiences. When the expected value of the bank's assets is low there exists an endogenous amplification effect associated with increasing the perceived quality of the bank's assets. Namely, more valuable assets induce long-term investors to pledge more funds. This increases the probability that the bank survives an eventual run of short-term creditors as the set of liquidity shocks that induce default is reduced. The increase in the probability of survival then allows long-term investors to offer a higher price for the bank's assets. The additional increase in the price feeds back and induces a larger probability of survival, and so on. As a result, when the long-term profitability of the bank's assets is low, the interaction between both audiences generates an amplification mechanism that implies that the probability that the bank survives is convex in the *perceived* long-term profitability of the assets. Therefore, when the quality of the assets is low, the policy-maker prefers finer disclosure policies over coarser rules, similar to a risk-lover decision-maker who prefers lotteries over deterministic outcomes.

In contrast, when the long-term profitability of the bank's assets is high, the bank may prevent default altogether by raising enough funds to persuade short-term creditors to keep rolling over the bank's debt. Using a more transparent disclosure policy in this case does not generate any benefits and, in fact, may hurt risk-sharing among banks with heterogeneous asset qualities. Thus, when the long-term profitability of the bank's assets is sufficiently good, the optimal asset quality review assigns a unique and hence opaque passing grade.

Consistent with the qualitative properties of the optimal disclosure found in this paper, the empirical literature on stress tests has consistently found evidence that banks with weaker fundamentals (i.e., riskier assets, more leverage, larger amounts of non-performing loans), are subject to more transparency than banks with stronger fundamentals (see e.g., Morgan et al. [2014], Flannery et al. [2017], and Ahnert et al. [2018a]). The analysis proposed in this paper suggests that larger revisions in prices for weaker banks, after the disclosure of their private information, should not be interpreted as an anomaly but, instead, as a feature of optimal disclosures in environments with multiple audiences.

Crucially, I find that imposing contingent recapitalizations is instrumental to implementing the optimal policy. Without recapitalizations information disclosure about the bank's fundamentals may be ineffective and the regulator may fail to help the bank to raise funds. As a matter of fact, a disclosure rule that is not complemented with recapitalizations may backfire and prove worse than a *laissez faire* policy.[10] I show that the policy proposed in the paper implements the optimal solution to a broader mechanism design problem in which the policy-maker possesses the authority to dictate the type of securities and price the bank should choose when approaching long-term investors. I show that conferring this authority to the policy-maker is not necessary because the same outcome can be implemented by combining appropriately designed information

[10] I assume that the policy-maker cannot commit ex-ante to the liquidity stress test before the liquidity shock materializes. That is, in case a liquidity shock occurs, the policy-maker runs the liquidity stress test that maximizes the probability of survival.

disclosures with recapitalizations requirements.

The intuition behind the former result is that, in the absence of government intervention, the threat of a run of short-term creditors serves as a discipline device toward the possibility that types with different asset qualities *separate* during the fund-raising stage, and hence may promote risk-sharing.[11] In fact, if the probability of default is small enough, banks with high quality assets may try to signal their private information by retaining a larger fraction of their assets on their balance sheets and, consequently, raising less precautionary funds to minimize default risk. The possibility of disclosing information about the bank's liquidity position reduces the subsequent probability of default and, as a result, increases the incentives to signal. This has a negative impact on risk-sharing. Recapitalizations thus substitute for the disciplining role served by short-term creditors' run by threatening the bank to reduce the dividends that can be distributed if the latter failed to raise the funds specified by the policy-maker.

In certain environments, the policy-maker may not be able to measure the variables that are private information to the bank. In the second part of the paper I consider a richer setting where the regulator cannot conduct liquidity stress tests in a timely manner, after a liquidity shock materializes and before short-term creditors decide whether to run on the bank. The policy-maker implements an *emergency lending* program that asks the bank to self-report the magnitude of the liquidity shortage and additional residual information about the quality of the bank's assets. The regulator may purchase some (claims on the bank's) assets and, additionally, may publicly communicate part of the information learned while dealing with bank to the rest of market participants. That is, the policy-maker may act as the LOLR and, at the same time, may try to persuade the market by disclosing relevant information regarding the bank's fundamentals.

The problem of designing an emergency lending program that elicits information about the bank's buffers is similar to the problem considered in Philippon and Skreta [2012] and Tirole [2012], in that a bank's outside options are endogenous to the choice of the government's program. A bank that refuses to participate in the program faces short-term creditors whose beliefs depend on the government's mechanism. The novelty with respect to those earlier models is that the policy-maker may engage in strategic information disclosure about the information elicited from the bank. These additional properties drastically change the set of equilibrium outcomes.

The optimal emergency lending program asks the bank to (confidentially) report its private information and promises in return to assign a pass-fail grade. Contingent on assigning a passing grade, the policy-maker purchases claims on the bank's assets. When the regulator announces the bank has passed the test, short-term creditors find it in their best interest to keep rolling over the bank's debt. In turn, when the policy-maker fails the bank, short-term creditors willingly stop pledging to it.

To induce all liquidity types to truthfully report their liquidity positions, the policy-maker needs to compensate those types that are passed with lower probability, as otherwise no bank would truthfully report its liquidity needs. This compensation is done by offering higher prices for the bank's assets. The optimal resolution program offers a passing grade to most illiquid banks with low probability but compensates them with

[11] The disciplining role served by short-term creditors has been described in the literature going back to Calomiris and Kahn [1991]. For recent developments, see Cheng and Milbradt [2012] and Eisenbach [2017].

higher prices for their assets, while more liquid banks are assigned a pass grade with higher probability and lower prices for their assets. In this manner, the government improves the average liquidity position of banks receiving the passing grade, which persuades creditors to keep pledging funds to the bank.

I use the characterization of the optimal resolution program to show that interventions that involve simultaneous pledging by both the private (i.e. long-term investors) and the public sector (i.e., the policy-maker) are suboptimal. Imposing recapitalizations undermines the effectiveness of the government's resolution programs. In fact, a bank that retains a smaller fraction of its assets can be promised fewer funds by the government under the natural constraint that the latter does not pay more than the fair price of the securities she purchases. Given that the effectiveness of the resolution program relies on compensating extremely vulnerable banks (which, again, receive a passing grade less often than more liquid banks) with higher prices for their assets, forcing the bank to sell a fraction of its assets, in a precautionary manner, decreases the amount of funds that can be provided within the emergency lending program. Moreover, having the bank raising funds from long-term investors intensifies incentive compatibility issues. As a result, the optimal emergency lending mechanism minimizes any prior recapitalizations by the private sector (i.e., long-term investors).

The policy-maker is thus confronted with the dilemma of choosing between private-sector financing, which maximizes the price of the bank's securities by selling them before the liquidity shock occurs, and the government's resolution program, which asks the bank to report information about its liquidity buffers and then reveals information to its short-term creditors. I show that optimal comprehensive interventions display a non-monotone pecking order. Institutions with high-quality assets are given a pass grade by the asset quality review and are required to raise enough capital from long-term investors. Banks with intermediate-quality assets are assigned one of multiple failing grades and are funded with the government's resolution program. Finally, institutions with extremely poor-quality assets are failed with one of multiple failing grades and are induced to seek private-sector funding. The paper thus shows that the non-monotonicity in funding strategies need not be proof of suboptimality. In fact, the non-monotone pecking order naturally arises when accounting for the strategic interaction of the multiple audiences.

The rest of the paper is organized as follows. Below, I wrap up the introduction with a brief review of the most pertinent literature. Section 2 presents the model. Section 3 describes the equilibria in the absence of government intervention. Section 4 studies the optimal comprehensive disclosure policy. Section 5 studies the case where the policy-maker designs an elicitation mechanism to learn the liquidity position of the bank. Proofs omitted in the text are in the Appendix or in the Supplementary Material.

Related literature. The paper is related to several strands of the literature. The first strand is the literature on *regulatory disclosures*. Close in spirit to this paper is the work by Faria-e Castro et al. [2016] who consider a model of information disclosure in an environment with runnable liabilities and asymmetric information. The paper focuses on the trade-off between the government's fiscal capacity and the degree of transparency of stress tests. Crucially, that paper assumes that there exists a one-to-one relationship between liquidity position and long-term profitability of bank's assets. In contrast, in the present paper, I relax the assumption that liquidity and asset quality perfectly correlates, which allows me to examine the role of disclosure to multiple audiences who care about different aspects of the bank's multi-dimensional fundamentals. In the second part of the paper, where I allow the policy-maker to purchase claims on the bank's assets, I find that the degree of transparency of the stress test affects the amount of funds the policy-maker can commit to use, generating a trade-off between coarser disclosure policies and the effectiveness of the regulator's program at eliciting information from the bank. In other words, I show that stronger financial capacity need not come with more information disclosure, contrary to what is established in Faria-e Castro et al. [2016].

Goldstein and Leitner [2018] consider the stress test design problem of a regulator who wishes to facilitate risk sharing among banks endowed with assets of heterogeneous qualities. My model complements theirs by analyzing an environment where the amount of additional funds needed by the bank is endogenously determined by the disclosure policy and the endogenous interaction between the multiple audiences which care about different aspects of the bank's balance sheet. Orlov et al. [2017] consider the joint design of stress tests and capital requirements in a setting where multiple banks have correlated exposures to an exogenous shock. Inostroza and Pavan [2019] explore optimal disclosure policies when the policy-maker faces multiple receivers endowed with heterogeneous information, under an adversarial approach. They show that optimal stress tests need not generate conformism in beliefs among market participants, but generate perfect coordination among their actions. Alvarez and Barlevy [2015] study the incentives of banks to disclose balance sheet (hard) information in a setting where the market is not able to observe the exposure to counterparty risks. In my model, banks cannot disclose hard information but may try to signal information through their funding strategy. Bouvard et al. [2015] study a credit rollover setting where a policy maker must choose between transparency (full disclosure) and opacity (no disclosure) but cannot commit to a disclosure policy. In contrast, I assume the policy maker can fully commit to her disclosure policy and allow for fully flexible information structures.

Optimal government interventions in markets plagued by adverse selection have been studied in Philippon and Skreta [2012], Tirole [2012], and Fuchs and Skrzypacz [2015]. These papers share the common feature that government interventions affect post-intervention outcomes and vice versa. The first two papers consider a static setting, and show that the policy-maker optimally chooses to purchase low quality assets to *jump-start* a frozen market, permitting banks with better assets to receive funding from the private sector. The third paper considers a dynamic model in which low quality assets are sold first, which gradually improves the pool of legacy assets. The paper shows that the regulator should subsidize trade early in the model, and then impose prohibitively high taxes that essentially shut-down the asset market. In the second part of the paper, I propose a model that shares the common feature of these papers. Namely, that the policy-maker's liquidity-provision program generates endogenous participation constraints. In my model, however, the policy-maker may also engage in information design when trading with the bank, and some banks are funded directly by the government, instead of the private sector.

The present paper also contributes to the extensive literature on security design under adverse selection, as in Myers and Majluf [1984], DeMarzo and Duffie [1999], and DeMarzo and Fishman [2007], among others. I adopt the framework of Nachman and Noe [1994], who consider the problem of a seller with private (but imperfect) information about the profitability of her assets, and who issues claims on them in exchange for funds that help her meet a former liability. I modify their setting by introducing a probability of default, which is determined in equilibrium. In contrast to their celebrated result, which shows existence of a unique equilibrium where all types of sellers pool over the same debt-like security, I show that in the current environment there exist multiple equilibria, and that when investors' prior beliefs about the subsequent liquidity shock are pessimistic, the existence of a bank type with poor-quality assets is enough to induce market freezing, regardless of the aggregate quality of the assets. Recent developments along these lines include Daley et al. [2016], who consider the effect of ratings on security issuance; Yang [2015], who studies security design when the buyer may acquire information about asset quality at a cost; Szydlowski [2018], who considers the problem of a firm that seeks financing and chooses both its information disclosure policy and the type of security it offers to external investors; and Azarmsa and Cong [2018] who study the role of information in relationship finance.

Finally, this paper relates to the literature on *information design*. This literature can be traced back to Myerson [1986], who introduced the idea that, in a general class of dynamic games of incomplete information, the designer can restrict attention to private incentive-compatible action recommendations to agents. Recent developments include Kamenica and Gentzkow [2011], Kamenica and Gentzkow [2016], and Ely [2017]. These papers consider persuasion with a single receiver. Persuasion with multiple receivers is less studied. Calzolari and Pavan [2006a] consider an auction setting in which the sender is the initial owner of a good and where the different receivers are bidders in an upstream market who then resell in a downstream market. Related to this paper is Dworczak [2016], who offers an analysis of persuasion in other mechanism design environments with aftermarkets.Alonso and Camara [2016a] and Bardhi and Guo [2017] consider persuasion in a voting context, whereas Mathevet et al. [2016] and Taneva [2016] study persuasion in more general multi-receiver settings. Bergemann and Morris [2016a] and Bergemann and Morris [2016b] characterize the set of outcome distributions that can be sustained as Bayes-Nash equilibria under arbitrary information structures consistent with a given common prior. Alonso and Camara [2016b] study public persuasion in a setting with multiple receivers with heterogeneous priors. Kolotilin et al. [2017] consider a screening environment whereby the designer elicits the agents' private information prior to disclosing further information. Basak and Zhou [2017] and Doval and Ely [2017] study dynamic games in which the designer can control both the agents' information and the timing of their actions.

2 Model

Players and Actions. The economy consists of a bank, short-term creditors, long-term investors, and a policy-maker. There are 3 periods, $T \equiv \{1,2,3\}$. The bank is risk-neutral and has two legacy assets: (i) a risky and illiquid asset and (ii) a safe and liquid asset.[12] Both assets mature in period 3. The risky asset delivers an observable stochastic cash flow, $y \in \mathbb{R}_+$, while the safe asset has a face value R. In period 1, in order to increase the amount of liquid funds available at the second period, the bank may sell claims on its assets to a competitive and risk-neutral set of long-term investors. At the beginning of period 2, the bank may suffer a temporary liquidity shock (described in detail below) that turns a fraction of the liquid asset illiquid, preventing the bank from selling a fraction of it.[13] Finally, on the liability side of the bank's balance sheet, a continuum of short-term creditors of mass one, uniformly distributed over $[0,1]$, has a claim of \$1 due in period 2, which they may redeem at this period (*early*), or equal to R if they decide to *roll it over* until $t=3$. Let $a_i \in \{0,1\}$ denote the action chosen by creditor i, where $a_i = 0$ represents the action of rolling over the bank's debt, and $a_i = 1$ the decision of withdrawing by the end of the second period. I denote by $A \in [0,1]$ the fraction of short-term creditors who chooses to stop pledging to the bank.

Fundamentals. The fundamentals of the bank's balance sheet are captured by the vector (ω, y). The variable y represents the risky asset's cash flows which are drawn from the absolutely contintuous cdf F^y with support $\mathbb{R}_+$. The variable ω represents the bank's short-term liquidity. More specifically, $\omega \in \Omega \equiv [0,1]$ represents the fraction of the safe asset that the bank can sell during the second period in order to obtain additional funds to repay its obligations. A value of $\omega < 1$ can be interpreted as an unexpected liquidity shock which turns a fraction of the safe asset illiquid (e.g., imposition of haircuts or off-balance sheet items that become due at $t=2$). I assume that the fraction of the asset that is not liquidated becomes available at $t=3$ and can be used to repay short-term creditors that have rolled over the bank's debt. Thus, ω represents a *temporary* liquidity shock. This last assumption that the shock is temporary is made for simplicity.[14]

Default. If the fraction of short-term creditors who decide not to roll over the bank's debt is large enough with respect to the bank's available cash, bankruptcy is triggered. In that case, the bank's risky asset is liquidated. The risky asset liquidation value is $l < 1$.[15]

Precautionary Fund Raising. To reduce the probability of default, the bank may raise funds at $t=2$ by selling claims on its risky asset to long-term investors. If the bank raises P units of funds, the amount of cash

[12] There are multiple ways to motivate the assumption that the risky asset is illiquid. In the case of loans, this assumption captures the idea that the bank has a technology to monitor the borrowers which cannot be easily transferred to external investors.

[13] Alternatively, there exists a stochastic obligation that needs to be paid during the second period in addition to the fraction of early withdrawals. Importantly, the bank will suffer a liquidity shortage with positive probability.

[14] A model where a fraction $1-\omega$ of the safe asset is destroyed during the interim period (i.e., a permanent liquidity shock) and cannot be used to repay late withdrawals can be captured in the present specification by assuming that the range of F^y is given by $[1-\omega, \infty)$.

[15] The rich environment proposed in this paper allows to treat the value of the assets upon banktuprtcy and the amount the bank can get by selling assets in the (secondary) market differently. The former represents the amount that can be raised once the bank has filed for bankruptcy (see, e.g., the FDIC resolution handbook https://www.fdic.gov/bank/historical/reshandbook/resolutions-handbook.pdf). The latter is captured with the fund-raising game described in detail below.

available to repay early withdrawals is given by $\omega + P$.

Exogenous Information. I assume that there is *gradual resolution of uncertainty.* At $t = 1$, the bank's long-term cash flows, y, are drawn from F^y. The bank then learns a private signal θ about y, and forms beliefs about the realization of y according to the conditional cdf F^y_θ (resp., pdf f^y_θ), where θ belongs to the set $\Theta = \{\theta_L, \theta_H\}$, with $\theta_H > \theta_L$. I refer to θ as the bank's asset quality type. The policy maker, investors and short-term creditors share a common prior $\mu_\theta \in \Delta\Theta$ about the bank's asset type. I assume that the conditional pdf f^y_θ satisfies log-supermodularity in (y, θ) (or, equivalently, that the realization of cash-flows of different types θ are ordered according to MLRP). The cash flow realization cannot be observed by any market participant until $t = 3$. The liquidity shock ω is drawn from $F^\omega \in \Delta\Omega$ at the beginning of the second period and is only observed by the bank. These assumptions are made to reflect the idea that the profitability of the bank's asset depends on investment decisions made in the past, while the bank's liquidity position is subject to unexpected contingencies and may vary precipitously. All market participants anticipate at $t = 1$ the possibility that a liquidity shock takes place in period 2 but do not know its severity. All agents in the economy share the prior belief F^ω about the bank's liquidity shock. I denote by λ the mass point associated to $\omega = 1$.[16]

Payoffs. For simplicity, I assume no discounting. If the bank raises P units of money during the second period, draws a liquidity shock ω, and a fraction A of short-term creditors withdraws early, it survives as long as the available funds are greater than its obligations: $\omega + P \geq A$. In such a case, the bank may use the remaining cash to buy a bond and obtain a payoff of $R(P + \omega - A)$ at $t = 3$. Thus, the bank's payoff when it raises P units of cash in period 2, cash flows are $\tilde{y}$ during period 3, the liquidity shock is ω, and faces a fraction A of early withdrawals, is given by:

$$\begin{aligned} U(P, \tilde{y}, \omega, A) &= R\left((P + \omega - A) + ((1-\omega) - (1-A)) + \frac{\tilde{y}}{R}\right) \times 1\{P + \omega \geq A\} \\ &= (PR + \tilde{y}) \times 1\{P + \omega \geq A\}. \end{aligned} \tag{1}$$

Short-term creditors' payoffs depend on their actions as follows. The utility from withdrawing early is normalized to 0. The utility of a short-term creditor who decides to withdraw late is denoted by $u_i(\tilde{\omega}, A)$, where $\tilde{\omega}$ represents the total amount of available cash held by the bank at $t = 2$. The utility $u_i(\tilde{\omega}, A)$ is measurable with respect to the bank's fate:

$$u_i(\tilde{\omega}, A) = g(\tilde{\omega}, A; l, R)\, 1\{\tilde{\omega} \geq A\} + b(\tilde{\omega}, A; l, R)\, 1\{\tilde{\omega} < A\},$$

where $g(\tilde{\omega}, A; l, R) > 0 > b(\tilde{\omega}, A; l, R)$ for all $\tilde{\omega}, A, l, R$, with g and b are non-decreasing in $\tilde{\omega}, l, R$. I supress the dependence of $u_i(\cdot)$ on l, R for notational conveniece.[17]

[16] The implicit assumption that y and ω are independent does not mean that the bank's liquidity and asset quality are uncorrelated. In fact, the price P that the bank is able to obtain by selling claims on its asset correlates with its underlying quality. Thus, banks with a better assets, in the absence of information frictions, should be able to secure more liquid funds at short notice and, therefore, should be more liquid ex-post.

[17] A natural example of such a function might be $u_i(\tilde{\omega}, A) \equiv \frac{\tilde{\omega} + l}{A} \times 1\{\tilde{\omega} < A\} + R \times 1\{\tilde{\omega} \geq A\} - 1$. That is, if the amount of

Finally, I assume large social costs associated with the default of the *systemically important* bank. The policy maker obtains a positive payoff $W_0(A)$ when default is successfully avoided, and a payoff of 0 when that is not the case, with $W_0(\cdot)$ non-increasing.[18]

$$U^P(\tilde{\omega}, A) = W_0(A) \times 1\{\tilde{\omega} \geq A\}.$$

Asset Market. After observing its asset quality type θ in period 1, the bank proposes to long-term investors a security $s[\theta]$, which corresponds to a claim on future cash flows realizations of the risky asset. Formally, $s[\theta]$ belongs to $S \equiv \{s : \mathbb{R}_+ \to \mathbb{R}_+ \text{ s.t: (LL),(M),(MR)}\}$ where:

$$\begin{array}{rcl} \text{(LL)} & 0 \leq s(y) \leq y & \forall y \geq 0 \\ \text{(M)} & s(y) & \text{non-decreasing} \\ \text{(MR)} & y - s(y) & \text{non-decreasing.} \end{array}$$

These assumptions are standard in the literature of security design.[19] The market observes the security $s[\theta]$ and prices it according to the available public information. Importantly, I assume that the claims promised to long-term investors are subordinated to the ones of short-term creditors, and hence are repaid only if the bank avoids default.[20]

Intervention Policies. The policy-maker concerned with the possibility that the bank defaults may choose to intervene. The policy maker possesses a technology that allows her to disclose information to all market participants and to give recommendations to the bank about the amount of funds to raise from long-term investors. The assumption of *gradual resolution of uncertainty* implies that the designer may disclose information about the cash-flows at $t = 1$, after y has been determined, but can disclose information about the liquidity shock ω only at $t = 2$, after ω has been drawn. I denote by Γ^y the disclosure policy about the profitability of the bank's assets, y, and refer to it as the bank's *asset quality review,* and by Γ^ω the liquidity examination conducted in the second period about the bank's liquidity position, which I dub the *stress test.* In addition to the information revealed by the asset quality review, the policy-maker may impose a recapitalization requirements according to the rule $\mathscr{R}$ which specifies the amount that must be raised as a function of

withdrawals exceeds the bank's liquidity position, $\tilde{\omega}$, the bank defaults and short-term creditors obtain a proportional fraction of their claim $\frac{\tilde{\omega}+l}{A}$. Otherwise, if the bank survives, they receive R.

[18]Note that, in this model, all banks are solvent but may potentially become illiquid. A richer model that accomodates for insolvent banks can be easily contructed by allowing negative realizations of y. The predictions obtained in the present specification easily extend to the richer environment as long as the policy maker obtains a negative utility from having an insolvent bank surviving.

[19]The first constraint represents *limited liability* and states that a security $s \in S$ is in fact a sharing rule of the asset's cash-flows. The second constraint, the *monotonicity* condition, requires that the security is non-decreasing in the asset's cash-flows, since otherwise the bank would have the option of requesting (risk free) credit to a third party to boost the cash-flow realization and thus decrease the amount owed to the initial investors. Finally, the last constraint imposes that the share of cash-flows kept by the bank is non-decreasing for, otherwise, the bank would have incentives to *burn* part of the cash-flows to improve her payoff.

[20]See Ahnert et al. [2018b] for a model of asset encumbrance where banks may choose the fraction of secured (or *enuncumbered*) funding they request from long-term investors.

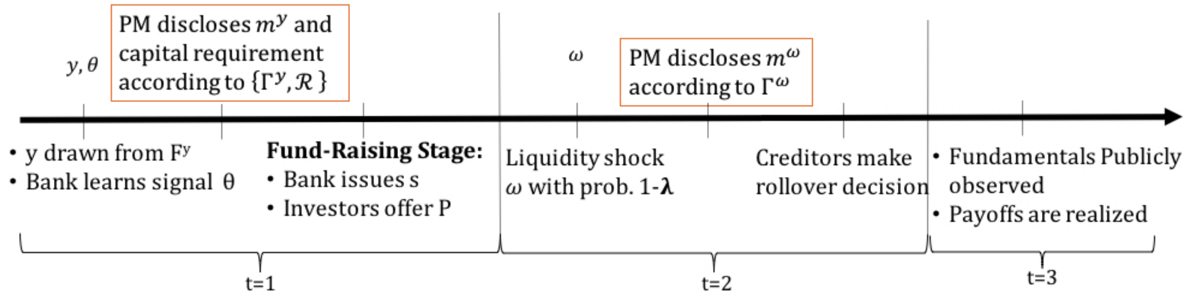

Figure 2: Timing.

the information disclosed by Γ^y. I make the implicit assumption that the technology needed to conduct the asset quality review Γ^y is time-demanding and cannot be postponed until the liquidity shock takes place, since then the policy-maker might not be able to disclose information on time, before short-term creditors choose whether to keep pledging to the bank. Moreover, I assume that any information learned while conducting the asset quality review Γ^y becomes public. That is, the policy-maker cannot choose to learn information about y and not share it with market participants.[21] Finally, I assume that the policy-maker cannot commit at $t = 1$ to the stress test Γ^ω she will conduct in period 2.[22] This assumption is made on practical grounds. Given that the bank is a sistemically important financial institution, promises made in period 1 to conduct a specific stress test Γ^ω during period 2 are not credible. The policy-maker chooses in period 2 the stress test that maximizes her payoff. [23]

Timing. The sequence of events is as follows:

Period 0. The policy maker designs the asset quality review and recapitalization policy, $\{\Gamma^y, \mathscr{R}\}$, and publicly announces it.

Period 1. (a) y is drawn from F^y. (b) The bank observes a private signal θ about y. (c) The policy-maker discloses information m^y and recapitalizations according to the joint policy $\{\Gamma^y, \mathscr{R}\}$. (d) The bank sells a security $s \in S$ to long-term investors at price $P \geq 0$. I refer to (d) as the *fund-raising stage.*

Period 2. (a) The policy maker designs the stress test Γ^ω and publicly announce it; (b) ω is drawn from F^ω; (c) The policy-maker conducts the stress test Γ^ω and discloses information m^ω; (d) Short-term creditors observe P and m^ω, and decide whether withdraw early or late; (e) The bank liquidates a fraction of her bond and its fate is determined according to whether $\omega + P$ is greater than the fraction of early withdrawals, A. Any

[21] I assume that any information produced by the regulator leaks and, therefore, if the policy-maker wants the rest of market participants not to learn some information she should not produce it in the first place. A similar assumption is made by Faria-e Castro et al. [2016].

[22] In other words, the policy-maker has *intra-temporal* commitment power but not *inter-temporal* commitment power. That is, the policy-maker *can* commit to disclose information according to asset quality review Γ^y (resp. Γ^ω) in period 1 (resp. 2), but not to the liquidity stress test she will conduct in period 2, at $t = 1$.

[23] This assumption is also consistent with a narrative where the policy maker designing the asset quality review is different from the one designing the stress test. Such a narrative resonates well with the European experience where the policy maker conducting the asset quality reviews (European Central Bank) differs from the one in charge of running the stress tests (European Banking Authority, in cooperation with the European Single Resolution Board).

excess of liquid funds is reinvested.

Period 3. Conditional on the bank's survival, (a) y is materialized and $s(y)$ is paid to long-term investors; (b) The fraction of bond not liquidated early, and any amount reinvested at period 2, is collected with interests and late short-term creditors are paid back.

3 Laissez Faire

3.1 Raising Capital to Persuade Short-Term Creditors

In this section, we study the case where the policy maker does not intervene. I show that when the liquidity shock is severe the market for the bank's asset may freeze leaving the bank vulnerable to short-term creditors runs. Moreover, under stringent conditions this *market freeze* may become the unique equilibrium outcome of the fund-raising stage.

The bank after observing its asset quality type, θ, enters the *fund-raising stage* by approaching long-term investors. The bank offers them claims on its asset in order to raise funds to pay its obligations and avoid default. I follow an adversarial approach, and assume that, when multiple action profiles are consistent with equilibrium play, short-term creditors coordinate on the most aggressive outcome from the perspective of the bank.

Let $\mathbb{E}\left(u\left(\omega+P,A\right)\right)$ be the expected utility of a short-term creditor who chooses to pledge when the fraction of early withdrawals is given by A, the seller has successfully raised P units of capital, and the liquidity shock is ω. The adversarial approach then implies that all short-term creditors choose to stop rolling over the bank's debt whenever withdrawing early is a best response to everyone withdrawing early. That is, each short-term creditor withdraws early when

$$\mathbb{E}\left(u\left(\omega+P,1\right)\right)=\int_0^1 u\left(\omega+P,1\right)F^{\omega}(d\omega)\leq 0. \tag{2}$$

In what follows I assume that λ, the probability with which no liquidity shock occurs, is small enough so that, if the bank does not raise additional funds, short-term creditors withdraw early (i.e., inequality (2) holds for $P=0$).

Assumption 1. $\mathbb{E}\left(u\left(\omega,1\right)\right)\leq 0$.

Next, let $K\geq 0$ be the minimum amount of capital that the bank needs to raise to persuade short-term creditors to keep rolling over its debt under adversarial coordination. That is,

$$K\equiv\inf\left\{P\geq 0:\mathbb{E}\left(u\left(\omega+P,1\right)\right)>0\right\}.$$

The bank may convince short-term creditors that it is liquid by raising K units of capital in the asset market. Define $\bar{A}\left(P\right)$ as the most aggressive fraction of early withdrawals, for a given recapitalization level, P. From the definition of K above, we have that $\bar{A}(P)=1\{P\leq K\}$. To make the problem interesting we assume that the type L-bank has an asset with expected, discounted cash flows below K, while type H's expected discounted cash flows are above it.

Assumption 2. $\frac{1}{R}\mathbb{E}_L(y) < K \leq \frac{1}{R}\mathbb{E}_H(y)$.

By the end of period 2, short-term creditors perfectly observe the bank's recapitalization and decide whether to rollover the bank's debt. If the bank raises at least K units of capital, then no short-term creditor withdraws early, allowing the bank to survive. On the other hand, if the amount raised is smaller than K, then all short-term creditors withdraw early, in which case the survival of the bank depends on the amount of capital raised and on the realization of the liquidity shock ω. Thus, the maximal price that long-term investors are willing to pay for any security s placed by the bank is

$$P(s;\mu) \equiv \sup\left\{ p \geq 0 : \frac{\mathbb{E}_\mu(s)}{R}\mathbb{P}\{\omega + p \geq A(p)\} \geq p \right\}, \tag{3}$$

where $\mathbb{E}_\mu(s)$ is the expected value of security s when the market holds beliefs $\mu \in \Delta\Theta$ about the bank's type. Note that the definition of $P(s,\mu)$ implies that, in case the equation

$$\frac{\mathbb{E}_\mu(s)}{R}\mathbb{P}\{\omega + p \geq A(p)\} = p, \tag{4}$$

admits multiple solutions, the selected one is the one associated with the largest price.[24] The next assumption will be used for certain results, for it favors tractability.

Assumption 3. *The prior distribution of the liquidity level ω, F^ω, is concave.*

Assumption 3 reflects the idea that the liquidity problem is severe. Intuitively, when F^ω is concave (i.e., when the density f^ω is non-increasing), low liquidity levels are more likely to occur. When this is the case, and additionally $\lambda = 0$ (the shock occurs w.p. 1), investors refuse to fund any project with NPV below K. To see this, note that in this case the LHS of inequality in (3), $\mathbb{E}_\mu(s)\mathbb{P}\{\omega + p \geq A(p)\}/R$, is smaller than the RHS, p, meaning that the expected payoff an investor obtains from purchasing security s is no greater than what he pays. As a consequence, the market refuses to purchase security s because it expects a high probability of default. The intuition behind this result is that, under an adversarial approach, investors believe that short-term creditors will overreact to the inability of the bank of raising enough capital. This generates a negative feedback cycle since it invites the market to offer a lower price for the security issued by the bank. The bank's inability to raise funds then makes a massive early withdrawal more likely, which in turn implies a higher probability of default and thus a lower price, and so on. Hence, when $\lambda = 0$ and assumption 3 holds, the bank survives only if the price collected is at least K.

[24] This selection has a game-theoretic foundation similar in spirit to the one encountered in *Bertrand* competition models. Namely, if the market reached a price $\tilde{P} < P(s;\mu)$ satisfying 4, any buyer could deviate and offer a greater price $\hat{P}$ for which the LHS of equation 4 is strictly greater than the RHS, and obtain a positive gain in the process. Such deviation would be willingly accepted by the bank. As a result, $\tilde{P}$ would be inconsistent with equilibrium play. $P(s;\mu)$ is thus the unique price consistent with competitive markets and immune to such deviations.

3.2 Solution Concept

I assume that renegotiation between short-term creditors and the bank is not feasible. Given the speed of events and the dispersion of short-term creditors, renegotiation is, in most cases, unviable in practice.[25] The government's most preferred outcome consists of having all bank types issuing securities that allow them to survive the liquidity shock, and hence avoid bankruptcy. As is usually the case with signaling games, the fund raising game may be plagued with multiple equilibria. In order to focus on equilibria which take into account the propensity of bank types to deviate, I restrict attention to PBE satisfying the D1 criterion, and I refer to them hereafter as equilibria.

Let $V(P,s,\theta)$ be the utility of a bank of type θ, selling a security s and receiving funds in the amount of P. Without government intervention, the bank's payoff can be written as:

$$\begin{aligned} V(P,s,\theta) &\equiv \mathbb{E}_\theta\left((PR+y-s)\,1\{\omega+P\geq A(P)\}\right) \\ &= (PR+\mathbb{E}_\theta(y-s))\,\mathbb{P}\{\omega\geq A(P)-P\}. \end{aligned} \tag{5}$$

I will say that $\left\{\{s^\star_\theta\}_{\theta\in\Theta},\mu^\star,P^\star,A^\star\right\}$ is an equilibrium of the fund-raising game if:

$$\begin{aligned}
&\text{[Sequential Rationality]:} && s^\star_\theta \in && \arg\max_s V\left(P^\star(s),\theta,s\right) \\
&\text{[Competitive Investors]:} && P^\star(s) = \sup && \left\{p\geq 0: \frac{\mathbb{E}_{\mu^\star(s)}(s)}{R}\mathbb{P}\{\omega+p\geq A^\star(p)\}\geq p\right\} \\
&\text{[Adversarial Coordination]:} && A^\star(P) = && 1\{P<K\},\ \forall P\geq 0 \\
&\text{[Belief Consistency]:} && \mu^\star(s) && \text{computed according to Bayes rule on-path}
\end{aligned}$$

Additionally, I impose that off-path beliefs associated with securities not observed in equilibrium, assign all probability weight to the asset quality type with the greatest propensity to deviate to them. This refinement is usually known as the D1 criterion (see the Appendix).

3.3 Equilibrium Characterization.

Next, we characterize the set of equilibria that arise in the fund-raising game under a laissez faire policy. The main result of this section shows that, when the bank's liquidity problem is severe, the market for assets may freeze. The bank's liquidity problem is severe when $\lambda = 0$ (i.e., when the liquidity shock occurs w.p. 1) and assumption 3 holds. I show that, under these two assumptions, the only type of equilibria in the fund-raising game are pooling equilibria, where both bank types issue the same debt contract. If the expected profitability of the asset of the type L bank is low enough, then there exists an equilibrium where the asset market freezes and no security is issued by any bank type. Furthermore, when in addition the average quality of the bank's asset is low, then market freezing is the unique equilibrium of the fund-raising game. As a result, all bank types default w.p. 1 (recall that $\lambda = 0$). Finally, I show that when the expected profitability of a type L-bank is

[25] Landier and Ueda [2009] make a similar assumption.

good enough, the unique equilibrium of the game has both bank types placing a debt contract which collects enough funds to dissuade short-term creditors from running.

Proposition 1. *The following properties are true:*

1. *Any pooling equilibrium must be in debt contracts (i.e.,* $s_{pool} = \min\{y,d\}$ *for some* $d \geq 0$*). Moreover,* $P\left(s_{pool}\right) \leq K$.

2. *If assumption (2) holds, then in any separating equilibrium, the security issued by type H satisfies that* $P\left(s_H^{sep}\right) \leq \mathbb{E}_L(y) < KR$.

3. *Suppose that assumption (3) holds. Then,*

   (a) *(Market freeze) If, in addition, assumption (2) also holds, then there is an equilibrium where* $s_\theta = \mathbf{0}$ *for all* $\theta \in \Theta$*. Moreover, if* $\frac{\mathbb{E}(y)}{R} < K$*, this is the unique equilibrium.*

   (b) *(Risk-Sharing) If* $\frac{\mathbb{E}(y)}{R} \geq K$*, then there exists an equilibrium where* $s_\theta = \min\left\{y, D^{pool}\right\}$ *with* D^{pool} *defined as the unique solution to* $\frac{1}{R}\mathbb{E}\left(\min\left\{y, d^{pool}\right\}\right) = K$*. Moreover, if* $\frac{\mathbb{E}_L(y)}{R} \geq K$*, this is the unique equilibrium.*

Proposition1 extends the results in Nachman and Noe [1994] to the current environment where the probability of default is endogenously determined by the interaction between the two audiences, short-term creditors and long-term investors.[26] The proposition shows that the celebrated uniqueness result of the former paper breaks in the current environment. Nonetheless, some properties remain true. Part (1) of the proposition shows that pooling equilibria necessarily occur over debt debt contracts. Part (2) states that, unlike Nachman and Noe [1994]'s environment, the current model admits the possibility of separating equilibria. However, in any such an equilibrium type-H raises an amount below K and hence remains vulnerable to rollover risk despite the fact that the value of its asset is above the cutoff K.

Finally, part (3) shows that, when the bank expects a severe liquidity shock, the presence of a bank type with sufficiently poor assets is enough to guarantee the existence of an equilibrium where the market for the bank's assets freezes over, preventing the bank from raising funds to avoid the run of short-term creditors. Long-term investors' ability to foresee the possibility of a run and to price assets accordingly, together with the incentives of the banks of type H to separate from type L, induce a *fire sale* effect so severe that the bank is unable to raise funds. As a consequence of this property, any security which a type H bank may try to issue is also issued by type L, generating contagion among bank types and inducing the freeze of the asset market.

4 Disclosure Policies

The policy maker, concerned with the potential freeze of the asset market may choose to intervene by conducting a *comprehensive assessment*. In period 1, the policy maker has the technology to conduct an *asset quality*

[26]The model in Nachman and Noe [1994] assumes that the seller of the asset (i.e., the bank in our environment) survives w.p. 1 if the latter raises an exogenous amount K, and defaults, also with certainty, if the bank does not raise this ammount.

review, $\Gamma^y = \{M^y, \pi^y\}$, characterized by a disclosure policy, $\pi^y : \mathbb{R}_+ \to \Delta M^y$, where M^y is an arbitrary set of scores. Conditional on the score $m^y \in M^y$ disclosed by the asset quality review, the policy maker specifies a *recapitalization* rule $\mathscr{R}(\cdot|m^y) : \mathbb{R}_+ \to [0,1]$, where for any $P \in \mathbb{R}_+$, $\mathscr{R}(P|m^y)$ represents the maximal fraction of the bank's payoff the latter is allowed to distribute as dividends, as a function of the the level of capital raised during the fund-raising stage, P.[27],[28] The solution presented below *implements* the optimal solution in an environment where the policy maker has the authority to choose the bank's recapitalization strategy (i.e., the security sold to investors). The recapitalization rule emphasized below resonates well with the *capital conservation buffers* recommended by Basel III which restrict the amount of dividends that can be distributed as a function of the bank's capitalization.

Consider the following family of recapitalization rules satisfying

$$\mathscr{R}(P) = \begin{cases} 1 & P > C \\ \alpha & P \leq C, \end{cases}$$

with $\alpha, C \in [0,1]$. I interpret any such a rule as imposing a *minimal recapitalization requirement*, C, which if not met limits the amount of dividends the bank is allowed to distribute to a fraction α of the bank's total profit. The optimal recapitalization policy can be described as a minimal recapitalization requirement. The decision of allowing shareholders to distribute only a fraction of the bank's profit serves the purpose of enforcing the policy-maker's recommendation.[29] Note that although I confer the designer the authority to impose recapitalizations, I do not allow her to repudiate the contracts the bank agrees upon with the investors. That is, investors preserve their claims on the future cash flows of the asset even if the bank does not comply with the recapitalization requirement. Importantly, the government commits to not inject any type of funds to insulate short-term creditors from the liquidity shock and, hence, taxpayers' money is not at stake. I relax this assumption in the next section. I show next that imposing recapitalizations serves as a *discipline device* to control separation incentives among bank types during the fund-raising stage.

In period 2, the policy-maker conducts a stress test, $\Gamma^\omega[P] = \{M^\omega, \pi^\omega[P]\}$, and discloses information about the bank's liquidity shock according to the rule $\pi^\omega[P] : \Omega \to \Delta M^\omega$.[30] Hereafter, I refer to the combination of an asset quality review, a recapitalization rule, and a stress test, $\Psi = \{\Gamma^y, \mathscr{R}, \Gamma^\omega\}$, as a *comprehensive assessment*.

[27]The assumption that $\mathscr{R}(P|m^y)$ takes only deterministic values is without loss of optimality as it will become clear later on.

[28]Assuming that $\mathscr{R}$ does not depend directly on y is wlog. I make this assumption so that the induced beliefs about the quality of the bank's asset depend only on m^y, and not on $\mathscr{R}$. A similar assumption is made in Orlov et al. [2017].

[29]Imposing recapitalizations can be interpreted in different ways in this *one-shot* framework (as opposed to a repeated game setup). The favored interpretation is that it represents a limit on the amount that can be distributed as dividends if the bank fails to raise the required level of capital. It could also represent the decision of selling the firm to another institution, and α in that case, represents the discount applied to the value of the bank.

[30]Given that the ownership of asset's claims and the true realization of y are irrelevant for short-term creditors, who care about the liquidity shock and the amount of funds collected by the bank, P, restricting attention to policies π^ω that only depend on P is without loss.

4.1 Period 1

During the first period, y is determined. The policy maker then discloses score m^y according to the policy π^y. Given the score m^y the policy maker then specifies a recapitalization rule $\mathscr{R}[m^y]$. The bank then approaches long-term investors and offers a security s. The latter, after observing the security issued by the bank, form beliefs $\mu \in \Delta\Theta$ about its asset quality type. I denote by $P_\mu(s;m^y)$ the competitive price offered to the bank. Suppose that investors, which hold beliefs F^ω about the bank's liquidity shock, expect the designer to disclose information about ω according to $\Gamma^\omega[P] = \{M^\omega, \pi^\omega[P]\}$. Then,

$$P_\mu(s;m^y) \equiv \sup\left\{P : \frac{\mathbb{E}_\mu(s;m^y)}{R}\int_{\Omega\times M^\omega} 1\{\omega + P \geq A(P,m^\omega)\}\,\pi^\omega(dm^\omega|\omega;P)\,F^\omega(d\omega) \geq P\right\}, \tag{6}$$

where $\bar{A}(P,m^\omega)$ represents the most aggressive fraction of early withdrawals when the bank is able to raise P units of additional capital and the designer discloses information m^ω about ω.

4.2 Period 2

After the liquidity shock ω materializes and the amount of capital raised by the bank P has been observed by all market participants, the designer conducts the stress test Γ^ω. Assume that score $m^\omega \in M^\omega$ is publicly disclosed. Let $F^{\omega|m}(\cdot|m^\omega)$ be the posterior probability distribution characterizing the beliefs about the liquidity shock ω induced.

$$F^{\omega|m}(\Lambda|m^\omega) = \frac{\int_\Lambda \pi^\omega(m^\omega|\omega)\,F^\omega(d\omega)}{\int_\Omega \pi^\omega(m^\omega|\omega)\,F^\omega(d\omega)},\ \ \forall\Lambda\subseteq\Omega.$$

Denote by $\mathbb{E}(u(\omega+P,1)|m^\omega)$ the expected posterior utility of a short-term creditor when he observes the public score m^ω. That is,

$$\mathbb{E}(u(\omega+P,1)|m^\omega) \equiv \int_\Omega u(\omega,P,1)\,F^{\omega|m}(d\omega|m^\omega).$$

The most aggressive fraction of early withdrawals faced by the bank is then given by

$$\bar{A}(P,m^\omega;\Gamma^\omega) = 1\{P \leq \bar{K}(m^\omega)\}.$$

where $\bar{K}(m^\omega) \equiv \inf\{P \geq 0 : \mathbb{E}(u(\omega+P,1)|m^\omega) > 0\}$ is defined as the minimal amount of capital needed to persuade short-term creditors to keep pledging after score m^ω is disclosed. Thus,

$$\bar{A}(P,m^\omega;\Gamma^\omega) = 1\{\mathbb{E}(u(\omega+P,1)|m^\omega) \leq 0\}.$$

As a result, the payoff that a type θ-bank obtains when it issues a security s at price P, when information m^y is disclosed at $t = 1$, and recapitalizations are specified by the policy $\mathscr{R}$, is given by:

$$\begin{aligned} V(s,P,\theta;m^y,\mathscr{R}) &= \mathscr{R}(P|m^y) \times (PR + \mathbb{E}_\theta(y-s|m^y)) \times \\ &\quad \times \left(\int_{\Omega\times M^\omega} 1\left\{\omega + P \geq \bar{A}(P,m^\omega)\right\} d\pi^\omega(m^\omega|\omega;P)\,F^\omega(d\omega)\right). \end{aligned}$$

4.3 Stress tests as convex functions

I now characterize the optimal comprehensive assessment $\Psi = \{\Gamma^y, \mathscr{R}, \Gamma^\omega\}$. We proceed by backward induction. Assume that an amount P has been raised during the fund-raising stage. The approach below borrows from Gentzkow and Kamenica [2016] who characterize arbitrary disclosures by the induced distribution of posterior expectations. The approach is described in detail in Appendix A.

Consider *any* stress test $\Gamma^\omega = \{M^\omega, \pi^\omega\}$. Each score m^ω disclosed by stress test Γ^ω induces a posterior distribution over ω, $F^\omega(\cdot|m^\omega)$. Hence, every score m^ω disclosed with positive probability generates a posterior expectation of $u(\omega+P,1)$, $\mathbb{E}(u(\omega+P,1)|m^\omega)$, the expected payoff of a short-term creditor, who expects all other short-term creditors to withdraw early, after score m^ω is disclosed. That is, each score m^ω induces a new posterior expectation:

$$\mathbb{E}(u(\omega+P,1)|m^\omega) \quad = \quad \int_\Omega u(\omega+P,1)\, F^{\omega|m}(d\omega|m^\omega).$$

Denote by G^{Γ^ω} the distribution of posterior expectations $\mathbb{E}(u(\omega+P,1)|m^\omega)$ induced by an arbitrary stress test Γ^ω. Let $G^\omega_{\text{FD}}(\cdot;P)$ be the distribution of posterior expectations of $u(\omega+P,1)$ induced by policy the full-disclosure policy (i.e., the policy that follows the rule $\Gamma^\omega_{\text{FD}} \equiv \{M^\omega = \Omega, \pi^\omega_{\text{FD}}\}$, with $\pi^\omega_{\text{FD}}(m^\omega|\omega) = 1\{m^\omega = \omega\}$).

The next proposition shows that the problem of finding the optimal stress test is equivalent to finding the distribution of posterior expectations that maximizes the weight assigned to the event $\{\omega \in \Omega : \mathbb{E}(u(\omega+P,1)|m^\omega) > 0\}$. Intuitively, under adversarial coordination, when short-term creditors have homogenous beliefs the policy maker's task needs to convince short-term creditors that their expected payoff if they choose to rollover the bank's debt is positive, even if the rest of short-term creditors choose to run on the bank.[31]

Proposition 2. *Fix $P \geq 0$. The stress test that maximizes the designer's payoff, given by the solution to*

$$\begin{aligned} \max_{\Gamma^\omega=\{\pi^\omega,M^\omega\}} \quad & \mathbb{E}\left(W_0\left(\bar{A}(P,m^\omega)\right) \times 1\left\{\omega+P \geq \bar{A}(P,m^\omega)\right\}\right) \\ s.t.: \quad & \bar{A}(P,m^\omega) = 1\{\mathbb{E}(u(\omega+P,1)|m^\omega) \leq 0\}, \end{aligned}$$

is also characterized by the solution to the problem of finding the distribution of posterior expectations $\mathbb{E}(u(\omega+P,1)|m^\omega)$, G^{Γ^ω}, among all mean preserving contractions of the full-disclosure distribution of posterior expectations, G^ω_{FD}, that solves

$$\begin{aligned} \max_{G^{\Gamma^\omega}} \quad & 1 - G^{\Gamma^\omega}(0) \\ s.t.: \quad & G^\omega_{FD} \succeq_{MPS} G^{\Gamma^\omega}. \end{aligned}$$

[31] Inostroza and Pavan [2019] show, in an environment with heterogeneous beliefs, that the optimal disclosure policy perfectly coordinates short-term creditors actions. The current model can be modified to accomodate heterogeneous beliefs. The specification in the current model retains the same qualitative properties as the former paper (e.g., perfect coordination) while simplifying the intrincancies of characterizing the optimal policy in the richer environment.

Next, for any stress test, Γ^{ω}, and any amount $P \geq 0$ raised by the bank during the fund-raising game, define the integral function $\mathscr{G}^{\Gamma^{\omega}}(t;P) \equiv \int_{\tilde{u}=u(0,P,1)}^{t} G^{\Gamma^{\omega}}(\tilde{u};P)\,d\tilde{u}$. Let $\mathscr{G}^{\omega}_{\text{FD}}$ and $\mathscr{G}^{\omega}_{\emptyset}$ be the integral functions associated with the full-disclosure policy, $\Gamma^{\omega}_{\text{FD}}$, and no-disclosure policy, $\Gamma^{\omega}_{\emptyset}$, respectively. The set of feasible critical stress tests Γ^{ω}, coincides with the set of convex functions that lie between $\mathscr{G}^{\omega}_{\text{FD}}$ and $\mathscr{G}^{\omega}_{\emptyset}$.

Lemma 1. *Consider an arbitrary stress test Γ^{ω}. Then $\mathscr{G}^{\Gamma^{\omega}}(t;P)$ is convex and satisfies $\mathscr{G}^{\omega}_{FD}(t) \geq \mathscr{G}^{\Gamma^{\omega}}(t) \geq \mathscr{G}^{\omega}_{\emptyset}(t)$ for all $t \in [u(P,1), u(1+P,1)]$. Conversely, any convex function $h(\cdot)$, satisfying $\mathscr{G}^{\omega}_{FD}(t) \geq h(t) \geq \mathscr{G}^{\omega}_{\emptyset}(t)$ for all $t \in [u(0,P,1), u(1,P,1)]$ corresponds to the integral distribution function of some disclosure policy Γ^{ω}.*

The designer's problem is thus equivalent to finding the policy Γ^{ω} which generates the convex function $\mathscr{G}^{\Gamma^{\omega}}$, between $\mathscr{G}^{\omega}_{\emptyset}$ and $\mathscr{G}^{\omega}_{\text{FD}}$, with minimal slope at $t = 0$. As can be seen from Figure 3, the solution to the designer's problem is thus given by the monotone-binary policy $\Gamma^{\omega}_{\star} = (\{0,1\}, \pi^{\omega}_{\star})$ so that:

$$\pi^{\omega}_{\star}(0|\omega) = 1\{u(\omega+P,1) \geq \bar{u}(P)\} = 1\{\omega \geq \bar{\omega}(P)\},$$

where $\bar{u}(P)$ corresponds to the point at which $\mathscr{G}^{\omega}_{\text{FD}}$ is tangent to the line with minimal slope to the left of 0 that respects the convexity of $\mathscr{G}^{\Gamma^{\omega}_{\star}}$. The value of $\bar{u}(P)$ can also be characterized by $\bar{u}(P) = u(\bar{\omega}(P)+P,1)$, where $\bar{\omega}(P)$ represents the liquidity cutoff satisfying

$$\bar{\omega}(P) \equiv \inf\{\tilde{\omega} \geq 0 : \mathbb{E}(u(\omega+P,1)\,|\,\omega \geq \tilde{\omega}) > 0\}. \tag{7}$$

To see this last point, note that the policy $\Gamma^{\omega}_{\star}$ induces a distribution of posterior means $G^{\Gamma^{\omega}_{\star}}$ which assigns positive probability to only two points, which coincide with the points at which $\mathscr{G}^{\Gamma^{\omega}_{\star}}$ changes slope. To see that the first point at which $\mathscr{G}^{\Gamma^{\omega}_{\star}}$ changes slope coincides with

$$\mathbb{E}(u(\omega+P,1)\,|\,\omega < \bar{\omega}(P)),$$

note that the tangency condition implies that $G^{\Gamma^{\omega}_{\star}}(\bar{u}(P)) = G^{\omega}_{\text{FD}}(\bar{u}(P))$ where the RHS corresponds to $\mathbb{P}\{\omega \in \Omega : u(\omega+P,1) \leq$
or equivalently, $F^{\omega}(\bar{\omega}(P))$.

The optimal stress test can thus be interpreted as a pass-fail announcement, where given the level of recapitalization, P, the policy-maker assigns a *pass* grade when the liquidity of the bank is above the cutoff $\bar{\omega}(P)$. Proposition 3 summarizes the above findings.

Proposition 3. *Fix the amount of capital $P \geq 0$. Then, the optimal liquidity stress test $\Gamma^{\omega}_{\star}[P]$ consists of a monotone pass-fail test. That is, there exists a decreasing function $\bar{\omega}(\cdot)$, such that $\Gamma^{\omega}_{\star}(P) = (\{0,1\}, \pi^{\omega}_{\star}[P])$, with $\pi^{\omega}_{\star}(0|\omega;P) = 1_{\{\omega \geq \bar{\omega}(P)\}}$.*

Under the optimal policy, when the policy maker announces that the bank passed the stress test (i.e., when $\Gamma^{\omega}[P]$ discloses $m^{\omega} = 0$), all short-term creditors roll over the bank's debt, and hence survival occurs with certainty. When, instead, the bank fails the stress test $\Gamma^{\omega}[P]$ (i.e., when $\Gamma^{\omega}[P]$ discloses $m^{\omega} = 1$), all short-term creditors withdraw early and the bank defaults. To see this last point, note that $\omega \leq \bar{\omega}(P) < 1-P$ and therefore failing the stress test implies bank failure with certainty.

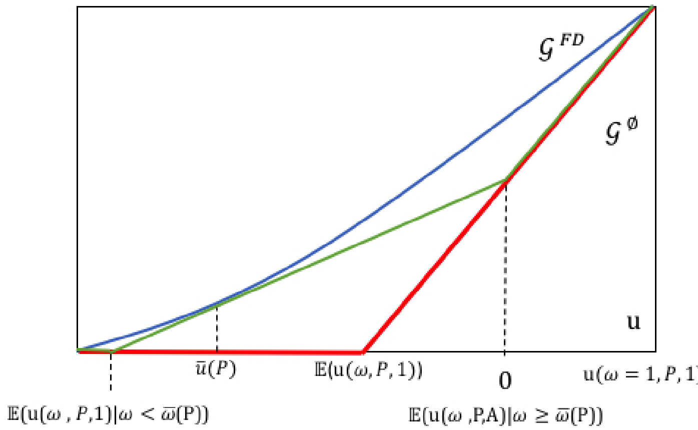

Figure 3: Optimal Stress Test.

4.4 Asset Quality Review

We now proceed to characterize the optimal policy $\{\Gamma^y, \mathscr{R}\}$ conducted in period 1, taking into account the optimal stress test $\Gamma^\omega[P]$ that follows in period 2. The optimal policy $\{\Gamma^y, \mathscr{R}\}$ takes a very simple form: it combines a recommendation to the bank about the minimal amount of capital to raise during the first period, along with a disclosure about the returns of the bank's asset y. To make sure that the recapitalization is followed, the policy maker chooses a minimal recapitalization rule with α small enough to limit the bank's ability to distribute dividends if the amount of capital falls short of the minimal level required.

As the next result shows, the policy maker asks the bank to raise an amount equivalent to the minimum between the capital cutoff which prevents posterior runs, K, and the expected price of the entire asset $\bar{P}(\mathbb{E}(y|m^y))$, where

$$\bar{P}(z) \equiv \sup\left\{P \geq 0 : \frac{z}{R}\mathbb{P}\{\omega \geq \bar{\omega}(P)\} \geq P\right\} \tag{8}$$

represents the (maximal) fair price consistent with selling a security with expected value of $z \geq 0$, accounting for the endogenous probability of default. Given the policy maker's commitment to limit the bank's ability to distribute dividends when the bank does not meet the capital cutoff, the game played by the bank and long-term investors becomes similar to the one in Proposition 1 for values of α small enough and, therefore, under the best continuation equilibrium, both asset quality types pool and offer a debt security $s_\star^{\text{pool}}$ satisfying:

$$\frac{1}{R}\mathbb{E}\left(s_\star^{\text{pool}}|m^y\right) = \min\{K, \bar{P}(\mathbb{E}(y|m^y))\}$$

On-path, recapitalization requirements are always obeyed.

At $t = 1$, the designer discloses information about the realization of future cash-flows y according to the rule $\pi^y : \mathbb{R}_+ \to \Delta M^y$. Proposition 4 below shows that recapitalizations are necessary to minimize the probability of default. By introducing recapitalizations the policy-maker mitigates separation incentives among bank types during the fund-raising game. In fact, type H has an incentive to separate from type L, so as to avoid *underpricing*. If the probability of default is low, high-quality banks may prefer to expose themselves to rollover risk, by raising less funds than K, and signal their type. The imposition of a minimal recapitalization requirement makes this type of strategies unprofitabl. The following proposition shows that, whenever possible, the optimal policy asks the bank to raise at least K to persuade short-term creditors to keep rolling over the bank's debt. Whenever this is not possible (i.e., whenever the value of the assets falls below K) the regulator asks the bank to sell the whole asset.

Proposition 4. *For any score m^y disclosed with positive probability under the asset quality review $\Gamma^y = \{M^y, \pi^y\}$, consider the recapitalization rule:*

$$\mathscr{R}(P|m^y) = \begin{cases} 1, & P > \min\{K, \bar{P}(\mathbb{E}(y|m^y))\} \\ \alpha, & \textit{otherwise.} \end{cases} \quad (9)$$

(a) If $\alpha < \alpha^*(\mathbb{E}(y|m^y))$, then $s_L^* = s_H^* = \min\left(y, \tilde{D}\right)$, with $\tilde{D}$ such that $\mathbb{E}\left(\min\left(y, \tilde{D}\right)\right) = K$, is an equilibrium of the fund-raising stage.

(b) If $\alpha = 0$, then $s_L^* = s_H^* = \min\left(y, \tilde{D}\right)$ is the unique equilibrium of the fund-raising stage.

Recapitalizations are instrumental to implement the optimal comprehensive policy. An asset quality review that reveals that the asset's expected cash-flows are greater than K (i.e., a test that discloses information m^y, such that $\frac{1}{R}\mathbb{E}(y|m^y) \geq K$), but does not impose recapitalizations, need not guarantee that all bank types raise enough funds to prevent default. In fact, in the absence of recapitalizations requirements, default may occur with positive probability, across all equilibria, even if, without government intervention, the bank would have survived with certainty. The reason behind this surprising observation is that short-term creditors impose *market discipline* on the bank during the fund-raising stage and mitigate, to some extent, separation incentives among different bank types. Indeed, when the bank and long-term investors expect the policy-maker to disclose information about the liquidity shock, their assessment about short-term creditors' expected response becomes more optimistic. This, in turn, makes it easier for type H to separate from type L, since rollover risk is mitigated. As a result, risk-sharing incentives dissipate. Imposing contingent recapitalizations substitute for the disciplining role of short-term creditors' run by limiting the bank's dividends if the minimal capital cut-off is not met. This implies that disclosing information about the liquidity shock, without imposing recapitalizations, may prove ineffective and even counterproductive at preventing the disruption of capital markets.

The next proposition shows that a policy maker endowed with a technology to conduct liquidity stress tests but does not impose recapitalizations may fare worse than a policy-maker that does not intervene at all.

Condition 1. The distribution of liquidity shocks F^ω and the short-term creditors' payoff functions g and b satisfy:

(a) F^ω is convex over $[0,1]$.

(b) f^ω is (right) continuous at $\omega = 0$, with $f^\omega(0) = 0$.

(c) $\lim_{P \to K^-} F^\omega(1-P) < \bar{\phi} \equiv \frac{\mathbb{E}_H(y-s_D)-\mathbb{E}_L(y-s_D)}{\mathbb{E}_H(y)-\mathbb{E}_L(y)}$.

Proposition 5. *Assume Condition 1 holds, then without recapitalizations, under the sequentially optimal stress test* Γ^{ω}*, default occurs with positive probability across all equilibria of the fund-raising stage. In contrast, under the laissez-faire policy, the probability of default reduces to* 0.

Assumptions (a) and (b) in Condition 1 imply that, under the sequentially optimal stress test Γ^{ω}, both bank types find it optimal to deviate from the pooling equilibrium where both banks raise K funds from long-term investors (recall that, as proved in Proposition 1, banks raise at most K when pooling, and type H always raise strictly less than K in any separating equilibrium). Intuitively, assumptions (a) and (b) guarantee that by raising slightly less than the amount K the probability of default barely increases above 0. Such a deviation is always interpreted by the market as coming from type H. Given that type H has a better asset than type L, the former is relatively more willing to risk defaulting in the next round to signal its quality. However, if the probability of default increases significantly when the bank does not raise the amount K, the benefits from signaling may be outweighed by the likelihood of default. Assumptions (a) and (b) guarantee that the probability of default behaves smoothly around the threshold K. Assumption (c), on the other hand, implies that, in the absence of any disclosure, the probability of default substantially increases if the bank does not meet the cutoff K. This effect imposes discipline on the banks and acutally forces them to raise enough funds to dissuade short-term creditors from running. Under assumption (c), both bank types pool over the debt contract $s_D = \min\{y, D\}$, with $\frac{\mathbb{E}(s_D)}{R} = K$, and therefore avoid default with certainty.[32]

Surprisingly, then, having the technology to conduct stress tests need not be optimal if these disclosures are not accompanied by additional policies to complement them. To prevent separation among asset quality types the policy maker has to punish banks that, despite being able to raise K, choose not to do so.[33] By imposing recapitalizations, the policy maker retains the benefits of having a technology to conduct liquidity stress tests, and avoids the costs of dissipating pooling incentives.

4.5 Optimal Comprehensive Assessment

The analysis so far characterizes the optimal recapitalization policy $\mathscr{R}[m^y]$ for any score m^y disclosed under Γ^y. We now proceed to the characterization of the optimal asset quality review Γ^y_* taking into account the optimal policies $\{\mathscr{R}, \Gamma^{\omega}\}$.

Any score m^y disclosed with positive probability induces a posterior expectation of y, $\mathbb{E}(y|m^y)$. Let G^{Γ^y} be the distribution of posterior expectations induced by an arbitrary asset quality review Γ^y. The set of possible distributions of posterior expectations that can be induced with a disclosure policy coincides with the set

[32]To have a better sense with respect to how restrictive Condition 1 really is, note that, e.g., when short-term creditors' payoff functions g and b are invariant in ω, assumption (c) is equivalent to requiring that the ratio $\frac{g}{g+|b|} < \bar{\phi}$.

[33]If the policy maker were able to commit to the liquidity stress test Γ^{ω} when designing the asset quality review Γ^y, she might threaten the bank to conduct an adversarial liquidity stress test if the latter were to raise less funds than what she envisions. These threats, however, would require the policy maker to minimize the probability of survival (off-path) if the bank failed to raise enough capital. The approach followed in this paper (i.e., lack of intertemporal commitment) implies that the optimal policy will not be sustained with non-credible threats.

of distributions which are a mean-preserving contraction of the prior F^y (Blackwell [1953], Gentzkow and Kamenica [2016]).

Next, fix a score m^y. Proposition 4 implies that the policy maker chooses a minimal recapitalization rule so that the cutoff defining whether the bank survives or not is given by $\bar{\omega}\left(\bar{P}\left(\mathbb{E}\left(y|m^y\right)\right)\right)$. Recall that the function $\bar{\omega}(P)$ identifies the critical value of the liquidity shock below which the bank defaults when the capital raised at $t=1$ is equal to P. This value is equal to 0 for any $P \geq K$ since at these prices the probability of default is 0.

4.5.1 Convexity and the Probability of Survival

Define $\phi(\tau)$ as the probability that the bank survives conditional on selling a security with expected value of τ. That is,

$$\phi(\tau) \equiv \mathbb{P}\{\omega \geq \bar{\omega}(\bar{P}(\tau))\} = 1 - F^{\omega}(\bar{\omega}(\bar{P}(\tau))).$$

The next lemma shows that, under some conditions, ϕ is convex over the critical region $(0, KR)$. The convexity of ϕ is a consequence of the interaction of the two audiences: short-term creditors and long-term investors. When the expected value of the security sold to long-term investors, τ, increases by $\varepsilon > 0$, the probability that the bank survives increases, as the bank becomes resilient to more stringent liquidity shocks and hence more resilient to potential runs of short-term creditors. The increase in the probability of survival feeds back and increases the expected value of the bank's security to long-term investors and hence the price they are willing to offer. This increase in the amount collected from long-term investors increases further the probability of survival, which again increases the price paid by long-term investors, and so on. As a result, in the absence of other forces, this amplification mechanism implies that the probability that the bank survives is convex in the expected quality of the bank's security.

However, there exists another force at play. Increasing the amount collected by the bank decreases the critical threshold above which the bank will pass the stress test in period 2. Decreasing the critical threshold then increases the probability of survival, but this effect need not occur in a convex manner. The next lemma identifies primitive sufficient conditions under which the combination of both forces induces a convex probability of survival.

Assumption 4. *Short-term creditors' payoff functions $g(\omega, A; l, R)$ and $b(\omega, A; l, R)$ are invariant in ω.*

Lemma 2. *Under assumptions 3 and 4, ϕ is convex in $(0, KR)$.*

Although assumption (4) is somewhat demanding, it still encompasses many cases of interest. That the payoff of withdrawing late and seeing the bank survive, $g(\omega, A; l, R)$, does not depend on ω is a natural assumption as short-term creditors' payoff in the long-run should not be associated to short-term liquidity shocks. That the payoff of withdrawing late and seeing the bank default, $b(\omega, A; l, R)$, does not depend on ω is more restrictive but may be justified on the grounds of large bankruptcy costs. Note that this assumption is implicitly assumed in the banking literature that follows Rochet and Vives [2004] where short-term creditors' decision of withdrawing early or late only depends on their assessment of the probability of default. Assumption (4) generalizes the assumption on those earlier models by allowing the payoffs g and b to depend on the size of early withdrawals A, the liquididation value l, and the returns on short-term creditors' contracts R.

4.5.2 The Optimal Asset Qulity Review

The designer's objective function can be written as:

$$\mathbb{E}\left(W_0\left(1\left\{\omega < \bar{\omega}\left(\bar{P}\left(\mathbb{E}\left(y|m^y\right)\right)\right)\right\}\right) \times 1\left\{\omega \geq \bar{\omega}\left(\bar{P}\left(\mathbb{E}\left(y|m^y\right)\right)\right)\right\}\right),$$

or equivalently,

$$W_0\left(0\right) \times \phi\left(\mathbb{E}\left(y|m^y\right)\right).$$

Thus, the policy-maker's problem reduces to:

$$\max_{G^{\Gamma_y}} \quad \int_0^\infty \phi\left(\tau\right) G^{\Gamma^y}\left(d\tau\right)$$
$$\text{s.t:} \quad F^y \succeq_{\text{MPS}} G^{\Gamma^y}.$$

Define Z^{Γ^y} as the auxiliary function that allows to take mean-preserving contractions of F^y. That is, for any mean-preserving contraction G^{Γ^y}, Z^{Γ^y} is defined so that $G^{\Gamma^y} = F^y + Z^{\Gamma^y}$. Any such $Z^{\Gamma^y}: \mathbb{R}_+ \to \mathbb{R}$ must respect the following conditions: $F^y + Z^{\Gamma^y}$ *(i) belongs to* $[0,1]$*, (ii) is non-decreasing, (iii) is right-continuous, and satisfies*

$$(iv) \int_0^{\tilde{y}} Z^{\Gamma^y}(y)dy \leq 0 \;\; (\forall \tilde{y} \geq 0), \; \int_0^\infty Z^{\Gamma^y}(y)dy = 0, \; Z^{\Gamma^y}(\infty) = 0.$$

I define the set of all such auxiliary functions as

$$\mathscr{Z}\left(F^y\right) \equiv \left\{Z : \mathbb{R}_+ \to \mathbb{R} : (i) - (iv)\right\}.$$

We can thus rewrite the policy maker's period 1 problem in terms of Z^{Γ^y} as

$$\max_{Z^{\Gamma^y}} \quad \int_0^\infty \phi\left(\tau\right) Z^{\Gamma^y}\left(d\tau\right)$$
$$\text{s.t:} \quad Z^{\Gamma^y} \in \mathscr{Z}\left(F^y\right).$$

As the next theorem shows, the optimal asset quality review, Γ_*^y, consists of a monotone partition signal, where different values of y are pooled (if at all) with adjacent realizations (i.e., within the same interval). I show that when the long-term profitability of the asset is good enough, above a cutoff y^+, the asset quality review Γ_*^y assigns a unique, and hence *opaque,* passing grade. When the long-term profitability is low, i.e., falls below the cutoff y^+, Γ_*^y is more *transparent* and assigns one of multiple failing grades. Moreover, I show that whenever the probability that the bank survives, $\phi\left(\tau\right)$, is convex in $(0, KR)$, then Γ_*^y takes a simple form. It fully discloses the realization of y for any realization below a cutoff y^+, and pool all realizations above y^+ under a single message, say m_+^y. The posterior expectation induced by message m_+^y satisfies $\frac{1}{R}\mathbb{E}\left(y|y \geq y^+\right) \geq K$. Thus, y^+ corresponds to the lowest cutoff that allows the bank to raise enough capital to persuade short-term creditors to rollover the bank's debt, under the prior beliefs characterized by F^ω.

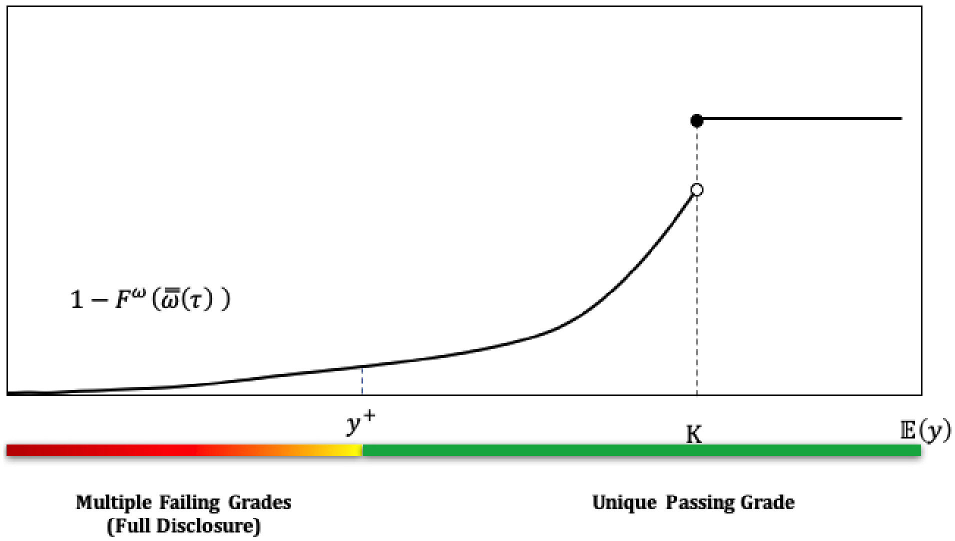

Figure 4: Optimal Asset Quality Review under Assumption 3.

Theorem 1. *The optimal asset quality review* Γ_*^y *consists of a monotone partitional signal. That is, there exists a monotone partition* $\mathscr{P} = \left\{m_i^y \equiv [y_i, y_{i+1})\right\}_{i\in I}$ *of* $\mathbb{R}_+$ *such that* $\Gamma_*^y = \left\{\left\{m_i^y\right\}_{i\in I}, \pi^y\right\}$*, with* $\mathbb{E}(y|m_i^y) < \mathbb{E}(y|m_j^y)$ *for all* $i<j$*, and* $\mathbb{E}\left(y|m_{I+1}^y\right) \geq K$*. Furthermore, if* $\phi(\cdot)$ *is convex over* $(0, KR)$*, then* $\Gamma_*^y = \left\{\left\{[0,y^+] \cup m_{pass}^y\right\}, \pi^y\right\}$*, with* $\pi^y(\tilde{y}|y) = 1\{\tilde{y} = y\}$ *and* $\pi^y\left(m_{pass}^y|y\right) = 1\{y \geq y^+\}$ *for all* $\tilde{y} \in [0, y^+]$*, and all* $y \geq 0$*, where* y^+ *is defined by:*

$$y^+ \equiv \inf\left\{y \geq 0 : \int_y^{\max\{KR,\mathbb{E}(y)\}} (F^y(y) - F^y(\tau))\, d\tau + \int_{\max\{KR,\mathbb{E}(y)\}}^{\infty} (1 - F^y(\tau))\, d\tau \geq 0\right\}. \tag{10}$$

Congruent with the qualitative properties of the disclosure policy found in this paper, the empirical literature has consistently found evidence that banks with weaker fundamentals (i.e., riskier assets, more leverage, and larger amounts of non-performing loans), are subject to more transparency than banks with stronger fundamentals when conducting stress tests (see Morgan et al. [2014], Flannery et al. [2017], and Ahnert et al. [2018a]). As a result of this, larger revisions in prices of weaker banks, after disclosure of their private information, should not be interpreted as an anomaly but, instead, as a feature of optimal disclosures with multiple audiences.

Theorem 1, along with the former results, imply that under assumptions 3 and 4, the optimal comprehensive policy $\Psi = \{\Gamma^y, \mathscr{R}, \Gamma^\omega\}$ has a simple structure. The policy Ψ assigns a single grade to all banks that meet a minimum standard in terms of profitability of their assets. This grade should be thought of as passing the policy-maker test on the quality of the bank's asset. Any bank failing to meet this minimal standard receives a grade that fully reveals the quality of its assets. The policy Ψ also specifies a recapitalization rule that asks the bank to either raise enough funds to prevent a short-term creditors' run, or to sell the whole asset to long-term investors when its quality is low. Finally, the optimal policy entails a follow-up stress test on the bank's liquidity position which takes the form of a monotone pass-fail test that fails all banks with a liquidity

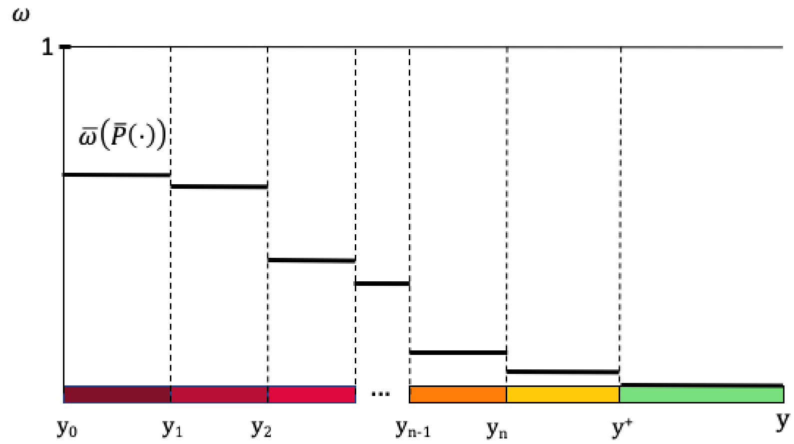

Figure 5: Optimal Comprehensive Assessment $\Psi = \{\Gamma^y, \mathscr{R}, \Gamma^\omega\}$.

position below an optimal cut-off, and passes the other.

Corollary 1. *The optimal comprehensive policy $\Psi = \{\Gamma^y, \mathscr{R}, \Gamma^\omega\}$ can be sequentially implemented by:*

(1) Conducting an asset quality review which (i) assigns a passing grade (m^y_{pass}) to all banks with assets generating cash-flow above y^+, and assigns a failing grade m^y_i to any assets delivering cash-flows $y \in (y_i, y_{i+1}]$, and (ii) imposing recapitalizations which dictates that the bank raises K when receiving the passing grade, and to sell the asset when falling below cut-off y^+.

(2) Conducting a liquidity stress test that informs short-term creditors of wether the liquidity shock is above the cut-off $\bar{\omega}(\bar{P}(\mathbb{E}(y|m^y)))$.

The optimal asset quality review Γ^y pools all cash-flows realizations above y^+ so that the induced posterior mean, $\mathbb{E}(y|\{y > y^+\})$, is greater than K and, hence, all short-term creditors are dissuaded from running. Using a more transparent disclosure policy for high values of y does not generate any benefits and, in fact, may hinder risk-sharing among banks with heterogeneous asset qualities. Thus, when the long-term profitability of the assets of a bank is above y^+, the optimal asset quality review assigns an *opaque* (and unique) pass grade.

In contrast, when the profitability of the assets, y, falls below y^+, the optimal policy specifies multiple failing grades. The intuition for this result is that there exist two forces that shape the bank's probability of survival: (a) an endogenous amplification effect associated with increasing the perceived value of the bank's assets due to the interaction between multiple audiences, and (b) the prior distribution of the liquidity shocks, characterized by F^ω. To understand the the first effect, note that by promising more valuable securities to investors, the latter may pledge a bigger amount of funds to the bank. This increases the probability of survival since the set of liquidity shocks that induce default shrinks. The increase in the probability of survival then allows long-term investors to offer a higher price for the bank's securities. The additional increase in the price offered to the bank feeds back and induces a larger probability of survival, and so on. This implies that, starting from a uniform prior distribution about the liquidity shock, the posterior probability of survival increases more than proportionally with increments in the value of the securities placed by the bank. In other words, the probability of survival is *convex* in the perceived quality of the bank's assets. The second effect, which is given by the prior distribution of the liquidity shock, may then reinforce the first effect or dissipate it.

If the probability density function of the liquidity shock, f^{ω}, does not increase too fast, as it is the case for instance under Assumption 3, the amplification effect in (a) dominates and the induced probability of survival is convex. Whenever this is the case, the policy maker prefers to separate asset profitability levels rather than pooling them together. That is, the policy-maker's objectives prefers finer information disclosures. As a result, and perhaps surprisingly, the optimal asset quality review is *more transparent* when banks have poor quality assets.

4.6 Comparison with Single Audience - Environment

Below I provide an example that shows that the optimality of multiple failing grades is a consequence of the interaction between multiple audiences. I prove that when long-term investors are protected against the potential default of the bank (which dissipates the amplification effect in (a)) and the prior distribution of the liquidity shock is uniform over $[0,1]$, the optimal asset quality review is given by a monotone pass-fail test.

Example 1. Suppose that long-term investors' claims are ring-fenced, or *encumbered*, so that they remain available to investors even in the event of default. Moreover, assume that the prior distribution of the liquidity shock, F^{ω}, is uniformly distributed over $[0,1]$. Then, the optimal asset quality review is characterized by the monotone pass-fail test $\Gamma^{y}_{\text{P-F}} = \left\{\{0,1\}, \pi^{y}_{\text{P-F}}\right\}$, with $\pi^{y}_{\text{P-F}}(1|y) \equiv 1\left\{y < y^{+}_{\text{P-F}}\right\}$ and where $\mathbb{E}\left\{y | y \geq y^{+}_{\text{P-F}}\right\} = \min\left\{\mathbb{E}\{y\}, KR\right\}$.

When the bank is able to ring-fence the claims promised to long-term investors, the amplification effect described above, induced by the interaction of both audiences, evaporates. That is, long-term investors are no longer concerned about short-term creditors' beliefs about the bank's liquidity position since their claims are protected even in the case of default. If, in addition, the prior distribution of the liquidity shock assigns equal weight to all possible realizations, the bank's probability of survival becomes linear on the value of the claims promised to long-term investors. That is, increasing the value of the security issued by the bank increases the probability of survival proportionally. This, in turn, implies that the policy-maker is indifferent between pooling different profitability levels together, under a unique failing grade, or using a more transparent disclosure policy. As a result, the *opaque* policy $\Gamma^{y}_{\text{P-F}}$ is optimal. In the presence of a single audience (or alternatively multiple unrelated audiences), the optimal disclosure rule consists of a pass-fail message.

In the remainder of the paper I extend the analysis to cases where the policy-maker is not able to measure all the aspects that are private information to the bank. If the policy-maker desires this information to be disclosed to the rest of market participants, she must incentivize the bank to self-report it.

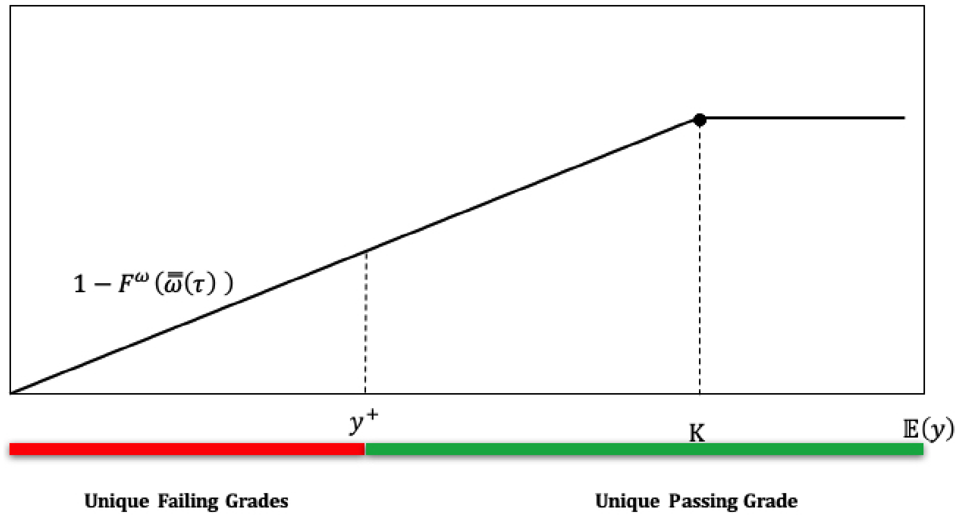

Figure 6: Optimal Asset Quality Review with a Single Audience.

5 Information and the Lender of Last Resort

5.1 Screening Liquidity Position

In this section, I study the interplay between *information disclosure* and the policy maker's role as a *lender of last resort* (LOLR). To tackle this problem, I consider interventions wherein the policy maker does not have access to a disclosure technology that allows her to respond to liquidity shocks in a timely manner. Conducting stress tests usually takes policy makers several months while liquidity shocks that compromise the bank's buffers may transpire precipitously. I thus assume that the policy maker needs to rely on information directly reported by the bank when such liquidity shocks occur. Additionally, I relax the assumption that the policy maker cannot use public funds to help banks under distress. I assume instead that the policy maker may purchase securities from the bank using taxpayers' money, but under the natural budget constraint that the price paid not exceed the fair price of the securities, taking into consideration the probability of default. This is a natural constraint often found in practice that prevents policy makers from giving away taxpayers' funds.

The timing of the game remains identical to the one in Section 4, with the single modification that, instead of allowing the policy maker to conduct the stress test Γ^{ω} at $t = 2$, the policy-maker runs an emergency lending mechanism $\Upsilon^{\omega,\theta}$ which asks the bank to self-report its private information (ω, θ) and, conditional on the report, offers funding in exchange for a claim on the bank's asset, and specifies a public disclosure about the bank's information $\pi^{\omega,\theta} : \Omega \times \Theta \to \Delta M^{\omega,\theta}$. I further assume that the policy maker cannot force the bank to accept the deals she offers. This assumption is made to rule out solutions that involve *confiscation* by the policy maker. An intervention $\Psi = \{\Gamma^{y}, \Upsilon^{\omega,\theta}\}$ thus consists of an asset quality review Γ^{y}, and an emergency lending mechanism $\Upsilon^{\omega,\theta}$. Hereafter, I refer to Ψ as a *persuasion mechanism* as it combines a *bayesian persuasion* component with the usual tools from the mechanism design literature.[34]

5.2 Period 2: Emergency Lending Mechanism

Suppose that the policy-maker has disclosed information m^y in period 1 according to the asset quality review Γ^y, and that the bank has successfully raised P units of capital by selling securities to long-term investors. Recall that, if the recapitalization P is such that

$$\int_0^{1-P} b(\omega+P,1)f^{\omega}(\omega)d\omega + g\times(1-F^{\omega}(\omega)) \leq 0, \tag{11}$$

then, under adversarial coordination, all short-term creditors withdraw early in the absence of the policy maker's intervention. In this case, the policy maker offers the bank the emergency lending mechanism $\Upsilon^{\omega,\theta} = \left\{\left\{M^{\omega,\theta},\pi^{\omega,\theta}\right\},t,s\right\}$, which asks the bank to report its asset quality type θ and its liquidity position ω, and as a function of the report $\left(\tilde{\theta},\tilde{\omega}\right)$, offers to purchase a claim on the bank's asset $s\left[\tilde{\omega},\tilde{\theta}\right]$ (with $s\left(y|\tilde{\omega},\tilde{\theta}\right)\in[0,y-s^{\star}(y)]$ for all y), at a price $t\left(\tilde{\omega},\tilde{\theta}\right)\geq 0$. In addition, $\Upsilon^{\omega,\theta}$ discloses a score $m^{\omega,\theta}$ to all market participants according to the disclosure policy $\pi^{\omega,\theta}\left[\tilde{\omega},\tilde{\theta}\right]\in\Delta M^{\omega,\theta}$. I assume that the mechanism $\Upsilon^{\omega,\theta}$ must be (interim) (i) *incentive compatible* and (ii) *individually rational*. That is, (i) the bank must be at least as well-off by disclosing its private information than by reporting any other value of (ω,θ), and (ii) the bank must be at least as well-off by participating in the designer's resolution mechanism than by opting out of it. Given that the designer can always induce the same conditions that the bank would face when opting out of the program, it is wlog to assume that all bank types participate in the policy-maker's program[35].

An argument similar to the one establishing the *Revelation Principle* implies that it is without loss of optimality to restrict attention to emergency lending mechanisms where the scores announced to short-term creditors take the form of action recommendations.[36] This means that we can restrict the analysis to disclosure mechanisms with $M^{\omega,\theta}=\{0,1\}$, where message $m^{\omega,\theta}=0$ is interpreted as the recommendation to rollover the bank's debt or *passing* the policy maker's test, and $m^{\omega,\theta}=1$ as the recommendation to stop pledging funds, or *failing* the policy maker's assessment. We distinguish between the security and price offered by the policy maker when she discloses score $m^{\omega,\theta}=0$, $\left(t_0\left(\tilde{\omega},\tilde{\theta}\right),s_0\left(\tilde{\omega},\tilde{\theta}\right)\right)$, and the contract offered when announcing $m^{\omega}=1$, $\left(t_1\left(\tilde{\omega},\tilde{\theta}\right),s_1\left(\tilde{\omega},\tilde{\theta}\right)\right)$. Short-term creditors then *obey* the policy maker's announcement as long as

$$\mathbb{E}\left(u\left(P+t_0(\omega,\theta),\omega,1\right)|m^{\omega,\theta}=0\right) = \frac{\sum_\theta \mu_\theta \int_\Omega u\left(\omega,P+t_0(\omega,\theta),1\right)\pi^{\omega,\theta}(0|\omega,\theta)F^{\omega}(d\omega)}{\sum_\theta \mu_\theta \int_\Omega \pi^{\omega,\theta}(0|\omega,\theta)F^{\omega}(d\omega)} > 0, \tag{12}$$

[34] The persuasion mechanism term was first used by Kolotilin et al. [2017] in a single-receiver environment.

[35] Philippon and Skreta [2012] and Tirole [2012] study a similar problem, but focus on indirect mechanisms where the bank has to decide whether to participate in the government program or not. This leads them to a mechanism design problem with endogenous participation constraints. In contrast, this paper follows a direct mechanism approach, where it is wlog to assume participation by all types. This distinction is obviously inconsequential for the allocations that are induced on-path. What makes my analysis fundamentally different from these earlier works is that I enrich the designer's problem by allowing her to disclose information in addition to purchasing assets.

[36] This is a consequence of the assumptions of (a) adversarial coordination and (b) homogeneity in short-term creditors' beliefs. Together these assumption imply that, for any arbitrary score disclosed by the policy maker $\tilde{m}$, the size of withdrawals $A\left(\tilde{m}\right)\in\{0,1\}$.

and

$$\mathbb{E}\left(u\left(P+t_1(\omega,\theta),\omega,1\right)|m^{\omega,\theta}=1\right)=\frac{\sum_\theta \mu_\theta \int_\Omega u\left(\omega,P+t_1(\omega,\theta),1\right)\pi^{\omega,\theta}(1|\omega,\theta)F^\omega(d\omega)}{\sum_\theta \mu_\theta \int_\Omega \pi^{\omega,\theta}(1|\omega,\theta)F^\omega(d\omega)}\leq 0. \tag{13}$$

Hereafter, I refer to conditions (12) and (13) as *obedience constraints.* As shown in the former section, the policy-maker's optimal disclosure policy, absent incentive compatibility constraints, consists of failing all banks with a liquidity position below the cutoff $\bar{\omega}(P)$, so that banks with liquidity positions above $\bar{\omega}(P)$ may survive. However, no bank vulnerable to runs (i.e., banks with $\omega<1-P$) would ever choose to report its true type if this leads the designer to recommend short-term creditors to run with certainty.

In order to solve this conflict, the policy-maker may offer less liquid banks to purchase their assets at better terms in exchange of a lower *passing* probability. This implies that more liquid (but still vulnerable) banks have to receive lower prices for their remaining claims on the asset. The fact that these banks would default in the absence of a deal with the policy-maker then makes the mechanism incentive compatible. In what follows I provide a proof to the arguments explained above.

Let $U_P\left(\tilde{\omega},\tilde{\theta},\omega,\theta\right)$ be the utility of a bank with private information (ω,θ) which has successfully raised P units of capital in period 1 and reports $(\tilde{\omega},\tilde{\theta})$. That is,

$$\begin{aligned}U_P\left(\tilde{\omega},\tilde{\theta},\omega,\theta\right) &= \sum_{m^{\omega,\theta}\in\{0,1\}}\pi^{\omega,\theta}\left(m^{\omega,\theta}|\tilde{\omega},\tilde{\theta}\right)1\left\{\omega+P+t_{m^{\omega,\theta}}\left(\tilde{\omega},\tilde{\theta}\right)\geq A\left(P,m^{\omega,\theta}\right)\right\}\times\\ &\quad\times\left(\left(P+t_{m^{\omega,\theta}}\left(\tilde{\omega},\tilde{\theta}\right)\right)R+\mathbb{E}_\theta\left(y-s^\star-s_m\left[\tilde{\omega},\tilde{\theta}\right]\right)\right)\end{aligned}$$

where $A\left(P,m^{\omega,\theta}\right)$ corresponds to the most aggressive fraction of withdrawals consistent with observing the bank raising P units of capital and the policy-maker disclosing message $m^{\omega,\theta}\in\{0,1\}$. That the mechanism satisfies incentive compatibility then translates to:

$$(\omega,\theta)\in\arg\max_{\tilde{\omega},\tilde{\theta}} U_P\left(\tilde{\omega},\tilde{\theta},\omega,\theta\right). \tag{14}$$

I impose that the price paid by the policy-maker not exceed the fair price of the security purchased from the bank. This means that:

$$\begin{aligned}\int_{\Omega\times\Theta} t_{m^{\omega,\theta}}(\omega,\theta)d\Lambda\left(\omega,\theta|m^{\omega,\theta}\right) &\leq \int_{\Omega\times\Theta}\frac{\mathbb{E}\left(s\left(y;\omega,\theta\right)|m^y,\theta\right)}{R}\times\\ &\quad\times 1\left\{\omega+P+t_{m^{\omega,\theta}}(\omega,\theta)\geq A\left(P,m^{\omega,\theta}\right)\right\}d\Lambda\left(\omega,\theta|m^{\omega,\theta}\right),\\ &\quad\text{for all } m^{\omega,\theta}\in\{0,1\}\end{aligned} \tag{15}$$

where $\Lambda\left(\omega,\theta|m^{\omega,\theta}\right)$ represents the probability measure over (ω,θ), conditional on announcing $m^{\omega,\theta}$. That is,

$$\Lambda\left(B|m^{\omega,\theta}\right)=\frac{\int_B \pi^{\omega,\theta}\left(m^{\omega,\theta}|\omega,\theta\right)F^\omega(d\omega)\mu_0(d\theta)}{\int_B F^\omega(d\omega)\mu_0(d\theta)},\quad \text{all } B\subset\Omega\times\Theta, m^{\omega,\theta}\in\{0,1\}.$$

Intuitively, constraint (15) captures the idea that the policy-maker should always pay a fair price for the securities she purchases from the bank, accounting for all the information publicly available at the moment of the transaction. This constraint prevents that the policy-maker consistently runs losses within her resolution mechanism. I refer to (15) as the *fair price constraint.*

The policy-maker's problem can then be summarized as finding the disclosure policy $\left\{\{0,1\},\pi^{\omega,\theta}(\cdot|\cdot,\cdot)\right\}$ and transfers $\{t_{m^{\omega,\theta}}(\cdot,\cdot)\}_{m^{\omega,\theta}\in\{0,1\}}$ that maximize the ex-ante probability of saving the bank under obedience constraints, incentive compatibility constraints, and the fair price constraint.

$$\max_{\left\{\{\{0,1\},\pi^{\omega,\theta}\},\{t_{m^{\omega,\theta}}\}_{m^{\omega,\theta}\in\{0,1\}}\right\}} \quad \sum_{\theta\in\Theta}\mu_\theta\int_0^{1-P}\pi^{\omega,\theta}(0|\omega,\theta)\,F^{\omega}(d\omega)$$
$$\text{s.t:} \quad (12),(13),(14),(15). \tag{16}$$

5.3 Incentive Compatibility Constraints

Purchasing claims on the bank's asset when the policy maker has assigned the failing score makes obedience constraint (13) and incentive compatibility constraints harder to satisfy and does not provide any benefit. Thus, it is without loss of optimality to set $t_1(\omega,\theta)=0$ and $s_1[\omega,\theta]=\mathbf{0}$ for all ω and θ. Furthermore, given that any vulnerable bank will fail if not helped by the government, I restrict attention to mechanisms that set $s_0[\omega,\theta_L]=y-s^\star$ for all $\omega<1-P$, since this allows the policy-maker to offer higher prices for the bank's securities.

The last property need not be satisfied for type-H banks. To see this, note that it might be in the interest of the policy-maker to offer type-H banks to retain a fraction of their asset. This might be useful to alleviate incentive constraints. The precise type of securities purchased by the policy maker is irrelevant. The only thing that matters is the fraction of the expected value of the security retained by the bank.

Let $z_\theta\equiv\mathbb{E}_\theta\left(y-s^\star_\theta\right)$ be the value of the claims of a type-θ bank net the cash flows promised to long-term investors under security $s^\star_\theta$. Let $\phi_\theta(\omega)$ denote the fraction of z_θ the bank retains on its balance sheet when its type is (θ,ω). As explained above, it is without loss of optimality to impose that $\phi_L(\omega)=0$ for all $\omega\leq 1-P$.

5.3.1 Vulnerable banks

Incentive compatibility requires that banks do not have incentives to pretend to have neither a different liquidity position nor a different asset quality type. This implies that the utility of vulnerable banks must be equalized across all $\omega<1-P$, for a given asset quality type, since otherwise the bank would report the message that yields best terms. That is, $\forall\omega<1-P$,

$$\pi^{\omega,\theta}(0|\omega,\theta_L)\times\left(\left(P+t_0^{\omega,\theta}(\omega,\theta_L)\right)R\right)=V_L, \tag{17}$$

and

$$\pi^{\omega,\theta}(0|\omega,\theta_H)\times\left(\left(P+t_0^{\omega,\theta}(\omega,\theta_H)\right)R+\phi_H(\omega)z_H\right)=V_H. \tag{18}$$

Additionally, banks must not have incentives to simultaneously deviate in both dimensions. That is, to pretend to have a different asset quality type and liquidity type. This means that a vulnerable type L-bank must not want to pretend to be a type H bank, for any level of liquidity:

$$V_L \geq \pi^{\omega,\theta}(0|\omega,\theta_H) \times \left(\left(P + t_0^{\omega,\theta}(\omega,\theta_H) \right) R + \phi_H(\omega) z_L \right), \ \ \forall \omega \geq 1-P. \tag{19}$$

Similarly, a vulnerable type H-bank must not have incentives to mimic a type L-bank:

$$V_H \geq V_L, \ \ \forall \omega \geq 1-P. \tag{20}$$

Likewise, safe banks must not have incentives to pretend to have a different liquidity position. Given that safe banks survive regardless of the signal disclosed by the policy maker, it must be the case that, $\forall \omega \geq 1-P$,

$$\pi^{\omega,\theta}(0|\omega,\theta_L) \times \left(P + t_0^{\omega,\theta}(\omega,\theta_L) \right) R + \left(1 - \pi^{\omega,\theta}(0|\omega,\theta_L) \right) \times PR = S_L,$$

and

$$\pi^{\omega,\theta}(0|\omega,\theta_H) \times \left(\left(P + t_0^{\omega,\theta}(\omega,\theta_H) \right) R + \phi_H(\omega) z_H \right) + \left(1 - \pi^{\omega,\theta}(0|\omega,\theta_H) \right) \times PR = S_H.$$

Given that the fair price constraint (15) requires that

$$t_0^{\omega,\theta}(\omega,\theta_L) \leq \frac{z_L}{R} \ \text{ and } \ t_0^{\omega,\theta}(\omega,\theta_H) \leq \frac{z_H}{R},$$

we obtain that $S_\theta \leq PR + z_\theta$, for all $\theta \in \Theta$. Moreover, given that a safe bank θ can always secure a payoff of $PR + z_\theta$ by retaining its asset, that

$$S_\theta = PR + z_\theta, \quad \text{all } \theta \in \Theta. \tag{21}$$

Next, we make sure that safe banks do not want to mimic vulnerable banks (those with $\omega < 1-P$), and vice versa. To do this, consider the conditions preventing that safe banks pretend to be vulnerable. Safe banks, by definition, will not fail regardless of short-term creditors's actions and, hence, regardless of the signal disclosed by the policy-maker. Additionally, liquid banks with high quality assets (i.e., a bank with $\omega \geq 1-P$ and $\theta = \theta_H$) will not accept a deal to sell any security $\hat{s}$ at a price less than $\mathbb{E}_H(\hat{s}|m^y)$. However, any deal that pays a security $\hat{s}$ at least $\mathbb{E}_H(\hat{s}|m^y)$ would prompt safe banks with a low-quality asset, and potentially vulnerable banks of both asset quality types, to pretend to be safe and to have a high-quality asset. The *fair-price* constraint mentioned above, however, implies that the policy-maker cannot afford to pay all types of banks as if they were type-H. The combination of the IC constraints with the fair-price constraint then imply that the policy-maker must not purchase any non-trivial security from safe banks with high-quality a asset. That is, $s_0[\omega,\theta_H] = \mathbf{0}$ and $t_0^{\omega,\theta}(\omega,\theta_H) = 0$ for any $\omega \geq 1-P$.

Next, consider the incentives problem for safe banks with a low-quality asset. By pretending to be a vulnerable bank $(\tilde{\omega},\theta)$ such banks would receive a payment $t_0^{\omega,\theta}(\tilde{\omega},\tilde{\theta})$ in case they receive a passing grade but would never fail, irrespective of the grade they receive. For a safe bank with a type-L asset (i.e., a bank (ω,θ_L) with $\omega \geq 1-P$) to not have incentives to claim to be vulnerable it must then be that:

$$S_L \geq \max_{\tilde{\omega}} \pi^{\omega,\theta}(0|\tilde{\omega},\theta_L) \left(P + t_0^{\omega,\theta}(\tilde{\omega},\theta_L) \right) R + \left(1 - \pi^{\omega,\theta}(0|\tilde{\omega},\theta_L) \right) (PR + z_L),$$

and

$$S_L \geq \max_{\tilde{\omega}} \pi^{\omega,\theta}\left(0|\tilde{\omega},\theta_H\right)\left(\left(P+t_0^{\omega,\theta}\left(\tilde{\omega},\theta_H\right)\right)R+\phi_H\left(\tilde{\omega}\right)z_L\right)+\left(1-\pi^{\omega,\theta}\left(0|\tilde{\omega},\theta_H\right)\right)\left(PR+z_L\right).$$

These constraints impose a bound on the amount that the policy-maker can pay to vulnerable banks. In fact, the above constraints, together, imply that

$$t_0^{\omega,\theta}\left(\omega,\theta_L\right) \leq \frac{z_L}{R}, \ \ \forall \omega < 1-P, \tag{22}$$

and

$$t_0^{\omega,\theta}\left(\omega,\theta_H\right) \leq \left(1-\phi_H\left(\omega\right)\right)\frac{z_L}{R}, \ \ \forall \omega < 1-P. \tag{23}$$

Additionally, if the designer were to pass *safe* banks with probability one, then all vulnerable bank types would claim to be safe. In particular, vulnerable banks with high-quality assets would claim to be safe, thus avoiding default and being pooled with low-quality types. To overcome this problem the policy-maker must fail safe banks with positive probability. Let π^s be the probability with which the policy-maker passes a safe bank. Incentive compatibility then requires that $V_L \geq \pi^s \times (P+z_L)R$, and that $V_H \geq \pi^s \times (P+z_H)R$, so that no vulnerable bank type has incentives to claim to be safe. [37]We can restate both inequalities as:

$$\pi^s \leq \min\left\{\frac{V_L}{PR+z_L}, \frac{V_H}{PR+z_H}\right\}. \tag{24}$$

The fact that liquid banks cannot be offered the passing grade with high probability makes obedience constraint (13) hard to satisfy. Obedience constraints (12) and (13) imply that $A\left(P,m^{\omega,\theta}\right) = m^{\omega,\theta}$, for all $m^{\omega,\theta} \in \{0,1\}$.

Summarizing, the policy-maker's problem can be reduced to finding a passing probability $\pi(0|\cdot,\cdot)$ and transfer $t_0(\cdot,\cdot)$, which maximize the probability of passing vulnerable banks, subject to the obedience constraints (12) and (13), incentive constraints among vulnerable banks (17), (18), incentive compatibility constraints guaranteeing that safe banks do not want to mimic vulnerable ones and vice versa (24), (22), and (23), and the constraint that imposes that the policy-maker does not pay more than the fair price (**??**) for the security she purchases from the bank:

$$\max_{\left\{\left\{\{0,1\},\pi^{\omega,\theta}\right\},t_0^{\omega,\theta}\right\}} \quad \sum_{\theta\in\Theta}\mu_\theta \times \left(\int_0^{1-P}\pi^{\omega,\theta}\left(0|\omega,\theta\right)F^{\omega}(d\omega)\right)$$
$$\text{s.t:} \quad (12),(13),(15),(17),...,(24). \tag{25}$$

Let $\bar{U}_{\text{LF}}(P) \equiv \int_0^{1-P} b(\omega+P,1)f^{\omega}(\omega)\,d\omega + g\times\left(1-F^{\omega}\left(1-P\right)\right)$ be a short-term creditor's (ex-ante) expected payoff under the *Laissez Faire* regime, when the bank successfully raises P units of capital during the fund raising stage and the rest of short-term creditors choose to stop rolling over the bank's debt. I restrict attention to the case where the expected quality of the bank's asset is depressed to the point that the policy-maker cannot set recapitalization levels to dissuade short-term creditors from running on the bank.

[37]Note that π_s does not depend on θ since if it were to differ across different asset quality types, vulnerable types, would end up mimicking the one with the highest passing probability.

Assumption 5. $\mathbb{E}(y) \leq K$.

Assumption (5) means that $\bar{U}_{\mathrm{LF}}(P) < 0$ for any $P < \mathbb{E}(y)$. That is, the bank is unable to persuade short-term creditors to keep rolling over its debt even if it sold the whole asset. This implies, in particular, that imposing forced recapitalizations in period 1 will not suffice to prevent bank failure if a liquidity shock materializes during $t = 2$. Under assumption (5), the policy-maker's ability to prevent the bank's default thus depends on her capacity to elicit information about the bank's liquidity position, and her ability to persuade short-term creditors to keep pledging to the bank.

The next result characterizes the optimal mechanism under an alternative (relaxed) setting wherein the policy-maker perfectly *observes* the bank's asset quality type during the second period when conducting the screening mechanism. The optimal mechanism under the new setting will be instrumental to characterize the optimal persuasion mechanism under the original environment.

5.4 Observable Asset Quality Type

Suppose that at $t = 2$ the policy-maker is able to (privately) observe the bank's asset quality type. [38] The optimal screening mechanism under this new setting has interest on its own as it sheds light on the trade-off faced by a policy-maker that elicits information before purchasing assets and disclosing of information, abstracting from the difficulties associated with screening additional private information on the asset quality dimension. Let $\Upsilon^{\omega,\theta}_{\mathrm{OAQ}}[\theta]$ represent the optimal screening mechanism when the policy-maker observes that the bank's asset quality type is θ. Clearly, the policy-maker's (ex-ante) expected payoff under $\sum_{\tilde{\theta}} \mu_0(\tilde{\theta})\,\Upsilon^{\omega,\theta}_{\mathrm{OAQ}}[\tilde{\theta}]$ (weakly) dominates the payoff under the original setting. This is a consequence of the fact that the set of incentive compatibility constraints shrinks. The next result characterizes the optimal screening mechanism under this alternative setting, $\Upsilon^{\omega,\theta}_{\mathrm{OAQ}}[\theta]$. As I show below, the characterization of $\Upsilon^{\omega,\theta}_{\mathrm{OAQ}}[\theta]$ will be instrumental to find the optimal persuasion mechanism under the original environment.

Proposition 6. *Assume that the policy-maker perfectly observes θ at $t = 2$. Suppose that the bank raises $P < K$ after the asset quality review Γ^y discloses m^y. Then, the optimal screening mechanism when the bank's asset quality type is θ, $\Upsilon^{\omega,\theta}_{OAQ}[\theta;P]$, is characterized by:*

$$t^{\omega,\theta}_{0\ \mathrm{OAQ}}(\omega;\theta,P) = \begin{cases} \frac{z_\theta}{R} & \omega < \hat{\omega}_\theta \\ 1-P-\omega & \omega \in [\hat{\omega}_\theta, \check{\omega}_\theta] \\ 1-P-\check{\omega}_\theta & \omega \in (\check{\omega}_\theta, 1-P) \\ 0 & \omega \geq 1-P \end{cases}, \quad \pi^{\omega,\theta}_{\mathrm{OAQ}}(0|\omega;\theta,P) = \begin{cases} \frac{\bar{V}_\theta}{PR+z_\theta} & \omega < \hat{\omega}_\theta \\ \frac{\bar{V}_\theta}{(1-\omega)R} & \omega \in [\hat{\omega}_\theta, \check{\omega}_\theta] \\ \frac{\bar{V}_\theta}{(1-\check{\omega}_\theta)R} & \omega \in (\check{\omega}_\theta, 1-P) \\ \frac{\bar{V}_\theta}{PR+z_\theta} & \omega \geq 1-P \end{cases}$$

[38] Under the new setting the policy-maker may observe the bank's asset quality type and the information she learns does not leak to long-term investors. This assumption contrasts with the assumption made in the previous section that any information the policy-maker learns during the first period about the quality of the bank's asset cannot be concealed from the market. I show below that the optimal persuasion mechanism under this new setting is constant accross asset quality types, turning this assumption innocuous.

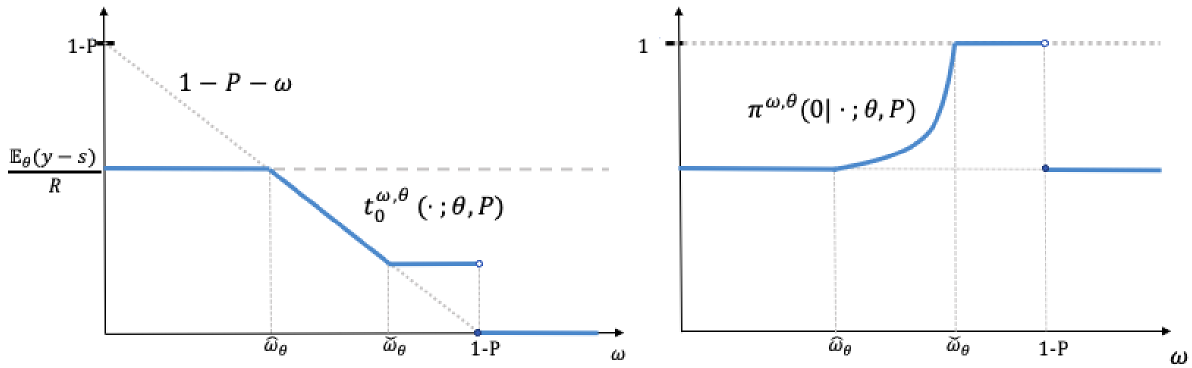

Figure 7: Optimal *liquidity screening* mechanism under perfect observability of asset quality.

where $\hat{\omega}_\theta \equiv 1-P-\frac{z_\theta}{R}$,

$$\bar{V}_\theta \equiv \min\left\{(1-\check{\omega})R, \frac{|\bar{U}_{\text{LF}}(P)|}{\int_0^{1-P}\frac{|b(\omega+P,1)|F^\omega(d\omega)}{\left(P+t_{0\ \text{OAQ}}^{\omega,\theta}(\omega;\theta)\right)R}-g\times\frac{(1-F^\omega(1-P))}{PR+z_\theta}}\right\},$$

and $\check{\omega}_\theta$ is chosen so that:

$$\int_{\hat{\omega}_\theta}^{\check{\omega}_\theta}\frac{F^\omega(d\omega)}{(1-\omega)R}+\frac{F^\omega(1-P)-F^\omega(\check{\omega})}{(1-\check{\omega}_\theta)R}=\frac{1}{g}\times\int_0^{\hat{\omega}}\frac{|b(\omega+P,1)|F^\omega(d\omega)}{PR+z_\theta}-\frac{(1-F^\omega(1-P))}{PR+z_\theta}.$$

The optimal screening mechanism characterized in Proposition 6 is illustrated in Figure 7. To persuade short-term creditors to follow the recommendation to rollover the bank's debt, the policy-maker has to modify the likelihood of the bank's survival. The bound on the price that can be pledged by the policy-maker implies that banks with a buffer smaller than $\hat{\omega}_\theta+P+\frac{z_\theta}{R}$ default when all short-term creditors withdraw early. The policy-maker then minimizes the passing probability assigned to these liquidity types and compensates them by paying them the maximal price consistent with constraints (22, 23).

All banks with liquidity positions above $\hat{\omega}_\theta$ receive enough funds to prevent default under an adversarial withdrawal. Incentive compatibility among vulnerable banks impose a negative relationship between the passing probability and the price paid by the policy-maker. Banks with a liquidity shock $\omega \in [\hat{\omega}_\theta, \check{\omega}_\theta)$ receive the smallest price that allows them to survive a massive withdrawal in order to maximize the probability of assigning a passing grade. The level $\check{\omega}_\theta$ is chosen so that obedience constraint is satisfied. Intuitively, the smaller the value of $\check{\omega}_\theta$, the more liquidity-types receive the maximal passing probability and, hence, the larger the aggregate survival probability. The optimal liquidity screening mechanism chooses the minimal value of $\check{\omega}_\theta$ consistent with obedience constraint (12).

I use the construction of the optimal screening mechanism when asset quality is observable to the policy-maker (but not to the asset market), $\Upsilon_{\text{OAQ}}^{\omega,\theta}$, to characterize the optimal screening mechanism under the original setting. I show below that the latter has a simple characterization. In fact, at the optimum, the policy-maker does not screen the quality of the bank's asset, and only solicits information about the bank's liquidity position. I make precise the last statement below.

Definition. A screening mechanism $\Upsilon^{\omega,\theta} = \{\{\{0,1\},\pi\},t\}$ is said to be a *non discriminatory* (ND) screening mechanism if:

$$t_0(\omega,\theta_L) = t_0(\omega,\theta_H) = t_0(\omega),\ \ \pi(0|\omega,\theta_L) = \pi(0|\omega,\theta_H) = \pi(0|\omega),\ \phi_H(\cdot) = \mathbf{0}\ \ \forall \omega \in \Omega. \tag{26}$$

The optimal screening mechanism will be non-discriminatory. Observe that a ND mechanism need not satisfy *incentive compatibility.* As a matter of fact, a ND mechanism might satisfy local incentive constraints but will most likely fail to satisfy global incentive constraints (24), (22), and (23). These are constraints that prevent safe liquidity types to mimic vulnerable types, and vice versa. I show that the optimal screening mechanism corresponds to a ND mechanism that respects all incentive constraints. Intuitively, the bank's private information regarding the quality of its asset hurts the policy-maker's ability to run its liquidity provision program. In fact, as the next lemma shows, a policy-maker concerned with the potential default of the bank would do strictly better if the bank did not have private information regarding the quality of its asset in the first place. In order to avoid that safe banks with poor quality assets mimic vulnerable banks, the policy-maker needs to constraint the price she pays within its liquidity provision program. Moreover, private information about the quality of the assets implies that the policy-maker needs to decrease the probability with which she passes safe banks, since otherwise vulnerable banks with high quality assets would claim to be safe, as can be seen in (24). As a result, the policy-maker is strictly better off if banks do not possess private information.

Rigorously, let $\mathbb{E}_{\mu_0}(\theta) \equiv \mu_0\theta_H + (1-\mu_0)\theta_L$ and let

$$\Upsilon^{\omega,\theta}_{\text{OAQ}}\left[\mathbb{E}_{\mu_0}(\theta)\right] = \left\{\left\{\left\{0,1\right\},\pi^{\omega,\theta}_{\text{OAQ}}\left[\mathbb{E}_{\mu_0}(\theta)\right]\right\},t^{\omega,\theta}_{\text{OAQ}}\left[\mathbb{E}_{\mu_0}(\theta)\right]\right\}$$

be the optimal screening mechanism when the bank is known to have asset quality $\mathbb{E}_{\mu_0}(\theta)$. [39]The next lemma shows that the ND mechanism $\Upsilon^{\omega,\theta}_{\text{OAQ}}\left[\mathbb{E}_{\mu_0}(\theta)\right]$ represents an upper bound of what can be accomplished under the original setting.

Proposition 7. *Let* $\hat{\Upsilon}^{\omega,\theta} = \{\{\{0,1\},\hat{\pi}\},\hat{t}\}$ *be any feasible mechanism satisfying (12)-(**??**), then* $\Upsilon^{\omega,\theta}_{OAQ}\left[\mathbb{E}_{\mu_0}(\theta)\right] \succeq_{PM} \hat{\Upsilon}^{\omega,\theta}$*. That is:*

$$\int_0^{1-P} \pi^{\omega,\theta}_{OAQ}\left(0|\omega;\mathbb{E}_{\mu_0}(\theta)\right)F^{\omega}(d\omega) \geq \sum_{\theta\in\Theta}\mu_\theta \times \left(\int_0^{1-P}\hat{\pi}^{\omega,\theta}(0|\omega,\theta)\,F^{\omega}(d\omega)\right).$$

I show in the next section that the policy-maker can always implement a screening mechanism that reaches the same likelihood of survival than $\Upsilon^{\omega,\theta}_{\text{OAQ}}[\mathbb{E}(\theta)]$, under the optimal persuasion mechanism Ψ.

[39] Alternatively, $\Upsilon^{\omega,\theta}_{\text{OAQ}}[\mathbb{E}(\theta)]$ represents the optimal mechanism when the bank does not possess additional information with respect to the quality of its asset.

5.5 Period 1: Asset Quality Review

In this section, I study the joint design of the optimal asset quality review Γ^y, and recapitalization requirements, that precede the choice of the screening mechanism $\Upsilon^{\omega,\theta}$. As I show below, the policy-maker faces an important trade-off when designing the recapitalizations rule to impose in the first period: On the one hand, smaller recapitalizations allow the bank to retain a greater fraction of the asset on its balance sheet. This increases the price that the policy-maker may offer to the bank, thus, enhancing the effectiveness of the liquidity provision program $\Upsilon^{\omega,\theta}$. On the other hand, more stringent recapitalizations permit the bank to raise capital before the liquidity shock materializes. This helps decrease the premium the bank has to pay to compensate for rollover risk.

In order to implement successful liquidity provision programs (i.e., for the policy-maker to be able to assign informative grades about the bank's liquidity position), the bank needs to own remaining claims on its asset with a value above a minimum threshold at the end of $t = 1$. When this cut-off is not met, the regulator can not induce the bank to self-report private information regarding its liquidity buffers. As discussed previously, the key trade-off that allows the regulator to induce the bank to self report its liquidity position involves a negative relation between the amount of funds offered to the bank, and the probability of assigning a passing grade. When the value of the remaining claims on the bank's asset is small, the maximal amount than can be pledged by the policy-maker is too low to discourage most vulnerable banks from mimicking more liquid ones and, hence, information elicitation about ω does not take place. Let $\underline{E}$ be the minimal expected value of the bank's remaining claims necessary for information elicitation.

$$\underline{E} \equiv \inf_{E} \left\{ E \geq 0 : \frac{b(1,1)}{E} F^{\omega}\left(1 - \frac{E}{R}\right) + \int_{1-\frac{E}{R}}^{1} \frac{g}{(1-\omega)R} F^{\omega}(d\omega) \geq 0 \right\}. \tag{27}$$

Theorem 2 characterizes the optimal recapitalization policy and liquidity provision program for any message disclosed by the asset quality review $\Gamma^y = \{M^y, \pi^y\}$. I show that for intermediate ranges of asset quality y the policy-maker induces the bank to report its liquidity position and discloses information to the bank's short-term creditors according to a stochastic rule which assigns a pass grade in a monotone manner (that is, more liquid banks are passed with higher probability). The price the policy-maker pays for the bank's assets is decreasing in the bank's liquidity. Moreover, in this case the regulator does not impose recapitalizations during the first period, and effectively asks the bank not to approach long-term investors. For low values of y the policy-maker, instead, is unable to elicit information about the bank's liquidity position. In that case, the policy-maker recommends the bank to raise capital from long-term investors before the liquidity shock materializes, which helps the bank maximize the amount of funds it gets in exchange for claims on its asset. Similarly, when the value of y is large, the policy-maker asks the bank to seek private sector financing (i.e., from long-term investors). In this case the bank is asked to raise enough funds to persuade short-term creditors to rollover.

Theorem 2. *Fix a message m^y disclosed with positive probability under Γ^y. The optimal recapitalization policy and resolution program can be characterized as a function of the expected value of the asset's cash-flows, $\bar{z} \equiv \mathbb{E}(y|m^y)$, as follows:*

(i) If $\bar{z} \geq K$, the optimal recapitalization policy is given by $R_{\alpha}(P) = 1\{P < K\}$ for some $\alpha > 0$, and no liquidity-provision program is required.

(ii) If $\underline{E} < \bar{z} < K$, then the bank is asked to not raise capital from long-term investors, and the policy-maker uses the following liquidity-provision program to solicit information about ω :

$$t_0^{\star}(\omega;\bar{z}) \equiv \begin{cases} \frac{\bar{z}}{R} & \omega < 1-\frac{\bar{z}}{R} \\ 1-\omega & \omega \in \left[1-\frac{\bar{z}}{R}, \check{\omega}\right] \\ 1-\check{\omega} & \omega \in (\check{\omega},1] \end{cases}, \quad \pi^{\star}(0|\omega;\bar{z}) \equiv \begin{cases} \frac{(1-\check{\omega})R}{z} & \omega < 1-\frac{\bar{z}}{R} \\ \frac{1-\check{\omega}}{(1-\omega)} & \omega \in \left[1-\frac{\bar{z}}{R}, \check{\omega}\right] \\ 1 & \omega \in (\check{\omega},1] \end{cases} \tag{28}$$

with $\check{\omega}$ implicitly defined by:

$$g \times \left(\int_{1-\frac{z}{R}}^{\check{\omega}} \frac{f^{\omega}(\omega)}{(1-\omega)R} d\omega + \frac{1-F^{\omega}(\check{\omega})}{(1-\check{\omega})R} \right) = \frac{\int_0^{1-\frac{z}{R}} b\left(\frac{\bar{z}}{R},1\right) f^{\omega} d\omega}{\bar{z}}.$$

(iii) If $z \leq \underline{E}$, the bank is asked to seek funding from long-term investors and the recapitalization policy $R_{\tilde{\alpha}}(P) = 1\{P < \bar{P}(z)\}$ for some $\tilde{\alpha} > 0$ is imposed.

Theorem 2 shows that interventions inducing simultaneous pledging by the market and the government are sub-optimal. The intuition behind this result is that inducing the bank to raise capital from long-term investors reduces the effectiveness of the policy-maker's liquidity-provision program. Recall that a bank that retains a smaller fraction of its asset can be offered less funds by the government under the *fair price* constraint. Given that the effectiveness of the liquidity-provision program relies on compensating extremely vulnerable banks, which are passed less often than more liquid ones, with higher prices for the remaining claims on their assets, requiring that the bank sells a fraction of its asset to long-term investors decreases the elicitation capacity of the policy-maker once the liquidity shock materializes. Additionally, having the bank raising funds from long-term investors intensifies incentive compatibility issues in the regulator's elicitation program. In fact, any amount of capital $P > 0$ raised during the fund-raising game makes the bank safe against runs for all $\omega > 1-P$, regardless of the policy-maker's program. The larger P is, the larger the set of liquidity shocks under which the bank survives. Furthermore, the larger P is, the smaller the amount of cash the policy-maker can pay to to vulnerable banks and the smaller the probability a pass grade can be assigned to highly liquid safe banks. At the optimum, the policy-maker either maximizes P and then forgoes using a liquidity-provision program, or sets $P = 0$ (thus asking the bank to refrain from raising funds from long-term investors) and then uses a non discriminatory liquidity screening mechanism.

The formal proof that the optimal intervention has this *bang-bang* structure is in the Appendix. The strategy used to prove this result consists of solving a relaxed version of the policy-maker's problem where the bank does not receive private information about the quality of its asset. As shown above in lemma (7), the solution to this relaxed problem (weakly) dominates the solution under the original problem. I show that the solution to the alternative problem either maximizes the capital raised from the private sector, or sets $P = 0$ and then uses a liquidity-provision program. Whenever the policy-maker chooses the latter, setting $P = 0$ implies that the optimal liquidity-provision program satisfies all incentive compatibility constraints under the original problem and, hence, must be optimal.

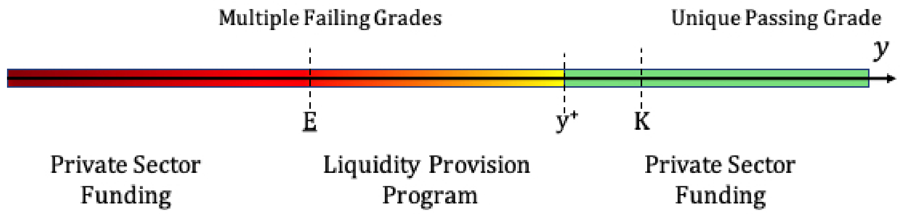

Figure 8: Structure optimal comprehensive intervention.

The next theorem completes the analysis by characterizing the structure of the optimal persuasion mechanism as a function of the quality of the bank's asset.

Theorem 3. *The optimal comprehensive policy $\hat{\Psi} = (\Gamma^y, \mathscr{R}, \Upsilon^\omega)$ is characterized by a monotone partition $\mathscr{P} = \{(y_i, y_{i+1}]\}_{i\in I}$ of $\mathbb{R}_+$ such that the optimal asset quality review $\Gamma^y = \left\{\left\{m_i^y\right\}_{i\in I}, \pi^y\right\}$ satisfies $\mathbb{E}(y|m_i^y) < \mathbb{E}(y|m_j^y)$ for all $i < j$. Moreover, the highest interval always include y^+. Furthermore,*

(1) If $y \geq y^+$, the policy-maker passes the bank and sets recapitalizations according to the policy $R_\alpha(P) = 1\{P < K\}$ for some $\alpha > 0$.

(2) If $y \in (y_i, y_{i+1}]$ with $\mathbb{E}(y|m_i^y) \in (\underline{E}, K)$, either the bank is funded only by the private sector, in which case $R_\alpha(P) = 1\{P < \bar{P}(z)\}$, or the bank is funded only by the government only the government through the liquidity-provision program characterized by $t_0^\star\left(\omega; \mathbb{E}(y|m_i^y)\right), \pi_0^\star\left(\omega; \mathbb{E}(y|m_i^y)\right)$, where $t_0^\star$ and $\pi_0^\star$ are as defined in (28).

(3) If $y \in (y_i, y_{i+1}]$ with $\mathbb{E}(y|m_i^y) \leq \underline{E}$, the bank is asked to seek external funding, the government imposes recapitalizations according to $R_{\tilde{\alpha}}(P) = 1\{P < \bar{P}(z)\}$ for some $\tilde{\alpha} > 0$, and no liquidity program is used.

Theorem 3 shows that the optimal comprehensive policy features a non-monotone pecking order. Institutions with high-quality assets are given a passing grade by the asset quality review Γ^y, and are required to raise enough capital from the private sector to persuade short-term creditors to rollover its debt. Banks with intermediate-quality assets, in turn, are assigned one of multiple failing grades and are funded with the government's optimal liquidity provision program. Finally, institutions with extremely poor-quality assets, are failed with multiple failing grades and are induced to seek funding from the private sector.

Theorem 2 informs the policy debate by showing that non-monotone relations between long-term asset profitability and the source of funding that institutions receive, need not be a proof of sub-optimality. In fact, they are expected to arise in these type of environments. In contrast, as highlighted before, simultaneous pledging by both the public and the private sector is, indeed, evidence of sub-optimality. Furthermore, the analysis shows that recapitalization rules need not be part of an optimal policy. In fact, in opposition to the results found in section 4 that advocate for the use of recapitalization policies, theorems 2 and 3 offer a message of caution. If the policy-maker believes she will not be able to react in a quickly manner to liquidity events, and implement a liquidity stress test to alleviate pessimistic assessment of short-term creditors, then recapitalization policies are costly and undesirable. Such rules deplete the amount of assets that the bank may use as collateral to obtain emergency lending from liquidity provision programs run by the policy-maker, negatively affecting her capacity to elicit information about the bank's liquidity needs, and therefore her ability to persuade short-term creditors to keep pledging to the bank.

6 Conclusions

This paper studies government interventions aimed at stabilizing financial institutions subject to rollover risk. I consider a rich environment which emphasizes the interaction among multiple audiences who care about different aspects of the bank's multi-dimensional fundamentals. I show that complementing disclosure policies with minimal recapitalizations is essential to maximizing the probability of the bank's survival. By combining appropriately designed information disclosures with recapitalizations, the policy-maker is able to implement the optimal solution to a broader mechanism design problem where she has the authority to dictate the type of securities and the price the bank should choose when approaching long-term investors. Conferring such authority to the policy-maker is however not necessary. Perhaps surprisingly, the optimal review is *opaque* when the institution has high-quality assets and assigns a unique pass grade. In contrast, the optimal review is more *transparent* with banks with low-quality assets, in which case multiple failing grades are assigned to the bank as a function of the precise quality of the assets, which also triggers a follow-up stress test on the bank's liquidity position.

When the policy-maker lacks the ability to examine the bank's liquidity position and, hence, needs to elicit information from the bank, the initial asset quality review is followed by an emergency lending program, whereby the government offers to buy assets from the bank, in exchange of cash and a public disclosure of the bank's liquidity. I show that, in this case, imposing recapitalizations undermines the effectiveness of the government's liquidity program. I also show that simultaneous pledging by the government and the private sector is suboptimal. I find that optimal comprehensive policies feature a non-monotone pecking order: Institutions with high-quality assets are given a pass grade by the asset quality review that assess the long-term profitability of the bank's assets and are required to raise enough capital from the private sector to persuade short-term creditors to rollover its debt. Banks with intermediate-quality assets, in turn, are assigned one of multiple failing grades, and are funded with the government's emergency lending program. Finally, institutions with extremely poor-quality assets are failed with multiple failing grades and are induced to seek private sector financing.

The above results are worth extending in several directions. The analysis presented assumes the policy maker knows the distribution of future liquidity shocks when she designs the optimal comprehensive policy. Such knowledge may come from previous experience with banks of similar fundamentals. While this is a natural starting point, there are many environments in which it is more appropriate to assume that the designer lacks information about the joint distribution of the underlying fundamentals. In future work, it would be interesting to investigate the optimal disclosure policy in such situations. One idea is to apply a robust approach to the policy maker's problem, whereby the designer expects nature to select the information structure that minimizes her payoff. The characterization of the optimal policy in this environment is highly relevant both from a theoretical standpoint and for the associated policy implications.

The analysis also assumes that uncertainty regarding the bank's liquidity is resolved after the bank approaches long-term investors. However, short-term creditors' runs are intrinsically dynamic phenomena. If the fundamentals are partially persistent over time, the optimal policy must also specify the timing of disclosures. In future work, it would be interesting to extend the analysis in this direction.

Appendix A: Laissez Faire

D1 Refinement. Define first the set of *best responses* to an arbitrary security s, $BR(s)$, as the set of prices which are consistent with rationality of the investors under some belief about the asset quality type of the bank:[40]

$$BR(s) \equiv \left\{ P : \frac{\mathbb{E}_H(s)}{R} \mathbb{P}\left\{\omega + P \geq A^\star(P)\right\} \geq P \right\}.$$

Define then,

$$\mathscr{D}(\theta|s) \equiv \left\{P \in BR(s) : V(P, s, \theta) > V\left(P^\star\left(s^\star_\theta\right), s^\star_\theta, \theta\right)\right\}$$

$$\mathscr{D}^0(\theta|s) \equiv \left\{P \in BR(s) : V(P, s, \theta) = V\left(P^\star\left(s^\star_\theta\right), s^\star_\theta, \theta\right)\right\}.$$

The profile $\left\{\left\{s^\star_\theta\right\}_{\theta\in\Theta}, \mu^\star, P^\star, A^\star\right\}$ satisfies the D1 criterion if for any security $s \in S$ with $s \neq s_*(\theta)$ all $\theta \in \Theta$, $\mu_*(s)$ is such that $\forall \theta, \theta' \left(\mathscr{D}(\theta|s) \cup \mathscr{D}^0(\theta|s)\right) \subset \mathscr{D}(\theta'|s)\right) \Rightarrow \mu_*(\theta|s) = 0$.

Definition 1. We say a function $g : Y \subseteq \mathbb{R} \to \mathbb{R}$ satisfies *single crossing from above* (SCFA), if the following holds true: if there exists some $y \in Y$ such that $g(y) < 0$, then $\forall \tilde{y} > y$, $g(\tilde{y}) \leq 0$. Similarly, we say that $h : Y \subseteq \mathbb{R} \to \mathbb{R}$ satisfies *single crossing from below* (SCFB), if the following holds true: if there exists some $y \in Y$ such that $h(y) > 0$, then $\forall \tilde{y} > y$, $h(\tilde{y}) \geq 0$.

Lemma 3. *Suppose that $g : Y \subseteq \mathbb{R} \to \mathbb{R}$ satisfies SCFA and that $f(y,t)$ is log-supermodular for all $(y,t) \in Y \times T \subseteq \mathbb{R}^2$. Define $\phi(t) \equiv \int_Y g(y) f(y,t) dy$ and let $y_0 \equiv \inf\{y \in Y : g(y) < 0\}$.Then, $\forall \tilde{t} > t \in T$:*

$$\phi(\tilde{t}) = 0 \Rightarrow \phi(t) > 0.$$

Proof. That $f(y,t)$ is log-SM implies that $\frac{f(\cdot,t)}{f(\cdot,\tilde{t})}$ is non-increasing. Then,

$$\begin{aligned} \phi(t) &= \int_Y 1\{y \leq y_0\} g(y) \frac{f(y,t)}{f(y,\tilde{t})} f(y,\tilde{t}) dy + \int_Y 1\{y > y_0\} g(y) \frac{f(y,t)}{f(y,\tilde{t})} f(y,\tilde{t}) dy \\ &\geq \left(\frac{f(y_0,t)}{f(y_0,\tilde{t})}\right) \phi(\tilde{t}) \end{aligned}$$

which implies the result. □

Proof of Proposition 1.

The proof offered below is general in that it applies regardless of whether the designer has disclosed information about the fundamentals (y, ω).

[40] First-order stochastic dominance (which is implied by MLRP) means that

$$\left\{P > 0 : \frac{\mathbb{E}_H(s)}{R} \times \mathbb{P}\left\{\omega + P \geq A^\star(P)\right\} \geq P\right\} = \bigcup_{\mu\in\Delta\Theta} \left\{P > 0 : \frac{\mathbb{E}(s;\mu)}{R} \times \mathbb{P}\left\{\omega + P \geq A^\star(P)\right\} \geq P\right\}$$

Assume that the probability that the bank survives can be written as $\mathbb{P}\left\{\omega \geq \omega^{\sharp}(\tau)\right\}$, where $\omega^{\sharp}(\cdot)$ represents a non-increasing function, continuously differentiable for all $\tau < K$, and with $\omega^{\sharp}(\tau) = 0$, for all $\tau \geq K$. In the context of section 3, $\omega^{\sharp}(P) = A^{\star}(P) - P$, while in the context of section 4, $\omega^{\sharp} = \bar{\omega}$. Define $\Phi(\tau)$ as the set of prices which induce a non-negative profit to any investor when a security of expected value τ is purchased. That is

$$\Phi(\tau) \equiv \left\{P \geq 0 : \frac{\tau}{R}\mathbb{P}\left\{\omega \geq \omega^{\sharp}(P)\right\} \geq P\right\}.$$

Part 1. Suppose that there exists an equilibrium of the fund-raising game, $\{\{\sigma_\theta\}_\theta, \mu, P, A\}$, and any non-trivial security $\hat{s} \in S$ with $\sigma_\theta(\hat{s}) > 0$, for all $\theta \in \Theta$. We prove that any such security needs to be a debt contract. To see this, suppose that $\hat{s}$ is not a debt contract. Define the debt security $s_D \equiv \min\{y, D\}$ where D is such that $\mathbb{E}_H(s_D - \hat{s}) = 0$. Note that $s_D - \hat{s}$ satisfies *single crossing from above* (SCFA) and hence lemma 3 implies that $\mathbb{E}_L(s_D - \hat{s}) > 0 = \mathbb{E}_H(s_D - \hat{s})$. Thus,

$$\mathbb{E}_H(y - s_D) - \mathbb{E}_L(y - s_D) > \mathbb{E}_H(y - \hat{s}) - \mathbb{E}_L(y - \hat{s}). \tag{29}$$

Next, let $P^{\sharp}(\tau) \equiv \sup \Phi(\tau)$ and define $\Delta V_\theta(P)$ as the difference in payoffs for bank θ obtained by switching to security s_D, and sell it at price P, instead of issuing security $\hat{s}$ and receiving the market price $P(\hat{s}) = P^{\sharp}\left(\mathbb{E}_{\hat{\mu}}(\hat{s})\right)$, with $\hat{\mu} = \frac{\sigma_H(\hat{s})}{\sigma_L(\hat{s}) + \sigma_H(\hat{s})} \in (0,1)$. That is,

$$\begin{aligned}
\Delta V_\theta\left(\tilde{P}\right) &= V\left(\tilde{P}, s_D, \theta\right) - V\left(P(\hat{s}), \hat{s}, \theta\right) \\
&= \left(\tilde{P}R + \mathbb{E}_\theta(y - s_D)\right)\mathbb{P}\left\{\omega \geq \omega^{\sharp}\left(\tilde{P}\right)\right\} - \left(P(\hat{s})R + \mathbb{E}_\theta(y - \hat{s})\right)\mathbb{P}\left\{\omega \geq \omega^{\sharp}\left(P(\hat{s})\right)\right\},
\end{aligned}$$

Inequality (29) together with the fact that $y - s_D$ and $y - \hat{s}$ are monotone then imply that:

$$\begin{aligned}
\Delta V_H\left(\tilde{P}\right) - \Delta V_L\left(\tilde{P}\right) &= \left(\mathbb{E}_H(y - s_D) - \mathbb{E}_L(y - s_D)\right)\mathbb{P}\left\{\omega \geq \omega^{\sharp}\left(\tilde{P}\right)\right\} + \\
&\quad - \left(\mathbb{E}_H(y - \hat{s}) - \mathbb{E}_L(y - \hat{s})\right)\mathbb{P}\left\{\omega \geq \omega^{\sharp}\left(P(\hat{s})\right)\right\} \\
&> 0, \ \forall \tilde{P} \geq P(\hat{s}).
\end{aligned} \tag{30}$$

Note then that the continuity properties of $\omega^{\sharp}$ and the fact that F^{ω} is a cdf imply that $\Phi(\tau)$ is compact and strictly increasing for any $\tau \geq 0$.[41] Thus,

$$\begin{aligned}
P(\hat{s}) &= \max\ \Phi\left(\mathbb{E}_{\hat{\mu}}(\hat{s})\right) \\
&< \max\ \Phi\left(\mathbb{E}_H(\hat{s})\right) \\
&= \max\ BR(s_D),
\end{aligned}$$

where the first equality follows from the compacity of Φ and the definition of $P(\hat{s})$. The inequality arises from the strict monotonicity of Φ and the MLRP order of signals θ. The second equality is by definition of $BR(\cdot)$, the construction of s_D, and the compacity of Φ.

[41] We say that a correspondance $\varphi : \mathbb{R}_+ \to 2^{\mathbb{R}_+}$ is strictly increasing if, for any τ, $\tau' \in \mathbb{R}_+$, with $\tau < \tau'$, $\varphi(\tau) \subsetneq \varphi(\tau')$.

Finally, notice that $\mathbb{E}_L(y-\hat{s}) > \mathbb{E}_L(y-s_D)$, and therefore $\Delta V_L(P(\hat{s})) < 0$. On the other hand, by construction we have that $\Delta V_H(P(\hat{s})) = 0$, which together with the fact that $P(\hat{s}) < P^\sharp(\mathbb{E}_H(s_D))$ and the result in (30) imply that

$$\mathscr{D}(\theta_L|s_D) \cup \mathscr{D}^0(\theta_L|s_D) \subset \mathscr{D}(\theta_H|s_D) = \left[P(\hat{s}), P^\sharp(\mathbb{E}_H(s_D))\right].$$

As a consequence, market beliefs consistent with D1 must necessarily assign $\mu(\theta_H|s_D) = 1$. This implies that $P(s_D) = P^\sharp(\mathbb{E}_H(s_D)) > P(\hat{s})$, since bank H is not pooled with L when placing s_D, and therefore by definition of s_D we have that $\Delta V_H\left(P^\sharp(s_D)\right) > 0$, which contradicts the assumption that $\{\{\sigma_\theta\}_\theta, \mu, P, A\}$ is an equilibrium.

Next, we prove that the price of any debt security s_d placed by both types in equilibrium, cannot be larger than K. Assume by contradiction that $P(s_d \equiv \min\{y,d\}) > K$. Consider the alternative debt contract $s_\varepsilon = \min\{y, d-\varepsilon\}$ with $\varepsilon > 0$ small so that $P(s_\varepsilon) > K$ and $\mathbb{E}_H(s_\varepsilon) > \mathbb{E}(s_d)$. We show that type H can profitably deviate from equilibrium play and issue s_ε instead. Observe that $s_d - s_\varepsilon$ is an increasing function. FOSD then implies that:

$$\mathbb{E}_H(s_d - s_\varepsilon) > \mathbb{E}_L(s_d - s_\varepsilon),$$

or equivalently,

$$\mathbb{E}_H(y-s_\varepsilon) - \mathbb{E}_L(y-s_\varepsilon) > \mathbb{E}_H(y-s_d) - \mathbb{E}_L(y-s_d). \tag{31}$$

Similar to the analysis above, let $\Delta V_\theta\left(\tilde{P}\right) \equiv V\left(\tilde{P}, s_\varepsilon, \theta\right) - V(P(s_d), s_d, \theta)$. Inequality 31 implies that:

$$\begin{aligned}\Delta V_H\left(\tilde{P}\right) - \Delta V_L\left(\tilde{P}\right) &= (\mathbb{E}_H(y-s_\varepsilon) - \mathbb{E}_L(y-s_\varepsilon)) \times \mathbb{P}\left\{\omega \geq \omega^\sharp\left(\tilde{P}\right)\right\} \\ &\quad - (\mathbb{E}_H(y-s_d) - \mathbb{E}_L(y-s_d)) \times \underbrace{\mathbb{P}\left\{\omega \geq \omega^\sharp(P(s_d))\right\}}_{=1} \\ &> 0, \ \forall \tilde{P} \geq K. \end{aligned} \tag{32}$$

Next, for small values of ε we have:

$$\Phi(\mathbb{E}(s_d)) \subsetneq \Phi(\mathbb{E}_H(s_\varepsilon)) = BR(s_\varepsilon),$$

and hence $P(s_d) = \max \Phi(\mathbb{E}(s_d))$ is contained in $BR(s_\varepsilon)$. Moreover, given that s_ε is smaller than s_d, we must have that $\Delta V_\theta(P(s_d)) > 0$ for both $\theta \in \Theta$. Finally, by choosing ε small enough, and using inequalities (31) and (32), there must exist some $\tilde{P} \in [K, P(s_d))$ for which $\Delta V_H\left(\tilde{P}\right) > 0 > V_L\left(\tilde{P}\right)$. Thus, $\mathscr{D}(\theta_L|s_\varepsilon) \cup \mathscr{D}^0(\theta_L|s_\varepsilon) \subset \mathscr{D}(\theta_H|s_\varepsilon)$, and consequently market beliefs consistent with D1 must assign $\mu(\theta_H|s_\varepsilon) = 1$, which implies that type H can profitably deviate and separate from type L. This is a contradiction and therefore any debt contract under which both types pool must have a price no larger than K.

Part 2. In any equilibrium in which there exists a security s_H issued only by type H (i.e., $\sigma_H(s_H) > 0 = \sigma_L(s_H)$), we must have that $P(s_H) \leq \mathbb{E}_L(y) < KR$.

To see this, assume by contradiction that $P(s_H) > \frac{1}{R}\mathbb{E}_L(y)$. Denote by s_L any security issued with positive probability by type L. Observe that the separating nature of the equilibrium requires that:

$$P(s_L) = \max \Phi(\mathbb{E}_L(s_L)) \leq \frac{\mathbb{E}_L(s_L)}{R},$$

as $\mathbb{E}_L(s_L) < KR$ and therefore $\mathbb{P}\left\{\omega \geq \omega^\sharp(\mathbb{E}_L(s_L))\right\} \leq 1$. Hence, the amount collected by type H must be such that:

$$P(s_H)R > P(s_L)R + \mathbb{E}_L(y - s_L). \tag{33}$$

As a result, type L has incentives to mimic type H. To see this, let $P(s_\theta)$ be the obtained obtained when issuing and observe that:

$$\begin{aligned} V(P(s_H), s_H, \theta_L) - V(P(s_L), s_L, \theta_L) &= (P(s_H)R + \mathbb{E}_L(y - s_H)) \times \mathbb{P}\left\{\omega \geq \omega^\sharp(P(s_H))\right\} \\ &\quad - (P(s_L)R + \mathbb{E}_L(y - s_L)) \times \mathbb{P}\left\{\omega \geq \omega^\sharp(P(s_L))\right\} \\ &> (P(s_H)R + \mathbb{E}_L(y - s_H) - (P(s_L)R + \mathbb{E}_L(y - s_L))) \times \\ &\quad \times \mathbb{P}\left\{\omega \geq \omega^\sharp(P(s_L))\right\} \\ &> 0, \end{aligned}$$

where the first inequality arises from the fact that $P(s_H) > P(s_L)$ and the monotonicity of $\omega^\sharp$. The second inequality, in turn, is a consequence of equation (33). This is a contradiction and hence $P(s_H) \leq \frac{1}{R}\mathbb{E}_L(y|m^y)$. □

Part 3. To see part (a), suppose that $\frac{1}{R}\mathbb{E}_L(y) < K$. Consider the deviation to any security $\hat{s}$ satisfying $\frac{\mathbb{E}_\mu(\hat{s})}{R} \geq K$ for some $\mu \in (0,1]$, which is the only relevant case since the market would never fund the low type. Observe that $BR(\hat{s}) = \left[K, \frac{1}{R}\mathbb{E}_H(\hat{s})\right]$, since any price below K induces default with certainty when assumption (3) holds and also $\lambda = 0$, and any $P \geq K$ dissuades all short-term creditors from running, and hence prevents default w.p. 1. As a consequence, type L can profitably deviate and place security $\hat{s}$ for any price $P \in BR(\hat{s})$:

$$V(P, \theta_L, \hat{s}) = (PR + \mathbb{E}_{\theta_L}(y - s)) \times \underbrace{\mathbb{P}\left\{\omega \geq \omega^\sharp(P)\right\}}_{=1} > 0.$$

Thus, $\mathscr{D}(\theta_L; \hat{s}) = BR(\hat{s})$, implying that market beliefs that assign $\mu(\theta_L, s) = 1$ for any such $s \in S$ are consistent with D1. This amounts to say that any feasible deviation is always attributed to type L, and therefore no seller type gets funded. If $\frac{1}{R}\mathbb{E}(y) < K$ instead, then part 1 and 2 above, together with the observation that, when assumption (3) holds, $\Phi(\mathbb{E}(y)) = \{0\}$ imply that $s_\theta = \mathbf{0}$ for all $\theta \in \Theta$ is the unique equilibrium.

Finally, to see part(b), assume that $\frac{1}{R}\mathbb{E}(y) \geq K$. The result follows directly from Theorem 4 in Nachman and Noe [1994].

Appendix B: Comprehensive Assessment

Proof of Proposition 2.

Below I prove a sequence of lemmas that induce the result.

Lemma 4. *Fix the amount raised by the bank, $P \geq 0$. The problem of designing a stress test that maximizes the policy maker's payoff :*

$$\max_{\Gamma^{\omega}=\{\pi^{\omega},M^{\omega}\}} \quad \mathbb{E}\left(W_0\left(\bar{A}\left(P,m^{\omega}\right)\right)\times 1\left\{\omega+P\geq \bar{A}\left(P,m^{\omega}\right)\right\}\right)$$
$$s.t.: \quad \bar{A}\left(P,m^{\omega}\right)=1\left\{\mathbb{E}\left(u\left(\omega+P,1\right)|m^{\omega}\right)\leq 0\right\},$$

is equivalent to the problem of maximizing the probability that short-term creditors refrain from withdrawing early under the most aggressive strategy profile (i.e., maximizing $\mathbb{P}\{\mathbb{E}\left(u\left(\omega+P,1\right);\Gamma^{\omega}\right)>0\}$). The policy maker's problem can thus be written as

$$\max_{\Gamma^{\omega}=\{\pi^{\omega},M^{\omega}\}}\sum_{m^{\omega}\in M^{\omega}} 1\left\{\mathbb{E}\left(u\left(\omega+P,1\right)|m^{\omega}\right)>0\right\}\times \underbrace{\int_{\Omega}\pi^{\omega}\left(m^{\omega}|\omega\right)F^{\omega}\left(d\omega\right)}_{\equiv \pi^{\omega}\left(m^{\omega}\right)}. \tag{34}$$

Proof. Consider an arbitrary stress test $\Gamma^{\omega}=\{\pi^{\omega},M^{\omega}\}$. Assume that there exists some score $\bar{m}$ disclosed with positive probability under Γ^{ω} for which (i) $\bar{A}\left(P,\bar{m}\right)=1$, and (ii)

$$\mathbb{P}\left\{\omega:\omega+P\geq 1\wedge \pi^{\omega}\left(\bar{m}|\omega\right)>0\right\}>0.$$

That is, score $\bar{m}$ induces all short-term creditors to withdraw early and satisfies that the set of realizations of ω in which the bank survives even if all short-term creditors withdraw early, has positive measure. Consider then the alternative policy $\hat{\Gamma}^{\omega}=\left\{\hat{\pi}^{\omega},\hat{M}^{\omega}=M^{\omega}\cup\{\bar{m}_0,\bar{m}_1\}\right\}$ constructed as follows: for any $m\in M^{\omega}$ different from $\bar{m}$, $\hat{\pi}^{\omega}\left(m|\cdot\right)=\pi^{\omega}\left(m|\cdot\right)$. Additionally, $\hat{\pi}^{\omega}\left(\bar{m}_0|\omega\right)=\pi^{\omega}\left(\bar{m}|\omega\right)\times 1_{\{\omega+P\geq 1\}}$ and $\hat{\pi}^{\omega}\left(\bar{m}_1|\omega\right)=\pi^{\omega}\left(\bar{m}|\omega\right)\times 1_{\{\omega+P<1\}}$ for all $\omega\in\Omega$. Policy $\hat{\Gamma}^{\omega}$ preserves the probability that the bank survives and decreases the number of short-term creditors who withdraw early. Therefore, $\hat{\Gamma}^{\omega}$ weakly dominates Γ^{ω}. As a result, assuming that the optimal policy maximizes the probability that short-term creditors refrain from attacking is without loss. □

This lemma shows that the problem of maximizing the policy maker's payoff by means of a stress test Γ^{ω} is equivalent to maximizing the probability that short-term creditors withdraw late. I thus focus on maximizing the expression in (34).

Consider then *any* stress test $\Gamma^{\omega}=\{M^{\omega},\pi^{\omega}\}$. Each score m^{ω} disclosed by stress test Γ^{ω} induces a posterior distribution over ω, $F^{\omega}(\cdot|m^{\omega})$. Hence, every score m^{ω} disclosed with positive probability generates a posterior expectation of $u\left(\omega+P,1\right)$, the utility of a short-term creditor withdrawing late obtains when the bank raises P units of capital and when all other short-term creditors withdraw early. That is, each message m^{ω} induces a new assessment:

$$\mathbb{E}\left(u\left(\omega+P,1\right)|m^{\omega}\right) \quad = \quad \int_{\Omega}u\left(\omega+P,1\right)F^{\omega}(d\omega|m^{\omega}).$$

The optimal stress test Γ^{ω} can then be characterized by the distribution of posterior expectations of $u\left(\omega,P,1\right)$ induced. Let $G^{\Gamma^{\omega}}\left(\cdot;P\right)$ be the distribution of posterior means of $u\left(\omega,P,1\right)$ induced by policy Γ^{ω}.

The next lemma shows that the distribution of posterior expectations associated with any stress test Γ^{ω}, $G^{\Gamma^{\omega}}$, corresponds to a mean-preserving contraction of the distribution associated with the full-disclosure policy $\Gamma^{\omega}_{\text{FD}}$, G^{ω}_{FD}, and a mean-preserving spread of the no-disclosure policy, $G^{\omega}_{\emptyset}$. That is, $G^{\omega}_{\text{FD}} \succeq_{\text{MPS}} G^{\Gamma^{\omega}} \succeq_{\text{MPS}} G^{\omega}_{\emptyset}$, where the partial order $\succeq_{\text{MPS}}$ is defined as follows:

Definition 2. Let F and G be distribution functions with support in $X \subseteq \mathbb{R}$. We say that F dominates H in the MPS order, $F \succeq_{\text{MPS}} H$, if $\int_X \varphi(x)F(dx) \geq \int_X \varphi(x)G(dx)$ for any convex function φ in X.

Lemma 5. [Blackwell] *Let $\Gamma^{\omega}_1 = (M^{\omega}_1, \pi^{\omega}_1)$ and $\Gamma^{\omega}_2 = (M^{\omega}_2, \pi^{\omega}_2)$ be two stress tests. Assume that there exists $z : M^{\omega}_1 \times M^{\omega}_2 \to [0,1]$ such that:*

(i) $\pi^{\omega}_2(m_2|\omega) = \sum_{M^{\omega}_1} z(m_1, m_2)\, \pi^{\omega}_1(m_1|\omega)$, $\forall \omega \in [0,1], \forall m_2 \in M^{\omega}_2$

(ii) $\sum_{M^{\omega}_2} z(m_1, m_2) = 1$, $\forall m_1 \in M^{\omega}_1$.

Then the distributions of posterior expected utility of short-term creditors, $\mathbb{E}(u(\omega + P, 1))$, induced by Γ^{ω}_1 and Γ^{ω}_2 are such that $G^{\Gamma^{\omega}_1} \succeq_{MPS} G^{\Gamma^{\omega}_2}$.

Proof. Let $f^{m_i} \in \Delta[0,1]$ be the posterior pdf after observing message $m_i \in M^{\omega}_i$, and $\pi^{\omega}_i(m_i) = \int \pi^{\omega}_i(m_i|\omega)\, f^{\omega}(\omega) d\omega$ the total probability of observing disclosure m_i, under policy Γ^{ω}_i, $i \in \{1,2\}$. Observe that *bayesian updating* together with property (i) imply that for any message $m_2 \in M^{\omega}_2$ with $\pi^{\omega}_2(m_2) > 0$ we have: □

$$f^{m^2}(\omega) = \sum_{m_1 \in M^{\omega}_1} \left(\frac{\pi^{\omega}_1(m_1)\, z(m_1, m_2)}{\pi^{\omega}_2(m_2)} \right) f^{m^1}(\omega).$$

This implies that, for any convex function φ,

$$\begin{aligned}
\sum_{m_2 \in M^{\omega}_2} \pi^{\omega}_2(m_2)\, \varphi\left(\int_0^1 \omega f^{m_2}(\omega) d\omega \right) &= \sum_{m_2 \in M^{\omega}_2} \pi^{\omega}_2(m_2)\, \varphi\left(\sum_{m_1 \in M^{\omega}_1} \left(\frac{\pi^{\omega}_1(m_1)\, z(m_1, m_2)}{\pi^{\omega}_2(m_2)} \right) \int_0^1 \omega f^{m_1}(\omega) d\omega \right) \\
&\leq \sum_{m_2 \in M^{\omega}_2} \sum_{m_1 \in M^{\omega}_1} \pi^{\omega}_1(m_1)\, z(m_1, m_2)\, \varphi\left(\int_0^1 \omega f^{m_1}(\omega) d\omega \right) \\
&= \sum_{m_1 \in M^{\omega}_1} \pi^{\omega}_1(m_1)\, \varphi\left(\int_0^1 \omega f^{m_1}(\omega) d\omega \right),
\end{aligned}$$

where the second inequality comes from Jensen's inequality and the last equality from using property (ii). As a result, $G^{\Gamma^{\omega}_1} \succeq_{\text{MPS}} G^{\Gamma^{\omega}_2}$.□

Lemma 5 shows that stress tests that are more informative (in the Blackwell sense) induce distributions of posterior expected utility (of late short-term creditors), $\mathbb{E}(u(\omega + P, 1))$, that dominate in the MPS order defined above. As a result, $G^{\omega}_{\text{FD}} \succeq_{MPS} G^{\Gamma^{\omega}} \succeq_{MPS} G^{\omega}_{\emptyset}$.

Consider then the problem of maximizing the likelihood that short-term creditors keep pledging to the bank. Using lemmas (4)-(5), the policy-maker's problem can be reformulated as maximizing

$$\mathbb{P}\{\mathbb{E}(u(\omega + P, 1); \Gamma^{\omega}) > 0\} = 1 - G^{\Gamma^{\omega}}(0; P)$$

among all possible disclosure policies over ω. That is,

$$\max_{G^{\Gamma^\omega}} \quad 1 - G^{\Gamma^\omega}(0)$$
$$\text{s.t:} \quad G^{\omega}_{\text{FD}} \succeq_{\text{MPS}} G^{\Gamma^\omega}.$$

This concludes the proof of Proposition 2. □

Proof of Lemma 1

Under full-disclosure, each message generates a degenerate posterior distribution with all weight assigned to $u(\omega,P,1)$ when ω is realized, which also coincides with the posterior mean induced by the message. As a result, $\mathscr{G}^{\omega}_{\text{FD}}(t;P) = \int_{u(0,P,1)}^{t} G^{\omega}_{\text{FD}}(\tilde{u};P)\,d\tilde{u}$, where

$$G^{\omega}_{\text{FD}}(\tilde{u};P) = \int_{u(0,P,A=1)}^{\tilde{u}} \frac{f_\omega\left(u^{-1}(z;P,1)\right)}{\partial_\omega u\left(u^{-1}(z;P,1),\tau,1\right)}dz$$

corresponds to the distribution of $u(\omega,P,1)$ under full-disclosure. Next, notice that under no-disclosure, the posterior mean remains unchanged and equal to $\mathbb{E}(u(\omega+P,1)\,|\emptyset)$. Thus, $\mathscr{G}^{\omega}_{\emptyset}(t;P) = \int_{u(0,P,1)}^{t} 1\{\tilde{u} \geq \mathbb{E}(u(\omega,P,1)\,|\emptyset)\}\,d\tilde{u}$. To save on notation, hereafter we will omit the dependence on P of all disclosure policies and associated distributions. Any disclosure policy Γ^ω, induces a function $\mathscr{G}^{\Gamma^\omega}(t) \equiv \int_{u(0,P,1)}^{t} G^{\Gamma^\omega}(\tilde{u})\,d\tilde{u}$. That $G^{\omega}_{\text{FD}} \succeq_{\text{MPS}} G^{\Gamma^\omega} \succeq_{\text{MPS}} G^{\omega}_{\emptyset}$ implies that $\mathscr{G}^{\omega}_{\text{FD}}(t) \geq \mathscr{G}^{\Gamma^\omega}(t) \geq \mathscr{G}^{\omega}_{\emptyset}(t)$ for all $t \in [u(P,1),u(1+P,1)]$, which can be seen from applying the definition of $\succeq_{\text{MPS}}$ to the convex function $\max\{\omega - t, 0\}$. Moreover, $\mathscr{G}^{\Gamma^\omega}$ is convex since G^{Γ^ω} is non-decreasing. Conversely, any non-decreasing, convex function h in $[u(P,1),u(1+P,1)]$, which satisfies that $\mathscr{G}^{\omega}_{\text{FD}}(t) \geq h(t) \geq \mathscr{G}^{\omega}_{\emptyset}(t)$ can be induced by some policy Γ^ω. To see this note that h is differentiable almost everywhere and its right derivative is always well-defined since it is convex. Let $G(\tilde{u}) \equiv h'(\tilde{u}^+)$ be the right-derivative of h at $\tilde{u}$. Observe next that $\lim_{\tilde{u}\to\infty} G(\tilde{u}) = 1$, and thus G is a distribution. Finally, note that G^{ω}_{FD} is a mean-preserving spread of G and therefore there must exist a policy that induces it by Strassen's Theorem (See Theorem 1.5.20 in Müller and Stoyan [2002]).□

Proof of Proposition 5.

First, we prove that under the laissez-faire policy there exists an equilibrium of the fund-raising stage where both bank types pool over the debt contact $s_D \equiv \min\{y, D\}$, with $D > 0$ such that $\frac{\mathbb{E}(s_D)}{R} = K$. At this equilibrium, the market keep its prior belief about θ, μ, when observing security s_D and thus offers a payoff equal to K for s_D.

To see that this is, in fact, an equilibrium fix an arbitrary security $\tilde{s}(\cdot)$ and define $\Delta V_\theta(P|\tilde{s})$ as the differential payoff obtained by type θ by switching from security s_D to $\tilde{s}$ and receiving a price P for the latter. That is,

$$\Delta V_\theta(P|\tilde{s}) \equiv (PR + \mathbb{E}_\theta(y - \tilde{s}))\,\phi(P) - (KR + \mathbb{E}_\theta(y - s_D)),$$

where $\phi(P) = \mathbb{P}\{\omega \geq 1 - P\} = 1 - F^\omega(1-P)$. We show that beliefs that assign probability 1 to the type being $\theta = L$ are consistent with D1. Clearly, under such beliefs no bank type deviates. The next claim reduces the set of deviations that need to be considered.

Claim 1. For any security $s \in \mathscr{S}$, let $s_d \equiv \min\{y,d\}$ be such that $\mathbb{E}_H(s-s_d)=0$. Then, $\Delta V_L(P|s_d) < \Delta V_L(P|s)$.

Proof. Note that, for any $s_0 \in \mathscr{S}$,

$$\Delta V_H(P|s_0) - \Delta V_L(P|s_0) = (\mathbb{E}_H(y-s_0) - \mathbb{E}_L(y-s_0))\,\phi(P) - (\mathbb{E}_H(y-s_D) - \mathbb{E}_L(y-s_D))\,.$$

By virtue of Lemma 3 and the definition of s_d, wethen have that $\mathbb{E}_L(s-s_d)<0$. As a result, $\Delta V_L(P|s_d) < \Delta V_L(P|s)$. □

Claim 1 implies that the only deviations that need to be considered are those to debt contracts. Indeed, for any security $s \in \mathscr{S}$, the *equivalent debt* security s_d reduces the set of prices that would induce type L to deviate while keeping the set of prices for type H unchanged (i.e., $\Delta V_H(P|s_d) = \Delta V_H(P|s)$). Under the D1 criterion, off-path beliefs, at any security s, must assign all weight to the bank type with the largest set of prices consistent with a profitable deviation.[42] Claim 1 then shows that it is enough to restrict attention to debt contracts as these are the securities that minimize the set of price consistent with a profitable deviation for type L.

Consider first deviations to debt contracts $\tilde{s}=\min\{y,\tilde{d}\}$ with $\mathbb{E}_H(\tilde{s}-s_D)>0$. In this case, for any $P\geq K$,

$$\Delta V_\theta(P|\tilde{s}) = (P-K)R - \mathbb{E}_\theta(\tilde{s}-s_D)\,,\ \theta\in\{L,H\}\,.$$

The fact that $\tilde{s}$ is a debt contract implies that $\tilde{s}-s_D$ is weakly increasing and therefore FOSD (implied by MLRP) means that $\mathbb{E}_H(\tilde{s}-s_D) > \mathbb{E}_L(\tilde{s}-s_D)$. As a result, there exists a price $\hat{P}>K$ for which

$$\Delta V_L\left(\hat{P}|\tilde{s}\right) > 0 > \Delta V_H\left(\hat{P}|\tilde{s}\right).$$

This implies that beliefs satisfying $\mu(\tilde{s}) = 1\{\theta=\theta_L\}$ are consistent with D1.

Now consider the case where $\tilde{s}$ is a debt contract with $\frac{\mathbb{E}_H(s_D)}{R} \geq \frac{\mathbb{E}_H(\tilde{s})}{R}$. For any $P\geq K$, we have that

$$\Delta V_\theta(P|\tilde{s}) = (P-K)R + \mathbb{E}_\theta(s_D-\tilde{s})\,,\ \theta\in\{\theta_L,\theta_H\}\,.$$

The fact that $\tilde{s}$ is a debt contract implies that $s_D-\tilde{s}$ is positive and weakly increasing. Thus, $\Delta V_\theta(K|\tilde{s})\geq 0$ for all θ. Next, for any $P<K$,

$$\begin{aligned}
\Delta V_\theta(P|\tilde{s}) &= (\mathbb{E}_H(y-\tilde{s}) - \mathbb{E}_L(y-\tilde{s}))\,\phi(P) - (\mathbb{E}_H(y-s_D) - \mathbb{E}_L(y-s_D)) \\
&< (\mathbb{E}_H(y-\tilde{s}) - \mathbb{E}_L(y-\tilde{s}))\,\bar{\phi} - (\mathbb{E}_H(y-s_D) - \mathbb{E}_L(y-s_D)) \\
&= \left(\frac{\mathbb{E}_H(y-\tilde{s}) - \mathbb{E}_L(y-\tilde{s})}{\mathbb{E}_H(y) - \mathbb{E}_L(y)} - 1\right)(\mathbb{E}_H(y-s_D) - \mathbb{E}_L(y-s_D)) \\
&< 0,
\end{aligned}$$

where the first inequality follows from assumption (c) in Condition 1. The second equality is by definition of $\bar{\phi}$. The last inequality follows from noting that $\mathbb{E}_H(\tilde{s}) - \mathbb{E}_L(\tilde{s}) < 0$ since $\tilde{s}$ is monotone and signals are ordered

[42] To be precise, the set of relevant prices are those in $BR(s) = \left\{P\geq 0 : \frac{\mathbb{E}_H(s)}{R}\phi(P) \geq P\right\}$ (see the equilirum definition in the Appendix). This set remains unchanged when considering the equivalent debt security s_d, by construction.

according to MLRP. As a result, $\mathscr{D}(\theta_L|\tilde{s}) \supseteq \mathscr{D}(\theta_H|\tilde{s})$ and, therefore, beliefs satisfying $\mu(\tilde{s}) = 1\{\theta = \theta_L\}$ are consistent with D1. This completes the proof that s_D is an equilibrium of the fund-raising stage.

Next, we prove that, under the sequentially optimal stress test Γ^{ω}, having both bank types pooling over the security s_D cannot be an equilibrium outcome. To show this, we prove that there exists a profitable deviation. In fact, consider the security $s_{\varepsilon} = \min\{y, D-\varepsilon\}$ with $\varepsilon > 0$ small. Similarly to the analysis above, define $\Delta V_{\theta}^{\Gamma^{\omega}}(P|\tilde{s})$ as the differential payoff obtained by type θ by switching from security s_D to s_{ε} and receiving a price P, when the policy maker runs the sequentially optimal stress test Γ^{ω}. That is,

$$\Delta V_{\theta}^{\Gamma^{\omega}}(P|s_{\varepsilon}) \equiv (PR + \mathbb{E}_{\theta}(y-\tilde{s}))\hat{\phi}(P) - (KR + \mathbb{E}_{\theta}(y-s_D)),$$

where $\hat{\phi}(P) = \mathbb{P}\{\omega \geq \bar{\omega}(P)\} = 1 - F^{\omega}(\bar{\omega}(P))$. For any $P \geq K$, we have that

$$\Delta V_{\theta}^{\Gamma^{\omega}}(P|s_{\varepsilon}) = (P-K)R + \mathbb{E}_{\theta}(s_D - s_{\varepsilon}),\ \theta \in \{\theta_L, \theta_H\}.$$

Thus, $\Delta V_{\theta}^{\Gamma^{\omega}}(P|s_{\varepsilon}) > 0$ for any $P \geq K$, and any θ. Next, note that

$$\Delta V_{H}^{\Gamma^{\omega}}(K|s_{\varepsilon}) - \Delta V_{L}^{\Gamma^{\omega}}(K|s_{\varepsilon}) = \mathbb{E}_H(s_D - s_{\varepsilon}) - \mathbb{E}_L(s_D - s_{\varepsilon}) > 0,$$

as $s_D - s_{\varepsilon}$ is non-decreasing. We prove that, under the assumptions in Condition 1, there exists a price $P_{\varepsilon} < K$ satisfying that $\Delta V_{H}^{\Gamma^{\omega}}(P_{\varepsilon}|s_{\varepsilon}) > 0 > \Delta V_{L}^{\Gamma^{\omega}}(P_{\varepsilon}|s_{\varepsilon})$. To see this, let $\tilde{P}_{\varepsilon} < K$ be defined as the unique solution to $\Delta V_{H}^{\Gamma^{\omega}}(P|s_{\varepsilon}) = 0$. Note that the definition of $\bar{\omega}(\cdot)$ implies that $\lim_{P\to K^-} \hat{\phi}(P) = 0$ and, therefore, $\lim_{\varepsilon\to 0^+} \tilde{P}_{\varepsilon} = K$. Next, we rewrite $\Delta V_{\theta}^{\Gamma^{\omega}}(\tilde{P}_{\varepsilon}|s_{\varepsilon})$ using the first-order Taylor expansion as

$$\Delta V_{\theta}^{\Gamma^{\omega}}(\tilde{P}_{\varepsilon}|s_{\varepsilon}) = \Delta V_{\theta}^{\Gamma^{\omega}}(K|s_{\varepsilon}) + \partial_P^- \Delta V_{\theta}^{\Gamma^{\omega}}(K|s_{\varepsilon})(\tilde{P}_{\varepsilon} - K) + o(\tilde{P}_{\varepsilon} - K),$$

where $\partial_P^- \Delta V_{\theta}^{\Gamma^{\omega}}(K|s_{\varepsilon}) \equiv \lim_{P\to K^-}\lim_{\xi\to 0} \frac{\Delta V_{\theta}^{\Gamma^{\omega}}(P|s_{\varepsilon}) - \Delta V_{\theta}^{\Gamma^{\omega}}(P-\xi|s_{\varepsilon})}{\xi}$ represents the *left* derivative of $\Delta V_{\theta}^{\Gamma^{\omega}}(P|s_{\varepsilon})$ at K. Thus, we can express

$$\Delta V_{L}^{\Gamma^{\omega}}(\tilde{P}_{\varepsilon}|s_{\varepsilon}) = \Delta V_{L}^{\Gamma^{\omega}}(K|s_{\varepsilon}) - \partial_P^- \Delta V_{L}^{\Gamma^{\omega}}(K|s_{\varepsilon}) \underbrace{\frac{\Delta V_{H}^{\Gamma^{\omega}}(K|s_{\varepsilon}) + o(\tilde{P}_{\varepsilon} - K)}{\partial_P^- \Delta V_{H}^{\Gamma^{\omega}}(K|s_{\varepsilon})}}_{=K-\tilde{P}_{\varepsilon}} + o(\tilde{P}_{\varepsilon} - K).$$

Next, assumption (b) in Condition 1, together with the fact $\lim_{P\to K^-} \bar{\omega}(P) = 0$, imply that

$$\lim_{P\to K^-} \hat{\phi}'(P) = \lim_{P\to K^-} f^{\omega}(\bar{\omega}(P))\bar{\omega}'(P) = 0,$$

which in turn implies that

$$\frac{\partial_P^- \Delta V_{L}^{\Gamma^{\omega}}(K|s_{\varepsilon})}{\partial_P^- \Delta V_{H}^{\Gamma^{\omega}}(K|s_{\varepsilon})} = \lim_{P\to K^-} \frac{R\phi(P) + (KR + \mathbb{E}_L(y - s_{\varepsilon}))\hat{\phi}'(P)}{R\phi(P) + (KR + \mathbb{E}_H(y - s_{\varepsilon}))\hat{\phi}'(P)} = 1 > \frac{\Delta V_{L}^{\Gamma^{\omega}}(K|s_{\varepsilon})}{\Delta V_{H}^{\Gamma^{\omega}}(K|s_{\varepsilon})}.$$

Thus, by choosing $\varepsilon = \tilde{\varepsilon}$ sufficiently close to 0 we obtain that $\Delta V_{L}^{\Gamma^{\omega}}(\tilde{P}_{\tilde{\varepsilon}}|s_{\tilde{\varepsilon}}) < 0 = \Delta V_{H}^{\Gamma^{\omega}}(\tilde{P}_{\tilde{\varepsilon}}|s_{\tilde{\varepsilon}})$.

Finally, consider $\varepsilon = \tilde{\tilde{\varepsilon}}$ sufficiently small so that $\frac{\mathbb{E}_H(s_{\tilde{\tilde{\varepsilon}}})}{R} > K$. Note that assumption (a) in Condition 1 then implies that $BR(s_{\tilde{\tilde{\varepsilon}}}) = \left[0, \frac{\mathbb{E}_H(s_{\tilde{\tilde{\varepsilon}}})}{R}\right]$. By picking $\varepsilon = \min\{\tilde{\varepsilon}, \tilde{\tilde{\varepsilon}}\}$ we then have that $\mathscr{D}(\theta_H|s_{\varepsilon}) \supsetneq \mathscr{D}(\theta_L|\varepsilon)$.

As a consequence, beliefs consistent with D1 necessarily assign $\mu(s_\varepsilon) = 1\{\theta = \theta_H\}$ and therefore such a deviation receives a price $P = \frac{\mathbb{E}_H(s_{\tilde{\varepsilon}})}{R} > K$ from long-term investors leading both types to choose s_ε over s_D. This proves that s_D cannot be an equilibrium. The rest of the proof follows from results (1) and (2) in Proposition 1 which show that (i) any pooling contract always delivers a price weakly smaller than K, and that (ii) in any separating equilibrium, type H always raises less than K.[43] □

Proof of Lemma *2*.

Assume first that $\bar{P}(\tau) > 0$ for $\tau > 0$ (this is always the case if $\lambda > 0$). We can then use equation 7, which implicitly defines $\bar{\omega}$, and compute:

$$\phi'(\tau) = \underbrace{\left(\frac{g+|b|}{|b|}\right)}_{\equiv c} f^{\omega}(1-\bar{P}(\tau))\bar{P}'(\tau) = \frac{cf^{\omega}(1-\bar{P}(\tau))\phi(\tau)}{R-cf^{\omega}(1-\bar{P}(\tau))\tau} \geq 0, \ \ \forall \tau \in [0, KR) \tag{35}$$

where $\bar{P}'$ can be obtained from its definition in equation (8) and equals:

$$\bar{P}'(\tau) = \frac{\tau\phi'(\tau) + \phi(\tau)}{R}.$$

Differentiating equation (35) once more and using assumptions 3 and 4 we obtain that the sign of ϕ'' coincides with the sign of:

$$1 - \frac{c}{R} f^{\omega}(1-\bar{P}(\tau))\tau = 1 - F^{\omega}(\bar{\omega}(\bar{P}(\tau))) \geq 0,$$

which proves the lemma. □

Proof of Theorem 1.

We first prove that ϕ satisfies the following properties: ϕ is (a) continuous, (b) non-decreasing, and (c) satisfies $\phi(0) = 0$, and $\phi(y) = 1$ for all $y \geq KR$. That ϕ is continuous comes from the fact that (i) $\bar{\omega}(\cdot)$ is continuously differentiable, (ii) $F^{\omega}(\cdot)$ admits a density and has at most one mass point at $\omega = 1$, and (iii) $\bar{P}$ is continuous. To see this last point, we apply the *maximum theorem* to the definition of $\bar{P}$:

$$\begin{aligned} \bar{P}(\tau) &= \max P \\ &\text{s.t:} \quad P \in \Gamma(\tau) \equiv \left\{P \geq 0 : \frac{\tau}{R}\mathbb{P}\{\omega \geq \bar{\omega}(P)\} \geq P\right\} \end{aligned}$$

where $\Gamma(\cdot)$ is a compact valued and continuous correspondence. To see (b), we note that $\bar{P}$ is non-decreasing and that $\bar{\omega}$ is non-increasing which implies the result. Finally, (c) is by definition of functions $\bar{P}$ and $\bar{\omega}$. Conditions (a)-(c) guarantee that ϕ satisfies the *regularity* assumption in Dworczak and Martini [2019]. That

[43] Note that the proof of Proposition 1 is general and works not only for the laissez faire policy but also under the sequentially rational stress test Γ^{ω}.

the optimal disclosure policy consists of monotone partitions thus follows proposition 2 in their paper. Next, to prove that the highest partition includes KR, we observe that using integration by parts, we can rewrite the policy maker's objective function as:

$$\int_0^{\infty}\phi(y)Z(dy) \quad = \quad -\left(\left(1-\lim_{y\to KR^-}\phi(y)\right)Z(KR)+\int_0^{\infty}\phi'(y)Z(y)dy\right).$$

As a result, the designer's problem is equivalent to:

$$\min_{Z} \quad \left(1-\lim_{y\to KR^-}\phi(y)\right)Z(KR)+\int_0^{\infty}\phi'(y)Z(y)\,dy \tag{36}$$
$$\text{s.t:} \quad Z\in\mathscr{X}(F^y).$$

Conditions (b) and (c) then imply that it is optimal to choose $Z(y)=1-F^y(y)$ for all $y\geq KR$. This implies that KR will be included in the highest partition cell.

Next, we show that when ϕ is convex, the optimal policy takes a simple form: there exists a threshold $y^+\geq 0$ and Γ_*^y fully disclose y for any $y\in[0,y^+)$ and pools under the same score all $y\geq y^+$.

Lemma 6. *ϕ convex implies that the optimal choice of Z is given by:*

$$Z_*(y)=\begin{cases}0 & y<y^+\\ F^y(y^+)-F^y(y) & y\in[y^+,KR)\\ 1-F^y(y) & y\geq KR,\end{cases} \tag{37}$$

where

$$y^+=\inf\left\{y\geq 0:\int_y^{KR}(F^y(y)-F^y(\tau))\,d\tau+\int_{KR}^{\infty}(1-F^y(\tau))\,d\tau\geq 0\right\}.$$

Proof. That ϕ is convex implies that ϕ is differentiable for *almost all* $y\geq 0$, with ϕ' non-decreasing over $[0,KR)$. Moreover, the definition of K implies that $\phi'=0$ for all $y\geq KR$.

Next, take any function $X\in\mathscr{X}(F^y)$. We prove that either (a) $X=Z$ for (almost) all $y\geq 0$, or else (b) X is dominated. Assume that $X\neq Z$. The constraint that $F^y(y)+X(y)\leq 1$ for all $y\geq 0$, together with the requirement that $\int_0^{\infty}X(y)dy=0$, impose a lower bound on $\int_0^{KR}X(y)dy$. In fact, we must have that $\int_{KR}^{\infty}X(y)dy\leq\int_{KR}^{\infty}(1-F^y(y))\,dy$, and hence $\int_0^{KR}X(y)dy\geq-\int_{KR}^{\infty}(1-F^y(y))\,dy$. We prove that the last inequality must be binding for any non-dominated policy X.

Step 1. *If $\int_0^{KR}X(y)dy>-\int_{KR}^{\infty}(1-F^y(y))\,dy$, then X is necessarily dominated.*

If $\int_0^{KR}X(y)dy>-\int_{KR}^{\infty}(1-F^y(y))\,dy$ then there exist alternative feasible policies that allocate more (negative) mass to the interval $[0,KR)$, which improves the objective function since ϕ' is positive (recall the policy maker's optimization problem in (36)). One of such improvements is

$$X^{\delta}(y)\equiv\begin{cases}(1-\delta)X(y)+\delta(-F^y(y)) & y<KR\\ X(y) & KR\leq y<KR+\xi(\delta),\\ \varepsilon(\delta)X(y)+(1-\varepsilon(\delta))(1-F^y(y)) & y\geq KR+\xi(\delta)\end{cases}$$

where, for each $\delta > 0$, $\varepsilon(\delta), \xi(\delta) > 0$ are chosen so that[44]

$$\int_0^\infty X^\delta(y)\,dy = 0.$$

Note that $X^\delta \in \mathscr{X}(F^y)$ for δ small. In fact, $F^y + X^\delta = (1-\delta)(X + F^y)$ *(i) belongs to* $[0, 1-\delta] \subset [0,1]$*;* (ii) is right-continuous (inherited from X); (iii) non-decreasing, which follows from $X \in \mathscr{X}(F^y)$ and the fact that

$$\lim_{y \to KR^-} (1-\delta)(X(y) + F^y(y)) < (1-\delta)(X(KR) + F^y(KR));$$

and (iv) $X^\delta(y)$ satisfies

$$\int_0^{\tilde{y}} X^\delta(y)\,dy \le \int_0^{\tilde{y}} X(y)\,dy \le 0, \text{ for all } y,$$

and

$$X^\delta(\infty) = 0 = \int_0^\infty X^\delta(y)\,dy,$$

by construction.

Clearly,

$$\int_0^\infty \phi'(y) X^\delta(y)\,dy < \int_0^\infty \phi'(y) X(y)\,dy,$$

which, together with the fact that $X^\delta(KR) = X(KR)$, imply that X^δ dominates X. Which proves Step 1.

Assume then that $\int_0^{KR} X(y)dy = -\int_{KR}^\infty (1 - F^y(y))\,dy$. For any $Z \in \mathscr{X}(F^y)$, let

$$y_0(Z) \equiv \sup\{y \in [0, KR) : Z(y) \ge 0\}.$$

For convenience, I omit hereafter the dependence of y_0 on Z. Next, fix any $\varepsilon, \Delta > 0$ small, and construct the alternative policy

$$X^{\varepsilon,\Delta}(y) \equiv \begin{cases} 0 & y < y_0 + \Delta + \varepsilon \\ F^y(y_0 + \Delta + \varepsilon) - F^y(y) & y \in [y_0 + \Delta + \varepsilon, \hat{y}(\varepsilon)) \\ F^y(y_0 + \Delta + \varepsilon) - F^y(\hat{y}(\varepsilon)) & y \in \left[\hat{y}(\varepsilon), \hat{\hat{y}}(\varepsilon)\right) \\ X(y) & y \ge \hat{\hat{y}}(\varepsilon), \end{cases}$$

where $\hat{y}(\varepsilon), \hat{\hat{y}}(\varepsilon)$ are implicitly defined by

$$F^y(y_0 + \Delta + \varepsilon) - F^y(\hat{y}(\varepsilon)) = X\left(\hat{\hat{y}}(\varepsilon)\right) \tag{38}$$

$$\int_0^\infty X^{\varepsilon,\Delta}(y)\,dy = 0. \tag{39}$$

Assume that ε and Δ are small enough so that $\hat{\hat{y}}(\varepsilon) < KR$. [45]

[44] $\varepsilon(\delta)$ and $\xi(\delta)$ are well defined for $\delta > 0$ small as $\int_0^{KR} X(y)dy > -\int_{KR}^\infty (1 - F^y(y))\,dy$, and therefore $\left|\int_0^{KR} X(y)dy\right| < \int_{KR}^\infty (1 - F^y(y))\,dy$, which implies that $\int_{KR}^\infty X(y)dy < \int_{KR}^\infty (1 - F^y(y))\,dy$.

[45] Obviously, $\hat{y}(\varepsilon)$ and $\hat{\hat{y}}(\varepsilon)$ are also functions of Δ. I omit this dependence as Δ is kept fixed throughout the proof. To see that both functions are well defined, note that the definition of y^+, together with Step1 in the proof, imply that $y_0 < y^+$. The requirement that $X + F^y$ is increasing then implies that there must exist a point within (y_0, KR) where $X^{\varepsilon,\Delta}$ crosses from below X. As a result, $\hat{y}(\varepsilon)$ and $\hat{\hat{y}}(\varepsilon)$ are well defined provided that ε and Δ are small enough.

Step 2. *For ε, Δ small, $X^{\varepsilon,\Delta}$ dominates X.*

To see this, consider the policy maker's payoff under policy $X^{\varepsilon,\Delta}$,

$$\bar{U}^{\varepsilon,\Delta} \equiv \left(1 - \lim_{y \to KR^-} \phi(y)\right) X^{\varepsilon,\Delta}(KR) + \int_0^\infty \phi'(y) X^{\varepsilon,\Delta}(y)\, dy.$$

Next, differentiating (39) with respect to ε we obtain

$$\begin{aligned} 0 &= \left(F^y(y_0+\Delta+\varepsilon) - F^y(\hat{y}(\varepsilon))\right)\hat{y}'(\varepsilon) + f^y(y_0+\Delta+\varepsilon)\left(\hat{y}(\varepsilon) - (y_0+\Delta+\varepsilon)\right) + \\ &\quad + \left(f^y(y_0+\Delta+\varepsilon) - f^y(\hat{y}(\varepsilon))\hat{y}'(\varepsilon)\right)\left(\hat{\hat{y}}(\varepsilon) - \hat{y}(\varepsilon)\right) + \\ &\quad + \left(F^y(y_0+\Delta+\varepsilon) - F^y(\hat{y}(\varepsilon))\right)\left(\hat{\hat{y}}'(\varepsilon) - \hat{y}'(\varepsilon)\right) - X\left(\hat{\hat{y}}(\varepsilon)\right)\hat{\hat{y}}'(\varepsilon) \\ &= f^y(y_0+\Delta+\varepsilon)\left(\hat{\hat{y}}(\varepsilon) - (y_0+\Delta+\varepsilon)\right) - f^y(\hat{y}(\varepsilon))\hat{y}'(\varepsilon)\left(\hat{\hat{y}}(\varepsilon) - \hat{y}(\varepsilon)\right), \end{aligned} \tag{40}$$

where the second equality obtains from (38). Therefore,

$$\begin{aligned} \frac{\partial}{\partial\varepsilon}\bar{U}^{\varepsilon,\Delta} &= \left(F^y(y_0+\Delta+\varepsilon) - F^y(\hat{y}(\varepsilon))\right)\phi'(\hat{y}(\varepsilon))\hat{y}'(\varepsilon) + f^y(y_0+\Delta+\varepsilon)\int_{y_0+\Delta+\varepsilon}^{\hat{y}(\varepsilon)} \phi'(y)\, dy + \\ &\quad + \left(f^y(y_0+\Delta+\varepsilon) - f^y(\hat{y}(\varepsilon))\hat{y}'(\varepsilon)\right)\left(\phi\left(\hat{\hat{y}}(\varepsilon)\right) - \phi(\hat{y}(\varepsilon))\right) + \\ &\quad + \left(F^y(y_0+\Delta+\varepsilon) - F^y(\hat{y}(\varepsilon))\right)\left(\phi'\left(\hat{\hat{y}}(\varepsilon)\right)\hat{\hat{y}}'(\varepsilon) - \phi'(\hat{y}(\varepsilon))\hat{y}'(\varepsilon)\right) + \\ &\quad - X\left(\hat{\hat{y}}(\varepsilon)\right)\phi'\left(\hat{\hat{y}}(\varepsilon)\right)\hat{\hat{y}}'(\varepsilon) \\ &= f^y(y_0+\Delta+\varepsilon)\left(\phi\left(\hat{\hat{y}}(\varepsilon)\right) - \phi(y_0+\Delta+\varepsilon)\right) - f^y(\hat{y}(\varepsilon))\hat{y}'(\varepsilon)\left(\phi\left(\hat{\hat{y}}(\varepsilon)\right) - \phi(\hat{y}(\varepsilon))\right) \\ &= f^y(y_0+\Delta+\varepsilon)\left\{\frac{\left(\phi\left(\hat{\hat{y}}(\varepsilon)\right) - \phi(y_0+\Delta+\varepsilon)\right)}{\left(\hat{\hat{y}}(\varepsilon) - (y_0+\Delta+\varepsilon)\right)} - \frac{\phi\left(\hat{\hat{y}}(\varepsilon)\right) - \phi(\hat{y}(\varepsilon))}{\left(\hat{\hat{y}}(\varepsilon) - \hat{y}(\varepsilon)\right)}\right\} \times \\ &\quad \times \left(\hat{\hat{y}}(\varepsilon) - (y_0+\Delta+\varepsilon)\right) \\ &< 0. \end{aligned}$$

The first equality comes from the definition of $\bar{U}^{\varepsilon,\Delta}$ and $X^{\varepsilon,\Delta}$. The second equality obtains from using (38). The third equality arises from plugging in the identity found in (40). Finally, the inequality is a consecuence of the convexity of ϕ. As a result, any policy X different from Z_* is dominated. This proves the lemma.

That $Z_*(y) = 0$ for all $y \le y^+$ implies that $G(y) = F^y(y)$ for any such y, or equivalently, that G coincides with the full-disclosure policy for all $y \le y^+$. On the other hand, that $G(y) = F^y(y^+) - F^y(y)$ for all $y \in (y^+, KR)$, and $G(y) = 1$ for all $y \ge \max KR$, means that the optimal policy pools all the realizations of y above y^+ under a single message, so that the induced posterior mean is at least KR. □

Appendix C: Elicitation Mechanisms

Proof of Proposition 6.

Fix a message m^y disclosed with positive probability under Γ^y. Suppose that during the second period the policy-maker perfectly observes the bank's asset quality type, θ. The policy-maker's problem in (25) can then be written as:

$$
\begin{aligned}
\max_{\{V_\theta, t_0(\cdot;\theta), \pi(0|\cdot;\theta)\}} \quad & \int_0^{1-P} \pi(0|\omega;\theta) F^\omega(d\omega) \\
\text{s.t:} \quad (i) \quad & \int_0^{1-P} \left((b-g)\times 1_{\{P+t_0(\omega;\theta)+\omega<1\}} + g\right)\pi(0|\omega;\theta)F^\omega(d\omega) + \\
& +\pi^s_\theta \times g \times (1-F^\omega(1-P))) \geq 0 \\
(ii) \quad & V_\theta \times \left(\int_0^{1-P} |b|\,\pi(0|\omega;\theta)F^\omega(d\omega) - g\times\pi^s_\theta \times (1-F^\omega(1-P))\right) \leq |\bar{U}_{\text{LF}}(P)| \\
(iii) \quad & \pi(0|\omega;\theta)\times((P+t_0(\omega;\theta))R) = V_\theta,\ \forall\omega \leq 1-P, \forall\theta\in\Theta \\
(iv) \quad & \pi(0|\omega;\theta) = \pi^s_\theta \leq \frac{V_\theta}{PR+z_\theta},\ \ \forall\omega \geq 1-P, \forall\theta\in\Theta \\
(v) \quad & t_0(\omega;\theta) \leq \frac{z_\theta}{R},\ \ \forall\theta\in\Theta
\end{aligned}
$$

where the first two constraints are the obedience constraints associated with messages 0 and 1, respectively, and the last three correspond to incentive compatibility constraints: (iii) imposes that the payoff of any bank reporting a liquidity position below $1-P$ must be the same, (iv) guarantees that vulnerable banks do not have incentives to mimic safe banks, and (v) simultaneously guarantees that (a) safe banks do not want to mimic vulnerable banks, and (b) that the fair-price constraint is satisfied.

Let $\hat{\omega}_\theta \equiv 1-P-\frac{z_\theta}{R}$. Define next the auxiliary variable ρ_θ as follows:

$$
\rho_\theta \equiv \int_0^{\hat{\omega}_\theta} \frac{\left|\frac{b}{g}\right| \times F^\omega(d\omega)}{PR+z_\theta} - \frac{1-F^\omega(1-P)}{PR+z_\theta}.
$$

I characterize the optimal screening mechanism as a function of the value of ρ_θ. Assume first that

$$
\rho_\theta \in \left(\frac{F^\omega(1-P)-F^\omega(\hat{\omega}_\theta)}{PR+z_\theta}, \int_{\hat{\omega}_\theta}^{1-P} \frac{F^\omega(d\omega)}{(1-\omega)R}\right).
$$

Note that inequality (iv) must bind since this relaxes (i) and (ii), does not affect neither (iii) nor (v), and therefore allows to improve the policy-maker's objective function. Next, constraint (iii) implies that we can write the policy-maker's problem as a function only of V_θ and t_0. Thus, the set of relevant constraints is given by:

$$
\begin{aligned}
(i') \quad & \left(\int_0^{1-P} \left(\frac{(b-g)\times 1_{\{P+t_0(\omega;\theta)+\omega<1\}} + g}{(P+t_0(\omega;\theta))R}\right) F^\omega(d\omega) + g\times\frac{(1-F^\omega(1-P))}{PR+z_\theta}\right) \geq 0 \\
(ii') \quad & V_\theta \times \int_0^{1-P} \frac{|b|\times F^\omega(d\omega)}{(P+t_0(\omega;\theta))R} \leq |\bar{U}_{\text{LF}}(P)| + V_\theta \times g \times \frac{(1-F^\omega(1-P))}{PR+z_\theta} \\
(iv') \quad & \pi^s_\theta = \frac{V_\theta}{PR+z_\theta},\ \ \forall\theta\in\Theta \\
(v) \quad & t_0(\omega;\theta) \leq \frac{z_\theta}{R},\ \ \forall\theta\in\Theta \\
(vi) \quad & \frac{V_\theta}{(P+t_0(\omega;\theta))R} \leq 1\ \ \forall\omega\leq 1-P, \forall\theta\in\Theta,
\end{aligned}
$$

where the new constraint (vi) is added so that probabilities are well defined.

Next, I characterize the optimal mechanism $\Upsilon^{\omega,\theta}[\theta] = \left\{t_m(\cdot;\theta), \pi^{\omega,\theta}(m|\cdot;\theta)\right\}_{m\in\{0,1\}}$. I assume existence, which I then verify by constructing the optimal mechanism.

Claim 0: Constraint (vi) must *essentially* bind for at least some $\omega \in \Omega$, and for all $\theta \in \Theta$. Rigorously,

$$\sup_{\omega\in\Omega} \frac{V_\theta}{(P+t_0(\omega;\theta))R} = 1. \tag{41}$$

I show that, under $\Upsilon^{\omega,\theta}$, if equation (41) is not satisfied, then there exists another feasible mechanism $\tilde{\Upsilon}^{\omega,\theta}$ that dominates $\Upsilon^{\omega,\theta}$. To prove this, suppose by contradiction that, under $\Upsilon^{\omega,\theta}$,

$$\delta_\theta \equiv 1 - \sup_{\omega\in\Omega} \frac{V_\theta}{(P+t_0(\omega;\theta))R} > 0.$$

Define then the mechanism $\tilde{\Upsilon}^{\omega,\theta}[\theta] = \left\{\tilde{t}_m(\cdot;\theta), \tilde{\pi}^{\omega,\theta}(m|\cdot;\theta)\right\}_{m\in\{0,1\}}$ such that (a) $\tilde{t}_m(\cdot;\theta) \equiv t_m(\cdot;\theta)$ for all $m \in \{0,1\}$, and all $\theta \in \Theta$, (b) $\tilde{\pi}^{\omega,\theta}(0|\cdot;\theta) \equiv \frac{\pi^{\omega,\theta}(0|\cdot;\theta)}{1-\delta_\theta}$ for all $\theta \in \Theta$, and (c) $\tilde{\pi}^s_\theta = \frac{\pi^s_\theta}{1-\delta_\theta}$. The new mechanism $\tilde{\Upsilon}^{\omega,\theta}[\theta]$ satisfies (i)-(v), and does strictly better than $\Upsilon^{\omega,\theta}[\theta]$. This is a contradiction.

Claim 1: $t_0(\omega;\theta) = \frac{z_\theta}{R}$ for all $\omega < \hat{\omega}_\theta$.

To see this, let $\Upsilon^{\omega,\theta}[\theta] = \left\{t_m^{\omega,\theta}(\cdot;\theta), \pi^{\omega,\theta}(m|\cdot;\theta)\right\}_{m\in\{0,1\}}$ be the optimal screening mechanism when the quality type is known to be θ. Suppose, by contradiction, that the claim is not true. I show that then it is possible to find another mechanism which strictly improves upon $\Upsilon^{\omega,\theta}$.

Consider the alternative mechanism $\Upsilon^\varepsilon[\theta] = \left\{t_m^\varepsilon(\cdot;\theta), \pi^{\omega,\theta}(m|\cdot;\theta)\right\}_{m\in\{0,1\}}$ which offers the alternative price t_0^ε that modifies the value of $t_0^{\omega,\theta}$ for values of $\omega \le \hat{\omega}_\theta$ in the following way:

$$t_0^\varepsilon(\omega;\theta) \equiv \begin{cases} \varepsilon z_\theta + (1-\varepsilon)t_0(\omega;\theta) & \omega \le \hat{\omega}_\theta \\ t_0(\omega;\theta) & \omega > \hat{\omega}_\theta. \end{cases}$$

Let V_θ^ε be the value of V_θ which preserves the value of the LHS in (ii'). That is,

$$V_\theta^\varepsilon \times \left(\int_0^{1-P} \frac{|b|F^\omega(d\omega)}{\left(P+t_0^\varepsilon(\omega;\theta)\right)R}\right) = V_\theta \times \left(\int_0^{1-P} \frac{|b|F^\omega(d\omega)}{(P+t_0(\omega;\theta))R}\right).$$

This perturbation relaxes (i') since $b < 0$ and $t_0^\varepsilon(\omega;\theta) \ge t_0^{\omega,\theta}(\omega;\theta)$ for all $\omega \le \hat{\omega}_\theta \le 1-P-t_0^{\omega,\theta}(\omega;\theta)$.[46] The perturbation also increases the value of V_θ, which then relaxes (ii') since the RHS increases while the LHS remains constant (by construction). Constraint (v) is never affected by this perturbation, while (vi) is strictly relaxed as $t_0^\varepsilon(\omega;\theta) \ge t_0^{\omega,\theta}(\omega;\theta)$ for all ω. As a result, constraint (vi) does not longer bind and, hence, by the result in claim 0, the designer can do strictly better with another mechanism. This is a contradiction. Thus, we must have that $t_0(\omega;\theta) = \frac{z_\theta}{R}$ for all $\omega < \hat{\omega}$, and all θ.

Claim 2: $\exists \check{\omega}_\theta \in [\hat{\omega}_\theta, 1-P]$ so that $t_0(\omega;\theta) = \max\{1-\omega-P, 1-\check{\omega}_\theta-P\}$ for all $\omega \in [\hat{\omega}_\theta, 1-P]$.

Consider an arbitrary pricing policy $\tilde{t}_0$. Construct the alternative policy $t_0(\omega) \equiv \max\{1-\omega-P, 1-\tilde{\omega}-P\}$ for all $\omega \in [\hat{\omega}, 1-P]$, where $\tilde{\omega}$ is chosen so that:

$$\int_{\hat{\omega}_\theta}^{1-P} \frac{F^\omega(d\omega)}{(P+\tilde{t}_0(\omega;\theta))R} = \int_{\hat{\omega}_\theta}^{1-P} \frac{f^\omega(\omega)}{\max\{1-\omega, 1-\tilde{\omega}\}R} d\omega.$$

[46]Note that, for any $\omega \in \Omega$, $1-P-t_0^{\omega,\theta}(\omega;\theta) \ge 1-P-\frac{z_\theta}{R} = \hat{\omega}_\theta$

I claim that the policy induced by t_0 dominates the one induced by $\tilde{t}_0$. To see this this, note that constraints (i'), (ii'), (v) remain unchanged under the alternative policy, but constraint (vi) relaxes. In fact,

$$\sup_{\omega\in[\hat{\omega}_\theta,1-P]}\left\{\frac{V_\theta}{(P+t_0(\omega))R}\right\}\leq\sup_{\omega\in[\hat{\omega}_\theta,1-P]}\left\{\frac{V_\theta}{(P+\tilde{t}_0(\omega))R}\right\}\leq 1.$$

The first inequality is strict if $F^{\omega}\left(\left\{\omega\in[\hat{\omega}_\theta,1-P]:\tilde{t}_0(\omega;\theta)\neq t_0(\omega)\right\}\right)>0.\square$

Claim 3: Constraint (i') must bind.

This constraint corresponds to obedience constraint (12), and requires that short-term creditors have an incentive to follow the recommendation to keep rolling over the bank's debt. By contradiction, assume that this constraint does not bind. Then,

$$\int_0^{\hat{\omega}_\theta}\frac{b\times F^{\omega}(d\omega)}{(P+t_0(\omega;\theta))R}+\int_{\hat{\omega}_\theta}^{1-P}\frac{g\times F^{\omega}(d\omega)}{(P+t_0(\omega;\theta))R}+g\times\frac{(1-F^{\omega}(1-P))}{(PR+z_\theta)}>0. \tag{42}$$

This implies that either (ii') or (vi) must be binding, or otherwise the policy-maker would strictly increase the passing probability $\pi^{\omega,\theta}(0|\omega;\theta)=\frac{V_\theta}{(P+t_0(\omega;\theta))R}$.

Suppose first that (ii') is the binding constraint. Consider the following deviation from the optimal mechanism $\Upsilon^{\omega,\theta}$: We modify t_0 between $[\hat{\omega}_\theta,1-P]$ for some θ, so that the new price can be written as $\tilde{t}_0^{\varepsilon}=\max\left\{1-\omega-P,1-\breve{\omega}_\theta^{\varepsilon}-P\right\}$, where $\breve{\omega}_\theta^{\varepsilon}<\breve{\omega}_\theta$ satisfies that

$$\int_{\hat{\omega}_\theta}^{1-P}\frac{F^{\omega}(d\omega)}{P+\tilde{t}_0^{\varepsilon}(\omega;\theta)}=\int_{\hat{\omega}_\theta}^{1-P}\frac{F^{\omega}(d\omega)}{P+t_0(\omega;\theta)}-\varepsilon,$$

for some $\varepsilon>0$ small enough so that the inequality above is respected[47]. Next, let $\tilde{V}_\theta(\varepsilon)$ be the maximal value that V_θ may take under the new policy so that (ii') remains unchanged. That is,

$$\tilde{V}_\theta(\varepsilon)\times\left(\int_0^{\hat{\omega}_\theta}\frac{|b|\times F^{\omega}(d\omega)}{(P+t_0(\omega;\theta))R}+\int_{\hat{\omega}_\theta}^{1-P}\frac{|b|\times F^{\omega}(d\omega)}{\left(P+\tilde{t}_0^{\varepsilon}(\omega;\theta)\right)R}-g\times\frac{(1-F^{\omega}(1-P))}{PR+z_\theta}\right)=C_\theta, \tag{43}$$

where $C_\theta>0$ is a constant. Next, differentiating (43) against ε and then taking the limit from the right as ε goes to 0, we get:

$$\lim_{\varepsilon\downarrow 0}\tilde{V}_\theta'(\varepsilon)=\frac{\tilde{V}_\theta(0)\times|b|}{\left(|b|\times\int_0^{1-P}\frac{f^{\omega}(\omega)}{(P+t_0(\omega;\theta))R}d\omega-g\times\frac{(1-F^{\omega}(1-P))}{PR+z_\theta}\right)}.$$

This allows us to compute the effect of such a perturbation on the policy-maker's payoff

$$W_\theta=\tilde{V}_\theta\int_0^{1-P}\frac{F^{\omega}(d\omega)}{P+t_0(\omega;\theta)}+F^{\omega}(1-P)$$

[47] The existence of such ε comes from (42), since this inequality implies:

$$\int_{\hat{\omega}}^{1-P}\frac{f^{\omega}(\omega)}{P+t_0(\omega)}d\omega>\rho>\frac{F^{\omega}(1-P)-F^{\omega}(\hat{\omega})}{P+z_L}.$$

for small values of ε. In fact,

$$\begin{aligned}\lim_{\varepsilon\to0^+}\frac{dW_\theta}{d\varepsilon} &\propto \left(\lim_{\varepsilon\to0^+}\tilde{V}'_\theta(\varepsilon)\right)\cdot\frac{W(0)}{\tilde{V}_\theta(0)}-\tilde{V}_\theta(0)\\ &= \frac{V_\theta^2}{C}\times(g-b)\times\frac{(1-F^\omega(1-P))}{PR+z_\theta}\\ &> 0.\end{aligned}$$

which contradicts the optimality of $\Upsilon^{\omega,\theta}$.

Next, assume that (vi) is the binding constraint for some θ (which determines the value of V_θ). Consider the alternative policy

$$\widehat{t}_0^\varepsilon(\omega;\theta)\equiv\begin{cases}\tilde{b}_\theta & \omega\le\hat{\omega}_\theta\\ \max\left\{1-\omega-P,1-\check{\omega}_\theta^\varepsilon-P\right\} & \omega\in(\hat{\omega}_\theta,1-P],\\ 0 & \omega>1-P\end{cases}$$

with $\check{\omega}_\theta^\varepsilon=\check{\omega}_\theta-\varepsilon$ and ε small enough so that (ii') is still satisfied. Let $\check{V}_\theta^\varepsilon$ be the maximal value that V may take under the new policy so that (vi) is still satisfied. That is,

$$\frac{\check{V}_\theta^\varepsilon}{1-\check{\omega}_\theta^\varepsilon}=\frac{V_\theta}{1-\check{\omega}_\theta}.$$

This implies that $\check{V}_\theta^\varepsilon>V_\theta$ and hence $\widehat{\pi}^\varepsilon(0|\omega;\theta)\equiv\frac{\check{V}_\theta^\varepsilon}{\left(P+\widehat{t}_0^\varepsilon(\omega)\right)R}>\pi(0|\omega;\theta)$ for all $\omega\le\check{\omega}_\theta$ and $\widehat{\pi}^\varepsilon(0|\omega;\theta)=\pi(0|\omega;\theta)$ for all $\omega>\check{\omega}_\theta$, and hence the policy-maker's payoff must increase. This is a contradiction and hence (i') must be satisfied with equality. □

This means that

$$\int_{\hat{\omega}_\theta}^{1-P}\frac{F^\omega(d\omega)}{(P+t_0(\omega;\theta))R}=\rho_\theta\in\left(\frac{F^\omega(1-P)-F^\omega(\hat{\omega}_\theta)}{PR+\frac{z_\theta}{R}},\int_{\hat{\omega}_\theta}^{1-P}\frac{F^\omega(d\omega)}{(1-\omega)R}\right),$$

which is feasible.

Therefore, we choose $t_0(\omega;\theta)$ in $[\hat{\omega}_\theta,1-P]$ among all the policies satisfying (i') so that V_θ is largest. Let $\check{\omega}_\theta$ be implicitly defined by:

$$\int_{\hat{\omega}_\theta}^{\check{\omega}_\theta}\frac{F^\omega(d\omega)}{(1-\omega)R}+\frac{F^\omega(1-P)-F^\omega(\check{\omega}_\theta)}{(1-\check{\omega}_\theta)R}=\int_0^{\check{\omega}_\theta}\frac{|b|\times F^\omega(d\omega)}{g\times\left(P+\frac{z_\theta}{R}\right)R}-\frac{1-F^\omega(1-P)}{PR+z_\theta}.$$

That is, $\check{\omega}_\theta$ is the cutoff defining the price t_0 which maximizes $\min_{\omega\le1-P}(P+t_0(\omega;\theta))R$ while still respecting (i'). The optimal policy is thus given by:

$$t_0(\omega;\theta)=\begin{cases}\frac{z_\theta}{R} & \omega<\hat{\omega}_\theta\\ 1-P-\omega & \omega\in[\hat{\omega}_\theta,\check{\omega}_\theta]\\ 1-P-\check{\omega}_\theta & \omega\in(\check{\omega}_\theta,1-P)\\ 0 & \omega\ge1-P\end{cases},\quad \pi(0|\omega;\theta)=\begin{cases}\frac{\bar{V}_\theta}{PR+z_\theta} & \omega<\hat{\omega}_\theta\\ \frac{\bar{V}_\theta}{(1-\omega)R} & \omega\in[\hat{\omega}_\theta,\check{\omega}_\theta]\\ \frac{\bar{V}_\theta}{(1-\check{\omega}_\theta)R} & \omega\in(\check{\omega}_\theta,1-P)\\ \frac{\bar{V}_\theta}{PR+z_\theta} & \omega\ge1-P\end{cases}$$

where $\bar{V}_\theta$ is chosen so that (ii) and (vi) hold:

$$\bar{V}_\theta \equiv \min\left\{(1-\check{\omega}_\theta)R, \frac{|\bar{U}_{\text{LF}}(P)|}{\int_0^{1-P}\frac{|b|\times f^\omega(\omega)}{(P+t_0(\omega;\theta))R}d\omega - g\times\frac{(1-F^\omega(1-P))}{PR+z_\theta}}\right\}.$$

Finally, assume that for some $\theta\in\Theta$

$$\rho_\theta \geq \int_{\hat{\omega}_\theta}^{1-P}\frac{F^\omega(d\omega)}{(1-\omega)R}. \tag{44}$$

Then, the designer in unable to successfully dissuade short-term creditors from running on the bank with positive probability. In other words, $\pi(0|\cdot)=\mathbf{0}$. To see this, rewrite the inequality 44 as:

$$\int_0^{\hat{\omega}_\theta}\frac{|b|\times F^\omega(d\omega)}{(PR+z_\theta)} - \frac{g\times(1-F^\omega(1-P))}{PR+z_\theta} \geq g\times\int_{\hat{\omega}_\theta}^{1-P}\frac{f^\omega(\omega)}{(1-\omega)R}d\omega,$$

or equivalently,

$$\mathbb{E}\left(u(\omega,P,1)|0\right) = \mathbb{E}_{\mu_0}\left(\int_0^{\hat{\omega}_\theta}\frac{b\times F^\omega(d\omega)}{(PR+z_\theta)} + \int_{\hat{\omega}_\theta}^{1-P}\frac{g\times F^\omega(d\omega)}{(1-\omega)R} + \frac{g\times(1-F^\omega(1-P))}{PR+z_\theta}\right) \leq 0.$$

That is, short-term creditors obtain a negative payoff if they pledge to the bank (and the rest does not), even if the designer were to offer enough funds so that every bank with $\omega>\hat{\omega}_\theta$ survives the liquidity shortage caused by all short-term creditors refraining from rolling over the bank's debt. As a result, under the most adversarial equilibrium all short-term creditors run on the bank. The policy-maker thus cannot engage in disclosing informative messages about the bank's liquidity position, and may only try to increase the likelihood of the bank's survival by purchasing claims on its asset. The optimal strategy for the policy-maker consists of purchasing the totality of the remaining claims on the asset at the largest price allowed by *fair price* constraint. Thus, the government purchases $y-s^\star$ at price $t^\emptyset$ defined by:

$$t^\emptyset \equiv \sup\left\{\tau\leq B : \frac{\sum_\theta \mu_\theta \mathbb{E}_\theta(y-s^\star)}{R}\times\mathbb{P}\{\omega+P+\tau\geq 1\}\geq\tau\right\}.$$

□

Proof of Lemma 7

First, I claim that for any arbitrary mechanism $\Upsilon^{\omega,\theta}$, we have that $\Upsilon_{\text{OAQ}}[\theta_L]\succeq_{\text{PM}}\Upsilon^{\omega,\theta}$. To see this, suppose we relax constraint (24) and assume instead that:

$$\pi^s \leq \frac{V_L}{PR+z_L}. \tag{45}$$

Clearly, the optimal screening mechanism of the relaxed problem weakly dominates $\Upsilon^{\omega,\theta}$, which satisfies the original constraint (24). Moreover, constraint (23) requires that:

$$t(\omega,\theta_H) \leq (1-\phi_H(\omega))\times\frac{z_L}{R}\leq\frac{z_L}{R},\ \ \forall\omega.$$

As a consequence, the policy-maker may not pledge more than the expected discounted value of the asset for a type-L bank. This, in turn, implies that there is no benefit associated with telling apart type-H banks from type-L ones. As a result, the optimal mechanism of the relaxed problem sets $\phi_H = 0$. The optimal mechanism of the relaxed problem then is given by $\Upsilon_{\text{OAK}}[\theta_L]$:

$$t(\omega, \theta_H) = t(\omega, \theta_L) = t_{\text{OAQ}}(\omega; \theta_L), \;\; \pi(\omega, \theta_H) = \pi(\omega, \theta_L) = \pi_{\text{OAQ}}(\omega; \theta_L) \quad \forall \omega \in \Omega.$$

Finally, the conclusion obtains from the fact that $\Upsilon_{\text{OAQ}}\left[\mathbb{E}_{\mu_0}(\theta)\right] \succeq_{\text{PM}} \Upsilon_{\text{OAQ}}[\theta_L]$. That is, the optimal mechanism when the bank has, with certainty, the average type $\mathbb{E}_{\mu_0}(\theta)$ (or alternatively, the optimal mechanism when the bank does not have private information), dominates the optimal mechanism that emerge when the bank possesses pessimistic information about its asset, and this information is observed by the policy-maker.□

Proof of Theorem 2

Proof. Fix a message m^y disclosed with positive probability under Γ^y, and assume that the bank successfully raises P units of capital after the asset quality review Γ^y discloses m^y. Let $\pi_{\text{OAQ}}\left[\mathbb{E}_{\mu_0}(\theta)\right]$, $t_{\text{OAQ}}\left[\mathbb{E}_{\mu_0}(\theta)\right]$ be the disclosure and pricing policy associated with the screening mechanism $\Upsilon^{\omega,\theta}_{\text{OAQ}}\left[\mathbb{E}_{\mu_0}(\theta)\right]$, the optimal screening mechanism under the alternative setting wherein the bank does not possess private information regarding the quality of its asset. That is, $\Upsilon^{\omega,\theta}_{\text{OAQ}}\left[\mathbb{E}_{\mu_0}(\theta)\right]$ corresponds to the optimal screening mechanism characterized in proposition (6), when the bank has a unique, average, asset quality type $\mathbb{E}_{\mu_0}(\theta)$. This implies that there exist constants $\bar{V}$ and $\bar{\pi}^s$ so that the following constraints are satisfied:

$$\begin{aligned}
&(i) \quad \int_0^{1-P} \left((b-g) \cdot 1_{\left\{P + t_{\text{OAQ}}(\omega; \mathbb{E}_{\mu_0}(\theta)) + \omega < 1\right\}} + g \right) \pi_{\text{OAQ}}\left(0|\omega; \mathbb{E}_{\mu_0}(\theta)\right) F^{\omega}(d\omega) + \\
&\qquad + \bar{\pi}^s \times g \times \left(1 - F^{\omega}(1-P)\right) \geq 0 \\
&(ii) \quad \left(\int_0^{1-P} |b| \times \pi_{\text{OAQ}}\left(0|\omega; \mathbb{E}_{\mu_0}(\theta)\right)(d\omega) \right) - g \times \bar{\pi}^s \times \left(1 - F^{\omega}(1-P)\right) \leq |\bar{U}_{\text{LF}}(P)| \\
&(iii) \quad \pi_{\text{OAQ}}\left(0|\omega; \mathbb{E}_{\mu_0}(\theta)\right) \times \left(\left(P + t_{\text{OAQ}}\left(\omega; \mathbb{E}_{\mu_0}(\theta)\right)\right) R \right) = \bar{V}, \; \forall \omega \leq 1-P \\
&(iv) \quad \bar{\pi}^s \leq \frac{\bar{V}}{PR + \bar{z}} \\
&(v) \quad t_{\text{OAQ}}\left(\omega; \mathbb{E}_{\mu_0}(\theta)\right) \leq \frac{\bar{z}}{R}, \;\; \forall \omega.
\end{aligned}$$

That the securities purchased by the government are not penalized with a premium to compensate for rollover risk follows from the fact that the policy-maker only purchases when assigning the passing grade, in which case the probability that the bank fails equals 0. The proof shows that even if we assume that long-term investors pay the default-free price of the claims during the first period, the designer still prefers to minimize the claims that are sold to the asset market at $t = 1$. I show that when $\mathbb{E}(y|m^y)$ is high enough so that elicitation is in fact possible, the policy-maker prefers to minimize the amount raised by the bank during the fund-raising game in order to increase the value of $\bar{z}$, which provides her with more elicitation capacity during the second period. Assume that $\mathbb{E}(y|m^y) \geq \underline{\text{E}}$. This means, as it will become clear below, that elicitation of information about ω is possible. I characterize the optimal recapitalization and subsequent screening mechanism that follows the disclosure m^y.

Note that although $\Upsilon^{\omega,\theta}_{\text{OAQ}}[\mathbb{E}_{\mu_0}(\theta)]$ satisfies (i)-(v), it does not respect incentive compatibility under the original setting. In fact, $\Upsilon^{\omega,\theta}_{\text{OAQ}}[\mathbb{E}_{\mu_0}(\theta)]$ fails to satisfy constraint (22). That is, under this alternative mechanism safe banks (i.e., those with $\omega > 1-P$), but with a low quality asset, have incentives to claim to be illiquid and receive a price for its asset above its fair value. I show that under the optimal persuasion mechanism, $P^* = 0$ whenever $\underline{\mathrm{E}} < \mathbb{E}(y) < K$. That is, the policy-maker minimizes the recapitalization rule in order to boost her elicitation capacity during the second period.

Claim 1: $\mathbb{E}(y|m^y) \geq \underline{\mathrm{E}}$ implies that the set of potential policies satisfying (i)-(v) is non-empty.

By definition of $\underline{\mathrm{E}}$, when $\mathbb{E}(y|m^y) \geq \underline{\mathrm{E}}$ there exist transfers $t(\cdot)$ and probability π_s so that (i) holds. Moreover, there always exist policies satisfying constraint (ii)-(iv), which can be seen by choosing V small enough, and then choosing $\pi(0|\cdot)$ consistently. $\square$

Following proposition (6), the optimal screening mechanism in the absence of bank's private information about asset quality, $\Upsilon^{\omega,\theta}_{\text{OAQ}}\left[\mathbb{E}_{\mu_0}(\theta)\right]$, can be characterized as a function of P and $\bar{z}$ as follows:

$$(t_{\text{OAQ}}(\omega), \pi_{\text{OAQ}}(0|\omega)) = \begin{cases} \frac{\mathbb{E}(y|m^y)}{R},\ \frac{\bar{V}}{\mathbb{E}(y|m^y)} & \omega < \hat{\omega} \\ 1-P-\omega,\ \frac{\bar{V}}{(1-\omega)R} & \omega \in [\hat{\omega}, \breve{\omega}] \\ 1-P-\breve{\omega},\ \frac{\bar{V}}{(1-\breve{\omega})R} & \omega \in (\breve{\omega}, 1-P) \\ 0,\ \frac{\bar{V}}{\mathbb{E}(y|m^y)} & \omega \geq 1-P \end{cases}$$

with $\breve{\omega}$ and $\bar{V}$ are chosen so that:

$$\int_{1-\frac{\mathbb{E}(y|m^y)}{R}}^{\breve{\omega}} \frac{F^{\omega}(d\omega)}{(1-\omega)R} + \frac{F^{\omega}(1-P) - F^{\omega}(\breve{\omega})}{(1-\breve{\omega})R} = \int_0^{\frac{\mathbb{E}(y|m^y)}{R}} \frac{|b|\,F^{\omega}(d\omega)}{g \times \mathbb{E}(y|m^y)} - \frac{(1-F^{\omega}(1-P))}{\mathbb{E}(y|m^y)}. \tag{46}$$

$$\bar{V} \equiv \min\left\{(1-\breve{\omega})R, \frac{|\bar{U}_{\text{LF}}(P)|}{\int_0^{1-P} \frac{|b|F(d\omega)}{(P+t_{\text{OAQ}}(\omega))R} - g \times \frac{(1-F^{\omega}(1-P))}{\mathbb{E}(y|m^y)}}\right\}. \tag{47}$$

Claim 2: $\bar{V} = (1-\breve{\omega})R$.

To see this, note that constraint (ii) is satisfied with strict inequality. In fact, that $\bar{U}_{\text{LF}}(P) < 0$ for all

$P < \frac{\mathbb{E}(y|m^y)}{R}$ implies that

$$\begin{aligned}
& \int_0^{1-P} b \times \left(1-\pi_{\text{OAQ}}\left(0|\omega;\mathbb{E}_{\mu_0}(\theta)\right)\right) F^{\omega}(d\omega) + g \times (1-\bar{\pi}^s)\left(1-F^{\omega}(1-P)\right) \\
= & \int_0^{1-\frac{\mathbb{E}(y|m^y)}{R}} \left(1-\frac{\bar{V}}{\mathbb{E}(y|m^y)}\right) bF(d\omega) + \int_{1-\frac{\mathbb{E}(y|m^y)}{R}}^{\breve{\omega}} \left(1-\frac{\bar{V}}{(1-\omega)R}\right) bF^{\omega}(d\omega) \\
& + \int_{\breve{\omega}}^{1-P} \left(1-\frac{\bar{V}}{(1-\breve{\omega})R}\right) bF^{\omega}(d\omega) + g \times \left(1-\frac{\bar{V}}{\mathbb{E}(y|m^y)}\right)\left(1-F^{\omega}(1-P)\right) \\
< & \left(1-\frac{\bar{V}}{\mathbb{E}(y|m^y)}\right) \times \left(\int_0^{1-\frac{\mathbb{E}(y|m^y)}{R}} b \times F(d\omega) + g \times \left(1-F^{\omega}\left(1-\frac{\mathbb{E}(y|m^y)}{R}\right)\right)\right) \\
= & \left(1-\frac{\bar{V}}{\mathbb{E}(y|m^y)}\right) \times \bar{U}_{\text{LF}}\left(\frac{\mathbb{E}(y|m^y)}{R}\right) \\
\leq & \; 0.
\end{aligned}$$

As a consequence, the constraint defining the value of $\bar{V}$ is (iii). The result follows from the implicit restriction that $\pi_{\text{OAQ}}\left[\mathbb{E}_{\mu_0}(\theta)\right]$ is a probability measure:

$$(vi)\pi_{\text{OAQ}}\left(0|\omega;\mathbb{E}_{\mu_0}(\theta)\right) = \frac{\bar{V}}{\left(P+t_{\text{OAQ}}\left(\omega;\mathbb{E}_{\mu_0}(\theta)\right)\right)R} \leq 1 \;\; \forall \omega \leq 1-P.$$

Thus, at the optimum:

$$\bar{V} = \inf\left\{\left(P+t_{\text{OAQ}}\left(\omega;\mathbb{E}_{\mu_0}(\theta)\right)\right)R : \omega \leq 1-P\right\} = (1-\breve{\omega})R.$$

□

Next, I show that the designer can improve her payoff by decreasing P, and increasing $\bar{z}$ accordingly, so that $PR+\bar{z} \leq \mathbb{E}(y|m^y)$.

Claim 3: The optimal persuasion mechanism sets either $P=0$, or $P=\bar{P}(\mathbb{E}(y|m^y))$ (i.e., optimal interventions either involves the government, or the private sector, but not both) for any $\lambda \in [0,1]$.

To see this, assume that $\lambda = 1$, so that the liquidity shock is a 0-probability event. This assumption exacerbates the incentives to let long-term investors (the private sector) purchase securities from the bank during the fund-raising game at $t=1$ as the bank avoids discounts (haircuts) on its asset to compensate for default risk.

Consider the following function

$$\begin{aligned}
\varphi^{+}(P,\breve{\omega}) \equiv & \int_0^{1-P} \left((b-g)\cdot 1_{\left\{P+t_{\text{OAQ}}(\omega;\mathbb{E}_{\mu_0}(\theta))+\omega<1\right\}} + g\right) \pi_{\text{OAQ}}\left(0|\omega;\mathbb{E}_{\mu_0}(\theta)\right) F^{\omega}(d\omega) + \\
& + \bar{\pi}^s \times g \times \left(1-F^{\omega}(1-P)\right) \\
= & \frac{\int_0^{1-\frac{\mathbb{E}(y|m^y)}{R}} b\left(\frac{\mathbb{E}(y|m^y)}{R},1\right)}{\mathbb{E}(y|m^y)} F(d\omega) + \\
& + g \times \left(\int_{1-\frac{\mathbb{E}(y|m^y)}{R}}^{\breve{\omega}} \frac{F^{\omega}(d\omega)}{(1-\omega)R} + \frac{F^{\omega}(1-P)-F^{\omega}(\breve{\omega})}{(1-\breve{\omega})R} + \frac{\left(1-F^{\omega}(1-P)\right)}{\mathbb{E}(y|m^y)}\right).
\end{aligned}$$

φ^{+} corresponds to the expected payoff of a short-term creditor, at the optimal elicitation mechanism, under message $m^{\omega,\theta}=0$ (pass). Function φ^{+} decreases with P (or equivalently, increases with $\bar{z}$) if we keep the rest of variables (other than $\bar{z}$) constant, since $(1-\breve{\omega})R<\mathbb{E}(y|m^{y})$. The case in which $(1-\breve{\omega})R=\mathbb{E}(y|m^{y})$ corresponds to the situation in which the policy-maker can avoid default altogether (with certainty) and thus is not considered here. This implies that (i) is relaxed when we decrease the value of P, or equivalently, when we increase the value $\bar{z}$. Decreasing P (and therefore increasing $\bar{z}$) also relaxes (ii) and (v), and does not affect neither (iii), nor (iv). To see the first point, consider the following function:

$$\begin{aligned}\varphi^{-}(P,\breve{\omega},\bar{V}) &\equiv \int_{0}^{1-P} b\times\left(1-\pi_{\text{OAQ}}\left(0|\omega;\mathbb{E}_{\mu_0}(\theta)\right)\right)F^{\omega}(d\omega)+\\ &\quad +g\times\left(1-F^{\omega}(1-P)\right)\times\left(1-\bar{\pi}^{s}\right)\\ &= \int_{0}^{1-\frac{\mathbb{E}(y|m^{y})}{R}} b\times\left(1-\frac{\bar{V}}{\mathbb{E}(y|m^{y})}\right)F(d\omega)+\int_{1-\frac{\mathbb{E}(y|m^{y})}{R}}^{\breve{\omega}} b\times\left(1-\frac{\bar{V}}{(1-\omega)R}\right)F^{\omega}(d\omega)\\ &\quad +\left(\int_{\breve{\omega}}^{1-P} b\times\left(1-\frac{\bar{V}}{(1-\breve{\omega})R}\right)F^{\omega}(d\omega)+g\times\left(1-F^{\omega}(1-P)\right)\times\left(1-\frac{\bar{V}}{\mathbb{E}(y|m^{y})}\right)\right).\end{aligned}$$

φ^{-} corresponds to the expected payoff of short-term creditors, at the optimal elicitation mechanism, under message $m^{\omega,\theta}=1$ (fail). Function φ^{-} increases with P if we keep the rest of variables (other than z) constant. As a result, reducing P relaxes constraint (ii). Finally to see that (iii) is not affected by reductions of P, observe that for every reduction of P in the amount of Δ, the maximal price that may be pledged by the policy-maker (determined by constraint (v)) increases by Δ. Thus, the policy-maker can replicate the effect of P by increasing the price paid by the securities, t_{OAQ}, in the same amount.

Next, define $\breve{\omega}(P)$ as the optimal cutoff associated with any price $P\in\left[0,\frac{E}{R}\right]$, as in (46). That is, $\breve{\omega}(P)$ is chosen so that $\varphi^{+}(P,\breve{\omega}(P))=0$. Consider the case where $P=0$. The optimal elicitation mechanism is then given by:

$$\left(t_{\text{OAQ}}^{P=0}(\omega),\pi_{\text{OAQ}}^{P=0}(0|\omega)\right)=\begin{cases}\frac{\mathbb{E}(y|m^{y})}{R},\ \frac{(1-\breve{\omega}(0))R}{\mathbb{E}(y|m^{y})} & \omega<1-\frac{\mathbb{E}(y|m^{y})}{R}\\ 1-\omega,\ \frac{1-\breve{\omega}(0)}{(1-\omega)} & \omega\in\left[1-\frac{\mathbb{E}(y|m^{y})}{R},\breve{\omega}(0)\right]\\ 1-\breve{\omega}(0),\ 1 & \omega\in(\breve{\omega}(0),1].\end{cases}$$

Choose any alternative policy in which the bank raises a price $\tilde{P}\in(0,1-\breve{\omega}(0))$ from long-term investors. That φ^{+} decreases with P implies that $\breve{\omega}\left(\tilde{P}\right)>\tilde{\omega}(0)$, since $\breve{\omega}\left(\tilde{P}\right)$ satisfies $\varphi^{+}\left(\tilde{P},\breve{\omega}\left(\tilde{P}\right)\right)=0$. This means that $\pi^{P=0}(0|\omega)>\pi^{\tilde{P}}(0|\omega)$ for all $\omega\leq\breve{\omega}\left(\tilde{P}\right)$, and $\pi^{P=0}(0|\omega)=\pi^{\tilde{P}}(0|\omega)=1$ for all $\omega>\breve{\omega}\left(\tilde{P}\right)$. As a result, the policy-maker's payoff is strictly greater at $P=0$. Finally, consider the case where $\tilde{P}\leq 1-\breve{\omega}(0)$. We note that:

$$\begin{aligned}\frac{\int_{0}^{1-\frac{\mathbb{E}(y|m^{y})}{R}} b\left(\frac{\mathbb{E}(y|m^{y})}{R},1\right)f^{\omega}(\omega)}{\mathbb{E}(y|m^{y})}d\omega+g\cdot\left(\int_{1-\frac{\mathbb{E}(y|m^{y})}{R}}^{1-P}\frac{f^{\omega}(\omega)}{(1-\omega)R}d\omega+\frac{(1-F^{\omega}(1-P))}{\mathbb{E}(y|m^{y})}\right) &< \varphi^{+}(0,\breve{\omega}(0))\\ &= 0,\end{aligned}$$

which means that the policy-maker is unable to convince short-term creditors to keep pledging to the bank, regardless of her chosen elicitation mechanism. Clearly, if best elicitation mechanism does not require long-term investors funding under $\lambda=1$, it won't require it for $\lambda<1$. Thus, the best liquidity provision program sets $P = 0$, which confirms that the optimal intervention will never involve the government and the private sector at the same time.$\square$

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