Stress Testing and Bank Lending*

Joel Shapiro† Jing Zeng‡

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Abstract

Stress tests can affect banks' lending behavior. Since regulators care about lending, banks' reactions affect the test's design and create a feedback loop. We demonstrate that there may be multiple equilibria due to strategic complementarity, possibly leading to excess default or insufficient lending to the real economy. The stress tests may be too soft or too tough. Banking supervision exams have similar properties. When the recapitalization of banks becomes more difficult, stress tests are less informative. However, when a bank is more systemic, the stress test will be more informative.

Keywords: Bank regulation, stress tests, bank lending, feedback effect

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†Saïd Business School, University of Oxford, Park End Street, Oxford OX1 1HP. Email: Joel.Shapiro@sbs.ox.ac.uk

‡Frankfurt School of Finance and Management, Adickesallee 32-35, 60322 Frankfurt am Main, Germany. University of Vienna, Oskar-Morgenstern-Platz 1, 1090 Wien, Austria. Email: Jing.Zeng@univie.ac.at.

1 Introduction

Stress tests, a new policy tool for bank regulators, were first used in the recent financial crisis and have become regular exercises since the crisis. They assess a bank's ability to withstand adverse shocks and are generally accompanied by requirements intended to boost the capital of banks that are found to be at risk.

Naturally, bank behavior changes in response to stress testing exercises. Acharya, Berger, and Roman (2018) find that all banks that underwent the U.S. SCAP and CCAR tests reduced their risk by raising loan spreads and decreasing their commercial real estate credit and credit card loan activity.[1]

Regulators must take banks' reactions into account when conducting the tests. One might posit that if regulators want to boost lending, they might make stress tests softer. Indeed, in the case of bank ratings, Agarwal et al. (2014) show that state-level banking regulators gave banks higher ratings than federal regulators (due to concerns over the local economy), which led to more bank failures.

In this paper, we study the *feedback* effect between stress testing and bank lending. Banks may take too much risk or not lend enough. Regulators anticipate this by designing a stress test that is either tough or soft. Nevertheless, the regulator may fail to maximize surplus (its objective) because the interaction between the bank and the regulator may be self-fulfilling and result in coordination failures, leading to either excess default or inefficiently low levels of lending to the real economy.

In the model, there are two sequential stress testing exercises. For simplicity, there is one bank that is tested in both exercises. Each period, the bank decides whether to originate a risky loan or to invest in a risk-free asset. The regulator can observe the quality of the risky loan and may require the bank to raise capital (which we refer to as "failing" the stress test). Therefore, stress tests in the model are about gathering information and taking actions based on that information (while considering the reaction of banks), rather than optimally choosing how to reveal information to the market.[2] This is in line with the annual exercises during non-crisis times, when runs are an

[1] Cortés et al. (2019), Berrospide and Edge (2019), Connolly (2017) and Calem, Correa, and Lee (2020) have similar findings.

[2] The theoretical literature mostly focuses on this latter point; we discuss the literature in the next section.

unlikely response to stress test results.

The regulator may be one of three types: lenient, strict, or strategic. The regulator knows its type, and all other agents are uncertain about it. Lenient- and strict-type regulators are behavioral; the lenient type always passes banks and the strict type always fails them. A strategic regulator maximizes surplus. Its decision to fail a bank depends on the trade-off between the cost of forgone credit and the benefit of reducing costly default. After the first stress test result, the bank updates its beliefs about the regulator's type, decides whether to make a risky loan, and undergoes a second stress test. Thus, the regulator's first stress test serves two purposes: to possibly boost capital for the bank in the first period *and* to signal the regulator's willingness to force the bank to raise capital in the second period.

Banks may take too much or too little risk from the regulator's point of view. On the one hand, the bank may take too much risk due to its limited downside. On the other hand, the bank's owners may take too little risk to avoid being diluted by a capital raising requirement.[3] The bank's anticipated choice affects the toughness of the stress test, and the toughness of the stress test affects the bank's choice.

The regulator faces a natural trade-off in conducting the first stress test:

First, the strategic regulator may want to build a reputation for being lenient, to try to increase the bank's lending in the second period. Since the lenient regulator does not require banks to raise capital, there is a "soft" equilibrium in which the strategic regulator builds the perception that it is lenient by passing a bank that should fail. This is reminiscent of the EU's 2016 stress test, which eliminated the pass/fail grading scheme, found only one bank to be undercapitalized, and allowed one bank to get around one of the rules of the test.[4]

Second, the strategic regulator may want to build a reputation for being strict, which can prevent future excess risk-taking. This leads to a "tough" equilibrium in which the regulator builds the reputation that it is strict by failing a bank that should pass. The U.S. has routinely been

[3]Thakor (1996) provides evidence that the adoption of risk-based capital requirements under Basel I and the passage of FDICIA in 1991 led to banks substituting risky lending with Treasury investments, potentially prolonging the economic downturn.

[4]The bank found to be undercapitalized, Monte dei Paschi di Siena, had already failed the 2014 stress test and was well known by the market to be in distress. It was also revealed that Deutsche Bank was given an exception to the stress test rules, resulting in it appearing to have more capital ("Deutsche Bank received special treatment in EU stress tests," by Laura Noonan, Caroline Binham and James Shotter, *Financial Times*, October 10, 2016). We note that the 2018 test also had no pass/fail requirement and all banks tested were judged to be well capitalized.

criticized for being too tough: imposing very adverse scenarios, not providing the stress testing model to banks, accompanying the test with asset quality reviews, and conducting qualitative reviews all combine to create a stringent test.[5]

Finally, there is one more type of equilibrium - one in which the regulator doesn't engage in reputation building and rates the bank in accordance with the bank's quality.

There may be multiple equilibria that coexist, leading to a natural coordination failure. This occurs due to a subtle strategic complementarity between the strategic regulator's choice of toughness in the first stress test and the bank's second-period risk choice. In the case where the strategic regulator prefers less risk, the less likely the strategic regulator is to pass the bank in the first period, the more risk the bank takes in the second period when it observes a pass as it believes the regulator is more likely to be lenient. This prompts the strategic regulator to be even tougher (as it prefers less risk), and leads to a self-fulfilling equilibrium. In the case where the strategic regulator prefers more lending, the more likely the strategic regulator is to pass the bank in the first period, the less risk the bank takes in the second period when it observes a fail as it believes the regulator is more likely to be strict. This prompts the strategic regulator to be even softer (as it prefers more lending), and leads to a self-fulfilling equilibrium.

This implies that the presence of stress tests may result in too many defaults or suboptimal levels of lending to the real economy. We formally demonstrate that when multiple equilibria exist, the equilibrium with reputation building (tough in the first case above, soft in the second case) has lower surplus than the informative equilibrium it coexists with.

We show that when recapitalization becomes more difficult, stress tests are less informative. Recapitalization may become more difficult because of either the scarcity of capital or lucrative alternative uses for capital. In this situation, passing a bad bank or failing a good bank is less costly since the possibility of recapitalization is lower in any case.

When the bank is more systemic, stress tests are more informative, thus indicating that regulators tailor stress tests to bank size and linkages. Regulators frequently debate and revise criteria for deciding which banks should be included in stress tests.

The multiplicity of equilibria naturally raises the issue of how a particular equilibrium may be

[5] A discussion of this and the recent tilt towards leniency is in "Banks rest hopes for lighter regulatory burden on Fed's Quarles," by Pete Schroeder and Michelle Price, *Reuters*, October 25, 2017.

chosen. One way might be if regulators could commit, ex-ante, to a way to use the information that they collect. We analyze a version of model allowing for commitment. Indeed, there is no multiplicity of equilibria, but the regulator may still be soft or tough in order to induce more or less risk-taking. In practice, commitment is difficult to accomplish, as regulators and/or pressures on regulators may change. Banking regulators do attempt to restrict their choices by announcing stress test scenarios in advance and undertaking costly audits of bank data (e.g., asset quality reviews).

In stress testing exercises, by examining the banking system, a regulator may uncover information about liquidity and systemic linkages that individual banks may be unaware of. In our model, this is the motivation for why the regulator has private information about bank risk. However, we also analyze the case in which the regulator uncovers only the information about quality of the bank's assets that the bank already knows. This is similar to banking supervision exams or "bottom-up" stress tests in which the regulator allows the bank to perform the test (as in Europe). We find that these exams have qualitatively similar properties to stress tests where the regulator has private information, including the presence of multiple equilibria.

In the model, uncertainty about the regulator's type plays a key role. Given that (i) increased lending may come with risk to the economy, and (ii) bank distress may have systemic consequences, there is ample motivation to keep this information/intention private. This uncertainty may also arise from the political process. Decision-making may be opaque, bureaucratic, or tied up in legislative bargaining.[6] Meanwhile, governments with a mandate to stimulate the economy may respond to lobbying by various interest groups or upcoming elections.[7]

There is little direct evidence, but much indirect evidence, of regulators behaving strategically during disclosure exercises. The variance in stress test results to date seem to support the idea of regulatory discretion.[8] Beyond Agarwal et al. (2014), cited above, Bird et al. (2020) show that U.S. stress tests were soft towards large banks and tough with poorly capitalized banks, affecting bank equity issuance and payout policy. The recent Libor scandal revealed that Paul Tucker, deputy governor of the Bank of England, made a statement to Barclays' CEO that was

[6]Shapiro and Skeie (2015) provide examples of related uncertainty around bailouts during the financial crisis.

[7]Thakor (2014) discusses the political economy of banking.

[8]The 2009 U.S. SCAP was widely perceived as a success (Goldstein and Sapra, 2014), with subsequent U.S. tests retaining credibility. European stress tests have varied in perceived quality (Schuermann, 2014), with the early versions so unsuccessful that Ireland and Spain hired independent private firms to conduct stress tests on their banks.

interpreted as a suggestion that the bank lower its Libor submissions.[9] Hoshi and Kashyap (2010) and Skinner (2008) discuss accounting rule changes that the government of Japan used to improve the appearance of its financial institutions during the country's crisis.[10]

The goal of "stress tests [is to] help ensure that banks will have enough capital to keep lending even under highly adverse circumstances" (Bernanke, 2013). Our model thus focuses on the effect of stress tests on bank lending. Another tool regulators have to affect bank lending is monetary policy, through the bank lending channel. Nevertheless, there are drawbacks to this approach. First, as monetary policy applies uniformly to banks, it may foster moral hazard and excess risk taking (e.g., Farhi and Tirole (2012) and Jiminez, Ongena, Peydro, and Saurina (2014)). Second, monetary policy may not be effective at incentivizing lending (e.g. Ashcraft (2006) and Peydro, Polo, and Sette (forthcoming)).

Theoretical Literature

Our paper identifies the regulator's reputation concern as a source of feedback effects in the banking sector. In a different context, Ordoñez (2013, 2018) shows that banks' reputation concerns, which provide discipline to keep banks from taking excessive risk, can lead to fragility and a crisis of confidence in the market.[11] Other theories have predicted self-fulfilling banking lending freezes due to interdependence of banks' lending opportunities (Bebchuk and Goldstein, 2011) and fear of future fire sales (Diamond and Rajan, 2011).

There are a few papers on reputation management by a regulator. Morrison and White (2013) argue that a regulator may choose to forbear when it knows that a bank is in distress because liquidating the bank may give the regulator the reputation of being unable to screen, triggering contagion in the banking system.[12] Boot and Thakor (1993) also find that bank closure policy may be inefficient due to reputation management by the regulator, but this is due to the regulator being self-interested rather than being worried about social welfare consequences, as in Morrison

[9]The CEO of Barclays wrote notes at the time on his conversation with Tucker, who reportedly said, "It did not always need to be the case that [Barclays] appeared as high as [Barclays has] recently." This quote and a report on what happened appeared in the *Financial Times* (B. Masters, G. Parker, and K. Burgess, Diamond Lets Loose Over Libor, *Financial Times*, July 3, 2012).

[10]Nevertheless, stress tests do contain significant information that is valued by markets (Flannery, Hirtle, and Kovner (2017) demonstrate this and survey recent evidence).

[11]Piccolo and Shapiro (2020) show that a credit rating agency's reputation concern creates a feedback effect with the CDS and bond markets.

[12]Morrison and White (2005) also model reputation as the ability of the regulator to screen, but do not consider the effect of ex-post learning about the type of the regulator.

and White (2013). Shapiro and Skeie (2015) show that a regulator may use bailouts to stave off depositor runs and forbearance to stave off excess risk-taking by banks. Our paper uses the reputation management modelling framework and derives a feedback effect not present in these papers; the strategic complementarity between the regulator's stress test and the bank's lending decision leads to multiple equilibria.

There are several recent theoretical papers on stress tests.[13] Quigley and Walther (2020) show that more disclosure by a bank regulator decreases the amount of information that banks provide to the public, and that the regulator may take advantage of this to stop runs. Bouvard, Chaigneau, and de Motta (2015) show that transparency is better in bad times and opacity is better in good times. Goldstein and Leitner (2018) find a similar result in a very different model, in which the regulator is concerned about risk sharing (the Hirshleifer effect) between banks. Williams (2017) looks at bank portfolio choice and liquidity in this context. Orlov, Zryumov, and Skrzypacz (2018) show that the optimal stress test will test banks sequentially. Faria-e-Castro, Martinez, and Philippon (2017) demonstrate that stress tests will be more informative when the regulator has a strong fiscal position (to stop runs). In contrast to these papers, in our model, *reputational incentives* drive the regulator's choices, not commitment to a disclosure rule.[14] In addition, we focus on capital requirements and banks' endogenous choice of risk as key elements of stress testing; the papers listed above focus on information revelation to prevent bank runs.[15]

2 The model

We consider a model with three risk-neutral agents: the regulator, the bank and a capital provider. The model has two periods $t \in \{1, 2\}$ and the regulator conducts a stress test for the bank in each period. We assume that the regulator has a discount factor $\delta \geq 0$ for the payoffs from the second period, where δ may be larger than 1 (as, e.g., in Laffont and Tirole, 1993). The discount

[13] There are a few papers on regulatory disclosure. Goldstein and Sapra (2014) survey the disclosure literature to describe the costs and benefits of information provision for stress testing. Prescott (2008) argues that more information disclosure by a bank regulator decreases the amount of information that banks provide to the regulator. Bond, Goldstein, and Prescott (2010) and Bond and Goldstein (2015) analyze government interventions that rely on and endogenously determine market information.

[14] Quigley and Walther (2018) and Bouvard, Chaigneau, and de Motta (2015) do not have commitment or reputation.

[15] In Dogra and Rhee (2018), the regulator commits to a disclosure rule but banks may choose their risk profile to satisfy the stress testing regime, leading to 'model monoculture'.

factor captures the relative importance of the future of the banking sector for the regulator. For simplicity, we do not allow for discounting within a period.

We now provide a very basic timeline of each period. In each period t, where $t = \{1, 2\}$, there are three stages:

1. Bank investment choice;
2. Stress test and (possible) recapitalization;
3. Payoffs realize.

We proceed in the following subsections to discuss each aspect in detail: the bank, the stress test, recapitalization, the preferences of the regulator, and the regulator's reputation.

2.1 The bank

At stage 1, the bank raises one unit of fully insured deposits, which mature at stage 3.[16] The bank can choose between two possible investments. The first is a safe asset that returns $R_0 > 1$ at stage 3. The second is a risky loan, whose quality q_t can be good (g) or bad (b). The prior probability that the loan is good is denoted by α. A good loan ($q_t = g$) repays R with probability 1 at stage 3, whereas a bad loan ($q_t = b$) repays R with probability $1 - d$ and 0 otherwise at stage 3. We assume that the expected return of the risky loan is higher than that of the safe investment, representing the risk-return trade-off:

Assumption 1. $[\alpha + (1-\alpha)(1-d)]\, R > R_0$.

At stage 3, the bank uses the payoff of its investment to repay the deposits, and pays out the residual profit (if there is any) to its owners as dividends.

In order to focus on the regulator's reputation building incentives when conducting the stress test in the first period, we make the simplifying assumption that in the first period, the bank has extended the risky loan.[17]

[16] In an earlier version of this paper, we remove the assumption that deposits are fully insured and allow the bank's liabilities to be priced by the market. All of our qualitative results remain. The version with deposit insurance is presented for simplicity. The results with no deposit insurance are available upon request.

[17] Allowing endogenous loan origination effort in the first period does not alter the reputation-building incentives we demonstrate in Section 4.

2.2 Stress testing

We assume that only the regulator learns the credit quality of the risky loan (through the stress test). In Section 7, we demonstrate that the main results do not change if the bank also knows this information. The regulator could have generated private information from having done a stress test on many banks. In this case, it may have gathered more information on asset values and liquidity than individual banks had. Given this, the regulator may understand more about systemic risk and tail risk (not modeled here). This is an element of the macroprudential role of stress tests.

At stage 2, the regulator conducts the stress test. It first observes the quality q_t of the bank's risky loan and then decides whether to require the bank to raise capital. We will henceforth refer to the regulatory action of requiring the bank to raise capital as "failing", and not requiring the bank to raise capital as "passing."[18]

The stress test in the model, therefore, is not primarily about conveying information to the market about the health of the bank. The test provides the regulator with information on the bank's health, which the regulator uses in its decision to recapitalize the bank. The stress test accompanied by the recapitalization does convey additional information;[19] this information is about the type of the regulator, which is private information (this is defined below). In the second period, the bank reacts to this information inferred from the first-period stress test, forming the basis of the reputation mechanism.

2.3 Recapitalization

If a bank fails the stress test, we assume that the bank is required to raise one unit of capital, kept in costless storage with zero net return, so that the bank with the risky loan will not default at stage 3 even if its borrower does not repay.[20] There is a capital provider who can fund the bank. We assume that the capital provider's outside option for its funding is an alternative investment that produces a total return of $\rho > 1$, which we call the opportunity cost of capital. The opportunity cost of capital is high ($\rho = \overline{\rho}$) with probability γ, and low ($\rho = \underline{\rho}$) with probability $1 - \gamma$. We

[18] To be precise, a "fail" is an announced requirement for the bank to recapitalize. We will allow for recapitalizations to be attempted but not to work out, which we still consider a fail.

[19] The stress test results themselves are cheap talk in the model, but the recapitalizations incur costs (and benefits) for the regulator, making signaling possible.

[20] For simplicity, we assume that capital earns zero net return. The results do not change if capital is reinvested in the safe investment with a return $R_0 > 1$.

assume that the ρ is realized after the stress test, when the bank approaches the capital provider for funds, and is publicly observable.

We make the following assumption about the expected return on the risky loan:

Assumption 2. *$R < \overline{\rho}$ and $(1-d)R \geq \underline{\rho}$.*

This assumption implies that recapitalization is feasible only with probability $1-\gamma$, when the opportunity cost of capital is low, regardless of the risky loan's quality. First, if the opportunity cost of capital is high, then the expected value of a good loan is lower than the capital provider's outside option. This also implies that recapitalization is infeasible for the bad loan. Second, if the opportunity cost of capital is low, then the expected value of a bad loan is higher than the capital provider's outside option. This also implies that recapitalization is feasible for the good loan.

We assume that the capital provider has some bargaining power β due to the scarcity of capital, enabling it to capture a fraction of the expected surplus of the bank. Thus, raising capital results in a (private) dilution cost for the bank's owners. The banking literature generally views raising equity capital as costly for banks (for a discussion, see, e.g., Acharya, Mehran, and Thakor (2016)). We model this cost as dilution due to the bargaining power of a capital provider, which fits our scenario of a public requirement by a regulator, though other mechanisms that impose a cost on the bank when trying to shore up capital would also work.[21] We make the following assumption on the effect of recapitalization:

Assumption 3. *$\gamma\left[\alpha+(1-\alpha)(1-d)\right](R-1) < R_0 - 1$.*

This assumption looks at the decision of the bank at stage 1 of whether to choose the risky loan or the safe asset. The bank considers its expected payoff given its priors and expectations about the regulator's actions. The assumption implies that if the expected dilution from recapitalization was sufficiently large, the bank's owners will not find it worthwhile to originate the risky loan. More specifically, the left-hand side of the above expression takes into account that, if the bank originates a risky loan, the expected equity value of the bank is given by $\left[\alpha+(1-\alpha)(1-d)\right](R-1)$, which the bank's owners receive if recapitalization is infeasible (with probability γ). Otherwise, when

[21]For example, the bank may be forced to sell assets at fire-sale prices. This is a loss in value for the bank. And those who are purchasing the assets are distorting their investment decisions, as in our model. Hanson, Kashyap, and Stein (2011) discuss this effect and review the literature on fire sales.

recapitalization is feasible, the worst thing that could happen to the bank's owners is that they surrender all of their equity and have a payoff of zero.

2.4 Regulatory preferences

In the next section, we will formally define a regulator who is free to choose its stress testing strategy as strategic. In this section we focus on payoffs for the strategic regulator. The strategic regulator's objective function is to maximize social surplus. This includes the net payoff of the asset chosen by the bank and externalities from the bank's risky lending. We now detail these externalities.

There are two social costs of risky lending. The first is the cost to society of a bank default at stage 3. Specifically, if a bank operates without being recapitalized and the borrower repays 0 at stage 3, the bank defaults and a social cost to society D is incurred. The cost of bank default may represent the cost of financing the deposit insurance payout,[22] the loss of value from future intermediation that the bank may perform, the cost to resolve the bank, or the cost of contagion.

The second social cost of risky lending is the capital provider's opportunity cost: the alternative investment that goes unfunded when the capital provider recapitalizes the bank. This is incurred only if $\rho = \underline{\rho}$, as implied by Assumption 2.

We make the following assumption about the social costs of risky lending.

Assumption 4. $dD > \underline{\rho} - 1 > 0$.

This assumption states that a strategic regulator finds it beneficial to recapitalize a bank whose risky loan is known to be bad, but not a bank whose risky loan is known to be good.

Finally, we add one more potential externality, which we call the social benefit of risky lending: if the bank originates a risky loan at stage 1, it generates a positive externality equal to B. Broadly, increased credit is positively associated with economic growth and income for the poor (across both countries and U.S. states; see Demirgüç-Kunt and Levine (2018)).[23] Nevertheless, despite the ample evidence that U.S. stress tests reduced risky lending cited in the introduction, Cortés et al. (2019)

[22] The deposit insurance payout would be costly if (i) deposit insurance weren't fairly priced, or (ii) there were a cost (e.g., political) of using the deposit insurance fund.

[23] Garmaise and Moskowitz (2006) provide causal evidence of the social effects of credit allocation, such as reduced crime.

and Berrospide and Edge (2019) indicate stress tests do not change aggregate lending. This may imply that the benefit B is low.

2.5 Regulator reputation

The regulator can be one of three types: strategic, lenient, or strict.[24] The strategic type trades off the social benefits and costs associated with recapitalization when deciding whether to fail a bank. The lenient and the strict types are behavioral: The lenient type always passes the bank, and the strict type always fails the bank. The behavioral types can also be considered uninformative (or uninformed) types, as their test does not screen banks.[25]

The regulator knows its own type, but during the stress test in period t (where $t = \{1, 2\}$), the owners of the bank and the capital provider are uncertain about the regulator's type. These agents believe that, the regulator is lenient with probability z_t^L, strict with probability z_t^S, and strategic with probability $1 - z_t^L - z_t^S$. In our model, z_1^L and z_1^S are the probabilities with which nature chooses the regulator to be a lenient and a strict type, respectively. The pair (z_2^L, z_2^S) is the updated belief that the regulator is a lenient and a strict type, respectively, after the first-period stress test. Our model thus allows us to endogenize the regulator's incentives to build a reputation of leniency or strictness.

2.6 Summary of timing

The regulator conducts stress testing of the bank in the first period, and again in the second period if the bank has not defaulted in the first period. If the bank defaults in the first period, the bank is closed down and does not continue into the second period. At the beginning of the second period, the beliefs about the regulator's type are updated depending on the result of the bank's stress test and the realized payoff of the bank in the first period. The timing is illustrated in Figure 1.

We assume that the probability that the risky loan opportunity is good in the second period is independent of whether the risky loan opportunity is good in the first period, and that the type of

[24] Frenkel (2015) also considers a reputational setting with a strategic agent and two behavioral agents. However, his focus is on multiple audiences for rating agencies and hence his results are substantially different.

[25] The behavior of different types of the regulator can be microfounded in a model in which they have different net social benefits of risky lending. Specifically, if the lenient (strict) type regulator has a low (high) cost of bank default D, a high (low) social cost of capital ρ, and/or a large (small) benefit from risky lending B, then passing (failing) the bank with certainty is, indeed, the dominant strategy.

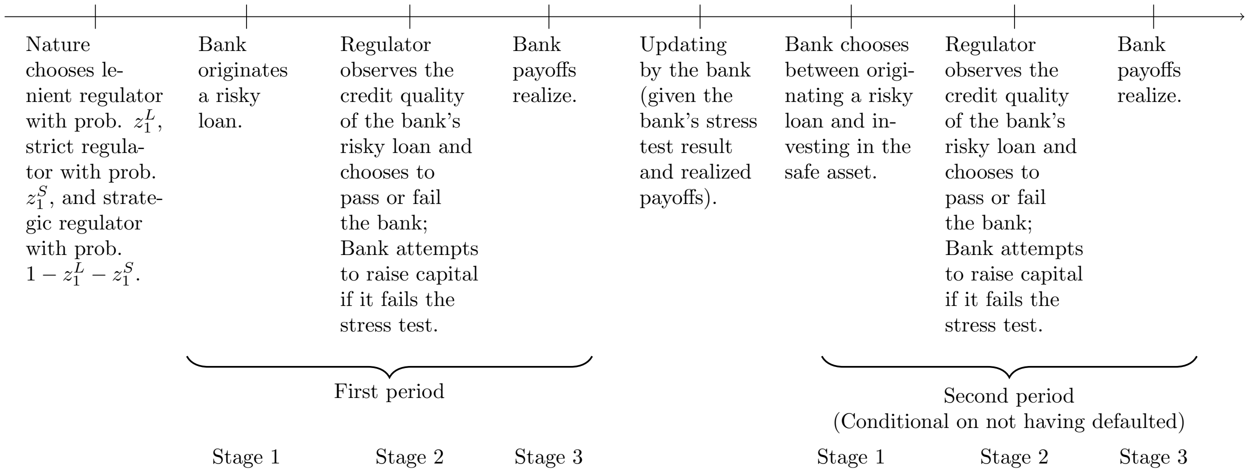


Figure 1: Timeline of events

the regulator is independent from the quality of the bank's risky loans. Furthermore, the regulator's type remains the same in both periods.

We use the equilibrium concept of Perfect Bayesian equilibrium to solve the game.

3 Stress testing in the second period

We begin the analysis of the model by using backward induction, and characterize the equilibrium in the second period. We first characterize the strategic regulator's stress test strategy at stage 2, taking as given the bank's investment decision at stage 1.

If the bank invests in the safe asset at stage 1, it is clear that the bank will not default and, therefore, requires no capital at stage 2. We will focus on describing the equilibrium stress test outcome given that the bank extends a risky loan at stage 1.

Since the game does not continue after the second period, the regulator has no reputational incentives. The stress test strategy of the strategic regulator at stage 2 depends on the quality of the bank's risky loan $q_2 \in \{g, b\}$. Specifically, the strategic regulator passes the bank if and only if the loan is good, as Assumption 4 implies.

The bank attempts to raise one unit of capital if it fails the stress test. The bank fails either because the regulator is strategic and the bank is bad, or because the regulator is the strict type. The total value of a good and bad bank's equity (including the capital provider's equity) post-recapitalization, is R and $(1-d)R$, respectively, taking into account that the one unit of capital raised will all be paid out to the depositors at maturity. The expected value of bank equity post-recapitalization is thus equal to:

$$\frac{\alpha z_2^S + (1-\alpha)(1-z_2^L)(1-d)}{\alpha z_2^S + (1-\alpha)(1-z_2^L)} R. \tag{1}$$

The capital provider's outside option is equal to the expected return on the forgone alternative investment, ρ. Assumption 2 implies that the total surplus is positive if and only if the opportunity cost of capital is low ($\rho = \underline{\rho}$).

If recapitalization is feasible, we define $1-\phi$ as the fraction of equity that the bank's owners retain. In order to determine this fraction, we now examine how the surplus is split between the capital provider and the bank's owners. When recapitalization is feasible, the capital provider's outside option is $\underline{\rho}$. We assume that the bank's outside option is 0, as the regulator compels the bank to be recapitalized. The total surplus is, therefore, equal to Eq. 1 minus $\underline{\rho}$. We use the Nash bargaining solution to define the split of the surplus, where the capital provider gets a fraction of the surplus determined by its bargaining power β. The transfer from the bank's owners to the capital provider is given by the right-hand side of Eq. 2 below, which is equal to the capital provider's outside option plus the fraction of surplus it obtains through bargaining power. Therefore, the equity given to the capital providers is a fraction ϕ, determined by:

$$\phi \frac{\alpha z_2^S + (1-\alpha)(1-z_2^L)(1-d)}{\alpha z_2^S + (1-\alpha)(1-z_2^L)} R = \underline{\rho} + \beta \left[\frac{\alpha z_2^S + (1-\alpha)(1-z_2^L)(1-d)}{\alpha z_2^S + (1-\alpha)(1-z_2^L)} R - \underline{\rho} \right]. \tag{2}$$

We can now analyze the bank's investment decision at stage 1. At stage 1, the bank anticipates the fraction of equity ϕ it will need to sell to the capital provider in exchange for capital. The bank originates a risky loan if and only if:

$$\begin{aligned} &\underbrace{\left(\alpha \left[(1-z_2^S) + z_2^S \gamma\right] + (1-\alpha)\left[z_2^L + (1-z_2^L)\gamma\right](1-d)\right)(R-1)}_{\text{pass, or fail but recapitalization infeasible}} \\ &\qquad + \underbrace{(1-\gamma)(1-\phi)\left[\alpha z_2^S + (1-\alpha)(1-z_2^L)(1-d)\right] R}_{\text{fail and recapitalized}} \geq R_0 - 1. \end{aligned} \tag{3}$$

The bank originates a risky loan if and only if the expected payoff to the bank's owners is higher when it originates a risky loan (represented by the left-hand side of Eq. 3) than when it invests in the safe investment (represented by the right-hand side of Eq. 3). Notice that the expected payoff to the bank's owners when it originates a risky loan consists of two terms. First, if the bank passes or is unable to raise capital after a fail and there is no default, it receives the net payoff $R-1$ at stage 4. Second, if the bank fails the stress test and is recapitalized, the bank's owners face dilution during recapitalization, and, thus, their payoff is only the retained share $1-\phi$ of the bank's equity. The equity is priced after the stress test and reflects the equilibrium choice of the regulator in the second period.

Proposition 1. *In the second period, there exists a unique equilibrium, in which the bank originates a risky loan if and only if* $\Delta(z_2^L, z_2^S) \geq 0$, *where:*

$$\begin{aligned}\Delta(z_2^L, z_2^S) \equiv{} & \underbrace{[\alpha + (1-\alpha)(1-d)]\,(R-1) - (R_0 - 1)}_{\text{profit differential without recapitalization}} \\ & \underbrace{- (1-\gamma)\left(\begin{array}{r}\left[\alpha z_2^S + (1-\alpha)(1-z_2^L)\right]\left(\underline{\rho}-1\right) \\ + \left[\alpha z_2^S + (1-\alpha)(1-z_2^L)\right]\beta\left[\frac{\alpha z_2^S + (1-\alpha)(1-z_2^L)(1-d)}{\alpha z_2^S + (1-\alpha)(1-z_2^L)}R - \underline{\rho}\right] \\ + (1-\alpha)(1-z_2^L)d\end{array}\right)}_{\text{dilution cost of recapitalization}}\end{aligned} \tag{4}$$

If the bank extends a risky loan at stage 2, the lenient regulator passes the bank with certainty, the strict regulator fails the bank with certainty, and the strategic regulator passes the bank with certainty if and only if the bank's loan is good.

Moreover, $\Delta(z_2^L, z_2^S)$ *is strictly increasing in* z_2^L *and strictly decreasing in* z_2^S. *In addition,* $\Delta(z_2^L, z_2^S)$ *is strictly decreasing in* β, *and there exists* $\underline{\beta} < 1$, *such that* $\Delta(0,1) \geq 0$ *if and only if* $\beta \leq \underline{\beta}$.

This proposition states that the bank's incentive to originate a risky loan takes two factors into account. On the one hand, the bank benefits from originating the risky loan because it produces a higher expected profit than the safe investment (the profit differential term in $\Delta(z_2^L, z_2^S)$). On the other hand, the bank incurs an expected dilution cost whenever recapitalization is feasible, which consists of three terms. First, capital is costly because of the forgone net return on its outside option, $(\underline{\rho}-1)$. Second, the capital provider extracts rents, as discussed above. Third, recapitalization increases repayments to depositors. Consider the situation where the bank's risky loan is bad and fails at stage 3. After recapitalization, the bank repays the depositors in full. In comparison, without recapitalization, the bank defaults with probability d and does not repay the depositors.[26] Since the bank faces the possibility of failing the stress test and, thus, having to raise capital, the bank originates the risky loan only if the gains from higher expected profit outweigh the potential dilution cost of recapitalization.

Importantly, the bank's incentive to originate a risky loan is increasing in the regulator's reputation for being the lenient type, z_2^L, and decreasing in the regulator's reputation for being the strict type, z_2^S. This is the case because the lenient type regulator is less likely to require the bank to raise capital, while the strict type is more likely to do so. In particular, if the regulator's reputation is sufficiently lenient, the bank strictly prefers to originate a risky loan, i.e., $\Delta(1,0) > 0$. If the regulator's reputation is sufficiently strict, however, Proposition 1 states that the bank still originates a risky loan (i.e., $\Delta(0,1) \geq 0$) if the dilution cost of capital is not too high (i.e., $\beta \leq \underline{\beta}$). For the remainder of the baseline model analysis, we impose the following additional assumption to restrict attention to the interesting set of the parameter space in which the bank's project choice indeed varies depending on the regulator's reputation.

Assumption 5. *$\beta > \underline{\beta}$, where $\underline{\beta}$ is defined in Proposition 1.*

We now consider how the strategic regulator's payoff (surplus) depends on reputation (z_2^L, z_2^S) and the bank's lending choice. Let U_R and U_0 denote the strategic regulator's expected surplus from the bank in the second period when the bank originates a risky loan and invests in the safe asset, respectively. We can express the expected surplus as follows:

$$U_R = [\alpha + (1-\alpha)(1-d)]\, R - 1 + X, \tag{5}$$

$$U_0 = R_0 - 1, \tag{6}$$

where X represents the net social benefits of risky lending, given by:

$$X \equiv B - (1-\alpha)\left[\gamma d D + (1-\gamma)(\underline{\rho} - 1)\right]. \tag{7}$$

When the bank extends a risky loan, the strategic regulator internalizes the net social benefits

[26] These increased repayments are indeed a cost to the bank because they are not ex ante priced due to the presence of deposit insurance.

of risky lending X. The first part of X is the positive externality of bank lending B. Conditional on a bad loan (with probability $1-\alpha$), X must also account for the social costs of a potential bank default, consisting of the expected cost of bank default dD if recapitalization is infeasible (with probability γ) and the forgone net return from the capital providers' alternative investment $\underline{\rho}-1$ when the bank is recapitalized (with probability $1-\gamma$). Notice that X encapsulates all of the externalities from risky lending; the following analysis will use only X rather than the individual components.

It then follows from Proposition 1 that the strategic regulator's expected surplus, for a given reputation (z_2^L, z_2^S), denoted by $U(z_2^L, z_2^S)$, is given by

$$U(z_2^L, z_2^S) = \begin{cases} U_R, & \text{if } \Delta(z_2^L, z_2^S) > 0, \\ U_0, & \text{if } \Delta(z_2^L, z_2^S) < 0, \\ \lambda U_R + (1-\lambda)U_0 \text{ for some } \lambda \in [0,1], & \text{if } \Delta(z_2^L, z_2^S) = 0, \end{cases} \tag{8}$$

where we have taken into account that, if $\Delta(z_2^L, z_2^S) = 0$, the bank is indifferent between originating a risky loan and investing in the safe asset. Thus, the bank may employ a mixed strategy and randomize between the two investment choices with some probability λ.

The regulator internalizes the social benefits and costs of risky lending, whereas the bank cares only about the private cost of recapitalization. Therefore, the bank's investment choice characterized in Proposition 1 generally differs from the socially optimal choice. The more the bank expects the regulator to be the lenient (strict) type, the more the bank is willing to originate a risky loan (invest in the safe asset). However, originating a risky loan (investing in the safe asset) is socially preferred only if the net social benefits of risky lending X are sufficiently high (low). In the following analysis, we demonstrate that the divergence in preferences between the strategic regulator and the bank leads to reputation-building incentives for the strategic regulator that depend on the net social benefits of risky lending X.

4 Stress testing in the first period

In this section, we analyze the regulator's equilibrium stress testing strategy for the bank in the first period, given the equilibrium in the second period. In particular, we consider the strategic regulator's incentives to pass the bank in the first period at stage 2. These incentives are driven by concerns for the bank in the first period and the reputational consequences of the regulator's observable decision to pass or fail the bank.

At stage 2, the regulator takes the posterior beliefs held by the bank in the second period about the regulator's type as given. These posterior beliefs depend on the stress test result and the bank's realized payoff in the first period. Let (z_R^L, z_R^S) denote the posterior beliefs about the probability that the regulator is the lenient type and the strict type, given that the bank passes the stress test in the first period and realizes a payoff of R. If the bank passes the stress test in the first period and realizes a payoff of 0 instead, the bank defaults and does not continue to the second period. Let (z_f^L, z_f^S) denote the posterior beliefs about the probability that the regulator is the lenient type and the strict type, given that the bank fails the stress test in the first period. As will become clear, this posterior belief does not depend on the bank's realized payoff in the first period.

The strategic regulator's incentive to pass the bank, given the quality of the bank's risky loan $q_1 \in \{g, b\}$, can be described by the net gain of passing the bank relative to failing the bank, which we denote by $G_{q_1}\left((z_R^L, z_R^S), (z_f^L, z_f^S)\right)$:

$$\begin{aligned}
G_g\left((z_R^L, z_R^S), (z_f^L, z_f^S)\right) &= \underbrace{(1-\gamma)(\underline{\rho}-1)}_{\text{first period bank surplus effect}} + \underbrace{\delta\left[U(z_R^L, z_R^S) - U(z_f^L, z_f^S)\right]}_{\text{reputation effect}}, \\
G_b\left((z_R^L, z_R^S), (z_f^L, z_f^S)\right) &= \underbrace{(1-\gamma)\left[(\underline{\rho}-1) - dD\right]}_{\text{first period bank surplus effect}} \\
&\quad + \underbrace{\delta\left[(1-d)U(z_R^L, z_R^S) - \left[(1-d) + d(1-\gamma)\right]U(z_f^L, z_f^S)\right]}_{\text{reputation effect}}. \qquad (9)
\end{aligned}$$

In both expressions, the first term represents the net gain in terms of the expected surplus in the first period, and the second term represents the reputation concern in terms of the expected surplus in the second period. The first term takes into account that, when the bank fails, recapitalization is feasible only with probability $1-\gamma$. In that case, the first-period bank surplus effect of passing the bank relative to failing it is equal to the capital provider's alternative investment (which can now be realized with a pass) less the expected cost of a bank default (which may also be realized with a pass) if the quality of the investment is bad.

The first-period bank surplus effect is positive if the first bank's risky loan is good, as there is no risk of default. This effect is negative if the risky loan is bad, given Assumption 4.

The reputation effect depends on the regulator's posterior reputation after it grades the first-period bank and the payoffs are realized. Given that the lenient type regulator passes the bank with certainty, if the strategic regulator fails the bank in the first stress test, it is pooled only with the strict type regulator ($z_f^L = 0$); the bank will then realize that it will be recapitalized in the second period if its risky investment is of bad quality.

Then the posterior probability that the regulator is the strict type, given that the bank fails the first stress test, is

$$z_f^S(\pi_g, \pi_b) = \frac{z_1^S}{z_1^S + [\alpha(1-\pi_g) + (1-\alpha)(1-\pi_b)](1 - z_1^L - z_1^S)}, \tag{10}$$

where π_g and π_b denote the strategic regulator's probability of passing the bank in the first stress test, given that the bank's risky loan is good or bad, respectively.

In contrast, given that the strict type regulator fails the bank with certainty, if the strategic regulator passes the bank in the first stress test, it is pooled only with the lenient type regulator ($z_R^S = 0$); the bank will then realize that it is less likely to be recapitalized in the second period when its risky investment is of bad quality. As a result, passing or failing the bank in the first period affects the reputation of the regulator and may lead to different investment decisions by the bank in the second period.

Thus the posterior probability that the regulator is the lenient type, given that the bank passes the first stress test and then realizes a payoff of R, is given by

$$z_R^L(\pi_g, \pi_b) = \frac{[\alpha + (1-\alpha)(1-d)]z_1^L}{[\alpha + (1-\alpha)(1-d)]z_1^L + [\alpha\pi_g + (1-\alpha)(1-d)\pi_b](1 - z_1^L - z_1^S)}. \tag{11}$$

The following lemma establishes the set of possible equilibrium stress testing strategies, which narrows down our analysis.

Lemma 1. *In any equilibrium, the stress testing strategy of the strategic regulator is one of the following:*

Informative: *it passes the bank if and only if the bank's risky loan is good;*

Tough: *it passes the bank with probability $\pi_g^* < 1$ if the bank's risky loan is good and fails the bank with certainty if the loan is bad; or*

Soft: *it passes the bank with certainty if the bank's risky loan is good and passes the bank with positive probability $\pi_b^* > 0$ if the loan is bad.*

To understand this lemma, let us compare the incentives for the strategic regulator to pass a bank with a good risky loan and a bad risky loan, given by G_g and G_b in Eq. 9, respectively. We rearrange G_b as

$$G_b\left((z_R^L, z_R^S), (z_f^L, z_f^S)\right) = (1-\gamma)(\underline{\rho}-1) + \delta(1-d)\left[U(z_R^L, z_R^S) - U(z_f^L, z_f^S)\right] \\ - d(1-\gamma)\left[D + \delta U(z_f^L, z_f^S)\right] \qquad (12)$$

The third term in the above expression captures the most important difference compared to G_g, stemming from the fact that passing the bank with a good risky loan does not lead to default, but passing the bank with a bad risky loan results in an increase in the probability of default by $(1-\gamma)d$, which takes into account that recapitalization may not be feasible when the bank fails.[27] Passing a bank with a bad risky loan is thus costlier than passing a bank with a good risky loan due to two costs that result from bank default—a social cost of default D in the first period, and a loss of expected surplus $U(z_f^L, z_f^S)$ in the second period. Overall, the greater incentives to pass a bank with a good risky loan than a bank with a bad risky loan implies that, first, it might be the case that the strategic regulator passes a bank with a good risky loan but fails a bank with a bad risky loan. In addition, this also implies that the strategic regulator may pass a bank with a bad risky loan with positive probability only if it passes a bank with a good risky loan with certainty, and that it may fail a bank with a good risky loan with positive probability only if it fails a bank with a bad risky loan with certainty. Thus, Lemma 1 represents all possible equilibrium strategies

[27] The second term of Eq. 12 also differs from the second term of G_g given in Eq. 9 by a factor of $(1-d)$. This reflects the fact that, in the absence of recapitalization, the bank with a bad risky loan continues to the second period only if it does not default in the first period. In the appendix we formally prove the Lemma, taking into account the difference in both terms.

of the strategic regulator in the first period.

We now show that for intermediate levels of the net social benefits of risky lending X, there is a unique equilibrium in which the strategic regulator's stress testing strategy in the first period is identical to its strategy in the second period (and is informative).

Proposition 2. *There exist cutoffs for the net social benefits of risky lending $\overline{X}$ and $\underline{X}$, with $\underline{X} < \overline{X}$, such that for $X \in [\underline{X}, \overline{X}]$, there exists a unique equilibrium in the first period in which the stress testing strategy of the strategic regulator in the first period is identical to that in the second period, described in Proposition 1, and is informative.*

For intermediate levels of the net social benefit of risky lending ($X \in [\underline{X}, \overline{X}]$), the expected surplus for the strategic regulator is not too sensitive to the bank's investment decision in the second period. That is, for intermediate X, the social values of the risky project and the safe asset are close, so the bank's investment choice in the second period has little effect on the regulator's surplus, and the regulator can choose its static optimal stress testing strategy in the first period. Proposition 2 shows that, in this case, the equilibrium is unique and is informative.

In the following sections, we will show that the other two types of equilibria described in Lemma 1 can arise if the net social benefits of risky lending X is either low or high, and depend on the regulator's reputation-building incentives.

4.1 Low net social benefits of risky lending $X < \underline{X}$

When the net social benefit of risky lending X is low (e.g., due to a large cost of bank default D), the expected surplus of the strategic regulator from the bank in the second period is higher when the bank invests in the safe asset. If the concerns about excessive risk-taking by the bank in the second period are sufficiently large, the strategic regulator may want to fail the bank with a risky loan in the first period even when the loan is good, in order to reveal its willingness to fail a bank during the second stress test.

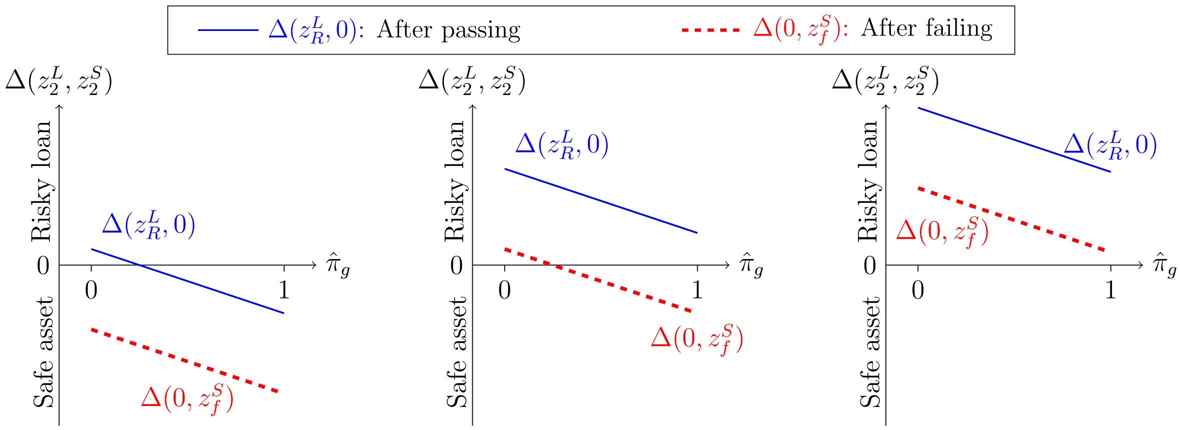
In the following proposition, we demonstrate that the strategic regulator's reputation-building incentives to reduce excessive risk-taking by the bank can produce a tough equilibrium in addition to the informative equilibrium (which was previously the unique outcome when X was moderate ($X \in [\underline{X}, \overline{X}]$)).

Proposition 3. *For low net social benefits of risky lending $X < \underline{X}$, there exists an equilibrium in the bank's first period stress test that is either informative or tough (as described in Lemma 1). Specifically, there exist thresholds $\bar{\beta}$, where $\bar{\beta} > \underline{\beta}$, $\bar{z}_1^L(z_1^S)$, $\bar{z}_1^S(z_1^L)$, and δ_g, such that*

- *If $\beta \geq \bar{\beta}$ and $z_1^L \leq \bar{z}_1^L(z_1^S)$,*
  - *for $\delta < \delta_g$, the unique equilibrium is informative,*
  - *for $\delta \geq \delta_g$, the informative equilibrium coexists with tough equilibria.*
- *If $\beta \geq \bar{\beta}$ and $z_1^L > \bar{z}_1^L(z_1^S)$, or $\beta \leq \bar{\beta}$ and $z_1^S > \bar{z}_1^S(z_1^L)$,*
  - *for $\delta < \delta_g$, the unique equilibrium is informative,*
  - *for $\delta > \delta_g$, the unique equilibrium is tough,*
  - *for $\delta = \delta_g$, the informative equilibrium coexists with tough equilibria.*
- *If $\beta \leq \bar{\beta}$ and $z_1^S \leq \bar{z}_1^S(z_1^L)$, the unique equilibrium is informative.*

In the informative equilibrium, the strategic regulator passes the bank with a good risky loan in the first period, which maximizes the first period expected surplus. Failing the bank in this case could result in a costly recapitalization of the bank with no benefit, since the good loan will not default. However, by failing this bank, the strategic regulator is able to reveal its willingness to recapitalize the bank (i.e., that it is not lenient ($z_f^L = 0$)), whereas passing this bank would have signaled reluctance to recapitalize the bank (i.e., that it is not strict ($z_R^S = 0$)). As a result, there are parameter values for which failing the bank induces the bank to invest in the safe asset in the second period, while passing the bank leads it to invest in the risky loan in the second period. In other words, by failing the bank, the strategic regulator both fosters a reputation for strictness and avoids a reputation for leniency, reducing excessive risk-taking. Reputation building to reduce excessive risk-taking can be worthwhile when the regulator's reputation concern (δ) is sufficiently high, so that the regulator's reputational benefit outweighs the short-term efficiency loss.

When the regulator's reputation concern is sufficiently high ($\delta > \delta_g$), Proposition 3 demonstrates that the tough equilibrium may coexist with the informative equilibrium, or the equilibrium may be unique. This is driven by a strategic interaction between the strategic regulator's stress test in the first period and the bank's lending behavior in the second period.



(a) $\beta \geq \bar{\beta}$ and $z_1^L \leq \bar{z}_1^L(z_1^S)$

(b) $\beta \geq \bar{\beta}$ and $z_1^L > \bar{z}_1^L(z_1^S)$, or $\beta \leq \bar{\beta}$ and $z_1^S > \bar{z}_1^S(z_1^L)$

(c) $\beta \leq \bar{\beta}$ and $z_1^S \leq \bar{z}_1^S(z_1^L)$

Figure 2: The bank's incentive to originate a risky loan in the second period after passing and failing the stress test in the first period, given its conjectured probability $\hat{\pi}_g$ that the strategic regulator passes a good bank in the first period and give that the strategic regulator fails a bad bank in the first period with certainty ($\pi_b = 0$).

Consider the first case in Proposition 3, where the bank has a high expected cost of recapitalization, i.e., β is high and z_1^L is low relative to z_1^S. In this case, the bank invests in the safe asset in the second period after failing the stress test in the first period regardless of the strategic regulator's conjectured stress test strategy $\hat{\pi}_g$. However, after passing the stress test in the first period, the bank may originate a risky loan or invest in the safe asset in the second period depending on its conjecture $\hat{\pi}_g$, as illustrated in Fig. 2a. This leads to a *strategic complementarity* between the strategic regulator's stress test in the first period and the bank's lending behavior in the second period *after passing* the stress test in the first period. This is driven by the bank's belief updating process about the regulator's perceived leniency (z_R^L), since after passing the stress test in the first period, the regulator's perceived strictness is zero ($z_R^S = 0$). In this case, if the bank conjectures that the strategic regulator adopts a tougher stress testing strategy (lower $\hat{\pi}_g$), the bank infers that the regulator who passes the bank in the first period is more likely to be lenient (higher z_R^L). Consequently, the bank's incentive to originate the risky loan after a pass result is larger, which decreases the net gain for the strategic regulator from passing the bank in the first period, justifying a tougher testing strategy. It is, indeed, this strategic complementarity that leads to equilibrium multiplicity.

Equilibrium multiplicity implies a potential loss of surplus. One issue is that the inefficient equilibrium may be played. In Section 5, we demonstrate that the tough equilibrium has a lower surplus than the informative equilibrium for these parameters. Another issue is miscoordination - the strategic regulator may play the equilibrium strategy for one equilibrium, while the bank plays the equilibrium strategy for the another equilibrium. For example, the strategic regulator may play its informative strategy, and the bank may originate the risky loan after a pass. That is, in both of these situations, there is excess risk-taking due the lack of ability to coordinate.

Now consider the second case in Proposition 3, where the bank has an intermediate expected cost of recapitalization. In this case, the bank originates a risky loan in the second period after passing the stress test in the first period regardless of the strategic regulator's conjectured stress test strategy $\hat{\pi}_g$. However, after failing the stress test in the first period, the bank may originate a risky loan or invest in the safe asset in the second period depending on its conjecture $\hat{\pi}_g$, as illustrated in Fig. 2b. This leads to a *strategic substitutability* between the strategic regulator's stress test in the first period and the bank's lending behavior in the second period *after failing* the stress test in the first period. This is driven by the bank's belief updating process about the regulator's perceived strictness (z_f^S), since after failing the stress test in the first period, the regulator's perceived leniency is zero ($z_f^L = 0$). In this case, if the bank conjectures that the strategic regulator adopts a tougher stress testing strategy (lower $\hat{\pi}_g$), the bank infers that the regulator who fails the bank in the first period is less likely to be strict (lower z_f^S). Consequently, the bank's incentive to originate the risky loan after a fail result is larger, which decreases the net gain for the strategic regulator from failing the bank in the first period, diminishing the incentive to adopt a tougher testing strategy. This strategic substitutability leads to equilibrium uniqueness.[28]

In the third case in Proposition 3, the bank has a low expected cost of recapitalization. In this case, the bank originates a risky loan in the second period after both passing and failing the stress test in the first period, regardless of its conjecture about the strategic regulator's stress test strategy $\hat{\pi}_g$, as illustrated in Fig. 2c. This results in no strategic interaction between the strategic

[28]Note that Proposition 3 states that there also exists multiplicity in the knife-edge case in which $\delta = \delta_g$. This is for a different reason than the strategic complementarity discussed above. Here, δ_g is defined such that the regulator's incentive to pass rather than fail a bank with a good quality asset (G_g in Eq. 9) is equal to zero; i.e., the regulator is indifferent between passing and failing and can mix with probability π_g. We note that the first term of G_g in Eq. 9, the first period bank surplus effect, is positive. For $G_g = 0$ and $\pi_g \in (0, 1)$ to be consistent, this implies that the reputation effect must be negative to offset the first period bank surplus effect. For the reputation effect to be negative, it must be the case that (i) the bank will originate a risky loan in the second period after a pass in the first period, and (ii) the bank will invest in the safe asset after a fail in the first period. There are multiple π_g that produce beliefs that induce such a strategy by the bank.

regulator's stress test in the first period and the bank's lending behavior in the second period, resulting in a unique informative equilibrium.

U.S. stress tests have generally been regarded as much tougher than European ones. First, the Federal Reserve performs the stress test itself on data provided by the banks (and does not provide the model to the banks), whereas in Europe, it has been the case that the banks themselves perform the test. Second, the U.S. stress tests have regularly been accompanied by Asset Quality Reviews, whereas this has been infrequent for European stress tests. Third, one of the most feared elements of the U.S. stress tests has been the fact that there is a qualitative element that can be (and has been) used to fail banks.[29] A possible reason for this is that European authorities prioritized stimulating lending more, given the slow recovery after the crisis.

4.2 High net social benefits of risky lending $X > \overline{X}$

When the net social benefits of risky lending X is high (e.g., due to a large externality benefit from lending B), the expected surplus to the strategic regulator from the bank in the second period is higher when the bank invests in the risky asset. If the concerns about too little lending by the bank in the second period are sufficiently large, the strategic regulator may want to pass the bank with a risky loan in the first period even when the loan is bad, in order to reveal its willingness to pass a bank during the second stress test.

In the following proposition, we demonstrate that for high X, there is still an equilibrium in which the strategic regulator's stress testing strategy in the first period is identical to its strategy in the second period (informative). However, reputation-building incentives to encourage lending by the second-period bank can lead to a soft equilibrium for the bank's stress test.

Proposition 4. *For high net social benefits of risky lending $X > \overline{X}$, there exists an equilibrium in the first bank's stress test that is either informative or soft (as described in Lemma 1). Specifically, there exists a threshold δ_b, such that*

- *If $\beta \geq \bar{\beta}$ and $z_1^L \leq \bar{z}_1^L(z_1^S)$, the unique equilibrium is informative.*
- *If $\beta \geq \bar{\beta}$ and $z_1^L > \bar{z}_1^L(z_1^S)$, or $\beta \leq \bar{\beta}$ and $z_1^S > \bar{z}_1^S(z_1^L)$,*

[29] The qualitative element for domestic banks was removed in March 2019 (see "US financial regulators relax Obama-era rules," by Kiran Stacey and Sam Fleming, Financial Times, March 7, 2019).

- *for $\delta < \delta_b$, the unique equilibrium is informative,*
- *for $\delta > \delta_b$, the unique equilibrium is soft,*
- *for $\delta = \delta_b$, the informative equilibrium coexists with tough equilibria.*

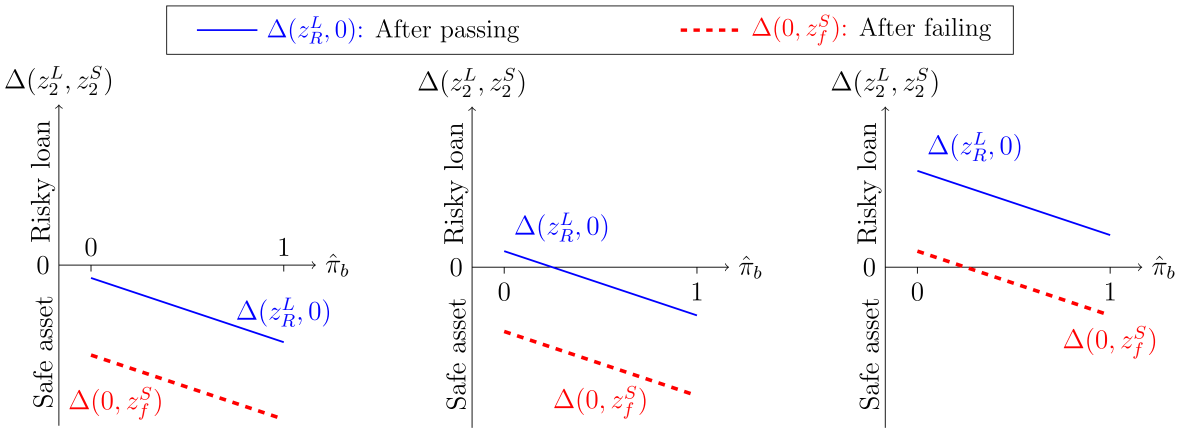
- *If $\beta \leq \bar{\beta}$ and $z_1^S \leq \bar{z}_1^S(z_1^L)$,*
  - *for $\delta < \delta_b$, the unique equilibrium is informative,*
  - *for $\delta \geq \delta_b$, the informative equilibrium coexists with soft equilibria.*

Passing the bank with a bad risky loan is costly, as it incurs an expected cost of default that is higher than the opportunity cost of capital. However, in the soft equilibrium, by passing the bank, the strategic regulator is able to signal its willingness to tolerate the bank's risky lending without requiring recapitalization (i.e., that it is not strict ($z_R^S = 0$)), whereas failing the bank would have revealed its readiness to recapitalize the bank (i.e., that it is not lenient ($z_f^L = 0$)). Thus passing the bank can be useful to the strategic regulator when it induces the bank to originate a risky loan in the second period. In other words, by passing the bank, the strategic regulator both fosters a reputation for leniency and avoids a reputation for strictness, encouraging lending. Reputation building to encourage lending can be worthwhile when the regulator's reputation concern (δ) is sufficiently high, so that the regulator's reputational benefit outweighs the short-term efficiency loss.

When the regulator's reputation concern is sufficiently high ($\delta > \delta_b$), Proposition 4 demonstrates that the soft equilibrium may coexist with the informative equilibrium, or the equilibrium may be unique, analogous to Proposition 3.[30] This is again driven by a strategic interaction between the strategic regulator's stress test in the first period and the bank's lending behavior in the second period.

In the third case in Proposition 4, the bank has a low expected cost of recapitalization and originates a risky loan after passing the stress test in the first period. However, after failing the stress test in the first period, the bank may originate a risky loan or invest in the safe asset in the second period depending on the conjectured probability that the strategic regulator passes the

[30]We note that in Proposition 4, strategic complementarity occurs in the third case (where the expected cost of recapitalization is low) rather than in the first case (where the expected cost of recapitalization is high) as in Proposition 3. This is due to the changes in the strategic regulator's incentives from X.



(a) $\beta \geq \bar{\beta}$ and $z_1^L \leq \bar{z}_1^L(z_1^S)$

(b) $\beta \geq \bar{\beta}$ and $z_1^L > \bar{z}_1^L(z_1^S)$, or $\beta \leq \bar{\beta}$ and $z_1^S > \bar{z}_1^S(z_1^L)$

(c) $\beta \leq \bar{\beta}$ and $z_1^S \leq \bar{z}_1^S(z_1^L)$

Figure 3: The bank's incentive to originate a risky loan in the second period after passing and failing the stress test in the first period, given its conjectured probability $\hat{\pi}_b$ that the strategic regulator passes a bad bank in the first period and given that the strategic regulator passes a good bank in the first period with certainty ($\pi_g = 1$).

bank with a bad risky loan $\hat{\pi}_b$, as illustrated in Fig. 3c. This leads to a *strategic complementarity* between the strategic regulator's stress test in the first period and the bank's lending behavior in the second period *after failing* the stress test in the first period. Here, if the bank conjectures that the strategic regulator adopts a more lenient stress testing strategy (higher $\hat{\pi}_b$), the bank infers that the regulator who fails the bank in the first period is more likely to be strict (higher z_f^S). Consequently, the bank's incentive to originate the risky loan after a fail result is smaller, which decreases the net gain for the strategic regulator from failing the bank in the first period, justifying a more lenient testing strategy. This strategic complementarity thus leads to equilibrium multiplicity.

As in the previous section, equilibrium multiplicity implies a potential loss of surplus. In Section 5, we demonstrate that the soft equilibrium has a lower surplus than the informative equilibrium for these parameters. Miscoordination may also occur - the strategic regulator may play the equilibrium strategy for one equilibrium, while the bank plays the equilibrium strategy for the another equilibrium. For example, the strategic regulator may play its informative strategy, and the bank invests in the safe asset after a fail. That is, in both of these situations, there is not enough lending due the lack of ability to coordinate.

In the second case in Proposition 4, the bank has an intermediate expected cost of recapitalization. In this case, the bank invests in the safe asset in the second period after failing the stress test the first period. However, after passing the stress test in the first period, the bank may originate a risky loan or invest in the safe asset in the second period depending on the conjecture $\hat{\pi}_b$, as illustrated in Fig. 3b. This leads to a *strategic substitutability* between the strategic regulator's stress test in the first period and the bank's lending behavior in the second period *after passing* the stress test in the first period. If the bank conjectures that the strategic regulator adopts a more lenient stress testing strategy (higher $\hat{\pi}_b$), the bank infers that the regulator who passes the bank in the first period is less likely to be lenient (lower z_R^L). Consequently, the bank's incentive to originate the risky loan after a pass result is smaller, which decreases the net gain for the strategic regulator from passing the bank in the first period, diminishing the incentive to adopt a softer testing strategy. This strategic substitutability leads to equilibrium uniqueness.[31]

In the first case in Proposition 4, the bank has a high expected cost of recapitalization. In this case, the bank invests in the safe asset in the second period after both passing and failing the stress test in the first period, regardless of its conjecture about the strategic regulator's stress test strategy $\hat{\pi}_b$, as illustrated in Fig. 3a. This results in no strategic interaction between the strategic regulator's stress test in the first period and the bank's lending behavior in the second period, resulting in a unique informative equilibrium.

While the initial European stress tests performed poorly (e.g., passing Irish banks and Dexia), one might argue that during crisis times, the main focus was preventing runs - and without a fiscal backstop it was hard to maintain credibility (Faria-e-Castro, Martinez, and Philippon, 2017). We argue that in normal times, a stress test may be soft to incentivize banks to lend to the real economy. This may explain the 2016 EU stress test, which eliminated the pass/fail criteria, reduced the number of banks stress tested by about half, used less-adverse scenarios than did the U.S. and UK, and singled out only one bank as undercapitalized - Monti dei Paschi di Siena, which had failed the previous (2014) stress test and was well known to be in distress. The press also revealed that Deutsche Bank was given a special exception to one of the stress test rules, boosting its recorded capitalization.[32]

[31] As in Proposition 3, there exists multiplicity in the knife-edge case in which $\delta = \delta_b$.

[32] See "Deutsche Bank received special treatment in EU stress tests," by Laura Noonan, Caroline Binham and James Shotter, *Financial Times*, October 10, 2016. We also note that the 2018 test also had no pass/fail requirement, included only 48 banks, and all banks tested were judged to be well capitalized.

5 Discussion

Having characterized the equilibria of the model, we discuss the implications of the model in this section. First, we show how the existence of multiple equilibria leads to a loss of surplus. Second, we consider how stress tests may vary with the availability of capital. Third, we explore the implications of the model for stress test design when banks are systemic.

5.1 Multiplicity and inefficiency

Propositions 3 and 4 have shown that strategic complementarity between the strategic regulator's stress test and the bank's lending choice can lead to equilibrium multiplicity. The following proposition shows that when a tough or soft equilibrium coexist with an informative equilibrium, their surplus is *lower* than the surplus in the informative equilibrium.

Proposition 5. *For parameter values such that the informative equilibrium coexists with tough or soft equilibria, equilibrium surplus is higher in the informative equilibrium than in a tough or soft equilibrium.*

To see this, consider first the case with low net social benefits of risky lending ($X \leq \underline{X}$). In this case, the informative equilibrium coexists with tough equilibria when the bank's expected cost of recapitalization is sufficiently high, priors about the regulator being the lenient type are sufficiently low, and the discount factor is sufficiently large. In the informative equilibrium, the bank invests in the safe asset with certainty in the second period, whereas in a tough equilibrium, the bank may originate a risky loan in the second period if it passes the stress test in the first period. The tough equilibrium is therefore inefficient, as it is associated with both the possible costly recapitalization of the bank with a good risky asset in the first period *and* inefficient risk taking by the bank in the second period.

Similarly, in the case with high net social benefit of risky lending ($X \geq \overline{X}$), the informative equilibrium coexists with soft equilibria when the bank's expected cost of recapitalization is sufficiently low, priors about the regulator being the strict type are sufficiently low, and the discount factor is sufficiently large. In the informative equilibrium, the bank originates a risky loan with certainty in the second period, whereas in a soft equilibrium, the bank may invest in the safe asset in the second period if it fails the stress test in the first period. In this case, the soft equilibrium is inefficient, as it is associated with both the possible costly default of the bank with a bad risky loan in the first period due to a lack of recapitalization *and* inefficiently low lending by the bank in the second period.

This result shows that the reputation concerns of the regulator can be a source of inefficiency, contrary to the conventional wisdom that reputation can be beneficial as it enhances commitment power (e.g., Kreps and Wilson (1982) and Milgrom and Roberts (1982)).[33] This is driven by a strategic complementarity that produces an inefficient reputation-building equilibrium (in addition to the static equilibrium) not present in standard reputation models.[34]

5.2 Availability of capital

Let γ_1 denote the probability that recapitalization is infeasible in the first period. The following corollary assesses how the availability of capital in the first period affects the regulator's stress testing strategy.

Corollary 1. *When net social benefits of risky lending are:*

- *Low ($X < \underline{X}$): an increase in the probability that recapitalization is infeasible (γ_1) strictly shrinks the parameter space ($\beta, z_1^L, z_1^S, \delta$) for which an informative equilibrium exists, and strictly enlarges the parameter space for which a tough equilibrium exists.*
- *High ($X > \overline{X}$): an increase in the probability that recapitalization is infeasible (γ_1) strictly shrinks the parameter space ($\beta, z_1^L, z_1^S, \delta$) for which an informative equilibrium exists, and strictly enlarges the parameter space for which a soft equilibrium exists.*

The corollary demonstrates that when there is a higher probability that recapitalization is infeasible in the first period, the strategic regulator's reputation-building incentives are exacerbated, and the stress test becomes less informative. This is because the strategic regulator trades off the cost/benefit of recapitalizing the bank in the first period against the cost/benefit of affecting the

[33] Ely and Valimaki (2003) and Ely, Fudenberg, and Levine (2008) demonstrate that "bad reputation" can lead to market breakdown. In our model, the reputational incentive to separate from a behavioral type (from the lenient type when X is low and from the strict type when X is high) is similar to "bad reputation". Unlike their setting, where participation is optional, we obtain the coexistence of an equilibrium with lower surplus due to strategic complementarity.

[34] Ordoñez (2013, 2018) also demonstrate that reputation concerns can produce strategic compelmentarity that leads to multiple equilibria. However, in his setting, reputation building improves efficiency by providing discipline to keep banks from taking excessive risk.

bank's investment decision in the second period. While the reputation effect depends only on the bank's updated belief about the regulator's type, the cost of passing a bad bank or failing a good bank in the first period is smaller if recapitalization is less feasible in the first period.

This result is related to that of Faria-e-Castro, Martinez, and Philippon (2017) in that, as recapitalizing the bank becomes more difficult, the test becomes less informative. Nevertheless, we demonstrate this link through a dynamic reputation model, whereas they have the regulator committing upfront to the informativeness of the stress test.

The U.S.'s swifter recovery from the crisis means that capital raising for banks was likely to be easier. Our model implies that stress tests will be more informative in this situation, which appears consistent with reality.

5.3 Stress tests of systemic banks

Let D_1 denote the social cost of a potential bank default in the first period. The following corollary assesses how the the social cost of a potential bank default in the first period affects the regulator's stress testing strategy.

Corollary 2. *When net social benefits of risky lending are:*

- *Low ($X < \underline{X}$): an increase in D_1 does not affect the parameter space $(\beta, z_1^L, z_1^S, \delta)$ for which either an informative equilibrium or a tough equilibrium exist.*
- *High ($X > \overline{X}$): an increase in D_1 strictly enlarges the parameter space $(\beta, z_1^L, z_1^S, \delta)$ for which an informative equilibrium exists, and strictly shrinks the parameter space for which a soft equilibrium exists.*

The corollary demonstrates that when the social cost of a potential bank failure in the first period is higher, the strategic regulator becomes less soft/more informative when facing reputation-building incentives to incentivize lending ($X > \overline{X}$). This occurs because, when considering whether to pass a bad bank in the first period, the strategic regulator trades off the cost/benefit of recapitalizing the bank in the first period against the cost/benefit of affecting the bank's investment decision in the second period. While the reputation effect depends only on the bank's updated belief about the regulator's type, the cost of passing a bad bank in the first period is larger if the cost of a potential bank failure in the first period is larger. The strategic regulator's stress testing strategy when facing reputation-building incentives to curb excessive risk-taking ($X < \underline{X}$) is unaffected since, in this case, the main focus is whether to pass or fail a good bank, which does not run the risk of default.

In both the U.S. and Europe, since the inception of stress tests, there have been ongoing debates about how large/systemic a bank must be in order to be included in the stress testing exercise. To the extent that larger and more systemic banks have a higher expected cost of default, our model predicts that they should be subject to (weakly) more informative tests.

6 Stress tests with commitment

In this section, we alter the model to one in which a strategic regulator can publicly commit to a stress testing strategy. As this eliminates the reputation mechanism, we focus only on one period and only on the strategic regulator. We demonstrate that with commitment, the strategic regulator may design a stress test that is either tough, in order to discourage excessive risk-taking; soft, in order to incentivize lending; or informative. However, in contrast to the baseline model, the strategic regulator's equilibrium toughness or softness does not lead to inefficient equilibrium multiplicity, as it is driven by direct incentive provision.

Specifically, consider the model of stress testing described in Section 2 with only one period. We assume that the strategic regulator can publicly commit to a stress testing strategy of passing a bank with a good risky loan with probability π_g and passing a bank with a bad risky loan with probability π_b at the beginning of the game.

We solve this model by first characterizing the bank's investment decision after observing the strategic regulator's committed stress testing strategy (π_g, π_b). It follows that the bank originates a risky loan if and only if $\tilde{\Delta}(\pi_g, \pi_b) \geq 0$, where

$$\begin{aligned}\tilde{\Delta}(\pi_g, \pi_b) \equiv{}& [\alpha + (1-\alpha)(1-d)](R-1) - (R_0 - 1) \\ &- (1-\gamma)\left(\begin{array}{l}[\alpha(1-\pi_g) + (1-\alpha)(1-\pi_b)](\underline{\rho} - 1) \\ \quad [\alpha(1-\pi_g) + (1-\alpha)(1-\pi_b)]\,\beta\left[\frac{\alpha(1-\pi_g)+(1-\alpha)(1-\pi_b)(1-d)}{\alpha(1-\pi_g)+(1-\alpha)(1-\pi_b)} R - \rho\right] \\ \hfill + (1-\alpha)(1-\pi_b)d\end{array}\right).\end{aligned} \quad (13)$$

This expression is analogous to Eq. 4, where the second term captures the bank's expected dilution cost of recapitalization.

We can then express the strategic regulator's expected surplus given the strategic regulator's strategy (π_g, π_b), which we denote by $\tilde{U}(\pi_g, \pi_b)$, as follows, analogous to Eq. 8:

$$\tilde{U}(\pi_g, \pi_b) = \begin{cases} \tilde{U}_R(\pi_g, \pi_b), & \text{if } \tilde{\Delta}(\pi_g, \pi_b) > 0, \\ U_0, & \text{if } \tilde{\Delta}(\pi_g, \pi_b) < 0, \\ \lambda \tilde{U}_R(\pi_g, \pi_b) + (1-\lambda) U_0 & \\ \qquad\qquad \text{for some } \lambda \in [0,1], & \text{if } \tilde{\Delta}(\pi_g, \pi_b) = 0, \end{cases} \tag{14}$$

where U_0 captures the expected surplus of the bank who invests in the safe asset and is defined in Eq. 6. The term $\tilde{U}_R(\pi_g, \pi_b)$ captures the strategic regulator's expected surplus when the bank originates a risky loan, given the regulator's stress testing strategy, and is defined by:

$$\tilde{U}_R(\pi_g, \pi_b) = [\alpha + (1-\alpha)(1-d)]\, R - 1 + \tilde{X}(\pi_g, \pi_b), \tag{15}$$

where $\tilde{X}(\pi_g, \pi_b)$ represents the net social benefits of risky lending given the stress testing strategy (π_g, π_b):

$$\tilde{X}(\pi_g, \pi_b) \equiv B - (1-\alpha)\,[\gamma + \pi_b(1-\gamma)]\, dD - [\alpha(1-\pi_g) + (1-\alpha)(1-\pi_b)]\,(1-\gamma)(\underline{\rho} - 1). \tag{16}$$

Eq. 16 is analogous to Eq. 7, while additionally taking into account the costs associated with failing a bank with a good risky loan (if $\pi_g < 1$) and passing a bank with a bad risky loan (if $\pi_b > 0$). Notice that $\tilde{X}(\pi_g, \pi_b)$ is maximized at $(\pi_g, \pi_b) = (1, 0)$, in which case it coincides with X in the baseline model. As a result, X represents the maximal net social benefit of risky lending, which is obtained if the strategic regulator adopts the informative stress testing strategy.

We can now characterize the strategic regulator's optimal stress testing strategy, which solves $\max_{(\pi_g, \pi_b)} \tilde{U}(\pi_g, \pi_b)$. Since the strategic regulator's optimal stress testing strategy becomes inconsequential if the bank invests in the safe asset, the strategic regulator's optimal stress testing strategy may not be unique. We thus restrict attention to the strategies that are trembling-hand perfect in order to rule out unreasonable off-equilibrium actions.[35]

Proposition 6. *In a one-period model of stress test with commitment, there exists a unique trembling-hand perfect equilibrium, in which the strategic regulator's stress testing strategy is as follows.*

- *If the maximal net social benefits of risky lending is low $X \leq \underline{X}$, the strategic regulator passes the bank with a good risky loan with probability $\pi_g^* \leq 1$, and fails the bank with a bad risky loan with certainty.*
- *If the maximal net social benefits of risky lending is high $X \geq \underline{X}$, the strategic regulator passes the bank with a good risky loan with certainty, and passes the bank with a bad risky loan with probability $\pi_b^* \geq 0$.*

This proposition shows that, if the maximal net social benefits of risky lending is low, the strategic regulator may find it optimal to commit to a tough stress testing strategy in order to induce the bank to invest in the safe asset. If, on the other hand, the maximal net social benefits of risky lending is high, the strategic regulator may find it optimal to commit to a soft stress testing strategy in order to induce the bank to originate a risky loan. In these cases, direct incentives through announced toughness and softness are provided in order to influence bank lending behavior, rather than through regulator reputation as in the baseline model.

The proposition identifies a single threshold $\underline{X}$ of the maximal net social benefit such that strategic regulator switches from a regime of possibly failing the bank with a good risky loan (being tough or informative) to a regime of possibly passing the bank with a bad risky loan (being soft or informative). There is only one threshold here, in contrast to the baseline model where there were two thresholds, $\underline{X}$ and $\overline{X}$. In the baseline model, the threshold $\overline{X}$ arises because the bank could default between periods one and two if the risky loan in the first period was bad. This dynamic effect does not exist in the static commitment model. However, the threshold $\underline{X}$ is still relevant; it reflects only the static preference of the strategic regulator between a risky loan or safe investment (which was also important in the baseline model when the risky loan in the first period was good, as it did not default).

[35]Notice that the all equilibria characterized in the baseline model are trembling-hand perfect, because they concern only stress testing strategies that are sequentially rational.

Commitment by the regulator can eliminate multiple equilibria and improve efficiency. Of course, committing to future actions (such as what to do when learning the bank has bad risky loans) may not be feasible. This requires substantial independence from political pressure and processes that are well-defined. Regulators do specify stress test scenarios in advance, which can be more soft or less soft, given the effect desired. Asset quality reviews also commit more resources and reveal more information about bank positions.

7 The bank and the regulator both learn the asset quality

In this section, we demonstrate that our results are robust when the stress test does not give the regulator more information than the bank has. Specifically, consider a stress test in which the signal about the quality q_t of the bank's risky loan observed by the regulator during the test is also observed by the bank. This could be the case because:

- the stress test uncovers only the private information that the bank already has about its loan quality.[36] This is, indeed, the case for banking supervision examinations. These exams are conducted on a regular basis by collecting information and assessing the health of banks on multiple dimensions, and they have real effects.[37] They do not use information from the entire banking system to assess the position of each bank (which can be the source of the regulator's private information in the baseline model). In the United States, this has historically been conducted using the CAMEL rating system, though, in recent years, variations on this rating system have been implemented;[38] or

- the stress test produces/uncovers new information, but regulators share that information with the bank. This second case resembles the European stress test exercises, which use an approach whereby the regulator provides the model and basic parameters to banks, which

[36]For example, in Walther and White (Forthcoming), the regulator and the bank both observe the bank's asset value, while creditors do not. They consider the effectiveness of bail-ins in this scenario.

[37]Agarwal et al. (2014) demonstrate real effects of exams: leniency leads to more bank failures; a higher proportion of banks unable to repay TARP money in the crisis; and a larger discount on assets of banks liquidated by the FDIC. Hirtle, Kovner, and Plosser (2018) demonstrate real effects of more banking supervision effort (measured by hours).

[38]The RFI/C(D) system was recently supplanted by the LFI system for large financial institutions. See (https://www.davispolk.com/files/2018-11-06_federal_reserve_finalizes_new_supervisory_ratings_system_for_large_financial_institutions.pdf).

then perform the test themselves.[39] In contrast, the U.S. uses an approach whereby the regulators perform the test and do not provide all of the information about the model or the results.[40]

We now highlight the key differences in this extension in relation to the baseline model, and defer the formal derivation of the results to the appendix. As in the baseline model, the equilibrium in the second period for a given set of beliefs of the bank (that the regulator is lenient or strict) is as described in Proposition 1. Unlike in the baseline model, here, in the second period, the bank forms posterior beliefs $(z^L_{R,q_1}, z^S_{R,q_1})$ and $(z^L_{f,q_1}, z^S_{f,q_1})$, given that the bank passes or fails the stress test in the first period, respectively, *and* the loan quality in the first period is $q_1 \in \{g, b\}$. The strategic regulator must take into account these beliefs when deciding whether to pass the bank in the first period. The following proposition summarizes the equilibria when the bank and the regulator both learn the asset quality.

Proposition 7. *When the bank and the regulator both learn the risky loan's quality q_t, the equilibria are characterized as follows.*

- *For intermediate levels of net social benefits of risky lending $X \in [\underline{X}, \overline{X}]$, there exists a unique informative equilibrium (as described in Proposition 2).*
- *For low net social benefits of risky lending $X < \underline{X}$, there exists an equilibrium for the first bank's stress test, and any equilibrium is either informative or tough. There exists a threshold $\hat{z}^L_1(z^S_1)$, such that for $\delta > \delta_g$, the informative equilibrium coexists with tough equilibria if and only if $\beta \geq \bar{\beta}$ and $z^L_1 \leq \hat{z}^L_1(z^S_1)$.*
- *For high net social benefits of risky lending $X > \overline{X}$, there exists an equilibrium for the first bank's stress test, and any equilibrium is either informative or soft. There exists a threshold $\hat{z}^S_1(z^L_1)$, such that for $\delta > \delta_b$, the informative equilibrium coexists with soft equilibria if and only if $\beta \leq \bar{\beta}$ and $z^S_1 \leq \hat{z}^S_1(z^L_1)$.*

[39] Note that we do not model the inherent moral hazard problem when a bank is permitted to do its own stress test.

[40] These are sometimes referred to as "bottom-up" (banks do the test) and "top-down" (regulator does the test) approaches, but definitions of these terms vary. See Baudino et al. (2018) and Niepmann and Stebunovs (2018) for a discussion of top-down vs. bottom-up approaches. The U.S. recently made more information available about its test after complaints about opacity by banks (https://uk.reuters.com/article/uk-usa-fed-stresstests/fed-gives-u-s-banks-more-stress-test-information-unveils-2019-scenarios-idUKKCN1PU2GE).

Proposition 7 shows that the two main results of the baseline model remain qualitatively similar. First, if the regulator's reputation concern δ is sufficiently high, there can be a tough equilibrium when the strategic regulator wishes to discourage excessive risk-taking by the bank (for $X < \underline{X}$), and there can be a soft equilibrium when the strategic regulator wishes to encourage bank lending (for $X > \overline{X}$). Second, the strategic regulator's reputation concerns can lead to a strategic complementarity between the strategic regulator's stress test in the first period and the bank's lending behavior in the second period, and thus result in equilibrium multiplicity. Specifically, as before, this is the case for a low net social benefit of bank lending ($X < \underline{X}$) when the bank has a high expected cost of recapitalization (i.e., β is high and z_1^L is low relative to z_1^S), and this is the case for a high net social benefit of bank lending ($X > \overline{X}$) when the bank has a high expected cost of recapitalization (i.e., β is low and z_1^S is low relative to z_1^L).[41]

8 Conclusion

A recent addition to the regulatory toolkit, stress tests provide assessments of bank risk in adverse scenarios. Regulators respond to negative information by requiring banks to raise capital. However, regulators have incentives to be tough, by asking even some safe banks to raise capital, or to be soft, by allowing some risky banks to get by without raising capital. These incentives are driven by the weight that the regulator places on lending in the economy versus stability. Banks respond to the softness of the stress test by altering their lending policies. This may lead to multiple equilibria, and the regulator may get trapped in one of them, leading to a loss of surplus. This inefficiency will also be present in banking supervision exams.

It would be interesting to extend the model to study stress testing with multiple banks in a macroprudential setting.

[41] We note that the cutoffs for z_1^L and z_1^S are different than in the baseline model.

A Proofs

A.1 Proof of Proposition 1 (Second-period bank choice)

$\Delta(z_2^L, z_2^S)$ given by Eq. 4 is obtained by substituting Eq. 2 into Eq. 3 to eliminate ϕ and rearranging.

It follows from Eq. 4 that $\Delta(z_2^L, z_2^S)$ is strictly increasing in z_2^L and strictly decreasing in z_2^S. In particular, we have:

$$\Delta(1,0) = [\alpha + (1-\alpha)(1-d)]\,(R-1) - (R_0 - 1) > 0, \tag{17}$$

where the inequality follows from Assumption 1, and

$$\begin{aligned}\Delta(0,1) = \gamma\,[\alpha + (1-\alpha)(1-d)]\,(R-1) - (R_0-1) \\ + (1-\gamma)(1-\beta)\left([\alpha + (1-\alpha)(1-d)]\,R - \underline{\rho}\right).\end{aligned} \tag{18}$$

In addition, Eq. 4 implies that $\Delta(z_2^L, z_2^S)$ is strictly decreasing in β. Assumption 3 implies that $\Delta(0,1) < 0$ for $\beta = 1$. Therefore, there exists a unique $\underline{\beta} < 1$, such that $\Delta(z_2^L, z_2^S) \geq 0$ if and only if $\beta \leq \underline{\beta}$, where $\underline{\beta}$ is defined by $\Delta(0,1) = 0$, i.e.,

$$\gamma\,[\alpha + (1-\alpha)(1-d)]\,(R-1) - (R_0-1) + (1-\gamma)(1-\underline{\beta})\left([\alpha + (1-\alpha)(1-d)]\,R - \underline{\rho}\right) = 0. \tag{19}$$

Notice that, Eq. 19 allows the threshold $\underline{\beta}$ to be negative. If so, then Assumption 5 is satisfied for all $\beta \in [0,1]$. ∎

A.2 Proof of Lemma 1

From Eq. 9 and Eq. 12, and using the reasoning in the text immediately after this lemma, we have the following properties:

- For all (z_R^L, z_R^S) and (z_f^L, z_f^S) such that $U(z_R^L, z_R^S) - U(z_f^L, z_f^S) \geq 0$, we have $G_g\left((z_R^L, z_R^S), (z_f^L, z_f^S)\right) > 0$, and $G_g\left((z_R^L, z_R^S), (z_f^L, z_f^S)\right) > G_b\left((z_R^L, z_R^S), (z_f^L, z_f^S)\right)$, where $G_b\left((z_R^L, z_R^S), (z_f^L, z_f^S)\right)$ may be greater than or smaller than 0.

- For all (z_R^L, z_R^S) and (z_f^L, z_f^S) such that $U(z_R^L, z_R^S) - U(z_f^L, z_f^S) < 0$, we have $G_b\left((z_R^L, z_R^S), (z_f^L, z_f^S)\right) <$

0, whereas $G_g\left((z_R^L, z_R^S), (z_f^L, z_f^S)\right)$ may be greater than or smaller than 0.

Used the fact that $z_R^S = z_f^L = 0$ for all (π_g, π_b), these properties therefore implies that, in any equilibrium,

- if $\pi_b > 0$, which implies that $G_b\left((z_R^L(\pi_g, \pi_b), 0), (0, z_f^S(\pi_g, \pi_b))\right) \geq 0$, then we also have $G_g\left((z_R^L(\pi_g, \pi_b), 0), (0, z_f^S(\pi_g, \pi_b))\right) > 0$ and thus $\pi_g = 1$;
- if $\pi_g < 1$, which implies that $G_g\left((z_R^L(\pi_g, \pi_b), 0), (0, z_f^S(\pi_g, \pi_b))\right) \leq 0$, then we also have $G_b\left((z_R^L(\pi_g, \pi_b), 0), (0, z_f^S(\pi_g, \pi_b))\right) < 0$ and thus $\pi_b = 0$.

This implies that any equilibrium must be one of the three types of equilibrium stated in this lemma. Formally, the three types of equilibrium stated in this lemma are defined as follows, having used the fact that $z_R^S = z_f^L = 0$ for all (π_g, π_b):

- The informative equilibrium, i.e. one with $\pi_g = 1$ and $\pi_b = 0$, exists if and only if

$$G_g\left((z_R^L(1,0), 0), (0, z_f^S(1,0))\right) > 0 > G_b\left((z_R^L(1,0), 0), (0, z_f^S(1,0))\right). \tag{20}$$

- A tough equilibrium, i.e., one with $\pi_g < 1$ and $\pi_b = 0$, exists if and only if there exists $\pi_g \in [0, 1)$ such that

$$G_g\left((z_R^L(\pi_g,0), 0), (0, z_f^S(\pi_g,0))\right) \leq 0 \text{ and } G_b\left((z_R^L(\pi_g,0), 0), (0, z_f^S(\pi_g,0))\right) < 0. \tag{21}$$

- A soft equilibrium, i.e., one with $\pi_g = 1$ and $\pi_b > 0$, exists if and only if there exists $\pi_b \in (0, 1]$ such that

$$G_g\left((z_R^L(\pi_g,0), 0), (0, z_f^S(\pi_g,0))\right) > 0 \text{ and } G_b\left((z_R^L(\pi_g,0), 0), (0, z_f^S(\pi_g,0))\right) \geq 0. \tag{22}$$

∎

A.3 Proof of Proposition 2 (First-period equilibrium for $X \in [\underline{X}, \overline{X}]$ is informative)

Let $\underline{X}$ be defined such that $U_R = U_0$, i.e.,

$$[\alpha + (1-\alpha)(1-d)]\, R - 1 + \underline{X} = R_0 - 1. \tag{23}$$

Let $\overline{X}$ be defined such that $(1-d)U_R = [(1-d) + d(1-\gamma)]\, U_0$, i.e.,

$$(1-d)\left([\alpha + (1-\alpha)(1-d)]\, R - 1 + \overline{X}\right) = [(1-d) + d(1-\gamma)]\,(R_0 - 1). \tag{24}$$

It is straightforward to show that $\overline{X} > \underline{X}$.

We now consider the case in which $X \in [\underline{X}, \overline{X}]$. Notice that $X \geq \underline{X}$ and the fact that $z_f^L = z_R^S = 0$ for all (π_g, π_b) imply that $U_R \geq U(z_R^L(\pi_g, \pi_b), 0) \geq U(0, z_f^S(\pi_g, \pi_b)) \geq U_0$. It follows that:

$$\begin{aligned} G_g\left((z_R^L(\pi_g,\pi_b),0),(0,z_f^S(\pi_g,\pi_b))\right) &= (1-\gamma)(\underline{\rho}-1) + \delta\left[U(z_R^L(\pi_g,\pi_b),0) - U(0, z_f^R(\pi_g,\pi_b))\right] \\ &\geq (1-\gamma)(\underline{\rho}-1) > 0. \end{aligned} \tag{25}$$

Moreover, they also imply that:

$$\begin{aligned} &G_b\left((z_R^L(\pi_g,\pi_b),0),(0,z_f^S(\pi_g,\pi_b))\right) \\ =&(1-\gamma)\left[(\underline{\rho}-1) - dD\right] + \delta\left[(1-d)U(z_R^L(\pi_g,\pi_b),0) - [(1-d)+d(1-\gamma)]\, U(0,z_f^S(\pi_g,\pi_b))\right] \\ \leq&(1-\gamma)\left[(\underline{\rho}-1) - dD\right] + \delta\left[(1-d)U_R - [(1-d)+d(1-\gamma)]\, U_0\right] < 0, \end{aligned} \tag{26}$$

where the second inequality follows because of Assumption 4 and the fact that $X \leq \overline{X}$.

Since $G_g\left((z_R^L(\pi_g,\pi_b),0),(0,z_f^S(\pi_g,\pi_b))\right) > 0 > G_b\left((z_R^L(\pi_g,\pi_b),0),(0,z_f^S(\pi_g,\pi_b))\right)$ for all (π_g, π_b), in this case, there exists a unique equilibrium in which the strategic regulator passes the bank in the first period if and only if the risky loan is good. ∎

A.4 Proof of Proposition 3 (First-period equilibrium for $X < \underline{X}$ can be informative or tough)

Before we proceed to prove this proposition, it is useful to define a number of thresholds. Let us define $\bar{\beta}$ such that $\Delta(0,0) = 0$, i.e.,

$$\begin{aligned}[\alpha + (1-\alpha)(1-d)]\,(R-1) - (R_0 - 1) \\ - (1-\gamma)(1-\alpha)\left((\underline{\rho} - 1) + \bar{\beta}\left[(1-d)R - \underline{\rho}\right] + d\right) = 0.\end{aligned} \tag{27}$$

We can show that $\bar{\beta} > \underline{\beta}$ exists and is unique, because i) $\Delta(z_2^L, z_2^S)$ is strictly decreasing in β for all $\beta \in \mathbb{R}$, ii) $\Delta(0,0)$ is strictly positive at $\beta = \underline{\beta}$, as $\underline{\beta}$ is defined such that $\Delta(0,1) = 0$, and iii) $\Delta(0,0)$ is strictly negative as $\bar{\beta} \to \infty$.

We can now define a threshold of the prior reputation that the regulator is lenient, $\bar{z}_1^L(z_1^S) \in [0, 1 - z_1^S)$, such that $\Delta(z_R^L(1,0), 0) \geq 0$ for all $z_1^L \geq \bar{z}_1^L(z_1^S)$:

- If $\beta < \bar{\beta}$, $\bar{z}_1^L(z_1^S) = 0$ for all $z_1^L \in [0,1)$. This is because, in this case, for all $z_1^L \geq 0$, we have $\Delta(z_R^L(1,0), 0) \geq \Delta(0,0) > 0$.

- If $\beta \geq \bar{\beta}$, $\bar{z}_1^L(z_1^S)$ is defined by

$$\begin{aligned}[\alpha + (1-\alpha)(1-d)]\,(R-1) - (R_0 - 1) \\ - (1-\gamma)(1-\alpha)(1 - \frac{[\alpha + (1-\alpha)(1-d)]\,\bar{z}_1^L(z_1^S)}{[\alpha + (1-\alpha)(1-d)]\,\bar{z}_1^L(z_1^S) + \alpha(1 - \bar{z}_1^L(z_1^S) - z_1^S)}) \\ \times\left((\bar{\rho} - 1) + \beta\left[(1-d)R - \bar{\rho}\right] + d\right) = 0.\end{aligned} \tag{28}$$

  In this case, $\bar{z}_1^L(z_1^S) \in [0, 1 - z_1^S)$ exists and is unique because the left-hand side of the above expression i) is strictly increasing in $\bar{z}_1^L$ for all $\bar{z}_1^L \in [0, 1 - z_1^S)$, ii) is strictly positive for $\bar{z}_1^L = 1 - z_1^S$, as it coincides with $\Delta(1,0)$, and iii) is negative for $\bar{z}_1^L = 0$ and strictly negative if and only if $\beta > \bar{\beta}$, as it coincides with $\Delta(0,0)$.

Analogously, let us define a threshold of the prior reputation that the regulator is strict, $\bar{z}_1^S(z_1^L) \in [0, 1 - z_1^L)$, such that $\Delta(0, z_f^S(1,0)) \leq 0$ for all $z_1^S \geq \bar{z}_1^S(z_1^L)$:

- If $\beta \leq \bar{\beta}$, $\bar{z}_1^S(z_1^L)$ is defined by

$$
\begin{aligned}
&[\alpha + (1-\alpha)(1-d)]\,(R-1) - (R_0 - 1) \\
&\quad - (1-\gamma)\left[\alpha \frac{\bar{z}_1^S(z_1^L)}{\bar{z}_1^S(z_1^L) + (1-\alpha)(1-z_1^L-\bar{z}_1^S(z_1^L))} + (1-\alpha)\right] \\
&\qquad \times \left((\underline{\rho}-1) + \beta \left[\frac{\alpha \frac{\bar{z}_1^S(z_1^L)}{\bar{z}_1^S(z_1^L)+(1-\alpha)(1-z_1^L-\bar{z}_1^S(z_1^L))} + (1-\alpha)(1-d)}{\alpha \frac{\bar{z}_1^S(z_1^L)}{\bar{z}_1^S(z_1^L)+(1-\alpha)(1-z_1^L-\bar{z}_1^S(z_1^L))} + (1-\alpha)} R - \underline{\rho} \right] \right. \\
&\qquad\qquad \left. + \frac{(1-\alpha)d}{\alpha \frac{\bar{z}_1^S(z_1^L)}{\bar{z}_1^S(z_1^L)+(1-\alpha)(1-z_1^L-\bar{z}_1^S(z_1^L))} + (1-\alpha)} \right) = 0. \qquad (29)
\end{aligned}
$$

  In this case, $\bar{z}_1^L(z_1^S) \in [0, 1 - z_1^L)$ exists and is unique, because the left-hand side of the above expression i) is strictly decreasing in $\bar{z}_1^S$ for all $\bar{z}_1^S \in [0, 1 - z_1^L)$, ii) is strictly negative for $\bar{z}_1^S = 1 - z_1^L$ by Assumption 3, as it coincides with $\Delta(0, 1)$, and iii) is positive for $\bar{z}_1^S = 0$ and strictly positive if and only if $\beta < \bar{\beta}$, as it coincides with $\Delta(0, 0)$.

- If $\beta > \bar{\beta}$, $\bar{z}_1^S(z_1^L) = 0$ for all $z_1^L \in [0, 1)$. This is because, in this case, for all $z_1^S \geq 0$, $\Delta(0, z_f^S(1, 0)) \leq \Delta(0, 0) < 0$.

We now proceed to prove this proposition. First, we show that the equilibrium is either informative or tough, i.e., $\pi_b = 0$. Recall that $\underline{X}$ is defined such that $U_R = U_0$ (and is given by Eq. 23), and $z_f^L = z_R^S = 0$ for all (π_g, π_b). Therefore, for all $X < \underline{X}$, we have $U_R \leq U(z_R^L(\pi_g, \pi_b), 0) \leq U(0, z_f^S(\pi_g, \pi_b)) \leq U_0$, with at least one strict inequality. It follows that:

$$
\begin{aligned}
&G_b\left((z_R^L(\pi_g, \pi_b), 0), (0, z_f^S(\pi_g, \pi_b))\right) \\
=&(1-\gamma)\left[(\underline{\rho}-1) - dD\right] + \delta(1-d)\left[U(z_R^L(\pi_g, \pi_b), 0) - U(0, z_f^S(\pi_g, \pi_b))\right] \\
&\qquad - \delta d(1-\gamma) U(0, z_f^S(\pi_g, \pi_b)) < 0. \qquad (30)
\end{aligned}
$$

This then implies that $\pi_b = 0$.

We next characterize the two possible types of equilibria: the informative equilibrium (i.e., one with $\pi_g = 1$) exists if and only if $G_g((z_R^L(1, 0), 0), (0, z_f^S(1, 0))) \geq 0$, and a tough equilibrium (i.e., one with $\pi_g < 1$) exists if and only if $G_g((z_R^L(\pi_g, 0), 0), (0, z_f^S(\pi_g, 0))) \leq 0$ for some $\pi_g < 1$. It is useful to notice that $\Delta(z_R^L(\pi_g, 0), 0)$ and $\Delta(0, z_f^S(\pi_g, 0))$ are both strictly decreasing in π_g. This follows because i) $\Delta(z_2^L, z_2^S)$ is strictly increasing in z_2^L and strictly decreasing in z_2^S, and ii)

$z_R^L(\pi_g, 0)$ is strictly decreasing in π_g and $z_f^S(\pi_g, 0)$ is strictly increasing in π_g.

- Suppose $\beta \geq \bar{\beta}$ and $z_1^L \leq \bar{z}_1^L(z_1^S)$. This case is illustrated in Fig. 2a (here, we impose the equilibrium condition $\hat{\pi}_g = \pi_g$): After passing the stress test in the first period, the bank prefers to originate a risky loan if and only if π_g is sufficiently low. That is, there exists a unique $\bar{\pi}_g \in (0, 1]$, such that $\Delta(z_R^L(\pi_g, 0), 0) \geq 0$ if and only if $\pi_g \leq \bar{\pi}_g$. This follows because $\Delta(z_R^L(\pi_g, 0), 0)$ is strictly decreasing in π_g and $\Delta(z_R^L(0, 0), 0) = \Delta(1, 0) > 0 \geq \Delta(z_R^L(1, 0), 0)$, where the first inequality follows from Assumption 1 and the second inequality follows from $z_1^L \leq \bar{z}_1^L(z_1^S)$. After failing the stress test in the first period, the bank strictly prefers to invest in the safe asset in the second period, i.e., $\Delta(0, z_f^S(\pi_g, 0)) < \Delta(0, 0) \leq 0$ for all $\pi_g \in [0, 1]$, where the last inequality follows from $\beta \geq \bar{\beta}$. Using Eq. 8, we then have

$$
\begin{aligned}
&G_g((z_R^L(\pi_g, 0), 0), (0, z_f^S(\pi_g, 0))) \\
=&(1-\gamma)(\underline{\rho}-1) + \begin{cases} 0, & \text{if } \pi_g > \bar{\pi}_g, \\ (1-\lambda)\delta\left(U_R - U_0\right) \text{ for some } \lambda \in [0,1], & \text{if } \pi_g = \bar{\pi}_g, \\ \delta\left(U_R - U_0\right), & \text{if } \pi_g < \bar{\pi}_g. \end{cases}
\end{aligned} \tag{31}
$$

  Note that the first term of the above expression is strictly positive by Assumption 4. From this expression, we have that, first, the informative equilibrium with $\pi_g = 1$ exists for all δ. Second, consider the tough equilibria. Recall that the definition of $\underline{X}$, which is given by Eq. 23, implies that $U_R < U_0$ for all $X < \underline{X}$. A tough equilibrium with $\pi_g < 1$ thus exists if and only if $\delta \geq \delta_g$, where δ_g is defined by

$$
(1-\gamma)(\underline{\rho}-1) + \delta_g\left(U_R - U_0\right) = 0. \tag{32}
$$

  More specifically, if $\delta > \delta_g$ and $\beta > \bar{\beta}$, which implies that $\bar{\pi}_g < 1$, then two tough equilibria exist with $\pi_g = \bar{\pi}_g$ and $\pi_g = 0$; if $\delta > \delta_g$ and $\beta = \bar{\beta}$, then $\bar{\pi}_g = 1$ and only one tough equilibrium exists with $\pi_g = 0$; if $\delta = \delta_g$, then a continuum of equilibria with $\pi_g \in [0, \bar{\pi}_g]$ exist.

- Suppose $\beta \geq \bar{\beta}$ and $z_1^L > \bar{z}_1^L(z_1^S)$, or $\beta \leq \bar{\beta}$ and $z_1^S > \bar{z}_1^S(z_1^L)$. This case is illustrated in

Fig. 2b (here, we impose the equilibrium condition $\hat{\pi}_g = \pi_g$): After passing the stress test in the first period, the bank strictly prefers to originate a risky loan, i.e., $\Delta(z_R^L(\pi_g, 0), 0) \geq \Delta(z_R^L(1, 0), 0) > 0$ for all $\pi_g \in [0, 1]$, where the first inequality follows because $\Delta(z_R^L(\pi_g, 0), 0)$ is decreasing in π_g and the second inequality holds for all $z_1^L > \bar{z}_1^L(z_1^S)$ if $\beta \geq \bar{\beta}$ and for all $z_1^L \in (0, 1)$ if $\beta < \bar{\beta}$. After failing the stress test in the first period, the bank prefers to originate a risky loan if and only if π_g is sufficiently low. That is, there exists a threshold $\underline{\pi}_g < 1$, such that $\Delta(0, z_f^S(\pi_g, 0)) \geq 0$ if and only if $\pi_g \leq \underline{\pi}_g$. This follows because $\Delta(0, z_f^S(\pi_g, 0))$ is strictly decreasing in π_g and $\Delta(0, z_f^S(1, 0)) < 0$ holds for all $z_1^S \in (0, 1)$ if $\beta > \bar{\beta}$ and for all $z_1^S > \bar{z}_1^S(z_1^L)$ if $\beta \leq \bar{\beta}$. Notice that, the threshold may be strictly negative, in which case we have $\pi_g > \underline{\pi}_g$ for all $\pi_g \in [0, 1]$. Specifically, $\underline{\pi}_g \geq 0$ if and only if $\Delta(0, z_f^S(0, 0)) \geq 0$. Using Eq. 8, we then have

$$\begin{aligned} &G_g((z_R^L(\pi_g, 0), 0), (0, z_f^S(\pi_g, 0))) \\ =&(1-\gamma)(\underline{\rho} - 1) + \begin{cases} \delta\left(U_R - U_0\right), & \text{if } \pi_g > \underline{\pi}_g, \\ (1-\lambda)\delta\left(U_R - U_0\right) \text{ for some } \lambda \in [0, 1], & \text{if } \pi_g = \underline{\pi}_g, \\ 0, & \text{if } \pi_g < \underline{\pi}_g. \end{cases} \end{aligned} \tag{33}$$

From this expression, we have that, first, if $\delta < \delta_g$, the unique equilibrium is informative; second, if $\delta > \delta_g$, the unique equilibrium is tough with $\pi_g = \max\{\underline{\pi}_g, 0\}$; third, if $\delta = \delta_g$, then a continuum of equilibria with $\pi_g \in \left[\max\{\underline{\pi}_g, 0\}, 1\right]$ exist.

- Suppose $\beta \leq \bar{\beta}$ and $z_1^S \leq \bar{z}_1^S(z_1^L)$. This case is illustrated in Fig. 2c (here, we impose the equilibrium condition $\hat{\pi}_g = \pi_g$): After both passing and failing the stress test in the first period, the bank prefers to originate a risky loan, i.e., $\Delta(z_R^L(\pi_g, 0), 0) > \Delta(0, z_f^S(\pi_g, 0)) \geq \Delta(0, z_f^S(1, 0)) \geq 0$ for all $\pi_g \in [0, 1]$, where the second inequality follows because $\Delta(0, z_f^S(\pi_g, 0))$ is strictly decreasing in π_g, and the last inequality follows from $z_1^S \leq \bar{z}_1^S(z_1^L)$. Using Eq. 8, we then have

$$
\begin{aligned}
&G_g((z_R^L(\pi_g,0),0),(0,z_f^S(\pi_g,0)))\\
=&(1-\gamma)(\underline{\rho}-1)+\begin{cases}(1-\lambda)\delta\left(U_R-U_0\right) \text{ for some } \lambda\in[0,1], & \text{if } \pi_g=1 \text{ and } z_1^S=\bar{z}_1^S(z_1^L),\\ 0, & \text{if } \pi_g<1 \text{ or } z_1^S<\bar{z}_1^S(z_1^L).\end{cases}
\end{aligned}
\tag{34}
$$

From this expression, we have that the unique equilibrium is informative with $\pi_g = 1$.

∎

A.5 Proof of Proposition 4 (First-period equilibrium for $X > \overline{X}$ can be informative or soft)

Since $X > \overline{X} > \underline{X}$, we have $G_g(z_2^R(\pi_g, \pi_b), 0) > 0$ for all (π_g, π_b), as shown in the proof of Proposition 2. Therefore, $\pi_g = 1$ in any equilibrium, i.e., the equilibrium is either informative or soft.

We next characterize the two possible types of equilibria: the informative equilibrium (i.e., one with $\pi_b = 0$) exists if and only if $G_b((z_R^L(1,0),0),(0,z_f^S(1,0))) \leq 0$, and a soft equilibrium (i.e., one with $\pi_b > 0$) exists if and only if $G_b((z_R^L(1,\pi_b),0),(0,z_f^S(1,\pi_b))) \geq 0$ for some $\pi_b \in (0,1]$. It is useful to notice that $\Delta(z_R^L(1,\pi_b),0)$ and $\Delta(0,z_f^S(1,\pi_b))$ are both strictly decreasing in π_b. This follows because i) $\Delta(z_2^L, z_2^S)$ is strictly increasing in z_2^L and strictly decreasing in z_2^S, and ii) $z_R^L(1,\pi_b)$ is strictly decreasing in π_g and $z_f^S(1,\pi_b)$ is strictly increasing in π_b.

- Suppose $\beta \geq \bar{\beta}$ and $z_1^L \leq \bar{z}_1^L(z_1^S)$. This case is illustrated in Fig. 3a (here, we impose the equilibrium condition $\hat{\pi}_b = \pi_b$): After both passing and failing the stress test in the first period, the bank prefers to originate a risky loan, i.e., $\Delta(0,z_f^S(1,\pi_b)) < \Delta(z_R^L(1,\pi_b),0) \leq \Delta(z_R^L(1,0),0) \leq 0$ for all $\pi_b \in [0,1]$, where the second inequality follows because $\Delta(z_R^L(1,\pi_b),0)$ is strictly decreasing in π_b and the last inequality follows from $z_1^L \leq \bar{z}_1^L(z_1^S)$. Using Eq. 8, we have that,

$$G_b((z_R^L(1,\pi_b),0),(0,z_f^S(1,\pi_b)))$$

$$
\begin{aligned}
=&(1-\gamma)\left[(\underline{\rho}-1)-dD\right] \\
&+\begin{cases}
\delta\left[-d(1-\gamma)U_0\right], & \text{if } \pi_b>0 \text{ or } z_1^L<\bar{z}_1^L(z_1^S), \\
\lambda\delta\left[(1-d)U_R-\left[(1-d)+d(1-\gamma)\right]U_0\right] & \\
\quad+(1-\lambda)\delta\left[-d(1-\gamma)U_0\right] \text{ for some } \lambda\in[0,1], & \text{if } \pi_b=0 \text{ and } z_1^L=\bar{z}_1^L(z_1^S),
\end{cases}
\end{aligned}
\tag{35}
$$

where the first term of the above expression is strictly negative by Assumption 4. From this expression, we have that the unique equilibrium is informative with $\pi_b=0$.

- Suppose $\beta\geq\bar{\beta}$ and $z_1^L>\bar{z}_1^L(z_1^S)$, or $\beta\leq\bar{\beta}$ and $z_1^S>\bar{z}_1^S(z_1^L)$. This case is illustrated in Fig. 3b (here, we impose the equilibrium condition $\hat{\pi}_b=\pi_b$): After passing the stress test in the first period, the bank prefers to originate a risky loan if and only if π_b is sufficiently low. That is, there exists $\bar{\pi}_b>0$, such that $\Delta(z_R^L(1,\pi_b),0)\geq 0$ if and only if $\pi_b\leq\bar{\pi}_b$. This follows because $\Delta(z_R^L(1,\pi_b),0)$ is strictly decreasing in π_b and $\Delta(z_R^L(1,0),0)>0$ holds for all $z_1^L>\bar{z}_1^L(z_1^S)$ if $\beta\geq\bar{\beta}$ and for all $z_1^L\in(0,1)$ if $\beta<\bar{\beta}$. Notice that, the threshold may be strictly above 1, in which case we have $\pi_b<\bar{\pi}_b$ for all $\pi_b\in[0,1]$. Specifically, $\bar{\pi}_b\leq 1$ if and only if $\Delta(z_R^L(1,1),0)\leq 0$. After failing the stress test in the first period, the bank strictly prefers to invest in the risky asset, i.e., $\Delta(0,z_f^S(1,\pi_b))\leq\Delta(0,z_f^S(1,0))<0$ for all $\pi_b\in[0,1]$, where the first inequality follows because $\Delta(0,z_f^S(1,\pi_b))$ is strictly decreasing in π_b and the second inequality holds for all $z_1^S\in(0,1)$ if $\beta>\bar{\beta}$ and for all $z_1^S>\bar{z}_1^S(z_1^L)$ if $\beta\leq\bar{\beta}$. Using Eq. 8, we then have

$$
\begin{aligned}
&G_b((z_R^L(1,\pi_b),0),(0,z_f^S(1,\pi_b))) \\
=&(1-\gamma)\left[(\underline{\rho}-1)-dD\right] \\
&+\begin{cases}
\delta\left[-d(1-\gamma)U_0\right], & \text{if } \pi_b>\bar{\pi}_b, \\
\lambda\delta\left[(1-d)U_R-\left[(1-d)+d(1-\gamma)\right]U_0\right] & \\
\quad+(1-\lambda)\delta\left[-d(1-\gamma)U_0\right] \text{ for some } \lambda\in[0,1], & \text{if } \pi_b=\bar{\pi}_b, \\
\delta\left[(1-d)U_R-\left[(1-d)+d(1-\gamma)\right]U_0\right], & \text{if } \pi_b<\bar{\pi}_b,
\end{cases}
\end{aligned}
\tag{36}
$$

Notice that the first term of the above expression is negative by Assumption 4. Recall that the definition of $\overline{X}$, which is given by Eq. 24, implies that $(1-d)U_R > [(1-d)+d(1-\gamma)]\,U_0$ for all $X > \overline{X}$. Let us therefore define $\delta_b > 0$ by:

$$(1-\gamma)\left[(\underline{\rho}-1)-dD\right] + \delta_b\left[(1-d)U_R - [(1-d)+d(1-\gamma)]\,U_0\right] = 0. \tag{37}$$

From these expressions, we have that, first, if $\delta < \delta_b$, the unique equilibrium is informative; second, if $\delta > \delta_b$, the unique equilibrium is soft with $\pi_b = \min\{\bar{\pi}_b, 1\}$; third, if $\delta = \delta_b$, then a continuum of equilibria with $\pi_b \in [0, \min\{\bar{\pi}_b, 1\}]$ exist.

- Suppose $\beta \leq \bar{\beta}$ and $z_1^S \leq \bar{z}_1^S(z_1^L)$. This case is illustrated in Fig. 3c (here, we impose the equilibrium condition $\hat{\pi}_b = \pi_b$): After passing the stress test in the first period, the bank strictly prefers to invest in the risky asset, i.e., $\Delta(z_R^L(1,\pi_b),0) > \Delta(0,0) \geq 0$ for all $\pi_b \in [0,1]$, where the last inequality follows from $\beta \leq \bar{\beta}$. After failing the stress test, the bank prefers to originate a risky loan if and only if π_b is low. That is, there exists $\underline{\pi}_b \in [0,1)$, such that $\Delta(0, z_f^S(1,\pi_b)) \geq 0$ if and only if $\pi_b \leq \underline{\pi}_b$. This follows because $\Delta(0, z_f^S(1,\pi_b))$ is strictly decreasing in π_g and $\Delta(0, z_f^S(1,0)) \geq 0 > \Delta(0,1) = \Delta(0, z_f^S(1,1))$, where the first inequality follows from $z_1^S \leq \bar{z}_1^S(z_1^L)$ and the second inequality follows from Assumption 5. Using Eq. 8, we then have

$$\begin{aligned}
&G_b((z_R^L(1,\pi_b),0),(0,z_f^S(1,\pi_b)))\\
=&(1-\gamma)\left[(\underline{\rho}-1)-dD\right]\\
&+\begin{cases}
\delta\left[(1-d)U_R - [(1-d)+d(1-\gamma)]\,U_0\right], & \text{if } \pi_b > \underline{\pi}_b,\\[2ex]
\lambda\delta\left[(1-d)U_R - [(1-d)+d(1-\gamma)]\,U_0\right] & \\
\quad +(1-\lambda)\delta\left[-d(1-\gamma)U_R\right] \text{ for some } \lambda \in [0,1], & \text{if } \pi_b = \underline{\pi}_b,\\[2ex]
\delta\left[-d(1-\gamma)U_R\right], & \text{if } \pi_b < \underline{\pi}_b.
\end{cases}
\end{aligned} \tag{38}$$

From this expression, we have that, first, the informative equilibrium with $\pi_b = 0$ exists for all δ. Second, consider the soft equilibria. A soft equilibrium with $\pi_b > 0$ thus exists if and only if $\delta \geq \delta_b$. More specifically, if $\delta > \delta_b$ and $\beta < \bar{\beta}$, which implies that $\underline{\pi}_b > 0$, then two soft equilibria exist with $\pi_b = 1$ and $\pi_b = \underline{\pi}_b$; if $\delta > \delta_b$ and $\beta = \bar{\beta}$, then $\underline{\pi}_b = 0$ and only one soft equilibrium exists with $\pi_b = 1$; if $\delta = \delta_b$, then a continuum of equilibria with $\pi_b \in [\underline{\pi}_b, 1]$ exist.

∎

A.6 Proof of Proposition 5 (Multiplicity and inefficiency)

Before we proceed to prove this proposition, it is useful to define the following objects. Let us denote by $V_p(z_R^L, z_R^S)$ and $V_f(z_f^L, z_f^S)$ the strategic regulator's expected surplus over two periods if it adopts a strategy of passing or failing the bank in the first period with certainty, respectively, taking as given the bank's posterior beliefs in the second period:

$$
\begin{aligned}
V_p(z_R^L, z_R^S) = & \left[\alpha + (1-\alpha)(1-d)\right] R - 1 - (1-\alpha) dD \\
& + \delta \left[\alpha + (1-\alpha)(1-d)\right] U(z_R^L, z_R^S), \qquad (39) \\
V_f(z_f^L, z_f^S) = & \left[\alpha + (1-\alpha)(1-d)\right] R - \left[\gamma + (1-\gamma)\underline{\rho}\right] - (1-\alpha)\gamma dD \\
& + \delta \left(\alpha + (1-\alpha)\left[(1-d) + d(1-\gamma)\right]\right) U(z_f^L, z_f^S). \qquad (40)
\end{aligned}
$$

In each of the above expressions, the first line of the expression represents the expected surplus of the bank in the first period, which consists of the expected payoff of its risky loan, minus the expected cost of funding that includes the probability of successful recapitalization, and minus the expected cost of bank default. The second line captures the expected surplus of the bank in the second period, taking as given the regulator's reputation that depends on the stress test results in the first period.

We now proceed to compute the equilibrium surplus.

- Consider first the case with $X < \underline{X}$, in which the equilibrium is either informative or tough. Therefore for any equilibrium stress testing strategy $(\pi_g, 0)$, we can express surplus as follows:

$$
V(\pi_g, 0) = V_f(0, z_f^S(\pi_g, 0)) + \alpha \pi_g G_g\left(z_R^L(\pi_g, 0), 0), (0, z_f^S(\pi_g, 0))\right). \qquad (41)
$$

The first term represents the expected surplus for the strategic regulator over two periods if the regulator fails the bank in the first period with certainty. The second term then captures the expected net gain from passing the bank in the first period with a good risky loan with probability π_g. This expression takes into account the regulator's equilibrium reputation, with $z_R^S = z_f^L = 0$.

Notice that $V_f(0, z_f^S(\pi_g, 0))$ is increasing in π_g, because $V_f(z_f^L, z_f^S)$ is increasing in z_f^S and $z_f^S(\pi_g, 0)$ is increasing in π_g. We then have, equilibrium surplus is higher in the informative equilibrium with $\pi_g = 1$ than in a tough equilibrium with $\pi_g = \pi_g^* < 1$:

$$\begin{aligned} V(1,0) &= V_f(0, z_f^S(1,0)) + \alpha G_g\left(z_R^L(1,0),0),(0,z_f^S(1,0))\right) \\ &\geq V_f(0, z_f^S(\pi_g^*,0)) + \alpha \pi_g^* G_g\left(z_R^L(\pi_g^*,0),0),(0,z_f^S(\pi_g^*,0))\right) = V(\pi_g^*,0). \end{aligned} \tag{42}$$

The inequality follows because i) $V_f(0, z_f^S(1,0)) \geq V_f(0, z_f^S(\pi_g^*, 0))$, and ii) in the informative equilibrium with $\pi_g^* = 1$, we have $G_g\left(z_R^L(1,0),0),(0,z_f^S(1,0))\right) \geq 0$, whereas in a tough equilibrium with $\pi_g^* \in [0,1)$, we have $G_g\left(z_R^L(\pi_g^*,0),0),(0,z_f^S(\pi_g^*,0))\right) \leq 0$.

- Consider next the case with $X > \overline{X}$, in which the equilibrium is either informative or soft. Therefore, for any equilibrium stress testing strategy $(1, \pi_b)$, we can express surplus as follows:

$$V(1,\pi_b) = V_p(z_R^L(1,\pi_b),0) - (1-\alpha)(1-\pi_b)G_b\left(z_R^L(1,\pi_b),0),(0,z_f^S(1,\pi_b))\right).$$

The first term represents the expected surplus for the strategic regulator over two periods if the regulator passes the bank in the first period with certainty. The second term then captures the expected net loss from failing the bank in the first period with a bad risky loan with probability $(1 - \pi_b)$. This expression takes into account the regulator's equilibrium reputation, with $z_R^S = z_f^L = 0$.

Notice that $V_p(z_R^L(1, \pi_b), 0)$ is decreasing in π_b, because $V_p(z_R^L, z_R^S)$ is increasing in z_R^L, and $z_R^L(1, \pi_b)$ decreasing in π_b. We then have, equilibrium surplus is higher in the informative equilibrium with $\pi_b = 0$ than in a soft equilibrium with $\pi_b = \pi_b^* > 0$:

$$\begin{aligned} V(1,0) &= V_p(z_R^L(1,0),0) - (1-\alpha)G_b\left(z_R^L(1,0),0),(0,z_f^S(1,0))\right) \\ &\geq V_p(z_R^L(1,\pi_b^*),0) - (1-\alpha)(1-\pi_b^*)G_b\left(z_R^L(1,\pi_b^*),0),(0,z_f^S(1,\pi_b^*))\right) = V(1,\pi_b^*). \end{aligned} \tag{43}$$

The inequality follows because i) $V_p(z_R^L(1,0),0) \geq V_p(z_R^L(1,\pi_b^*),0)$, and ii) in the informative equilibrium with $\pi_b^* = 0$, we have $G_b\left(z_R^L(1,0),0),(0,z_f^S(1,0))\right) \leq 0$, whereas in a soft equilibrium with $\pi_b^* > 0$, we have $G_b\left(z_R^L(1,\pi_b^*),0),(0,z_f^S(1,\pi_b^*))\right) \geq 0$.

To summarize, whenever the informative equilibrium coexists with a tough or a soft equilibrium, surplus is higher in the informative equilibrium. ∎

A.7 Proof of Corollary 1 (Availability of capital)

This corollary follows immediately from Propositions 3–4, the implicit function theorem, and the observation that $G_g(\cdot)$ is decreasing in γ_1, and $G_b(\cdot)$ is increasing in γ_1. ∎

A.8 Proof of Corollary 2 (Stress test of systemic banks)

This corollary follows immediately from Propositions 3–4, the implicit function theorem, and the observation that $G_g(\cdot)$ is independent of D_1, and $G_b(\cdot)$ is decreasing in D_1. ∎

A.9 Proof of Proposition 6 (One-period model with commitment)

We characterize the strategic regulator's optimal stress testing strategy.

- Suppose $X \leq \underline{X}$. This implies that $U_0 \geq U_R \geq \tilde{U}_R(\pi_g,\pi_b)$ for all (π_g,π_b).

  Notice that $\tilde{\Delta}(0,0) = \Delta(0,1) < 0$, where the inequality follows from Assumption 5. This implies that inducing the bank to invest in the safe asset is always feasible. Therefore the strategic regulator's optimal strategy is $(\pi_g,0)$ for any $\pi_g \leq 1$ such that $\tilde{\Delta}(\pi_g,0) \leq 0$, so that the bank invests in the safe asset, resulting in $\tilde{U}^* = U_0$. Notice that, in all equilibria, the regulator's stress test is off-equilibrium, since the bank does not originate a risky loan.

  Due to the presence of multiple equilibria that differ only in the strategic regulator's stress test off-the-equilibrium-path, we apply the trembling-hand refinement. Suppose the bank originates a risky loan with probability ϵ. Since the expected surplus from a risky loan, $\tilde{U}_R(\pi_g,0)$, is strictly increasing in π_g, as $\epsilon \to 0$, the strategic regulator's unique optimal stress testing strategy is $(\pi_g^*,0)$, where $\pi_g^* \in [0,1]$ is the maximum value such that $\tilde{\Delta}(\pi_g,0) \leq 0$. This implies that, if $\tilde{\Delta}(1,0) \leq 0$, then the strategic regulator's uniquely optimal trembling-hand perfect strategy is $(\pi_g^*,\pi_b^*) = (1,0)$; if $\tilde{\Delta}(1,0) > 0$, then the strategy regulator's uniquely optimal trembling-hand perfect strategy is $(\pi_g^*, 0)$, where π_g^* is given by $\tilde{\Delta}(\pi_g^*, 0) = 0$. $\pi_g^* \in (0,1)$ exists and is unique, because $\tilde{\Delta}(\pi_g, 0)$ is strictly increasing in π_g and we have $\tilde{\Delta}(0,0) < 0 < \tilde{\Delta}(1,0)$.

To summarize, the strategic regulator's uniquely optimal trembling-hand perfect strategy is $(\pi_g^*, 0)$, where $\pi_g^* \leq 1$.

- Suppose $X > \underline{X}$. This implies that $U_0 < U_R = \tilde{U}_R(1,0)$. Notice that $\tilde{\Delta}(1,1) = \Delta(1,0) > 0$, where the inequality is implied by Assumption 1. This implies that inducing the bank to originate a risky loan is always feasible. Let us consider the following two cases.

  - Suppose $\tilde{U}_R(1,1) \geq U_0$, which implies that $\tilde{U}_R(1,\pi_b) \geq U_0$ for all $\pi_b \in [0,1]$, so that inducing the bank to originate a risky loan is always preferred. Since the expected surplus from a risky loan, $\tilde{U}_R(1,\pi_b)$, is strictly decreasing in π_b, the strategic regulator's optimal strategy is $(1,\pi_b^*)$, where $\pi_b^* \in [0,1]$ is the minimum value such that $\tilde{\Delta}(1,\pi_b) \geq 0$. This implies that, if $\tilde{\Delta}(1,0) \geq 0$, then the strategic regulator's optimal strategy is $(\pi_g^*, \pi_b^*) = (1,0)$; if $\tilde{\Delta}(1,0) < 0$, then the strategic regulator's optimal strategy is $(1,\pi_b^*)$, where π_b^* is given by $\tilde{\Delta}(1,\pi_b^*) = 0$. $\pi_b^* \in (0,1]$ exists and is unique, because $\tilde{\Delta}(1,\pi_b)$ is strictly increasing in π_b and we have $\tilde{\Delta}(1,0) < 0 \leq \tilde{\Delta}(1,1)$. Given the regulator's optimal strategy, the bank originates a risky loan, resulting in $\tilde{U}^* = \tilde{U}_R(1,\pi_b^*)$.
  - Suppose $\tilde{U}_R(1,1) < U_0$. This implies that there exists $\tilde{\pi}_b \in (0,1)$, such that $\tilde{U}_R(1,\tilde{\pi}_b) = U_0$.

    First, consider parameter values such that $\tilde{\Delta}(1,\tilde{\pi}_b) > 0$, so that inducing the bank to originate a risky loan is both preferred by the strategic regulator and feasible. Since the expected surplus from a risky loan, $\tilde{U}_R(1,\pi_b)$, is strictly decreasing in π_b, the strategic regulator's optimal strategy is $(1,\pi_b^*)$, where π_b^* is the minimum value such that $\tilde{\Delta}(1,\pi_b) \geq 0$. This implies that, if $\tilde{\Delta}(1,0) \geq 0$, then the strategic regulator's optimal strategy is $(\pi_g^*, \pi_b^*) = (1,0)$; if $\tilde{\Delta}(1,0) < 0$, then the strategic regulator's optimal strategy is $(1,\pi_b^*)$, where π_b^* is given by $\tilde{\Delta}(1,\pi_b^*) = 0$. $\pi_b^* \in (0,\tilde{\pi}_b]$ exists and is unique, because $\tilde{\Delta}(1,\pi_b)$ is strictly increasing in π_b and we have $\tilde{\Delta}(1,0) < 0 \leq \tilde{\Delta}(1,\tilde{\pi}_b)$. Given the regulator's optimal strategy, the bank originates a risky loan, resulting in $\tilde{U}^* = \tilde{U}_R(1,\pi_b^*)$.

Second, consider parameter values such that $\tilde{\Delta}(1, \tilde{\pi}_b) \leq 0$, so that inducing the bank to originate a risky loan is feasible but not preferred. Therefore the strategic regulator's optimal strategy is any (π_g, π_b) such that $\tilde{\Delta}(\pi_g, \pi_b) \leq 0$, and the bank invests in the safe asset, resulting in $\tilde{U}^* = U_0$. This is because, $\tilde{\Delta}(1, \tilde{\pi}_b) \leq 0$ implies that any (π_g, π_b) such that $\tilde{\Delta}(\pi_g, \pi_b) > 0$ has $\pi_g = 1$ and $\pi_b > \tilde{\pi}_b$, since $\tilde{\Delta}(\pi_g, \pi_b)$ is increasing in π_g and π_b. Yet, for any (π_g, π_b) such that $\tilde{\Delta}(\pi_g, \pi_b) > 0$, this also implies that $\tilde{U}_R(1, \pi_b) < \tilde{U}_R(1, \tilde{\pi}_b) = U_0$. In this case, there are again multiple equilibria that differ only in the strategic regulator's stress test off-equilibrium, since the bank does not originate a risky loan. We therefore apply the trembling-hand refinement. Suppose the bank originates a risky loan with probability ϵ. Since the expected surplus from a risky loan is strictly decreasing in π_g and decreasing in π_b, as $\epsilon \to 0$, the strategic regulator's unique optimal trembling-hand perfect strategy is $(\pi_g^*, \pi_b^*) = (1, 0)$.

To summarize, the strategic regulator's uniquely optimal trembling-hand perfect strategy is $(1, \pi_b^*)$, where $\pi_b^* \geq 0$.

∎

A.10 Proof of Proposition 7 (Bank and regulator both learn asset quality)

As discussed in the text preceding this proposition, the bank's lending behavior in the second period, given the regulator's reputation (z_2^L, z_2^S), is determined by $\Delta(z_2^L, z_2^S)$, which is characterized by Proposition 1.

We now describe the bank's belief updating process. In equilibrium, as in the baseline model, the posterior belief that the regulator is strict, given that the bank passes the stress test in the first period, is $z^S_{R,q_1} = 0$ since the strict regulator always fails the bank; the posterior belief that the regulator is lenient, given that the bank fails the stress test in the first period, is $z^L_{f,q_1} = 0$ since only a lenient regulator always passes the bank. In addition, the posterior probability that the regulator is the strict type, given that the bank had a loan of quality q_1 and fails the first stress test is given by:

$$z^S_{f,q_1}(\pi_{q_1}) = \frac{z_1^S}{z_1^S + (1 - \pi_{q_1})(1 - z_1^L - z_1^S)}, \tag{44}$$

and the posterior probability that the regulator is the lenient type, given that the bank had a loan of quality q_1 and passes the first stress test is given by:

$$z^L_{R,q_1}(\pi_{q_1}) = \frac{z^L_1}{z^L_1 + \pi_{q_1}(1 - z^L_1 - z^S_1)}. \tag{45}$$

Finally, taking the bank's posterior beliefs described above as given, the strategic regulator's incentives to pass the bank with loan quality q_1 in the first period are characterized by $G_{q_1}\left((z^L_{R,q_1}, z^S_{R,q_1}), (z^L_{f,q_1}, z^S_{f,q_1})\right)$, where $G_{q_1}(\cdot)$ is defined by Eq. 9.

Before we proceed to prove this proposition, it is useful to define the following thresholds. First, let us define a threshold of the prior reputation that the regulator is lenient, $\hat{z}^L_1(z^S_1) \in [0, 1 - z^S_1)$, such that $\Delta(z^L_{R,g}(1), 0) = \Delta(z^L_{R,b}(1), 0) \geq 0$ for all $z^L_1 \geq \hat{z}^L_1(z^S_1)$ for all $q_1 \in \{g, b\}$, i.e.,

$$\hat{z}^L_1(z^S_1) \begin{cases} \text{is defined by } \Delta(\frac{\hat{z}^L_1(z^S_1)}{1 - z^S_1}, 0) = 0, & \text{if } \beta \geq \bar{\beta}, \\ = 0, & \text{if } \beta < \bar{\beta}. \end{cases} \tag{46}$$

If $\beta \geq \bar{\beta}$, $\hat{z}^L_1(z^S_1) \in [0, 1 - z^S_1)$ exists and is unique because $\Delta(\frac{\hat{z}^L_1(z^S_1)}{1 - z^S_1}, 0)$ is i) strictly increasing in $\hat{z}^L_1$ for all $\hat{z}^L_1 \in [0, 1 - z^S_1]$, ii) strictly positive for $\hat{z}^L_1 = 1 - z^S_1$, as it coincides with $\Delta(1, 0) > 0$, and iii) negative for $\hat{z}^L_1 = 0$, as it coincides with $\Delta(0, 0)$, and strictly negative if and only if $\beta > \bar{\beta}$.

Recall that the threshold $\bar{z}^L_1(z^S_1)$, defined in the proof of Proposition 3, is such that $\Delta(z^L_R(1, 0), 0) \geq 0$ for all $z^L_1 \geq \bar{z}^L_1(z^S_1)$. Notice that both $\hat{z}^L_1(z^S_1)$ and $\bar{z}^L_1(z^S_1)$ are equal to 0 if and only if $\beta \leq \beta$. This is because both $\Delta(z^L_{R,q_1}(1), 0)$ for all $q_1 \in \{g, b\}$ and $\Delta(z^L_R(1, 0), 0)$ are minimized at $z^L_1 = 0$ when they are equal to $\Delta(0, 0)$, which is non-negative if and only if $\beta \leq \beta$. Moreover, Eqs. 28 and 46 imply that $\hat{z}^L_1(z^S_1) \geq \bar{z}^L_1(z^S_1)$, with strict inequality for all $\beta > \bar{\beta}$.

Next, let us define a threshold of the prior reputation that the regulator is strict, $\hat{z}^S_1(z^L_1) \in [0, 1 - z^L_1)$, such that $\Delta(0, z^S_{f,g}(0)) = \Delta(0, z^S_{f,b}(0)) \leq 0$ for all $z^S_1 \geq \hat{z}^S_1(z^L_1)$, i.e.,

$$\hat{z}^S_1(z^L_1) \begin{cases} \text{is defined by } \Delta(0, \frac{\hat{z}^S_1(z^L_1)}{1 - z^L_1}) = 0, & \text{if } \beta \leq \bar{\beta}, \\ = 0, & \text{if } \beta > \bar{\beta}. \end{cases} \tag{47}$$

If $\beta \leq \bar{\beta}$, $\hat{z}^S_1(z^L_1) \in (0, 1 - z^L_1)$ exists and is unique because $\Delta(0, \frac{\hat{z}^S_1(z^L_1)}{1 - z^L_1})$ is i) strictly decreasing in

$\hat{z}_1^S$ for all $\hat{z}_1^S \in [0, 1 - z_1^L]$, ii) positive for $\hat{z}_1^S = 0$, as it coincides with $\Delta(0,0)$, and strictly positive if and only if $\beta < \bar{\beta}$ and iii) strictly negative for $\hat{z}_1^S = 1$, as it coincides with $\Delta(0,1) < 0$.

Recall that the threshold $\bar{z}_1^S(z_1^L)$, defined in the proof of Proposition 3, is such that $\Delta(0, z_f^S(1,0)) \leq 0$ for all $z_1^S \geq \bar{z}_1^S(z_1^L)$. Notice that both $\hat{z}_1^S(z_1^L)$ and $\bar{z}_1^S(z_1^L)$ are equal to 0 if and only if $\beta \geq \beta$. This is because both $\Delta(0, z_{f,q_1}^S(0))$ for all $q_1 \in \{g, b\}$ and $\Delta(0, z_f^S(1,0))$ are maximized at $z_1^S = 0$ when they are equal to $\Delta(0,0)$, which is non-positive if and only if $\beta \geq \beta$. Moreover, Eqs. 29 and 47 imply that $\hat{z}_1^S(z_1^S) \geq \bar{z}_1^S(z_1^L)$, with strict inequality for all $\beta < \bar{\beta}$.

We now prove this proposition by analyzing the three regions of X separately. It is useful to notice that $\Delta(z_{R,q_1}^L(\pi_{q_1}), 0)$ and $\Delta(0, z_{f,q_1}^S(\pi_{q_1}))$ are both strictly decreasing in π_{q_1}. This follows because i) $\Delta(z_2^L, z_2^S)$ is strictly increasing in z_2^L and strictly decreasing in z_2^S, and ii) $z_{R,q_1}^L(\pi_{q_1})$ is strictly decreasing in π_{q_1} and $z_{f,q_1}^S(\pi_{q_1})$ is strictly increasing in π_{q_1}.

- $X \in [\underline{X}, \overline{X}]$. The proof is identical to the proof of Proposition 2.

- $X < \underline{X}$. The characterization of the equilibrium follows the logic of the proof of Proposition 3. We have that $G_b\left((z_{R,b}^L(\pi_b), 0), (0, z_{f,b}^S(\pi_b))\right) < 0$ for all π_b, and, therefore, $\pi_b = 0$ in any equilibrium. The equilibrium is, thus, either informative or tough. The informative equilibrium (i.e., one with $\pi_g = 1$) exists if and only if $G_g\left((z_{R,g}^L(1), 0), (0, z_{f,g}^S(1))\right) \geq 0$, whereas a tough equilibrium (i.e., one with $\pi_g < 1$) exists if and only if $G_g\left((z_{R,g}^L(\pi_g), 0), (0, z_{f,g}^S(\pi_g))\right) \leq 0$ for some $\pi_g < 1$.

  - Suppose $\beta \geq \bar{\beta}$ and $z_1^L \leq \hat{z}_1^L(z_1^S)$. This case is analogous to that illustrated in Fig. 2a: After passing the stress test in the first period, the bank prefers to originate a risky loan if and only if π_g is sufficiently low. That is, there exists a unique $\check{\pi}_g \in (0, 1]$, such that $\Delta(z_{R,g}^L(\pi_g), 0) \geq 0$ if and only if $\pi_g \leq \check{\pi}_g$. This follows because $\Delta(z_{R,g}^L(\pi_g), 0)$ is strictly decreasing in π_g and $\Delta(z_{R,g}^L(0), 0) = \Delta(1, 0) > 0 \geq \Delta(z_{R,g}^L(1), 0)$ where the first inequality follows from Assumption 1 and the second inequality follows from $z_1^L \leq \hat{z}_1^L(z_1^S)$. After failing the stress test in the first period, the bank strictly prefers to invest in the safe asset in the second period, i.e., $\Delta(0, z_{f,g}^S(\pi_g)) < \Delta(0,0) \leq 0$ for all $\pi_g \in [0, 1]$, where the last inequality follows from $\beta \geq \bar{\beta}$. Almost identical to the proof of the first case of Proposition 3, we can show that in this case, the informative equilibrium with

$\pi_g = 1$ exists for all δ, and a tough equilibrium with $\pi_g < 1$ exists if and only if $\delta \geq \delta_g$. More specifically, if $\delta > \delta_g$ and $\beta > \bar{\beta}$, then two tough equilibria exist; if $\delta > \delta_g$ and $\beta = \bar{\beta}$, then one tough equilibrium exists; if $\delta = \delta_g$, then a continuum of tough equilibria exist.

– Suppose $\beta \geq \bar{\beta}$ and $z_1^L > \hat{z}_1^L(z_1^S)$, or $\beta \leq \bar{\beta}$. This case is analogous to that illustrated in Fig. 2b: After passing the stress test in the first period, the bank strictly prefers to originate a risky loan, i.e., $\Delta(z_{R,g}^L(\pi_g), 0) \geq \Delta(z_{R,g}^L(1), 0) > 0$, where the first inequality follows because $\Delta(z_{R,g}^L(\pi_g), 0)$ is strictly decreasing in π_g and the second inequality holds for all z_1^L if $\beta < \bar{\beta}$ and for all $z_1^L > \hat{z}_1^L(z_1^S)$ if $\beta \geq \bar{\beta}$. After failing the stress test in the first period, the bank prefers to originate a risky loan if and only if π_g is sufficiently low. That is, there exists a threshold $\breve{\pi}_g < 1$ such that $\Delta(0, z_{f,g}^S(\pi_g)) \geq 0$ if and only if $\pi_g \leq \breve{\pi}_g$. This follows because $\Delta(0, z_{f,g}^S(\pi_g))$ is strictly decreasing in π_g and $\Delta(0, z_{f,g}^S(1)) = \Delta(0, 1) < 0$ by Assumption 5. Notice that the threshold may be strictly negative, in which case we have $\pi_g > \breve{\pi}_g$ for all $\pi_g \in [0, 1]$. Specifically, $\breve{\pi}_g \geq 0$ if and only if $\Delta(0, z_{f,g}^S(0)) \geq 0$. Almost identical to the second case of Proposition 3, we can show that in this case, if $\delta < \delta_g$, the unique equilibrium is informative; if $\delta > \delta_g$, the unique equilibrium is tough; if $\delta = \delta_g$, the informative equilibrium coexists with a continuum of tough equilibria.

Notice that, there is no case that is analogous to the third case in the baseline model (illustrated in Fig. 2c). Compared to the baseline model, the perceived probability that the regulator is strict given a fail is lower when the quality of the asset is known to be good: $z_{f,g}^S(\pi_g) \leq z_f^S(\pi_g, 0)$ for all $\pi_g \in [0, 1]$, with strictly inequality for all $\pi_g > 0$. This is because the strategic regulator is more likely to pass a bank with a good risky loan than a bank with a bad risky loan. This then implies that when the quality of the asset is known to be good, the bank's incentive to originate a risky loan after failing the stress test in the first period is smaller: $\Delta(0, z_{f,g}^S(\pi_g)) \leq \Delta(0, z_f^S(\pi_g, 0))$, with strict inequality for all $\pi_g > 0$. Indeed, $\Delta(0, z_{f,g}^S(1)) = \Delta(0, 1) < 0$ implies that the third case of the baseline model does not occur.

- $X > \overline{X}$. The characterization of the equilibrium follows the logic of the proof of Proposition 4. We have that $G_g\left((z_{R,g}^L(\pi_g), 0), (0, z_{f,g}^S(\pi_g))\right) > 0$ for all π_g, and, therefore, $\pi_g = 1$ in any

equilibrium. The equilibrium is thus either informative or soft. The informative equilibrium (i.e., one with $\pi_b = 0$) exists if and only if $G_b\left((z^L_{R,b}(0), 0), (0, z^S_{f,b}(0))\right) \leq 0$, whereas a soft equilibrium (i.e., one with $\pi_b > 0$) exists if and only if
$G_b\left((z^L_{R,b}(\pi_b), 0), (0, z^S_{f,b}(\pi_b))\right) \geq 0$ for some $\pi_b > 0$.

– Suppose $\beta \geq \bar{\beta}$, or $\beta \leq \bar{\beta}$ and $z^S_1 > \hat{z}^S_1(z^L_1)$. This case is analogous to that illustrated in Fig. 3b: After passing the stress test in the first period, the bank prefers to originate a risky loan if and only if π_b is sufficiently low. That is, there exists $\check{\pi}_b > 0$, such that $\Delta(z^L_{R,b}(\pi_b), 0) \geq 0$ if and only if $\pi_b \leq \check{\pi}_g$. This follows because $\Delta(z^L_{R,b}(\pi_b), 0)$ is strictly decreasing in π_b and $\Delta(z^L_{R,b}(0), 0) = \Delta(1, 0) > 0$ by Assumption 1. Notice that the threshold may be strictly above 1, in which case we have $\pi_b < \check{\pi}_b$ for all $\pi_b \in [0, 1]$. Specifically, $\check{\pi}_b \leq 1$ if and only if $\Delta(z^L_{R,b}(1), 0) \leq 0$. After failing the stress test in the first period, the bank strictly prefers to invest in the safe asset, i.e., $\Delta(0, z^S_{R,b}(\pi_b)) \leq \Delta(0, z^S_{R,b}(0)) < 0$ for all $\pi_b \in [0, 1]$, where the first inequality follows because $\Delta(0, z^S_{R,b}(\pi_b))$ is strictly decreasing in π_b and the second inequality holds for all z^S_1 if $\beta \geq \bar{\beta}$ and for all $z^S_1 > \hat{z}^S_1(z^L_1)$ if $\beta \leq \bar{\beta}$. Almost identical to the second case of Proposition 4, we can show that in this case, if $\delta < \delta_b$, the unique equilibrium is informative; if $\delta > \delta_b$, the unique equilibrium is soft with $\pi_b > 0$; if $\delta = \delta_b$, the informative equilibrium coexists with a continuum of soft equilibria.

  Notice again that there is no case that is analogous to the first case in the baseline model (illustrated in Fig. 3a). Compared to the baseline model, the perceived probability that the regulator is lenient given a pass is higher when the quality of the asset is known to be bad: $z^L_{R,b}(\pi_b) \geq z^L_R(1, \pi_b)$ for all $\pi_b \in [0, 1]$, with strict inequality for all $\pi_b < 1$. This is because the strategic regulator is more likely to pass a bank with a good risky loan than a bank with a bad risky loan. This then implies that when the quality of the asset is known to be bad, the bank's incentive to originate a risky loan is higher: $\Delta(z^L_{R,b}(\pi_b), 0) \geq \Delta(z^L_R(1, \pi_b), 0)$, with strict inequality for all $\pi_b < 1$. Indeed, $\Delta(z^L_{R,b}(0), 0) = \Delta(1, 0) > 0$ implies that the first case of the baseline model does not occur.

– Suppose $\beta \leq \bar{\beta}$ and $z^S_1 \leq \hat{z}^S_1(z^L_1)$. This case is analogous to that illustrated in Fig. 3c:

After passing the stress test in the first period, the bank strictly prefers to originate a risky loan, i.e., $\Delta(z^L_{R,b}(\pi_b), 0) > \Delta(0, 0) \geq 0$ for all $\pi_b \in [0, 1]$, where the last inequality follows from $\beta \leq \bar{\beta}$. After failing the stress test in the first period, the bank prefers to originate a risky loan if and only if π_b is sufficiently low. That is, there exists $\breve{\pi}_b \in [0, 1)$, such that $\Delta(z^L_{R,b}(\pi_b), 0) \geq 0$ if and only if $\pi_b \leq \breve{\pi}_b$. This follows because $\Delta(0, z^S_{f,b}(\pi_b))$ is strictly decreasing in π_b and $\Delta(0, z^S_{f,b}(0)) \geq 0 > \Delta(0, 1) = \Delta(0, z^S_{f,b}(1))$, where the first inequality follows from $z^S_1 \leq \hat{z}^S_1(z^L_1)$ and the second inequality follows from Assumption 5. Almost identical to the proof of the third case of Proposition 4, we can show that in this case, the informative equilibrium with $\pi_b = 0$ exists for all δ, and a soft equilibrium with $\pi_b > 0$ exists if and only if $\delta \geq \delta_b$. More specifically, if $\delta > \delta_b$ and $\beta > \bar{\beta}$, then two soft equilibria exist; if $\delta > \delta_b$ and $\beta = \bar{\beta}$, then one soft equilibrium exists; if $\delta = \delta_b$, then a continuum of soft equilibria exist.

$\blacksquare$

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