Designing Stress Scenarios

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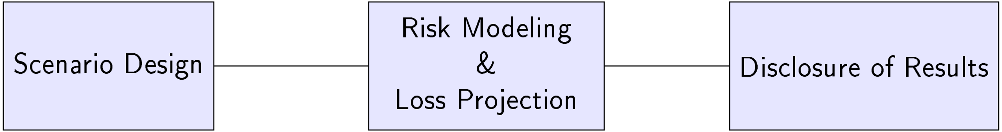
- Three components



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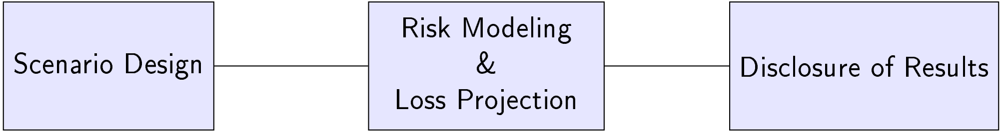

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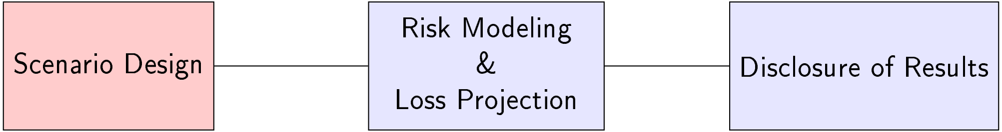

- Literature focuses on disclosure of results
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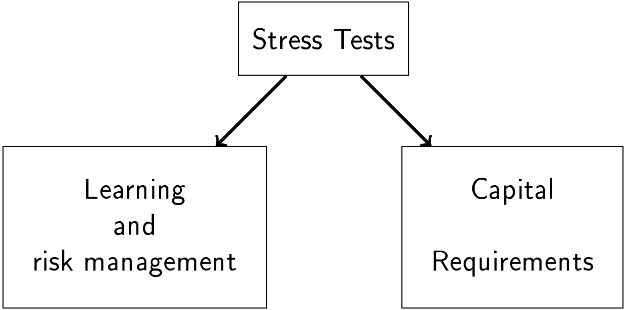
- **This paper:** Optimal **scenario design**

What are stress tests used for?



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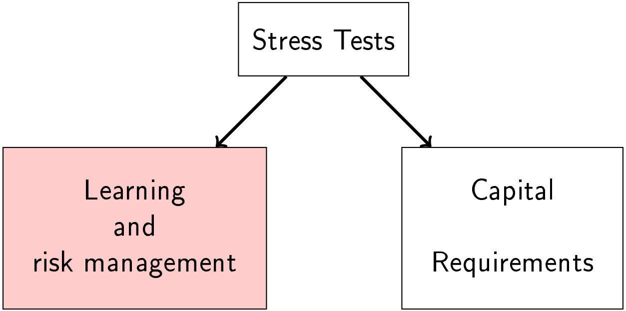

This paper: model stress tests as a ***learning mechanism***

- Learn to manage risk and take a remedial action

Main results

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   - Stress few factors vs. many factors
   - Depends on the cost of the ex-post remedial action and priors

Main results

1. Scenario choice as signal design problem
    - Informational content of stress test depends on stress scenarios
    - Endogenous information processing constraint
2. Specialization vs. diversification in learning
    - Stress few factors vs. many factors
    - Depends on the cost of the ex-post remedial action and priors
3. How much to stress each factor depends on
    - cost of remedial action, beliefs about exposures, how systemic the factor is

Environment

- J macroeconomic factors, $s = [s_1, ..., s_J]$
- N banks, $i = 1, \ldots, N$
  - Losses of bank i given s

    $$y_i = x_i \cdot s + \eta_i,$$

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- One regulator with preferences over aggregate wealth
  $$W = \overline{\omega} - \sum_i y_i$$
  - *Remedial action* $a_{i,j}$ to reduce i's exposure to factor s_j at a convex cost $c_j\left(a_{i,j}\right)$

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- The regulator does not know the exposures $\{x_i\}_i$ and can learn from ***stress tests***

Stress tests

A ***stress test*** is

1. a set of M scenarios $\hat{S} = \left[\hat{s}^{(1)\prime}, ..., \hat{s}^{(M)\prime}\right]'$

2. reported losses $\hat{\boldsymbol{y}} \equiv \left\{\hat{y}_i^{(m)}\right\}_i$ for each scenario m for each bank i

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$$\hat{y}_i^{(m)} = \hat{s}^{(m)} \cdot x_i^{\prime} + \alpha_i \left(M\right) \varepsilon_i^0 + \sigma_{\varepsilon,i}\left(\hat{s}^{(m)}\right) \cdot \varepsilon_i,$$

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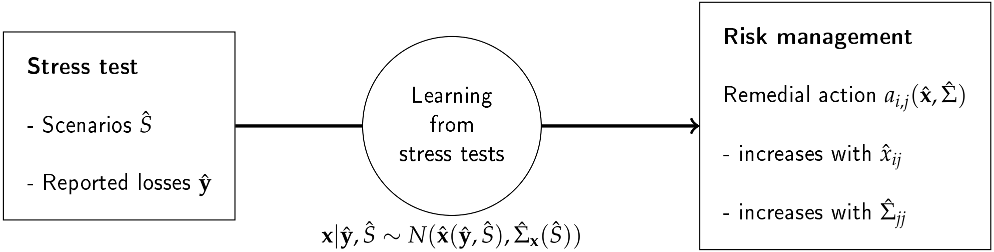
- Reported losses are noisy signals about exposures that depend on $\hat{S}$
  - Weight on exposures is determined by $\hat{s}$
  - Harder to predict losses under more extreme scenarios: $\sigma_{\varepsilon,i}$ increasing in $\left\|\hat{s}^{(m)}\right\|$
  - Costly to have more scenarios. Today: Fixed $M = 1$

Learning and risk management



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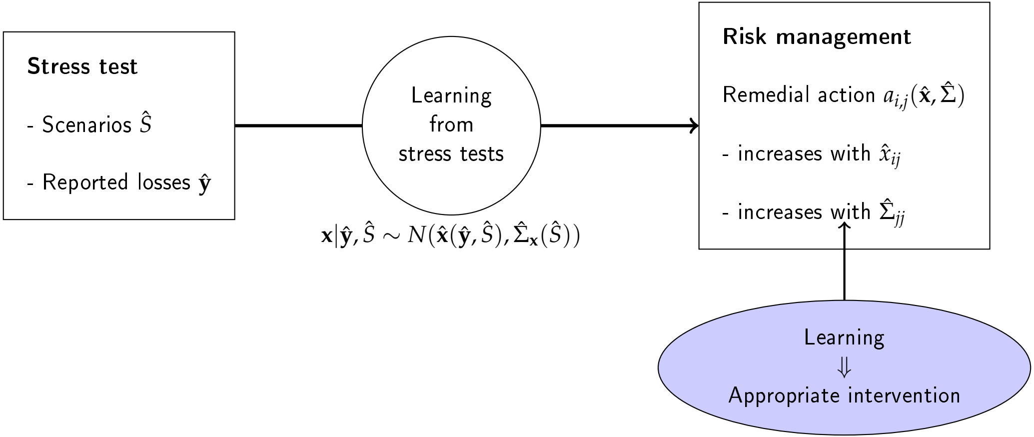

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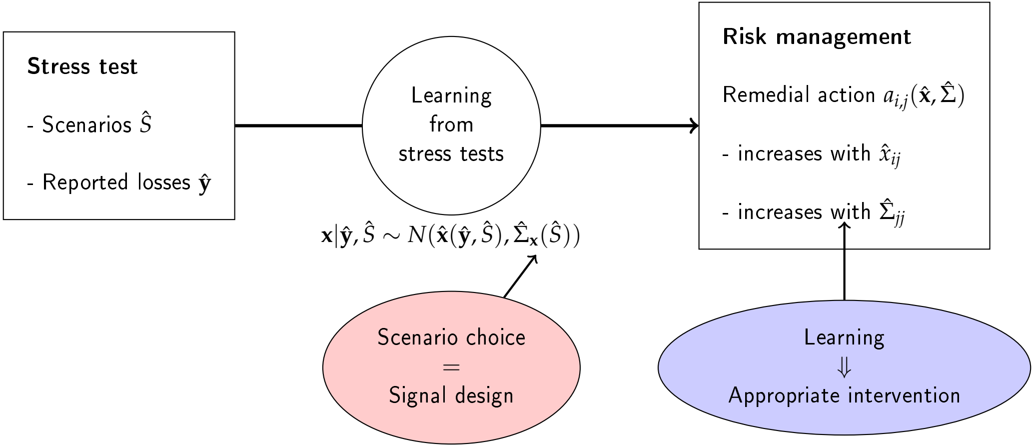

Learning from stress tests

- Stress test results (signals)

$$
\begin{aligned}
\hat{y}_1^{(1)} &= \hat{s}^{(1)} \cdot x_1' + e_1^{(1)} \\
&\vdots \\
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$N \times M$ signals

Learning from stress tests

- Stress test results (signals)

$$\hat{\mathbf{y}} = \left(\mathbf{I}_N \otimes \hat{S}\right)\mathbf{x} + \mathbf{e},$$

where $\mathbf{e} \sim N\left(0, \mathbf{\Sigma_e}\left(\hat{S}\right)\right)$ is the vector of errors

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- Applying the Kalman filter, the regulator's posterior is

$$\mathbf{x} |\, \hat{\mathbf{y}} \sim N\left(\bar{\mathbf{x}} + K\left(\hat{S}\right)\left(\hat{\mathbf{y}} - \bar{\mathbf{x}}\right), \hat{\Sigma}_{\mathbf{x}}\left(\hat{S}\right)\right)$$

where

$$K\left(\hat{S}\right) = \Sigma_{\mathbf{x}}\left(\mathbf{I}_N \otimes \hat{S}\right)' \left(\left(\mathbf{I}_N \otimes \hat{S}\right) \Sigma_{\mathbf{x}} \left(\mathbf{I}_N \otimes \hat{S}\right)' + \boldsymbol{\Sigma}_{\mathbf{e}}\left(\hat{S}\right)\right)^{-1}$$

$$\hat{\Sigma}_{\mathbf{x}}\left(\hat{S}\right) = \left(\mathbf{I}_{NJ} - K\left(\hat{S}\right)\left(\mathbf{I}_N \otimes \hat{S}\right)\right) \Sigma_{\mathbf{x}}$$

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Posterior variance
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how much information

Scenario choice as signal design

- A scenario choice maps to a posterior precision
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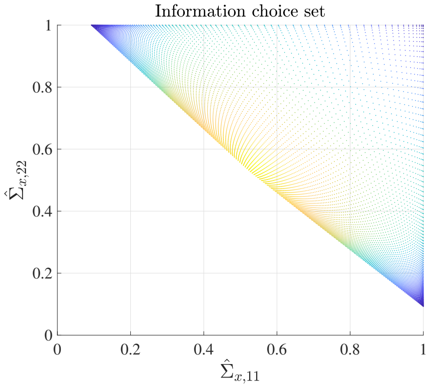

Prior correlation in exposures $\Sigma_{x,12} = 0$

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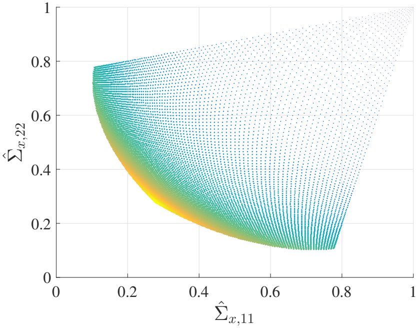

Prior correlation in exposures $\Sigma_{x,12} = 0.5$

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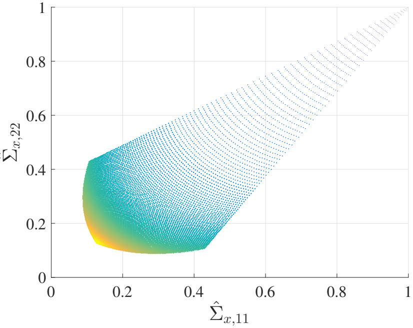

Prior correlation in exposures $\Sigma_{x,12} = 0.8$

Scenario choice as signal design

- Regulator's problem

$$\max_{\hat{\Sigma}_{\mathbf{x}} \in \mathbf{\Sigma}} \mathbb{E}\left[\mathbb{E}\left[U\left(W\left(\left\{a^{\star}_{i,j}\left(\hat{\mathbf{x}}, \hat{\Sigma}_{\mathbf{x}}\right)\right\}_{i,j}\right)\right) - \sum_{i,j} c_j\left(a^{\star}_{i,j}\left(\hat{\mathbf{x}}, \hat{\Sigma}_{\mathbf{x}}\right)\right) \middle| \hat{\mathbf{x}}, \hat{\Sigma}_{\mathbf{x}}\right]\right]$$

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- Two ways to reduce risk: learning (ex-ante) vs. intervening (ex-post)
  - Increasing returns to learning: More learning $\leftrightarrow$ intervention responds more to $\hat{y}$
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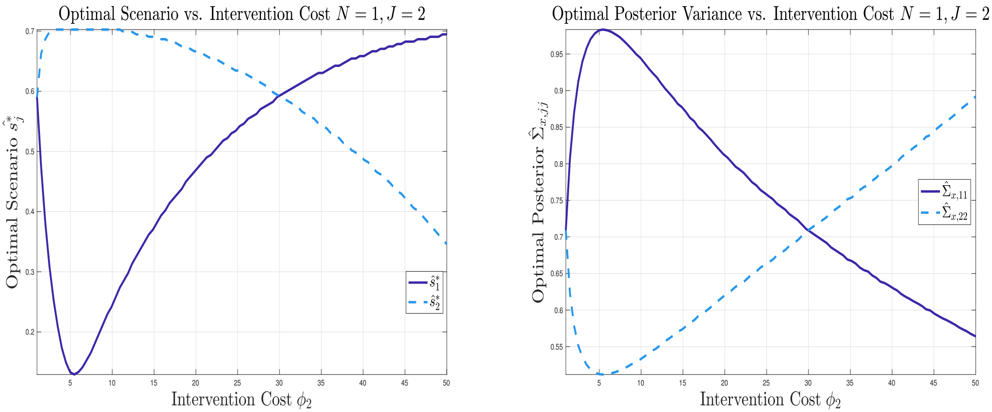
- Optimal learning policy
  - Specialization if increasing returns $>$ convexity in costs $\Rightarrow$ stress few factors
  - Diversification if increasing returns $<$ convexity in costs $\Rightarrow$ stress many factors

Optimal Scenario

- Example: Mean variance preferences + quadratic costs + one scenario
- The weight of a factor in the optimal scenario
  - is non-monotone with respect to its ex-post intervention cost
  - is non-monotone with respect to its expected mean
  - increases with its prior uncertainty
  - increases with the correlation with exposures within the bank
  - increases with the correlation with exposures across banks (systemic factors)

Intervention costs

- One representative bank $N = 1$, two risk factors $J = 2$



Higher expected exposure to a risk factor

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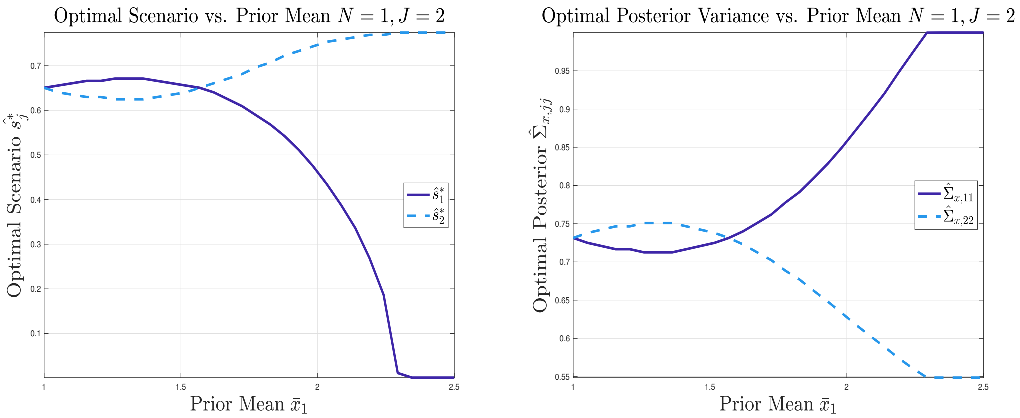
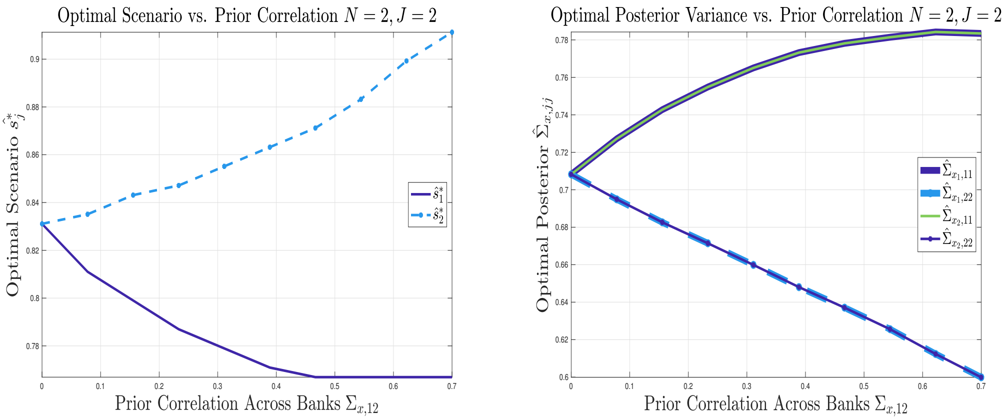


Systemic factors

- Two banks $N = 2$, two factors $J = 2$



Summary

- Scalable and implementable framework to design stress scenarios
  - Inputs: Regulator's beliefs and preferences
  - Extensions: non-separable intervention costs, other preferences
- Going forward:
  - Dynamic stress testing: multiple rounds of learning through stress tests
  - Strategic exposures: Endogeneize bank exposures (moral hazard, time inconsistency)