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**Discussion of:  
“Exploring the sources of loan default  
clustering using survival analysis with frailty”**

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## Summary

- The paper documents that models with grouped frailty outperform models without frailty for the modeling of loan credit risk of SME's
  - When frailties are grouped by the systemic importance of issuing bank
  - After accounting for loan, firm, and macro variables
  - Especially for the smallest Mexican firms
- It proposes an approach to construct internally consistent estimates of conditional default probabilities over multiple periods

## My take

- The paper provides further evidence of the relevance of frailty, especially for smaller firms
- It uses an alternative modeling approach, providing validation to existing literature
- My comments will focus primarily on the contribution to literature
  - Where does the literature stand right now?
  - How does the paper fit in?
  - Open questions

## Broader picture

- Credit risk modeling goes back a long way!
  - **Altman (1968)** already used hazard-like models to establish the relevance of firm-specific variables for credit risk
- Clustering in default times was observed early already as well
  - **Lang and Stulz (1992)** documented clustering of defaults within industries
- Big progress was made by **Das et al. (2007)**
  - Rejected the null hypothesis that firm default times are **conditionally independent**; i.e., that they are correlated just through exogenous firm & macro risk factors
- **Duffie et al. (2009)** introduced the concept of **frailty** to capture the excess default clustering in U.S. data

## Pre Duffie et al. (2009)

- Suppose that  $T_i$  is the default time of Entity  $i$  (which may be a firm or a loan, more on this later)
- Most models prior to Duffie et al. (2009) assumed that the survival function of Entity  $i$  is

$$S_i(t) = \mathbb{P}_t(T_i > t) = \exp\left(-\int_0^t \lambda_{i,s} ds\right),$$

where  $\lambda_{i,s}$  is the intensity (i.e., the conditional default rate; the higher  $\lambda_{s,i}$ , the more likely that Entity  $i$  will default early)

- Typically,  $\lambda_s$  is assumed to be a function of a vector of observable firm variables  $X_{i,s}$  and macro variables  $Y_s$ :

$$\lambda_{i,s} = f(X_{i,s}, Y_s)$$

- This specification was rejected by Das et al. (2007)

## Post Duffie et al. (2009)

- What Duffie et al. (2009) did is to introduce a **latent** factor that is common across all entities and drives excess default correlation:
  - In particular, Duffie et al. (2009) assume that the default intensity is

$$\lambda_{i,s} = f(X_{i,s}, Y_s, Z_s),$$

where  $Z$  is not latent

- Initially, frailty was mostly a statistical artifact
  - $Z$  can be viewed as a residual, capturing any default correlation that cannot be explained by observable factors
- More recently, frailty is understood to capture **information contagion** (Koopman et al. (2011), Benzoni et al. (2015))
  - Investors extrapolate financial distress across firms, affecting the financing options of surviving firms

## Where is the literature now?

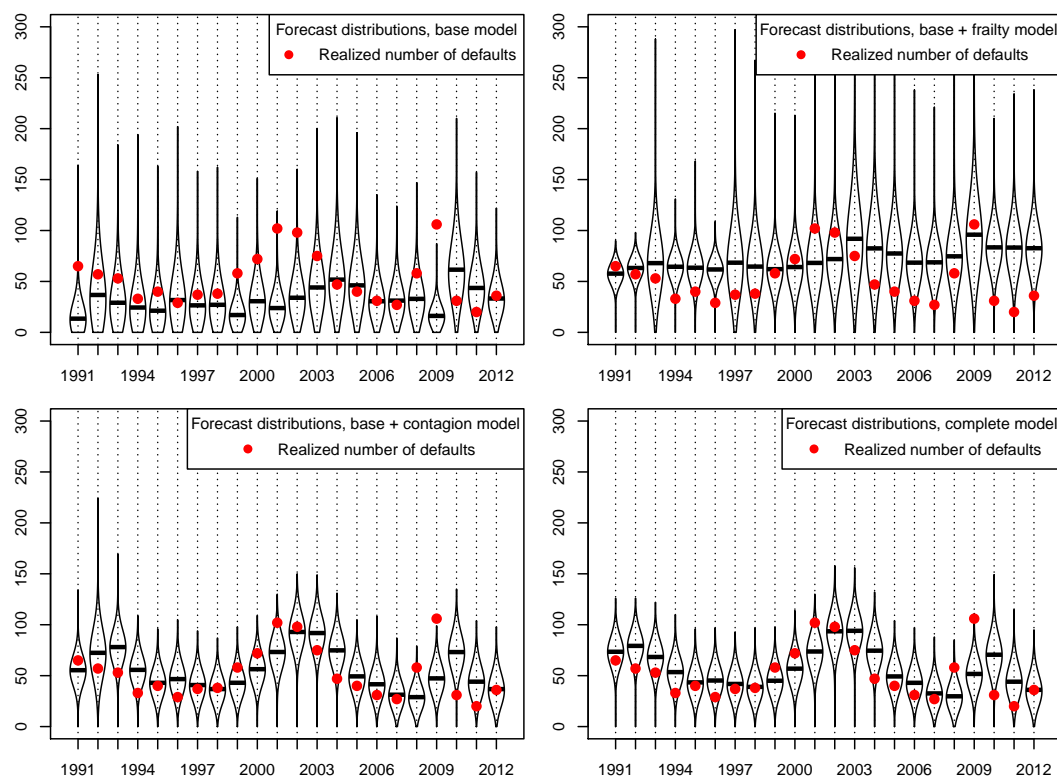
- Most recently, [Azizpour et al. \(2018\)](#) reject the null hypothesis that the frailty formulation of [Duffie et al. \(2009\)](#) is sufficient to explain the degree of default correlation in U.S. data
  - Instead, we show that a specification of the type

$$\lambda_{i,s} = f(X_{i,s}, Y_s, Z_s, T_{-i}),$$

where  $T_{-i}$  are the default times of all other firms, **cannot** be rejected by the data

- The results of [Azizpour et al. \(2018\)](#) highlight **network effects**
  - Cascades of failures among interconnected firms, as in [Acemoglu et al. \(2012\)](#), [Eisenberg and Noe \(2001\)](#), and others
- Both network and frailty channels are critical

## Models without frailty or contagion



**Azizpour et al. (2018)** establish that models that neglect frailty tends to understate credit risk out-of-sample, while models that neglect network effects generate too dispersed credit risk forecasts



## How the paper fit in the literature?

- The paper assumes the following specification

$$\lambda_{i,s} = f(X_{i,s}, Y_s, Z_{\text{Group}(i)}),$$

where  $T_{-i}$  are the default times of all other firms, cannot be rejected by the data

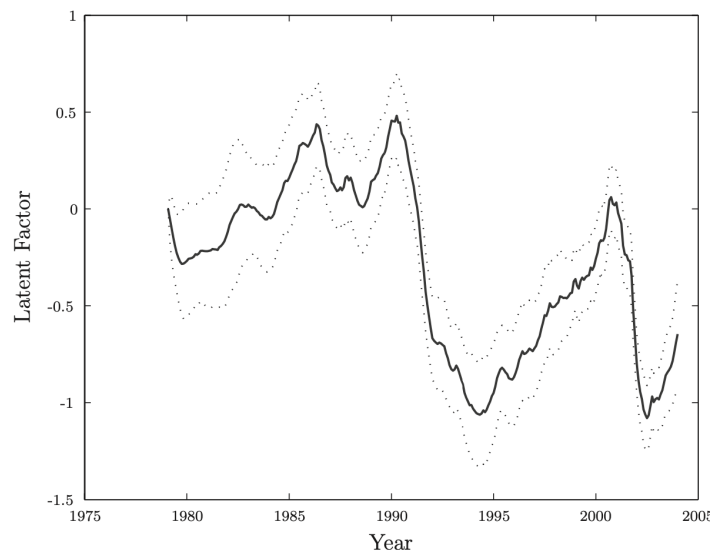
- Compared to the specification of **Azizpour et al. (2018)**:

$$\lambda_{i,s} = f(X_{i,s}, Y_s, Z_s, T_{-i})$$

- The paper focuses on loan defaults while **Azizpour et al. (2018)** focuses on firm failures
- What consequences might these differing approaches have?

## Frailty choices

- The frailty model of the paper is more granular but less dynamic than the one of [Azizpour et al. \(2018\)](#)
  - Time-variation appears to be critical



Posterior mean of frailty of [Duffie et al. \(2009\)](#)

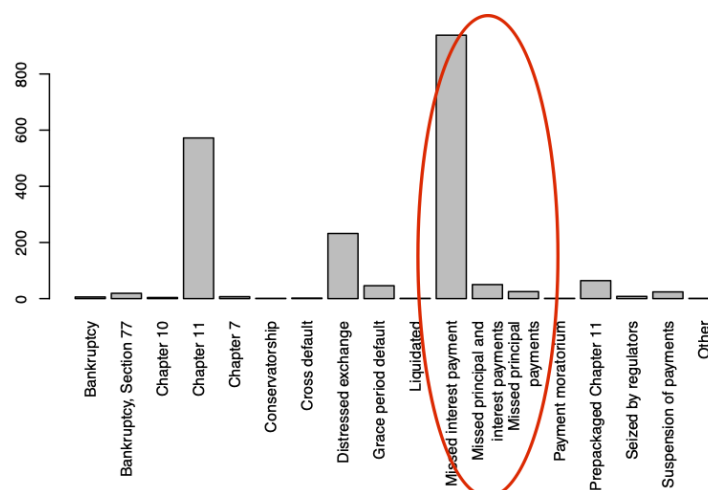
- Granularity also appears important ([Chava et al. \(2011\)](#))
- Open question: interaction of frailty granularity & time variation?

## Neglecting the network channel

- The paper neglects the network channel of contagion
- But the network channel of credit risk appears to be critical
  - Azizpour et al. (2018) reject all models that do not include network effects
- Open question: to which degree does a more granular frailty capture network effects?
  - Data on network interconnections is hard to get, but critical for loan-level modeling (see Schwenkler and Zheng (2019) for a recent attempt at extracting network data from the news).

## Loan vs firm defaults

- The authors argue that they differ from the literature because they study loan-level credit risk rather than firm credit risk
- But most papers in this space consider several types of defaults besides Chapter 7 & Chapter 11. Some of them coincide with the definition of default of the paper!



Types of defaults in [Azizpour et al. \(2018\)](#)

- These types of events are considered to be defaults by Moody's

## Loan vs firm defaults

- Most existing papers exclude subsequent events of this type within a certain time frame (1 year)
- This is because, once the rating agency has assigned a default label to a firm, it causes all sorts of issues for its financing options
- But the paper considers the study of the clustering of many of these events for the same firm to be main contribution
  - Can the authors flesh out more clearly why this would lead to different insights?
- It may be that many of these SME loans are not rated, so that loan-level modeling may be more relevant than firm-level modeling
  - Is this then more a study of rated vs non-rated bonds?

## Minor comments

- Hazard  $\neq$  intensity
  - Actually, hazard = posterior mean of intensity under an equivalent probability measure (see [Giesecke and Schwenkler \(2018\)](#))
  - Interpretation of estimates?
- Internally consistent multi-period default probability forecasting also possible for reduced form models as in [Azizpour et al. \(2018\)](#)
- [Christoffersen and Matin \(2019\)](#) introduce a machine-learning credit risk model with multivariate frailties, similar to the approach in the paper

## Conclusion

- New evidence in support of frailty in an interesting context!
- The paper contributes to a rich existing literature
- The authors can push their paper a bit further, fleshing out their contributions relative to the field
- Good luck with the paper!

**Thank you!**



## References

Acemoglu, Daron, Vasco M. Carvalho, Asuman Ozdaglar and Alireza Tahbaz-Salehi (2012), 'The network origins of aggregate fluctuations', *Econometrica* **80**(5), 1977–2016.

Altman, Edward I. (1968), 'Financial ratios, discriminant analysis and the prediction of corporate bankruptcy', *Journal of Finance* **23**(4), 589–609.

Azizpour, S., K. Giesecke and G. Schwenkler (2018), 'Exploring the sources of default clustering', *Journal of Financial Economics* **129**(1), 154–183.

Benzoni, Luca, Pierre Collin-Dufresne, Robert Goldstein and Jean Helwege (2015), 'Modeling credit contagion via the updating of fragile beliefs', *Review of Financial Studies* **28**(7), 1960–2008.

Chava, Sudheer, Catalina Stefanescu and Stuart Turnbull (2011), 'Modeling expected loss', *Management Science* **57**(7), 1267–1287.

Christoffersen, Benjamin and Rastin Matin (2019), Modeling frailty correlated defaults with multivariate latent factors. Working Paper. Available at SSRN: <https://ssrn.com/abstract=3339981>.

Das, Sanjiv, Darrell Duffie, Nikunj Kapadia and Leandro Saita (2007), 'Common failings: How corporate defaults are correlated', *Journal of Finance* **62**, 93–117.

Duffie, Darrell, Andreas Eckner, Guillaume Horel and Leandro Saita (2009), 'Frailty correlated default', *Journal of Finance* **64**, 2089–2123.

Eisenberg, Larry and Thomas H. Noe (2001), 'Systemic risk in financial systems', *Management Science* **47**(2), 236–249.

Giesecke, Kay and Gustavo Schwenkler (2018), 'Filtered likelihood for point processes', *Journal of Econometrics* **204**(1), 33–53.

Koopman, Siem Jan, Andre Lucas and Bernd Schwaab (2011), 'Modeling frailty-correlated defaults using many macroeconomic covariates', *Journal of Econometrics* **162**(2), 312–325.

Lang, Larry and Rene Stulz (1992), 'Contagion and competitive intra-industry effects of bankruptcy announcements', *Journal of Financial Economics* **32**, 45–60.

Schwenkler, Gustavo and Hannan Zheng (2019), The network of firms implied by the news. Working Paper.