Optimal Stress Tests in Financial Networks*

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Abstract

I study the information design on a financial network to maximize its stability, where banks' default outcomes are determined by a fixed point payment problem that accounts for both fundamentals and interbank contagion. In addition to the cross-state risk sharing in single-bank stress test models, the system-level design highlights the novel cross-bank risk sharing: passing banks together reduces the counterparty risks among them, but at a cost of reporting the same information despite their independent fundamentals. When the expected bank profitability is high, or interbank exposure is large, the optimal policy is less discriminatory across banks. For specific network structures, I find: 1) in a ring network, banks at least a specific distance away from the nearest shock may pass, at states where shocks are connected on adjacent banks; 2) an interconnected structure is not necessarily more stable under the optimal policy; 3) typically more connected banks receive preferred treatment.

JEL Classification: D82, D83, G21, G28

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1 Introduction

Disclosing information about bank qualities is crucial in resolving a banking crisis. Consequent to the Great Recession, bank supervisors around the world conduct periodic stress tests to ensure the banking system's resilience to economic uncertainty. Importantly, the test results are disclosed so as to restore market confidence, which is a distinguishing feature from policies that provide direct financial support. As a result, a growing literature studies how a cash constrained regulator designs the informativeness of stress tests to improve financial stability via affecting market participants' beliefs.[1]

However, this literature thus far examines *single bank* stress tests where banks are stand-alone, and hence remains essentially micro-prudential.[2] This contrasts with the 2007-09 financial crisis, which vividly demonstrated the significance of *interbank contagion* among systematically important financial institutions (SIFIs). In addition, despite significant heterogeneity across banks, the regulator seems to be reluctant to treat banks differently. For instance, during the recent COVID-19 pandemic, U.S. stress test restricted the payout policies of all banks who were tested.[3] Hence, it is critical to take a holistic perspetive regarding the broader macroprudential stress test design on the banking system. The network theory is well accepted to characterize the structure of interdependence among banks, and thus a natural framework to examine system-level stress testing.

I study the optimal stress test in given financial network structures that maximizes the expected system stability. A cash-constrained regulator commits to a disclosure rule that communicates each bank's quality via its test result, taking into account the implied counterparty risks on other banks. As in the stress test disclosure literature, a passing result that signals good quality is not perfectly informative, to allow for risk sharing across states. However, in stark contrast with single bank stress tests, in my model with an interconnected banking network, the disclosure about one bank is informative about its payments to the other banks. Therefore, disclosure about individual banks are interdependent due to the endogenous counterparty risk.

To the best of my knowledge, this is the first paper to account for interbank contagion in stress test design. It is of direct policy relevance: for instance, in 2017 the Federal Reserve added the "Counterparty Default Component" which assesses the bank's financial health assuming that its largest counterparty defaults.[4] Yet the largest counterparty assumption may underestimate risk, while the exogenous counterparty failure for each single bank may exaggerate risk. By designing stress test on the financial network as a whole, this paper provides a complete characterization of

[1] See Goldstein and Leitner (2018); Faria-e Castro et al. (2016); Leitner and Williams (2017); Orlov et al. (2018); Inostroza and Pavan (2018); Inostroza (2019); and others.

[2] Orlov et al. (2018) discuss macro-prudential recapitalization of banks, in which bad banks are separated to liquidate their assets first.

[3] In 2020, besides the annual stress test in June, the Federal Reserve conducted a second round of stress test in December. In the June stress test, the Federal Reserve suspended the share repurchases and limited the dividends payout for all the 34 banks; in the December stress test, the Federal Reserve extended the earlier restrictions on distributions, with modifications that dividends and share repurchases will be limited based on income.

[4] Eight financial institutions are subject to this component: Bank of America Corporation; The Bank of New York Mellon Corporation; Citigroup Inc.; The Goldman Sachs Group, Inc.; JPMorgan Chase & Co.; Morgan Stanley; State Street Corporation; and Wells Fargo & Co.

interbank spillovers and endogenizes the effects of disclosure on counterparty risk. Further, my paper is of theoretical interest to the information design literature. For instance, prior consistency in this literature requires disclosure to be on average correct; in this study, this average quality depends on the counterparty risk that is affected by the disclosure about other banks.

In the single bank stress testing literature (e.g., Goldstein and Leitner, 2018), the policy highlights cross-state risk sharing: a passing signal is not perfectly informative but pools some bad states, as long as it represents acceptable quality on average. The cross-state risk sharing is still at work in this paper. However, my analysis highlights the novel *cross-bank risk sharing*: passing a particular set of banks *together* helps them deliver to each other as counterparties, and hence improve their average quality in a collective way.[5] In general, if bank profitability is high, or complementarity among banks is high, the optimal policy tends to be less discriminatory and report the same signal across banks, despite the independence of the banks' stand-alone assets.

I introduce uncertainty to the financial network framework of Eisenberg and Noe (2001) by applying the information design approach (Bergemann and Morris, 2016a,b), where the regulator maximizes system stability via affecting the beliefs about shocks to banks. To be more specific, Eisenberg and Noe (2001) characterize the interbank payments at contingent states in a given network structure. In my paper, before bank qualities (state) are revealed, each bank has an option to exchange its risky asset side for cash, with which to settle liabilities later. As the key part of the model, the value of this refinancing opportunity is affected by disclosure: before state realization, the regulator commits to a disclosure rule that specifies how likely different signals are observed at each state, based on which market participants learn about the underlying bank qualities.

In my model, there are three dates $0, 1, 2$. Bank balance sheet are exogenously given, and the positions are fixed except for possible refinancing at $t = 1$. Banks are connected by interbank liabilities, the collection of which corresponds to the network structure which is exogenously given. Each bank also has a risky loan project that realizes at $t = 1$; this is the source of external shocks in the system. Project shocks are independent across banks, and the state corresponds to the collection of project realizations. A bank also has external liabilities senior to interbank debts. The state is reavealed at $t = 2$, when all liabilities are due. These outside-of-network cash flows, i.e., loan projects and senior liabilities, ensure that interbank payments are a well defined set of fixed points—what banks pay creditor banks depend on what they receive from borrower banks.

The regulator maximizes the weighted number of banks that are solvent, by influencing the interim ($t = 1$) beliefs about banks and their refinacing opportunities. Specifically, at $t = 0$ the regulator commits to a disclosure policy that specifies how likely each bank is reported with h or l at each state. At $t = 1$ when the state realizes but is not revealed, a public signal of all banks' test results (h or l) is released. The market hence updates beliefs about banks, which influences the

[5] This point is related to endogenous risk sharing between agents in a connected structure. For instance, Ambrus et al. (2014) show in social networks that connections enforce informal insurance payments, and result in strong comovement in consumption of insured individuals. Here, risk sharing is enabled by information design, and a public signal that passes banks together results in reduced counterparty risks and comovement in these banks' qualities.

amount that a bank could raise if it exchanges its risky asset side—both the loans and interbank receivables—for cash. The default penalty is sufficiently large, so a bank refinances if and only if the amount to be raised exceeds liabilities; otherwise, the bank stays put at $t = 1$ low market value in hope for resurrection tomorrow.

The regulator's problem could be represented in Bergemann and Morris (2016a,b)'s framework: a signal recommends actions to banks, h for refinancing and l for waiting; at the same time, the disclosure policy should satisfy *obedience constraints*, which ensure that banks would indeed take the recommended actions. Hence, h should convey good bank quality, upon which the bank refinances at $t = 1$ with enough cash and never defaults at $t = 2$; l induces low interim market value at $t = 1$, and the eventual bank outcome at $t = 2$ still depends on the underlying state.

A lenient test that is not perfectly informative allows banks to share risk across states. Specifically, a bank may also be reported with h at states where it would otherwise default. Via refinancing upon h, the bank essentially borrows liquidity with itself from good states to bad states, which improves its chance of survival. In addition, there is no counterparty risk from a bank that is reported with h. Cross-bank risk sharing arises when the public signal reports h on multiple banks at the same time. On one hand, reduced counterparty risk improves bank quality and facilitates refinancing. On the other hand, the same signal h is reported on banks with different underlying cash flows from independent projects, and the regulator needs to convince the market that the weaker banks are also on average healthy.

In solving the optimal stress test design, I derive an index which characterizes the efficiency of any particular way of risk sharing, the collection of which makes up a feasible policy. This is one of the major contribution of the paper. In general, the efficiency is a function of some general distribution of the signal across states.[6] I first show that the relevant obedience constraints are those of h to ensure high interim market value. More importantly, under very general conditions of low priors and idiosyncratic shocks to banks, all obedience constraints of h are binding without loss of generality. The binding constraints allow me to compute the index directly.

There are two aspects of the efficiency index, the *maximum attainable payoff* and the *marginal value* of risk sharing. Intuitively, if bank profitability is high and thus cross-state borrowing is less constrained, a less discriminatory signal that reports h on many banks naturally results in higher system stability. The marginal value, on the other hand, becomes important when cross-state borrowing is constrained. It equals the ratio of the improvement in system stability due to refinancing, over the underlying liquidity shortage at states that borrow liquidity. The liquidity shortage captures how sensitive investors adjust their beliefs about h downward and hence restricts the extent of cross-state risk sharing. For a signal that reports only one bank as h and the rest as l, the ratio reduces to the gain-to-cost ratio in the single-bank stress test by Goldstein and Leitner (2018). For relatively large interbank exposure, counterparty risk becomes important;

[6]The linear programming problem itself is finite, and the optimal solution lies on one of the vertices. The problem with this approach is that we only know which constraints are binding when we have the solution, so it does not allow for comparative statics and policy predictions. Here, we argue that an economically meaningful index is a function of the signal distribution, which has infinite dimension.

despite independent project fundamentals, a less discriminatory signal is preferred so as to reduce counterparty risk and liquidity shortage.

Then I use this efficiency index as a powerful tool to study the properties of the optimal policy for different primitive parameters and network structures. In general connected networks, the optimal policy is less discriminatory across banks when 1) the expected bank profitability is high, or 2) counterparty exposure is large.[7] Under some conditions, all banks can always refinance and perfect risk sharing is achieved. If banks have balanced interbank claims and liabilities, and outside investors demand zero rent, perfect risk sharing is achieved, and decentralized refinancing implements the same outcome as centralized clearing. Conditional on a small number of fundamental shocks, perfect risk sharing could also be achieved.

Consistent with my model implications, stress test in practice exhibits reluctance to separate banks. For example, in years after the Great Recession, the Federal Reserve separated several banks and restricted their payout plans. In recent years of good economic growth, almost all banks pass their stress tests. During the recent COVID-19 pandemic, bank stress tests again did not separate, and restricted payouts of all tested banks. Taking an international perspective, as banks involved in the European-wide stress test are more heterogeneous and less connected banks, the European stress test results are more discriminatory compared with the U.S. practice.

Then I study the implications of network structures on the optimal disclosure. First, I examine the effects of interconnectedness in symmetric networks. In the complete network where banks are all connected to each other, the optimal policy is mainly driven by the potential improvement in system stability. Liquidity is borrowed to the critical states where the network transitions from stable to fragile. As a result, information design increases the threshold number of shocks that results in system failure. In the ring network where a bank is only connected to the neighboring lender and borrower, the optimal policy is mainly driven by minimizing the liquidity shortage of refinancing banks (reported with h). It features a distance-based signal and "quarantine" effect. Specifically, the signal allows risk sharing if banks are over a specific distance away from the nearest impaired bank; the underlying project shocks are on adjacent banks, which is similar to a "quarantine" practice.

Connecting back to the network literature on connectivity, Acemoglu et al. (2015) find that interconnectedness may be "robust" or "fragile" depending on shock size, while the ring network is always the least stable. I find that under the optimal information structure, the ring network may be more more stable. In my paper, as the contagion at bad times may be reduced via cross state risk sharing, shock size is not consequential. Instead, the mechanism highlights the economic trade-off of cross-state risk sharing that concerns outside liquidity premium and bank profitability, among others. When cross-state risk sharing is constrained, the debt overhang problem in a connected network—the raised liquidity easily spreads to counterparties' senior creditors—becomes more expensive. In a less connected structure, shocks may be quarantined locally to

[7] We emphasize that the all-l signal does not necessarily lead to system failure, but instead all banks are denied the opportunity of cross-state risk sharing and wait in hope for the best. One example of this signal is that in the worst time of 2008, the Federal Reserve required all key banks to participate in TARP instead of self saving.

be absorbed by senior creditors. When cross-state risk sharing is constrained, interconnectedness becomes favorable for easier coordination in banks' refinancing to reduce contagion.

Next, I examine the core-periphery network, which is a widely studied empirically relevant structure(see Hojman and Szeidl, 2008; Craig and Von Peter, 2014; Farboodi, 2017, among others). Typically the core banks receive preferred treatment, because core banks have large spillovers to periphery banks and better risk sharing with more counterparties. As the outcome of optimal risk sharing, the preferred treatment should qualify relatively small cost of cross-state borrowing as compared with the complementarity between the core bank and its connected peripheries. As a counter example, when cross-state borrowing is extremely expensive, only some periphery banks may refinance; this is because compared with their connected core bank, they are one distance further from distant project shocks.

Finally, I compare the Counterparty Default Component (CDC) used in practice with the optimal policy. The CDC assumes that each bank's largest counterparty fails, and so is essentially a single-bank stress test conducted independently on individual banks with exogenous counterparty contagion. Compared with the system-level optimal policy, due to a bank's dependence on the largest counterparty, the CDC practice is more lenient in interconnected structures and harsher in sparse structures, as a result of the importance of the neglected counterparty risks. In addition, without coordination in the system-level, the CDC practice is more discriminatory both in good times and bad times.

Related Literature

My paper is related to several strands of literature. The first strand is the literature on *bank stress test*. Goldstein and Sapra (2014) and Leitner (2014) give overall discussions on the benefits and costs of regulatory disclosure on banks. Several papers study stress test disclosure as applications of Bayesian persuasion (Kamenica and Gentzkow (2011)) and information design (Bergemann and Morris, 2016a, 2019). In Goldstein and Leitner (2018), the optimal disclosure reports some bad banks to prevent market breakdown, and pools as many bad banks with good banks for cross-state risk sharing. Faria-e Castro et al. (2016) argue that fiscal capacity affects the optimal disclosure policy because of the costly backstops that ensue. Williams (2017) examines the effects of disclosure on a bank's ex ante asset choice. Leitner and Williams (2017) discuss whether to disclose stress test models to banks as a tradeoff between bank's gaming under model disclosure and socially undesirable investments under model secrecy. Orlov et al. (2018) consider the design of macro-prudential stress tests with capital requirements on banks to avoid future default. A dynamic disclosure which forces weak banks to raise capital first leads to efficient recapitalization of stronger banks. Inostroza and Pavan (2018) study the information design with multiple receivers in the global games framework of regime change.[8] The optimal policy removes strategic uncertainty, whereas the structural uncertainty of disagreement on fundamentals persists. Inostroza (2019) incorporates rollover risks, and the optimal policy first discloses the banks with good assets, and

[8] Goldstein and Huang (2016) also consider a global game model of regime change.

additionally test the banks with bad assets on liquidity positions with contingent recapitalization requirements. All these papers examine single bank stress test where banks are *stand-alone*. To the best of our best knowledge, this is the first stress test paper to account for interbank contagion.

My paper is also related to the more general literature of information disclosure on financial institutions. The classic concern about perfect disclosure is the Hirshleifer effect (Hirshleifer (1971)) that reduces risk sharing opportunities. Dang et al. (2017) explain why banks should be opaque, which is consistent with our implication of coordinating regulatory disclosures across banks. Bouvard et al. (2015) study how regulatory disclosure affects the possibility of bank runs. Other related contributions include Bouvard et al. (2015), Shapiro and Skeie (2015).

My paper is an application of *Bayesian persuasion* with multiple receivers, and adopts the framework of *information design*. This literature traces to Myerson (1986) who argues that the designer can restrict to action recommendations to the agents, in a general class of multi-stage games of incomplete information. Kamenica and Gentzkow (2011) study the optimal persuasion between a sender and a single receiver. Other contributions on the single-sender, single receiver persuasion include Brocas and Carrillo (2007), Rayo and Segal (2010), Gentzkow and Kamenica (2014). Dworczak and Martini (2019) present a price-theoretic approach to Bayesian persuasion, and characterize the conditions for monotone partitional signaling, when payoffs depend only on the mean of posterior. Ely et al. (2015), Ely (2017), Xandri (2016) and Doval and Ely (2016) study persuasions in dynamic settings, and Gentzkow and Kamenica (2016) allow for multiple senders. Bergemann and Morris (2016a), Bergemann and Morris (2016b) and Bergemann and Morris (2019) present the information design framework with multiple receivers that unifies communication in games and Bayesian persuasion. I build on the notion of Bayes-correlated equilibrium (BCE) in Bergemann and Morris (2016a), Bergemann and Morris (2016b), who argue that the set of Bayes-Nash equlibria that can arise correspond to the BCEs that are obedient—the agents take the recommended actions under the resulting posteriors. Taneva (2019), Mathevet et al. (2020), Alonso and Camara (2016a), Alonso and Camara (2016b), Bergemann et al. (2015) and Bardhi and Guo (2018) also study the information design with multiple receivers. Galperti and Perego (2018) formalize the dual problem of the Bergemann and Morris (2016a) information design framework.

The micro-foundation of the payoffs in this paper relates to the *financial network* literature.[9] In the seminal work of Allen and Gale (2000), interbank lending networks allow banks in different regions to share liquidity risk (a la Diamond and Dybvig (1983)), and a complete network provides the most efficient risk sharing. Eisenberg and Noe (2001) introduce a basic framework to study financial contagion in exogenous networks determined by interbank liabilities, and show that the set of fixed points of interbank repayments exists and is generically unique. Acemoglu et al. (2015) study the extent of financial contagion under different network structures. We introduce uncertainty, and study the information design that influences the distribution of interbank payment equilibrium across contingent states so as to maximize expected system stability. Other

[9]For surveys of the financial network literature, see Allen and Babus (2009), Glasserman and Young (2016) and Summer (2013).

contributions on financial contagion and system stability in exogenous network structures include Dasgupta (2004), Caballero and Simsek (2013), Elliott et al. (2014), Glasserman and Young (2015) and Gârleanu et al. (2015). Recent work on network interventions include Alvarez and Barlevy (2015), Galeotti et al. (2017), Kanik (2018), Bernard et al. (2017), Erol (2018) and Bernard et al. (2019). The closest to our paper is Alvarez and Barlevy (2015), who discuss whether mandatory disclosure can improve welfare as opposed to voluntary disclosures by banks who have strategic substitutability or complementarity in equity levels due to contagion. Other papers that study the role of network structures in strategic behaviors include Ballester et al. (2006), Galeotti et al. (2010), Bramoullé et al. (2014), Choi et al. (2017), Babus and Kondor (2018). Babus and Kondor (2018) micro-found a Kyle (1989) model in connected intermediaries and analyze how decentralization affect information diffusion. There is also a growing literature that considers endogenous interbank linkages: Goyal and Vega-Redondo (2005), Farboodi (2017), Erol and Ordoñez (2017), Wang (2016) and others. Galperti and Perego (2019) and Egorov and Sonin (2020) study persuasion on social networks. Galperti and Perego (2019) assume that the designer can communicate with only a limited number of agents, who then share the information with neighbors. In Egorov and Sonin (2020), agents can either directly buy information from the sender, or rely on social network with diminishing communication.

The rest of the paper is organized as follows. Model setting is presented in Section 2. Section 3 solves the problem and provides general properties of the optimal policy. Section 4.1 discusses symmetric networks and the effect of connectivity. Section 4.2 discusses asymmetric networks. Section 5 concludes.

2 The Model

After presenting the model, this section provides some preliminary analysis after formulating the regulator's problem in the approach of Bergemann and Morris (2016a).

2.1 Model Setup

The economy lasts for three dates $t = 0, 1, 2$, and is populated by three types of risk-neutral agents: banks, a regulator and investors. At $t = 0$ the regulator commits to a disclosure policy before the state is realized. At $t = 1$, the state realizes, and a public signal is released according to the disclosure policy. The signal influences a bank's refinancing opportunity which may help with clearing debts later at $t = 2$. The key difference from single-bank stress test is that disclosure is informative about interbank payments. I provide a notation table for all variables in Appendix A.

Banks and Financial Network I introduce uncertainty and information design to the financial network model of Eisenberg and Noe (2001) and Acemoglu et al. (2015), who focus on the payment equilibrium at contingent states. Bank balance sheet is exogenously given at $t = 0$, and the positions

Bank i's Balance Sheet (Book Value)

Assets:	**Liabilities:**
Loan project $\tilde{A}_i$	Senior Liabilities v_i
Interbank claims $y_i^{in} \equiv \sum_{j \neq i} y_{ij}$	Interbank debts $y_i^{out} \equiv \sum_{j \neq i} y_{ji}$
	Equity

Figure 1: Bank Balance Sheet

remain unchanged except for possible refinancing at $t = 1$. A typical bank i's balance sheet is as follows,

On the asset side, each bank has a risky loan project that at $t = 1$ delivers binary payoff $g : \tilde{A}_i = A_i > 0$ or $b : \tilde{A}_i = 0$, where g and b are labels about the bank's project shock. $\tilde{A}_i$ realizes independently across banks, with common prior $\mathbb{P}\left(\tilde{A}_i = A_i\right) = p_i \in \left[\frac{1}{2}, 1\right]$. The state of nature $\theta \in \Theta$ is the collection of project realizations:

$$\theta \equiv \tilde{A}_1 \times \tilde{A}_2 \times \cdots \times \tilde{A}_n \in \Theta. \tag{1}$$

For the network with n banks, there are 2^n possible states. Bank i also has interbank claims of $y_i^{in} \equiv \sum_{j \neq i} y_{ij}$ to collect at $t = 2$, where y_{ij} is the face value that borrower bank j owes to lender bank i (thus incoming for bank i). Throughout this paper, I refer to "asset" as the collection of project and interbank claims. This differs from the studies of single-bank stress test, where typically "asset" means the loan project alone.

On the liability side, each bank i has senior liabilities of face value v_i, and junior interbank liabilities of $y_i^{in} \equiv \sum_{j \neq i} y_{ji}$; both take the form of standard debt contracts.[10] The residual value is bank equity. All liability obligations must be cleared at $t = 2$. Senior liabilities must be paid fully before any payments to interbank liabilities, and interbank liabilities are pari passu across creditor banks.

The collection of interbank liabilities $\{y_{ij}\}$ corresponds to the network structure. As a bank's outgoing payments depend on the payments that it receives from the other banks, interbank payments are a set of fixed points which I will formally characterize later in Subsection 2.2.

A bank i is *solvent* if it repays all liabilities $v_i + y_i^{out}$ in full at $t = 2$, and otherwise it *defaults*. Let $x_{ij} \in [0, y_{ij}]$ be the actual repayments that bank i receives from bank j at $t = 2$. In a model

[10] The interbank liabilities include both on-balance-sheet interbank lending and off-balance-sheet derivative exposures, and hence are a significant proportion for the large banks who are subject to stress tests.

without information design, a bank is solvent if and only if

$$\tilde{A}_i + \sum_{j \neq i} x_{ij} \geq v_i + y_i^{out}.$$

Information Design and Refinance Desicions At $t = 0$, the regulator commits to an information structure $\left\{\hat{\mathbf{S}}, \boldsymbol{\pi}\right\}$ which consists of the *signal space* $\hat{\mathbf{S}}$ and *distribution of signals* $\boldsymbol{\pi}$. A typical signal $s = s_1 \times \cdots \times s_n$ is an n-element vector that reports on all banks. The signal space $\hat{\mathbf{S}} \equiv \hat{\mathbf{S}}_1 \times \cdots \times \hat{\mathbf{S}}_n$ is the collection of available signals, where $\hat{\mathbf{S}}_i$ is the set of available signals for bank i and is finite. Distribution of signals $\boldsymbol{\pi} : \Theta \to \triangle\hat{\mathbf{S}}$ specifies the conditional probability of any signal $s \in \hat{\mathbf{S}}$ at any contingent state $\theta \in \Theta$. Thus $\boldsymbol{\pi}$ is a $|\hat{\mathbf{S}}| \times |\Theta|$ matrix, with a typical element

$$\pi\left(s\left|\theta\right.\right) \equiv \mathbb{P}\left(s\left|\theta\right.\right) \in [0,1],$$

and $\sum_{s\in\hat{\mathbf{S}}} \pi\left(s\left|\theta\right.\right) = 1$ for all θ. The information structure summarizes stress test rules in terms of the informativeness about the underlying bank conditions. For example, the most stringent test rule reports the true state exactly, while the most lenient test rule is uninformative and babbles the same signal.

The disclosure policy affects the default or solvency outcomes via banks' interim refinancing opportunities at $t = 1$. At the beginning of $t = 1$, banks' risky projects $\left\{\tilde{A}_i\right\}$ (state θ) realize, which nobody observes.[11] Given θ, a public signal s is released according to pre-specified conditional distribution $\boldsymbol{\pi}$. Then each bank i chooses an action a_i from a binary set. Bank i could "raises funds", or $a_i = 1$, by pledging its total assets—project with random payoff $\tilde{A}_i$ and repayments from borrower banks $\sum_{j\neq i} x_{ij}$—to outside investors,[12] and use the newly raised cash to repay liabilities at $t = 2$; in this case, outside investors' claim becomes the most senior, and at $t = 2$ they seize bank i's incoming cash flows from its total assets.[13] Or, bank i could also "wait" till $t = 2$, or $a_i = 0$, and clear debts with project payoff $\tilde{A}_i$ and whatever other banks repay $\sum_{j\neq i} x_{ij}$.

Let m_i be the amount of cash that could be raised from risk neutral investors at $t = 1$ against bank i's total assets. As a standard assumption in asset market with frictions, these investors are specialized and hence earn some rent.(e.g., DeMarzo and Duffie, 1999; He and Krishnamurthy, 2013) I assume that investors apply an exogenous discount factor $\delta \in (0, 1]$ on any $t = 2$ cash flows. Throughout the paper, I refer to δ as capturing *outside liquidity premium*. Given signal realization $s \equiv s_1 \times \cdots \times s_n$ and equilibrium bank actions $a \equiv a_1 \times \cdots \times a_n$, investors value the bank's incoming

[11] Under the parameter assumptions that will be introduced in Subsection 2.4, my analysis is robust if $\tilde{A}_i$ is observed by bank i itself, because counterparty uncertainty makes it unfavorable to signal via refinancing choices. In Goldstein and Leitner (2018), when the bank's reservation value on asset is private information, selling assets has signaling effects. As an application of Bergemann and Morris (2016a)'s framework, my model is a special case where the obedience constraints are degenerate in agent types.

[12] For simplicity I assume that banks, if they decide to raise funds, pledge the entire portfolio of assets to outside investors. In the market equilibrium under the optimal information design, it is indeed in the interest of banks to do so, as opposed to pledging a proportion of total assets. See discussion in footnote 20.

[13] This could be interpreted as derivative contracts that create super-seniority for the investors, or directly selling all the assets as in Goldstein and Leitner (2018).

cash flows at the Bayes' posterior. Hence, the bank can raise the following amount of cash when refinancing:

$$m_i(s,a) \equiv \mathbb{E}\left[\delta\left(\tilde{A}_i + \sum_{j\neq i} x_{ij}\right)\middle| s,a\right]. \tag{2}$$

To summarize, essentially, before meeting its obligations at $t=2$, a bank has the option at $t=1$ to exchange its *risky* asset for an m_i amount of *riskless* cash, whose value is determined by disclosure and bank actions.[14]

Let $e_i(s,a,\theta)$ be bank i's cash flows at $t=2$,

$$e_i(s,a,\theta) \equiv \underbrace{a_i m_i}_{\text{refinance}} + \underbrace{(1-a_i)\left(\tilde{A}_i(\theta) + \sum_{j\neq i} x_{ij}\right)}_{\text{wait}}. \tag{3}$$

I assume that the bank is subjective to an exogenous default punishment $\lambda_i > 0$ if it defaults on any liabilities. Bank i's utility at $t=2$ is

$$u_i(s,a,\theta) = \underbrace{\left[e_i(s,a,\theta) - v_i - y_i^{out}\right]^+}_{\text{residual cash flow}} - \underbrace{\lambda_i \cdot \mathbf{1}_{\left\{e_i < v_i + y_i^{out}\right\}}}_{\text{punishment if default}}. \tag{4}$$

I also assume that the default penalty λ_i is sufficiently large (relative to the residual cash flows), so that a bank's objective is to minimize the chance of default. This gives rise to a simple strategy on the bank's refinancing decision, which allows me to focus on the interbank spillovers in the information design problem. The bank will seize the opportunity to refinance whenever the interim market valuation of its total assets m_i is enough to repay its liabilities, and hence ensures no default at $t=2$. This is to say,

$$a_i = \begin{cases} 1 \text{ (raise funds)}, & \text{if } m_i \geq v_i + y_i^{out}, \\ 0 \text{ (wait)}, & \text{if } m_i < v_i + y_i^{out}. \end{cases} \tag{5}$$

As it will be clear when I characterize interbank payments in Subsection 2.2, a bank that waits *does not necessarily default.* Despite a low interim market value, its underlying incoming cash flows may turn out to be sufficient.

Therefore, the financial market allows for risk sharing at a pooling price, a role similar to that in Goldstein and Leitner (2018) where risk sharing incentives result from ex post idiosyncratic project shocks. The major difference here is the spillover effects of full payments to counterparties. In contrast, in Orlov et al. (2018), banks sell projects in the financial market to build up cash buffers against future loss as required by the regulator.

Timeline The model timeline is summarized as follows:

[14] I do not allow netting of interbank liabilities on the part of investors. See Donaldson and Piacentino (2017) for why banks do not net gross liabilities.

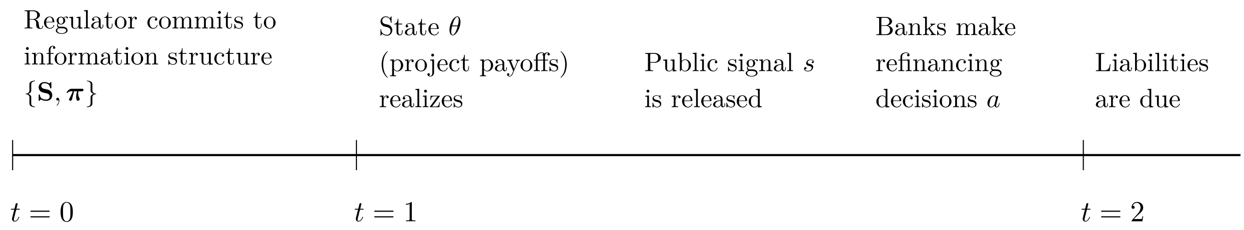

Figure 2: Model Timeline

2.2 Formulating the Information Design Problem

I argue that because a typical bank's action a_i is binary, without loss of generality, I can use the following signal space $\mathbf{S}$ where the signal for each individual bank s_i is binary—h or l,

$$\mathbf{S} \equiv \{s = s_1 \times \cdots \times s_n \,|s_i \in \{h, l\}\,,\ \forall i\}\,. \tag{6}$$

The simplification to binary signal is natural when there is only one agent with binary action. For the multiple banks in this paper, I also need to verify that possible coordinations among banks are not excluded from the signal space $\mathbf{S}$. As will be shown in the next paragraph, in the Bergemann and Morris (2016a) framework that I apply, any coordination among banks could be implemented by some $s \in \mathbf{S}$.

Obedience Constraints I use the Bergemann and Morris (2016a) framework that reformulates signals as action recommendations and adds *obedience constraints* which ensure that each bank does follow the recommended action. Specifically, $s_i = h$ recommends bank i to "raise funds," and $s_i = l$ recommends to "wait." Then the obedience constraints of s say, for each bank i, given that other banks follow the recommendations of s_{-i}, the cash that it could raise against assets is enough to clear debts if and only if $s_i = h$. Put it in a more succinct way, for all i,

$$\begin{cases} s_i = h, s_{-i}: & m_i \geq v_i + y_i^{out}, \\ s_i = l, s_{-i}: & m_i < v_i + y_i^{out}; \end{cases} \tag{7}$$

given (5), the bank i would refinance if and only if $s_i = h$. If the prior profitability of banks is low, an example of violating the obedience constraints is always reporting all banks are h. As obedience constraints assume bank coordination (other banks follow s_{-i}), this framework focuses on the best equilibrium from the designer's perspective. Because I am studying a social planner's problem, the assumption is reasonable here. [15]

As bank actions a are implied by the signal realization s in the Bergemann and Morris (2016a) framework, the notation a could be discarded hereafter. In addition, as banks that choose to

[15] Bergemann and Morris (2016a) focuses on the Bayes Nash Equilibrium that is "obedient." In general information design problem with multiple receivers, equilibrium multiplicity arises from agents' action choices.

refinance must be solvent ex post, henceforth I interchangeably use "reported as h", "passing", "allow refinancing" and "allow risk sharing across states" to refer to some bank i with $s_i = h$.

Interbank Payments At $t = 2$, the state is revealed, and liabilities are due. Recall that $x_{ij} \in [0, y_{ij}]$ is the actual payment from bank j to bank i. I follow Eisenberg and Noe (2001) to determine interbank payments $\{x_{ij}\}$, taking into account banks' refinancing choices at $t = 1$. Recall in (3) that $e_i(s, a, \theta)$ is bank i's cash flows at $t = 2$ as a function of refinancing choices a, and then

$$x_{ij}(s, \theta) = \left\{ \min \left[y_{ij}, \frac{y_{ij}}{y_j^{out}} \left(e_i(s, a, \theta) - v_j \right) \right] \right\}^+ .$$

As shown in the "min" operator, bank j repays bank i up to the face value y_{ij}. If bank j cannot make full payments and defaults, it pays out whatever is left after senior liability payments, $e_i(s, a, \theta) - v_j$, to junior creditors in proportion of the face values owed (pari passu interbank liabilities). The superscript "plus" sign on the outer bracket says payments are subject to limited liabilities.

Recall that a bank refinances if and only if it could raise enough cash to be solvent (see (5)), and the obedience constraints (7) guarantee that banks follow the recommendation of signal s. Hence,

$$x_{ij}(s, \theta) = \begin{cases} y_{ij}, & s_j = h, \\ \left\{ \min \left[y_{ij}, \frac{y_{ij}}{y_j^{out}} \left(\tilde{A}_j(\theta) + \sum_{k \neq j} x_{jk}(s, \theta) - v_j \right) \right] \right\}^+, & s_j = l. \end{cases} \tag{8}$$

If $s_j = h$, under (7), bank j would have raised enough cash at $t = 1$, and makes full payment $x_{ij} = y_{ij}$ at $t = 2$. If instead $s_j = l$, it stays put at $t = 1$. Depending on the realization of the risky incoming cash flows $\tilde{A}_j(\theta) + \sum_{k \neq j} x_{jk}(s, \theta)$, bank j may be solvent or default. Therefore, reporting h enables a bank to share risk across states via refinancing, while reporting l leaves the bank's payments risky.

As a bank's outgoing payments depend on the incoming payments from borrower banks, interbank payments $\{x_{ij}(s, \theta)\}$ are a set of fixed points such that the above payment rule (8) is simultaneously satisfied for every interbank liability.

Regulator's Payoff Let $w(s, \theta)$ be the weighted number of banks that survive at $t = 2$, where default penalty λ_i serves as the exogenous weight on bank i; i.e.,

$$w(s, \theta) \equiv \sum_i \lambda_i \mathbf{1}_{\left\{ \sum_{j \neq i} x_{ji}(s, \theta) = y_i^{out} \right\}}, \tag{9}$$

where $\sum_{j \neq i} x_{ji}(s, \theta) = y_i^{out}$ indicates that bank i is solvent (as it pays junior liabilities in full). The solvent banks include those reported with h, and some that have high realizations of risky incoming cash flows but reported with l (see 8).

I show in Lemma 5 in Appendix B.1 that the total welfare at $t = 2$ is affine in $w(s, \theta)$. As a result, I use $w(s, \theta)$ as the contingent payoff to the regulator at $t = 2$. Then the regulator's payoff at $t = 0$ is the expected weighted number of banks that survive, i.e.,

$$W(\boldsymbol{\pi}) \equiv \mathbb{E}\left[w(s,\theta)\right] = \sum_{\theta} \mathbb{P}(\theta) \sum_{s \in \mathbf{S}} \pi(s\,|\theta)\, w(s,\theta). \tag{10}$$

In the general analysis, the weights $\{\lambda_i\}$ are given exogenously without any restrictions. However, weights become important in specific applications, where I make reasonable assumptions on λ_i. For example, in Section 4, when studying symmetric networks, I assume equal weights; when studying asymmetric networks, I weight banks by their sizes.

Regulator's Problem

Definition 1. Given an information structure $\{\mathbf{S}, \boldsymbol{\pi}\}$ that satisfies the obedience constraints (7), the *market equilibrium* is the collection of refinancing decisions $a(s) = a_1(s) \times \cdots \times a_n(s)$ and interbank payments $\{x_{ij}(s,\theta)\}$, such that

1. at $t = 1$, for any signal s, banks' refinancing decisions $a(s)$ follow signal recommendations; and
2. at $t = 2$, for any (s,θ), interbank payments $\{x_{ij}(s,\theta)\}$ satisfy the payment rule (8) for every interbank liability.

For notational convenience, I introduce $L_i(s,\theta)$ as bank i's state-contingent incoming cash flows at $t = 2$,

$$L_i(s,\theta) \equiv \tilde{A}_i(\theta) + \sum_{j \neq i} x_{ij}(s,\theta). \tag{11}$$

I refer to $L_i(s,\theta)$ as *underlying liquidity* throughout the paper. Given the market equilibrium induced by (s,θ), the regulator's problem $\hat{\mathcal{P}}$ at $t = 0$ is

$$\left(\hat{\mathcal{P}}\right) \qquad W^* = \max_{\boldsymbol{\pi}} \sum_{\theta} \mathbb{P}(\theta) \sum_{s \in \mathbf{S}} \pi(s\,|\theta) \sum_{i} \lambda_i \mathbf{1}_{\{\sum_{j \neq i} x_{ji} = y_i^{out}\}} \tag{12}$$

$$\text{(Obedience: h)} \qquad s.t. \sum_{\theta} \left[\frac{\mathbb{P}(\theta)\pi(s\,|\theta)}{\sum_{\theta} \mathbb{P}(\theta)\pi(s\,|\theta)} \cdot \delta L_i(s,\theta)\right] \geq v_i + y_i^{out}, \ (\forall s, \forall i \text{ with } s_i = h) \tag{13}$$

$$\text{(Obedience: l)} \qquad \sum_{\theta} \left[\frac{\mathbb{P}(\theta)\pi(s\,|\theta)}{\sum_{\theta} \mathbb{P}(\theta)\pi(s\,|\theta)} \cdot \delta L_i(s,\theta)\right] < v_i + y_i^{out}, \ (\forall s, \forall i \text{ with } s_i = l) \tag{14}$$

$$\text{(Prior Consistency)} \qquad \sum_{s} \pi(s\,|\theta) = 1, \qquad (\forall \theta) \tag{15}$$

$$\text{(Probability)} \qquad 0 \leq \pi(s\,|\theta) \leq 1. \qquad (\forall \theta, \mathbf{s}) \tag{16}$$

At $t = 0$, the regulator commits to the optimal information structure that maximizes $W(\boldsymbol{\pi})$, subject to the obedience constraints for both $s_i = h$ and $s_i = l$, the prior consistency constraints that at each state the conditional probabilities of reporting all possible signals add up to 1, and the constraints of $\pi(s|\theta)$ being a probability measure.

The obedience constraints (13) and (14) are derived from (2) and (7). For example, (13) says, for any bank i reported as $s_i = h$ under some signal s, when all other banks follow their action recommendation s_{-i}, the posterior expectation of bank i's incoming cash flows is enough to repay liabilities, so bank i would indeed raise funds. Similarly, (14) says that any bank reported with $s_i = l$ strictly prefers to "wait."

In the left hand side of (13) and (14), the discount rate of outside investors $\delta \leq 1$ says outside capital is expensive.[16] Under (13), *riskless* cash injection from *outside of the network* to replace the *risky in-network* cash flow of interbank payments increases the expected value of the latter, which is reminiscent of valuable liquidity injection into the distressed banking system during 2007-09 financial crisis. As a result, refinancing, which is essentially a risk sharing scheme, may increase bank value even if $\delta < 1$ in my model. The information design problem explores the optimal way to share risk (report which banks with favorable signal h) given the network structure $\{y_{ij}\}$ and the cost of outside liquidity δ.

2.3 Autarky

I introduce the market outcome absent information design as the *autarky benchmark*. As will be shown later in Section 3, it is convenient to characterize the incremental payoff from the outcome at prior as the incentive of influencing beliefs. In the autarky benchmark, bank i's refinancing decision a_i is determined by other banks' refinancing decisions a_{-i}, and the investors' prior about the project $\tilde{A}_i$. Depending on the refinancing choices, interbank payments at contingent states $\left\{x^0_{ij}(a,\theta)\right\}$ are defined similarly as in (8), where superscript "0" represents autarky. Let $w_0(\theta)$ and W_0 denote respectively the contingent state and ex ante expected number of banks that survive in autarky:

$$w_0(\theta) \equiv \sum_i \lambda_i \mathbf{1}_{\left\{\sum_{j\neq i} x^0_{ij}(a,\theta) = y^{out}_i\right\}},$$
$$W_0 \equiv \mathbb{E}\left[w_0(\theta)\right] = \sum_\theta \mathbb{P}(\theta)\, w_0(\theta).$$

Market equilibria in autarky could be implemented by null information structures that always reports the same signal. In general, the exact signal that is babbled does not matter. In the Bergemann and Morris (2016a) framework, however, the babbled signal additionally serves to coordinate bank actions, so null information structures with different signals lead to different market equilibria. Hence, the set of feasible null information structures corresponds to all the autarky equilibria, where each signal implements one equilibrium.

[16] As explained, it can be justified as the premium or rent earned by specialized intermediary investors such as hedge funds.

Under the low prior parameter assumption that will be introduced in Subsection 2.4, there is a unique equilibrium in autarky where no bank refinances. Then autarky could be represented by a policy that always reports all banks with l; i.e., for all $\theta \in \Theta$, we have $\pi\left(s_l \,|\theta\right) = 1,$ where $s_l \equiv (l, \cdots, l)$. Therefore, in this paper, s_l could be viewed as some default signal. Any reported signal $s \neq s_l$ involves refinancing for some banks and improves the autarky outcome.

2.4 Preliminary Analysis

Now I show some regularity results on the solution to the information design problem over the financial network.

Proposition 1. *The solution to the regulator's problem $\hat{\mathcal{P}}$ in (12) exists and is generically unique.*

First, Eisenberg and Noe (2001) and Acemoglu et al. (2015) show that the set of interbank payments exists and is generically unique. The key is that the out-of-network cash flows (loan projects and senior liabilities) pin down the fixed points of payments.[17] I argue that the result extends to my payment rule (8) that incorporates interim refinancing. Refinancing is similar to altering the network structure such that a bank that refinanced becomes some "source" node to provide riskless cash to other banks. Second, in the Bergemann and Morris (2016a)'s framework, given the interbank payments that may result, the refinancing equilibrium is specified by signal s and hence also unique. Last, in the information design problem, the regulator essentially chooses a distribution of market equilibrium $\{a\left(s\right), \{x_{ij}(s,\theta)\}\}$ via influencing beliefs. Generically, the solution to the linear programming problem is unique. When for example, banks are symmetric, multiple optimal policies exist and deliver the same payoff. In that case, without loss of generality, I focus on the symmetric policy.

Lemma 1. *The following relaxed problem $\mathcal{P}$ without the obedience constraints of $s_i = l$ (14) has the same solution as the original problem $\hat{\mathcal{P}}$:*

$$(\mathcal{P}) \qquad W^* = \max_{\boldsymbol{\pi}} \sum_{\theta} \mathbb{P}\left(\theta\right) \sum_{s \in \mathbf{S}} \pi\left(s\,|\theta\right) \sum_{i} \lambda_i \mathbf{1}_{\{\sum_{j \neq i} x_{ji} = y_i^{out}\}} \tag{17}$$

$$(\textit{Obedience: } h\,) \qquad s.t. \sum_{\theta} \left[\frac{\mathbb{P}(\theta)\pi\left(s\,|\theta\right)}{\sum_{\theta}\mathbb{P}(\theta)\pi\left(s\,|\theta\right)} \cdot \delta L_i\left(s,\theta\right)\right] \geq v_i + y_i^{out}, \ (\forall s, \forall i \textit{ with } s_i = h) \tag{18}$$

$$(\textit{Prior Consistency}) \qquad \sum_{s} \pi\left(s\,|\theta\right) = 1, \qquad (\forall \theta) \tag{19}$$

$$(\textit{Probability}) \qquad 0 \leq \pi\left(s\,|\theta\right) \leq 1. \qquad (\forall \theta, s) \tag{20}$$

Lemma 1 says that I can instead solve the relaxed problem $\mathcal{P}$. Intuitively, refinancing not only guarantees the bank's own survival, but also improves the other banks' incoming cash flows. Hence,

[17] I take senior liabilities as an example for illustration. Suppose the network consists of two symmetric banks with $y_{12} = y_{21} = y$ and $v_1 = v_2 = 0$. If $\tilde{A}_1 = \tilde{A}_2 = 0$ realizes, then any set of payments with $x_{12} = x_{21} \in [0, y]$ is an equilibrium. If instead $v_1 > 0$, then $x_{12} = x_{21} > 0$ cannot be an equilibrium because any payments of x_{12} received should be paid to senior debt first. Hence, the out-of-network flows pinns down the unique set of payments.

the regulator prefers a bank's refinancing to waiting, and would let a bank refinance whenever possible. So the optimal solution to the relaxed problem $\mathcal{P}$ satisfies the obedience constraints of $s_i = l$ (14) in the original problem $\hat{\mathcal{P}}$. Henceforth, whenever I mention obedience constraints, I refer to those associated with h (13).

Throughout this paper, I impose the following parameter assumptions.

Assumption 1. *The network's total net cash flow is positive in expectation, i.e.,*

$$\sum_i p_i A_i \geq \sum_i v_i.$$

Assumption 1 says that the financial network as a collection has positive NPV. The inequality involves only the outside-of-network cash flows $\left\{\tilde{A}_i\right\}$ and $\{v_i\}$, simply because by definition the aggregate interbank claims and interbank liabilities are balanced, $\sum_i y_i^{in} = \sum_i y_i^{out}$.

Lemma 2. *Suppose $\delta = 1$. If for each bank, interbank liabilities and claims are balanced $y_i^{in} = y_i^{out}$, and NPV is positive $p_i A_i \geq v_i$, then the optimal policy $\boldsymbol{\pi}^*$ always reports all banks h, i.e., for all $\theta \in \Theta$, $\pi^*(s_h | \theta) = 1$.*

In the class of networks that satisfy the conditions in Lemma 2, when all other banks are reported with $s_{-i} = (h, \cdots, h)$, bank i receives full payments y_i^{in} and is solvent in expectation. Hence, it is optimal to report $s_i = h$ as well. Therefore, the optimal policy implements centralized clearing on the interbank liabilities, with the participation of decentralized investors.

The illustrative examples in Section 4 satisfy the conditions in Lemma 2. To rule out the uninteresting centralized clearing result, I assume that the outside liquidity premium is sufficiently high, as specified in the following assumption.

Assumption 2. *For every $i \in \{1, \cdots, n\}$,*

$$\delta\left(pA_i + y_i^{in}\right) < v_i + y_i^{out}. \tag{21}$$

Under this condition, banks do not refinance in autarky.

(21) is a sufficient condition for no refinancing in autarky,[18] which is satisfied when prior is low or the outside liquidity premium is high. Focusing on the distress scenario greatly simplifies the analysis. Under Assumption 2, autarky is implemented by the null information structure that always reports $s_l \equiv (l, \cdots, l)$, i.e.,

$$\forall \theta \in \Theta,\ \pi(s_l | \theta) = 1,\ \text{where } s_l \equiv (l, \cdots, l).$$

Therefore, any reported signal $s \neq s_l$ leads to some risk sharing and improves the autarky outcome, whereas s_l could be viewed as a default signal.

[18] A sufficient and necessary condition should be that there is only one feasible null information structure, with $s_l \equiv (l, \cdots, l)$.

3 General Solution Properties

In this part, I first discuss the spillover effects of disclosure on counterparty banks. Then I introduce an index that summarizes the value of any particular way of risk sharing, based on which I introduce some general properties of the optimal disclosure policy.

3.1 Cross-state and Cross-bank Risk Sharing

In this paper's context, an example of single-bank stress test is a class of restricted policies in which the regulator designs the disclosure on only one bank i (the signal space for any other bank is a singleton, say $\{l\}$). Another example is the Counterparty Default Component in practice. Disclosure on each bank is separately determined, assuming that only the bank's largest counterparty defaults. I evaluate this policy in Section 4.3.

In the single-bank stress test literature, the optimal (restricted) policy features the *cross-state risk sharing*: the passing signal h is not fully informative, but instead "pools" good states—where the bank is solvent—with bad states, to the extent that h means being solvent on average. As explained in Section 2.1, the "pooling" is via refinancing, i.e., exchanging the risky asset payoffs that are contingent on the underlying states for riskless cash. In contrast, reporting l induces waiting and hence does not involve any cross-state borrowing.

Contingent on a bad state where bank i would otherwise default in autarky, if $s_i = h$ realizes, the bank raises enough cash to make full payments, which improves the system solvency. However, investors discount their beliefs about $s_i = h$, knowing that it may be reported at this bad state. Goldstein and Leitner (2018) summarize the efficiency of reporting h at each state by a gain-over-cost ratio. The optimal policy reports h at all states with ratios above a threshold that is determined by the prior consistency condition.[19]

Missing in single-bank stress tests is that disclosure about one bank is informative about the counterparty risks faced by other banks, which further affects the disclosure policy about them. Novel in this paper, I study the disclosure policy on the system as a whole. This leads to a *cross-bank risk sharing* effect that is new to the literature. Specifically, when the public signal reports h on multiple banks, these banks refinance and deliver to each other as counterparties. Their asset qualities (project and interbank claims) improve in a collective way, as compared with the case where some of these banks are reported with l and make risky counterparty payments.

On one hand, the regulator would like to pass as many banks at the same time as possible to minimize counterparty risk. On the other hand, there is cost to report the same signal $s_i = h$ across banks whose cash flows are not completely aligned (due to independent project returns), as the regulator has to convince investors that the weakest bank in the group is on average solvent. In the Subsection 3.3, I introduce an index that summarizes the efficiency of both cross-state and cross-bank risk sharing.

[19] For more general persuation problems where only the posterior mean matters, Dworczak and Martini (2019) characterize the conditions for monotone partitional signaling.

3.2 Dual Problem and Binding Obedience Constraints

The regulator's relaxed problem $\mathcal{P}$ in (17) is linear programming with finite vertices. One can always calculate and compare the value at each vertex. This brute-force search method has two apparent drawbacks. First, for relatively large n, the dimensionality could be prohibitively high. Second, it is difficult to draw economic implications or conduct comparative statics, because I only know which constraints are binding when the optimal policy is found.

Instead, I introduce an index to characterize the efficiency for any particular way of risk sharing. The index summarizes the tradeoffs of cross-state and cross bank risk sharing, and implies some general properties of the optimal policy. The difficulty comes from the multiple obedience constraints (13) imposed on the banks whomever the signal reports with h, which essentially say that the posterior asset quality of these banks is above liabilities so that they refinance. Which subset of these obedience constraints bind depends on both the collection of states where s is reported, and the distributions of the other signals on these states. As a result, unlike in single-bank stress tests a la Goldstein and Leitner (2018), here the value of risk sharing under a specific signal is generally not separable across states but depends on the exact distribution.

To deal with this difficulty, I show that all obedience constraints regarding h, i.e., (13), must bind for banks that are reported as h with positive probabilities in the optimal policy. This allows me to calculate the efficiency index directly. As a preparation, I first introduce the dual problem of $\mathcal{P}$ (17) to connect the optimal policy with the shadow values.

Dual Problem For notational convenience, let

$$I_h(s) \equiv \{i \in \{1, 2, \cdots, n\} \,|s_i = h\} \tag{22}$$

denote the collection of banks that signal s reports with $s_i = h$. Let $\mu_i(s)$ denote the multiplier of the obedience constraint (13) for bank $i \in I_h(s)$ under signal s, and $q(\theta)$ denote the multiplier of the prior consistency constraint (15) at state θ. Intuitively, μ reflects the value of risk sharing across states, and q reflects the value of influencing belief at a particular state. I show in Appendix B.4 that strong duality holds, and solving $\mathcal{P}$ in (17) is equivalent to solving the following dual problem $\mathcal{D}$:

$$(\mathcal{D}) \quad W^* = \min_{\substack{\forall s:\ \boldsymbol{\mu}^s \in \mathbb{R}^{I_h(s)} \\ \boldsymbol{q} \in \mathbb{R}^{\Theta}}} \sum_{\theta} \mathbb{P}(\theta)\, q(\theta) \tag{23}$$

$$s.t \;\; q(\theta) \geq w(s, \theta) + \sum_{i \in I_h(s)} \mu_i(s) \left(\delta L_i(s, \theta) - v_i - y_i^{out}\right), \;(\forall \theta, \forall s) \tag{24}$$

$$\mu_i(s) \geq 0, \qquad\qquad (\forall s_i = h \text{ in } \forall s)$$

where $\boldsymbol{\mu}^s$ and $\boldsymbol{q}$ are the vector-form of multipliers, and the layout of condition (24) follows Galperti and Perego (2018) for better economic interpretations.

As stated in the dual problem $\mathcal{D}$ in (23), the optimal information structure minimizes the expected cost of influencing beliefs across states, $\sum_{\theta}\mathbb{P}(\theta)\,q(\theta)$. According to Condition (24), the marginal cost at a particular state $q(\theta)$, or the shadow value of reporting signals, is no less than the marginal value of reporting any signal s. Specifically, as shown in the right hand side of (24), the value of reporting a particular signal s consists of two parts: the resulting system stability $w(s,\theta)$, and the value of cross-state borrowing of banks whoever reported with h,

$$\sum_{i\in I_h(s)}\underbrace{\mu_i(s)}_{\text{price}}\underbrace{\left(\delta L_i(s,\theta)-v_i-y_i^{out}\right)}_{\text{quantity}}.$$

Intuitively, the shadow value of (13) $\mu_i(s)$ is the *price* of cross-state borrowing; depending on its sign, $\delta L_i(s,\theta)-v_i-y_i^{out}$ is the contingent state surplus or shortfall compared with total liabilities, and thus measures the *quantity* lent or borrowed at θ.

For signals that are reported at θ with positive probability, i.e. $\pi(s\,|\theta)>0$, the marginal cost equals marginal value and therefore (24) takes equality; for signals that are not reported, i.e. $\pi(s\,|\theta)=0$, their associated values must be smaller than the marginal cost $q(\theta)$, so the inequality in (24) is strict generically. These are summarized by the following complementary-slackness condition, where under strong duality, the shadow value of (24) is $\mathbb{P}(\theta)\,\pi(s\,|\theta)$:

$$\underbrace{\mathbb{P}(\theta)\,\pi(s\,|\theta)}_{\text{multiplier of (24)}}\cdot\left\{q(\theta)-\left[w(s,\theta)+\sum_{i\in I_h(s)}\mu_i(s)\left(\delta L_i(s,\theta)-v_i-y_i^{out}\right)\right]\right\}=0.$$

Therefore, (24) says that the regulator influences belief at state θ using the signal s with the highest value.

Recall that the autarky case absent information design could be implemented by always reporting $s_l\equiv(l,\cdots,l)$. s_l does not involve any cross-state borrowing, and thus the value of reporting s_l at some state is $w_0(\theta)$. Then as long as the optimal policy specifies some banks to refinance at certain state θ, the value at that state exceeds autarky, $q(\theta)>w_0(\theta)$.

Binding Obedience Constraints Recall in Lemma 1 that it is without loss of generality to solve the relaxed problem $\mathcal{P}$ in (17) without the obedience constraints of l (14), and henceforth the obedience constraints refer to those of h, i.e., (13).

Proposition 2, which is one of the key results in the paper, shows formally that under Assumption 2 (which guarantees no bank refinances at prior), (13) are binding for all banks that are reported with $s_i=h$. Economically, the binding obedience constraints of h means that the regulator exhausts all resources when designing information to allow for risk sharing.[20] This is a non-trivial result as the signal s may report h on multiple banks: if the obedience constraint for some bank is slack, a simple replication argument that increases the signal's probability at bad states might violate the

[20]This also implies that the equilibrium under the optimal information design policy is robust to the possibility of partial refinancing by banks, i.e., a bank raises funds against a proportion of its total assets. Under the optimal policy, banks who refinance under h has no residual cash flow. Hence, in face of large default penalty, these banks have no incentive to deviate to pledge only a proportion of their assets (which leads to default for sure).

obedience constraints of other banks that *this signal* reports as h.

Proposition 2. *In the optimal solution $\boldsymbol{\pi}^*$ to the problem $\mathcal{P}$ in (17), the obedience constraints (13) must bind for every bank i that s reports with $s_i = h$ on the support, i.e., the multiplier of (13) satisfies $\mu_i(s) > 0$ for all $i \in I_h(s)$ generically.*

Proposition 2 only depends on banks' idiosyncratic shocks and low priors (more specifically, Assumption 2), and hence extends to a very general class of information design problems. Here is the sketch of the proof (by contradition); for details, see Appendix B.5. For the illustration purpose here, say there exists only one violation to Proposition 2: $\mu_{i'}(s) = 0$ for some signal s that reports $s_{i'} = h$ on bank i'. First, I show that s will be reported disproportionately more at states where bank i has a bad project $\tilde{A}_{i'} = 0$. Second, this disproportionate reporting of s contradicts with Assumption 2 of relatively low prior and/or relatively high premium (of outside liquidity), completing the proof.

While the second claim is more transparent, the first claim, whose formal proof relies on the dual problem formulated above, is helpful in understanding the model mechanism. To see this, I show that at any pair of mirror image states that have the same project realizations for all banks except i' (same $\tilde{A}_{-i'}$), the value of reporting s is the same. Intuitively, this is because the additional project shock $\tilde{A}_{i'} = 0$ at the worse state is absorbed by outside investors and thus has no effect on the network. In addition, s is preferred at such worse states with $\tilde{A}_{i'} = 0$, because for other signals s', either it is costly for bank i' to refinance (i.e., $\mu_{i'}(s') > 0$), or absent refinancing bank i' defaults (i.e., $s'_{i'} = l$).

3.3 Risk Sharing Strategies

The binding obedience constraints established in Proposition 2 allow me to define the smallest elements—I call risk sharing strategies—that make up a feasible disclosure policy $\boldsymbol{\pi}$. In addition, they allow me to directly characterize the value of any particular risk sharing strategy by an index, which sheds light on the properties of the optimal solution.

Let θ_g^s denote a "good" state for signal $s \neq s_l \equiv (l, \cdots, l)$, where banks whomever are reported with h—$I_h(s) \equiv \{i \,|s_i = h\}$ have excess net inflows, i.e.,

$$\theta_g^s \in \left\{\theta \in \Theta \middle| \forall i \in I_h(s),\ \Delta L_i\left(s, \theta_g^s\right) \equiv \delta L_i\left(s, \theta_g^s\right) - v_i - y_i^{out} \geq 0\right\} \equiv \Theta_g^s. \tag{25}$$

Let $\tilde{\Theta}^s$ denote a collection of other states, such that $\tilde{\Theta}^s \cap \Theta_g^s = \emptyset$ and $\left|\tilde{\Theta}^s\right| = |I_h(s)|$, i.e., the number of states $\left|\tilde{\Theta}^s\right|$ is the same as that of banks in $I_h(s)$.

Definition 2. I call the tuple $\left(s, \tilde{\Theta}^s; \theta_g^s\right)$ a *risk sharing strategy*: reporting s at both a "good" state θ_g^s and some "bad" states $\tilde{\Theta}$, under which banks in $I_h(s)$ borrow liquidity from θ_g^s to $\tilde{\Theta}^s$ via interim refinancing, such that each bank is just solvent at the posterior, i.e., binding obedience constraints in Eq. (18). The probability weights over $\tilde{\Theta}^s$ is proportional to some $\left|\tilde{\Theta}^s\right| \times 1$ vector[21]

$$\left(\boldsymbol{\kappa}^{s,\tilde{\Theta}^s;\theta_g^s}\right)_{\left|\tilde{\Theta}^s\right|\times 1} \equiv \underbrace{\left(-\Delta\boldsymbol{\mathcal{L}}^{s,\tilde{\Theta}^s}\right)^{-1}_{|I_h(s)|\times\left|\tilde{\Theta}^s\right|}}_{\text{shortage}} \underbrace{\left(\Delta\boldsymbol{\mathcal{L}}^{s,\theta_g^s}\right)_{|I_h(s)|\times 1}}_{\text{excess net inflows}}. \tag{26}$$

I denote each element of $\boldsymbol{\kappa}^{s,\tilde{\Theta}^s;\theta_g^s}$ by $\kappa\left(s,\theta\left(\tilde{\Theta}^s\right);\theta_g^s\right)$.

For example, consider a three-bank complete network where all banks are symmetric and each borrows equally from the other banks. One risk sharing strategy is $(hhh, \{bgg, gbg, ggb\}; ggg)$, in which hhh is reported both at ggg and across $\tilde{\Theta} = \{bgg, gbg, ggb\}$ with structure $\boldsymbol{\kappa} \propto (1,1,1)^\top$ by symmetry. With respect to this strategy, when observing hhh, investors know that there may be one bad project equally likely in one of the banks, such that all banks are just solvent at posterior.

The risk sharing strategy ensures the number of bad states $\left|\tilde{\Theta}^s\right|$ to be the same as the number of banks $I_h(s)$ that refinance (generically), so that I can invert the binding obedience constraints in Eq. (18). In Equation (26), $\Delta\boldsymbol{\mathcal{L}}^{s,\theta_g^s}$ is the amount of excess liquidity accumulated when the regulator increases the probability of s at θ_g^s marginally, and $-\Delta\boldsymbol{\mathcal{L}}^{s,\tilde{\Theta}}$—the shortage from liabilities—is the marginal amount borrowed when reporting s at $\tilde{\Theta}^s$. Hence, $\boldsymbol{\kappa}^{s,\tilde{\Theta}^s,\theta_g^s}$ is the induced probability distribution of s across $\tilde{\Theta}^s$ when the regulator increases the probability of s at θ_g^s marginally.

The following Proposition 3 shows that I can focus on the strategies with $\boldsymbol{\kappa}^{s,\tilde{\Theta}^s,\theta_g^s} \geq \mathbf{0}$ or $\boldsymbol{\kappa}^{s,\tilde{\Theta}^s,\theta_g^s} \leq \mathbf{0}$. Then the probability measure constraints of s on θ_g^s and $\tilde{\Theta}^s$ imply that strategy $\left(s,\tilde{\Theta}^s;\theta_g^s\right)$ has the maximum probability

$$\overline{\eta}\left(s,\tilde{\Theta}^s;\theta_g^s\right) \equiv \max\left\{\mathbb{P}\left(\theta_g^s\right), \max_{\theta\in\tilde{\Theta}^s}\left|\frac{\mathbb{P}(\theta)}{\kappa\left(s,\theta;\theta_g^s\right)}\right|\right\}.$$

The next definition characterizes the value of risk sharing strategies.

Definition 3. The value of a risk sharing strategy $\left(s,\tilde{\Theta}^s;\theta_g^s\right)$, denoted by $\xi\left(s,\tilde{\Theta}^s;\theta_g^s\right) \in \mathbb{R}$, is defined as

$$\xi\left(s,\tilde{\Theta}^s;\theta_g^s\right) \equiv \underbrace{\overline{\eta}\left(s,\tilde{\Theta}^s;\theta_g^s\right)}_{\text{maximum probability}} \underbrace{\left(\Delta\boldsymbol{w}^{s,\tilde{\Theta}}\right)^\top}_{\text{incremental stability}} \underbrace{\underbrace{\left(-\Delta\boldsymbol{\mathcal{L}}^{s,\tilde{\Theta}}\right)^{-1}}_{\text{shortage}} \underbrace{\left(\Delta\boldsymbol{\mathcal{L}}^{s,\theta_g^s}\right)}_{\text{excess net inflows}}}_{=\boldsymbol{\kappa}^{s,\tilde{\Theta}^s,\theta_g^s}}, \tag{27}$$

where $\Delta\boldsymbol{\mathcal{L}}^{s,\theta_g^s}$ and $\Delta\boldsymbol{\mathcal{L}}^{s,\tilde{\Theta}}$ are the vector or matrix form of $\Delta L_i(s,\theta)$ for banks in $I_h(s)$ at states θ_g^s and $\tilde{\Theta}$ respectively, and $\Delta\boldsymbol{w}^{s,\tilde{\Theta}}$ is the vector-form incremental system stability $w(s,\theta) - w_0(\theta)$ given signal s across states $\tilde{\Theta}$.

[21]Generally $\boldsymbol{\kappa}$ is not necessarily non-negative. If for some $\theta\left(\tilde{\Theta}^s\right) \in \tilde{\Theta}^s$ we have $\kappa\left(s,\theta\left(\tilde{\Theta}^s\right);\theta_g^s\right) < 0$, then banks $I_h(s)$ accumulate liquidity at $\theta\left(\tilde{\Theta}^s\right)$. If $\boldsymbol{\kappa}^{s,\tilde{\Theta}^s;\theta_g^s} \leq \mathbf{0}$, then there exists some probability weights over $\tilde{\Theta}^s$ with which $\tilde{\Theta}^s$ as a collection resembles a good state, where banks $I_h(s)$ accumulate liquidity. Proposition 3 shows that the optimal solution could be decomposed as a weighted sum of strategies with $\boldsymbol{\kappa} \leq \mathbf{0}$ or $\boldsymbol{\kappa} \geq \mathbf{0}$.

In (27), $\left(\Delta\boldsymbol{w}^{s,\tilde{\Theta}}\right)^{\top}\boldsymbol{\kappa}^{s,\tilde{\Theta}^s,\theta_g^s}$ measures the strategy's *marginal value.* To see this, recall that $\boldsymbol{\kappa}^{s,\tilde{\Theta}^s,\theta_g^s}$ is the induced probability of refinancing across $\tilde{\Theta}^s$ if the regulator increases the probability of s at θ_g^s marginally: $\Delta\boldsymbol{\mathcal{L}}^{s,\theta_g^s}$—the amount of excess liquidity—is the marginal amount lent, and $-\Delta\boldsymbol{\mathcal{L}}^{s,\tilde{\Theta}}$—the shortage from liabilities—is the marginal amount borrowed. The other part $\Delta\boldsymbol{w}^{s,\tilde{\Theta}}$ is the improvement in system stability when banks reported as h $(I_h(s))$ refinance at $\tilde{\Theta}^s$. Hence, $\left(\Delta\boldsymbol{w}^{s,\tilde{\Theta}}\right)^{\top}\boldsymbol{\kappa}^{s,\tilde{\Theta}^s,\theta_g^s}$ is the improvement in system stability when the regulator marginally increases the probability of the risk sharing strategy $\left(s,\tilde{\Theta}^s;\theta_g^s\right)$. When s reports only one bank being h and all other banks being l, $\left(\Delta\boldsymbol{w}^{s,\tilde{\Theta}}\right)^{\top}\boldsymbol{\kappa}^{s,\tilde{\Theta}^s,\theta_g^s}$ is the gain-to-cost ratio in Goldstein and Leitner (2018).

Eq. (27) additionally incorporates the strategy's maximum probability $\overline{\eta}\left(s,\tilde{\Theta}^s;\theta_g^s\right)\leq\mathbb{P}\left(\theta_g^s\right)$. Therefore, $\xi\left(s,\tilde{\Theta}^s;\theta_g^s\right)$ measures a strategy's maximum payoff. When bank profitability is high and there is ample liquidity to borrow, $\overline{\eta}\left(s,\tilde{\Theta}^s;\theta_g^s\right)=\frac{\mathbb{P}(\theta)}{\kappa\left(s,\tilde{\theta};\theta_g^s\right)}$ for some $\tilde{\theta}\in\tilde{\Theta}^s$, so banks reported as h could fully refinance at $\tilde{\theta}$ by borrowing only a fraction $\frac{\overline{\eta}\left(s,\tilde{\Theta}^s;\theta_g^s\right)}{\mathbb{P}\left(\theta_g^s\right)}<1$ of liquidity from the good state θ_g^s; in this case, ξ is constrained by how much s improves stability. Intuitively, a signal that reports more banks as h results in higher improvement. In contrast, when bank profitability is low and cross-state borrowing is constrained, $\overline{\eta}\left(s,\tilde{\Theta}^s;\theta_g^s\right)=\mathbb{P}\left(\theta_g^s\right)$ and the strategy exhausts the liquidity the good state θ_g^s before banks $I_h(s)$ could fully refinance at any state in $\tilde{\Theta}$; in this case, ξ is additionally constrained by the probability of refinancing $\boldsymbol{\kappa}$.

These two cases can be illustrated by the following example where $\tilde{\Theta}^s=\left\{\tilde{\theta}\right\}$ is a singleton (therefore both $\Delta\mathcal{L}^{s,\theta_g^s}$ and $\Delta\mathcal{L}^{s,\tilde{\theta}}$ are scalars):

$$\xi\left(s,\left\{\tilde{\theta}\right\};\theta_g^s\right)=\begin{cases}\frac{\Delta\mathcal{L}^{s,\theta_g^s}}{-\Delta\mathcal{L}^{s,\tilde{\theta}}}\mathbb{P}\left(\theta_g^s\right)\left[w\left(s,\tilde{\theta}\right)-w_0\left(\tilde{\theta}\right)\right], & \overline{\eta}\left(s,\left\{\tilde{\theta}\right\};\theta_g^s\right)=\mathbb{P}\left(\theta_g^s\right),\\ w\left(s,\tilde{\theta}\right)-w_0\left(\tilde{\theta}\right), & \overline{\eta}\left(s,\left\{\tilde{\theta}\right\};\theta_g^s\right)<\mathbb{P}\left(\theta_g^s\right).\end{cases}\tag{28}$$

Intuitively, when cross-state risk sharing is unconstrained, a signal that passes more banks is preferred due to large improvement in outcome; otherwise, such a signal may be unfavorable if the underlying liquidities across passing banks $\left(-\Delta\boldsymbol{\mathcal{L}}^{s,\tilde{\Theta}}\right)$ are misaligned, so the worse banks that are reported as h restrict the refinancing probability.

The next proposition shows that the optimal policy is a weighted sum of risk sharing strategies, so one can view the risk sharing strategy $\left(s,\tilde{\Theta}^s;\theta_g^s\right)$ to be the smallest building block of the optimal solution.

Proposition 3. *Let* $\boldsymbol{R}\equiv\left\{\left(s,\tilde{\Theta}^s;\theta_g^s\right)\middle|\boldsymbol{\kappa}^{s,\tilde{\Theta}^s,\theta_g^s}\geq\mathbf{0}\text{ or }\boldsymbol{\kappa}^{s,\tilde{\Theta}^s,\theta_g^s}\leq\mathbf{0}\right\}$ *where* $\boldsymbol{\kappa}^{s,\tilde{\Theta}^s,\theta_g^s}$ *is defined in (26).*
1) The optimal policy $\boldsymbol{\pi}^*$ *is the solution to the following problem that finds weights* $\left\{\eta\left(s,\tilde{\Theta}^s;\theta_g^s\right)\right\}$ *for strategies* $\left(s,\tilde{\Theta}^s;\theta_g^s\right)\in\boldsymbol{R}$*:*

$$\max_{\eta} \sum_{(s,\tilde{\Theta}^s;\theta_g^s)\in \boldsymbol{R}} \frac{\eta\left(s,\tilde{\Theta}^s;\theta_g^s\right)}{\overline{\eta}\left(s,\tilde{\Theta}^s;\theta_g^s\right)} \xi\left(s,\tilde{\Theta}^s;\theta_g^s\right)$$

$$s.t.\ (PC1)\forall\theta: \quad \sum_{s\neq s_l:\theta\in\Theta_g^s} \sum_{\tilde{\Theta}^s} \eta\left(s,\tilde{\Theta}^s;\theta_g^s\right) + \sum_{s\neq s_l:\theta\notin\Theta_g^s} \sum_{\tilde{\Theta}^s:\theta\in\tilde{\Theta}^s} \eta\left(s,\tilde{\Theta}^s;\theta_g^s\right)\kappa\left(s,\theta;\theta_g^s\right) \leq \mathbb{P}(\theta), \quad (29)$$

$$\eta\left(s,\tilde{\Theta}^s;\theta_g^s\right)\boldsymbol{\kappa}\left(s,\tilde{\Theta}^s;\theta_g^s\right) \geq \boldsymbol{0}; \quad (30)$$

2) Suppose $\delta < \overline{\delta}$ for some $\overline{\delta} < 1$ such that there is no strategy with $\boldsymbol{\kappa} \leq \boldsymbol{0}$. In the optimal policy $\boldsymbol{\pi}^*$, the number of states where $s_l \equiv (l, \cdots, l)$ is not reported ((29) binds) equals the number of strategies that make up $\boldsymbol{\pi}^*$.

The proposition connects the index ξ with the optimal policy $\boldsymbol{\pi}^*$. Intuitively, as the objective is linear, the optimal policy in general will "prioritizes" the most valuable risk sharing strategies until some prior consistency constraint (29) binds. Hence, the number of strategies equals the number of binding (29).

3.4 Illustrating Examples

While it is difficult to characterize the exact decomposition of $\boldsymbol{\pi}^*$ in general,[22] in the next subsections I present a three-bank complete network example to illustrate the connection between the relative efficiencies of strategies and the properties of the solution structure. Then I the extend these properties to more general networks.

Example 1. Banks are ex ate identical, and each bank borrows equally from all other banks, i.e., $A_i = A$, $v_i = v$ and $y_{ji} = \frac{y_i^{out}}{n-1} = \frac{y}{n-1}$. Suppose that whenever there is a bad project, all banks default in autarky, i.e., $w_0(\theta) = 0$ for any $\theta \neq \overline{\theta} \equiv ggg$. This also implies that $\overline{\theta}$ is the only "good" state to lend liquidity. I examine two sets of parameters—interbank exposure $\underline{y} < \overline{y}$, and the probability of a good project $\underline{p} < \overline{p}$ which affects bank profitability.

First, I present the reported signals at each state in the optimal policy $\boldsymbol{\pi}^*$ in Figure 3, to show that $\boldsymbol{\pi}^*$is a weighted sum of the risk sharing strategies introduced in Definition 2.

As an illustration of Proposition 3 to decompose the optimal policy into risk sharing strategies, I elaborate the case of $\overline{p}$ in the third subfigure of Figure 3 as an example (the decomposition when $p = \underline{p}$ is straightforward and involves only one strategy, or symmetric strategies that are equivalent). With high profitability $\overline{p}$, $\boldsymbol{\pi}^*$ involves essentially two states where $\pi^*(lll\,|\theta) = 0$ and constraint (29) binds, i.e., ggg and the symmetric states $\{ggb, gbg, bgg\}$. Consistent with Proposition 3, $\boldsymbol{\pi}^*$ is a weighted sum of two risk sharing strategies $(hhh, \{ggb, gbg, bgg\}; ggg)$, $(hhh, \{gbb, bgb, bbg\}; ggg)$, and signal $s_l = lll$ which does not involve any risk sharing.

[22] Generally it is challenging to analytically characterize the solution. Specifically, the value of reporting some signal depends on the distribution of other signals across the same states. Hence, it is difficult to cleanly summarize the effects of a positive weight of one risk sharing strategy on the potential payoffs from other strategies. This concern is similar to that in the knapsack problem.

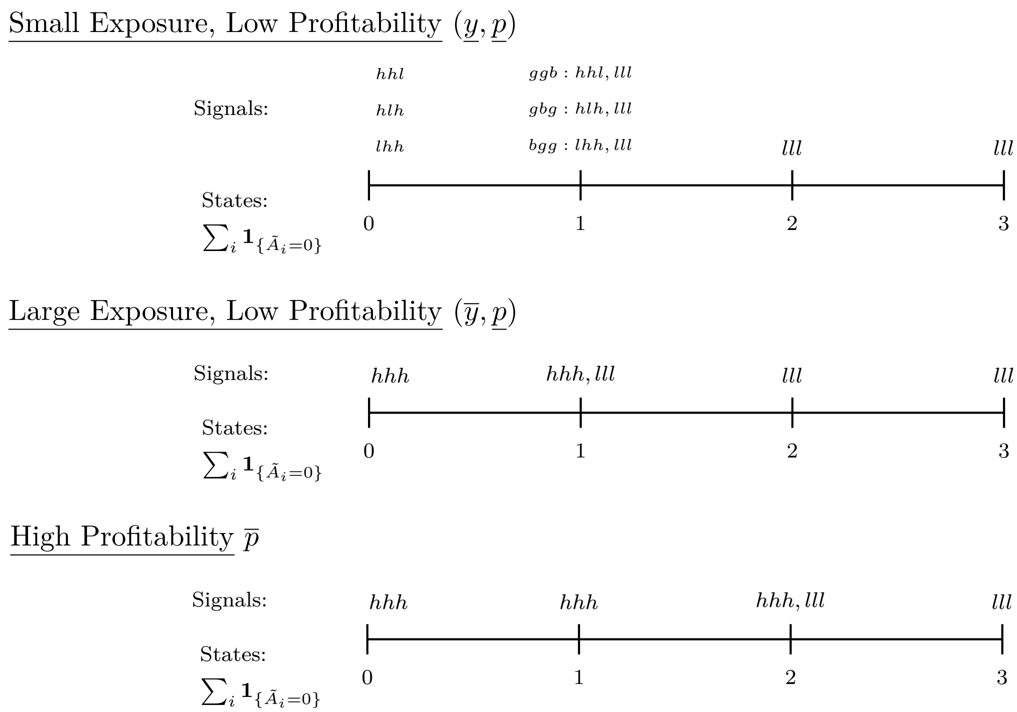

Figure 3: Optimal Policy Structure

Now I illustrate the efficiency of the strategies as summarized by ξ, and show its relation to $\boldsymbol{\pi}^*$. For expositional convenience, in this example I use some $\theta \in \tilde{\Theta}^s$ to represent $\tilde{\Theta}^s$. The most relevant indices are

$$\xi\left(lhh, bgg; \overline{\theta}\right) = \underbrace{\overline{\eta}\left(lhh, bgg; \overline{\theta}\right)}_{\text{maximum probability}} \cdot \underbrace{\left[\delta A - v - (1-\delta)\, y\right]}_{\text{excess net inflows} \mathcal{L}^{lhh,\overline{\theta}}} \cdot \frac{w\,(lhh, bgg) - w_0\,(bgg)}{\underbrace{v + y - \delta\,(2A - v)}_{\text{shortage} - \mathcal{L}^{lhh,bgg}}}, \tag{31}$$

$$\xi\left(hhh, bgg; \overline{\theta}\right) = \underbrace{\overline{\eta}\left(hhh, bgg; \overline{\theta}\right)}_{\text{maximum probability}} \cdot \underbrace{\left[\delta A - v - (1-\delta)\, y\right]}_{\text{excess net inflows} \mathcal{L}^{hhh,\overline{\theta}}} \cdot \frac{w\,(hhh, bgg) - w_0\,(bgg)}{\underbrace{v + (1-\delta)\, y - \frac{2\delta A}{3}}_{\text{shortage} - \mathcal{L}^{hhh,bgg}}}. \tag{32}$$

Recall that $\overline{\eta}$ is determined by the probability constraints on $\overline{\theta}$ and $\tilde{\Theta}$. $\overline{\eta} = \mathbb{P}\left(\theta_g^s\right)$ implies that the liquidity at the good state $\overline{\theta}$ is scarce and exhausts before $\tilde{\Theta}$ could be insured fully by borrowing from $\overline{\theta}$. Otherwise, $\overline{\eta} < \mathbb{P}\left(\theta_g^s\right)$ implies the cross-state risk sharing via this strategy is not constrained, as banks could fully refinance at $\tilde{\Theta}$ by borrowing a fraction of liquidity at $\overline{\theta}$. In this example,

$$\frac{\xi\left(s, \theta; \overline{\theta}\right)}{\mathbb{P}\left(\theta\right)} = \begin{cases} \frac{\mathbb{P}\left(\overline{\theta}\right)}{\mathbb{P}\left(\theta\right)} \frac{\Delta \mathcal{L}^{s,\overline{\theta}}}{-\Delta \mathcal{L}^{s,\theta}} \left[w\,(s,\theta) - w_0\,(\theta)\right], & \overline{\eta}\left(lhh, bgg; \overline{\theta}\right) = \mathbb{P}\left(\overline{\theta}\right), \\ w\,(s,\theta) - w_0\,(\theta)\,, & \overline{\eta}\left(lhh, bgg; \overline{\theta}\right) < \mathbb{P}\left(\overline{\theta}\right). \end{cases} \tag{33}$$

where I scale by $\mathbb{P}\left(\theta\right)^{-1}$ to denote the maximum conditional probability of refinancing at bad states.

Figure 4 plots $\frac{\xi\left(s,\theta;\overline{\theta}\right)}{\mathbb{P}(\theta)}$ as a function of the good project probability p. The examples with scarce

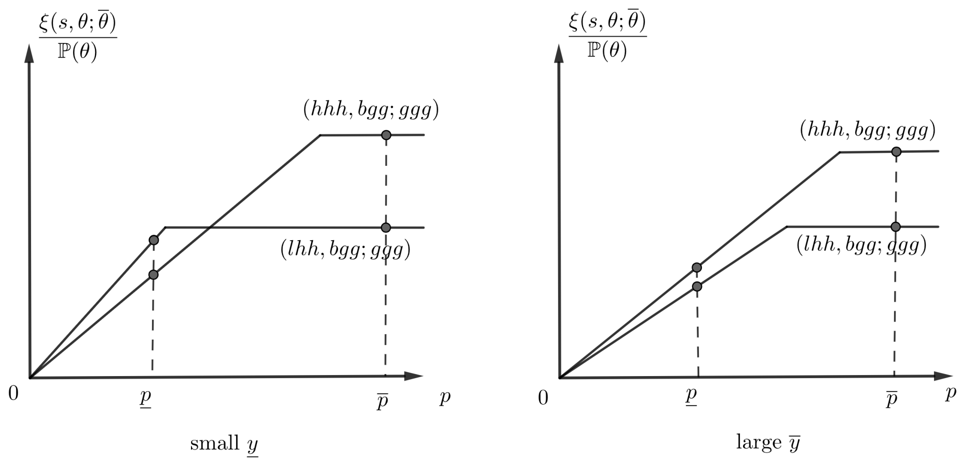

Figure 4: Index Comparison

Both panels plot $\frac{\xi\left(s,\theta;\overline{\theta}\right)}{\mathbb{P}(\theta)}$ against the good project probability p which affects $\overline{\eta}$. The left panel illustrates a relatively small interbank exposure $\underline{y}$, and the right panel illustrates a relatively large $\overline{y}$. When p is relatively large, $\tilde{\Theta}$ is insured fully, and the value of the horizontal line is $\frac{\xi\left(s,\theta;\overline{\theta}\right)}{\mathbb{P}(\theta)} = w\left(s,\theta\right) - w_0\left(\theta\right)$.

liquidity $\underline{p}$ fall on the increasing part of $\frac{\xi\left(s,\theta;\overline{\theta}\right)}{\mathbb{P}(\theta)}$ where $\overline{\eta} = \mathbb{P}\left(\overline{\theta}\right)$. The regulator prioritizes the strategy with the highest *marginal value*, which corresponds to the slopes. In the left panel with small interbank exposure $\underline{y}$, $\xi\left(hhh, bgg; \overline{\theta}\right) < \xi\left(lhh, bgg; \overline{\theta}\right)$ at $\underline{p}$, because passing the bank with project shock together is costly when interbank spillovers, or the counterparty risk that could be reduced, is small. In contrast, in the right panel with large $\overline{y}$, the marginal value of $\left(hhh, bgg; \overline{\theta}\right)$ is higher than $\left(lhh, bgg; \overline{\theta}\right)$ from reducing high counterparty risk. In examples with $\overline{p}$, cross-state risk sharing is not constrainted, so the regulator always prefers passing more banks regardless of interbank spillover.

3.5 Comparative Statics

In this subsection, I show that the key insights in the illustrating example extend to more general networks. I focus on the networks that satisfy the following two assumptions.

Assumption 3. *For every bank i,*

1) zero repayments from borrower banks result in default,

$$A_i < v_i + y_i^{out};$$

2) bad project results in senior default,

$$y_i^{in} < v_i.$$

The first part says interbank contagion is relevant, as a bank cannot meet liabilities without any repayments from counterparty banks. The second part is mainly for simplification: a bank with bad project defaults on senior debts and thus repays zero on junior liabilities, even if it receives full payments on interbank claims.

Assumption 4. *There exist constants r_1, r_2^{in}, $r_2^{out} > 0$ such that for every bank i,*

$$v_i = r_v A_i, \;\; y_i^{in} = r_y^{in} A_i, \;\; y_i^{out} = r_y^{out} A_i.$$

Assumption 4 says that banks share the same leverage ratios. Note that since the total interbank claims and interbank liabilities are balanced by construction, i.e., $\sum_i y_i^{in} = \sum_i y_i^{out}$, Assumption 4 implies that every bank has balanced interbank claims and liabilities, i.e., $y_i \equiv y_i^{in} = y_i^{out}$ and $r_y = r_y^{in} = r_y^{out}$.

A *non-discriminatory policy* reports the same signal for all banks, i.e., either reports $s_h \equiv (h, \cdots, h)$ or $s_l \equiv (l, \cdots, l)$. On the other hand, a *discriminatory policy* separates only a subset of banks with h from the rest of banks that are reported with l, given the underlying liquidity levels at each state. The next proposition shows that r_v and r_y determine how discriminatory the optimal policy is.

Proposition 4. *For any number of banks $k \leq n$, there exists a sequence of thresholds $\left\{\overline{r}^k\right\}$ and $\left\{\underline{r}_y^k\right\}$ such that*
1). when $r_v + r_y \leq \overline{r}^k$, any $s \neq s_l$ that is reported with positive probability $\sum_\theta \mathbb{P}(\theta)\pi^*(s|\theta) > 0$ reports $s_i = h$ on at least k banks;
2). when $r_y \geq \underline{r}_y^k$, signals that are reported at $\overline{\theta} \equiv (A_1, \cdots, A_n)$ reports $s_i = h$ on at least k banks.

Proposition 4 extends the key results of the illustrating example to a very general class of network structures. The first part says, if the payoff of good project is high relative to liabilities (small $r_v + r_y$), cross-state borrowing is less constrained. Then the maximum payoff at bad states $\Delta w(s, \theta)$ becomes more important, and regulator prefers a less discriminatory signal that reports h on more banks.

Instead, when liquidity is scarce (large $r_v + r_y$) and cross-state borrowing is constrained, the second part of the proposition provides a sufficient condition for a less discriminatory policy. The probability of refinancing becomes important, which is negatively related to the underlying liquidity shortage from liabilities $\left(-\Delta\boldsymbol{\mathcal{L}}^{s,\tilde{\Theta}}\right)$. A less discriminatory signal has two effects on $\left(-\Delta\boldsymbol{\mathcal{L}}^{s,\tilde{\Theta}}\right)$. On the one hand, it reduces the counterparty risk among the banks that refinance, which decreases $\left(-\Delta\boldsymbol{\mathcal{L}}^{s,\tilde{\Theta}}\right)$; on the other hand, it allows weaker banks to refinance, which increases shortage $\left(-\Delta\boldsymbol{\mathcal{L}}^{s,\tilde{\Theta}}\right)$. When interbank exposure is large (large r_y), the first effect dominates and the optimal policy is less discriminatory.

Intuitively, under a discriminatory policy, signals report h on the subset of banks who have better underlying cash flows, and strong complementarity among themselves. However, without detailed network structures, it is difficult to give prediction on what these banks are. In the next section, I present three representative network structures to illustrate how the optimal policy depends on the interbank complementarity shaped by the structure.

Before specific network structures, I present a general observation that simplifies the optimal policy characterization. Let $B(\theta)$ be the number of project shocks at state θ:

$$B(\theta) \equiv \sum_i \mathbf{1}_{\left\{\tilde{A}_i(\theta)=0\right\}}. \tag{34}$$

Conditional on the states with a small $B(\theta)$, if the regulator does not identify the banks with project shocks, the inferred probability of project risk in a typical bank is small. If in addition there is no counterparty risk, conditional on the states with $B(\theta)$ project shocks, a typical bank is perceived to be solvent if

$$B(\theta) \leq \overline{B}^{NCP} \equiv \frac{n\left[\delta(A+y)-v-y\right]}{\delta A}. \tag{35}$$

Note that the argument takes into account that conditional on s_h, the states with the same number of project shocks are symmetric.

Lemma 3. *There exists some $\underline{p}<1$, such that if $p \geq \underline{p}$, we have $\pi^*(s_h|\theta)=1$ for any states with $1 \leq B(\theta) \leq \overline{B}^{NCP}$.*

At states with only a few project shocks, it is a dominate strategy to report s_h, which results in the maximum payoff $\Delta w(s_h, \theta)$ at these states. One can check that the obedience constraints for s_h are satisfied: first, conditional on s_h, there is no counterparty risk as other banks are insured with outside liquidity under s_h; second, conditional on a small number of project shocks $1 \leq B(\theta) \leq \overline{B}^{NCP}$ which are not identified under s_h, the risk of a bad project for an individual bank is low enough. Actually, conditional on s_h, banks have excess net inflows under s_h at states with $1 \leq B(\theta) \leq \overline{B}^{NCP}$, i.e., $\left\{\theta : 1 \leq B(\theta) \leq \overline{B}^{NCP}\right\} \subset \Theta_g^{s_h}$.

In the Appendix, I show that when the shock probability is within $\left[\underline{p}, \overline{p}\right]$, the optimal information structure $\boldsymbol{\pi}^*$ could be simply constructed from the strategy associated of the highest index ξ^*. Under this simplification, the key tradeoffs of cross-bank risk sharing shaped by the network structures still remain. Hence, in Section 4, when characterizing the optimal solution, I apply this result for clean exposition.

4 Network Structure Implications

Now I impose some stylized network structures to study the implications of optimal stress tests and their mechanisms. In this section, I first present three representative networks to illustrate how the optimal policy depends on the interbank complementarity. I examine the effects of *connectivity* in both symmetric structures that vary in the number of a typical bank's counterparties (Section

4.1), and one asymmetric structure where within the same network banks vary in the number of counterparties (Section 4.2). Last, in Section 4.3, I evaluate the counterparty default component in practice as compared with the optimal policy.

4.1 Connectivity in Symmetric Networks

This subsection examines the effect of interconnectedness under symmetric networks. Specifically, I study the complete network with the highest connectivity and the ring network with the lowest connectivity. Naturally, in this subsection I set equal bank weights $\lambda_i = 1$.

I connect with the results in Acemoglu et al. (2015) which feature these two structures at contingent states absent refinancing and thus correspond to my autarky case. They show that when the interbank exposure is sufficiently large, the ring network is always the least stable structure, while the complete network is "robust-yet-fragile"—the most stable structure under small shocks but the least stable structure under large shocks; in later discussions I call this "phase transition."

First I show that the optimal disclosure policies echo these key insights in Acemoglu et al. (2015). In the complete network, phase transition at the critical states creates incentive (captured by Δw) for the regulator to allow refinancing at these critical states. In the ring network, the policy minimizes the belief discount on passing banks (mainly captured by shortage $-\Delta \mathcal{L}$). Next I highlight that although the complete network is more stable in autarky as in Acemoglu et al. (2015), under information design, the ring network may be more stable due to the flexibility to quarantine shocks and let senior creditors absorb the losses.

Complete Network Similar to the illustrating example in subsection 3.4, a symmetric complete network corresponds to

$$A_i = A,\ v_i = v,\ y_i^{in} = y_i^{out} = y, \quad \text{for all } i;\ y_{ij} = \frac{y}{n-1} \text{ for all } i \neq j.$$

As any project shock is equally transmitted to other banks, the state-contingent liquidity levels $L_i(s, \theta)$ are relatively aligned across banks, despite their independent projects.

Under symmetry, I summarize the states by their number of negative project shocks

$$B(\theta) \equiv \sum_i \mathbf{1}_{\left\{\tilde{A}_i(\theta)=0\right\}},$$

which serves as the uni-dimensional state. Let $\theta^B \in \{\theta \,|B(\theta) = B\}$ denote a typical state with B project shocks. In the complete network, there is a threshold number of project shocks $\overline{B}^C$, such that when $B(\theta) \leq \overline{B}^C$, only banks with bad projects default in the autarky case absent information design; while when $B(\theta) > \overline{B}^C$, all banks default in autarky due to contagion. Following Acemoglu et al. (2015), one can show that

$$\overline{B}^C = \frac{(n-1)(A-v)}{y}.$$

Without loss of generality, I focus on symmetric information structures. Loosely speaking, symmetry requires that the conditional probability $\pi(s|\theta)$ is the same when the regulator passes the same number of banks with good projects and the same number of banks with bad projects, at states with the same number of project shocks. Hence, I evaluate a typical passing bank's excess cash flows $\Delta\mathcal{L}$ using its average value across the symmetric states. Note that letting a bank with $\tilde{A}_i = A$ refinance both improves system stability w and reduces a passing bank's average shortage across symmetric states $-\Delta\mathcal{L}$. So in the optimal policy, any signal $s \neq s_l$ reports h on all banks with good projects. I introduce

$$H \equiv \sum_i \mathbf{1}_{\{s_i = h\}}$$

as the number of banks that signal s reports as h. Therefore, a typical risk sharing strategy could be represented as $\left(s(H), \theta^B; \theta_g^s\right)$, in which H banks refinance by borrowing from θ_g^s to a subset of states with B shocks. In this risk sharing strategy, $n - B$ banks that are passed have good projects and $H - (n - B)$ banks are with bad projects.[23]

I calculate a typical passing bank's average excess cash flows across symmetric states $\Delta\mathcal{L}^{s(H),\theta^B}$ to be

$$\Delta\mathcal{L}^{s(H),\theta^B} = \delta\overline{L_i\left(s(H), \theta^B\right)} - v - y = \delta\left(\frac{n-B}{H}A + \frac{H-1}{n-1}y\right) - v - y.$$

Here, $\overline{L_i\left(s(H), \theta^B\right)}$ denote a representative passing bank's *average* liquidity across the symmetric signal-state pairs that satisfy $\left(s(H), \theta^B\right)$. In expectation, at $\frac{n-B}{H}$ fraction of states the bank has a good project that generates additional cash flows A. Furthermore, the bank receives $H - 1$ counterparties' full payments of $\frac{y}{n-1}$, hence the second term $\frac{H-1}{n-1}y$. One can also see that the underlying cash flows are relatively aligned across banks: the contagion from $n - H$ default banks is equally born by the rest of banks, causing a shortage of $(n - H) \cdot \frac{y}{n-1}$ to each. Recall that $\overline{\theta} \equiv (A, \cdots, A)$ denote the state where all banks have good projects. According to Definition 3, the efficiency of the strategy $\left(s(H), \theta^B; \overline{\theta}\right)$ is

$$\xi\left(s(H), \theta^B; \overline{\theta}\right) = \eta\left(s(H), \theta^B; \overline{\theta}\right) \cdot \underbrace{\left[H - (n-B)\,\mathbf{1}_{B \leq \overline{B}^C}\right]}_{\Delta w(s(H),\theta^B)} \cdot \frac{\overbrace{\delta(A+y) - v - y}^{\Delta\mathcal{L}^{\overline{\theta}}}}{\underbrace{v + y - \delta\left(\dfrac{H-1}{n-1}y + \dfrac{n-B}{H}A\right)}_{-\Delta\mathcal{L}^{s(H),\theta^B}}}, \tag{36}$$

where the improvement at a bad state from autarky, $\Delta w\left(s(H), \theta^B\right)$, depends on $\overline{B}^C$—the threshold number of shocks above which the complete network becomes fragile.

In (36), the second term $\Delta w\left(s(H), \theta^B\right)$ reflects the robust-yet-fragile nature of the complete network: the gain of insuring one project shock is much larger if it is the critical shock that causes all banks to default. If the regulator allows one additional bank with bad project to refinance

[23] In an example with three banks, $\left(s(2), \theta^2; \overline{\theta}\right)$ says that at the states with two shocks that borrow liquidity, the signal passes the bank with good project and one of the bank with bad project; hence it summarizes the symmetric strategies $(hhl, \{bgb, gbb\}; ggg)$, $(hlh, \{bbg, gbb\}; ggg)$ and $(lhh, \{bgb, bbg\}; ggg)$ under the original definition.

(increasing H), there are two effects on the shortage from liabilities $-\Delta\mathcal{L}^{s(H),\theta^B}$: the counterparty risk is reduced but there is a higher chance that a passing bank has bad project. Hence, the policy will be less discriminatory if y is relatively large or A is relatively small, an insight to be confirmed in the next proposition.

To simplify the exposition of the optimal policy $\boldsymbol{\pi}^*$, I focus on certain parametrization as detailed in Proposition5. Especially, I require the outside liquidity to be sufficiently expensive, so that the optimal policy prioritizes the risk sharing strategies with large ξ. The following proposition highlights the key features in the optimal policy of the complete network.

Proposition 5. *Suppose $\delta \leq \overline{\delta}^c$, $\underline{p} \leq p < 1$ and $\frac{y}{A} \geq \underline{r}_y^c$ for some constants $\overline{\delta}^c < 1$ and $0 < \underline{p} < 1$. 1). The optimal policy $\boldsymbol{\pi}^*$ is nondiscriminatory and only reports $s_h = (h, \cdots, h)$ and $s_l = (l, \cdots, l)$; 2). if the critical number of project shocks that causes system failure is relatively large $\overline{B}^C > \overline{B}^{NCP}$, refinancing is prioritized at these critical states; i.e., the reported signals at each states are:*

$$\{s\,|\pi^*\,(s\,|\theta\,) > 0\} = \begin{cases} s_h, & 1 \leq B\,(\theta) \leq \lfloor \overline{B}^{NCP} \rfloor, \\ s_l, & \lfloor \overline{B}^{NCP} \rfloor + 1 \leq B\,(\theta) < \lfloor \overline{B}^{C} \rfloor + 1, \\ s_h, s_l & B\,(\theta) = \lfloor \overline{B}^{C} \rfloor + 1, \\ s_l & B\,(\theta) > \lfloor \overline{B}^{C} \rfloor + 1; \end{cases}$$

otherwise, when $\overline{B}^C \leq \overline{B}^{NCP}$, refinancing is prioritized at states with fewer project shocks:

$$\{s\,|\pi^*\,(s\,|\theta\,) > 0\} = \begin{cases} s_h, & 1 \leq B\,(\theta) \leq \lfloor \overline{B}^{NCP} \rfloor, \\ s_h, s_l & B\,(\theta) = \lfloor \overline{B}^{NCP} \rfloor + 1, \\ s_l & B\,(\theta) > \lfloor \overline{B}^{NCP} \rfloor + 1. \end{cases}$$

When $\frac{y}{A} \geq \underline{r}_y^c$, counterparty risk becomes more important and idiosyncratic project risk becomes less important. Hence, the optimal policy is nondiscriminatory. The distribution of s_h across states is consistent with Lemma 3, which shows that at states with only a few project shocks $1 \leq B\,(\theta) \leq \overline{B}^{NCP}$, it is optimal to report s_h so that all banks refinance thanks no counterparty risk and a low potential project risk.

Reporting s_h at states with more shocks, however, reduces the posterior mean about passing banks. The regulator weighs the improvement in system stability Δw from refinacing, against the bank liquidity shortage $-\Delta\mathcal{L}$ which captures how sensititive investors reduce their beliefs about passing. As shown in the efficiency index (36), the variation in a typical passing bank's shortage $-\Delta\mathcal{L}$ is small under different risk sharing strategies, because it burdens only a small fraction of the risk. In contrast, whether banks refinance at the critical states with $\lfloor \overline{B}^C \rfloor + 1$ shocks makes a significant difference in the improvement of system stability Δw. Therefore, when $\overline{B}^C > \overline{B}^{NCP}$, the optimal policy prioritizes risk sharing at the critical states.

As a result, banks are solvent when the number of shocks is smaller than the phase transition threshold $\overline{B}^C$, but at these critical states the system may still survive due to cross-state risk sharing under the optimal information design. In general, when outside liquidity is less expensive, banks could refinance at states with even more shocks. Therefore, under the optimal disclosure policy, the complete network is still "robust-yet-fragile", but the threshold at which stability changes is higher.

Ring Network In a symmetric ring network, banks are symmetric, and bank $i+1$ (1) is the sole lender bank of i (n):[24]

$$A_i = A,\ v_i = v,\ y_i^{in} = y_i^{out} = y,\quad \text{for all } i;\ y_{i+1,i} = y_{1n} = y \text{ for all } i \geq 1.$$

In contrast to the complete network where any shock is shared equally among all other banks, here the shock is transmitted *one-to-one* from the borrower bank to the lender bank. Hence, banks of different *distance* to the nearest project shock suffers from different levels of counterparty contagion. I introduce $d \in \{0, 1, 2, \cdots, n-1\}$ to denote this distance.

For a given state θ, suppose a typical bank i is $\hat{d} \geq 1$ distance from the nearest borrower bank with bad project; i.e., the $\left(i - \hat{d}\right)_{th}$ bank suffers a negative project shock but not for banks in between, so that $\tilde{A}_{i-\hat{d}}(\theta) = 0$ and $\tilde{A}_j(\theta) = A$ for $j \in \left\{i - \hat{d} + 1, \cdots, i-1\right\}$. Recall the second part of Assumption 3 that banks with bad projects make zero counterparty payments. Then bank i's net cash flows at a contingent state is

$$\Delta L_i(s,\theta) = \delta \left\{ \tilde{A}_i(\theta) + \underbrace{\prod_{j=i-\hat{d}}^{i-1} \mathbf{1}_{s_j=l} \cdot \min\left\{(d-2)(A-v), y\right\}}_{\text{no bank b/w } i-\hat{d} \text{ and } i-1 \text{ refinance}} + \underbrace{\left(1 - \prod_{j=i-\hat{d}}^{i-1} \mathbf{1}_{s_j=l}\right) y}_{\text{some bank b/w i-}\hat{d} \text{ and } i-1 \text{ refinances}} \right\} - v - y. \tag{37}$$

If there is no refinancing for banks between the bank with the source of shock $(i - \hat{d})$ and bank i's neighboring borrower bank $i-1$, the counterparty risk faced by bank i that starts from bank $i - \hat{d}$'s project shock diminishes as the borrower banks between $i - \hat{d} + 1$ and $i-1$ each accumulates $A - v$ cash flows in the payment to the next lender bank, until some bank is solvent; this scenario is characterized by the indicator function $\prod_{j=i-\hat{d}}^{i-1} \mathbf{1}_{s_j=l}$ that all of these borrower banks are reported with l. On the other hand, if some bank $j \in \left\{i - \hat{d}, \cdots, i-1\right\}$ is reported with h and refinances, bank i's counterparty risk is eliminated.

We learn two things from Eq. (37). First, compared to the complete network studied above, in the ring network liquidity is much less aligned across banks because distance d causes a greater heterogeneity in counterparty risks. Second, more importantly, information design significantly changes the *effective distance* to shock as shown in the indicator function $\left(1 - \prod_{j=i-\hat{d}}^{i-1} \mathbf{1}_{s_j=l}\right)$. As

[24] In the rest of this subsection, because banks could be renumbered under symmetry, I assume that any bank indicator is between 1 and n.

long as one of i's borrowing banks refinances, the banks in the downstream will be free from contagion.

The intuition illustrated in the individual bank's excess liquidity in (37) carries over to the liquidity shortage matrix $-\Delta\boldsymbol{\mathcal{L}}$ when multiple banks refinance at the same time. In contrast to the complete network where the shortage does not change much across signals and states (each bank burdens only a small fraction of counterparty risks), here the effective distance to shocks—and hence counterparty risks—crucially depends on the signal and state.

I highlight that the optimal policy has a structure that increases the effective distance to shock of passing banks whenever possible, thereby reducing the shortage $-\Delta\boldsymbol{\mathcal{L}}$ and increasing the probability of cross-state risk sharing. As will be shown in the following proposition, first, the regulator uses a distance-based signal that reports $s_i = h$ on banks no less than a threshold distance $\underline{d}$ away from an impaired bank. Hence, except for the first passing banks with the exact distance $\underline{d}$, passing banks in the downstream are not subject to counterparty risks, whose effective distance could be normalized as n. Second, the signal is reported at a state where project shocks are on adjacent banks, which maximizes the effective distance of the first passing bank all else equal.[25]

For simplicity, I assume interbank exposure to be sufficiently large to rule out the case of "safe" distance, in which a bank is free of counterparty risk in autarky if its distance from the nearest impaired borrower bank exceeds some "safe" distance:

$$y > \left(n - \overline{B}^{NCP} - 1\right)(A - v)\,. \tag{38}$$

Given Lemma 3, for states with $1 \leq B(\theta) \leq \overline{B}^{NCP}$, it is a dominant strategy to report s_h. Hence, (38) says, at the relevant states with more than $\overline{B}^{NCP}$ project shocks, in the ring network all banks default in autarky.

Without loss of generality, I focus on the symmetric equilibrium and normalize the numbering of banks for expositional convenience. Let θ^B_{Conn} denote the state with B project shocks on adjacent banks $1, 2, \cdots, B$, i.e.,

$$\theta^B_{Conn}: \ \tilde{A}_i(\theta) = \begin{cases} 0, & 1 \leq i \leq B, \\ A, & B+1 \leq i \leq n. \end{cases}$$

I call $s(\underline{d}, B)$ distance-based signals for $\underline{d} \in \{0, 1, \cdots, n-B\}$, which satisfy

$$s(\underline{d}, B) : s_i(\underline{d}, B) = \begin{cases} h, & \text{if } B + \underline{d} \leq i \leq n, \\ l, & \text{otherwise.} \end{cases}$$

Hence, when $s(\underline{d}, B)$ is reported at θ^B_{Conn}, it is a distance-based signal that passes banks that are at least $\underline{d}$ banks away from the nearest bank with asset impairment, which is bank B. Figure 5 illustrates an example with $n = 6,\ B = 2,\ \underline{d} = 2$.

[25] Conditional on signals that pass the same number of banks at states with the same number of project shocks.

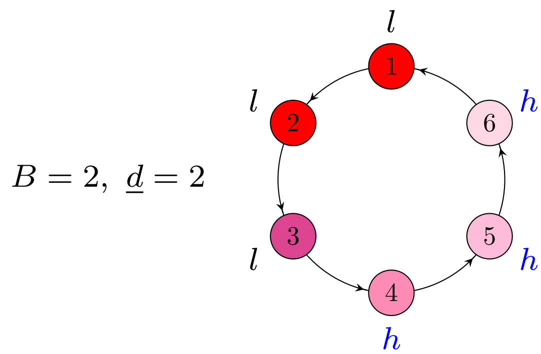

Figure 5: Distance-based Signal with Adjacent Shocks
The figure illustrates a ring network of six banks, where the arrow illustrates the direction of interbank payments. The colors illustrates a contingent state where the red color represents the two project shocks on adjacent banks 1 and 2. The pink color of different shades correspond to the fact that the other four banks default in aurtarky under Condition (38) but banks have higher underlying inflows as they become far away from shocks. The signal then corresponds to reporting only on banks that are at least two distance from the nearest shock at bank 2, i.e., bank 4, 5, and 6. Note the graph corresponds to a sequence of six symmetric signals of threshold distance reported at states with shocks, where we normalize the numbering of banks for illustration.

The following proposition summarizes the structure of the optimal policy:

Proposition 6. *There exist constants $0 < \underline{p} < \overline{p}^r < 1$, such that if $p \in \left[\underline{p}, \overline{p}^r\right]$ and (38), the optimal policy $\boldsymbol{\pi}^*$ reports the following signals across states:*

$$\{s \,|\pi (s\,|\theta) > 0\} = \begin{cases} s\left(\underline{d}^*, \lfloor \overline{B}^{NCP} \rfloor + 1\right) & \theta = \overline{\theta}, \\ s_h, & \theta : 1 \le B(\theta) \le \overline{B}^{NCP}, \\ s\left(\underline{d}^*, \lfloor \overline{B}^{NCP} \rfloor + 1\right), s_h, s_l, & \theta \in \tilde{\Theta}^{*s}\left(\theta_{Conn}^{\lfloor \overline{B}^{NCP} \rfloor + 1}\right), \\ s_l, & Otherwise. \end{cases} \tag{39}$$

where $\theta_{Conn}^{\bar{b}_1+1} \in \tilde{\Theta}^{*s}\left(\theta_{Conn}^{\bar{b}_1+1}\right)$, and $\tilde{\Theta}^{*s}$ is a collection of states with $\bar{b}_1 + 1$ shocks specified in Appendix B.10. The threshold distance $\underline{d}^*$ decreases in counterparty exposure y.

As in Lemma 3, the regulator always reports s_h at states with a small number of project shocks $1 \le B(\theta) \le \overline{B}^{NCP}$: conditinal on these states and s_h, a typical bank has excess net cash flows on average, thanks to a low risk of bad project and no counterparty risk. Hence, s_h is further reported at states with $\lfloor \overline{B}^{NCP} \rfloor + 1$ project shocks to lend liquidity to those states.

For other states, the optimal poliy prioritizes the class of symmetric risk sharing strategies with the highest efficiency as captured by index $\xi^*\left(s, \tilde{\Theta}^s; \overline{\theta}\right)$, because liquidity is relatively scarce under condition $p \le \overline{p}^r$. First, as shown in (37), lender banks with good projects are solvent if some borrower bank refinances. The signal $s\left(\underline{d}^*, \lfloor \overline{B}^{NCP} \rfloor + 1\right)$ when reported at $\theta_{Conn}^{\lfloor \overline{B}^{NCP} \rfloor + 1}$ exploits this spillover effect by reporting whether banks are $\underline{d}^*$ distance away from the nearest bank with asset impairment. Second, conditional on the number of project shocks B, the contagion effects on the rest of the network are minimized at states where these shocks are on adjacent banks, i.e., θ^{B}_{Conn}. By reporting $s\left(\underline{d}^*, \lfloor \overline{B}^{NCP} \rfloor + 1\right)$ at $\theta_{Conn}^{\lfloor \overline{B}^{NCP} \rfloor + 1}$, project shocks are "quarantined" locally, and the losses in banks $\left\{1, 2, \cdots, \lfloor \overline{B}^{NCP} \rfloor\right\}$ are only burdened by their senior creditors.

The optimal threshold distance $\underline{d}^*$ depends on the counterparty exposure y. As the counterparty exposure y increases, the benefit of cross-bank risk sharing—the spillover effects on lender banks—becomes larger. Consequently, $\underline{d}^*$ decreases, leading to a less discriminatory disclosure policy.

Connectivity and Stability Outcomes Once we understand how the two (stylized) network structures determine the properties of the optimal policy, I now examine the role of information design on the network stability.

As no bank refinances in autarky, I have the following key take-away by borrowing results from Acemoglu et al. (2015) that study contingent-state stability. When interbank exposure is sufficiently high, with a small number of project shocks, the complete network is the most stable structure while the ring network is the least stable structure; with a large number of project shocks, both networks are the least stable structures. Therefore, absent information design, the complete network is more stable than the ring network in expectation.

In contrast, I highlight that under the optimal disclosure policy, the ring network may be more stable than the complete network, suggesting a greater value brought on by information design under the ring network structure. On one hand, in the complete network shocks are burdened equally among banks and liquidity is more aligned across banks, which increases the efficiency of cross-bank risk sharing. On the other hand, as shown in Proposition 6, in the ring network the optimal policy has a "quarantine" effect to allow refinancing at states where shocks are connected on adjacent banks. As a result, the senior creditors who are outside of the network absorb a larger proportion of the losses, and banks that are far way from the project shocks may have higher liquidity levels than banks in the complete network.

Proposition 7. *There exists a non-empty set of parameters, under which absent information design, the complete network is more stable than the ring network, $W_0^C \geq W_0^R$; while with information design, the ring network is more stable than the complete network, $W^R\left(\boldsymbol{\pi}^{R*}\right) > W^C\left(\boldsymbol{\pi}^{C*}\right)$.*

The proposition highlights the flexibility of designing beliefs about shocks in a less connected structure, in which the shocks could be quarantined locally and absorbed by senior creditors. This "bail-in" effect alleviates debt overhang greatly and persuade outside investors to inject liquidity to the more distant healthier banks. In contrast, in a connected structrure, any injected cash flow first guarantees payments to senior creditors (senior creditors of both the refinancing bank and the other banks who receive payments from the refinancing bank). Consistent with my emphasis on separating healthier banks when cross-state borrowing is constrained, the flexibility to quarantine may dominate the more efficient cross-bank risk sharing under interconnectedness. The following numerical example illustrates the proposition.

Example 2. Consider parameters $n = 4,\ A = 1,\ v = 0.75,\ y = 0.54,\ p = 0.2,\ \delta = 1$. In autarky, the complete structure is more stable, $W_0^C > W_0^R$. Under the optimal information design, $W^R\left(\boldsymbol{\pi}^{R*}\right) > W^C\left(\boldsymbol{\pi}^{C*}\right)$. Under this parameterization, the improvement from information design depends critically on states with two shocks, where the quarantine effect shows up and dominates. To see this, first, note that $\overline{B}^{NCP} = 1$, so conditional on states with one project shock $B(\theta) = 1$, s_h is always reported in both networks. Second, one can verify that in the ring network, pooling $\overline{\theta}$ and $\theta^2_{Conn} = bbgg$ under signal $s\left(\underline{d} = 2, \theta^2_{Conn}\right) = lllh$, results in a higher $W^R(\boldsymbol{\pi})$ in the ring network than that achieved under any policy in the complete network.

Connecting back to Acemoglu et al. (2015), the two counterforces of connectivity under information design relates to the "robust-yet-fragile" result of interconnectedness in the network literature, but the underlying mechanisms are quite different. Previous studies focus on contagion cascades under ex post shocks, and a connected network enjoys better risk sharing in good times but is subject to worse contagion in bad times. In this paper, banks can share risk across states, and counterparty risk is reduced if banks are coordinated to borrow from good times to bad times together. Instead, my result highlights the economic trade-off of cross-state risk sharing—for example, outside liquidity premium, bank profitability, and etc: when cross-state risk sharing is less constrained, connectivity becomes favorable under which it is easier to coordinate banks' refinancing to reduce contagion; when cross-state risk sharing is more constrained, however, the injected liquidity absorbed by senior creditors becomes costlier and it is more favorable to quarantine the shocks locally and separate healthier banks to allow for refinancing.

4.2 Asymmetric Networks and Preferred Treatment

This subsection discusses how the optimal policy treats banks with different number of counterparties in an asymmetric network. The network intervention literature shows that capital should be injected to banks with a greater centrality, for the larger the spillovers to other banks. The key difference in my paper is that banks borrow from themselves across states. I show that the more connected banks receive preferred treatment with some qualifications.

Core-periphery networks are the most empirically relevant structure, where there are a few highly interconnected core banks and many sparsely connected periphery banks. I study a class of symmetric core-periphery networks where a typical periphery bank is only connected to some core bank, and banks within the core or periphery group are ex ante symmetric.[26]

Specifically, suppose there are n_c core banks and n_p periphery banks. Each core bank is connected to the same number of periphery banks, which is $\frac{n_p}{n_c}$, and each periphery bank is connected to one core. One example is the star network where $n_c = 1$; another example is a symmetric complete network as the core part, and each core bank is connected to $\frac{n_p}{n_c}$ additional periphery banks via double-sided interbank debt contracts. The same leverage ratio across banks as stated in

[26] The rich state space and cross-bank risk sharing greatly complicates the exposition. For clean exposition of centrality in more general networks, I need restrictions on the state space or signal structure. Nevertheless, the analysis here highlights the key tradeoffs regarding asymmetric connectivity across banks within the same network.

Assumption 4 allows me to focus on the effect of bank connectivity. As a result, the core banks with more counterparties effectively have larger sizes. Hence, I weight a bank's by its size by assuming

$$\frac{\lambda_i}{\lambda_j} = \frac{A_i}{A_j}, \ \forall i, \forall j \neq i.$$

Proposition 8. *There exists constants* $\underline{r}_y$ *and* $\overline{r}$*, such that if* $\frac{y_i}{A_i} \geq \underline{r}_y$ *or* $\frac{y_i+v_i}{A_i} \leq \overline{r}$*, core banks receive preferred treatment; i.e., they are more likely to refinance than the periphery banks.*

Because a core bank is connected to multiple counterparties who have independent projects, it is subject to a smaller contagion risk. Hence, a core bank is on average healthier than a periphery bank which has only one counterparty. In addition, given the large spillover effect from core to periphery, if some periphery banks are reported as h, the regulator might as well report h on their connected core bank provided it has good project. Nevertheless, it is possible that at some states the periphery banks are healthier than their connected core bank: if the contagion risk originates from some distant bank, a periphery bank will be one distance away than its connected core bank. The parameter restriction in Proposition 8 guarantees that cross-state borrowing is not too constrained such that the strong complementarity between the connected core and peripheries dominates the the benefit of separating healthier peripheries at a few states.

4.3 Counterparty Default Component

In 2017, the Federal Reserve added the Counterparty Default Component to bank stress test, which examines the bank's status assuming that its largest counterparty default. In this subsection, I formalize the information structure that corresponds to this policy in practice, and compare with the optimal policy $\boldsymbol{\pi}^*$.

To model the counterparty default component, I conduct single-bank stress test for each bank separately. For a typical bank i, the regulator assumes that its largest counterparty i' defaults and repays $x_{ii'} = 0$, while bank i's other counterparties make full payments $\sum_{j\neq i} x_{ij} = \sum_{j\neq i,i'} y_{ij}$. For simplicity, I assume that all participants naively take such interbank payments as given. Hence, the information design is only about bank i's project shock, and the distribution of each bank's signal s_i are independent. Specifically, the information structure consists of signal space

$$\mathbf{S}^{CDC} = \mathbf{S} \equiv \{ s = (s_1, s_2, \cdots, s_n) \,|s_i = h \text{ or } l \},$$

and a typical signal's conditional distribution is

$$\pi^{CDC}(s\,|\theta) = \prod_{i=1}^{n} \mathbb{P}\left(s_i \middle| \tilde{A}_i\right).$$

Remark 1. Let $\pi_i^{CDC}(h\,|g) \equiv \mathbb{P}\left(s_i = h \middle| \tilde{A}_i = A_i\right)$ and $\pi_i^{CDC}(h\,|b) \equiv \mathbb{P}\left(s_i = h \middle| \tilde{A}_i = 0\right)$. Then in the Counterparty Default Component practice, the optimal information structure is summarized by

$$\pi_i^{CDC}\left(h\,|g\right)=1,\ \pi_i^{CDC}\left(h\,|b\right)=\frac{p_i}{1-p_i}\left[\frac{\delta\left(A_i+y_i^{in}-y_{ii'}\right)-v_i-y_i^{out}}{v_i+y_i^{out}-\delta\left(y_i^{in}-y_{ii'}\right)}\right]^+,$$

where bank $i'$is bank i's largest counterparty $i'\equiv\arg\max_{j\neq i}y_{ij}$.

The disclosure about each bank is independent, and directly follows from the result of binary-state persuasion problem. The l signal is informative about the bank's bad project. In contrast, the h signal pools the bad state with good state such that h implies decent quality on average, under which outside investors enable the bank to refinance and share risk across states.

Hence, the extent of the cross-state risk sharing of each bank is determined by on its dependence on the largest counterparty. In addition, the regulator cannot directly coordinate banks to refinance, and cross-bank risk sharing is absent. In light of these observations, the following remark summarizes the comparison between $\boldsymbol{\pi}^{CDC}$ and $\boldsymbol{\pi}^*$.

Remark 2. Compared with the system-level optimal information structure $\boldsymbol{\pi}^*$, the Counterparty Default Component practice $\boldsymbol{\pi}^{CDC}$
1) is more lenient on banks that are more connected, and harsher on banks that have few counterparties;
2) is more discriminatory both at good times with a few shocks and at bad times with a lot of shocks.

The first point results from the consequences of neglecting counterparty risks in different network structures. Intuitively, in a very connected structure such as the complete network, banks are passed with a high probability under $\boldsymbol{\pi}^{CDC}$, because the default of the largest counterparty affects little of the overall healthiness of the bank. In a very sparse structure such as the ring network, however, contagion from the largest counterparty is decisive. Under Assumption 3, $\pi_i^{CDC}\left(h\,|b\right)=0$ in the ring network and the disclosure is perfectly informative. The second point highlights the importance of coordination and cross-bank risk sharing. As the disclosure on banks are independent, s_i is only informative about the bank's project quality, regardless of contagion risks from counterparties. This is in sharp contrast with $\boldsymbol{\pi}^*$: when the number of shocks is small, as shown in Lemma 3, the regulator passes all banks thanks to no counterparty risk and low project risk; when there are a lot of shocks, all banks are reported as l, because risk sharing is prioritized at states with fewer shocks where banks have smaller shortfall from liabilities.

5 Conclusion

I study the optimal stress test disclosure that maximizes the total solvency rate in financial networks. The network structure is exogenous, and characterizes how banks are connected via interbank liabilities. A passing stress test result that signals good quality but not perfectly informative enables the bank to share risk across states, by raising enough outside liquidity to clear liabilities. As the key feature in the model, disclosure has spillover effects among banks: a passing result on one bank reduces the counterparty risk faced by the other banks. I hence highlight that in addition to cross-state risk sharing studied in single-bank stress test, the system-level stress test involves the novel *cross-bank risk sharing*: banks who are passed together become healthier as they deliver to each other as counterparties. Hence, cross-bank risk sharing resembles netting counterparty risk.

On the other hand, when passing banks together, the regulator needs to convince that the weaker banks in the group are on average healthy. When bank profitability is high, cross-state risk sharing is less constrained, the optimal policy is less discriminatory and passes more banks together. The optimal policy is also less discriminatory when interbank exposure is large, so that the cash flows across banks are relatively aligned despite independent fundamental risks. Otherwise, the optimal policy separates the healthier banks and let them refinance.

Network structures determine the complementarity between banks and thus the efficiency of cross-bank risk sharing. In the complete network, the optimal policy is less discriminatory and let banks refinance when the system transitions from a stable structure to a fragile structure. In the ring network, the optimal policy passes banks that are far from the impaired banks, at states where project shocks are quarantined locally on adjacent banks. In asymmetric networks, the more connected banks with more counterparties receive preferred treatment. Under the optimal information structure, mre connected structures have a "robust-yet-fragile" feature that arises from the trade-off of cross-state risk sharing, which is different from that in the network literature. Although it is more efficient to reduce counterparty risk in a more connected network by passing banks together, in a less connected network, there is the flexibility to design the beliefs about shocks to be concentrated locally, such that senior creditors absorb the losses.

In this paper, I highlight the importance of coordination in the system-level stress test design. Single-bank stress test that takes counterparty contagion as given—for example the counterparty default component in practice, may be too harsh because coordination could reduce counterparty risk.

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A Summary of Variables

Notation	*Definition and Meaning*	*Characterization*
n	Number of banks	
$\tilde{A}_i$	Loan project payoff of bank i	$\tilde{A}_i \in \{A_i > 0, 0\}$
g, b	Label for project realization	$g : \tilde{A}_i = A_i;\ b : \tilde{A}_i = 0$
p_i	Probability of good project for bank i	$p_i \equiv \mathbb{P}\left(\tilde{A}_i = A_i\right) \geq \frac{1}{2}$
θ	State of nature	$\theta \equiv \tilde{A}_1 \times \cdots \times \tilde{A}_n \in \Theta$
v_i	Face value of senior debts owed by bank i	
y_{ij}	Face value of interbank debts borrowed by bank j from bank i	
y_i^{in}, y_i^{out}	Total interbank claims (incoming), liabilities (outgoing) of bank i	$y_i^{in} \equiv \sum_{j \neq i} y_{ij},\ y_i^{out} \equiv \sum_{j \neq i} y_{ji}$
$\mathbf{S}$	Signal space	$\mathbf{S} = \mathbf{S_1} \times \cdots \times \mathbf{S_n},\ \mathbf{S}_i = \{h, l\}$
s	A typical signal	$s \equiv s_1 \times \cdots \times s_n \in \mathbf{S},\ s_i \in \{h, l\}$
$\boldsymbol{\pi}$	Signal distribution	$\boldsymbol{\pi} : \Theta \to \triangle \mathbf{S},\ \pi\left(s \mid \theta\right) \equiv \mathbb{P}\left(s \mid \theta\right)$
a_i	Bank i's action given signal realization	$a_i \in \{\text{raise funds},\ \text{wait}\}$
λ_i	Default punishment $-\lambda_i < 0$ for bank i; bank weights	
δ	Discount factor of outside liquidity	
m_i	Investors' valuation of bank i's total assets	$m_i : \mathbf{S} \times \boldsymbol{\pi} \times X \to \mathbb{R}$
$x_{ij}\left(s, \theta\right)$	Actual interbank payment from j to i	See (8)
$w\left(s, \theta\right)$	Total weighted number of banks that are solvent at contingent state	See (9)
$W\left(\boldsymbol{\pi}\right)$	Expected weighted number of banks that are solvent	See (10)
$y_{ij}^0(\theta)$	Actual interbank payment from j to i in autarky	
$w_0\left(\theta\right),\ W_0$	Contingent state and expected number of solvent banks in autarky	
$L_i\left(s, \theta\right)$	Bank i's inflows at contingent state	See (11)
$\Delta L_i\left(s, \theta\right)$	Bank i's excess inflows at contingent state	
$\mu_i\left(s\right)$	Shadow value of bank i's obedience constraint under $s_i = h$	
$q\left(\theta\right)$	Shadow value of the prior consistency constraint	
$I_h\left(s\right)$	Banks that are reported with $s_i = h$ under s	See (**??**)
$\left(s, \tilde{\Theta}^s; \theta_g^s\right)$	A risk sharing strategy	See Definition 2
$\xi\left(s, \tilde{\Theta}^s; \theta_g^s\right)$	Efficiency of strategy	See Definition 3
$\eta\left(s, \tilde{\Theta}^s; \theta_g^s\right)$	Weight of risk sharing strategy	
$B\left(\theta\right)$	Number of project shocks	
$H\left(s\right)$	Number of banks reported as $s_i = h$ under s	

B Mathematical Appendix

B.1 Preliminary Lemmas

Lemma 4. *If all banks default on junior liabilities, generically some banks default on senior liability.*

Proof. Let $T_i \leq v_i$ be bank i's payments to its senior creditor. A default bank pays out all cash flows to creditors and does not hold excess cash flows. As a result, if all banks default, the aggregate outside-of-network incoming cash flows, i.e., total project payoffs $\sum_i \tilde{A}_i$, must equal the aggregate outside-of-network outgoing cash flows, i.e., total payments to senior creditors $\sum_i T_i$.

To see this, for a typical default bank i,

$$\sum_{j \neq i} x_{ji} = \tilde{A}_i + \sum_{j \neq i} x_{ij} - T_i.$$

Take sum over all banks, and then

$$\sum_i \sum_{j \neq i} x_{ji} = \sum_i \tilde{A}_i - \sum_i T_i + \sum_i \sum_{j \neq i} x_{ij},$$

which says $\sum_i \tilde{A}_i = \sum_i T_i$.

As generically $\sum_i \tilde{A}_i \neq \sum_i v_i$, then at least some bank defaults on senior liability, $T_i < v_i$. $\square$

Lemma 5. *The contingent state social welfare $\hat{w}(s, \theta)$ is affine in the weighted number of banks that are solvent.*

Proof. Total welfare includes bank utilities $\{u_i\}$, investors' profits and payments received by banks' senior creditors $\{T_i\}$ (notation introduced in the proof of Lemma 4). Conditional on (s, θ) realization, among the banks that are reported with $s_i = l$, I use I_{ls}, I_{ld}, I_{lf} to denote respectively the sets of banks that are solvent, that default on junior liabilities but fully repay senior creditors, and that default on senior creditors. Recall in (22) that $I_h(s)$ denote the banks that signal s reports as $s_i = h$. Then at $t = 2$,

$$\begin{aligned}
\hat{w}(s, \theta) &= \underbrace{\sum_i u_i}_{\text{bank}} + \underbrace{\sum_{i \in I_h} \left(\tilde{A}_i + \sum_{j \neq i} x_{ij} - m_i \right)}_{\text{investors}} + \underbrace{\sum_i T_i}_{\text{senior creditors}} \\
&= \sum_{i \in I_h, I_{ls}} \left(\tilde{A}_i + \sum_{j \neq i} x_{ij} - y_i^{out} \right) + \sum_{i \in I_{ld}} \left(\underbrace{\tilde{A}_i + \sum_{j \neq i} x_{ij} - \sum_{j \neq i} x_{ji}}_{= v_i} - \lambda_i \right) + \sum_{i \in I_{lf}} \left(\underbrace{\tilde{A}_i + \sum_{j \neq i} x_{ij}}_{= T_i < v_i} - \lambda_i \right) \\
&= \sum_i \tilde{A}_i - \sum_{i \in I_{ld}, I_{lf}} \lambda_i.
\end{aligned}$$

The second equality aggregates the surplus across different types of agents. The surplus generated by an individual bank is its net cash flows subtracting the default punishment if it defaults. The third equality follows from $\sum_i \sum_j x_{ij} = \sum_i \sum_j x_{ji}$. Hence, the expected total welfare is

$$\hat{W} \equiv \mathbb{E}\left[\hat{w}\left(s, \theta\right)\right] = \sum_i p_i A_i + \underbrace{\mathbb{E}\left[\sum_{i \in I_h, I_{ls}} \lambda_i\right]}_{=W} - \sum_i \lambda_i.$$

$\hat{W}$ is linear in expected number of banks that survive W, where banks are weighted by the size of their default penalties. □

B.2 Proof of Lemma 1

Proof. Suppose in the optimal information structure $\boldsymbol{\pi}^*$ of the relaxed problem $\mathcal{P}$ in (17), for some signal s' that is reported with positive probability, the obedience constraint of l (14) in the original problem $\hat{\mathcal{P}}$ in (12) is violated for bank $\tilde{i}$ with $s'_{\tilde{i}} = l$, i.e.,

$$\sum_\theta \mathbb{P}\left(\theta\right) \pi^*\left(s' \left| \theta\right.\right) \left[\delta L_{\tilde{i}}\left(s', \theta\right) - v_{\tilde{i}} - y_{\tilde{i}}^{out}\right] \geq 0. \tag{40}$$

Consider another information structure $\tilde{\boldsymbol{\pi}}$ that replaces s' in $\boldsymbol{\pi}^*$ with $s'' \equiv \left(s_{\tilde{i}} = h, s'_{-\tilde{i}}\right)$—a mirror image signal that replaces $s'_{\tilde{i}}$ with h. Specifically,

$$\tilde{\pi}\left(s \left| \theta\right.\right) = \begin{cases} 0, & \text{if } s = s', \\ \pi^*\left(s \left| \theta\right.\right) + \pi^*\left(s' \left| \theta\right.\right), & \text{if } s = s'', \\ \pi^*\left(s \left| \theta\right.\right), & \text{if } s \neq s', s''. \end{cases}$$

One can verify that $\tilde{\boldsymbol{\pi}}$ is feasible in the relaxed problem $\mathcal{P}$, and delivers higher expected payoff. For feasibility, it is sufficient to check the h obedience constraints (18) associated with signal s''. For $s''_{\tilde{i}} = h$,

$$\begin{aligned} & \sum_\theta \mathbb{P}\left(\theta\right) \tilde{\pi}\left(s'' \left| \theta\right.\right) \left[\delta L_{\tilde{i}}\left(s'', \theta\right) - v_{\tilde{i}} - y_{\tilde{i}}^{out}\right] \\ \geq & \underbrace{\sum_\theta \mathbb{P}\left(\theta\right) \pi^*\left(s' \left| \theta\right.\right) \left[\delta L_{\tilde{i}}\left(s', \theta\right) - v_{\tilde{i}} - y_{\tilde{i}}^{out}\right]}_{(40):\geq 0} + \underbrace{\sum_\theta \mathbb{P}\left(\theta\right) \pi^*\left(s'' \left| \theta\right.\right) \left[\delta L_{\tilde{i}}\left(s'', \theta\right) - v_{\tilde{i}} - y_{\tilde{i}}^{out}\right]}_{(13) \text{ for } s''_{\tilde{i}} = h: \geq 0} \\ \geq & 0, \end{aligned}$$

where the first inequality follows from $L_{\tilde{i}}\left(s'', \theta\right) \geq L_{\tilde{i}}\left(s', \theta\right)$. For the other banks that s'' reports with h, the obedience constraints are satisfied as $L_i\left(s'', \theta\right) \geq L_i\left(s', \theta\right)$. For system stability, notice that $w\left(s'', \theta\right) \geq w\left(s', \theta\right)$ as bank $\tilde{i}$ refinances and makes full payments. Hence, from the construction of $\tilde{\boldsymbol{\pi}}$, I have $W\left(\tilde{\boldsymbol{\pi}}\right) \geq W\left(\boldsymbol{\pi}^*\right)$. □

B.3 Proof of Proposition 1

Proof. I first show that for any (s, θ), the set of interbank payments $\{x_{ij}(s,\theta)\}$ exists and is generically unique. I show that for any signal s, the interbank payments $\{x_{ij}(s,\theta)\}$ with refinancing could be implemented by interbank payments $\{\hat{x}_{ij}(\theta)\}$ without refinancing in a new network structure. Then I can apply the results from Acemoglu et al. (2015) that $\{\hat{x}_{ij}(\theta)\}$ exist and are generically unique.

Given s, I redefine the network by incorporating the interbank liabilities of the banks that refinance into their creditor banks' project payoffs:

$$\hat{\tilde{A}}_i = \begin{cases} \tilde{A}_i + \sum_{j \in I_h(s)} y_{ij}, & \forall i \notin I_h(s) \\ \tilde{A}_i, & \forall i \in I_h(s) \end{cases}, \hat{y}_{ji} = \begin{cases} y_{ji}, & \forall i \notin I_h(s) \\ 0, & \forall i \in I_h(s) \end{cases},$$

where $I_h(s)$ is the set of banks that signal s reports with h. The interbank payments $\{\hat{x}_{ij}(\theta)\}$ in the new structure are such that the following holds simultaneously for every i and j:

$$\hat{x}_{ij} = \min \left[\hat{y}_{ij}, \frac{\hat{y}_{ij}}{\sum_{k \neq j} \hat{y}_{kj}} \left(\hat{\tilde{A}}_j + \sum_{j \neq i} \hat{x}_{ji} - v_j \right)^+ \right],$$

where I normalize $\frac{\hat{y}_{ij}}{\sum_{k \neq j} \hat{y}_{kj}}$ to $\frac{1}{n-1}$ for any $\sum_{k \neq j} \hat{y}_{kj} = 0$. From Proposition 1 in Acemoglu et al. (2015), the set of $\{\hat{x}_{ij}(\theta)\}$ exists and is generically unique. Then the property also holds for $\{x_{ij}(s,\theta)\}$, which satisfy $x_{ij}(s,\theta) = \mathbf{1}_{j \in I_h(s)} \cdot y_{ij} + \mathbf{1}_{j \notin I_h(s)} \cdot \hat{x}_{ij}(\theta)$.

Hence, the feasible set of the relaxed problem $\mathcal{P}$ in (17) is well defined. One feasible information structure is to always "truthfully report". At each state θ, the regulator always allows a bank to refinance if and only if enough cash could be raised against its underlying cash flows; i.e.,

$$s_i(\theta) = \begin{cases} h, & \text{if } \delta \left[\tilde{A}_i(\theta) + \sum_{j \neq i} x_{ij}^0(s(\theta), \theta) \right] \geq v_i + y_i^{out}, \\ l, & \text{otherwise,} \end{cases}$$

and $\pi(s(\theta) | \theta) = 1$ for all θ. Therefore, by standard extreme value theorem, the optimal solution $\boldsymbol{\pi}^*$ exists.

For uniqueness, the optimal solution to the linear programming problem lies on the vertex of the feasible set. If there exists multiple optimal solutions, then the projection of the object function is parallel to some constraint, or some constraints are identical, which do not hold under a generic set of parameters. $\square$

B.4 Dual Approach

Let $\chi(s,\theta) \equiv \mathbb{P}(\theta)\pi(s|\theta)$ be the joint probability. For notational convenience, I introduce the following vector and matrix representations, where the involved banks are those reported as h

under the signal s, or $i \in I_h(s)$:

$$
\begin{aligned}
&(\boldsymbol{\chi^s})_{|\Theta|\times 1} \equiv \begin{bmatrix} \cdots & \chi(s,\theta) & \cdots \end{bmatrix}^\top, \ (\boldsymbol{\chi})_{|\Theta||S|\times 1} = \begin{bmatrix} \cdots & (\boldsymbol{\chi^s})^\top & \cdots \end{bmatrix}^\top,\\
&(\boldsymbol{w^s})_{|\Theta|\times 1} \equiv \begin{bmatrix} \cdots & w(s,\theta) & \cdots \end{bmatrix}^\top_{|\Theta|\times 1}, \ (\boldsymbol{w})_{|\Theta||S|\times 1} = \begin{bmatrix} \cdots & (\boldsymbol{w^s})^\top & \cdots \end{bmatrix}^\top,\\
&(\boldsymbol{y^s})_{|I_h(s)|\times|\Theta|} \equiv \begin{bmatrix} \cdots, & \left(y_i^{out}\mathbf{1}_{|\Theta|\times 1}\right) & ,\cdots \end{bmatrix}^\top,\\
&(\boldsymbol{v^s})_{|I_h(s)|\times|\Theta|} \equiv \begin{bmatrix} \cdots, & \left(v_i\mathbf{1}_{|\Theta|\times 1}\right) & ,\cdots \end{bmatrix}^\top,\\
&(\boldsymbol{L_i^s})_{|\Theta|\times 1} \equiv \begin{bmatrix} \cdots & L_i(s,\theta) & \cdots \end{bmatrix}^\top, \ (\boldsymbol{L^s})_{|I_h(s)|\times|\Theta|} \equiv \begin{bmatrix} \cdots & (\boldsymbol{L_i^s})^\top & \cdots \end{bmatrix},\\
&\Delta\boldsymbol{\mathcal{L}^s} \equiv \delta\boldsymbol{L^s} - \boldsymbol{v^s} - \boldsymbol{y^s},\\
&(\boldsymbol{P})_{|\Theta|\times 1} \equiv \begin{bmatrix} \cdots & \mathbb{P}(\theta) & \cdots \end{bmatrix}^\top,\\
&\boldsymbol{b} \equiv \begin{bmatrix} \mathbf{0}_{1\times\left(\sum_s |I_h(s)|\right)}, & \boldsymbol{P}^T & ,-\boldsymbol{P}^T \end{bmatrix}^\top.
\end{aligned}
$$

Proposition 9. *The dual problem $\mathcal{D}$ of the regulator's problem $\mathcal{P}$ is*

$$
\begin{aligned}
D^* = &\min_{\substack{\forall s,\, i\in I_h(s):\, \boldsymbol{\mu}(s)\in\mathbb{R}^{I_h(s)}\\ \boldsymbol{q}\in\mathbb{R}^{\Theta}}} \sum_\theta \mathbb{P}(\theta)\, q(\theta)\\
s.t\ \ & q(\theta) \geq w(s,\theta) + \sum_{i\in I_h(s)} \mu_i(s)\left(\delta L_i(s,\theta) - v_i - y_i^{out}\right),\ (\forall\theta,\forall s) \qquad (41)\\
&\mu_i(s) \geq 0. \qquad\qquad (\forall s_i = h\ \ in\ \forall s)
\end{aligned}
$$

1. *(Strong Duality) $W(\boldsymbol{\chi}^*) = \mathcal{D}^*(\boldsymbol{\mu}^*, \boldsymbol{q}^*)$, where $\boldsymbol{\mu}^*, \boldsymbol{q}^*$ ($\boldsymbol{\chi}^*$) are the multipliers of the constraints of the primal (dual) problem.*

2. *(Complementary Slackness) As an implication of strong duality, if $\pi(s'|\theta) > 0\,(=0)$ for some signal s', condition (41) evaluated at θ takes equality (is strict) for s'.*

Proof. First, I rewrite the primal problem $\mathcal{P}$ in vector and maxtrix form:

$$
\begin{aligned}
W^* = &\max_{\boldsymbol{\chi}\in\mathbb{R}^{S\times\Theta}} \boldsymbol{w}^\top\boldsymbol{\chi}\\
s.t.&\boldsymbol{\mathcal{A}\chi} \leq \boldsymbol{b},\\
&\boldsymbol{\chi} \geq \mathbf{0},
\end{aligned}
$$

where

$$
\boldsymbol{\mathcal{A}} \equiv \begin{bmatrix}
\cdots & & & & \\
& \Delta\boldsymbol{\mathcal{L}}^{s'} & & & \\
& & \Delta\boldsymbol{\mathcal{L}}^{s''} & & \\
& & & \cdots & \\
\cdots & \boldsymbol{I}_{|\Theta|} & \boldsymbol{I}_{|\Theta|} & \cdots & \boldsymbol{I}_{|\Theta|}\\
\cdots & -\boldsymbol{I}_{|\Theta|} & -\boldsymbol{I}_{|\Theta|} & \cdots & -\boldsymbol{I}_{|\Theta|}
\end{bmatrix}
$$

is a $\left(\sum_s |I_h(s)| + 2|\Theta|\right)$ by $|\Theta||S|$ matrix, where $\boldsymbol{s}', \boldsymbol{s}''$ are two examples of signals. Note that the prior consistency constraints $\sum_\theta \chi(s,\theta) = \mathbb{P}(\theta)$ are rewritten as $\sum_\theta \chi(s,\theta) \leq \mathbb{P}(\theta)$ and $-\sum_\theta \chi(s,\theta) \leq -\mathbb{P}(\theta)$ in the standardized form.

Let $\mu_i(s)$, $q^1(\theta)$ and $q^2(\theta)$ denote respectively the Lagrangian multiplier of the obedience constraint (13) of $s_i = h$ for some s, and the multipliers the rewritten prior consistency constraints. Rewriting the dual problem in vector and matrix form,

$$
\begin{aligned}
D^* &\equiv \min \boldsymbol{b}^\top \mathbf{z} \\
s.t.\,\boldsymbol{A}^\top \mathbf{z} &\geq \boldsymbol{w}, \\
\mathbf{z} &\geq \mathbf{0},
\end{aligned}
$$

where $\mathbf{z} \equiv \begin{bmatrix} \boldsymbol{\mu^T} & (\mathbf{q^1})^\top & (\mathbf{q^2})^\top \end{bmatrix}$, and $\boldsymbol{\mu^s} \equiv \begin{bmatrix} \cdots & \mu_i(s) & \cdots \end{bmatrix}^\top_{|I_h(s)|\times 1}$, $\boldsymbol{\mu} \equiv \begin{bmatrix} \cdots & (\boldsymbol{\mu^s})^\top & \cdots \end{bmatrix}^\top_{(\sum|I_h(s)|)\times 1}$, $\boldsymbol{q^1} \equiv \begin{bmatrix} \cdots & q^1(\theta) & \cdots \end{bmatrix}^\top_{|\Theta|\times 1}$ and $\boldsymbol{q^2} \equiv \begin{bmatrix} \cdots & q^1(\theta) & \cdots \end{bmatrix}^\top_{|\Theta|\times 1}$ are the vector representations. In scalar form, the first set of constraints says,

$$
\sum_{i\in I_h(s)} \mu_i(s)\left(v_i + y_i^{out} - \delta L_i(s,\theta)\right) + q^1(\theta) - q^2(\theta) \geq w(s,\theta),
$$

for each (s,θ) combination. Introduce $q(\theta) \equiv q^1(\theta) - q^2(\theta)$, and thus $\mathbf{z} \geq \mathbf{0}$ is equivalent to $\mu_i(s) \geq 0$ and unrestricted $q(\theta) \in \mathbb{R}$.

As both the primal and the dual are bounded, strong duality holds: $W(\boldsymbol{\chi}^*) = \mathcal{D}^*(\boldsymbol{\mu}^*, \boldsymbol{q}^*)$. $\boldsymbol{\chi}^*$ are the multipliers of the first set of constraints in the dual problem. Hence, the complementary-slackness condition of the dual is

$$
\chi(s,\theta)\left[q(\theta) - w(s,\theta) - \sum_{i\in I_h(s)} \mu_i(s)\left(\delta L_i(s,\theta) - v_i - y_i^{out}\right)\right] = 0. \tag{42}
$$

□

B.5 Proof of Proposition 2

Lemma 6. *Under the optimal disclosure policy $\boldsymbol{\pi}^*$, suppose for some signal s' that is reported with positive probability, the obedience constraint of some bank i that s' reports as $s'_i = h$ is slack. Let $\tilde{S}$ denote the collection of signals with such slack obedience constraint for bank i,*

$$
\tilde{S} \equiv \left\{ s | \sum_\theta \mathbb{P}(\theta)\,\pi^*(s\,|\theta) > 0,\ s_i = h,\ \mu_i(s) = 0 \right\}. \tag{43}
$$

For any signal $s' \in \tilde{S}$, and any state θ' with $\tilde{A}_i > 0$ where s' is reported—$\pi^*(s'\,|\theta') > 0$, let θ'' be the mirror-image state of θ' such that $\tilde{A}_i(\theta') > 0$, $\tilde{A}_i(\theta'') = 0$ and $\tilde{A}_{-i}(\theta'') = \tilde{A}_{-i}(\theta'')$. Then

1. $\pi^*(s'\,|\theta'') > 0$;

2. If $\pi^*(\hat{s}\,|\theta'')>0$ for some signal $\hat{s}$, then $\hat{s}\in\tilde{S}$.

Proof. Recall that $\Delta L_i(s,\theta)$ is bank i's contingent-state net cash flows,

$$\Delta L_i(s,\theta)\equiv\delta L_i(s,\theta)-v_i-y_j^{out}$$

For the first result, suppose for some $s'\in\tilde{S}$ and $\pi^*(s'\,|\theta')>0$, I have $\pi^*(s'\,|\theta'')=0$. Let $s''\neq s'$ denote some signal that is reported at θ'', i.e., $\pi^*(s''\,|\theta'')>0$. The complementary-slackness condition of the dual problem (42) at θ'' implies,

$$q(\theta'')=w(s'',\theta'')+\sum_{j\in I_h(s'')}\mu_j(s'')\,\Delta L_j(s'',\theta''),\tag{44}$$

$$q(\theta'')>w(s',\theta'')+\sum_{j\in I_h(s')}\mu_j(s')\,\Delta L_j(s',\theta''),\tag{45}$$

Similarly, at θ', $\pi^*(s'\,|\theta')>0$ implies

$$q(\theta')=w(s',\theta')+\sum_{j\in I_h(s')}\mu_j(s')\,\Delta L_j(s',\theta'),\tag{46}$$

$$q(\theta')\geq w(s'',\theta')+\sum_{j\in I_h(s'')}\mu_j(s'')\,\Delta L_j(s'',\theta').\tag{47}$$

The two states θ', θ'' differ in only bank i's project realization. Contingent on signal s' which reports $s'_i=h$, bank i refinances and makes full payments. Then except for bank i's underlying inflows that satisfy $L_i(s',\theta')>L_i(s',\theta'')$, the outcome at $t=2$ at the two states are identical. i.e., interbank payments $x_{ij}(s',\theta')=x_{ij}(s',\theta'')$ for any i, j, other banks' inflows $L_j(s',\theta')=L_j(s',\theta'')$ for $j\neq i$, and system stability $w(s',\theta')=w(s',\theta'')$. As $\mu_i(s')=0$, there is no cost for bank i to borrow across states under s', so $L_i(s',\theta')>L_i(s',\theta'')$ does not matter. Then

$$w(s',\theta')+\sum_{j\in I_h(s')}\mu_j(s')\,\Delta L_j(s',\theta')=w(s',\theta'')+\sum_{j\in I_h(s')}\mu_j(s')\,\Delta L_j(s',\theta'').\tag{48}$$

From equations (44)-(47) and (48), I have

$$w(s'',\theta')+\sum_{j\in I_h(s'')}\mu_j(s'')\,\Delta L_j(s'',\theta')<w(s'',\theta'')+\sum_{j\in I_h(s'')}\mu_j(s'')\,\Delta L_j(s'',\theta'').\tag{49}$$

The inequality says that the value of reporting s'' is strictly higher at the worse state θ''. However, the additional project shock of bank i decreases any bank's inflows $L_j(s,\theta')\geq L_j(s,\theta'')$ and hurts system stability $w(s'',\theta')\geq w(s'',\theta')$. Contradiction. Hence, $\pi^*(s'\,|\theta'')>0$.

If $\pi^*(s'\,|\theta'')=1$, the second result holds. If $\pi^*(s'\,|\theta'')<1$, let $\hat{s}$ denote some signal that is reported at θ'', i.e., $\pi^*(\hat{s}\,|\theta'')>0$. Now I show that $\hat{s}\in\tilde{S}$. Apply (42) to $\pi^*(\hat{s}\,|\theta')>0$,

$\pi^*(s'|\theta') > 0$ and $\pi^*(s'|\theta'') > 0$, and I have

$$q(\theta') = w(s',\theta') + \sum_{j\in I_h(s')} \mu_j(s')\,\Delta L_j(s',\theta') \geq w(\hat{s},\theta') + \sum_{j\in I_h(\hat{s})} \mu_j(\hat{s})\,\Delta L_j(\hat{s},\theta'),$$
$$q(\theta'') = w(s',\theta'') + \sum_{j\in I_h(s)} \mu_j(s')\,\Delta L_j(s',\theta'') = w(\hat{s},\theta'') + \sum_{j\in I_h(\hat{s})} \mu_j(\hat{s})\,\Delta L_j(\hat{s},\theta'').$$

(46) and (48) imply $q(\theta') = q(\theta'')$, and hence

$$w(\hat{s},\theta') + \sum_{j\in I_h(\hat{s})} \mu_j(\hat{s})\,\Delta L_j(\hat{s},\theta') \leq w(\hat{s},\theta'') + \sum_{j\in I_h(\hat{s})} \mu_j(\hat{s})\,\Delta L_j(\hat{s},\theta''). \tag{50}$$

I discuss by different cases of $\hat{s}_i$. If $\hat{s}_i = l$, bank i pays $\sum_{j\neq i} x_{ji} = 0$ to creditor banks at θ'', but pays some amount $\sum_{j\neq i} x_{ji} > 0$ at θ'. Hence $L_j(\hat{s},\theta') > L_j(\hat{s},\theta'')$ for all banks, and $w(\hat{s},\theta') \geq w(\hat{s},\theta'')$. Then (50) fails. If $\hat{s}_i = h$, bank i makes full payments $\sum_{j\neq i} x_{ji} = y_i^{out}$ regardless of $\tilde{A}_i$, and thus $L_j(\hat{s},\theta') = L_j(\hat{s},\theta'')$ for bank $j \neq i$, $w(\hat{s},\theta') = w(\hat{s},\theta')$, but $L_i(\hat{s},\theta') > L_i(\hat{s},\theta'')$. If additionally $\mu_i(\hat{s}) > 0\,(=0)$, (50) fails (takes equality). Therefore, (50) implies $\hat{s} \in \tilde{S}$. □

Proof of Proposition 2

Proof. Recall in (43) that $\tilde{S}$ denotes the collection of reported signals with slack (h) obedience constraint for bank i. Let $\mu_i(s') = 0$ denote such a violation, for some $s' \in \tilde{S}$ that reports $s'_i = h$.

First, I apply Lemma 6 to argue the following: for any mirror-image signal pair θ', θ'' with $\tilde{A}_{-i}(\theta') = \tilde{A}_{-i}(\theta'')$ and $\tilde{A}_i(\theta') > 0, \tilde{A}_i(\theta'') = 0$, if $\pi^*(s'|\theta') > 0$, then

$$\sum_{s\in\tilde{S}} \pi^*(s,\theta') \leq \sum_{s\in\tilde{S}} \pi^*(s,\theta'') = 1. \tag{51}$$

Then for the slack obedience constraints to hold in the first place, $s \in \tilde{S}$ being reported disproportionately at states with $\tilde{A}_i = 0$ (see Equation (51)) implies that bank i has a level of high profitability, which contradicts 2.

To see this, for any signal $s \in \tilde{S}$ (s is reported with positive probability but $\mu_i(s) = 0$ for some bank with $s_i = h$), bank i's obedience constraint says,

$$\begin{aligned}
&\sum_{\theta} \mathbb{P}(\theta)\,\pi^*(s,\theta)\,\Delta L_i(s,\theta)\\
=&\sum_{\tilde{A}_{-i}} \mathbb{P}\left(\tilde{A}_{-i}\right)\{\mathbb{P}\left(\tilde{A}_i > 0\right)\pi^*\left(s\left|\tilde{A}_i > 0, \tilde{A}_{-i}\right.\right)\Delta L_i\left(s, \tilde{A}_i > 0, \tilde{A}_{-i}\right)\\
&+\mathbb{P}\left(\tilde{A}_i = 0\right)\pi^*\left(s\left|\tilde{A}_i = 0, \tilde{A}_{-i}\right.\right)\Delta L_i\left(s, \tilde{A}_i = 0, \tilde{A}_{-i}\right)\}\\
>&0
\end{aligned}$$

As $\sum_{j\neq i} x_{ij}(s,\theta) \leq y_i^{in}$, the obedience constraint implies

$$\sum_{\tilde{A}_{-i}} \mathbb{P}\left(\tilde{A}_{-i}\right)\left\{p_i\pi^*\left(s\left|\tilde{A}_i>0,\tilde{A}_{-i}\right.\right)\delta\left(A_i+y_i^{in}\right)+(1-p_i)\,\pi^*\left(s\left|\tilde{A}_i=0,\tilde{A}_{-i}\right.\right)\delta y_i^{in}-v_i-y_i^{out}\right\}>0.$$

Sum over $s \in \tilde{S}$ and use (51),

$$\begin{aligned}
0 &< \sum_{s\in\tilde{S}}\sum_{\tilde{A}_{-i}} \mathbb{P}\left(\tilde{A}_{-i}\right)\left\{p_i\pi^*\left(s\left|\tilde{A}_i>0,\tilde{A}_{-i}\right.\right)\delta\left(A_i+y_i^{in}\right)+(1-p_i)\,\pi^*\left(s\left|\tilde{A}_i=0,\tilde{A}_{-i}\right.\right)\delta y_i^{in}-v_i-y_i^{out}\right\}\\
&\leq \sum_{\tilde{A}_{-i}} \mathbb{P}\left(\tilde{A}_{-i}\right)\left\{p_i\sum_{s\in\tilde{S}}\pi^*\left(s\left|\tilde{A}_i=0,\tilde{A}_{-i}\right.\right)\delta\left(A_i+y_i^{in}\right)+(1-p_i)\sum_{s\in\tilde{S}}\pi^*\left(s\left|\tilde{A}_i=0,\tilde{A}_{-i}\right.\right)\delta y_i^{in}-v_i-y_i^{out}\right\}\\
&= \sum_{\tilde{A}_{-i}} \mathbb{P}\left(\tilde{A}_{-i}\right)\sum_{s\in\tilde{S}}\pi^*\left(s\left|\tilde{A}_i=0,\tilde{A}_{-i}\right.\right)\left(p_i\delta A_i+\delta y_i^{in}-v_i-y_i^{out}\right),
\end{aligned}$$

which contradicts with Assumption 2. □

B.6 Proof of Proposition 3

Proof. Step 1.

I argue that the optimal policy $\boldsymbol{\pi}^*$ is a weighted sum of risk sharing strategies. Recall $\boldsymbol{\kappa}\left(s,\tilde{\Theta}^s;\theta_g^s\right)_{\left|\tilde{\Theta}^s\right|\times 1}$ in (26) characterizes the signal distribution over $\tilde{\Theta}^s$ for a given strategy, and $\eta\left(s,\tilde{\Theta}^s;\theta_g^s\right)$ is the strategy's probability. Then there exists a set of $\left\{\eta\left(s,\tilde{\Theta}^s;\theta_g^s\right)\right\}$ such that

$$\mathbb{P}(\theta)\,\pi\left(s'\left|\theta\right.\right)=\begin{cases}\sum_{\tilde{\Theta}^{s'}}\eta\left(s',\tilde{\Theta}^{s'};\theta_g^{s'}\right), & \theta=\theta_g^{s'},\\ \sum_{\theta_g^{s'}}\sum_{\tilde{\Theta}:\theta\in\tilde{\Theta}}\eta\left(s',\tilde{\Theta}^{s'};\theta_g^{s'}\right)\kappa\left(s,\theta\left(\tilde{\Theta}^{s'}\right);\theta_g^{s}\right), & \theta\notin\Theta_g^{s'}.\end{cases}\tag{52}$$

To see this, (52) corresponds to $\{\theta\left|\pi^*\left(s'\left|\theta\right.\right)>0\right.\}-\left|I_h(s')\right|$ linear equations of $\left|\Theta_g^{s'}\right|\cdot\binom{k-\left|\Theta_g^{s'}\right|}{\left|I_h(s')\right|}$ variables,

Let $z(s')\equiv\left|\{\theta\left|\pi^*\left(s'\left|\theta\right.\right)>0\right.\}\right|$ denote the number of states where s' is reported. Then there are $\underbrace{\left|\Theta_g^{s'}\right|}_{\text{num of }\theta_g^{s'}}\cdot\underbrace{\binom{z(s')-\left|\Theta_g^{s'}\right|}{\left|I_h(s')\right|}}_{\text{num of }\tilde{\Theta}^{s'}}$ variables to satisfy $z(s')-\left|I_h(s')\right|$ linear equations as in (52), where $-\left|I_h(s')\right|$ corresponds to the binding obedience constraints as restrictions on $\{\pi^*\left(\cdot\left|s'\right.\right)\}$. As $z(s')\geq\left|\Theta_g^{s'}\right|+\left|I_h(s')\right|$, such $\left\{\eta\left(s,\tilde{\Theta}^s;\theta_g^s\right)\right\}$ exist.

Step 2.

I argue that the decompsition is "well behaved": I can focus on strategies with $\boldsymbol{\kappa}\geq 0$ or $\boldsymbol{\kappa}\leq 0$, and the corresponding weights satisfy $\left\{\eta\left(s,\tilde{\Theta}^s(\theta);\theta_g^s\right)\kappa\left(s,\tilde{\Theta}^s(\theta);\theta_g^s\right)\geq 0\right\}$. The idea is, if there exists some $\eta\left(s,\tilde{\Theta}^s(\theta);\theta_g^s\right)\kappa\left(s,\tilde{\Theta}^s(\theta);\theta_g^s\right)<0$, there must exist some other strategy with $\eta\left(s',\tilde{\Theta}^{s'}(\theta);\theta_g^{s'}\right)\kappa\left(s',\tilde{\Theta}^{s'}(\theta);\theta_g^{s'}\right)>0$ at the same state θ. Then I redefine a strategy by combinding some parts of each.

For expositional convenience, I aggregate all the good states for a given signal, and introduce the signal's aggregate excess liquidity as $\boldsymbol{c}^s \equiv \Delta\boldsymbol{\mathcal{L}}^{s,\Theta_g^s}\boldsymbol{\chi}^{*\Theta_g^s}$. It is without loss of generality to work with strategy weights $\left\{\hat{\eta}\left(s,\tilde{\Theta}^s\right)\right\}$ and structures $\left\{\hat{\kappa}\left(s,\tilde{\Theta}^s\right)\right\}$ that satisfy

$$\sum_{\eta}\hat{\eta}\left(s,\tilde{\Theta}^s\right)\boldsymbol{c}^s = \sum_{\eta}\hat{\eta}\left(s,\tilde{\Theta}^s\right)\left(-\Delta\boldsymbol{\mathcal{L}}^{s,\tilde{\Theta}}\right)\hat{\boldsymbol{\kappa}}^{s,\tilde{\Theta}}.$$

Suppose for some $\left(s',\tilde{\Theta}'^{s'}\right)$, I have $\hat{\eta}\left(s',\tilde{\Theta}^{s'}\right)<0$. As the optimal policy satisfy $\pi^*\left(s'\,|\cdot\right)\geq 0$, there exists a sequence of strategies with signal s' and bad states $\tilde{\Theta}_1^{s'},\tilde{\Theta}_2^{s'},\cdots,\tilde{\Theta}_{k\leq|I_h(s')|}^{s'}$, who have positive weights $\hat{\eta}\left(s',\tilde{\Theta}_j^{s'}\right)>0$, and $\tilde{\Theta}^{s'}\subset\cup_{j\leq k}\tilde{\Theta}_j^{s'}$.

Then I redefine the strategies to eliminate $\hat{\eta}\left(s',\tilde{\Theta}^{s'}\right)<0$. First, I construct a strategy that lends liquidity to states $\tilde{\Theta}'^{s'}$ via signal s', with structure $\hat{\boldsymbol{\kappa}}^{s',\tilde{\Theta}'}\propto\boldsymbol{\chi}^{*s',\tilde{\Theta}'^{s'}}$, where $\chi^*\left(s,\theta\right)$ is the joint probability in the optimal policy. It is easy to show that there exists some $\hat{\Theta}$ with $\hat{\Theta}\cap\tilde{\Theta}'^{s'}=\emptyset$, which lends liquidity in the constructed strategy. Specifically, there exists some $\Delta\hat{\boldsymbol{\chi}}^{s',\Theta_g^s\cup\hat{\Theta}}\leq\hat{\boldsymbol{\chi}}^{*s',\Theta_g^s\cup\hat{\Theta}}$ that provides $\Delta\boldsymbol{\mathcal{L}}^{s',\Theta_g^s\cup\hat{\Theta}}\Delta\hat{\boldsymbol{\chi}}^{s',\Theta_g^s\cup\hat{\Theta}}=\left(-\Delta\boldsymbol{\mathcal{L}}^{s',\tilde{\Theta}'^{s'}}\right)\boldsymbol{\chi}^{*s',\tilde{\Theta}'^{s'}}$ liquidity. Hence, I have constructed a strategy and it has a positive weight $\hat{\eta}$.

Second, I decompose $\boldsymbol{\pi}^*$ conditional on the constructed strategy and weight being included in the decomposition. Specifically, let $\hat{\boldsymbol{\chi}}$denote the joint distribution when $\boldsymbol{\chi}^{*s,\tilde{\Theta}}$ and $\Delta\hat{\boldsymbol{\chi}}^{s',\Theta_g^s\cup\hat{\Theta}}$ are substracted from $\boldsymbol{\chi}^*$, under which s' is reported at $|I_h\left(s'\right)|$ less states. Then I find the new weights $\left\{\hat{\eta}\left(s,\tilde{\Theta}^s\right)\right\}$ and structures $\left\{\hat{\kappa}\left(s,\tilde{\Theta}^s\right)\right\}$ given $\hat{\boldsymbol{\chi}}$.

I repeat the previous two steps whenever some $\hat{\eta}<0$, until for all $\tilde{\Theta}^{s'}$with $\left(-\Delta\boldsymbol{\mathcal{L}}^{s',\tilde{\Theta}}\right)\hat{\boldsymbol{\kappa}}^{s',\tilde{\Theta}}>0$, I have $\left|\cup\tilde{\Theta}^{s'}\right|\leq|I_h\left(s'\right)|$. Due to the binding obedience constraints, equality holds and non-negative weight for the last strategy in the decomposition is ensured.

By a similar argument, I could focus on $\hat{\eta}\left(s,\tilde{\Theta}^s\left(\theta\right)\right)\hat{\kappa}\left(s,\tilde{\Theta}^s\left(\theta\right)\right)\geq 0$.

Step 3.

I prove the second part of the proposition. The idea is that $\boldsymbol{\pi}^*$ prioritizes more efficient strategies in a linear problem, so the number of risk sharing strategies should be reflected in the shadow values of influencing beliefs at specific states, $q\left(\theta\right)$. Let $\hat{\Theta}\left(\boldsymbol{q}^*\right)\equiv\left\{\theta\in\Theta\,|q\left(\theta\right)>w_0\left(\theta\right)\right\}$, which corresponds to the states where $\pi^*\left(s_l\,|\theta\right)=0$ and so involve some risk sharing. I argue that $\left|\hat{\Theta}\right|=k$.

"$k\leq\left|\hat{\Theta}\right|$": let $\Theta_G\left(\boldsymbol{\pi}^*\right)\equiv\left\{\theta\,\middle|\theta\in\Theta_g^s\text{ and }\pi^*\left(s\,\middle|\theta_g^s\right)>0\right\}$ denote the good states involved in $\boldsymbol{\pi}^*$. Suppose for $\theta_g\in\Theta_G\left(\boldsymbol{\pi}^*\right)$, there are $r\left(\theta_g\right)\geq 1$ risk sharing strategies that borrow from θ_g. Let $\left(s',\tilde{\Theta}';\theta_g\right)$ denote a typical strategy that is involved. Its cross-state borrowing price $\left(\boldsymbol{\mu}^{s'}\right)^\top=\left[\cdots,\mu_i\left(s'\right),\cdots\right]_{i\in I_h(s')}^\top$ is implied by $q\left(\theta'\right)$ over $\theta'\in\tilde{\Theta}'$:

$$q\left(\theta'\right)=w\left(s',\theta'\right)+\sum_{i\in I_h(s')}\mu_i\left(s'\right)\Delta L_i\left(s',\theta'\right)\Rightarrow\left(\boldsymbol{\mu}^{s'}\right)^\top=\left(\boldsymbol{w}^{s',\tilde{\Theta}'}-\boldsymbol{q}^{\tilde{\Theta}'}\right)^\top\left[\left(-\Delta\boldsymbol{\mathcal{L}}^{s',\tilde{\Theta}'}\right)^\top\right]^{-1},$$

which further implies

$$q\left(\theta_g\right) = w\left(s',\theta_g\right) + \underbrace{\left(\boldsymbol{w}^{s',\tilde{\Theta}'} - \boldsymbol{q}^{\tilde{\Theta}'}\right)^{\top}\left[\left(-\Delta\boldsymbol{\mathcal{L}}^{s',\tilde{\Theta}'}\right)^{\top}\right]^{-1}}_{=\left(\boldsymbol{\mu}^{s'}\right)^{\top}}\left(\Delta\boldsymbol{\mathcal{L}}^{s',\theta_g}\right). \tag{53}$$

This holds for all $r\left(\theta_g\right)$ strategies, which corresponds to $r\left(\theta_g\right)$ restrictions on $q\left(\theta_g\right)$ and $\cup\boldsymbol{q}^{\tilde{\Theta}'}$.

"$k \geq \left|\hat{\Theta}\right|$": suppose $k < \left|\hat{\Theta}\right|$. Note that $s_l \equiv (l, \cdots, l)$ is not reported on $\hat{\Theta}$. From the dual problem constraint (24), I have $q(\theta) \geq w(s_l, \theta) = w_0(\theta)$, where the multipliers of (14) are zero as shown in Lemma 1, and s_l implements the autarky market equilibrium under Assumption 2. Hence, I have $\left|\hat{\Theta}\right|$ constraints on k variables $\left\{\eta\left(s, \tilde{\Theta}^s; \theta_g^s\right)\right\}$, which contradicts with $k < \left|\hat{\Theta}\right|$.

Therefore, $k = \left|\hat{\Theta}\right|$. □

B.7 Proof of Proposition 4

Proof. Step 1.

I represent interbank payments in vector and matrix form, and show that they are essentially linear functions of the liabilities y_i^{out} of solvent banks, and the out-of-network cash flows $A_i - v_i$ of banks that default on junior liabilities (hence $\tilde{A}_i = A_i$).

For any joint (s, θ), let $Solv$, $Dflt$ and $Fail$ respectively denote the subset of banks that are solvent, default on junior liabilities and default on senior liabilities. Let $\boldsymbol{C}(s,\theta)$ be a selection matrix that selects the subset of banks as in the superscript; for example,

$$\boldsymbol{C}_{ij}^{Solv} \equiv \begin{cases} 1, & \text{if } i = j \text{ and } i \in Solv, \\ 0, & \text{if } i = j \text{ and } i \notin Solv, \\ 0, & \text{if } i \neq j. \end{cases}$$

Let $\boldsymbol{K}$ denote the interbank liability weight matrix, where $\boldsymbol{K}_{ij} \equiv \frac{y_{ij}}{y_j^{out}}$ for $i \neq j$ denotes the proportion of lender bank i's claim on borrower bank j in borrower bank's total interbank liabilities; I normalize $\boldsymbol{K}_{ii} \equiv 0$.

Hence, according to (8), I have

$$\boldsymbol{C}^{Dflt}\boldsymbol{x} = \underbrace{\boldsymbol{C}^{Dflt}\left(\tilde{\boldsymbol{A}} - \boldsymbol{v}\right)}_{\text{out-of-network net inflow}} + \underbrace{\boldsymbol{K}\boldsymbol{C}^{Dflt}\boldsymbol{x}}_{\text{payments from } Dflt} + \underbrace{\boldsymbol{K}\boldsymbol{C}^{Solv}\boldsymbol{y}}_{\text{full payments from } Solv}, \tag{54}$$

where $\boldsymbol{x}$, $\boldsymbol{y}$, $\tilde{\boldsymbol{A}}$ and $\boldsymbol{v}$ are all $n \times 1$, and are respectively the vector forms of bank's total actual interbank payments $\boldsymbol{x}_i \equiv \sum_{j\neq i} x_{ji}$, interbank liability y_i, project payoff $\tilde{A}_i$ and senior liability v_i. Solving for $\boldsymbol{C}^{Dflt}\boldsymbol{x}$ in (54), a bank's underlying excess cash flow $\Delta L_i(s,\theta)$ satisfies

$$\Delta L_i(s,\theta) = \delta\left\{\tilde{A}_i + (\boldsymbol{K}_i)^{\top}\boldsymbol{C}^{Solv}\boldsymbol{y} + (\boldsymbol{K}_i)^{\top}(\boldsymbol{I} - \boldsymbol{K})^{-1}\left[\boldsymbol{C}^{Dflt}\left(\tilde{\boldsymbol{A}} - \boldsymbol{v}\right) + (\boldsymbol{K}_i)^{\top}\boldsymbol{C}^{Solv}\boldsymbol{y}\right]\right\} - v_i - y_i.$$

Step 2.

Suppose in the optimal information structure, s is reported at bad states $\tilde{\Theta}^s$. I argue that a sufficient condition for replacing s with s' which additionally reports $s'_{i'} = h$ at $\tilde{\Theta}^s$ is

$$\Delta L_{i'}\left(s', \theta\right) \geq 0, \text{ or } \min_{i \in I_h(s)} \frac{A_{i'}}{A_i} \frac{\Delta L_i\left(s, \theta\right)}{\Delta L_{i'}\left(s', \theta\right)} \geq \underline{\rho}\left(i', s\right). \tag{55}$$

First, I focus on risk sharing strategies with $\theta_g^s = \overline{\theta}$ to prove the second part of the proposition. For a given strategy $\left(s, \tilde{\Theta}^s; \overline{\theta}\right)$ with positive weight $\eta > 0$ in $\boldsymbol{\pi}^*$, the obedience constraint for $i \in I_h(s)$ says

$$\Delta L_i\left(s, \overline{\theta}\right) + \sum_{\theta \in \tilde{\Theta}^s} \kappa\left(s, \theta; \overline{\theta}\right) \Delta L_i\left(s, \theta\right) = 0.$$

Note that for any $i' \neq I_h(s)$, and a signal that additionally reports i' with h, i.e., $s' \equiv \left(s'_{i'} = h, s'_{-i'} = s_{-i'}\right)$, the above condition is violated for i' under s'. Instead there exists some

$$\rho\left(i', s\right) \equiv \min_{\theta \in \tilde{\Theta}^s} \min \left\{1, \left[\frac{\Delta L_i\left(s, \theta\right)}{\Delta L_i\left(s, \overline{\theta}\right)} \left(\min_{i \in I_h(s)} \frac{\Delta L_{i'}\left(s', \theta\right)}{\Delta L_{i'}\left(s', \overline{\theta}\right)}\right)^{-1}\right]^+\right\} \in (0,1)$$

that scales down the probability of s' such that

$$\Delta L_{i'}\left(s, \overline{\theta}\right) + \rho\left(i', s\right) \sum_{\theta \in \tilde{\Theta}^s} \kappa\left(s, \theta; \overline{\theta}\right) \Delta L_{i'}\left(s', \theta\right) \geq 0.$$

As for associated payoff, the optimality of the strategy implies that $(\rho\boldsymbol{\kappa})^\top \boldsymbol{w}^{s', \tilde{\Theta}^s} < (\boldsymbol{\kappa})^\top \boldsymbol{w}^{s, \tilde{\Theta}^s}$. Let $\underline{\rho}\left(i', s\right)$ be the threshold value such that the scaled down probability of s' delivers the same payoff as the original strategy with s, i.e. $\left(\underline{\rho}\boldsymbol{\kappa}\right)^\top \boldsymbol{w}^{s', \tilde{\Theta}^s} = (\boldsymbol{\kappa})^\top \boldsymbol{w}^{s, \tilde{\Theta}^s}$. Under the symmetric leverage Assumption 4, I have $\frac{\Delta L_{i'}\left(s', \overline{\theta}\right)}{\Delta L_i\left(s, \overline{\theta}\right)} = \frac{A_{i'}}{A_i}$ and the sufficient condition boils down to (55).

Step 3.

I show that the sufficient condition (55) is related to r_y and $r_v + r_y$ as stated in the proposition.

Let $\zeta_i \equiv \frac{A_i}{A} = \frac{y_i}{y} = \frac{v_i}{v}$ characterize the size of bank i, and $\boldsymbol{\zeta}$ the vector form. Let selection matrix $\boldsymbol{C}^G$ denote the set of banks with good projects $\tilde{A}_i > 0$. Then the sufficient condition could be written in matrix form as

$$\min_{i \in I_h(s)} \frac{v + y - \frac{\delta}{\zeta_i}\left\{\tilde{A}_i + (\boldsymbol{K}_i)^\top \boldsymbol{C}^{Solv}\boldsymbol{\zeta} y + (\boldsymbol{K}_i)^\top (\boldsymbol{I} - \boldsymbol{K})^{-1}\left[\boldsymbol{C}^{Dflt}\left(\boldsymbol{C}^G\boldsymbol{\zeta}A - \boldsymbol{\zeta}v\right) + (\boldsymbol{K}_i)^\top \boldsymbol{C}^{Solv}\boldsymbol{\zeta}y\right]\right\}}{v + y - \frac{\delta}{\zeta_{i'}}\left\{\tilde{A}_{i'} + (\boldsymbol{K}_{i'})^\top \boldsymbol{C}'^{Solv}\boldsymbol{\zeta} y + (\boldsymbol{K}_{i'})^\top (\boldsymbol{I} - \boldsymbol{K})^{-1}\left[\boldsymbol{C}'^{Dflt}\left(\boldsymbol{C}^G\boldsymbol{\zeta}A - \boldsymbol{\zeta}v\right) + (\boldsymbol{K}_{i'})^\top \boldsymbol{C}'^{Solv}\boldsymbol{\zeta}y\right]\right\}} \tag{56}$$

$$\geq \underline{\rho}\left(i', s\right) \tag{57}$$

where $\boldsymbol{C}'^{Solv}$ and $\boldsymbol{C}'^{Dflt}$ refer to respectively the set of banks that are solvent and default on junior liabilities under (s', θ).

From the construction of s', I have $\boldsymbol{C}'^{Solv} \geq \boldsymbol{C}^{Solv}$, $\boldsymbol{C}'^{Dflt} \leq \boldsymbol{C}^{Dflt}$ and $\boldsymbol{C}'^{Solv} - \boldsymbol{C}^{Solv} \geq \boldsymbol{C}^{Dflt} - \boldsymbol{C}'^{Dflt}$; in addition, $\boldsymbol{C}^{Solv}$, $\boldsymbol{C}'^{Solv}$ decrease in y and $\boldsymbol{C}^{Dflt}$, $\boldsymbol{C}'^{Dflt}$ increase in y. As a result, $\frac{\zeta_{i'}}{\zeta_i}\frac{L_i(s,\theta)}{L_{i'}(s,\theta)}$ decreases in y. This implies that the left hand side of (56) increases in y, which completes the second part of the proposition.

As for the first part of the proposition, note that with a relatively large A, some probability constraint of the original risk sharing strategy $\left(s, \tilde{\Theta}^s; \overline{\theta}\right)$ binds at $\theta \in \tilde{\Theta}^s$, which leads to a smaller $\underline{\rho}(i', s)$ as determined in the condition for same payoff, $\left(\underline{\rho}\overline{\boldsymbol{\kappa}}\right)^\top \boldsymbol{w}^{s', \tilde{\Theta}^s} = (\overline{\boldsymbol{\kappa}})^\top \boldsymbol{w}^{s, \tilde{\Theta}^s}$. On the other hand, with a relatively large A, the good states for s also becomes good states for s' that additionally reports $s'_{i'} = h$. □

B.8 Simplification of Solution Structure

Proposition 10. *There exists constants $\underline{p}$ and $\overline{p}$, such that when $p \in \left[\underline{p}, \overline{p}\right]$, the optimal policy $\boldsymbol{\pi}^*$ has the following structure*

$$\{s \,|\pi(s\,|\theta) > 0\} = \begin{cases} s^* & \overline{\theta}, \\ s_h, & \theta : 1 < B(\theta) \leq \overline{B}^{NCP}, \\ s^*, s_h, s_l & \theta \in \tilde{\Theta}^{*s}, \\ s_l & \textit{Otherwise.} \end{cases}$$

where $\left(s^*, \tilde{\Theta}^{*s}; \overline{\theta}\right)$ has the highest index ξ among all strategies that borrow to states with $B(\theta) > \overline{B}^{NCP}$.

Proof. Suppose $p_i = p$. I argue that there exists an $\underline{p} \in (0, 1)$ such that when $p \geq \underline{p}$, I can construct $\boldsymbol{\pi}^*$ by finding the maximum $\xi^*\left(s, \theta; \overline{\theta}\right)$ under $\theta_g^s = \overline{\theta}$. The idea is, cross-bank signal choice is only relevant when banks have low profitability on average and belief updating is sensitive. Either all excess inflows are exhausted (OB bind) at the maximum ξ^*, or they allow banks to refinance at worse states with more shocks, where cross-state borrowing is much less expensive and s_h dominates when p is relatively large.

First, I argue that if some state $\hat{\theta} \neq \overline{\theta}$ has excess inflows for all banks, i.e., $\Delta L_i\left(s_h, \hat{\theta}\right) \geq y_i^{out}$ for all i, then $\pi^*\left(s_h \middle| \hat{\theta}\right) = 1$ and the optimal policy implements perfect cross-bank risk sharing at these states. Suppose not, and $\pi^*\left(s' \middle| \hat{\theta}\right) > 0$ for some signal $s' \neq s_h$ and s' is also reported at $\hat{\Theta}^{s'}$. Let $B(\theta) \equiv \sum_i \mathbf{1}_{\left\{\tilde{A}_i(\theta)=0\right\}}$ denote the number of project shocks. Under Assumption 4, banks are balanced $y_i^{in} = y_i^{out}$ as $\sum_i y_i^{in} = \sum y_i^{out}$. It follows that s_h implements complete netting of all interbank liabilities. Thus for all states θ with $B(\theta) = B\left(\hat{\theta}\right)$, I have $\Delta L_i(s_h, \theta) \geq 0$ for all i, and $\pi(s_h\,|\theta) = \pi\left(s_h \middle| \hat{\theta}\right) = 1$ results in the maximum possible payoff $\sum_{\theta:B(\theta)=b(\hat{\theta})} \mathbb{P}(\theta)\left[\sum_i \lambda_i - w_0(\theta)\right]$ at theses states. For the reported signal s', if s' consumes liquidity at $\theta : B(\theta) = B\left(\hat{\theta}\right)$, the associated payoff is striclty smaller. If s'accumulates liquidity, then s' is reported at significantly less states with $B\left(\hat{\theta}\right)$ shocks each with $\mathbb{P}(\theta)\left[\sum_i \lambda_i - w_0(\theta)\right]$ less payoff than s_h. When p is relatively large, this effect dominates any extra payoff of reporting s' at states with more shocks $B > B\left(\hat{\theta}\right)$.

Second, I argue that for all other "good states", $\overline{\theta}$ and $\theta_g^{s \neq s_h}$, it is without loss of generality to focus on $\overline{\theta}$. For any $\theta_g^s \neq \overline{\theta}$, $\pi\left(s\,|\theta\right)$ for $\theta \in \hat{\Theta}^s$ can be replicated by reporting s at $\overline{\theta}$, and the resulting $\pi\left(s\,\middle|\overline{\theta}\right) \leq \pi\left(s\,\middle|\theta_g^s\right)$. Hence, I consume the excess liquidity at $\theta_g^s \neq \overline{\theta}$ when that of $\overline{\theta}$ is exhausted.

Last, when p is sufficiently large, either $\pi\left(s^*\,\middle|\overline{\theta}\right) = 1$ for (s^*, θ^*) with the highest index, or $\pi\left(s^*\,\middle|\overline{\theta}\right) < 1$ and s is reported at $\overline{\theta}$ to extend cross-state risk sharing for other states. If these states have more project shocks than $b\left(\theta^*\right)$, where cross-state borrowing is much less expensive and s_h dominates. □

B.9 Proof of Proposition 5

Proof. First, I formalize the symmetric equilibrium in the context of complete network. Recall that $H\left(s\right) \equiv \left|I_h\left(s\right)\right|$ and $B\left(\theta\right) \equiv \sum_i \mathbf{1}_{\left\{\tilde{A}_i(\theta)=0\right\}}$ denote respectively the number of banks that s reports as h, and the number of underlying project shocks at θ. Let $H_B\left(s, \theta\right) \equiv \sum_i \mathbf{1}_{\left\{\tilde{A}_i=0, s_i=h\right\}}$ denote the number of banks with bad projects that signal s reports as h. As ex post banks fall into four categories depending on $\tilde{A}_i$ and s_i realizations, in a symmetric equilibrium, conditional probability $\pi\left(s\,|\theta\right)$ is the same for any pair of $\left(s, \theta\right)$ that share the same number of banks in each category. Specifically,[27]

$$\forall\left(s,\theta\right),\left(s',\theta'\right) \text{ with H(}s\text{)=H(}s'\text{), B(}\theta\text{)=B(}\theta'\text{), } H_B\left(s,\theta\right) = H_B\left(s',\theta'\right): \; \pi\left(s\,|\theta\right) = \pi\left(s'\,|\theta'\right). \quad (58)$$

Second, I argue that in the optimal solution $\boldsymbol{\pi}^*$, any signal $s \neq s_l$ reports $s_i = h$ on banks with good project, i.e., $H_B\left(s,\theta\right) = H\left(s\right) - \left(n - B\left(\theta\right)\right)$. Taking into account the symmetric structure in (58), a typical passing bank's average total inflows across the symmetric states is

$$\overline{L_i\left(s,\theta\right)} = \underbrace{\frac{H\left(s\right) - H_B\left(s,\theta\right)}{H\left(s\right)}A}_{\text{project payoff}} + \underbrace{\frac{H\left(s\right)-1}{n-1}y}_{\text{payments from banks with } h} + \underbrace{\frac{\left(n - B\left(\theta\right)\right) - \left(H\left(s\right) - H_B\left(s,\theta\right)\right)}{n-1}\underbrace{\mathbf{1}_{\left\{s_j=l, \tilde{A}_j>0\right\}} x_{\cdot j}}_{\leq y}}_{\text{payments from banks with } l},$$

which increases in $H\left(s\right)$. In addition, $w\left(s,\theta\right)$ increases in $H\left(s\right)$. Hence, any signal $s \neq s_l$ reports banks with good project as h. I summarize a class of symmetric risk sharing strategies as $\left(s\left(H\right), \theta^B; \theta_g^s\right)$.

Next, I calculate the efficiency index $\xi\left(s\left(H\right), \theta^B; \overline{\theta}\right)$, based on which to solve for the optimal policy when the simplification conditions hold as in Proposition 10. According to Lemma 3, for states with $1 \leq B\left(\theta\right) \leq \overline{B}^{NCP}$, I have $\pi^*\left(s_h\,|\theta\right) = 1$. Hence, I focus on states with $B\left(\theta\right) > \overline{B}^{NCP}$

[27] Note that $\left(s, \theta\right)$ and $\left(s', \theta'\right)$ may share the same signal $s = s'$ or the same state $\theta = \theta'$.

that borrow liquidity from $\overline{\theta}$. I have

$$\xi\left(s\left(H\right),\theta^B;\overline{\theta}\right)=\eta\left(s\left(H\right),\theta^B;\overline{\theta}\right)\cdot\underbrace{\left[H-\left(n-B\right)\mathbf{1}_{B\leq\overline{B}^C}\right]}_{\Delta w\left(s(H),\theta^B\right)}\cdot\frac{\overbrace{\delta\left(A+y\right)-v-y}^{\Delta\mathcal{L}^{\overline{\theta}}}}{\underbrace{v+y-\delta\left(\frac{H-1}{n-1}y+\frac{n-B}{H}A\right)}_{-\Delta\mathcal{L}^{s(H),\theta^B}}}.$$

For any class of symmetric strategies that includes multiple (s,θ), the liquidity available to borrow from $\overline{\theta}$ is scarce when $\delta\leq\overline{\delta}^c$, and $\eta\left(s\left(H\right),\theta^B;\overline{\theta}\right)=\mathbb{P}\left(\overline{\theta}\right)$. Then I solve for the (H,B) that delivers the highest ξ. With some calculation, there exists a threshold $\hat{A}$ such that when $A\leq\hat{A}$, I have $\frac{\partial\xi\left(s(h),\theta^B;\overline{\theta}\right)}{\partial B_h}>0$. Hence, the optimal policy either reports s_h or s_l. Plugging in s_h, for $B>\overline{B}^{NCP}$ I have

$$\xi\left(s_h,\theta^B;\overline{\theta}\right)=\begin{cases}\mathbb{P}\left(\overline{\theta}\right)n\cdot\frac{\delta(A+y)-v-y}{v+(1-\delta)y-\frac{n-B}{n}\delta A}, & \overline{B}^{NCP}\geq\overline{B}^C,\\ \mathbb{P}\left(\overline{\theta}\right)\left[B+\left(n-B\right)\mathbf{1}_{B>\overline{B}^C}\right]\cdot\frac{\delta(A+y)-v-y}{v+(1-\delta)y-\frac{n-B}{n}\delta A}, & \overline{B}^{NCP}<\overline{B}^C.\end{cases}$$

If $\overline{B}^{NCP}\geq\overline{B}^C$, the index decreases in B. If $\overline{B}^{NCP}<\overline{B}^C$ and A is relatively small say $A\leq\hat{A}'$, $\xi\left(s_h,\theta^B;\overline{\theta}\right)$ has the largest value when $B=\lfloor\overline{B}^C\rfloor+1$. As both the optimality of s_h and $\lfloor\overline{B}^C\rfloor+1$ rely on a relatively small A, I can find a threshold $\underline{r}_y^c$ with $\frac{y}{A}\geq\underline{r}_y^c$ as stated in the proposition. □

B.10 Proof of Proposition 6

Lemma 7. *Suppose $p\in\left[\underline{p},\overline{p}^R\right]$ and (38) hold. Let $\left(s^*,\tilde{\Theta}^{*s};\overline{\theta}\right)$ denote the strategy with the highest index $\xi\left(s,\theta;\overline{\theta}\right)$. Then the optimal policy $\boldsymbol{\pi}^*$ reports the following signals across states:*

$$\{s\,|\pi\left(s\,|\theta\right)>0\}=\begin{cases}s^* & \overline{\theta},\\ s_h, & \theta:1<B\left(\theta\right)\leq\overline{B}^{NCP},\\ s^*,s_h,s_l & \theta\in\tilde{\Theta}^{*s},\\ s_l & \textit{Otherwise.}\end{cases}$$

Proof. When $p\geq\underline{p}^R$, Lemma 3 applies and $\mathbb{P}\left(s_h\middle|1\leq B\left(\theta\right)\leq\overline{B}^{NCP}\right)$. For states with more project shocks $B\left(\theta\right)>\overline{B}^{NCP}$, under condition (38), all banks default in autarky and $w_0\left(\theta\right)=0$. There are two implications. First, for any signal $s\neq s_l$, the relevant good state is $\theta_g=\overline{\theta}$. Second, since $\Delta w\left(s_h,\theta\right)=n$ when $B\left(\theta\right)>\overline{B}^{NCP}$, the regulator prefers to lend the liquidity accumulated $1\leq B\left(\theta\right)\leq\overline{B}^{NCP}$ to states with less shocks, so s_h is reported at states with $B\left(\theta\right)=\lfloor\overline{B}^{NCP}\rfloor+1$.

When $p\leq\overline{p}^R<1$, the cross-state risk sharing is constrained and $\overline{\eta}<\mathbb{P}\left(\theta_g^s\right)$. Note that the symmetry in the ring structrure makes this circumstance more likely to arise. According to Proposition 10, the optimal policy has the structure presented in the lemma. I argue that $\theta\in\tilde{\Theta}^{*s}$ have project shocks of $\overline{B}^{NCP}+1$ because the states with more shocks are dominated. For expositional convenience, I assume that s_h is also reported at $\theta \in \tilde{\Theta}^{*s}$instead of other states with $B(\theta) = \overline{B}^{NCP} + 1$, despite some violation of equilibrium symmetry. $\square$

Lemma 8. *(Distance Strategy) Suppose (38) and $\left[\underline{p}^R, \overline{p}^R\right]$ hold. Among the states with B project shocks, strategies with the highest ξ is distance-based: there exists a $\tilde{\theta} \in \tilde{\Theta}^s$ and distance $d \geq 1$, such that starting from any bank with negative project shock (included) till the next bank with project shock (shock bank excluded), the signal reports l on the first $d-1$ banks, and h on the rest of the banks.*

Proof. The idea is for any violation against the structure stated in the lemma, there is improvement from switching the signal h downstream to lender banks and the project shocks upstream on borrower banks.

First, I rule out cases where for a sequence of banks have good projects, a signal reports h on some borrower banks (upstream) but l on some lender banks (downstream). Specifically, suppose $\pi(s' | \theta') > 0$ for

$$\theta' : \tilde{A}_j > 0 \text{ for } i \leq j \leq i+d \text{ where } d > 1, \text{ and } \tilde{A}_{i+d+1} = 0;$$

$$s' : s'_j \equiv \begin{cases} h, & i \leq j \leq i+d_1 < i+d, \\ l, & i+d_1 < j \leq i+d, \\ \text{unrestricted}, & \text{otherwise.} \end{cases}$$

I move the project shock of bank $i+d$ at θ' one bank downstream, and call the resulting state θ'', i.e.,

$$\theta'' : \tilde{A}_j(\theta'') \equiv \begin{cases} \tilde{A}_{j+1}(\theta'), & j \in \{j | i+d \leq j < i+d_2, \ s_j = l, \ s_{i+d_2} = h\} \\ \tilde{A}_j(\theta'), & \text{otherwise.} \end{cases}$$

Then I adjust the probability distribution by moving the probability of s' on θ' to θ'':

$$\hat{\pi}(s' | \theta) = \begin{cases} \pi(s' | \theta') + \pi(s' | \theta), & \theta = \theta'', \\ 0, & \theta = \theta', \\ \pi(s' | \theta), & \theta \neq \theta', \theta''. \end{cases}$$

One can verify that the new information structure is feasible (the obedience constraint for bank i could be slack while those for other banks are equivalent) and delivers the same payoff to the regulator.

Second, I rule out cases where for a sequence of banks have good projects, a signal reports h on some borrower banks (upstream) and lender banks (downstream), but reports l on some banks in between. Specifically, suppose $\pi\left(s'\left|\theta'\right.\right)>0$ for

$$\theta':\tilde{A}_j>0\text{ for }i<j\leq i+d\text{ where }d>1\text{ and }\tilde{A}_{i+d+1}=0;$$

$$s':s'_j\equiv\begin{cases}h, & j=i,i+d_1+1,\cdots,i+d,\\ l, & j=i+1,\cdots,i+d_1,\\ \text{unrestricted}, & \text{otherwise.}\end{cases}$$

As the obedience constraints associated with $s'$are binding, the above structure implies that for any bank j in $i<j\leq i+d_1$, there exists a state θ with $\pi\left(s'\left|\theta\right.\right)>0$ and $\tilde{A}_j\left(\theta\right)=0$; otherwise, there exists a bank $\hat{j}$ in $i<\hat{j}\leq i+d_1$ that is reported with $s'_{\hat{j}}=l$ but $\Delta L_{\hat{j}}>0$ at all states with $\pi\left(s'\left|\theta\right.\right)>0$. Then I propose the adjustment by "postponing" the $s'_i=h$ to its lender bank (downstream). Specifically, I replace s' with s''

$$s'':s''_j\equiv\begin{cases}s'_j, & j\neq i,i+1,\\ h, & j=i+1,\\ l, & j=i,\end{cases}$$

and adjust the cross-state distribution such that the adjustment is feasible and delivers the same payoff to the regulator. Note that some obedience constraints become slack. Let

$$\hat{i}\equiv\sup_k\left\{k<i\left|\tilde{A}_{k+1}(\theta')=0,s'_{k+1}=l\right.\right\},$$

and

$$\hat{\pi}\left(s''\left|\theta\right.\right)=\begin{cases}\pi\left(s'\left|\theta\right.\right), & \theta\in\left\{\theta\left|\tilde{A}_{i+1}=A\right.\right\}\cup\left\{\theta\left|\tilde{A}_{i-1}=\tilde{A}_i=\tilde{A}_{i+1}=0\right.\right\},\\ 0, & \theta:\tilde{A}_{i-1}=A,\tilde{A}_{i+1}=0,\\ \pi\left(s'\left|\theta\right.\right)+\pi\left(s'\left|\hat{\theta}\right.\right), & \theta:\tilde{A}_i=A,\tilde{A}_{\hat{i}}=0,\ \text{otherwise}\tilde{A}_j(\theta)=\tilde{A}_j(\hat{\theta});\ \hat{\theta}:\tilde{A}_{i-1}=A,\tilde{A}_i=\tilde{A}_{i+1}=0,\\ \pi\left(s'\left|\theta\right.\right)+\pi\left(s'\left|\tilde{\theta}\right.\right), & \theta:\tilde{A}_{i+1}=A,\tilde{A}_{\hat{i}}=0,\ \text{otherwise}\tilde{A}_j(\theta)=\tilde{A}_j(\tilde{\theta});\ \tilde{\theta}:\tilde{A}_{i-1}=\tilde{A}_i=A,\tilde{A}_{i+1}=0.\end{cases}$$

By similar argument, I know the adjustment holds for any $d_1>1$. □

Lemma 9. *(Connected Shocks)Suppose (38) and* $\left[\underline{p}^R,\overline{p}^R\right]$ *hold. For states with* $B\left(\theta\right)\geq\overline{B}^{NCP}+1$, $\tilde{\Theta}^{*s}$ *include the state where project shocks are on adjacent banks.*

Proof. This lemma discusses at which states to lend liquidity. Without loss of generality, for any states with $B\left(\theta\right)\geq\overline{B}^{NCP}+1$ of interest, I assume that there is only one area of adjacent project shocks, and all other project shocks are disjoint.

First, I argue that I can focus on cases where for any two non-adjacent banks with project shocks, say $\tilde{A}_{i_1}\left(\theta'\right)=\tilde{A}_{i_2}\left(\theta'\right)=0$, at least one bank i in between—$i_1<i<i_2$ and i_2—is reported with h with positive probability. Otherwise, I denote the violation as follows: for any $s'\neq s_l$

reported at θ', $\pi(s'|\theta') > 1$, I have $s'_{i_1+1} = s'_{i_1+2} = \cdots = s'_{i_2-1} = l$. Then the regulator prefers reporting s' at another state θ'' where the project shock at i_2 is switced upstream to bank i_1+1. i.e., $\tilde{A}_{i_1+1}(\theta'') = 0$, $\tilde{A}_{i_2}(\theta'') = A$, and $\tilde{A}_j(\theta'') = \tilde{A}_j(\theta')$ for $j \neq i_1+1$ and i_2; I make the adjustment of $\hat{\pi}(s'|\theta') = 0$ and $\hat{\pi}(s'|\theta'') = \pi(s'|\theta') + \pi(s'|\theta'')$.

Next, conditional on states with B shocks and the distance-based structures in Lemma 8, I compare the efficiencies of lending to states with different shock distributions which statisfy the first point above. I show that it is most efficient to lend to states $\tilde{\Theta}^s$ that includes θ^B_{Conn} where shocks are on adjacent banks.

Given the distance-based strategy structure, for each state of interest, it is convenient to define a bank *sequence as* the set of banks in between two impaired banks, where the first impaired bank is included and the impaired bank in the downstream is not included. If only one bank is reported with $s_i = h$ in each sequence, one can use a simple replication argument to show that θ^B_{Conn} is prefered. Otherwise, the distance threshold $\underline{d}$ and the number of bank sequences $z(\theta)$ are sufficient statistics for index $\xi\left(s(\underline{d}), \theta; \overline{\theta}\right)$. I show that the index

$$\xi\left(s(\underline{d}), \theta; \overline{\theta}\right) \leq X \cdot \max\left\{\frac{|I_h(s)| + [z(\theta)-1](\underline{d}-1)}{y + (1-\delta)v - \delta\underline{d}(A-v)}, \frac{|I_h(s)| + z(\theta)(\underline{d}-1)}{v}\right\},$$

where X is a constant that captures the excess liquidity to borrow from the good state, and $\underline{d}-1$ is the upper bound of banks that turn out to be solvent but are reported $s_i = l$.

Then I show that conditional on the distance-based strategy in Lemma 8 and distance $\underline{d}$, the index is higher if liquidity is lent to state project shocks are on adjacent banks. Let

$$\theta^B_{Conn} : \tilde{A}_1 = \tilde{A}_2 = \cdots = \tilde{A}_B = 0, \ \tilde{A}_{B+1} = \tilde{A}_{B+2} = \cdots = \tilde{A}_n = A.$$

Depending on the asset realization of the critical bank $\hat{i}$ that is just over $\underline{d}$ distance from shock, I have

$$\xi\left(s(\underline{d}), \theta^B_{Conn}; \overline{\theta}\right) > \begin{cases} \frac{n-B-\underline{d}+1}{y+(1-\delta)v-\delta\underline{d}(A-v)} > \frac{|I_h(s)|+[z(\theta)-1](\underline{d}-1)}{y+(1-\delta)v-\delta\underline{d}(A-v)}, & \tilde{A}_{\hat{i}} > 0, \\ \frac{n-B+z(\theta)}{v} > \frac{|I_h(s)|+z(\theta)(\underline{d}-1)}{v}, & \tilde{A}_{\hat{i}} = 0. \end{cases}$$

□

Proof of Proposition 6

Proof. The proofs for the structure of the signal distribution are shown in Lemma 7, 8 and 9. This part examines the optimal choice of $\underline{d}$ and characterizes the collection of bad states $\tilde{\Theta}^*$ that borrow liquidity. For illustration purpose, I analyze the choice of $\underline{d}$ when $\underline{d} \geq 1$, and the discussion is similar for the $\underline{d} = 0$ case with a little difference in equation expositions.

From Lemma 8 and 9, I discuss the index $\xi\left(s(\underline{d}), \theta^B_{Conn}; \overline{\theta}\right)$, which represents letting each bank $i = B + \underline{d}, \cdots, n$ borrow from its own excess liquidity at $\overline{\theta}$ to itself at a collection of states $\tilde{\Theta}^{s(\underline{d})}$ that includes θ^B_{Conn} where $\tilde{A}_1(\theta) = \cdots = \tilde{A}_B(\theta) = 0$, $\tilde{A}_{B+1}(\theta) = \cdots = \tilde{A}_n(\theta) = 0$. Note that there are $n-1$ other states that are symmetric to θ^B_{Conn}, and it is without loss of generality to focus on θ^B_{Conn} to analyze the index.

To begin with, I solve for the optimal $\tilde{\Theta}^{s(\underline{d})}$ for a given signal $s(\underline{d})$. Because all obedience constraints are binding, I know for any bank $i = B + \underline{d} + 1, \cdots, n$ with $\tilde{A}_i\left(\theta^B_{Conn}\right) = A$ and are reported $s_i(\underline{d}) = h$, there exists some $\theta \neq \theta^B_{Conn}$ and $\theta \in \tilde{\Theta}^{s(\underline{d})}$ with $\tilde{A}_i(\theta) = 0$. Then for given $\left(s(\underline{d}), \theta^B_{Conn}\right)$, sufficient statistics for ξ are

$$B_h \equiv \sum_i \mathbf{1}_{\left\{\tilde{A}_i(\theta)=0, s_i=h\right\}} = \sum_{i>B+\underline{d}}^{n} \mathbf{1}_{\left\{\tilde{A}_i=0\right\}}.$$

By an argument similar to Lemma 9, I consider $\theta \in \tilde{\Theta}^{s(\underline{d})}$ with $\tilde{A}_1 = \cdots = \tilde{A}_{B-B_h} = 0$. For notational convenience, let $\theta^{(B,B_h)}$denote a typical state of $\tilde{\Theta}^{s(\underline{d})}$, and $\theta^B_{Conn} = \theta^{(B,0)}$. Depending on whether bank $B + \underline{d}$'s borrower bank defaults at $\theta^{(B,B_h)}$,

$$L_{B+\underline{d}}\left(s(\underline{d}), \theta^{(B,B_h>0)}\right) = \begin{cases} (\underline{d} + H_B)(A - v), & x_{B+\underline{d},B+\underline{d}-1} < y, \\ A + y, & x_{B+\underline{d},B+\underline{d}-1} = y. \end{cases}$$

In addition, as some of banks $B - B_h + 1, \cdots, B + \underline{d} - 1$ that are reported l but may turn out to be solvent, I have

$$w\left(s(\underline{d}), \theta^{(B,B_h>0)}\right) = \begin{cases} \left[B_h - \lfloor \frac{y+(1-\delta)v}{A-v} - (\underline{d}-1) \rfloor\right]^+ + |I_h(s)|, & 1 \leq B_h < B, \\ n, & B_h = B. \end{cases}$$

For other banks $i > B + \underline{d}$, I have $L_{i>B+\underline{d}}\left(s(\underline{d}), \theta^{(B,B_h)}\right) = A$ or 0. The obedience constraints of bank $B + \underline{d}$ and a typical bank $i > B + \underline{d}$ pinns down the cross-state distribution of $\pi(s(\underline{d})|\cdot)$ over $\tilde{\Theta}^{s(\underline{d})}$, where states $\theta^{(B,B_h>0)}$ are symmetric.

As a result, the index $\xi\left(s(\underline{d}), \theta^B_{Conn}; \overline{\theta}\right)$ has the following three possible representations:

$$\xi\left(s(\underline{d}), \theta^B_{Conn}; \overline{\theta}\right) \propto \begin{cases} \left\{\left[(1-\delta)v + y - \delta\left(\bar{d}-1\right)(A-v)\right] \frac{\frac{A}{|I_h(s)|-1}}{A - \frac{v}{\delta} + \frac{A}{|I_h(s)|-1}} - (\delta A - v)\right\}^{-1}, \\ \left\{\left[(1-\delta)v + y - \delta\left(\bar{d}-1\right)(A-v)\right] \frac{\frac{\delta B_h A}{|I_h(s)|-1}}{(1-\delta)v+y-\delta\left(\bar{d}-1\right)(A-v)+\frac{\delta b_h A}{|I_h(s)|-1}} - (\delta A - v)\right\}^{-1}, \\ \left\{ \begin{array}{l} \left[(1-\delta)v + y - \delta\left(\bar{d}-1\right)(A-v)\right] \dfrac{\frac{B_h A}{|I_h(s)|-1} + \frac{\Delta w\left(s(\underline{d}), \theta^{(B,B_h>0)}\right)}{|I_h(s)|}(\delta A - v)}{\frac{\delta B_h A}{|I_h(s)|-1} + \frac{\Delta w\left(s(\underline{d}), \theta^{(B,B_h>0)}\right)}{|I_h(s)|}\left[(1-\delta)v+y-\delta\left(\bar{d}-1\right)(A-v)\right]} \\ - (\delta A - v) \end{array} \right\}^{-1}. \end{cases}$$

From their monotonicity in B_h, I know[28]

$$B_h^* \in \left\{ \lceil \frac{(1+\delta)\, y + \delta v}{A - v} \rceil - \underline{d}, \lceil \frac{(1+\delta)\, y + \delta v}{A - v} \rceil - \underline{d} + 1, B \right\}.$$

Conditional on the choice of B_h, $\xi \left(s\left(\underline{d}\right), \theta^B_{Conn}; \overline{\theta} \right)$ decreases in y, so the choice of $\underline{d}$ decreases in y. □

B.11 Proof of Proposition 7

Proof. The key relies on the non-emptiness of good project return A (conditional on other parameters n, δ, p, v, y). I need A to be in an intermediate level: bank profitability is low and cross-state risk sharing is constrained, under which debt overhang of senior creditors is expensive; but A is not too low relative to interbank spillover, under which the benefit of cross-bank risk sharing in the complete structure is dominated.

First, I need $W_0^C \geq W_0^R$ in autarky. According to Acemoglu et al. (2015), conditional on only one shock, the complete structure is always weakly more stable when the ring structure. Conditional on multiple shocks, the result applies when y is relatively large. Hence, a sufficient condition for $W_0^C \geq W_0^R$ is

$$y > (n-2)(A - v).$$

Under this condition, the first part of Assumption 3 is satisfied, and (38) holds if $\overline{B}^{NCP} \geq 1$.

Second, I construct conditions under which $W^R\left(\boldsymbol{\pi}^{*R}\right) > W^C\left(\boldsymbol{\pi}^{*C}\right)$. I propose a feasible policy $\boldsymbol{\pi}^R$ in the ring network, and construct conditions under which the resulting system stability $W^R\left(\boldsymbol{\pi}^R\right)$ is higher than that under any feasible policy in the complete network, i.e., $W^R\left(\boldsymbol{\pi}^R\right) > W^C(\boldsymbol{\pi})$. I show that this essentially requires $A \geq \underline{A}(v, y, n, \delta)$.

To show this, I add conditions under which the comparison of $W^R\left(\boldsymbol{\pi}^R\right)$ and $W^C(\boldsymbol{\pi})$ is equivalent to the comparison between indices ξ. Essentially, this requires $\underline{p} \leq p \leq \overline{p}^C$, under which Proposition 10 applies and thus $W(\boldsymbol{\pi})$ is a linear function of the largest ξ. In the ring structure, I propose strategy $\left(s\left(\underline{d}, \lfloor B^{NCP} \rfloor + 1\right), \tilde{\Theta}\left(\theta_{Conn}^{\lfloor B^{NCP} \rfloor + 1}\right); \overline{\theta} \right)$ as in Proposition 6. The associated

$$\xi^R > P\left(\overline{\theta}\right) H^R\left(s\left(\underline{d}, \lfloor B^{NCP} \rfloor + 1\right)\right) \frac{\delta(A+y) - v - y}{-\Delta L^R_{\lfloor B^{NCP} \rfloor + \underline{d}}\left(s\left(\underline{d}, \lfloor B^{NCP} \rfloor + 1\right), \theta_{Conn}^{\lfloor B^{NCP} \rfloor + 1}\right)},$$

where the inequality is because the posterior expectation of $\Delta L_{\lfloor B^{NCP} \rfloor + \underline{d}}(s, \theta)$ over $\tilde{\Theta}\left(\theta_{Conn}^{\lfloor B^{NCP} \rfloor + 1}\right)$ must be larger, if all the obedience constraints of the passing banks bind. Then a sufficient condition

[28] The third equation is monotone in B_h, and the first equation does not depend on B_h, in which case I pick $B_h = \lceil \frac{(1+\delta)y + \delta v}{A - v} \rceil - \underline{d}$.

for $\xi^R > \xi^C\left(s\left(H\right), \theta^B; \overline{\theta}\right)$ is

$$\frac{H^R\left(s\left(\underline{d}, \lfloor B^{NCP}\rfloor + 1\right)\right)}{v + y - \delta\left(A + \left(\underline{d} - 1\right)\left(A - v\right)\right)} \geq \frac{H^C}{v + y - \delta\left(\frac{H^C - 1}{n-1} y + \frac{n-B}{H^C} A\right)}. \tag{59}$$

There exists some $\underline{A}\left(v, y, n, \delta\right)$ such that the above condition is satisfied when $A \geq \underline{A}\left(v, y, n, \delta\right)$.

In sum, I need $\underline{p} \leq p \leq \overline{p}^C$ and $y \geq \underline{y}\left(A, v, n, \delta\right)$ for scarce liquidity and simplified characterization of regulator payoff, $A \geq \underline{A}\left(v, y, n, \delta\right)$ for $W^R\left(\boldsymbol{\pi}^R\right) > W^C\left(\boldsymbol{\pi}\right)$, and the previous general parameter assumptions 2 and 3. Instead of explicitly characterizing the joint set of the conditions, I refer to the numerical example in the paper to argue the non-emptyness. □

B.12 Proof of Proposition 8

Proof. First, I introduce some notations to refer to a core bank and its connected periphery banks. Let i^c denote a typical core bank, and i^p a typical periphery bank. To illustrate connection, I use $c(i^p)$ to refer to the core bank that periphery bank i^p is connected to, and $p(i^c)$ refer to a typical periphery bank that is connected to core bank i^c.

The idea to find conditions under which whenever a signal s reports $s_{i^p} = h$ on a periphery bank, it also reports $s_{c(i^p)} = h$ on the connected core bank due to the strong complementarity. Combining with the core banks' better risk sharing with more counterparties, I show that the core banks receive prefered treatment.

I introduce ζ_i as the size-adjusted liquidity level, where the size refers to a bank's excess liquidity at $\overline{\theta}$

$$\zeta_i\left(s, \theta\right) \equiv \frac{\Delta L_i\left(s, \theta\right)}{\delta A_i - v_i - \left(1 - \delta\right) y_i}.$$

As banks share the same leverage (see Assumption 4), from the interbank payment rule, ζ across the connected banks satisfy

$$\zeta_i(s_l, \theta) = \frac{\delta \tilde{A}_i - v_i - y_i}{\delta A_i - v_i - \left(1 - \delta\right) y_i} + \sum_{j \neq i} \frac{y_{ij}}{\sum_{s \neq i} y_{is}} \left\{\min\left[\zeta_j(s_l, \theta), \frac{\delta y_i}{\delta A_i - v_i - \left(1 - \delta\right) y_i}\right]\right\}^+.$$

For simplicity, I use (s, θ) to refer to a risk sharing strategy where $\theta \in \tilde{\Theta}$. Without loss of generality, I can focus on strategies that satisfy the following: if $s_i = h$, then $s_j = h$ for all $j \neq i$ with $\zeta_j(s, \theta) \geq \zeta_i(s, \theta)$. Hence, I have the following scenarios:

1. If $\pi\left(s' \,|\theta'\right) > 0$, where $s'_{i^c} = h$ and $\tilde{A}_{p(i^c)}\left(\theta'\right) > 0$, I have $s'_{p(i^c)} = h$.
2. If $\pi\left(s' \,|\theta'\right) > 0$, where $s'_{i^p} = h$ and $\tilde{A}_{i^p}\left(\theta'\right) > 0$ $\tilde{A}_{c(i^p)}\left(\theta'\right) > 0$, I have $s'_{c(i^p)} = h$ under the following conditions.

On the one hand, there is strong externality of $c(i^p)$ on i^p: if $s'_{c(i^p)} = h$, I have

$$\zeta_{i^p} = \sup \zeta = 1 + \frac{y_i}{\delta A_i - v_i - \left(1 - \delta\right) y_i};$$

if instead $s'_{c(i^p)} = l$, I have

$$\zeta_{i^p} = \frac{\delta A_{i^p} - v_i - \delta y_i}{\delta A_{i^p} - v_i - (1-\delta)\, y_i} + \zeta_{c(i^p)} > \zeta_{c(i^p)}.$$

Hence, cross-bank risk sharing is more efficient under large y, there exists threshold $\hat{y}$ such that when $y > \hat{y}$, the regulator reports h on both $i^p, c(i^p)$.

3. If $\pi\,(s'\,|\theta')\, > 0$ and $\tilde{A}_{i^p}\,(\theta')\,\tilde{A}_{c(i^p)}\,(\theta') = 0$. A typical core bank is more efficient in risk sharing as it has more counter-parties and project shocks are i.i.d. across banks. Specifically,

$$\mathbb{P}\left(\zeta_{i^p}(s_l, \theta) \geq \frac{\delta A - v - \delta y}{\delta A - v - (1-\delta)\, y}\right) = p^2,$$
$$\mathbb{P}\left(\zeta_{i^c}(s_l, \theta) \geq \frac{\delta A - v - \delta y}{\delta A - v - (1-\delta)\, y}\right) = p[1-(1-p)^{t(i^c)}],$$

where $t(i^c) = \sum_{j \neq i^c} \mathbf{1}_{\{R_{i^c,j} > 0\}}$ is the number of bank i^c's counter-parties. Moreover, For any $\zeta' < \sup \zeta(s_l, \theta)$,

$$\mathbb{P}(\zeta_{i^p} \geq \zeta') \leq \mathbb{P}(\zeta_{c(i^p)} \geq \zeta' - 1).$$

Hence, when the marginal (s, θ) moves to $\zeta_i(s, \theta) = 1 - \frac{R}{A-D}$, I have $\mathbb{P}(s_{i^p} = h) \leq \mathbb{P}(s_{c(i^p)} = h)$. By the similar arguement of risk sharing with multiple counterparties, $\mathbb{P}(s_{c(j^p)} = h|\tilde{A}_{j^p} = 0) \geq \mathbb{P}(s_{j^p} = h|\tilde{A}_{j^p} = 0)$.

In sum, $\mathbb{P}(s_{i^p} = h) \leq \mathbb{P}(s_{c(i^p)} = h)$.

□