# Stablecoin Self-Regulation*

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**Abstract**

This paper analyzes the fragility of stablecoin issuers in an economy where they coexist with traditional financial institutions, such as banks. We fully characterize a self-enforcing mechanism aimed at improving stablecoin resiliency. Such mechanism relies on two essential components: (i) a voluntary loss mutualization fund, and (ii) costly participation to the fund in the form of one-period titles. We compare this mechanism with the regulatory proposals advanced by policy makers and academics alike, uncovering direct and indirect effects of stablecoin regulation on the fragility of traditional financial institutions.

# 1 Introduction

Stablecoins are cryptocurrencies that peg their value to a reference asset, typically a fiat currency such as the US dollar. The stabilization mechanisms supposed to maintain the peg, however, are all imperfect, as revealed by the collapse of various stablecoin initiatives over the past several years.[1] Indeed, stablecoin issuers have been likened to banks and money market funds for their susceptibility to runs and flight-to-safety dynamics.[2] Regulators' concerns for contagion to the traditional financial system have prompted several legislative initiatives calling for a unified regulatory framework for stablecoins.

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[1]For example, the collapse of Terraform Labs' Terra USD in May 2022, and that of IRON, on the Polygon and Binance blockchains, that occurred in June 2021.

[2]See Baughman *et al.* (2022), Anadu *et al.* (2023), Ma *et al.* (2023).

This paper formalizes the key frictions identified by economists and policy makers as responsible for the need of regulation, and shows that a voluntary loss mutualization fund with costly participation would avoid the need for regulation. Importantly, by modeling together both traditional financial institutions and stablecoin issuers, this paper is the first contribution to analyzing the impact of legislative initiatives to regulate stablecoin on the stability of the traditional financial system.

We provide a simple model of stablecoin issuance and redemption, where issuers are subject to limited commitment to redeem stablecoins, and would do so only if it is in their best interest. As in Kehoe and Levine (1993), this gives rise to an endogenous limit on stablecoin issuance, which can restrict stablecoins' ability to promote economic activity below its efficient level.

We propose a mechanism to improve the resiliency of stablecoin issuers by relaxing the limited commitment problem and the endogenous limit on issuance that ensues. Two elements are key for the effectiveness of such mechanism in reducing issuers' incentive to renege on their obligations. A loss mutualization fund, in which contributions are voluntary, paired with costly titles to membership of the fund for each obligation insured by the fund. Both elements act on issuers' incentives to honor their obligations by affecting the amount of the payment necessary to prevent a default and the values of paying and defaulting, through different channels. The voluntary nature of the contributions to the fund makes this mechanism particularly powerful, as it is self-enforcing, thus not relying on regulatory oversight.

The proposed mechanism is then used as a benchmark to assess the plans for stablecoin regulation advanced by policy makers and academics. These plans miss at least one of the two essential elements identified by our proposed mechanism to discipline default incentives. In addition, the plan of bringing stablecoin issuers under the same regulatory umbrella as banks, often recommended in the policy arena, results in banks substituting away from issuing insured deposits and into issuing uninsured deposits.

## 1.1 Background and contribution

Stablecoins have garnered much attention over the past several years, as they experienced extraordinary growth with market capitalization increasing from $5 billion in 2019 to about $130 billion in November 2023, roughy equivalent to the GDP of Slovakia. Stablecoins offer a potential way to pay for goods and services, and facilitate the trading of other volatile crypto assets by allowing market participants to avoid inefficiencies stemming from converting back to fiat currency for crypto trades.

Stablecoins essentially serve as both a means of payment and store of value for these transactions. To do so, stablecoins aim to maintain a stable value with respect to a reference asset, such as a fiat currency, using various stabilization mechanisms. The stabilization mechanism adopted by the largest stablecoin issuers relies on the issuer honoring its obligation to redeem on demand the stablecoins for the reference asset or for other assets that can be easily turned into the reference asset. For example, Circle states that "USDC is always redeemable 1:1 for US dollars".[3]

Unless the reference asset itself is held in custody against the issuance of stablecoins, the

[3]See Circle USDC Terms, Legal & Privacy.

issuer might be unable to honor the redemption promise. Indeed, issuers have an incentive to invest in possibly illiquid assets to earn high yields, thus compromising the reliability of their redemption obligation. Such incentive makes stablecoins issuers very similar to banks, which issue debt in the form of on-demand deposits. The analogy with banks has been long discussed by both policy makers and economists, with some advocating to regulate stablecoins issuers with banks. The report of the President's Working Group on Financial Markets (2021) recommended that "legislation should limit stablecoin issuance, and related activities of redemption and maintenance of reserve assets, to entities that are insured depository institutions". Jackson *et al.* (2022) similarly argue for the regulation of stablecoins as subsidiaries of insured depository institutions, though as separate legal entities.

The main ideas behind such proposals for regulation are (i) the absence of oversight for stablecoins, (ii) the effectiveness of current banking regulation in preventing widespread bank failures in almost a century. The susceptibility of banks to failing is often referred to as fragility, and it is intrinsic to the nature of their redemption promise: depositors are allowed to withdraw their funds on demand at par. Stablecoin issuers offer a similar promise (eg Circle) and are similarly fragile.

This paper rigorously analyzes these ideas and proposes a framework to formalize the key frictions identified by economists and policy makers as responsible for the need of regulation. In particular, the weakness of oversight for stablecoin issuers is captured in the model by a parameter indicating the issuer's opportunity to default on the redemption obligation without being detected. In the model, the only difference between banks and stablecoins issuers is the likelihood with which defaulting on their obligation will go undetected. Stablecoin issuers are more opaque than banks, as their disclosure requirements are non-existent and they are exempt from supervision.

In other respects, stablecoin issuers are similar to banks, who offer depositors the option to withdraw their funds on demand but operate with a fractional reserve model, thereby risking to be illiquid at a time when many depositors withdraw. Both stablecoin issuers and banks are subject to a limited commitment friction, in the sense that they will honor the redemption promise only if it is in their best interest to do so. This gives rise to an endogenous repayment constraint, as in Kehoe and Levine (1993), which caps the amount of stablecoins or deposits that can be issued in equilibrium, and captures the fragility of the issuer's promise. A tighter constraint restricts the ability of stablecoin issuers and banks alike to invest in productive opportunities, thus any arrangement that relaxes the constraint is welfare improving.

We propose an arrangement with two essential elements to discipline the severity of the limited commitment friction: first, a voluntary contribution to a loss mutualization fund, which serves as a guarantee of payment for stablecoin holders and banks' depositors, resulting in larger capacity to issue stablecoins and deposits. Hence, issuers have more skin in the game, as they stand to lose more were they to default and give up the franchise value of their firm. Second, costly membership titles to the loss mutualization fund, which are rights to have liabilities insured by the fund. An issuer (bank) wanting stablecoins (deposits) to be insured by the loss mutualization fund needs to pay a cost that entitles the creditors of those stablecoins (deposits) to be repayed by the fund. Costly membership titles also relax the repayment constraint. The intuition lays in the endogeneity of the repayment constraint and acts through multiple channels, described in detail in section **??**. Loosely speaking, we can interpret the first element of the proposed arrangement as a survivors' pay rule to allocate losses, and the second as initial margins, which are direct functions of the trading positions.

The voluntary nature of the proposed arrangement is crucial, as the contributions to the loss mutualization fund are subject to the same incentive constraint as repayments of obligations among individual agents. Hence, no assumption is made about a third party having better enforcement power than private agents. If agents have incentives to renege on their obligations to repay other agents, so they have incentives to renege on their obligations to pay any other party, including the loss mutualization fund. As in Kehoe and Levine (1993), the main economic mechanism responsible for agent's incentives to honor their promises works through agents' value for reputation: when detected, an issuer who defaulted will not receive any funds in the future.

Importantly, the voluntary nature of the proposed arrangements, together with the two key features outlined above, closely resembles the organization and governance of central counterparties (CCPs), which originated endogenously as financial market participants in the late 1700s attempted to reduce counterparty risk and to optimize the provision of services linked to various trading activities.[4] Moreover, the resiliency of CCPs in times of stress has been praised after the 2007-9 financial crisis and has been at the root of the policy proposals that followed it. Thus, in reality as in the paper, the proposed arrangement can be purely private and self-enforcing.

The analysis shows that such a voluntary arrangement would relax the repayment constraints of traditional financial institutions and stablecoin issuers, thus reducing the fragility inherent in the redemption promise. Lower fragility results in a larger set of economies where the equilibrium is efficient, and in higher welfare when the equilibrium is not efficient. In the paper, we refer to this arrangement as segregated regulation, as traditional financial institutions and stablecoin issuers participate to their own self-enforcing regulatory mechanism, respectively. When implementing our proposed arrangement by connecting stablecoin issuers to banks in some way, the latter effectively subsidize the former. Stablecoin issuers are relatively more risky as they can more often renege on their redemption obligations without being detected, and gain by being pooled with less risky traditional financial institutions. Banks' best response is to partially exit the loss mutualization fund and issue uninsured deposits. Thus, such implementation of stablecoin regulation would have a double edged impact on financial stability, improving it by reducing stablecoins' fragility on one hand, but also endangering it by increasing banks' fragility on the other hand.

Current plans for stablecoin regulation advanced in policy and legal circles lack the essential features of our proposed mechanism. In particular, these plans are silent about the establishment of a loss mutualization mechanism for stablecoins, while they clearly call for stablecoin issuers not to enjoy FDIC insurance. Moreover, they suggest linking stablecoin issuers to banks. Despite they do so mostly for oversight purposes, in an attempt to improve stablecoin issuers' transparency, any such link should be carefully designed due to the indirect effects it may have on the resiliency of the traditional financial system.

[4] See Kroszner (2006), Kroszner (1999), Bernanke (2011).

## 1.2 The current state of stablecoin regulation in the US

The US regulatory landscape for stablecoins is marked with uncertainty, particularly at the federal level. With no comprehensive, nationwide regulatory framework for stablecoins, a variety of regulatory frameworks have emerged at the state level.[5] Numerous states currently regulate virtual currency activity through their money transmission laws, though few offer specific guidance regarding stablecoins, with Texas being an exception.[6] Other States have options for companies to receive licenses for stablecoin activities as well.[7]

Furthermore, some federally insured banks have announced plans to issue stablecoins under the assumption that stablecoins are within the scope of products that such banks have the authority to issue.[8] Importantly, traditional bank protections, such as FDIC insurance, do not cleanly cover stablecoins.[9]

On November 1, 2021, the President's Working Group on Financial Markets, the FDIC and the OCC collectively issued a Report on Stablecoins, which did not contain any specific new rules or guidance but outlined recommendations with broad implications for existing stablecoin markets. The most significant and specific recommendation of the report was that Congress should enact legislation to "limit stablecoin issuance, and related activities of redemption and maintenance of reserve assets, to entities that are insured depository institutions", thus prohibiting other entities from issuing payment stablecoins and suggesting both (i) that issuing stablecoins is the kind of activity that can be fully performed by banks, and (ii) that stablecoins are deposits under the Federal Deposit Insurance Act and Section 21 of the Glass-Steagall Act.[10] Shortly afterwards, however, the OCC and the FDIC issued interpretive letters announcing that national and FDIC-supervised institutions must notify their supervirosy office of their intention to engage in crypto related activities, including the issuance of stablecoins and holding stablecoins reserves.

Between 2022 and 2023 several agencies issued reports and statements emphasizing the need of effective regulation of crypto assets, and calling for Congressional action to expand regulators' powers to prevent the misuse of customer assets, strengthen disclosure requirements and provide severe penalties for violation of illicit finance rules.[11]

[5] While federal agencies agree that stablecoins need regulatory oversight to minimize risk to the financial system, the CFTC has gone a step further, initiating enforcement actions against stablecoin issuers for violations of the Commodity Exchange Act. For example, the CFTC settled charges with the companies that created the stablecoin Tether for alleged misrepresentations regarding the reserves backing the stablecoin. See Landy *et al.* (2023).

[6] Texas has taken the position that stablecoins backed by a sovereign currency are regulated by its money transmission laws because they "may be considered a claim that can be converted into currency and thus fall within the definition of money or monetary value" under Texas law. See Landy *et al.* (2023).

[7] Such as Nebraska, where a digital asset depository is authorized to issue stablecoins and hold deposits at FDIC insured financial institutions with a main chartered office in the state. See Nebraska Revised Statute 8-3024.

[8] This authority was confirmed by the Office of the Comptroller of the Currency (OCC), then later partially walked back to require pre-authorization by banks before engaging in these activities. See Landy *et al.* (2023).

[9] Paxos makes clear that, while the primary deposit account that holds fiat cash reserves is FDIC insured, "USD Stablecoins themselves are not FDIC insured". See Landy *et al.* (2023).

[10] See Landy *et al.* (2023).

[11] Among these agencies there are the Federal Reserve, the FDIC, the OCC, the White House's National Economic Council.

The Clarity for Payment Stablecoins Act, introduced in the House of Representatives in July 2023, offers guidance for both Federal and State qualified issuers of payment stablecoins. It provides ruling on certification of issuers, examinations, supervision, restrictions on assets, commingling or segregation of assets. With respect to the links between stablecoin issuers and banks, the Clarity for Payment Stablecoin Act: (i) treats stablecoin issuers as subsidiaries of insured depository institutions; (ii) requires reserves backing the stablecoins to be, among other things, funds at insured depository institutions.

On August 8, 2023 the Federal Reserve issued two letters announcing a Novel Activities Supervision Program focusing on banks' participation in crypto related activities, including stablecoin/dollar token issuance and distribution. The new program would "ensure that the risks associated with innovation are appropriately addressed" and "enhance the supervision" of stablecoin activities conducted by banks supervised by the Federal Reserve.

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- this paper takes as given the existence of a gap in the opacity level between SC and banks

- The motivation for this is twofold: first, as of Oct 2023, SC activity has been increasingly occurring through entities not licensed in the US despite the US is still the largest crypto market –see Chainalysis report Oct 2023. Therefore, even if the degree of opacity currently enjoyed by SC issuers diminished, if oversight rules were deemed too stringent by SC issuers, they would still find ways to operate outside those boundaries, as they currently do. Hence, a voluntary regulatory scheme would be far more effective. Second, we know that oversight is not perfect. Even if we were to succeed in designing an architecture for the supervision of SC issuers, we know that such supervision is imperfect (recall SVB and the Barr investigation among others). Therefore, to the extent that incentives to renege on the redemption promise exist, a voluntary regulatory scheme would be far more effective in this case as well.[12]

- The mechanism proposed here does have an element of insurance, with the segregated insurance dominating the merged for traditional financial institutions. Hence, this paper contributes to this specific side of the debate on the regulation of SC by showing that (i) loss mutualization is an essential part of a mechanism designed to reduce the fragility of SC; (ii) merging loss mutualization mechanisms across intrinsically different institutions has redistributive effects, by increasing welfare of the riskier institution at the expense of the less risky one. Notice, however, that in this paper SC issuers are not

[12]Further to this argument, evidence that SC issuers have such strong incentives to renege on the redemption promise abounds and recently culminated with Circle, the SC issuer with the loosest restrictions on the set of customers allowed to trade directly on the primary market, has announced that it will only allow corporate customers on the primary market. While no reason for this decision has been officially provided, it isn't hard to argue that Circle is attempting to place restrictions on the redemption activity, similarly to Tether's business model, thus effectively allowing Circle to suspend convertibility of its SCs in times of stress. Such restrictions on the possibility to redeem a SC will likely affects the performance of the SC with respect to the reference asset, possibly breaking the peg more often than they currently do, as the underlying stabilization mechanism for fiat backed stablecoins relies on arbitrage with the issuer.

intrinsically undesirable. To the contrary, fostering their activity is welfare improving. Therefore, a question for extensions would be to analyze other dimensions along which SC are not analogous to banks.

# 2 Model

The model economy is a version of Lagos and Wright (2005), or Rocheteau and Wright (2005). Time is discrete and infinite, with each period divided into two subperiods in which trade occurs: a centralized market (CM) where all agents are in the same location, and a decentralized market (DM) where they meet and trade bilaterally. The economy is divided into two separate sectors, a crypto and a traditional sector. Each sector is populated by a continuum of infinitely lived agents with mass 2, half of whom are buyers, or issuers as they optimally issue some form of debt, with the other half being sellers, or holders as they optimally hold issuers' debt.

There are three types of consumption goods: a perishable consumption good in the CM, denoted $X_t$ and used as numeraire, and two perishable consumption goods in the DM, one in each sector, denoted $x_t^i$ with $i = c$ for the crypto sector and $i = t$ for the traditional sector.

All agents can consume and produce the CM good with a linear production technology using labor, $H_t$, while issuers in sector $i$ can consume the DM good $x_t^i$, which is produced with a linear production technology by holders in sector $i$ using labor, $h_t$. Issuers' preferences in sector $i$ are represented by

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t [X_t - H_t + u(x_t^i)]$$

where $\beta \in (0, 1)$ is the discount factor across periods and $u$ a strictly increasing and strictly concave utility function satisfying Inada conditions.[13] Holders' preferences are represented by

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t [X_t - h_t]$$

During the DM, each issuer is randomly matched with a holder in each sector. A fraction $\rho^i$ of DM meetings are limited information meetings, in which the holder does not have access to the issuer's history. However, the interaction between the issuer and the holder in the meeting is publicly recorded. The remaining fraction $1 - \rho^i$ of DM meetings are full-information meetings, in which the holder has access to the public record. Thus, information about all trades and all interactions in each DM meeting is perfectly recorded, but agents may not have access to it sometimes. For banks and stablecoin issuers, such records represent the outcome of audits, of disclosure statements and supervision if any. Importantly, we assume that $\rho^c > \rho^t$.[14]

[13] There is no discounting across subperiods.

[14] While we focus on transparency, reliability of attestations and audits, and, broadly speaking, supervision, this model is suitable to extensions allowing for differences between banks and stablecoin issuers along other dimensions, such as volatility of assets in which the issuer invests the reserves.

Another key credit friction, in addition to imperfect record keeping, is limited commitment (Kehoe and Levine (1993), Carapella and Williamson (2015)), in that economic agents in the model cannot be forced to work. Thus, a private debt will be repaid only if it is in the debtor's interest to do so. This is key in determining the voluntary nature of the regulatory schemes proposed.

In a DM meeting between an issuer and a holder, the issuer makes a take-it-or-leave-it (TIOLI) offer to the holder, which will depend on the information available to the holder, such as whether the meeting is full or limited information, and on what the issuer would lose by defaulting on obligations stemming from the contract.

In this economy, agents' heterogeneity introduces a simple motive for intertemporal exchange. Random matching in the DM permits the coexistence of credit based contracts with both poor information and with good information about an agent's history. Finally, the assumption of quasilinear preferences for issuers eliminates wealth effects, and makes the CM a period when debts are settled and the problem restarts. As a result, some elements of decision making have a two-period structure while maintaining an infinite horizon, critical for supporting the credit arrangements. Linear preferences for holders, combined with TIOLI offers by issuers in the DM imply that behaviour by sellers is trivial, simplifying our analysis by allowing us to focus on the behaviour of issuers.

# 3 Unregulated economy: punishments and equilibria

In models with limited commitment, the punishment for default is key in determining equilibrium outcomes, even if it never occurs in equilibrium, as agents have equilibrium beliefs about how punishment occurs off-equilibrium. Throughout the paper we focus on individual punishments, under which a default by one agent triggers retribution directed only against the individual defaulter, off-equilibrium. We also focus on symmetric equilibria, in that each issuer and each holder choose the same strategy.

This section analyzes equilibria in an unregulated economy for each sector. Issuers and holders are physically together in the CM, where debt contracts agreed upon in the previous period, $l_{t-1}$, are settled: bank depositors receive (or withdraw) their funds and stablecoin holders redeem their tokens with the issuers.[15] In the DM issuers and holders meet bilaterally at random. Meetings are either full or limited information, and debt issuance is subject to limited commitment. Debt contracts arise as equilibrium outcomes in this environment given the mismatch between the timing of consumption and production across agents.

Let $v_t^i$ denote the value of an issuer in sector $i$ at the end of the CM at time $t$, and $\hat{v}_t^i$ the value of an issuer who defaults on his/her obligations. As a visual aid, Figure 1 shows the sequence of activities during a period in the model. Notice that in symmetric equilibria the consumption allocation in sector $i$ at time $t$, $x_t^i$, is the same regardless of whether the DM meeting is full or limited information, as all debt issuers in a sector behave the same.[16]

[15] Deposit withdrawals are simplified as the goal of the paper is to study debtors' incentives to repay rather than run dynamics. We assume that banks either pay or do not pay their depositors.

[16] Hence, there is no adverse selection in limited information meetings, because all issuers will repay on equilibrium path. In asymmetric equilibria, instead, some issuers default on path, thus introducing adverse selection in limited information meetings, and, with it, the possibility that pooling or separating contracts

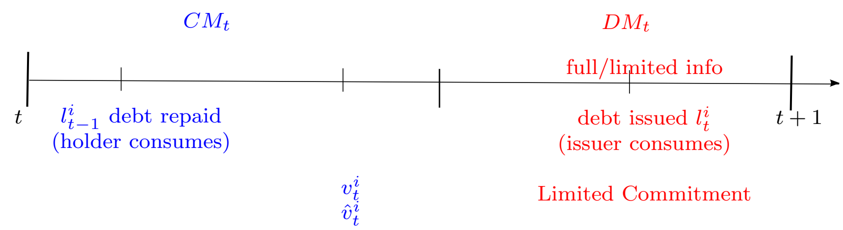


Figure 1: Timing of activities during a period.

Hence, $v^i_t$ is

$$
\begin{array}{rll}
v^i_t & = \max_{\{x^i_t, l^i_t, H_{t+1}\}} & u(x^i_t) - \beta H_{t+1} + \beta v^i_{t+1} \\
& s.t. & x^i_t \leq l^i_t \\
& & l^i_t \leq \beta H_{t+1} \\
& & \beta H_{t+1} \leq \beta(v^i_{t+1} - \hat{v}^i_{t+1})
\end{array}
$$

where the first constraint is a resource constraint within the DM meeting, as the issuer's consumption, $x^i_t$, can't exceed the holder's production, $l^i_t$, to be interpreted as the funds transferred by the holder/depositor to the issuer/bank. The second constraint is the holder's participation constraint, stating that DM production can't exceed the present value of consumption in the next CM, which, in turn equals the labor effort of the issuer due to the linear production technology in the CM. The third constraint is the issuer's incentive constraint, stating that the present value of his labor effort can't exceed the present value of the value of being an issuer net of the value of being a defaulter. Due to our assumptions of TIOLI offer by the issuer, of strictly increasing $u$, and focusing on stationary equilibria, we can simplify the issuer's decision problem as

$$
\begin{array}{rll}
v^i & = \max_{\{x^i\}} & u(x^i) - x^i + \beta v^i \\
& s.t. & x^i \leq \beta(v^i - \hat{v}^i)
\end{array}
$$

If an issuer chooses to default, then off-equilibrium he will not receive a deposit in meeting a holder in a full information meeting in the DM. If an issuer who has defaulted were in a limited information meeting in the DM during any period after default occurs, the holder does not know that this individual buyer defaulted. Hence, the holder will receive a TIOLI offer from the issuer of consumption for the issuer in the DM, $x^i_t$, in return for a payment $\frac{x^i_t}{\beta}$ in the next CM. The holder is just indifferent to accepting this offer, knowing that the offer is optimal for the issuer among offers on which he has not incentive to default. We

might be optimal.

assume that the holder believes that any offer other than $(x_t^i, \frac{x_t^i}{\beta})$ comes from an issuer who has defaulted in the past. Thus, off-equilibrium, any issuer who has defaulted will make the offer $(x_t^i, \frac{x_t^i}{\beta})$ to get the same allocation as other issuers receive in equilibrium. Thus, the continuation utility of default is

$$\hat{v}_t^i = \frac{\rho^i u(x_t^i)}{1-\beta}$$

### 3.1 Unregulated economy: equilibrium with IC slack

Let $x^*$ denote the efficient allocation in the DM, satisfying $u'(x^*) = 1$.[17]

Notice that $v^i = \frac{u(x^i)-x^i}{1-\beta}$ and $\hat{v}^i = \frac{\rho^i u(x_t^i)}{1-\beta} > 0$. Thus, the incentive constraint is $x^i \leq (1-\rho^i)\,\beta u\,(x^i)$. There exists an equilibrium with $x^*$ satisfying the incentive constraint if and only if $x^* \leq (1-\rho^i)\,\beta u\,(x^*)$. The higher $\rho^i$, the smaller the set of economies, indexed by primitives, where the efficient allocation is part of an equilibrium.

### 3.2 Unregulated economy: equilibrium with IC binding

There exists an equilibrium with the incentive constraint binding if and only if $x^* > (1-\rho^i)\,\beta u\,(x^*)$, and if there exists $x_E^i \in (0, x^*)$ such that $(1-\rho^i)\,\beta u\,(x_E^i) = x_E^i$.

Notice that an issuer's consumption is decreasing is $\rho^i$. Hence, the more severe the limited commitment friction, represented by the probability of default going undetected, the fewer resources an issuer can get for issuing a unit of debt. In other words, the fragility of an issuer is represented by the severity of his commitment problem, and debt holders are reluctant to fund fragile issuers.

## 4 Segregated self-regulation

Key elements of the analysis are the study of issuers' incentives to default on their obligations, and of how such incentives affect the liquidity of the liability issued, be it stablecoins or deposits. In this section we propose and characterise a mechanism that relaxes the severity of such incentive problem along multiple dimensions. An important feature of this mechanism is its self-enforcing nature, as any payments by issuers to the mechanism will be subject to the same incentive constraint to which issuers' obligations are subject.

As a benchmark we consider a mechanism for each type of issuer, and refer to it as segregated self-regulation. Intuitively, this modeling choice does not preclude any connection that might exist in reality between the crypto and the traditional financial sector, rather it should be interpreted as limiting the connections between policies adopted for the crypto and for the traditional financial sector, as will be clear in the next section.[18]

The key elements of the proposed mechanism are (i) a loss mutualization fund in each sector $i \in \{c, t\}$, with lump-sum contributions by each issuer in the CM, denoted $\tau^i$; (ii) costly

[17]With linear disutility of DM labor for the holder, this condition characterizes the first best level of DM exchange between an issuer and a holder.

[18]This includes the understanding that the design of such policies might affect and should account for common markets across the crypto and for the traditional financial sector.

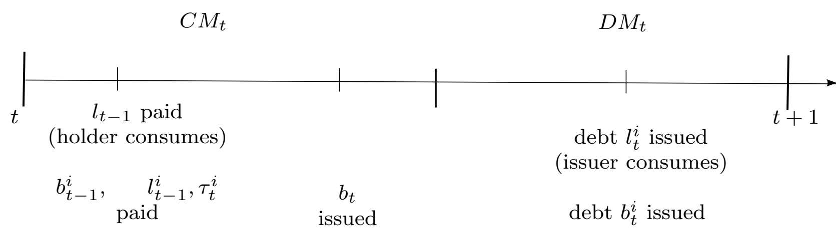


Figure 2: Elements of regulation and timing of activities during a period.

titles to membership of the fund, denoted $b^i$, purchased in the CM in the same amount as the obligations chosen by issuers to be insured by the fund. In section 9 we show that both elements are essential.

The contributions to the loss mutualization fund are voluntary: issuers can default on the payment of contributions to the mechanisms as well as on their obligations to other agents. Both defaults will be publicly recorded.

The membership titles are one-period lived. Hence, it is not feasible for an issuer to sell unused membership titles to other issuers, nor to store unused membership titles to the next CM and use them next period.[19] If an issuer desires to insure its liabilities in the amount $b^i$, then that issuer will need to purchase membership titles in the same amount, as each membership title grants the issuer the right to have the mechanism cover one unit of his liabilities. Hence, $b^i$ denotes the units of CM good paid by the mechanism to creditors of issuer $i$, as well as the amount of membership titles demanded by issuer $i$. Membership titles are issued by the mechanism in the CM and sold on a Walrasian market in each sector $i$ at price $q_t^i$. The revenue from the sales of membership titles is thus available in the CM to cover insured liabilities of issuers of type $i$. Letting $B_t^i$ denote the supply of membership titles, the resource constraint for the mechanism is

$$\tau_t^i + q_t^i B_t^i \quad \geq \quad B_{t-1}^i$$

which, in a stationary equilibrium, is simply $\tau^i = B^i(1 - q^i)$. As a visual aid, Figure 2 shows the sequence of activities during a period, as well as payments into and out of the mechanism.

[19]This design feature allows to discipline default incentives further than by allowing trading or storage of unused membership titles. If fact, an issuer's value of defaulting, $\hat{v}^i$, would need to account for the revenue from selling the membership titles, or for the gain in saving on the labor effort otherwise necessary to pay for the membership titles next period.

The value of an issuer of type $i$ at the end of the CM is

$$\begin{aligned} v^i &= \max\left\{-q^i b^i + u\left(x^i\right) - l^i - \beta\tau^i + \beta v^i\right\} \\ & \quad x^i \leq l^i + \beta b^i & (1) \\ & \quad l^i + \beta\tau^i \leq \beta\left(v^i - \hat{v}^i\right) & (2) \end{aligned}$$

An issuer's consumption $x^i$ can be funded by insured liabilities, $b^i$, and uninsured liabilities, $l^i$. While the former will be covered by the mechanism, the latter remain the issuer's obligations.[20] As in the previous section, the issuer makes a TIOLI offer to the holder, resulting in the holder's participation constraint (1) binding: the holder requires a payment $\frac{l^i}{\beta}$ to hold an issuer's uninsured liabilities $l^i$, but will be paid by the mechanism when holding an issuer's insured liabilities $b^i$.[21] Since the mechanism pays one-to-one for each title $b^i$ issued, the DM present value of the holder's repayments in the next CM is $l^i + \beta b^i$.

Constraint (2) is the repayment constraint for the issuer, also referred to as the incentive constraint: the DM present value of the payments he will have to make in the next CM is $l^i + \beta\tau^i$. The issuer has an incentive to pay his obligations towards the holder and towards the mechanism if and only if his payoff from doing so exceeds that from defaulting: $-(l^i + \beta\tau^i) + \beta v^i \geq \hat{v}^i$.

The issuer's objecting function shows that, when looking to insure $b^i$ of his liabilities $x^i$, the issuer will need to purchase titles in the same amount at price $q^i$ in the CM. Then the issuer enjoys utility from consuming $x^i$ in the DM (also interpreted as the payoff from investing the funds received by issuing liabilities $x^i$), and the has to repay his obligations in the next CM. The present value of those obligations is the sum of $l^i$, stemming from uninsured liabilities, and $\beta\tau^i$, stemming from contributions to the mutualization fund.

A key difference between the equilibrium in the economy with a mechanism and that in an unregulated economy, is that now $\hat{v}^i = 0$ is possible. A defaulting issuer will have to make the same offer as a repaying issuer in limited information meetings, otherwise he will be detected and not receive $x^i$ from the holder. To imitate a repaying issuer, a defaulting one will have to purchase the same amount of membership titles. Hence, the value of a defaulting issuer is

$$\hat{v}^i = \frac{-q^i b^i + \rho^i u(x^i)}{1-\beta} \tag{3}$$

with $\hat{v}^i = 0$ if and only if $-q^i b^i + \rho^i u(x^i) \leq 0$.

The introduction of costly membership titles has multiple effects on the incentive constraint (2), with one of them going through the value of default. With respect to an unregulated economy, the fact that defaulters have to pay a cost in order to disguise themselves as repaying issuers off-path, can push $\hat{v}^i$ to its lowest feasible value. Indeed, it is always possible for a defaulting issuer not to trade in the DM, thus securing $\hat{v}^i = 0$. Therefore, the

[20] To help interpretation, consider the clearing of over-the-counter (OTC) trades that are eligible for central clearing but not required to be such. Then, $l^i$ can be thought of as the volume of trade that a financial market participant choose to clear bilaterally, while $b^i$ the trades that he submits for central clearing, as they are a liability of the central counterparty.

[21] To fix ideas, consider a stablecoin issuer: $l^s$ denotes the uninsured stablecoins purchased by a holder, while $b^s$ denotes the insured stablecoins puchased by a holder.

possibility that $\hat{v}^i = 0$ in equilibrium relaxes the incentive constraint and improves welfare, as our characterization of equilibria below will show.

## 4.1 IC slack

First, consider the case where the repayment constrain does not bind. Then, in equilibrium, $x^i = x^* = l^i + \beta b^i$.The first order condition with respect to $b^i$ implies $q^i = \beta$, as shown in Appendix A. With respect to an unregulated economy we shall now consider equilibria both with $\hat{v}^i > 0$ and with $\hat{v}^i = 0$.

**Proposition 1** *There exists an equilibrium with $x^*$ and $\hat{v} > 0$ if and only if*

$$\begin{aligned} \beta B^i &< \rho^i u(x^*) && (4) \\ x^* - \beta^2 B^i &< \beta\left(1-\rho^i\right) u(x^*) && (5) \\ \beta B^i &\leq x^* && (6) \end{aligned}$$

*If $u(x) = \frac{x^{1-\sigma}}{1-\sigma}$ then these are*

$$\begin{aligned} \beta B^i &< \frac{\rho^i}{1-\sigma} \\ \frac{1}{\beta} - \frac{(1-\rho^i)}{1-\sigma} &< \beta B^i \\ \beta B^i &\leq 1 \end{aligned}$$

*There exists an equilibrium with $x^*$ and $\hat{v} = 0$ if and only if $\beta B^i \leq x^*$ and*

$$\begin{aligned} x^* &< \beta u(x^*) && (7) \\ \rho^i u(x^*) &\leq \beta B^i && (8) \end{aligned}$$

*If $u(x) = \frac{x^{1-\sigma}}{1-\sigma}$ then these are $\beta B^i \leq 1$ and*

$$\frac{\rho^i}{\beta B^i} \leq 1 - \sigma \quad < \quad \beta$$

*for which it is necessary that $\rho^i < \beta^2 B^i$.*

**Proof.** See Appendix A. ■

Proposition 1 shows that the set of economies where the incentive constraint is slack, and the consumption allocation is efficient, is larger with segregated regulation than without regulation. To see this, consider the case where $\hat{v}^i > 0$ first. For any $B^i > 0$ inequality (5) describes a larger set of economies, indexed by primitives, than the necessary and sufficient condition $x^* \leq (1-\rho^i)\beta u(x^*)$ derived in section 3.1. The same argument applies for the case where $\hat{v}^i = 0$, by comparing inequality (7) with the necessary and sufficient condition $x^* \leq (1-\rho^i)\beta u(x^*)$ derived in section 3.1.

Moreover, the mechanism can choose the supply of membership titles, $B^i$, to maximize welfare. Membership titles are costly to purchase in the CM, and their cost needs to be sufficiently large for $\hat{v}^i = 0$ in equilibrium. However, since the incentive constraint is slack in the equilibria we are constructing in this section, the mechanism will implement the efficient allocation with $\hat{v}^i > 0$, if feasible. To do so, the mechanism will choose the smallest $B^i$ to satisfy (4)-(6), as long as such $B^i > 0$ exists given $(\beta, \rho^i, \sigma)$ and $x^*$. If, instead, $x^*$ is too large for any $B^i > 0$ to satisfy (4)-(6) given $(\beta, \rho^i, \sigma)$, then the mechanism will choose the smallest $B^i$ to satisfy (8), thus implementing the efficient allocation $x^*$ together with $\hat{v}^i = 0$.

Equilibria with $\hat{v}^i = 0$ are generally characterized by a less tight incentive constraint, as the value of a defaulting issuer is pushed to its lower bound.[22] As a result, equilibria with $\hat{v}^i = 0$ can support higher DM consumption. When DM consumption is already at its first best level, $x^*$, however, the mechanism will not find it optimal to further relax the incentive constraint if it is costly to do so – as membership titles are costly to purchase.

## 4.2 IC binds

Consider now equilibria where the incentive constraint binds, and focus first on the case where $\hat{v}^i = 0$. The following proposition fully characterizes such equilibrium.

**Proposition 2** *Assume $u(x) = \frac{x^{1-\sigma}}{1-\sigma}$. There exists a unique equilibrium with (i) $x_E = l^i + \beta B^i < x^*$ solving $x_E = \beta u(x_E)$, (ii) $v^i = -q^i b^i + u(x^i)$, and (iii) $\hat{v}^i = 0$, if and only if $1 > \sigma + \beta$ and $B^i > \underline{B} = \rho^i \left( \frac{\beta(1-\sigma)}{(1-\sigma)^{1+\sigma}} \right)^{\frac{1}{\sigma}}$.*

**Proof.** See Appendix A. ■

Notice that the mechanism could choose the optimal quantity of membership titles. Because $x_E$ is independent of $B^i$ but $v^i$ decreases in $B^i$ as $q^i > 0$, then the mechanism maximizes welfare by choosing the smallest $B^i$ consistent with this equilibrium. The mechanism will choose $B^i$ to satisfy the necessary and sufficient condition for $\hat{v} = 0$, that is $\rho^i U(x^i) = qB^i = \beta u'(x^i) B^i$. Thus, at the optimal mechanism $v^i = (1 - \rho^i) u(x^i)$. Effectively, this means that the punishment for default is worst when good buyers fully pay for the rents that defaulters would accrue due to the information friction in the environment.

Consider now equilibria where $\hat{v}^i > 0$, characterized in the following proposition.

**Proposition 3** *Assume $u(x) = \frac{x^{1-\sigma}}{1-\sigma}$. There exists a unique $x_E^i = l^i + \beta B^i < x^*$ solving*

$$x_E^i - u'(x_E^i) \beta^2 B^i = \beta (1 - \rho^i) u(x_E^i) \tag{9}$$

*with $v^i = \frac{-q^i B^i}{1-\beta} + \left(1 + \frac{\beta \rho^i}{1-\beta}\right) u(x^i)$ and $\hat{v}^i > 0$, if and only if*

$$\frac{1}{\beta} - (1 - \rho^i) \frac{1}{1 - \sigma} > \beta B^i > 0 \tag{10}$$

*and*

$$(\beta B^i)^\sigma < \beta (1 - \rho^i) (\rho^i)^\sigma (1 - \sigma)^{-(1+\sigma)} + \beta \left( \frac{\rho^i}{(1 - \sigma)} \right)^{(1+\sigma)} \tag{11}$$

[22] It is always possible for a defaulting issuer to not trade in the DM, thus securing a payoff of zero. Indeed, when not trading in the DM, a defaulting issuer would not need to imitate an issuer who honors his obligations.

**Proof.** See Appendix A. ■

As in the previous case, the mechanism can choose the quantity of membership titles to maximize welfare. If $\hat{v}^i > 0$, however, $x_E^i$ depends on $B$, so $\frac{\partial x_E^i}{\partial B}$ is obtained from differentiating (21). Without risking confusion, let us momentarily drop superscript $i$ to ease notation:

$$\begin{aligned}(1+\sigma)(x)^{-\sigma}\frac{\partial x}{\partial B}\beta^2 B-(x)^{-(1+\sigma)}\beta^2 &= -\sigma\frac{\beta\left(1-\rho^i\right)}{1-\sigma}(x)^{-\sigma-1}\frac{\partial x}{\partial B}\\ \frac{\partial x}{\partial B}\left[\sigma\frac{\beta\left(1-\rho^i\right)}{1-\sigma}(x)^{-\sigma-1}+(1+\sigma)(x)^{-\sigma}\beta^2 B\right] &= (x)^{-(1+\sigma)}\beta^2\end{aligned}$$

from which, with $x > 0$, it follows that $\frac{\partial x}{\partial B} > 0$. The effect on $v$, however, depends also on the price of $B$, which in equilibrium is $\beta u'(x)$:

$$(1-\beta)v = -\beta B u'(x) + u(x)\left[1-\beta(1-\rho)\right]$$

If $u(x) = \frac{x^{1-\sigma}}{1-\sigma}$ this is

$$(1-\beta)v = -\beta B x^{-\sigma} + \frac{x^{1-\sigma}}{1-\sigma}\left[1-\beta(1-\rho)\right]$$

Hence, $\frac{\partial v}{\partial B}$ can be characterized from:

$$(1-\beta)\frac{\partial v}{\partial B^i} = x^{-\sigma}\left\{-\beta+\frac{\partial x}{\partial B}\left(\sigma\beta B x^{-1}+\left[1-\beta(1-\rho)\right]\right)\right\}$$

then $\frac{\partial v}{\partial B} > 0$ if and only if

$$\frac{\partial x}{\partial B}\left(\sigma\beta B x^{-1}+\left[1-\beta(1-\rho)\right]\right) > \beta \tag{12}$$

which can be rearranged as

$$\frac{1}{\beta}-(1-\rho) > \frac{\sigma\left(1-\rho^i\right)}{\beta(1-\sigma)}+\left[(1+\sigma)x-\frac{\sigma}{x}\right]B \tag{13}$$

Notice that the right hand side of the last inequality in increasing in $x$. Therefore, if the inequality is weakly satisfied at $x^*$ then it is also satisfied at $x_E$. At $x^*$ , the last inequality is

$$\frac{1}{\beta}-\left(1-\rho^i\right) \geq \frac{\sigma\left(1-\rho^i\right)}{\beta(1-\sigma)}+B \tag{14}$$

Thus, if (14) is satisfied, then the optimal choice of $B$ by the mechanism is the largest value of $B$ consistent with the necessary and sufficient conditions for this equilibrium to exist. Such conditions are (10) and (11). Now notice that if (14) is satisfied then so is (10). Therefore, if (14) then the mechanism chooses the largest value of $B$ so that (11) is satisfied.

Notice that (14) is a sufficient condition for $\frac{\partial v}{\partial B} > 0$. Therefore, if (14) is violated, it is still possible that $\frac{\partial v}{\partial B} > 0$ as (13) might be satisfied.

### 4.2.1 Comparison across equilibria with a binding incentive constraint.

In this section we deepen our analysis of the equilibria with a binding incentive constraint. With the optimal choice of $B$ by the mechanism possibly affecting welfare, it is reasonable to compare the consumption allocation and issuers' values across equilibria.

Consider the consumption allocation in the equilibrium with $\hat{v} = 0$ and the equation defining the consumption allocation in the equilibrium with $\hat{v} > 0$, (21). As shown in Appendix A, the left hand side of (21) is monotonically increasing in consumption, while the right hand side is monotonically decreasing in consumption. Therefore, if the equilibrium consumption with $\hat{v} = 0$ is such that the left hand side of (21) exceeds the right hand side, then such consumption allocation is larger than in the equilibrium with $\hat{v} > 0$, which is the case if and only if:

$$\frac{1}{(1-\sigma)^{\sigma}} \frac{\rho^i}{(1-\sigma)} \left( \frac{\beta}{(1-\sigma)} \right)^{\frac{1}{\sigma}} \quad > \quad \beta B^{i(>0)} \tag{15}$$

Proposition 3 shows that necessary and sufficient conditions for an equilibrium with $\hat{v}^i > 0$ are (10) and (11), which we can rearrange as

$$\beta B^{i(>0)} \quad < \quad \left( \frac{\beta}{(1-\sigma)} \right)^{\frac{1}{\sigma}} \frac{\rho^i}{(1-\sigma)}$$

Because $\frac{1}{(1-\sigma)^{\sigma}} > 1$ under our assumption that $\sigma < 1$, then if (11) is satisfied also (15) is satisfied and the consumption allocation in the equilibrium with $\hat{v} = 0$ is larger than that with $\hat{v} > 0$.

Intuitively, with a harsher punishment for default when $\hat{v}^i = 0$, the incentive constraint is less tight, thus supporting larger consumption by the issuer. The larger consumption allocation, however, comes at a cost for issuers, who have to pay a sufficiently high price to purchase membership titles to push the continuation utility of default to zero. Therefore, whether the mechanism would choose to issue the right amount of membership titles to implement the equilibrium with $\hat{v} = 0$ or with $\hat{v} > 0$ isn't as straightforward as determining the relative size of the consumption allocation, as the general equilibrium effect of a higher price of membership titles needs to be taken into account.

## 4.3 Discussion

The economic mechanism by which the proposed self-regulation alleviates the incentive problem works through various channels. First, the loss mutualization fund provides skin in the game for issuers, as it acts as a guarantee of payment for holders. Holders are more willing to acquire deposits in sector $i = t$ and stablecoins in sector $i = c$, which results in issuers' increased ability to issue debt. Because a defaulting issuer only benefits from increased debt issuance when undetected, the value of a repaying issuer, $v^i$, increases more than the value of a defaulting issuer, $\hat{v}^i$. Thus, the incentive constraint is relaxed.

Second, the membership titles to the loss mutualization fund act on the incentive constraint through multiple channels. First, the revenue from the sales of membership titles can be used to pay for the obligations of defaulting issuers. Thus, the cost of such default is spread also among defaulters themselves rather than only among repaying issuers through their contribution to the loss mutualization fund, $\tau^i$. As a consequence, a lower contribution to the fund is necessary to cover insured debt, relaxing the incentive constraint. Moreover, the cost of membership titles on the continuation value of defaulting exceeds that on the value of repaying. A defaulting issuer needs to purchase membership titles and mimic the behavior of a repaying issuer in order not to be detected in limited information meetings. In full information meetings, however, a defaulting issuer is always identified as such, and won't be able to issue debt. On the contrary, a repaying issuer is able to issue SC in every state of the world. Therefore, a defaulter pays the membership cost upfront but gets to enjoy its benefits less often than a repaying issuer. Thus, the incentive constraint is relaxed.

# 5 Merged self-regulation

In this section we characterize the set of equilibria and their properties when the mechanism for reducing the severity of the commitment friction has common elements across heterogeneous issuers, such as banks and stablecoin issuers. We refer to such mechanism as merged self-regulation, and interpret it as a formalization of the proposals for regulating stablecoin issuers that have been advanced in policy circles.[23]

With respect to the mechanism described in section 4, merged self-regulation connects banks and stablecoin issuers by a common loss mutualization fund, or membership titles, or both. Here, we consider both elements being common: all issuers are taxed the same lump-sum contribution $\tau$, if they are willing to pay, and face the same price $q$ to purchase membership titles on the Walrasian market in the CM. Thus, the decision problem of issuer $i = c, t$ is

$$\begin{aligned} v^i &= \max\left\{-qb^i + u\left(x^i\right) - l^i - \beta\tau + \beta v^i\right\} \\ & \quad x^i \leq l^i + \beta b^i \\ & \quad l^i + \beta\tau \leq \beta\left(v^i - \hat{v}^i\right) \end{aligned}$$

The mechanism resource constraint accounts for the population of issuers to insure in both sectors, with each type $i = c, t$ continuously distributed over $[0, 1]$:

$$2\tau = B(1-q)$$

where $B$ still denotes the total supply of membership titles by the mechanism. The market clearing condition for membership titles is

$$B = b^c + b^t.$$

## 5.1 Solution case both IC slack

**Case $\hat{v}^i = \hat{v}^j = 0$.**

[23]Specifically, one can interpret such merged regulation as a the principle of *same activity, same risk, same regulation*, as highlighted by the Financial Stability Board for stablecoins in its July 2023 report.

A necessary condition for the existence of an equilibrium with $\hat{v}^i = \hat{v}^j = 0$ is $b^i > 0$, $b^j > 0$, as $\hat{v} = \frac{-qb+\rho u(x)}{1-\beta}$. Hence, in this section we construct equilibria with $\hat{v}^i = \hat{v}^j = 0$ and $b^i > 0$,$b^j > 0$.

**Proposition 4** *There exist equilibria with incentive constraints slack both in the crypto and traditional sector if and only if for all i:*

$$\begin{aligned} \rho^i U(x^*) &\leq \beta b^i \\ U(x^*) &\geq \frac{x^*}{\beta} + \left(\frac{B}{2} - b^i\right)(1-\beta) \end{aligned}$$

*One equilibrium allocation is $x^*, b^i = b^j = \frac{B}{2}, l^i = l^j \geq 0$, in which case the mechanism must set $\frac{B}{2} \geq \rho^c U(x^*)$. For this equilibrium it is necessary that $\beta \geq \rho^c > \rho^t$.*

*If $u(c) = \frac{x^{1-\sigma}}{1-\sigma}$ then these conditions are $\frac{\rho^c}{1-\sigma} \leq \beta \frac{B}{2} \leq 1 \leq \frac{\beta}{1-\sigma}$.*

**Proof.** See Appendix B. ■

It is worth pointing out, however, that also other portfolio allocations with $b^i \neq b^j$ and $l^i \neq l^j$ can be part of an equilibrium but are not all payoff equivalent due to the cost to acquire membership titles, which reduces the value of a buyer. The highest value allocation consistent with this equilibrium would be one where

$$\begin{aligned} b^c &= \rho^c U(x^*) \\ b^t &= \rho^t U(x^*) \\ B &= b^c + b^t \\ l^c &= x^* - \beta b^c \\ l^t &= x^* - \beta b^t \end{aligned}$$

Therefore, with $\rho^c > \rho^t$, it follows that $b^c > b^t$ and $l^c < l^t$ since buyers in both sectors consume the same allocation.

**Case $\hat{v}^i > 0$ and $\hat{v}^j > 0$.**

In this section we focus on the equilibrium with $b^i > 0, b^j > 0$. If $b^i = 0$ ($b^j = 0$), the equilibrium allocation is equivalent to that with segregated regulation for type $i$ ($j$) and no regulation for type $j$ ($i$).

**Proposition 5** *Assume $u(c) = \frac{x^{1-\sigma}}{1-\sigma}$. There exist equilibria with both incentive constraints slack and $\hat{v}^i > 0, \hat{v}^j > 0$ if and only if for $k = i, j$*

$$\begin{aligned} x^* - \beta b^k + \beta \frac{B}{2}(1-\beta) &\leq \beta\left(1-\rho^k\right) U(x^*) \\ \rho^k U(x^*) &> \beta b^k \end{aligned}$$

**Proof.** See Appendix B. ■

Notice that, as in the case where $\hat{v}^i = \hat{v}^j = 0$, several portfolio allocations are part of this equilibrium as long as these inequalities are satisfied for all $i$. However, they are not all payoff equivalent because the membership titles are costly to purchase. The highest value portfolio allocation, and thus the one that the mechanism would choose $B$ to implement, is such that

$$\begin{aligned} x^* + \beta\frac{B}{2}(1-\beta) - \beta(1-\rho^c)U(x^*) &= \beta b^c \\ x^* + \beta\frac{B}{2}(1-\beta) - \beta\left(1-\rho^t\right)U(x^*) &= \beta b^t \end{aligned}$$

with $b^c > b^t$ and with market clearing condition

$$\begin{aligned} B &= b^c + b^t \\ &= 2\frac{x^*}{\beta} + B(1-\beta) - \left(2-\rho^c-\rho^t\right)U(x^*) \\ \beta\frac{B}{2} &= \frac{x^*}{\beta} - \left(1-\frac{\rho^c}{2}-\frac{\rho^t}{2}\right)U(x^*) \end{aligned}$$

and $\hat{v}^i > 0, \hat{v}^j > 0$ iff

$$\begin{aligned} x^* + \beta\frac{B}{2}(1-\beta) - \beta(1-\rho^c)U(x^*) &< \rho^c U(x^*) \\ x^* + \beta\frac{B}{2}(1-\beta) - \beta\left(1-\rho^t\right)U(x^*) &< \rho^t U(x^*) \end{aligned}$$

and the incentive constraints are slack by construction. If $u(c) = \frac{x^{1-\sigma}}{1-\sigma}$ these are

$$\beta\frac{B}{2} = \frac{1}{\beta} - \left(1-\frac{\rho^c}{2}-\frac{\rho^t}{2}\right)\frac{1}{(1-\sigma)}$$

and for $i = c, t$

$$\sigma > (1-\beta)\left[\beta\frac{B}{2}(1-\sigma) + \left(1-\rho^i\right)\right]$$

we can then substitute from the first into the second to get

$$\begin{aligned} \sigma &> (1-\beta)\left[\frac{1-\sigma}{\beta} - \left(1-\frac{\rho^c}{2}-\frac{\rho^t}{2}\right) + \left(1-\rho^i\right)\right] \\ \frac{\sigma}{\beta} &> (1-\beta)\left[\frac{1}{\beta} + \frac{\rho^c}{2} + \frac{\rho^t}{2} - \rho^i\right] \\ \sigma &> (1-\beta)\left[1 + \beta\left(\frac{\rho^c}{2} + \frac{\rho^t}{2} - \rho^i\right)\right] \end{aligned}$$

The last inequality has to hold for $i = c, t$. Since the tighter of the two is for $i = t$ then it is sufficient that

$$\sigma > (1-\beta)\left[1 + \beta\left(\frac{\rho^c - \rho^t}{2}\right)\right]$$

**Case $\hat{v}^j = 0$ and $\hat{v}^i > 0$**

**Proposition 6** *There exist equilibria with both incentive constraints slack and $\hat{v}^i > 0$ and $\hat{v}^j = 0$ if and only if*

$$\begin{aligned} x^* + \beta \frac{B}{2}(1-\beta) - \beta\left(1-\rho^i\right)U(x^*) &\leq \beta b^i < \rho^i U(x^*) \\ \max\left(\rho^j U(x^*), \frac{x^* + \beta\frac{B}{2}(1-\beta) - \beta U(x^*)}{(1-\beta)}\right) &\leq \beta b^j. \end{aligned}$$

*If $u(c) = \frac{x^{1-\sigma}}{1-\sigma}$ these conditions are:*

$$\begin{aligned} 1 + \beta \frac{B}{2}(1-\beta) - \beta\frac{(1-\rho^i)}{1-\sigma} &\leq \beta b^i < \frac{\rho^i}{1-\sigma} \\ \max\left(\frac{\rho^j}{1-\sigma}, \beta\frac{B}{2} + \frac{1-\frac{\beta}{1-\sigma}}{(1-\beta)}\right) &\leq \beta b^j \end{aligned}$$

*for which it is necessary that $1 + \beta\frac{B}{2}(1-\beta) < \frac{\beta + \rho^i(1-\beta)}{1-\sigma}$.*

**Proof.** See Appendix B. ■

Notice that, as intuitive because the IC is slack:

$$v^i = v^j = \frac{-qb^i + U(x^*) - x^* + \beta b^i - \beta\frac{B}{2}(1-q)}{1-\beta}$$

that are simply

$$v^i = \frac{U(x^*) - x^*}{1-\beta} - \beta\frac{B}{2}$$

So the mechanism can set $b^i = B - b^j$ so that

$$\begin{aligned} x^* + \beta \frac{B}{2}(1-\beta) - \beta\left(1-\rho^i\right)U(x^*) &= \beta b^i \\ \max\left(\rho^j U(x^*), \frac{x^* + \beta\frac{B}{2}(1-\beta) - \beta U(x^*)}{(1-\beta)}\right) &= \beta b^j \end{aligned}$$

and as long as $\beta b^i < \rho^i U(x^*)$ then this is an equilibrium:

$$x^* + \beta\frac{B}{2}(1-\beta) < \left(\beta\left(1-\rho^i\right) + \rho^i\right)U(x^*)$$

If $u(c) = \frac{x^{1-\sigma}}{1-\sigma}$ the necessary and sufficient conditions are

$$\begin{aligned} 1 + \beta \frac{B}{2}(1-\beta) - \beta\frac{(1-\rho^i)}{1-\sigma} &\leq \beta b^i < \frac{\rho^i}{1-\sigma} \\ \max\left(\frac{\rho^j}{1-\sigma}, \beta\frac{B}{2} + \frac{1-\frac{\beta}{1-\sigma}}{(1-\beta)}\right) &\leq \beta b^j \end{aligned}$$

and for the first it is necessary that

$$1 + \beta\frac{B}{2}(1-\beta) < \frac{\beta + \rho^i(1-\beta)}{1-\sigma}$$

### 5.1.1 Discussion on ICs slack

With respect to unregulated economy, segregated regulation implements the efficient allocation in a larger set of economies.

In the case of Merged regulation, let us partition the three cases.

First, consider $\hat{v}^i > 0$ for all $i$: here merged regulation is equivalent to segregated regulation for slack ICs in terms of the set of economies where it implements the efficient allocation. However, it is more costly for buyers in the traditional sector and less costly for buyers in the crypto sector to implement the efficient allocation under merged regulation than under segregated regulation.

To see this, recall that membership titles are costly to purchase for buyers, so the smallest amount of membership titles consistent with the efficient allocation the higher the value of buyers at the end of the CM, $v^i, v^j$. Consider then the necessary and sufficient condition for the IC to be binding under segregated and merged regulations when $\hat{v}^i > 0$ for all $i$:

$$\begin{aligned} x^* - \beta\left(1-\rho^i\right)U\left(x^*\right) &\leq \beta^2 B^i \\ x^* - \beta\left(1-\rho^i\right)U\left(x^*\right) &\leq \beta^2\frac{B}{2} + \beta b^i - \beta\frac{B}{2} \end{aligned}$$

Then recall that market clearing in merged regulation requires $B = b^i + b^j$ and that the mechanism always chooses the smallest amount of membership titles to support the efficient allocation. Hence, in segregated regulation $B^i = \frac{x^*-\beta\left(1-\rho^i\right)U(x^*)}{\beta^2}$ and in merged regulation

$$\frac{x^* - \beta\left(1-\rho^i\right)U\left(x^*\right)}{\beta^2} = \frac{b^i}{2}\left(1+\frac{1}{\beta}\right) - \frac{b^j}{2}\left(\frac{1}{\beta}-1\right)$$

If $b^i = b^j$ then $b^i = B^i$. However, $b^c > b^t$, so for $i = c, j = t$ the necessary $b^c$ to satisfy the IC at equality is smaller than $B^i$, and viceversa for $i = t, j = c$.

In conclusion, the cost to implement the efficient allocation in terms of value of buyers is larger for the traditional sector with merged regulation than with segregated regulation. Conversely, such cost is smaller for the crypto sector with merged regulation than with segregated regulation.

Second, consider the case $\hat{v}^i = 0$ for all $i$, and compare it with the analogous case in merged regulation. For $\hat{v}^i = 0$ in both regulations we need enough membership titles to satisfy

$$\begin{aligned} \rho^i U\left(x^*\right) &\leq \beta B^i \\ \rho^i U\left(x^*\right) &\leq \beta b^i \end{aligned}$$

implying that $b^c > b^t$in both regulations. The necessary and sufficient condition for the IC to be slack, however, is

$$\begin{aligned} U\left(x^*\right) &\geq \frac{x^*}{\beta} \\ U\left(x^*\right) &\geq \frac{x^*}{\beta} + \left(\frac{B}{2} - b^i\right)(1-\beta) \end{aligned}$$

Because $b^c > b^t$ with merged regulation, then $\frac{B}{2} - b^c < 0$ and $\frac{B}{2} - b^t > 0$. Hence, the necessary and sufficient condition for the IC to be slack is satisfied in a larger set of economies than with segregated regulation for the crypto sector, and in a smaller set of economies for the traditional sector.

Third, for the case where $\hat{v}^j = 0$ and $\hat{v}^i > 0$,we have that the conclusions of either case above will carry through, depending on whether segregated regulation is with $\hat{v} > 0$ or $\hat{v} = 0$. Thus the cost of merged regulation relative to segregated regulation is either in terms of cost of membership titles to buyers or in terms of set of economies where the IC is slack.

## 5.2 Solution case both incentive constraints bind

### 5.2.1 Case $\hat{v} = 0$ for all $i$

A necessary condition for the existence of an equilibrium with $\hat{v}^i = \hat{v}^j = 0$ is $b^i > 0$, $b^j > 0$, as $\hat{v} = \frac{-qb+\rho u(x)}{1-\beta}$. Hence, in this section we construct equilibria with $\hat{v}^i = \hat{v}^j = 0$ and $b^i > 0$,$b^j > 0$.

**Proposition 7** *There exists an equilibrium with both incentive constraints binding and* $\hat{v}^i = 0$ *for all* $i$ *if and only if* $\beta U(x^*) < x^*$ *and* $\beta U'(x_E) b^i \geq \rho^i U(x_E)$. *The consumption allocation* $x_E$ *solves* $x = \beta U(x)$. *If* $u(x) = \frac{x^{1-\sigma}}{1-\sigma}$ *the necessary and sufficient conditions are*

$$\begin{aligned} x_E &= \left(\frac{\beta}{(1-\sigma)}\right)^{\frac{1}{\sigma}} \\ 1 &> \sigma + \beta \\ b^i &\geq \rho^i \frac{\beta^{\frac{1}{\sigma}-1}}{(1-\sigma)^{\frac{1}{\sigma}+1}} \end{aligned}$$

**Proof.** See Appendix B. ■

The portfolio allocation between $l^i$ and $b^i$ is indeterminate as long as all constraints are satisfied – in particular $qb^i \geq \rho^i U(x_E)$, which set a lower bound on $b^i$.

Notice that $v^i$ is decreasing in $b^i$. So, if the mechanism were to choose the optimal level of $B$ it would have to be such that $q(B - b^t) = \rho^c U(x_E)$ and then $qb^t = \rho^t U(x_E)$. Therefore, $b^c > b^t$ and $l^c < l^t$.

In general, a bound on $B$ is defined by

$$\beta U'(x_E) B \geq \left(\rho^c + \rho^t\right) U(x_E)$$

which, with $u(x) = \frac{x^{1-\sigma}}{1-\sigma}$ is

$$\beta B \geq \left(\rho^c + \rho^t\right) \frac{\overline{x}}{1-\sigma}$$

or $\frac{\beta B(1-\sigma)}{(\rho^c+\rho^t)} \geq x_E$, that is to say, using the equation defining $x_E$:

$$\frac{\beta}{1-\sigma} \geq \left[\frac{\beta B(1-\sigma)}{(\rho^c+\rho^t)}\right]^{\sigma}$$

Also, with $u(x) = \frac{x^{1-\sigma}}{1-\sigma}$ $\hat{v}^i = 0$ if an only if:

$$\beta\left(1-\sigma\right) b^i \quad \geq \quad \rho^i x_E$$

that, using $x_E = \left(\frac{\beta}{(1-\sigma)}\right)^{\frac{1}{\sigma}}$, is simply

$$\frac{b^i}{\rho^i} \quad \geq \quad \frac{\beta^{\frac{1-\sigma}{\sigma}}}{\left(1-\sigma\right)^{\frac{1+\sigma}{\sigma}}}$$

### 5.2.2 Case $\hat{v} > 0$, $b^i > 0$ for all $i$

In this section we focus on the equilibrium with $b^i > 0, b^j > 0$. If $b^i = 0$ ($b^j = 0$), the equilibrium allocation is equivalent to that with segregated regulation for type $i$ ($j$) and no regulation for type $j$ ($i$).

**Proposition 8** *Assume $u(x) = \frac{x^{1-\sigma}}{1-\sigma}$. There exists an equilibrium with both $b^i > 0$, a unique $x_E < x^*$ solving*

$$2x^{1+\sigma} \quad = \quad \beta^2 B + \beta\frac{x\left(2-\rho^j-\rho^i\right)}{1-\sigma}$$

*if $B < \frac{2^{1+\sigma}}{\beta^2} - \frac{\left(2-\rho^j-\rho^i\right)}{\beta(1-\sigma)}$ and if either one of the following conditions is satisfied:*

1. *$\rho^i < 1-\sigma$ and*

$$B \quad < \quad 2\frac{\left(1-\rho^i\right)}{\left(1-\beta\right)}\left[\beta\frac{\left(1-\frac{\rho^j}{2}-\frac{\rho^i}{2}\right)}{\left(1-\sigma\right)^2}\right]^{\frac{1}{1+\sigma}}$$

2. *$\rho^i \geq 1-\sigma$ and $\left(\frac{\beta B}{\rho^i}\right)^{\sigma} \leq \frac{\left(2-\rho^j\right)}{2(1-\sigma)^{1+\sigma}}$.*

**Proof.** See Appendix B. ■

Notice, however, that $x_E$ is increasing in $B$, therefore it is not obvious that the mechanism will choose the smallest $B$ satisfying all the conditions.

### 5.2.3 Case $\hat{v}^i > 0, \hat{v}^j = 0$

**Proposition 9** *Assume $u(x) = \frac{x^{1-\sigma}}{1-\sigma}$, and*

1. *for the IC to bind*

$$\frac{\beta B}{2} \quad > \quad \frac{2\left(1-\sigma-\beta\right)+\beta\sigma}{\beta\left(1-\beta\right)\left(1-\sigma\right)} - \frac{\left(1-\rho^i\right)}{1-\sigma}$$

*2. for $\hat{v}^i > 0$*

$$\frac{\beta B}{2} \quad > \quad \left(\frac{\beta}{1-\sigma}\right)^{\frac{1}{\sigma}} \frac{\rho^i}{1-\sigma}$$

*Then there exists an equilibrium with a unique $x_E < x^*$ solving (26).*

**Proof.** See Appendix B. ■

### 5.2.4 Discussion

When both IC bind there can be equilibria with punishments being both zero, both positive, and positive for the traditional sector while zero for the crypto sector (possibly also the reverse but necessary conditions may be harder to satisfy).

Across these equilibria, if $\beta B^{>>} < \beta^{\frac{1}{\sigma}} \frac{(\rho^i+\rho^j)}{(1-\sigma)^{\frac{1}{\sigma}+1}}$, where $B^{>>}$ denotes the membership titles when $\hat{v}^i > 0\ \forall i$, then the consumption allocation with $\hat{v}^i = 0$ for all $i$ is larger than that with $\hat{v}^i > 0$ for all $i$ , though the value of buyers might not be due to the larger holdings of membership titles that are necessary to drive $\hat{v}^i = 0$. The conclusion that the consumption allocation with $\hat{v}^i = 0$ for all $i$ is larger follows from comparing the equations defining the equilibrium consumption allocations. With $\hat{v}^i = 0$ for all $i$ we have $x = \beta U(x)$, while with $\hat{v}^i > 0$ for all $i$ we have

$$2x - \beta^2 U'(x) B^{>>} \quad = \quad \beta U(x)\left(2 - \rho^j - \rho^i\right).$$

Therefore, if at $x = \left(\frac{\beta}{(1-\sigma)}\right)^{\frac{1}{\sigma}}$ we have that

$$2x - \beta^2 U'(x) B^{>>} \quad > \quad \beta U(x)\left(2 - \rho^j - \rho^i\right)$$

then $x_E < \left(\frac{\beta}{(1-\sigma)}\right)^{\frac{1}{\sigma}}$. The above inequality evaluated at $x = \left(\frac{\beta}{(1-\sigma)}\right)^{\frac{1}{\sigma}}$ is indeed $\beta B^{>>} < \beta^{\frac{1}{\sigma}} \frac{(\rho^i+\rho^j)}{(1-\sigma)^{\frac{1}{\sigma}+1}}$. [24]

Finally, the case where $\hat{v}^i > 0 = \hat{v}^j$ can arise (mostly for $i = t$) when the merged regulation pushes the traditional sector to save on membership titles, due to cross subsidization, to the point that traditional banks prefer to live with a tighter incentive constraint (due to $\hat{v}^i > 0$).

Let us denote by $x_E^{>=}$ the solution to the case where $\hat{v}^i > 0$ and $\hat{v}^j = 0$, and by $x_E^{>>}$ the solution to the case where $\hat{v}^i > 0$ for all $i$.

To compare such $x_E^{>>}$ with $x_E^{>=}$, it will be convenient to rewrite the equation for $x_E^{>>}$ as

$$2x - \beta U(x)\left(2 - \rho^i\right) + \rho^j \beta U(x) \quad = \quad \beta^2 U'(x) B^{>>}$$

[24] Specifically, it is

$$\rho^j + \rho^i \quad > \quad \beta^2 \left(\frac{\beta}{(1-\sigma)}\right)^{-\frac{1+\sigma}{\sigma}} B^{>>}$$

where $B^{>>}$ denotes the membership titles issued by the mechanism in the case where $\hat{v}^i > 0$ for all $i$.

Consider now the case where $\hat{v}^i > 0$ and $\hat{v}^j = 0$ and the equation defining the consumption allocation $x_E^{>=}$. Divide both sides by $x$ to get:

$$2 - \left(2 - \rho^i\right)\beta\frac{x^{-\sigma}}{1-\sigma} \quad = \quad -\rho^i\frac{x^{-\sigma}}{1-\sigma}\beta + \beta^2 B^{>=}x^{-\sigma-1} + x^{\sigma}\frac{4}{\beta} - \frac{\beta B^{>=}}{x} - 2\frac{\left(2-\rho^i\right)}{1-\sigma}.$$

Suppose that $x_E^{>=} = x_E^{>>}$. Then this equation should hold also for $x_E^{>>}$. If instead the right hand side is strictly positive, then $x_E^{>=} > x_E^{>>}$ because the left hand side of this equation is increasing in $x$. To compare this with the consumption allocation when $\hat{v}^i > 0$ for all $i$, $x_E^{>>}$, divide both sides of the equation defining $x_E^{>>}$ by $x$ and rearrange it as:

$$2 - \left(2 - \rho^i\right)\beta\frac{x^{-\sigma}}{1-\sigma} \quad = \quad \beta^2 x^{-\sigma-1}B^{>>} - \rho^j\beta\frac{x^{-\sigma}}{1-\sigma}$$

Thus, $x_E^{>=} > x_E^{>>}$ if and only if the following inequality is satisfied:

$$\begin{aligned} -\rho^i\frac{x^{-\sigma}}{1-\sigma}\beta + \beta^2 B^{>=}x^{-\sigma-1} + x^{\sigma}\frac{4}{\beta} - \frac{\beta B^{>=}}{x} - 2\frac{\left(2-\rho^i\right)}{1-\sigma} \quad &> \\ \beta^2 x^{-\sigma-1}B^{>} - \rho^j\beta\frac{x^{-\sigma}}{1-\sigma} & \end{aligned}$$

which can be rearranged as

$$\begin{aligned} \left(\rho^j - \rho^i\right)\beta U\left(x\right) - \beta\left(b^{i>>} + b^{j>>}\right)q^{>>} + \left(q^{>>} - 1\right)\beta B^{>=} + & \\ x^{\sigma+1}\frac{4}{\beta} - 2\left(2-\rho^i\right)\frac{U\left(x\right)}{U'\left(x\right)} \quad &> \quad 0 \end{aligned}$$

Even if we know that $\left(q^{>>} - 1\right) < 0$ and that $\rho^j\beta U\left(x\right) - \beta b^{j>}q^{>>} > 0$, establishing whether this inequality is satisfied or not isn't straightforward.

# 6 Solution case one IC binds

Without loss of generality, let $i$ denote the type whose IC is slack. We can make some general observations.

1. It must be that either $b^i > 0$ and $b^j = 0$, or $b^j > 0$ and $b^i = 0$. Otherwise, the FOC for $b$ implies that the consumption allocation is the same across types, but then we can't have that the IC binds for one type only, while for the type who has it slack $x^i = x^*$. Notice, importantly, that in this case $\tau = B\left(1-q\right)$ as it is charged only to the issuers who participate into it, that are the types who hold $b$. In other words, with one IC slack, it must be that one type only issues uninsured liabilities: $b^i = 0$.

2. We cannot have $\hat{v}^i = 0$ for the type with $b^i = 0$. In fact for this type $\hat{v}^i = \frac{\rho^i U\left(x^i\right)}{1-\beta} > 0$.

3. As a corollary, there is no equilibrium with both $\hat{v}^i = 0$. Indeed, with one IC binding and one slack it must be that $b^i > 0$ if and only if $b^j = 0$, or $b^i = 0$ if and only if $b^j > 0$. This, however, implies that either $\hat{v}^j > 0$ or $\hat{v}^i > 0$.

## 6.1 Case $\hat{v}^i > 0, \hat{v}^j > 0$

### 6.1.1 Case $b^i = 0$

**Proposition 10** *There exists an equilibrium with $x^i = x^* > x^j$ wherex$^j$ solves*

$$x^j - \beta^2 BU'\left(x^j\right) \quad = \quad \beta\left(1-\rho^j\right)U\left(x^j\right)$$

*if and only if, with $u(x) = \frac{x^{1-\sigma}}{1-\sigma}$:*

$$1 - \beta^2 B\left(x^*\right)^{-(1+\sigma)} \quad > \quad \beta\left(1-\rho^j\right)\frac{\left(x^*\right)^{-\sigma}}{1-\sigma}$$

*and*

$$\frac{\beta B}{\rho^j}\left(1-\sigma\right) - \beta^2 B\left(\frac{\beta B}{\rho^j}\left(1-\sigma\right)\right)^{-\sigma} \quad < \quad \beta\left(1-\rho^j\right)\frac{\left(\frac{\beta B}{\rho^j}\left(1-\sigma\right)\right)^{1-\sigma}}{1-\sigma}$$

**Proof.** See Appendix B. ■

### 6.1.2 Case $b^i > 0$

**Proposition 11** *Assume $u(x) = \frac{x^{1-\sigma}}{1-\sigma}$. There exists an equilibrium with only one incentive constraint binding for type $j$ and $b^i > 0$ if and only if*

$$\begin{aligned} 1 &\leq \beta\left[\frac{(1-\rho^i)}{(1-\sigma)} + \beta B\right] \\ 1 &> \beta\frac{(1-\rho^j)}{(1-\sigma)} \\ \beta B &< \frac{\rho^i}{(1-\sigma)} \end{aligned}$$

*and $x^j$ solves $l^j = \beta\left(1-\rho^j\right)U\left(l^j\right)$.*

**Proof.** See Appendix B. ■

Notice that by comparing the equation defining $x^j$ here and when $b^j > 0$ it follows that consumption is higher when $b^j > 0$.

## 6.2 Case $\hat{v}^j = 0, \hat{v}^i > 0$ or $\hat{v}^i = 0, \hat{v}^j > 0$

Recall that we let $i$ denote the type with a slack incentive constraint.

**Proposition 12** *There is no equilibrium with only one incentive constraint binding and $\hat{v}^j = 0, \hat{v}^i > 0$. There exists an equilibrium with only one incentive constraint binding and $\hat{v}^i = 0, \hat{v}^j > 0$ if and only if*

$$\begin{aligned} \beta(1-\rho^j)u(x^*) &< x^* < \beta u(x^*) \\ \beta B &\geq \rho^i u(x^*) \end{aligned}$$

**Proof.** See Appendix B. ■

# 7 Merged regulation: general results

1. With merged regulation, any equilibrium with $b^i > 0$ $\forall i$ is such that $x^c = x^t$. This follows from the FOC for $b$: $-q + \beta U'(x^i) \leq 0$.

2. With merged regulation, If $l^i = 0$ for all $i$ then IC is slack for all $i$ and $b^c = b^t = \frac{x^*}{\beta} = \frac{B}{2}$. Hence, if IC binds, it must be that $l^i > 0$ for at least one $i$.

3. With merged regulation and IC slack, $b^i$ rather than $B$ enters explicitly the necessary and sufficient conditions for the equilibrium to exist, apart from the case when it may be $\frac{B}{2}$. The reason why $b^i$ appears is that the portfolio allocation is irrelevant, so $b^i$ is not pinned down.

4. With merged regulation and both IC bind and $\hat{v} > 0$, it must be that $l^t > l^c \geq 0$. Everything follows from IC binding, as it is not possible to have a common $\tau$ and different $\rho^i$.

5. Portfolio allocation is irrelevant

   (a) If the incentive constraint is slack, the portfolio composition between $b > 0$ and $l$ is irrelevant. However, $B$ affects the set of economies where there exists an equilibrium with a slack incentive constraint. However, if in equilibrium $\hat{v} = 0$ then $b > 0$.

   (b) If in equilibrium $\hat{v}^i = 0$ for all $i$ then the portfolio allocation is irrelevant between $b > 0$ and $l$, and both types of issuers enjoy the same consumption and utility. When instead $\hat{v}^i > 0$ then the holdings of membership titles matter both for the consumption allocation and for the set of economies where equilibria with certain features exist.

   (c) The case $\hat{v}^i = 0$ for all $i$ is not consistent with an equilibrium where only one incentive constraint binds.

# 8 Comparison across regulation types

## 8.1 Unregulated, IC binds

This equilibrium exists if and only if

$$x^* \quad > \quad \beta \left(1 - \rho^i\right) U\left(x^*\right)$$

where $x_E^i : \{x > 0 : x = \beta\left(1 - \rho^i\right) U(x)\}$.

## 8.2 Unregulated, IC slack

This equilibrium exists if and only if

$$x^* \quad \leq \quad \beta \left(1 - \rho^i\right) U\left(x^*\right).$$

Notice that $\hat{v} > 0$.

## 8.3 Segregated regulation, IC binds

### 8.3.1 Case $\hat{v} > 0$.

This equilibrium exists if and only if

$$x^* > \beta\left(1-\rho^i\right)U\left(x^*\right)+\beta^2 B^i U'\left(x^*\right)$$

where $x_E^i = \{x > 0 : x = \beta\left(1-\rho^i\right)U\left(x\right)+\beta^2 B^i U'\left(x\right)\ \}$.

And

$$\beta B^i U'\left(x_E^i\right) < \rho^i U\left(x_E^i\right)$$

Notice that in this case the consumption allocation is increasing in $B^i$ so it is reasonable to expect that the mechanism would set $B^i$ to its largest feasible value compatible with both NSC above (also the IC bind condition can be stated as an upper bound on $B^i$).

Intuition: Conjecture is that the reason why consumption is increasing in $B^i$ is the spreading out of default costs over defaulters themselves. The more membership titles are required to issue insured liabilities the more defaulters themselves need to pay to imitate non defaulters.

### 8.3.2 Case $\hat{v} = 0$.

This equilibrium exists if and only if

$$x^* > \beta U\left(x^*\right)$$

and $x_E^i = \{x > 0 : x = \beta U\left(x\right)\}$.

And

$$\beta B^i U'\left(x_E^i\right) \geq \rho^i U\left(x_E^i\right)$$

From the previous section we can prove that if $u(x) = \frac{x^{1-\sigma}}{1-\sigma}$ then $x_E^{=0} > x_E^{>0}$, where $x_E^{>=}$ denotes the consumption allocation when $\hat{v}^i = 0$, and $x_E^{>0}$ denotes the consumption allocation when $\hat{v}^i > 0$.

## 8.4 Segregated regulation, IC slack

### 8.4.1 Case $\hat{v} > 0$.

This equilibrium exists if and only if

$$x^* \leq \beta\left(1-\rho^i\right)U\left(x^*\right)+\beta^2 B^i$$

And

$$\beta B^i < \rho^i U\left(x^*\right)$$

Here as well, as in the case with IC binding, the reason why $B^i$ relaxes the necessary and sufficient conditions for the incentive constraint to be slack is the spreading out of the cost of default also over defaulters themselves.

### 8.4.2 Case $\hat{v} = 0$.

This equilibrium exists if and only if

$$x^* \leq \beta U(x^*)$$

And

$$\beta B^i \geq \rho^i U(x^*)$$

Comparing this case with that of $\hat{v} > 0$, and considering the bounds on $B^i$ consistent with $\hat{v} > 0$, we can also conclude that the set defined by the necessary and sufficient condition for the incentive constraint to be slack, is larger when $\hat{v} = 0$.

## 8.5 Merged regulation, both incentive constraints bind

### 8.5.1 Case both $\hat{v} > 0$.

There exists an equilibrium with both $b^i > 0$, a unique $x_E < x^*$ solving

$$2x^{1+\sigma} = \beta^2 B + \beta \frac{x\left(2 - \rho^j - \rho^i\right)}{1-\sigma}$$

if and only if

$$\begin{aligned} x^* &> \beta\left(1 - \frac{\rho^j}{2} - \frac{\rho^i}{2}\right) U(x^*) + \beta^2 \frac{B}{2} U'(x^*) \\ \rho^i U(x_E) &> \beta b^i U'(x_E) \end{aligned}$$

With $u(x) = \frac{x^{1-\sigma}}{1-\sigma}$, sufficient conditions for these inequalities to be satisfied are $B < \frac{2^{1+\sigma}}{\beta^2} - \frac{\left(2-\rho^j-\rho^i\right)}{\beta(1-\sigma)}$ and either one of the following:

1. $\rho^i < 1 - \sigma$ and

$$B < 2\frac{(1-\rho^i)}{(1-\beta)}\left[\beta \frac{\left(1 - \frac{\rho^j}{2} - \frac{\rho^i}{2}\right)}{(1-\sigma)^2}\right]^{\frac{1}{1+\sigma}}$$

2. $\rho^i \geq 1 - \sigma$ and $\left(\frac{\beta B}{\rho^i}\right)^{\sigma} \leq \frac{\left(2-\rho^j\right)}{2(1-\sigma)^{1+\sigma}}$.

Notice that in this case $b^c > b^t$ as both types consume the same allocation but they finance it differently. The fact that the regulation is merged pushes the $t$ type partially out of the mutualization fund. In terms of consumption, with respect to segregated regulation and $\hat{v} > 0$, the $c$ type consumes more here with merged regulation while the $t$ type consumes more with segregated regulation.

### 8.5.2 Case both $\hat{v} = 0$.

This equilibrium exists if and only if

$$x^* \;>\; \beta U(x^*)$$

and

$$\beta b^i U'(x_E) \;\geq\; \rho^i U(x_E)$$

with $x_E = \{x > 0 : x = \beta U(x)\}$. If $u(x) = \frac{x^{1-\sigma}}{1-\sigma}$, then $x_E = (\frac{\beta}{1-\sigma})^{\frac{1}{\sigma}}$ and the above inequalities are

$$\begin{aligned} 1 &> \beta + \sigma \\ b^i \frac{(1-\sigma)^{\frac{1}{\sigma}+1}}{\beta^{\frac{1}{\sigma}-1}} &\geq \rho^i \end{aligned}$$

With respect to segregated regulation and $\hat{v} = 0$ the consumption allocation is the same. Depending on whether $B^i \gtreqless b^i$, where the former denotes the holdings of membership titles in segregated regulation and the latter the holdings of membership titles by type $i$ issuer, then the set of economies where merged regulation is feasible can be smaller than segregated regulation.

In a setting where membership titles are not costly to issue (or the membership criteria are not costly to maintain), there is no good reason to keep $B$ or $B^i$ to their smallest feasible level. However, if there was an incentive to do so, then in the economy with segregated regulation and $\hat{v} = 0$ we would have $\beta B^i U'(x_E) = \rho^i U(x_E)$, where $x_E$ is the same as in this care with merged regulation and $\hat{v} = 0$.

### 8.5.3 Case $\hat{v}^j = 0 < \hat{v}^i$.

If an equilibrium exists it is unique if and only if

$$\frac{\beta B}{2} \;>\; \frac{2(1-\sigma-\beta)+\beta\sigma}{\beta(1-\beta)(1-\sigma)} - \frac{(1-\rho^i)}{1-\sigma}$$

which also guarantees that the IC binds.[25] Then the equilibrium allocation, $x_E$, solves

$$\frac{\beta B}{2} + x\left(1 + \frac{(2-\rho^i)}{1-\sigma}\right) \;=\; U(x)\,\beta\left(1-\rho^i\right) + \beta^2 \frac{B}{2} U'(x) + \frac{x}{U'(x)}\frac{2}{\beta}$$

The equilibrium conjecture $\hat{v}^i > 0$ is verified if and only if $x_E < (\frac{\beta}{1-\sigma})^{\frac{1}{\sigma}}$, for which a sufficient condition is

$$\frac{\beta B}{2} \;>\; \left(\frac{\beta}{1-\sigma}\right)^{\frac{1}{\sigma}} \frac{\rho^i}{1-\sigma}$$

while a sufficient condition for $\hat{v}^j = 0$ is ****

[25] If this inequality is violated there may exist no solution for the consumption allocation.

Notice that it is not straightforward to assess whether the consumption allocation with merged regulation in this case is smaller or larger than the consumption allocation with segregated regulation and $\hat{v} > 0$ and also with $\hat{v} = 0$.[26]

## 8.6 Merged regulation, only one IC binds

### 8.6.1 Case $\hat{v}^i > 0, \forall i$ and $x^i = x^* > x^j = x_E$

**Case $b^i = 0$.**

$$x^* > \beta\left(1-\rho^j\right)U\left(x^*\right) + \beta^2 BU'\left(x^*\right)$$

where $x_E$ solves this at $=$.

$$x^* \leq \beta\left(1-\rho^i\right)U\left(x^*\right)$$

for IC slack for type $i$. But these two inequalities are consistent only for $j = c$ $i = t$. And with respect to segregated regulation or unregulated economy, it must be that type $i$ had already IC slack in the unregulated economy, while type $j$ had segregated regulation and IC binding, and also IC binding in the unregulate economy.

And

$$\beta BU'\left(x_E\right) < \rho^j U\left(x_E\right)$$

which in CRRA are

$$\begin{aligned} 1-\beta B &> \frac{\beta\left(1-\rho^j\right)}{1-\sigma} \\ (1-\sigma)^{1+\sigma} &< \beta^2 B\left(\frac{\beta B}{\rho^j}\right)^{-(1+\sigma)} + \beta\left(1-\rho^j\right)\left(\frac{\beta B}{\rho^j}\right)^{-\sigma} \end{aligned}$$

This case, for $j = c$ $i = t$, is only consistent with segregated regulation IC binds and $\hat{v} > 0$ for $j$, and unregulated IC slack for $i$.

**Case $b^i > 0 = b^j$.**

$$x^* > \beta\left(1-\rho^j\right)U\left(x^*\right)$$

where $x_E$ solves this at $=$.

[26] The former solves

$$x = U(x)\beta\left(1-\rho^i\right) + \beta^2\frac{B}{2}U'(x)$$

and the latter solves

$$x = \beta U(x)$$

$$x^* \leq \beta\left(1-\rho^i\right)U\left(x^*\right)+\beta^2 B$$

And

$$\beta B < \rho^i U\left(x^*\right)$$

This case is equivalent to segregated regulation for $j$ and unregulated economy for $i$. There is no link between the two types in this case because only one is holding $b$ and he's the only type taxed.

## 8.7 Merged regulation, both IC slack

### 8.7.1 Case both $\hat{v} > 0$

$$\begin{aligned} x^* - \beta b^i + \beta\frac{B}{2}(1-\beta) &\leq \beta\left(1-\rho^i\right)U\left(x^*\right) \\ \rho^i U\left(x^*\right) &> \beta b^i \end{aligned}$$

for IC slack and for $\hat{v}^i > 0$.

Whether this allows for a larger set of economies than segregated regulation depends on the relative size of $B^i$ in segregated regulation and $\frac{B}{2}$ here, and whether $b^i \gtrless \frac{B}{2}$.

### 8.7.2 Case both $\hat{v} = 0$

$$\begin{aligned} x^* + \beta\left(\frac{B}{2}-b^i\right)(1-\beta) &\leq \beta U\left(x^*\right) \\ \rho^i U\left(x^*\right) &\leq \beta b^i \end{aligned}$$

for $\hat{v}^i = 0$ and for IC slack.

### 8.7.3 Case $\hat{v}^j = 0$ and $\hat{v}^i > 0$

$$\begin{aligned} x^* - \beta b^i + \beta\frac{B}{2}(1-\beta) &\leq \beta\left(1-\rho^i\right)U\left(x^*\right) \\ x^* + \beta\left(\frac{B}{2}-b^j\right)(1-\beta) &\leq \beta U\left(x^*\right) \end{aligned}$$

and any $b^i = B - b^j$ such that

$$\begin{aligned} \rho^i U\left(x^*\right) &> \beta b^i \\ \rho^j U\left(x^*\right) &\leq \beta b^j \end{aligned}$$

but with both IC slack then the two decision problems are the same and one might conjecture that $b^i = b^j = \frac{B}{2}$. But with IC slack the composition of uninsured and insured liabilities doesn't matter for consumption. Therefore the portfolio composition across types needs not be the same.

# 9 Extensions

Our results in sections 4 and 5 highlight the independent roles of the loss mutualization fund and the membership titles in relaxing the incentive constraint. While both directly affect the continuation values $v^i$ and $\hat{v}^i$, the latter also affects the incentive constraint indirectly, by reducing the contributions to the fund necessary to cover insured debt. A natural follow-up question is whether both elements are essential, or, under some conditions, one of them suffices to reduce the severity of the commitment friction. In this section we answer this question.

Considering our benchmark proposal for segregated self-regulation, first we study the possibility of equilibria where the price of the membership titles is sufficiently high to fund all insured debt, without the need to resort to contributions from issuers. Second, we dig deeper into the roles that membership titles play in relaxing the incentive constraint. We analyze a version of our mechanism where membership titles are (i) a fixed cost or (ii) a license to issue, both paid prior to issuance, and compare the resulting equilibria with those of an unregulated economy and of our benchmark mechanism. If the key role of membership titles works through affecting incentives through an *entry cost*, then these versions of our mechanism should also relax the incentive constraint.

## 9.1 Stationary equilibria with $q = 1$

In this section we characterize the conditions under which our proposed mechanism is effective even without the loss mutualization fund. In other words, we characterize stationary equilibria where $q = 1$, implying that $\tau = B(1-q) = 0$. These equilibria are consistent only with a binding incentive constraint. If the incentive constraint was slack then $u'(x^*) = 1$ and the FOC for $b$ imply $q = \beta$.

Consider our benchmark proposal of segregated self-regulation.

**Equilibrium with $\hat{v}^i > 0$.** Let us try to construct an equilibrium with IC binding and $q = 1$. With $\hat{v}^i = \frac{-q^i b^i + \rho^i U(x^i)}{1-\beta} > 0$, the binding IC is

$$v^i = -q^i B^i + U\left(x^i\right) + \beta \hat{v}^i$$

even if $l^i > 0$ in equilibrium. Then

$$\begin{aligned} v^i - \hat{v}^i &= -q^i B^i + U\left(x^i\right) - (1-\beta)\,\hat{v}^i \\ &= \left(1-\rho^i\right) U\left(x^i\right) \end{aligned}$$

By mkt clearing $b^i = B^i$, $l^i = x^i - \beta B^i$ and the binding IC, with $q = 1$, we have

$$x^i - \beta B^i = \beta\left(1-\rho^i\right) U\left(x^i\right)$$

which, using also the FOC for $b^i$, also implies $1 = \beta U'(x^i)$. We need to check that such $x^i < x^*$ and that $-B^i + \rho^i U(x^i) > 0$. So this is an equilibrium if and only if

$$\begin{aligned} x^i &= U'^{-1}\left(\frac{1}{\beta}\right) \\ \beta B^i &= x^i - \beta\left(1-\rho^i\right) U\left(x^i\right) \geq 0 \\ \beta \rho^i U\left(x^i\right) &> \beta B^i \end{aligned}$$

Hence, the second and third give:

$$\beta U\left(x^i\right) > x^i$$

and $B^i > 0$ requires

$$\beta U\left(x^i\right) > x^i \geq \beta\left(1-\rho^i\right) U\left(x^i\right)$$

So with CRRA we have $x = \beta^{\frac{1}{\sigma}}$

$$\frac{\beta}{1-\sigma}\left(\beta^{\frac{1}{\sigma}}\right)^{1-\sigma} > \beta^{\frac{1}{\sigma}} \geq \frac{\beta\left(1-\rho^i\right)}{1-\sigma}\left(\beta^{\frac{1}{\sigma}}\right)^{1-\sigma}$$

the first inequality is

$$\frac{1}{1-\sigma} > 1$$

which is always satisfied. While the second inequality is

$$1 \geq \frac{\left(1-\rho^i\right)}{1-\sigma}$$

in other words $\rho^i$ has to be large enough.

**Equilibrium with $\hat{v}^i = 0$.** The FOC for $b$ again implies $x^i = U'^{-1}\left(\frac{1}{\beta}\right)$. Using the binding IC, we have

$$v^i = -b^i + U\left(x^i\right)$$

which, plugged back into the IC to solve for $x^i$ yields

$$\begin{aligned} l^i &= -\beta b^i + \beta U\left(x^i\right) \\ x^i &= \beta U\left(x^i\right) \end{aligned}$$

but the last equation may be inconsistent with $x^i = U'^{-1}\left(\frac{1}{\beta}\right)$. In CRRA we have $x = \beta^{\frac{1}{\sigma}}$ and

$$1 = \frac{\beta}{1-\sigma} x^{i-\sigma}$$

## 9.2 A mechanism with only membership titles

Consider a version of our proposed mechanism for segregated self-regulation, described in section 4, where no loss mutualization fund exists, and where membership titles are an ex-ante cost granting the issuer the right to issue debt. We describe two formalization of such ex-ante cost: first, a fixed lump sum cost to be paid ex ante to access DM trading. Second, a license to issue DM debt, similarly to what described in section 4 but with no resources supplied to the loss mutualization fund.

### 9.2.1 Membership titles as entry cost

Focusing on stationary equilibria, if membership titles are a fixed entry cost to DM trading, the decision problem of an issuer is:

$$\begin{aligned} v \quad &= \max_{\{l,x\}} \quad -b + u(x) - l + \beta v \\ s.t. \quad & \quad x \leq l \\ & \quad l \leq \beta(v - \hat{v}) \end{aligned}$$

Notice that the resources gathered from CM payment of the entry cost do not appear in this decision problem, as we assume that those goods are disposed of. Because we aim to disentangle the role played by membership titles through the continuation values $v^i$ and $\hat{v}^i$, from their role in supplying resources that could be used to reward agents in the CM following DM trading. With respect to our modeling choice of $\tau^i$ in section 4 this entry cost is to be paid ex-ante rather than ex-post, as the latter is similar to a survivor's pay rule.

The continuation value of default is $\hat{v} = \frac{-b+\rho u(x)}{1-\beta}$.

Equilibria with the incentive constraint slack and $\hat{v} > 0$ are such that

$$\begin{aligned} v &= \frac{-b + u(x^*) - x^*}{1-\beta} \\ \hat{v} &= \frac{-b + \rho u(x^*)}{1-\beta} > 0 \\ x^* &\leq \beta(1-\rho)u(x^*) \end{aligned} \tag{16}$$

while equilibria with a binding incentive constraint and $\hat{v} > 0$ are such that the DM consumption allocation, $x_E^1$, satisfies $x = \beta(1-\rho)u(x)$ and

$$x^* > \beta(1-\rho)u(x^*).$$

Equilibria with the incentive constraint slack and $\hat{v} = 0$ are such that

$$\begin{aligned} x^* &\leq \beta[u(x^*) - b] \\ b &\geq \rho u(x^*) \end{aligned} \tag{17}$$

while equilibria with a binding incentive constraint and $\hat{v} = 0$ are such that the DM consumption allocation, $x_E^2$, satisfies $x = \beta[u(x)) - b]$ and

$$x^* > \beta[u(x^*) - b].$$

Comparing these results with those derived in sections 3.1 and 3.2, it is easy to see that the equilibrium allocation when $\hat{v} > 0$ is the same as that of the unregulated economy when the incentive constraint binds, and, when it doesn't, the necessary and sufficient conditions for the incentive constraint to be slack are the same.

With respect to the results obtained in section 4 when $\hat{v} > 0$, the consumption allocation $x_E^1$ is worse when the incentive constraint binds, and, when it doesn't, the necessary and sufficient conditions for the incentive constraint to be slack are stricter, in the sense that they are satisfied for a smaller set of primitives. Indeed, recall that the equation characterizing the equilibrium allocation in section 4.2 is

$$x = (1-\rho)u(x) + \beta^2 B u'(x),$$

which, for any $B > 0$, pins down $x > x_E^1$. Also, we showed in section 4.1 that the incentive constraint is slack if and only if $x^* \leq (1-\rho)u(x^*) + \beta^2 B u'(x^*)$, which, for any $B > 0$, is more easily satisfied than (16).

Comparing (17) with our results from sections 4.1 and 4.2 when $\hat{v} = 0$, the consumption allocation $x_E^2$ is worse when the incentive constraint binds, and, when it doesn't, the necessary and sufficient conditions for the incentive constraint to be slack are stricter, in the sense that they are satisfied for a smaller set of primitives. Indeed, recall that the equation characterizing the equilibrium allocation in section 4.2 is $x = \beta u(x)$, which pins down $x > x_E^2$. Also, we showed in section 4.1 that the incentive constraint is slack if and only if $x^* \leq \beta u(x^*)$, which is more easily satisfied than (17) for any $b > 0$.

### 9.2.2 Membership titles as licenses to issue debt

Focusing on stationary equilibria, consider membership titles as entitling their buyer to issue debt in the same amount as the titles purchased at a market clearing price $q$. In order to focus on the role played by membership titles through the continuation values $v^i$ and $\hat{v}^i$, we assume that the resources gathered from CM payment of the titles are disposed of, as in the previous section.

The decision problem of an issuer is:

$$\begin{aligned} v \;&= \max_{\{l,x\}} \quad -qx + u(x) - l + \beta v \\ &\quad s.t. \qquad x \leq l \\ &\qquad\qquad\quad l \leq \beta(v - \hat{v}) \end{aligned}$$

where $q$ clears the market for membership titles so that its demand, $x$, equals its supply $B$. It is easy to see that the optimal choice of $x$ satisfies $u'(x) = 1 + q$ and that the continuation value of default is $\hat{v} = \frac{-qx+\rho u(x)}{1-\beta}$.

Equilibria with the incentive constraint slack and $\hat{v} > 0$ are such that

$$\begin{aligned} v &= \frac{-qx + u(x^*) - x^*}{1-\beta} \\ \hat{v} &= \frac{-qx + \rho u(x^*)}{1-\beta} > 0 \\ x^* &\leq \beta(1-\rho)u(x^*) \qquad (18) \end{aligned}$$

while equilibria with a binding incentive constraint and $\hat{v} > 0$ are such that the DM consumption allocation, $x_E^3$, satisfies $x\beta(1-\rho)u(x)$ and $x^* > \beta(1-\rho)u(x^*)$. Hence, the equilibrium allocation $x_E^3 = x_E^1$, which is the same as that of the unregulated economy when the incentive constraint binds, as derived in section 3.2. When the incentive constraint is slack, however, the equilibrium allocation must satisfy the market clearing condition

$$x = u^{'-1}(1+q) = B$$

Because in equilibrium $q > 0$, any supply of membership titles $B$ chosen by the mechanism will implement an allocation, say $\tilde{x}$, such that $\tilde{x} < x^*$. Hence, when the incentive constraint is slack and $\hat{v} > 0$, welfare is higher in the unregulated economy of section 3.1.

Equilibria with the incentive constraint slack and $\hat{v} = 0$ are such that

$$\begin{aligned} v &= \frac{u(\tilde{x}) - (1+q)\tilde{x}}{1-\beta} \\ q\tilde{x} &\geq \rho u(\tilde{x}) \\ \tilde{x}(1+q\beta) &\leq \beta u(\tilde{x}) \end{aligned} \tag{19}$$

where $\tilde{x} < x^*$ and $q = u'(\tilde{x}) - 1$, as discussed above. Therefore, if the conditions for an equilibrium in the unregulated economy of section 3.1 are satisfied, welfare is higher with no regulation.

The equilibrium allocation with a binding incentive constraint and $\hat{v} = 0$, $x_E^4$, satisfies

$$x(1+q\beta) \quad = \quad \beta u(x) \tag{20}$$

with $q = u'(x) - 1$. In the specific case of $u(x) = \frac{x^{1-\sigma}}{1-\sigma}$, it is easy to see that $x_E^4 > x_E^3$, that is to say the equilibrium allocation with $\hat{v} = 0$ is larger than that with $\hat{v} > 0$. Since $x_E^3 = x_E^1$, which was the same consumption allocation as in the unregulated economy described in section 3.2, we conclude that consumption is higher with the proposed regulation via membership titles as licenses to issue debt when the incentive constraint binds and with $\hat{v} = 0$.

# 10 Conclusion

# Appendices

## A Segregated regulation

### A.1 Proof of Proposition 1

**Proof.**

Consider first the case where $\hat{v} > 0$. Then

$$\begin{aligned} v^i &= \frac{-q^i b^i + U(x^*) - (x^* - \beta B^i) - \beta\tau}{1-\beta} \\ & x^* \leq l^i + \beta B^i \\ & l^i + \beta B^i \left(1 - q^i\right) \leq \beta \left(v^i - \hat{v}^i\right) \end{aligned}$$

which, substituting the FOC for $b^i$ ($q^i = \beta$) yields:

$$v^i = \frac{U(x^*) - x^*}{1-\beta} - \beta B^i$$

and $\hat{v}^i = \frac{-q^i b^i + \rho^i U(x^*)}{1-\beta}$ so that the IC is slack if and only if

$$x^* - \beta b^i + \beta B^i \left(1 - q^i\right) \leq \beta \left(1 - \rho^i\right) U(x^*)$$

that is

$$x^* - q^i \beta B^i \leq \beta \left(1 - \rho^i\right) U(x^*)$$

If $l^i = 0$ then $\beta B^i = x^*$ and

$$v^i = \frac{U(x^*) - x^*}{1-\beta} - x^*$$

So, if $B^i < \frac{x^*}{\beta}$ then private uninsured liabilities are also issued. Otherwise the mechanism can choose $\beta B^i = x^*$ and attain the efficient allocation.

However, for $\hat{v}^i = \frac{-q^i b^i + \rho^i U(x^*)}{1-\beta}$ it must be that $\rho^i$ is sufficiently large or $B^i$ sufficiently small.

$$\rho^i U(x^*) > \beta B^i$$

Overall we have that the NSC are

$$\begin{aligned} \beta B^i &< \rho^i U(x^*) \\ x^* - q^i \beta B^i &< \beta \left(1 - \rho^i\right) U(x^*) \\ \beta B^i &\leq x^* \end{aligned}$$

where the last inequality just shows that the mechanism will choose $B^i$ at most to fund the efficient allocation.

In $U(x) = \frac{x^{1-\sigma}}{1-\sigma}$ these are

$$\begin{aligned} \beta B^i &< \frac{\rho^i}{1-\sigma} \\ \frac{1}{\beta} - \frac{(1-\rho^i)}{1-\sigma} &< \beta B^i \\ \beta B^i &\leq 1 \end{aligned}$$

Consider now the case where $\hat{v} = 0$. Then

$$v^i = \frac{U(x^*) - x^*}{1-\beta} - \beta B^i$$

but with $\hat{v}^i = 0$ the IC is slack if and only if

$$\begin{aligned} l^i + \beta B^i \left(1 - q^i\right) &< \beta \frac{U(x^*) - x^*}{1-\beta} - \beta^2 B^i \\ x^* &< \beta U(x^*) \end{aligned}$$

If $l^i = 0$ then $\beta B^i = x^*$ and

$$v^i = \frac{U(x^*) - x^*}{1-\beta} - x^*$$

So, if $B^i < \frac{x^*}{\beta}$ then private uninsured liabilities are also issued. Otherwise the mechanism can choose $\beta B^i = x^*$ and attain the efficient allocation.

However, for $\hat{v}^i = 0$ it must be that $\rho^i$ is sufficiently small.

$$\rho^i U(x^*) \leq \beta B^i$$

Overall the NSC are

$$\begin{aligned} x^* &< \beta U(x^*) \\ \rho^i U(x^*) &\leq \beta B^i \end{aligned}$$

and still $\beta B^i \leq x^*$.

These, in $U(x) = \frac{x^{1-\sigma}}{1-\sigma}$, are

$$\begin{aligned} 1-\sigma &< \beta \\ \rho^i &\leq (1-\sigma)\beta B^i \end{aligned}$$

and $\beta B^i \leq 1$.

■

## A.2 Proof of proposition 2

**Proof.**

The buyer's decision problem is

$$\begin{aligned} v^i &= \max\left\{-q^i b^i + U\left(x^i\right) - l^i - \beta\tau^i + \beta v^i\right\} \\ & x^i \leq l^i + \beta b^i \\ & l^i + \beta\tau^i \leq \beta v^i \end{aligned}$$

with mechanism resource constraint

$$\tau^i = B^i(1-q)$$

with $\hat{v}^i = 0 > \frac{-q^i b^i + \rho^i U\left(x^i\right)}{1-\beta}$. then, using binding IC, we have

$$v^i = -q^i b^i + U\left(x^i\right)$$

which, plugged back into the IC to solve for $x^i$ yields

$$\begin{aligned} l^i + \beta\tau^i &= -q^i\beta b^i + \beta U\left(x^i\right) \\ x^i &= \beta U\left(x^i\right) \end{aligned}$$

where verifying that the IC binds requires

$$x^* > \beta U(x^*)$$

and verifying that $\hat{v}^i = 0 \geq \frac{-q^i b^i + \rho^i U\left(x^i\right)}{1-\beta}$requires

$$\rho^i U\left(x^i\right) \leq \beta U'\left(x^i\right) B^i$$

in $U(x) = \frac{x^{1-\sigma}}{1-\sigma}$ these are

$$1 > \sigma + \beta$$

and

$$\rho^i \frac{\left(x^i\right)^{1-\sigma}}{1-\sigma} \leq \beta\left(x^i\right)^{-\sigma} B^i$$

that can be rearranged as

$$x^i \leq \beta B^i \frac{(1-\sigma)}{\rho^i}$$

which, using the equation defining $x^i$, is:

$$\beta B^i \frac{(1-\sigma)}{\rho^i} \geq \beta U\left(\beta B^i \frac{(1-\sigma)}{\rho^i}\right)$$

or, in $U(x) = \frac{x^{1-\sigma}}{1-\sigma}$

$$\left(\frac{B^i}{\rho^i}\right)^{\sigma} \geq \beta^{(1-\sigma)} \frac{1}{(1-\sigma)^{1+\sigma}}$$

that is $B^i \geq \underline{B} = \rho^i \left(\frac{\beta^{(1-\sigma)}}{(1-\sigma)^{1+\sigma}}\right)^{\frac{1}{\sigma}}$

Also, $x^i = \left(\frac{\beta}{(1-\sigma)}\right)^{\frac{1}{\sigma}}$. ■

## A.3 Proof of proposition 3

**Proof.**

$$\begin{aligned} v^i &= \max\left\{-q^i b^i + U\left(x^i\right) - l^i - \beta\tau^i + \beta v^i\right\} \\ & \quad x^i \leq l^i + \beta b^i \\ & \quad l^i + \beta\tau^i \leq \beta\left(v^i - \hat{v}^i\right) \end{aligned}$$

with mechanism resource constraint

$$\tau^i = B^i (1-q)$$

with $\hat{v}^i = \frac{-q^i b^i + \rho^i U\left(x^i\right)}{1-\beta} > 0$. With the binding IC we have

$$v^i = -q^i B^i + U\left(x^i\right) + \beta\hat{v}^i$$

then

$$\begin{aligned} v^i - \hat{v}^i &= -q^i B^i + U\left(x^i\right) - (1-\beta)\,\hat{v}^i \\ &= \left(1-\rho^i\right) U\left(x^i\right) \end{aligned}$$

By mkt clearing $b^i = B^i$, $l^i = x^i - \beta B^i$ and the binding IC we have

$$x^i - q\beta B^i = \beta\left(1-\rho^i\right) U\left(x^i\right)$$

which, using also the FOC for $b^i$, is

$$x^i - U'\left(x^i\right)\beta^2 B^i = \beta\left(1-\rho^i\right) U\left(x^i\right)$$

and we need to check that such $x^i < x^*$ and that $-q^i b^i + \rho^i U\left(x^i\right) > 0$.

In $U(x) = \frac{x^{1-\sigma}}{1-\sigma}$ case the equation pinning down $x^i$ is then

$$1 - \left(x^i\right)^{-(1+\sigma)} \beta^2 B^i = \beta\left(1-\rho^i\right) \frac{\left(x^i\right)^{-\sigma}}{1-\sigma} \tag{21}$$

Where the left hand side is monotonically increasing, with

$$\begin{aligned}\lim_{x\to 0} 1-\left(x^i\right)^{-(1+\sigma)}\beta^2 B^i &= -\infty \\ \lim_{x\to \infty} 1-\left(x^i\right)^{-(1+\sigma)}\beta^2 B^i &= 1\end{aligned}$$

while the right hand side is monotonically decreasing, with

$$\begin{aligned}\lim_{x\to 0} \beta\left(1-\rho^i\right)\frac{\left(x^i\right)^{-\sigma}}{1-\sigma} &= \infty \\ \lim_{x\to \infty} \beta\left(1-\rho^i\right)\frac{\left(x^i\right)^{-\sigma}}{1-\sigma} &= 0\end{aligned}$$

Therefore there exists a unique $x_E$ solving (21).

Checking that $x^i < x^*$ requires

$$1-\left(x^*\right)^{-(1+\sigma)}\beta^2 B^i > \beta\left(1-\rho^i\right)\frac{\left(x^*\right)^{-\sigma}}{1-\sigma}$$

that is to say

$$\frac{1}{\beta}-\left(1-\rho^i\right)\frac{1}{1-\sigma} > \beta B^i$$

for which it is necessary that $\frac{1}{\beta} > \left(1-\rho^i\right)\frac{1}{1-\sigma}$, that can be rearranged as

$$1 > \sigma+\beta\left(1-\rho^i\right)$$

while checking that $-q^i b^i+\rho^i U\left(x^i\right) > 0$ requires

$$\begin{aligned}\rho^i\frac{\left(x^i\right)^{1-\sigma}}{1-\sigma} &> \beta\left(x^i\right)^{-\sigma} B^i \\ x^i &> \beta B^i\frac{(1-\sigma)}{\rho^i}\end{aligned}$$

that is to say

$$1-\left(\beta B^i\frac{(1-\sigma)}{\rho^i}\right)^{-(1+\sigma)}\beta^2 B^i < \beta\left(1-\rho^i\right)\frac{\left(\beta B^i\frac{(1-\sigma)}{\rho^i}\right)^{-\sigma}}{1-\sigma}$$

which can be rearranged as

$$\left(\beta B^i\right)^{\sigma} < \beta\left(1-\rho^i\right)\left(\rho^i\right)^{\sigma}(1-\sigma)^{-(1+\sigma)}+\beta\left(\frac{\rho^i}{(1-\sigma)}\right)^{(1+\sigma)}$$

Together with the upper bound on $B^i$ set by $x^i < x^*$ we have

$$\left(\beta B^i\right)^{\sigma} < \min\left(\left(\frac{1}{\beta}-\left(1-\rho^i\right)\frac{1}{1-\sigma}\right)^{\sigma}, \beta\left(1-\rho^i\right)\left(\rho^i\right)^{\sigma}(1-\sigma)^{-(1+\sigma)}+\beta\left(\frac{\rho^i}{(1-\sigma)}\right)^{(1+\sigma)}\right)$$

■

# B Merged regulation

## B.1 Proof of proposition 4

**Proof.**

From The decision problem we have

$$v^i = \frac{-qb^i + U(x^i) - x^* + \beta b^i - \beta \frac{B}{2}(1-q)}{1-\beta}$$

$$x^* - \beta b^i + \beta \frac{B}{2}(1-q) \leq \beta v^i$$

and IC is slack IFF

$$\begin{aligned} x^* - \beta b^i + \beta \frac{B}{2}(1-q) &\leq \beta \frac{-qb^i + U(x^i) - x^* + \beta b^i - \beta \frac{B}{2}(1-q)}{1-\beta} \\ x^* - \beta b^i + \beta \frac{B}{2}(1-q) &\leq \beta \left[-qb^i + U(x^*)\right] \end{aligned}$$

where we can susbtitute $q = \beta$ to have simply

$$\frac{x^*}{\beta} + \left(\frac{B}{2} - b^i\right)(1-\beta) \leq U(x^*)$$

If $-\beta b^i + \rho^c U(x^*) \leq 0$ then $\hat{v}^i = 0\ \forall i$:

$$\rho^c U(x^*) \leq \beta b^i$$

One possible equilibrium allocation is for both sectors to choose the same portfolio composition between $b^i$ and $l^i$, in which case $b^i = \frac{B}{2}$ and the IC is slack if and only if $x^* \leq \beta U(x^*)$. Combining the necessary and sufficient conditions yields

$$\rho^i U(x^*) \leq \beta b^i \leq x^* \leq \beta U(x^*)$$

where the second inequality follows from buyers never purchasing more membership titles than what's necessary to purchase $x^*$.

So a necessary condition is $\beta \geq \rho^c > \rho^t$.

In $U(x) = \frac{x^{1-\sigma}}{1-\sigma}$ these conditions are

$$\frac{\rho^c}{1-\sigma} \leq \beta b^i \leq 1 \leq \frac{\beta}{1-\sigma}$$

■

## B.2 Proof of proposition 5

**Proof.** From The decision problem we have

$$v^i = \frac{-qb^i + U(x^*) - (x^* - \beta b^i) - \beta\tau}{1-\beta}$$

$$\left(x^* - \beta b^i\right) + \beta\tau \leq \beta\left(v^i - \hat{v}^i\right)$$

with $\hat{v}^i = \frac{-qb^i + \rho^i U\left(x^i\right)}{1-\beta}$

Then:

$$\left(v^i - \hat{v}^i\right) = \frac{\left(1-\rho^i\right) U\left(x^*\right) - \left(x^* - \beta b^i\right) - \beta\tau}{1-\beta}$$

So the IC is slack IFF

$$\left(x^* - \beta b^i\right) + \beta\tau \leq \beta\left(\frac{\left(1-\rho^i\right) U\left(x^*\right) - \left(x^* - \beta b^i\right) - \beta\tau}{1-\beta}\right)$$

that can be rearranged as

$$x^* - \beta b^i + \beta\tau \leq \beta\left(1-\rho^i\right) U\left(x^*\right)$$

and, using also the mechanism's resource constraint and $q = \beta$

$$x^* - \beta b^i + \beta\frac{B}{2}\left(1-\beta\right) \leq \beta\left(1-\rho^i\right) U\left(x^*\right)$$

which can be rearranged as

$$x^* + \beta\frac{B}{2}\left(1-\beta\right) - \beta\left(1-\rho^i\right) U\left(x^*\right) \leq \beta b^i$$

Then $\hat{v}^i > 0$ iff $-qb^i + \rho^i U\left(x^i\right) > 0$ that is

$$\rho^i U\left(x^*\right) > \beta b^i$$

Hence, combining them yields

$$x^* + \beta\frac{B}{2}\left(1-\beta\right) - \beta\left(1-\rho^i\right) U\left(x^*\right) \leq \beta b^i < \rho^i U\left(x^*\right)$$

For these to be satisfied it is necessary that

$$\begin{aligned} x^* + \beta\frac{B}{2}\left(1-\beta\right) - \beta\left(1-\rho^i\right) U\left(x^*\right) &< \rho^i U\left(x^*\right) \\ x^* + \beta\frac{B}{2}\left(1-\beta\right) &< \left(\left(1-\beta\right)\rho^i + \beta\right) U\left(x^*\right) \end{aligned}$$

■

## B.3 Proof of proposition 6

**Proof.** Without loss of generality let $-qb^i + \rho^i U\left(x^*\right) > 0$ and $-qb^j + \rho^j U\left(x^*\right) \leq 0$. The incentive constraints are slack if and only if:

$$\begin{aligned} x^* - \beta b^i + \beta\frac{B}{2}\left(1-\beta\right) &\leq \beta\left(1-\rho^i\right) U\left(x^*\right) \\ x^* + \beta\left(\frac{B}{2} - b^j\right)\left(1-\beta\right) &\leq \beta U\left(x^*\right) \end{aligned}$$

where $q = \beta$ has been used because the incentive constraint is slack, and now the incentive constraints are different from each other due to $\hat{v}^j = 0$ and $\hat{v}^i > 0$. The necessary and sufficient conditions for slack incentive constraints can be rearranged as

$$\begin{aligned} x^* + \beta \frac{B}{2}(1-\beta) - \beta\left(1-\rho^i\right) U(x^*) &\leq \beta b^i \\ x^* + \beta \frac{B}{2}(1-\beta) - \beta U(x^*) &\leq \beta b^j (1-\beta) \end{aligned}$$

The conjectures $\hat{v}^j = 0$ and $\hat{v}^i > 0$ are verified if and only if

$$\begin{aligned} \rho^i U(x^*) &> \beta b^i \\ \rho^j U(x^*) &\leq \beta b^j \end{aligned}$$

So, combining them, yields[27]

$$\begin{aligned} x^* + \beta \frac{B}{2}(1-\beta) - \beta\left(1-\rho^i\right) U(x^*) &\leq \beta b^i < \rho^i U(x^*) \\ \max\left(\rho^j U(x^*), \frac{x^* + \beta \frac{B}{2}(1-\beta) - \beta U(x^*)}{(1-\beta)}\right) &\leq \beta b^j \end{aligned}$$

We can rearranged these conditions as

$$\begin{aligned} x^* - \beta b^i + \beta \frac{B}{2}(1-\beta) &\leq \beta\left(1-\rho^i\right) U(x^*) \\ \beta b^i &< \rho^i U(x^*) \end{aligned}$$

for type $i$ and

$$\begin{aligned} x^* - \beta b^j (1-\beta) + \beta \frac{B}{2}(1-\beta) &\leq \beta U(x^*) \\ \beta b^j &\geq \rho^j U(x^*) \end{aligned}$$

for type $j$. ■

## B.4 Proof of proposition 7

**Proof.** In a symmetric equilibrium there is no distinction between the types of meetings.

[27] A necessary condition for the first set of inequalities is $\left(\rho^i + \beta\left(1-\rho^i\right)\right) U(x^*) > x^* + \beta \frac{B}{2}(1-\beta)$. If $U(c) = \frac{x^{1-\sigma}}{1-\sigma}$ this is

$$\begin{aligned} \left((1-\beta)\rho^i + \beta\right) \frac{1}{1-\sigma} &> 1 + \beta \frac{B}{2}(1-\beta) \\ \sigma &> (1-\beta)\left[\beta \frac{B}{2}(1-\sigma) + \left(1-\rho^i\right)\right] \end{aligned}$$

The binding IC is then $l^i + \beta\tau = \beta v^i$ with

$$v^i = -qb^i + U\left(x^i\right)$$

and $q = \beta U'\left(x^i\right)$. This implies $x^i = x^j = \overline{x}$.

We can then substitute into the binding IC to get

$$l^i + \beta\tau = \beta\left[-qb^i + U\left(\overline{x}\right)\right]$$

and using also $\tau = \frac{B}{2}\left(1-q\right)$ and $l^i + \beta b^i = x^i$ we have

$$\overline{x} - \beta b^i + \beta\frac{B}{2}\left(1-q\right) = \beta\left[-qb^i + U\left(\overline{x}\right)\right]$$

Summing then over $i$ yields

$$2\overline{x} - \beta B + \beta B\left(1-q\right) = \beta\left[-qB + 2U\left(\overline{x}\right)\right]$$

We can solve this equation for $\overline{x}$:

$$\overline{x} = \beta U\left(\overline{x}\right)$$

Notice that for this case to possibly arise (ie for $\hat{v} = 0$ to possibly be the case), we need $b$ – check with CW individual punishment without gov debt).

And for this to be an equilibrium it must be $qb^i \geq \rho^i U\left(\overline{x}\right)$ and $\overline{x} < x^*$. Using the equation defining $\overline{x}$, we have that $\overline{x} < x^*$ if and only if

$$\beta U\left(x^*\right) < x^*$$

■

## B.5 Proof of proposition 8

**Proof.**

Buyers' decision problem is:

$$\begin{aligned}
v^i &= -qb^i + U\left(x^i\right) + \beta\frac{-qb^i + \rho^i U\left(x^i\right)}{1-\beta} \\
\left(v^i - \hat{v}^i\right) &= -qb^i + U\left(x^i\right) - \left(1-\beta\right)\hat{v}^i \\
&= \left(1-\rho^i\right)U\left(x^i\right) \\
l^i + \beta\tau &= \beta\left(1-\rho^i\right)U\left(x^i\right)
\end{aligned}$$

with $\tau = \frac{B}{2}\left(1-q\right)$ and $\hat{v}^i = \frac{-qb^i + \rho^i U\left(x^i\right)}{1-\beta}$. From the first order condition for $b$ we know that $q = \beta U'\left(x\right)$ hence, if both $b^i > 0$ then consumption is the same. However, $v^i$ might differ because $b^i, l^i$ differ. Using also $\tau = \frac{B}{2}\left(1-q\right)$ we get

$$l^i + \beta\frac{B}{2}\left(1-q\right) = \beta\left(1-\rho^i\right)U\left(x^i\right)$$

$$x-\beta b^{i}+\beta\frac{B}{2}\left(1-q\right) \;=\; \beta\left(1-\rho^{i}\right)U\left(x\right)$$

We can do the same for $j$ and then add the two equations and use $\left(B=b^{i}+b^{j}\right)$. So we have

$$2x+\beta B\left(1-q\right)-\beta U\left(x\right)\left(2-\rho^{j}-\rho^{i}\right) \;=\; \beta B$$

which we can then solve for $x$:

**Case $U(x)=\frac{x^{1-\sigma}}{1-\sigma}$.** If $U(x)=\frac{x^{1-\sigma}}{1-\sigma}$ this is

$$\begin{aligned} 2x-\beta\frac{x^{1-\sigma}}{1-\sigma}\left(2-\rho^{j}-\rho^{i}\right) &= \beta^{2}x^{-\sigma}B \\ 2x^{1+\sigma} &= \beta^{2}B+\beta\frac{x\left(2-\rho^{j}-\rho^{i}\right)}{1-\sigma} \end{aligned}$$

We assumed $\sigma<1$ so the left hand side is a convex function and the right hand side a linear function of $x$. Let $fL\left(x\right)=2x^{1+\sigma}$ and $fR\left(x\right)=\beta^{2}B+\beta\frac{x}{1-\sigma}\left(2-\rho^{j}-\rho^{i}\right)$. Then

$$\begin{aligned} x=0 &\Rightarrow fL\left(x\right)=0, fR\left(x\right)=\beta^{2}B \\ x\rightarrow\infty &\Rightarrow fL\left(x\right)\rightarrow\infty, fR\left(x\right)\rightarrow\infty \end{aligned}$$

but as $x\rightarrow\infty$ $fL$ has steeper slope so it will grow to infinity faster:

$$\begin{aligned} fL'\left(x\right) &= \frac{\beta}{1-\sigma}\left(2-\rho^{j}-\rho^{i}\right) \\ fR'\left(x\right) &= 2\left(1+\sigma\right)x^{\sigma} \end{aligned}$$

with $\sigma<1$ $fL'$ is always positive, while $fR'\left(x\right)>0$ for all $x>0$, but $fR'\left(0\right)=0$, so let $x_{0}$ denoting the value of $x$ such that $fL'\left(x_{0}\right)=fR'\left(x_{0}\right)$. That is

$$x_{0} \;=\; \left[\frac{\beta}{1-\sigma}\frac{\left(2-\rho^{j}-\rho^{i}\right)}{2\left(1+\sigma\right)}\right]^{\frac{1}{\sigma}}>0$$

Hence, by the intermediate value theorem, there exists $x_{E}\in\left(0,\infty\right)$ such that $fL\left(x_{E}\right)=fR\left(x_{E}\right)$, that is to say that pins down the consumption allocation for this candidate equilibrium. We can then get $l^{i}$ from the IC

$$\left(l^{i}+\beta\tau\right) \;=\; \beta\left(1-\rho^{i}\right)U\left(x\right)$$

which implies that $l^{i}$ decreases in $\rho^{i}$, so $l^{t}>l^{c}\geq0$.

**Verify that IC binds** This requires

$$2\left(x^{*}\right)^{1+\sigma} \;>\; \beta^{2}B+\beta\frac{x^{*}\left(2-\rho^{j}-\rho^{i}\right)}{1-\sigma}$$

which, subsituting $x^{*}=1$, is

$$2^{1+\sigma} \;>\; \beta^{2}B+\beta\frac{\left(2-\rho^{j}-\rho^{i}\right)}{1-\sigma}$$

In terms of general $U$ it is

$$\frac{2x}{U'\left(x\right)} \;>\; \beta^{2}B+\beta\frac{U\left(x\right)}{U'\left(x\right)}\left(2-\rho^{j}-\rho^{i}\right)$$

**Verify $\hat{v}^i > 0$ for all $i$** We can then verify that $-qb^i + \rho^i U(x^i) > 0$ for all $i$

$$\begin{aligned}
\rho^i \frac{x_E^{1-\sigma}}{1-\sigma} &> \beta x_E^{-\sigma} b^i, \forall i \\
\rho^i \frac{x_E}{1-\sigma} &> \beta b^i, \forall i
\end{aligned}$$

which, substituting out $b^i$, is[28]

$$\begin{aligned}
\rho^i \frac{U(x)}{U'(x)} &> x - \beta\left(1-\rho^i\right) U(x) + \beta \frac{B}{2}\left(1 - \beta U'(x)\right) \\
\rho^i \frac{U(x)}{xU'(x)} &> 1 - \beta\left(1-\rho^i\right) \frac{U(x)}{x} + \beta \frac{B}{2}\left(\frac{1}{x} - \beta \frac{U'(x)}{x}\right) \\
\frac{\rho^i}{(1-\sigma)} + \beta\left(1-\rho^i\right) \frac{U'(x)}{(1-\sigma)} - 1 &> \beta \frac{B}{2}\left(\frac{1-\beta U'(x)}{x}\right) \\
\beta \frac{B}{2} &< \frac{\frac{\rho^i}{(1-\sigma)} + \beta\left(1-\rho^i\right)\frac{U'(x)}{(1-\sigma)} - 1}{\left(\frac{1-\beta U'(x)}{x}\right)}
\end{aligned}$$

that yields

$$\begin{aligned}
\beta \frac{B}{2} &< \frac{x \frac{\rho^i}{(1-\sigma)}}{(1-\beta U'(x))} + \beta \frac{(1-\rho^i)}{(1-\sigma)} \frac{xU'(x)}{(1-\beta U'(x))} - \frac{x}{(1-\beta U'(x))} \\
&< \frac{x}{(1-\beta U'(x))}\left[\frac{\rho^i}{(1-\sigma)} - 1\right] + \beta\left(1-\rho^i\right) \frac{U(x)}{(1-\beta U'(x))} \qquad (22)
\end{aligned}$$

Notice

1. that $\beta U'(x) \leq 1$ always, otherwise $\tau < 0$ and all buyers want to contribute, so the IC would not involve $\tau$.

2. if $\rho^i < 1 - \sigma$ then a sufficient condition for (22) is

$$\beta \frac{B}{2} \leq \beta\left(1-\rho^i\right) \frac{U(x)}{(1-\beta U'(x))} \qquad (23)$$

   as the first term in (22) is strictly positive.

[28] We derive $b^i$ from $x_E = l^i + \beta b^i$ and the IC binding, $l^i + \beta\tau = \beta\left(1-\rho^i\right) U(x)$, with $\tau = \frac{B}{2}\left(1 - \beta U'(x)\right)$. Hence:

$$x - \beta b^i + \beta \frac{B}{2}\left(1 - \beta U'(x)\right) = \beta\left(1-\rho^i\right) U(x)$$

from which we get

$$x - \beta\left(1-\rho^i\right) U(x) + \beta \frac{B}{2}\left(1 - \beta U'(x)\right) = \beta b^i.$$

3. that $\frac{1}{(1-\beta U'(x))}$ is decreasing in $x$, so a lower bound for it is $\frac{1}{(1-\beta U'(x^*))}$

Therefore, if $\rho^i < 1-\sigma$, a sufficient condition for (23) is

$$\frac{B}{2} \leq \frac{(1-\rho^i)}{(1-\sigma)(1-\beta)} x^{(1-\sigma)} \tag{24}$$

This can be further rearranged as a lower bound on $x_E$:

$$x_E \geq \left(\frac{B}{2}\frac{(1-\sigma)(1-\beta)}{(1-\rho^i)}\right)^{\frac{1}{1-\sigma}}$$

which, using the equation defining $x_E$, is equivalent to:

$$2\left(\frac{B}{2}\frac{(1-\sigma)(1-\beta)}{(1-\rho^i)}\right)^{\frac{1+\sigma}{1-\sigma}} \leq \beta^2 B + \beta\frac{(2-\rho^j-\rho^i)}{1-\sigma}\left(\frac{B}{2}\frac{(1-\sigma)(1-\beta)}{(1-\rho^i)}\right)^{\frac{1}{1-\sigma}}$$

A sufficient condition for this is

$$2\left(\frac{B}{2}\frac{(1-\sigma)(1-\beta)}{(1-\rho^i)}\right)^{\frac{1+\sigma}{1-\sigma}} \leq \beta\frac{(2-\rho^j-\rho^i)}{1-\sigma}\left(\frac{B}{2}\frac{(1-\sigma)(1-\beta)}{(1-\rho^i)}\right)^{\frac{1}{1-\sigma}}$$

that can be rearranged as

$$\begin{aligned}
2\left(\frac{B}{2}\right)^{1+\sigma}(1-\sigma)^{\frac{2}{1-\sigma}}\left(\frac{(1-\beta)}{(1-\rho^i)}\right)^{\frac{1+\sigma}{1-\sigma}} &\leq \beta\left(2-\rho^j-\rho^i\right)(1-\sigma)^{\frac{1}{1-\sigma}}\left(\frac{(1-\beta)}{(1-\rho^i)}\right)^{\frac{1}{1-\sigma}} \\
\left(\frac{B}{2}\right)^{1+\sigma}(1-\sigma)^2\left(\frac{(1-\beta)}{(1-\rho^i)}\right)^{1+\sigma} &\leq \beta\left(1-\frac{\rho^j}{2}-\frac{\rho^i}{2}\right)
\end{aligned}$$

that can be finally rewritten as an upper bound on $B$:

$$\frac{B}{2}\frac{(1-\beta)}{(1-\rho^i)} \leq \left[\beta\frac{\left(1-\frac{\rho^j}{2}-\frac{\rho^i}{2}\right)}{(1-\sigma)^2}\right]^{\frac{1}{1+\sigma}}$$

Alternatively, and especially if $\rho^i \geq 1-\sigma$, then we can use $\rho^i\frac{U(x_E)}{U'(x_E)} \geq \beta B$ as a sufficient condition for $\hat{v}^i > 0$, because $b^i \leq B$ by feasibility. With $U(x) = \frac{x^{1-\sigma}}{1-\sigma}$ this is simply

$$\rho^i\frac{x_E}{1-\sigma} \geq \beta B$$

which sets a lower bound on $x_E$: $x_E \geq \frac{\beta B}{\rho^i}(1-\sigma)$. Thus, using the equation defining $x_E$ we have

$$\begin{aligned}
2\left(\frac{\beta B}{\rho^i}(1-\sigma)\right)^{1+\sigma} &\leq \beta^2 B + \beta\frac{(2-\rho^j-\rho^i)}{1-\sigma}\frac{\beta B}{\rho^i}(1-\sigma) \\
\left(\frac{\beta B}{\rho^i}\right)^{\sigma} &\leq \frac{(2-\rho^j)}{2(1-\sigma)^{1+\sigma}}
\end{aligned}$$

which also sets an upper bound on $B$. ■

## B.6 Proof of proposition 9

**Proof.**

From the decision problem we have

$$\begin{aligned} v^i &= \frac{-qb^i + U(x^i) - l^i - \beta\tau}{1-\beta} \\ & l^i + \beta\tau = \beta\left(v^i - \hat{v}^i\right) \\ & l^j + \beta\tau = \beta v^j \end{aligned}$$

with $\tau = \frac{B}{2}(1-q)$ and $\hat{v}^i = \frac{-qb^i + \rho^i U\left(x^i\right)}{1-\beta}$ and $x^i = x^j = x$ because $b^i > 0$ for all $i$. Then

$$v^j = -qb^j + U(x)$$

and the IC for type $j$ is

$$l^j + \beta\tau = \beta\left[-qb^j + U(x)\right]$$

while the IC for type $i$ is

$$\begin{aligned} l^i + \beta\tau &= \beta\frac{\left[\left(1-\rho^i\right)U(x) - \left(l^i - \beta\tau\right)\right]}{1-\beta} \\ l^i + \beta\tau &= \beta\left(1-\rho^i\right)U(x) \end{aligned}$$

We can then use $l^i = x - \beta b^i$ for both types, to get that the ICs are[29]

$$\begin{aligned} x - \beta b^j + \beta\frac{B}{2}(1-q) &= \beta\left[-qb^j + U(x)\right] \\ x - \beta b^i + \beta\frac{B}{2}(1-q) &= \beta\left(1-\rho^i\right)U(x) \end{aligned}$$

we can rearrange these as

$$\beta b^j = \frac{x - \beta U(x)}{(1-q)} + \beta\frac{B}{2}$$

and

$$\beta b^i = x - \beta\left(1-\rho^i\right)U(x) + \beta\frac{B}{2}(1-q)$$

Then summing them up and using market clearing yields

$$\begin{aligned} \beta B\frac{q}{2} &= x\left(1 + \frac{1}{1-q}\right) - \beta U(x)\left[\left(1-\rho^i\right) + \frac{1}{1-q}\right] \\ \beta B\frac{q}{2} &= x\frac{2-q}{1-q} - \beta U(x)\frac{(1-q)\left(1-\rho^i\right)+1}{1-q} \\ x(2-q) &= \beta B\frac{q(1-q)}{2} + \beta U(x)\left[(1-q)\left(1-\rho^i\right)+1\right] \end{aligned}$$

[29] Notice that these two IC are identical if $-\beta\rho^i U(x) = -\beta q b^j$.

if $U(x)=\frac{x^{1-\sigma}}{1-\sigma}$ this is[30]

$$\begin{aligned}
\beta B\frac{\beta x^{-\sigma}\left(1-\beta x^{-\sigma}\right)}{2} &= x\left(2-\beta x^{-\sigma}\right)-\beta\frac{x^{1-\sigma}}{1-\sigma}\left[\left(1-\beta x^{-\sigma}\right)\left(1-\rho^{i}\right)+1\right] \\
\beta B\frac{\beta\left(1-\beta x^{-\sigma}\right)}{2} &= x^{1+\sigma}\left(2-\beta x^{-\sigma}\right)-\beta\frac{x}{1-\sigma}\left[\left(1-\beta x^{-\sigma}\right)\left(1-\rho^{i}\right)+1\right] \\
\frac{\beta B\left(1-\beta x^{-\sigma}\right)}{2} &= x^{1+\sigma}\left(\frac{2}{\beta}-x^{-\sigma}\right)-\frac{x}{1-\sigma}\left[\left(1-\beta x^{-\sigma}\right)\left(1-\rho^{i}\right)+1\right] \\
\frac{\beta B-\beta^{2}Bx^{-\sigma}}{2} &= x^{1+\sigma}\frac{2}{\beta}-x\left(1+\frac{\left(2-\rho^{i}\right)}{1-\sigma}\right)+\frac{x^{1-\sigma}}{1-\sigma}\beta\left(1-\rho^{i}\right) \\
\frac{\beta B-\beta^{2}Bx^{-\sigma}}{2}+x\left(1+\frac{\left(2-\rho^{i}\right)}{1-\sigma}\right)-x^{1+\sigma}\frac{2}{\beta} &= \frac{x^{1-\sigma}}{1-\sigma}\beta\left(1-\rho^{i}\right) \qquad (25)
\end{aligned}$$

which we can further rearrange as

$$\frac{\beta B-\beta^{2}Bx^{-\sigma}}{2}+x\left(1+\frac{\left(2-\rho^{i}\right)}{1-\sigma}\right) = \frac{x^{1-\sigma}}{1-\sigma}\beta\left(1-\rho^{i}\right)+x^{1+\sigma}\frac{2}{\beta} \qquad (26)$$

with left hand side starting at $-\infty$ and going to $+\infty$, and the right hand side starting from 0 and going to $+\infty$.[31] Moreover

$$\begin{aligned}
lhs' &= \frac{\sigma\beta^{2}Bx^{-\sigma-1}}{2}+1+\frac{\left(2-\rho^{i}\right)}{1-\sigma} \\
rhs' &= x^{-\sigma}\beta\left(1-\rho^{i}\right)+x^{\sigma}\left(1+\sigma\right)\frac{2}{\beta}
\end{aligned}$$

Further notice that

$$\begin{aligned}
lhs'' &= \left(-\sigma-1\right)\frac{\sigma\beta^{2}B}{2}\frac{1}{x^{\sigma+2}}<0 \\
rhs'' &= -\sigma\beta\left(1-\rho^{i}\right)x^{-\sigma-1}+\sigma x^{\sigma-1}\left(1+\sigma\right)\frac{2}{\beta} \\
&= \sigma x^{-1}\left[x^{\sigma}\left(1+\sigma\right)\frac{2}{\beta}-\beta\left(1-\rho^{i}\right)x^{-\sigma}\right]
\end{aligned}$$

---

[30] And for such $x$ it must be that $-qb^{i}+\rho^{i}U\left(x\right)>0$ and $-qb^{j}+\rho^{j}U\left(x\right)<0$, that is to say $\frac{b^{i}}{\rho^{i}}<\frac{U(x)}{q}<\frac{b^{j}}{\rho^{j}}$. Using the ICs to get

$$\frac{x+\beta\frac{B}{2}\left(1-q\right)-\beta\left(1-\rho^{i}\right)U\left(x\right)}{\beta\rho^{i}} < \frac{U(x)}{q} < \frac{x+\beta\frac{B}{2}\left(1-q\right)-\beta U\left(x\right)}{\left(1-q\right)\beta\rho^{j}}$$

[31] Equation (26) can be written in implicit terms as

$$\begin{aligned}
\beta B-\beta^{2}BU'\left(x\right)+2x\left(1+\frac{\left(2-\rho^{i}\right)}{1-\sigma}\right) &= U\left(x\right)2\beta\left(1-\rho^{i}\right)+\frac{x}{U'\left(x\right)}\frac{4}{\beta} \\
2x-\beta^{2}BU'\left(x\right)-\beta U\left(x\right)\left(2-\rho^{i}\right) &= -\rho^{i}U\left(x\right)\beta+\frac{x}{U'\left(x\right)}\frac{4}{\beta}-\beta B-2x\frac{\left(2-\rho^{i}\right)}{1-\sigma}
\end{aligned}$$

where the right hand side is concave as $x \to 0$ and convex at $x^* = 1$, as $\sigma \left[ (1+\sigma) \frac{2}{\beta} - \beta (1-\rho^i) \right] > 0$. Then we have that $rhs'' = 0$ for $x_r$solving

$$\begin{aligned} x^{\sigma} (1+\sigma) \frac{2}{\beta} &= \beta \left(1-\rho^i\right) x^{-\sigma} \\ x &= \left( \frac{\beta^2 (1-\rho^i)}{2(1+\sigma)} \right)^{\frac{1}{2\sigma}} \end{aligned}$$

So there can be no solution for $x_E$, two, or one if at $LHS(x^*) > RHS(x^*)$. The last case arises if and only if

$$\frac{\beta B (1-\beta)}{2} + 1 + \frac{(2-\rho^i)}{1-\sigma} > \frac{\beta (1-\rho^i)}{1-\sigma} + \frac{2}{\beta}$$

which can be rearranged as

$$\begin{aligned} \frac{\beta B}{2} &> \frac{1}{(1-\beta)} \left[ \frac{2}{\beta} - \frac{2-\sigma}{1-\sigma} \right] - \frac{(1-\rho^i)}{1-\sigma} \\ \frac{\beta B}{2} &> \frac{2(1-\sigma-\beta)+\beta\sigma}{\beta(1-\beta)(1-\sigma)} - \frac{(1-\rho^i)}{1-\sigma} \end{aligned} \tag{27}$$

is a lower bound on $B$. Notice that this inequality also verifies that the IC is binding.

Verifying then that $\hat{v}^i > 0$ requires $\rho^i U(x^i) > qb^i$,which, using the equation for $b^i$ derived above, is

$$\rho^i \frac{U(x)}{U(x)} > x - \beta \left(1-\rho^i\right) U(x) + \beta \frac{B}{2} (1-q)$$

Using then the equation defining $x_E$ , (26), for the right hand side of this inequality we have

$$\begin{aligned} \rho^i \frac{U(x)}{U'(x)} &> 2 \frac{x}{\beta U'(x)} - x \frac{2-\rho^i}{1-\sigma} \\ \frac{\rho^i U(x)}{x} &> \frac{2}{\beta} - U'(x) \frac{2-\rho^i}{1-\sigma} \\ \frac{\rho^i x^{-\sigma}}{1-\sigma} &> \frac{2}{\beta} - x^{-\sigma} \frac{2-\rho^i}{1-\sigma} \\ x^{-\sigma} &> \frac{1-\sigma}{\beta} \end{aligned}$$

or $x < \left(\frac{\beta}{1-\sigma}\right)^{\frac{1}{\sigma}}$, which is the consumption allocation in the case where $\hat{v}^i = 0$ for all $i$, and for which a sufficient condition is $LHS\left(\left(\frac{\beta}{1-\sigma}\right)^{\frac{1}{\sigma}}\right) > RHS\left(\left(\frac{\beta}{1-\sigma}\right)^{\frac{1}{\sigma}}\right)$. Notice that $LHS\left(\left(\frac{\beta}{1-\sigma}\right)^{\frac{1}{\sigma}}\right) > RHS\left(\left(\frac{\beta}{1-\sigma}\right)^{\frac{1}{\sigma}}\right)$ guarantee that $x < \left(\frac{\beta}{1-\sigma}\right)^{\frac{1}{\sigma}}$ because we already know that $\frac{\beta}{1-\sigma} < 1 = x^*$ (by assumption for the case where $\hat{v}^i = 0$ for all $i$) and because (27) guarantees the IC binds.[32] This sufficient condition is

$$\begin{aligned}
\frac{\sigma\beta B}{2}+\left(\frac{\beta}{1-\sigma}\right)^{\frac{1}{\sigma}}\left(\rho^{i}+\frac{\left(2-\rho^{i}\right)}{1-\sigma}\right) &> \left(\frac{\beta}{1-\sigma}\right)^{\frac{1+\sigma}{\sigma}}\frac{2}{\beta} \\
\frac{\sigma\beta B}{2} &> \left(\frac{\beta}{1-\sigma}\right)^{\frac{1}{\sigma}}\left[\frac{2}{1-\sigma}-\rho^{i}-\frac{\left(2-\rho^{i}\right)}{1-\sigma}\right] \\
\frac{\sigma\beta B}{2} &> \left(\frac{\beta}{1-\sigma}\right)^{\frac{1}{\sigma}}\rho^{i}\left(\frac{\sigma}{1-\sigma}\right) \\
\frac{\beta B}{2} &> \left(\frac{\beta}{1-\sigma}\right)^{\frac{1}{\sigma}}\frac{\rho^{i}}{1-\sigma}
\end{aligned}$$

Verifying then that $\hat{v}^{j}=0$ requires $\rho^{j}U\left(x\right)\leq qb^{j}$, which, using the equation for $b^{j}$ derived above, is:

$$\begin{aligned}
\rho^{j}\frac{U\left(x\right)}{U'\left(x\right)} &\leq \frac{x-\beta U\left(x\right)}{\left(1-\beta U'\left(x\right)\right)}+\beta\frac{B}{2} \\
\rho^{j}\frac{U\left(x\right)}{U'\left(x\right)}\left(1-\beta U'\left(x\right)\right) &\leq x-\beta U\left(x\right)+\beta\frac{B}{2}\left(1-\beta U'\left(x\right)\right)
\end{aligned}$$

Then we can use the equation defining $x_{E}$, which we can rearrange as

$$\frac{\beta B-\beta^{2}Bx^{-\sigma}}{2}+x-\frac{x^{1-\sigma}}{1-\sigma}\beta = -\rho^{i}\frac{x^{1-\sigma}}{1-\sigma}\beta+x^{1+\sigma}\frac{2}{\beta}-x\frac{\left(2-\rho^{i}\right)}{1-\sigma}$$

and substitute back to get

$$\begin{aligned}
\rho^{j}\frac{U\left(x\right)}{U'\left(x\right)}\left(1-\beta U'\left(x\right)\right) &\leq \frac{\beta B-\beta^{2}Bx^{-\sigma}}{2}+x-\frac{x^{1-\sigma}}{1-\sigma}\beta \\
\rho^{j}\frac{x}{1-\sigma}\left(1-\beta U'\left(x\right)\right) &\leq -\rho^{i}\frac{x^{1-\sigma}}{1-\sigma}\beta+x^{1+\sigma}\frac{2}{\beta}-x\frac{\left(2-\rho^{i}\right)}{1-\sigma} \\
\rho^{j}\left(1-\beta x^{-\sigma}\right) &\leq -\rho^{i}x^{-\sigma}\beta+x^{\sigma}\frac{2}{\beta}\left(1-\sigma\right)-\left(2-\rho^{i}\right) \\
2+\rho^{j}-\rho^{i} &\leq \beta x^{-\sigma}\left(\rho^{j}-\rho^{i}\right)+x^{\sigma}\frac{2}{\beta}\left(1-\sigma\right) \\
x^{\sigma}\left(2+\rho^{j}-\rho^{i}\right) &\leq \beta\left(\rho^{j}-\rho^{i}\right)+x^{2\sigma}\frac{2}{\beta}\left(1-\sigma\right)
\end{aligned}$$

which is a quadratic equation in $x^{\sigma}$. Then, with $\frac{2}{\beta}\left(1-\sigma\right)>0$, the above inequality is satisfied for $x\leq x_{1}$ and $x\geq x_{2}$ with $x_{1,2}$ solving

$$x^{2\sigma}\frac{2}{\beta}\left(1-\sigma\right)-x^{\sigma}\left(2+\rho^{j}-\rho^{i}\right)+\beta\left(\rho^{j}-\rho^{i}\right) = 0$$

[32] Otherwise, $LHS\left(\left(\frac{\beta}{1-\sigma}\right)^{\frac{1}{\sigma}}\right)>RHS\left(\left(\frac{\beta}{1-\sigma}\right)^{\frac{1}{\sigma}}\right)$ might be consistent with $\left(\frac{\beta}{1-\sigma}\right)^{\frac{1}{\sigma}}>x_{E}$.

Hence

$$
\begin{aligned}
x_{1,2} &= \frac{1}{\frac{4}{\beta}(1-\sigma)}\left[\left(2+\rho^j-\rho^i\right) \pm \sqrt{\left(2+\rho^j-\rho^i\right)^2 - 4\beta\left(\rho^j-\rho^i\right)\frac{2}{\beta}(1-\sigma)}\right] \\
&= \frac{\beta}{4(1-\sigma)}\left[\left(2+\rho^j-\rho^i\right) \pm \sqrt{4+\left(\rho^j-\rho^i\right)^2 + 4\left(\rho^j-\rho^i\right) - 8\left(\rho^j-\rho^i\right)(1-\sigma)}\right] \\
&= \frac{\beta}{4(1-\sigma)}\left[\left(2+\rho^j-\rho^i\right) \pm 2\sqrt{1+\left(\rho^j-\rho^i\right)^2 - \left(\rho^j-\rho^i\right) + 2\sigma\left(\rho^j-\rho^i\right)}\right] \\
&= \frac{\beta}{4(1-\sigma)}\left[\left(2+\rho^j-\rho^i\right) \pm 2\sqrt{1+\left(\rho^j-\rho^i-1+2\sigma\right)\left(\rho^j-\rho^i\right)}\right]
\end{aligned}
$$

Such $x_{1,2}$ exist if and only if

$$
1+\left(\rho^j-\rho^i\right)^2 + (2\sigma-1)\left(\rho^j-\rho^i\right) \geq 0
$$

which is always satisfied because $(2\sigma-1)\left(\rho^j-\rho^i\right) \in (-1,1)$. Notice, however, that $x_1 \geq 0$ if and only if $\left(\rho^j-\rho^i\right) \geq 0$, and that $x_2 < x^*$ when $\rho^j < \rho^i$ if and only if

$$
\begin{aligned}
&1+\left(\rho^j-\rho^i-1+2\sigma\right)\left(\rho^j-\rho^i\right) < \\
&4\left(\frac{(1-\sigma)}{\beta}\right)^2 + \left(1+\frac{\rho^j-\rho^i}{2}\right)^2 - 4\frac{(1-\sigma)}{\beta}\left(1+\frac{\rho^j-\rho^i}{2}\right).
\end{aligned} \tag{28}
$$

If this inequality is violated when $\rho^j < \rho^i$ then there is no equilibrium with $\hat{v}^i > 0, \hat{v}^j = 0$. Hence, if $\rho^j < \rho^i$ and if (28) is satisfied, then this equilibrium exist if and only if $LHS(x_2) \leq RHS(x_2)$. This is because inequality $LHS(x_2) \leq RHS(x_2)$ guarantees that $x_E \geq x_2$ when $x_2 < x^*$.

Alternatively, if $\rho^j \geq \rho^i$ and $\left(\rho^j-\rho^i\right)^2\left(\frac{1}{4}-1\right) + \left(\rho^j-\rho^i\right)2(1-\sigma) \geq 0$, then $x_E < \left(\frac{\beta}{1-\sigma}\right)^{\frac{1}{\sigma}} \leq x_1 < x_2$ and $\hat{v}^j = 0$ always. Hence, no additional conditions are necessary to characterize this equilibrium. If, instead, $\rho^j \geq \rho^i$ and $\left(\rho^j-\rho^i\right)^2\left(\frac{1}{4}-1\right)+\left(\rho^j-\rho^i\right)2(1-\sigma) < 0$, then $x_1 < \left(\frac{\beta}{1-\sigma}\right)^{\frac{1}{\sigma}}$, which implies that a sufficient condition for $\hat{v}^j = 0$ is $LHS(x_1) > RHS(x_1)$.[33] ■

## B.7 Proof of proposition 10

**Proof.**

If $b^i = 0$ then for type $i$ we have

$$
v^i = \frac{U(x^*) - x^*}{1-\beta}
$$

and it must be that

$$
\hat{v}^i = \frac{\rho^i U(x^*)}{1-\beta} > 0
$$

[33] Because $x_1 < \left(\frac{\beta}{1-\sigma}\right)^{\frac{1}{\sigma}} < x^*$, then $LHS(x_1) > RHS(x_1)$ guarantees that $x_E < x_1$.

and for the IC to be slack

$$x^* \leq \beta\left(\frac{\left(1-\rho^i\right)U\left(x^*\right)-x^*}{1-\beta}\right)$$

which can be rearranged as

$$x^* \leq \beta\left(1-\rho^i\right)U\left(x^*\right)$$

For type $j$

$$q = \beta U'\left(x^j\right) > \beta = \beta U'\left(x^*\right)$$

where the inequality follows from the fact that IC is binding for $j$, and

$$\begin{aligned} v^j &= \frac{-qb^j+U\left(x^j\right)-l^j-\beta\tau}{1-\beta} \\ l^j+\beta\tau &= \beta\left(v^j-\hat{v}^j\right) \\ \hat{v}^j &= \frac{-qb^j+\rho^j U\left(x^j\right)}{1-\beta} \end{aligned}$$

hence, the binding IC is

$$\begin{aligned} l^j+\beta\tau &= \beta\frac{\left(1-\rho^j\right)U\left(x^j\right)-l^j-\beta\tau}{1-\beta} \\ l^j+\beta\tau &= \beta\left(1-\rho^j\right)U\left(x^j\right) \end{aligned}$$

Also, $b^j = B$, $l^j = x^j-\beta B$, and $\tau = B(1-q)$. So we have

$$\begin{aligned} x^j-\beta B+\beta B(1-q) &= \beta\left(1-\rho^j\right)U\left(x^j\right) \\ x^j-\beta^2 BU'\left(x^j\right) &= \beta\left(1-\rho^j\right)U\left(x^j\right) \end{aligned}$$

which pins down $x^j$.

**Case with $U(x)=\frac{x^{1-\sigma}}{1-\sigma}$** If $U(x)=\frac{x^{1-\sigma}}{1-\sigma}$ this is

$$x-\beta^2 Bx^{-\sigma} = \beta\left(1-\rho^j\right)\frac{x^{1-\sigma}}{1-\sigma}$$

We can rearrange the equation pinning down $x^j$ as

$$1-\beta^2 Bx^{-(1+\sigma)} = \beta\left(1-\rho^j\right)\frac{x^{-\sigma}}{1-\sigma}$$

the left hand side starts at $-\infty$, is always increasing and converges to 1. The right hand side starts at $+\infty$, is always decreasing and converges to 0. Therefore, there exists a unique $x_E$ such that the IC of type $j$ binds. This is the case IFF

$$1-\beta^2 B\left(x^*\right)^{-(1+\sigma)} > \beta\left(1-\rho^j\right)\frac{\left(x^*\right)^{-\sigma}}{1-\sigma}$$

That is to say

$$1 \;\; > \;\; \beta^2 B + \beta \frac{(1-\rho^j)}{1-\sigma}$$

In this case, we also need to verify that t and that $-qb^j + \rho^j U(x^j) > 0$, which requires

$$\rho^j U\left(x^j\right) \;\; > \;\; \beta U'\left(x^j\right) B$$

which, if $U(x) = \frac{x^{1-\sigma}}{1-\sigma}$ is

$$\begin{aligned}
\rho^j \frac{x^{1-\sigma}}{1-\sigma} &> \beta x^{-\sigma} B \\
x^E &> \frac{\beta B}{\rho^j}(1-\sigma)
\end{aligned}$$

that is to say

$$\frac{\beta B}{\rho^j}(1-\sigma) - \beta^2 B\left(\frac{\beta B}{\rho^j}(1-\sigma)\right)^{-\sigma} \;\; < \;\; \beta\left(1-\rho^j\right)\frac{\left(\frac{\beta B}{\rho^j}(1-\sigma)\right)^{1-\sigma}}{1-\sigma}$$

that can be rearranged as

$$\begin{aligned}
1 - \beta^2 B\left(\frac{\beta B}{\rho^j}(1-\sigma)\right)^{-(1+\sigma)} &< \beta\left(1-\rho^j\right)\left(\frac{\beta B}{\rho^j}\right)^{-\sigma}(1-\sigma)^{-(1+\sigma)} \\
(1-\sigma)^{1+\sigma} &< \beta^2 B\left(\frac{\beta B}{\rho^j}\right)^{-(1+\sigma)} + \beta\left(1-\rho^j\right)\left(\frac{\beta B}{\rho^j}\right)^{-\sigma}
\end{aligned}$$

■

## B.8 Proof of proposition 11

**Proof.**

Then the portfolio allocation of $i$ is irrelevant (but $i$ needs to acquire $B$ at least) and $b^j = 0$. We still have

$$v^i \;\; = \;\; \frac{-qb^i + U(x^*) - (x^* - \beta b^i) - \beta\tau}{1-\beta}$$

but now

$$\hat{v}^i \;\; = \;\; \frac{-qb^i + \rho^i U(x^*)}{1-\beta}$$

which we need to verify it's such that $-qb^i + \rho^i U(x^*) > 0$, and for the IC to be slack

$$\begin{aligned}
x^* - \beta b^i + \beta\tau &\leq \beta\left(\frac{(1-\rho^i)\,U(x^*) - (x^* - \beta b^i) - \beta\tau}{1-\beta}\right) \\
x^* - \beta b^i + \beta\tau &\leq \beta\left(1-\rho^i\right)U(x^*) \\
x^* - q\beta B &\leq \beta\left(1-\rho^i\right)U(x^*)
\end{aligned}$$

where we also substituted out the market clearing condition $b^i = B$ since $b^j = 0$, and the mechanism's resource constraint. Also, the FOC for $b^i$ is $q = \beta U'(x^*) = \beta$. Hence, for the IC to be slack it is necessary and sufficient that

$$x^* \leq \beta\left[\left(1-\rho^i\right)U(x^*) + \beta B\right]$$

and for $\hat{v}^i > 0$ (which we require in this case as $\hat{v}^i > 0$ for all $i$:

$$\begin{aligned} -qb^i + \rho^i U(x^*) &> 0 \\ -\beta B + \rho^i U(x^*) &> 0 \end{aligned}$$

For type $j$ the binding IC is $l^j = \beta\left(v^j - \hat{v}^j\right)$ where

$$\begin{aligned} v^j &= \frac{U(l^j) - l^j}{1-\beta} \\ \hat{v}^j &= \frac{\rho^j U(l^j)}{1-\beta} > 0 \end{aligned}$$

Hence

$$\begin{aligned} l^j &= \beta\left(\frac{\left(1-\rho^j\right)U(l^j) - l^j}{1-\beta}\right) \\ l^j &= \beta\left(1-\rho^j\right)U\left(l^j\right) \end{aligned}$$

and for IC to bind for type $j$ it must be that $l^j < x^*$, that is to say $\beta\left(1-\rho^j\right)U(x^*) < x^*$.

If $U(x) = \frac{x^{1-\sigma}}{1-\sigma}$ the solution for $l^j$ is such that

$$\begin{aligned} (1-\sigma) &= \beta\left(1-\rho^j\right)\left(l^j\right)^{-\sigma} \\ l^j &= \left[\frac{\beta\left(1-\rho^j\right)}{(1-\sigma)}\right]^{\frac{1}{\sigma}} \end{aligned}$$

checking that IC for type $j$ binds requires

$$\beta\left(1-\rho^j\right) + \sigma < 1$$

And for the IC to be slack for type $i$ it is necessary and sufficient that

$$1 \leq \beta\left[\frac{\left(1-\rho^i\right)}{(1-\sigma)} + \beta B\right]$$

and for $\hat{v}^i > 0$ (which we require in this case as $\hat{v}^i > 0$ for all $i$:

$$\frac{\rho^i}{\beta(1-\sigma)} > B$$

■

## B.9 Proof of proposition 12

**Proof.**

**Case $b^j > 0, b^i = 0$.** The type with IC slack has $\hat{v}^i > 0$, and we conjecture also $b^i = 0$ in this equilibrium.[34] Then, for the unconstrained type we have:

$$v^i = \frac{U(x^*) - x^*}{1-\beta}$$

with the IC $x^* \leq \beta \left( v^i - \frac{\rho^i U(x^*)}{1-\beta} \right)$, and the mechanism resource constraint $\tau = B(1-q)$ as the IC slack type does not purchase $b$.

For IC slack we have

$$\begin{aligned} x^* &\leq \beta \left( \frac{(1-\rho^i) U(x^*) - x^*}{1-\beta} \right) \\ x^* &\leq \beta \left(1 - \rho^i\right) U(x^*) \end{aligned}$$

For the type $j$ with IC binding and $\hat{v}^j = 0$, $x^j$ solves the IC binding

$$l^j + \beta B (1-q) = \beta v^j$$

where we substitute $v^j = -qB + U(x^j)$ and $l^j + \beta B = x$ to get

$$l^j + \beta B (1-q) = \beta \left( -qB + U\left(x^j\right)\right)$$

that can be rearranged as

$$\begin{aligned} l^j + \beta B &= \beta U(x) \\ x &= \beta U(x) \end{aligned}$$

We can then check that $-qB + \rho^j U(x) < 0$ and that such $x < x^*$. For the latter, however it must be that

$$x^* > \beta U(x^*)$$

but this is inconsistent with the NSC for the IC to be slack for type $i$. Hence, this case never arises.

[34] Otherwise, with $b > 0$, both consumption allocations would need to be the same and unconstrained. Suppose IC slack is associated with $\hat{v}^i = 0$. Then

$$\begin{aligned} v^i &= U(x^*) - x^* + \beta v^i \\ &x^* \leq \beta v^i \end{aligned}$$

but it cannot be that $\hat{v}^i = 0$ for a type with IC slack, because $\hat{v}^i = \frac{\rho^i U(x^*)}{1-\beta}$.

**Case $b^i > 0, b^j = 0$.** If $b^j = 0$, then $\hat{v}^j > 0$. Indeed, if a buyer chooses zero holdings of membership titles then the punishment continuation value is strictly positive. Then it must be that for $i$ with IC slack: $\hat{v}^i = 0 < \hat{v}^j$ with $b^j = 0 < b^i$, and the IC is binding for type $j$.

Then, for the unconstrained type $i$ we have:

$$v^i = \frac{-qb^i + U(x^*) - x^* - \beta\tau + \beta b^i}{1-\beta}$$

and we need to check that $-qb^i + \rho^i U(x^*) \leq 0$. So, using also $q = \beta$ and market clearing, yields

$$\rho^i U(x^*) \leq \beta B$$

So $B$ has to be sufficiently large for $\hat{v}^i = 0$ (that is $B \geq \frac{\rho^i}{\beta(1-\sigma)}$ in CRRA) and the portfolio allocation doesn't matter. For IC slack we have

$$\begin{aligned} x^* - \beta b^i + \beta\tau &< \beta v^i = \beta\frac{-qb^i + U(x^*) - x^* - \beta\tau + \beta b^i}{1-\beta} \\ x^* - \beta b^i + \beta\tau &< \beta\left[-qb^i + U(x^*)\right] \\ x^* &< \beta U(x^*) \end{aligned}$$

For type $j$, the binding IC is

$$l^j = \beta\left(v^j - \hat{v}^j\right)$$

where

$$v^j - \hat{v}^j = \frac{(1-\rho^j)\, U(x^j) - l^j}{1-\beta}$$

so the IC binding pins down $x^j = l^j$ :

$$\frac{\overline{x}}{\beta} = \left(1-\rho^j\right) U(\overline{x})$$

and it must be that such $\overline{x} < x^*$that is to say $\frac{x^*}{\beta} > (1-\rho^j)\, U(x^*)$. And, for $b^j = 0$, it also must be that

$$-q + \beta U'(\overline{x}) < 0$$

which is equivalent to $\overline{x} > x^*$ as this inequality is $U'(\overline{x}) < 1$ at the price $q = \beta$. But this is impossible.

However, There can be an equilibrium with $b^i > 0, b^j = 0$ if $\hat{v}^j > 0, \hat{v}^i = 0$.[35] With $\hat{v}^j > 0, \hat{v}^i = 0$, with $b^i > 0, b^j = 0$ and IC slack for $i$ we have

$$\begin{aligned} x^i &= x^* \\ x^j &< x^* \end{aligned}$$

[35] Notice that only this case is possible as $b^i = 0$ would contradict $\hat{v}^i = 0$.

and

$$\begin{aligned} x^* - \beta b^i + \beta\tau &< \beta v^i \\ x^j &= \beta\left(v^j - \hat{v}^j\right) \end{aligned}$$

Also notice that $q = \beta$ and $\tau = B(1-q)$ because only $i$ purchase membership titles and their IC is slack. So

$$\begin{aligned} v^i &= -\beta b^i + U(x^*) - \left(x^* - \beta b^i\right) - \beta B(1-\beta) + \beta v^i \\ v^i &= \frac{U(x^*) - x^*}{1-\beta} - \beta B \end{aligned}$$

and

$$\begin{aligned} v^j &= \frac{U(x^j) - x^j}{1-\beta} \\ \hat{v}^j &= \frac{\rho^j U(x^j)}{1-\beta} \end{aligned}$$

so the IC is $x^j = \beta(1-\rho^j)U(x^j)$. So the IC is binding IFF $x^* > \beta(1-\rho^j)U(x^*)$.

And the IC is slack for $i$ IFF

$$\begin{aligned} x^* - \beta B + \beta B(1-\beta) &< \beta\left[\frac{U(x^*) - x^*}{1-\beta} - \beta B\right] \\ x^* - \beta^2 B &< \frac{\beta(U(x^*) - x^*)}{1-\beta} - \beta^2 B \\ x^* &< \beta U(x^*). \end{aligned}$$

Finally, $\hat{v}^i = 0$ if and only if $\beta B \geq \rho^i U(x^*)$. ■

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