Credit Card Competition and Naive Hyperbolic Consumers*

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Abstract

In this paper, we show that the consumer might be unresponsive to interest rate and credit limit in the credit card offers of the companies because of the combination of the consumer's time inconsistency, credit card companies' grace period offer and one period lag in using a new card. Consequently, we demonstrate that there might be no competition on interest rate and credit limit even if there is more than one firm in the market and even if the consumer accepts only one card. We determine whether the credit card companies can exploit time-inconsistent consumers and gain positive expected profits. We show that in fact there are circumstances in which there would be zero and positive expected profits possible.

1 Introduction

In this paper, we propose an explanation for some special features of the credit card market that have been pointed out in previous empirical studies. Our explanation uses the time inconsistency of consumers and we therefore also contribute to the debate about whether time-inconsistent consumers could be "money pumps" or whether competition will mitigate the excess-profit-generating effect of time inconsistency.

Ausubel (1991) finds that the credit card market is far from being competitive, with the rate of return three to five times higher than in banking based on the evidence from 1980's, despite the presence of factors which leads to competitive outcome such as homogenous good, existence of more than 4,000 companies/banks and the

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fact that ten largest firms account for only about two-fifths of the market. Evans & Schmalensee (2000) in a part of their book "Paying with Plastic" on criticism of Ausubel's findings argue that measuring profitability accurately especially in payment card industry (because of the risk factor) is very difficult. However, if there is supracompetitive profits in this industry as many researchers and consumer advocates support, it is worth analyzing the implications of the existence of this excessive profits. There are two theoretical papers which I mention in the following section provide alternative explanations for the phenomenon Ausubel (1991) points.

Ausubel also suggests possible theoretical explanations for this failure of competition in the credit card market. One of these explanations he gives is related to the time inconsistency of the consumer. In this paper, using Ausubel's (1991) idea of the time inconsistent consumer in the credit card market, we show that the consumer might be indifferent to differences on interest rate and credit limit offers of the companies. The reason for this indifference is the combination of the consumer's *time inconsistency*, credit card companies' *grace period offer* and *one period lag* in using a new card. Consequently, we demonstrate that there will be no competition on interest rate and credit limit even if there is more than one firm in the market and even if the consumer accepts only one card. The basic intuition behind our result is as follows:

Time inconsistency, grace period offer and *one period lag* all together cause the consumer to underestimate his borrowing at each period, and consequently to believe that his borrowing each period is less than his next period income. Therefore, he thinks that he is just a convenience user who borrows for only one period and pays back in the next period without interest. However, his actual borrowing at a period turns out be more than his income at the subsequent period and accordingly he cannot pay all of his debt in the subsequent period. Therefore, he pays the rest of his debt in the period after the subsequent period with interest since the grace period gives him the flexibility to delay the payment without interest for only one period. In this kind of a setting, the consumer is indifferent to difference on interest rate offers when deciding which contracts to accept even though he will pay interest. Moreover, he is indifferent to difference on credit limit offers as long as these offers are more than the consumer's believed amount of borrowing. We analyze the case in which there are two credit card companies and each of them simultaneously offers a credit card with interest rate and credit limit to this kind of a consumer. Since the consumer is unresponsive to interest rate, the companies will not compete on that. In addition, the companies will also not compete on the credit limit. As a result, the companies will be able to charge higher interest rates than the competitive level and will be able to make positive expected profits.

We investigate whether the intuition above implies that credit card companies can exploit time-inconsistent consumers and gain positive expected profits. We investigate this in the simplest possible setting with one time-inconsistent consumer and two credit cards. We show that in fact there are circumstances in which there would be zero and others in which there would be positive expected profits possible.

2 Related Literature and Contribution

Ausubel (1991) reported that "credit card interest rates have been exceptionally sticky relative to the cost of funds. Moreover, major credit card issuers have persistently earned from three to five times the ordinary rate of return in banking during the period 1983-1988". He also stated that the credit card market is far from being competitive despite the existence of more than 4,000 firms and no significant barriers to entry.

In order to provide an explanation for the high rates of return and for the lack of competition on interest rate in the credit card market, which are pointed by Ausubel (1991), we consider the time inconsistency of consumers. Strotz (1956) claims that people would not obey their optimal plan of the present moment if they are allowed to reconsider their plans at future periods. Because people are impatient, they give more weight to the earlier time as it gets closer, and this causes time inconsistent behavior (also known as hyperbolic behavior). There is a significant amount of evidence that shows that preferences are time inconsistent.[1] Phelps and Pollak (1968) and Laibson (1997) use the so called $\beta-\delta$ discount function which is given by $\{1,\ \beta\delta,\ \beta\delta^2,\ \beta\delta^3,\ ..\}$ to model hyperbolic behavior. Since the $\beta-\delta$ discount function is a common way to model discrete time hyperbolic discounting, we use this method in our model.

In the hyperbolic discounting literature, starting with Strotz (1956), two types of consumers discussed. The first kind of consumer is called "naive" as he is not aware of his time inconsistency. More specifically, he knows that his future discounting today is $\{1,\ \beta\delta,\ \beta\delta^2,\ \beta\delta^3,\ ..\}$, and believes that from tomorrow on it will be $\{1,\ \delta,\ \delta^2,\ \delta^3,\ ..\}$ although in reality it will be $\{1,\ \beta\delta,\ \beta\delta^2,\ \beta\delta^3,\ ..\}$ again. On the other hand, the "sophisticated" hyperbolic consumer is aware of his time inconsistency and knows that his future discounting today is $\{1,\ \beta\delta,\ \beta\delta^2,\ \beta\delta^3,\ ..\}$, and he believes that it will be $\{1,\ \beta\delta,\ \beta\delta^2,\ \beta\delta^3,\ ..\}$ again from tomorrow on. O'Donough & Rabin (2001) introduce a model to represent a partially naive hyperbolic consumer, who is aware of his time inconsistency but underestimates that. According to O'Donough & Rabin (2001), partially naive hyperbolic consumer knows that his future discounting today is $\{1,\ \beta\delta,\ \beta\delta^2,\ \beta\delta^3,\ ..\}$, and he believes that it will be $\{1,\ \beta'\delta,\ \beta'\delta^2,\ \beta'\delta^3,\ ..\}$ from tomorrow on such that $\beta < \beta'$.

A few theoretical papers provides alternative explanations for the phenomenon we consider. Parlour & Rajan (2001) construct a model such that the competing firms cannot sustain equilibria with zero profits under certain conditions. In their model, there are three stages. In the first stage, companies offer contracts with credit line and interest rate, while in the second stage the consumer decides which contracts to

[1] See Hausman (1979), Loewenstein and Thaler (1989), Ainslie (1991), and DellaVigna and Malmendier (2002)

accept and whether he is going to default. If he is going to default, he accepts all contracts offered in the second stage, and then he defaults in the third stage. Within this model, the consumer has an incentive to default given by $\alpha \geq 0$ such that αd is the amount of the consumer's shielded assets from bankruptcy when he defaults on total loans of d. They show that there are positive profits to lenders and that the interest rates are sticky and above the risk-adjusted cost of funds if the incentive to default, which is denoted by α, is high enough and if there is multiple contracting. In their model, the consumer decides to default or not at the second stage when he decides on which offers to accept. Therefore, in Parlour & Rajan (2001) default is strategic.

In our model, as opposed to Parlour & Rajan (2001), the consumer may decide to default in later periods than the contracting period, and therefore default is not strategic. He believes that he will use the credit card for convenience and he will utilize the grace period. Consequently, he believes he will not pay interest. We also include a cost of bankruptcy for the consumer since bankruptcy will adversely affect the consumer's credit score and, accordingly, the terms of any credit he will get later.

Dellavigna & Malmendier (2003) analyzes the firms' profit- maximizing contract design if they have partially naive time inconsistent consumers. They show that firms price leisure goods, which are goods with immediate benefits and delayed costs, like credit card financed consumptions, higher than the marginal cost. Although this marginal cost pricing does not depend on the assumption of monopoly, profits depend on this assumption and if there is competition, one expects to have zero expected profit.

Dellavigna & Malmendier (2003) do not consider the risk of bankruptcy in their model. The only similarity between their model and ours is the time inconsistency of the consumer. We include the risk of bankruptcy in our paper and show that positive expected profit equilibrium might be possible under some conditions even if there are competing firms.

In our model, we include the grace period feature of the credit card market such that if the consumer can pay his debt back within the next period, he does not pay interest. To our knowledge this feature of the credit card market is not included in the previous models. Including grace period lets us analyze the decision of a time inconsistent consumer who, because he thinks he is a convenience user, believes he is free of paying interest. If we did not have the grace period in our model, interest rate would always affect the decision of the consumer. In reality, however, interest rate does not affect the decision of a convenience user.

3 The Model

3.1 Environment

We studied an environment in which there are three periods of consumption preceded by an initial period in which contracting occurs but no consumption takes place. There is one good at each consumption period. There are three agents in the model. One agent is the consumer/borrower and two agents are the companies/lenders who compete against each other for the borrower.

3.2 Specification of the Consumer

Consumer chooses the contract at the initial period and consumes the consumption good in each consumption period. Consumer's utility is also affected by whether he defaults or not in the last period (although the period three is the terminal date in this model, this component of utility might be interpreted informally as reflecting the effects of future constraints on trade opportunities as a result of having defaulted). Therefore, the consumer's consumption set is $\{0,\ 1\}\times\{0,\ 1\}\times\mathbb{R}^8_+\times\{-1,\ 0\}\times\{-1,\ 0\}$.

At each period $t = 0,\ 1,\ 2$, the consumer has a utility over the consumption set and he aims to maximize U_t such that:

$$\begin{aligned} U_0 &= \beta\delta\left[u(c_1) + \delta u(c_2) + \delta^2 u(c_3) + \delta^2 v(d_1,\ d_2)\right] \\ U_1 &= u(c_1) + \beta\delta u(c_2) + \beta\delta^2 u(c_3) + \beta\delta^2 v(d_1,\ d_2) \\ U_2 &= u(c_2) + \beta\delta u(c_3) + \beta\delta v(d_1,\ d_2) \end{aligned}$$

Note that we do not need to write the total utility at period three since the trade which will take place at period three would already be determined at period two.

At each consumption period, the consumer receives an endowment amount of m of the consumption good. The consumer chooses a trade $y_j^t = (s_{j0},\ n_{j1}^t,\ p_{j2}^t,\ n_{j2}^t,\ p_{j3}^t,\ d_j^t)$ on each company's card $j = 1,\ 2$ and at each period $t = 0,\ 1,\ 2$ such that s_{j0} denotes whether the contract is signed at period zero, $n_{j\tau}^t$ and $p_{j\tau}^t$ represent respectively the decision about the quantity of new loan n and repayment p which the consumer makes at date t regarding company j and which affects the consumption level at date τ, and d_j^t represents the default decision:

$$\begin{aligned} s_{j0} &= 0 \text{ means no contract signed with lender } j \\ s_{j0} &= 1 \text{ means contract signed with lender } j \end{aligned}$$

$$\begin{aligned} d_j^t &= -1 \text{ represents planning default against the respective lender} \\ d_j^t &= 0 \text{ represents planning meeting contractual obligations} \end{aligned}$$

For each company j the set of vectors from which the consumer can choose a trade is denoted by $\Phi_j \subseteq \mathbb{R}^6_+$ such that $(0,0,0,0,0,0)$ belongs to Φ_j. The consumer's consumption at each period is the sum of his endowment and two new loans minus two payments. Define $\xi(y^t_j) = (s_{j0}n^t_{j1},\ s_{j0}(n^t_{j2} - p^t_{j2}),\ \ - s_{j0}p^t_{j3},\ s_{j0}d^t_j)$, $n^t_\tau = \sum_{j=1}^{2} n^t_{j\tau}$ and $p^t_\tau = \sum_{j=1}^{2} p^t_{j\tau}$.

The consumer chooses these trades sequentially. At date zero, he chooses $y^0_1 \in \Phi_1$ and $y^0_2 \in \Phi_2$ to maximize U_0 of $m + \xi(y^0_1) + \xi(y^0_2)$. The only way of these trades constraints the future ones is that, if $y^0_j = (0,0,0,0,0,0)$ then the next periods' trade with company j should be the same (the interpretation of this constraint is that if the consumer decides at date zero not to deal with a lender, then the decision is irreversible).

At date one, he chooses $y^1_1 \in \Phi_1$and $y^1_2 \in \Phi_2$ to maximize U_1 of $m + \xi(y^1_1) + \xi(y^1_2)$ subject to the constraint imposed by the previous period, that is if $s_{j0} = 0$ for $j = 1,\ 2$ then $y^1_j = (0,0,0,0,0,0)$.

At date two, he chooses $y^2_1 \in \Phi_1$and $y^2_2 \in \Phi_2$ to maximize U_2 of $m + \xi(y^2_1) + \xi(y^2_2)$ subject to the constraints given at period zero, that is if $s_{j0} = 0$ for $j = 1,\ 2$ then $y^1_j = (0,0,0,0,0,0)$, and at period one, which is $n^2_{j1} = n^1_{j1}$ for $j = 1,\ 2$. Note that the trade determined at period two does not have to be the same as the trade determined at date one. This is the idea of naive hyperbolic optimization described in the literature review.

Let S be the set of pairs $(\Phi_1,\ \Phi_2)$ of nonempty, compact subsets of $\{0,\ 1\} \times \mathbb{R}^4_+ \times \{-1,\ 0\}$. Define $Y : S \to [\{0,\ 1\} \times \mathbb{R}^4_+ \times \{-1,\ 0\}]^2$ such that Y_j is y^2_j resulting from the consumer's sequential optimization.

The consumer's utility function is strictly increasing and concave in consumption and the standard Inada conditions hold. There is an arbitrarily small cost for accepting a contract at period zero.

3.3 Specification of the Companies

Companies offer contracts at the initial period only. Companies are exponential discounted profit maximizers. Company $j's$ period zero discounted profit is based on the final decision of the consumer at date two, that is $\Pi_j(y^2_j) = \delta^3(n^2_{j1} - p^2_{j2})r_j$.

Informally, each credit card company j charges an interest rate of r_j for loans more than one period although it is not permitted to charge interest for only one period loans. The cost of lending money for one period for the companies is zero since we can think that the credit card companies' other sources of revenue, such as merchant discounts and the fees charged to the customers, cancels out the cost of lending money for one period[2]. If the consumer cannot pay his debt at period three

[2] Merchant discount is a fee a merchant pays to the acquiring bank for processing the credit card transaction.

he defaults. We assume that the credit card companies lose everything they lent at the time of default without loss of generality.

3.3.1 Legal Restrictions

The company's strategy set is not the set of all contracts but consists only of contracts specified by a credit limit l and interest rate r, that have the following form:

$$\begin{aligned} n_{j1} &\leq l_j \\ n_{j2} + n_{j1} - p_{j2} &\leq l_j \end{aligned}$$

which means that the consumer's total debt cannot be greater than his credit limit l_j at any period.

$$\begin{aligned} p_{j2} &\leq \min\{m,\ n_{j1}\} \\ p_{j3} &\leq \min\{m,\ (n_{j1} - p_{j2})(1 + r_j) + h_{j2}\} \end{aligned}$$

which means that the consumer's payment cannot be higher than his income and his total debt at any period.

$$\begin{aligned} d_j &= 0 \text{ if } p_{j3} = (n_{j1} - p_{j2})(1 + r_j) + n_{j2} \\ d_j &= -1 \text{ otherwise} \end{aligned}$$

which means that the consumer defaults if he cannot pay all his debt back. The cost of default for the consumer is exogenously given as C such that $v(-1, 0) = v(0, -1) = v(-1, -1) = -C$ and $v(0, 0) = 0$.

3.4 Strategic Interaction

The two companies make simultaneous contract offers and the consumer decides which ones to choose, then subsequently the consumer also makes two sequential decisions as described under section 3.1. Therefore, the only strategic game between the companies and the consumer takes place at period zero.

Let define the strategic-form game Γ of the following form

$$\Gamma = \left(N,\ \mathcal{H}_i,\ \mathcal{A},\ (S_i)_{i\in N}, (u_i)_{i\in N}\right)$$

where N is the set of players $i = 1,\ 2,\ 3$, namely two companies and one consumer, $\mathcal{H}_i$ denotes the collection of player i's information sets, $\mathcal{A}$ set of possible actions in the game, $S(H) \subset \mathcal{A}$ the set of actions possible at information set H and u_i is a function such that $u_i : \times_{i\in N} S_i \to \mathbb{R}$ which represents the expected utility payoff that player i would get in this game. A strategy for player i is a function $s_i : \mathcal{H}_i \rightarrow \mathcal{A}$ such that $s_i(H) \in S(H)$ for all $H \in \mathcal{H}_i$.

We focus on pure strategy subgame perfect equilibria of this game.[3] We can have two different kinds of equilibria depending on the consumer's behavior when he is indifferent between two contracts. If the consumer chooses the contract with lower interest when he is indifferent between two, then we call the consumer rate-sensitive in that equilibrium[4]. If the consumer randomizes when he is indifferent between two contracts, then we call the consumer rate-insensitive in that equilibrium.

From this point on I will examine only the subgame perfect equilibria which satisfy one condition on consumer and two conditions on companies. The condition on consumer is that he chooses a contract with half probability when he is indifferent, one condition on companies is that they offer the minimum credit limit when they are indifferent among credit limits which are higher than the consumer's income and the other condition on companies is that they offer the maximum credit limit when they are indifferent among credit limits which are lower than the consumer's income.

4 Analysis

4.1 Consumer Behavior

In this section, we will analyze the naive hyperbolic consumer's behavior once he accepts an offer. We can write c_τ, the consumption at period τ, as follows:

$$\begin{aligned} c_1 &= m + n_1 \\ c_2 &= m - p_2 + n_2 \\ c_3 &= m - p_3 \end{aligned}$$

It is possible write each period total payment in terms of the endowment and the total debt of the previous period. This is because we know that it is a dominant strategy for the consumer to pay as much of his debt as possible at any period and to utilize the grace period.

Remark 1 *The consumer's optimization implies the following relations:*

$$\begin{aligned} &if\ n_1 \leq m && \Rightarrow \quad p_2 = n_1 \\ & && \Rightarrow \quad p_3 = n_2 \\ &if\ n_1 > m && \Rightarrow \quad p_2 = m \\ & && \Rightarrow \quad p_3 = (n_1 - m)(1 + r_i) + n_{j1}(1 + r_j) \ \ if\ n_{i1} > m\ for\ r_i \geq r_j \\ & && and \\ & && \Rightarrow \quad p_3 = (n_1 - m)(1 + r_j) \ \ if\ n_{i1} \leq m\ for\ r_i \geq r_j \end{aligned}$$

[3]Note that not all equilibria are trembling hand perfect.

[4]Note that this will be trembling-hand perfect equilibrium.

At each period, naive hyperbolic consumer underestimates his future period borrowings and this may cause the consumer to believe that he can pay his credit card loans back in the following period without interest.

Proposition 1 *There exist a cutoff exponential discount factor δ' for the naive hyperbolic consumer such that*

$$\text{for all } \delta > \delta',\ n_1^0 < m$$

and accordingly the initial period self believes that he will not pay interest in the future.

Proof. See appendix for the proof of the proposition. ■

When the consumer comes to the first period, he realizes that he underestimated his first period borrowing and reconsiders his consumption plan. Note that $n_1^0 < n_1^1$ for all $\beta \in (0,\ 1)$.

Proposition 2 *There exist a cutoff exponential discount factor δ'' and a cutoff hyperbolic discount factor β' for the naive hyperbolic consumer such that*

$$\text{For all } (\delta,\ \beta) \text{ such that } \delta > \delta'' \text{ and } \beta < \beta',\ n_1^0 < m < n_1^1$$

accordingly the the initial period self believes that he will not pay interest in the future although in reality he will.

Proof. See appendix for the proof of the proposition ■

Remark 2 *Given that both of the credit limit offers in two contracts either higher or lower than n_1^0, the naive hyperbolic consumer with $\delta > \delta''$ and $\beta < \beta'$ will be indifferent between these two contracts since he believes that he will borrow only $n_1^0 < m$ in period one and he will not pay interest on his credit card loans. However, this consumer ends up borrowing $n_1^1 > m$ in period one and paying interest. In this model, we will consider the consumer with $\delta > \delta''$ and $\beta < \beta'$ only.*

Since there is an arbitrarily small cost for applying a card, the consumer will choose only one card if there is at least one company offering enough credit for the consumer according to his initial period self.

Remark 3 [5]

$$\begin{aligned}
\text{If } \max\{l_1,\ l_2\} &\geq n_1^0 > \min\{l_1,\ l_2\} \\
&\Rightarrow \text{consumer accepts only the contract with higher credit limit} \\
\text{If } \min\{l_1,\ l_2\} &\geq n_1^0 \Rightarrow \text{consumer accepts only one contract randomly} \\
\text{If } \max\{l_1,\ l_2\} &< n_1^0 \Rightarrow \text{consumer accepts both contracts}
\end{aligned}$$

[5]Note that there is always a subgame perfect equilibrium in which the consumer is rate-sensitive.

4.2 Credit Card Companies' Behavior

4.2.1 When the Consumer Accepts only one Contract at the Initial Period

We can write the objective function of a credit card company whose contract accepted by the consumer at the initial period:

$$\begin{aligned} & \max\ \delta^3(l-m)r && (1)\\ & s.t. && \\ l \ \geq\ & n_1^1 \geq m && (2)\\ C \ \geq\ & C'(l_1,\ l_2,\ r,\ n_1,\ n_2) && (3)\end{aligned}$$

The first constraint is because of the fact that there will be no interest revenue for the company if the consumer does not spend more than his income. There is an exogenously given cost of default C for the consumer. We can think that as the cost of having unfavorable terms in any contract in the future after declaring bankruptcy[6].We can think the consumer's cost of default C is inversely related to the riskiness of the consumer. Consider a consumer with a good credit history and another consumer with a bad credit history. The first consumer faces lower interest rates and more favorable terms in any contract (e.g. auto loans, mortgages, and other kinds of credit) than the other consumer because of the difference in their credit scores [7]. Therefore, the first kind of the consumer is going to lose more in bankruptcy than the other consumer and consequently has a higher cost of bankruptcy than the other one. As a result, as the consumer becomes more of a risky prospect (determined by lower credit score), the cost of bankruptcy will be lower for the consumer.

In order for consumer not to plan to default, the cost of default for the consumer should be higher than some cutoff value C' which is a function of the credit limits offered by two companies, of the interest rate of the chosen company and of the consumer's planned borrowing amounts. Since the consumer may plan to default at any period and since he is time inconsistent, we might have three different cutoff values namely $C_0,\ C_1$ and C_2 for each period respectively.

[6] "A consumer reporting company can report most accurate negative information for seven years and bankruptcy information for 10 years." (Federal Trade Commission)

[7] Consider a couple who are looking to buy their first house. Let's say they want a 30-year mortgage loan and their FICO credit scores are 720. They could qualify for a mortgage with a low 5.5 percent interest rate. But if their scores are 580, they probably would pay 8.5 percent of more-that's at least 3 full percentage points more in interest. On a $100,000 mortgage loan, that 3 point difference will cost them $2,400 dollars a year, adding up to $72,000 dollars more over the loan's 30-year lifetime. (CFA & FairIsaac 2005)

Lemma 1 *When only one contract accepted, cutoff cost of default for each period is as follows:*

$$
\begin{aligned}
C_0 &= \frac{1}{\delta^2}\left[\begin{array}{c} \max\limits_{n_1 \leq l_1 + l_2} \left[u(m+n_1) + \delta u(m + l_1 + l_2 - n_1) + \delta^2 u(m)\right] \\ -\max\limits_{n_1 \leq m,\ n_2} \left[u(m+n_1) + \delta u(m - n_1 + n_2) + \delta^2 u(m - n_2)\right] \end{array}\right] \\
C_1 &= \frac{1}{\beta\delta^2}\left[\begin{array}{c} \max\limits_{n_1 \leq l_1} \left[u(m+n_1) + \beta\delta u(m + l_1 - n_1) + \beta\delta^2 u(m)\right] \\ -\max\limits_{n_1 \leq l_1,\ n_2} \left[u(m+n_1) + \beta\delta u(n_2) + \beta\delta^2 u(m - (n_1 - m)(1+r) - n_2)\right] \end{array}\right] \\
C_2 &= \frac{1}{\beta\delta}\left[\begin{array}{c} \left[u(m + l_1 - n_1) + \beta\delta u(m)\right] \\ -\max\limits_{n_2} \left[u(n_2) + \beta\delta u(m - (n_1 - m)(1+r) - n_2)\right] \end{array}\right]
\end{aligned}
$$

Proof. See the appendix for the proof of the lemma. ■

Lemma 2 *We can show that* $C_1 < C_2$*, therefore* $C' = \max\{C_0,\ C_2\}$

Proof. See the appendix for the proof of the lemma. ■

4.2.2 When the Consumer accepts both Contracts at the Initial Period

Note that $n_1^0 < m$ for the consumer with $\delta > \delta''$ *and* $\beta < \beta'$ and the consumer accepts both contracts only if $\max\{l_1,\ l_2\} < n_1^0$. This means that none of the credit limit offers is higher than the consumer's income when the consumer accepts both contracts. Consequently, the consumer can pay his debt on the card with higher interest in full within the grace period. As a result, we can write the objective function of a credit card company i for $l_1 + l_2 \geq n_1^1$ and $n_1^1 > l_1 + l_2$ respectively as follows:

$$
\begin{aligned}
& \max\ (n_1^1 - m) r_i & (4) \\
& s.t & \\
l_1 + l_2 &\geq n_1^1 \geq m & (5) \\
r_i &< r_j & (6) \\
C &\geq C'(l_1,\ l_2,\ r,\ n_1,\ n_2) & (7)
\end{aligned}
$$

Note that the other company's credit limit and interest rate affect whether the consumer pays interest when the consumer accepts two contracts.

Lemma 3 *When both contracts are accepted, cutoff cost of default for each period is as follows:*

$$C_0 = \frac{1}{\delta^2}\left[\begin{array}{c} \max\limits_{n_1 \le l_1 + l_2}\left[u(m+n_1)+\delta u(m+l_1+l_2-n_1)+\delta^2 u(m)\right] \\ -\max\limits_{n_1 \le m,\ n_2}\left[u(m+n_1)+\delta u(m-n_1+n_2)+\delta^2 u(m-n_2)\right] \end{array}\right]$$

$$C_1 = \frac{1}{\beta\delta^2}\left[\begin{array}{c} \max\limits_{n_1 \le l_1}\left[u(m+n_1)+\beta\delta u(m+l_1+l_2-n_1)+\beta\delta^2 u(m)\right] \\ -\max\limits_{n_1 \le l_1,\ n_2}\left[u(m+n_1)+\beta\delta u(n_2)+\beta\delta^2 u(m-(n_1-m)(1+r)-n_2)\right] \end{array}\right]$$

$$C_2 = \frac{1}{\beta\delta}\left[\begin{array}{c} \left[u(m+l_1+l_2-n_1)+\beta\delta u(m)\right] \\ -\max\limits_{n_2}\left[u(n_2)+\beta\delta u(m-(n_1-m)(1+r)-n_2)\right] \end{array}\right]$$

Proof. The proof of this lemma is the same as the proof of 1 except the credit limit at period one and two will be $l_1 + l_2$ instead of only l_1. ■

Lemma 4 *We can show that* $C_0 < C_1 < C_2$, *therefore* $C' = C_2$

Proof. See the appendix for the proof of the lemma. ■

4.3 Description of the Equilibria

4.3.1 When the Consumer Accepts only one Contract at the Initial Period

In order to describe the equilibrium, we will fist analyze the case with only one company chosen at the initial period. Note that at least one company needs to offer more than n_1^0 in order only one company to be chosen at the initial period.

We will determine the best response functions in terms of credit limit. For a given credit limit offered by the second company, the best response of the first company can be found by solving the company's problem (1) within the given constraints (2) and (3). From the objective function, we can see that the first company will choose the profit maximizing credit limit and interest rate from the first order conditions. The question is how the optimal credit limit we find from the first order conditions change with the second company's offered credit limit, namely l_2.

Remark 4 *In the objective function of the first company, the other company's credit limit does not appear but it appears in the cost of bankruptcy constraint, specifically when* $C' = C_0 \ge C_2$. *Therefore:*

$$\begin{aligned} C_0 &< C_2 \Rightarrow\ l_2 \text{ does not affect the best response } l_1 \\ C_0 &\ge C_2 \Rightarrow l_2 \text{ may decrease the best response } l_1 \end{aligned}$$

Lemma 5 *There is a cutoff credit limit l_2' for the second company such that:*

$$\begin{aligned} \text{for } l_2 &< l_2' \Rightarrow C_0 < C_2 \\ \text{for } l_2 &\geq l_2' \Rightarrow C_0 \geq C_2 \end{aligned}$$

Proof. Let $l_2 = 0$, then it is easy to show that $C_0 < C_1$ as we did in the proof of 4, and therefore $C_0 < C_2$. Note that C_2 does not depend on l_2, but C_0 :

$$\frac{\partial C_0}{\partial l_2} = \frac{u'(m + l_1 + l_2 - n_1)}{\delta} > 0$$

Therefore, there will be l_2' such that

$$\begin{aligned} \text{for } l_2 &< l_2' \Rightarrow C_0 < C_2 \Rightarrow C' = C_2 \\ \text{for } l_2 &\geq l_2' \Rightarrow C_0 \geq C_2 \Rightarrow C' = C_0 \end{aligned}$$

■

Remark 5 [8]*As the second company's offered credit limit increases even more than l_2', there will be a cutoff credit limit l_2'' such that for $l_2 = l_2''$ the cost of default constraint will be binding ($C = C' = C_0$) for the current credit limit and interest rate offer of the first company. Note that C_0 is not affected by interest rate but by the first company's credit limit, specifically $\frac{\partial C_0}{\partial l_1} > 0$. Therefore, for $l_2 \geq l_2''$ the best response credit limit of the first company will be a decreasing line with a slope of -1, which is because of the fact that $\frac{\partial C_0}{\partial l_1} = \frac{\partial C_0}{\partial l_2}$, in order to satisfy the cost of default constraint. Moreover, there will be another cutoff credit limit l_2''' such that the first company's best response will be to offer zero credit limit. As a result:*

$$\begin{aligned} \text{for } l_2'' &> l_2 \Rightarrow \text{the first company's best response is a straight line} \\ \text{for } l_2''' &> l_2 \geq l_2'' \Rightarrow \text{the first company's best response is decreasing with } -1 \text{ slope} \\ \text{for } l_2 &\geq l_2''' \Rightarrow \text{ the first company's best response will be zero} \end{aligned}$$

Proposition 3 *In the credit card market, if the profit maximizing credit limit l_i of the company i is greater than n_1^0 for $0 < l_j < n_1^0$, then the following two different equilibria will be possible depending on the parameter values*

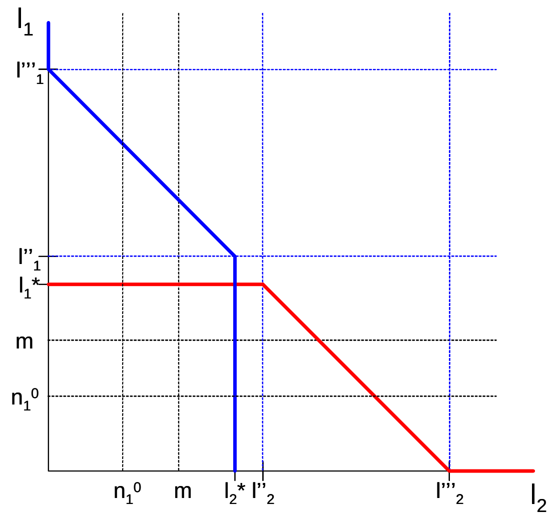
1. *Positive expected profit equilibria without competition on interest rate*

2. *Zero profit equilibria without competition on interest rate*

Proof. Let l_j^* is the profit maximizing credit limit for company j such that $C' = C_2 \geq C_0$ for company j given that $l_i < l_i'$.

The hypothetical best response curves will be as follows:

[8]Note that $l_2' = l_2''$ is possible.



1. If the companies' best response curves cross each other at a point in which at least one company's credit limit offer is greater than the consumer's income, then that point may represent a positive profit equilibrium. This is because the consumer will choose only one card at the initial period, and therefore there will not be competition in later periods. Since at least one of the company's credit limit offer is more than the consumer's income, this company will get interest revenue if it is chosen at the initial period.

2. If the companies' best response curves cross each other at a point in which one of the company's credit limit offer is between n_1^0 and m and the other's credit limit offer is between zero and m, then that point represents a zero profit equilibrium with no competition on interest. Since at least one of the companies offer a credit limit more than the consumer's believed amount of borrowing n_1^0, the consumer will accept only one card at the initial period, and therefore there will not be competition in later periods. Since the offered credit limit of the chosen company is less than the consumer's income, the consumer will not be able to borrow more than his income at any period. Therefore, he will be able to pay his each period borrowing back within the grace period without interest and he will not generate any interest revenue for the companies.

■

Proposition 4 *In the credit card market, if it is possible that the profit maximizing credit limit of the company i is greater than n_1^0 for $l_j = 0$ and less than n_1^0 for some l_j in $[0,\ n_1^0]$, then the following three cases might occur:*

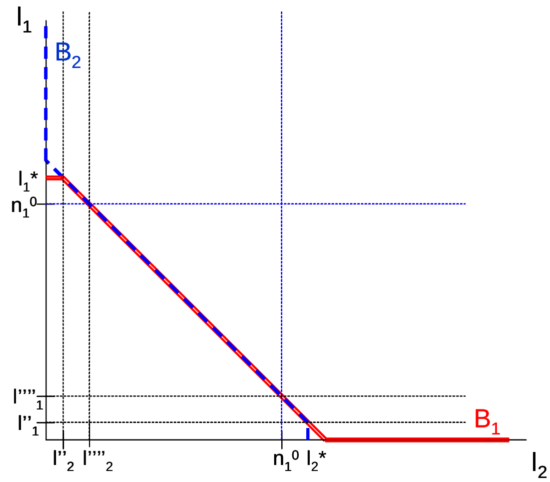
1. *Positive expected profit equilibria without competition on interest rate*

2. *Zero profit equilibria without competition on interest rate*

3. *Zero profit equilibria with competition on interest rate*

Proof. Let l_j^* is the profit maximizing credit limit for company j such that $C' = C_2 \geq C_0$ for company j given that $l_i < l_i'$

Let denote the hypothetical best response curves as B_1 and B_2 for company 1 and 2 respectively when the consumer accepts only one contract as in the following graph:



l_i'' for $i = 1, 2$ and n_1^0 are as defined before.

Note that $l_j < n_1^0$ for $l_i'''' < l_i < n_1^0$ for $i \neq j$. This means that the consumer should have accepted both contracts when $l_i'''' < l_i < n_1^0$, and consequently B_j cannot be the best response in that region. In order to find the best response in that region, we can draw an imaginary best response curve B_j^m under the following two conditions:

1. the interest rate is fixed to zero because of the competition

2. both contracts are accepted at the initial period

Let's use the following notation for different cutoff cost of default values:

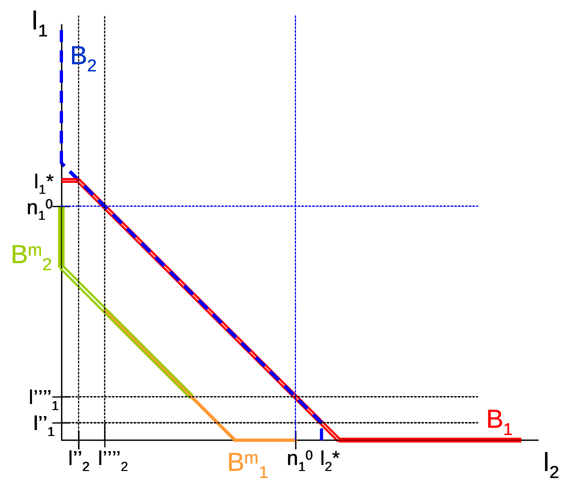
$$
\begin{aligned}
C_0^r &= \text{cutoff cost of default at time } t = 0 \text{ when } r > 0 \text{ with only one contract accepted} \\
C_0^0 &= \text{cutoff cost of default at time } t = 0 \text{ when } r = 0 \text{ with both contracts accepted} \\
C_2^r &= \text{cutoff cost of default at time } t = 2 \text{ when } r > 0 \text{ with only one contract accepted} \\
C_2^0 &= \text{cutoff cost of default at time } t = 2 \text{ when } r = 0 \text{ with both contracts accepted}
\end{aligned}
$$

When both contracts are accepted, $C_2^0 > C_0^0$ is always true from (3) and therefore $C' = C_2^0$

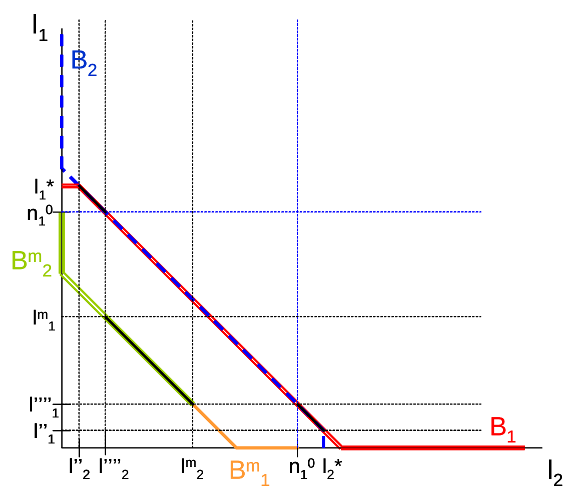
For $l_i \geq l_i''$, the imaginary best response curve should always be under the original best response curve. This is because of the following relation among the cutoff cost of defaults:

$$C_0^r = C_0^0 < C_2^0$$

Since the cost of default constraint is binding for C_0^r, C_2^0 will not satisfy the cost of default constraint unless the company j decreases its credit limit given the other company's credit limit l_i. As a result, we can draw the imaginary best response curves B_j^m for $l_i'''' < l_i < n_1^0$ as follows:[9]



Therefore, we can show the equilibrium points in black in the following graph:



For $i \neq j$:

1. if $l_j^* \geq m$

[9]Note that B_j^m for $j = 1, 2$ may start from a point lower than l_j'''' . In that case, we do not observe zero profit equilibria with competition on interest.

(a) for $(l_i,\ l_j)$ such that $m < l_j \leq l_j^*$ and $l_i'' \leq l_i < l_i'' + l_j^* - m$, there is positive expected profit equilibria without competition on interest

(b) for $(l_i,\ l_j)$ such that $n_1^0 < l_j \leq m$ and $l_i'' + l_j^* - m < l_i \leq l_i''''$, there is zero profit equilibria without competition on interest

(c) for $(l_i,\ l_j)$ such that $l_j'''' < l_j \leq l_j^m$ and $l_i'''' < l_i \leq l_i^m$, there is zero profit equilibria with competition on interest

2. if $l_j^* < m$

(a) for $(l_i,\ l_j)$ such that $n_1^0 < l_j \leq l_j^*$ and $l_i'' < l_i \leq l_i''''$, there is zero profit equilibria without competition on interest

(b) for $(l_i,\ l_j)$ such that $l_j'''' < l_j \leq l_j^m$ and $l_i'''' < l_i \leq l_i^m$, there is zero profit equilibria with competition on interest

■

4.3.2 When the Consumer Accepts both Contracts at the Initial Period

Now, we will analyze the case in which none of the companies offer more than n_1^0 at the initial period because of the cost of default constraint, and therefore the consumer accepts both credit cards.

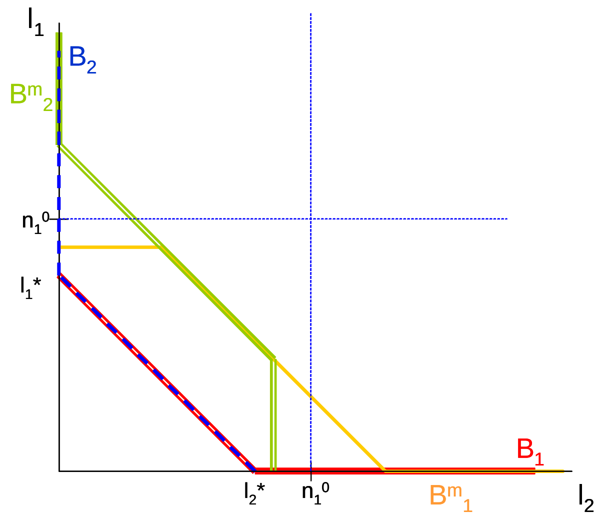
Once the consumer have two cards on hand, he will not pay interest to the company with higher interest but to the company with lower interest in order to minimize the cost of borrowing. Therefore, if the consumer accepts two cards at the initial period, then the competition among the companies will drive the interest rates to zero.

Remark 6 *When the consumer has two cards, we showed that $C' = C_2$ in lemma 4. Given that both companies will offer zero interest, the best response of the first company will be a decreasing curve since the only way to decrease C' will be by decreasing the credit limit as the second company offers more credit limit. Moreover, there will be a cutoff credit limit for the second company l_2''' such that for $l_2 \geq l_2'''$ the first company's best response credit limit will be zero.*

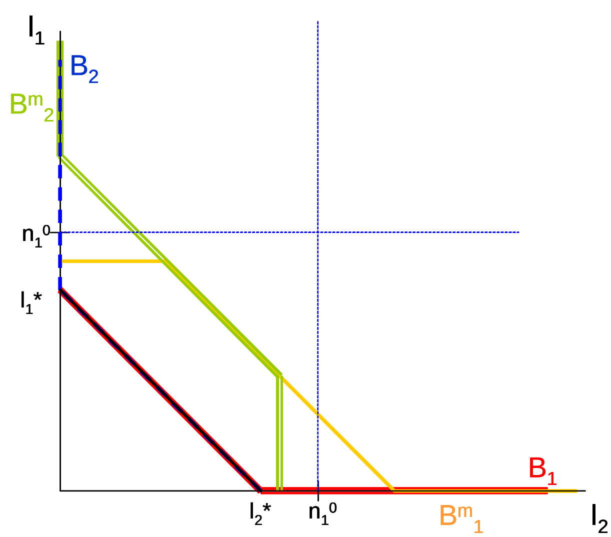
Proposition 5 *In the credit card market, if the profit maximizing credit limit for a company is less than n_1^0 given that the other company's credit limit offer is zero, then only zero profit equilibria with competition on interest will be possible.*

Proof. Let's define l_j^* as the profit maximizing credit limit for company j given that the other company's credit limit offer is zero. The reason for l_j^* to be less than n_0^1 is because of the binding cost of default constraint, namely $C = C' = C_2$ when $l_j = l_j^*$ and $l_i = 0$. From the expression for C_2 given in lemma 3, we can say that as l_i increases, l_j should decrease the same amount in order to keep satisfying the cost of default constraint. Therefore, the best response function for each company will be a line with a slope of -1 until it reaches zero and then a horizontal line at zero.

Let denote the hypothetical best response curves as B_1 and B_2 for company 1 and 2 respectively when the consumer accepts both contracts as in the following graph:



Note that the consumer should have accepted only one contract when $n_0^1 < B_j$, and consequently B_j cannot be the best response in that region. In order to find the best response in that region, we can draw an imaginary best response curve B_j^m under the condition that only one contract is accepted. Therefore, B_j^m for $j = 1,\ 2$ will be a horizontal line first, then a decreasing line with a slope of -1 and a horizontal line at zero as shown above.As a result, the zero profit equilibria will be as shown on the following graph in black:



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5 Discussion & Conclusion

In our model, we focus on two aspects of a credit card contract which are the interest rate and the credit limit. Cash back and reward points are other aspects of the credit card contracts which were not common 10 years ago. If we would include these aspects as well, then positive expected profits would not be possible. Because, there would be competition on these aspects of the credit card contracts which will drive the expected profits to zero even if there would not be any competition on interest rate.

The small cost of applying a credit card makes the consumer choose just one card randomly when he is not responsive to interest rate and when the offered credit limits are higher than the consumer's believed amount of borrowing. This cost of applying a credit card is essential to have positive profit equilibrium.

In our model, there is an exogenous variable depending on the bankruptcy law, namely the consumer's cost of bankruptcy C. With a change in bankruptcy law, this exogenous variable may change. If the consumer's cost of bankruptcy (C) increases, then the consumer will be less likely to default and credit card companies will be more likely to get positive expected profit. We define consumer's cost of bankruptcy as the cost of having unfavorable terms in any contract in the future after declaring bankruptcy. If the bankruptcy information stays in the consumer's credit report longer, than we expect a bigger C, and consequently higher possibility of positive expected profits.

Another feature of our model is that there are no credit card offers after the initial period. If we allow credit card offers after the initial period and if the consumer can start borrowing on a new card only one period after accepting the offer, our results will still hold in this three period of consumption model. We suspect that positive expected profits may still be possible depending on the severity of the consumer's time inconsistency in a more than three periods of consumption model. When the consumer realizes that he is going to borrow more than his income and will incur interest, the only way to decrease or eliminate the interest payment is to borrow on another card with a smaller interest rate. However, the consumer currently has only one card in that period, therefore in order to get another card he needs to accept an offer made at that period, but can not start borrowing immediately. In other words, to accept another credit card offer does not help to decrease the interest payment since the consumer will not be able to start borrowing on the new card immediately. Moreover, the only time the consumer accepts a new card offer with lower interest rate will be the time the consumer believes he is going to have debt more than his income in the next period and consequently pay interest. However, if the consumer underestimates his future borrowing because of his time inconsistency, he may believe that he will not have debt more than his income in the next period, and therefore he does not accept a new credit card offer with less interest rate at the current period. Note that this kind of behavior may occur without transaction costs. If we include transaction costs the consumer incurs when he accepts another card, then positive expected profit equilibrium exists with less time inconsistent consumers as well.

Our results hold for more than three periods of consumption models as well. If the consumer is partially naive as defined in O'Donoghue & Rabin (2001), a positive expected profit will still be possible but less likely.

In this paper, we investigate one of Ausubel's suggested explanations for his findings in Ausubel (1991) in a setting with one naive hyperbolic consumer and two exponential credit card companies. We show that there are circumstances in which there would be zero and positive expected profits possible.

6 Appendix

Proof of Proposition 1.

1. if $n_1 = \sum_{j=1}^{2} n_{j1} \leq m \Rightarrow \sum_{j=1}^{2} p_{j2} = n_1$ and $\sum_{j=1}^{2} p_{j3} = \sum_{j=1}^{2} n_{j2} = n_2$. Therefore, we can write the objective function of the consumer at $t = 0$ as follows:

$$\max_{n_1 \leq m} \beta\delta \left[u(m + n_1) + \delta u(m - n_1 + n_2) + \delta^2 u(m - n_2) \right] \tag{8}$$

FOCs:

$$u'(m - n_1^0 + n_2^0) = \delta u'(m - n_2^0) \tag{9}$$

$$u'(m + n_1^0) = \delta u'(m - n_1^0 + n_2^0) \tag{10}$$

We know that $n_1^0 = n_2^0 = 0$, when $\delta = 1$. In order to see how n_1^0 and h_2 change with δ, we can take the derivative of (9) and (10) with respect to δ :

$$u''(m - n_1^0 + n_2^0)(-\frac{\partial n_1^0}{\partial \delta} + \frac{\partial n_2^0}{\partial \delta}) = u'(m - n_2^0) + \delta u''(m - n_2^0)(-\frac{\partial n_2^0}{\partial \delta}) \tag{11}$$

$$u''(m + n_1^0)\frac{\partial n_1^0}{\partial \delta} = u'(m - n_1^0 + n_2^0) + \delta u''(m - n_1^0 + n_2^0)(-\frac{\partial n_1^0}{\partial \delta} + \frac{\partial n_2^0}{\partial \delta}) \tag{12}$$

- If $\frac{\partial n_1^0}{\partial \delta} > 0 \Rightarrow (-\frac{\partial n_1^0}{\partial \delta} + \frac{\partial n_2^0}{\partial \delta}) > 0$ *and* $\frac{\partial n_2^0}{\partial \delta} > 0$ by (12). However, the signs of these derivatives give a contradiction in (11). Therefore, $\frac{\partial n_1^0}{\partial \delta} < 0$ must be correct.

- Given that $\frac{\partial n_1^0}{\partial \delta} < 0$, if $\frac{\partial n_2^0}{\partial \delta} > 0 \Rightarrow (-\frac{\partial n_1^0}{\partial \delta} + \frac{\partial n_2^0}{\partial \delta}) > 0$. However, the sign of these derivatives give a contradiction in (11). Therefore, $\frac{\partial n_2^0}{\partial \delta} < 0$ must be also correct.

  From (9) and (10), for $\delta < 1$:

$$n_2^0 > \frac{n_1^0}{2} \textit{ and } n_1^0 > \frac{n_2^0}{2} \tag{13}$$

  As δ decreases, n_1^0 *and* n_2^0 increase. We know that n_2^0 will never be greater than m from the first order conditions. Moreover, if we do not allow n_2^0 to be greater than $m/2$, by (13) we can make sure that n_1^0 will not be greater than m. So, there is a lower bound for δ, namely δ^*, such that for $\delta > \delta^*$ the constraints given at the beginning of this case are satisfied in the optimal plan of the consumer in period zero and the consumer believes that he will not keep a positive balance on his credit card.

2. if $n_1 = \sum_{j=1}^{2} n_{j1} \geq m \implies \sum_{j=1}^{2} p_{j2} = m$ and $\sum_{j=1}^{2} p_{j3} = \left(\sum_{j=1}^{2} n_{j1} - m\right)(1 + r_j)$ assuming that $n_{i1} \leq m$ for $r_i \geq r_j$. Therefore the objective function is:

$$\max_{n_1 \geq m} \beta\delta \left[u(m + n_1) + \delta u(n_2) + \delta^2 u(m - (n_1 - m)(1 + r_j))\right] \tag{14}$$

   FOCs:

$$u'(n_2^0) = \delta u'(m(2 + r) - n_1^0(1 + r) - n_2^0) \tag{15}$$

$$u'(m + n_1^0) = \delta(1 + r)u'(n_2^0) \tag{16}$$

   If we take the derivative of (15) and (16) with respect to δ :

$$\begin{aligned} u''(n_2^0)\frac{\partial n_2^0}{\partial \delta} &= u'(m(2 + r) - n_1^0(1 + r) - n_2^0) \\ &\quad + \delta u''(m(2 + r) - n_1^0(1 + r) - n_2^0)(-\frac{\partial n_1^0}{\partial \delta}(1 + r) - \frac{\partial n_2^0}{\partial \delta}) \end{aligned} \tag{17}$$

$$u''(m + n_1^0)\frac{\partial n_1^0}{\partial \delta} = (1 + r)u'(n_2^0) + \delta(1 + r)u''(n_2^0)\frac{\partial n_2^0}{\partial \delta} \tag{18}$$

   - If $\frac{\partial n_1^0}{\partial \delta} > 0 \Rightarrow \frac{\partial n_2^0}{\partial \delta} > 0$ by (18). However, the sign of these derivatives give a contradiction in (17). Therefore, $\frac{\partial n_1^0}{\partial \delta} < 0$ must be correct.

- It is not possible to determine the sign of $\frac{\partial n_2^0}{\partial \delta}$. Luckily we know that $n_2^0 > n_1^0$ by (16) if $\delta = 1$. Since n_2^0 cannot be greater than m, $n_1^0 < m$ for $\delta = 1$. The only way to have $n_1^0 > m$ is by decreasing δ since $\frac{\partial n_1^0}{\partial \delta} < 0$.

As a result, we can say that there is cutoff δ value, namely δ^{**}, such that for $\delta < \delta^{**}$ the constraint given at the beginning of this case is not binding. For $\delta > \delta^{**}$, the constraint is binding such that $n_1^0 = m$ and the consumer's problem is:

$$\max \ \beta\delta \left[u(2m) + \delta u(n_2^0) + \delta^2 u(m - n_2^0)\right]$$

From the analysis we have made, for the consumers with $\delta > \delta' = \max\{\delta^*, \delta^{**}\}$, the constraint in problem (8) is not binding, but the constraint in problem (14) is binding. In the first problem, from the definition of maximum we can write:

$$\begin{aligned} & \max_{n_1 \leq m, \ n_2^0} \ \beta\delta \left[u(m + n_1) + \delta u(m - n_1 + n_2) + \delta^2 u(m - n_2)\right] \\ \geq \ & \max_{n_2^0} \beta\delta \left[u(2m) + \delta u(n_2) + \delta^2 u(m - n_2)\right] \end{aligned}$$

Since the constraint is binding in the second problem, the maximum for that problem is max $\beta\delta \left[u(2m) + \delta u(n_2) + \delta^2 u(m - n_2)\right]$. So we can incur from the previous inequality that the consumer with $\delta > \delta' = \max\{\delta^*, \delta^{**}\}$, will believe that he will not pay interest on his credit card borrowing.

■

Proof of Proposition 2. In proposition 1, we showed that a consumer with exponential discount factor $\delta > \delta' = \max\{\delta^*, \ \delta^{**}\}$ believes that he will not keep positive balance. In order to show that this consumer will keep positive balance and pay interest we will analyze the consumer with $\delta > \delta'$ only.

When the consumer comes to first period, his objective function will be as follows:

$$\max \ u(m + n_1) + \beta\delta u(m - p_2 + n_2) + \beta\delta^2 u(m - p_3)$$

1. if $n_1 < m \implies p_2 = n_1, \ p_3 = n_2$

   problem is

$$\max \ u(m + n_1) + \beta\delta u(m - n_1 + n_2) + \beta\delta^2 u(m - n_2) \tag{19}$$

   FOCs:

$$u'(m - n_1^1 + n_2^1) = \delta u'(m - n_2^1) \tag{20}$$

$$u'(m+n_1^1) = \beta\delta u'(m-n_1^1+n_2^1) \tag{21}$$

For $\delta > \delta'$, in order to determine how n_1^1 and n_2^1 change with β, we take the derivative of (20) and (21) with respect to β:

$$u''(m-n_1^1+n_2^1)(-\frac{\partial n_1^1}{\partial \beta} + \frac{\partial n_2^1}{\partial \beta}) = \delta u''(m-n_2)(-\frac{\partial n_2^1}{\partial \beta}) \tag{22}$$

$$u''(m+n_1^1)\frac{\partial n_1^1}{\partial \beta} = \delta u'(m-n_1^1+n_2^1) + \beta\delta u''(m-n_1^1+n_2^1)(-\frac{\partial n_1^1}{\partial \beta} + \frac{\partial n_2^1}{\partial \beta}) \tag{23}$$

- If $\frac{\partial n_1^1}{\partial \beta} > 0 \Rightarrow (-\frac{\partial n_1^1}{\partial \beta} + \frac{\partial n_2^1}{\partial \beta}) > 0$ *and* $\frac{\partial n_2^1}{\partial \beta} > 0$ by (23). However, these inequalities do not satisfy (22). Therefore, $\frac{\partial n_1^1}{\partial \beta} < 0$ must be true.
- Given that $\frac{\partial n_1^1}{\partial \beta} < 0$, if $\frac{\partial n_2^1}{\partial \beta} > 0$, these inequalities do not satisfy (22). Therefore, $\frac{\partial n_2^1}{\partial \beta} < 0$ must be also true.

As a result, as β decreases, n_1^1 and n_2^2 increases.

Write down the difference of the left and right hand side of the inside of the utility function in (20), as ε_1:

$-n_1^1 + n_2^1 + n_2^1 = \varepsilon_1 \implies n_1^1 = 2n_2^2 - \varepsilon_1$

For any β as $\delta \to 1$, then $\varepsilon_1 \to 0$, $n_1^1 \to 2n_2^1$.

So, we can write (21) as:

$u'(m+2n_2^1) = \beta u'(m-n_2^2)$

As $\beta \to 0$, $\frac{u'(m+2n_2^1)}{u'(m-n_2^1)} \to 0$, then $n_2^1 \to m$, because as n_2^1 goes to m , the denominator will be infinity and the numerator will be a finite number.

Therefore, we can say that there will be a $\widetilde{\delta}$ and β^* such that for $\delta > \widetilde{\delta}$ and $\beta < \beta^*$, $n_1^1 > m$ and this case will not be possible.

2. if $n_1 > m \implies p_2 = m$ & $p_3 = (n_1 - m)(1+r) + n_2$

   problem is

$$\max\ u(m+n_1) + \beta\delta u(n_2) + \beta\delta^2 u(m-(n_1-m)(1+r)-n_2) \tag{24}$$

FOCs:

$$u'(n_2^1) = \delta u'(m(2+r) - n_1^1(1+r) - n_2^1) \tag{25}$$

$$u'(m+n_1^1)=\beta\delta(1+r)u'(n_2^1) \quad (26)$$

For $\delta > \delta'$, in order to determine how n_1^1 and n_2^1 change with β, we take the derivative of (25) and (26) with respect to β:

$$u''(n_2^1)(\frac{\partial n_2^1}{\partial \beta})=\delta u''(m(2+r)-n_1^1(1+r)-n_2^1)(-\frac{\partial n_1^1}{\partial \beta}(1+r)-\frac{\partial n_2^1}{\partial \beta}) \quad (27)$$

$$u''(m+n_1^1)(\frac{\partial n_1^1}{\partial \beta})=\delta(1+r)u'(n_2^1)+\beta\delta(1+r)u''(n_2^1)\frac{\partial n_2^1}{\partial \beta} \quad (28)$$

- If $\frac{\partial n_1^1}{\partial \beta}>0 \Rightarrow \frac{\partial n_2^1}{\partial \beta}>0$ by (28). However, these inequalities do not satisfy (27). Therefore, $\frac{\partial n_1^1}{\partial \beta}<0$ must be true.
- Given that $\frac{\partial n_1^1}{\partial \beta}<0$, if $\frac{\partial n_2^1}{\partial \beta}<0$, the equation (27) gives a contradiction. Therefore, $\frac{\partial n_2^1}{\partial \beta}>0$ must be also true.

As a result, n_1^1 decreases and n_2^1 increases with β.

Write down the difference of the left and right hand side of the inside of the utility function in (25) as γ_1:

$n_2^1-m(2+r)+n_1^1(1+r)+n_2^1=\gamma_1 \implies n_1^1=\frac{\gamma_1-2n_2^1+m(2+r)}{1+r}$

For any β as $\delta\to 1,$ then $\gamma_1\to 0,\ n_1^1\to\frac{-2n_2^1+m(2+r)}{1+r}$

So, we can write (26) as:

$u'(m+\frac{-2n_2^1+m(2+r)}{1+r})=\beta(1+r)u'(n_2^1)$

As $\beta\to 0$, $\frac{u'\left(\frac{-2n_2^1+m(2+r)}{1+r}\right)}{u'(n_2^1)}\to 0$, then $n_2^1\to 0$, because as n_2^1 goes to 0, the denominator will be infinity and the numerator will be a finite number.

When $n_2^1\to 0$, then $n_1^1\to m\frac{2+r}{1+r}>m$.

Therefore, we can say that there will be a $\widehat{\delta}$ and β^{**} such that for $\delta>\widehat{\delta}$ and $\beta<\beta^{**},\ n_1^1>m$ and this case will be possible.

After analyzing the two cases above, for $\delta>\delta''=\max\{\delta',\ \widetilde{\delta},\ \widehat{\delta}\}$ and $\beta<\beta'=\min\{\beta^*,\ \beta^{**}\}$ we showed that the consumer will pay interest at the last period on his credit card borrowing as opposed to his belief in period zero.

■

Proof of Lemma 1.

1. At the initial period

   The consumer's total utility if he plans not to default and if he plans to default is as follows:

$$\max_{n_1 \le m,\ n_2} \left[u(m+n_1) + \delta u(m - n_1 + n_2) + \delta^2 u(m - n_2)\right]$$
$$\max_{n_1} \left[u(m+n_1) + \delta u(m + l_1 + l_2 - n_1) + \delta^2 u(m)\right]$$

   Therefore, the cutoff cost of default in order not to plan to default at the first period can be found as follows:

$$C_0 = \frac{1}{\delta^2}\left[\begin{array}{c} \max_{n_1} \left[u(m+n_1) + \delta u(m + l_1 + l_2 - n_1) + \delta^2 u(m)\right] \\ \max_{n_1 \le m,\ n_2} \left[u(m+n_1) + \delta u(m - n_1 + n_2) + \delta^2 u(m - n_2)\right] \end{array}\right]$$

2. At the first period

   When the consumer reaches the first period, he realizes that his actual borrowing is more than his income and consequently he will pay interest. In this case, his first period borrowing will be more than his income if he plans to default as well. Therefore, the consumer's total utility if he plans not to default and if he plans to default respectively as follows:

$$\max_{n_1 \le l_1,\ n_2} \left[u(m+n_1) + \beta\delta u(n_2) + \beta\delta^2 u(m - (n_1 - m)(1+r) - n_2)\right]$$
$$\max_{n_1 \le l_1} \left[u(m+n_1) + \beta\delta u(m + l_1 - n_1) + \beta\delta^2 u(m)\right]$$

   Therefore, the cutoff cost of default in order not to plan to default at the first period can be found as follows:

$$C_1 = \frac{1}{\beta\delta^2}\left[\begin{array}{c} \max_{n_1 \le l_1} \left[u(m+n_1) + \beta\delta u(m + l_1 - n_1) + \beta\delta^2 u(m)\right] \\ - \max_{n_1 \le l_1,\ n_2} \left[u(m+n_1) + \beta\delta u(n_2) + \beta\delta^2 u(m - (n_1 - m)(1+r) - n_2)\right] \end{array}\right]$$

3. At the second period

   When the consumer comes to the second period, the consumer's total utility if he plans not to default and if he plans to default respectively as follows:

$$\max_{n_2}\left[u(n_2)+\beta\delta u(m-(n_1-m)(1+r)-n_2)\right]$$
$$\left[u(m+l_1-n_1)+\beta\delta u(m)\right]$$

Therefore, we can find the cutoff cost of default in order not to plan to default at the second period as follows:

$$C_2=\frac{1}{\beta\delta}\left[\begin{array}{c}[u(m+l_1-n_1)+\beta\delta u(m)]\\ -\max\limits_{n_2}[u(n_2)+\beta\delta u(m-(n_1-m)(1+r)-n_2)]\end{array}\right]$$

■

Proof of Lemma 2. If β would be equal to 1 from the second period on as the first period self believes, then C_2 would be equal to C_1 and the consumer would not be time inconsistent. Therefore, it will be enough to look at how C_2 changes with β to find out the relation between C_1 and C_2 since we know that C_2 is equal to C_1 when $\beta=1$.

$$\frac{\partial C_2}{\partial\beta} = \frac{\left[\begin{array}{c}\left(u'\left(m+l_1-n_1\right)\left(-\frac{\partial n_1}{\partial\beta}\right)+\delta u(m)\right)\\ -\left(\begin{array}{c}\delta u(m-(n_1-m)(1+r)-n_2^*)\\ +\beta\delta u'\left(m-\left(n_1-m\right)\left(1+r\right)-n_2^*\right)\left(1+r\right)\left(-\frac{\partial n_1}{\partial\beta}\right)\end{array}\right)\end{array}\right]\beta\delta}{\left(\beta\delta\right)^2} -\frac{\left(\begin{array}{c}[u(m+l_1-n_1)+\beta\delta u(m)]\\ -[u(n_2^*)+\beta\delta u(m-(n_1-m)(1+r)-n_2^*)]\end{array}\right)\delta}{\left(\beta\delta\right)^2}$$

such that n_2^* represents the profit maximizing n_2 in case of planning not to default. If we simplify the previous equation:

$$\frac{\partial C_2}{\partial\beta} = \left(-\frac{\partial n_1}{\partial\beta}\right)\frac{\left[u'\left(m+l_1-n_1\right)-\beta\delta u'\left(m-\left(n_1-m\right)\left(1+r\right)-n_2\right)\left(1+r\right)\right]\beta\delta}{\left(\beta\delta\right)^2} -\frac{\left[u(m+l_1-n_1)-u(n_2^*)\right]\delta}{\left(\beta\delta\right)^2}$$

We know that $\left(-\frac{\partial n_1}{\partial\beta}\right)>0$ from the proof of proposition 2. From the definition of n_2^* :

$$u'(n_2^*)=\beta\delta u'\left(m-\left(n_1-m\right)\left(1+r\right)-n_2^*\right) \tag{29}$$

At period two, if the consumer plans to default he borrows more than the optimal amount if he plans not to default:

$$m + l_1 - n_1 \geq n_2^* \tag{30}$$

From (29) and (30):

$$\begin{aligned}
u'(m + l_1 - n_1) &< \beta\delta u'(m - (n_1 - m)(1 + r) - n_2^*) \\
&\Rightarrow [u'(m + l_1 - n_1) - \beta\delta u'(m - (n_1 - m)(1 + r) - n_2)(1 + r)] < 0 \\
&\Rightarrow \left(-\frac{\partial n_1}{\partial \beta}\right) \frac{\begin{bmatrix} u'(m + l_1 - n_1) \\ -\beta\delta u'(m - (n_1 - m)(1 + r) - n_2)(1 + r) \end{bmatrix} \beta\delta}{(\beta\delta)^2} < 0 \qquad (31) \\
&\quad and \\
u(m + l_1 - n_1) &\geq u(n_2^*) \\
&\Rightarrow -\frac{[u(m + l_1 - n_1) - u(n_2^*)]\,\delta}{(\beta\delta)^2} < 0 \qquad (32)
\end{aligned}$$

From (31) and (32):

$$\frac{\partial C_2}{\partial \beta} < 0$$

Accordingly;

$$C_2 > C_1 \text{ for } \beta < 1$$

Finally, we can define the cutoff C' as follows:

$$C' = \max\{C_0,\ C_2\}$$

■

Proof of Lemma 4. If $\beta = 1$, then C_1 would be equal to C_0 since the consumer would be time consistent. If we show how C_1 changes with β, then we can compare C_0 and C_1.

$$\begin{aligned}
\frac{\partial C_1}{\partial \beta} &= \frac{\begin{bmatrix} \left[\delta u(m + l_1 + l_2 - n_1^*) + \delta^2 u(m)\right] \\ -\left[\delta u(n_2^{**}) + \delta^2 u(m - (n_1^{**} - m)(1 + r) - n_2^{**})\right] \end{bmatrix} \beta\delta^2}{(\beta\delta^2)^2} \\
&\quad - \frac{\begin{pmatrix} \left[u(m + n_1^*) + \beta\delta u(m + l_1 + l_2 - n_1^*) + \beta\delta^2 u(m)\right] \\ -\left[u(m + n_1^{**}) + \beta\delta u(n_2^{**}) + \beta\delta^2 u(m - (n_1^{**} - m)(1 + r) - n_2^{**})\right] \end{pmatrix} \delta^2}{(\beta\delta^2)^2}
\end{aligned}$$

such that n_1^*, n_1^{**} and n_2^{**} represent the profit maximizing n_1 and n_2 in case of planning default and not to default respectively. If we simplify the previous equation:

$$\frac{\partial C_1}{\partial \beta} = -\frac{\left[u(m+n_1^*) - u(m+n_1^{**})\right]\delta^2}{\left(\beta\delta^2\right)^2} < 0$$

Accordingly;

$$C_1 > C_0 \text{ since } \beta < 1$$

Finally, we can write $C' = C_2$ since we already know that $C_2 > C_1$ from the previous proof. ■

7 References

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