Liquidity Management and Monetary Policy*

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Abstract

We develop a novel theoretical framework to study monetary policy in conjunction with an explicit role for banks. Banks are subject to a maturity mismatch problem that leads to a precautionary motive for holding Central Bank reserves. Monetary policy can have real and permanent effects by altering the trade-offs bank face between lending, holding reserves, holding deposits and paying dividends. We use this framework to analyze the macroeconomic effects of different monetary policy instruments and regulatory constraints. We also study how these policy tools interact with exogenous shocks to the volatility of withdrawals, equity losses and the demand for loans. We then use a calibrated version of our model to investigate quantitatively how the effectiveness of monetary policy may have changed in the aftermath of the 2008-2009 crisis, in response to these shocks and the regulatory constraints imposed by Basel-II.

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1 Introduction

The last five years have witnessed new challenges in the conduct of monetary policy. During the Great Recession, Central Banks were faced frozen inter-bank lending markets and dropped their interest rate targets all the way to its zero lower-bound. At the peak of the recession and even during its aftermath, Central Banks resorted to unconventional monetary tools. By this, they purchased private paper and expanded their balance sheets in an attempt to preserve financial stability and boost economic activity. These policies followed events that dramatically affected the banking system. The financial crisis was a time where the banking system saw unprecedented equity losses and banks responded by cutting back on lending. In the aftermath, banks seem to have accumulated central bank reserves without substantially resuming their lending in response to various monetary stimuli.[1]

These outcomes cast doubts about the effectiveness of monetary policy in stimulating lending when the banking system is facing a substantial disruption. Despite the evident connection between money and banking, there lacks a modern macroeconomic model that enables the study of monetary policy in conjunction with an explicit role for banks.

In this paper, we take a step towards filling this gap. We propose a new theoretical framework that focuses on some of the institutional details of banking to explain how monetary policy is implemented. We build a theory that enables us to answer a number of theoretical questions. How is the transmission monetary policy affected by the decisions of commercial banks? How is this interaction affected by bank solvency and liquidity ratios? What is the connection between monetary policy and financial regulation?[2] What shocks can affect the power of monetary policy?

We then use our theory to answer quantitative questions about the strength of monetary policy in the last five years. In particular, we calibrate our model and use it to uncover the type of shocks can explain the excess holdings of reserves by banks without a corresponding increase in lending.

In our model the effectiveness monetary policy critically depends on the actions undertaken by rational banks. Although the model is real, monetary policy carries real effects through the lending channel. The model relies on a mechanism by which bank lending reacts to monetary policy because policy instruments alter the trade-offs between making a profit on a loan against exposing a bank to more liquidity risk.

In essence, the heart of our model is a liquidity management problem. Banks choose the optimal mix between lending, deposit issuance and holding bank reserves to hedge liquidity

[1] A summary of these events is presented later in the paper.

[2] We refer to regulation such as the one put in place through the Dodd-Frank act or the Basel-III committee on bank supervision.

risk. Liquidity risks are associated with financial losses that follow when deposits are withdrawn and a bank does not have liquid reserves to comply with the command to transfer deposits to another bank. The mechanics are the following. When a bank grants a loan, it creates a liability in the form of a demand deposit. Granting a loan is profitable because a higher interest is charged on the loan. However, there is a trade-off. More lending relative to an amount reserves induces a potential maturity mismatch between bank assets, which are long term, and deposits, which are callable at any time. We assume that loans cannot be sold easily due to various frictions. Hence, banks hold central bank reserves (cash-like securities) to meet an unexpected deposit withdrawal. However, it may well be that the payment of bank liabilities exceeds their reserve levels. In such instances, banks must incur in financial losses. These losses occur because they must obtain expensive borrowing from the FED or other banks. Hence, holding more reserves, insures the bank against that risk.

We introduce this problem into a dynamic general equilibrium model with rational profit-maximizing heterogeneous banks. The bank liquidity management problem is captured by a portfolio with non-linear returns. The effects of monetary policy can be understood through that liquidity-management portfolio problem. Thus, monetary policy operates by altering the incentives that banks face when granting loans. Short-run monetary policy effects result from the ability to alter the return to this portfolio problem. Long-run monetary-policy effects are present because bank equity returns are affected. Consequently, the size of the financial sector is also altered. An important feature of this channel is that monetary policy can have real effects even if prices are fully flexible.

The implementation of monetary policy in our model is carried out through the use of different policy instruments. We study the effects of discount rates, open market operations (conventional and unconventional), reserve requirements and interests on reserves. All these instruments have a common effect. They can tilt the balance towards a more lax lending policy. The macroeconomic effects result from more lending and lower interest rates which help stimulate economic activity. However, and this is the point we wish to convey in this paper, is that as much as monetary policy can affect bank lending practices, in turn, its power can be affected by other various circumstances that affect banking decisions.

The model delivers a rich description for banking and monetary indicators. For an individual bank, it explains the behavior of their reserve holdings, lending policies, leverage and dividend policies. For the banking industry as a whole, it provides descriptions of lending volumes, interbank lending, excess reserve holdings. It also describes interbank borrowing and lending rates. It also provides a description of banking financial indicators such as return on loans, return on equity, banking dividend ratios as well as the book and market value of banks. It has predictions about the size of the financial sector relative to the rest of the economy. Finally, at the macroeconomic level, it provides a prediction about the evolution of monetary aggregates, M0, M1 and the money multiplier.

We use these descriptions to explain the dynamic effects of aggregate outcomes to changes in different monetary policy instruments and financial regulation. We use the model to explain the pass-through from policy interest rates to lending rates and interbank rates and overall lending, a measure of the effectiveness of monetary policy. We also study other shocks that we take as exogenous. These refer to the volatility of bank withdrawals, losses on equity and shocks that affect the demand for loans. We explain how these features depend on the conditions of bank liquidity, leverage and the maturity structure of loans.

We use these properties to calibrate the model. We apply our theory to answer some questions about banking during and after the 2008-2009 financial crisis. We ask, what type of shocks can explain the substantial holdings of bank excess reserves while lending has not resumed? We begin by fitting a sequence of shocks to our model that correspond to a common narrative of the events that occurred during the crisis. We argue that bank regulation or equally a weak demand for loans can equally explain the lack of strong lending post crisis.

In the practical world, together with credit management, liquidity management is the one of the main problems faced by banking institutions. Liquidity risks are present even if banks face no credit risk and are entirely solvent so. Hence, for this paper, we chose to abstract from any additional frictions. However, the paper is substantially rich in its predictions and lessons. It also presents a technical contribution in that it is highly tractable and can be solved quickly. Thus, we believe we can extend this model to incorporate richer features very easily in the future.

The paper is organized as follows. The following section provides an explanation of the liquidity management problem through the analysis of bank balance sheet. We then discusses where the model fits in the literature. Section 3, presents a partial equilibrium model of banks that takes a demand for loans as given. Section 4 presents the calibration and empirical analysis. We study the steady state and policy functions in sections 5 and 6. We use this environment to study the effects of deterministic shock paths in section 7. We use the environment to answer questions about monetary policy in the context of the US financial crisis in section 8.2 .

2 Overview of the Liquidity Management Problem

Bank Balance Sheets. To clarify the main mechanism in the paper, consider the balance sheet of a bank depicted in the left panel of Figure 1. Banks hold central bank reserves and loans as part of their assets. They hold deposits on demand and equity as liabilities.

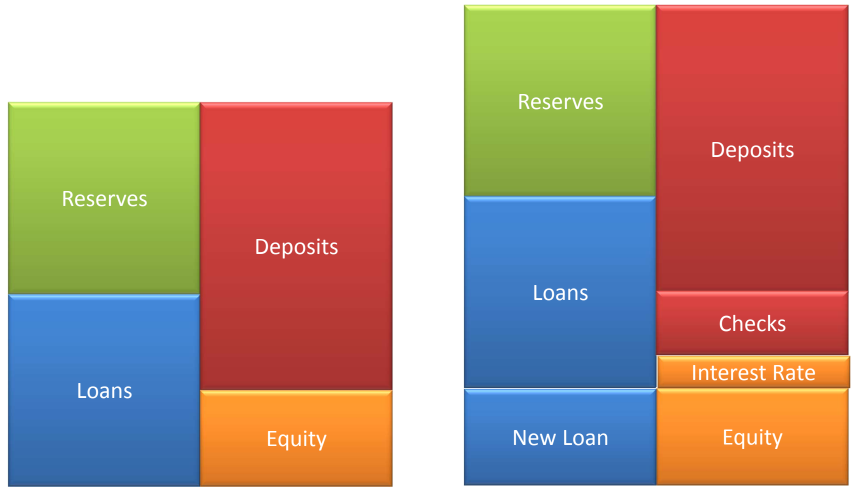

Figure 1: Bank Balance Sheet

When they provide loans, banks are effectively granting credit lines in the form of checks or an electronic account. These credit lines enable borrowers to make payments to purchase goods. After the payment is made, the receiver of those payments holds those bank liabilities and in turn can make further payments. What matters for the bank is that when it provides a loan, it also is simultaneously creating a liability in the form of a demand deposit. Of course, for every face-value dollar it lends, banks issue less than one-for-one in liabilities. They earn an interest. This increases the bank's equity. The right panel of Figure 1 shows the balance sheet of a bank after a loan expansion. Hence, granting a loan is profitable for the bank. Another thing to note is that the balance sheet expansion constitutes the creation of assets and liabilities, which in turns constitutes a form of monetary creation.

Liquidity Risk. A critical feature of our framework is that deposits can be withdrawn immediately causing a maturity mismatch. Often, large withdrawals will occur before a loan matures. So when a withdrawal occurs, deposits, a liability, are transferred from one bank to another. The bank that receives the deposits will take over those liabilities and will, therefore request the other bank an assets to take charge of those liabilities. The first bank could in theory transfer loans to the assets of the receiving bank. However, loans are often times subject to illiquidity. The issuing bank may be a specialist on the sector, there may be adverse-selection or moral-hazard concerns or transferring the loan monitoring may be costly or it may fear losing the client. Instead, banks rely on reserves to transfer funds to other banks.

If reserves are insufficient, banks must borrow reserves from other banks or from the central bank. Thus, it induces in additional costly borrowing. The left panel of Figure 3

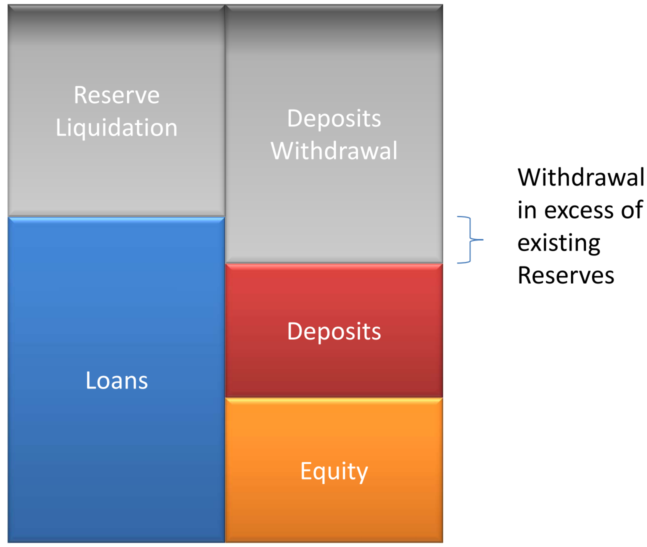

Figure 2: Equity losses from liquidity risk.

describes this possibility. Suppose there is a withdrawal that exceeds the level of currency reserves at the bank. In the model, banks must rely on costly overnight borrowing to compensate for the withdrawal. These costs induce a reduction in the banks equity, and an overall shrinkage of the banks balance sheet.

Comparing the equity before the loan expansion in the left panel of Figure 1 with the equity in the left panel of Figure 3, we see the losses that may occur if banks expand there assets too much and there's a deposit withdrawal. If withdrawal risk is independent of the size of the balance sheet, the higher the amount of lending, the more liquidity risk is the bank exposed to. To see this, notice that the relative size of reserves to deposits, the bank's liquidity ratio, falls with the amount of lending (see Figure 2). With less reserves, the bank is forced to use more borrowing to finance the shortage liquidity. Liquidity risk affects the bank's decisions ex-ante because the banks cash position will remain low after the cash obtained when loans mature are liquidated to repay overnight borrowing. Thus, when providing a loan, banks must take into account how this affects their liquidity ratio and how this increases the risk of incurring in additional costs. In other words, the value of additional lending may decrease if banks are being exposed to greater liquidity risk. This trade off is clear by comparing the size of equity in Figures 1 and 3.

Naturally, if banks become more cautious for whatever reason, this can have real effects because it may induce real effects in economic activity by decreasing the availability of

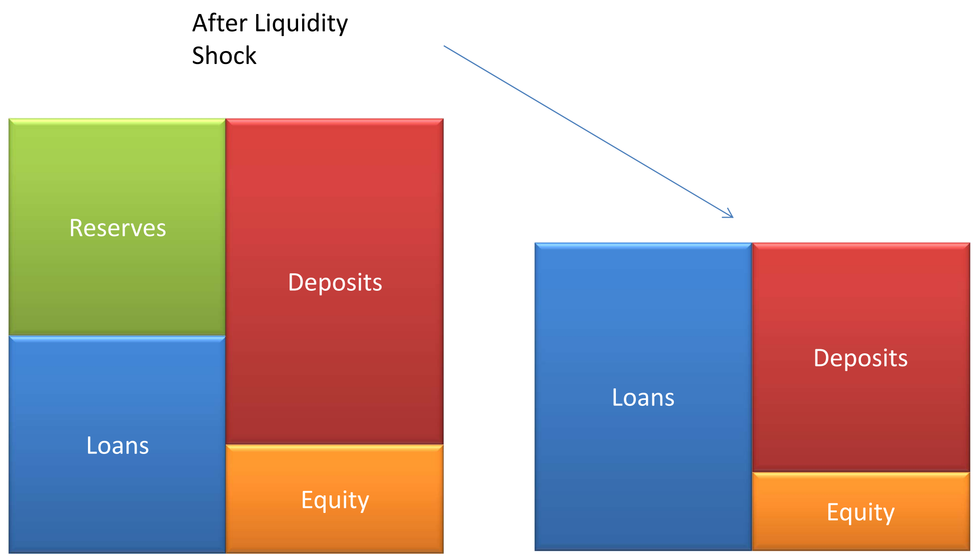

Figure 3: Equity losses from liquidity risk.

mediums of exchange in the economy.[3] One reason that affects this trade off is monetary policy.

Policy Instruments. In practice, there is a plethora of monetary policy instruments that can be used by central banks to alter the trade off described above. However, most Central Banks use only three: open market operations, discount window lending and imposing reserve requirements. These three tools of monetary policy affect the liquidity management trade-off in different ways, but they all carry the same effect.

Open market operations are large purchase of loans that substitute loans for currency reserves. In practice, these purchases are often times operations on Treasury Bonds (T-Bills) which can be seen as liquid loans. For now let's abstract from the presence of government paper. In our framework, the exchange of liquid assets for illiquid assets will cause a reduction the liquidity risk faced by those institutions participating of the bond sale. Since at least part of the banking system will have more reserves, some institution will be inclined to do more lending. Since currency reserves correspond to M0 and loans are granted by creating deposits, the response by banks corresponds to an endogenous monetary creation.

The discount window operates differently in the model. It affects the financial losses incurred after a withdrawal because banks that lack the reserves will be able to borrow at a lower rate. By reducing financial losses, a reduction in the borrowing rates, banks are more inclined to doing more lending. Finally, reserve requirements operate by determining what

[3] Assuming that the non-financial sector cannot create this with the same ease as the financial sector.

is the level of currency after which banks have to start borrowing reserves. Penalties on missing that target are another possible instrument.

2.1 Literature Review

There is a long tradition that dates at least Bagehot (1999) in describing the importance of banking for the transmission of monetary policy. A first formal attempt to present this in a model with a full description of households, firms and banks is Gurley and Shaw (1964). The approach of modeling banks in the implementation of monetary policy was practically abandoned from macroeconomics for many years.[4] Until the Great Recession, questions of how monetary policy affects the macroeconomic environment and how it is implemented through banks were treated independently. This simplification was a natural outcome. Banking didn't seem to matter much for the macro-economy in the US. The banking industry was amongst the most stable industries in terms of dividend ratios, solvency ratios and stock market value. The pass-through between policy rates and key rates seemed or between the monetary base and higher aggregates was seen as stable. Moreover, all post-war recessions in the US have had a name attached to it, but only one of these, the Savings and Loans crisis, has a name attached to financial system.

In the aftermath of the Great Recession, there has been numerous calls for writing models with an explicit role for banks in the determination of monetary policy. To name some few surveys, see for example Woodford (2010) and Mishkin (2011).

The profession has responded rapidly to these calls. We will undoubtedly make an unfair job of citing the most relevant papers to this one. For example, Gertler and Karadi (2009) and Curdia and Woodford (2009) study the effects of open market operations in environments where intermediaries face leverage-like constraints that invest directly in firm's equity. A notable distinction of our model is that we introduce a liquidity choice by banks, which allows us to endogeneize a critical aspect of banks' risk management. Moreover, this allows us to study how different shocks and policies affect the financial sector, by altering both solvency and liquidity positions.

Our paper tries to bring ideas from the banking literature into a fully-fledged dynamic macro model. In particular we focus on a maturity mismatch problem which leads to a demand for reserves. The idea is borrowed from the classical papers on banking, (e.g. Diamond and Dybvig (1983)) which makes banks subject to the risk of withdrawals. Part of the focus of this literatures is the of study bank runs.[5] For us, the maturity mismatch problem

[4] We are thinking of banking as a subset of models with macroeconomic models with financial frictions, which themselves where few.

[5] See Gertler and Kiyotaki (2013) for a recent model that incorporates bank runs into DSGE models.

simply yields a demand for central bank reserves as a hedging instrument to financial costs associated with these losses. Along that dimension, we also borrow ideas from the literature on the payments system in Freeman (1996) and a sequence of other related papers. The banking payments system leads to liquidity management problem which studied early on by Frost (1971).

The maturity mismatch problem hence yields a hedging demand for reserves which can explain how monetary policy effects bank lending. Also recently, Stein (2012) studies an environment where this demand emerges via an exogenous demand for safe assets in the utility function but abstracts from dynamics. A related model to ours in focusing on the dynamics of asset creation by banks is Brunnermeier and Sannikov (2012). They present an environment where banks create inside money and the central bank creates outside money. Outside money plays the same role as inside money as a medium of exchange that allows investment opportunities to be carried out. We share the spirit of having money as an asset that relaxes constraints but we differ in that outside money are FED reserves which are not used for commercial transactions. For us, reserves are special for the payments system.

Another paper with a special role for bank assets is Bolton and Freixas (2009). This paper introduces a differentiated role for different bank liabilities due to asymmetric information. Prior to the crisis, Stiglitz and Greenwald (2003) suggested the need of an explicit modeling of banks and asymmetric information problems to understand monetary economics. Stiglitz and Greenwald (2003) argued that monetary policy can have highly non-linear effects because of this feature. The demand for loans shocks that we study here have a motivation that is related with credit rationing. We are far from studying a rich lending problem here but we hope to head in that direction soon.

A model particularly relevant for us is Afonso and Lagos (2012) who focus on the market for bank reserves. These authors study the market for FED funds through a money search environment to explain the allocations and trades the intra-day money markets. We view are model as the counterpart to theirs in that we study what occurs during the rest of the day taking as given the outcomes in their marker. A demand for reserve emerges the reserve position will affect bank payoffs when they enter the Afonso and Lagos (2012) market.

In a sequence of papers, Corbae and D'Erasmo (2013a,b) study the industry dynamics of the banking industry focusing on the too-big-to-fail advantage by large banks. Our paper differs from all of this papers in that we stress a liquidity management problem for banks but we have in common that are models are fully dynamic. We also share common features with two other papers. The key friction in our model is like in Gertler and Kiyotaki (2012) and relates to the presence of withdrawal shocks. As them, we share the spirit of the classical work by Diamond and Dybvig (1983), Allen and Gale (1998) and Holmstrom and Tirole

(1997); Holmstrm and Tirole (1998).

Recent empirical papers include the work by Krishnamurthy and Vissing-Jørgensen (2011, 2012). Our model more closely relates to the study of Kashyap and Stein (2000) on the effects of monetary policy via the lending channel. In fact, we believe we model the credit channel they refer to. Kashyap and Stein (2012) study the optimality of interests on reserves.

Although not often found in macroeconomics, there are many textbooks on practical banking that touch the subject. See for example, Saunders and Cornett (2010) or Duttweiler (2009). Our model can be also seen as a model where withdrawal risk leads to a demand for reserves. Due to this problem, we break the Modigliani-Miller theorem for open market operations stressed in Wallace (1981).

3 Partial Equilibrium Model

We begin our description of with a partial equilibrium dynamic model of competing banks. The focus of this section is on explaining bank decisions as functions of policy variables. In particular we want to understand the supply of loans function as functions of bank states and aggregate shocks. We then close the model introducing a real sector which will have a demand for loans. We choose this organization of our model because there are many ways of closing the model and we don't need to impose a particular structure. It turns out that the way in which we close the model is by imposing assumptions such that monetary policy has real effects despite that the model is entirely real.

Time is discrete and indexed by t. There is an infinite horizon. Each period is divided into two stages: a lending stage (l) and a cash-balancing stage (b). A dollar plays the role of the numeraire. The economy is populated by a continuum of heterogenous banks whose identity is denoted by z. Banks face an exogenous demand for loans, an exogenous deterministic monetary policy and a vector of shocks that we describe later.

3.1 Banks

The goal of banks is to maximize dividend payment streams $\{DIV_t\}_{t\geq 0}$. The bank's preferences over dividend streams are evaluated via an expected utility criterion:

$$\mathbb{E}\left[\sum_{t\geq 0}\beta^t U\left(DIV_t\right)\right]$$

where $U\left(x\right)\equiv\frac{x^{1-\gamma}}{1-\gamma}$ and DIV_t is the banker's consumption at date t. Banks hold a portfolio of loans, B_t, and Central Bank reserves, C_t, as part of their assets and demand deposits,

D_t, as their liabilities. These are the bank's individual state variables. We describe some properties of these state variables.

Loans. When granting a loan, borrowers promise to repay the bank $I_t(1-\delta)\delta^n$ in period $t+n$ for all $n \geq 0$, in units of the numeraire.[6] Hence, loans constitute long-run assets which are a promise to a geometrically decaying stream of payments. Notice that the total coupon payments that a bank will receive by time t+T from a loan made at t is:

$$P_{t+T} = (1-\delta) I_t + (1-\delta) I_t\delta + (1-\delta) I_t\delta^2 + ... + (1-\delta) I_t\delta^T$$

so clearly,

$$\lim_{T\to\infty} P_{t+T} = I_t.$$

The state variable the total coupon payments at time t, which we denote by B_t. Using the same sequence of payments, total coupon payments received at any point in time are:

$$B_t = (1-\delta)I_{t-1} + (1-\delta)\delta I_{t-2} \quad + (1-\delta)\delta^2 I_{t-3} \text{ ......}$$

At $t+1$,

$$B_{t+1} = (1-\delta)I_t + (1-\delta)\delta^2 I_{t-1} \quad + (1-\delta)\delta^3 I_{t-2} \text{ ......}$$

so we can write the law of motion of loans as:

$$B_{t+1} = \delta B_t + (1-\delta)I_t.$$

Thus, B_{t+1}, is the value of all future coupon payments including the current ones. Banks grant new loans I_t at a market price q_t.[7] The inverse of this price, $\left(q_t^l\right)^{-1}$ is taken as given and represents the lending rate. In particular, when giving a loan, banks create demand deposits in the size of the loan $q_t^l I_t$. These deposits are given to the borrower who can use them to make payments. The rest of the loan, the amount $\left(1-q_t^l\right) I_t$, is a bank's immediate profits from intermediation.

An important assumption is that bank loans are illiquid. They can be sold to other banks but only during the lending stage.[8] The underlying assumption is that banks specialize in

[6] Payments begin at the period of issuance without loss of generality.

[7] This can be easily generalized to allow for some degree of market power. We use the price of the loan for convenience but one can easily go back and forth from prices to interest rates.

[8] Allowing loans to be sold only during the lending stage is done for convenience. By preventing loans from being sold during the balancing stage, we introduce a form of illiquidity that is essential to having loans being illiquid. Allowing loans to be transferable during the lending stage, is useful to reduce the state space of the model. In particular, we will show that it won't be necessary to keep track of the composition but only the size of the balance sheet thanks to this assumption.

their loans perhaps because they have particular expertise on their borrowers, or specialize in certain industries. Loans can be also illiquid due to adverse selection. For any of these reasons, banks would need to spend some time to analyze a loan before buying it. This assumption can be easily relaxed by allowing sales at a discount at the expense of requiring one additional state variable.

Finally, we assume there is a clearing house that will allow the bank to reduce its deposit levels as loans matures. New loans are granted during the lending stage.

Demand Deposits. Behind the scenes, banks have an implicit technology that is enabling transactions between third parties. Deposits are created when banks provide loans. As noted above, when granting loans, the borrower receives a credit line which enables him to purchase goods. Thus, when providing a new loan, banks are also creating deposits callable on demand. They are creating an asset (a liability for their borrower) and issuing a liability (an asset for a third party). The borrower uses these deposits to purchase goods. The holder of these deposits can, in turn, transfer the funds to others and, so on. Implicitly, bank deposits are playing a role as a medium of exchange. Simultaneously, banks are liable to the holder of those deposits.

Demand deposits are the bank's only form of liability. Deposits are reduced when borrowers make payments to the bank. Essentially, deposits are returned to the bank. Thinking of the financial system as a whole, it is providing lines of credit so that borrowers can perform transactions. Eventually, the borrower must obtain back deposits to repay previously issued loans.

During the balancing stage, banks face a random deposit-withdrawal shock w_t. The process for the stochastic withdrawals satisfies $w_t = \omega_t D_t$, where $\omega \sim F(\cdot, \phi_t)$ in $(-\infty, 1])$. The parameter ϕ_t is an exogenous process that affects the possible withdrawal risks. When w_t is realized, it decreases the deposits from a bank by that amount. In the background, over time, these deposits are transferred by there from one bank to another bank. This process seems a natural process since deposits are being constantly used for transactions. We think of w_t as capturing the complexity of transactions in the payments system which ultimately lead to randomness in withdrawals.

For the time being, we assume that deposits do not leave the banking sector:

Assumption 1 (Deposit Conservation)**.** *Deposits are preserved within the financial sector:*

$$\int_0^{\infty} \omega_t D_t F(d\omega, \phi_t) = 0, \ \forall \phi_t$$

Under the assumption above, we will see, that it is equivalent to saying that there are no withdrawals of reserves from the banking system or otherwise that there are no runs on the system.[9]

Since at the balancing stage, we treat bank loans as perfectly illiquid, when a deposit is transferred from one bank to another, the bank receiving the deposits will request exchange reserves to clear out the deposit transaction. Thus, reserves are special assets: they are completely liquid. Banks experience the withdrawal shocks during the lending stage.

Reserves. Banks use reserves to finance the transfer of deposits from one bank to the other. They often have sufficient reserves to meet there payments but if the shock is very large (small), reserves may be short (long) of funding the outflow (inflow) of deposits. If so, a Bank must head to the discount window or the interbank market and pay (earn) an interest on an short term loan. The cost (benefit) of borrowing (lending) reservers is determined by the function χ. We interpret χ is a policy parameter that captures a combination of the discount window and the overnight fed-funds market. They show how the discount window rate will affect the bank's discount rate.

Now, the point at which banks are in need of borrowing is not necessarily zero reserves. In particular, banks may need to borrow if after the withdrawal, they are below a reserve requirement which is determined by a policy parameter $\rho \in [0,1]$. Thus, if at any point $\rho D_t \geq C_t$, bank's face a penalty $\chi(\rho D_t - C_t)$ in the form of borrowing needs.

The function χ maps deviation from reserve requirements to a penalty:

$$\chi(x) = \begin{cases} \underline{\chi} x \ if \quad x \leq 0 \\ \overline{\chi} x \quad if \quad x > 0 \end{cases} .$$

An important parameter restriction is that $\underline{\chi} < \overline{\chi}$. In practice, central banks set up windows for lending and borrowing rates overnight. Afonso and Lagos (2012) provide a formal model for the overnight interbank market. The outcome of their analysis is that banks ending a day with positive balances lend out reserves to banks with negative balances with a certain probability. The balance that is not lend earns interest on deposits at the FED and the fraction that cannot be borrowed pays interests.[10] Our interpretation $\{\underline{\chi}, \overline{\chi}\}$ is as an average of repeated interactions in the FED funds market. The values for $\{\underline{\chi}, \overline{\chi}\}$ represent the average cost of ending with positive or negative balances considering that banks can borrow from the interbank market with a certain probability or otherwise they must lend or borrow from the FED. We take $\{\underline{\chi}, \overline{\chi}\}$ as policy parameters.

[9] We can extend the model easily assuming that the private sector can withdraw deposits in the form of currency. This can be easily incorporated disposing the assumption that deposits do not leave the banking sector.

[10] We can extend our model by allowing the opening of this market with possible interesting interactions.

Bank Equity. Bank equity is the sum of their assets minus their liabilities:

$$N_t = B_t + C_t - D_t$$

Equity evolves according to the realization of bank profits that stem from lending to customers and borrowing from the interbank market. Profits are realized during the lending stage. Also, during the lending stage, part of the bank's equity can be payed-out as dividends, DIV_t. Dividends are payed-out by issuing deposits to bank share holders. Finally, banks face a regulatory framework that constraints the amount of loans they can make and dividends they can payout.

3.2 Timing of Events and Laws of Motion

Notation. We use Z_t to denote the value of variable Z_t during the lending stage and $\tilde{Z}_t$ to denote its value at the beginning of the rebalancing stage.

Lending Stage: Banks enter the period during the lending stage with currency C_t, a portfolio of loans B_t, and a deposits, D_t as their individual states. The aggregate state includes monetary policy variables (for now treated as parameters), real economic activity observables (for now constant), and an exogenous demand for loans (for now exogenous). This aggregate states are summarized in the vector X_t**.**

During the lending stage, banks decide over the amount of new loans they provide, I_t, dividend payments DIV_t and, φ_t, their purchases of reserves. These interbank transactions, φ_t, occur during the lending stage so they are different from the overnight FED-funds market that cost $\bar{\chi}$. This borrowing occur at an interest rate of r so we think of these as the LIBOR rate. This interest is payed in the form of deposits.

Upon a loan banks give a checking account to the borrower, or equivalently a deposit account to whom ever is exchanging a physical good (resources) to the borrower (in exchange for that check). When borrowing (lending) reserves, they issue (are issued) deposits against those assets. Thus, since dividends are payed in the form of bank liabilities, we obtain the following intra-period law of motion for demand deposits:

$$\tilde{D}_t = D_t + qI_t + DIV_t + \varphi_t(1 + r_t) - B_t(1 - \delta). \tag{1}$$

Thus, a bank that begins with D_t as deposits at the beginning of the stage ends with $\tilde{D}_t$ at the end through the following sources. It credits by qI_t the account of his borrower (or whomever he trades with), after a loan of size I_t. It also pays dividends to shareholders in amount DIV_t. It issues $\varphi_t(1 + r_t)$ liabilities to other banks if it borrows φ_t in cash. Finally,

$-B_t(1-\delta)$ deposits are reduced by the payment of previously issued loans.

The evolution of bank reserves is given by the sum of the previous stock plus the interest on reserves $(r_c C_t)$ plus the cash purchases,

$$\tilde{C}_t = (1 + r_c)\, C_t + \varphi_t. \tag{2}$$

Interests on reserves r_c, are payed by the monetary authority in the form of more reserves. Finally, loans evolve according to the fraction of the original stock that has not matured yet plus the newly issued loans.

$$\tilde{B}_t = \delta B_t + I_t. \tag{3}$$

Banks make these choices subject to the following capital requirement constraint that puts an upper bound on the amount of leverage the bank can take

$$\tilde{D}_t \leq \kappa \tilde{N}_t, \tag{4}$$

and to a liquidity requirement,

$$\tilde{C}_t \geq \eta \tilde{N}_t \tag{5}$$

which differs from the monetary policy reserve requirement.

Balancing Stage. During the balancing stage, banks receive the random deposit withdrawal shock ω_t. This shock is like a random increase in the demand for cash. If by the end of the period, banks do not hold sufficient cash relative to the deposits, they face penalties $\chi(\rho\tilde{D}_t - \tilde{C}_t)$. Penalties are payed in the form of deposits.

Hence the law of motion for cash accounts for the withdrawal,

$$C_{t+1} = \tilde{C}_t - \omega_t \tilde{D}_t$$

and

$$D_{t+1} = \tilde{D}_t(1 - \omega_t) + \chi(\rho\tilde{D}_t - \tilde{C}_t)).$$

That is, cash at the end of the period are given, by cash selected at the beginning of the period minus withdrawn deposits at the end of the period. The second law of motion reflects the liquidity shock and the fact that the penalty χ is repaid with deposits.

3.3 Bank Problems

The model can be expressed recursively so for its statement, we drop time subscripts. It is understood that prices (q, r) and policy variables $\left(\kappa, \eta, r_c, \left\{\underline{\chi}, \overline{\chi}\right\}\right)$ as well as the distribution of shocks F are functions of the aggregate state X.

Lending Stage. The optimization problem for a bank during the lending stage in recursive form is as follows.

Problem 1 (Bank Lending Stage) *The bank's problem during the lending stage is:*

$$\begin{aligned} V^{l}(C,B,D;X) &= \max_{\{I,DIV,\varphi\}\in\mathbb{R}} U\left(DIV\right)+\mathbb{E}\left[V^{b}(\tilde{C},\tilde{B},\tilde{D};\tilde{X})\right] \\ \tilde{D} &= D+qI+DIV+\varphi(1+r)-B(1-\delta) \\ \tilde{C} &= C\left(1+r_{c}\right)+\varphi \\ B' &= \delta B+I \\ \tilde{D} &\leq \kappa(\tilde{B}+\tilde{C}-\tilde{D}),\tilde{D}\geq 0 \\ \tilde{C} &\geq \eta(\tilde{B}+\tilde{C}-\tilde{D}),\tilde{C}\geq 0 \end{aligned}$$

Banks choose loans I, dividends DIV, reserve holdings $\tilde{C}$ and deposits $\tilde{D}$ to maximize expected dividends. The solve this problem subject to the law's of motions for deposits, reserves and loans, equations (1), 2 and (3). In addition, they must satisfy the policy constraints (4) and (5). On the technical side, the leverage constraints bounds the problem of the banks and thus renders their problem feasible. It prevents a Ponzi-scheme. Moreover, since $\tilde{D}\geq 0$, equity will remain positive at all periods before the ω shock is realized.

It is important to note that if the bank arrives to a node with negative equity, the problem is not well defined. However, in choosing its policies, it will make decisions such that it is guaranteed that it doesn't run out of equity.

During the balancing state, banks make no decisions but rather experience the withdrawal shock ω. There problem is given by the following condition:

Problem 2 (Bank Balancing Stage) *The bank's problem during the balancing stage is:*

$$\begin{aligned} V^{b}(\tilde{C},\tilde{B},\tilde{D};\tilde{X}) &= \beta\mathbb{E}\left[V^{l}(C',B',D';X')|X\right] \\ C' &= \tilde{C}-\omega\tilde{D} \\ B' &= \tilde{B} \\ D\prime &= \tilde{D}(1-\omega)+\chi(\rho\tilde{D}-C') \end{aligned}$$

Loans remain unchanged as withdrawals do not affect the stock of loans. Instead, $\omega\tilde{D}$ are transferred to other banks in the form of reserves. The withdrawal reduces deposits by $(1-\omega)$. Finally, deposits change depending on the penalty faced by banks χ. We can collapse the model into a single period. Since there are no actions between periods, then there's no need to write-up two Bellman equations. Combining the lending and balancing stage problems we obtain a value function for the balancing stage which is recursive:

Problem 3 *The bank's problem during the lending stage is:*

$$\begin{aligned} V^{l}(C,B,D) &= \max_{\{I,DIV,\tilde{C},\tilde{D}\}\in\mathbb{R}^{4}_{+}} U\left(DIV\right)\ldots \\ &\quad +\beta\mathbb{E}\left[V^{l}(\tilde{C}-\omega'\tilde{D},\tilde{B},\tilde{D}(1-\omega')+\chi(\rho\tilde{D}-(\tilde{C}-\omega'\tilde{D}));X')|X\right] \\ \tilde{D} &= D+qI+DIV_{t}+\varphi(1+r)-B(1-\delta) \\ \tilde{B} &= \delta B+I \\ \tilde{C} &= \varphi+C\left(1+r_{c}\right) \\ \tilde{D} &\leq \kappa(\tilde{B}+\tilde{C}-\tilde{D}) \\ \tilde{C} &\geq \eta(\tilde{B}+\tilde{C}-\tilde{D}). \end{aligned}$$

In what follows, we characterize the bank's policies. This will provide some intuition about their decisions.

3.4 Characterization of Problems

The recursive problem of banks can be characterized with a single state variable. The complication is off course to find the correct state variable. Note that one can substitute for the cash borrowing, φ_t, in (2) to express the evolution of deposits during the lending stage, equation (1), in terms of a decisions about cash holdings:

$$\tilde{D}=D+qI+DIV+(\tilde{C}-C\left(1+r_{c}\right))(1+r)-B(1-\delta).$$

In addition, one can substitute I, from equation 3 into the expression above to obtain:

$$\tilde{D}=D+q(\tilde{B}-\delta B)+DIV+(\tilde{C}-C\left(1+r_{c}\right))(1+r)-B(1-\delta)$$

and rearranging terms one obtains:

$$(1+r)\tilde{C}+q\tilde{B}+DIV-\tilde{D}=C\left(1+r_{c}\right)\left(1+r\right)+q\delta B+B(1-\delta)-D.$$

This equation takes the form of a budget constraint for the banker in terms of his wealth. In particular, the bank's sources of funds stem from two sources. The one is the value of its cash holdings which can be lent at an interest, $C\left(1+r_{c}\right)\left(1+r\right)$. The other is the value of loans. The illiquid fraction of loans, δB, is valued at q because this is the replacement cost of the illiquid fraction. The rest, $B(1-\delta)$, is valued at the same amount as deposits. Deposits are subtracted from the bank's total loans. Funds are used to obtain cash for the following period, $\tilde{C}$, to fund new loans $\tilde{B}$ or to pay dividends, DIV. Naturally, these uses can be expanded by issuing more liabilities, $\tilde{D}$ today.

One can define the market value of equity as, $E \equiv C\,(1+r_c)\,(1+r)+q\delta B+B(1-\delta)-D$, corresponding to the right hand side of the bank's budget constraint. If we ignore the constraints that $\tilde{B} \geq \delta B$, something that can be ruled in equilibrium, we can express the Bellman equation without reference to the different states but only as a function of E. Thus,

Proposition 1 (Single-State Representation) *The problem of the bank can be written the following way:*

$$
\begin{aligned}
V^l(E) &= \max_{\tilde{C},B',\tilde{D},DIV\in\mathbb{R}^4_+} u(DIV) + \beta\mathbb{E}\left[V^l(E')|X\right] \\
E &= q\tilde{B} + \tilde{C}(1+r) + DIV - \tilde{D} \\
E' &= \left(q'\delta + (1-\delta)\right)\tilde{B} + \tilde{C}\,(1+r_c)\,(1+r') - \tilde{D}(1+(1+r_c)\,r'\omega') - \chi((\rho+\omega)\,\tilde{D} - \tilde{C})) \\
\tilde{D} &\leq \kappa(\tilde{B} + \tilde{C} - \tilde{D}) \\
\tilde{C} &\geq \eta(\tilde{B} + \tilde{C} - \tilde{D})
\end{aligned}
$$

The first constraint now collapse all of the laws of motion for the bank's assets. It now writes them as a budget constraint in term's of the bank's equity (wealth). This is a familiar budget constraint in that the bank must choose consumption, or dividends, and two assets, $\left(\tilde{B},\tilde{C}\right)$ and borrowing $\tilde{D}$ subject to a leverage constraint and liquidity constraint. He then makes this decision to maximize utility taking into account the law of motion for his equity after the liquidity shock.[11]

The continuation value of the value function, has also one argument, E', which is obtained by substituting the values of D',C' and B' as functions of current states. The budget constraint is linear in E and the objective is homothetic in dividends. Thus, by Alvarez and Stokey (1998) we have that the solution to this problem exists and is unique and we have that policy functions are linear.

One can guess and verify that the objective is:

Proposition 2 (Homogeneity-γ) *The value function $V^l(E;X)$ satisfies*

$$V^l(E;X) = v^l\,(X)\,E^{1-\gamma}$$

[11]With obvious abuse of notation, V henceforth denotes the value function in terms of E rather than (C,B,D).

where $v^l(X)$ is the value of

$$\max_{\tilde{c},\tilde{b},\tilde{d},div\in\mathbb{R}_+^4} u(div)+\beta\mathbb{E}\left[v^l(X')\,|X\right]\mathbb{E}_{\omega'}\left[(e')^{1-\gamma}\right]$$

subject to,

$$\begin{aligned}
1 &= q\tilde{b}+\tilde{c}(1+r)+div-\tilde{d}\\
e' &= \left[(q'\delta+(1-\delta))\,\tilde{b}+\tilde{c}\,(1+r_c')\,(1+r')-\tilde{d}(1+(1+r_c')\,r\omega')-\chi((\rho+\omega)\,\tilde{d}-\tilde{c}))^{1-\gamma}\right]\\
\tilde{d} &\leq \kappa(\tilde{b}+\tilde{c}-\tilde{d})\\
\tilde{c} &\geq \eta(\tilde{b}+\tilde{c}-\tilde{d}).
\end{aligned}$$

Moreover, all the policy functions of $V^l(E)$ satisfy $X=xE$. In the expression above, $\mathbb{E}_{\omega'}$ is the expectation under F.

Moreover, this problem satisfies a separation theorem whereby the policy function for dividends can be analyzed independently from the policy functions of deposit issuance, cash holdings and loans. Using the principle of optimality, we can break the Bellman equation in two parts. The first will take as given the choice of dividends.

Proposition 3 (Separation) *The value function $v^l(X)$ solves:*

$$v^l(X)=\max_{div\in\mathbb{R}_+} U(div)+\beta\mathbb{E}\left[v^l(X')\,|X\right]\Omega(X)^{1-\gamma}(1-div)^{1-\gamma}$$

where $\Omega(X)$ is the value of the following portfolio problem:

$$\max_{\dot{\delta},\dot{b},\dot{c}\in\mathbb{R}_+^3}\mathbb{E}_{\omega'}\left[(q'\delta+(1-\delta))\,\dot{b}+\dot{c}\,(1+r_c)\,(1+r')-\dot{d}(1+(1+r_c)\,r'\omega')-\chi((\rho+\omega)\,\dot{d}-\dot{c}))^{1-\gamma}\right]^{\frac{1}{1-\gamma}}$$

subject to

$$\begin{aligned}
1 &= q\dot{b}-\dot{d}+\dot{c}(1+r)\\
\dot{d} &\leq \kappa(\dot{b}+\dot{c}-\dot{d})\\
\dot{c} &\geq \eta(\dot{b}+\dot{c}-\dot{d}).
\end{aligned}$$

The solution to this portfolio problem is isomorphic to the solution to a portfolio in terms of portfolio shares invested in assets of different returns. We obtain this portfolio problem via change of variables that we can later invert to obtain the specific values of $\tilde{c},\tilde{b},\tilde{d}$. Define the following portfolio shares:

$$
\begin{aligned}
w_b &\equiv q\dot{b} \\
w_c &\equiv (1+r)\,\dot{c} \\
w_d &\equiv -\dot{d}.
\end{aligned}
$$

Return on Loans. Let $R^B(\delta)$ be there return on a loan given its maturity δ. This return depends on prices and the maturity of the loan and is adjusted for liquidity:

$$R_t^B(\delta) \equiv \frac{\delta q_{t+1} + (1-\delta)}{q_t},$$

where the return on the illiquid portion, δ, is q_{t+1}/q_t, the change in the value of illiquid bonds and the return on the liquid portion. $(1-\delta)$ is $\frac{1}{q_t}$, the one-period return on the loan.

Return on Reserves. The return on the cash and deposit components of the portfolios is determined jointly. These returns depend on the ω, the withdrawal shock. They can be separated into an independent return and a joint return component that follows from the penalty. The independent return on reserves is

$$R_t^C \equiv (1+r_{c,t})\left[\frac{1+r_{t+1}}{1+r_t}\right].$$

Since $1+r_t$ is the relative price of reserves relative to deposits, $\left[\frac{1+r_{t+1}}{1+r_t}\right]$ captures the revaluation component. Note that in a state where $r_{t+1} = r_t$, reserves have a return equal to $(1+r_{c,t})$ because they yield a constant relative price. Thus, the only return comes from the interest on reserves. When the interest on reserves and the prices change is zero, cash pay no independent return.

Return on Deposits. Deposits yield a return (cost) of R_t^D, given by

$$R_t^D \equiv (1+(1+r_{c,t})\,r_{t+1}\omega').$$

This returns are function of ω'. When this deposits leave, the bank losses reserves and deposits. The opportunity cost of loosing those reserves is $(1+r_{c,t})\,r_{t+1}$. The other term is 1 because deposits bear no interest.

Portfolio Illiquidity Cost. Finally, the joint return component is captured by potential cost (or benefit) of running out of reserves. This illiquidity cost is given by,

$$R^{\chi}(w_d, w_c) \equiv \chi\left((\rho+\omega)\,w_d - \frac{w_c}{(1+r)}\right).$$

In this expression, $(\rho+\omega)$ is the sum of the reserve requirement and the withdrawal fraction. The cost the actual amount of cash holdings for the bank are $(w_c/1+r)$, the pre-transformation amount of cash. So when the withdrawal and reserves are lower than the amount of cash, the agent has a negative account at the fed which activates the borrowing costs.

Portfolio Problem. Using the returns on the banks assets, we obtain the following liquidity management portfolio problem:

Proposition 4 (Portfolio) $\Omega(X)$ *is also the solution to the following liquidity-management portfolio problem:*

$$\max_{\{w_b,w_d,w_c\}} \mathbb{E}_{\omega'}\left[\left(R^B(\delta)\,w_b + R^C w_c - R^D w_d - R^\chi(w_d,w_c)\right)^{1-\gamma}\right]^{\frac{1}{1-\gamma}}$$

subject to,

$$\begin{aligned} 1 &= w_b + w_c - w_d \\ w_d &\leq \kappa\left(\frac{w_b}{q} + \frac{w_c}{(1+r)} - w_d\right), w_d, w_c, w_b \geq 0 \\ \frac{w_c}{(1+r)} &\geq \eta\left(\frac{w_b}{q} + \frac{w_c}{(1+r)} - w_d\right). \end{aligned}$$

This portfolio problem is not a standard portfolio problem. It features non-linear returns and has constraints on the portfolio weights. The intuition behind the main mechanism in this paper can be transparently understood by analyzing the strategies from this problem. What should be clear from this problem is that the constraints represent the budget constraint for the bank, the capital requirement and the liquidity requirement. The objective is the same as the objective in $\Omega(X)$ but expressed in terms of returns to the banks assets. We postpone the discussing its solution to the following section. Solving this problem, yields the solution to the banks policy functions. We can reverse the solution for $\left\{\tilde{c},\tilde{b},\tilde{d}\right\}$ via following formulas:

$$\begin{aligned} \tilde{b} &= (1-div)\,\frac{\omega_b}{q} \\ \tilde{c} &= (1-div)\,\frac{w_c}{(1+r)} \\ \tilde{d} &= -(1-div)\,w_d. \end{aligned}$$

The separation between the dividend-payment problem and the portfolio problem for the bank implies that the value function is given by:

$$v^l(X) = \max_{div} (div)^{1-\gamma} + \beta \mathbb{E}\left[v^l(X')\,|X\right] \Omega^*(X)(1-div)^{1-\gamma}.$$

We can characterize dividends and the value of the bank, without further reference to the choice of deposits, loans or reservers. We have the following proposition.

Proposition 5 (Portfolio) *Given the solution to $\Omega^*(X)$, the dividend ratio and value of bank equity are given by:*

$$div(X) = \frac{1}{1+\left[\beta \mathbb{E}\left[v^l(X')\,|X\right]\Omega^*(X)\right]^{1/\gamma}}$$

and

$$v^l(X)^{\frac{1}{\gamma}} = 1 + \left(\beta\Omega^*(X)\right)^{\frac{1}{\gamma}} \mathbb{E}\left[v^l(X')\,|X\right]^{\frac{1}{\gamma}}$$

for $\gamma > 0$. For the risk-neutral case, $\gamma = 0$,

$$div(X) = \begin{cases} 0 \; if & \beta\mathbb{E}\left[v^l(X')\,|X\right]\Omega^*(X) < 1 \\ 1 \;\; if & x\beta\mathbb{E}\left[v^l(X')\,|X\right]\Omega^*(X) > 0 \\ \in [0,1] & if\, \beta\mathbb{E}\left[v^l(X')\,|X\right]\Omega^*(X) = 1 \end{cases}.$$

The policy functions by banks will determine a demand for loans and a supply for central bank reserves.

Loan supply and Reserve Demand. During a given period, the total supply of new loans is given by:

$$I_t = \tilde{b}_t \int_0^1 E_t(z)\, dz - (1-\delta)\, \tilde{b}_{t-1} \int_0^1 E_{t-1}(z)\, dz.$$

Notice that the supply of new loans, which determines q_t, is the difference between current demand for the stock of loans minus the fraction of previously existing loans that has not matured yet. Similarly, the aggregate demand for central bank reserves is given by,

$$C_t = \int_0^1 \tilde{c}_t(z)\, E_t(z)\, dz.$$

Monetary Base and Aggregates. Given that the model does not have a role for coins or currency, banks reserves represent the entirety of the monetary base $M0_t$. The Central Bank choose its issuance of money at every point in time. Thus, the market clearing condition for the reserve market is given by:

$$M0_t = \int_0^1 c_t(z)\, E_t(z)\, dz.$$

Since deposits equal M1, M1 is affected by money creation by banks.

$$M1_t = \int_0^1 d_t(z)\, E_t(z)\, dz.$$

Thus, the model yields and endogenous money multiplier:

$$\mu_t = \frac{M1_t}{M0_t}.$$

Evolution of Bank Equity. The distribution of bank equity is evolving over time according to:

$$E_{t+1}(z) = \Psi_t E_t(z)$$

where $\Psi_t(\omega) = \left[\left(\frac{(q'\delta+(1-\delta))}{q} w_b^* + w_c^* + w_d^*(1+r'\omega') - \chi(-(\rho+\omega)\, w_d^* - \frac{w_c^*}{(1+r)}))\right)\right]$ is the growth rate of bank equity. We want to represent the evolution of the measure of Γ_t of equity holdings. Because the economy displays equity growth, equity is unbounded and thus, the support of this measure is the positive real line. Let $\mathcal{B}$ be the Borel σ-algebra on the positive real line. Then, define as $Q_t(e, \boldsymbol{E})$ as the probability that an individual bank with current equity e transits to the set $\boldsymbol{E}$ next period. Formally $Q_t : \mathbb{R}_+ \times \mathcal{B} \to [0,1]$, and

$$Q(e, \boldsymbol{E}) = \int_{-1}^{1} \mathbb{I}\left\{\Psi_t(\omega)\, e \in \boldsymbol{E}\right\} F(d\omega)$$

where $\mathbb{I}$ is the indicator function of the event in brackets. Then Q is a transition function and the associated T^* operator for the evolution of bank equity is given by:

$$\Gamma_{t+1}(\boldsymbol{E}) = \int_0^1 Q(e, \boldsymbol{E}) \Gamma_{t+1}(e)\, de.$$

The distribution of equity is fanning out and the operator is unbounded. Gibrat's law shows that for t large enough Γ_{t+1} is approximated well by a log-normal distribution. Moreover, by introducing more structure into the problem, we could easily obtain a Power law distribution for $\Gamma_{t+1}(\boldsymbol{E})$. We will use this properties in the calibrated version of the model.

Loan Demand. For now we assume an exogenous demand for loans of the form:

$$q_t = \Theta_t \left(I_t^D\right)^{\epsilon}. \tag{6}$$

Since, q is the inverse of the interest rate, this demand function is increasing so $\epsilon > 0$. In the following section, we present an environment for which there is a demand for real loans, we stems from properly micro-founded models.

In the appendix, we provide a microfoundation for the loan demand 6 that closes the model.

3.5 Equilibrium

We are now ready to characterize an equilibrium.

Definition. *An partial equilibrium is a sequence of government policies* $\left\{\rho_t, M0_t, \kappa_t, \eta_t, r_{c,t}, \underline{\chi}_t, \overline{\chi}_t\right\}_{t\geq 0}$, *bank policy rules* $\left\{\tilde{c}_t, \tilde{b}_t, \tilde{d}_t, div_t\right\}_{t\geq 0}$, *bank values* v_t *and equity distributions* Γ_t *and prices* $\{q_t, r_t\}_{t\geq 0}$,, *such that:*

1. *Given the price sequence* $\{q_t, r_t\}_{t\geq 0}$, *the policy* $\left\{\rho_t, M0_t, \kappa_t, \eta_t, r_{c,t}, \underline{\chi}_t, \overline{\chi}_t\right\}$ *the policy functions* $\left\{\tilde{c}_t, \tilde{b}_t, \tilde{d}_t, div_t\right\}$ *are solutions to Problem 3. Moreover,* v_t *is the value in Proposition 3.*

2. *Money Market Clears:*
$$\int_0^\infty \tilde{c}_t e \Gamma_t(e)\, de = M0_t.$$

3. *Loan Market Clears.*
$$I_t^D = \int_0^\infty \tilde{b}_t e \Gamma_t(e)\, de - \int_0^\infty \tilde{b}_{t-1} e \Gamma_{t-1}(e)\, de$$
and
$$I_t^D = \Theta_t^{-1}(q_t)^{\frac{1}{\epsilon}}$$

4. *Equity distribution evolves according to:*
$$\Gamma_{t+1}(\boldsymbol{E}) = \int_0^1 Q(e, \boldsymbol{E}) \Gamma_{t+1}(e)\, de.$$

3.6 Theoretical Analysis

To gain more intuition, it is convenient to analyze the portfolio problem of the bank derived in Proposition 4. This analysis should build the intuition on how a Central Bank can affect the behavior a bank's loans supply schedule. It should also illustrate the interplay between financial regulation and monetary policy.

Excess Returns in $\Omega^*(X)$. Fix any given state X. It is useful to re-write the portfolio problem $\Omega(X)$ by substituting out the weight on loans. We obtain:

$$\max_{\{w_d,w_c\}}\left(\mathbb{E}_{\omega'}\left(\underbrace{R^B}_{\text{Return to Equity}}\underbrace{-\left(R^B-R^C\right)w_c}_{\text{Cash Opportunity Cost}}+\underbrace{\left(R^B-R^D\right)}_{\text{Arbitrage}}w_d-\underbrace{R^{\chi}\left(w_d,w_c\right)}_{\text{Liquidity Cost}}\right)^{1-\gamma}\right)^{\frac{1}{1-\gamma}}$$

subject to,

$$\begin{aligned} w_d &\le \kappa\left(\frac{1-w_c+w_d}{q}+\frac{w_c}{(1+r)}-w_d\right), w_d, w_c \ge 0 \\ w_c &\ge \eta\left(\frac{1-w_c+w_d}{q}+\frac{w_c}{(1+r)}-w_d\right). \end{aligned}$$

The objective is clear. If banks hold all their equity on loans, they would obtain R^B per unit of equity. Issuing an additional deposit yields an arbitrage opportunity when the spread between return on loans and return on deposits is positive: $\left(R^B-R^D\right)$. Reserve holdings have the classical opportunity cost of cash, $\left(R^B-R^C\right)$ but they yield the benefit of reducing the exposure to liquidity risk by reducing the expected liquidity cost $R^{\chi}\left(w_d,w_c\right)$. To make progress in the analysis, we study some polar cases. For the time being, let's assume that the constraints don't bind so that $\kappa\to\infty$ and $\eta=0$ so that financial regulation constraints are not present. We now turn to study some polar cases that illustrate the main forces behind monetary policy.

3.6.1 Polar Cases

Polar Case I: Risk-Neutral Banks ($\gamma=0$). The case with risk neutrality illustrates how monetary policy acts like a tax. Hence, assume $\gamma=0$ so that bankers are risk neutral. The objective in the portfolio problem can be written as:

$$R^B+\max_{\{w_d,w_c\}}\left(R^B-R^D\right)w_d-\left(R^B-R^C\right)w_c-\mathbb{E}_{\omega'}\left[R^{\chi}\left(w_d,w_c\right)\right].$$

Then the expected liquidity cost $\mathbb{E}_{\omega'}\left[R^{\chi}\left(w_d,w_c\right)\right]$ takes the following form:

$$\mathbb{E}_{\omega'}\left[R^{\chi}\left(w_d,w_c\right)\right]=\bar{\chi}\int_{\bar{\omega}(w_d,w_c)}^{1}\left(\omega-\bar{\omega}\left(w_d,w_c\right)\right)f(\omega)\,d\omega-\underline{\chi}\int_{-\infty}^{\bar{\omega}(w_d,w_c)}\left(\bar{\omega}\left(w_d,w_c\right)-\omega\right)f(\omega)\,d\omega.$$

where $\bar{\omega}(w_d, w_c)$ is the threshold shock that leads the banker to experience a negative balance of reserves (relative to the required reserves). This cutoff can be written in term of the liquidity ratio defined by $L \equiv \frac{w_c}{w_d}$ is the liquidity ratio of the banker's portfolio choice. The cutoff is given by:

$$\bar{\omega}(w_d, w_c) = \bar{\omega}(1, L) = \left(L \frac{1}{(1+r)} - \rho\right).$$

The liquidity cost $R^{\chi}(w_d, w)$ can actually be written also as a linear function of the weight on deposits ω^d multiplied by a function of the liquidity ratio. This function takes the following form:

$$w_d \tilde{R}^{\chi}(1, L) = w_d \left(\bar{\chi} \int_{\bar{\omega}(L)}^{1} (\omega - L) f(\omega) d\omega - \underline{\chi} \int_{-\infty}^{\bar{\omega}^*(L)} (L - \omega) f(\omega) d\omega\right).$$

Introducing this representation into the original problem, yields a convenient reformulation.

$$\Omega(X) = R^B + \max_{w_d} w_d \left(\left(R^B - R^D\right) + \max_{L} \left\{-\left(R^B - R^C\right) L - \tilde{R}^{\chi}(1, L)\right\}\right).$$

This reformulation shows that the bank's problem is a linear function of the weight. Deposits have a direct return $\left(R^B - R^D\right)$, but the banker will choose an optimal amount of liquidity holdings L, per unit of deposits. The optimal liquidity ratio, weights the cost of obtaining liquidity against the reduction in the expected illiquidity cost.

For any given w_d, the optimality condition for L is the same. We have the following first order condition:

$$\frac{\partial \tilde{R}^{\chi}(1, L)}{\partial L} \leq \left(R^B - R^C\right) \text{ with equality if } L > 0.$$

The optimal liquidity ratio L^*, can be obtained by

$$\begin{aligned}
\left(R^B - R^C\right) &= \left(\bar{\chi} \int_{\bar{\omega}(L^*)}^{1} f(\omega) d\omega - \underline{\chi} \int_{-\infty}^{\bar{\omega}(L^*)} f(\omega) d\omega\right) \\
&= \bar{\chi}\left(1 - F\left(\bar{\omega}(L^*)\right)\right) - \underline{\chi} F\left(\bar{\omega}(L^*)\right) \\
&= \bar{\chi}\left(1 - F\left(\frac{L^*}{(1+r)} - \rho\right)\right) - \underline{\chi} F\left(\frac{L^*}{(1+r)} - \rho\right)
\end{aligned} \tag{7}$$

or is zero if the solution does not exist. The condition requires that the right hand side, which is a convex combination of $\bar{\chi}$ and $\underline{\chi}$ to be equal to $\left(R^B - R^C\right)$. To have an interior solution, the problem requires that:

$$\bar{\chi} > R^B - R^C > -\underline{\chi}.$$

The intuition is that for any value of w_d, the choice of L must equate the $R^B - R^C$ cost of cash to the reduction in the hazard of paying a penalty. To have an interior solution, it has to be the case that $\bar{\chi} > R^B - R^C$ since otherwise the bank would hold no reserves. The only reason why its willing to forgo the opportunity cost of not making a loan, $R^B - R^C$, is to reduce the likelihood of paying $\bar{\chi}$. If $-\underline{\chi} > R^B - R^C$, we are at the opposite extreme where the bank is willing to buy as many the reserves as possible because holding reserves is a better arbitrage opportunity than lending.

Let's now use the solution of the optimal liquidity ratio, L^*, which is interior. To back out the cost of liquidity for the banker, we substitute this optimal value and recompute the expected liquidity cost at the optimum.[12] The value of the objective for the banker as a function of L^* is:

$$\Omega(X) = R^B + \max_{w_d} w_d \left(\left(R^B - R^D\right) + \max_{L} \left\{ -\left(R^B - R^C\right) L^* - \tilde{R}^{\chi}(1, L^*) \right\} \right)$$

This polar example shows the key trade off in the model. Holding reserves is costly because the bank forgoes a unit of arbitrage, $\left(R^B - R^C\right)$. However, holding reserves reduces the risk of a liquidity cost, $\left(R^B - R^C\right)$. When choosing the optimal size for deposits ω^d, the banker incorporates into this decision for leverage, the amount of reserves it will hold, L^*. Thus, through the optimized liquidity ratio, monetary policy acts like a tax on financial intermediation measured by the cost of liquidity holdings $\left(R^B - R^C\right) L^* - \mathbb{E}_{\omega'}\left[R^{\chi}(1, L^*)\right]$.

This problem is linear in ω^d so the solution to the deposit decision would be pinned down entirely by the value of $\left(R^B - R^D\right)$ minus the optimal liquidity choice. In the case of risk neutrality, without constraints, ω^d solves a linear problem. So either ω^d is 0, unbounded or the solution indeterminate when the arbitrage on loans equals the expected liquidity cost at the optimal liquidity ratio L^*:

$$\underbrace{\left(R^B - R^D\right)}_{\text{Arbitrage on Loans}} = \underbrace{\left(R^B - R^C\right) L^* + R^{\chi}(1, L^*)}_{\text{Optimal Liquidity Ratio Cost}}.$$

In equilibrium, this condition must hold implying zero profits. Given that risk-neutrality implies a perfectly elastic, elasticity of intertemporal substitution, banks do not hold equity. This equilibrium condition is affected by monetary policy, but in turn imposes a restriction

[12] This amount becomes: $\mathbb{E}_{\omega'}\left[w_d R^{\chi}(L^*)\right] = -w_d \left(\bar{\chi}\mathbb{E}_{\omega}\left[\omega | \omega > \bar{\omega}(L^*)\right]\left(1 - F\left(\bar{\omega}(L^*)\right)\right) - \underline{\chi}\mathbb{E}_{\omega}\left[\omega | \omega < \bar{\omega}(L^*)\right] F\left(\bar{\omega}(L^*)\right)\right).$

on the set of equilibria imposed by monetary policy. Two additional polar cases lead to the very similar constraints.

Remark. *Under risk neutrality, $\gamma = 0$, the bank's portfolio problem may be written as a linear function of leverage. The return on this leverage is a function of an minimal liquidity ratio cost. Monetary policy introduces a liquidity ratio cost. Thus, monetary policy affects operates like a tax on financial intermediation.*

Polar Case II: No withdrawals ($\Pr(\omega = 0) = 1$). A special case is occurs when $\Pr(\omega = 0) = 1$. In this case, there's no uncertainty so there's is no difference between the portfolio decision of a risk-neutral and a risk-averse banker (only the dividend policy changes). Thus, following the steps before, the value of the portfolio problem is:

$$R^B + \max_{w_d} w_d \left(R^B - R^D\right) + w_d \left(\max_L - \left(R^B - R^C\right) L - \left(\bar{\chi} \left(\rho - L\right)^+ - \underline{\chi} \left(L - \rho\right)^+\right)\right).$$

To have an interior solution for the liquidity ratio, it is required that:

$$\bar{\chi} > R^B - R^C > -\underline{\chi},$$

since otherwise L is either 0 or infinity. When this condition is satisfied, the banker sets the liquidity ratio exactly to ρ, the reserve requirement in the model. So without risk, ρ determines the amount of reserves per unit of loan.

Assuming there is a market for reserves, incorporating this result into the objective yields:

$$\Omega\left(X\right) = R^B + \max_{w_d} w_d \left(\left(R^B - R^D\right) - \left(R^B - R^C\right) \rho\right).$$

This objective function shows that monetary policy acts, again, like a tax on loans. In equilibrium under certainty about withdrawals, a finite solution for the volume of deposits is given by:

$$\left(R^B - R^D\right) = \left(R^B - R^C\right) \rho.$$

Remark. *Without liquidity risk, $(\omega = 0)$, the bank's problem a linear function of the reserve requirements. This requires $\bar{\chi} > R^B - R^C > -\underline{\chi}$.*

Another thing to note about perfect foresight of bank about withdrawals is that the constraint that $\left(R^B - R^D\right) = \left(R^B - R^C\right) \rho$ implies that $R^C \leq R^D$, for partial reserve requirements. In other words that reserves lose value relative to deposits over time. This result is important. It shows without risk, banks have no precautionary motive for holding reserves. In an equilibrium with stability of interest rates, to have a determinate amount of loans and positive reserves the Central Bank would have to force $\rho \geq 1$ to have $R^C = R^D$. This would force banks to hold as many reserves as deposits. This is known as *narrow banking.* In this case, lending would not be influenced by the Central Bank and only affected by the amount of equity in the banking system. Alternatively, reserves would play no role, and $R^B = R^D$ with infinite leverage. Another possibility is to have $R^C \leq R^D$, and have $\rho \leq 1$, but this possibility leads to a fiscal cost.

The equilibrium in this polar case is given by:

$$B_t = D_t = (\rho_t)^{-1} M0_t$$

and

$$\left(R_t^B - R_t^D\right) = \left(R_t^B - R_t^C\right)\rho_t.$$

Hence, R_t^C is the outcome of the choice of $M0_t$, and ρ_t. We have the following,

Remark. *Without liquidity risk,* $(\omega = 0)$, *monetary policy can affect* $\{B_t\}$ *(provided that* B_t *is consistent with* $q_t \leq 1$*) by choosing* $\{\rho_t, M0_t\}$. *Different choices of* $\{\rho_t, M0_t\}$ *lead to different fiscal costs of monetary policy. In particular, when narrow banking is imposed,* $\rho_t = 1$, *monetary policy has the lowest fiscal cost.*

This result is interesting in itself. It shows that monetary policy has lower fiscal costs when there is a precautionary motive for holding reserves.

Case III: No Liquidity Cost . Let's now return to the case $\gamma \geq 0$ but assume there is no cost from a shortage of reserves. In such a case $\chi = 0$ (χ is the zero function). In this case, all risk is eliminated from the model. In such case reserves are not value for the reduction in the hazard rate of a liquidity cost. In this case, $\Omega(X)$ becomes a linear program since the solution to this problem is deterministic:

$$\Omega(X) = \underbrace{R^B}_{\text{Return to Equity}} + \max_{\{w_d, w_c\}} \underbrace{-\left(R^B - R^C\right) w_c}_{\text{Cash Opportunity Cost}} + \underbrace{\left(R^B - R^D\right) w_d}_{\text{Arbitrage}}$$

Now, an equilibrium with finite holdings will clearly require:

$$R^B = R^C = R^D = 1.$$

This polar example shows that if the Central Bank eliminates all the costs associated with liquidity risk, it has no effect on aggregate lending. The price of reserves r is pinned down by R^C and banks are indifferent between holding any mount of reserves. In this case, the amount of lending is entirely determined by the demand for loans at $q_t = 1$, the efficient amount of lending. Another way to say this, is that if monetary policy is to have any real effects, it must introduces distortions to the loans market.

Remark. *If the* $\bar{\chi} = \underline{\chi} = 0$*, monetary policy cannot affect lending.*

Power of Monetary Policy under Polar Cases. The first two polar cases show that monetary policy acts like a tax. In the third one, there is no monetary policy. However, a monetary policy $\left(M_t^0, \rho, \bar{\chi}, \underline{\chi}\right)$ faces some constraints. Under risk-neutrality (case I), the Central Bank can affect the cost of lending, and the amount of loans by increasing the costs of obtaining liquid funds. As noted above, under risk neutrality, there is no role for bank equity. To talk about ω^d and L we must use a limit argument where there is some minimal, non-binding amount of equity.

Under risk-neutrality, monetary policy has partial control over lending in this economy. At t, the system of equilibrium conditions is:

$$\left(R_t^B - R_t^C\right) = \bar{\chi}_t \left(1 - F_t\left(\frac{L_t^*}{(1+r_t)} - \rho_t\right)\right) - \underline{\chi}_t F_t\left(\frac{L_t^*}{(1+r_t)} - \rho_t\right)$$

$$B_t = D_t = \left(L_t^*\right)^{-1} M0_t$$

$$q_t = \Theta_t \left(B_t - (1-\delta) B_{t-1}\right)^{\epsilon}$$

$$R_t^B = 1 + (1-\delta)\frac{q_{t+1}}{q_t}$$

$$R_t^B = 1 + \left(R_t^B - R_t^C\right) L_t^* + R^{\chi}\left(L_t^*\right)$$

subject to:

$$\bar{\chi} > R_{_t}^B - R_{_t}^C > -\underline{\chi}_t.$$

Monetary policy can achieve any sequence of $\{B_t\}$ in the set of sequences that satisfies the constraints above. We are interested in understanding this problem furthermore to determine what shocks can reduce the set of equilibrium outcomes. A follow up version of this paper should provide a more detailed discussion.

Remark. *Under risk neutrality,* $\gamma = 0$*, monetary policy affects the supply of loans indirectly by affecting the liquidity ratio of banks* L^* *through its policy choice* $\left(M_t^0, \rho, \bar{\chi}, \underline{\chi}\right)$*. Shocks to* (Θ_t, F_t) *affect the set of equilibrium outcomes.*

Conjecture. *Negative shocks to the demand for* Θ_t *reduce the set of possible outcomes* *for B_t. However, monetary policy can undo the effects of withdrawal volatility shocks to F_t.*

Interaction between Monetary Policy and Regulatory Constraints. Another shock that affects the set of outcomes is the effect of regulatory constraint κ. We have yet to study the effects of such shocks on the set of monetary policy. With the bank regulation in place, the equilibrium conditions are altered because the constraint imposes a shadow value on various objects. So far, we can only conjecture the results.

Conjecture. *Negative shocks to the demand for κ_t reduce the set of possible outcomes for B_t.* Thus, the effects of monetary policy are affected by the capital requirements. However, the effects of η_t *can be undone by monetary policy.*

4 Calibration and Empirical Analysis

Calibrating our model requires an empirical estimate of the random-withdrawal process for deposits, F_t. We describe next our estimation procedure for this shock.

4.1 Analysis of Deposits, Liabilities and Equity

We use information from Call Reports collected by the Federal Deposit Insurance Corporation (FDIC). The Call Reports presents balance-sheet information for all Commercial Banks in the US. The data spans all the quarters from 1990 until 2011. Banks in our model have only one form of liability, demand deposits. In practice, however, Commercial Banks have other forms of liabilities such as bonds and long-term deposits (savings deposits). Thus, we document information on Total Liabilities (TL), Total Deposits (TD) and Demand Deposits (DD) and take the stance of calibrating F_t using information from the volatility of Total Deposits. In addition, we also use information on three forms of equity. We study the behavior of Equity (E), Tangible Equity (TE) and the same series adjusted for Loan Loss Allowances (RE and RTE respectively).

1990-2010 Sample Averages. The banking industry underwent a consolidation over the last two decades. Due to the substantial amount of mergers in the industry and measurement error, aggregating balance sheet information directly would be misleading. Hence, we isolates the effects of mergers on the increase in the volatility of different bank liabilities by eliminating observations with observations that lie more than four standard deviations or that exhibit negative entries.

The summary statistics for the quarterly growth rate of the aggregate time series is presented in Table 1.

Variable	Mean	Std. Dev.	N
TD	1.023	0.085	771410
DD	1.029	0.191	778467
TL	1.022	0.07	773629
TE	1.017	0.083	769077
RTE	1.017	0.097	766806
E	1.018	0.072	774407
RE	1.018	0.086	769338

Table 1: Summary statistics .

The data exhibits very similar patterns for the growth of total deposits and total liabilities. Demand deposits are 2.5 times more volatile than all the deposits. This may respond to a stronger seasonality in this variables. One of the reasons why we use use total deposits as our data counterpart for deposits in our model is that it is substantially volatile, the standard deviation is 8.5% per quarter, and it is close to the volatility of total liabilities, 7.0%. Moreover, demand deposits may be exchanged for deposits of longer maturity, which explains why total deposits are less volatile (being a sub-account), but through the lens of our model, this would be as a change in one account for another within the same bank. This can be observed from in the Table 2.

Variables	TD	DD	TL	TE	RTE	E	RE
TD	1.000						
DD	0.393	1.000					
TL	0.844	0.350	1.000				
TE	0.077	0.010	0.133	1.000			
RTE	0.046	0.002	0.096	0.858	1.000		
E	0.144	0.031	0.234	0.737	0.647	1.000	
RE	0.112	0.024	0.184	0.647	0.777	0.885	1.000

Table 2: Cross-Sectional correlation for Quarter-Bank observations

Figure 4 presents the evolution of the growth rates of these time series subtracting the growth rate of the GDP deflator. All the series show a strong seasonal component. The wider curve presents the Hodrick-Prescott filtered series. The trends reveal a decline in the growth rates towards the end of the sample, corresponding to the period of the Great Recession and onwards.

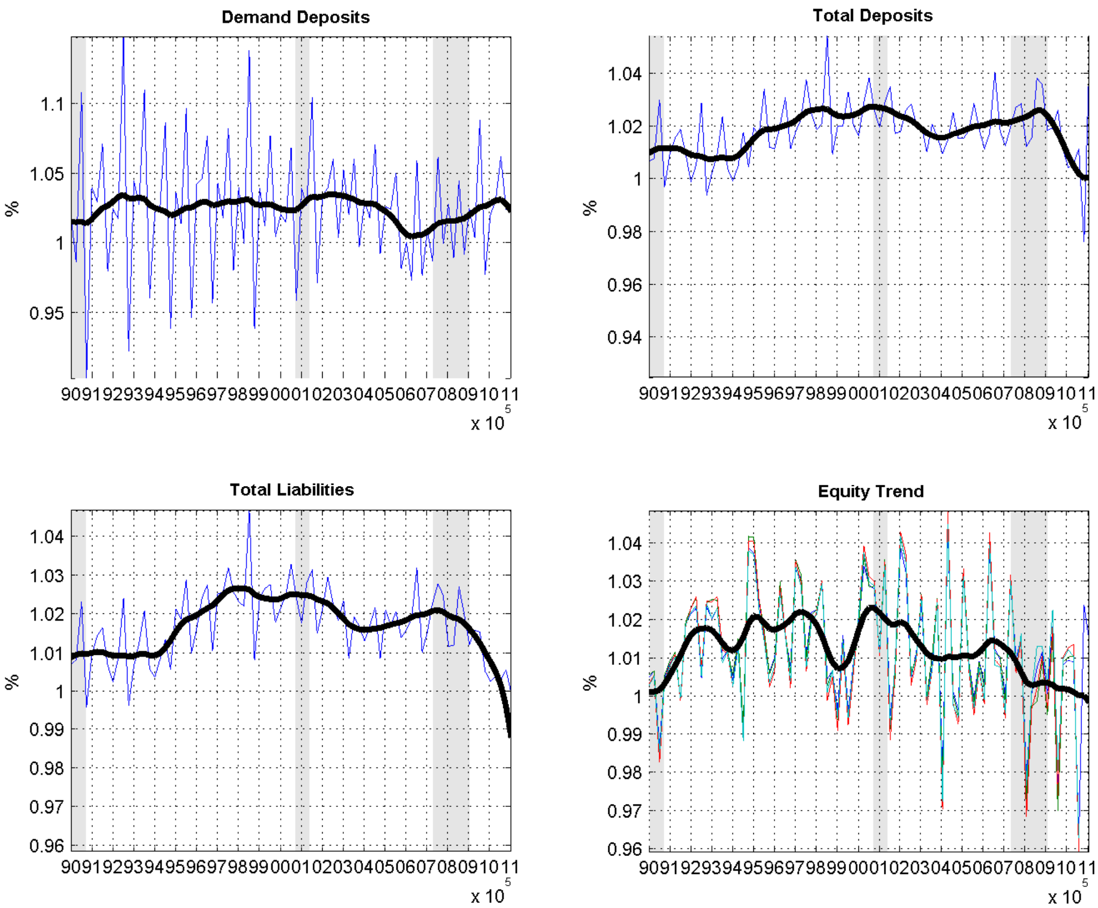

Figure 4: Cross-Sectional Average Growth Rates

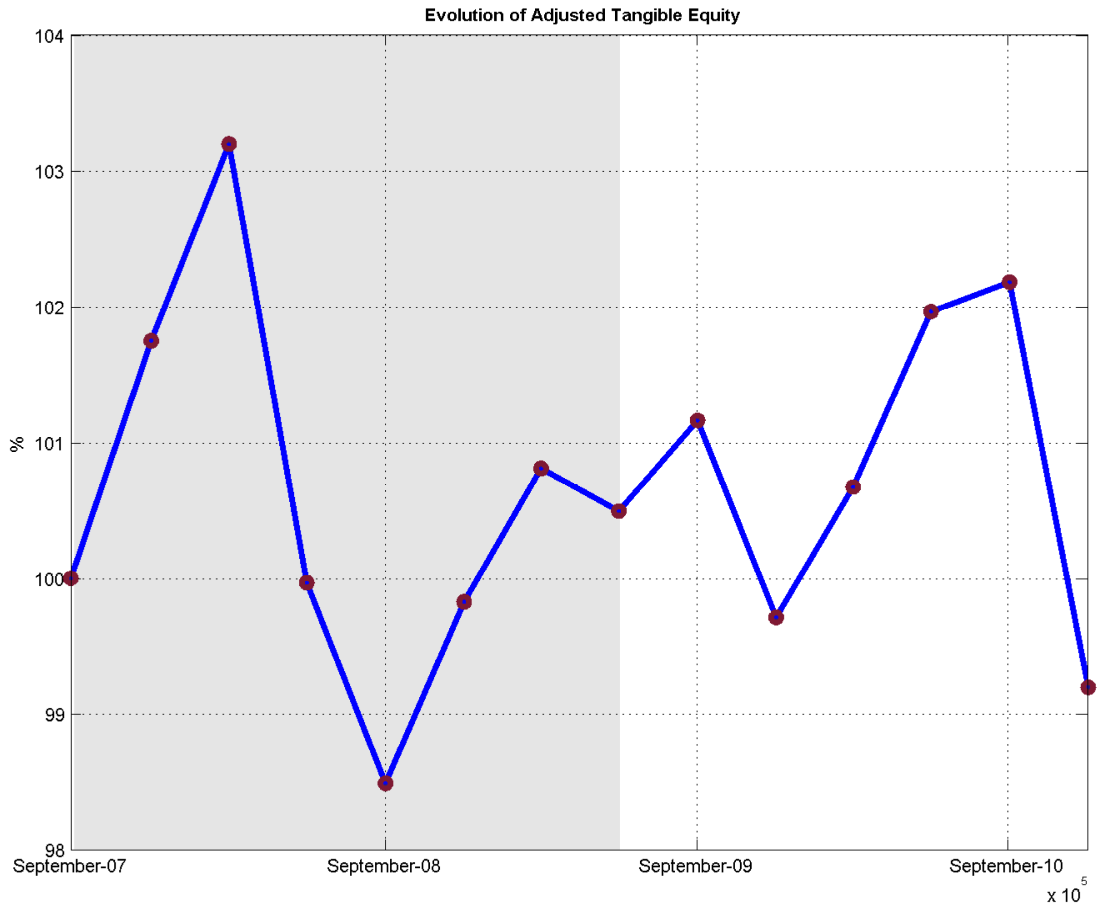

Figure 5: Evolution of RTE during the Great Recession.

For our quantitative analysis, we are particularly interested in the behavior of the series for bank equity. The figure shows a decline in filtered growth rates, but due to the strong volatility in the period, the filtered series does not show a decline in levels. One of the hypothesis that we consider in our quantitative analysis is the decline in equity during the Great Recession. A snapshot of the compounded growth rates reveals a mild decline in the book value of tangible equity even adjusting for Loan and Loss Allowances. Figure 5 shows the pattern. The evolution of the book value may be distorted by other factors such as TARP, and not materialized losses outside the book value. Hence, we will stretch the results and show use a benchmark of 5% for our book value equity losses.

Quarterly Cross-Sectional Deviations. Part of the variation in the bank-quarter statistics presented above have a common component, including seasonality, nominal changes in the time series and aggregate trends. To decompose the variation of these liabilities into their common component, we present the summary statistics in terms of deviations of these variables from their quarterly cross-sectional averages. Table 3 presents the results:

Variable	**Mean**	**Std. Dev.**	**N**
dev TD	0	0.084	771369
dev DD	0	0.183	777932
dev TL	0	0.069	773629
dev TE	0	0.081	769073
dev RTE	0	0.096	766803
dev E	0	0.071	774401
dev RE	0	0.085	769329

Table 3: Summary statistics

One thing one gathers from this table is that most of the variation is preserved even when one subtracts the evolution of aggregate averages. This shows the substantial amount of idiosyncratic volatility among banks. Except for the behavior of demand deposits, the volatility of the liabilities of banks is almost exclusively idiosyncratic. This can be seen from Table 4 which shows the behavior of the correlation in cross-sectional deviations from quarterly means. These correlations are essentially the same as the correlations for historical growth rates implying that the idiosyncratic component is quite high.

Variables	dev TD	dev DD G	dev TL	dev TE	dev RTE	dev E	dev RE
dev T D	1.000						
dev DD	0.389	1.000					
dev TL	0.844	0.345	1.000				
dev TE	0.082	0.027	0.135	1.000			
dev RTE	0.050	0.016	0.097	0.854	1.000		
dev E	0.152	0.052	0.238	0.727	0.635	1.000	
dev RE	0.118	0.040	0.187	0.635	0.769	0.881	1.000

Table 4: Correlation for Cross-Sectional Deviations from Means

The correlation in the data between deviation of tangible equity growth and the the deviation in the growth rate of total deposits ranges from 5% to 15% depending on the definition of

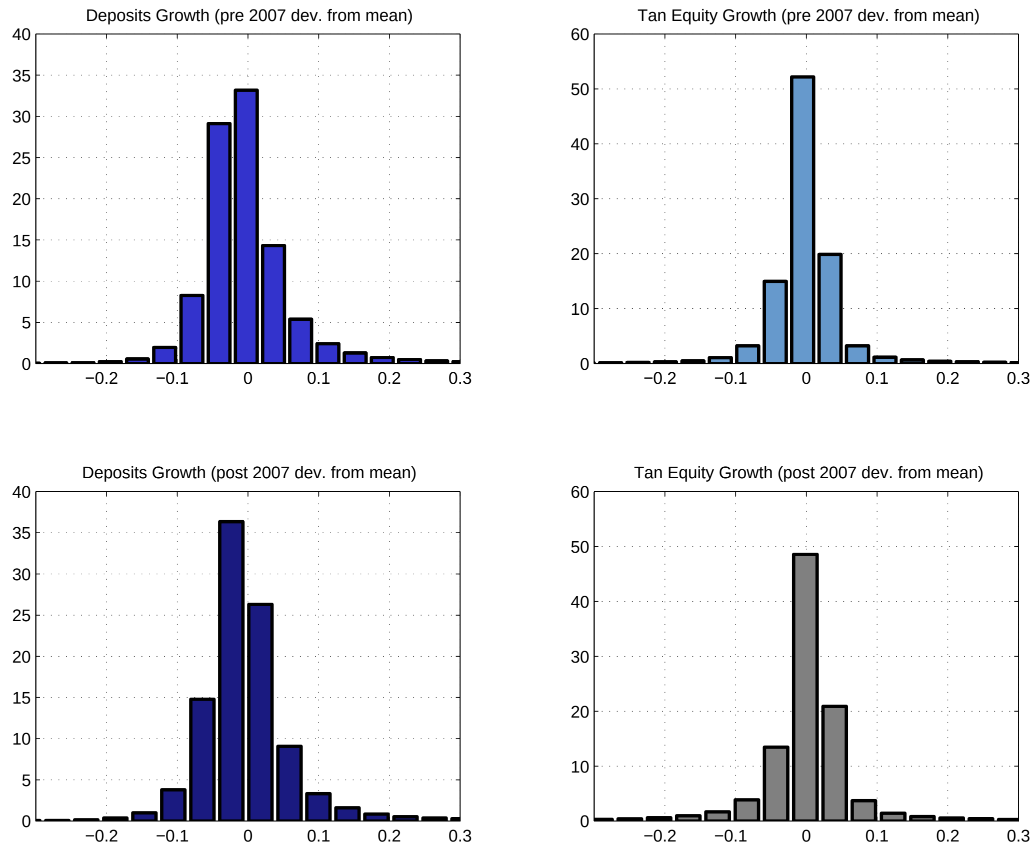

Figure 6: Cross-Sectional Distribution of Deviation from Cross-Sectional Average Growth Rates

equity that we use. In the model, this correlation will be very high (though not 1) because deposit volatility is the only source of risk for banks. In practice, banks face other important sources of risks such as loan risk, duration risk and trading risk. This figure however implies that deposit withdrawal risks are non-negligible risks for banks.

The following graph decomposes the data into two samples pre-crisis (1990Q1-2007Q4) and crisis (2008Q1-2010Q4). Figure 6 reports the empirical histograms for every quarter-bank growth observation.

We use the empirical histogram of the quarterly deviations of TD to calibrate F_t, the process for withdrawal shocks in our model. In the quantitative analysis, we contrast the behavior of

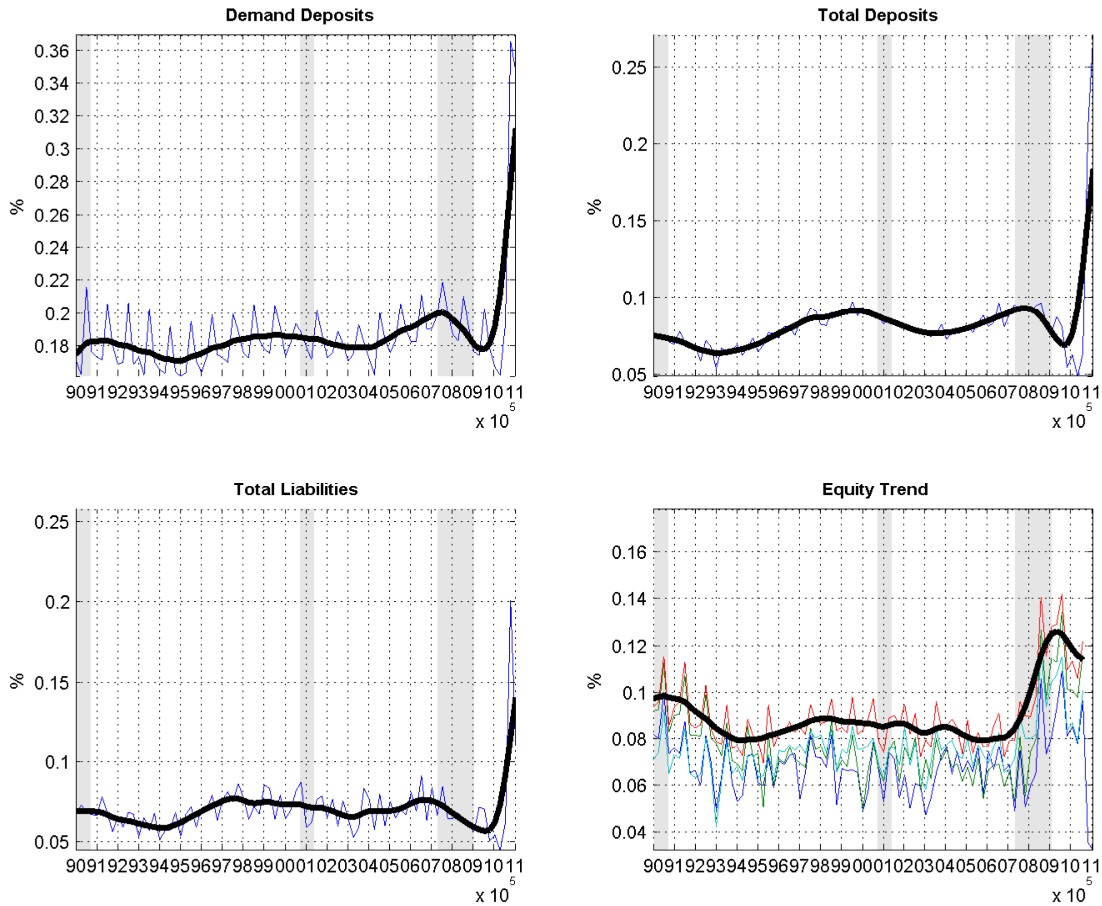

Figure 7: Quarterly Cross-Sectional Dispersions

equity volatility that results as an outcome with the corresponding histogram and correlation in the date for this variable.

We also analyze the evolution of the volatility in the variables. This analysis provides the basis for our calibration of the increase in withdrawals shocks during the Great Recession, one of the hypothesis that we test. Figure 7 shows the time series for cross-sectional dispersion in growth rates in all of the series that we study. As the cross-sectional averages these series display a high seasonal component. The Hodrick-Prescott filter of the series reveals non-negligible increases in cross-sectional dispersion, in particular after 2009. The cross-sectional dispersion of all the measures of equity show a 60% increase at the peak of the Great Recession.

Tests for Growth Independence. Our models assumes that the withdrawal process is i.i.d over time and banks. This assumption implies that if we subtract the common growth rates of all the balance sheet variables in our model, the residual should be serially uncorrelated. We test the independence of the deviations-from-means quarterly growth rates using an OLS estimation procedure. We run the deviations in quarterly growth rates from the cross-sectional averages against their lags. The evidence from OLS auto-regressions support the assumption that of time-independent growth. Tables 5 we report coefficients that are statistically identical to zero (with zeros lying within the two standard deviation bounds) for all of the variables of that we study.

Variable	**Coefficient**	**(Std. Err.)**
TD	0.001	(0.001)
DD	0.000	(0.001)
TL	-0.001	(0.001)
RTE	0.000	(0.001)
TE	-0.001	(0.001)
E	-0.001	(0.001)
RE	0.000	(0.001)

Table 5: Autocorrelation Estimates for First Lag of Growth Rates Cross-Sectional Deviations

4.2 Parameter Values.

The values of all parameters are listed in Table 6. We need to assign values to eleven parameters $\left\{\kappa, \beta, \delta, \gamma, \epsilon, \rho, \eta, r, r^f\right\}$. We set the capital requirement and the reserve requirement according to standard regulatory measures. In particular, we set $\kappa = 24$, which corresponds to the Tier 1 Capital Ratio, $\rho = 10$ percent. The discount factor is set to 0.99. The risk aversion is set to 1.

The parameter r, which captures the opportunity cost of holding cash, is set to 1.5 percent, which is in the range of the historical values for the real Libor rate. We set χ to be 80 percent of the interest rate so as to match the difference between the Fed Funds and the Libor rates. The loan average maturity is set to 2 years, implying a value for $\delta = 0.87$. The value of the loan demand elasticity $\epsilon = 8.0$. For now, we abstract from interest on reserves and the liquidity constraint in the lending stage, i.e, $r^f = 0, \eta = 0$. In addition, we set $\chi^L = 0$. Finally, for the withdrawal we use a non-parametric estimation as described above.

Table 6: Parameter Values

	Value
Capital requirement	$\kappa = 20$
Discount factor	$\beta = 0.995$
Risk aversion	$\gamma = 0.5$
Loan Maturity	$\delta = 0.7$
Interest rate (annualized)	$r = 0.03$
Liquidity Requirement	$\rho = 0.10$
Loand Demand Elasticity	$\epsilon = 1.0$
Interest on reserves	$r^f = 0$
Liquidity constraint	$\eta = 0$
Penalty	$\chi^L = 0.001$
Penalty	$\chi^H = 0.0$
Withdrawal-shock volatility	$\phi = 0.13$

5 Policy Functions

We start with a partial equilibrium analysis of the model by showing banks policy functions for different loan prices. Figure 8 reports decisions for cash, loans, dividend, as well as liquidity and leverage ratios, the value of the asset portfolio, liquidity risk, expected returns and expected equity growth. These policies correspond to the solution to the Bellman equation 3 for different values of price q.

A first observation that emerge from Figure 8 is that the supply of loans is decreasing in the loan price whereas dividends and cash ratios are increasing in the loan price. As loan prices decrease, loans become relatively more profitable leadings banks to keep a lower fraction of its assets in relatively low return assets, i.e., cash. Moreover, banks cut on dividend rate payments to allocate more funds to loan issuances and experience higher equity growth. The exposure to liquidity risk, measured as the expected penalty costs from falling short of cash holdings, is also decreasing in loan prices, reflecting the fact that banks asset portfolio becomes relatively more liquid.

Another key observation is that there is a kink in banks policies at the value of the loan price where the capital requirement ceases to bind. In particular, when the price of loans is sufficiently high, high profits from intermediation lead banks to reduce deposits to the point in which the capital requirement is not binding. In this region, a decrease in the loan price leads to a sharp increase in lending ratios. To the left of this point, decreases in the loan price lead to a less of a sharper increase in lending rates due to the fact that higher lending needs to be financed with either lower dividend payments or a reduction in cash holdings, which exposes the banks to more severe effects of withdrawal shocks. In other words, the capital requirement generates an asymmetric response to changes in the loan price depending on whether the capital requirement is binding or not.

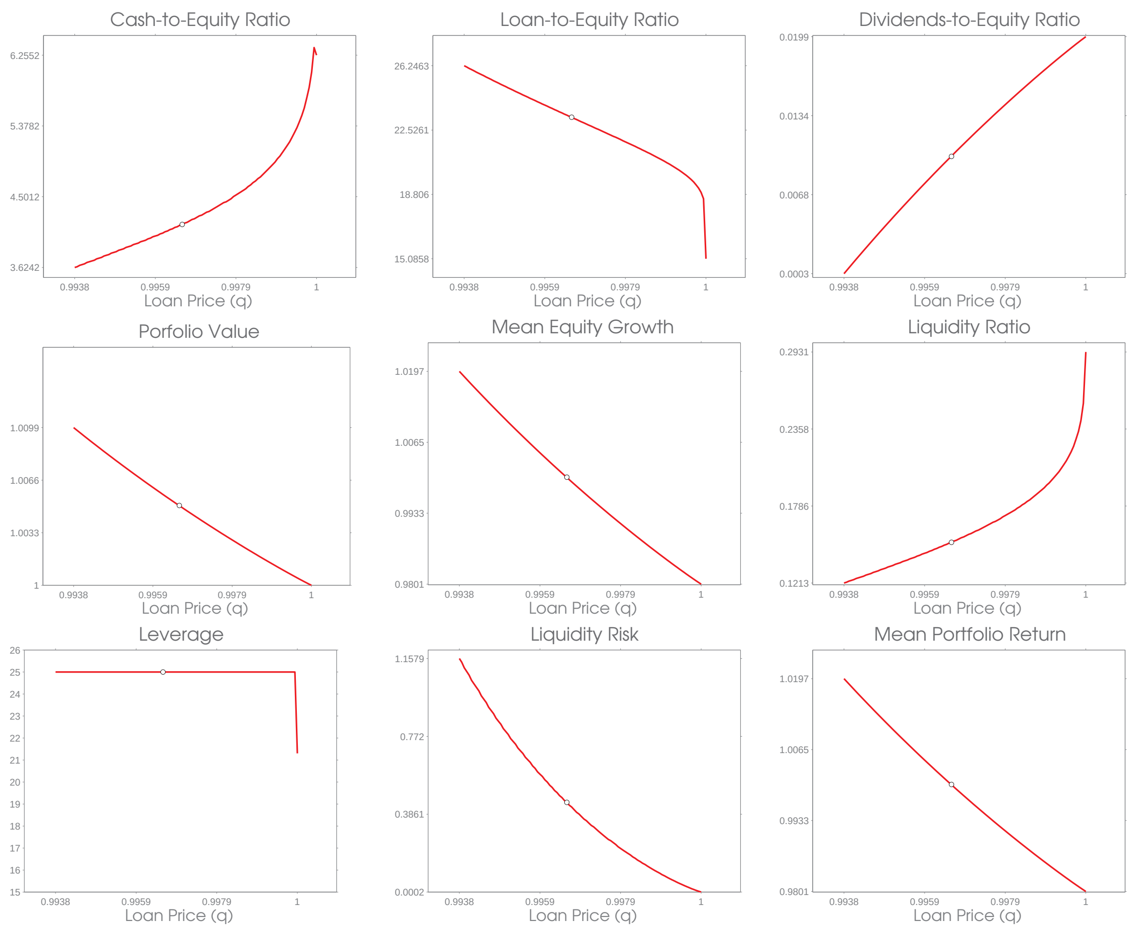

Figure 8: Policy function for different Loan Prices

The value of the interest rate plays an important role because it affects the cost of hedging liquidity risk. In fact, a high interest rate increases the opportunity cost of holding cash, and leads to lower lending ratios as figure 9 shows. In turn, this leads to higher dividend rates, lower growth and lower returns on the asset portfolio.

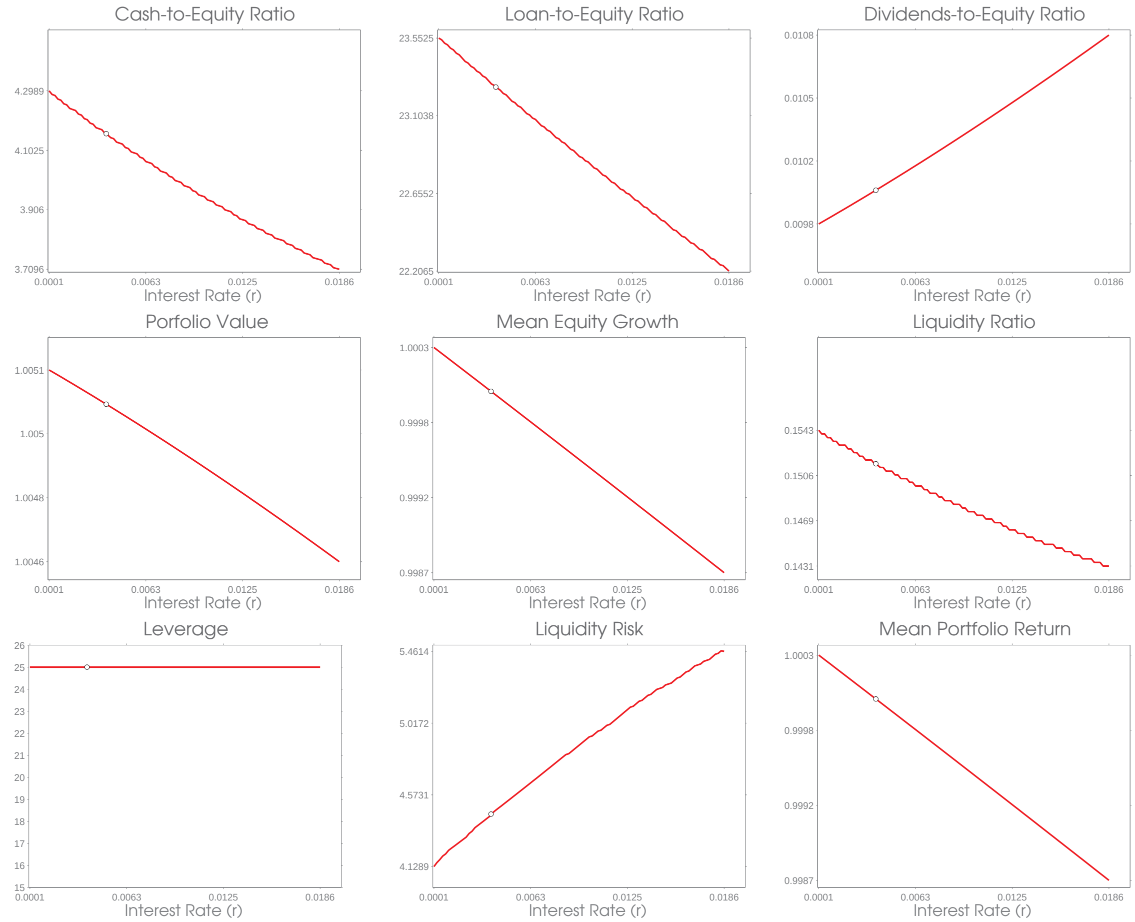

Figure 9: Policy function for different Interest Rates

6 Steady State Equilibrium Portfolio

The previous section describe portfolio decisions for an arbitrary loan price. We now analyze the equilibrium portfolio at steady state and investigate the effects of withdrawal shocks over the banks' balance sheets. The equilibrium portfolio corresponds to the solution of the Bellman equation 1 evaluated at the price that clears the loans market, according to condition 6. As it can be seen from the plot for equity growth in figure 8, the price that clears the market corresponds approximately to $q = 0.995$.

The top left panel of Figure 10 shows the value of the risk adjusted portfolio Ω_t for possible loans and cash weights w. The maximum value of the portfolio is reached at $\dot{c} = 2.4$ and $\dot{b} = 18.3$, which implies $\dot{d} = -19.8$ and a binding capital requirement constraint. The optimal portfolio allocations is a result of the trade-off banks face between return and liquidity risk: loans provide a high return whereas cash delivers a lower expected return but delivers the highest payoffs when an adverse withdrawal shock hits the banks.

The value of the return for each asset as a function of the withdrawal shock are shown in panel (d) of Figure 10. For low withdrawal shocks, the return on loans is higher than the return on loans. As the withdrawal shock reaches a value that makes the penalty requirement binding, cash provides extra liquidity benefits and delivers a higher return than loans. The discontinuity in the return for cash occurs at $\omega = 0.05$, the value of the withdrawal shock at which the liquidity penalty becomes strictly positive as illustrated in Panel (e). The return on deposits also fluctuate significantly depending on the withdrawal shock and also have a discontinuity at the value of ω at which banks start facing penalties from insufficient cash. In fact, the net return on deposits is given by $r\omega$ for values of ω lower than 0.04, and jump to $(r + \rho\chi)\omega$ for values of ω higher than 0.04.

The resulting return of the equilibrium portfolio as a function of the withdrawal shock is illustrated in Panel (c). This figure shows that in equilibrium the return of the portfolio is quite sensitive to withdrawal shocks. In fact, the range of portfolio returns can oscillate between 0.94 and 1.05 according to the realization of ω. In equilibrium, the banks that experience positive withdrawal shocks will increase the size of their equity whereas those that experience negative withdrawal shocks will shrink. Finally, panel (f) show the cash and deposit holdings at the end of the period that results from the portfolio choice in the lending stage and the effects of the withdrawal shock in the balancing stage.

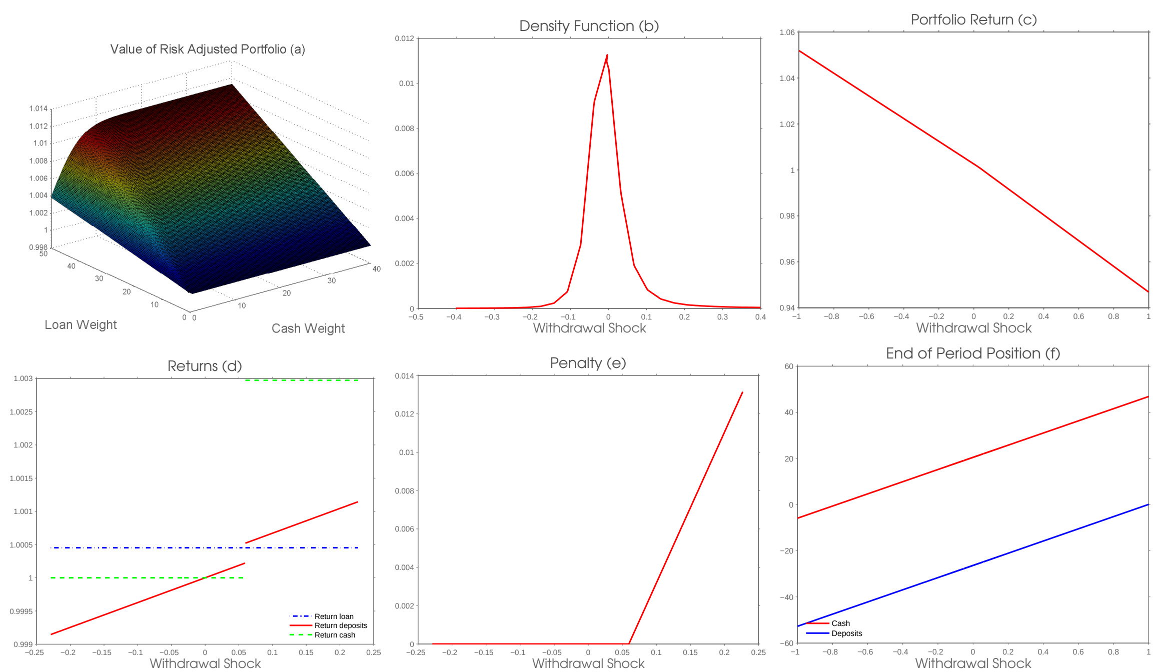

Figure 10: Portfolio Choices and Effects of Withdrawal Shocks

7 Transitional Dynamics

This section studies how the economy responds to different shocks. For this purpose, we consider an economy that is at steady state at period $t = 0$ and experience an unanticipated permanent shock. The shocks we consider are equity losses, credit demand shocks, a tightening of capital requirements, uncertainty shocks and rise in funding costs. For each shock, we analyze the transitional dynamics of banking and monetary indicators. The panels 11-15 in this section show the aggregate dynamics. Each panel shows that path of total lending, total cash, loan prices, the return on loans, total equity and new loan issuances. In addition, we also refer to the panels 23 -28 in the Appendix illustrate the dynamics of the individual banks policies. The superior panel for each figure shows the cash-to-equity ratio, loan-to-equity ratio and dividends-to-equity ratio. The intermediate panel shows the portfolio value, the mean equity growth and the liquidity ratio. Finally, the inferior panel shows the liquidity risk, bank value expressed in consumption units and the mean portfolio return.

7.1 Equity Losses

The first shock we consider is a sudden unexpected decline in bank equity, which can be interpreted as capturing an unexpected rise in non-performing loans or losses from other portions of the balance sheets. Figure 11 illustrates the response of the economy to equity losses of 5 percent. A key result is that the economy experiences an immediate drop in loan prices and converges back to the initial steady state. Hence, there is an amplification effect on the value of the bank through the decline in the value of asset holdings.

Let us analyze the transitional dynamics in more detail. What explains the immediate drop in loan prices? Suppose loan prices did not change at all. In this case, banks would keep the same lending ratios and total lending, computed as $E_1 b_1 - E_0 b_0 (1 - \delta)$, would fall. If loan prices do not change, however, the demand for loans would remain unchanged leading to an excess demand for loans. In equilibrium, this means that loan prices need to drop on impact. Low loan prices also mean that banks have high return from lending, and equity growth, as we analyzed in section 5. Hence, as the economy converges to the steady state, the increase in equity leads to a monotonic increase in loan prices.

The behavior of dividend rates reflect a trade-off in the model. On the one hand, equity losses lead to a rise in the return on loans that makes cutting dividend payments more attractive. On the other hand, general equilibrium effects generate wealth effects generating an increase in dividend rates. Overall, we find that that the first effect prevails.

The volume of bank lending follows a persistent decline. As the price of loans drop on impact, there is a sharp decline in the demand for new issuances, which is larger than the repayments of previous loans. This explains the initial fall in the stock of loans. As new issuances remain below repayments, the stock of loans continue to decline over the next 18 quarters. Once banks recover from about 50 percent of the initial losses, new issuances are higher than loan repayments, and credit lending recovers almost completely after about 7 years,

Equity losses also have implications for the evolution of other banking and monetary indicators. Low prices for loans during the transition to the steady state means that liquidity ratios and cash ratios are also below the steady state, whereas lending rates, portfolio value and portfolio returns are above the steady state. Moreover, liquidity risk is also higher during the transition reflecting the increase in lending ratios and the decrease in cash rates.

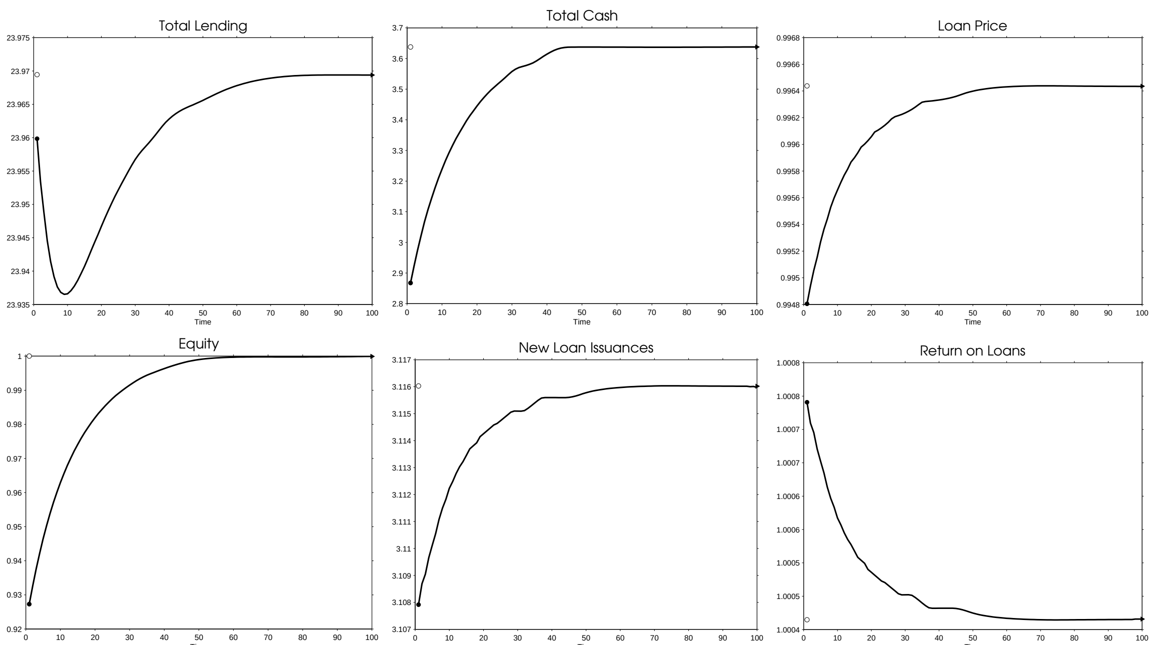

Figure 11: Shock to Equity Losses

7.2 Tightening of Capital Requirements

Next, we consider a sudden tightening of capital requirements, i.e., a reduction in κ.[13] This shock can be interpreted as tightening regulatory requirements or alternatively as reflecting solvency concerns. Figure 12 illustrates the effects.

Before analyzing the transition, let us first analyze the effects on the steady state of the economy. A central observation, apparent in figure 12, is that the new steady state features a lower loan price. The decrease in loan prices reflect the fact that in the new steady state,

[13]Recall that the capital requirement constraint is binding in the initial steady state.

banks are relatively more constrained in expanding their balance sheets. This leads to a lower supply for loans, lower loan-to-equity ratios, and consequently lower loan prices. Moreover, banks also cut dividend rates to increase the amount of loanable funds. Liquidity ratios are also lower in the new steady state, and this is associated with lower liquidity risk.

What does the transition to the new steady state look like? On impact, that there is a sharp decline in loan prices that exceed the long-run decline. This overshooting result is a key prediction of our model. To understand the intuition behind this result, it is useful to consider an alternative transition where the loan price adjusts immediately to the new steady state. If this were the case, the supply of loans on impact would be insufficient to meet the demand for loans. This would occur because a constant price for loans imply constant lending ratios and since the initial level of equity is below the long-run value, the short-run supply of loans is below the long-run supply of loans. As a result, this must imply that loan prices have to experience a sharp drop on impact and then increase towards the new steady state.

The overshooting in loan prices has implications for the dynamics of banks' balance sheet positions as well. In particular, lending ratios are characterized by a fall on impact followed by a continued fall. This gradual decline in lending rates is consistent with the behavior of loan prices. That is, the sharp drop on impact in loan prices mitigate the contraction in lending rates caused by the tightening of capital requirements. Over time, as loan prices recover, lending rates continue to fall. On the other hand, cash holdings measured as a fraction of total assets and equity fall more in the short run than in the long run. The asymmetry between the response of reserve and lending rates hinges on the general equilibrium effects. In fact, the sharp decline of loan prices leads banks to conduct a portfolio reallocation towards higher return assets. This explains why reserve rates overshoot whereas loan rates undershoot.

The asymmetric response of reserve and lending rates also have implications for liquidity risk along the transition path. In particular, the tightening of capital requirements generate a sharp rise of exposure to liquidity risk on impact, which coincides with a sharp drop in the liquidity ratio. Because banks face a tighter capital requirement constraint, they are effectively restricted in the growth of their balance sheets. Hence, banks naturally respond by shifting the portfolio composition towards more risky assets so that lending rates fall less than reserve rates. Moreover, this portfolio reallocation is reinforced by the decline in loan prices that results from the decline in total loanable funds.

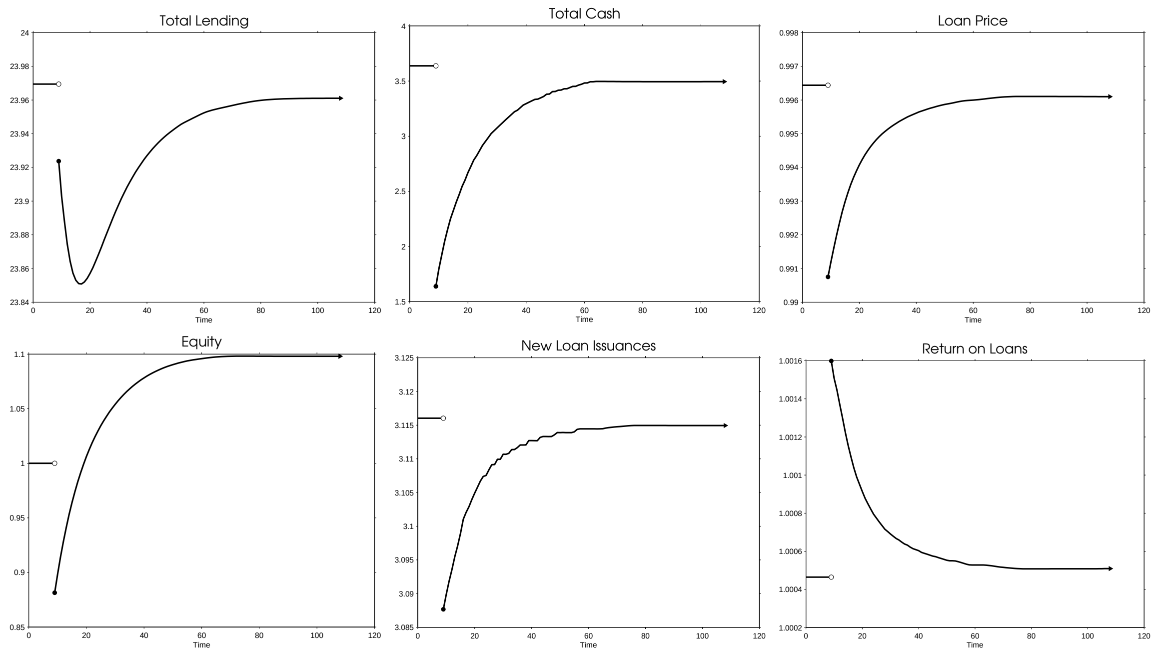

Figure 12: Tightening of Capital Requirements

7.3 Credit Demand Shocks

We now study the effects of a credit demand shock. In our model, this corresponds to a decline Θ_t, which can be rationalized by a decline in aggregate productivity, according to the microfoundations provided in section 4. Figure 13 illustrates the effects.

Let us first analyze the steady state effects. The price of loans at steady state remain unchanged and there is a decrease in the value of equity. The reason why the price of loans remain unchanged is that banks' optimization problem remain unchanged which implies that a different loan price would not be consistent with zero equity growth. Moreover, this also implies that banks ratios are identical in the new steady date. On the other hand, the level of equity is lower in the new steady state. Given that there is a decline in credit demand, and banks' lending ratios remain the same, this requires a drop in equity to restore the equilibrium in the loans market.

The transitional dynamics exhibit a jump in the price for loans that is followed by a gradual decline towards the same steady state value. Again, to understand the mechanism, suppose loan prices did not change at all. Given this price for loans, lending ratios would also remain unchanged throughout the transition. The decline in the demand for loans, however, implies that banks would an excess of loanable funds. Hence, in equilibrium, the price for loans must increase to clear the market, leading to a decline in total issuances.

How does the economy converge to the new steady state? Because the demand shock generate low portfolio returns, banks increase dividend rates and experience a reduction in equity. Moreover, as equity is reduced along the transition, loan prices fall leading to increasing portfolio returns until the steady state. Finally, an important observation is that there is on impact a sharp increase in the reserve rate and in the liquidity ratio. Intuitively, banks respond to the decline in credit demand by reallocating their portfolio towards cash and away from loans.

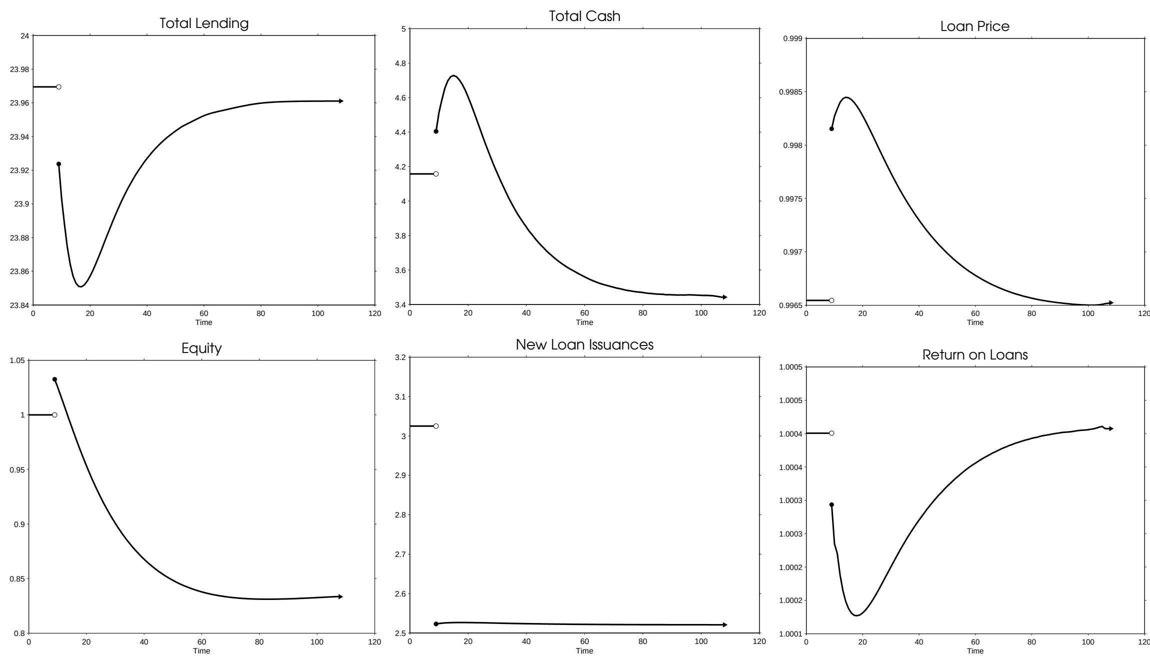

Figure 13: Credit Demand Shock

7.4 Rise in Inter-Bank Market Rates

Our next experiment is to consider shocks that affect directly the flow of liquidity through the inter-bank markets. This correspond in our model to an increase in χ. Figure 14 illustrates the effects of this shock.

The increase in χ produces a steady state with lower loan prices. The reason behind this result is that banks respond to a more dysfunctional inter-bank markets, by cutting lending rates and holding more cash. The decline in demand for loans requires a lower steady state value of the price for loans to restore equilibrium. The new steady state exhibits also a lower portfolio value and lower dividend rates.

As Figure **??** shows, the transitional dynamics exhibit an overshooting in the price of loans, as it was the case with the shock to the capital requirements. Following the same intuition, suppose that loan prices fall on impact to the steady state value. At those prices, the short run supply of loans would be below the long-run supply of loans, given that the level of initial equity is below the steady state value and the fact that lending ratios would be the same as well. Hence, to restore market clearing, the short run drop in loan prices has to be higher than the long-run decline in loan prices. Over time, high portfolio returns and low dividend rates result in equity growth, which eventually stabilizes the increase in loan prices.

Notice also that the overshooting in the price for loans mitigate the decline in lending ratios and moderate the increase in the liquidity ratio on impact. As equity and loan prices grow, banks reallocate their portfolio towards lower loans and more cash. In the new steady state, however, the direct effect of higher χ prevails and banks face higher liquidity risk.

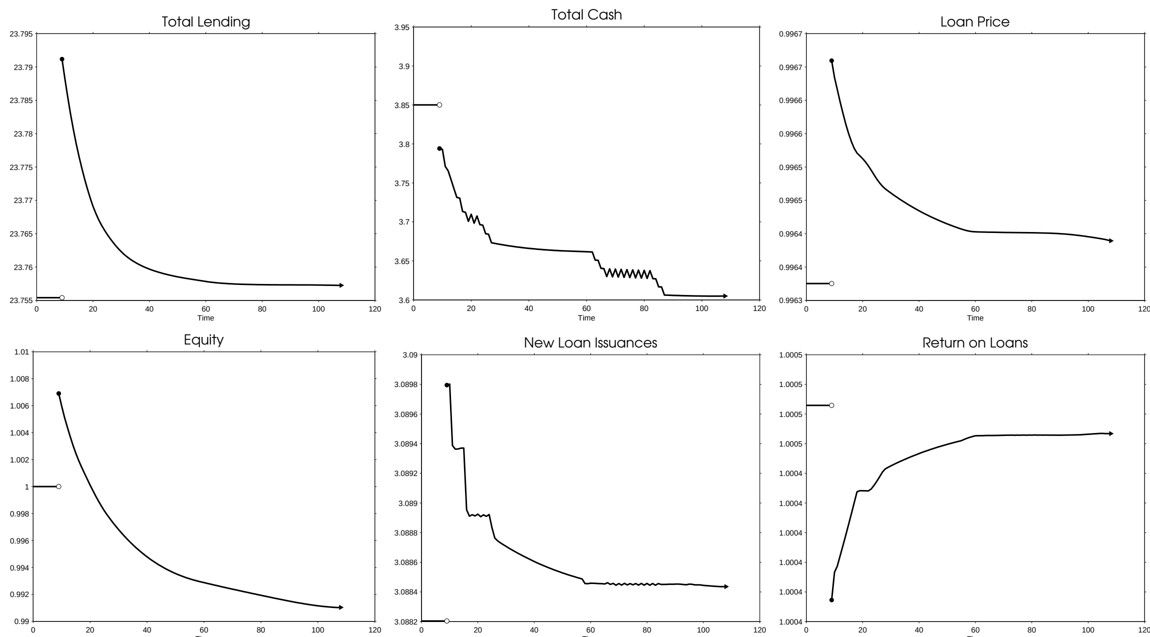

Figure 14: Rise in Inter-Bank Market Rates

7.5 Funding Liquidity Risk

The last shock we consider is an increase in funding liquidity risk. In particular, we analyze the response of the economy to an increase in the standard deviation of withdrawal shocks of 15 percent.[14]

An increase in funding liquidity risk generates a more precautionary behavior by banks. In particular, banks perceive lending to be more expensive because there is a higher risk

[14]For this experiment, we approximate the withdrawal shock using a symmetric beta distribution with the same volatility as the calibrated distribution and consider an increase in the standard deviation of this distribution.

that withdrawals would generate losses due to insufficient cash. As a result, banks restrict lending and cash holdings, as illustrated in Figure 28. The new steady features a higher return on loans, i.e. a lower loan price, to accommodate the decline in loan supply.

In the transition, there is again an overshooting in loan prices, as the short-run decline in loan supply is higher than the long-run decline. The increase in the volatility of withdrawals also generate overshooting in liquidity risk. On impact, there is a sharp increase in liquidity risk, but as the return on loans drop over time, banks also hoard more cash stabilizing in this way liquidity risk.

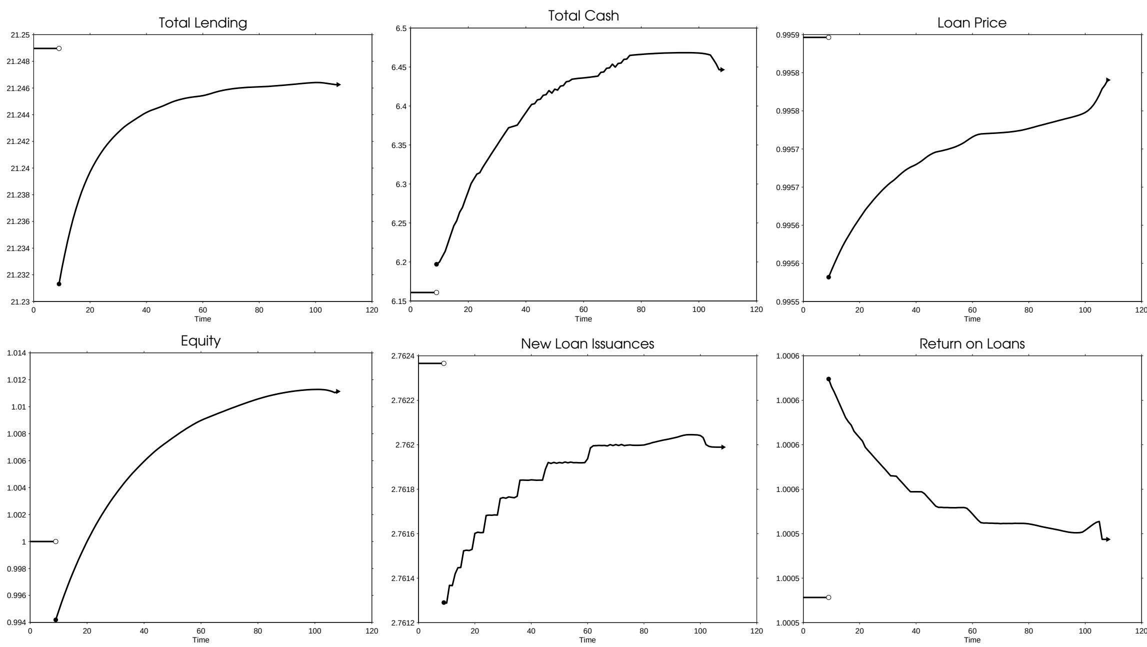

Figure 15: Shock to Funding Liquidity Risk

8 Monetary Policy during 2008-2013

8.1 Some Facts

We summarize widespread concerns about the ability of monetary policy to stimulate the economy via the following graphs.

Fact 1: Anomalous Interest Rate Behavior. Figure ?? shows three series. The volatile time series corresponds to the daily FED funds rate. The FED funds rate fluctuates around the corridor determined by the interest on the overnight rate and the borrowing rate, the policy instruments used by the FED to implement its policy. These are the analogue to χ_t in our model. The figure shows a reduction in these rates. The dashed line corresponds to the 11-month, Libor rate, the analogue of r_t in our model.

We can see, a very clear pattern. Prior to the Great Recession, the 11-month Libor rate used to track the FED Funds rate with a slight spread. This spread wideness during the Great Recession and has remained constantly higher. During the crisis (see zoom), the FED Funds market actually leaves the bands, a symptom that the FED lost control over its policy instruments. The fact that the 11-month Libor rate spread is despite the stimulus, perhaps banks are more cautious than before of lending to other banks.

Fact 2: Anomalous FED Balance-Sheet. Figure 17 shows the anomalous behavior of the Federal Reserve (FED) 's Balance Sheet. The picture shows the vast amount of assets on hands of the FED which correspond to all the open market operations carried out during the period that are held post recession. The corollary of this figure is the increase in FED liabilities. The various FED programs that explain this increase in the FED's balance sheet are documented by **Adrian et al**.

Fact 3: Excess Reserve Holdings. .

Figure 18 shows that the increment in excess reserves of the banking system. The counterpart to the increment in FED assets in Figure 17 are increases in Federal Reserve (FED) 's liabilities. Figure 18 shows an increment in excess reserves (and a slight increment in required reserves). This means that the monetary expansion has not been met with an equivalent expansion in lending by commercial bank institutions.

Fact 4: Depressed Lending Activities. .

Despite the increment in liquidity (FED funds) in the financial sector, lending actually contracted during the Great Recession. This is shown in the top left panel of Figure 19. This shows that the monetary expansion of the last 4 years has not been met with a counterpart in lending activities. In turn, bank liabilities have actually fallen also as seen from the top right panel. This is because banks create liabilities via lending. Despite the large amount of lending facilities, bank liabilities fall due to the strong effects on lending. A final corollary of this observations is the drop in the M1, money multiplier. This measure captures the above description.

Fact 5: Equity Losses.. Facts 1-4 are associated with a period of unprecedented equity losses for banks. Figure **??** shows the fall in market value of bank equity during the period. This fact should serve as a link between facts 1-4 and something that occurred in the banking sector. This list of facts opens some questions that we try to answer with our framework.

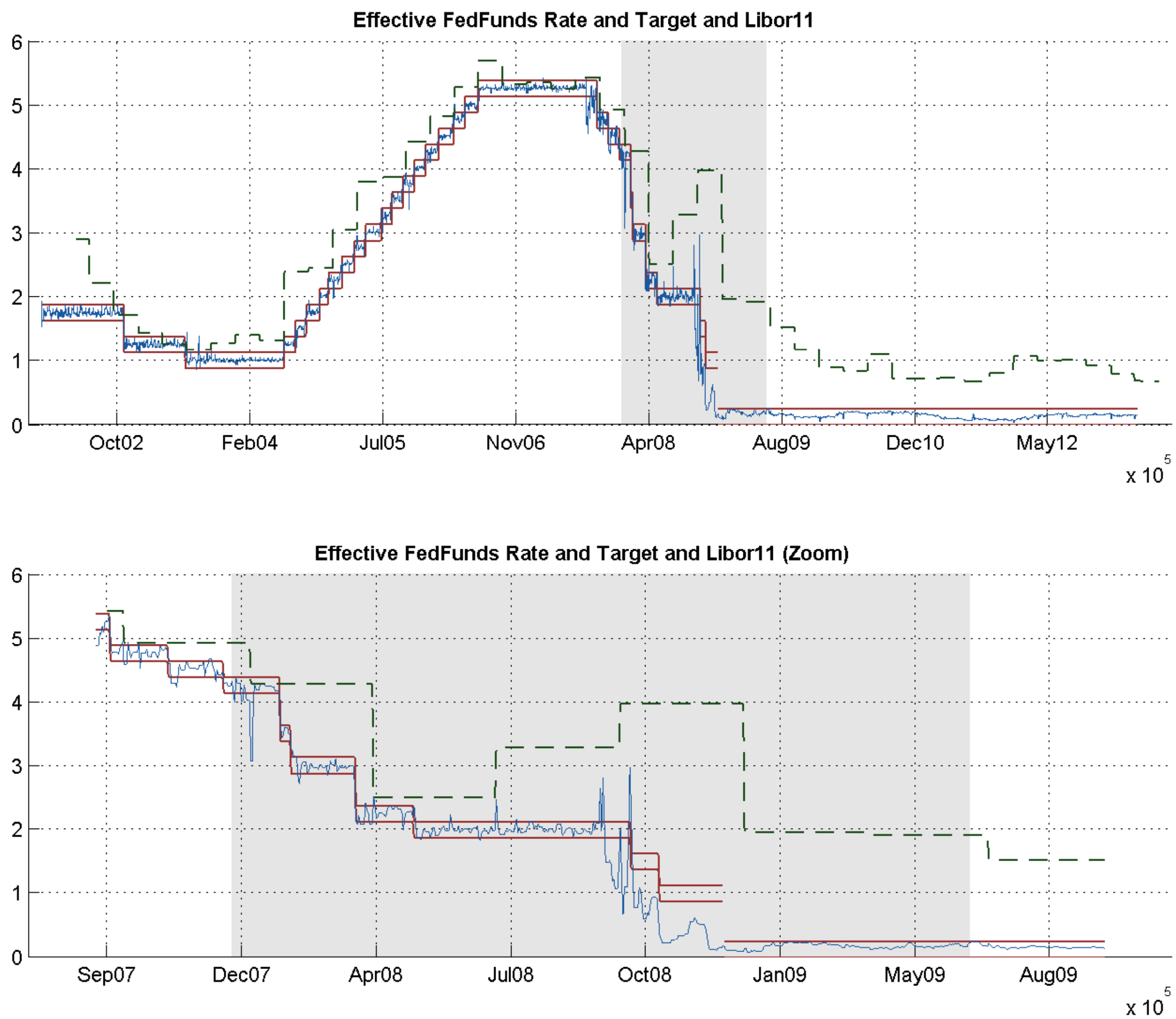

Figure 16: **Fed Funds Rate 2002-2012.** The figures plot the evolution of the FED Funds rate, the 11-month Libor rate and the overnight lending and borrowing rates.

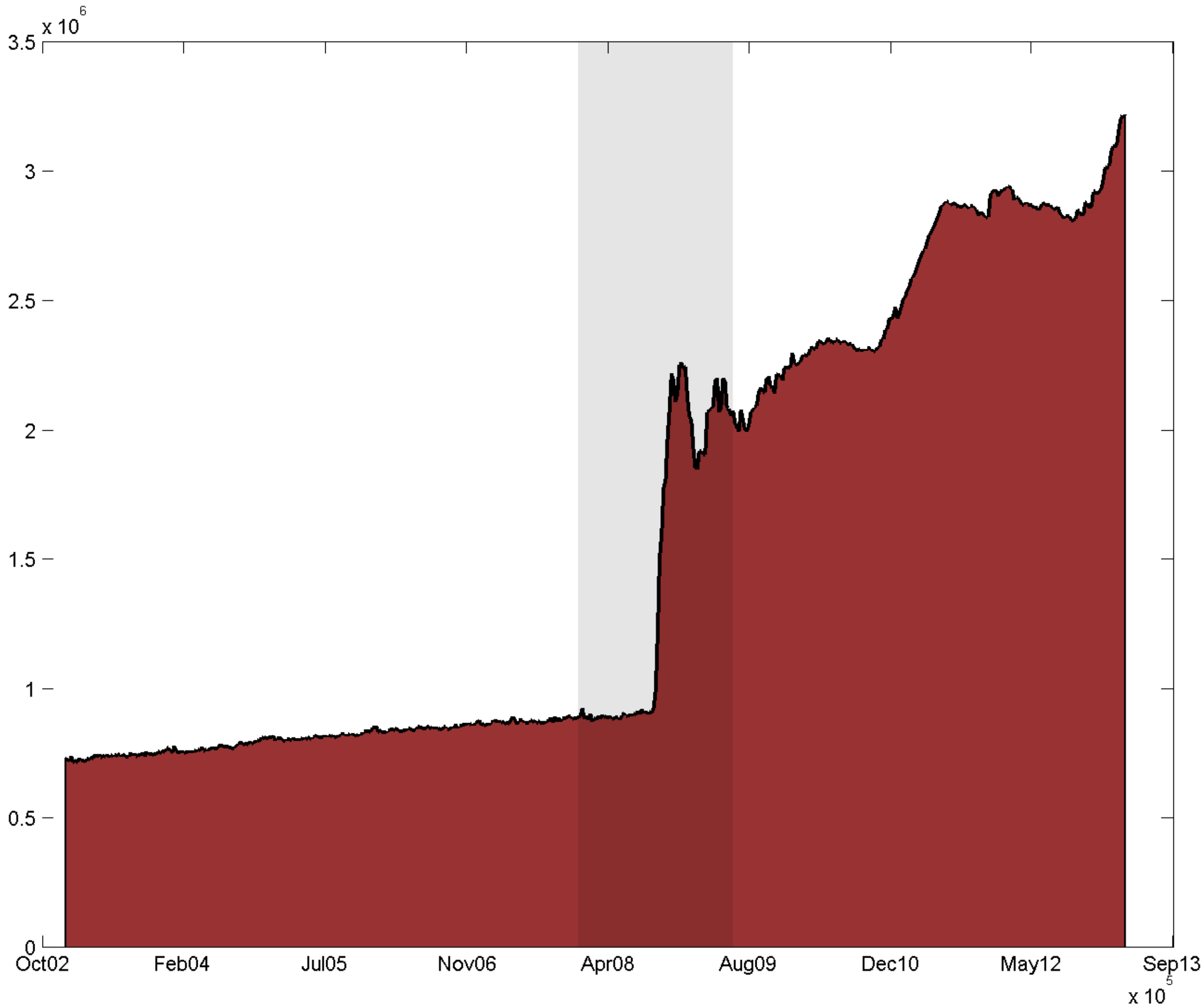

Figure 17: **Federal Reserve Assets: 2002-2012.** The figure shows the expansion in the FED's asset holdings. The magnitudes are in Millions of US$.

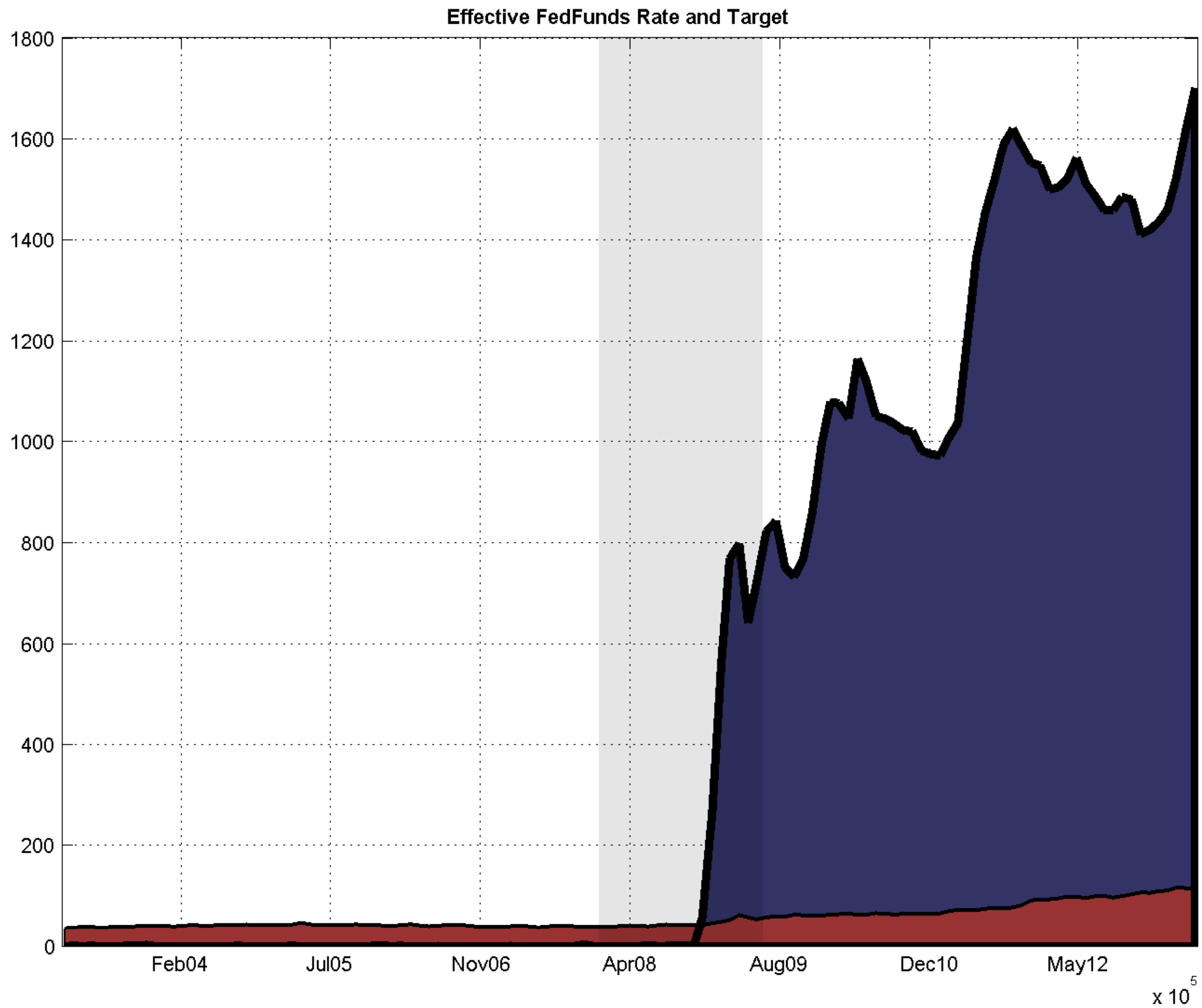

Figure 18: **Federal Reserve Assets: 2002-2012.** The figure shows the evolution of the Commercial Bank excess (blue) and required (red) reserve holding.

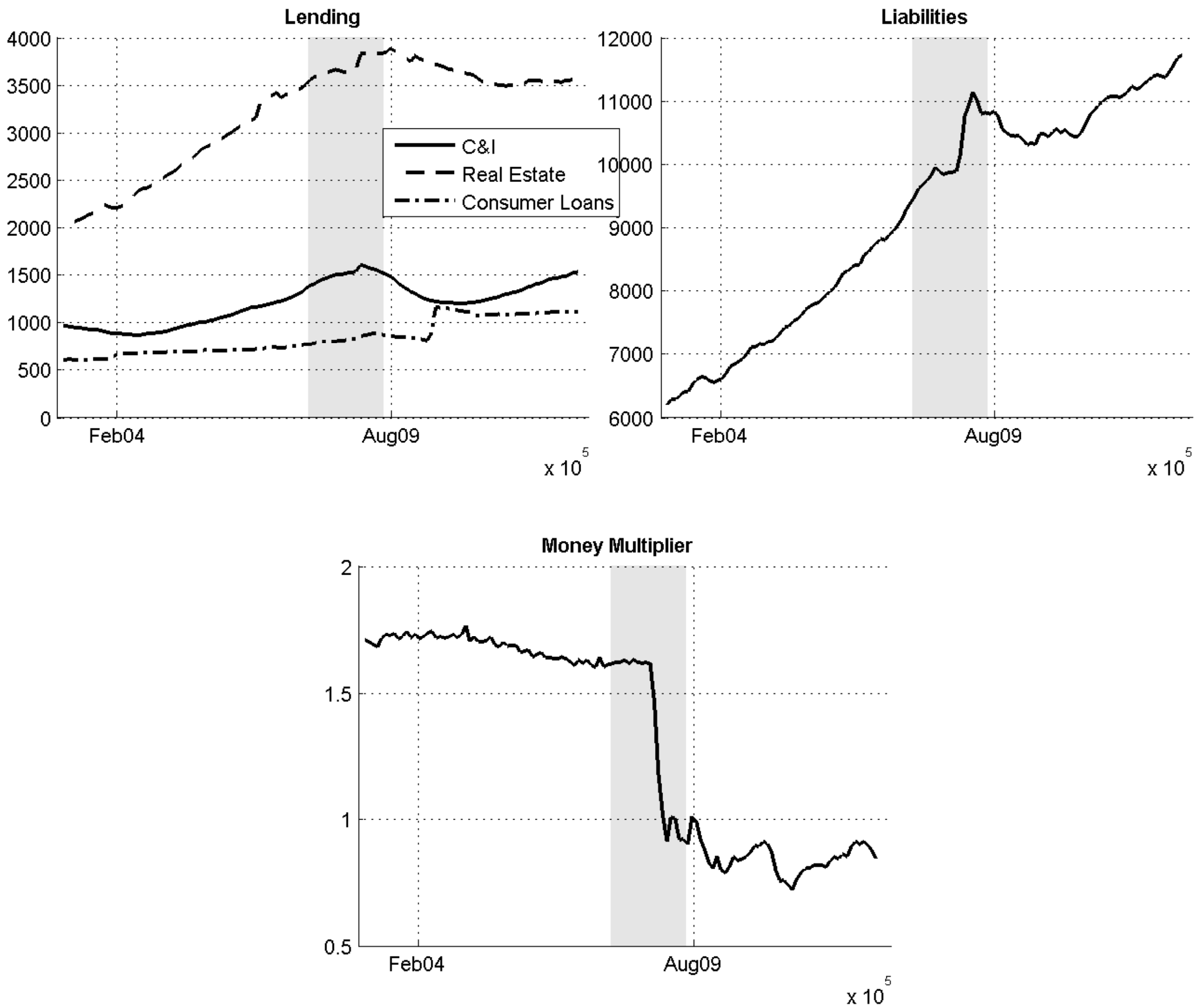

Figure 19: **Commercial Bank Assets: 2002-2012.** The figure shows several measures of commercial bank lending.

8.2 Some Questions

We use the model as a laboratory to answer whether monetary policy has been interrupted after the Great Recession. The model's answers will be based on the assumptions that we make for the real sector. In other words, they are conditional on having monetary policy working only through the lending channel. To answer these questions, we will again place the model within its perfect foresight environment. We assume all policy the path for interest rates is well defined.

This first questions concerns the equity losses experienced by banks. Bank equity losses will work by a risk aversion effect. Upon suffering unexpected losses, banks will reduce the amount of new lending relative to their equity. Thus it is natural to ask, can bank equity losses, of the magnitude of the Great Recession can render monetary policy inoperative?

An immediate policy response was to implement a sequence of new bank regulations via the Basel-III agreements. Basel-III tightens κ and η in our model. Hence, a direct policy implication is the tightening of bank leverage and the improvement of bank ratios. An indirect effect occurs via affecting bank incentives to pay dividends. Lower leverage will decrease bank returns and hence, induce a smaller banking sector. Can bank tighter bank regulation interrupt the lending channel?

During the Great Recession we saw that the FED funds rate was out of control for a small period. Moreover, the 11-month Libor spread is above its average historical level. Can greater withdrawal volatility explain this fact?

The fourth question is, can a reduction in loan demand explain the post recession patterns?

8.3 Model Answers: A Narrative of the last 5 Years

Building on the analysis from the previous section, we now explore if there are plausible shocks within our model that can lead to a severe contraction in the financial system similar to the one experienced recently in the US economy. In particular, we construct a counterfactual experiment where we hit the economy with various shocks, one at a time, in a cumulative fashion. The first part of the experiment consists of 3 shocks. First, we consider a shock to equity losses equivalent to 0.2 percent of total assets. The magnitude of this shock corresponds to the unexpected losses of AAA-rated subprime MBS tranches, estimated by Park (2011). Second, we consider a tightening of capital requirements along the new prescriptions of Basel III, by which Tier 1 Capital ratio increases from 4 percent to 6 percent over the course of 6 years. Third, we consider the introduction of a probability of a bank-run, which we take for now be 3 percent. Specifically, what we do is introduce a small probability event of banks losing the entire stock of deposits. This shock lasts for 1 year.[15] Finally, we consider a temporary demand shock to match an initial drop in loan issuances of 25 percent, in line with Ivashina and Scharfstein (2010). This shock is assumed to last for 6 years in order to capture the duration of the Great Recession.

Figure **??** shows the results of the counterfactual experiment. The thick broken line represent the economy which is hit only by equity losses. The straight line represents the economy hit by equity losses and by tighter capital requirements. Finally, the dashed line represents the economy hit by the these two shocks plus the increase in withdrawal risk. Notice that the first shock has only temporary effects while the second and third have permanent effects since the change in capital requirements is permanent.

The results suggest that a combination of shocks is essential to account for the dynamics of the US financial crisis. The drop in equity by itself is not enough to produce a large drop in new loan issuances. Moreover, the drop in equity results in a counterfactual decrease in cash holdings. Similarly, the tightening of capital requirements, contribute to a deepening of the drop in credit, but also leads banks to hold less cash, not more. On the other hand, the increase in withdrawal risk has the potential to increase cash holdings, unlike the shocks to equity losses and the tightening of capital requirements. As can be seen from Figure **??**, the increase in withdrawal risk generates significant increase in liquidity risk, but general equilibrium effects mitigate the effects over total reserves.

At this point, these shocks do not seem to be able to explain quantitatively the increase in liquidity holdings by banks or the decline in lending. Our preliminary investigations, however, suggest that large credit demand shocks are consistent with the increase in reserves and the decline in lending observed in the data. Interestingly, demand shocks also lead to increase in equity payouts, which suggest that credit demand shocks alone cannot explain the key features of the financial crisis.

[15] To keep the mean fixed, we also introduce the possibility of a large increase in deposits with the same probability.

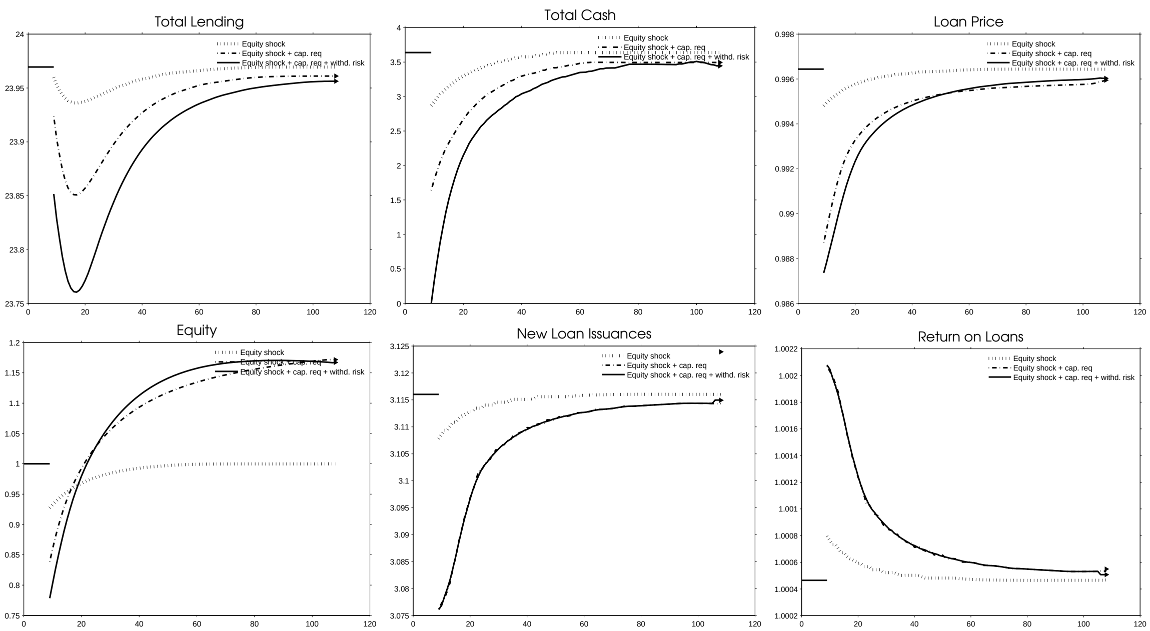

Figure 20: Financial Crisis

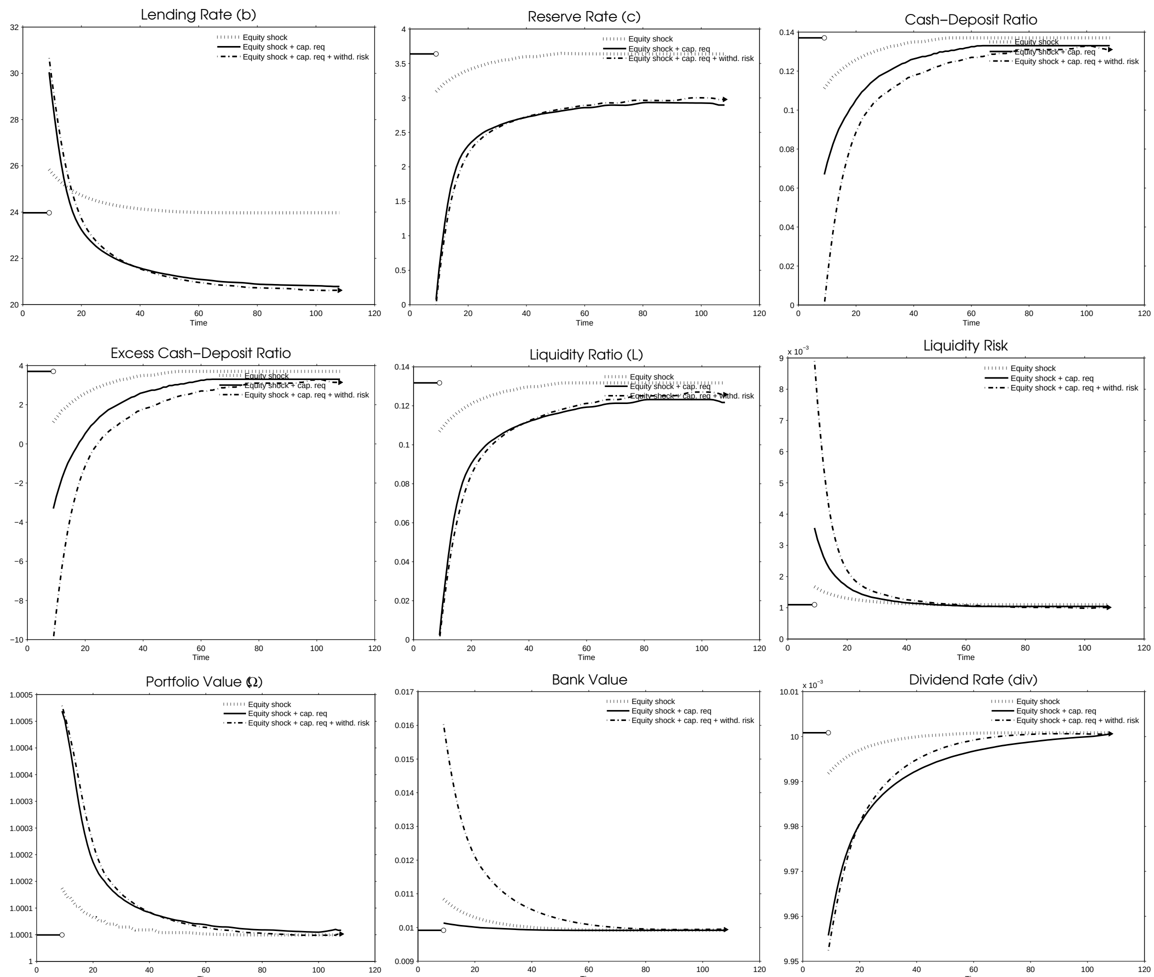

Figure 21: Financial Crisis

8.4 Effectiveness of Monetary Policy

INCOMPLETE

In this section, we study the effectiveness of monetary policy to stabilize financial markets. In particular, we consider how a temporary reduction in χ can offset the impact of a series of adverse shocks to financial markets studied above. The key question we address is whether monetary policy is more or less effective in times of distress.

8.5 Potential Extensions

Before concluding, there are several extensions that we want to discuss. These extensions will allow us a more thorough understanding of monetary transmission through this model.

Nominal contracts and other rigidities. We deliberately imposed assumptions in our model so that lending contracts are real. Our model can be enriched to incorporate features that may induce changes in prices, without corresponding movements in contracts. The introduction of nominal debt contracts and price movements may help us understand the role of Fisherian debt-deflation phenomena (see Krugman and Eggertson, for example.). Another form of nominal contracts can be introduced onto the labor market. This form of nominal friction, may help us evaluate the role of the interest rate channel.

Credit- Risk uncertainty. Although we have studied the effects of equity, these losses are unforseen. We believe our model may have interesting effects if there are periods where banks are aware, but uncertain, about potential equity losses.

A market for T-Bills. For tractability, we assumed that bank assets are either loans or reserves. Thus, the model neglects a role for Treasury Bills which are a very important part of bank balance sheets and are the main instrument for open market operations. Treasuries are a special type of asset. Unlike other forms of bank loans, T-Bills are highly liquid and homogeneous assets. Thus, they can play a role as a liquidity risk hedging instrument. However, they are not perfect substitutes for bank reserves for regulatory reasons and because they cannot be converted to currency immediately. Our model can be extended very easily to incorporate this assets. This analysis may help us understand the distinctions between conventional and unconventional policies through the lens of our model. Moreover, it can serve the purpose of allowing us to understand how changes in the aggregate supply of treasury bills has effects in conjunction with the monetary instruments we study in this paper.

Credit rationing. In our model, there's no feedback between lending and default probabilities. We believe our model may extended in this dimension allowing firms to suffer a form of credit rationing. One possibility is to introduce a problem of asymmetric information as in Bigio (2010). Alternatively, given the long-maturity of lending, our model can perhaps be used to explain how the lack of new loans affects the probability of older loans to payback the bank. This would stem from a combination of nominal contracts and long-term lending.

Fire sales and forced loan sales. We have assumed also that loans cannot be sold during the lending stage. In practice, banks may be able to sell loans in periods of stress suffering penalties. Allowing some flexibility in loans sales may allow us to understand the role of fire sales in this model. Fire sales may relax the illiquidity issues, but as stressed in work of Diamond and Rajan, they may lead to possible pecuniary externalities by lowering the market value of other banks assets and further tightening lending constraints. A richer model would introduce a concrete friction that would lead to an elastic demand for these assets.

System wide runs. Our model allows for withdrawals from one bank to another. However, the history the Great Depression was characterized by system wide withdrawals. Our model can be easily adapted to incorporate this feature in a way similar to the way done in Gertler and Kiyotaki (2012).

Credit rationing. In our model, there's no feedback between lending and default probabilities. We believe our model may extended in this dimension allowing firms to suffer a form of credit rationing. One possibility is to introduce a problem of asymmetric information as in Bigio (2010). Alternatively, given the long-maturity of lending, our model can perhaps be used to explain how the lack of new loans affects the probability of older loans to payback the bank. This would stem from a combination of nominal contracts and long-term lending.

9 Conclusions

The bulk of money-macro models has developed independently from the insights from literature on banking (Diamond and Dybvig (1983)). In doing so, the profession has lacked an explicit modeling of monetary policy through the financial system, and for good reasons. For a long time it did not seem to make much difference, monetary policy seemed to be carried with ease and with stable banks, equity growth, leverage, bank dividends and interest premia. Thus, in absence of crises, the activities of the financial sector can appear irrelevant for long stretches of time. The crisis, revealed that having a model that allows us to study banks in conjunction with banking may be a desired tool in our theoretical toolbox.

This paper has borrowed from the banking literature, to introduce a model where by maturity mismatch between assets and liabilities cause banks to demand reserves. A Central Bank can influence real activity by altering the trade-off in bank lending via various inputs. This model sheds light on what type of shocks may reduce the power of monetary policy. We have used this model to understand the effects of various shocks on the power of monetary policy.

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10 Appendix

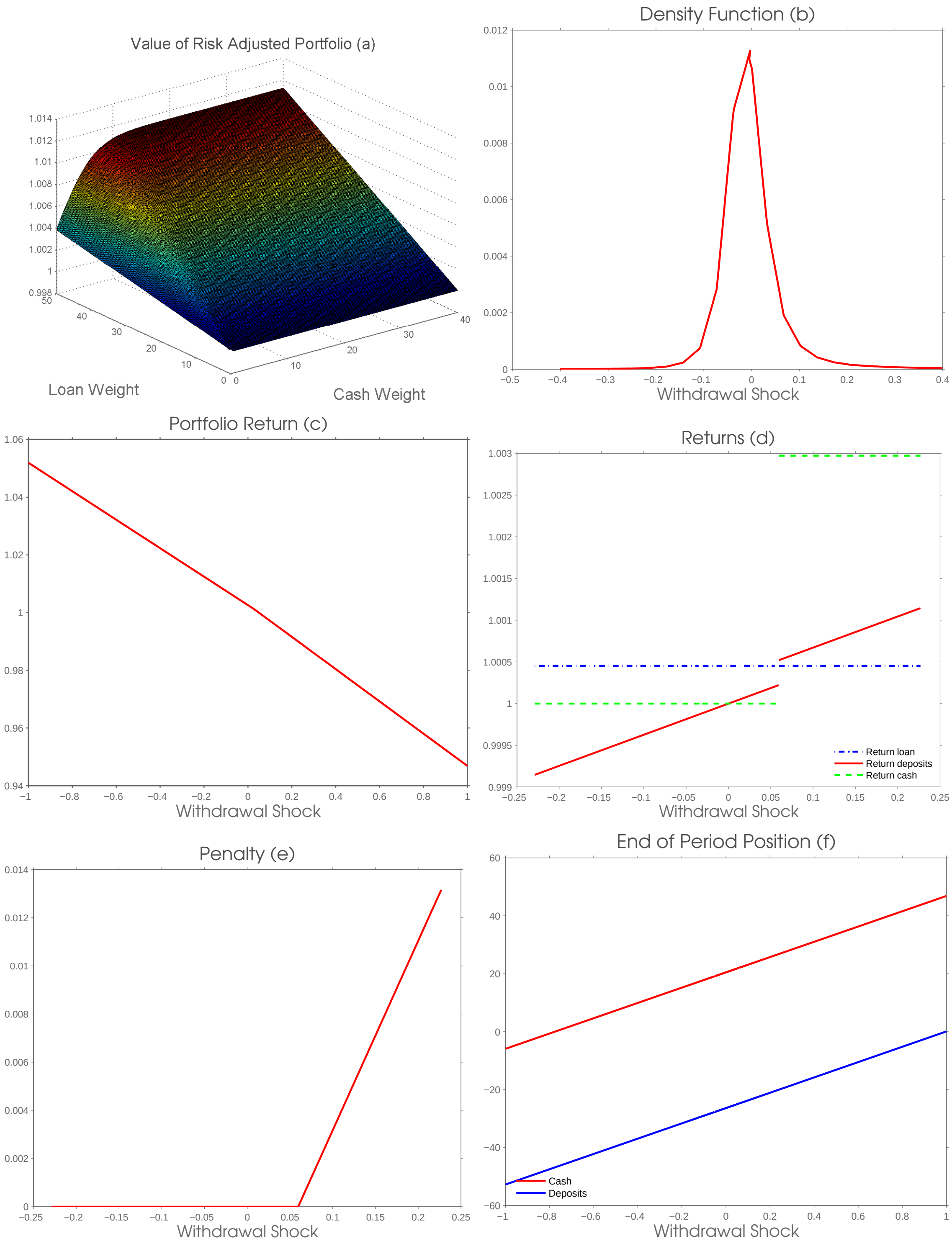

Figure 22: Portfolio Choices and Effects of Withdrawal Shocks

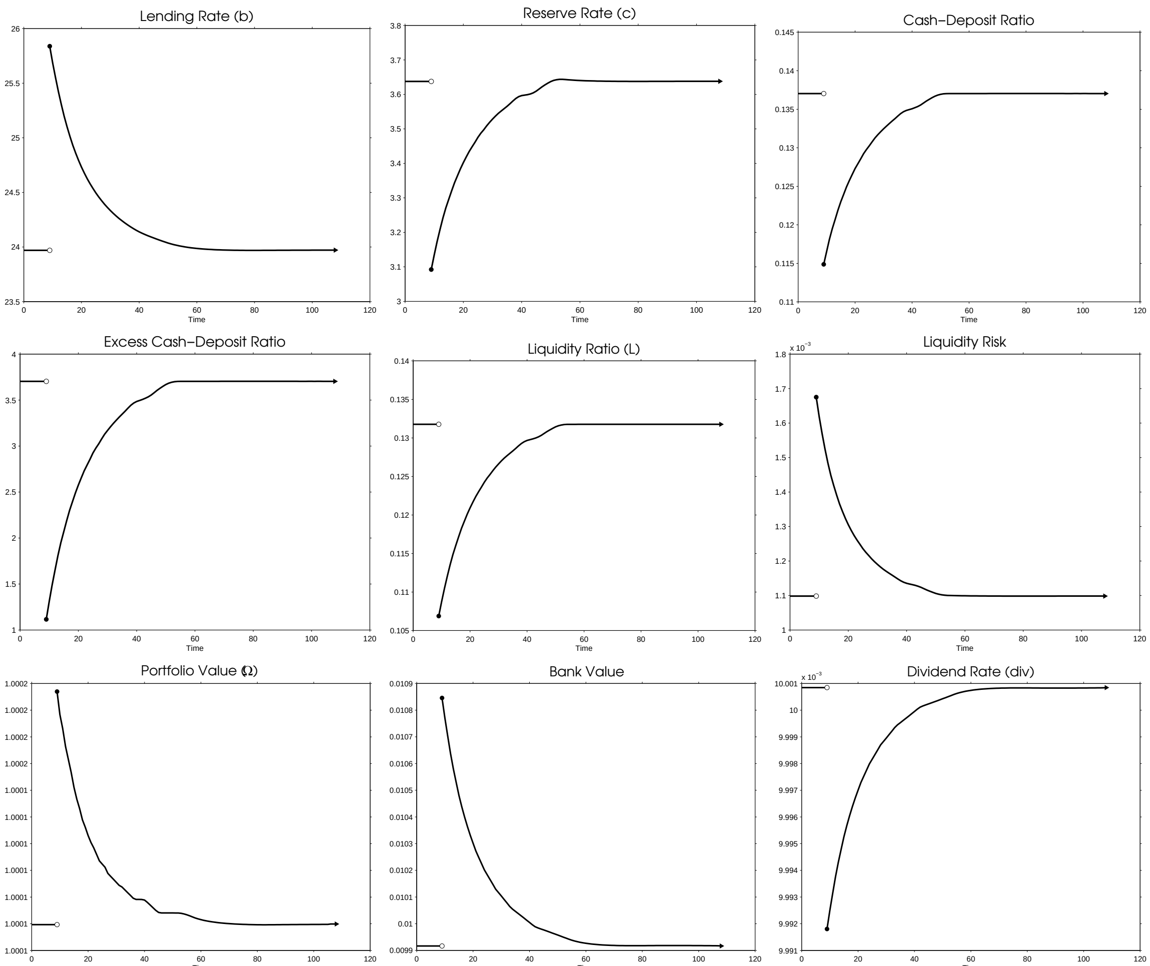

Figure 23: Equity Losses

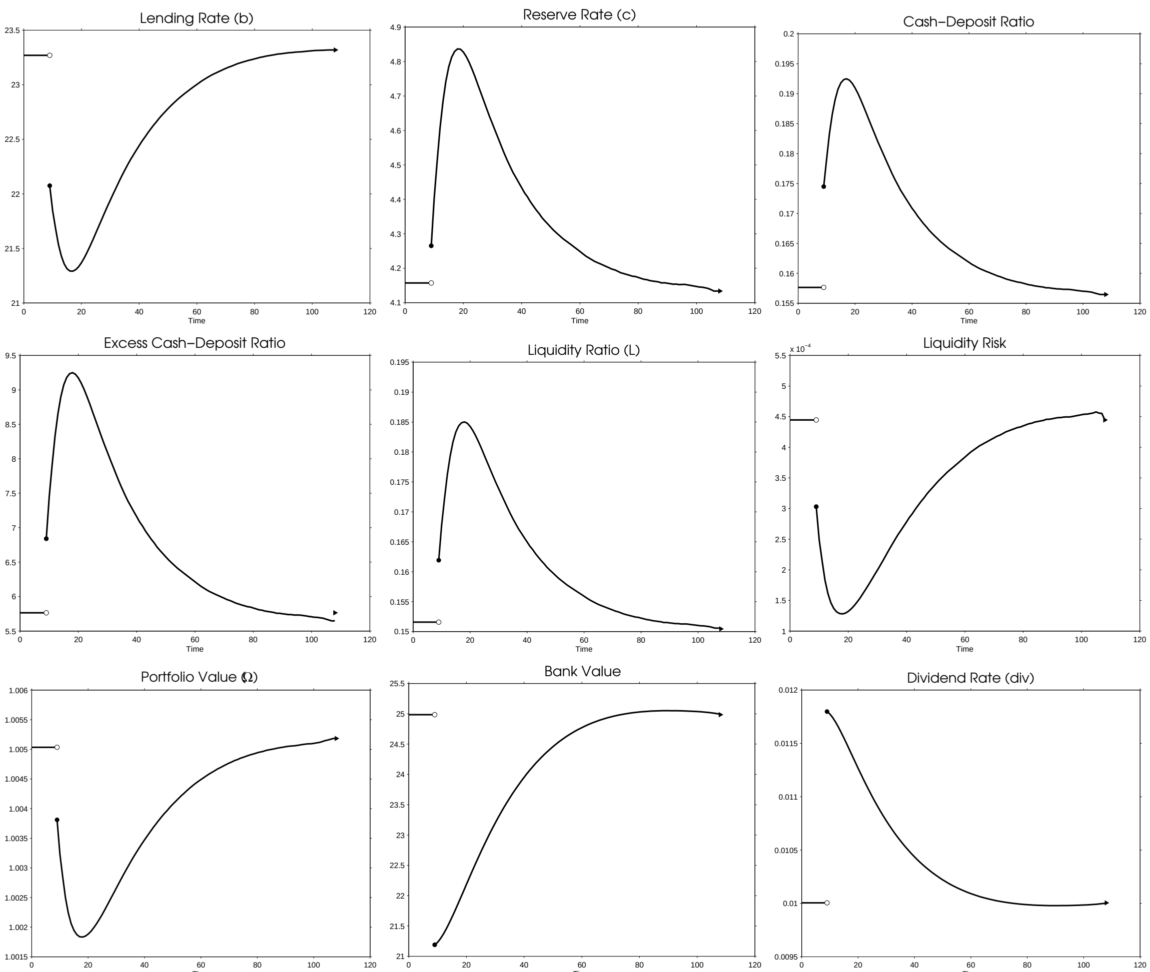

Figure 24: Demand Shock

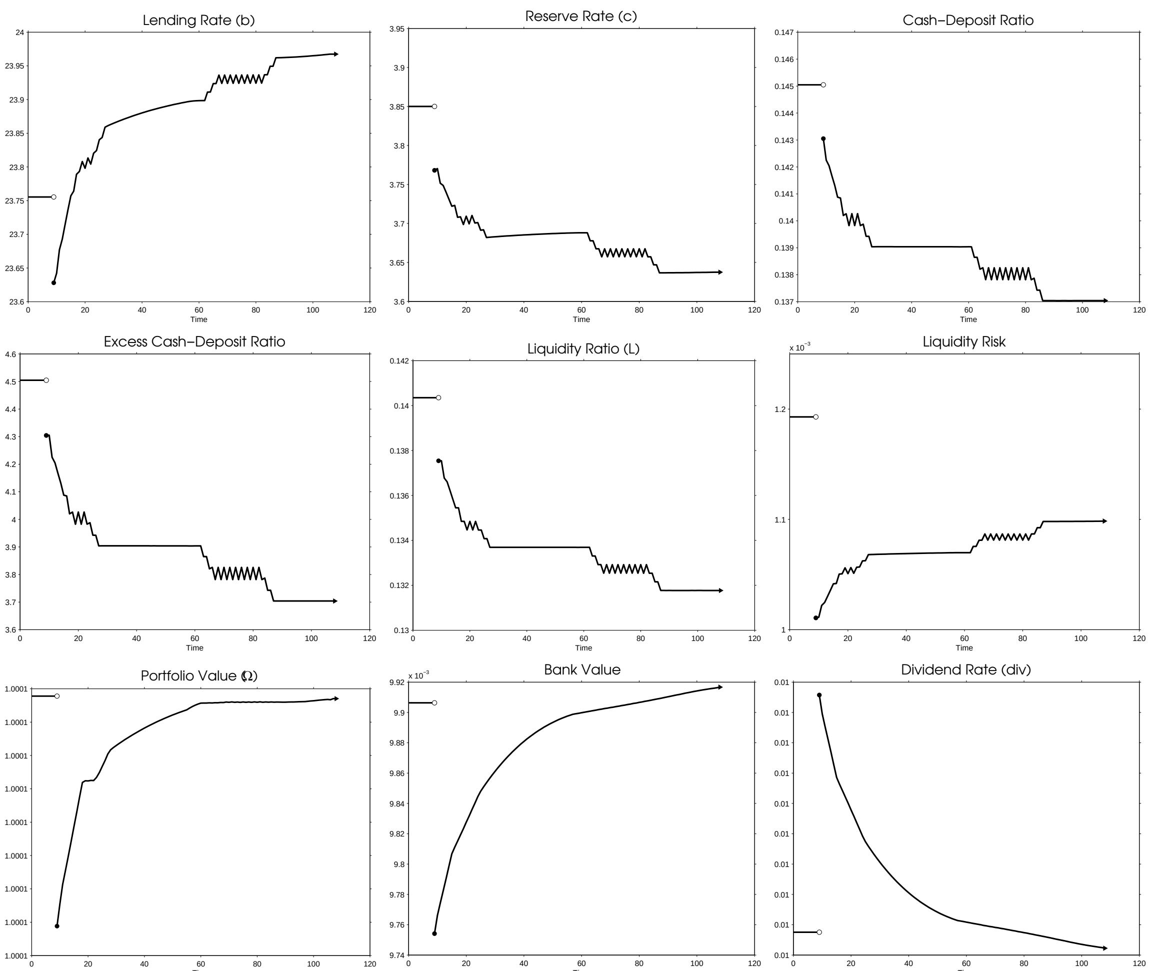

Figure 25: Rise in Inter-Bank Market Rates

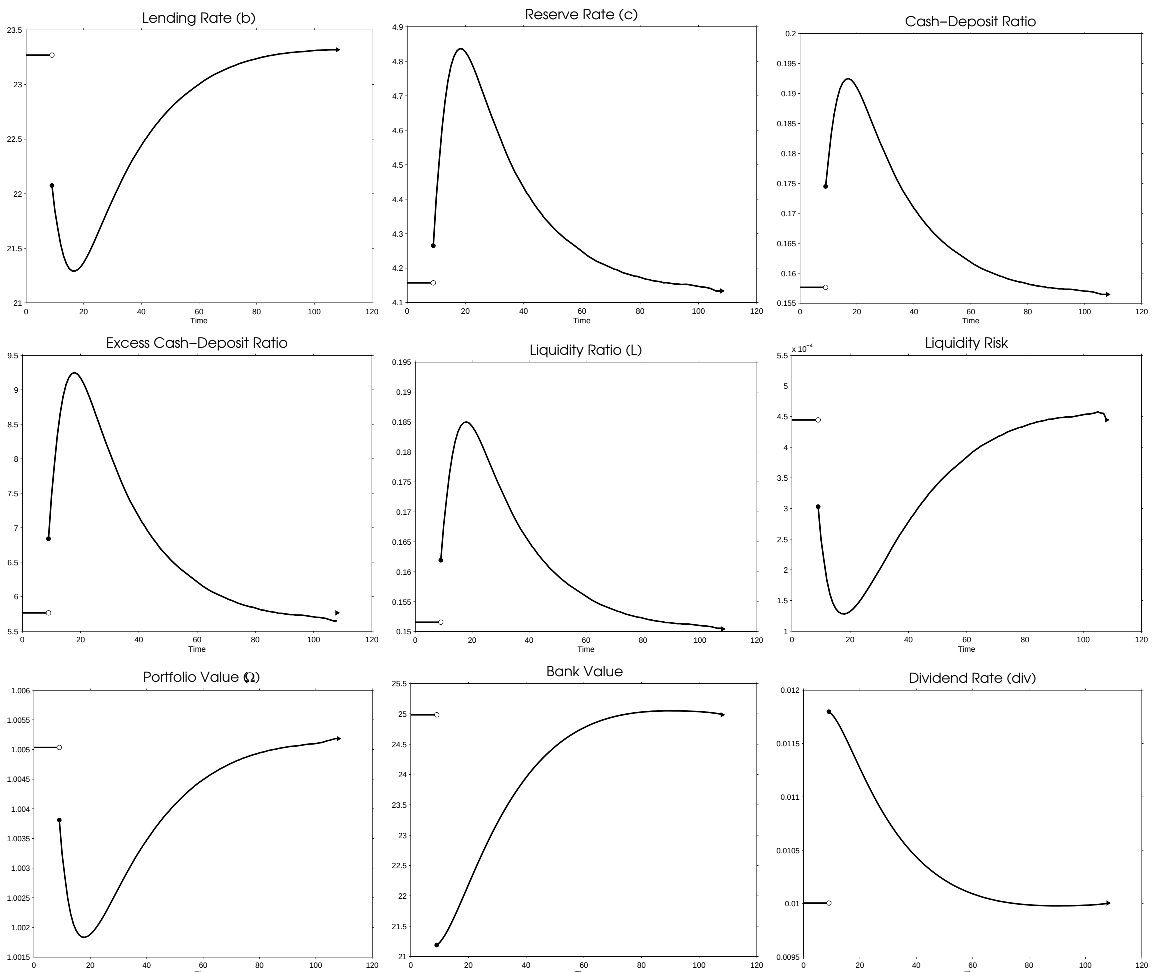

Figure 26: Loan Demand Shock

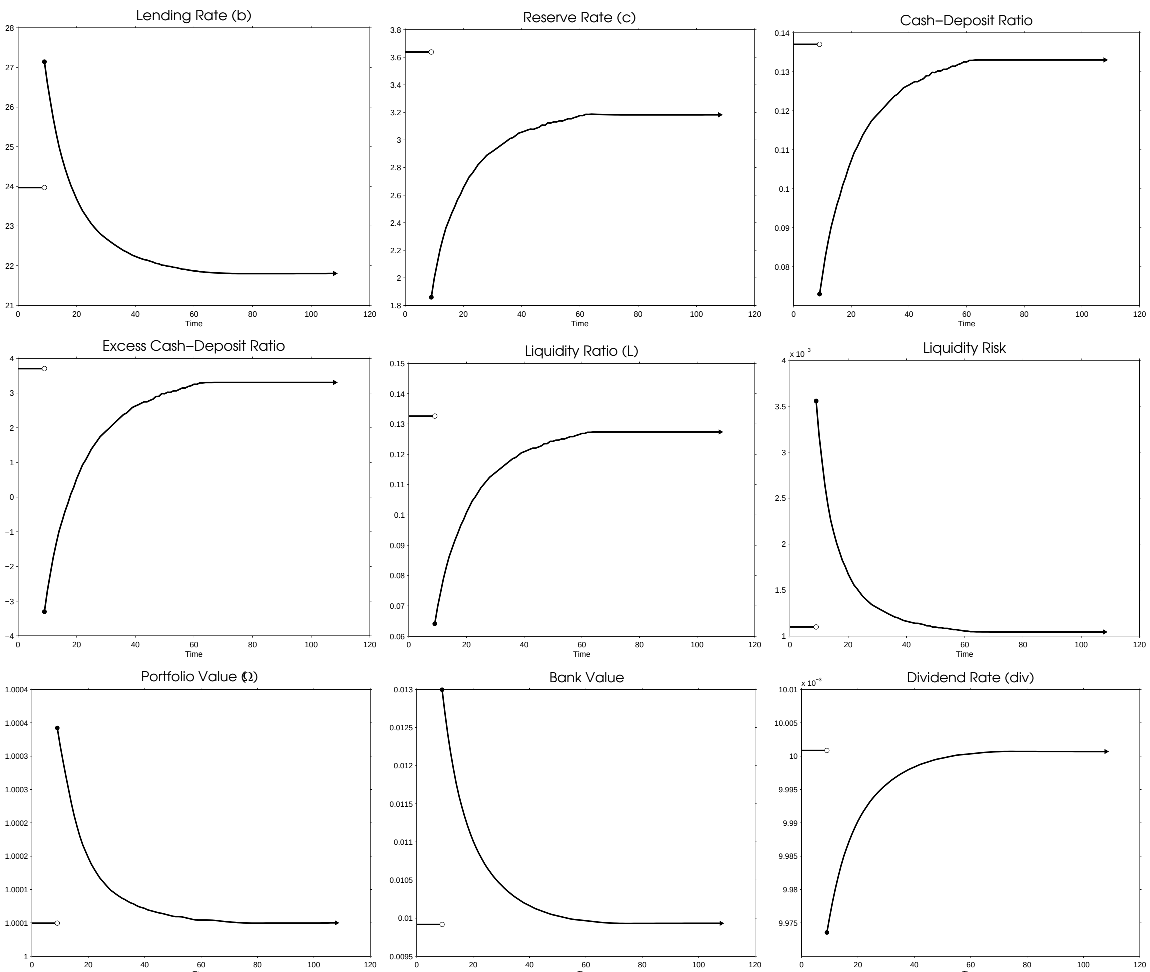

Figure 27: Tightening of Capital Requirements

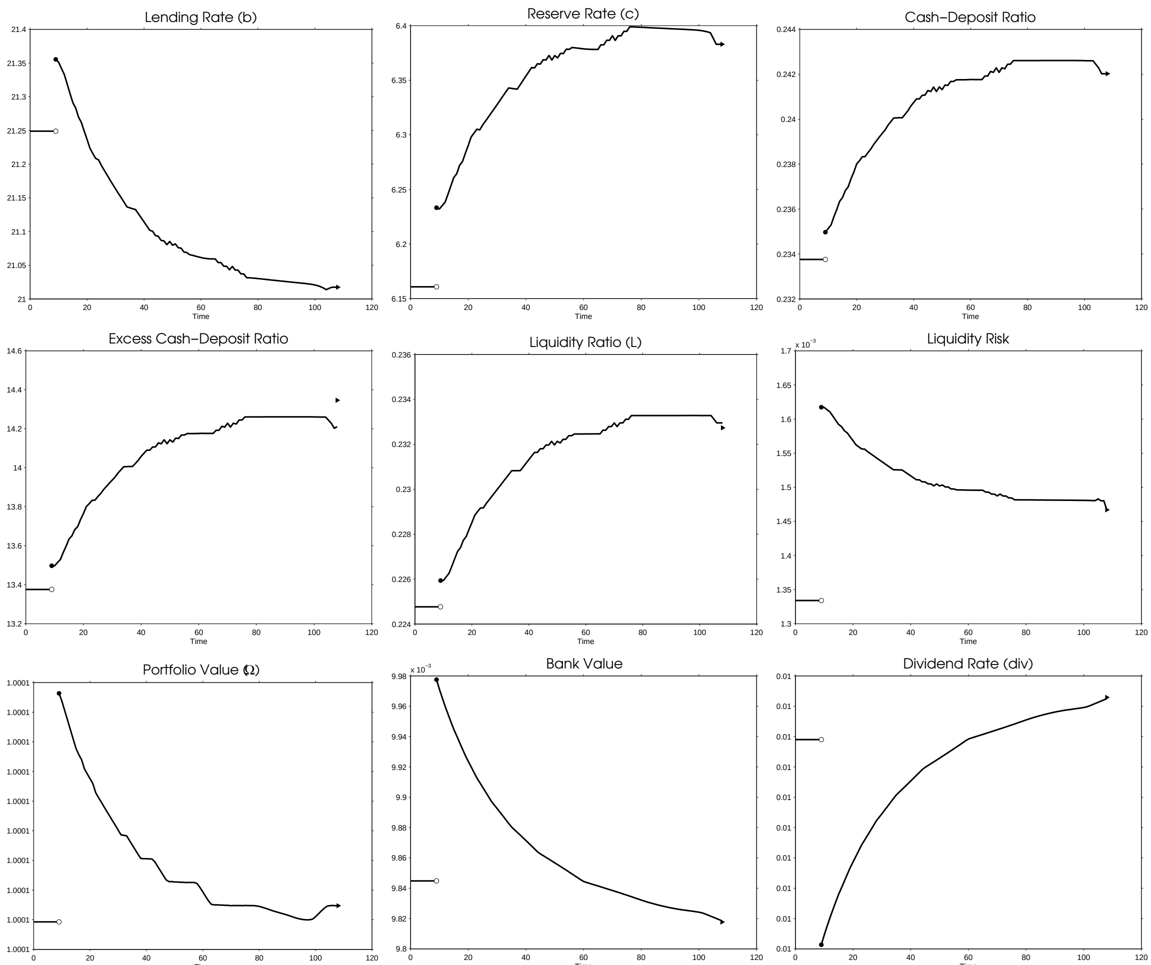

Figure 28: Rise in Liquidity Uncertainty

10.1 The Real Side - Closing the Model

We now turn to the real side of the model which is deliberately stylized. The idea is to present an environment where there is a real demand for loans, without complicating the analysis much. We make assumptions such that lending will have real effects despite that the model has no nominal rigidities. Thus, monetary policy carries out real effects in a fully real environment. In terms of the model, the goal here is to obtain a demand for loans like the one in (6).

Thus, the economy is composed of an discrete infinite amount of islands, indexed by τ. Each island is populated by workers and entrepreneurs. Islands are discovered in period τ.

Workers. Workers choose consumption, labor and deposit holdings. Workers don't have access to any additional savings technologies. Their period utility is given by:

$$\max_{\{c_t\}_{t\geq 0}\geq 0, h_0 \geq 0} \sum_{t=0}^{\infty} c_t - \frac{(h_t)^{1+\nu}}{(1+\nu)}$$

where h_0 is the labor supply and c consumption and ν is the inverse of the Frisch elasticity. The worker's budget constraint in period t is given by:

$$d^w_{t+1} + p_t c_t = d^w_t + w_t h_t$$

where d^w_{t+1} is the deposits held by the worker in period t, w_t is the wage and p_t the price of the good. It is the case that $d^w_t = 0$, for all $t \leq \tau$. Workers only work during the period of discovery τ, so without loss of generality $h_t = 0$ for $t \neq \tau$. The only role for workers is to provide the elastic labor-supply schedule. The only reason why they are long-lived is to allow us to talk about maturity in this model. Since deposits must be cleared out slowly, household's are long-lived. Also, notice that household's don't discount consumption over time. This is enough to argue that they are indifferent between consumption in any period so long as the price is constant. This allows us to eliminate any form of price variation in the model together with zero interest on reserves.[16]

Problem 4 (Workers) *The worker solves:*

$$U^w_t(\tau, d^w_t) = \max_{c_t, h_t} c_t - \frac{(h_t)^{1+\nu}}{(1+\nu)} + U^w_{t+1}(\tau, d^w_{t+1})$$

subject to

$$d^w_{t+1} + p_t c_t = d^w_t + w_t h_t$$

[16] The model could be modified to allow workers to save. With GHH preferences, this would not alter the labor supply. However, the model would require and additional state variable but the substance of the model would not change. GHH are commonly used to prevent counterfactual contractions in the labor supply.

with $d_0^w = 0$*, and* $h_t = 0$*, for* $t \neq \tau$*.*

Entrepreneurs. Entrepreneurs have utility

$$\max_{\{c_t\}_{t\geq 0}\geq 0} \sum_{t=0}^{\infty} c_t$$

They start their lives with a capital stock of k_t. In addition, the have access to a loan market. Their budget constraint in period t is given by:

$$p_t c_t = r_t^e k_t.$$

Production Technology. Production is carried out via k, combined with labor, h, using a Cobb-Douglas technology $F(k,h) \equiv k^{\alpha} h^{(1-\alpha)}$ to produce output. Profits are $AF(k,h) - wh$. Workers are hired from an elastic supply schedule, $w = (h^w)^{\nu}$ determined from the worker's problem.

Before production, an entrepreneur hires an amount of labor promising to pay wl. It is possible that the entrepreneur reneges on this promise and defaults on his entire payroll. This enforcement problem is not present between banks and firms. In particular, banks can fully enforce their loan contracts. The lack of commitment between firms and workers induces a financial need by firms. Firms can borrow from banks to finance payroll, issuing a liability to the banking sector, whereas banking institutions take liabilities in the form of deposit accounts with their workers.

Production is distributed over time via the following relationship. $y_t = (1-\delta)\,\delta^{t-\tau} R_{\tau}^e$ where R_{τ}^e is the production per entrepreneur in island τ.

This delivers a problem for banks similar to Christiano and Eichenbaum (1992). This problem is given by:

Problem 5 (Producer) *The entrepreneur solves:*

$$R_{\tau}^e\left(k; q_t^l, w_t\right) = \max_{l,h} Ak^{\alpha}h^{1-\alpha} - w_t h - (l - q_t^l l)$$

subject to:

$$(1 - tax)\, w_t h \leq q_t^l l \tag{8}$$

In this problem, ql is the amount of deposits (or credit) available to the firm. The firm uses this credit is used to finance the payroll of the firm. The tax allows us to introduce a labor market distortion that affects the real demand for loans. The solution to this problem is given by:

Proposition 6 (Loan Demand) *In equilibrium:*

$$q_t^l \equiv \left(\left(1-\tau\right)A\left(1-\alpha\right)k^{\alpha}\right)^{-\frac{(\nu+1)}{1-\alpha}} l^{-\frac{\alpha(1+2\nu)}{(1-\alpha)}}$$

so $\epsilon = -\frac{\alpha(1+2\nu)}{(1-\alpha)}$ *and* $\Theta \equiv \left(\left(1-\tau\right)A\left(1-\alpha\right)k^{\alpha}\right)$. *In addition,* $p_t = 1$ *without loss of generality.*

Equilibrium with Real Sector. A competitive equilibrium with the real sector is, a partial equilibrium where the loan demand is given by Proposition 6.

10.2 Proof of Propositions 1, 2 and 3

This section provides a proof of the optimal policies described in Section 3.4. The proof of Proposition 1 is straight forward by noticing that once E is determined, the banker does not care how he came-up with those resources. The proof of Propositions 2 and 3 is presented jointly and the strategy is guess and verify. Let X be the aggregate state. We guess the following. $V\left(E;X\right) = v\left(X\right)E^{1-\gamma}$ where $v\left(X\right)$ is the slope of the value function, a function of the aggregate state that will be solved for implicitly. Policy functions are given by: $DIV\left(E;X\right) = div\left(X\right)E$, $\tilde{B}\left(E;X\right) = \tilde{b}\left(E;X\right)E$, $\tilde{D}\left(E;X\right) = \tilde{d}\left(E;X\right)E$ and $\tilde{C}\left(E;X\right) = \tilde{c}\left(E;X\right)E$.

10.2.1 Proof of Proposition 2

Given the conjecture for functional form of the value function, the value function satisfies:

$$\begin{aligned}
V\left(E;X\right) &= \max_{DIV,\tilde{C},\tilde{B},\tilde{D}} U(DIV) + \beta\mathbb{E}\left[v\left(X'\right)\left(E'\right)^{1-\gamma})|X\right] \\
\text{Budget Constraint:} & \quad E = q\tilde{B} + \tilde{C}(1+r) + DIV - \tilde{D} \\
\text{Evolution of Equity} &: \quad E' = \left(q'\delta + (1-\delta)\right)\tilde{B} + \tilde{C}\left(1+r_c\right)\left(1+r'\right) - \tilde{D}(1+\left(1+r_c\right)r'\omega') - \chi((\rho+\omega) \\
\text{Capital Requirement} &: \quad \tilde{D} \leq \kappa(\tilde{B} + \tilde{C} - \tilde{D}) \\
\text{Liquidity Requirement} &: \quad \tilde{C} \geq \eta(\tilde{B} + \tilde{C} - \tilde{D})
\end{aligned}$$

where the form of the continuation value follows from our guess. We can express all of the constraints in the problem as linear constraints in the ratios of E. Dividing all of the constraints by E, we obtain:

$$
\begin{aligned}
1 &= div + q\tilde{b} + (1+r)\tilde{c} - \tilde{d} \\
E'/E &= \left(q'\delta + (1-\delta)\right)\tilde{b} + \tilde{c}\left(1+r_c\right)\left(1+r'\right) - \tilde{d}(1 + \left(1+r_c\right)r'\omega') - \chi((\rho+\omega)\,\tilde{d} - \tilde{c})) \\
\tilde{d} &\leq \kappa(\tilde{b} + \tilde{c} - \tilde{d}) \\
\tilde{c} &\geq \eta(\tilde{b} + \tilde{c} - \tilde{d})
\end{aligned}
$$

where $div = DIV/E$,$\tilde{b} = \tilde{B}/E, \tilde{c} = \tilde{C}/E$ and $\tilde{d} = \tilde{D}/E$. Since, E is given at the time of the decisions of B,C,D and DIV, we can express the value function in terms of choice of these ratios. Substituting the evolution of E' into the objective function, we obtain:

$$
\begin{aligned}
V\left(E;X\right) &= \max_{div,\tilde{c},\tilde{b},\tilde{d}} U(divE) + \beta\mathbb{E}\left[v\left(X'\right)\left(R\left(\omega,X,X'\right)E\right)^{1-\gamma})|X\right] \\
1 &= div + q\tilde{b} + (1+r)\tilde{c} - \tilde{d} \\
\tilde{d} &\leq \kappa(\tilde{b} + \tilde{c} - \tilde{d}) \\
\tilde{c} &\geq \eta(\tilde{b} + \tilde{c} - \tilde{d})
\end{aligned}
$$

where we use the fact that E' can be written as:

$$
E' = R\left(\omega, X, X'\right)E
$$

where $R\left(\omega, X, X'\right)$ is the realized return to the bank's equity and defined by:

$$
R\left(\omega,X,X'\right) \equiv \left(q\left(X'\right)\delta + (1-\delta)\right)\tilde{b} + (1 + r\left(X'\right))\tilde{c} - (1 + r\left(X'\right)\omega')\tilde{d} - \chi((\rho+\omega)\,\tilde{d} - \tilde{c})).
$$

We can do this factorization for E because the evolution of equity on hand is linear in all the term where prices appear. Moreover, it is also linear in the penalty χ also. To see this, observe that $\chi\left((\rho+\omega)\,\tilde{D} - \tilde{C}\right) = \chi\left(\left((\rho+\omega)\,\tilde{d} - \tilde{c}\right)E\right)$ by definition of $\left\{\tilde{d},\tilde{c}\right\}$. Since, $E \geq 0$ always, we have that

$$
(\rho+\omega)\,\tilde{D} - \tilde{C} \leq 0 \leftrightarrow \left((\rho+\omega)\,\tilde{d} - \tilde{c}\right) \leq 0.
$$

Thus, by definition of χ,

$$\chi((\rho+\omega)\tilde{D}-\tilde{C}) = \begin{cases} E\underline{\chi}\left((\rho+\omega)\tilde{d}-\tilde{c}\right) & \text{if } \left((\rho+\omega)\tilde{d}-\tilde{c}\right)\leq 0 \\ E\overline{\chi}\left((\rho+\omega)\tilde{d}-\tilde{c}\right) & \text{if } \left((\rho+\omega)\tilde{d}-\tilde{c}\right)>0 \end{cases}.$$
$$= E\chi\left((\rho+\omega)\tilde{d}-\tilde{c}\right).$$

Hence, the evolution of $R(\omega, X, X')$ is a function of the portfolio ratios b, c and d but not of the level of E. With this properties, one is allowed to factor out, $E^{1-\gamma}$ from the objective because it is a constant when decisions are made. Thus, the value function may be written as:

$$\begin{aligned} V(E;X) &= E^{1-\gamma}\left[\max_{div,\tilde{c},\tilde{b},\tilde{d}} U(div)+\beta\mathbb{E}\left[v(X')\,R(\omega,X,X')^{1-\gamma})|X\right]\right] \qquad (9)\\ 1 &= div+q\tilde{b}+(1+r)\tilde{c}-\tilde{d}\\ \tilde{d} &\leq \kappa(\tilde{b}+\tilde{c}-\tilde{d})\\ \tilde{c} &\geq \eta(\tilde{b}+\tilde{c}-\tilde{d}). \end{aligned}$$

Then, let an arbitrary $\tilde{v}(X)$ be the solution to:

$$\begin{aligned} \tilde{v}(X) &= \max_{div,\tilde{c},\tilde{b},\tilde{d}} U(div)+\beta\mathbb{E}\left[\tilde{v}(X')\,R(\omega,X,X')^{1-\gamma})|X\right]\\ 1 &= div+q\tilde{b}+(1+r)\tilde{c}-\tilde{d}\\ \tilde{d} &\leq \kappa(\tilde{b}+\tilde{c}-\tilde{d})\\ \tilde{c} &\geq \eta(\tilde{b}+\tilde{c}-\tilde{d}). \end{aligned}$$

We now shows that if $\tilde{v}(X)$ exists, $v(X)=\tilde{v}(X)$ verifies the guess to our Bellman equation. Substituting $v(X)$ for the particular choice of $\tilde{v}(X)$ in (9) allows us to write $V(E;X) = \tilde{v}(X)\,E^{1-\gamma}$. Note this is true because maximizing over $div, \tilde{c}, \tilde{b}, \tilde{d}$ yields a value of $\tilde{v}(X)$. Since, this also shows that $div, \tilde{c}, \tilde{b}, \tilde{d}$ and independent of E, and $DIV = divE$, $\tilde{B} = \tilde{b}E$, $\tilde{C} = \tilde{c}E$ and $\tilde{D} = \tilde{d}E$.

10.2.2 Proof of Proposition 3

We now solve for $\tilde{v}(X)$ and show that the value of equity on hand can be written as a consumption-savings problem and an independent portfolio problem. To do so, let $\dot{b}$, $\dot{c}$ and $\dot{d}$ be the fraction of loans, reserves and deposits invested by the bank after paying dividends.

To simplify notation, we suppress the reference to the aggregate state X. These functions are mathematically defined as:

$$\dot{b} \equiv \frac{\tilde{b}}{(1-div)}, \quad \dot{c} \equiv \frac{\tilde{c}}{(1-div)} \text{ and } \dot{d} \equiv \frac{\tilde{d}}{(1-div)}.$$

and collecting terms we obtain:

$$div + (1-div)\left(q\dot{b} + (1+r)\dot{c} - \dot{d}\right) = 1.$$

Thus, the resource constraint for the bank can also be written as,

$$q\dot{b} + (1+r)\dot{c} - \dot{d} = 1.$$

Similarly, we can multiply the capital and liquidity constraints by $(1-div)$ and express the constraints in :

$$\begin{aligned} \dot{d} &\leq \kappa(\dot{b} + \dot{c} - \dot{d}) \\ \dot{c} &\geq \eta(\dot{b} + \dot{c} - \dot{d}). \end{aligned}$$

To make further progress, we employ the principle of optimality and solve for $\left\{\dot{b}, \dot{c}, \dot{d}\right\}$, assuming we already know div. Let's assume an arbitrary div^o as the optimal choice in $v(X)$. Then $v(X)$ also satisfies:

$$v(X) = \max_{\dot{b},\dot{c},\dot{d}} U(div^o) + (1-div^o)^{1-\gamma}\, \beta\mathbb{E}\left[v(X')\, R(\omega, X, X')^{1-\gamma})|X\right]$$

subject to:

$$\begin{aligned} 1 &= q\dot{b} + \dot{c}(1+r) - \dot{d} \\ \dot{d} &\leq \kappa(\dot{b} + \dot{c} - \dot{d}) \\ \dot{c} &\geq \eta(\dot{b} + \dot{c} - \dot{d}). \end{aligned}$$

We note that $R(\omega, X, X') = (q(X')\,\delta + (1-\delta))\,\dot{b} + (1+r(X'))\dot{c} - (1+r(X')\,\omega')\dot{d} - \chi((\rho+\omega)\,\dot{d} - \dot{c}))\,(1-div^o)$. Since $R(\omega, X, X')$ only enterers in the continuation utility, then, $\left\{\dot{b}, \dot{c}, \dot{d}\right\}$ must solve:

$$\max_{\dot{b},\dot{c},\dot{d}} \mathbb{E}\left[v(X')\,(q(X')\,\delta + (1-\delta))\,\dot{b} + (1+r(X'))\dot{c} - \dot{d}(1+r(X')\,\omega') - \chi((\rho+\omega)\,\dot{d} - \dot{c}))^{1-\gamma})|X\right].$$

subject to

$$\begin{aligned} 1 &= q\dot{b} + \dot{c}(1+r) - \dot{d} \\ \dot{d} &\leq \kappa(\dot{b} + \dot{c} - \dot{d}) \\ \dot{c} &\geq \eta(\dot{b} + \dot{c} - \dot{d}) \end{aligned}$$

if it is part of a solution, otherwise there is a better solution to $v(X)$.

When, X' is deterministic $v(X')$ is know at stage X. In such cases, we can factor out this problem out of the max. However, we must be careful with the sign of $v(X')$ since the max operator switches to a min operator when we change signs. We will show that when $\gamma > 1$, $v(X')$ is negative, so we need to minimize term inside the brackets. To prevent changing the max operator to a min operator, we use the certainty equivalent operator. Thus, $\left\{\dot{b}, \dot{c}, \dot{d}\right\}$ are solutions to:

$$\Omega(X) = \max_{\dot{b},\dot{c},\dot{d}} \mathbb{E}_\omega \left[(q(X')\delta + (1-\delta))\dot{b} + (1 + r(X'))\dot{c} - \dot{d}(1 + r(X')\omega') - \chi((\rho+\omega)\dot{d} - \dot{c}))^{1-\gamma}) \right]^{\frac{1}{1-\gamma}}.$$

subject to

$$\begin{aligned} 1 &= q\dot{b} + \dot{c}(1+r) - \dot{d} \\ \dot{d} &\leq \kappa(\dot{b} + \dot{c} - \dot{d}) \\ \dot{c} &\geq \eta(\dot{b} + \dot{c} - \dot{d}). \end{aligned}$$

When, $(1-\gamma) < 0$, the solution to $\Omega(X)$ will be equivalent to minimizing the objective. For $\gamma \to 1$, the objective becomes:

$$\Omega(X) = \exp\{\mathbb{E}_\omega[\log(R(\omega, X, X'))]\}.$$

Note that we can only do this separation when X is deterministic because otherwise we need to account for the correlation between $v(X')$ and $R(\omega, X, X')$. However, for now we assume the problem is deterministic. Since, the solution to $\Omega(X)$ is the same for any div, and not just the optimal, div^o. The objective can be written as

$$v(X) = \max_{div} U(div) + (1-div)^{1-\gamma} \beta \mathbb{E}\left[v(X')\,\Omega(X)^{1-\gamma})|X\right],$$

which is the formulation in Proposition 3. Proposition 5 shows an explicit solution for div and $v(X)$.

10.3 Equivalence in Problems

We now show that using our guessed policy functions we recover the consumption savings problem above. In the original problem, the first order conditions for $\tilde{b}$ is:

$$\left(\tilde{b}\right) : q\left(X\right) U^{'}\left(div\left(X\right) E\right) E = \beta\left(\mathbb{E}\left[v\left(X\right) u'\left(E'\right) \frac{\partial E'}{\partial \tilde{b}}\right]\right) + \mu_{\kappa}\left(X\right) E^{1-\gamma}\kappa - \mu_{\eta}\left(X\right) E^{1-\gamma}\eta. \tag{10}$$

Now, we know from xxx that $\frac{\partial E'}{\partial \tilde{b}}$ is:

$$\left(q\left(X'\right)\delta + \left(1-\delta\right)\right) E.$$

Hence, the first order condition becomes:

$$q\left(X\right) U^{'}\left(div\left(X\right)\right) E^{1-\gamma} = \beta\mathbb{E}\left[v\left(X\right) u'\left(R\left(\omega, X, X'\right) E\right)\left(q\left(X'\right)\delta + \left(1-\delta\right)\right) E\right] + \mu_{\kappa}\left(X\right) E^{1-\gamma}\kappa - \mu_{\eta}\left(X\right) E^{1-\gamma}$$

and simplifying E this equation further more, we obtain:

$$q\left(X\right) U^{'}\left(div\left(X\right)\right) = \beta\mathbb{E}\left[v\left(X\right) R\left(\omega, X, X'\right)^{-\gamma}\left(q\left(X'\right)\delta + \left(1-\delta\right)\right)\right] + \mu_{\kappa}\left(X\right)\kappa - \mu_{\eta}\left(X\right)\eta.$$

Similarly, the first order condition for $\tilde{c}\left(X\right)$ yields,

$$\left(\tilde{c}\right) : \left(1+r\left(X\right)\right) U^{'}\left(div\left(X\right)\right) = \beta\mathbb{E}\left[v\left(X\right) R\left(\omega, X, X'\right)^{-\gamma} \frac{\partial R\left(\omega, X, X'\right)}{\partial \tilde{c}}\right] + \mu_{\kappa}\left(X\right)\kappa - \mu_{\eta}\left(X\right)\left(\eta - 1\right), \tag{11}$$

and a similar expression can be found for $\tilde{d}\left(X\right)$:

$$\left(\tilde{d}\right) : U^{'}\left(div\left(X\right)\right) = \beta\mathbb{E}\left[v\left(X\right) R\left(\omega, X, X'\right)^{-\gamma} \frac{\partial R\left(\omega, X, X'\right)}{\partial \tilde{d}}\right] + \mu_{\kappa}\left(X\right)\left(1-\kappa\right) + \mu_{\eta}\left(X\right)\eta. \tag{12}$$

Note that 10, 11 and 12 are independent of E. One can multiply equation 10 by $\dot{b}$, equation 11 by $\dot{c}$ and equation 12 by $\dot{d}$, and obtain:

$$U^{'}\left(div\right) = \left(1-div\right)^{1-\gamma}\beta\mathbb{E}\left[v\left(X\right)\Omega\left(X\right)^{1-\gamma}\right]$$

To see this, note that we are using the definitions

$$\mu_\kappa(X)\left[\kappa(\tilde{b}(E;X)+\tilde{c}(E;X)-\tilde{d}(E;X))-\tilde{d}(E;X)\right]=0$$

and

$$\mu_\eta(X)\left[\tilde{c}(E;X)-\eta(\tilde{b}(E;X)+\tilde{c}(E;X)-\tilde{d}(E;X))\right]=0$$

and that $q\dot{b}+(1+r)\dot{c}-\dot{d}=1$. Since, $U'(div)=(1-div)^{1-\gamma}\beta\mathbb{E}\left[v(X)\Omega(X)^{1-\gamma}\right]$ is the same first order condition for the for the problem in Proposition 5, we know that the optimal dividend choice satisfies the conditions in that problem also.

10.4 Proof of Proposition 5

Taking first order conditions we obtain:

$$div=\beta\mathbb{E}\left[v^l(X')\,|X\right]\Omega^*(X)^{-1/\gamma}(1-div)$$

and therefore one obtains:

$$div=\frac{1}{1+\left[\beta\mathbb{E}\left[v^l(X')\,|X\right]\Omega^*(X)\right]^{1/\gamma}}.$$

Substituting this expression for dividends, one obtains a functional equation for the value function:

$$\begin{aligned}
v^l(X) &= \frac{1}{\left(1+\left[\beta\mathbb{E}\left[v^l(X')\,|X\right]\Omega^*(X)\right]^{\frac{1}{\gamma}}\right)^{1-\gamma}} \\
&\quad +\beta\mathbb{E}\left[v^l(X')\,|X\right]\Omega^*(X)\left[\frac{\left[\beta\mathbb{E}\left[v^l(X')\,|X\right]\Omega^*(X)\right]^{\frac{1}{\gamma}}}{\left(1+\left[\beta\mathbb{E}\left[v^l(X')\,|X\right]\Omega^*(X)\right]^{\frac{1}{\gamma}}\right)}\right]^{1-\gamma} \\
&= \left(\frac{1+\left(\left[\beta\mathbb{E}\left[v^l(X')\,|X\right]\Omega^*(X)\right]^{\frac{\gamma+(1-\gamma)}{\gamma(1-\gamma)}}\right)^{1-\gamma}}{\left(1+\left[\beta\mathbb{E}\left[v^l(X')\,|X\right]\Omega^*(X)\right]^{\frac{1}{\gamma}}\right)^{1-\gamma}}\right) \\
&= \left(\frac{1+\left[\beta\mathbb{E}\left[v^l(X')\,|X\right]\Omega^*(X)\right]^{\frac{1}{\gamma}}}{\left(1+\left[\beta\mathbb{E}\left[v^l(X')\,|X\right]\Omega^*(X)\right]^{\frac{1}{\gamma}}\right)^{1-\gamma}}\right) \\
&= \left(1+\left[\beta\mathbb{E}\left[v^l(X')\,|X\right]\Omega^*(X)\right]^{\frac{1}{\gamma}}\right)^{\gamma}
\end{aligned}$$

Therefore, we obtain the following map:

$$v^{l}\left(X\right)^{\frac{1}{\gamma}}=1+\left(\beta\Omega^{*}\left(X\right)\right)^{\frac{1}{\gamma}}\mathbb{E}\left[v^{l}\left(X'\right)|X\right]^{\frac{1}{\gamma}}.$$

Performing a change of variables, and denoting $\upsilon\left(X\right)\equiv v^{l}\left(X\right)^{\frac{1}{\gamma}}$, we have the following functional equation:

$$\upsilon\left(X\right)=1+\left(\beta\Omega^{*}\left(X\right)\right)^{\frac{1}{\gamma}}\mathbb{E}\left[\upsilon\left(X'\right)^{\gamma}|X\right]^{\frac{1}{\gamma}}.$$

We can treat the right hand side of this functional equation as an operator. This operator will be a contraction depending on the values of $\left(\beta\Omega^{*}\left(X\right)\right)^{\frac{1}{\gamma}}$. Theorems in Alvarez and Stokey guarantee that this operator satisfies the dynamic programming arguments.

In a non-stochastic steady state we obtain:

$$v^{l}\left(X\right)=\left(\frac{1}{1-\left(\beta\Omega^{*}\left(X\right)\right)^{\frac{1}{\gamma}}}\right)^{\gamma}.$$

11 Derivation of Loan Demand (Incomplete)

The firm maximizes production minus its acquired financial liabilities. Substituting the constraint and taking first order conditions with respect to loans delivers:

$$A\left(1-\alpha\right)k^{\alpha}\left(\frac{q_{t}^{l}}{\left(1-\tau\right)w_{t}}\right)^{1-\alpha}l^{-\alpha}=1.$$

Now, one given the linearity of profits in bank loans per unit of capital, the total demand for h is given by:

$$\frac{q_{t}^{l}}{w_{t}}l=H^{D}.$$

Hence, in equilibrium,

$$\left(\frac{q_{t}^{l}}{\left(1-\tau\right)w_{t}}l\right)^{\nu}=w_{t}\rightarrow\left(\frac{q_{t}^{l}l}{\left(1-\tau\right)}\right)^{\frac{\nu}{\nu+1}}=w_{t}.$$

Back in the first order condition yields:

$$A\left(1-\alpha\right)k^{\alpha}\left(\frac{q_{t}^{l}}{\left(1-\tau\right)\left(q_{t}^{l}l\right)^{\frac{\nu}{\nu+1}}}\right)^{1-\alpha}l^{-\alpha}=1.$$

and this returns:

$$\frac{A\left(1-\alpha\right)k^{\alpha}}{\left(1-\tau\right)^{-\frac{1-\alpha}{\nu+1}}}\left(q_t^l\right)^{\frac{1-\alpha}{\nu+1}}l^{\frac{\alpha(1+2\nu)}{\nu+1}}=1.$$

This leads to a demand for loans given by:

$$q_t^l=\left(\left(1-\tau\right)A\left(1-\alpha\right)k^{\alpha}\right)^{-\frac{(\nu+1)}{1-\alpha}}l^{-\frac{\alpha(1+2\nu)}{(1-\alpha)}}$$

this expression pin's down the equilibrium condition. Efficiency requires $q_t^l=1$. Clearly, $q_t^l\in[0,1]$, since banks have no incentives to lend, if they will not obtain interest in exchange (bound on interest rates). This condition provides a demand for loans.

12 Analysis with Regulatory Constraints (Incomplete)

Interaction with Regulatory Constraints. Incomplete Section. There are more interesting interactions when regulatory constraints are in place:

subject to:

$$\begin{aligned}
w_d &\leq \kappa\left(\frac{1}{q}-\left(\frac{1}{q}-\frac{1}{(1+r)}\right)w_c+\left(\frac{1}{q}-1\right)w_d\right), w_d, w_c\geq 0\\
w_c &\geq \eta\left(\frac{1}{q}-\left(\frac{1}{q}-\frac{1}{(1+r)}\right)w_c+\left(\frac{1}{q}-1\right)w_d\right).
\end{aligned}$$

There are several cases of interest.

When constraints are non-trivial, the constraint set has the following properties:

$$w_d\leq\frac{\kappa}{1-\left(\frac{1}{q}-1\right)\kappa}\left(\frac{1}{q}-\left(\frac{1}{q}-\frac{1}{(1+r)}\right)w_c\right)\text{ and non-binding if }\left(\frac{1}{q}-1\right)\kappa\geq 1\text{ or }\left(\frac{1}{q}<\frac{1}{(1+r)}\right)$$

and

$$\left(\frac{1}{q}-1\right)w_d\leq-\eta\frac{1}{q}+\left(\eta\left(\frac{1}{q}-\frac{1}{(1+r)}\right)+1\right)w_c\text{ and non-binding if }\left(\frac{1}{q}-\frac{1}{(1+r)}\right)>0.$$

Thus, w_c will be chosen so to maximize the size of the constraint set, if the constraint set is non-empty. This would be given by:

$$\bar{w}^d(w_c) = \min\left(\frac{\kappa}{1-\left(\frac{1}{q}-1\right)\kappa}\left(\frac{1}{q}-\left(\frac{1}{q}-\frac{1}{(1+r)}\right)w_c\right), -\eta\frac{1}{q}+\left(\eta\left(\frac{1}{q}-\frac{1}{(1+r)}\right)+1\right)\right)$$

and since reserves have no cost:

$$\bar{w}_c = \max_{w_c} \bar{w}^d(w_c)$$

which is the solution to that maximizes the constraint set. If $R^B = R^C$, reserves have no cost. Then, $w_c = \bar{w}_c$ without loss of generality. If instead reserves have a cost, if reserves have a cost, $R^B > R^C$, the banker will compare between the benefit of relaxing the the constraint and not. Thus,

$$\left(R^B - R^C\right) \leq \frac{\partial \bar{w}^d(w_c)}{\partial w_c}\left(R^B - R^D\right)$$

A first thing to note is that if $\left(\frac{1}{q}-1\right)\kappa \geq 1$ and $w_c = 0$, the capital requirement constraint is never binding. Thus, if $\left(R^B - R^D\right) > 0$, w_d is infinite. If $R^B = R^D$ it is indeterminate and $R^B - R^D < 0$, $w_d = 0$. This would mean that prices adjust. Hence, in equilibrium $\left(\frac{1}{q}-1\right)\kappa < 1$, then w_d is binding and equal to the solution of:

$$w_d = \frac{\kappa}{1-\left(\frac{1}{q}-1\right)\kappa}\left(\frac{1}{q}-\left(\frac{1}{q}-\frac{1}{(1+r)}\right)w_c\right).$$

Thus, the solution of the program depends on whether w_c relaxes the constraint. If $\frac{1}{q} > \frac{1}{(1+r)}$, then w_c constraints the problem furthermore. Thus, it will be made as small as possible so that it the constraint bind. Once again, when regulatory constraints are in place, monetary policy can affect real activity but is constrained by the equilibrium conditions imposed by financial regulation.

With regulatory constraints in place, the choice of ω^d will be constrained by the capital requirement constraint or the liquidity requirement constraints. The decision of L^* and ω^d cannot be analyzed sequentially as both terms enter in the constraint. However, there is a clear trade off. The banker will want to increase his ω^d, provided its profitable to do so. Obtaining liquidity funds has an equity reduction cost so it will have an effect on the constraint. Thus, when constraints are binding there is an additional trade-off between increasing returns at the expense of increased liquidity exposure.

Remark. *Assume there's no financial regulation in place, so that* κ is finite *and* $\eta \geq 0$. *Monetary policy can affect the mix between* ω^d *and* ω^c *by affecting the cost of liquidity but* *its ability to affect outcomes is constrained by financial regulation.*

12.0.1 Steady-State

The analysis in the section above looks a the optimal portfolio decisions. This section studies the equilibrium properties in steady state. We break the problem into different cases. We substitute the t subscripts for ss to refer to a given steady state. An important thing to note is that in steady, prices for won't change but there may be a law of motion for M0 and E. In a steady state, the value of returns is given by:

$$R_{ss}^{B}(\delta) \equiv \delta + \frac{(1-\delta)}{q_{ss}}$$

$$R_{ss}^{C} \equiv 1 + r_{c,ss}$$

$$R_{ss}^{D} \equiv (1 + (1 + r_{c,ss})\, r_{ss}\omega')$$

and

$$R^{\chi}(w_d, w_c) \equiv \chi\left((\rho+\omega)\, w_d - \frac{w_c}{(1+r)}\right).$$

Risk-Neutral Banks. Thus, in a steady state, L^* in (7) solves:

$$\left(\delta + \frac{(1-\delta)}{q_{ss}} - (1 + r_{c,ss})\right) = \bar{\chi}\,(1 - F(L^*)) - \underline{\chi} F(L^*)\,.$$

An unconstrained solution requires a solution of the equation above to obtain a value for $L^*\left(q_{ss}, r_{ss}, \rho, \bar{\chi}, \underline{\chi}\right)$ and in order to have a finite value for ω^d, the following must also hold:

$$\delta + \frac{(1-\delta)}{q_{ss}} = \left(\delta + \frac{(1-\delta)}{q_{ss}} - (1 + r_{c,ss})\right) L^* + \left(\bar{\chi}\mathbb{E}_{\omega}\left[\omega|\omega > L^*\right](1 - F(L^*)) - \bar{\chi}\mathbb{E}_{\omega}\left[\omega|\omega < L^*\right] F(L^*)\right).$$

Substituting the solution of $L^*\left(q_{ss}, r_{ss}, \rho, \bar{\chi}, \underline{\chi}\right)$ in the equation above yields an implicit solution of q_{ss} as a function of policy parameters $\left\{r_{ss}, \rho, \bar{\chi}, \underline{\chi}\right\}$. Thus, the Central Bank has the ability to choose q_{ss}, and consequently the amount of loans in steady state with the restriction that $q_{ss} \in (0, 1]$, and $\bar{\chi} > \delta + \frac{(1-\delta)}{q_{ss}} - (1 + r_{c,ss}) > -\underline{\chi}$. We can say something more strong. Since the banker earns zero expected profits by the marginal deposit, his expected return to the portfolio is:

$$\Omega_{ss} = \delta + \frac{(1-\delta)}{q_{ss}}.$$

Under risk neutrality, positive equity holdings would require to solve:

$$1 \leq \beta \Omega_{ss},$$

since otherwise bankers would pay dividends immediately. This condition imposes a constraint on the set of steady state values of q_{ss} as positive equity is a required to have positive loans. Without this condition, leverage would have to be equal to infinity but this would violate the constraint of non-negative equity. If $\beta \Omega_{ss} < 1$, banks would retain equity and E_t would grow indefinitely with leverage decreasing over time.

When constraints are active, (κ, η) taking non-degenerate values, L^* and ω^d will fall within the constraint set imposed by financial regulation. The analysis above shows that at least the leverage constraint would be binding for implied values of q_{ss} such that the sequence of equity is shrinking. In such case, the quantity of loans will be determined by the constraints imposed by financial regulation.