Sticky Leverage

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Introduction

Models of monetary non-neutrality have traditionally emphasized the importance of sticky prices and/or wages

- This seems perhaps overdone

We focus on an alternative channel for monetary non-neutrality
Nominal debt that is both long-term and defaultable

- This is both large and quite costly to adjust (at least the principal)

This creates two problems for firms

- Default risk
- Debt overhang

Preview of Findings

- Debt deflation is a quantitatively powerful propagation mechanism
- Sticky or persistent leverage is the key
- Conventional Taylor rules can stabilize output in response to shocks

Related Literature

- Debt deflation: De Fiore, Teles, and Tristani (2011), Kang and Pflueger (2012), Christiano, Motto and Rostagno (2009), Bhamra, Fisher and Kuehn (2011)
- Debt overhang: Occhino and Pescatori (2012), Moyen (2007), Hennessy (2004), Chen and Manso (2010)
- Default, general equilibrium: Gomes and Schmid (2013), Gourio (2012), Miao and Wang (2010)
- No quantitative business cycle analysis with defaultable nominal, long-term debt

Model

Continuum of firms of measure one, firm j produces

$$y_t^j = A_t \left(k_t^j\right)^{\alpha} \left(n_t^j\right)^{1-\alpha}$$

with aggregate productivity $\ln A_t = \rho \ln A_{t-1} + \sigma\varepsilon_t$, and

$$k_{t+1}^j = \left(1 - \delta + i_t^j\right) k_t^j \equiv g\left(i_t^j\right) k_t^j$$

Define

$$R_t k_t^j \equiv \max_{n_t^j} A_t \left(k_t^j\right)^{\alpha} \left(n_t^j\right)^{1-\alpha} - w_t n_t^j$$

After-tax operational profits, with idiosyncratic IID shock z_t^j

$$(1-\tau)\left(R_t k_t^j - z_t^j k_t^j\right)$$

Debt

Nominal debt outstanding requires payment

$$(c+\lambda)\,\frac{b_t^j}{\mu_t}$$

where $b_t \equiv B_t / P_{t-1}$, c coupon, λ amort., μ_t inflation rate

After issuing s_t^j

$$b_{t+1}^j = (1-\lambda)\,\frac{b_t^j}{\mu_t} + \frac{s_t^j}{p_t^j}$$

p_t^j market value of debt

Equity Value and Default

Value to equity holders/owners

$$E\left(k_t^j, b_t^j, z_t^j, \mu_t\right) = \max\left[0, (1-\tau)\left(R_t - z_t^j\right)k_t^j - \left((1-\tau)c+\lambda\right)\frac{b_t^j}{\mu_t} + V\left(k_t^j, b_t^j, \mu_t\right)\right]$$

where

$$V\left(k_t^j, b_t^j, \mu_t\right) = \max_{b_{t+1}^j, k_{t+1}^j}\left\{p_t^j\left(b_{t+1}^j - (1-\lambda)\frac{b_t^j}{\mu_t}\right) - I_t^j + \tau\delta k_t^j + \mathrm{E}_t M_{t,t+1} E\left(k_t^j, b_t^j, z_t^j, \mu_t\right)\right\}$$

Firms default when

$$(1-\tau)\left(R_t k_t^j - z_t^j k_t^j\right) + V_t\left(k_t^j, b_t^j, \mu_t\right) < \left((1-\tau)c+\lambda\right)\frac{b_t^j}{\mu_t}$$

Debt Pricing

$$b^j_{t+1} p^j_t =$$

$$E_t M_{t,t+1} \left\{ \begin{array}{c} \Phi(z^{j*}_{t+1}) \left[c + \lambda\right] \frac{b^j_{t+1}}{\mu_{t+1}} + \\ \int_{z^{j,*}_{t+1}}^{\overline{z}^j} \left[\begin{array}{c} (1-\tau) \left(R_{t+1} k^j_{t+1} - z^j_{t+1} k^j_{t+1} \right) \\ +V \left(k^j_{t+1}, b^j_{t+1}, \mu_{t+1} \right) - \xi k_{t+1} \end{array} \right] d\Phi \left(z_{t+1} \right) \\ + (1-\lambda) \frac{p^j_{t+1} b^j_{t+1}}{\mu_{t+1}} \end{array} \right\}$$

Households and Equilibrium

Consumer/Investor preferences

$$\max_{\{C,N\}} E \sum_{t=0}^{\infty} \beta^t \left[(1-\theta) \ln C_t + \theta \ln (3 - N_t) \right]$$

Aggregate resource constraint

$$Y_t - \left[1 - \Phi\left(z^*\right)\right] \xi^r \xi K_t = C_t + I_t$$

Inflation Process

$$\ln \mu_t = (1 - \rho^{\mu}) \ln \mu + \rho^{\mu} \ln \mu_{t-1} + \varepsilon_t^{\mu}$$

Characterization

$$v\left(\omega,\mu\right)=$$
$$\max_{\omega',i}\left\{\begin{array}{c} p\left(\omega' g\left(i\right)-\left(1-\lambda\right)\frac{\omega}{\mu}\right)-i+\tau\delta+ \\ g\left(i\right)\mathrm{E}M'\int_{\underline{z}}^{z^{*\prime}}\left[\begin{array}{c}\left(1-\tau\right)\left(R'-z'\right) \\ -\left(\left(1-\tau\right)c+\lambda\right)\frac{\omega'}{\mu'}+v\left(\omega',\mu'\right)\end{array}\right]d\Phi\left(z'\right)\end{array}\right\}$$

with $\omega\equiv b/k$, $v\equiv V/k$

State of economy: $\left(\omega,K,\mu,A\right)$

Optimal Leverage

FOC for ω'

$$
\begin{aligned}
& pg\left(i\right) + \frac{\partial p}{\partial \omega'}\left(\omega' g\left(i\right) - \left(1-\lambda\right)\frac{\omega}{\mu}\right) \\
= \quad & g\left(i\right) \mathrm{E} M' \Phi\left(z^{*\prime}\right) \frac{1}{\mu'} \left[\begin{array}{c} \left(1-\tau\right) c + \lambda \\ + \left(1-\lambda\right) p' \end{array} \right]
\end{aligned}
$$

Sticky Leverage

One-period debt, $\lambda = 1$

$$p + \frac{\partial p}{\partial \omega'}\omega' = \mathrm{E}M'\Phi\left(z^{*\prime}\right)\left[\left(\left(1-\tau\right)c+1\right)\frac{1}{\mu'}\right]$$

Proposition:

- Assume μ i.i.d., $\xi^r = 0$, and no shocks for a long time so that $\mu_{t-1} = \bar{\mu}, \omega_t = \bar{\omega}$
- Then, shock on μ_t has no effect on $\omega_{t+1} = \bar{\omega}$

Sticky Leverage

Long-term debt, $\lambda < 1$

$$
\begin{aligned}
& pg(i) + \frac{\partial p}{\partial \omega'} \left(\omega' g(i) - (1-\lambda) \frac{\omega}{\mu} \right) \\
= \quad & g(i)\, \mathrm{E} M' \Phi \left(z^{*\prime} \right) \frac{1}{\mu'} \left[\left((1-\tau) c + \lambda \right) - p' (1-\lambda) \right]
\end{aligned}
$$

Proposition

- Assume μ i.i.d., $\xi^r = 0$, and no shocks for a long time so that $\mu_{t-1} = \bar{\mu}, \omega_t = \bar{\omega}$
- A negative shock on μ_t increases $\omega_{t+1} > \bar{\omega}$.

Sticky leverage: high $\omega/\mu \Rightarrow$ high ω'

Optimal Investment and Debt Overhang

FOC for i

$$1 - p\omega' = \mathrm{E}M' \int_{\underline{z}}^{z^{*\prime}} \left\{ \left[\begin{array}{c} (1-\tau)(R' - z') \\ -\left((1-\tau)c + \lambda\right) \frac{\omega'}{\mu'} \\ +v(\omega', \mu') \end{array} \right] \right\} d\Phi(z')$$

Proposition:

- Assume μ i.i.d., $\xi^r = 0$, and no shocks for a long time so that $\mu_{t-1} = \bar{\mu}, \omega_t = \bar{\omega}, R_{t+1} > \bar{R}$
- Then, shock on μ_t has no effect on R (and i) iff $\omega_{t+1} = \bar{\omega}$
- However if $\omega_{t+1} > \bar{\omega}$ then $R_{t+1} > \bar{R}$

Calibration

Parameter	Description	Value
β	Subjective Discount Factor	0.99
γ	Risk Aversion	1
θ	Elasticity of Labor	0.63
α	Capital Share	0.36
δ	Depreciation Rate	0.025

Calibration

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θ	Elasticity of Labor	0.63
α	Capital Share	0.36
δ	Depreciation Rate	0.025
λ	Debt Amortization Rate	0.06
τ	Tax Wedge	0.40
η_1	Distribution Parameter	0.6617
ξ	Default Loss	0.38
ξ^r	Fraction of Resource Cost	1

Idiosyncratic Shocks

Use general quadratic approximation to p.d.f.:

$$\phi(z) = \eta_1 + \eta_2 z + \eta_3 z^2$$

- Symmetry $\overline{z} = \underline{z} = 1$, and $E(z) = 0$
- One free parameter η_1

Shocks

VAR process for inflation and productivity

$$\begin{bmatrix} a_t \\ \mu_t \end{bmatrix} = \begin{bmatrix} \rho_a & \rho_{a,\mu} \\ \rho_{a,\mu} & \rho_\mu \end{bmatrix} \begin{bmatrix} a_{t-1} \\ \mu_{t-1} \end{bmatrix} + \begin{bmatrix} \varepsilon_t^a \\ \varepsilon_t^\mu \end{bmatrix}$$

Estimated values

$$\Gamma = \begin{bmatrix} 0.98 & -0.094 \\ 0.012 & 0.85 \end{bmatrix}$$

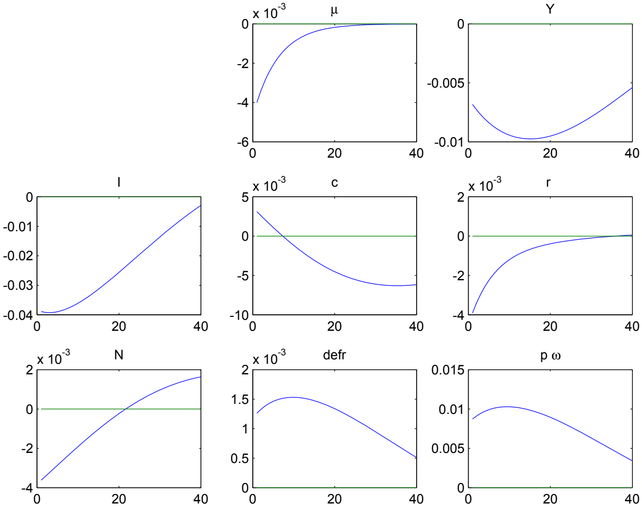
$$\sigma_a = 0.0074, \sigma_\mu = 0.0045, \rho_{\mu a} = -0.19$$

AR(1) version:

$$\begin{aligned} \rho_a &= 0.97, \qquad \sigma_a = 0.0070 \\ \rho_\mu &= 0.85, \qquad \sigma_\mu = 0.0040 \end{aligned}$$

$$\rho_{a,\mu} = \rho_{a,\mu} = \rho_{\mu a} = 0$$

Inflation shock



Key Moments

	Data	Model	Model
		AR(1)	VAR(1)
First Moments			
Investment/Output, I/Y	0.21	0.24	0.24
Leverage, ω	0.42	0.42	0.42
Default Rate, $1-\Phi(z^*)$	0.42%	0.42%	0.42%
Credit Spread	0.39%	0.39%	0.39%
Second Moments			
σ_Y	1.7%	1.6%	1.7%
σ_I/σ_Y	4.12	4.22	4.48
σ_C/σ_Y	0.54	0.41	0.43
σ_N/σ_Y	1.07	0.49	0.54
σ_ω	1.7%	1.5%	1.7%

Variance decomposition, AR(1)

	Y	Inv	Cons	Hrs	Lev	Default
Benchmark, $\bar{\omega} = 0.42$						
TFP shock a	0.63	0.37	0.39	0.60	0.10	0.05
Inflation shock μ	0.37	0.63	0.61	0.40	0.90	0.95
Low Leverage, $\bar{\omega} = 0.32$						
TFP shock a	0.84	0.65	0.83	0.89	0.10	0.01
Inflation shock μ	0.16	0.35	0.17	0.11	0.90	0.99
High Leverage, $\bar{\omega} = 0.52$						
TFP shock a	0.40	0.21	0.29	0.55	0.03	0.03
Inflation shock μ	0.60	0.79	0.71	0.45	0.97	0.97

Variance decomposition, AR(1)

	Y	Inv	Cons	Hrs	Lev	Default
Benchmark, $\lambda = 0.06$						
TFP shock a	0.63	0.37	0.39	0.60	0.10	0.05
Inflation shock μ	0.37	0.63	0.61	0.40	0.90	0.95
Long maturity, $\lambda = 0.03$						
TFP shock a	0.45	0.26	0.65	0.80	0.83	0.01
Inflation shock μ	0.55	0.74	0.35	0.20	0.17	0.99
One period debt, $\lambda = 1$						
TFP shock a	1	1	1	1	0.01	0.02
Inflation shock μ	0.00	0.00	0.00	0.00	0.99	0.98

Monetary policy rule

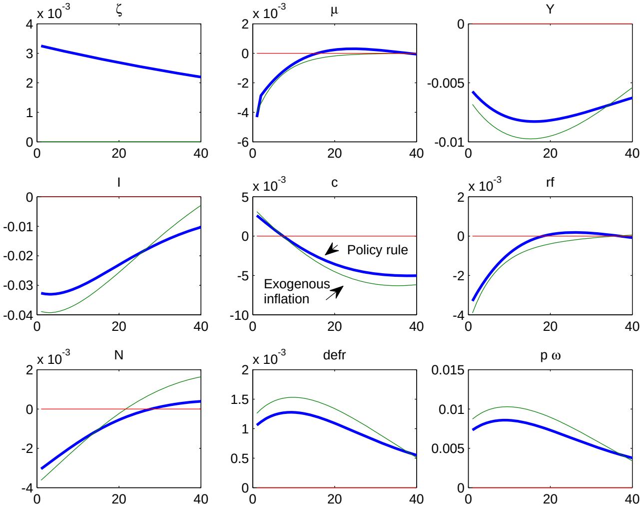
Taylor rule with interest rate smoothing

$$\hat{r}_t^f = \rho_R \hat{r}_{t-1}^f + (1 - \rho_R) \{\nu_m \hat{\mu}_t + \nu_y \hat{y}_t\} + \zeta_t$$

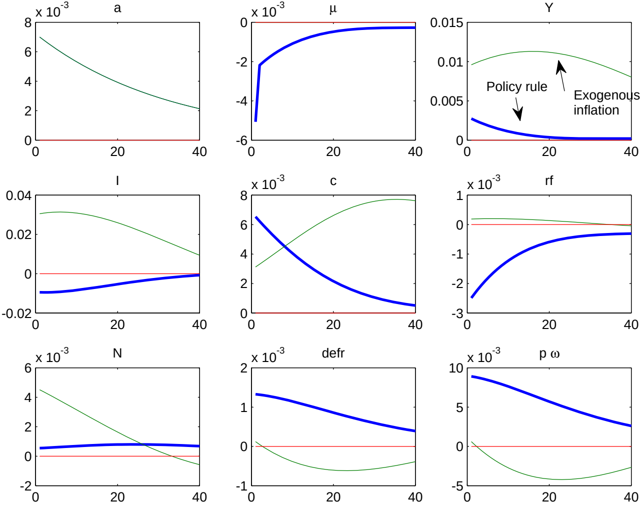
Calibration

$$\hat{r}_t^f = 0.6 \cdot \hat{r}_{t-1}^f + 0.4 \{1.5 \cdot \hat{\mu}_t + 0.5 \cdot \hat{y}_t\} + \zeta_t$$

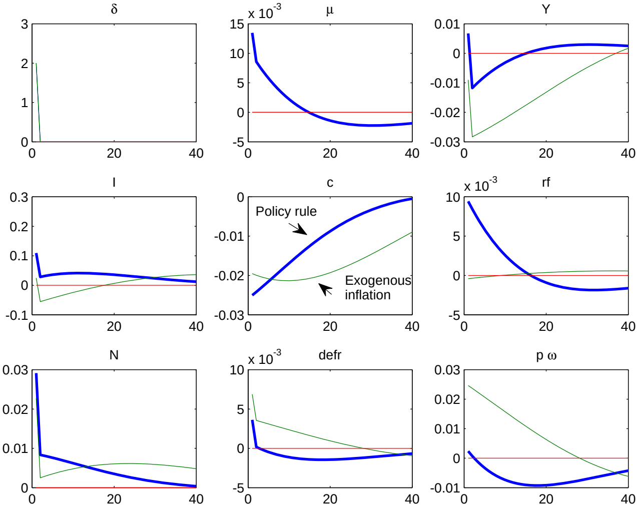
Monetary policy shock



Productivity shock



Wealth/capital shock



Conclusion

- Model with nominal long-term debt produces strong inflation non-neutrality without sticky prices
- Key mechanisms: sticky leverage and debt overhang
- Taylor rule implies a significant increase in inflation in response to both low productivity and wealth shocks